

Network Analysis

Dr. Yogesh A. Chauhan

GCET

Introduction

- Construction of a new building
- Introducing a new product
- Construction of a new plant including installation of machines
- Project management
- Installation of a pipeline project
- Huge amount of money, manpower and other resources are required
- CPM (Critical Path Method) and PERT (Programme Evaluation Review Technique)

CPM & PERT

- Planning & controlling the projects
- Helps to determine
 - expected project completion duration
 - The scheduled start and completion time for different activities
 - The key activities of the project which must be completed at the scheduled time
 - The time period by which the non key activities may be delayed without causing a delay in the completion of the whole project

- PERT is useful for analysing project scheduling problems in which the completion time of the different activities and therefore the whole project is not certain; It is probabilistic in nature; used in research and development projects
- CPM is used in projects in which the activity duration are known with certainty; deterministic in nature; used in construction projects

CPM & PERT can be employed for any situation where:

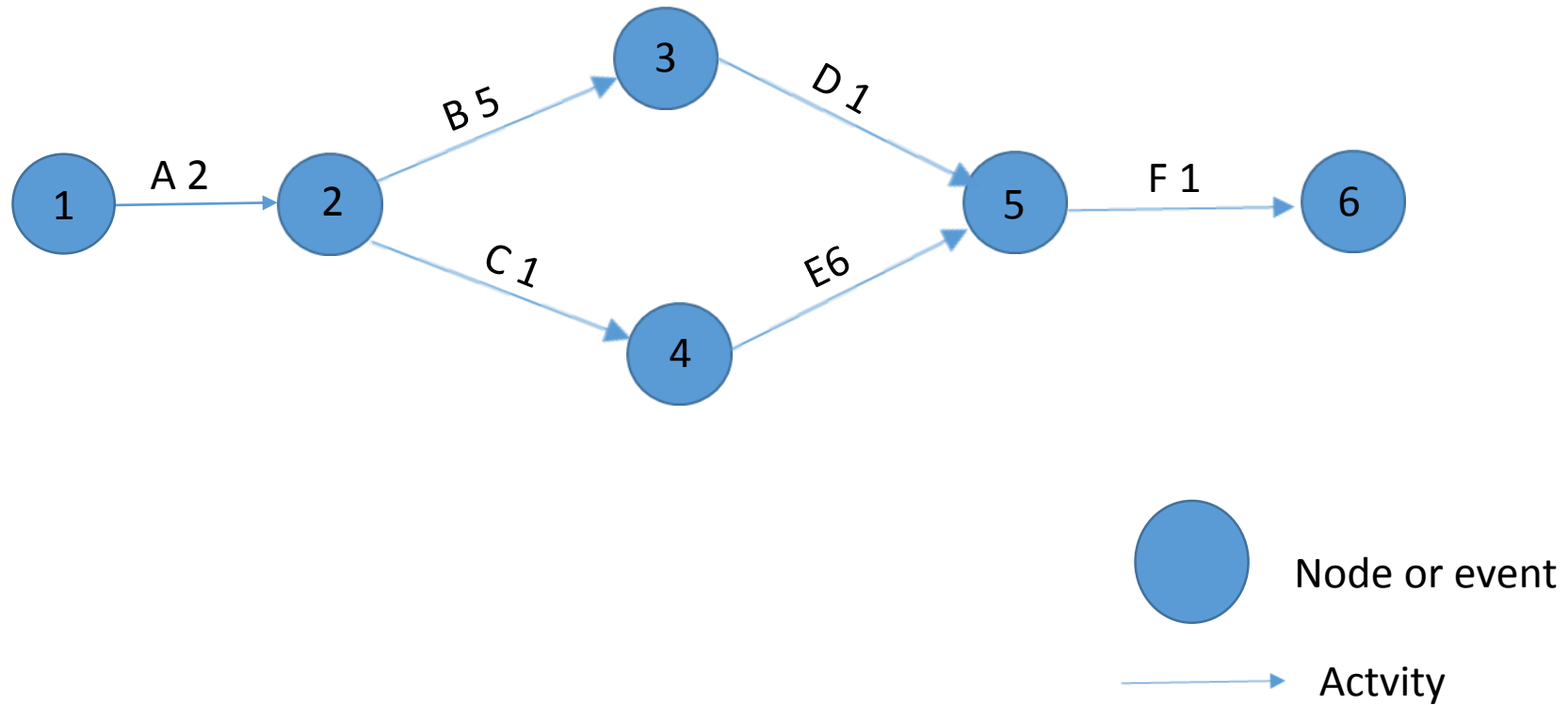
1. The project consists of well defined collection of activities or tasks.
2. The activities can be started and terminated independently of each other, even if the resources employed on the various activities are not independent.
3. The activities have predecessor/successor relationships

Network construction

- List of activities and precedence relationships

ACTIVITY	DESCRIPTION	DURATION (WEEKS)	IMMEDIATE PREDECESSOR (S)
A	Obtain the budget approval	2	-
B	Obtain the machine	5	A
C	Hire the operator	1	A
D	Install the machine	1	B
E	Train the operator	6	C
F	Produce the goods	1	D,E

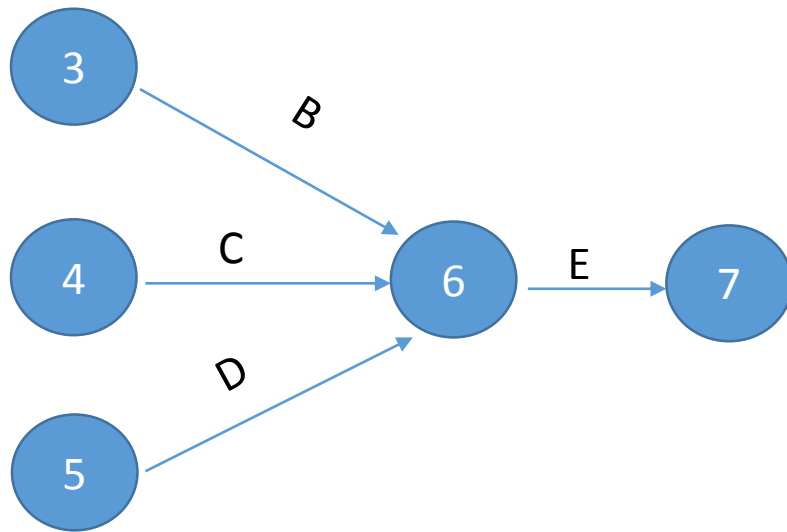
Network or Arrow Diagram



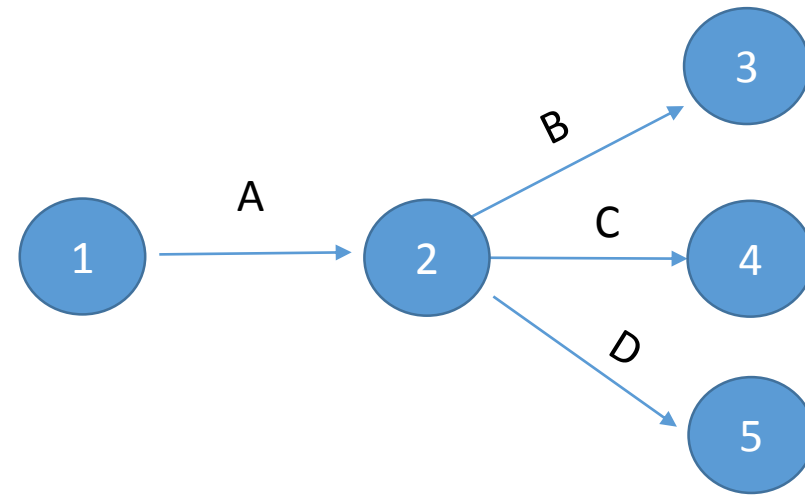
Rules of Network construction

- Each defined activity is represented by one and only one arrow.(no single activity can be represented more than once)
- Before an activity can be undertaken, all activities preceding it must be completed. (Predecessor-successor relationships)
- The arrows depicting various activities are indicative of logical precedence only. The length of the arrow has no significance.
- The arrow direction indicates the general progression in time. The arrow head represents the point in time at which the activity completion event takes place, while the arrow tail represents the point in time at which the event activity start occurs. The event marking the start of activities are called **tail events** while marking completion of activities are known as **head events**.

- When a number of activities terminate at one event, it indicates that no activity emanating from that event may start unless all activities terminating there have been completed.



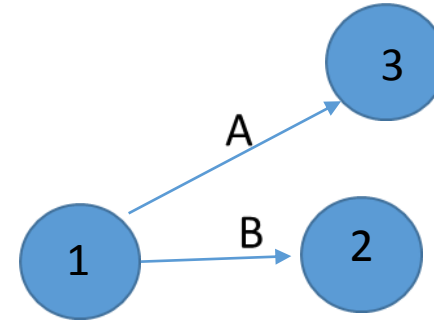
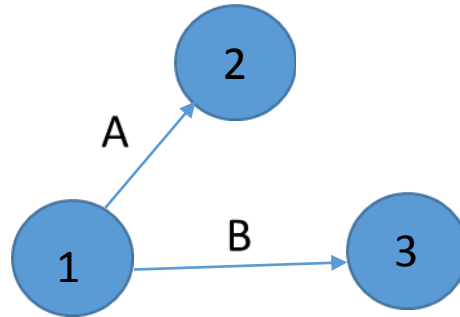
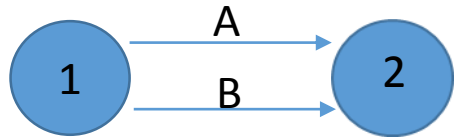
Merge Event



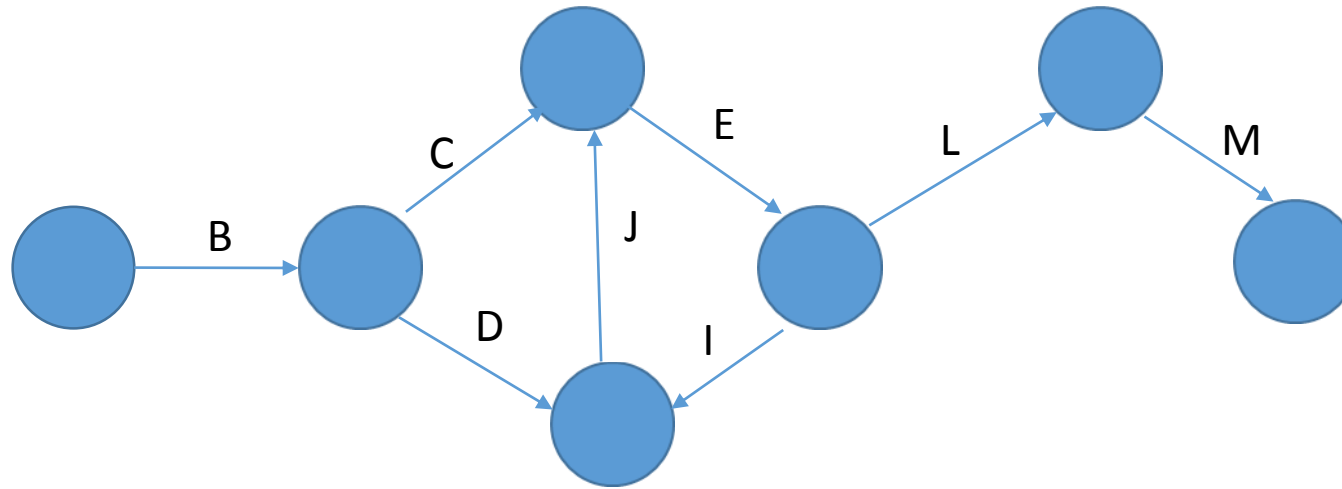
Burst Event

- Events are identified by numbers. No duplication is to be done.
- i-j activity, where i represents starting or initial node and j ending or terminal node.
- A network should have only one initial and one terminal node.
- An event which represents the joint completion of more than one activity is known as a **merge event**, while an event which portrays the initiation of more than one activity is called the **burst event**.
- Parallel activities between two events, without intervening events are prohibited. When two or more parallel activities in a project have the same head and tail events, dummy activities are needed to construct the network.

Dummy Activity



Loop in the network

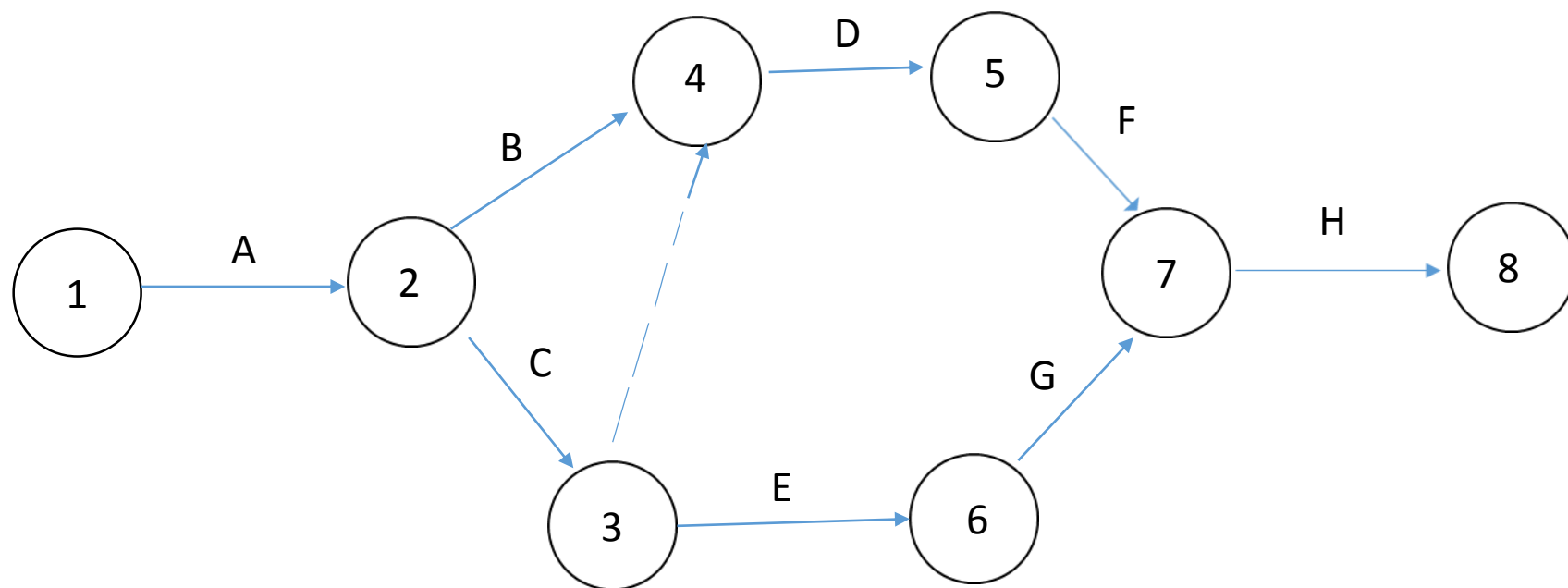


Looping is not permitted in a network. Thus if A precedes B, and B precedes C, then C cannot precede A. In developing a network it must be ensured that loops are not present. In the figure activities E, I and J form a loop. Activity E requires that activities C and J should be completed before it begins. But J depends on I which in turn depends on E. Thus, activity E can never start because it is dependent on itself!! The loop in a network suggest a fault in the logic, so that the definition of one or more of the dependency relationships is not valid.

Ex. Given the following information on a small project: A is the first activity of the project and precedes the activities B and C. the activity D succeeds both B and C whereas only C is required to start activity E. D precedes F while G succeeds E. H is the last activity of the project and succeeds F and G. Draw a network based on this information.

Activity	A	B	C	D	E	F	G	H
I.P. *	-	A	A	B,C	C	D	E	F,G

*I.P. – Immediate predecessor



Network Analysis

- Network Diagram: It is the graphical representation of logically and sequentially connected arrows and node, representing activities and events in a project.
- Activity: it is physically identifiable task of a project which consumes time and other resources.
- Node/Event: It marks beginning or completion of an activity
- Dummy activity: It shows the dependency of one activity with other, but does not consume any time or other resources.

Significance of Network Analysis:

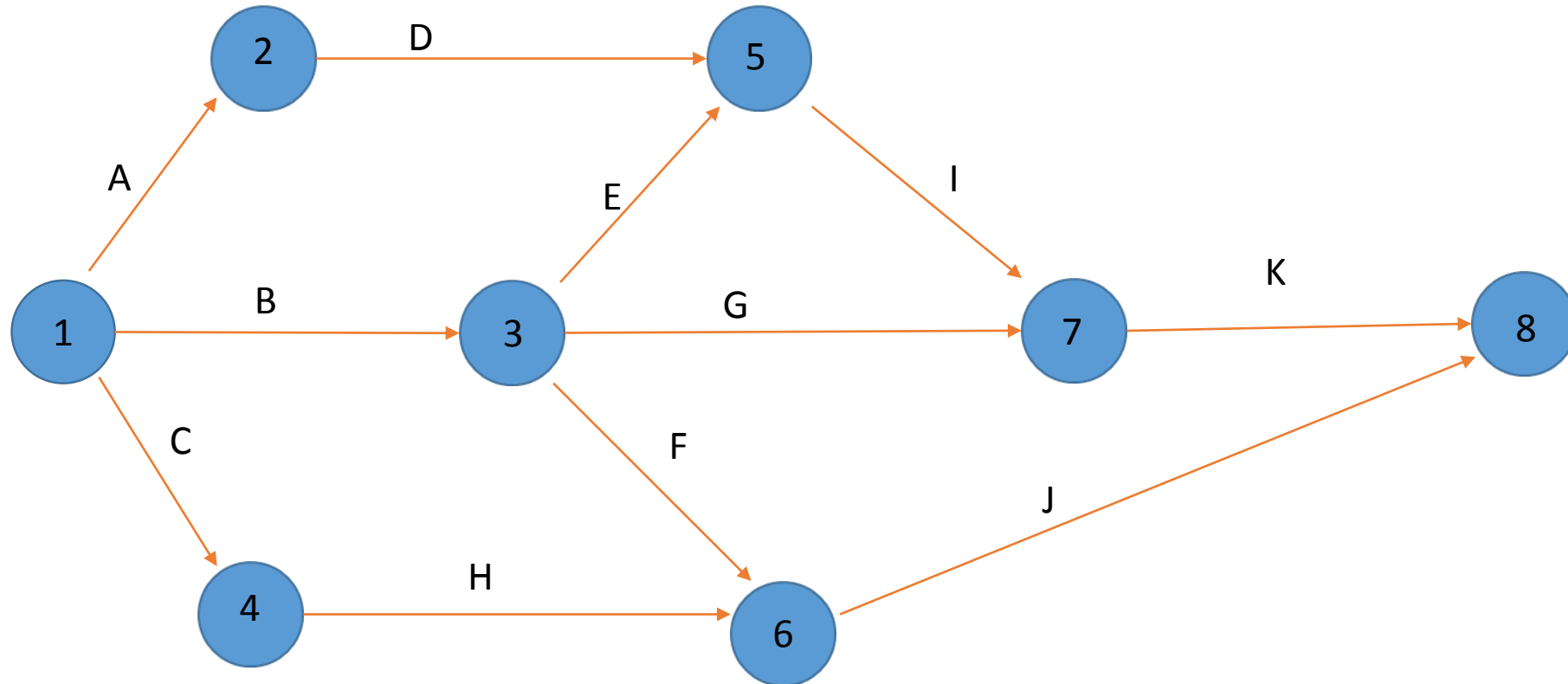
- When which activities can be scheduled
- How long it (project) will take to be completed
- How much buffer is available for performing the activities
- Which activities can be delayed and which cannot without affecting the project completion deadline

- Ex. Information on the activities required for a project is as follows:

Name	A	B	C	D	E	F	G	H	I	J	K
Activity Node	1-2	1-3	1-4	2-5	3-5	3-6	3-7	4-6	5-7	6-8	7-8
Duration (day)	2	7	8	3	6	10	4	6	2	5	6

Draw the network and calculate the earliest start (ES), earliest finish (EF), latest start (LS) and latest finish (LF) times of each of the activities.

Fig.1



- Ex. Draw a network corresponding to the following information.
Obtain the early and late start and completion times and determine the critical activities.

Activity	1-2	1-3	2-6	3-4	3-5	4-6	5-6	5-7	6-7
Duration	4	6	8	7	4	6	5	19	10

Float

- For critical activities earliest and latest times are the same. The critical activities cannot be scheduled later than their earliest schedule time, without delaying the project duration, they do not have any flexibility in scheduling.
- For non critical activities flexibility exists in scheduling. This is called as float or slack. The non critical activities can be postponed for sometime without affecting the project duration.
- Total float: The amount of time by which the activity can be delayed without delaying the project completion time.

Total float = Latest finish time - Earliest finish time

= Latest start time - Earliest start time

= Latest finish time - Earliest start time - duration of the activity

- Interfering float: Using the float of an activity may, affect the float times of the other activities. The part of the total float which causes a reduction in the float of the successor activities is called interfering float.
- Suppose the activity 1-4 in the network diagram is delayed by 2 days and it begins at $t=2$ instead of $t=0$, it would finish then at $t=10$. with $LS=11$ for it, the float for 4-6 is reduced to $11-10 = 1$ from $11-8=3$. Thus, using the float of 1-4 is seen to interfere into the float of its successor 4-6.

- Free float: It is that part of the total float which can be used without affecting the float of the succeeding activities. Thus, this is that value of the float which is consumable when the succeeding activities of the activity in question can be started at their earliest starting times.
- Eg. Activity 2-5 with a total float of 9 days. Suppose it is delayed by 7 days so that it starts at $t=9$ and completes at $t=12$. the succeeding activity, however can begin only at $t=13$. thus delay in 2-5 does not cause a delay in the start of its successor activity and hence does not interfere in its float.
- Free float = earliest start time for the following activity – earliest finish time of this activity.

- Independent float: The amount of float time which can be used without affecting either the head or the tail events. It represents the amount of float time available for an activity when its preceding activities are completed at their latest and its succeeding activities begin at their earliest time; leaving the minimum time available for its performance.
- Independent float = ES time for following activity- LF time for the preceding activity - Duration of the present activity
- Independent float is always either equal to or less than the free float of an activity. If it is negative, then it should be considered as Zero.

- Eg. For activity 2-5,
ES for following activity (5-7) = 13
LF for preceding activity (1-2) = 11
Duration of the activity = 3
Independent float = $13 - 11 - 3 = -1 \approx 0$

Floats

- Total Float, $F_t = LS_{ij} - ES_{ij}$
- Interfering Float, $F_{int} = \text{Max}(0, LF_{ij} - ES_{jk})$
- Free Float, $F_f = ES_{jk} - Ef_{ij}$
- Independent Float, $F_{ind} = \text{Max}(0, ES_{jk} - LF_{hi} - t_{ij})$

t_{ij} duration of the activity ij

jk succeeding activity

hi previous activity

ES,EF,LS,LF and Floats

Activity i-j	Duration t_{ij}	ES	EF	LS	LF	Total Float F_t	Int. Float F_{int}	Free Float F_f	Ind. Float F_{ind}
1-2	2	0	2	9	11	9	9	0	0
1-3	7	0	7	0	7	0	0	0	0
1-4	8	0	8	3	11	3	3	0	0
2-5	3	2	5	11	14	9	1	8	0
3-5	6	7	13	8	14	1	1	0	0
3-6	10	7	17	7	17	0	0	0	0
3-7	4	7	11	12	16	5	1	4	4
4-6	6	8	14	11	17	3	0	3	0
5-7	2	13	15	14	16	1	1	0	0
6-8	5	17	22	17	22	0	0	0	0
7-8	6	15	21	16	22	1	0	1	0

Time-cost Trade-off: Crashing

- It is based on the concept that the duration of some of the activities of a project can be cut down, if some additional resources- men, material and/or equipment are employed on them.
- Deliberate reduction of activity times by putting extra efforts is called crashing.
- The duration of an activity cannot be reduced indefinitely. The crash time represents the fully expedited or the minimum activity time duration that is possible.
- The activity cost corresponding to the crash time is called the crash cost which equals the minimum direct cost required to perform crash performance time.

- Normal activity time and normal activity cost
- Incremental cost/time period = $\frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$
- Eg. For an activity, if crash cost and time are Rs.700 and 8 days, and if the normal cost and time are Rs.500 and 12 days,
- Incremental cost/day = $\frac{700 - 500}{12 - 8} = \text{Rs. } 50 / \text{day}$
- In case of a choice, the activity with the least incremental cost of crashing would be chosen to be crashed.

Points to be considered while crashing:

1. Activities on the critical path only to be considered. Crashing of non critical activities would not reduce the project duration.
2. The critical path of the project may change while crashing.
3. If at any stage of crashing, there are multiple critical paths in a project, then activities from each of the paths should be chosen for crashing.

Example:

The following table shows for each activity needed to complete the project the normal time, the shortest time in which the activity can be completed of a building contract and the cost per day for reducing the time of each activity. The contract includes a penalty clause of Rs 100 per day over 17 days. The overhead cost per day is Rs. 160.

Activity	Normal time in days	Shortest time in days	Cost for reduction per day in Rs.
1-2	6	4	80
1-3	8	4	90
1-4	5	3	30
2-4	3	3	-
2-5	5	3	40
3-6	12	8	200
4-6	8	5	50
5-6	6	6	-

The cost of completing the eight activities in normal time is Rs. 6500.

- a) Calculate the normal duration of the project, its cost and the critical path.
- b) Calculate and plot on a graph the cost/time function for the project and state:
 - i. The lowest cost and associated time
 - ii. The shortest time and associated cost

PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

- PERT is useful for analysing project scheduling problems in which the completion time of the different activities and therefore the whole project is not certain; It is probabilistic in nature; used in research and development projects.
- It may be easier to complete the job/activity than expected, while at times due to unexpected obstacles there may be unplanned delays.
- This method uses three time estimates for an activity.
 1. Optimistic time (a): shortest time of completion of an activity. It represents an ideal estimate.

2. Most likely time (m): The time that would be expected to occur most often if the activity were frequently repeated under exactly the same conditions. It is the modal time.

3. Pessimistic time (b): Longest time the activity could take to complete. It is the worst time estimate and represents the time the activity would take if bad luck was faced.

$$\text{Expected time} = t_{ei} = \frac{a+4m+b}{6}$$

$$\text{Standard Deviation } \sigma_i = \frac{b-a}{6}$$

$$\text{Variance } \sigma_i^2 = \left(\frac{b-a}{6}\right)^2$$

- The critical path is to be found by using expected times.
- Aggregate of mean times and the summation of the variances of critical jobs would yield the project duration expected and its variance.
- Assumption- the sum of several independent activity duration will tend to be normally distributed, with a mean equal to the sum of their individual job times and the variance equal to the sum of their individual activity variances.
- The probability distribution of times for completing a project can be approximated by using a normal distribution curve which becomes more exact when the number of activities increases. (at least 30 activities will give a fairly exact curve.)

- For a given project, if the critical activities are 1,2,.....,k then

$$T_e = T_{e1} + t_{e2} + \dots + t_{ek} \text{ and}$$

$$V_T = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_k^2$$

The distribution of the project completion times is, then normally distributed with $\mu = T_e$ and $\sigma = \sigma_T$

With this probability of completing the project by a given date or during a given time interval can be determined.

Points to be kept in mind:

1. If there are two or more critical paths in a given network, then the one with the largest variance value should be used for determining T_e and V_T
2. The probability calculations by this way represent only the probability of completing the critical activities, assuming that other activities would be completed by the time the critical activities are over.

Example

- The owner of a chain of fast-food restaurants is considering a new computer system for accounting and inventory control. A computer company sent the following information about the system installation:

Activity	Activity Description	Immediate Predecessor	Most Optimistic time (days)	Most Likely time (days)	Most Pessimistic time (days)
A	Select the computer model	-	4	6	8
B	Design input/output system	A	5	7	15
C	Design monitoring system	A	4	8	12
D	Assemble computer hardware	B	15	20	25
E	Develop main programs	B	10	18	26
F	Develop input/output routines	C	8	9	16
G	Create database	E	4	8	12
H	Install the system	D,F	1	2	3
I	Test and implement	G,H	6	7	8

- a) Construct an arrow diagram for this problem, determine the critical path and state the expected project completion time.
- b) Determine the probability that the project will be completed in 55 days.
- c) If the company wants to be 90% sure that the system will be installed by a certain due date, how many days prior to that should it start the work?
- d) Suppose the company agrees to install the computer system in 50 days, failing which it would pay a penalty of Rs.500 per day. What is the probability that a penalty, but not exceeding Rs.2000, will be paid?
- e) Obtain in earliest and the latest scheduling times of the various activities.

TABLE A.2 Cumulative normal distribution (continued)

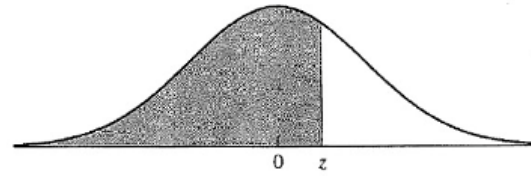
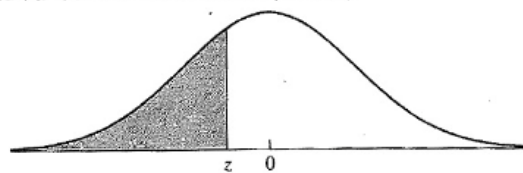
[illegible]

TABLE A.2 Cumulative normal distribution (z table)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

