Project Evaluation and review technique (PERT)

- each activity will have three time estimates.
  - Optimistic time
  - most likely time
  - Pessimistic time

Activity	Predecessor(s)	Duration (weeks)		
		O	m	P
A	-	5	6	7
B	-	1	3	5
C	-	1	4	7
D	A	1	2	3
E	B	1	2	9
F	C	1	5	9
G	C	2	2	8
H	E, F	4	4	10
I	D	2	5	8
J	H, G	2	2	8

J	H, G	2	2	8

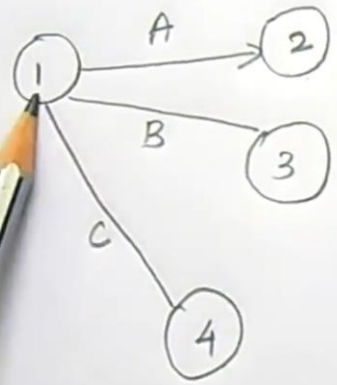
a) Construct the Project network.

b) Find the expected duration and variance of each activity.

c) Find the Critical path and expected Project-Completion time.

d) What is the Probability of Completing the project on or before 22 weeks?

a) The Project network :



A	-	5	6	7
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E	B	1	2	9
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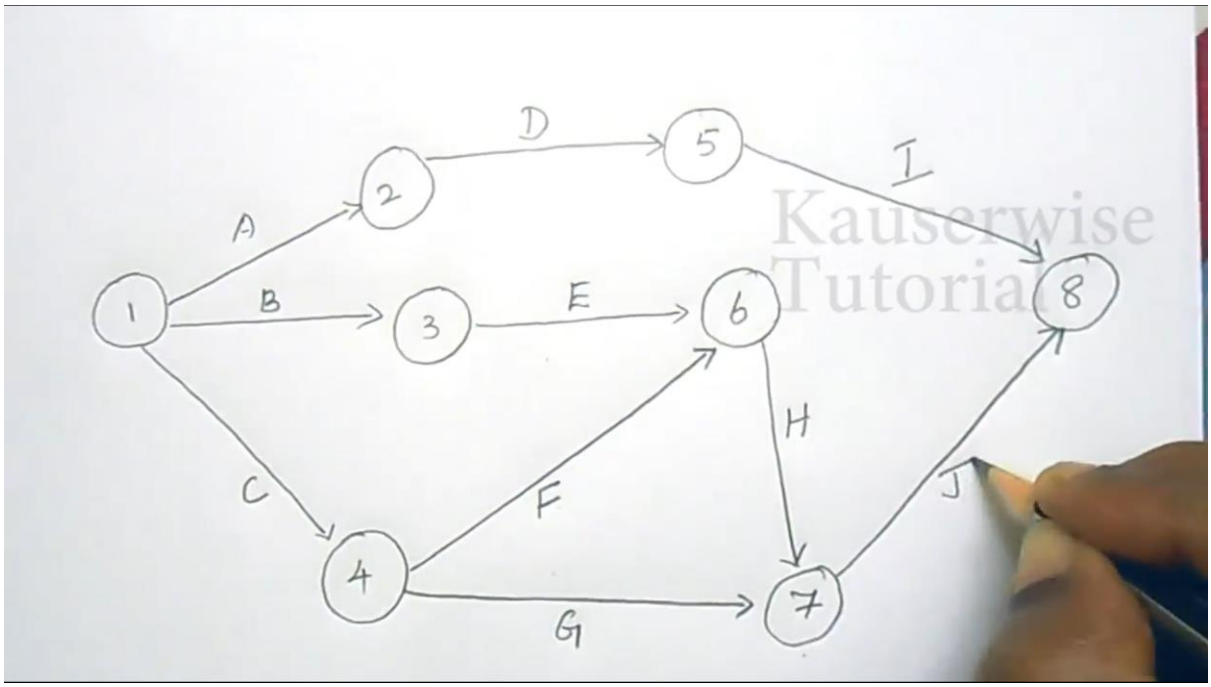

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I	D	2	5	8
J	H, G	2	2	8

2) Calculation of the expected duration and variance of each activity.

Activity	Duration (weeks)				
	o	m	p		
	5	6	7		
	1	3	5		
	1	4	7		
		2	3		
			9		

H	4	4	10
I	2	5	8
J	2	2	8

Mean t_e - Expected duration

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

J	2	2	8

$$A = \frac{5 + (4 \times 6) + 7}{6} = 6$$

$$B = \frac{1 + (4 \times 3) + 5}{6} = 3$$

I	2			
J	2	2	8	3

Mean t_e – Expected duration

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Variance $\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$

...ity	Duration (weeks)			Mean duration	Variance
	o	m	p		
	5	6	7	6	
	1	3	5	3	
	1	4	7	4	
	1	2	3	2	
	1	2	9	3	
	1	5	9	5	

C	1	4	7	2
D	1	2	3	2
E	1	2	9	3
F	1	5	9	5
G	2	2	8	3
H	4	4	10	5
I	2	5	8	5
J	2	2	8	3

I	2			
J	2	2	8	3

Mean t_e - Expected duration

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Variance $\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$

A	5	6	7	6	0.11
B	1	3	5	3	
				4	

$$A = \left(\frac{7-5}{6}\right)^2 = \left(\frac{2}{6}\right)^2 = 0.11$$

$$B = \left(\frac{5-1}{6}\right)^2 = \left(\frac{4}{6}\right)^2 = 0.44$$

C	1	4		2	0.11
D	1	2	3	3	1.78
E	1	2	9	5	1.78
F	1	5	9	3	1.00
G	2	2	8	5	1.00
H	4	4	10	5	1.00
I	2	5	8	3	1.00
J	2	2	8		

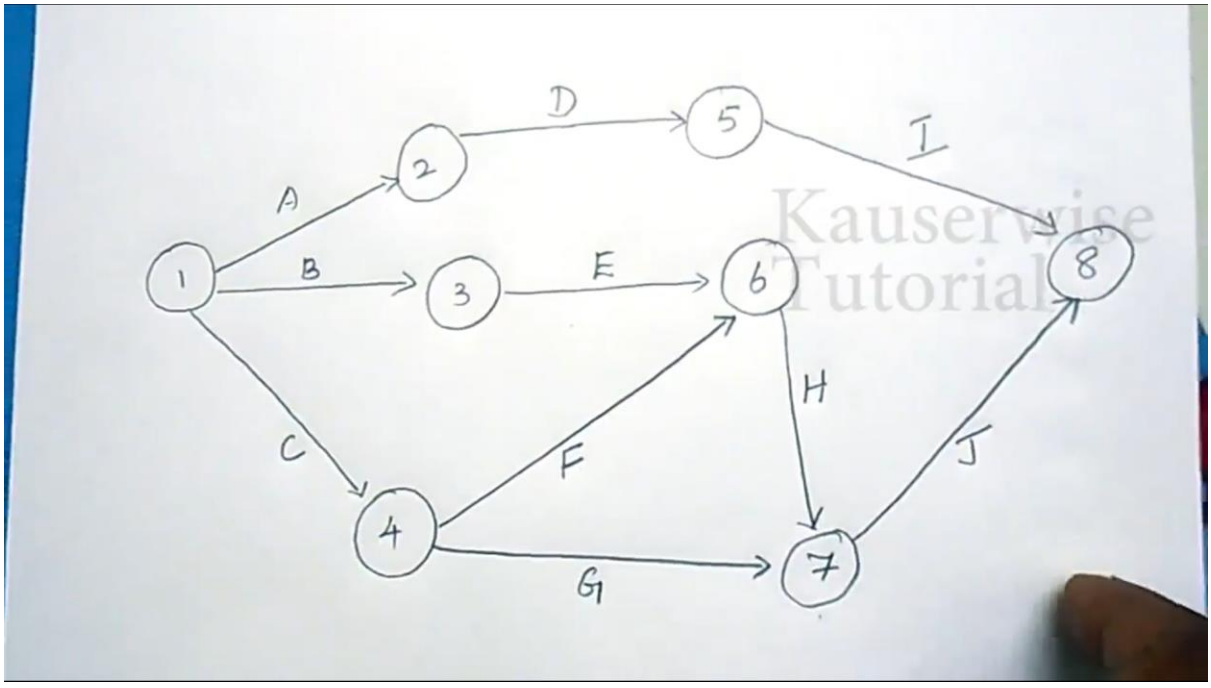
J	H, G	2	2	8

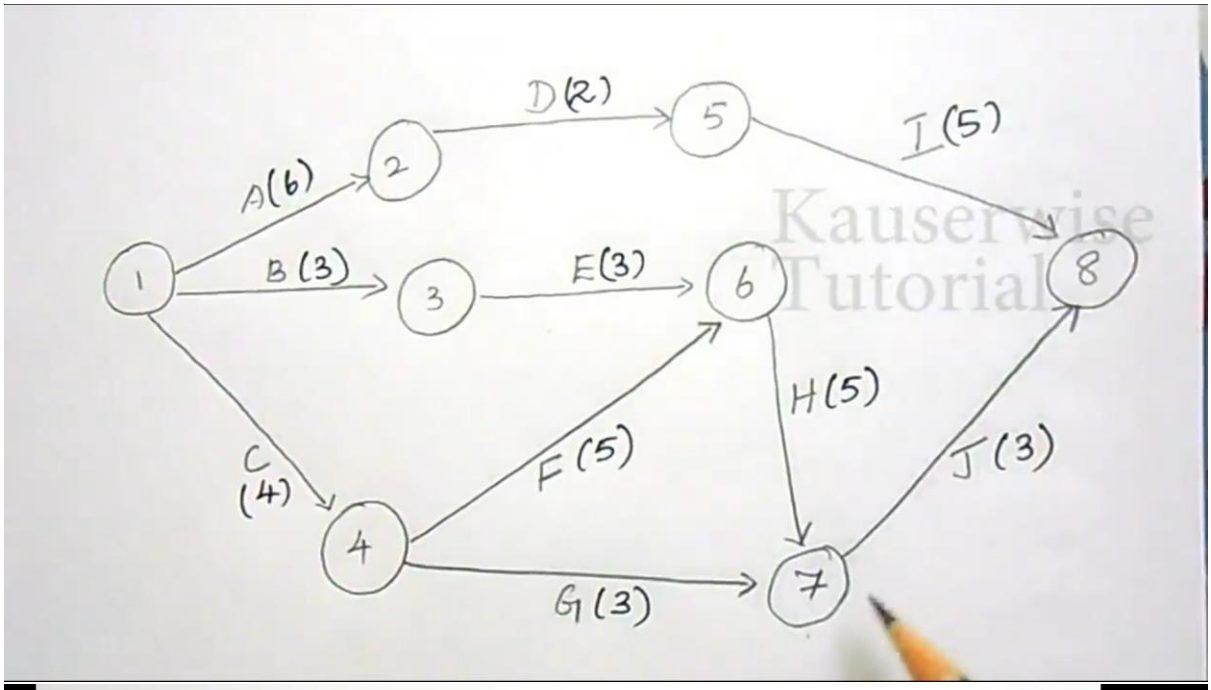
a) Construct the Project network.

b) Find the expected duration and variance of each activity.

c) Find the Critical path and expected project-completion time.

d) What is the probability of completing the project on or before 22 weeks?

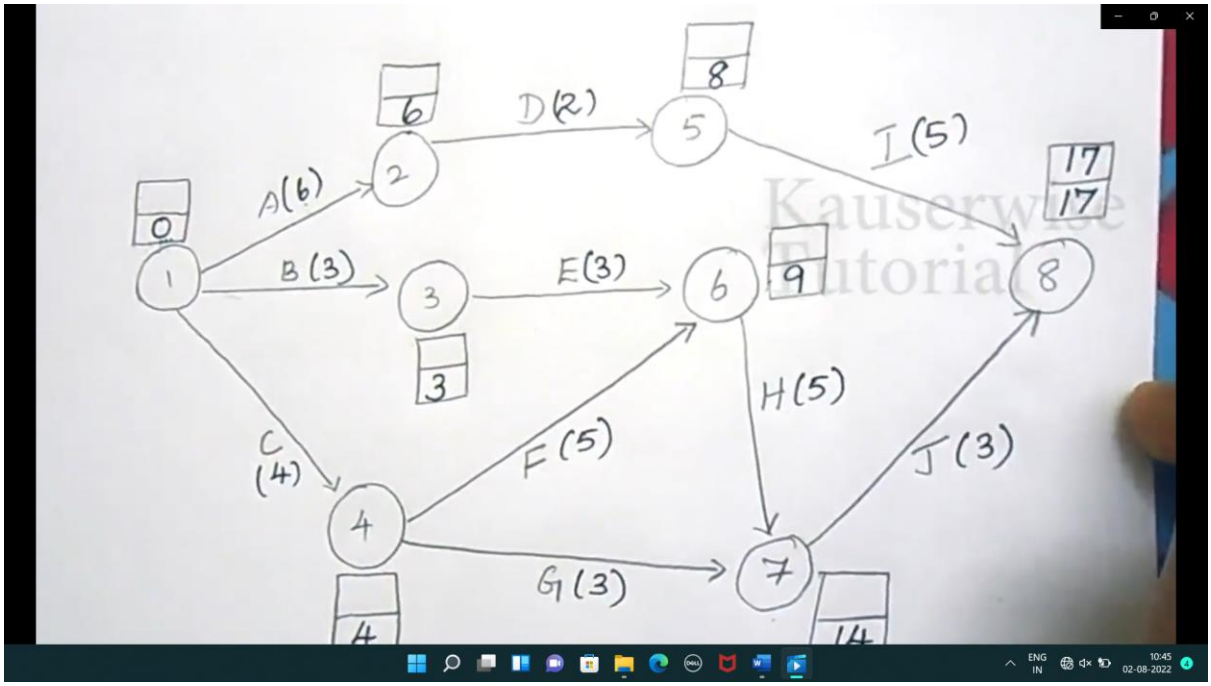


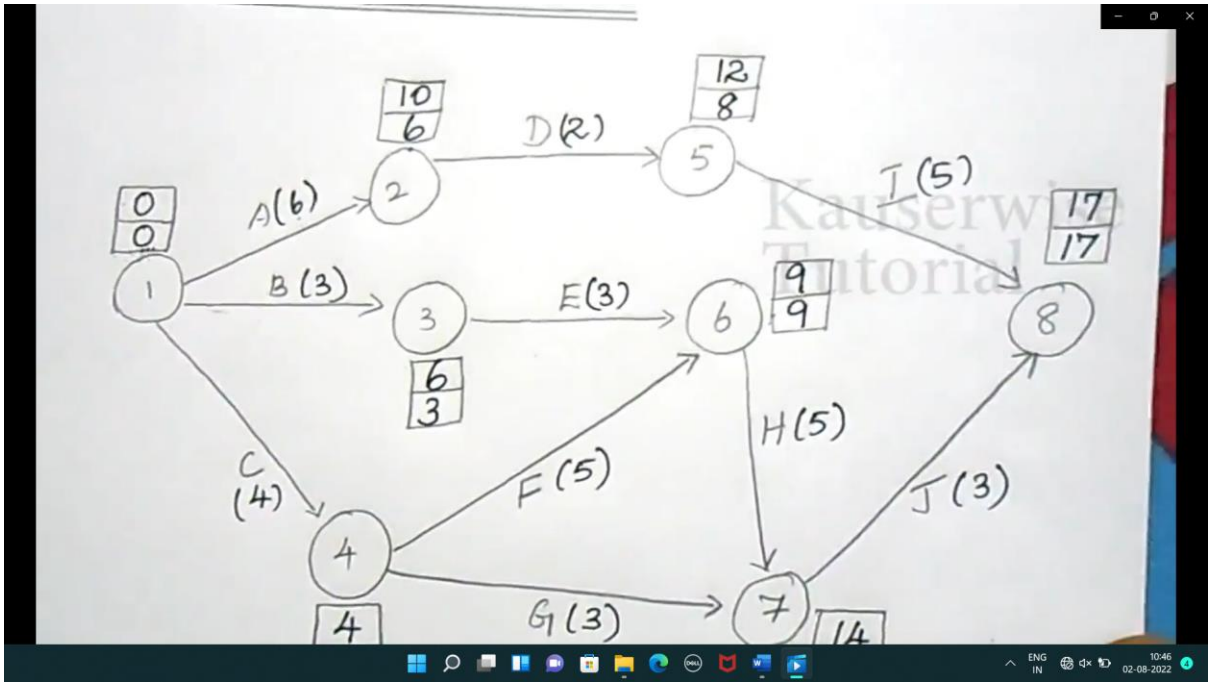


Earliest Start time – Forward Pass

Latest Completion time - backward pass

$= \max(ES_i + D_{ij})$


0
1
A(6)
2
6
D(2)
5
8
I(5)
17
17
8
B(3)
3
3
E(3)
6
9
H(5)
C
(4)
F(5)
J(3)
4
4
G(3)
7
14
Kauserwise Tutorials
ENG
IN
10:45
02-08-2022


0
0
1
A(6)
2
10
6
D(2)
5
12
8
I(5)
17
17
8
B(3)
3
6
3
E(3)
6
9
9
H(5)
C
(4)
F(5)
J(3)
4
4
G(3)
7
14
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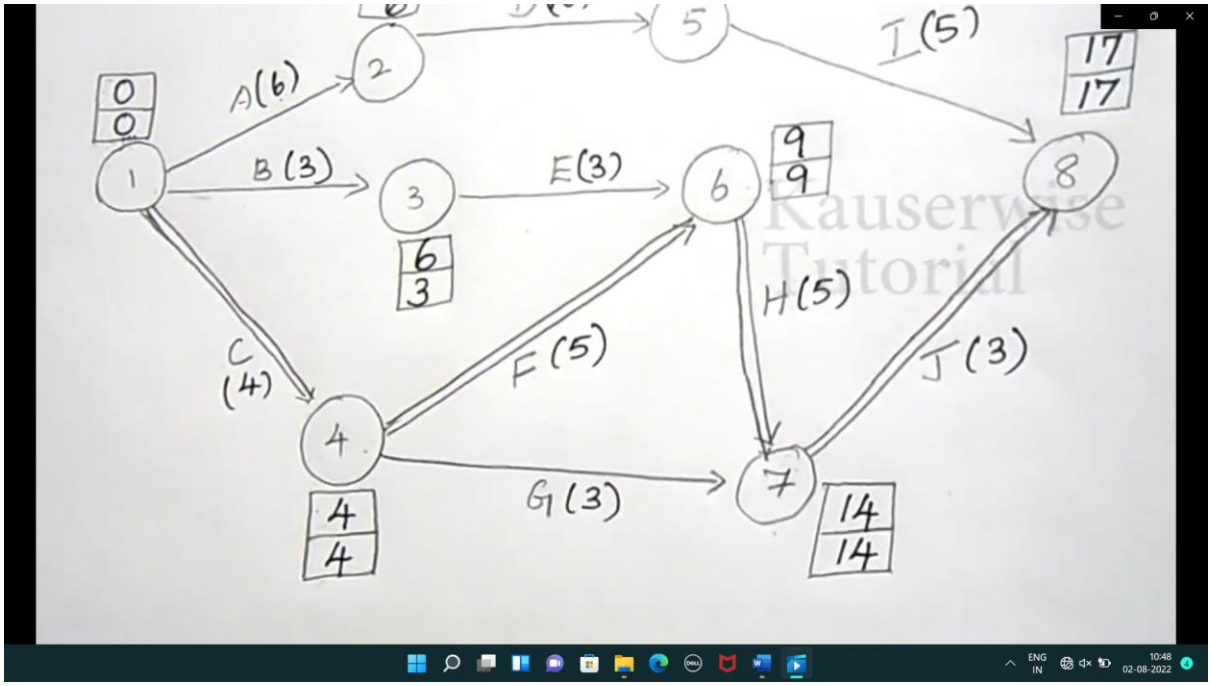
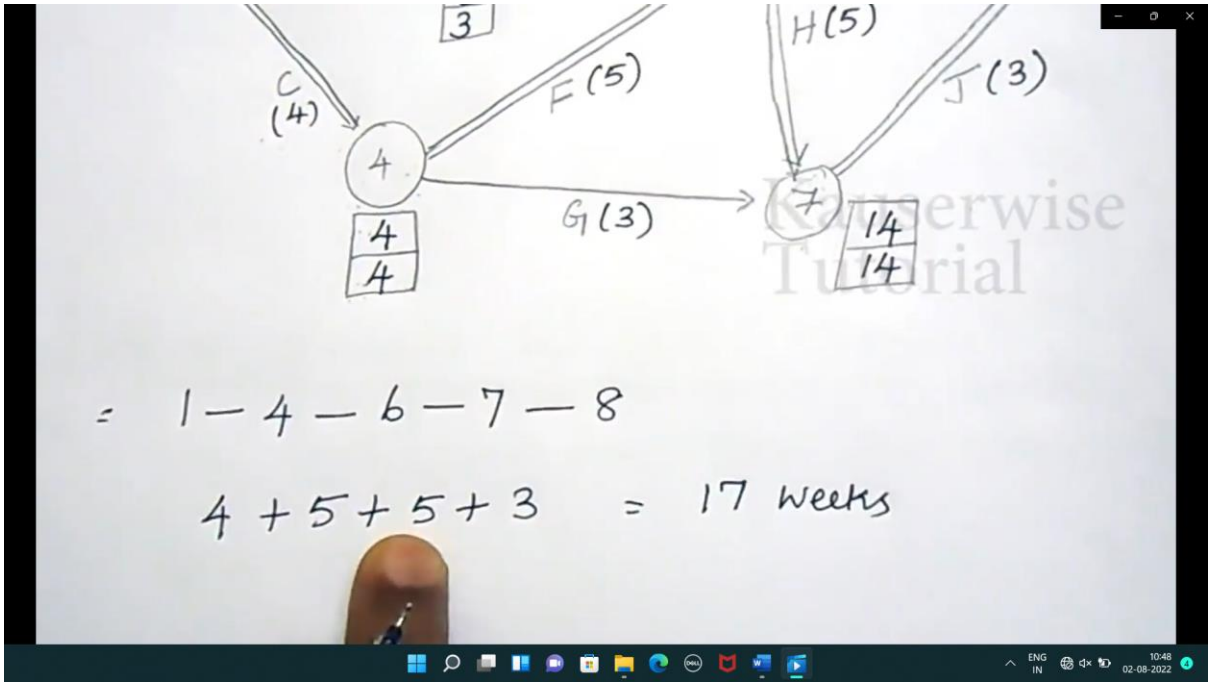
Conditions

1) $ES_i = LC_i$

2) $ES_j = LC_j$

3) $ES_j - ES_i = LC_j - LC_i = D_{ij}$





$= 1 - 4 - 6 - 7 - 8$

$4 + 5 + 5 + 3 = 17$ weeks

J	H, G	2	2	8

a) Construct the Project network.

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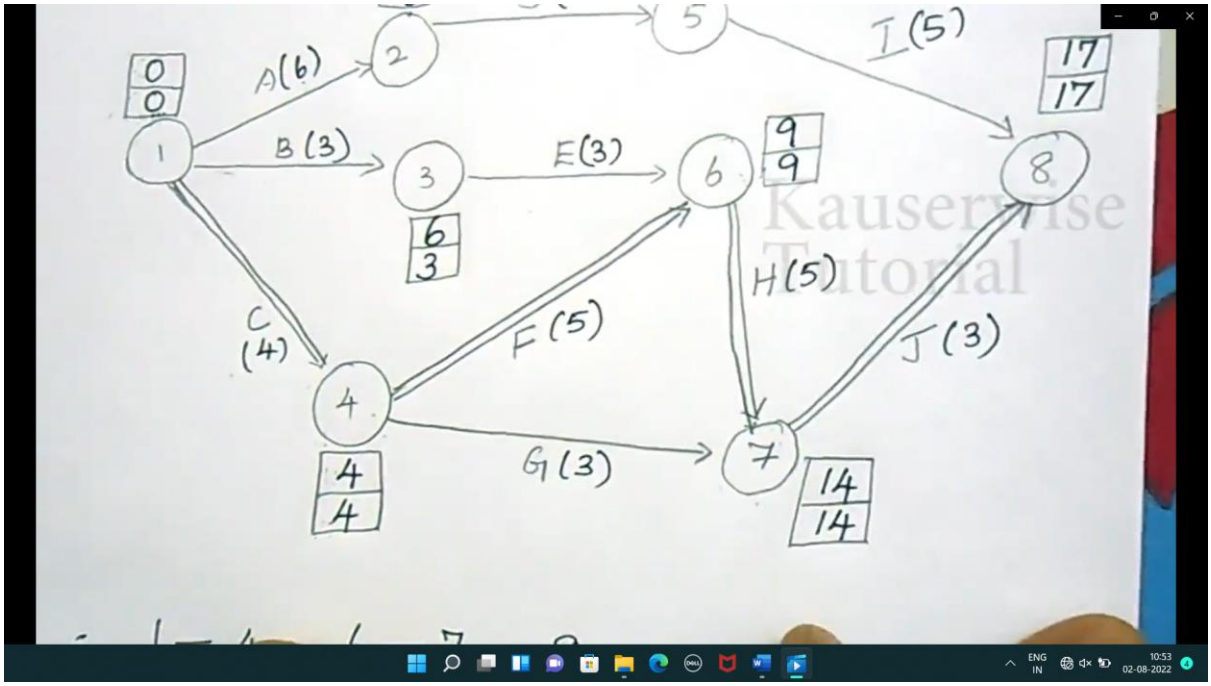
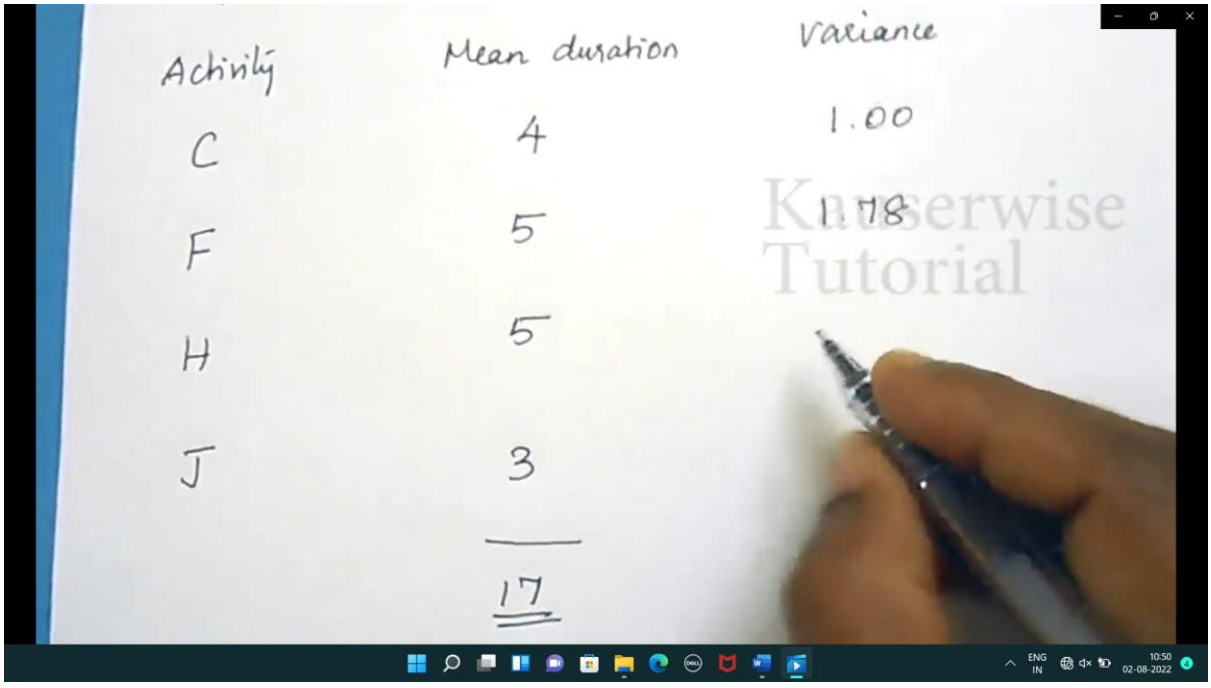
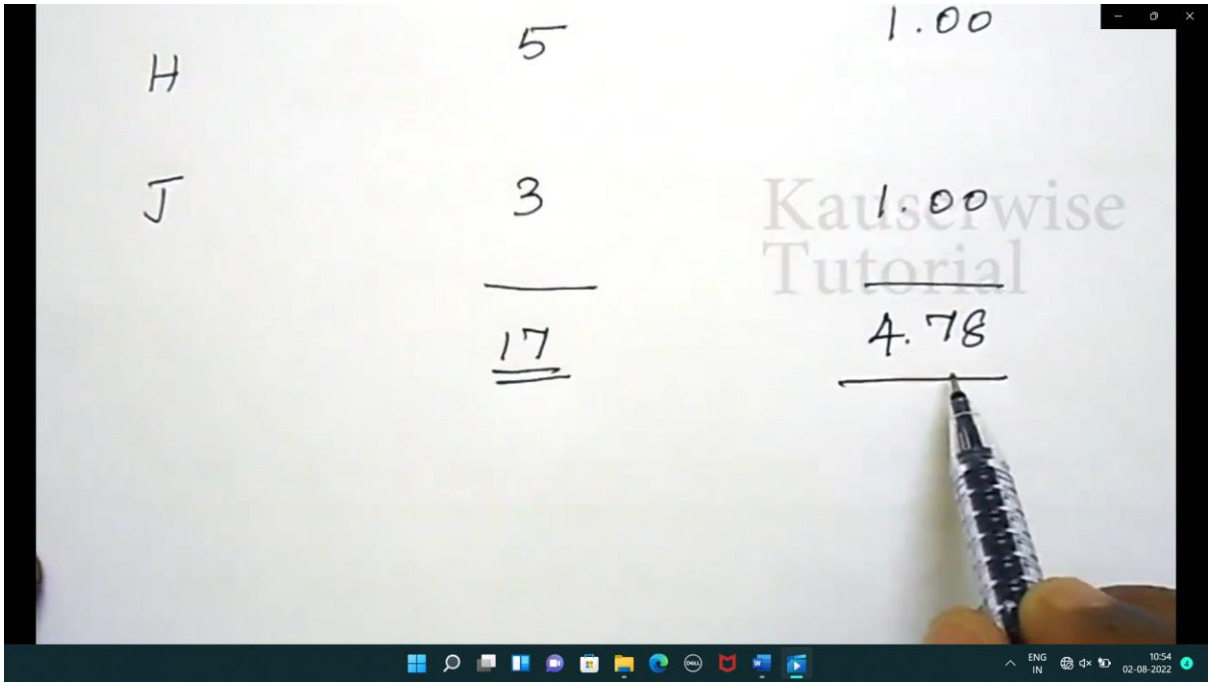
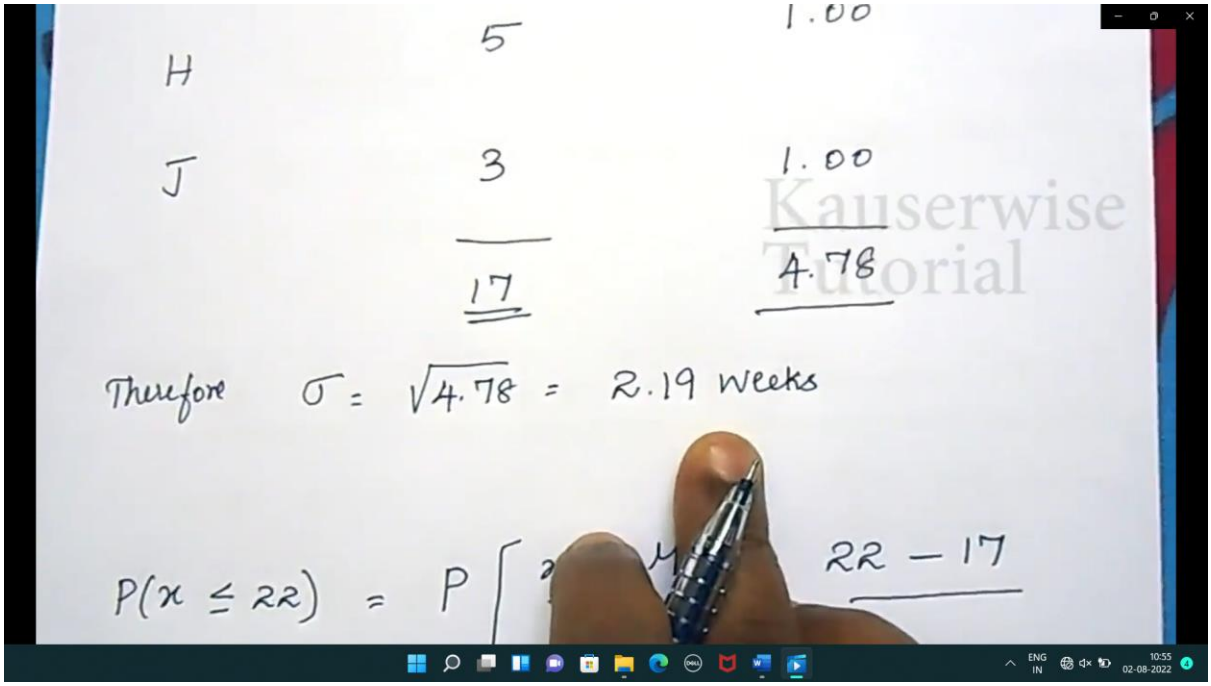
d) What is the Probability of Completing the project on or before 22 weeks?


A(6)
B(3)
C (4)
E(3)
F(5)
G(3)
H(5)
I(5)
J(3)
0 0
6 3
4 4
9 9
14 14
17 17
1
2
3
4
5
6
7
8
Kauserwise Tutorial


Activity
Mean duration
Variance
C
4
1.00
F
5
1.78
H
5
J
3
17
Kauserwise Tutorial


H
5
1.00
J
3
1.00
17
4.78
Kauserwise Tutorial


H
5
1.00
J
3
1.00
17
4.78
Kauserwise Tutorial
Therefore σ = √4.78 = 2.19 weeks
P(x ≤ 22) = P
22 − 17

17

Therefore $\sigma = \sqrt{4.78} = 2.19$ weeks

$$P(x \leq 22) = P\left[\frac{x - \mu}{\sigma} \leq \frac{22 - 17}{2.19}\right]$$

$$= P[z \leq 2.28]$$

Standard Normal Distribution table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370						

2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.99
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

2.28

2.2
0.08
2.28

0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.8133
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1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	
2.5	.9938	.9940	.9941	.9943	.9945					

Therefore $\sigma = \sqrt{4.78} =$ [illegible]

$$P(x \leq 22) = P\left[\frac{x - \mu}{\sigma} \leq \frac{22 - 17}{2.19}\right]$$

$$= P[z \leq 2.28] = 0.9887$$

This value is obtained from Std. normal distribution table. Therefore the probability of completing the Project on or before 22 weeks is 0.9887

i.e. 98.87 %