# Strassen Algorithm

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}.$$

$$\begin{aligned}
M_1 &= (A_{11} + A_{22})(B_{11} + B_{22}) \\
M_2 &= (A_{21} + A_{22})(B_{11}) \\
M_3 &= A_{11}(B_{12} - B_{22}) \\
M_4 &= A_{22}(B_{21} - B_{11}) \\
M_5 &= (A_{11} + A_{12})B_{22} \\
M_6 &= (A_{21} - A_{11})(B_{11} + B_{12}) \\
M_7 &= (A_{12} - A_{22})(B_{21} + B_{22})
\end{aligned}$$

$$\begin{aligned}
C_{11} &= M_1 + M_4 - M_5 + M_7 \\
C_{12} &= M_3 + M_5 \\
C_{21} &= M_2 + M_4 \\
C_{22} &= M_1 - M_2 + M_3 + M_6
\end{aligned}$$

- A, B an C are square metrices of size $N \times N$
- ~~a, b, c~~ $A_{11}$, $A_{12}$, $A_{21}$, $A_{22}$ are submatrices of A, of size $N/2 \times N/2$
- $B_{11}$, $B_{12}$, $B_{21}$, $B_{22}$ are submatrices of B, of size $N/2 \times N/2$
- $C_{11}$, $C_{12}$, $C_{22}$, $C_{21}$ are submatrices of C, of size $N/2 \times N/2$
- Addition & Multiplication of two matrices takes $O(N^2)$ time.

$$T(N) = 7T\left(\frac{N}{2}\right) + O(N^2)$$

Solving this with master theorem we get.

$a = 7$, $b = 2$ & $d = 2$

$\Rightarrow a > b^d$

$\therefore T(n) = n^{\log_2 7}$

$\approx n^{2.80}$

## Traditional Method

```
int i, j, k;
for (i = 0; i < n; i++)
 {
  for (j = 0; i < n; j++)
   {
    c[i][j] = 0;
     for (K = 0; K < n; K++)
      {
       c[i][j] += A[i][k] * B[k][j];
      }
   }
 }
```

Time Complexity is $O(n^3)$

$\Rightarrow 8T\left(\frac{n}{2}\right) + n^2$

Solving above with master theorem, we get

$$\boxed{T(n) = O(n^3)}$$