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Subject: Discrete Mathematics (3140708)
Tutorial-1

1	List the members of these sets. (a) $\{x \mid x$ is a real number such that $x^2 = 1\}$ (b) $\{x \mid x$ is a positive integer less than 12 $\}$ (c) $\{x \mid x$ is the square of an integer and $x < 100\}$
2	Determine whether each of these statements is true or false. $(a)\ 0 \in \varphi$ $(b)\ \varphi \subset \{0\}$ $(c)\ \{\varphi\} \subseteq \{\varphi\}$ $(d)\ \{0\} \in \{0\}$ $(e)\ \varphi \in \{\varphi\}$ $(f)\ \{\varphi\} \in \{\{\varphi\}\}$ $(g)\ \{\{\varphi\}\} \subset \{\varphi, \{\varphi\}\}$
3	Use set builder notation to give a description of each of these sets. $(a)\ \{0, 3, 6, 9, 12\}$ $(b)\ \{-3, -2, -1, 0, 1, 2, 3\}$
4	For each of the following sets, determine whether 2 is an element of that set. (a) $\{x \in \boldsymbol{R} \mid x$ is an integer greater than 1 $\}$ (b) $\{x \in \boldsymbol{R} \mid x$ is the square of an integer $\}$ $(c)\ \{2, \{2\}\}$ $(d)\ \{\{2\}, \{\{2\}\}\}$ $(e)\ \{\{2\}, \{2, \{2\}\}\}$ $(f)\ \{\{\{2\}\}\}$
5	Find two sets A and B such that $A \in B$ and $A \subseteq B$.
6	Use a Venn diagram to illustrate the following relationships: $(a)\ A \subseteq B$ *and* $B \subseteq C$ $(b)\ A \subset B$ *and* $B \subset C$ $(c)\ A \subset B$ *and* $A \subset C$.
7	What is the cardinality of each of these sets? $(a)\ \varphi$ $(b)\ \{\varphi\}$ $(c)\ \{\{a\}\}$ $(d)\ \{\varphi, \{\varphi\}\}$ $(e)\ \{\varphi, \{\varphi\}, \{\varphi, \{\varphi\}\}\}$.
8	How many elements does each of these sets have where a and b are distinct elements? $(a)\ P(\{a, b, \{a, b\}\})$ $(b)\ P(\{\varphi, a, \{a\}, \{\{a\}\}\})$ $(c)\ P(P(\varphi))$.
9	Determine whether each of these sets is the power set of a set, where a and b are distinct elements. $(a)\ \varphi$ $(b)\ \{\varphi, \{a\}\}$ $(c)\ \{\varphi, \{a\}, \{\varphi, \{a\}, \{\varphi, a\}\}\}$ $(d)\ \{\varphi, \{a\}, \{b\}, \{a, b\}\}$
10	Suppose that $A \times B = \varphi$, where A and B are sets. What can you conclude?
11	Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.
12	Translate each of these quantifications into English and determine its truth value. $(a)\ \forall x \in \boldsymbol{R}\ (x^2 \neq -1)$ $(b)\ \exists x \in \boldsymbol{Z}\ (x^2 = 2)$ $(c)\ \forall x \in \boldsymbol{Z}\ (x^2 > 0)$ $(d)\ \forall x \in \boldsymbol{R}\ (x^2 = x)$ $(e)\ \exists x \in \boldsymbol{R}\ (x^3 = -1)$ $(f)\ \exists x \in \boldsymbol{Z}\ (x + 1 > x)$

13	Find the truth set of each of these predicates where the domain is the set of integers. $(a)\ P(x): "x^2 < 3"$ $(b)\ Q(x): "x^2 > x"$ $(c)\ R(x): "2x + 1 = 0"$ $(d)\ P(x): "x^3 \geq 1"$ $(e)\ Q(x): "x^2 = 2"$ $(f)\ R(x): "x^2 < x"$
14	Draw the Venn Diagrams for each of these combinations of the sets A,B,C. • $A \cap (B \cup C)$ • $\bar{A} \cap \bar{B} \cap \bar{C}$ • $(A - B) \cup (A - c) \cup (B - c)$ • $A \cap (B - C)$
15	Find the power set of $\{\varphi, \{\varphi\}\}$.
16	100 of the 120 engineering students in a college take part in atleast one of the activity group discussion, debate and quiz. Also : 65 participate in group discussion, 45 participate in debate , 42 participate in quiz,20 participate in group discussion and debate, 25 participate in group discussion and quiz,15 participate in quiz and debate.Find the number of students who participate in : i) All the three activities ii) Exactly one of the activities. Ans: i) 8 ii) 56
17	Find the sets A and B if $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$, and $A \cap B = \{3,6,9\}$.
18	Can you conclude that $A = B$ if A, B and C are sets such that (a) $A \cup C = B \cup C$? (b) $A \cap C = B \cap C$? (C) $A \cup C = B \cup C$ and $A \cap C = B \cap C$? Give examples to justify your answer.
19	What can you say about the sets A and B if we know that $(a)\ A \cup B = A$ $(b)\ A \cap B = A$ $(c)\ A - B = A$ $(d)\ A \cap B = B \cap A$ $(e)\ A - B = B - A$
20	Show that if A is a subset of a universal set U, then $(a)\ A \oplus A = \varphi$ $(b)\ A \oplus \varphi = A$ $(c)\ A \oplus U = \bar{A}$
21	What subsets of a finite universal set do these bit strings represent? (a) the string with all zeros (b) the string with all ones
22	What is the bit string corresponding to the difference of two sets?

23	How can the union and intersection of n sets that all are subsets of the universal set U be found using bit strings?
24	Show that if A and B are sets, then $(A \oplus B) \oplus B = A$.
25	Find $(a)\ \bigcup_{i=1}^{n} A_i \qquad (b)\ \bigcap_{i=1}^{n} A_i \qquad (c)\ \bigcup_{i=1}^{\infty} A_i$ if for every positive integer i, $(a)\ A_i = \{1, 2, 3, \ldots, i\}$ $\quad (b)\ A_i = \{\ldots, -2, -1, 0, 1, \ldots, i\}$ $(d)\ A_i = \{i, i+1, i+2, \ldots\}$ $\quad (e)\ A_i = (0, i)$ $(g)\ A_i = [-i, i]$ $\quad (h)\ A_i = (i, \infty)$ $(j)\ A_i = \{-i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i\}$
26	Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express the sets $(a)\ \{3, 4, 5\} \qquad (b)\ \{1, 3, 6, 10\} \qquad (c)\ \{2, 3, 4, 7, 8, 9\}$ with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise. Also, find the set specified by each of the bit strings (a) 11 1100 1111 $\qquad (b)$ 01 0111 1000 $\qquad (c)$ 10 0000 0001