G H PATEL COLLEGE OF ENGINEERING & TECHNOLOGY, V V NAGAR

A.Y.2021-22: EVEN SEMESTER

102040405: DISCRETE MATHEMATICS

Assignment 4: Logic

1. Let p and q be the propositions
   p: You drive over 65 miles per hour.
   q: You get a speeding ticket.
   Write these propositions using p and q and logical connectives.
   (a) You do not drive over 65 miles per hour.
   (b) You drive over 65 miles per hour, but you do not get a speeding ticket.
   (c) You will get a speeding ticket if you drive over 65 miles per hour.
   (d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
   (e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
   (f) You get a speeding ticket, but you do not drive over 65 miles per hour.
   (g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

(a) $\neg p$
(b) $p \wedge \neg q$
(c) ~~$\neg p \wedge \neg q$~~ $q \rightarrow p$
(d) $\neg p \rightarrow \neg q$
(e) $p \rightarrow q$
(f) $q \wedge \neg p$
(g) $q \rightarrow p$

2. Let p, q and r be the propositions
   p: You get an A on the final exam.
   q: You do every exercise in this book.
   r: You get an A in this class.
   Write these propositions using p, q and r and logical connectives.
   (a) You get an A in this class, but you do not do every exercise in this book.
   (b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
   (c) To get an A in this class, it is necessary for you to get an A on the final.
   (d) You get an A on the final, but you don't do every exercise in this book; nevertheless you get an A in this class.
   (e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
   (f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

(a) $r \wedge \neg q$
(b) $p \wedge q \wedge r$
(c) $r \rightarrow p$
(d) $p \wedge \neg q \wedge r$
(e) $(p \wedge q) \rightarrow r$
(f) $r \leftrightarrow (p \vee q)$

3. Let Let p, q and r be the propositions
p: You have the flu.
q: You miss the final examination.
r: You pass the course.
Express each of these propositions as an English sentence.

(a) $p \to q$ (b) $\neg q \to r$ (c) $q \to \neg r$
(d) $p \vee q \vee r$ (e) $(p \to \neg r) \vee (q \to \neg r)$ (f) $(p \wedge q) \vee (\neg q \wedge r)$

(a) If you have flu then you will miss the final exam
(b) You won't miss the final examination if and only if you pass the course.
(c) If you miss the examination then you will be failing the course.
(d) You have the flu or you miss the exam or you pass the course
(e) If you have the flu then you'll not pass the course or if you miss the final exam then you'll fail the course.
(f) You have the flue and you miss the exam or you will not miss the final exam and you pass the course.

4. Determine whether the following statements are true or false.
(a) $1 + 1 = 3$ if and only if monkeys can fly. True - first is false & second is false so it makes biconditional true
(b) $0 > 1$ if and only if $2 \geq 1$. False - first is F and second is T, so everything false
(c) If $1 + 1 = 3$, then $2 + 2 = 4$. False
(d) If $2 + 2 = 4$, then $1 + 2 = 3$. True
First true & second → true, so ans true

5. For each of these sentences, determine whether an "inclusive or" or an "exclusive or" is intended.
(a) Coffee or tea comes with dinner.
(b) A password must have at least three digits or be at least eight characters long.
(c) Experience with C++ or Java is required.
(d) Lunch includes soup or salad.
(e) The prerequisite for the course is a course in number theory or a course in cryptography.
(f) School is closed if more than 2 feet of snow falls or if the wind chill is below −100.

(a) exclusive OR
(b) inclusive OR
(c) inclusive OR
(d) exclusive OR
(e) inclusive OR
(f) inclusive OR

6. Write each of these sentences in the form "if p, then q" in English.
(a) Willy gets caught whenever he cheats.
(b) Getting elected follows from knowing the right people.
(c) It snows whenever the wind blows from the northeast.
(d) That you get the job implies that you had the best credentials.
(e) It is necessary to have a valid password to log on to the server.
(f) You will reach the summit unless you begin your climb too late.

(a) If willy ~~the~~ cheats then he gets caught.

(b) If ~~you~~ it is knowing right people then ~~you will be~~ it is getting elected.

(c) If wind blows from the northeast then it snows.

(d) If you had the best credentials then you get the job

(e) If you want to log on to the server then it is necessary to have a valid password.

(f) If you begin you climb early then you will reach the summit.

7. Write each of these propositions in the form "p if and only if q" in English.
(a) For you to win the contest it is necessary and sufficient that you have the only winning ticket.
(b) If you watch television your mind will decay, and conversely.
(c) The trains run late on exactly those days when I take it.
(d) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.

(a) It is necessary and sufficient for you if you have the only winning ticket.

(b) Your mind will decay and conversely if you watch television.

(c) The train run later if I take it on those days.

(d) It is neccessary and sufficient that you learn how to solve discrete mathematics problem if you get an A.

(d) converse: If it is a sunny summer day, then I go to the beach.

Contrapositive: If it is a sunny summer day, then I go to beach

- inverse: I do not go the beach whenever it is not a sunny summer day.

8. State the converse, contrapositive and inverse of each of these conditional statements.
   (a) If it snows tonight, then I will stay at home.
   (b) I come to class whenever there is going to be a quiz.
   (c) A positive integer is a prime only if it has no divisors other than 1 and itself.
   (d) I go to the beach whenever it is a sunny summer day.

(a) Converse: If I stay at home, then it snows tonight.
Contrapositive: If I don't stay at home tonight then it won't be snowing.
Inverse: If it does not snow tonight, then I will not stay at home

(b) converse: If I come to class, the quiz is going to be held.
contrapositive: If I don't come to class, quiz is not going to held.
inverse: If the quiz is not going ~~to~~ be held, ~~s~~ I will not come to class.

(C) converse: If a positive integer has no divisor other than 1 & itself, then it is a prime number.
contrapositive: If a positive integer has divisor other than 1 and itself then it is not prime.

9. How many rows appear in a truth table for each of these compound propositions?
   (a) $p \to \neg p$ (b) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$ (c) $(p \vee \neg t) \wedge (p \vee \neg s)$

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10. Construct a truth table for each of these compound propositions.
   (a) $(p \oplus q) \to (p \oplus \neg q)$ (b) $(p \wedge q) \to (p \vee q)$

P	q	$P \oplus q$	$\bar{q}$	$P \oplus \bar{q}$	$(P \oplus q) \to (P \oplus \neg q)$
0	0	0	1	1	T
0	1	1	0	0	F
1	0	1	1	0	F
1	1	0	0	1	T

(b)

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \to (p \vee q)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

11. Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.
(*a*) 101 1110, 010 0001 (*b*) 00 0111 0001, 10 0100 1000

(a) OR ⇒ 111 1111

AND ⇒ 000 0000

XOR ⇒ 111 1111

(b) OR ⇒ 1001111001

AND ⇒ 0001000000

XOR ⇒ 1000111001

12. Evaluate each of these expressions.
(*a*) $(0\,1111 \wedge 1\,0101) \vee 0\,1000$ (*b*) $(0\,1010 \oplus 1\,1011) \oplus 0\,1000$

(a) $(00101) \vee (01000)$

$= 01101$

(b) $(10001) \oplus (01000)$

$= 11001$

13. Use a truth table to verify the De Morgan's law $\neg(p \wedge q) = \neg p \vee \neg q$.

p	q	$\neg p$	$\neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
F	F	T	T	T	T
F	T	T	F	T	T
T	F	F	T	T	T
T	T	F	F	F	F

By Truth Table $\neg(p \wedge q) = \neg p \vee \neg q$

14. Show that each of these conditional statements is a tautology without using truth tables.
(*a*) $(p \wedge q) \to (p \to q)$ (*b*) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

(a) $(p \wedge q) \to (p \to q) = (p \wedge q) \to (\neg p \vee q)$

$= (p \wedge q) \vee (\neg p \vee q)$

$= (\neg p \vee \neg q) \vee (\neg p \vee q)$

$= (\neg p \vee \neg q) \vee (\neg q \vee q)$

$= \neg p \vee T$

$= T$

(b) $[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$

$=\neg[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\vee r$

$=[\neg[(p\vee q)\wedge(p\to r)\vee\neg(q\to r)]\vee r$

$=[\neg(p\vee q)\vee\neg(\neg p\vee r)\vee\neg(\neg q\vee r)]\vee r$

$=[(\neg p\wedge\neg q)\vee(p\wedge\neg r)\vee(q\wedge\neg r)]\vee r$

$=[(\neg p\vee p)\wedge(\neg q\vee\neg r)\vee(q\wedge\neg r)]\vee r$

$=[(T\wedge(\neg q\vee q)\wedge(\neg r\wedge\neg r)]\vee r$

$=(T\wedge T\wedge\neg r)\vee r \quad = [T\wedge T\wedge(\neg r\vee r)] = T$

15. Show that each of the following pairs of compound propositions are logically equivalent.
(a) $\neg(p\oplus q)$, $p\leftrightarrow q$ (b) $\neg p\to(q\to r)$, $q\to(p\vee r)$

(a)

p	q	$p\oplus q$	$\neg(p\oplus q)$	$p\leftrightarrow q$
0	0	0	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	1	1

(b) $\neg p\to(q\to r)\equiv\neg p\to(\neg q\vee r)$

$=p\vee(\neg q\vee r)$

$=\neg q\vee(p\vee r)$

$=q\to(p\vee r)$

are logically equivalence

16. Show that each of the following pairs of compound propositions are not logically equivalent.
(a) $(p\wedge q)\to r$, $(p\to r)\wedge(q\to r)$ (b) $(p\to q)\to r$, $p\to(q\to r)$

(a) $(p\wedge q)\to r\equiv\neg(p\wedge q)\vee r$

$=(\neg p\vee\neg q)\vee r$

$=(\neg p\vee r)\vee(\neg q\vee r)$

$\not\equiv(p\to r)\vee(q\to r)$

(b)

p	q	r	$p\to q$	$(p\to q)\to r$	$q\to r$	$p\to(q\to r)$
F	F	F	T	F	T	T
F	F	T	T	T	T	T
F	T	F	T	F	F	T
F	T	T	T	T	T	T
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	T	F	F	F
T	T	T	T	T	T	T

$(p\to q)\to r\not\equiv p\to(q\to r)$

17. Find the dual of each of these compound propositions.

(a) $p \wedge (q \vee (r \wedge T))$ (b) $(p \vee F) \wedge (q \vee T)$

(a) $p \vee (q \wedge (r \vee F)$ (b) $(p \wedge F) \vee (q \wedge T)$

18. Let $P(x)$ be the statement "c can speak Russian" and let $Q(x)$ be the statement "x knows $C++$". Express each of the following sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

(a) There is a student at your school who can speak Russian and who knows $C++$.
(b) There is a student at your school who can speak Russian but doesn't know $C++$.
(c) Every student at your school either can speak Russian or knows $C++$.
(d) No student at your school can speak Russian or knows $C++$.

(a) $\exists x\,(P(x)) \wedge q(x)$ (c) $\forall x\,(P(x) \vee q(x))$

(b) $\exists x\; P(x) \wedge \neg q(x)$ (d) $\forall\; \neg(P(x) \vee q(x))$

19. Let $P(x)$ be the statement "$x = x^2$". If the domain consists of the integers, what are the truth values?

(a) $P(0)$ True (b) $P(1)$ True (c) $P(2)$ False
(d) $P(-1)$ False (e) $\exists x\, P(x)$ True (f) $\forall x\, P(x)$ False
(g) $\exists x\, \neg P(x)$ True (h) $\forall x\, \neg P(x)$ False

20. Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 2, 3, 4, 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

(a) $\exists x\, P(x)$ (b) $\forall x\, P(x)$
(c) $\neg\exists x\, P(x)$ (d) $\neg\forall x\, P(x)$
(e) $\forall x\,((x \neq 3) \to P(x)) \vee \exists x\, P(x)$

(a) $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

(b) $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

(c) $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

(d) $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$

(e) $(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)) \vee (P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

21. Suppose that the domain of the propositional function $P(x)$ consists of the integers $-5, -3, -1, 1, 3, 5$. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

(c) $\forall x\,((x \neq 1) \to P(x))$ (d) $\exists x\,((x \geq 0) \wedge P(x))$

(e) $\exists x\,(\neg P(x)) \wedge \forall x\,((x < 0) \to P(x))$

(c) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(-3) \wedge P(5)$

(d) $[P(1) \wedge P(3) \wedge P(5)] \wedge P(-5) \vee P(-3) \vee P(1) \vee P(1) \vee P(3) \vee P(5)]$

(e) $[\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee P(1)\, \neg P(3) \vee \neg P(5)] \wedge [P(-1) \wedge P(-3) \wedge P(-5)]$

22. Show that $\forall x\, P(x) \vee \forall x\, Q(x)$ and $\forall x\,(P(x) \vee Q(x))$ are not logically equivalent.

Let $P(x) = $ "$x^2 \to 1$"
$Q(x) = x^2 < 1$. Then $\forall x\, P(x)$ & $\forall x\, Q(x)$ are false

- It follow that he statement $\forall x\,(P(x)) \cup \forall x\,(Q(x))$
- For every element x in the domain, either $P(x)$ is true or $Q(x)$ is true.
- Therefore, the statement $P(x) \vee Q(x)$ is true for every x, Hence $\forall x\,(P(x) \cup Q(x))$ is true,

23. Show that $\exists x\, P(x) \wedge \exists x\, Q(x)$ and $\exists x\,(P(x) \wedge Q(x))$ are not logically equivalent.

Let $P(x) = $ "$x^2 \geq 1$" where the domain is the set of all real number
$Q(x) = x^2 < 1$. Then both the statement $\exists x\, P(x)$ and $\exists Q(x)$ are true
It follows that the statement $\exists x P(x)$ and $\exists x\, Q(x)$ ~~are true~~ is also true on $Q(x)$ is true but not both. Therefore, the statement $P(x) \wedge Q(x)$ is not true for any element. Hence $\exists x\,(P(x) \cap Q(x))$ is not ~~tru~~ true.
Too every element x in the domain either $P(x)$ is true or $Q(x)$ is true but not both.

24. If the domain consists of all the integers, what are the truth values of these statements?

(a) $\exists!x\,(x > 1)$ False (b) $\exists!x\,(x^2 = 1)$ False

(c) $\exists!x\,(x + 3 = 2x)$ True (d) $\exists!x\,(x = x + 1)$ False

25. What are the truth values of these statements?

(a) $\exists!x\, P(x) \to \exists x\, P(x)$ True (b) $\forall x\, P(x) \to \exists!x\, P(x)$ False

(c) $\exists!x\, \neg P(x) \to \neg\forall x\, P(x)$ True

26. Write our $\exists!x\, P(x)$, where the domain consists of the integers 1, 2, and 3, in terms of negations, conjunctions, and disjunctions.

$(P(1) \wedge \neg P(2) \wedge \neg P(3)) \vee (\neg P(1) \wedge P(2) \wedge \neg P(3)) \vee (\neg P(1) \wedge \neg P(2) \wedge P(3))$

27. Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a professor", "x is ignorant", and "x is vain", respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people.

(a) No professors are ignorant.
(b) All ignorant people are vain.
(c) No professors are vain.
(d) Does (c) follow from (a) and (b) ?

(a) $\forall x: P(x) \to \neg Q(x))$

(b) $\forall x: Q(x) \to R(x)$

(c) $\forall x: P(x) \to \neg R(x)$

(d) To check if a follow from c and b. We can build a truth table. If there would be at least one row where statements a & b would truth while c would be false then c does not follow from a & b.

28. Let $P(x)$, $Q(x)$, $R(x)$ and $S(x)$ be the statements "x is a baby", "x is logical", "x is able to manage a crocodile", and "x is despised", respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$ and $S(x)$, where the domain consists of all people.

(a) Babies are illogical.
(b) Nobody is despised who can manage a crocodile.
(c) Illogical persons are despised.
(d) Babies cannot manage crocodiles.
(e) Does (d) follow from (a), (b) and (c)?

(a) $\forall x (P(x)) \to \neg Q(x))$

(b) $\neg \exists x (S(x) \wedge R(x))$

(c) $\forall x (\neg Q(x) \to S(x))$

(d) $\forall x (P(x) \to \neg R(x))$

29. Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is a clear explanation", "x is satisfactory", and "x is an excuse", respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$.

(a) All clear explanations are satisfactory.
(b) Some excuses are unsatisfactory.
(c) Some excuses are not clear explanations.
(d) Does (c) follow from (a) and (b)?

(a) $\forall x (P(x) \to Q(x))$

(b) $\exists x (R(x) \wedge \neg Q(x))$

(c) $\exists x (R(x) \wedge \neg P(x))$

30. Let $F(x, y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express each of these statements.

(a) Everybody can fool Fred.
(b) Evelyn can fool everybody.
(c) Everybody can fool somebody.
(d) There is no one who can fool everybody.
(e) Everyone can be fooled by somebody.
(f) No one can fool both Fred and Jerry.
(g) Nancy can fool exactly two people.
(h) There is exactly one person whom everybody can fool.
(i) No one can fool himself or herself.
(j) There is someone who can fool exactly one person besides himself or herself.

(a) $\forall x\, F(x, \text{Fred})$

(b) $\forall y\, F(\text{Evelyn}; y)$

(c) $\forall x \exists y\, F(x, y)$

(d) $\forall y \exists x\, \neg F(x, y)$

(e) $\forall y \exists x((\neg x - y) \wedge F(x, y))$

(F) $\forall x\, \neg(F(x, \text{fred}) \wedge F(x, \text{jerry}))$

(g) $\exists x \exists y\, ((\neg x = y) \wedge F(\text{Nane}(y, x) \wedge F(\text{Nan}(y, y) \wedge \forall w(F(\text{Nan}(y; w) \rightarrow (w = x \vee w = y))$

(i) $\forall x\, \neg F(x, x)$

31. Let $S(x)$ be the predicate "x is a student", $F(x)$ the predicate "x is a faculty member", and $A(x, y)$ the predicate "x has asked y a question", where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.

(a) Lois has asked Professor Michaels a question.
(b) Every student has asked Professor Gross a question.
(c) Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
(d) Some student has not asked any faculty member a question.
(e) There is a faculty member who has never been asked a question by a student.
(f) Some student has asked every faculty member a question.
(g) There is a faculty member who has asked every other faculty member a question.
(h) Some student has never been asked a question by a faculty member.

(a) $A(S(\text{lois}); F(\text{michaels}))$

(b) $\forall x\, A(S(x), F(\text{Gross}))$

(c) $\forall x[A(F(x), f(\text{miller})) \vee F(\text{miller}), f(x)]$

(d) $\exists x\, \neg A(S(x), f(x))$

32. Express each of these statements using mathematical and logical operators, predicates, and quantifiers, where the domain consists of all integers.

(a) The sum of two negative integers is negative.
(b) The difference of two positive integers is not necessarily positive.
(c) The sum of the squares of two integers is greater than or equal to the square of their sum.
(d) The difference of a real number and itself is zero.

(a) $\forall x \forall y((a<0) \wedge (y<0) \rightarrow (x+y<0))$

(b) $\exists x \exists y((x>0) \wedge (y>0) \wedge (x-y) \leq 0))$

(c) $\forall x \forall y(x^2+y^2 \geq (x+y)^2)$

(d) $\forall x(x-x=0)$

(e) The absolute value of the sum of two integers does not exceed the sum of the absolute values of these integers.
(f) Every positive integer if the sum of the squares of four integers.
(g) There is a positive integer that is not the sum of three squares.

(e) $\forall x \forall y ((x+y) \Delta |x| + |y|)$
(f) $\forall x \exists a \exists b \exists c \exists d ((x>0) \rightarrow x = a^2+b^2+c^2+d^2)$
(g) $\exists x \exists a \exists b \exists c ((x>0) \rightarrow x \neq a^2+b^2+c^2)$

33. Let $Q(x, y)$ be the statement "$x + y = x - y$". If the domain for both variables consists of all integers, what are the truth values?

(a) $Q(1,1)$ False
(b) $Q(2,0)$ True
(c) $\forall y\, Q(1,y)$ False
(d) $\exists x\, Q(x,2)$ False
(e) $\exists x \exists y\, Q(x,y)$ True
(f) $\forall x \exists y\, Q(x,y)$ True
(g) $\exists y \forall x\, Q(x,y)$ False
(h) $\forall y \exists x\, Q(x,y)$ False
(i) $\forall x \forall y\, Q(x,y)$ False

34. Determine the truth value of each of these statements if m, n are integers and x, y are real numbers.

(a) $\exists n \forall m\, (n < m^2)$ True
(b) $\forall n \exists m\, (n + m = 0)$ True
(c) $\exists n \exists m\, (n^2 + m^2 = 6)$ False
(d) $\exists n \exists m\, (n + m = 4 \land n - m = 2)$ True
(e) $\forall x \exists y\, (x = y^2)$ False
(f) $\exists x \exists y\, (x + 2y = 2 \land 2x + 4y = 5)$ False
(g) $\exists x \forall y\, (y \neq 0 \rightarrow xy = 1)$ False
(h) $\forall x \forall y \exists z\, (z = (x + y)/2)$ True

35. Suppose the domain of the propositional function $P(x, y)$ consists of the pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

(a) $\forall x \forall y\, P(x,y)$
(b) $\exists x \exists y\, P(x,y)$
(c) $\exists x \forall y\, P(x,y)$
(d) $\forall y \exists x\, P(x,y)$

(a) $P(1,1) \land P(1,2) \land P(1,3) \land P(2,1) \land P(2,2) \land P(2,3) \land P(3,1) \land P(3,2) \land P(3$
(b) $P(1,1) \lor P(1,2) \lor P(1,3) \lor P(2,1) \lor P(2,2) \lor P(2,3) \lor P(3,1) \lor P(3,2) \lor P(3,3)$
(c) $[P(1,1) \land P(1,2) \land P(1,3)] \lor [P(2,1) \land P(2,2) \land P(2,3)] \lor \{P(3,1) \land P(3,2) \land P(2,$
d) $[P(1,1) \lor P(1,2) \lor P(1,3)] \land (P(2,1) \lor P(2,2) \lor P(2,3)) \land [P(3,1) \lor P(3,2) \lor P(3,3)]$

36. Rewrite each of these statements so that negation appears only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives.)

(a) $\neg \exists y \left(\exists x\, R(x,y) \lor \forall x\, S(x,y)\right)$
(b) $\neg y \left(\forall x \exists z\, T(x,y,z) \land \exists x \forall z\, U(x,y,z)\right)$

(a) $\forall y (\forall x \neg R(x,y) \land \exists x \neg S(x,y))$
(b) $\forall y (\exists x \forall z \neg T(x,y,z) \lor \forall x \exists z \neg U(x,y,z))$

37. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

(a) $\forall x \exists y\, (P(x,y) \rightarrow Q(x,y))$ (b) $\exists x \exists y\, (Q(x,y) \leftrightarrow Q(y,x))$

(c) $\forall y \exists x \exists z\, (T(x,y,z) \vee Q(x,y))$

(a) $\exists x\, \forall y\, [P(x,y) \wedge \neg Q(x,y)]$

(b) $\forall x \forall y\, [\neg Q(x,y) \leftrightarrow \neg Q(x,y)]$

(c) $\exists y \forall x \forall z\, (T\neg(x,y,z) \wedge \neg Q(x,y)]$

38. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

(a) $\forall x \exists y\, (y^2 = x)$ (b) $\forall x \exists y\, (y^2 - x < 100)$ (c) $\forall x \forall y\, (x^2 \neq y^3)$

(a) Let's take $x=5$, $y^2=15$

$\therefore y = \pm\sqrt{5}$: Here y is not integer

(b) $x <= 100$ for any

(C) take $x=1$ & $y=\pm 1$

$x^2=1$ & $y^3=\pm 1$

So here $x^2=y^3$

39. Use quantifiers to express the distributive laws of multiplication over addition for real numbers.

40. Determine the truth value of

(a) $\forall x \exists y\, (xy = 1)$ (b) $\exists x \forall y\, (x \leq y^2)$

if the domain for the variables consists of

(a) the nonzero real numbers.

(b) the nonzero integers.

(c) the positive real numbers.

(a)	(b)
1. True	1. False
2. False	2. True
3. True	3. False

41. What rule of inference is used in each of these arguments?

(a) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

(b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

(c) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand this material.

(d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will enjoy the holidays.

(a) P: It is shows today

q: The university will close

(a) $P \rightarrow q$

$\neg q$

$\overline{\neg P}$ avele medo tollers

(b) P: It is hotter than 100 degrees today

q: Pollution is dangerous

(b) $P \vee q$

$\neg P$

$\overline{q}$ - Disjoint syll.

(c) P: I worth all night on this homework

q: I can answer all the exercise

h: I will understand this material

(C) $P \rightarrow q$, $P \rightarrow r$

$q \rightarrow r$

My pothetial syllogism

(d) P: Steve will worth of a comp company this summer,
q: he will enjoy the holidays
$p \rightarrow p \vee q$ → addition

42. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
(a) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."
(b) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."
(c) "All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You do not eat Tofu." "Cheeseburgers are not healthy to eat."
(d) "I am either dreaming or hallucinating." "I am not dreaming." "If I am hallucinating, I see elephants running down the road."
(e) "All insects have six legs." "Dragonflies are insects." "Spiders do not have six legs." "Spiders eat dragonflies."

(a) $P(x)$: x is a Computer Science major
$Q(x)$: x has a personal computer
Ralph is not a computer Science major.
We can't conclude anything about Ann

$\forall x (P(x) \rightarrow Q(x))$
$\neg Q(\text{Ralph}), Q(\text{Ann})$
rule: modus tollens

(b) $P(x)$: I take the x off
$Q(x)$: It rains on x
$R(x)$: It snows on x.
$P_1: \forall x (P(x) \rightarrow (Q(x) \vee R(x)))$
$P_2: P(\text{Tue}) \vee P(\text{Thu})$
$P_3: \neg R(\text{Tue}) \wedge \neg Q(\text{Tue})$
$P_4: \neg R(\text{Thu})$

43. Show that the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$ and conclusion $q \rightarrow r$ is valid.

44. For each of these arguments, explain which rules of inference are used for each step.
(a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."
(b) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."
(c) "Each of five roommates, Melissa, Aaron, Ralph, Veneesha and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.

(a). $P(x)$: x knows how to write program in Java
$Q(x)$: x can get a high-Paying Job
$P_1: \forall x (P(x) \rightarrow Q(x))$
$P_2: P(\text{Doug})$

modus ponens: $Q(\text{Doug})$
existential generalization: $\exists x Q(x)$

(b) $P(x)$: x enjoys whale watching
$Q(x)$: x cares about ocean Pollution
$P_1: \exists x\, P(x)$, $P_2: \forall x\,(P(x) \to Q(x))$

Step	Reason
1) $\forall x\,(P(x) \to Q(x))$	Premise
2) $P(a)$	Existential instantiation
3) $P(a) \to Q(a)$	from the premise $\exists x\, P(x)$
4) $Q(a)$	from (3)
5) $\exists x\, Q(x)$	Existential generalization

45. For each of these arguments, determine whether the argument is correct or incorrect and explain why.
(a) Every computer science major takes discrete mathematics course. Natasha is taking discrete mathematics course. Therefore, Natasha is a computer science major.
(b) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.
(c) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

a) incorrect, $P(x)$: x is a Computer Science major $P_1: \forall x(P(x) \to Q(x$
$Q(x)$: x takes discrete mathematics course $P_2: Q$(Natash
$P(\text{Natasha}) \to Q(\text{Natasha})$ (Universal instantiation from P_1)
b) $P(x)$: x is parrot $P_1: \forall x(P(x) \to Q(x)) \to P(\text{Pet bird}) \to$ pet bird
$Q(x)$: x likes fruit $P_2: \neg P(\text{Pet bird}) \to \neg P(\text{pet bird})$
c) $P(x)$: car of x is convertible car $\Rightarrow$ incorrect.
$Q(x)$: Car of x is fun to drive $\to$ incorrect.

46. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?
(a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.
(b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.
(c) If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

a) not a valid. It uses the fallacy of affecting the conclusion
b) $p \to q$
$\underline{q}$
$\therefore p$ False

b. valid $p: n > 3$ $\quad q: n^2 > 9$ $\quad$ $p \to q$, $\underline{\neg q}$, $\therefore \neg p$. $\therefore$ modus tollens

c.) invalid : $p: n > 2$ $\quad q: n^2 > 4$ $\quad$ $p \to q$, $\neg p$, $\neg q$ $\therefore$ False. it uses the fallacy of denying the hypothesis

47. Identify the error or errors in this argument that shows that if $\exists x\, P(x) \wedge \exists x\, Q(x)$ is true then $\exists x\,(P(x) \wedge Q(x))$ is true.
$\exists x\, P(x) \wedge \exists x\, Q(x)$ Premise ✓
$\exists x\, P(x)$ Simplification ✓
$P(c)$ Existential instantiation ✓
$\exists x\, Q(x)$ Simplification ✓

$Q(c)$ Existential instantiation ×
$P(c) \wedge Q(c)$ Conjunction ✓
$\exists x\,(P(x) \wedge Q(x))$ Existential generalization ✓

48. Identify the error or errors in this argument that shows that if $\forall x\,(P(x) \vee Q(x))$ is true then $\forall x\, P(x) \vee \forall x\, Q(x)$ is true.

$\forall x\,(P(x) \vee Q(x))$ Premise ✓
$P(c) \vee Q(c)$ Universal Instantiation ✓
$P(c)$ Simplification ✓
$\forall x\, P(x)$ Universal generalization ×
$Q(c)$ Simplification ✓
$\forall x\, Q(x)$ Universal generalization ×
$\forall x\, P(x) \vee \forall x\, Q(x)$ Conjunction ✓

49. Use resolution to show the hypotheses "It is not raining or Yvette has her umbrella.", "Yvette does not have her umbrella or she does not get wet", and "It is raining or Yvette does not get wet" imply that "Yvette does not get wet."

r: It is raining
u: Yvette has her umbrella
w: Yvette does not get wet
rule of resolution $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$

$P_1: \neg r \vee u$
$P_2: \neg u \vee w$
$P_3: r \vee w$

50. Consider the following two assumptions:

1. Logic is difficult or not many students like logic.
2. If mathematics is easy, then logic is not difficult.

Determine whether each of the following are valid conclusions of these assumptions:

(a) That mathematics is not easy, if many students like logic.
(b) That not many students like logic, if mathematics is not easy.
(c) That mathematics is not easy or logic is difficult.
(d) That logic is not difficult or mathematics is not easy.
(e) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

p: logic is difficult
q: many students like logic
r: Mathematics is easy

1) $p \vee \neg q$
2) $r \rightarrow \neg p$
3) $\neg r \vee \neg p$ (Logical equivalence from 2)
4) $\neg(r \wedge p)$ (de morgan's law from 3)
5) $\neg p \vee \neg r$ (commutative law from 3)
6) $p \rightarrow \neg r$ (logical equivalence from 5)
7) $\neg q \vee \neg r$ (resolution from 1 & 3)
8) $q \rightarrow \neg r$ (~~resolution from~~ logical i.e. from 7
9) $\neg r \vee \neg q$ (commutative law from 7
10) $r \rightarrow \neg q$ (logical equivalence from 9)
11) $(p \vee \neg q) \wedge (\neg r \vee \neg q)$ conjuction from 1 & 9)

$\neg q \vee (p \wedge \neg r)$ (de Morgan's Laws from 11
$q \rightarrow (p \wedge \neg r)$ (logical equivalence from 12

a. $q \to \neg r$, valid

b. $\neg r \to \neg q$, invalid

c. $\neg r \vee p$, invalid

d. $\neg p \vee \neg r$, valid

e. $\neg q \to (\neg p \vee \neg r)$, invalid

pls.