# Lab Assignment-7

1. Write a function from scratch to implement the conjugate gradient for solving a system of linear equations. Use this function to solve the following system for $x$ :

$$\begin{aligned} 4x + 3y &= 24 \\ 3x + 4y - z &= 30 \\ -y + 4z &= -24 \end{aligned}$$

   with $x^0 = (0, 0, 0)^T$ and no preconditioning. Also, compare the number of iterations till convergence with the Gauss-Seidel method.

2. Consider a linear system $Ax = b$ with

$$A = \begin{bmatrix} 0.2 & 0.1 & 1 & 1 & 0 \\ 0.1 & 4 & -1 & 1 & -1 \\ 1 & -1 & 60 & 0 & -2 \\ 1 & 1 & 0 & 8 & 4 \\ 0 & -1 & -2 & 4 & 700 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

   a. Find the condition number of the matrix $A$ and the preconditioned matrix of $A$. (Use any conditioning matrix of your choice)
   b. Make a table showing the final solution of the above system and the number of iterations required for convergence using the following methods:
      i. - Jacobi
      ii. - Gauss-Seidel
      iii. - Conjugate Gradient
      iv. - Conjugate Gradient (Preconditioned)

3. Let

$$A_1 = \begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

   and $\mathbb{I} =$ Identity matrix of size $4 \times 4$, and $\mathbb{O} = 4 \times 4$ matrix of all zeros. Form a $16 \times 16$ matrix $A$ in partitioned form as:

$$A = \begin{bmatrix} A_1 & -\mathbb{I} & \mathbb{O} & \mathbb{O} \\ -\mathbb{I} & A_1 & -\mathbb{I} & \mathbb{O} \\ \mathbb{O} & -\mathbb{I} & A_1 & -\mathbb{I} \\ \mathbb{O} & \mathbb{O} & -\mathbb{I} & A_1 \end{bmatrix}$$

Let $b = (1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6)^T$. Solve the linear system $Ax = b$ using the Conjugate Gradient method without and with preconditioning (choose any conditioning matrix). And, make a table of convergence as indicated in question 2 .