

Fractional Knapsack (Greedy)

- In this problem we have a knapsack that has a weight limit W .
- There are items i_1, i_2, \dots, i_n each having weight w_1, w_2, \dots, w_n and some benefits associated with it v_1, v_2, \dots, v_n .
- Our objective is to maximize the benefit such that total weight inside the knapsack is at most W .
- And we are allowed to take an item in fractional part.

Example

Assume that we have a knapsack with max weight capacity

$$W = 16$$

Our objective is to fill the knapsack with items such that the benefit (value or profit) is maximum.

- Consider the following items and their associated weight and value.

Item	Weight	Value
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i ₁	6	6
i ₂	10	2
i ₃	3	1
i ₄	5	8
i ₅	1	3
i ₆	3	5

Steps:-

- Calculate value per weight for each item (we can call this value density)
- Sort the items as per the value density in descending order.
- Take as much item as possible not already taken in the knapsack
- Add density column to the table

$$\text{density} = (\text{value} / \text{weight})$$

Item	Weight	Value	Density
i ₁	6	6	1.000
i ₂	10	2	0.200
i ₃	3	1	0.333
i ₄	5	8	1.600
i ₅	1	3	3.000
i ₆	3	5	1.667

- Sort the item as per density in decreasing order

Item	Weight	Value	Density
i5	1	3	3.000
i6	3	5	1.667
i4	5	8	1.600
i1	6	6	1.000
i3	3	1	0.333
i2	10	2	0.200

- Now pick the item such that the benefit is maximum and total weight of the selected item is at most W .

→ Check is

$$\text{isWeight}(i_5) + \text{Total Weight} \leq W$$

(Inside Knapsack)

$$1 + 0 \leq 16$$

$$1 \leq 16$$

YES

So, take whole item

→ Check is

$$\text{isWeight}(i_6) + \text{Total Weight} \leq W$$

$$3 + 1 \leq 16$$

$$4 \leq 16$$

YES

So, take whole item

- Check i_4

$$\text{isWeight}(i_4) + \text{Total Weight} \leq w$$

$$5 + 4 \leq 16$$

$$9 \leq 16$$

YES

So we take the whole item

- Check i_1

$$\text{isWeight}(i_1) + \text{Total Weight} \leq w$$

$$6 + 9 \leq 16$$

YES

So, we take the whole item

- Check i_3

$$\text{isWeight}(i_3) + \text{Total Weight} \leq w$$

$$3 + 15 \leq 16 \quad (\text{YES})$$

No

So we will take fraction

- We will fill the knapsack with $\frac{1}{3}$ part of i_3 item having value $\left(\frac{1}{3}\right) = 0.333$

Now,

$$\frac{1}{3} \text{ part of } i_3 + \text{Total weight} \leq w$$

$$1 + 15 \leq w$$

$$16 \leq 16$$

Total weight is 16 so we will stop.

KNAPSACK

Item	Weight	Value	Total Weight	Benefit
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i ₅	1	3	1	3.000
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i ₆	3	5	4	8.000
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i ₄	5	8	9	16.000
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i ₁	6	6	15	22.000
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i ₃	1	0.333	16	22.333
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So, the maximum benefit is 22.333