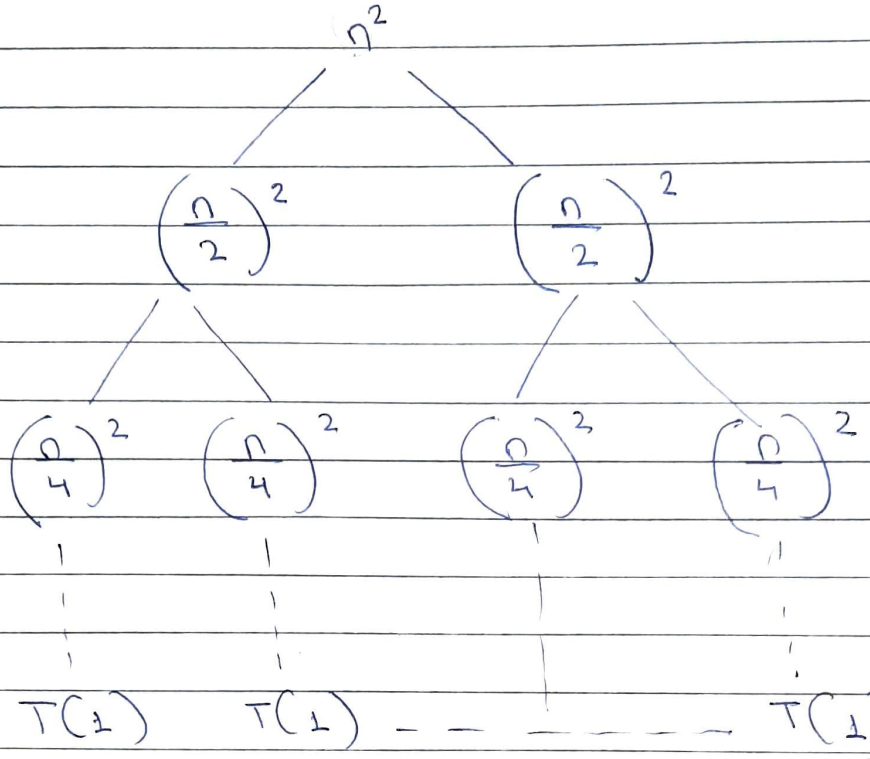
Recursion Tree Method

(1) $T(n) = 2T\left(\frac{n}{2}\right) + n^2$



$$T(n) = n^2 + 2\left[\left(\frac{n}{2}\right)^2\right] + 4\left[\left(\frac{n}{4}\right)^2\right] + 8\left[\left(\frac{n}{8}\right)^2\right] + \dots$$

$$= n^2 + 2\frac{n^2}{4} + 4\frac{n^2}{16} + 8\frac{n^2}{64} + \dots$$

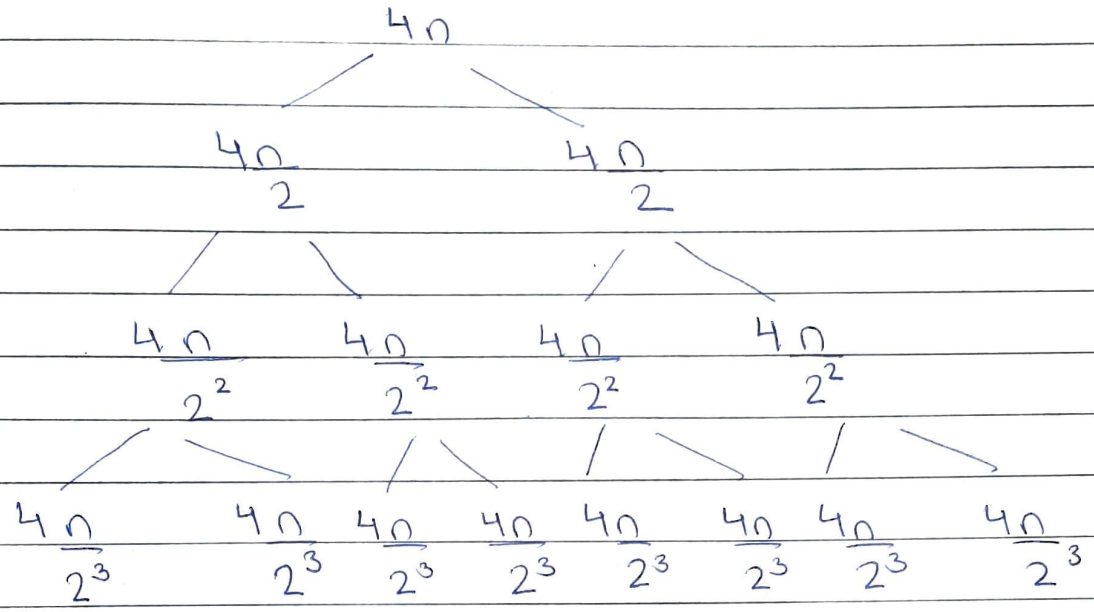
$$= n^2 + \frac{n^2}{2} + \frac{n^2}{4} + \frac{n^2}{8} + \dots$$

$$= n^2\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right)$$

$$= n^2\left(\frac{1}{1-1/2}\right) = 2n^2 \quad \left[\because \frac{a}{a-r}\right]$$

$$\boxed{T(n) = n^2}$$

(2) $T(1) = 4$ $\quad T(n) = 2T\left(\frac{n}{2}\right) + 4n$



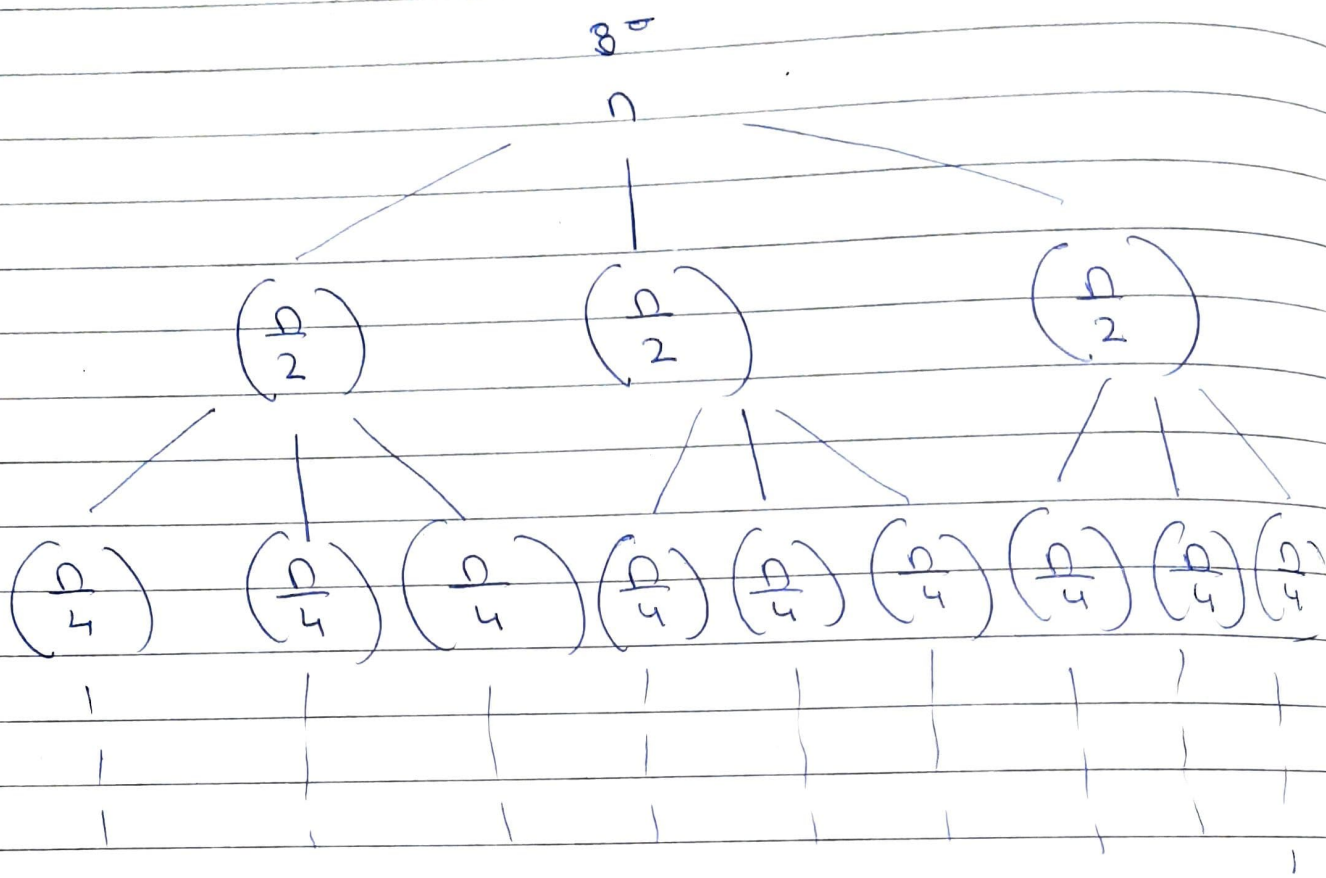
$$T(n) = 4n + 2\left[\frac{4n}{2}\right] + 4\left[\frac{4n}{2^2}\right] + 8\left[\frac{4n}{2^3}\right]$$
$$+ \text{ ----}$$

$$T(n) = 4n + 4n + 4n + 4n + \text{ ----}$$

$$= n(4 + 4 + 4 + 4 - \text{---})$$

$$\boxed{T(n) = n \log n}$$

(3) $T(n) = 3T\left(\frac{n}{2}\right) + n$



$$T(n) = n + 3\left(\frac{n}{2}\right) + 9\left(\frac{n}{4}\right) + 27\left(\frac{n}{8}\right) + \cdots$$

$$= 3n\left(1 + \frac{n}{2} + \frac{n}{2^2} + \frac{n}{2^3} + \frac{n}{2^4} + \cdots\right)$$

$$\boxed{T(n) = \log_2 n^3} \quad \log_2 3^n$$