TUTORIAL - 7

Date :- 18/04/22

1\. Find dual of each of these compound propositions.

a) $p \wedge (q \vee (r \wedge T))$ b) $(p \vee F) \wedge (q \vee T)$

→ a) $p \vee (q \wedge (r \vee F))$ b) $(p \wedge T) \vee (q \wedge F)$

2\. Let $P(x)$ be statement "x can speak Russian" & let $Q(x)$ be statement "x knows C++". Express each of following sentences in terms of $P(x)$, $Q(x)$, quantifiers & logical connectives. The domain for quantifiers consists of all students at your school. a) There is a student at your school who can speak Russian & who knows C++. ⇒ $\exists x (P(x) \wedge Q(x))$

b) There is a student at your school who can speak Russian but doesn't know C++. ⇒ $\exists x (P(x) \wedge \neg Q(x))$

c) Every student at your school either can speak Russian or knows C++. ⇒ $\forall x (P(x) \vee Q(x))$

d) No student ~~can~~ at your school can speak Russian or knows C++. ⇒ $\neg \exists x (P(x) \vee Q(x))$

3\. Let $P(x)$ be statement "$x = x^2$". If domain consists of integers, what are the truth values?

a) $P(0) = T$, b) $P(1) = T$, c) $P(2) = F$, d) $P(-1) = F$

e) $\exists x\ P(x) = T$, f) $\forall x\ P(x) = F$, g) $\exists x \neg P(x) = T$, h) $\forall x \neg P(x) = T$

4\. Suppose the domain of propositional function $P(x)$ consists of integers 1, 2, 3, 4, 5. Express statements without using quantifiers, instead using only negations, disjunctions & conjunctions.

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a) $\exists x\, P(x) = P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b) $\forall x\, P(x) = P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c) $\neg \exists x\, P(x) = \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5)$

d) $\neg \forall x\, P(x) = \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4) \wedge \neg P(5)$

e) $\forall x\, ((x \neq 3) \to P(x)) \vee \exists x\, P(x) = (P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

5. Suppose that domain of propositional function P(x) consists of integers −5, −3, −1, 1, 3, 5. Express these statements without using quantifiers, instead using negations, disjunctions & conjuctions.

a) $\forall x\, ((x \neq 1) \to P(x))$

$\to = P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

b) $\exists x\, ((x \geq 0) \wedge P(x))$

$\to = P(1) \vee P(3) \vee P(5)$

c) $\exists x\, (\neg P(x)) \wedge \forall x\, ((x < 0) \to P(x))$

$\to (\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-1))$

11. Let P(x), Q(x) & R(x) be statements "x is professor", "x is ignorant", & "x is vain" respectively. Express each of statements using quantifiers; logical connectives; & P(x), Q(x) & R(x) where domain consists of all people.

a) No professors are ignorant. $\Rightarrow \neg \exists x\, (P(x) \wedge Q(x))$

b) All ignorant people are vain. $\Rightarrow \forall x\, (Q(x) \to R(x))$

c) No professors are vain $\Rightarrow \forall x\, (P(x) \to \neg R(x))$

d) Does (c) follow from a) & b) ? $\Rightarrow$

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6. Show that $\forall x\, P(x) \vee \forall x\, Q(x)$ & $\forall x\, (P(x) \vee Q(x))$ are not logically equivalent.

→ Let $P(x) : x^2 < 1$ & $Q(x) : x^2 \geq 01$, $x \in Z$.

→ Here $\forall x\, P(x)$ = false & $\forall x\, Q(x)$ = false.
So $\forall x\, P(x) \vee \forall x\, Q(x)$ = False.

→ Now, $\forall x\, (P(x) \vee Q(x))$ = True as for all x, any of $P(x)$ or $Q(x)$ will be true.

→ So, $\forall x\, P(x) \vee \forall x\, Q(x)$ and $\forall x\, (P(x) \vee Q(x))$ are not logically equivalent.

7. Show that $\exists x\, P(x) \wedge \exists x\, Q(x)$ and $\exists x\, (P(x) \wedge Q(x))$ are not logically equivalent.

→ Let $P(x) : 2x = 4$ & $Q(x) : 3x+1 \leq 5$, $x \in Z$.

→ Here $\exists x\, P(x)$ = True & $\exists x\, Q(x)$ = True.
So, $\exists x\, P(x) \wedge \exists x\, Q(x)$ = True.

→ ~~Now, $\exists x\, (P(x) \wedge Q(x))$ = False as for all x, any of $P(x)$ & or $Q(x)$ will be false~~

So, $\exists (x)\, P(x) \wedge \exists x\, Q(x)$ and $\exists x\, (P(x) \wedge Q(x))$ are not logically equivalent.

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8. If domain consists of all integers, what are truth values of statements ?

a) $\exists! x\ (x > 1)$ = False

b) $\exists! x\ (x^2 = 1)$ = False

c) $\exists! x\ (x+3 = 2x)$ = True

d) $\exists! x\ (x = x+1)$ = False.

9. What are truth values of these statements

a) $\exists! x\ P(x) \rightarrow \exists x\ P(x)$ = True

b) $\forall x\ P(x) \rightarrow \exists! x\ P(x)$ = False

c) $\exists! x\ \neg P(x) \rightarrow \neg \forall x\ P(x)$ = $\exists! x\ \neg P(x) \rightarrow \exists x\ P(x)$

= True.

10. Write our $\exists! x\ P(x)$ where domain consists of integers 1, 2 & 3 in terms of negation conjunctions & disjunctions.

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12. Let $P(x)$, $Q(x)$, $R(x)$ & $S(x)$ be statements "x is baby", "x is logical", "x is able to manage crocodile" & "x is despised", respectively. Express each of statements using quantifiers; logical statements connectives; & $P(x)$, $Q(x)$, $R(x)$ & $S(x)$ where domain consists of all people.

a) Babies are illogical. $\Rightarrow \forall x\,(P(x) \rightarrow \neg Q(x))$

b) Nobody is despised who can manage a crocodile $\Rightarrow \forall x\,(R(x) \rightarrow \neg S(x))$

c) Illogical persons are despised. $\Rightarrow \forall x\,(\neg Q(x) \rightarrow S(x))$

d) Babies can't manage crocodiles. $\Rightarrow \forall x\,(P(x) \rightarrow \neg R(x))$

e) Does d) follow from a), b) & c)?

→ Suppose x is a baby, from premise a) x is illogical, & from premise c, x is despised. So, x can't manage a crocodile. So d follows i.e babies can't manage crocodiles.

3.

a)→ $\forall x\,(P(x) \rightarrow Q(x))$

b)→ $\exists x\,(R(x) \rightarrow \neg Q(x))$

c)→ $\exists x\,(R(x) \rightarrow \neg P(x))$

d)→ Suppose x is a clear explaination, from premise a, x is satisfactory. Now, let x be excuse, from premise b, some of x are unsatisfactory. So, c follows i.e some excuses are not clear explainations.

Q - 14.

a) Everybody can fool Fred.

→ $\forall x \exists y \quad F(x, \text{Fred})$

b) Evelyn can fool everybody.

→ $\exists x \forall y \quad F(\text{Evelyn}, y)$

c) Everybody can fool somebody

→ $\forall x \forall y \quad F(x, y)$

d) There is no one who can fool every

→ $\exists x \forall y \quad \neg(F(x, y))$

e) Everyone can be fooled by somebody.

→ $\forall x \exists y \quad F(x, y)$

f) No one can fool both Fred & Jerry

→

g) Nancy can fool exactly two people

→ $\exists x \exists y \quad F(\text{Nancy}, 2)$

h) There is exactly one person whom ... can fool

→ $\forall x \exists y \quad F(x, 1)$

i) No one can fool himself or herself.

→

j) There is someone who can fool exact... person besides himself or herself.

→ $\exists x \exists y \quad F(x, 1)$

1. a) No professors are ignorant. $\equiv \forall x (\neg P(x) \rightarrow \phi(x))$
b) All ignorant people are vaim. $= \forall x (\phi(x) \rightarrow P(x))$
c) No professors are vaim. $= \forall x (\neg P(x) \rightarrow \phi(x))$
d) Does c follows from a & b.

→ Suppose x is a professor, from premise a, x is not ignorant. Now let x is a person, from premise b, if x is ignorant x is vaim. So premise c follows.

5. $S(x)$: "x is student", $F(x)$: "x is faculty member" &
$A(x,y)$: "x has asked y a question".

a)

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