Differential Calculus-I

Unit: 2

Click to add text
Click to add text
Click to add text

ENGINEERING MATHEMATICS-I

B Tech 1st Sem

Rajnish Pandey

Department of Mathematics

Brief Introduction

Name: HarendraSinghal

Qualification: M.Sc , M.Phil (A.M.U, Aligarh), Ph.D(P)

Total Exp: 19+ Yrs

NIET Exp: 12+ Yrs

Course Taught: Maths-I, Maths-II, Maths-III

Evaluation Scheme

SEMESTER I

Sl. No.	Subject Codes	Subject	Periods			Evaluation Schemes				End Semester		Total	Credit
			L	T	P	CT	TA	TOTAL	PS	TE	PE		
3 WEEKS COMPULSORY INDUCTION PROGRAM													
1	AAS0103	Engineering Mathematics-I	3	1	0	30	20	50		100		150	4
2	AAS0101A	Engineering Physics	3	1	0	30	20	50		100		150	4
3	ACSE0101	Problem Solving using Python	3	0	0	30	20	50		100		150	3
4	AASL0101	Professional Communication	2	0	0	30	20	50		100		150	2
5	AAS0151A	Engineering Physics Lab	0	0	2				25		25	50	1
6	ACSE0151	Problem Solving using Python Lab	0	0	2				25		25	50	1
7	AASL0151	Professional Communication Lab	0	0	2				25		25	50	1
8	AME0151	Digital Manufacturing Practices	0	0	3				25		25	50	1.5
9		MOOCs (For B.Tech. Hons. Degree)											
		TOTAL										800	17.5

L: Lecture, T: Tutorial, P: Practical, CT: Class Test, TA: Teacher Assessment, PS: Practical Sessional, TE: Theory End Semester Exam., PE: Practical End Semester Exam.

SYLLABUS

B. TECH. FIRST YEAR (CS/IT/EC/ME/IOT/CSE/M.Tech(Integrated))		
Course code	**AAS0103**	L T P
Course title	**ENGINEERING MATHEMATICS-I**	3 1 0
Course objective: The objective of this course is to familiarize the graduate engineers with techniques in linear algebra, differential calculus-I, differential calculus-II and multivariable calculus. It aims to equip the students with standard concepts and tools from intermediate to advanced level that will enable them to tackle more advanced level of mathematics and applications that they would find useful in their disciplines.		
Pre-requisites: Knowledge of Mathematics up to 12thstandard.		
Course Contents / Syllabus		
UNIT-I	**Matrices**	**8 hours**
Types of Matrices: Symmetric, Skew-symmetric and Orthogonal Matrices; Complex Matrices, Inverse and Rank of matrix using elementary transformations, System of linear equations, Characteristic equation, Cayley-Hamilton Theorem and its application, Eigen values and eigenvectors; Diagonalisation of a Matrix.		

SYLLABUS

UNIT-II	**Differential Calculus-I**	**8 hours**
Successive Differentiation (nth order derivatives), Leibnitz theorem and its application, Asymptotes, Curve tracing: Cartesian and Polar co-ordinates. Partial derivatives, Total derivative, Euler's Theorem for homogeneous functions.		
UNIT-III	**Differential Calculus-II**	**8 hours**
Taylor and Maclaurin's theorems for a function of one and two variables, Jacobians, Approximation of errors. Maxima and Minima of functions of several variables, Lagrange Method of Multipliers.		
UNIT-IV	**Multivariable Calculus**	**10 hours**
Multiple integration: Double integral, Triple integral, Change of order of integration, Change of variables, Application: Areas and volumes, Centre of mass and centre of gravity (Constant and variable densities),Improper integrals, Beta & Gama function and their properties, Dirichlet's integral and its applications.		
UNIT-V	**Aptitude-I**	**8 hours**
Simplification , Percentage , Profit, loss & discount , Average, Number & Series, Coding & decoding		

Text Books

- S.N. Mishra, Engineering Mathematics-I, Cengage Learning, 2018.
- B. V. Ramana, Higher Engineering Mathematics, Tata Mc Graw-Hill Publishing Company Ltd., 2008.
- B. S. Grewal, Higher Engineering Mathematics, Khanna Publisher, 2005.
- R K. Jain & S R K. Iyenger , Advance Engineering Mathematics, Narosa Publishing House 2002.

Reference Books

- E. Kreyszig, Advance Engineering Mathematics, John Wiley & Sons, 2005.
- Peter V. O'Neil, Advance Engineering Mathematics, Thomson (Cengage) Learning, 2007.
- Veerarajan T., Engineering Mathematics for first year, Tata McGraw-Hill, New Delhi, 2008.
- P. Sivaramakrishna Das and C. Vijayakumari, Engineering Mathematics, 1st Edition, Pearson India Education Services Pvt. Ltd. Advanced Engineering Mathematics. Chandrika Prasad, Reena Garg, 2018.

Course Objective

- The objective of this course is to familiarize the graduate engineers with techniques in linear algebra.

- It aims to equip the students with standard concepts and tools from intermediate to advanced level of mathematics and applications that they would find useful in their disciplines

Course Outcomes 2021-22 (B. Tech. – 1st Sem)

Course Name: Engineering Mathematics-I (AAS0103)	
CO1	Apply the concept of matrices to solve linear simultaneous equations
CO2	Apply the concept of successive differentiation and partial differentiation to solve problems of Leibnitz theorems and total derivatives .
CO3	Apply partial differentiation for evaluating maxima, minima, Taylor’s series and Jacobians.
CO4	llustrate multiple integral to find area, volume, centre of mass and centre of gravity.
CO5	Demonstrate the basic concept of Profit, Loss, Number & Series, Coding & decoding.

Program Outcomes (POs)

1. **Engineering knowledge**
2. **Problem analysis**
3. **Design/development of solutions**
4. **Conduct investigations of complex problems**
5. **Modern tool usage**
6. **The engineer and society**

Program Outcomes (POs)

7. Environment and sustainability

8. Ethics

9. Individual and team work

10.Communication

11.Project management and finance

12. Life-long learning

CO-PO Mapping 2021-22 (B. Tech. – 1st Sem)

Course Name: Engineering Mathematics-I (AAS0103)												
CO	**PO-1**	**PO-2**	**PO-3**	**PO-4**	**PO-5**	**PO-6**	**PO-7**	**PO-8**	**PO-9**	**PO-10**	**PO-11**	**PO-12**
CO1	**3**	**2**	**1**	**1**	**3**	**2**	**-**	**-**	**-**	**2**	**2**	**3**
CO2	3	3	2	3	3	-	-	-	-	2	3	3
CO3	3	2	3	3	3	2	-	-	-	2	3	3
CO4	3	2	3	3	2	2	-	-	-	2	2	3
CO5	1	1	1	1	1	-	-	-	-	2	-	3
Mean	**2.6**	**2**	**2**	**2.2**	**2.4**	**2.0**	**-**	**-**	**-**	**2**	**2.5**	**3**

Result Analysis 2020-21 (Odd Semester)

INSTITUTE RESULT	**86.8%**
DEPARTMENT RESULT	**95.8%**

FACULTY NAME	BRANCH/SECTION	RESULT
Harendra Singhal	CS	100%
	EC	94.7%
	IOT	92.5%
	AI	93.5%
	DS	95.7%

END SEM. QUESTION PAPER PATTERN FOR OFFLINE MODE

Printed page: 02 **Subject Code: AAS0103**

Roll No: | | | | | | | | | | | | | |

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B. Tech.

(SEM: FIRST SEMESTERTHEORY EXAMINATION (2020-2021))

Subject Name: Engineering Mathematics-I

Time: 3Hours Max. Marks: 100

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 02pages & 8questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- Section B - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- Section C -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10marks each. You need to attempt any one part *a or b.*
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

END SEM. QUESTION PAPER PATTERN FOR OFFLINE MODE

SECTION – A

1.	**Attempt all the parts.**	**[10×1=10]**	**CO**
a.	A is a singular matrix of order 3 with eigen values 2 and 3. The third eigen value is (a) 1 (b) 0 (c) 4 (d) -1	**(1)**	**CO1**
b.	The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is (a) 0 (b) 1 (c) 2 (d) 3	**(1)**	**CO1**
c.	If $u = \frac{x^2}{a} + \frac{y^2}{b} - 7$ then $\frac{\partial u}{\partial x}$ is...............	**(1)**	**CO2**
d.	If $u = x^2$and $x = t^3$ then $\frac{du}{dt}$ is...............	**(1)**	**CO2**
e.	If $x = rcos\emptyset$ and $y = rsin\emptyset$ then $\frac{\partial(x,y)}{\partial(r,\emptyset)}$ is...............	**(1)**	**CO3**
f.	The function $z = y^2 + x^2y + x^4$ has a minimum at (0,0). (T/F)	**(1)**	**CO3**
g.	The value of the double integral $\int_{x=0}^{3}\int_{y=0}^{1}(x^2 + 3y^2)dydx$ is 12. (T/F)	**(1)**	**CO4**
h.	The value of $\int_0^\infty e^{-x^2}dx$ is $\sqrt{\pi}$. (T/F)	**(1)**	**CO4**
i.	The value of $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$ is..........	**(1)**	**CO5**
j.	Insert the missing number: 11, 13, 17, 19, 23, 29, 31, 37, 41, (...).	**(1)**	**CO5**

NIET Greater Noida

END SEM. QUESTION PAPER PATTERN FOR OFFLINE MODE

2.	**Attempt all the parts.**	**[5×2=10]**	**CO**
a.	Find a and b such that $A = \begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as eigen values.	**(2)**	**CO1**
b.	Find the n^{th} derivative of $y = sin\ (ax + b)$.	**(2)**	**CO2**
c.	The radius of a sphere is found to be 10 meter with a possible error of 0.02 meter. What is the relative error in calculating the volume of sphere?	**(2)**	**CO3**
d.	Prove that Beta function is symmetric.	**(2)**	**CO4**
e.	If 50% of $(x - y)$ is 30% of $(x + y)$ then what percent of x is y?	**(2)**	**CO5**

SECTION – B

3. Answer any five of the following- **[5×6=30]** **CO**

a.	Show that the system of equations $\left.\begin{aligned} 3x+4y+5z &= \alpha \\ 4x+5y+6z &= \beta \\ 5x+6y+7z &= \gamma \end{aligned}\right\}$ is consistent only if α, β and γ are in arithmetic progression.	**(6)**	**CO1**
b.	Trace the curve $a^2y^2 = x^2(a^2 - x^2)$.	**(6)**	**CO2**
c.	Prove that $\frac{1}{(1-x)} = \frac{1}{3} + \frac{(x+2)}{3^2} + \frac{(x+2)^2}{3^3} + \frac{(x+2)^3}{3^4} + \cdots$.	**(6)**	**CO3**
d.	Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\, dydx$.	**(6)**	**CO4**
e.	Using the transformation $x + y = u$ and $y = uv$, show that $\int_0^1 \int_0^{1-x} e^{\left(\frac{y}{x+y}\right)} dydx = \frac{1}{2}(e-1)$.	**(6)**	**CO4**
f.	The selling price of 20 articles is equal to the cost price of 25 articles. Find the profit percent.	**(6)**	**CO5**
g.	If the word LEADER is coded as 20-13-9-12-13-26, how would you write LIGHT?	**(6)**	**CO5**

SECTION – C

4	**Answer any one of the following-**		**[5×10=50]**	**CO**
	a.	State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$ and hence find A^{-1}.	**(10)**	**CO1**
	b.	Find eigen values and corresponding eigen vectors of the matrix $A=\begin{bmatrix}3&10&5\\-2&-3&-4\\3&5&7\end{bmatrix}$.	**(10)**	**CO1**
5.	**Answer any one of the following-**			
	a.	If $y=x^n \log x$, prove that $(i)\ y_{n+1}=\frac{n!}{x}$ $(ii)\ x^2 y_{p+2}+(2p-2n+1)xy_{p+1}+(p-n)^2 y_p=0$.	**(10)**	**CO2**
	b.	State and prove Euler's theorem for homogeneous function. Also prove that if $u=\tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$.	**(10)**	**CO2**
6.	**Answer any one of the following-**			
	a.	Expand x^y in powers of $(x-1)$ and $(y-1)$ upto the third degree terms.	**(10)**	**CO3**
	b.	Find a point on the paraboloid $z=x^2+y^2$ nearest to the point (3,-6,4).	**(10)**	**CO3**

7. Answer any one of the following-

a. Prove by the method of double integration that the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **(10)** **CO4**

b. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ using Dirichlet's theorem. **(10)** **CO4**

8. Answer any one of the following-

a. A batsman makes a score of 87 runs in the 17^{th} inning and thus increases his average by 3. Find his average after 17^{th} inning. **(10)** **CO5**

b. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers. **(10)** **CO5**

Offline Sample Question Paper [NIET 2020-21]	AAS0103.docx

END SEM. QUESTION PAPER PATTERN FOR ONLINE MODE

Printed page: ....

Subject Code: ....

Roll No: | | | | | | | | | | | | | |

<u>NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA</u>

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech./MBA/MCA/M.Tech (Integrated)

(SEM:......THEORY EXAMINATION(2020-2021)

Subject ....

Time: 2 Hours

Max. Marks: 100

END SEM. QUESTION PAPER PATTERN FOR ONLINE MODE

		SECTION – A	**[30]**	**CO**
1.	**Attempt all parts- (MCQ, True False)Three Question From Each Unit**		**[15×2=30]**	
		UNIT-1		
	1-a.	**Question-**	**(2)**	
	1-b.	**Question-**	**(2)**	
	1-c.	**Question-**	**(2)**	
		UNIT-2		
	1-d.	**Question-**	**(2)**	
	1-e.	**Question-**	**(2)**	
	1-f.	**Question-**	**(2)**	
		UNIT-3		
	1-g.	**Question-**	**(2)**	
	1-h.	**Question-**	**(2)**	
	1-i.	**Question-**	**(2)**	
		UNIT-4		
	1-j.	**Question-**	**(2)**	
	1-k.	**Question-**	**(2)**	
	1-l.	**Question-**	**(2)**	
		UNIT-5		
	1-m.	**Question-**	**(2)**	
	1-n.	**Question-**	**(2)**	
	1-o.	**Question-**	**(2)**	

END SEM. QUESTION PAPER PATTERN FOR ONLINE MODE

		SECTION – B	[20×2=40]	CO
2.	Attempt all Four parts. Fill in The Blanks, Match the pairs (From the Data Given in Glossary)) Question from Unseen passage - Four Question From Unit-I		[4×2=08]	CO
	Glossary- (Required words to be written)			
	2-a.	Question-	(2)	
	2-b.	Question-	(2)	
	2-c.	Question-	(2)	

	2-d.	Question-	(2)	
3.	Attempt all Four parts. Fill in The Blanks, Match the pairs (From the Data Given in Glossary) Question from Unseen passage - Four Question From Unit-II		[4×2=08]	CO
	Glossary- (Required words to be written)			
	3-a.	Question-	(2)	
	3-b.	Question-	(2)	
	3-c.	Question-	(2)	
	3-d.	Question-	(2)	
4.	Attempt all Four parts. Fill in The Blanks, Match the pairs (From the Data Given in Glossary) Question from Unseen passage - Four Question From Unit-III		[4×2=08]	CO
	Glossary- (Required words to be written)			
	4-a.	Question-	(2)	
	4-b.	Question-	(2)	
	4-c.	Question-	(2)	
	4-d.	Question-	(2)	

END SEM. QUESTION PAPER PATTERN FOR ONLINE MODE

5.	Attempt all Four parts. Fill in The Blanks, Match the pairs (From the Data Given in Glossary) Question from Unseen passage - Four Question From Unit-IV		[4×2=08]	CO
		Glossary- (Required words to be written)		
	5-a.	Question-	(2)	
	5-b.	Question-	(2)	
	5-c.	Question-	(2)	
	5-d.	Question-	(2)	
6.	Attempt all Four parts. Fill in The Blanks, Match the pairs (From the Data Given in Glossary) Question from Unseen passage - Four Question From Unit-V		[4×2=08]	CO
		Glossary- (Required words to be written)		
	6-a.	Question-	(2)	
	6-b.	Question-	(2)	
	6-c.	Question-	(2)	
	6-d.	Question-	(2)	
		SECTION – C		
7	Answer any 10 out 15 of the following, Subjective Type Question, Three Question from Each Unit		[10×3=30]	CO
		UNIT-1		
	7-a.	-Question-	(3)	
	7-b.	-Question-	(3)	
	7-c.	-Question-	(3)	
		UNIT-2		
	7-d.	-Question-	(3)	
	7-e.	-Question-	(3)	

END SEM. QUESTION PAPER PATTERN FOR ONLINE MODE

	7-f.	-Question-	(3)	
		UNIT-3		
	7-g.	-Question-	(3)	
	7-h.	-Question-	(3)	
	7-i.	-Question-	(3)	
		UNIT-4		
	7-j.	-Question-	(3)	
	7-k.	-Question-	(3)	
	7-l.	-Question-	(3)	
		UNIT-5		
	7-m	-Question-	(3)	
	7-n.	-Question-	(3)	
	7-o.	-Question-	(3)	

Online Sample Q. Paper [NIET 2020-21]	**Q.PTemplate (100M) 05-07-2021.docx**

Content

- Successive Differentiation(nth order derivatives)
- Leibnitz’s theorem and its application
- Asymptotes.
- Curve tracing: Cartesian and Polar Co-ordinates
- Partial derivatives
- Total derivatives
- Euler’s theorem for homogeneous functions

Objective of the topic,(CO2..)

Successive differentiation and Leibniz’s theorem:

In this section we will learn the following topic

- Definition of successive derivatives.
- The notion of successive differentiation.
- The Leibniz’s formula.
- The related problems

Successive Differentiation,(CO2..)

The process of finding the differential coefficient of a function again and again is called successive differentiation.

If $y = f(x)$ then, First differential coefficient is $\frac{dy}{dx}$

Second differential coefficient is $\frac{d^2y}{dx^2}$

Third differential coefficient is $\frac{d^3y}{dx^3}$

....

n^{th}differential coefficient of y $\frac{d^ny}{dx^n}$

Successive Differentiation,(CO2..)

Thus, if $y = f(x)$,the successive differential co-efficients of $f(x)$ are

$$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}, \dots \dots \quad \dots \frac{d^ny}{dx^n}$$

$$y_1, y_2, y_3, \quad \dots \dots \dots \dots y_n$$

$$y', y'', y''' \dots \dots \dots \dots y'^n$$

$$\mathrm{D}y, D^2y, D^3y \dots \dots \dots \dots D^ny$$

$$f'(x), f''(x), f'''(x) \dots \dots \dots \dots f'^n(x)$$

Successive Differentiation,(CO2..)

- **n^th derivative of some elementary functions-**

(1) If $y = e^{ax}$ then $y_n = a^n . e^{ax}$

(2) If $y = a^x$ then $y_n = a^x (\log a)^n$

(3) If $y = \dfrac{1}{ax+b}$ then $y_n = \dfrac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

(4) If $y = \log(ax+b)$ then $y_n = \dfrac{(-1)^{n-1} (n-1)! a^n}{(ax+b)^n}$

Successive Differentiation,(CO2..)

(5) If $y = \sin(ax+b)$ then $y_n = a^n \sin(ax+b+n\frac{\pi}{2})$

$(NIET\,2020-2021)$

(6) If $y = \cos(ax+b)$ then $y_n = a^n \cos(ax+b+n\frac{\pi}{2})$

(7) If $y = e^{ax}\sin(\mathrm{b}\,x+c)$ then

$$y_n = (a^2+\mathrm{b}^2)^{n/2}\ \mathrm{e}^{ax}\sin(\mathrm{b}\,x+c+n\tan^{-1}\frac{b}{a})$$

(8) If $y = e^{ax}\cos(\mathrm{b}\,x+c)$ then

$$y_n = (a^2+\mathrm{b}^2)^{n/2}\ \mathrm{e}^{ax}\cos(\mathrm{b}\,x+c+n\tan^{-1}\frac{b}{a})$$

Successive Differentiation,(CO2..)

(9) if $y=(ax+b)^m$

then $y_n = m(m-1)(m-2)(m-3).......(m-n+1)\,(\mathrm{ax+b})^{m-n}\,a^n$

Case (i) If m is a positive integer

$$y_n = \frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$$

Case (ii) If $\mathrm{m=n}$

then $y_n = n!\mathrm{a}^n$ = a constant

Case(iii) From case (ii)

$$y_{n+1}=0,\, y_{n+2}=0$$

$i.e\ y_n = 0$ when $n>m$

Successive Differentiation,(CO2..)

(10) If $y = (ax+b)^{-m}$ then

$$y_n = (-1)^n \frac{(m+n-1)!}{(m-1)!} (ax+b)^{-m-n} a^n$$

Click to add text
Click to add text
Click to add text

Successive Differentiation,(CO2..)

$$Q.1.\ \text{If } y=\frac{1}{1-5x+6x^2}, \text{find } y_n$$

$$Ans:(-1)^n n!\left[\frac{2^{n+1}}{(2x-1)^{n+1}}-\frac{3^{n+1}}{(3x-1)^{n+1}}\right]$$

$$Q.2.\ \text{If } y=\frac{2x+1}{(2x-1)(2x+3)}, \text{find } y_n$$

$$Ans:(-1)^n n!2^{n-1}\left[\frac{1}{(2x-1)^{n+1}}+\frac{1}{(2x+3)^{n+1}}\right]$$

$$Q.3.\ \text{If } y=\frac{ax+b}{cx+d}, \text{find } y_n$$

$$Ans:\frac{bc-ad}{c}\left[\frac{(-1)^n n!c^n}{(cx+d)^{n+1}}\right]$$

Successive Differentiation,(CO2..)

$$Q.4.\ If\ \ y = x\log(1+x),\ \text{Prove that}\ y_n = (-1)^{n-2}(n-2)!\left[\frac{(x+n)}{(x+1)^n}\right]$$

$$Q.5.\ If\ \ y = \sin \mathrm{p}\, x + \cos \mathrm{p}\, x, \text{Prove that}\ y_n = p^n[1+(-1)^n \sin 2px]^{1/2}$$

Hence show that $\mathrm{y}_8(\pi) = (1/2)^{31/2}$

where $p = 1/4$.

$$\text{Hint: } y_n = p^n[\sin(\mathrm{p}\, x + \frac{n\pi}{2}) + \cos(\mathrm{p}\, x + \frac{n\pi}{2})]$$

$$= p^n[\{\sin(\mathrm{p}\, x + \frac{n\pi}{2}) + \cos(\mathrm{p}\, x + \frac{n\pi}{2})\}^2]^{1/2}$$

$$= p^n[1 + 2\sin(\mathrm{p}\, x + \frac{n\pi}{2})\cos(\mathrm{p}\, x + \frac{n\pi}{2})]^{1/2}$$

Successive Differentiation,(CO2..)

- Q.6. y_n if $y = \frac{x^n - 1}{x-1}$.
- Q.7. The (n-1)th derivative of x^n is........
- Q.8. $y_n = \frac{d^n}{dx^n}(x^n logx)$ then prove that $y_n = ny_{n-1} + (n-1)$

  And Hence show that $y_n = n!\,(logx + 1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n})$
- Q.9. $y = e^x sin^2 x$ then find y_n.
- Q.10. If $y = xlog\frac{x-1}{x+1}$,

  show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

Leibnitz theorem,(CO2..)

This helps us to find the nth differential co-efficient of the product of two functions in terms of the successive derivatives of the functions.

Statement:

If u and v are two functions of x, having derivatives of the n^{th} order, then

$$\frac{d^n}{dx^n}(u\ v) = u_n\ v + {}^nc_1 u_{n-1}\ v_1 + {}^nc_2 u_{n-2}\ v_2 + {}^nc_3 u_{n-3}\ v_3 +$$

$$\ldots\ldots + {}^nc_r u_{n-r}\ v_r + \ldots\ldots + {}^nc_n u\ v_n\ ,$$

where suffixes of u and v denote the differentiation w.r.t. x.

Leibnitz theorem,(CO2..)

Q.1. Find the n$^{\text{th}}$ derivative of $x^3 \cos x$.

$$y = x^3 \cos x$$

Differentiating n times by Leibnitz theorem

$$\mathrm{D}^n y = (D^n \cos x).x^3 +{}^n C_1(D^{n-1} \cos x).Dx^3 +{}^n C_2(D^{n-2} \cos x).D^2x^3$$

$$+{}^n C_3(D^{n-3} \cos x).D^3x^3$$

$$= x^3 \cos(x + n\pi/2) + 3nx^2 \cos(x + (n-1)\pi/2) +$$

$$6x\frac{n(n-1)}{2!}\cos(x+(n-2)\pi/2) + 6\frac{n(n-1)(n-2)}{3!}\cos(x+(n-3)\pi/2)$$

$$= x^3 \cos(x + n\pi/2) + 3nx^2 \cos(x + (n-1)\pi/2) + 3xn(n-1)\cos(x+(n-2)\pi/2)$$

$$+ n(n-1)(n-2)\cos(x+(n-3)\pi/2).$$

Leibnitz theorem,(CO2..)

Q.2. If $y = \sin(m\sin^{-1} x)$, *prove that*

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$$

$$y_1 = \cos(m\sin^{-1} x)\frac{m}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,y_1 = m\cos(m\sin^{-1} x)$$

squaring both sides

$$(1-x^2)y_1^2 = m^2\cos^2(m\sin^{-1} x) = m^2(1-\sin^2(m\sin^{-1} x))$$

$$(1-x^2)y_1^2 = m^2(1-y^2)$$

again differentiating w.r.t. x

Leibnitz theorem,(CO2..)

Differentiating n times by Leibnitz theorem

$$(1-x^2)y_{n+2}+{}^nC_1(-2x)y_{n+1}+{}^nC_2(-2)y_n-(xy_{n+1}+{}^nC_1y_n)+m^2y_n=0.$$

$$(1-x^2)y_{n+2}+n(-2x)y_{n+1}+\frac{n(n-1)}{2!}(-2)y_n-(xy_{n+1}+ny_n)+m^2y_n=0.$$

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0.$$

NIET Greater Noida

Leibnitz theorem,(CO2..)

Q.3. Determine $y_n(0)$ where $y = e^{m\cos^{-1}x}$. (1)

$$y_1 = \frac{-my}{\sqrt{1-x^2}} \qquad (2)$$

$$(1-x^2)y_2 - xy_1 = m^2 y. \qquad (3)$$

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0. \qquad (4)$$

On putting $x = 0$ in above equations

$$y(0) = e^{m\pi/2}; \qquad y_1(0) = -me^{m\pi/2}$$

$$y_2(0) = m^2 e^{m\pi/2};$$

$$y_{n+2}(0) = (n^2+m^2)y_n(0) \qquad (5)$$

Leibnitz theorem,(CO2..)

$$\text{From equation (5)}, \ for\ n = 1, 2, 3, 4..........$$

$$y_3(0) = (1^2 + m^2)y_1(0) = -m(1^2 + m^2)e^{\frac{m\pi}{2}}$$

$$y_4(0) = (2^2 + m^2)y_2(0) = m^2(2^2 + m^2)e^{\frac{m\pi}{2}}$$

$$y_5(0) = (1^2 + m^2)y_3(0) = -m(1^2 + m^2)(3^2 + m^2)e^{\frac{m\pi}{2}}$$

$$y_6(0) = (2^2 + m^2)y_2(0) = m^2(2^2 + m^2)(4^2 + m^2)e^{\frac{m\pi}{2}}$$

Leibnitz theorem,(CO2..)

when n is even

$$y_n(0) = m^2(2^2 + m^2)(4^2 + m^2)\ldots\ldots\ldots\ldots\left((n-2)^2 + m^2\right)e^{\frac{m\pi}{2}}$$

when n is odd

$$y_n(0) = -m(1^2 + m^2)(3^2 + m^2)\ldots\ldots\ldots\ldots\left((n-2)^2 + m^2\right)e^{\frac{m\pi}{2}}$$

Leibnitz theorem,(CO2..)

Q.4. If $y=(\sin^{-1}x)^2$, find $y_n(0)$.

Q.5. If $y=(x+\sqrt{1+x^2})^m$, find $y_n(0)$.

Q.6. If $y=\sin(a\sin^{-1}x)$, find $y_n(0)$.

Recap

In this chapter, we discussed the following points

1. **Successive Differentiation** i.e.

The process of finding the differential coefficient of a function again and again is called successive differentiation.

2. **Leibnitz theorem i.e.**

$$\frac{d^n}{dx^n}(u\ v) = u_n\ v + {}^nc_1 u_{n-1}\ v_1 + {}^nc_2 u_{n-2}\ v_2 + {}^nc_3 u_{n-3}\ v_3 +$$

$$\ldots\ldots + {}^nc_r u_{n-r}\ v_r + \ldots\ldots + {}^nc_n u\ v_n\ ,$$

Assignment-2.1

Q.1. Find y_n, where $y = \frac{ax+b}{cx+d}$

Q.2. Find nth derivative of $y \ = \ x^2 e^x$at x = 0.

Q.3. If $y = x^n logx$, prove that $y_{n+1} = \frac{n!}{x}$ (NIET-2020-2021)

Weekly Assignment,co2

Assignment-2.2

Q.1. If $y = sin(asin^{-1}x)$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - a^2)y_n = 0$.

Q.2. If $y = (sin^{-1}x)^2$then findy_n (0).

Q.3. If $y = e^{mcos^{-1}x}$Show that
$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ *and calculate* $y_n(0)$.

Q.4. If $y = \left[x + \sqrt{1 + x^2}\right]^m$ *find* $y_n(0)$.

Q.5. If $x = sin\sqrt{y}$ *find* $y_n(0)$.

MCQ s

Q.1. If $y = (ax + b)^m$ then y_n at$n = \mathrm{m}$

(i) $n!\,a^n$ (ii) $n!\,n^n$ (iii) $na!\,a^n$ (iv) 0

Q.2. If$y = sin3x.sin2x$ then y_n

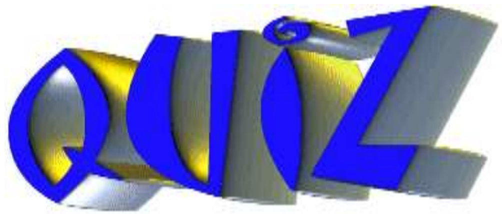
(i) $\frac{1}{2}\left[cos\left(x + \frac{n\pi}{2}\right) - 5^n cos\left(5x + \frac{n\pi}{2}\right)\right]$

(ii) $\frac{1}{2}\left[cos\left(x + \frac{n\pi}{2}\right) + 5^n cos\left(5x + \frac{n\pi}{2}\right)\right]$

(iii)$\frac{1}{2}\left[cos\left(x - \frac{n\pi}{2}\right) - 5^n cos\left(5x - \frac{n\pi}{2}\right)\right]$

(iv) $\frac{1}{2}\left[cos\left(5x + \frac{n\pi}{2}\right) - 5^n cos\left(5x + \frac{n\pi}{2}\right)\right]$

Leibnitz theorem,(CO2..)



Q.1. n[th]derivative of $y = \frac{1}{ax+b}$.

Q.2 n[th] derivative of $y = \log(ax + b)$

Q.3.4[th]derivative of $y = x^3$

Q.4.write n[th]term of Leibnitz's theorem

Leibnitz theorem,(CO2..)

FAQ

Q.1.If $y = a.\cos(logx) + b.\sin(logx)$ then prove that
$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$

Q.2.If $(y)^{\frac{1}{m}}+(y)^{\frac{-1}{m}}= 2x$, prove that
$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0.$

Q.3.If $y = x^n logx$ prove that (NIET2020-2021)

(i) $y_{n+1} = \frac{n!}{x}$

(ii) $x^2y_{p+2} + (2p-2n+1)xy_{p+1} + (p-n)^2y_p = 0.$

Expected Questions for University Exam

- **Q.1.** Find the n^{th} derivative of $x^2 \sin x$ at $x = 0$. **Ans:** $(n - n^2) \sin \frac{n\pi}{2}$

Q.2. If $y = e^{-x} cosx$, then find the value of $y_4 + 4y$. **Ans:** 0

Q.3. Find the 8^{th} derivative of $x^2 e^x$. **Ans:** $e^x (x^2+16x+56)$

Q.4. If $x = sin\left(\frac{logy}{a}\right)$,
prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0.$

Q.5. If $y = (x^2 - 1)^n$,

prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0$. Hence

if $P_n = \frac{d^n}{dx^n}(x^2 - 1)^n, show\ that\ \frac{d}{dx}\left\{(1 - x^n)\frac{dP_n}{dx}\right\} + n(n + 1)P_n = 0.$

Expected Questions for University Exam

Q.6. If $y^{1/m} + y^{-1/m} = 2x$,

Prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

Q.7. If $y = x^n \log x$, prove that

a. $y_{n+1} = \frac{n!}{x}$.

b. $x^2 y_{p+2} + (2p - 2n + 1)xy_{p+1} + (p-n)^2\ y_p = 0$.

Q.8. If $y = \left[x + \sqrt{1+x^2}\,\right]^m, find\ y_n(0)$.

Ans: $\begin{cases} When\ n\ is\ even, \{m^2(m^2-2^2)(m^2-4^2)\ \ldots\ldots\ldots\ [m^2-(n-2)]^2\} \\ When\ n\ is\ odd, \{m(m^2-1^2)(m^2-3^2)\ \ldots\ldots\ldots\ [m^2-(n-2)]^2\} \end{cases}$

ASSIGNMENT-2.1

..\Unit-2 (Differential Calculus-I , Assig.-2.1).docx

Successive Differentiation(nth order derivatives)-

https://www.youtube.com/watch?v=PkuPGKSacu0&list=PL2FUpm_Ld1Q3H00wVFuwjWOo1gtMXk1eb

Leibnitz's theorem and its application-

1.https://www.youtube.com/watch?v=QeWrQ9Fz3Wo&t=22s

2. https://www.youtube.com/watch?v=5dFrWCE6bHg

Asymptotes-

https://www.youtube.com/watch?v=WX6O9TiFYsA&t=110s

Objective of the topic,(CO2..)

Asymptotes and Curve Tracing

An **asymptote** of a **curve** is a line such that the distance between the **curve** ... information about the behavior of curves in the large, and determining the **asymptotes** of a function is an important step in **sketching** its graph.

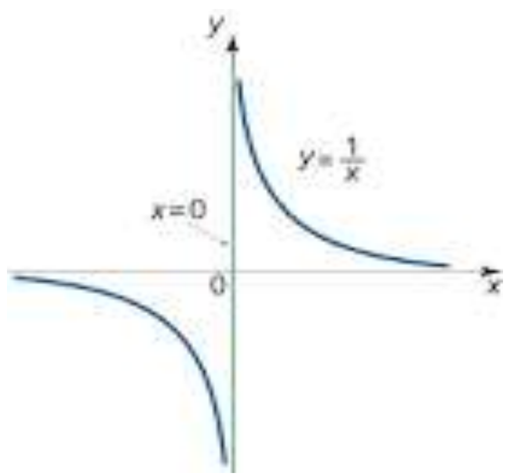
Asymptotes,(CO2..)

➢ Asymptotes-

An asymptotes of a curve is a line such that the distance between the curve and the line approaches zero as one or both of the X or Y coordinates tends to infinity.

OR

Asymptotes is a straight line which touches the curve $y = f(x)$ at infinity.



Asymptotes,(CO2..)

Working Rule-

1. Let $f(x, y) = 0$, be the given curve of nth degree.

2. Put $y = mx + c$, into $f(x, y) = 0$ and simplify $(x = 1, y = m)$.

3. Put the coefficient of x^n, x^{n-1} and x^{n-2} to zero. Now name them as $\emptyset_n(m)$, $\emptyset_{n-1}(m)$, $\emptyset_{n-2}(m)$ respectively.

4. Put $\emptyset_n(m) = 0$ and solve for m, say $m = m_1, m_2, \ldots m_n$ are its roots.

Asymptotes,(CO2..)

5.Find c by using the formula$c = -\frac{\emptyset_{n-1}(m)}{\emptyset'_n(m)}$, (provided $\emptyset'_n(m) \neq 0$).

6.Using the formula, we find c_1,c_2.....c_n.

7.Put the value of m_1, m_2,...m_n and c_1, c_2..... $c_n$in$y = mx + c$ then we get all asymptotes of the given curve$y = f(x)$.

Hence the asymptotes are

$y = m_1 x + c_1$,$y = m_2 x + c_2$,........, $y = m_n x + c_n$

Asymptotes,(CO2..)

Q.1.Find the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$$

Solution- we get

$$\emptyset_3(m) = 1 + 2m - m^2 - 2m^3 = 0$$

$$\Longrightarrow m = -\frac{1}{2}, 1, -1$$

And $$\emptyset_3'(m) = 2 - 2m - 6m^2$$

And $$\emptyset_2(m) = m - m^2$$

Therefore $$c = -\frac{\emptyset_{n-1}(m)}{\emptyset_n'(m)}$$

$$c = -\frac{m - m^2}{2 - 2m - 6m^2}$$

Asymptotes,(CO2..)

$c = \frac{m^2 - m}{2 - 2m - 6m^2}$ then

If $m_1 = -\frac{1}{2}$ and $c_1 = \frac{1}{2}$

If $m_2 = 1$ and $c_2 = 0$

If $m_3 = -1$ and $c_3 = -1$

Therefore, the asymptotes are

$$y = m_1 x + c_1 \Rightarrow y = -\frac{1}{2}x + \frac{1}{2}$$
$$y = m_2 x + c_2 \Rightarrow y = x$$
$$y = m_3 x + c_3 \Rightarrow y = -x - 1$$

Procedure for Tracing Curves having Cartesian Equation-

- **I. Symmetry-**

1. If the equation of the curve contains only even powers of y then curve is symmetrical about the X axis.

Ex. $y^2 = 4ax$

2. If the equation of the curve contains only even powers of x then curve is symmetrical about the Y axis.

Ex. $x^2 = 4ay$

3. If the equation of the curve contains even powers of both x and y then curve is symmetrical about the both axis.

Ex. $x^2 + y^2 = a^2$

Curve Tracing,(CO2..)

4. If the equation of the curve remains unchanged when x is changed to y and y is changed to x then curve is symmetrical about the line $y = x$

Ex. $x^3 + y^3 = 3axy$

5. If the equation of the curve remains unchanged when x is changed to -y and y is changed to -x then curve is symmetrical about the line $y = -x$

Ex. $x^4 + y^4 = 3a^2xy$

6. If the equation of the curve remains unchanged when x is changed to -x and y is changed to -y then curve is symmetrical about in opposite quadrants.

Ex. $x^5 + y^5 = 5ax^2y$

- **II.Origin-**

If the constant term is missing from the equation of the curve then it passes through the origin.

- **IF CURVE PASSES THROUGH THE ORIGIN THEN**

Find the tangents at origin for this we equate to zero the lowest degree terms appear in equation.

1. If the two tangents are real and distinct then origin is NODE.
2. If the two tangents are real and coincide then origin is CUSP.
3. If the two tangents are conjugate (or isolated) then origin are IMAGINARY.

NOTE-

1. A cusp is called a single cusp or a double cusp according as the two branches of the curve lie entirely on one side or both sides of the common normal.

2. A cusp single or double is said to be of first kind or second kind according as the two branches of the curve, lie on opposite or same side of the common tangent.

- **III. Asymptotes-**

Find all the asymptotes of the curve. The curve will not go beyond its asymptotes.

1.Asymptotes parallel to x-axis-

We equate to zero the coefficients of the highest power of x . If the coefficients of the highest power of x is constant then there is no asymptotes parallel to x-axis.

2.Asymptotes parallel to y-axis-

We equate to zero the coefficients of the highest power of y. If the coefficients of the highest power of y is constant then there is no asymptotes parallel to y-axis.

3.Oblique asymptotes- write the equation of the form

$y = mx + c + \frac{A}{x} + \frac{B}{x^2} + \cdots$, then $y = mx + c$ is the asymptote to the curve.

- **IV. Points of intersections-** Find intersections of the curve

(i)With the x-axis (put y = 0)

(ii)With the y-axis (put x = 0)

(iii)With the line y = x particularly if the curve is symmetrical aboutit.

(iv) With the line y = - x particularly if the curve is symmetrical about it.

(v) With the asymptotes (if necessary)

Curve Tracing,(CO2..)

- Note-

If curve cuts the axis at the point (a,0) then find the equation of the tangent at the point (a,0).

For this we put x = X + a and y = Y + 0 in given equation of curve and equate to zero the lowest degree terms.

- V. Region-Find regions in the four quadrants to which the curve is limited.
- VI. **Result-** Considering all these points ,found the rough shape of curve .

Curve Tracing,(CO2..)

Q.1.Trace the curve $y^2(2a - x) = x^3$.

Solution- Given equation of curve$y^2 2a - y^2 x = x^3$

(i)Symmetry-Clearly the given curve is only even power of y. Therefore curve is symmetrical about x-axis.

(ii)Position of origin-Clearly the given curve passes through origin (0,0) then equation of tangent at origin (0,0),for this we equate to zero the lowest degree term i.e.

$$2ay^2 = 0 \text{ i.e.} y = 0,0$$

$\Longrightarrow$ there are two real tangent but identical so origin is a cusp.

Curve Tracing,(CO2..)

(iii) Asymptotes- Asymptotes parallel to y-axis i.e.

$2a - x = 0 \implies x = 2a$,thus$x = 2a$ is only real Asymptotes of the curve.

(iv) Intersection with the axes- Clearly curve does not intersect axes any where except at origin.

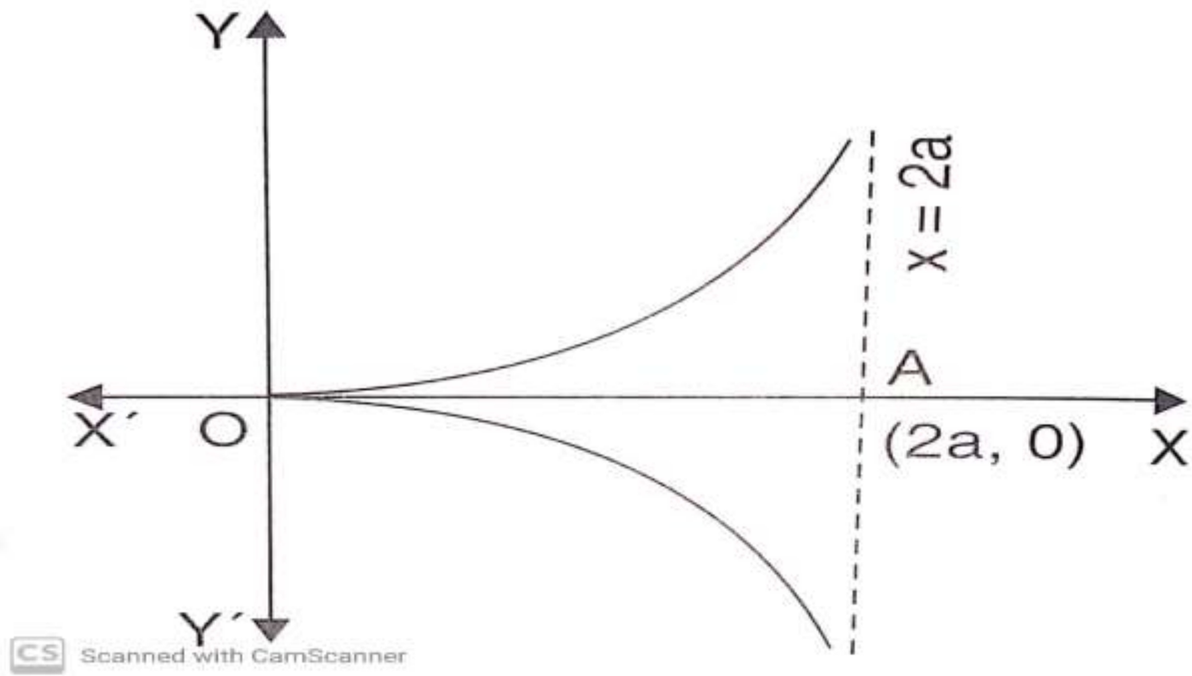
(v)Region- We have $y = \sqrt{\frac{x^3}{(2a-x)}}$

(a) When $x < 0, y$ is imaginary therefore no portion of the curve lies to the left of the line $x = 0$ i.e. y axis.

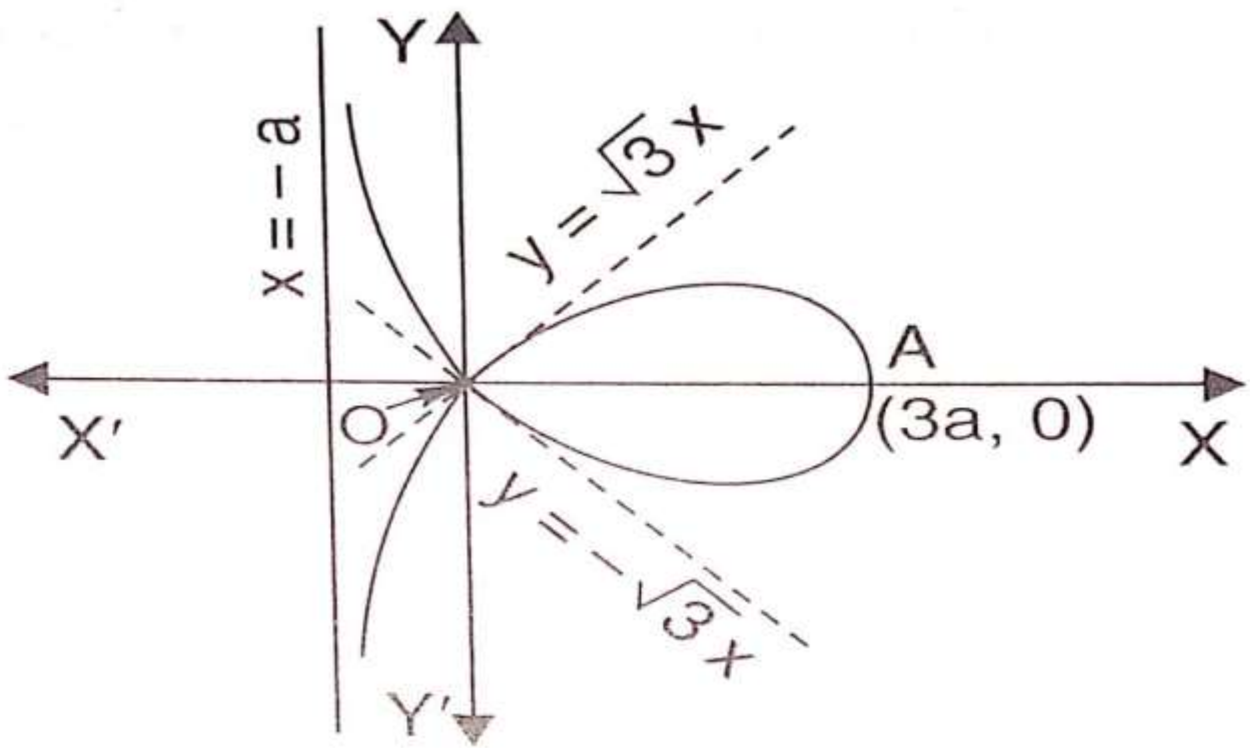
(b) When $0 < x < 2a$,y is real.

(c) When $x > 2a$,y is imaginary therefore no portion of the curve lies to the right of the line $x = 2a$.

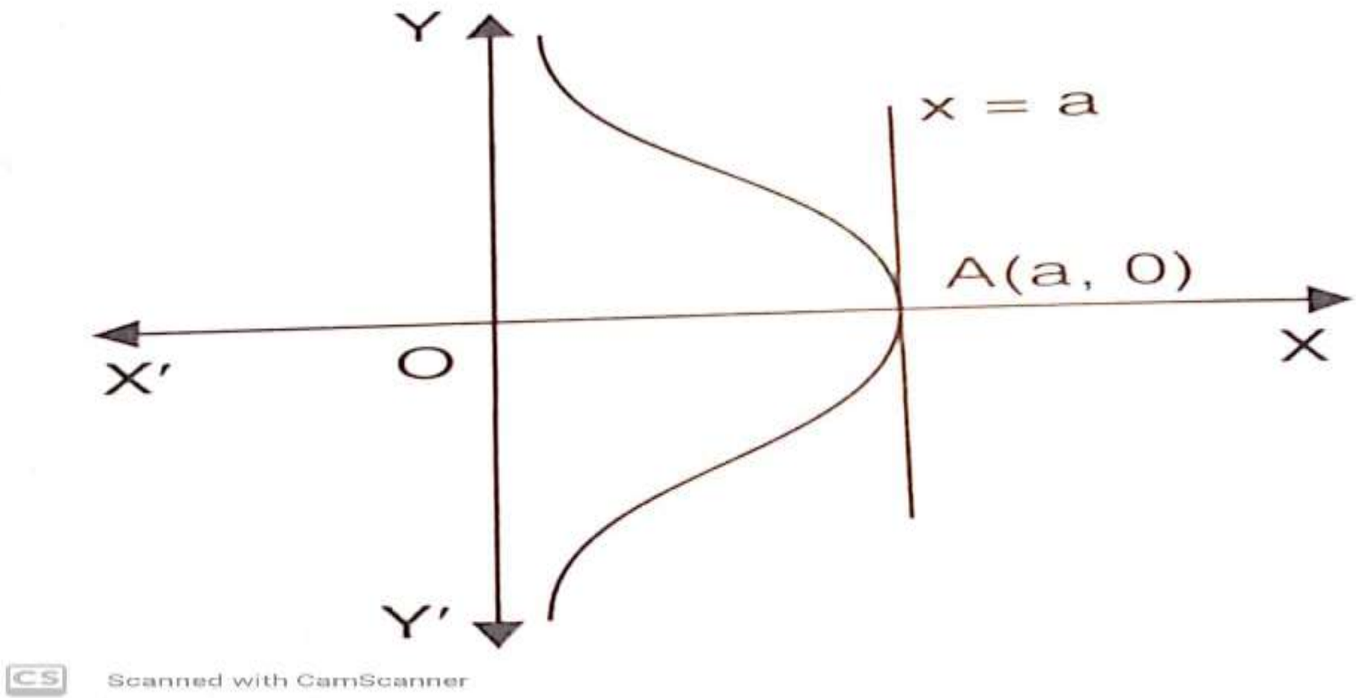
(vi) Result- Considering all these points ,found the rough shape of curve is as shown in figure.


Y
x = 2a
A
X´
O
(2a, 0)
X
Y´
Scanned with CamScanner

Q.2. Trace the curve $y^2(a+x) = x^2(3a-x)$.



Q.3. Trace the curve $xy^2 = a^2(a - x)$.



Procedure for Tracing Curves having Cartesian Equation-

1.Symmetry-

(i) The equation of the curve does not change by changing the sign of θ,curve is symmetrical about the initial line i.e. x-axis.

(ii) The equation of the curve does not change by putting$r = -r$, curve is symmetrical about the pole.

(iii) The equation of the curve does not change by putting

$\theta = \pi - \theta$ curve is symmetrical about the line$\theta = \frac{\pi}{2}$ i.e. y-axis.

(iv) The equation of the curve does not change by putting

$\theta = \frac{\pi}{2} - \theta$ curve is symmetrical about the lineθ=$\frac{\pi}{4}$ i.e. y = x line.

Curve Tracing(CO2..)

2.Pole or Origin- Find whether the curve passes through the pole or not.

For this we put r = 0 then we get some real value of θ,then curve passes through the pole.

- (i) Find the tangents at pole-

For this we put r = 0, the real value of θ gives the tangents at the pole.

- (ii)Find the points ,where the curve cuts the initial line and the line$\theta = \frac{\pi}{2}$.

3.Asymptotes-If r$\rightarrow \infty$ as$\theta \rightarrow \theta$, then there is an asymptote . We find all the asymptote of the curve.

4.Points of intersection-

Find some points on the curve for convenient values ofθ.

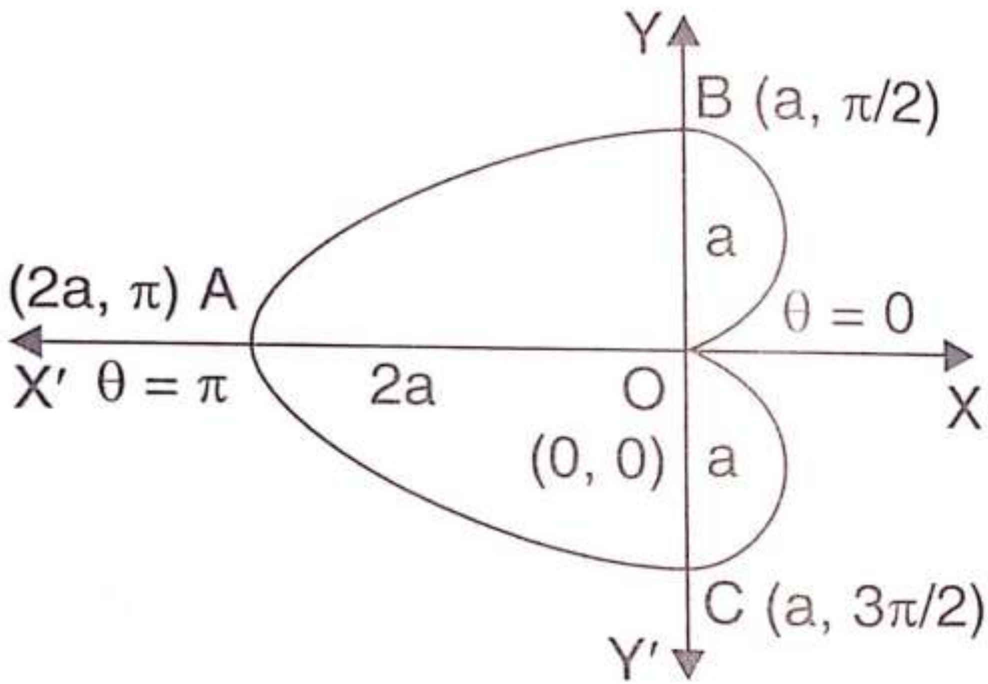
5.Region-

Solve the given equation for r and θ(if possible).Find the region in which the curve does not lie.

6.**Result-**

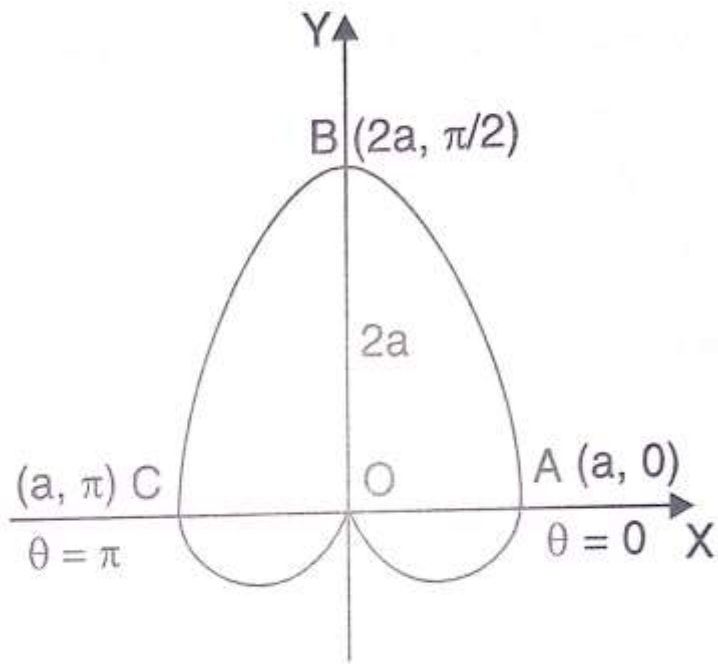
Considering all these points ,found the rough shape of curve .

Q.1. Trace the curve $r = a(1 - cos\theta)$.(Cardioid)



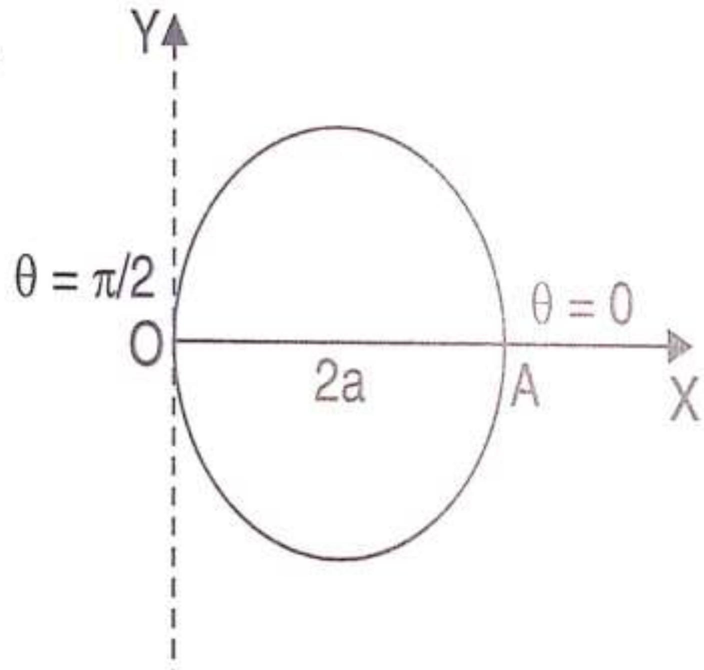
Curve Tracing(CO2..)

Q.2. Trace the curve $r = a(1 + cos\theta)$.(Cardioid)

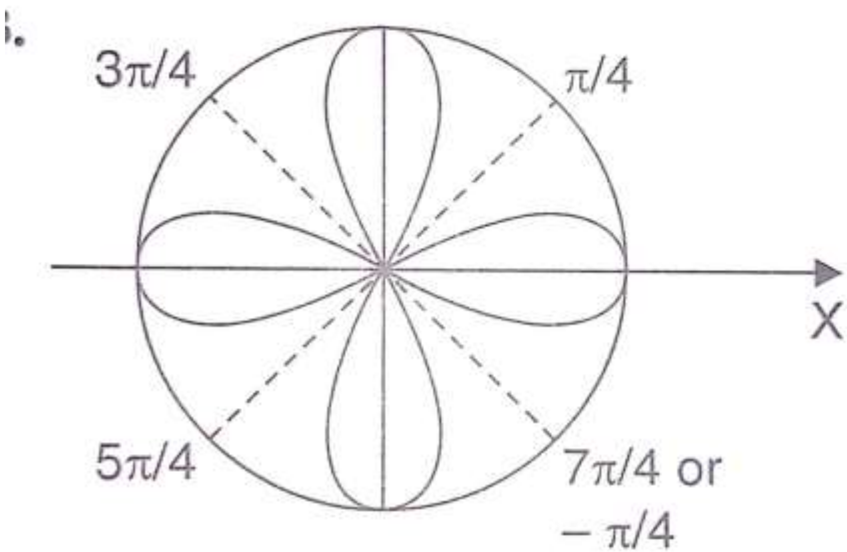


Curve Tracing(CO2..)

Q.3. Trace the curve $r = 2a\ cos\theta$.(Circle)



Q.4. Trace the curve $r = a\ cos2\theta$.



Recap

In this chapter, we discussed the following points

1. Asymptotes i.e. Asymptotes is a straight line which touches the curve $y = f(x)$ at infinity.

2. Curve Tracing **i.e.**

(i) Tracing Curves in Cartesian co-ordinates

(ii) Tracing Curves in Polar co-ordinates

Weekly Assignment,co2

Assignment-2.3

Q.1.Find the asymptotes of the curve , $x^3 + y^3 = 3axy$

Ans- $x + y + a = 0$ is a real asymptote.

Q.2. Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. (NIET2020-2021)

Q.3.Trace the curve $y^2(x - a) = x^2(x + a)$

Q.4. Trace the curve $r = a(1 + sin\theta)$.(Cardioid)

Q.5.Trace the curve : $y^2(a + x) = x^2(3a - x)$.

MCQ s

Q.1.Curve is symmetrical about x-axis then

(i)Even power of x (ii) Even power of y(iii) Even power of both

(iv) Odd power of x.

Q.2. Curve is symmetrical about y-axis then

(i)Even power of x (ii) Even power of y(iii) Even power of both

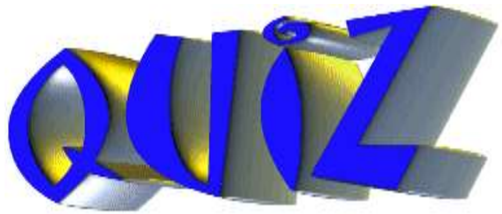
(iv) Odd power of x.

Q.3. Curve is symmetrical about both axes then

(i)Even power of x (ii) Even power of y(iii) Even power of both

(iv) Odd power of x.

Curve Tracing(CO2..)



Q.1. If even power of y then curve is symmetrical.....

Q.2. If even power of x then curve is symmetrical.....

Q.3. If highest power of x are constant then the Asymptote....

FAQ

Q.1. Find the asymptotes of the curve

$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0.$

Ans- $2xy + y = 0, x - y + 1 = 0$ and $x + y + 1 = 0$

Q.2. Trace the curve $y^2(a - x) = x^2(a + x)$.

Q.3. Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. (NIET2020-2021)

Expected Questions for University Exam

Q.1. Find the asymptotes of the curve , $x^3 + y^3 = 3axy$

Ans- $x + y + a = 0$ is a real asymptote.

Q.2. Trace the curve $r = a(1 + sin\theta)$.(Cardioid)

Q.3. Trace the curve $y^2(2a - x) = x^3$.

ASSIGNMENT-2.2

[..\Unit-2 (Differential Calculus-I, Assign.-2.2).docx]

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details

Curves Tracing (Cartesian co-ordinates) –
https://www.youtube.com/watch?v=GIl1ssdR2cg&list=PLhSp9OSVmeyK2yt8hdoo3Qze3O0Y67qaY

Curves Tracing (Polar co-ordinates) -
https://www.youtube.com/watch?v=0Nv_uSvwh_I&t=3s

Topic Objective

Partial derivatives

- Student will be able to understand partial derivatives and solve problems related to partial derivatives and concepts of homogeneous function and apply the Euler's theorem.

- It is useful to study the mathematical theorem that establishes a relationship between a homogeneous function and its partial derivatives.

Partial derivatives (CO2)

Function of two variables:

➢If three variables *x, y, z* are so related that value of *z* depends upon the values of *x* and *y,* then z is called function of *x* and *y* and is denoted as

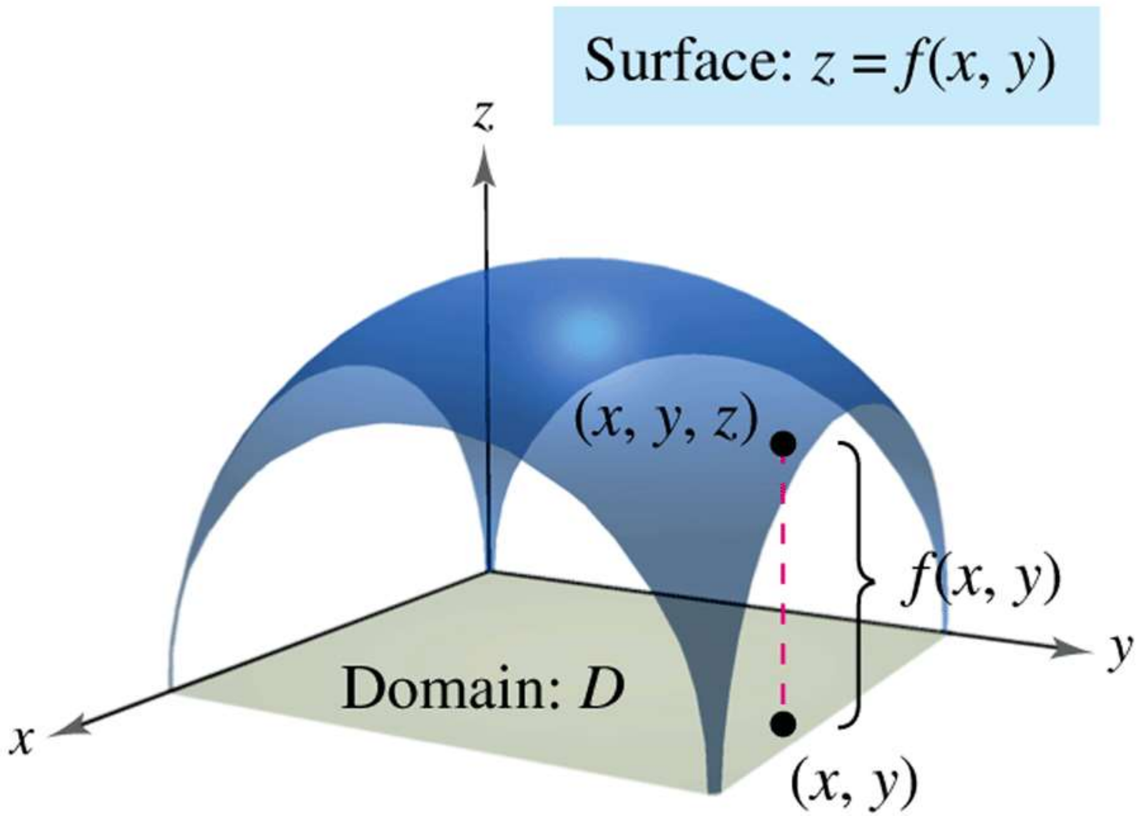
$$z = f(x, y)$$

➢The set of points (*x, y*) in the *x-y* plane for which *f(x, y)* is defined is called the domain of *f(x, y)* and denoted as D.

➢The domain may be entire *x-y* plane or a part of the *x-y* plane

➢The collection of corresponding values of z is called the range of *f(x, y)*

Geometrical Interpretation (CO2)



First Order Partial Derivatives (CO2)

- Let z = f(x, y) be a function of two independent variables x and y.

- The partial derivative of z with respect to x treating y as constant is denoted as

$$\frac{\partial z}{\partial x} \text{ or } \frac{\partial f}{\partial x} \text{ or } z_x \text{ or } f_x$$

$$\frac{\partial z}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

First Order Partial Derivatives (CO2)

- Let z = f(x, y) be a function of two independent variables x and y.

- The partial derivative of z with respect to y treating x as constant is denoted as

$$\frac{\partial z}{\partial y} \text{ or } \frac{\partial f}{\partial y} \text{ or } z_y \text{ or } f_y$$

$$\frac{\partial z}{\partial y} = \lim_{k \to 0} \frac{f(x, y+k) - f(x, y)}{k}$$

Higher Derivatives (CO2)

- If $z = f(x, y)$, then for second order partial derivatives we use the following notation:

$$f_{xx} = \frac{\partial}{\partial x}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$

$$f_{xy} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 z}{\partial y \partial x}$$

$$f_{yx} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y}$$

$$f_{yy} = \frac{\partial}{\partial y}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$

Mixed Partial Derivatives (CO2)

Clairaut's Theorem Suppose f is defined on a disk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are both continuous on D, then

$$f_{xy}(a, b) = f_{yx}(a, b)$$

- Partial derivatives of order 3 or higher can also be defined.
- For instance,and using Clairaut's Theorem we can show that $f_{xyy} = f_{yxy} = f_{yyx}$ if these functions are continuous.

$$f_{xyy} = (f_{xy})_y = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y\,\partial x}\right) = \frac{\partial^3 f}{\partial y^2\,\partial x}$$

Partial derivatives (CO2)

Q.1. $z = \sqrt{1 - x^2 - y^2}$; z is real.

- The domain is $(1 - x^2 - y^2) \geq 0 \Rightarrow x^2 + y^2 \leq 1$

  i.e. all x, y such that $x^2 + y^2 \leq 1$ is domain.

- The range is set of all real positive numbers.

Partial derivatives (CO2)

Q.2. If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2, 1)$ and $f_y(2, 1)$.

Solution: Holding *y* constant and differentiating with respect to *x*, we get

$$f_x(x, y) = 3x^2 + 2xy^3$$

$$f_x(2, 1) = 3 \cdot 2^2 + 2 \cdot 2 \cdot 1^3 = 16$$

Holding *x* constant and differentiating with respect to *y*, we get

$$f_y(x, y) = 3x^2y^2 - 4y$$

$$f_y(2, 1) = 3 \cdot 2^2 \cdot 1^2 - 4 \cdot 1 = 8$$

Partial derivatives (CO2)

Q.3. If $f(x,y)=\sin\left(\dfrac{x}{1+y}\right)$, calculate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

Solution: Using the Chain Rule for functions of one variable, we have

$$\frac{\partial f}{\partial x}=\cos\left(\frac{x}{1+y}\right).\frac{1}{1+y}$$

$$\frac{\partial f}{\partial y}=\cos\left(\frac{x}{1+y}\right).\left[-\frac{x}{(1+y)^2}\right]$$

Partial derivatives (CO2)

Q.4. Calculate f_{xxyz} $\;if\;$ $f(x, y, z) = sin(3x + yz)$.

Solution:

$$f_x = 3\cos(3x + yz)$$

$$f_{xx} = -9\sin(3x + yz)$$

$$f_{xxy} = -9z\cos(3x + yz)$$

$$\boxed{f_{xxyz} = -9\left[\cos(3x + yz) - yz\sin(3x + yz)\right]}$$

Partial derivatives (CO2)

Q.5. Find the second partial derivatives of $f(x,y) = x^3 + x^2y^2 - 2y^2$

Solution:

$$f_x(x,y) = 3x^2 + 2xy^3 \qquad f_y(x,y) = 3x^2y^2 - 4y$$

Therefore

$$f_{xx}(x,y) = 6x + 2y^3$$

$$f_{yy}(x,y) = 6x^2y - 4$$

$$f_{xy}(x,y) = 6xy^2$$

$$f_{yx}(x,y) = 6xy^2$$

Partial derivatives (CO2)

Q.6. if $u=\left(x^2+y^2+z^2\right)^{-1/2}$, Then evaluate $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}$

Solution: $\dfrac{\partial u}{\partial x}=-\dfrac{1}{2}\left(x^2+y^2+z^2\right)^{-3/2}.2x$

$$\Rightarrow \frac{\partial u}{\partial x}=-x\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2}=(-1)\left(x^2+y^2+z^2\right)^{-3/2}+(-x)\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}.2x$$

$$=-\left(x^2+y^2+z^2\right)^{-3/2}+3x^2\left(x^2+y^2+z^2\right)^{-5/2}$$

Partial derivatives (CO2)

$$\frac{\partial^2 u}{\partial y^2} = -\left(x^2+y^2+z^2\right)^{-3/2} + 3y^2\left(x^2+y^2+z^2\right)^{-5/2}$$

$$\frac{\partial^2 u}{\partial z^2} = -\left(x^2+y^2+z^2\right)^{-3/2} + 3z^2\left(x^2+y^2+z^2\right)^{-5/2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -3\left(x^2+y^2+z^2\right)^{-3/2} + 3(x^2+y^2+z^2)\left(x^2+y^2+z^2\right)^{-5/2}$$

$$= -3\left(x^2+y^2+z^2\right)^{-3/2} + 3\left(x^2+y^2+z^2\right)^{-3/2} \qquad = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Homogeneous Function (CO2)

- Consider the function

$$f(x,y) = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \dots\dots\dots + a_n y^n$$

The degree of each term in x and y is n.

- A function $f(x, y)$ of two independent variables x and y is said to be homogenious of degree n if $f(x, y)$ can be written in the form $x^n \phi\left(\frac{y}{x}\right)$ or $y^n \phi\left(\frac{x}{y}\right)$

- Examples

(1) $F(x, y) = x^n \sin(\frac{y}{x})$ (2) $F(x, y) = x^3 - 3xy^2 + y^3$

(3) $F(x, y) = \dfrac{\left(\sqrt{y} - \sqrt{x}\right)}{y - x}$

Euler's Theorem on Homogeneous Function (CO2)

- If $z = F(x, y)$ be a homogeneous function of x, y

of degree n in x and y then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = nz$.

(NIET2020-2021)

- Corollary

  If z = $f(x, y)$ is a homogeneous function of x and y of degree n,

  then $$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

Deductions From Euler's Theorem (CO2)

If $f(u) = V(x, y)$ be a homogeneous function of x, y of degree n then

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = n\dfrac{f(u)}{f'(u)}$.

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \varphi(u)[\varphi'(u) - 1]$

where $\varphi(u) = n\dfrac{f(u)}{f'(u)}$

Euler's Theorem (CO2)

Q.1. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{x}{y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Solution: Let $v = \sin^{-1}\frac{x}{y}$ and $w = \tan^{-1}\frac{x}{y}$

$$\therefore \qquad u = v + w$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \left(x\frac{\partial v}{\partial x} + x\frac{\partial w}{\partial x}\right) + \left(y\frac{\partial v}{\partial y} + y\frac{\partial w}{\partial y}\right)$$

$$= \left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) + \left(x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y}\right)$$

= 0 + 0 (since are homogenous functions of degree zero)

Euler's Theorem (CO2)

Q.2. If $u = \dfrac{x^2 y^2}{x+y}$, Then find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$

Solution:

$$u(tx,ty) = \frac{(tx)^2 (ty)^2}{tx+ty} = t^3 \frac{x^2 y^2}{x+y}$$

Here u is a homogenous functions of degree 3

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u = 3\frac{x^2 y^2}{x+y}$$

Euler's Theorem (CO2)

Q.3.

If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, $x \neq y$ then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \left(1-4\sin^2 u\right)\sin 2u$$

Solution: We have $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$

$$Let\ z = \frac{x^3+y^3}{x-y} \quad \text{then} \quad \tan u = z$$

$$\text{where z} = \frac{x^3+y^3}{x-y} = x^2\,\frac{1+\dfrac{y^3}{x^3}}{1-\dfrac{y}{x}}$$

is a homogeneous function of degree two.

Euler's Theorem (CO2)

By Eular's theorem ,we have $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y} = \text{n z} = 2\text{ z}$

but $\quad \frac{\partial z}{\partial x}=\sec^2 u\frac{\partial u}{\partial x} \quad$ and $\frac{\partial z}{\partial y}=\sec^2 u\frac{\partial u}{\partial y}$

$$\text{Also}\quad \frac{\partial^2 z}{\partial x^2}=\sec^2 u\frac{\partial^2 u}{\partial x^2}+2\ \sec^2 u\ \tan u\left(\frac{\partial u}{\partial x}\right)^2$$

$$\frac{\partial^2 z}{\partial y^2}=\sec^2 u\frac{\partial^2 u}{\partial y^2}+2\ \sec^2 u\ \tan u\left(\frac{\partial u}{\partial y}\right)^2$$

$$\frac{\partial^2 z}{\partial x\partial y}=\sec^2 u\frac{\partial^2 u}{\partial x\partial y}+2\ \sec^2 u\ \tan u\ \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}$$

Also by corollary of Eular's theorem,

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=2(2-1)z$$

Euler's Theorem (CO2)

$$\Rightarrow \sec^2 u\left(x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}\right)$$

$$+2\sec^2 u\ \tan u\left(x^2\left(\frac{\partial u}{\partial x}\right)^2 + 2xy\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y} + y^2\left(\frac{\partial u}{\partial y}\right)^2\right) = 2\tan u$$

$$\Rightarrow x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + 2\ \tan u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)^2 = 2\sin u\ \cos u$$

$$\Rightarrow x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u - 2\ \tan u \sin^2 2u$$

$$= \sin 2u\left(1 - 2\ \tan u \sin 2u\right)$$

$$= \sin 2u\left(1 - 4\sin^2 u\right)$$

Composite Function (CO2)

If $u = f(x, y)$ is a differentiable function of x and y, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then,

- u is said to be a composite function of t.

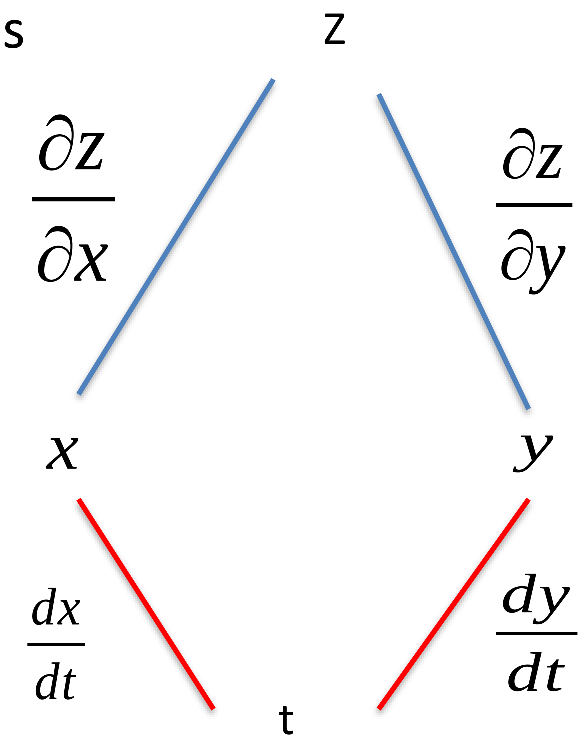
If $z = f(x, y)$ is a differentiable function of x and y, where $x = g(u,v)$ and $y = h(u,v)$ are both differentiable functions of u,v . Then,

- z is said to be a composite function of u,v.

Differentiation of Composite Function (CO2)

Chain Rule - Case 1

Suppose that *z* = *f*(*x*, *y*) is a differentiable function of *x* and *y*, where *x* = *g*(*t*) and *y* = *h*(*t*) are both differentiable functions of *t*. Then, *z* is a differentiable function of *t* and



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

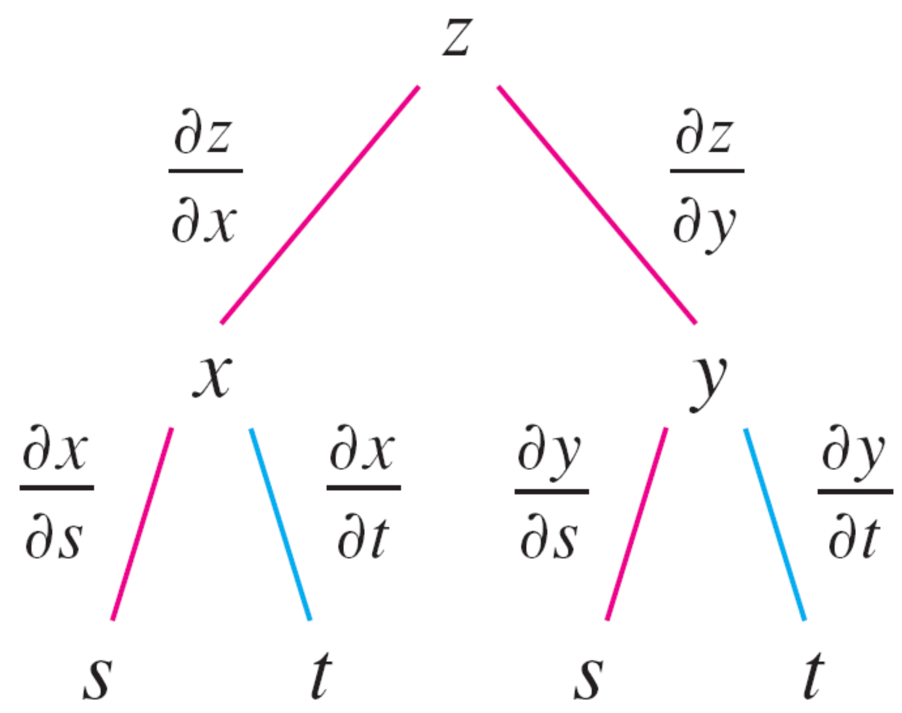
- $\frac{dz}{dt}$ is called the total differential coefficient of z with regard to t.

Chain Rule - Case 2

- Suppose $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$



Deduction (CO2)

- If both *x* and *y* are functions of *x then*,

$$\frac{\partial F}{\partial x}\frac{dx}{dx}+\frac{\partial F}{\partial y}\frac{dy}{dx}=0$$

- We obtain:

$$\frac{dy}{dx}=-\left(\frac{\partial F}{\partial x}\bigg/\frac{\partial F}{\partial y}\right)=-\frac{F_x}{F_y}$$

Total Derivative (CO 2)

Q.1. If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find dz/dt .

Solution: The Chain Rule gives

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= (2xy + 3y^4)(2\cos 2t) + (x^2 + 12xy^3)(-\sin t)$$

Total Derivative (CO 2)

Q.2. If $u = x^4 y + y^2 z^3$, where $x = rse^t, y = rs^2 e^{-t}, z = r^2 s \sin t$
find the value of $\partial u / \partial s$ when $r = 2, s = 1, t = 0$

Solution:

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial s}$$

$$= (4x^3 y)(re^t) + (x^4 + 2yz^3)(2rse^{-t}) + (3y^2 z^2)(r^2 \sin t)$$

When r = 2, s = 1, and t = 0, we have: x = 2, y = 2, z = 0
Thus,

$$\frac{\partial u}{\partial s} = (64)(2) + (16)(4) + (0)(0) = 192$$

Total Derivative (CO 2)

Q.3. Find dy/dx, if $x^3 + y^3 = 6xy$.

Solution: Let $F(x, y) = x^3 + y^3 - 6xy = 0$

By deduction, $$\frac{dy}{dx} = -\left(\frac{\partial F}{\partial x} / \frac{\partial F}{\partial y}\right) = -\frac{F_x}{F_y}$$

So, $$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{3x^2 - 6y}{3y^2 - 6x} = -\frac{x^2 - 2y}{y^2 - 2x}$$

Total Derivative (CO 2)

Q.4. If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$

Solution: Let $e^{y-z} = r, \quad e^{z-x} = s, \quad e^{x-y} = t$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x} \qquad = \frac{\partial u}{\partial r}\cdot 0 + \frac{\partial u}{\partial s} e^{z-x}(-1) + \frac{\partial u}{\partial t} e^{x-y}$$

$$\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial s} e^{z-x} + \frac{\partial u}{\partial t} e^{x-y} \text{..........}(1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y} \qquad = \frac{\partial u}{\partial r}\cdot e^{y-z} + \frac{\partial u}{\partial s}\cdot 0 + \frac{\partial u}{\partial t} e^{x-y}(-1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot e^{y-z} - \frac{\partial u}{\partial t} e^{x-y} \text{..........}(2)$$

Total Derivative (CO 2)

$$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial z}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial z}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial z} \quad =\frac{\partial u}{\partial r}\cdot e^{y-z}(-1)+\frac{\partial u}{\partial s}\cdot e^{z-x}+\frac{\partial u}{\partial t}\cdot 0$$

$$\frac{\partial u}{\partial z}=-\frac{\partial u}{\partial r}\cdot e^{y-z}+\frac{\partial u}{\partial s}\cdot e^{z-x} \text{...........}(3)$$

On adding eq. (1) , (2) and (3)

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$$

Recap

- ✓ Definition of partial derivative
- ✓ Higher derivatives
- ✓ Euler's Theorem on Homogeneous Function
- ✓ Composite function
- ✓ Total Derivatives
- ✓ Chain rule case 1 and 2

Weekly Assignment (CO2)

Partial derivatives

1. If $u(x, y, z) = \log(\tan x + \tan y + \tan z)$, prove that

   $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$

2. If $z = \log(e^x + e^y)$ show that $rt - s^2 = 0$ where $r = \frac{\delta^2 z}{\delta x^2}$, $t = \frac{\delta^2 z}{\delta y^2}$ and $s = \frac{\delta^2 z}{\delta x \delta y}$.

3. If $z = f(x + ct) + \emptyset(x - ct)$ show that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$

4. If $u = e^{xyz}$ then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2)u$

5. If $e^{\frac{-z}{x^2 - y^2}} = x - y$ then show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x^2 - y^2$

Weekly Assignment (CO2)

Euler's theorem based question

1. Verify Euler's theorem for the functions :(i) $z = \frac{x^{1/3}+y^{1/3}}{\sqrt{x}+\sqrt{y}}$ (ii) $u = \log \frac{x^2+y^2}{xy}$.

2. If $u = x\sin^{-1}\left(\frac{x}{y}\right) + y\sin^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2\frac{\delta^2 u}{\delta x^2} + 2xy\frac{\delta^2 u}{\delta x \delta y} + y^2\frac{\delta^2 u}{\delta y^2}$. Ans. 0.

3. If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -3\tan u$

4. If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $xu_x + yu_y = \sin 2u$

Total derivatives and change of variable

1. If $v = f(2x-3y, 3y-4z, 4z-2x)$ prove that $6v_x + 4v_y + 3v_z = 0$.

2. $If\ u = f(e^{y-z}, e^{z-x}, e^{x-y})$ Then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

3. $If\ u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$

Quiz

Q.1. If $u = e^{x^2+y^2+z^2}$, then $\frac{\partial^3 u}{\partial x \partial y \partial z}$.........

Q.2. $\frac{\partial^2 u}{\partial x \partial y} = \ldots\ldots\ldots,$ if $u = x^2 + y^2.$

Q.3. If $w = \frac{y}{z} + \frac{z}{x}$, then $xw_x + yw_y + zw_z = \ldots\ldots\ldots\ldots$

Quiz

Q.1. *If* $u(x,y)=\left(\sqrt{x}+\sqrt{y}\right)^5$, then value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ is ....

Q.2. If $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ ....

Q.3. If $u=\log\left(\frac{x^2}{y}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ ....

Q.4. If $u=x^2$ and $x=t^3$ then $\frac{du}{dt}$ is .... (NIET2020-2021)

Q.5. If $u=\frac{x^2}{a}+\frac{y^2}{b}-7$ then $\frac{\partial u}{\partial x}$ is ...... (NIET2020-2021)

FAQ

Q.1. Verify Euler's theorem if $u = \dfrac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$.

Q.2. If u = $\tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

$(NIET\,2020-2021)$

Q.3. If $z = \log\dfrac{x^2 + y^2}{x + y}$, show that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 1$

FAQ

1. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial x}\right)^2 u = \frac{-9}{(x+y+z)^2}$

2. If $v = \left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$

3. If $z(x+y) = \left(x^2 + y^2\right)$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$

4. If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$

5. Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$, where $u = \log\left(\frac{x^2 + y^2}{xy}\right)$

FAQ

Q.1. If $u = f(r)$, where $r^2 = x^2 + y^2$,

prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r} f'(r)$

Q.2. If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, show that

$$e^{-2u}\left[\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2\right] = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2.$$

Q.3. If $u = f(y - z, z - x, x - y)$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0.$

ASSIGNMENT-2.3

[..\Unit-2 (Differential Calculus-I , Assign.-2.3).docx]

Partial derivatives:

- NPTEL Series: Lecture 05 - Partial Derivatives- https://www.youtube.com/watch?v=6tQTRlbkbc8

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details

- **Euler's theorem**

- Lecture 12: Euler's theorem for homogeneous function
  https://www.youtube.com/watch?v=btLWNJdHzSQ

Faculty Video Links, Youtube & NPTEL Video Links and Online Courses Details

- **Total derivatives**
- Lecture 09 – Chain rule 1

https://www.youtube.com/watch?v=McT-UsFx1Es

- Lecture 10 – Chain rule 2

https://www.youtube.com/watch?v=_1TNtFqiFQo

- **Change of variables**

- Lecture 10 – Chain rule 2

https://www.youtube.com/watch?v=X6kp2o3mGtA

Summary

In this unit(Differential Calculus-I), we discussed the following points :

- Course Objective
- COs and POs of subject
- Mapping of COs and Pos
- Prerequisite and Recap

- Introduction(Calculus)

Summary

- Successive Differentiation(nth order derivatives)
- Leibnitz’s theorem and its application
- Asymptotes.
- Curve tracing: Cartesian and Polar Co-ordinates
- Partial Differentiation
- Euler’s Theorem

Thank You