3.4 Multiplying Large Integers Problems

We are familiar with the multiplication performed using positional numeral system. This method of multiplying two numbers has taught us in schooling. In this method multiply the multiplicand by each digit of multiplier and then add up all the properly shifted results. This method is also called **grade-school multiplication.**

For example :

$$\begin{array}{r} 42 \\ \times\ 34 \\ \hline 168 \\ +\ 1260 \\ \hline 1428 \end{array}$$

$\leftarrow$ padding 0 at units position

But this method is not convenient for performing multiplication of large integers. Hence let us discuss an interesting algorithm of multiplying large integers. **For example :** Consider multiplication of two integers 42 and 34. First let us represent these numbers according to positions.

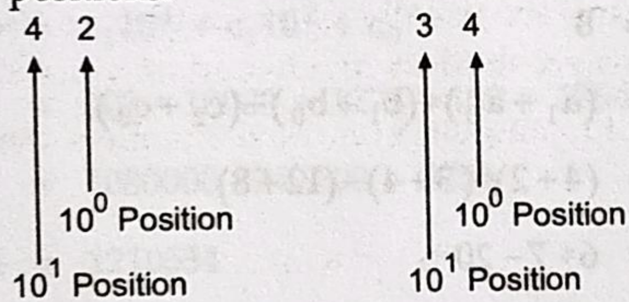

Fig. 3.3

i.e.

$$\begin{aligned} 42 \times 34 &= (4\times10^1 + 2\times10^0) * (3\times10^1 + 4\times10^0) \\ &= (4\times3)10^2 + (4\times4 + 2\times3)10^1 + (2\times4)10^0 \\ &= 1200 + 220 + 8 \\ &= 1428 \end{aligned}$$

Let us formulate this method -

Let, $c = a * b$

$$c = c_2\ 10^2 + c_1\ 10^1 + c_0$$

where
$$c_2 = a_1 * b_1$$
$$c_0 = a_0 * b_0$$
$$c_1 = (a_1 + a_0) * (b_1 + b_0) - (c_2 + c_0)$$

The 2 digit numbers are

$$a = a_1\ a_0$$
$$b = b_1\ b_0$$

Let perform multiplication operation with the help of formula given in equation (1).

$$c = a * b$$
$$= 42 \times 34$$

where $a_1 = 4,\ a_0 = 2$

$b_1 = 3,\ b_0 = 4$

Let us obtain c_0, c_1, c_2 values

$$c_2 = a_1 * b_1$$
$$= 4 * 3$$
$$c_2 = 12$$
$$c_0 = a_0 * b_0$$
$$= 2 * 4$$
$$c_0 = 8$$
$$c_1 = (a_1 + a_0) * (b_1 + b_0) - (c_2 + c_0)$$
$$= (4+2) * (3+4) - (12+8)$$
$$= 6 * 7 - 20$$
$$c_1 = 22$$
$$\therefore \quad a * b = c_2\ 10^2 + c_1\ 10^1 + c_0$$
$$= 12 * 10^2 + 22 * 10^1 + 8$$
$$= 1200 + 220 + 8$$
$$a * b = 1428$$

Consider a multiplication of 981 * 1234, as 981 us a three digit number, we will make it four digit by padding a zero to it. Now 0981 * 1234 makes both the operands as 4 digit values.

We will write

$$c = a * b$$

$$= 0981 * 1234$$

Divide the numbers in equal halves.

$$\therefore \quad a_1 = 09 \quad a_0 = 81$$

$$b_1 = 12 \quad b_0 = 34$$

Let us obtain c_0, c_1, c_2 Values

$$c_2 = a_1 * b_1$$

$$= 09 * 12$$

$$c_2 = 108$$

$$c_0 = a_0 * b_0$$

$$= 81 * 34$$

$$c_0 = 2754$$

$$c_1 = (a_1 + a_0) * (b_1 + b_0) - (c_2 + c_0)$$

$$= (9+81)*(12+34)-(108+2754)$$

$$= 4140 - 2862$$

$$c_1 = 1278$$

$$\therefore \quad a * b = c_2 10^4 + c_1 10^2 + c_0$$

$$= 108 * 10000 + 1278 * 100 + 2754$$

$$= 1080000 + 127800 + 2754$$

$$0981 * 1234 = 1210554$$

We can generalize this formula as -

> $c = a * b$
> $c = c_2\ 10^n + c_1\ 10^{n/2} + c_0$
> where, n is total number of digits in the integer
> $c_2 = a_1 * b_1$
> $c_0 = a_0 * b_0$
> $c_1 = (a_1 + a_0) * (b_1 + b_0) - (c_2 + c_0)$

Clearly, this algorithm can be implemented using recursion. This recursion can be stopped when n reaches to 0.