Date: 02/02/22

TUTORIAL : 01

Q1. List the members of sets.

(a) $\{x \mid x$ is a real no. s.t. $x^2 = 1\}$

$B = \{-1, +1\}$

(b) $\{x \mid x$ is a positive int less than $12\}$

$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(c) $\{x \mid x$ is the square of an int & $x < 100\}$

$C = \{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

Q2. True or false

(a) $0 \in \phi$ False (b) $\phi \subset \{0\}$ True (c) $\{\phi\} \subseteq \{\phi\}$ True

(d) $\{0\} \in \{0\}$ False (e) $\phi \in \{\phi\}$ True (f) $\{\phi\} \in \{\{\phi\}\}$ True

(g) $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$ True

Q3 Use a set builder notation to give a description of each set :

(a) $\{0, 3, 6, 9, 12\} = \{3x \mid x \in W ; 0 < x < 4\}$

(b) $\{-3, -2, -1, 0, 1, 2, 3\} = \{x \mid x \in Z ; -3 \le x \le 3\}$

Q4. For the foll^{g} sets, determine if 2 is an element

(a) $\{x \in R \mid x$ is an int greater than $1\}$

$A = \{2, 3, 4, \ldots\}$ $\Rightarrow$ 2 is an element as $2 \in A$

(b) $\{x \in R \mid x$ is the sq. of an int$\}$

$B = \{1, 4, 9, \ldots\}$ $\Rightarrow$ 2 is not an element

(c) $\{2, \{2\}\}$ $\Rightarrow$ 2 is an element

(d) $\{\{2\}, \{\{2\}\}\}$ $\Rightarrow$ 2 is not an element

(e) $\{\{2\}, \{2, \{2\}\}\}$ $\Rightarrow$ 2 is not an element

(f) $\{\{\{2\}\}\}$ $\Rightarrow$ 2 is not an element

Q5 Find two sets A & B st $A \in B$ & $A \subseteq B$

$A = \{1\}$ $B = \{\{1\}, \{1, 2, 3\}\}$

$\Rightarrow A \in B$ & $A \subseteq B$

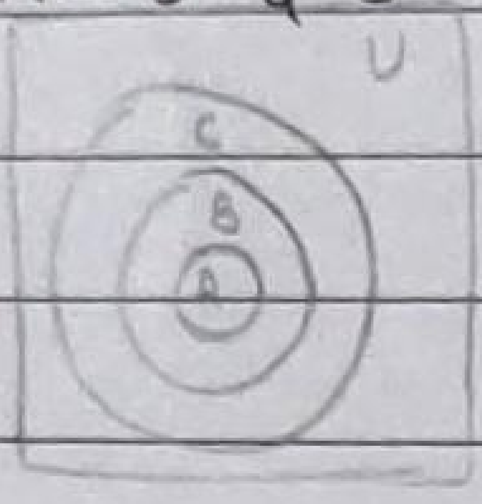
CROWN ABS

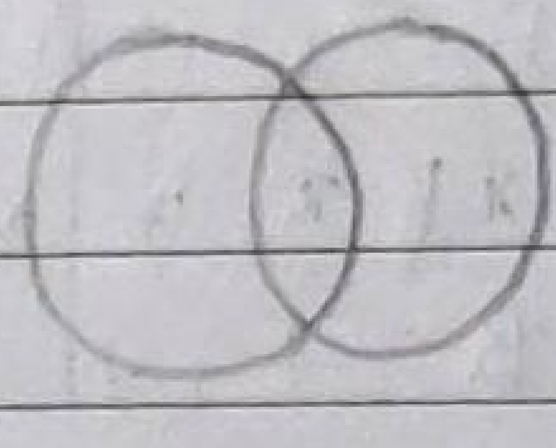
Scanned by TapScanner

Date :

Q6. Use a Venn diagram to illustrate the relationships:

(a) $A \subseteq B$ & $B \subseteq C$ (b) $A \subset B$ & $B \subset C$ (c) $A \subset B$ & $A \subset C$





Q7. What is the cardinality?

(a) $A = \phi$ $|A| = 0$ (b) $B = \{\phi\}$ $|B| = 1$ (c) $C = \{\{a\}\}$ $|C| = 1$

(d) $D = \{\phi, \{\phi\}\}$ $|D| = 2$ (e) $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ $|E| = 3$

Q8 How many elements does each of these sets have?

(a) $P(\{a, b, \{a, b\}\}) = P(A) = 2^n = 2^3 = 8$

$P = \{\phi, \{a\}, \{b\}, \{\{a, b\}\}, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\}$

(b) $P(\{\phi, a, \{a\}, \{\{a\}\}\}) = P(B) = 2^n = 2^4 = 16$

(c) $P(P(\phi)) = P(2^0) = P(1) = 2^1 = 2$

$P(P(\phi)) = \{\phi, \{1\}\}$

Q9 Determine if these sets are power set of a set.

(a) $P(A) = \phi$ (b) $P(B) = \{\phi, \{a\}\}$

$A = \{\ \}$ $\Rightarrow$ Yes $B = \{a\}$ $\Rightarrow$ Yes

(c) $P(C) = \{\phi, \{a\}, \{\phi, \{a\}\}, \{\phi, a\}\}$ (d) $P(D) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$C = \{a\}$ $\Rightarrow$ No $D = \{a, b\}$ $\Rightarrow$ Yes

Q10. If $A \times B = \phi$, where A & B are sets. What can you conclude?

A & B are empty sets

Q11 Explain why $A \times B \times C \neq (A \times B) \times C$?

Let $A = \{1, 2\}$

$B = \{a, b\}$

$C = \{A, B\}$

CROWN ABS

Date :

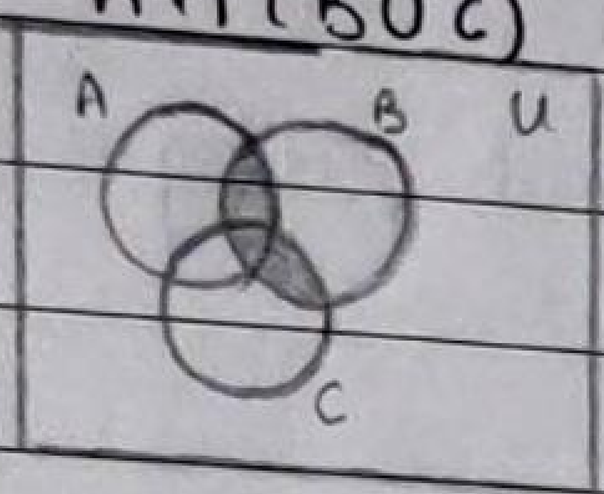
$A \times B = \{(1,a), (1,b), (2,a), (2,b)\}$

$A \times B \times C = \{(1,a,A)\ (1,a,B)\ (1,b,A)\ (1,b,B)\ (2,a,A)\ (2,a,B)\ (2,b,A)\ (2,b,B)\}$

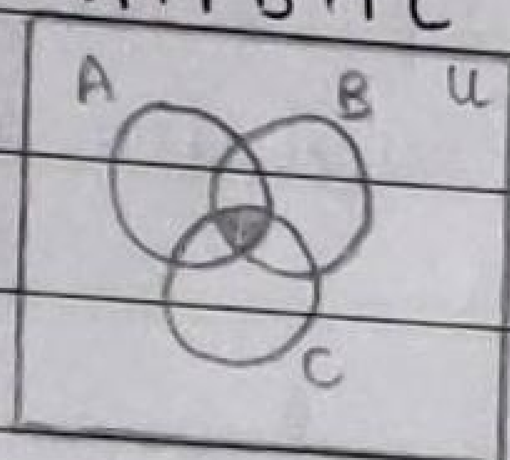
$(A \times B) \times C = \{((1,a),A), ((1,a),B), ((1,b),A), ((1,b),B), ((2,a),A), ((2,a),B), ((2,b),A), ((2,b),B)\}$

Q14 Venn diagram

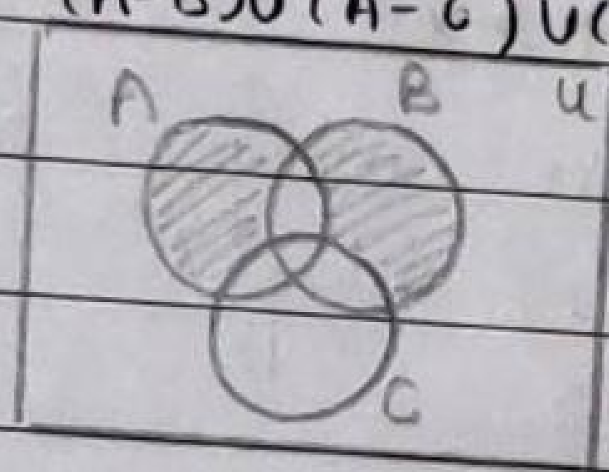
$A \cap (B \cup C)$



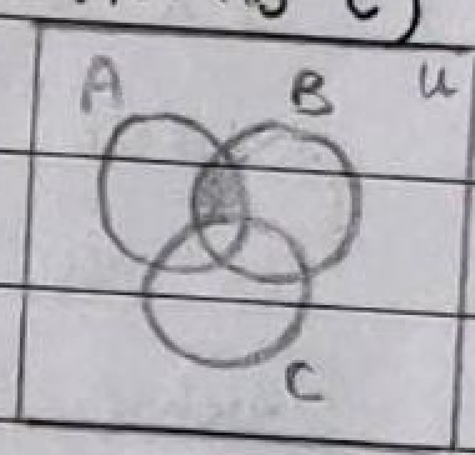
$\bar{A} \cap \bar{B} \cap \bar{C}$



$(A-B) \cup (A-C) \cup (B-C)$



$A \cap (B-C)$



Q15. Find power set of $\{\phi, \{\phi\}\}$

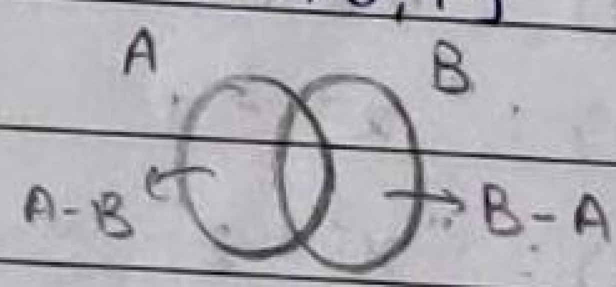
$P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Q17 Find sets A & B if $A-B = \{1,5,7,8\}$, $B-A = \{2,10\}$ & $A \cap B = \{3,6,9\}$

$A-B = \{1,5,7,8\}$ $B-A = \{2,10\}$ $A \cap B = \{3,6,9\}$

$A = \{1,3,5,6,7,8,9\}$

$B = \{2,3,6,9,10\}$



Q12 Translate each of these quantifications into english & determine its true value.

(a) $\forall x \in R\ (x^2 \neq -1)$

Square of all real number is not equal to -1.

True.

(b) $\exists x \in Z\ (x^2 = 2)$

There exists an integer st square of the number is equal to 2.

False

(c) $\forall x \in Z\ (x^2 > 0)$

Square of all integers is greater than 0.

True

Date :

(d) $\forall x \in R \ (x^2 = x)$

Square of all real number is equal to number itself.

False

(e) $\exists x \in R \ (x^3 = -1)$

There exists a real number st cube of number is equal to -1

True.

(f) $\exists x \in Z \ (x+1 > x)$

There exists an integer st successor of that integer is equal to the number itself.

~~False~~. True

Q13. Find truth set of each predicates where the domain is the set of integers

(a) $P(x)$: "$x^2 < 3$" $P = \{-1, 0, 1\}$

(b) $Q(x)$: "$x^2 > x$" $Q = Z - \{0, 1\}$

(c) $R(x)$: "$2x + 1 = 0$" $R = \phi$

(d) $P(x)$: "$x^3 \geq 1$" $P: Z^+$ OR $Z - \{Z^-, 0\}$

(e) $Q(x)$: "$x^2 = 2$" $Q = \phi$

(f) $R(x)$: "$x^2 < x$" $R = \phi$

Q18. Can you conclude $A = B$ if A, B & C are sets st

(a) $A \cup C = B \cup C$?

$A = \{1, 2, 3\}$

$B = \{7, 8, 9\}$

$C = \{4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 6\}$

$B \cup C = \{4, 5, 6, 7, 8, 9\}$

LHS $\neq$ RHS

$\Rightarrow A \neq B$

(b) $A \cap C = B \cap C$?

$A = \{1, 2, 3\}$

$C = \{1\}$

$B = \{1, 4, 5\}$

$A \cap C = \{1\}$

$B \cap C = \{1\}$

LHS = RHS

but $A \neq B$

(c) $A \cup C = B \cup C$ & $A \cap C = B \cap C$?

From previous examples,

$A \cup C \neq B \cup C$

& $A \cap C = B \cap C$

$\Rightarrow A \neq B$

Date :

Q19. What can you say about sets A & B if

(a) $A \cup B = A \Rightarrow B \subset A$

(b) $A \cap B = A \Rightarrow A \subset B$

(c) $A - B = A \Rightarrow B = \phi$

(d) $A \cap B = B \cap A \Rightarrow$ no specifications

(e) $A - B = B - A \Rightarrow A = B$

Q20. PT if A is subset of a universal set U, then

(a) $A \oplus A = \phi$

LHS $= A \oplus A$

$= (A \cup A) - (A \cap A)$

$= A - A$

$= \phi$

$=$ RHS

Hence, proved

(b) $A \oplus \phi = A$

LHS $= A \oplus \phi$

$= (A \cup \phi) - (A \cap \phi)$

$= A - \phi$

$= A$

$=$ RHS

Hence, proved

(c) $A \oplus U = \bar{A}$

LHS $= A \oplus U$

$= (A \cup U) - (A \cap U)$

$= U - A$

$= \bar{A}$

$=$ RHS

Hence, proved.

Q21. What subsets of a finite universal set do these bit strings represent?

(a) the string with all zeroes? $A = \phi$

(b) the string with all ones? $A = U$

Q22. What is the bit string corr to the diffⁿ of two sets?

$A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6, 7\}$ $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

A = 1111 0000 B = 0011 1110

$A - B = \{1, 2\} \Rightarrow$ 1100 0000

Q23. How can the union & intersection of n sets that all are subsets of the universal set U be found using bit strings?

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 3, 5, 7, 9\}$ $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\Rightarrow A \subset U$ & $B \subset U$

A = 11111 0000 B = 1 0101 0101

$A \cup B$ = 1 1111 0101

$A \cap B$ = 1 0101 0000

Date :

Q24. PT if A & B are the sets, then $(A \oplus B) \oplus B = A$.

$LHS = (A \oplus B) \oplus B$

$= [(A \cup B) - (A \cap B)] \oplus B$

$A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

$A \oplus B = \{1, 2, 4, 5\}$

$(A \oplus B) \oplus B = \{1, 2, 3\} = A.$

$LHS = (A \oplus B) \oplus B$

$= [(A - B) \cup (B - A)] \oplus B$

$= [((A-B) \cup (B-A)) - B] \cup [B - ((A-B) \cup (B-A))]$

$= [(A-B) \cup A] \cup [A \cup A]$

$= A \cup A = A = RHS$

Hence, proved.

Q25. Find $\bigcup_{i=1}^{n} A_i$ & $\bigcap_{i=1}^{n} A_i$

(i) $A_i = \{1, 2, 3, \dots i\}$

$A_1 = \{1\}$

$A_2 = \{1, 2\}$

$\vdots$

$A_n = \{1, 2, \dots n\}$

$\bigcup_{i=1}^{n} A_i = \{1, 2, 3, \dots\}$

$\bigcap_{i=1}^{n} A_i = \phi$

(ii) $A_i = \{\dots, -2, -1, 0, 1, 2, \dots\}$

$A_1 = \{-1, 0, 1\}$

$A_2 = \{-2, -1, 0, 1, 2\}$

$A \vdots$

$A_n = \{-n, \dots n\}$

$\bigcup_{i=1}^{n} A_i = \{-n, \dots -1, 0, 1, \dots n\}$

$\bigcap_{i=1}^{n} A_i = \{-1, 0, 1\}$

(iii) $A_i = \{i, i+1, i+2, \dots\}$

$A_1 = \{1, 2, 3, \dots\}$

$A_2 = \{2, 3, \dots\}$

$A \vdots$

$A_n = \{n, n+1, \dots\}$

$\bigcup_{i=1}^{n} A_i = \{1, 2, 3, \dots n+1, n+2, \dots\}$

$\bigcap_{i=1}^{n} A_i = \{n, n+1, \dots\}$

(iv) $A_i = (0, i)$

$A_1 = (0, 1)$

$A_2 = (0, 2)$

$\vdots$

$A_n = (0, n)$

$\bigcup_{i=1}^{n} A_i = (0, n)$ $\bigcap_{i=1}^{n} A_i = (0, 1)$

Date :

(v) $A_i = [-i, i]$

$A_1 = [-1, 1]$

$A_2 = [-2, 2]$

$\vdots$

$A_n = [-n, n]$

$\bigcup_{i=1}^{n} A_i = [-n, n]$

$\bigcap_{i=1}^{n} A_i = [-1, 1]$

(vi) $A_i = \{-i, -i+1, \ldots -1, 0, 1, i-1, i\}$

$A_1 = \{-1, 0, 1\}$

$A_2 = \{-2, -1, 0, 1, 2\}$

$\vdots$

$A_n = \{-n, \ldots -1, 0, 1, \ldots n\}$

$\bigcup_{i=1}^{n} A_i = \{-n, -n+1, \ldots -1, 0, 1, \ldots n\}$

$\bigcap_{i=1}^{n} A_i = \{-1, 0, 1\}$

Q26. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express the sets with bit strings where ith bit in the string is 1 if i is in the set & 0 otherwise.

(a) $\{3, 4, 5\}$ 00 1110 0000

(b) $\{1, 3, 6, 10\}$ 10 1001 0001

(c) $\{2, 3, 4, 7, 8, 9\}$ 01 1100 1110

Also, find the set specified by each of the bit strings.

(a) 11 1100 1111 $\{1, 2, 3, 4, \cancel{7}, 8, 9, 10\}$

(b) 01 0111 1000 $\{2, 4, 5, 6, 7\}$

(c) 10 0000 0001 $\{1, 10\}$