Lab Assignment-8

1. Let a system of linear equations Ax = b, where

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}, b = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

Solve this using conjugate gradient (with and without preconditioning) and with steepest gradient descent. Plot contours of the objective function

$$\left(\text{quadratic form } f(x) = \frac{1}{2}x^T Ax - b^T x \right)$$

and the path taken by each method to reach the solution.
Explain the concept of 'conjugate directions' and how it is used in the conjugate gradient method.

2. Generate a 60 × 60 matrix A with 1's on the main diagonal and in the last column, with −1's below the main diagonal, and with 0's everywhere else. Compute the condition number of the matrix A. Set a random vector x of length 60. Compute

$$b = A*x$$

Solve the linear system $Ax' = b$ using different methods taught in the class.
Compute the 2-norm of the difference between the computed vector x' and the true solution x generated previously. The relative difference is given by:

$$\frac{||x' - x||}{||x||}$$

Compare the relative errors of all the methods.