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A.Y.2019-20: EVEN SEMESTER

3140708: DISCRETE MATHEMATICS

Assignment 1: Set Theory

1. List the members of these sets.
(a) $\{x \mid x$ is a real number such that $x^2 = 1\}$
(b) $\{x \mid x$ is a positive integer less than $12\}$
(c) $\{x \mid x$ is the square of an integer and $x < 100\}$
(d) $\{x \mid x$ is an integer such that $x^2 = 2\}$

(a) $\{-1, 1\}$ (b) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

(c) $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$ (d) ~~Any~~ ϕ

2. Use set builder notation to give a description of each of these sets.
(a) $\{0, 3, 6, 9, 12\}$ (b) $\{-3, -2, -1, 0, 1, 2, 3\}$

(a) $\{x / x = 3n, n = 0, 1, 2, 3, 4\}$

(b) $\{x \in Z / -3 \leqslant x \leqslant 3x\}$

3. For each of the following sets, determine whether 2 is an element of that set.
(a) $\{x \in \mathbf{R} \mid x$ is an integer greater than $1\}$ (b) $\{x \in \mathbf{R} \mid x$ is the square of an integer $\}$
(c) $\{2, \{2\}\}$ (d) $\{\{2\}, \{\{2\}\}\}$ (e) $\{\{2\}, \{2, \{2\}\}\}$ (f) $\{\{\{2\}\}\}$

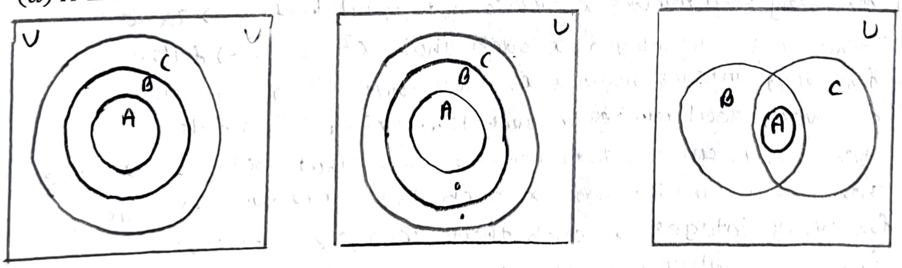
(a) yes (b) NO (c) yes (d) NO (e) NO (f) NO

4. Determine whether each of these statements is true or false.
(a) $0 \in \varphi$ (b) $\varphi \subset \{0\}$ (c) $\{\varphi\} \subseteq \{\varphi\}$ (d) $\{0\} \in \{0\}$
(e) $\varphi \in \{\varphi\}$ (f) $\{\varphi\} \in \{\{\varphi\}\}$ (g) $\{\{\varphi\}\} \subset \{\varphi, \{\varphi\}\}$

(a) false (b) false (c) True (d) False

(e) True (f) True (g) True

5. Use a Venn diagram to illustrate the following relationships:
(a) $A \subseteq B$ and $B \subseteq C$ (b) $A \subset B$ and $B \subset C$ (c) $A \subset B$ and $A \subset C$.



6. Find two sets A and B such that $A \in B$ and $A \subseteq B$.

$A = \{\}$ or ϕ

$B = \{\phi, \{1\}\}$

→ $A \subset B$, we can easily see that $\{\}$ is an empty set which is a subset of B.

→ $A \in B$, since we can easily see that ϕ is an element of B

7. What is the cardinality of each of these sets?

(a) φ (b) $\{\varphi\}$ (c) $\{\{a\}\}$ (d) $\{\varphi, \{\varphi\}\}$ (e) $\{\varphi, \{\varphi\}, \{\varphi, \{\varphi\}\}\}$.

(a) 0 (b) 1 (c) 1 (d) 2 (e) 3

8. Find the power set of $\{\varphi, \{\varphi\}\}$.

$= \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

9. How many elements does each of these sets have where a and b are distinct elements?

(a) $P(\{a, b, \{a, b\}\})$ (b) $P(\{\varphi, a, \{a\}, \{\{a\}\}\})$ (c) $P(P(\varphi))$.

(a) $2^n = 2^3 = 8$ (b) $2^4 = 16$ (c) $2^0 = 1$

10. Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

(a) φ (b) $\{\varphi, \{a\}\}$ (c) $\{\varphi, \{a\}, \{\varphi, \{a\}, \{\varphi, a\}\}\}$ (d) $\{\varphi, \{a\}, \{b\}, \{a, b\}\}$

(a) No (b) Yes (c) No (d) Yes

11. Suppose that $A \times B = \varphi$, where A and B are sets. What can you conclude?

→ $A = \emptyset$ or $B = \emptyset$ as if any one the sets is empty. the cartesian product of the two sets becomes $\emptyset$.

12. Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

→ $A \times B \times C = \{(a, b, c) \mid a \in A, b \in B, c \in C\}$ is the set of all triplets whereas $(A \times B) \times C = \{((a, b), c) \mid a \in A, b \in B, c \in C\}$ is the set all ordered pairs in which the first element is itself an ordered pair

13. Translate each of these quantifications into English and determine its truth value.

(a) $\forall x \in \mathbf{R}\ (x^2 \neq -1)$
(b) $\exists x \in \mathbf{Z}\ (x^2 = 2)$
(c) $\forall x \in \mathbf{Z}\ (x^2 > 0)$
(d) $\forall x \in \mathbf{R}\ (x^2 = x)$
(e) $\exists x \in \mathbf{R}\ (x^3 = -1)$
(f) $\exists x \in \mathbf{Z}\ (x + 1 > x)$
(g) $\forall x \in \mathbf{Z}\ (x - 1 \in Z)$
(h) $\forall x \in \mathbf{Z}\ (x^2 \in Z)$

(a) for every real number x, x^2 is not equal to -1. → True
(b) There exists an integer x such that $x^2 = 2$. → false
(c) for every integer number x, such that $x^2 > 0$ → True
(d) for every real number x such that $x^2 = x$ → false
(e) There exists an real number x such that $x^3 = -1$ → True
(f) There exists an integer x such that $x + 1 > x$ → True
(g) for every integer x such that $x - 1 \in Z$ → True
(h) for every integer number x, such that $x^2 \in Z$ → True

14. Find the truth set of each of these predicates where the domain is the set of integers.

(a) $P(x)$: "$x^2 < 3$"
(b) $Q(x)$: "$x^2 > x$"
(c) $R(x)$: "$2x + 1 = 0$"
(d) $P(x)$: "$x^3 \geq 1$"
(e) $Q(x)$: "$x^2 = 2$"
(f) $R(x)$: "$x^2 < x$"

(a) $\{-1, 0, 1\}$ (b) $\{-1, \pm 2, \pm 3 \ldots\}$ $= Z - \{0, 1\}$ (c) $\emptyset$

(d) $\{1, 2, 3, \ldots\}$ $= N$ (Natural Number) (e) $\emptyset$ (f) $\emptyset$

15. Try to understand the following proof of a distributive law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and then prove it using membership table.

First we show that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Let $x \in A \cup (B \cap C)$.

$$\therefore x \in A \text{ or } x \in B \cap C$$

Case 1: If $x \in A$, then $x \in A \cup B$ as well as $x \in A \cup C$.

$$\therefore x \in (A \cup B) \cap (A \cup C).$$

This proves that, in this case, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Case 2: If $x \notin A$, then x must belong to $B \cap C$.

$$\therefore x \in B \text{ as well as } x \in C.$$

$$\therefore x \in A \cup B \text{ as well as } x \in A \cup C.$$

This proves that, in this case also, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$.

Thus, it is proved that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$. (1)

Next, we have to show that $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$.

Let $x \in (A \cup B) \cap (A \cup C)$.

$$\therefore x \in A \cup B \text{ as well as } x \in A \cup C.$$

Case 1: If $x \in A$, then $x \in A \cup (B \cap C)$.

This proves that, in this case, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$.

Case 2: If $x \notin A$, then x must belong to B as well as C.

So, $x \in B \cap C$ and hence must belong to $A \cup (B \cap C)$.

This proves that, in this case also, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$.

Thus, it is proved that $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$. (2)

From (1) and (2), we can conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

→ first we show that, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cap C)$

→ Let $x \in A \cup (B \cap C)$ $\therefore$ $x \in A$ or $x \in B \cap C$

→ Case 1: If $x \in A$, then $x \in A \cup B$ as well as $x \in A \cup C$.

$\therefore x \in (A \cup B) \cap (A \cup C)$

This proves that, in this case, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cap C)$

→ Case 2: If $x \notin A$, then x must belong to $B \cap C$.

$\therefore x \in B$ as well as $x \in C$.

$\therefore x \in A \cup B$ as well as $x \in A \cup C$.

This proves that, in this case also, $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$

Thus, it is proved that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ — 1

Next, we have to show that $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$

Now, continue in file page 1

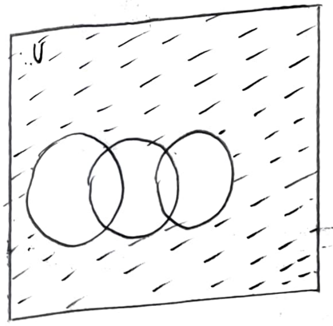
16. Draw the Venn diagrams for each of these combinations:

(a) $\bar{A} \cap \bar{B} \cap \bar{C}$ (b) $(A - B) \cup (A - C) \cup (B - C)$ (c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$

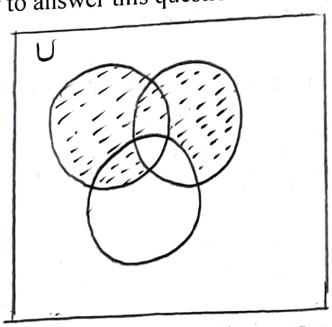
(d) $(A \cap B) \cup (C \cap D)$ (e) $A - (B \cap C \cap D)$

Use separate sheet of paper to answer this question.

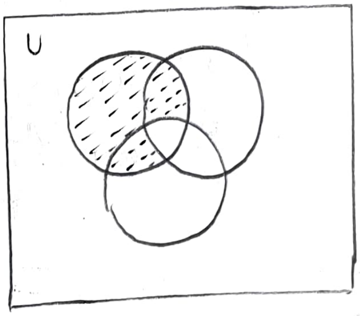
(a)



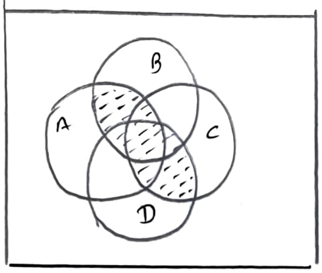
(b)



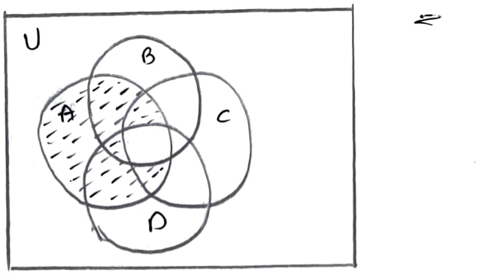
(c)



(d)



(e)



17. In a recent survey, people were asked if they took a vacation in the summer, winter or spring in the last year. The results were: 73 took a vacation in the summer, 51 took a vacation in the winter, 27 took a vacation in the spring, and 2 had taken no vacation. Also, 10 had taken vacations at all three times, 33 had taken both a summer and a winter vacation, 18 had taken only a winter vacation and 5 had taken both a summer and a spring but not a winter vacation.

(a) How many people had been surveyed?
(b) How many people had taken vacations at exactly two times of the year?
(c) How many people had taken vacations during at most one time of the year?
(d) What percentage had taken vacations during both summer and winter but not spring?

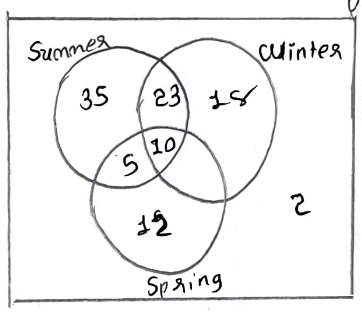
Ans: (*a*) **105** (*b*) **28** (*c*) **67** (*d*) **21.9048 %**

(a) $35 + 23 + 18 + 10 + 5 + 12 + 2 = 105$

(b) $23 + 5 = 28$

(c) $35 + 18 + 12 + 2 = 67$

(D) $\frac{23 \times 100}{105} = 21.9048\ \%$



18. Find the sets A and B if $A - B = \{1,5,7,8\}$, $B - A = \{2,10\}$, and $A \cap B = \{3,6,9\}$.

$A = \{1, 3, 5, 6, 7, 8, 9\}$
$B = \{2, 3, 6, 9, 10\}$

19. Can you conclude that $A = B$ if A, B and C are sets such that
(a) $A \cup C = B \cup C$? (b) $A \cap C = B \cap C$? (C) $A \cup C = B \cup C$ *and* $A \cap C = B \cap C$?
Give examples to justify your answer.

(a) The answer is No.
Ex $A = \{1,2\}$
$B = \{1,2,3\}$
$C = \{1,2,3,4\}$
$\Rightarrow A \cup C = B \cup C$
$= \{1, 2, 3, 4\}$,
$\Rightarrow$ but $A \neq B$,

(b) The answer is No.
Ex $A = \{1,2\}$
$B = \{1,2,3\}$
$C = \{1\}$
$\Rightarrow A \cap C = B \cap C$
$= \{1\}$
$\Rightarrow$ but, $A \neq B$

(c) suppose $A \cup C = B \cup C$ and $A \cap C = B \cap C$
$\rightarrow$ Let $x \in A$, Then $x \in A \cup C$, so by assumption, $x \in B \cup C$. Thus, $x \in B$ or $x \in C$.
$\rightarrow$ If $x \in B$, then we are done.
$\rightarrow$ If $x \in C$, then we have $x \in A \cap C$. By assumption, this means $x \in B \cap C$, which means $x \in B$.
So, in every case, $A \subseteq B$. A similar argument will show that $B \subseteq A$, So $A = B$.

20. What can you say about the sets A and B if we know that
(a) $A \cup B = A$ (b) $A \cap B = A$ (c) $A - B = A$
(d) $A \cap B = B \cap A$ (e) $A - B = B - A$

(a) $B \subseteq A$ (b) $A \subseteq B$ (c) $A \cap B = \emptyset$

(d) The given statement is the commutative law (e) $A = B$

21. Show that if A is a subset of a universal set U, then
(a) $A \oplus A = \varphi$ (b) $A \oplus \varphi = A$ (c) $A \oplus U = \bar{A}$ (d) $A \oplus \bar{A} = U$.

(a) $A \oplus A = (A \cup A) - (A \cap A) = A - A = \emptyset$

(b) $A \oplus \emptyset = (A \cup \emptyset) - (A \cap \emptyset) = A - \emptyset = A$

(c) $A \oplus U = (A \cup U) - (A \cap U) = U - A = \bar{A}$

(d) $A \oplus \bar{A} = (A \cup \bar{A}) - (A \cap \bar{A}) = U - \emptyset = U$

22. Show that if A and B are sets, then $(A \oplus B) \oplus B = A$.

L.H.S $= (A \oplus B) \oplus B = A \oplus (B \oplus B) = A \oplus ((B \cup B) - (B \cap B))$
$= A \oplus (B - B) = A \oplus \emptyset = A$

23. What can you say about the sets A and B if $A \oplus B = A$?

→ If $B = \emptyset$ then $A \oplus B = (A \cup B) - (A \cap B) = (A \cup \emptyset) - (A \cap \emptyset)$
$= A - \emptyset = A$

24. If A, B and C are sets such that $A \oplus C = B \oplus C$, can we conclude that $A = B$?

$A \oplus C = B \oplus C$

$\therefore (A \oplus C) \oplus C = (B \oplus C) \oplus C \Rightarrow A \oplus (C \oplus C) = B \oplus (C \oplus C) \Rightarrow A \oplus \emptyset = B \oplus \emptyset$
$\boxed{\Rightarrow A = B}$

25. Find

(a) $\bigcup_{i=1}^{n} A_i$ (b) $\bigcap_{i=1}^{n} A_i$ (c) $\bigcup_{i=1}^{\infty} A_i$ (d) $\bigcap_{i=1}^{\infty} A_i$

if for every positive integer i,
(a) $A_i = \{1, 2, 3, \ldots, i\}$ (b) $A_i = \{\ldots, -2, -1, 0, 1, \ldots, i\}$ (c) $A_i = \{0, i\}$
(d) $A_i = \{i, i+1, i+2, \ldots\}$ (e) $A_i = (0, i)$ (f) $A_i = \{-i, i\}$
(g) $A_i = [-i, i]$ (h) $A_i = (i, \infty)$ (i) $A_i = [i, \infty]$
(j) $A_i = \{-i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i\}$

(a) $A_i = \{1, 2, 3 \ldots i\}$

(a) $A_1 = \{1\}$
$A_2 = \{1, 2\}$
$A_3 = \{1, 2, 3\}$
$A_i = \{1, 2, 3 \ldots i\}$

(b) $\bigcap_{i=1}^{n} A_i = \{1\}$

(c) $A_i = \{1, 2, 3 \ldots\}$

(D) $A_i = \{1\}$

(b) $A_i = \{\ldots -2, -1, 0, 1 \ldots i\}$

(a) $A_i = \{\ldots -1, 0, 1, \ldots n\}$

(b) $A_i = \{\ldots -1, 0, 1\}$

(c) $A_i = \{\ldots -1, 0, 1 \ldots\}$

(d) $A_i = \{\ldots -1, 0, 1\}$

(c) $A_i = \{0, i\}$
$A_1 = \{0, 1\}$
$A_2 = \{0, 2\}$

(a) $A_i = \{0, 1, 2, 3, \ldots n\}$

(b) $A_i = \{0\}$

(c) $A_i = \{0, 1, 2, 3 \ldots\}$

(D) $A_i = \{0\}$

(d) $A_i = \{i, i+1, i+2, \ldots\}$
$A_1 = \{1, 2, 3 \ldots\}$
$A_2 = \{2, 3, 4, \ldots\}$

(a) $A_i = \{1, 2, 3, 4, 5 \ldots\}$

(b) $A_i = \{n, n+1, \ldots\}$
$= A_n$

(c) $A_i = \{1, 2, 3, 4 \ldots\}$

(d) $A_i = \emptyset$

(e) $A_i = (0, i)$
$A_1 = (0, 1)$
$A_2 = (0, 2)$

(a) $A_i = (0, n)$

(b) $A_i = (0, 1) = A_1$

(c) $A_i = (0, \infty)$

(D) $A_i = (0, 1) = A_1$

(f) $A_i = \{-i, i\}$
$A_1 = \{-1, 1\}$
$A_2 = \{-2, 2\}$

(a) $A_i = \{-n, \ldots, -1, 0, 1, \ldots n\}$

(b) $A_i = \emptyset$

(c) $A_i = \{-\infty \ldots -1, 0, 1, \ldots \infty\}$

(D) $A_i = \emptyset$

(g) $A_i = [-i, i]$

$A_1 = [-1, 1]$

$A_2 = [-2, 2]$

(a) $A_i = [-n, n]$

(b) $A_i = [-1, 1]$
$= A_1$

(c) $A_i = [-\infty, \infty]$

(d) $A_i = [-1, 1]$
$= A_1$

(h) $A_i = (i, \infty)$

$A_1 = (1, \infty)$

$A_2 = (2, \infty)$

(a) $A_i = (1, n)$

(b) $A_i = (n, \infty)$

(c) $A_i = (1, \infty)$

(d) $A_i = \phi$

(i) $A_i = [i, \infty]$

$A_1 = [1, \infty]$

$A_2 = [2, \infty]$

(a) $A_i = [1, \infty]$

(b) $A_i = [n, \infty)$
$= A_n$

(c) $A_i = [1, \infty]$

(d) $A_i = \phi$

(j) $A_i = \{-i, -i+1 .. -1, 0, 1 ... i-1, i\}$

$A_1 = \{-1, 0, 1\}$

$A_2 = \{-2, -1, 0, 1, 2\}$

(a) $A_i = \{-n ..., -1, 0, 1, ... n\}$

(b) $A_i = \{-1, 0, 1\}$

(c) $A_i = \{-\infty ... -1, 0, 1, ... \infty\}$

(d) $A_i = \{-1, 0, 1\}$

26. Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express the sets
$(a)\ \{3, 4, 5\}$ $(b)\ \{1, 3, 6, 10\}$ $(c)\ \{2, 3, 4, 7, 8, 9\}$
with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.
Also, find the set specified by each of the bit strings
(a) 11 1100 1111 (b) 01 0111 1000 (c) 10 0000 0001

(a) 0011100000 (b) 1010010001 (c) 0111001110

(a) $\{1, 2, 3, 4, 7, 8, 9, 10\}$ (b) $\{2, 4, 5, 6, 7\}$ (c) $\{1, 10\}$

27. What subsets of a finite universal set do these bit strings represent?
(a) the string with all zeros (b) the string with all ones

(a) A is the empty set (b) $A = U$

28. How can the union and intersection of n sets that all are subsets of the universal set U be found using bit strings?

→ If for two sets A and B we have to do union operation we have to do the process like if i^{th} bit of both set are 0 then only it will be 0 in string bit otherwise 1.

→ for Intersection operation, if the i^{th} bit of A and B both are 1 then only the resultant bit will be one, if anyone of it is 0 then it will be 0.

→ Similar for n sets, union can be done through like if i^{th} bit of n sets have anyone of the set 1 then the resultant string bit will be one.

→ Similar for intersection operation, if the i^{th} bit of all sets are 1. then only the resultant string bit will be 1 otherwise 0.

→ Let $x \in (A \cup B) \cap (A \cup C)$

$\therefore x \in A \cup B$ as well as $x \in A \cup C$.

Case 1 : If $x \in A$, then $x \in A \cup (B \cap C)$

This proves that, in this case, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$

Case 2 : If $x \notin A$, then x must belong to B as well as C

So, $x \in B \cap C$ and hence must belong to $A \cup (B \cap C)$

This proves that, in this case also, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$

Thus, it is proved that $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ -- ②

from ① and ②, we can conclude that $A \cup (B \cap C)$

$= (A \cup B) \cap (A \cup C)$.

→ Membership table

A	B	C	B ∪ C	A ∩ (B ∪ C)	A ∩ B	A ∩ C	(A ∩ B) ∪ (A ∩ C)
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

$\therefore$ we can prove $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ in above table