G H PATEL COLLEGE OF ENGINEERING & TECHNOLOGY, V V NAGAR

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3140708: DISCRETE MATHEMATICS

Assignment 3: Relations

1. List the ordered pairs in the relation R from $A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$, where $(a,b) \in R$ if and only if
(a) $gcd(a,b) = 1$.
(b) $lcm(a,b) = 2$.

(a) $\{(0,1),(1,0),(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(4,1),(4,3)\}$

(b) $\{(1,2),(2,1),(2,2)\}$

2. List all the ordered pairs in the relation $R = \{(a,b) \mid a \text{ divides } b\}$ on $A = \{1,2,3,4,5,6\}$. Display this relation graphically as well as in tabular form.

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)\}$

R	1	2	3	4	5	6
1	×	×	×	×	×	×
2		×		×		×
3			×			×
4				×		
5					×	
6						×

3. A relation R is called asymmetric if $(a,b) \in R$ implies that $(b,a) \notin R$.
For each of these relations on the set $\{1,2,3,4\}$, decide whether it is reflexive, irreflexive, symmetric, antisymmetric, asymmetric, transitive.
(a) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$
(b) $\{(1,1),(2,2),(3,3),(4,4)\}$
(c) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$
(d) $\{(1,2),(2,3),(3,4)\}$
(e) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$
(f) $\{(2,4),(4,2)\}$

Use ✓ in the respective cell of the following table if the property holds for that relation.

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Transitive
(a)	×	×	×	×	×	✓
(b)	✓	×	✓	✓	×	✓
(c)	✓	×	✓	×	×	✓
(d)	×	✓	×	✓	✓	×
(e)	×	✓	×	×	×	×
(f)	×	✓	✓	×	×	×

4. Determine whether the relation R on the set of all Web pages is reflexive, irreflexive, symmetric, antisymmetric, asymmetric, transitive, where $(a,b) \in R$ if and only if
(a) everyone who has visited Web page a has also visited Web page b.
(b) there are no common links found on both Web page a and Web page b.
(c) there is at least one common link on Web page a and Web page b.
(d) there is a Web page that includes links to both Web page a and Web page b.

a) reflexive: ✓
irreflexive: ×
symmetric: ×
antisymmetric: ×
asymmetric: ✓
Transitive: ✓

b) reflexive: ×
irreflexive: ✓
symmetric: ✓
antisymmetric: ×
asymmetric: ×
Transitive: ×

c) reflexive: ×
irreflexive: ×
symmetric = ✓
antisymmetric: ×
Transitive: ×
asymmetric: ×

d) reflexive:
irreflexive:
sym: ✓
antisym: ×
asym: ×
tran: ×

5. Determine whether the relation R on the set of all real numbers is reflexive, irreflexive, symmetric, antisymmetric, asymmetric, transitive, where $(x, y) \in R$ if and only if

(a) $x - y \in Q$.
(b) $x = 1 \text{ or } y = 1$.
(c) $x = y + 1 \text{ or } x = y - 1$.
(d) $x \geq y^2$.
(e) $xy \geq 1$.
(f) $x \equiv y \pmod 7$.
(g) x is a multiple of y.
(h) $xy = 0$.

Use ✓ in the respective cell of the following table if the property holds for that relation.

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Transitive
(a)	✓	×	✓	×	×	✓
(b)	×	×	✓	×	×	×
(c)	×	✓	✓	×	×	×
(d)	×	×	×	✓	✓	×
(e)	×	×	✓	×	×	×
(f)	✓	×	✓	×	×	✓
(g)	✓	×	✓	×	×	✓
(h)	×	×	✓	×	×	×

6. Give an example of a relation on a set that is
(a) symmetric and antisymmetric.
(b) neither symmetric nor antisymmetric.

$A = \{1, 2, 3, 4\}$

a) $R: \{(1,1), (2,2), (3,3), (4,4)\}$

b) $R: \{(1,2), (2,1), (3,4)\}$

7. Use quantifiers to express what it means for a relation to be
(a) irreflexive.
(b) asymmetric.

a) $\forall a \in A, (a, a) \notin R$

b) $\forall (a,b) \in R$ if $(a,b) \in R$ then $(b,a) \notin R$

8. Can a relation on a set be neither reflexive not irreflexive? Give an example.

Yes, $A = \{1, 2\}$ relation R on A

$R = \{(1,1)\}, \cancel{(1,2)}\}$

R is not reflexive because $(2,2) \notin R$

R is not irreflexive because $(1,1) \in R$

9. Must an asymmetric relation also be antisymmetric? Must an antisymmetric relation be asymmetric? Give examples to justify your answers.

→ Let $(a, b) \in R$ and $(b, a) \notin R$ (∵ R asymmetric)

∴ Relation is also antisymmetric because $(a,b) \in R$ but $(b,a) \notin R$ so we can say relation is antisymmetric

→ $A = \{1,2,3\}$

$R: \{(1,1), (2,2), (3,3)\}$

R is antisymmetric but R is not asymmetric

10. Let $R = \{(1,2), (1,3), (2,3), (2,4), (3,1)\}$ and $S = \{(2,1), (3,1), (3,2), (4,2)\}$ on the set $A = \{1, 2, 3, 4\}$. Find $S \circ R$.

$ROS = \{(2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4)\}$

$SOR = \{(1,1), (1,2), (2,1), (2,2)\}$

11. Let $R = \{(1,1), (2,1), (3,2), (4,3)\}$. Find the powers $R^n, n = 2, 3, 4, \ldots$

$R^2 = ROR = \{(1,1), (2,1), (3,2), (4,4)\} \circ \{(1,1), (2,1), (3,2), (4,3)\}$

$= \{(1,1)\ (2,1)\ (3,1)\ (4,2)\}$

$R^3 = R^2 \circ R = \{(1,1), (2,1), (3,1), (4,2)\} \circ \{(1,1)\ (2,1)\ (3,2)\ (4,3)\}$

$= \{(1,1)\ (2,1), (3,1)\ (4,1)\}$

$R^4 = R^3 \circ R = \{(1,1)\ (2,1)\ (3,1)\ (4,1)\} \circ \{(1,1)\ (2,1)\ (3,2)\ (4,3)\} = \{(1,1)\ (2,1)$
$(3,1)\ (4,1)\}$

12. Let R be the relation on the set of people consisting of pairs (a, b), where a is a parent of b. Let S be the relation on the set of people consisting of pairs (a, b), where a and b are siblings. What are $S \circ R$ and $R \circ S$?

$SOR = \{(a,b) \mid a \text{ is parent of } b\}$

$ROS = \{(a,b) \mid a \text{ is uncle or aunt of } b\}$

13. Consider the following relations:

$$R_1 = \{(a, b) \in R^2 \mid a > b\}, \quad R_2 = \{(a, b) \in R^2 \mid a \geq b\},$$
$$R_3 = \{(a, b) \in R^2 \mid a < b\}, \quad R_4 = \{(a, b) \in R^2 \mid a \leq b\},$$
$$R_5 = \{(a, b) \in R^2 \mid a = b\}, \quad R_6 = \{(a, b) \in R^2 \mid a \neq b\}.$$

Find (a) $R_1 \cup R_3$ (b) $R_2 \cap R_4$ (c) $R_1 \oplus R_3$ (d) $R_2 \oplus R_4$ (e) $R_4 \cap R_6$ (f) $R_3 - R_6$

$R_1 \cup R_3 = \{(a,b) \in R^2 \mid a > b\}$

for all pairs of real number $a \neq b$ to hold true condition $a > b$ or $a < b$

i.e. $(R_1 \cup R_3 = \{(a,b) \in R^2 \mid a \neq b\} = R_6$

(c) $R_1 \oplus R_3 = \{(a,b) \mid (a > b \text{ or } a < b)$ and not $(a > b \text{ and } a < b)\}$

$= \{(a,b) \mid a \neq b \text{ and not } F\}$

$= \{(a,b) \in R^2 \mid a \neq b \text{ and } T\}$

$= \{(a,b) \in R^2 \mid a \neq b\} = R_6$

(b) $R_2 \cap R_4 \neq \{(a,b) \in R^2 \mid a \geq b$ and

$= \{(a,b) \in R^2 \mid a = b\}$

$= R_5$

(d) $R_4 = \{(a,b) \mid (a \geq b)$ or $(a \leq b)$ and not $(a \geq b$ and $a \leq b)\}$
$= \{(a,b) \mid T$ and not $a = b\}$ ($\because T =$ True)
$= \{(a,b) \in R^2 \mid a \neq b$ and $T\}$
$= \{(a,b) \in R^2 \mid a \neq b\}$
$= R_6$

(e) $R_4 \cap R_6 = \{(a,b) \in R^2 \mid a \leq b$ and $\}$
$= \{(a,b) \in R^2 \mid a < b\}$
$= R_3$

(f) $R_6 - R_3 = \{(a,b) \in R^2 \mid a \neq b$ and not $a < b\}$
$= \{(a,b) \in R^2 \mid a \neq b$ and $a \geq b\}$
$= \{(a,b) \in R^2 \mid a \neq b$ and $a \geq b\}$
$= \{(a,b) \in R^2 \mid a > b\}$
$= \{(a,b) \in R^2 \mid a > b\} = R$

14. Let R_1 and R_2 be the "divides" and "is a multiple of" relations on the set of all integers, respectively. That is,
$R_1 = \{(a,b) \mid a \text{ divides } b\}$ and $R_2 = \{(a,b) \mid a \text{ is a multiple of } b\}$.
Find (a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 - R_2$ (d) $R_2 - R_1$ (e) $R_1 \oplus R_2$

(a) $R_1 \cup R_2 = \{(a,b) \in R^2 \mid a$ divides b or b divides $a\}$
(b) $R_1 \cap R_2 = \{(a,b) \in R^2 \mid a = \pm b$ and $a \neq 0\}$
(c) $R_1 - R_2 = \{(a,b) \in R^2 \mid a$ divides b & $a \neq \pm b\}$
(d) $R_2 - R_1 = \{(a,b) \in R^2 \mid b$ divides a & $a \neq \pm b\}$
(e) $R_1 \oplus R_2 \{(a,b) \in R \mid$ (a divides b or b divides a) & $a \neq \pm b\}$

15. Let R_1 and R_2 be the "congruent modulo 3" and "congruent modulo 4" relations, respectively, on the set of all integers. That is, $R_1 = \{(a,b) \mid a \equiv b \ (mod\ 3)\}$ and $R_2 = \{(a,b) \mid a \equiv b \ (mod\ 4)\}$. Find
(a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 - R_2$ (d) $R_2 - R_1$ (e) $R_1 \oplus R_2$

(a) $R_1 \cup R_2 = \{(a,b) \mid a-b \equiv 0, 3, 6, 9 \pmod{12}$ or $a-b \equiv 0, 4, 8 \pmod{12}\}$
(b) $R_1 \cap R_2 = \{(a,b) \mid a \equiv b \pmod{12}\}$
(c) $R_1 - R_2 = \{(a,b) \mid [a-b \equiv 3, 6$ or $9 \pmod{12}\}$
(d) $R_2 - R_1 = \{(a,b) \mid [a-b] \equiv 4$ or $8 \pmod{12}]\}$
(e) $R_1 \oplus R_2 = \{(a,b) \mid [(a-b) \equiv 3, 4, 6, 8$ or $9 \pmod{12}]\}$

16. Suppose that R and S are reflexive relations on a set A. Prove or disprove each of these statements.
(a) $R \cup S$ is reflexive. (b) $R \cap S$ is reflexive. (c) $S \circ R$ is reflexive.
(d) $R - S$ is irreflexive. (e) $R \oplus S$ is irreflexive.

(a) True (b) True
→ Let $(a,a) \in R$ & $(a,a) \in S \ \forall x \in A$
Since $R \subseteq R \cup S$, we have $(a,a) \in R \cup S, \forall a \in A$, Also, $(a,a) \in R \cap S, \forall a \in A$, Thus $R \cup S$ & $R \cap S$ are reflexive

(c) True, as $(a,a) \in R$ & $(a,a) \in S$, $\forall a \in A$
We have $(a,a) \in S \circ R$, so $S \circ R$ is reflexive
d) True: $R - S = R - (R \cap S)$ & $(a,a) \in R \cap S, \forall a \in A$, such pairs will not belong to $R - S$
∴ irreflexive

(e) $R \oplus S = (R \cup S) - (R \cap S)$ & $(a,a) \in R \cap S, \forall a \in A$ such pairs will not belong to $R \oplus S$ ∴ irreflexive

17. Let R be a relation. Let n be any positive integer. Prove the following statements:
(a) R is symmetric if and only if $R = R^{-1}$.
(b) R is antisymmetric if and only if $R \cap R^{-1} \subseteq \Delta$.
(c) R is reflexive if and only if $\bar{R}$ is irreflexive.
(d) If R is reflexive and transitive, then $R^n = R$.
(e) If R is reflexive (symmetric), then R^n is reflexive (symmetric).

a) assume R is symmetric
$(a,b) \in R$ $(b,a) \in R$ $(b,a) \in R$
$(a,b) \in R^{-1}$
$(a,b) \in R^{-1} \Rightarrow (b,a) \in R^{-1}$
$(a,b) \in R$ ($\because R$ is symmetric

assume that $R - R^{-1}$
$(a,b) \in R$ c) $(a,b) \in R^{-1}$
$(b,a) \in R$
∴ R is symmetric

18. Suppose that the relation R is irreflexive. Is R^2 necessarily irreflexive? Justify.

Suppose we have ~~tree~~ relation R such that $(a,b) \in R$ & $(b,a) \in R$ $(a \neq b)$ then by defination of composite $(a,b) \in R^2$ so R is irreflexive R^2 is not necessarily irreflexive

19. The following matrices represent relations on the set $A = \{1, 2, 3\}$:

$(a)\ M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad (b)\ M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad (c)\ M_{R_3} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) List the ordered pairs in the relations.
(b) Draw the directed graphs representing these relations.
(c) Determine whether the relations are reflexive, irreflexive, symmetric, antisymmetric and/or transitive by inspection of the matrices.

$\boxed{a}$

(a) $R_1 = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

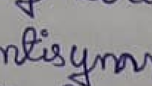
(b) $Q_1 \rightleftarrows Q_3 \quad Q_2$

(C) reflexive. As all diagonal elements are 1.
Symmetric as $u_{ij} = u_{ji}$ $(i \neq j)$
not ~~st~~ antisymmetric because $u(1,2) = 0\ u(2,1)$ & $u(1,3) = 1 = u(3,1)$

$\boxed{b}$

(a) $R_2 = \{(1,2), (2,2), (3,2)\}$

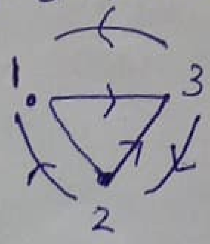
(b)



(C) irreflexive ~~all~~ as all diagonal entry are not 1
Assymetric as $u_{ij} \neq u_{ji}$ $(i \neq j)$
Antisymmetric as $u_{ij} = 1$ with $i \neq j$ than $u_{ji} = 0$

$\boxed{c}$ a) $R_3 = \{(1,1)(1,2)(1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

b)



(C) irreflexive as $u_{22} \neq 1$ not antisymmetric as $u(1,2) = 1 = M_1$ (2,2) $u(1,3) = 1 = u(3,$

20. How can the matrix representing a relation R on a set A be used to determine whether the relation is (a) irreflexive? (b) asymmetric?

$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (a) irreflexive as $M(3,3) = 0$

(b) asymmetric as $M(1,3) \neq M(3,1)$ and $M(2,3) \neq M(3,2)$

21. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad and \quad M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

(a) R_1^{-1} (b) $\bar{R}_1$ (c) $R_1 \cup R_2$ (d) $R_1 \cap R_2$
(e) $R_2 \circ R_1$ (f) $R_1^2 = R_1 \circ R_1$ (g) $R_1 \oplus R_2$

(a) $R_1^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $M_{\bar{R}_1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (c) $M_{R_1 \cup R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(d) $M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (e) $M_{R_2 \circ R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (f) $M_{R_1^2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(g) $M_{R_1 \oplus R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

22. Let R be a relation on a set A with n elements. If there are k nonzero entries in M_R, how many nonzero entries are there in (a) $M_{R^{-1}}$? (b) $M_{\bar{R}}$? 2

a) k because data remain same (b) n-k zero replace with 1 and 1 with 0.

23. How many nonzero entries does the matrix representing the relation R on the set $A = \{1, 2, 3, \dots, 100\}$ have if R is

(a) $\{(a, b) \mid a \leq b\}$ (b) $\{(a, b) \mid a \neq 0\}$
(c) $\{(a, b) \mid ab = 1\}$ (d) $\{(a, b) \mid a + b = 100\}$
(e) $\{(a, b) \mid a = b \pm 1\}$ (f) $\{(a, b) \mid a + b \leq 101\}$

$= 100 + 99 + \dots + 1$
$= \frac{100(100+1)}{2}$
$= 5050$

(b) $100 \times 100 = 1$
$= 10000$

(c) 1 pair possible
$(1, 1)$

(d) $= 99$

(e) for 1 & 100 1 possibility
2 to 99 2 possibility
$= 98 \times 2 + 1 + 1$
$= 198$

(f) $= 100 + 99 + \dots + 1$
$= 5050$

a	b
1	99
2	98
⋮	⋮
99	1

24. How can the directed graph of a relation R on a finite set A be used to determine whether a relation is (a) asymmetric? (b) irreflexive?

(a) In directed graph loop represent symmetricity. So there must be no loop in directed graph to be asymmetric relation

(b) The directed graph should ~~&~~ not have loop that related with itself to irreflexive.

25. Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

(a) $\{(0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3)\}$

(b) $\{(0,0), (1,1), (1,2), (2,1), (2,2), (3,3)\}$

(a) given ~~realtion~~ relation is not equivalance relation, As it is not reflexive. ($\because (1,1) \notin R$)

(b) Give Relation is equivalance relation. As it is reflexive, symmetric & transitive

26. Which of these relations on the set of all functions from Z to Z are equivalence relations? Determine the properties of an equivalence relation that the others lack.

(a) $\{(f, g) \mid f(1) = g(1)\}$

(b) $\{(f, g) \mid f(0) = g(0)\ or\ f(1) = g(1)\}$

(c) $\{(f, g) \mid f(0) = g(1)\ and\ f(1) = g(0)\}$

(d) $\{(f, g) \mid f(x) - g(x) = 1\ for\ all\ x \in Z\}$

(e) $\{(f, g) \mid for\ some\ C \in Z, for\ all\ x \in Z, f(x) - g(x) = C\}$

(a) equivalance relation

(b) not transitive some can have $f(0)=g(0)$ and $g(1)\neq h(1)$ ~~but~~ $f(0)\neq$ ~~$g(0)$~~ $h(0)$ they ~~and~~ then $(f,h)\notin R$ so not equivalance relation.

d) reflexive as $\forall x \in Z$ $f(x)-f(x)=0$

→ ~~so~~ symmetric as $f(x)$ & $g(x) \in Z$ then $f(x)-g(x)=C_1$ & $g(x)-f(x)=C_2$ C_1 & $C_2 \in Z$

→ transitive as $f(x)-g(x)=C_1$, $g(x)-h(x)=C_2$, $f(x)-h(x)=C_3 \rightarrow C_1, C_2, C_3$ $\therefore$ equivalance

(c) not reflexive as $f(x)-f(x)=0\neq 1$

→ not symmetric $f(x)-g(x)=1$ but $g(x)-f(x)\neq 1$

→ not transitive $f(x)-g(x)=1$ & $g(x)-h(x)=1$ then $f(x)-h(x)=2\neq 1$

$\therefore$ not equivalance relation

(e) not reflexive $f(0)=f(1)$ may not true

→ not transitive $f(0)=g(0)=h(1)$ $\therefore f(0)=h(0)$ & $f(C)=h(1)$ less not mean that $f(1)=h(0)$

$\therefore$ not equivalance relation

27. Show that the relation R on the set of all differentiable functions from R to R consisting of all pairs (f, g) such that $f'(x) = g'(x)$ for all real numbers x is an equivalence relation. Which functions are in the same equivalence class as the function $f(x) = x^2$?

Reflexive = let $f \in A$ since for every f, $f'(x)=f'(x)$ is always true

Symmetric = $f'(x)=g'(x)$ $\forall x \in R$ By symmetric of eq^n $g'(x)=f'(x)$ will be ~~&~~ always true

Transitive = If $f'(x)=g'(x)$ & $g'(x)=f'(x)$ $\forall x \in R$ then $f'(x)=h'(x)$ $\forall x \in R$ always true

Thus R is equivalance by reflexive, symmetric, Transitive property (b) $f(x)=x^2, \forall x \in R$

Any func^n with derivative $2x$ is in same class with $F(x)$ $g'(x)=2x$, $g(x)=x^2+c$ (c = constant)

Any function of type x^2+c is in same equivalance class

28. Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that $f(x) = f(y)$. Show that R is an equivalence relation on A. What are the equivalence classes of R?

reflexive: Let $x \in A$ $F(x)=F(x)$ will be true ∴ reflexive relation

Symmetric $F(x)=f(y)$ will be true for $x,y \in A$ ∴ Symmetric relation

Transitive $(x,y) \in R, (y,z) \in R$ $f(x)=f(y)$ & $f(y)=f(z)$ will be True ∴ Transitive

Equivalance class of R $[x]_R = \{y \in A / f(x)=f(y)\}$

29. Suppose that A is a nonempty set and R is an equivalence relation on A. Show that there is a function f with A as its domain such that $(x, y) \in R$ if and only if $f(x) = f(y)$.

$F: A \Rightarrow A \Rightarrow f(x) = x_0$ where x_0 represents $[x]_R \forall x \in A$

If $(x,y) \in R \Rightarrow x, y \in [x]_R \Rightarrow f(x) = x_0 = f(y)$

If $f(x) \cdot f(y) \Rightarrow x_0 = y_0 \Rightarrow [x]_R = [y]_R \Rightarrow y \in [x]_R \Rightarrow (x,y) \in R$

from that last statement we can conclude that the relation is equivalent

30. Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree except perhaps in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

reflexive: $x \in A$ $(x, x \in R)$ ∴ reflexive

Symmetric: $(x,y) \in R$ all bits of x, y are same except first 3 ∴ $(y,x) \in R$ ∴ Symmetric

Transitive: $(x,y) \in R$ & $(y,z) \in R$ all bits are x & y are same except first 3. all bits of y & z are same except first 3 ∴ $(x,z) \in R$ ∴ transitive. R is equivalance Relation.

31. Let R be the relation consisting of all pairs (x, y) such that x and y are strings of uppercase and lowercase English letters with the property that for every positive integer n, the nth characters in x and y are the same letter, either uppercase or lowercase. Show that R is an equivalence relation.

for reflexive: Since all letters of x is same as x itself their n^{th} letter has to be same $x_n = x_n \Rightarrow (x,x) \in R$ ∴ Reflexive

for Symmetric: Let $(x,y) \in R$ the n^{th} letter of x & y has to be the same $x_n = y_n \Rightarrow y_n = x_n \Rightarrow (y, x \in R)$ ∴ R is symmetric

for Transitive: Let $(x,y) \in R$ & $(y,z) \in R$. n^{th} letter of x & y are same n^{th} letter of y & z are same $x_n = y_n$ & $y_n = z_n$ n^{th} letter of x & z will be same $x_n = z_n$ $(x,z) \in R$

∴ R is transitive

• Thus R is an equivalance relation.

32. Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation. What is the equivalence class of $(1,2)$ in this relation?

reflexive: lets take $((a$ & $a=b)$
then $ad = ab$ & $bc = ba$
$\therefore ((a,b),(a,b)) \in R$
Symmetric $(a,b)(c,a) \in R \Rightarrow ad = bc$
$ab = aa$
$c((c,a),(a,b)) \in R$

Transitive
$A = \{(a,b) / a,b \in N - \{0\}\}$
$R = \{(a,b)(c,d) / ad = bc\}$
Transitivity $((a,b)(c,d)) \in R$
& $((c,d)(e,f)) \in R$
then $((a,b),(e,f)) \in R$
$((a,b)(c,d)) \in R = aacbc$

(C) $(c,d)(e,f) \in R_2)$
$(d = de)$
$a, b = c/d$ &
$c/d = e/f$
$a/b = e/f$
$af = be$

33. Let m be a positive integer with $m > 1$. Define the relation $R = \{(a,b) \mid a \equiv b \ (mod\ m)\}$ on the set of integers. Recall that $a \equiv b \ (mod\ m)$ if $a - b$ is divisible by m. The equivalence class of an element a in this relation is denoted as $[a]_m$. They are also called congruence classes modulo m. Show that R is an equivalence relation. Also, find $[n]_5$ where n is (a) 2? (b) 3? (c) 6? (d) -3?

Symmetric: aRb is divisible by m
✓. $a - b = km$ as $b - a = -km$
we get integer $r - k$ when $b - a$
is divided by m
Reflexive: $a - a = 0 \Rightarrow aRa$

Transitive: let a & b & bRc then $a-b$ & $b-c$ are both divisible by m $a-b = km$
& $b - c = k_2 m$
$[n]_5 = \{a \in Z \mid a - n$ is divided by $5\}$ $[2]_5 =$
$= \{a \in Z \mid a - n, -5k,$ for $k \in Z\}$ $\{5k+2 / k \in Z\}$
$= \{a \in Z \mid a < 5k + n,$ for $k \in Z\} = \{5k + n / k \in Z\}$

34. Let R be the relation on the set of real numbers such that xRy if x and y are real numbers that differ by less than 1, that is $|x - y| < 1$. Show that R is not an equivalence relation.

Reflexive: ✓
Symmetric: ✓
Transitive: ✗

$(4,3) \in R$, $(3,2) \in R$ $\therefore$ R is not
but $(4,2) \in R$ equivalence Relation

35. Show that the relation R on the set of all bit strings such that $s\,R\,t$ if and only if s and t contain the same number of 1s is an equivalence relation. What is the equivalence class of the bit string 011 for this relation?

Here, $(s,s) \in R$ so reflexive
$(s,t) \in R \Rightarrow (t,s) \in R$ so symmetric
$(s,t) \in R \Rightarrow (t,4) \in R \Rightarrow (s,4) \in R$ so transitive
$\therefore$ R is ~~a~~ equivalence
$[0,11]\ R = \{110, 101,$ ~~00~~$0110, 01100, 01100, 10100 \ldots\}$

36. Which of these collections of subsets are partitions on the set of bit strings of length 8?
(a) the set of bit strings that begin with 1, the set of bit strings that begin with 00, and the set of bit strings that begin with 01.
(b) the set of bit strings that contain the string 00, the set of bit strings that contain the

Every bit string will either starts from one or 0: The string with first bit 0 can be further divided into & disjoint sets, namely these with first & bits 00 & those with first two bits 01, These 3 sets are clearly disjoint & their union. with be the set of all bit string not a partition

(b) String 01010101 belong to both B & C

$B \cap C \neq 0$

(d) partition = True

string 01, the set of bit strings that contain the string 10, and the set of bit strings that contain the string 11.

(c) the set of bit strings that end with 00, the set of bit strings that end with 01, the set of bit strings that end with 10, and the set of bit strings that end with 11.

(d) the set of bit strings that have $3k$ ones, where k is a nonnegative integer; the set of bit strings that have $3k+1$ ones, where k is a nonnegative integer; the set of bit strings that have $3k+2$ ones, where k is a nonnegative integer.

(C) $A = \{S \mid S$ ends with $00\}$ ~~there~~ Here, $A \cup B \cup C \cup D$ contains all bit strings.

$B = \{S \mid S$ ends with $01\}$

$C = \{S \mid S$ ends with $10\}$ Here, $A \cap B = \Phi$, $B \cap C = \Phi$, $C \cap D = \Phi$, $A \cap D = \Phi$

$D = \{S \mid S$ ends with $11\}$ $B \cap D = \Phi$, $A \cap C = \Phi$ $\therefore$ Partion

37. Which of these collections of subsets are partitions of the set of integers?

(a) the set of even integers and the set of odd integers

(b) the set of positive integers and the set of negative integers

(c) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3.

(d) the set of integers less than -100, the set of integers with absolute value not exceeding 100, and the set of integers greater than 100.

(a) True $A \cup B = Z$ & $A \cap B = \Phi$

(b) False $A \cup B \neq Z$ (not includes 0)

(c) True $A \cup B = Z$ & $A \cap B = \Phi$

(d) True $A = \{n \in Z \mid n < -100\}$, $C = \{n \in Z \mid n > 100\}$

$B = \{n \in Z \mid |n| \leq 100\}$

$\Rightarrow A \cap B \cap C = \Phi$, $A \cap B = \Phi$, $B \cap C = \Phi$ $A \cap C = \Phi$

38. Which of these are partitions of the set of real numbers?

(a) the negative real numbers, $\{0\}$, the positive real numbers.

(b) the set of irrational numbers, the set of rational numbers.

(c) the set of intervals $[k, k+1]$, $k = \cdots, -2, -1, 0, 1, 2, \ldots$

(d) the set of intervals $(k, k+1)$, $k = \cdots, -2, -1, 0, 1, 2, \ldots$

(e) the set of intervals $(k, k+1]$, $k = \cdots, -2, -1, 0, 1, 2, \ldots$

(f) the sets $\{x + n \mid n \in Z\}$ for all $x \in [0, 1)$.

(a) True (b) True (c) For $k=1$, $[1,2]$

for $k=2$, $[2,3]$

so, $[1,2] \cap [2,3] = \{2\} \neq 0$

$\Rightarrow$ Not partitions

(d) $k=1 \Rightarrow (1,2)$

$k=2 \Rightarrow (2,3)$

doesn't contain any integers $\Rightarrow$ not partitions

(e) True

$k=0 \Rightarrow (0,1]$

$k=1 \Rightarrow (1,2]$

$k=2 \Rightarrow (2,3]$

$\Rightarrow$ Partitions

(f) True

It is a partition.

39. Which of these are partitions of the set $Z \times Z$ of ordered pairs of integers?

(a) the set of pairs (x, y), where x or y is odd; the set of pairs (x, y), where x is even; and the set of pairs (x, y), where y is even.

(b) the set of pairs (x, y), where both x and y are odd; the set of pairs (x, y), where exactly one of x and y is odd; and the set of pairs (x, y), where both x and y are even.

(c) the set of pairs (x, y), where x is positive; the set of pairs (x, y), where y is positive; and the set of pairs (x, y), where both x and y are negative.

(d) the set of pairs (x, y), where $3 \mid x$ and $3 \mid y$; the set of pairs (x, y), where $3 \mid x$ and $3 \nmid y$; the set of pairs (x, y), where $3 \nmid x$ and $3 \mid y$; and the set of pairs (x, y), where $3 \nmid x$ and $3 \nmid y$.

(e) the set of pairs (x, y), where $x \neq 0$ and $y \neq 0$; the set of pairs (x, y), where $x = 0$ and $y \neq 0$; and the set of pairs (x, y), where $x \neq 0$ and $y = 0$.

(a) not a partition, subsets are not pairwise disjoint, (2,3) is in both.

(b) This is a partition

(c) not a partition subset are not pairwise disjoint (2,3) is in both (0,0) is in non of the ~~bo~~ subsets.

(d) Partition

(e) not a partition, union of these subsets is not all of 2×2 (00) is in none of the pairs.

40. List the ordered pairs in the equivalence relations produced by these partitions of $\{0, 1, 2, 3, 4, 5\}$.

(a) $\{0\}, \{1, 2\}, \{3, 4, 5\}$

(b) $\{0, 1\}, \{2, 3\}, \{4, 5\}$

(c) $\{0, 1, 2\}, \{3, 4, 5\}$

(d) $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}$

(a) R = { (0,0) (1,1) (2,2) (1,2) (2,1) (3,3) (4,4) (5,5), (3,4), (4,3) (3,5), (5,3) (4,5) (5,4) }

(b) R = { (0,0) (1,1) (0,1) (1,0) (2,2) (3,3) (2,3) (3,2) (4,4) (5,5) (4,5) (5,4) }

(c) R = { (0,0) (1,1) (2,2) (0,1) (1,0) (0,2), (2,0) (1,2) (2,1) (3,3) (4,4) (5,5) (3,4) (4,3) (3,5) (5,3) (4,5) (5,4) }

(d) R = { (0,0) (1,1) (2,2) (3,3) (4,4) (5,5) }

41. Find the smallest equivalence relation on the set $\{a, b, c, d, e\}$ containing the relation $\{(a, b), (a, c), (d, e)\}$.

→ { (a,a) (b,b) (c,c) (d,d) (e,e), (a,b) (a,e), (d,e) (b,e) (b,a) (c,a) (e,d) (c,b) (b,c) }

42. Suppose that R_1 and R_2 are equivalence relations on the set S. Determine whether each of these combinations of R_1 and R_2 must be an equivalence relation.
(a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 \oplus R_2$

(a) False
if R_1 & R_2 Transitive
$R_1 = \{(1,1)(2,2)(3,3)(1,2)(2,1)\}$
$R_2 = \{(1,1)(2,2)(3,3)(1,3)(3,1)\}$
$R_1 \cup R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,1)(3,1)(1,3)\}$
Here, $(4,1) \in R$ $(1,3) \in R$ but $(4,3) \notin R$

(b) True
$(\forall x \in R)$
$(a,a) \in R$ & $(a,a) \in R_1$
$(a,a) \in R_1 \cap R_2$
$(a,b) \in R \Rightarrow (b,a) \in R_1$
$(a,b) \in R_2 \Rightarrow (b,a) \in R_2$
if $(a,b) \in R_1 \cap R_2$
$(b,a) \in R_1 \cap R_2$

$(a,b) \in R_1$ & $(b,c) \in R_1$
then $(a,c) \in R_1$
$(a,b) \in R_2 \Rightarrow (b,c) \in R_2$
then $(a,c) \in R_2$
if $(a,b) \in R_1 \cap R_2$
$(b,c) \in R_1 \cap R_2$
then $(a,c) \in R_1 \cap R_2$

43. Determine the number of different equivalence relations on a set with three elements by listing them. $\{1,2,3,4\}$

→ 1. $\{\{1,2,3\}\} = \{(1,1)(2,2)(3,3)(1,2)(2,1)(1,3)(3,1)(2,3)(3,2)\}$
2. $\{\{1\},\{2\},\{3\}\} = \{(1,1)(2,2)(3,3)\}$
3. $\{\{1,2\},\{3\}\}$, $\{1,3\}\{2\}\}$ & $\{\{2,3\},\{1\}\}$
$\{(1,1)(2,2)(1,2)(2,1)(3,3)\}$ $\{(2,2)(3,3)(2,3)(3,2)(1,1)\}$
4. $\{(1,1)(3,3),(1,3)(3,1)(2,2)\}$

ANS : 5

44. Do we necessarily get an equivalence relation when we form the transitive closure of the symmetric closure of the reflexive closure of a relation?

Yes

45. Do we necessarily get an equivalence relation when we form the symmetric closure of the reflexive closure of the transitive closure of a relation?

Yes

46. Let R be the relation $\{(a,b) \mid a \neq b\}$ on the set of integers. Find the reflexive closure of R.

→ $R^* = \{(a,b) \mid a \neq b\} \cup \{(a,b) \mid a = b\}$
$= R \cup \{(a,b) \mid a = b\}$

47. Find the smallest relation containing the relation $R = \{(a,b) \mid a > b\}$ on the set of positive integers that is both reflexive and symmetric.

$R \cup \{(a,a) \mid a \in A\} \cup \{(a,b) \mid a < b\} = R^2$

48. Suppose that the relation R on the finite set A is represented by the matrix M_R. Show that the matrix that represents the reflexive closure of R is $M_R \vee I_n$ and the matrix that represents the symmetric closure of R is $M_R \vee M_R'$.

let ~~a~~ Relation $R = \{(1,1)(2,3)(1,2)(2,1)(2,2)(3,2)\}$ OR

Set $A = \{1,2,3\}$

$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\therefore M_R \vee I_n = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$R' = \{(1,1)(2,3)(1,2)(2,1)(2,2)(3,2)(3,3$

$M_R \vee M_R' = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$; $R' = \{(1,1)(2,2)(3,3$
$(2,1)(1,3)(3,1$
$(3,2)\}$

49. When is it possible to define the "irreflexive closure" of a relation R?

50. Show that the closure of the relation $R = \{(0,0),(0,1),(1,1),(2,2)\}$ on the set $\{0,1,2\}$ with respect to the property P does not exist if P is the property

(a) "is not reflexive".

(b) "has an odd number of elements".

51. Let $R = \{(1,3),(2,4),(3,1),(3,5),(4,3),(5,1),(5,2),(5,4)\}$ be the relation on the set $\{1,2,3,4,5\}$. Find R^3 and R^*.

$R = \{(1,3)(2,4)(3,1)(3,5)(4,3)(5,1)(5,2)(5,4)\}$

$R^2 = R \circ R = \{(1,1)(1,5)(2,3)(3,3)(3,1)(3,2)(3,4)(4,1)(4,5)(5,3)(5,4)\}$

$R^3 = R^2 \circ R = \{(1,3)(1,1)(1,2)(1,4)(2,1)(2,5)(3,1)(3,5)(3,3)(3,4)(4,3)($
$(4,4)(5,1)(5,5)(5,3)(5,5)\}$

52. Let R be the relation. Prove the following:

(a) If R is reflexive, then R^* is reflexive.
(b) If R is symmetric, then R^* is symmetric.

53. Find the smallest relation containing the relation $\{(1,2),(1,4),(3,3),(4,1)\}$ that is
(a) reflexive and transitive.
(b) symmetric and transitive.
(c) reflexive, symmetric and transitive.

(a) {(1,1) (2,2) (4,4) (1,2) (1,4) (3,3) (4,1) (4,2)}
(b) {(1,2) (2,1) (1,4) (4,1) (3,3) (1,1) (2,2) (2,4), (1,1)}
(c) {(1,1) (2,2) (3,3) (4,4) (1,2) (2,1) (1,4) (4,1) (3,3) (2,4)}

54. There are two algorithms to find the transitive closure of a relation. One algorithm finds the zero-one matrix of the transitive closure R^* by using the expression

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee \dots \vee M_R^{[n]}$$

and the other is Warshall's algorithm.
Find the transitive closures of these relations on $\{1,2,3,4\}$.
(a) $\{(1,2),(2,1),(2,3),(3,4),(4,1)\}$
(b) $\{(2,1),(2,3),(3,1),(3,4),(4,1),(4,3)\}$
(c) $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$
(d) $\{(1,1),(1,4),(2,1),(2,3),(3,1),(3,2),(3,4),(4,2)\}$
Use separate sheets of paper to solve this example.

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad M_R^{[2]} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_R[3] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$M_R[4] = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Using Warshall's algo
Column 1 : (2,1) (4,1)
Row 1 : (1,2)
=> (2,1) (1,2) (4,1) (1,1) (2,2) (4,2)

Column 2 : (1,2) (2,2) (4,2)
Row 2 : (2,1) (2,2) (2,3)
=> (1,2) (2,1) (2,2) (2,3) (1,3) (1,1) (4,2) (4,3)

$$W_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad W_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[3]} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[4]} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{+} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

⇒ Warshall's Algo.

→ $W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Column 2: (3,2) (4,2)
Row 2: (2,1) (2,3) (2,4)
→ (3,2) (2,1) (3,1) (4,2) (2,3) (4,3) (3,3)
(2,2) (4,1) (2,4) (3,4) (4,4) (4,3)

→ $W_0 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Column 1: (1,1) (2,1) (3,1)
Row: (1,1) (1,4)
⇒ (1,1) (1,4) (2,1) (2,4) (3,1) (3,4)

→ $W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Column 3: (2,3) (3,3) (4,3)
Row: (3,1) (3,2) (3,3) (3,4)
→ (2,3) (2,1) (2,2) (2,4) (3,3) (4,3) (4,1),
(4,2) (4,4) (3,1) (3,2) (3,3) (3,4)

→ $W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Column 4: (1,4) (2,4) (3,4) (4,4)
Row 4: (4,1) (4,2) (4,3) (4,4)
⇒ (1,4) (1,1) (1,2) (1,3) (2,4) (2,1) (2,2) (2,3)
(3,4) (3,1) (3,2) (3,3) (4,4) (4,1) (4,2) (4,3)

→ $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

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$$M_R^{[4]} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^{+} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Warshall

$$W_0, W_1, W_2, W_3, W_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Column 1 : ϕ

Row 1 : (1,2) (1,3) (1,4)

Column 2 : (2,1)

Row 2 : (2,3) (2,4)

Column 3 : (1,3) (2,3)

Row 3 : (3,4)

→ (1,3) (1,4) (2,3) (2,4) (3,4)

Column 4 : (1,4) (2,4) (3,4)

Row 4 : ϕ

$\therefore W_0 = W_1 = W_2 = W_3 = W_4$

$\therefore$ Transitive closure : {(1,2) (1,3) (1,4) (2,3) (2,4) (3,4)}

(d) R = {(1,1) (1,4) (2,1) (2,3) (3,1) (3,2) (3,4) (4,2)}

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_R^{[2]} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

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(b) $M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

$$M_R^{[2]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_R^{[3]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M_R^{[4]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M_R^{[5]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_R^{+} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

using Warshall's algorithm,

$$W_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Column 1 : (2, 1) (3, 1) (4, 1)
Row 1 : ϕ
Column : ϕ
Row 2 : (2, 1) (2, 3)

$$W_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Column 3 : (2, 3) (4, 3)
Row 3 : (3, 1) (3, 4)
→ (2, 3) (3, 1) (3, 4) (2, 1) (2, 4)
(4, 3) (4, 1) (4, 4)

$$W_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Column 4 : (2, 4) (3, 4) (4, 4)
Row 4 : (4, 1) (4, 3) (4, 4)
→ (2, 4) (4, 1) (2, 1) (4, 3) (2, 3) (3, 4)
(3, 1) (3, 3) (4, 4)

$R = \{(1,2) (1,3) (1,4) (2,3) (2,4) (3,4)\}$

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^{[0]} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans. $M_R^{[2]} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$