Master's theorem to solve recurrence eqⁿ

Standard eqⁿ = $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

where $n \geq d$ ~~ad~~ and d is some constant.

Formulas for masters theorem

① $T(n) = \Theta(n^d)$ if $a < b^d$

② $T(n) = \Theta(n^d \log n)$ if $a = b$

③ $T(n) = \Theta(n^{\log_b a})$ if $a > b^d$

Examples

① $T(n) = 6T\left(\frac{n}{3}\right) + n$

Compare it with standard eqⁿ

Here $f(n) = n$ i.e. n^1

$$\boxed{\therefore d = 1}$$

Here $a = 6$ and $b = 3$

Now $b^d \Rightarrow 3^1 = 3$

i.e. $a > b^d$ i.e. $6 > 3^1$

$$\boxed{\text{Thus, } T(n) = \Theta(n^{\log_3 6})}$$

(2) $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

Here $d = 1$, $f(n) = n\log n$

$a = 2$, $b = 2$

$\therefore b^d = 2^1 = 2$

Here $a = b^d$ i.e. $2 = 2$

$\boxed{\text{Thus, } T(n) = \Theta(n\log n)}$

(3) $T(n) = 4T\left(\frac{n}{3}\right) + n^3$

Here $d = 3$, $f(n) = n^3$

$a = 4$, $b = 3$

$\therefore b^d = 3^3 = 9$

Here $a < b^d$

Thus, $T(n) = \Theta(n^3)$

More e.g.

(1) $T(n) = 8T(n/2) + n^2$

(2) $T(n) = 2T\left(\frac{n}{4}\right) + n$

(3) $T(n) = 9T\left(\frac{n}{3}\right) + n^3$

(4) $T(n) = T\left(\frac{n}{2}\right) + 1$