

⇒ Dynamic Programming

Binomial Coefficient

For calculating binomial coefficient

$$\binom{n}{k} = \begin{cases} 1 & \text{if } k=0 \text{ or } k=n \\ \binom{n-1}{k-1} + \binom{n-1}{k} & \text{if } 0 < k < n \\ 0 & \text{Otherwise} \end{cases}$$

Algorithm function

function $c(n, k)$

if $(k=0 \text{ || } k=n)$
{

return 1;

}

else

return $c(n-1, k-1) + c(n-1, k)$

Example

$$(1) \quad c\left(\begin{matrix} 4 \\ 2 \end{matrix}\right)$$

$$= c\left(\begin{matrix} 3 \\ 1 \end{matrix}\right) + c\left(\begin{matrix} 3 \\ 2 \end{matrix}\right) \quad c\left(\begin{matrix} 3 \\ 1 \end{matrix}\right)$$

$$\begin{aligned} \text{Now, for } c\left(\begin{matrix} 3 \\ 1 \end{matrix}\right) \\ &= c\left(\begin{matrix} 2 \\ 0 \end{matrix}\right) + c\left(\begin{matrix} 2 \\ 1 \end{matrix}\right) \\ &= 1 + c\left(\begin{matrix} 2 \\ 1 \end{matrix}\right) \end{aligned}$$

$$\begin{aligned} \text{For } c\left(\begin{matrix} 2 \\ 1 \end{matrix}\right) \\ &= c\left(\begin{matrix} 1 \\ 0 \end{matrix}\right) + c\left(\begin{matrix} 1 \\ 1 \end{matrix}\right) \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$\therefore c(3,1) = 1 + 1 + 1 = 3$$

$$\begin{aligned}\text{Now for } c(3,2) \\ &= c(2,1) + c(2,2) \\ &= c(2,1) + 1\end{aligned}$$

$$\begin{aligned}\text{For } c(2,1) \\ &= c(1,0) + c(1,1) \\ &= 1 + 1 \\ &= 2\end{aligned}$$

$$c(3,2) = 3$$

$$\begin{aligned}c(4,2) &= 3 + 3 \\ &= \underline{\underline{6}}\end{aligned}$$