Bubble Sort Analysis

Algorithm

```
for i<-0 to n do
{
  for j<-i+1 to n do
  {
    if(a[j] < a[i])
    {
      temp = a[i];
      a[i] = a[j];
      a[j] = temp;
    }
  }
}
```

Analysis

$T(n)$ = Outer for loop with variable i × Inner for loop with variable j × Basic Operation

$$I(n) = \sum_{i=0}^{n} \times \sum_{j=i+1}^{n} \times 1$$

$$= \sum_{i=0}^{n} \sum_{j=i+1}^{n}$$

Formula $\sum_{i=0}^{n} 1 = (n - 0 + 1)$

$$= \sum_{i=0}^{n} (n - (i+1) + 1)$$

$$= \sum_{i=0}^{n} (n - i)$$

$$= \sum_{i=0}^{n} n - \sum_{i=0}^{n} i$$

Formula : $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

$$= \sum_{i=0}^{n} n - \frac{n(n+1)}{2}$$

$$= n \sum_{i=0}^{n} 1 - \frac{n(n+1)}{2}$$

$$= n(n - 0 + 1) - \frac{n(n+1)}{2}$$

$$= n^2 + 1 - \frac{n^2 + n}{2}$$

$$= \frac{2n^2 + 1 - n^2 + n}{2}$$

$$= \frac{n^2 + n + 1}{2}$$

$$T(n) = \Theta(n^2)$$