Matrix Chain Multiplication

①   A = 4×10

   B = 10×3

   C = 3×12

   D = 12×20

   E = 20×7

Based on example,

$$P_0 = 4, \quad P_1 = 10, \quad P_2 = 3, \quad P_3 = 12, \quad P_4 = 20, \quad P_5 = 7$$

$$j \longrightarrow$$

	1	2	3	4	5	
	0					1
		0				2
			0			3 ↓ i
				0		4
					0	5

Formula :-

1) if $i = j$ then $M[i][j] = 0$

2) if $i < j$ then $M[i][j]$

$$= \min \left(M[i][k] + M[k+1][j] + P_{i-1} * P_k * P_j \right)$$

$$\text{with } i \leq k \leq j$$

① $M[1, 2]$

$= \min (M[i][k] + M[k+1][j] + P_{i-1} * P_k * P_j)$
$$\text{with } i \leq k < j$$

$= \min (M[1][1] + M[2][2] + 4 * 10 * 3)$

$= \min (0 + 0 + 120)$

$= 120$

② $M[2, 3]$

$= \min (M[2][2] + M[3][3] + P_{i-1} * P_k * P_j)$

$= \min (0 + 0 + 10 * 3 * 12)$

$= \min (360)$

$= 360$

③ $M[3,4]$

$= \min \left(M[3][3] + M[4][4] + P_2 * P_3 * P_4 \right)$

$= \min \left(0 + 0 + 3 * 12 * 7 \right)$

$= \min \left(1680 \right)$

$= 1680$

④ $M[4,5]$

$= \min \left(M[4][4] + M[5][5] + P_3 * P_4 * P_5 \right)$

$= \min \left(0 + 0 + 12 * 20 * 7 \right)$

$= \min \left(1680 \right)$

$= 1680$

⑤ $M[1,3]$

K can have 2 Values   1 & 2.

For K=1

$M[1,3] = (M[1][1] + M[2][3] + 4*10*12)$

$\quad\quad\quad = (0 + 360 + 480)$

$\quad\quad\quad = 840$

For K=2

$M[1,3] = (M[1][2] + M[3][3] + 4*3*12)$

$\quad\quad\quad = (120 + 0 + 144)$

$\quad\quad\quad = 264$

$\boxed{M[1,3] = 264}$

⑥ $M[2,4]$

Here we have 2 values of K i.e. 2 & 3

For K=2

$M[2,4] = (M[2,2] + M[3][4] + 10*3*20)$

$\quad\quad\quad = (0 + 720 + 600)$

$\quad\quad\quad = 1320$

For K = 3

$$M[2,4] = (M[2][3] + M[4][4] + 10 * 12 * 20)$$

$$= (360 + 0 + 2400)$$

$$= 2760$$

Ans of $M[2,4]$ = 1320

⑦ $M[3,5]$

values of K = 3 & 4

For K = 3 ⟶

Ans $M[3,5]$ = 1140

⑧ $M[1,4]$

Value of K = 1, 2, 3.

Ans = 1080

⑨ $M[2,5]$

Value of K = 2, 3, 4

Ans = 1350

(10)   M[1,5]

Value of K = 1, 2, 3, 4

Ans = 1344

	1	2	3	4	5	
	0	120	264	1080	1344	1
		0	360	1320	1350	2
			0	720	1140	3
				0	1680	4
					0	5