Lab Assignment-1

1) The following code generates the cat's picture.

```
import numpy as np
import matplotlib.pyplot as plt
topx=np.array([1, .98, .8, .83, .8, .77, .5, .25, 0, -.25, -.5, -.8, -.8, -.98, -1])
topy=np.array([0, .15, .7, 1.15, 1.03, 1.15, .85, .95, 1, .95, .85, 1.2, .7, .15, 0])
botx=-np.cos(np.pi*np.arange(1,10)/10)
boty=-np.sin(np.pi*np.arange(1,10)/10)
wiskx=np.array([.2, 1.3, .2, 1.4, .2, 1.4, .2, 1.3, .2, .17, .13, .08, .03, 0])
wisky=np.array([ 0, .3, 0, .1, 0, -.1, 0, -.3, 0, .1, -.1, .1, -.1, 0]) -.2
xeye=np.array([0, .2, .3, .4, .43, .45, .43, .4, .37, .35, .37, .4, .5, .6, .5, .4, .3, .2,
0])
yeye=np.array([.5, .5, .43, .4, .42, .5, .58, .6, .58, .5, .42, .4, .43, .5, .57, .6, .57,
.5,.5])-.2
x=np.concatenate((topx,botx,wiskx,xeye,-xeye,-wiskx[::-1]))
y=np.concatenate((topy,boty,wisky,yeye, yeye,wisky[::-1]))
cat = np.array([x,y]).T
plt.plot(cat[:,0], cat[:,1],'b')
plt.xlim(-2, 2)
plt.ylim(-2, 2)
plt.show()
```

Do the following for each transformation below:

A. create the matrix associated to the given transformation
B. show the effect of applying this matrix to the picture.

The transformations are:

a. Reflection though the x-axis
b. Reflection though the y-axis
c. Reflection though the axis y=x
d. Reflection though the axis y=-x
e. Reflection though the origin
f. Horizontal Contraction by a factor ⅓
g. Vertical Contraction by a factor ⅕
h. Horizontal Expansions by a factor 3
i. Vertical Expansions by a factor of 4
j. Horizontal Shears by a factor of -1.5 then +1.5
k. Vertical Shears by a factor of -1.5 then +1.5
l. Projection onto the x-axis
m. Projection onto the y-axis

n. Rotation by an angle of π/ 3 counterclockwise
o. Rotation by an angle of π/3 clockwise

2) Let v be a vector, rotate this vector by given angle θ
   a) v = [1, 2] , $\theta = 30^o$ anticlockwise
   b) v = [1, 2], $\theta = 45^o$ clockwise
   c) v = [2, 4], shear along x-axis

3) Let T: R3 → R3. (Use vector v = [2, 4, 1] for the visualization)
   a) If T(a, b, c)=(a, b, 0), show that T is the projection on the xy plane along the z-axis.
   b) Find a formula for T(a, b, c), where T represents the scaling along the x and z-axis by a factor of 3.
   c) If T(a, b, c)=(a − c, b, 0), show that T is the projection on the xy-plane along the line L = {(a, 0, a): a ∈ R}.
   d) Find a formula for T(a, b, c), where T represents 3D shearing in the X axis?

4) Consider a 2 × 2 square matrix A given by:

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

   e) Calculate the eigenvalues and corresponding eigenvectors of matrix A.
   f) Create a visualization that shows the original and transformed eigenvectors:
      i) In the first subplot, display the original eigenvectors as arrows on a 2D plane.
      ii) In the second subplot, show the transformed eigenvectors after applying matrix A as arrows on the same 2D plane.
      iii) Ensure that both subplots have appropriate axis labels, titles, and legends.
   g) Explain what happens to the eigenvectors when matrix A is applied as a linear transformation based on your visualization.
   h) Describe how the concept of eigenvectors being stretched (scaled) but not rotated as illustrated in the visualization.