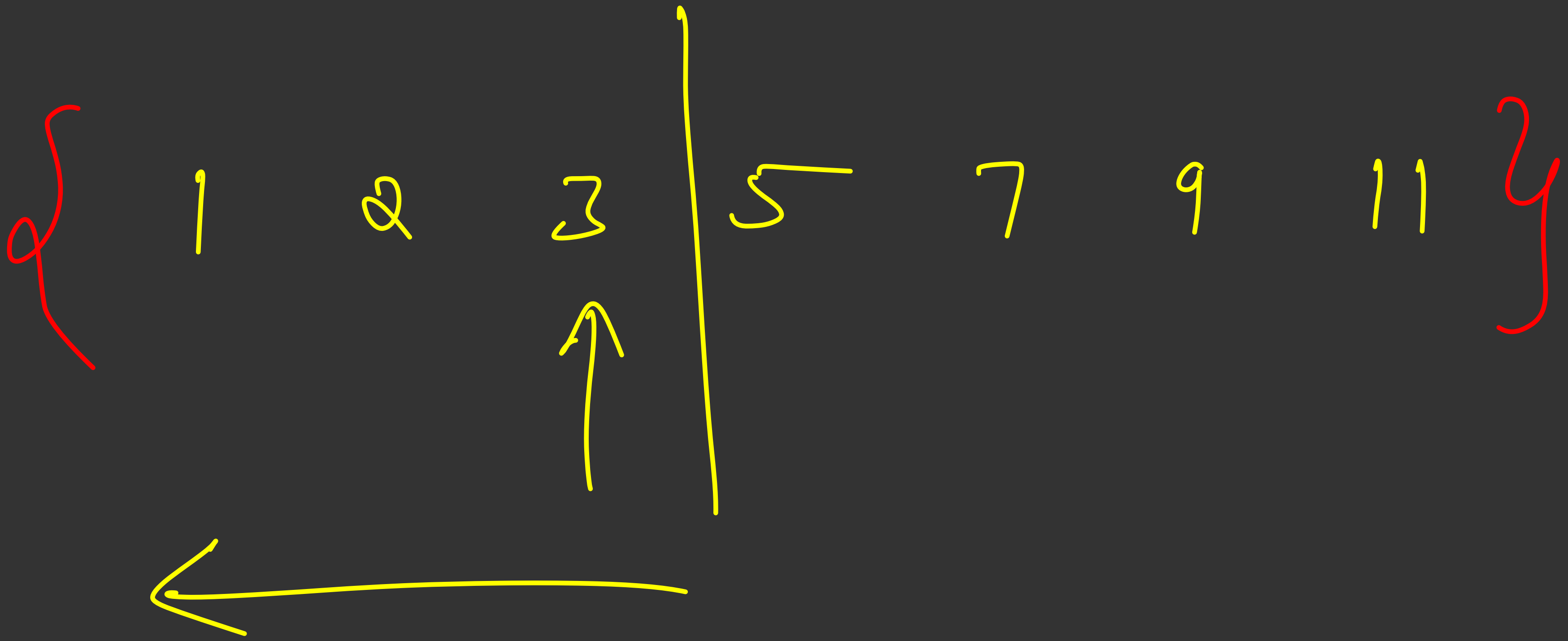
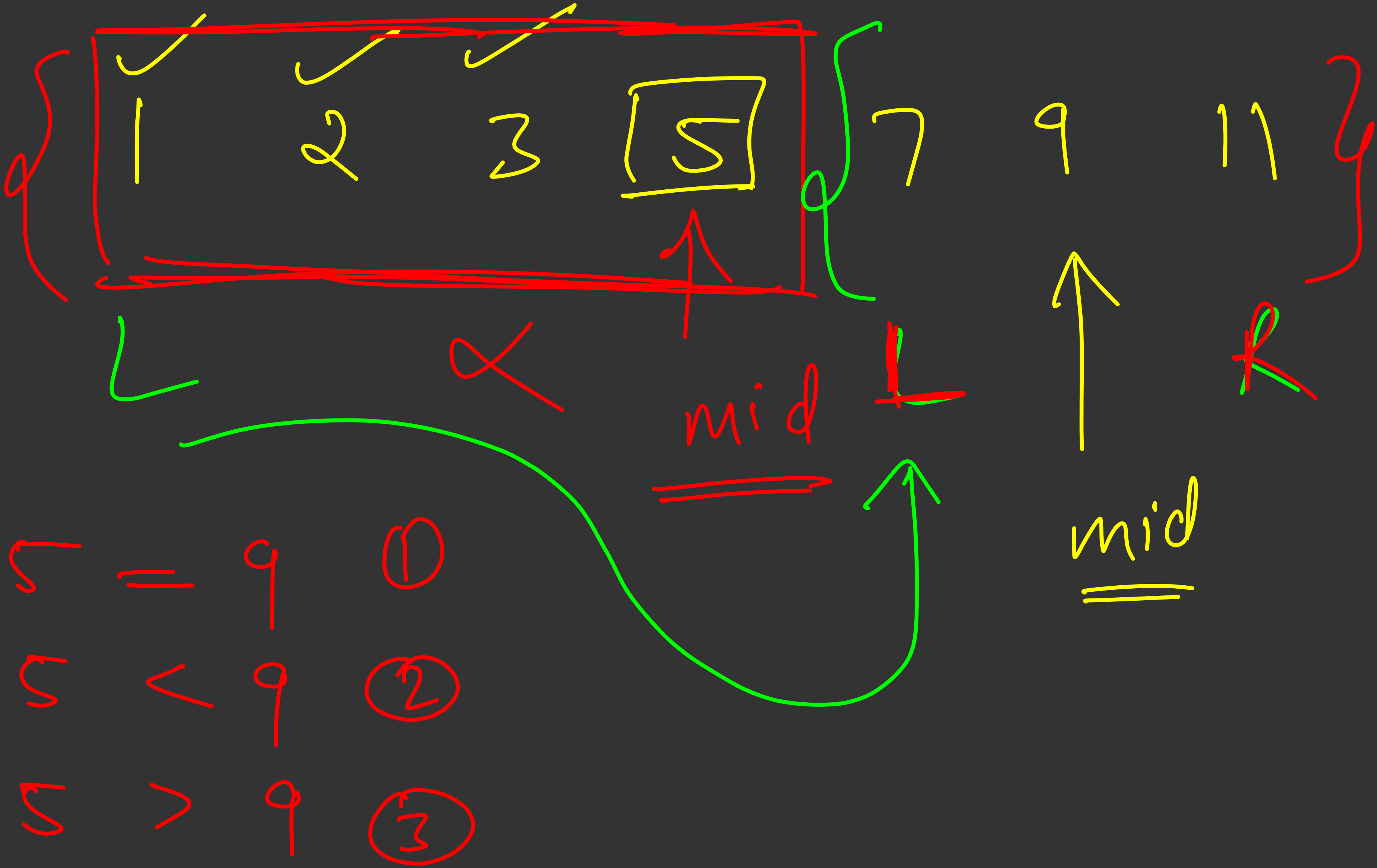
Binary Searching on Answer

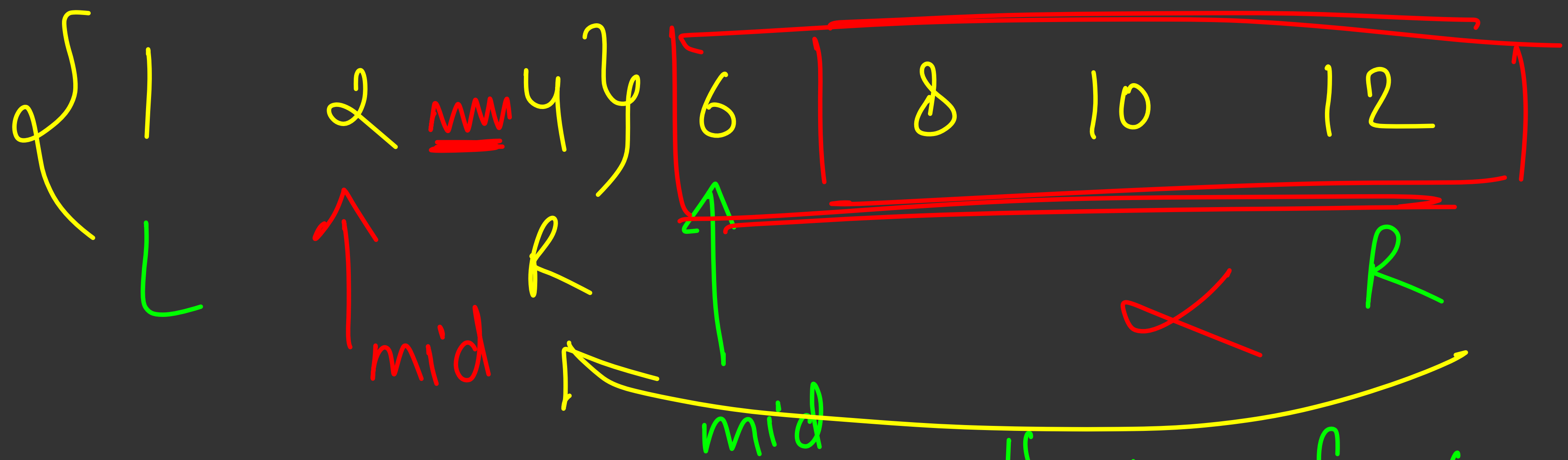
Advanced Binary Search

① sorted array → find $\underline{\underline{x}}$



$\boxed{9}$


✓ ✓ ✓
1 2 3 5 7 9 11
L
mid
L
mid
R
5 = 9 ①
5 < 9 ②
5 > 9 ③



$6 < 4$ ①

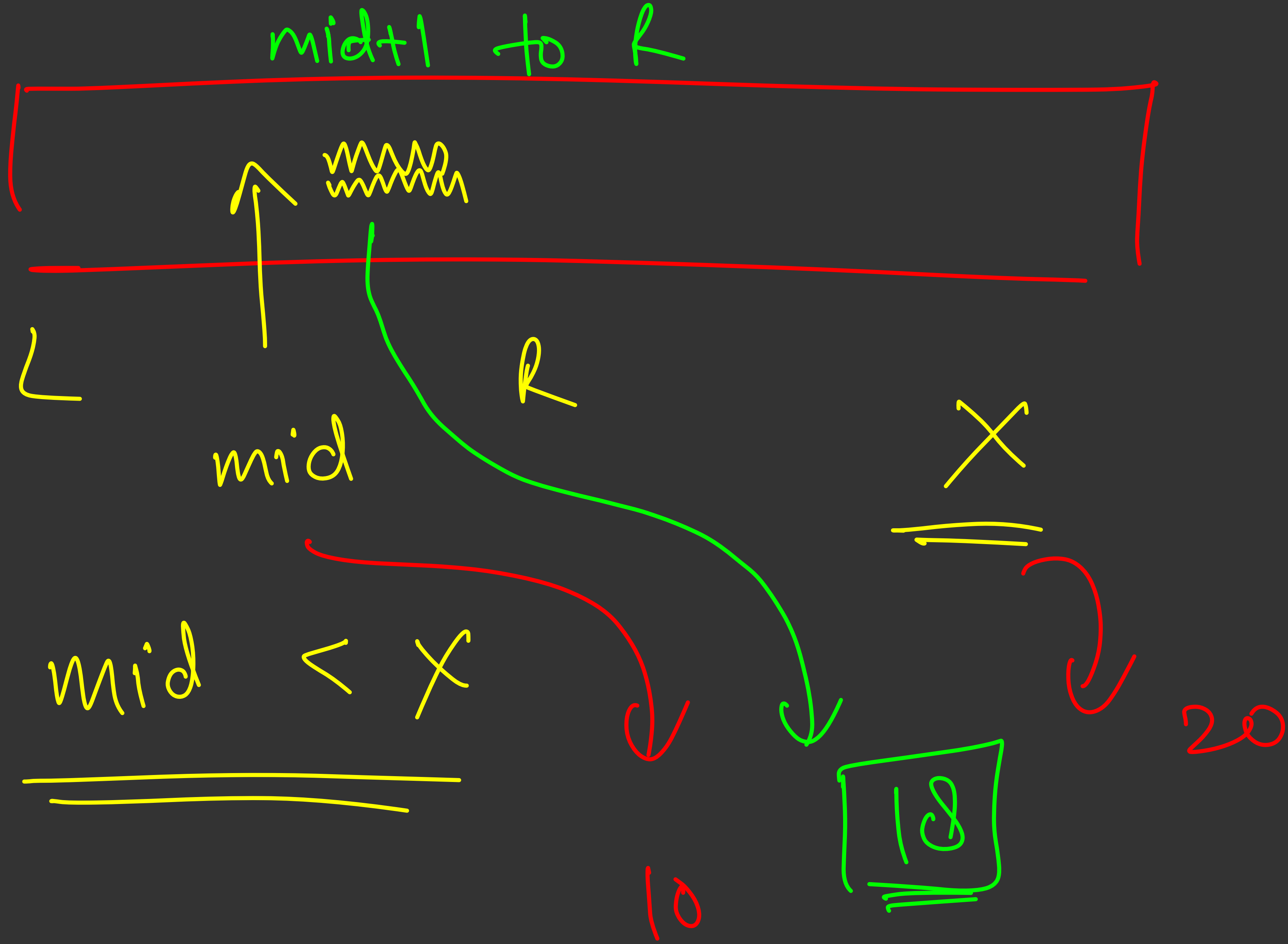
$6 > 4$ ②

$6 = 4$ ③

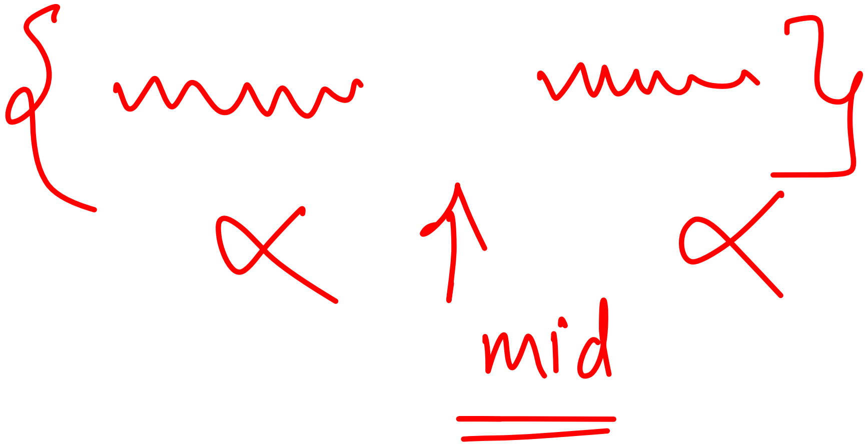
# no. of elements smaller than 4

index of largest element smaller than 4

$2 > 4$ ① $2 < 4$ ② $2 = 4$ ③ 4


mid+1 to R
L
mid
R
X
mid < X
10
18
20

Binary Search Revision



Time Complexity Discussion

We are reducing our search space at every step into half of current search space

$$\underline{\underline{10}} \rightarrow \underline{\underline{5}} \rightarrow \underline{\underline{2}} \rightarrow \textcircled{1}$$

Recurrence:

$$T(n) = T(n / 2) + 1$$

$$\underline{\underline{n}} \rightarrow \frac{n}{2} \rightarrow \frac{n}{4} \rightarrow \frac{n}{8} ---- \textcircled{1}$$

$$T(1) = 1$$

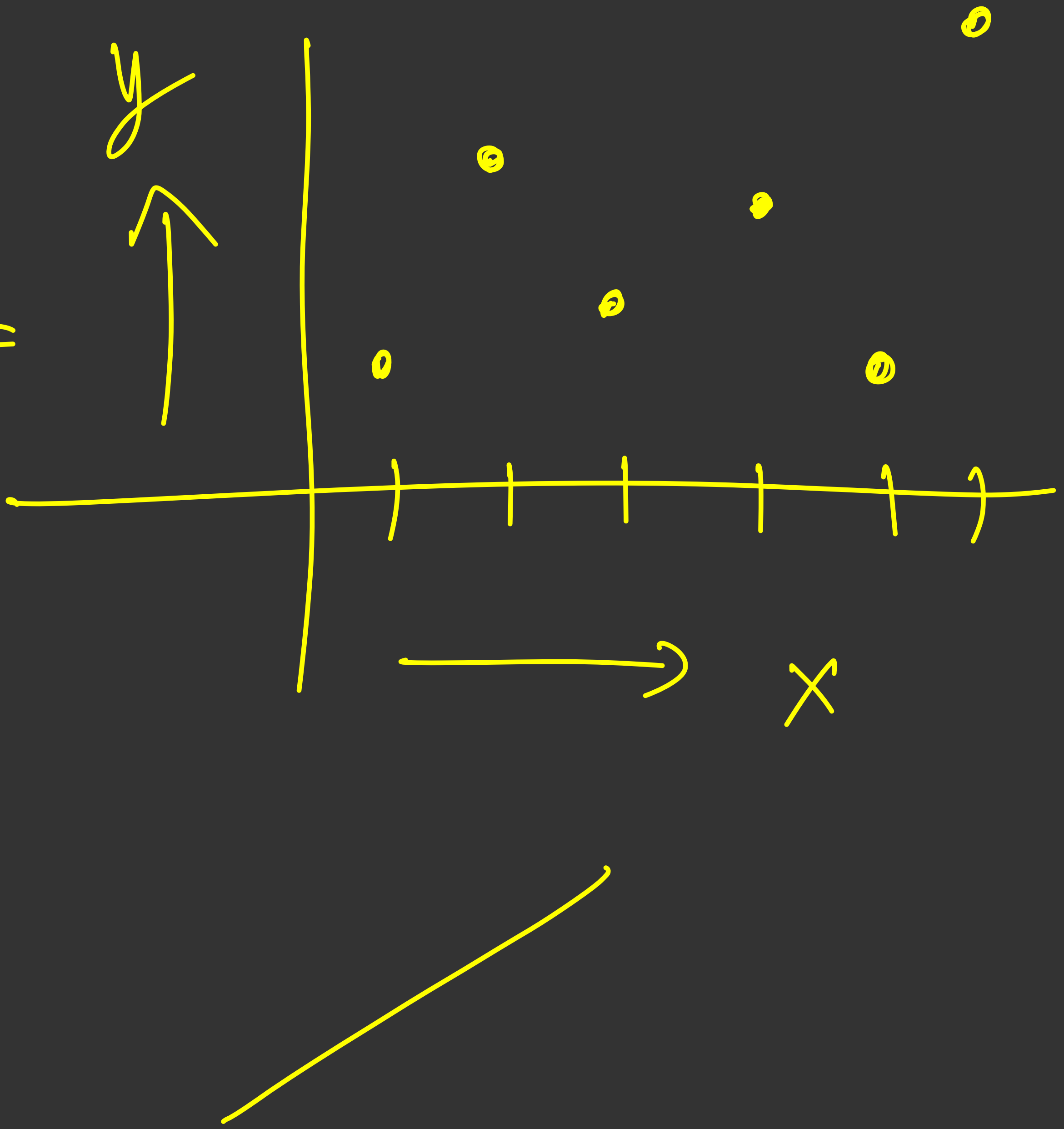
Time Complexity: O(logn)

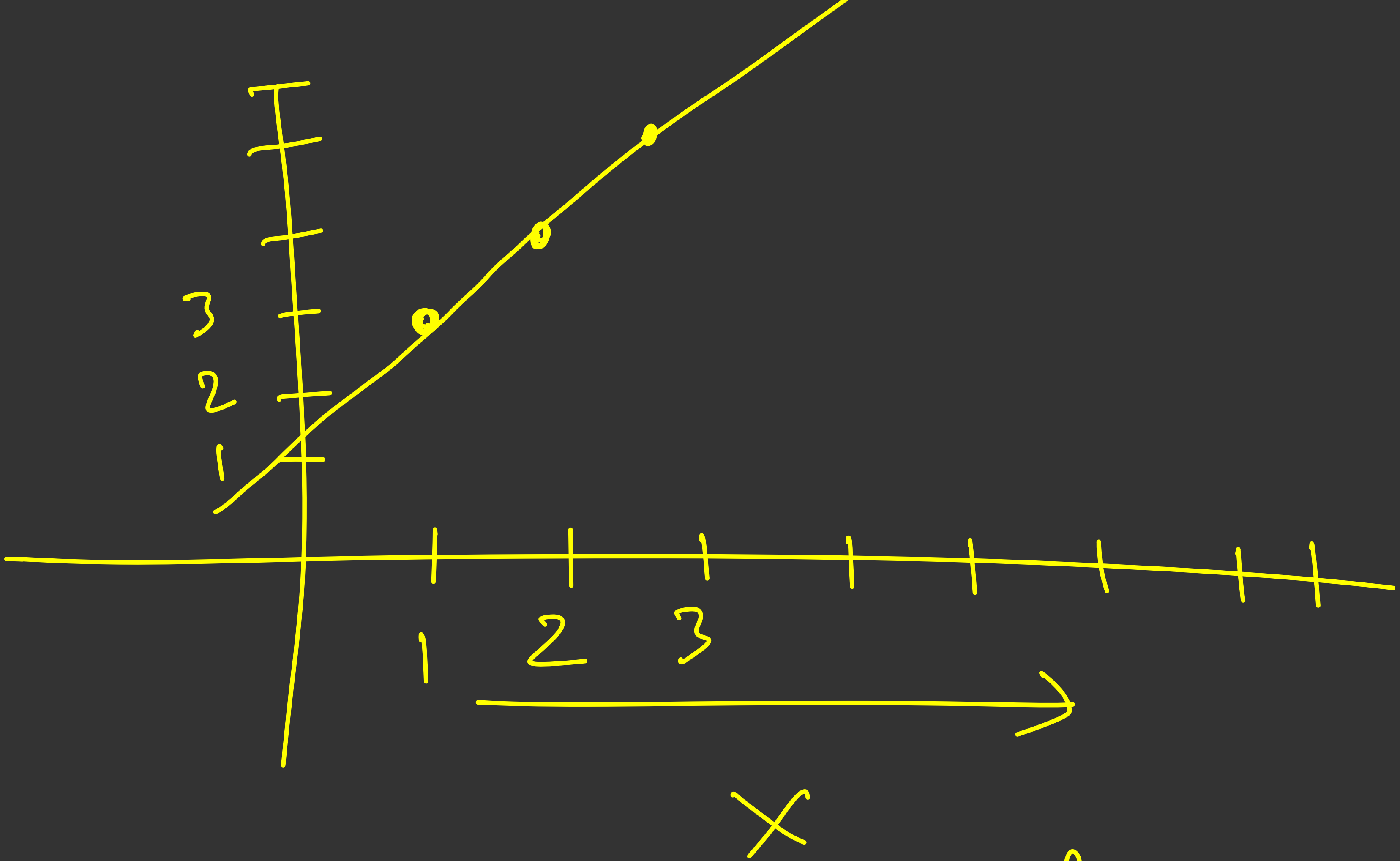
Monotonicity function

$$f(x) \longrightarrow y$$

x = input

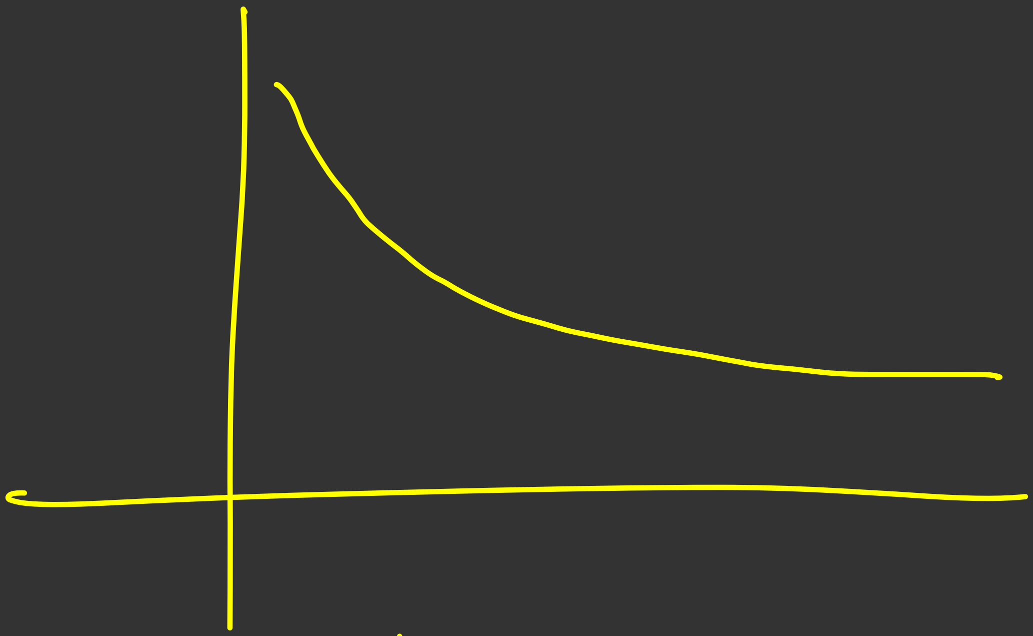
y = output



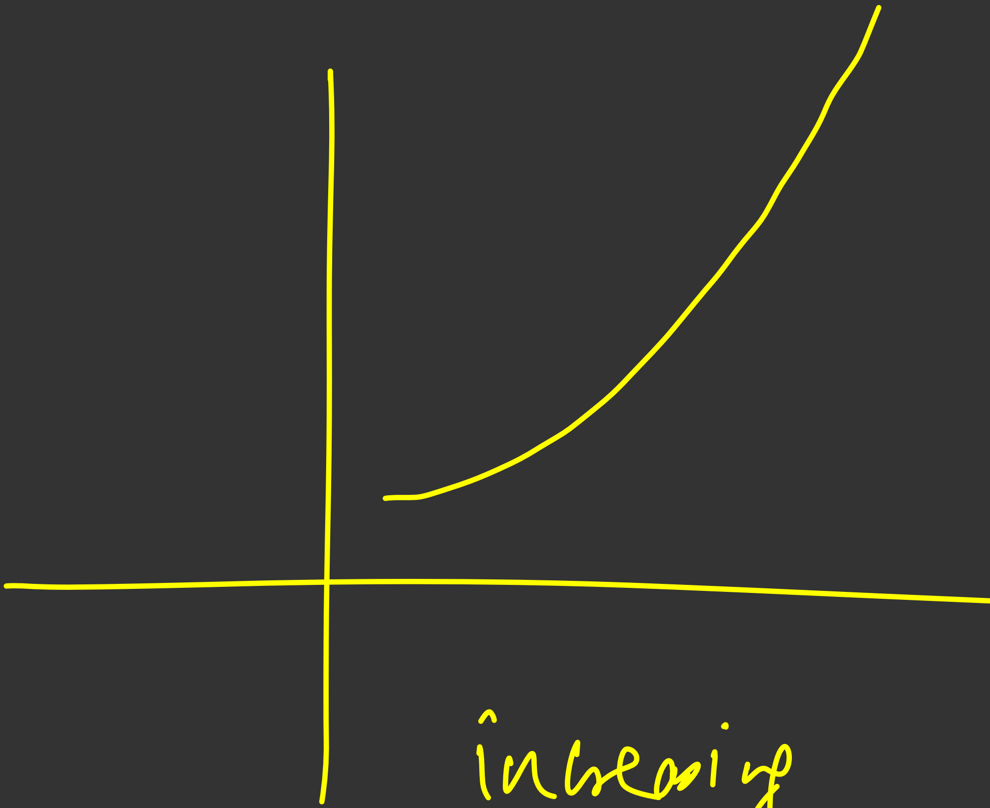


① $x \uparrow$ $y \uparrow$ $f(x) = x+2$

② $x \uparrow$ $y \downarrow$ $f(x) = 1/x$



decreasing monotonic function



increasing monotonic function

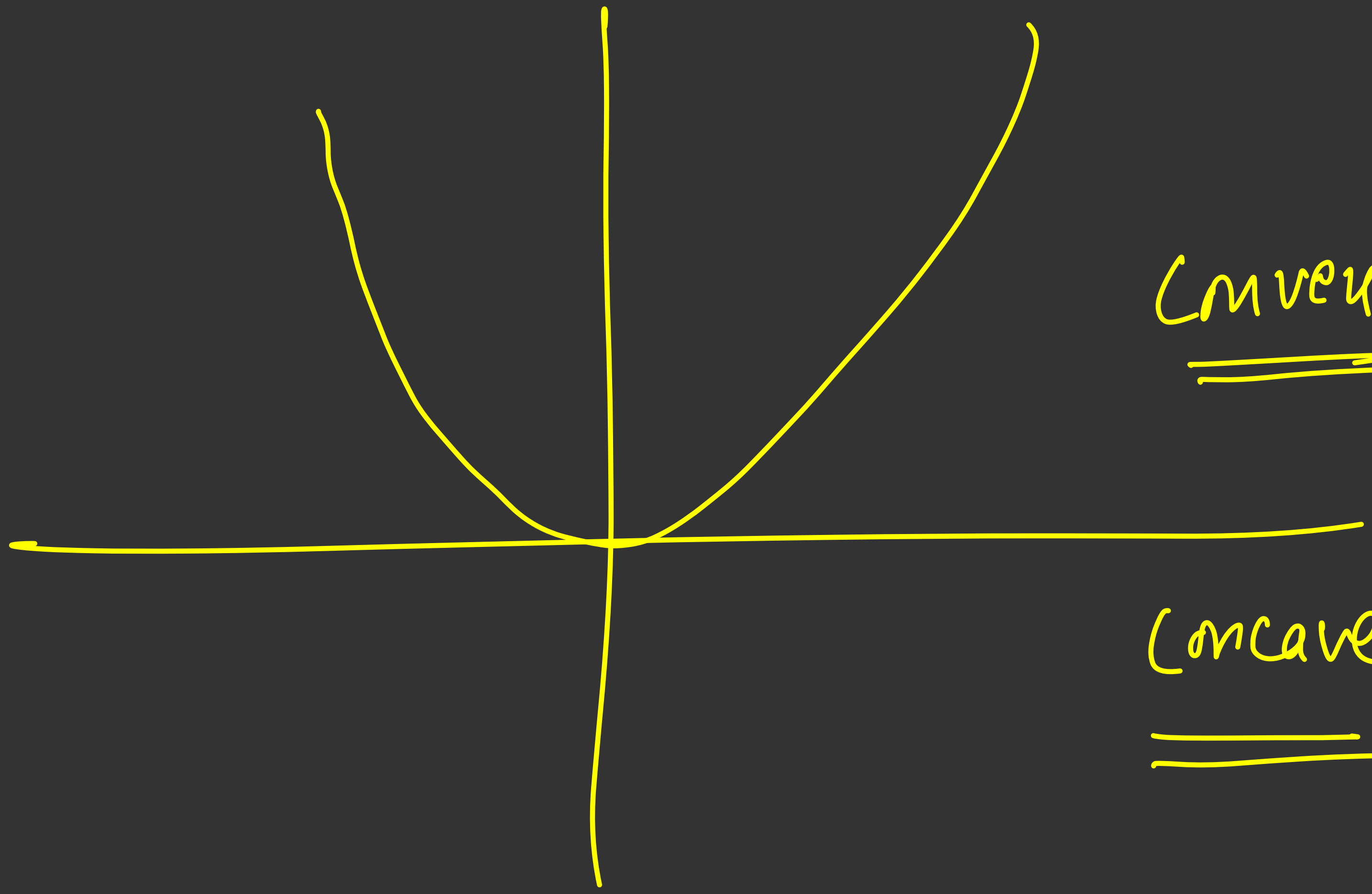
$\boxed{0\cdot 1 = 0\cdot 01}$

$\boxed{0\cdot 2 = 0\cdot 04}$

$1/x \Rightarrow$

$|x| \rightarrow$

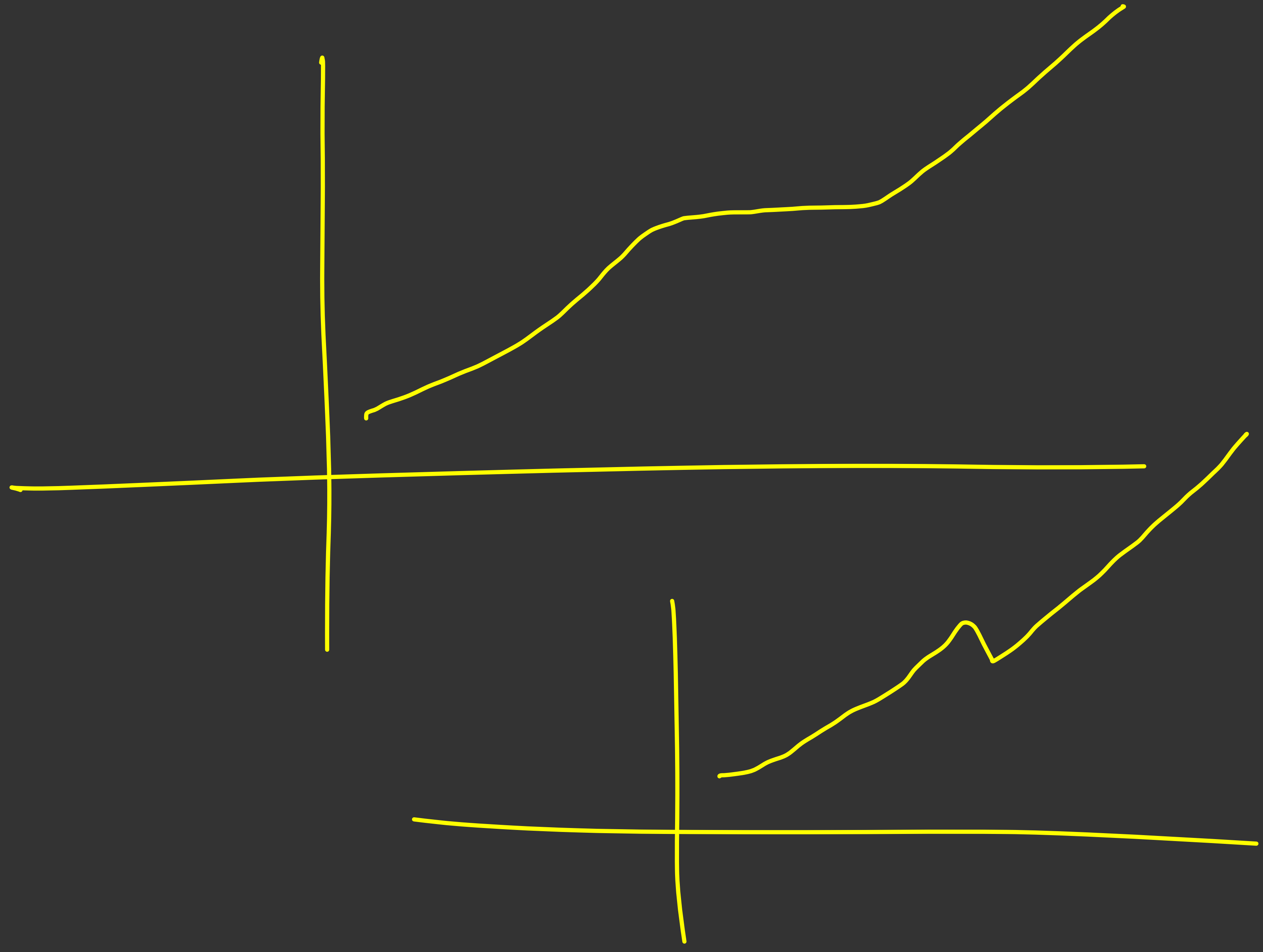
$x \uparrow \qquad f(x) \uparrow$



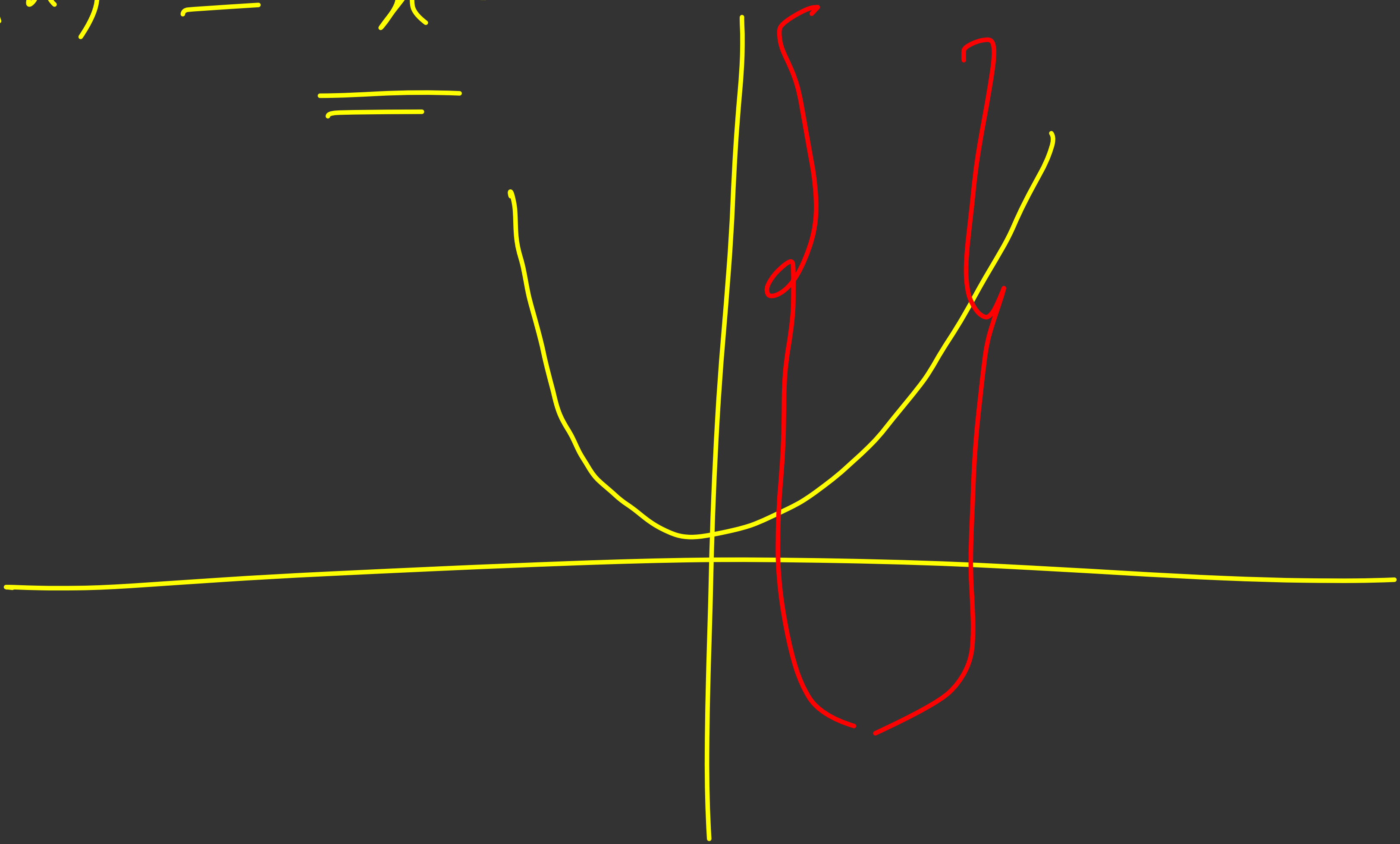
Convex

Concave

$x \uparrow \qquad \underline{f(x)} \uparrow$



$f(x) = x^2$



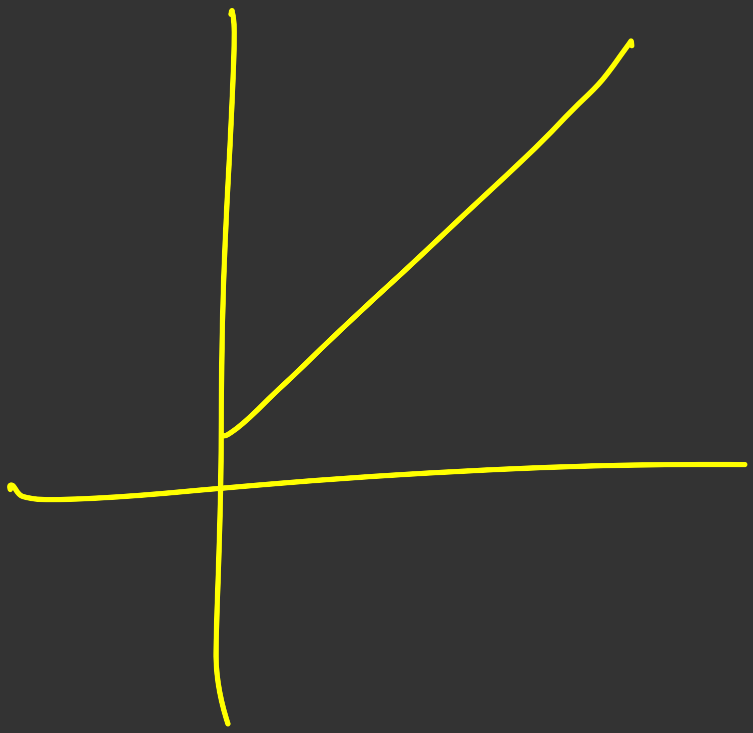
Binary Search Revision

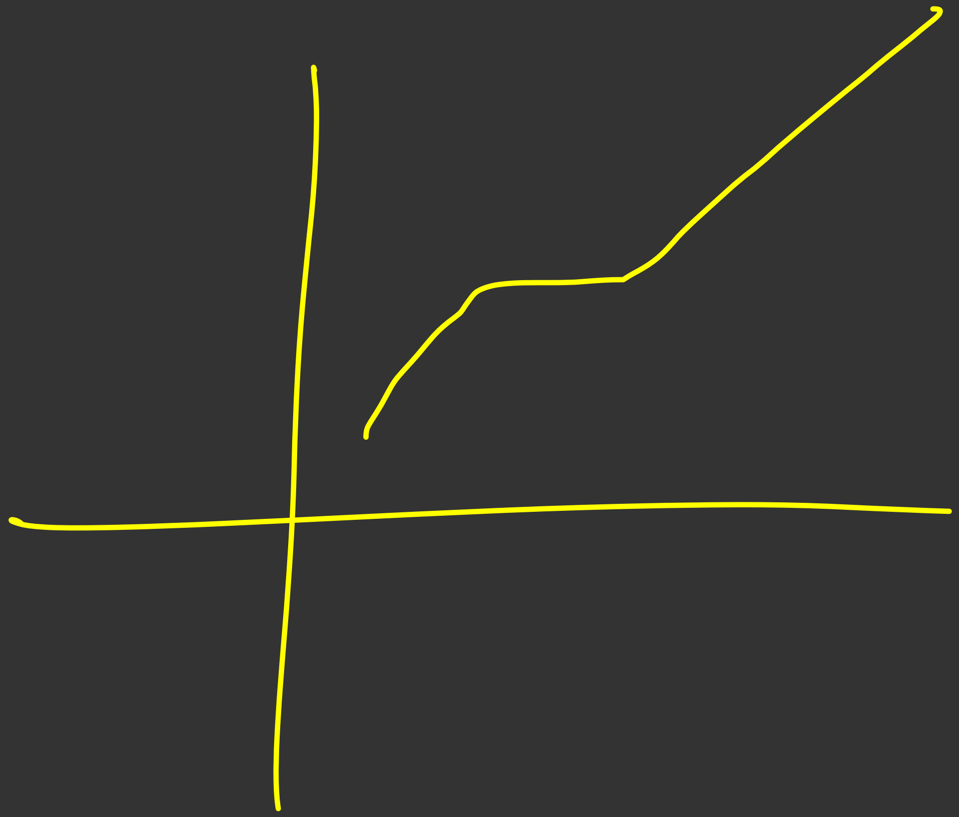
Requirement for using Binary Search

Monotonicity

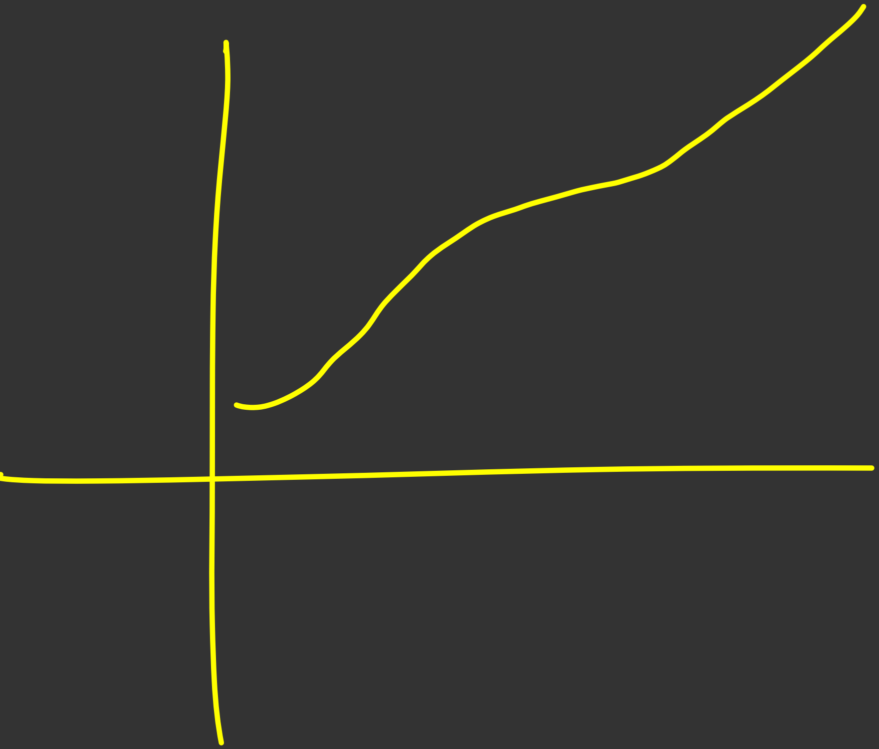
- $f(x) \geq f(y)$ iff $x > y$ (increasing monotonic) non decreasing
- $f(x) \leq f(y)$ iff $x > y$ (decreasing monotonic) non increasing

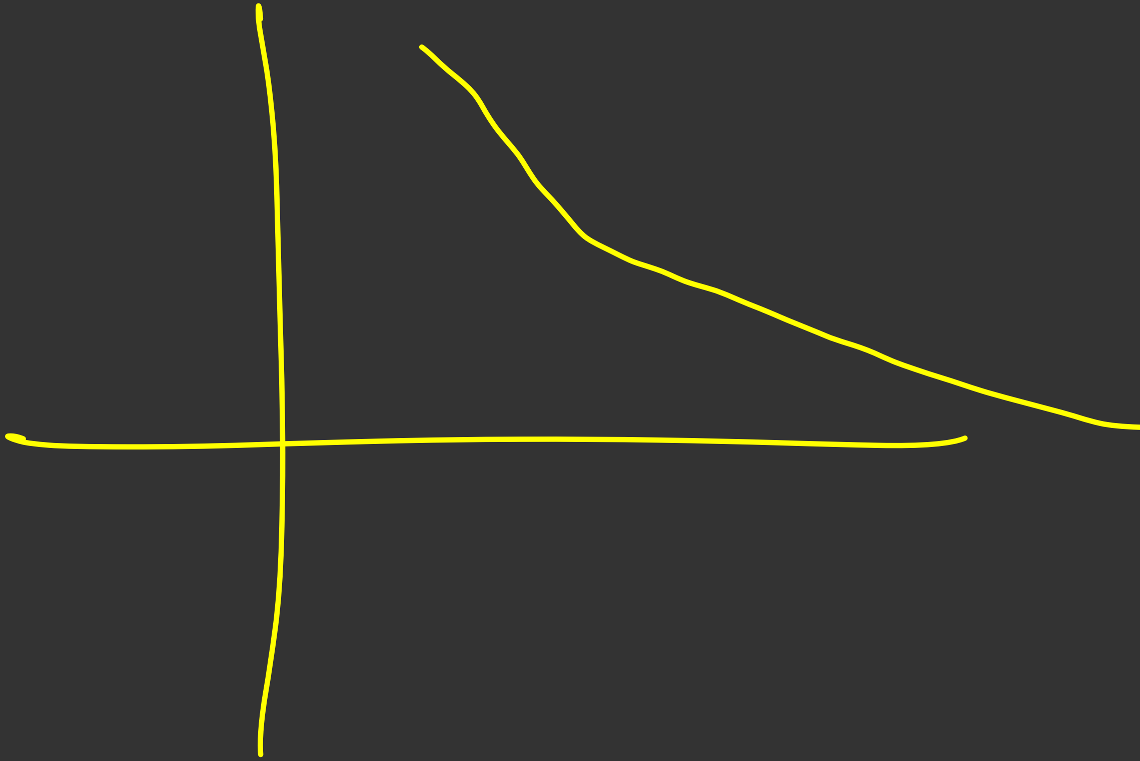
increasing | non decreasing





decreasing | non increasing





$$f(x) = x^2 + 2$$

tell me the value of x for which ↑

$$f(x) = \underline{\underline{66}}$$

x	$f(x)$
$x = 0$	2
$x = 1$	3
$x = 2$	6
$x = 3$	11
$x = 4$	18
$x = 5$	27
$x = 6$	38
$x = 7$	51
$x = 8$	66

$$f(x) = \underline{\underline{1026}}$$

$$\underline{\underline{10^{16}}}$$

$x =$	0	1	2	3	4	5	6	7	8
$y =$	2	3	6	11	18	27	38	51	66
	T	T	T	T	T	T	T	F	F
	T	T	T	T	T	F	F	F	F

$y = 18$

$$x^2 + 2 \le 39$$

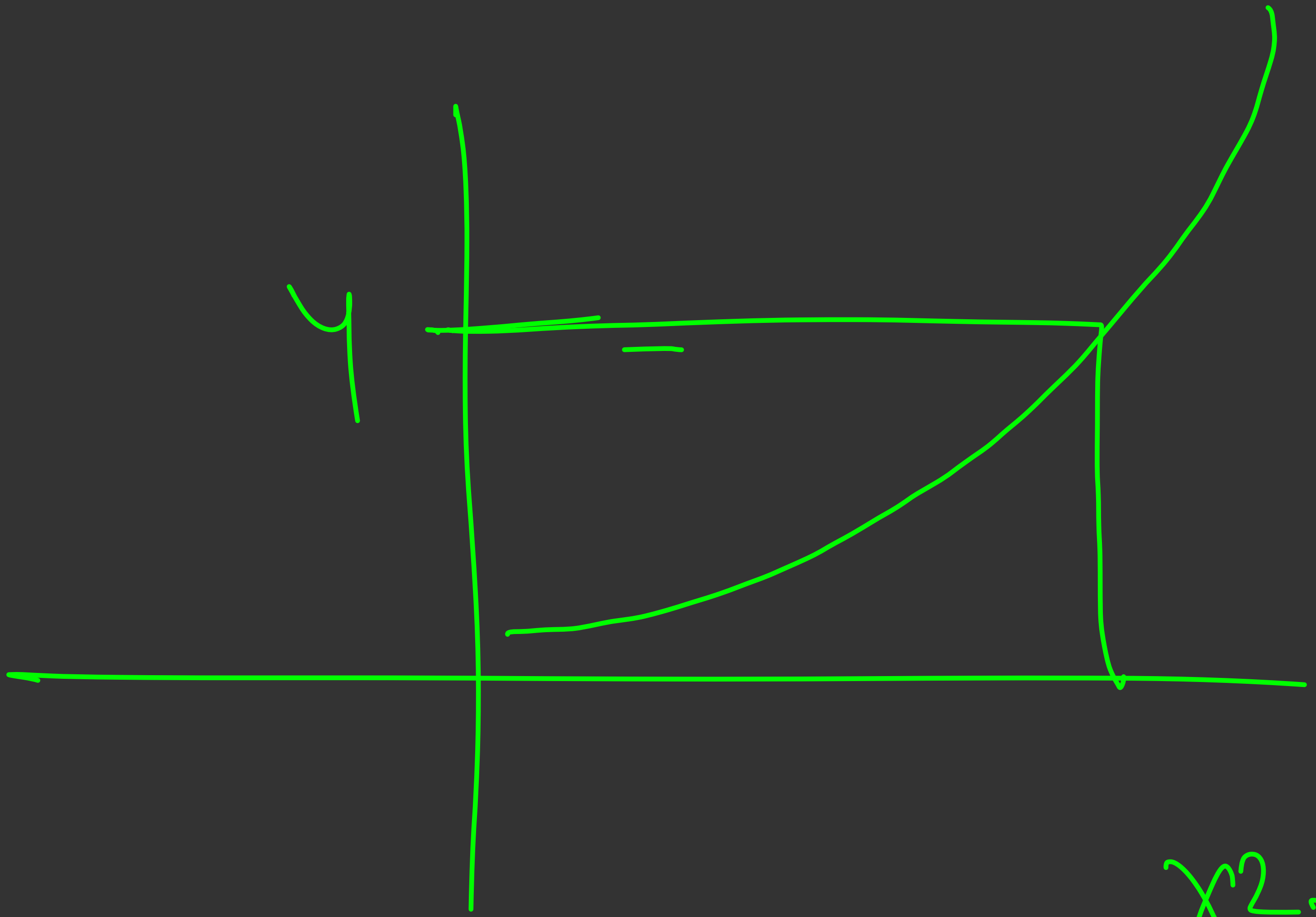
39

$x = 4$

$$6^2 + 2 \le 39$$

$4^2 + 2 = 18$

$$38 \le 39$$



x^2+2

$$f(x) = \underline{\underline{x^2 + 2}}$$

find out the value of x for which $f(x) = y$

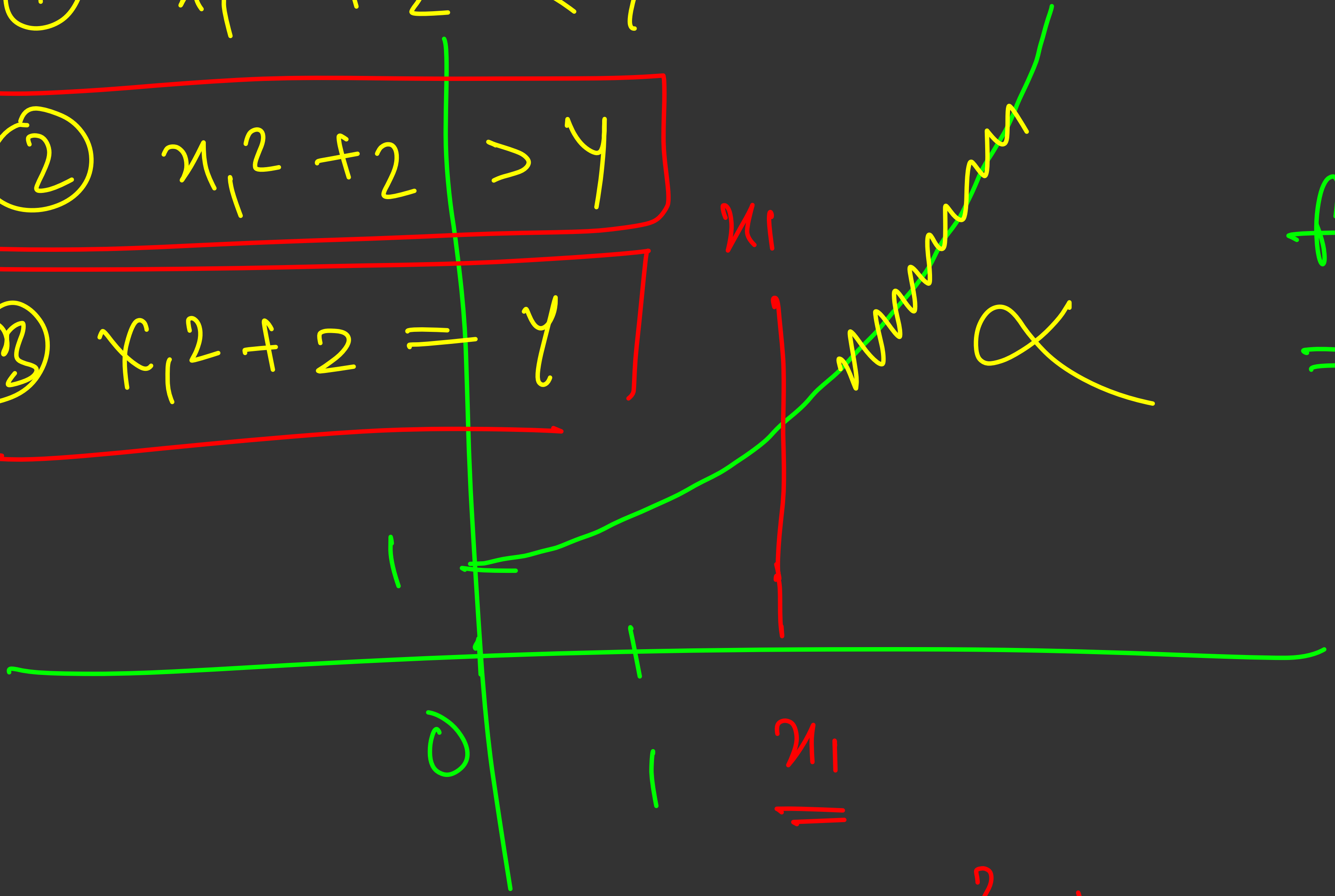
$$y = x^2 + 2$$

$$x = \sqrt{y - 2}$$

① $x_1^2 + 2 < y$

$\boxed{② \; x_1^2 + 2 > y}$

$\boxed{③ \; x_1^2 + 2 = y}$



$f(x)$

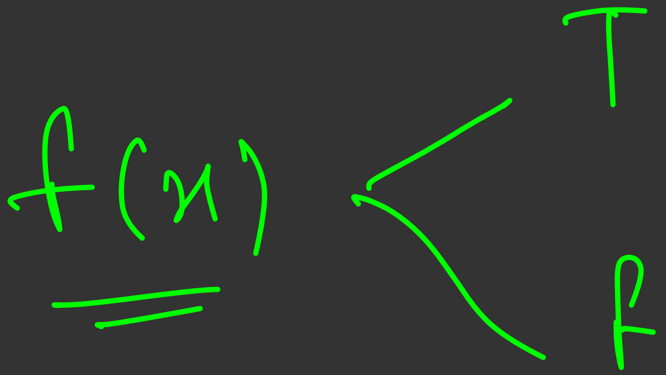
$= x^2 + 2$

$\underline{\underline{y}}$

$\underline{\underline{10^{16}}}$

$x_1^2 + 2 = y$

Predicate function



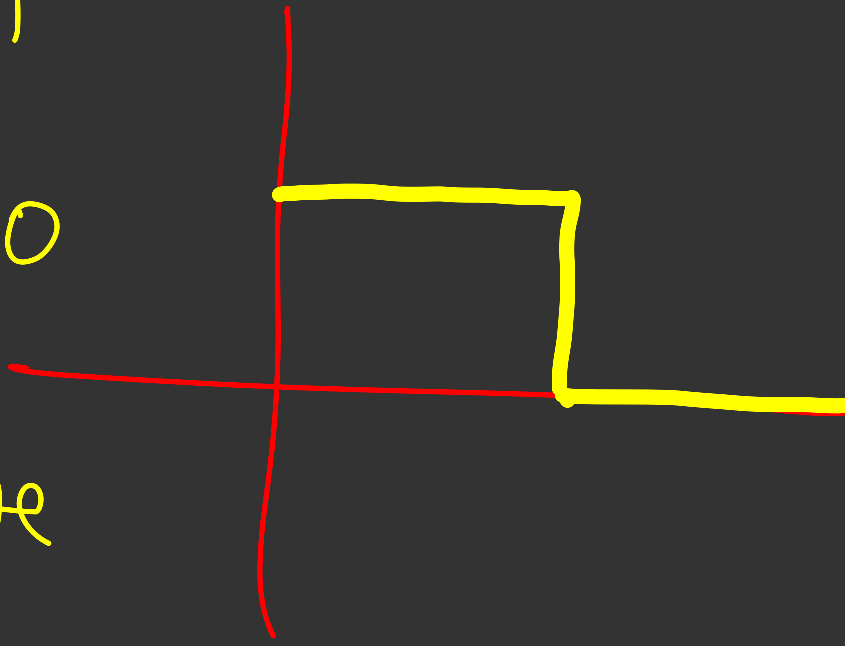
$$f(x) = \begin{cases} T & \text{if } x < 3 \\ F & \text{if } x \geq 3 \end{cases}$$

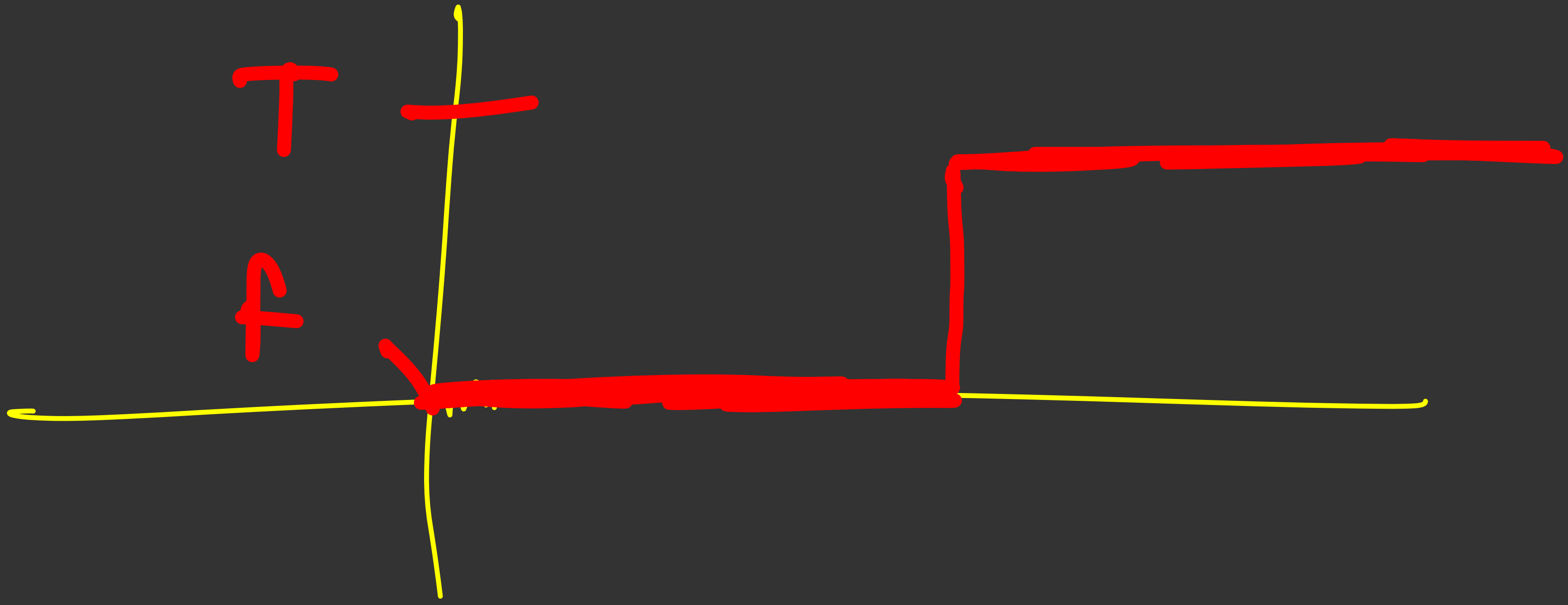
$f(x)$

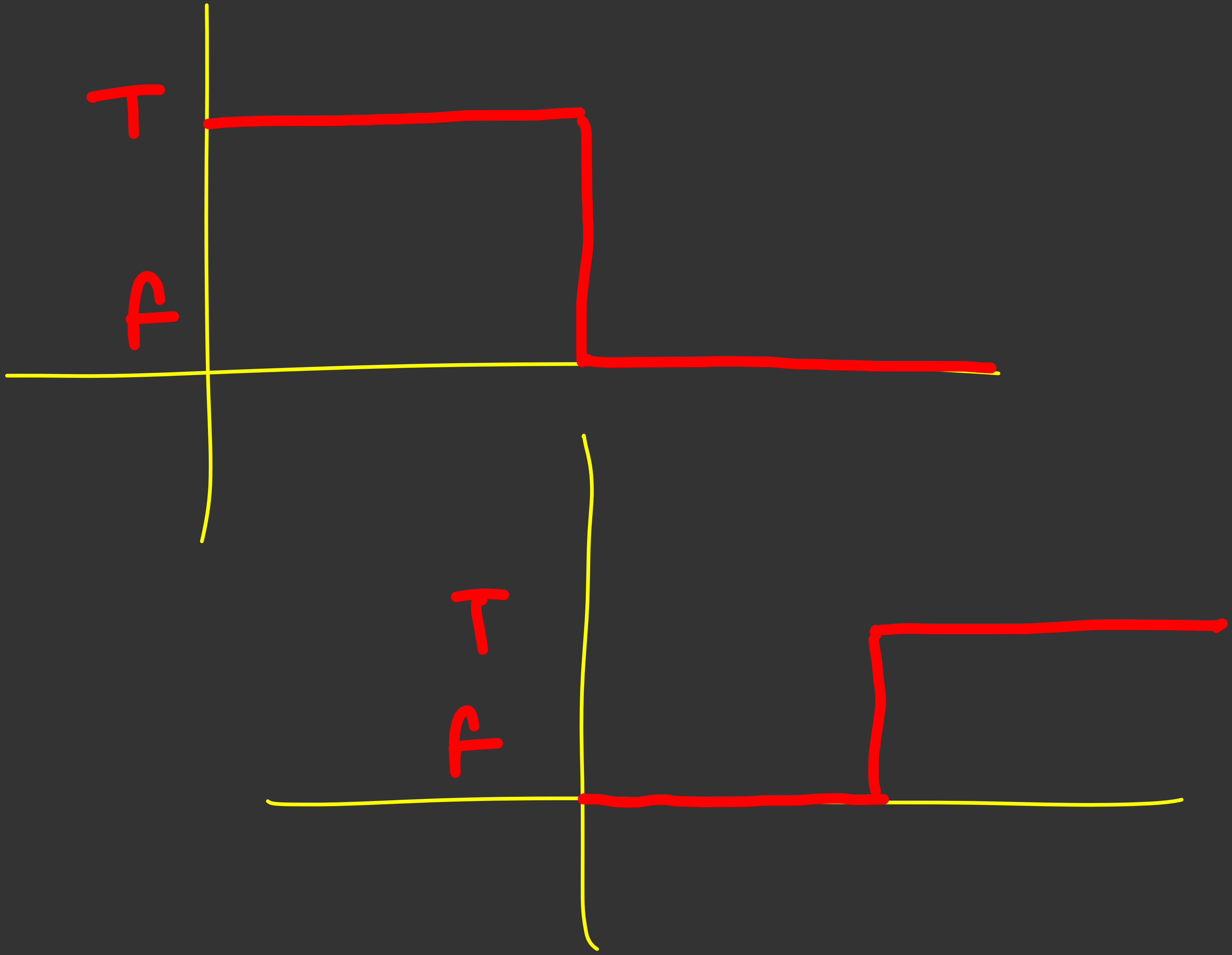
$T \to 1$

$f \to 0$

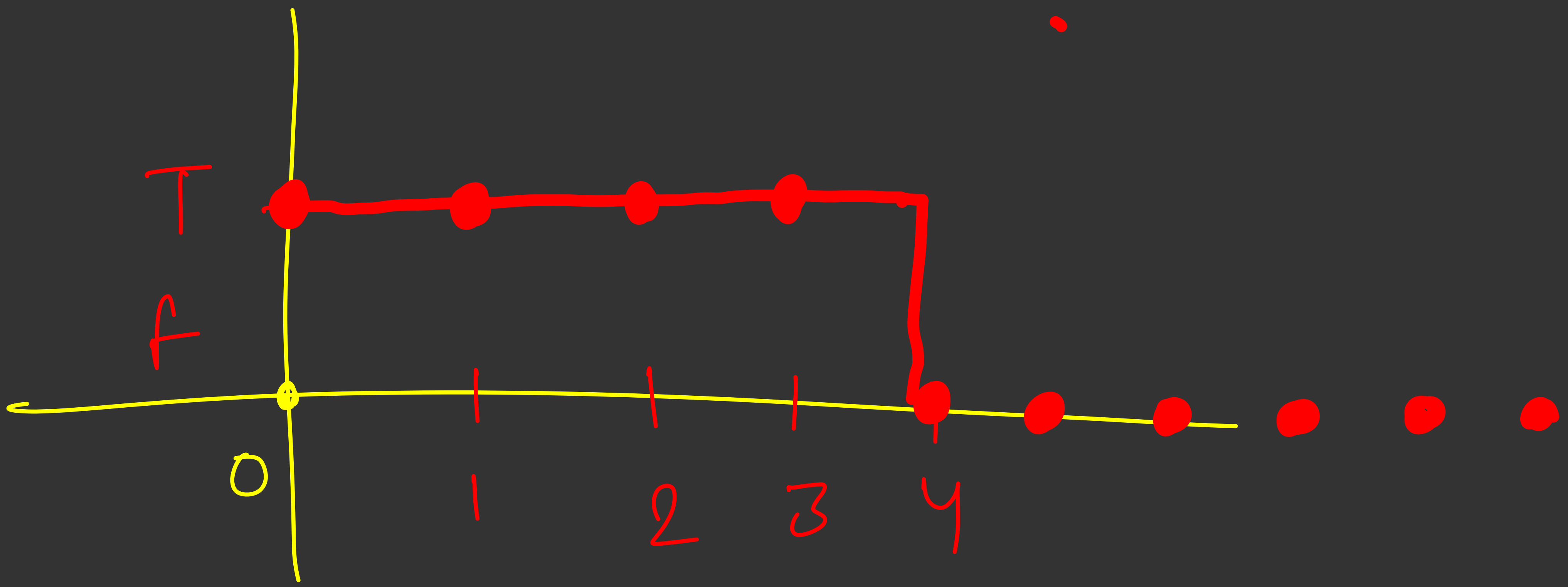
monotonic + predicate





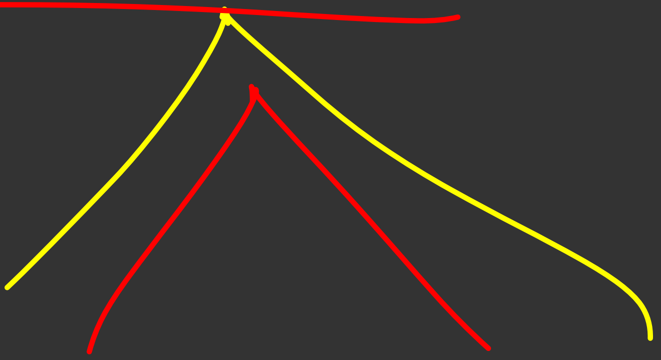

T
F
T
F

$$f(x) = \begin{cases} T & \text{if } \underline{x + 2 \leq 5} \\ f & \text{if } x + 2 > 5 \end{cases}$$



$\underline{f(x) = x^2 + 2}$ $\underline{\underline{y}}$

$$\boxed{x^2 + 2 = y}$$

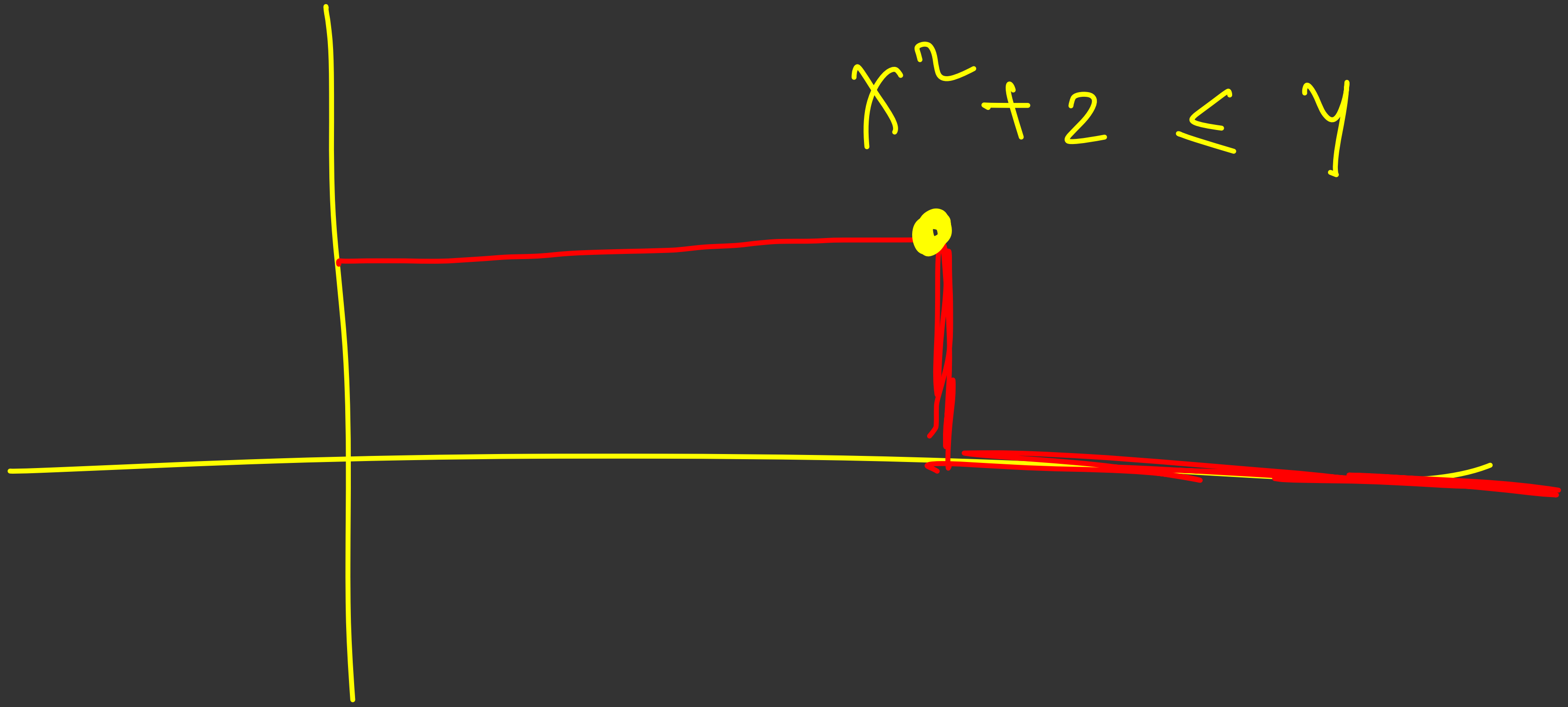


$$\boxed{x^2+2 \leq y} \qquad x^2+2 > y$$

T

F

$x^2 + 2 \leq y$

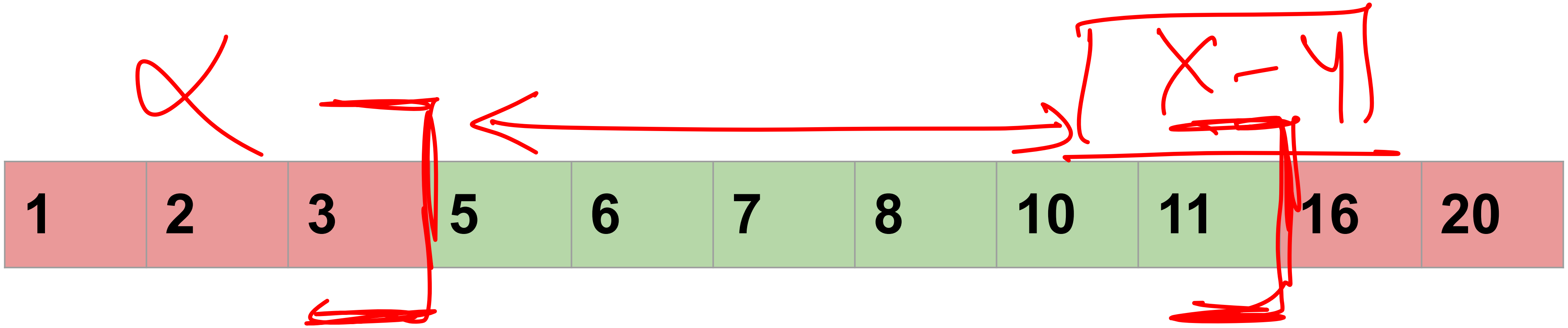


Binary Search Revision

$X \quad \boxed{\leq r}$

$Y \quad \boxed{< l}$

Find Number of elements in the range of l to r in a sorted array



1	2	3	5	6	7	8	10	11	16	20

Ex: Number of Elements in the range 4 to 12 are 6

L=4

R=12

Predicate Functions

Predicate functions are functions that return a single TRUE or False. You use predicate functions to check if your input meets some condition or not.

Examples:

- F(x) = True if x > 10 otherwise False
- F(x) = True if x is a character otherwise False
- F(x) = True if x^2 is an odd number otherwise False

Binary Searching on Answer

monotonic predicate

- Consider a predicate P defined over some ordered set S (the search space). The search space consists of candidate answers to the problem. In our case, a predicate is a function which returns TRUE or FALSE. We use the predicate to verify if a candidate answer is legal or not.

- Example: We have the set of numbers {1, 2, 3, 4, 5}. Our predicate function could be following:
  Return TRUE if the number is less than 3 and FALSE otherwise

Monotonic Predicate Function

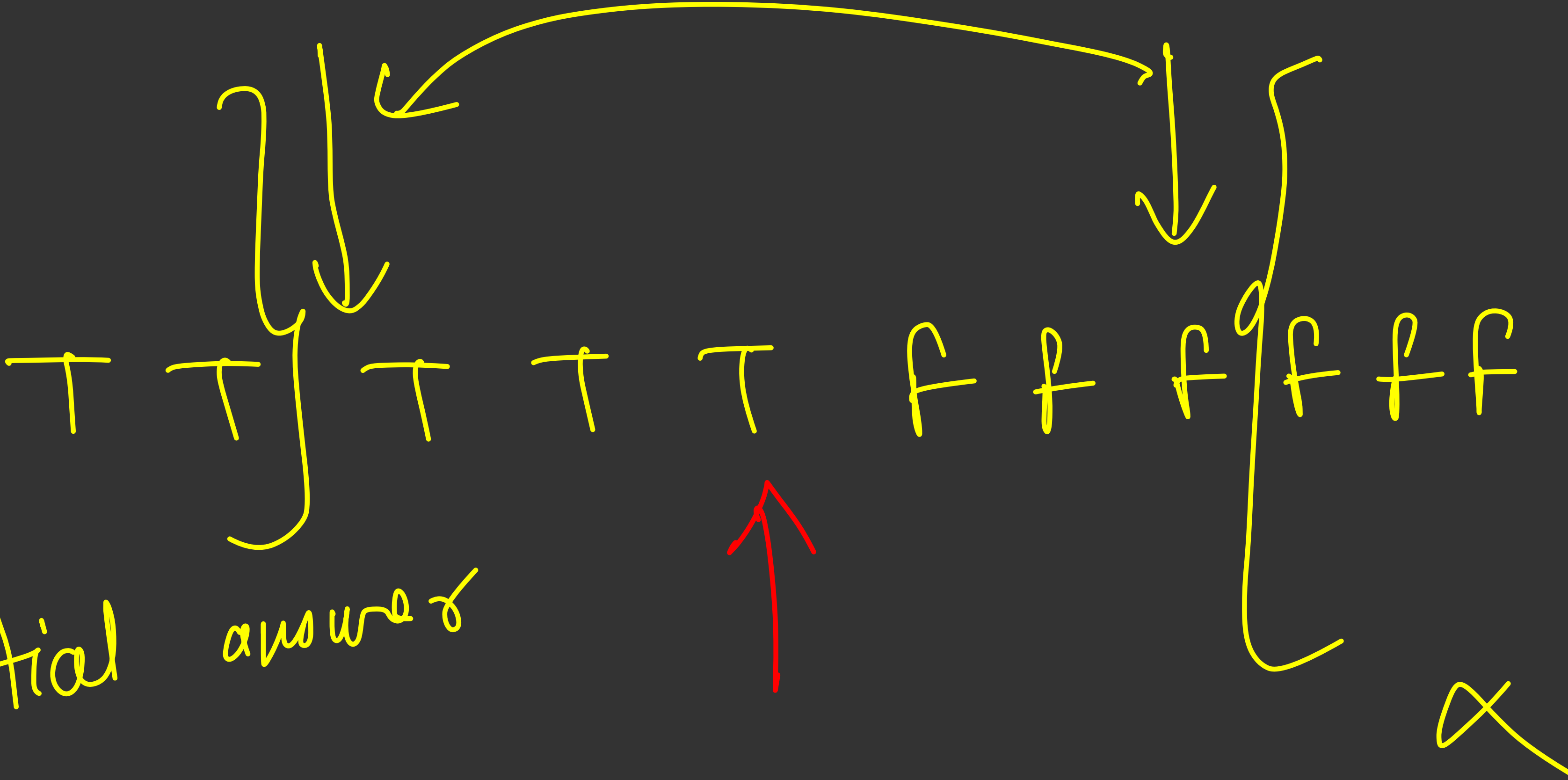
1. TTTTTTTTTFFFFFFFF ✓

T	T	T	T	F	F	F	F	F

2. FFFFFFFFTTTTTTTTT ✓

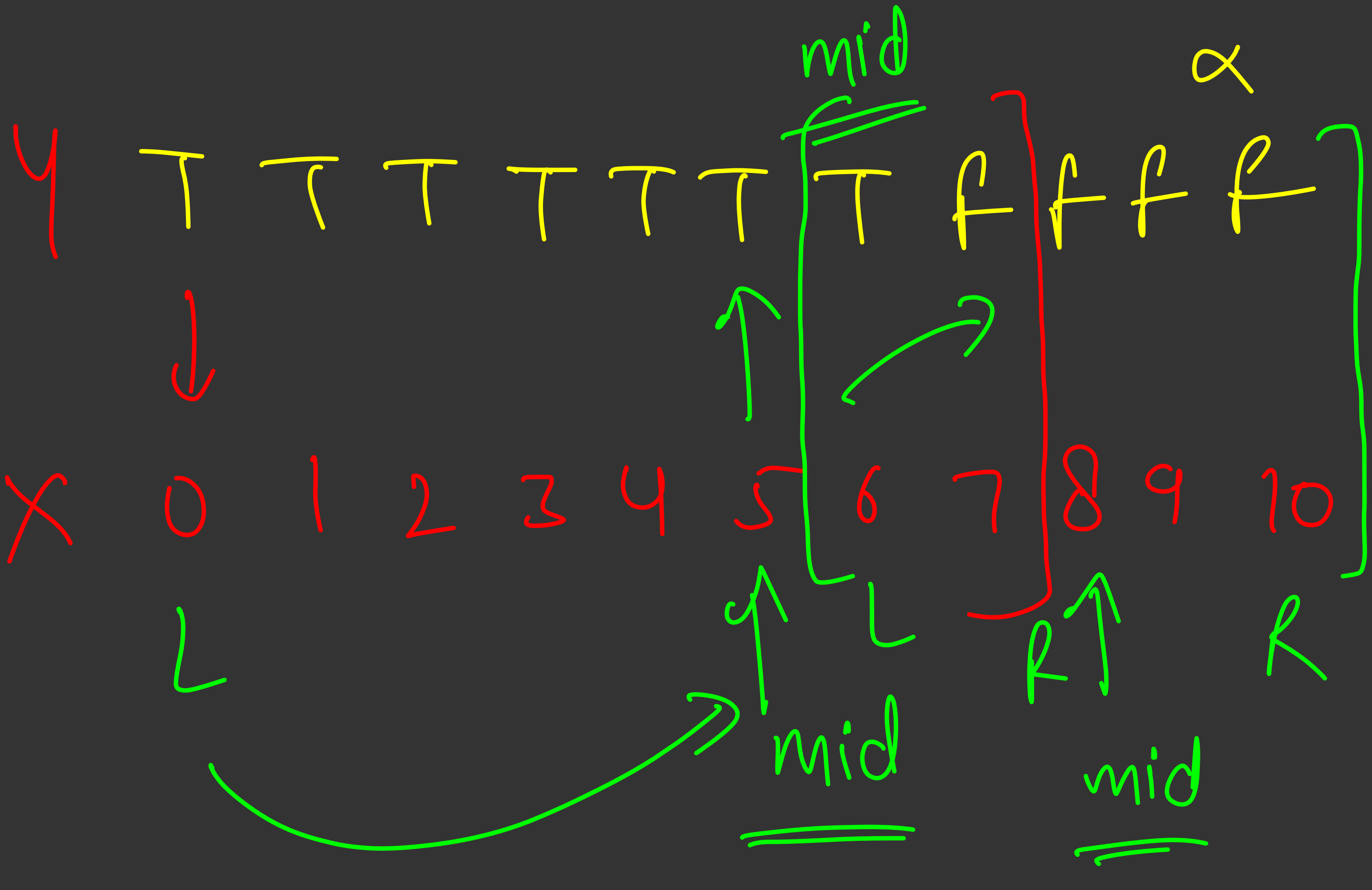
F	F	F	F	F	F	T	T	T

f(x) =



① Potential answer

② We can find a → better answers on the right



$$\boxed{\frac{L+R}{2}}$$

Ans = 5

Ans = 6

$$\boxed{6}$$

① finding out last true

② finding out first false

FairWorkload Problem

Given an array of workloads, split it among ‘k’ workers, such that the maximum work that any worker has to do is minimised (can’t change order of workloads).

Eg. [10, 20, 30, 40, 50, 60, 70, 80, 90]

$\boxed{x}$ $x+10$

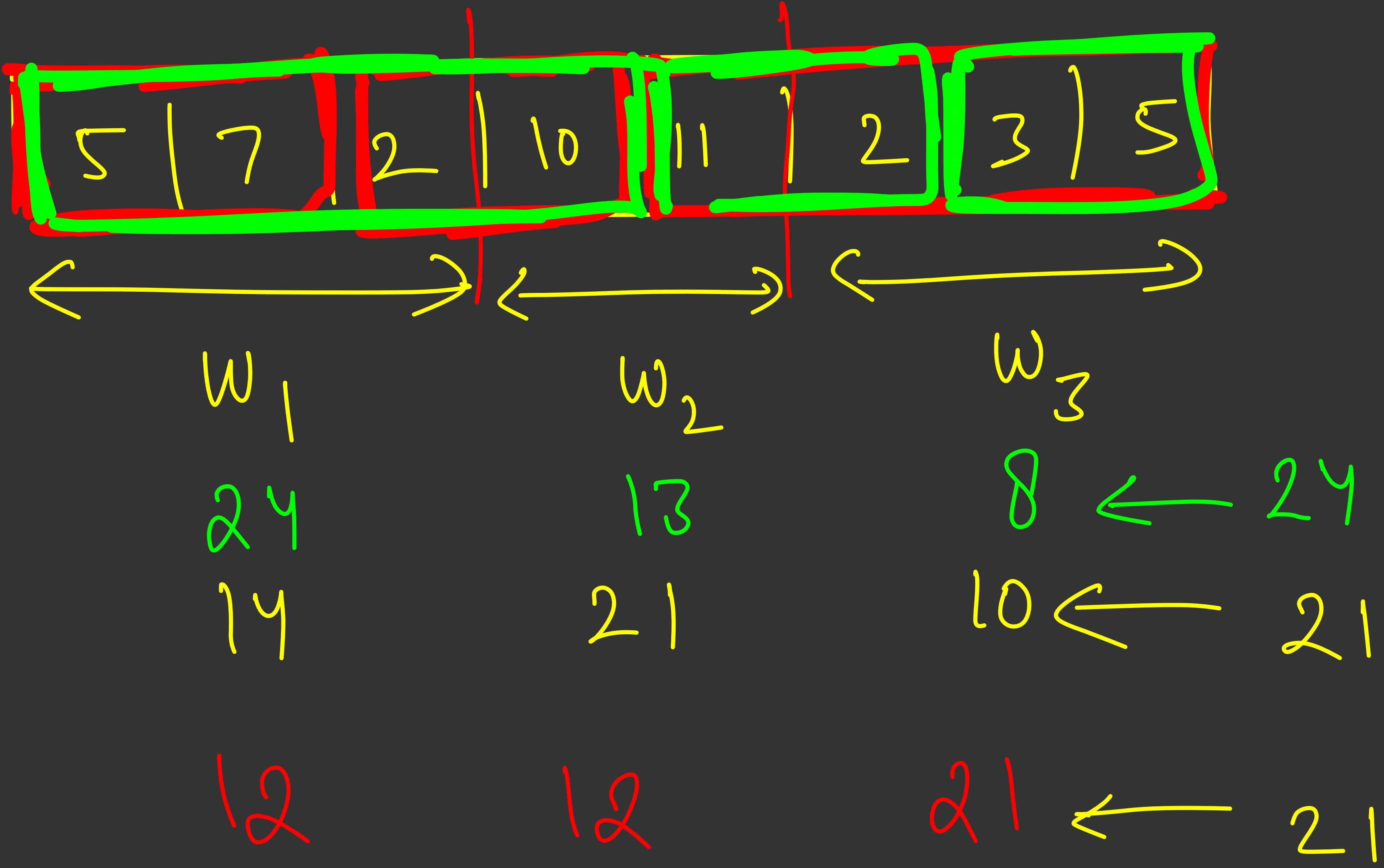
- Solution : 10 20 30 40 50 | 60 70 | 80 90

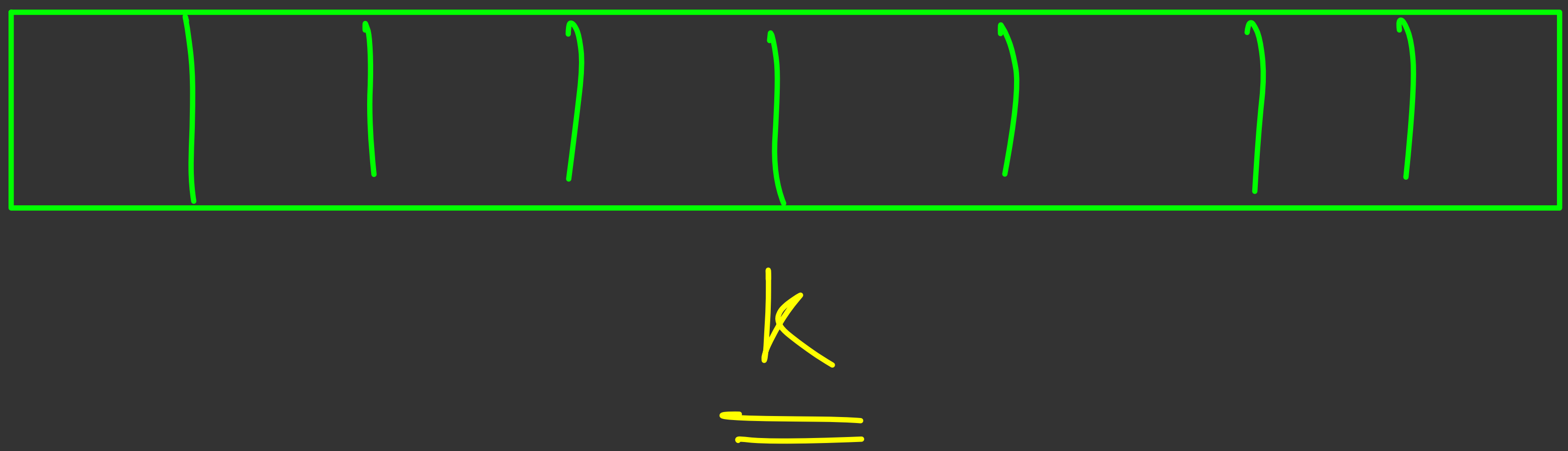
  First worker - 150,

  Second worker - 130,

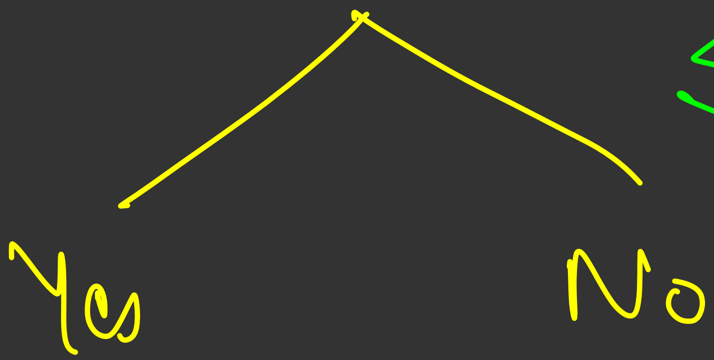
  Third worker - 170

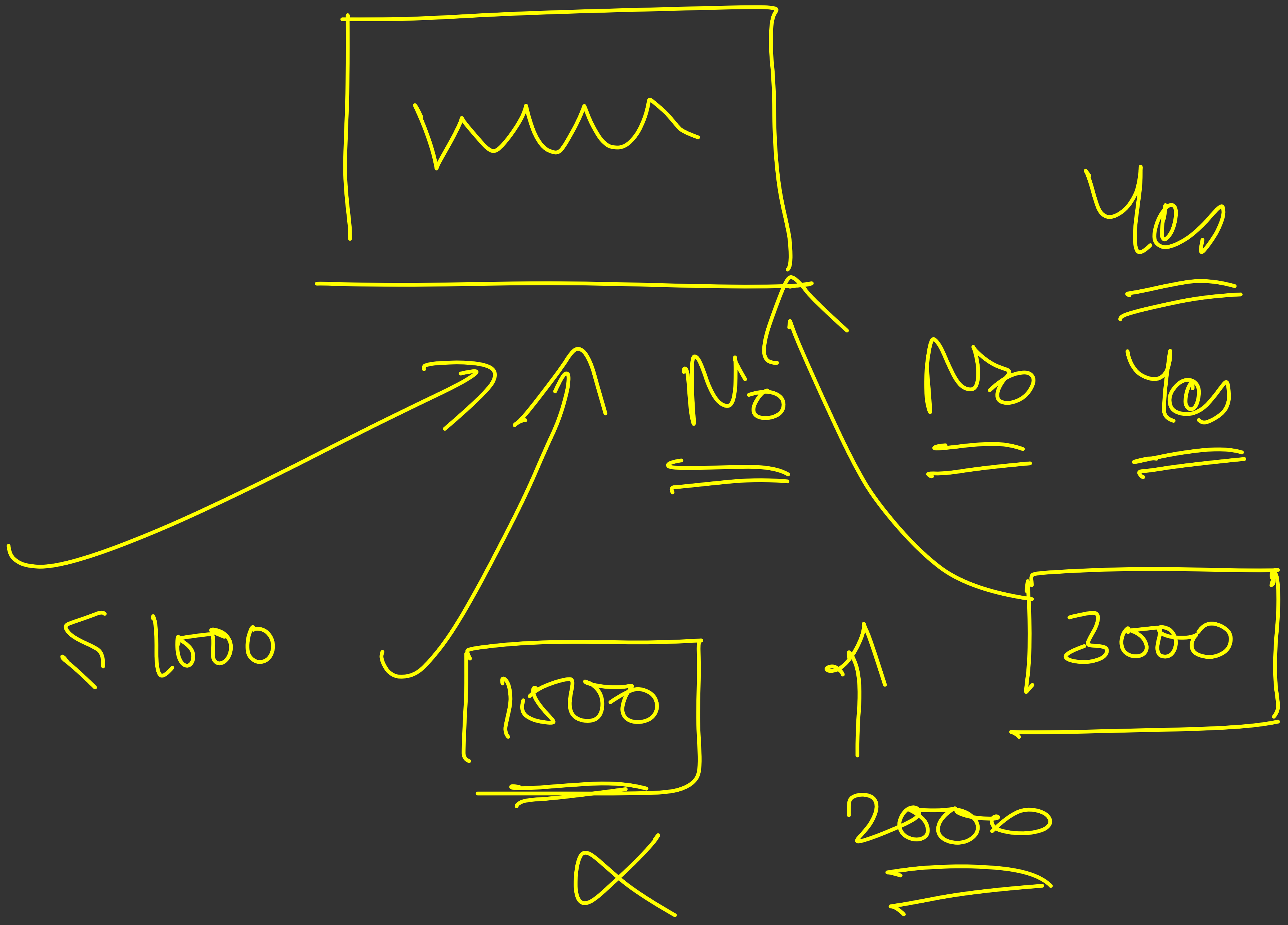
Is it possible to partition workload in a way that the highest workload of any worker is less than 170? Hence, answer is 170.


5
7
2
10
11
2
3
5
w1
w2
w3
24
13
8
24
14
21
10
21
12
12
21
21

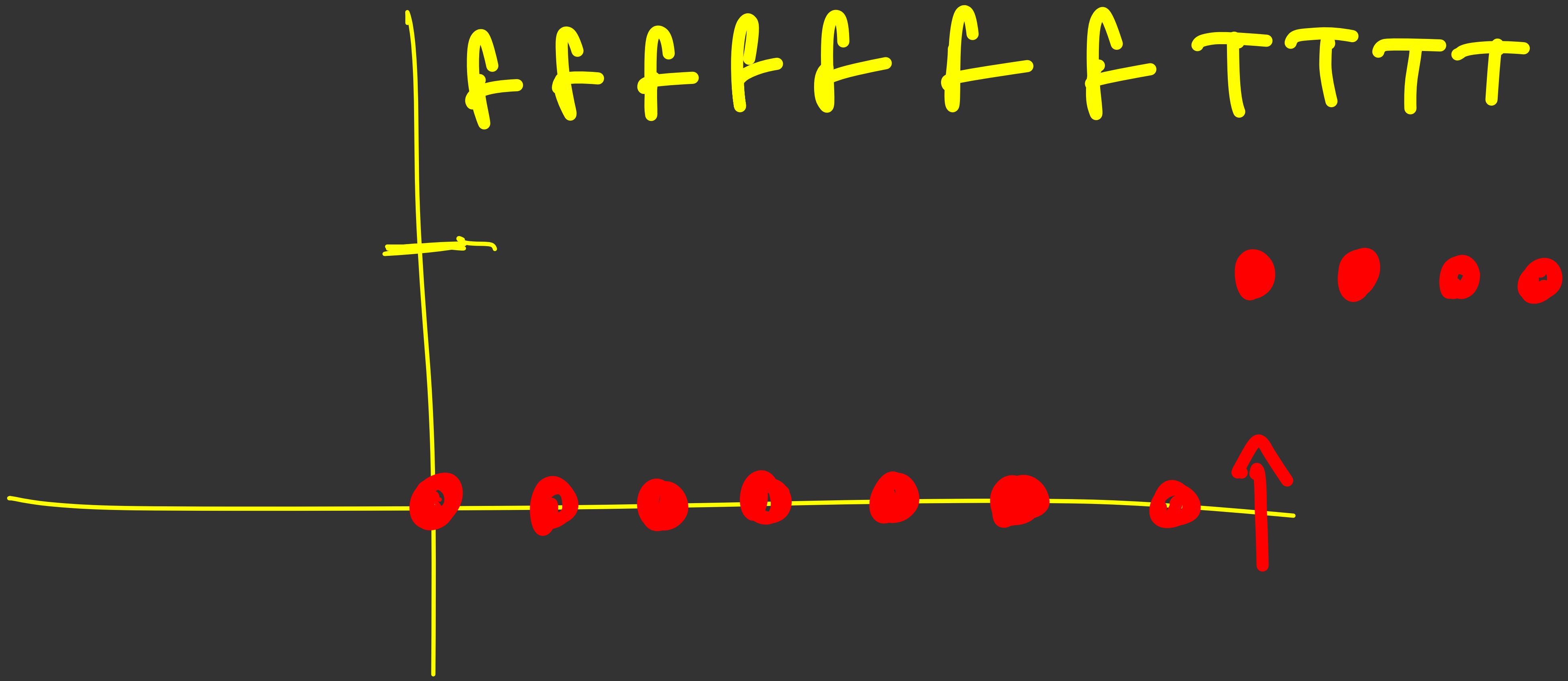


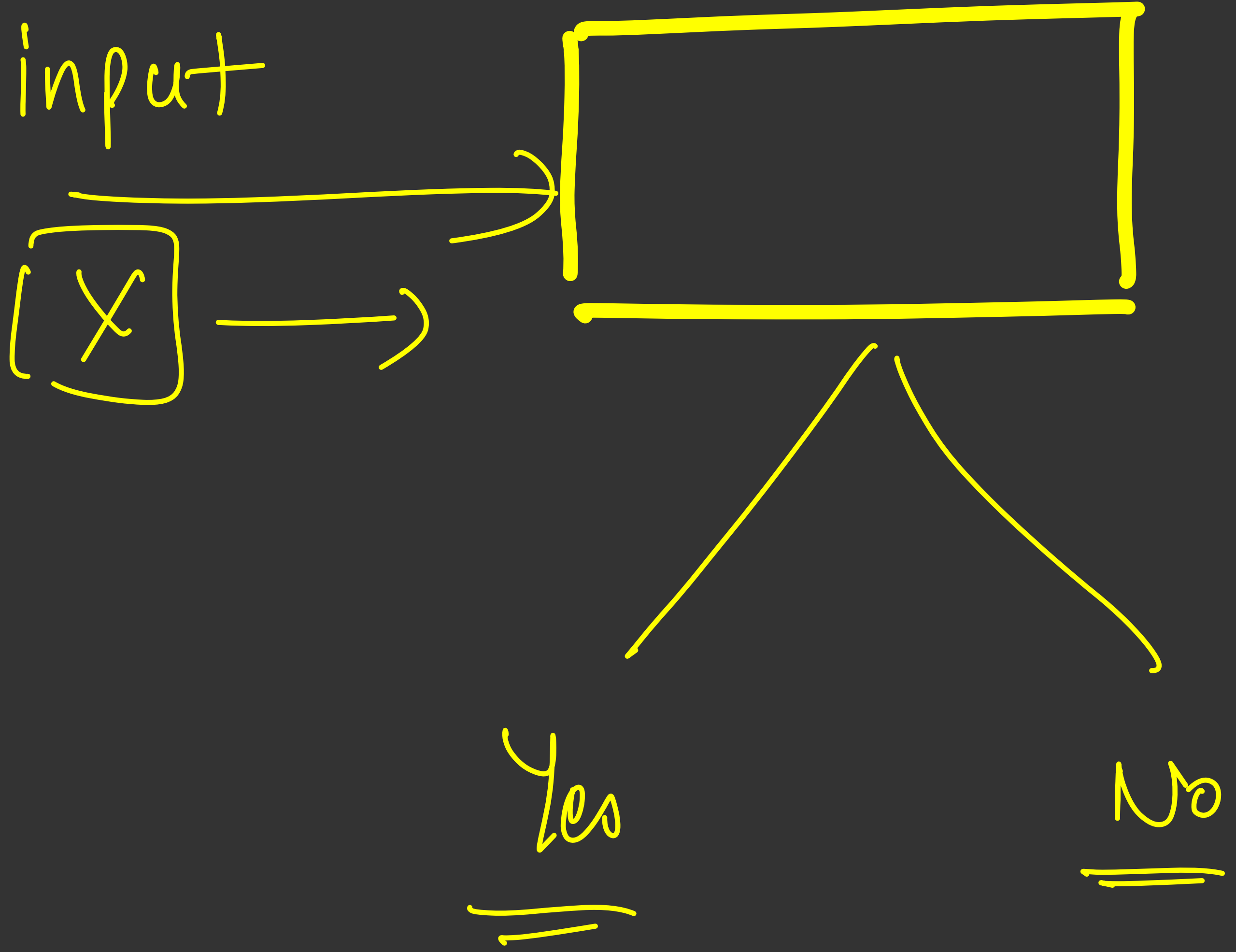
Can you divide the array into k parts such that the maximum workload on any worker is $\leq$ 1000




No
No
Yes
Yes
₹ 1000
1500
2000
3000
∝

Can I divide s.t highest workload $\leq X$





Checker

whether or not you can split the array into k subarrays such that max workload $\leq X$

2	3	10	6	5	9	2	10
✓	✓	✓	✓	✓	↑	↑	↑

$X = 28$

2 subarray

Yes

$w_1 = 2+3+10+6+5$

$w_2 = 9+2+10$

K

X

```
int  count = 1  ;
                                    w[i] > x
int  sum = 0

for ( int i=0 ; i<n ; i++) {
        if ( sum + w[i] <= x )
                sum += w[i]
        else
            sum = w[i]  }  }
            count ++
}
```

```
if ( count > k )
        return false
else
        return true
```

L

20	30	40	50	60

$k = 5$

$x \leq 10$

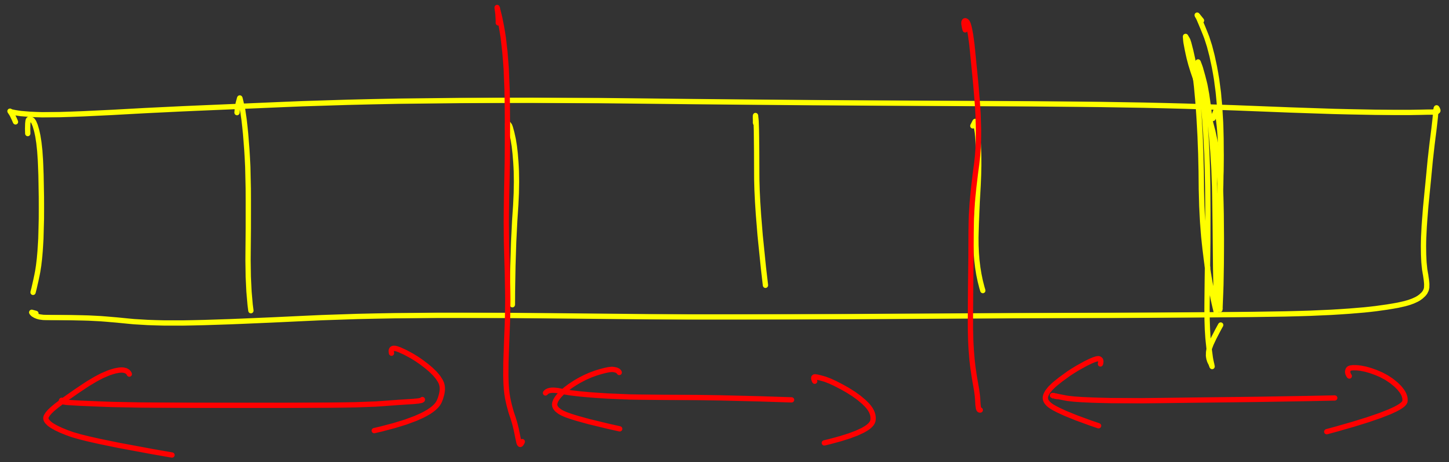
$\geq$

if you can asign to

5 worker

Can you also asign to

6 workers



3

4

$$x \to \{\ 1 \text{ ———————— } 10^{16}\ \}$$

$$\underbrace{\underline{x}}$$

$\underline{O(n)}$ o $\underline{O(\log(\text{search space}))}$

$\underline{1 \text{ ———— } 10^{16}}$

$$\boxed{\underline{\text{Sqrt}(X)}} \longrightarrow \underline{k}$$

$$x \Rightarrow \underline{1 \text{ to } 10^{16}}$$

$$10^{8}$$

$$X \leftarrow \underline{10}$$

$$\text{Sqrt}(x) = 3$$

$$\underline{k^2 \leq x}$$

$$\underline{r = x}$$

$$(10^{16})^2$$

k =	0	1	2	3	4	5	6
	T	T	T	T	f	f	f

x = 10

```
while (l ≤ r) {
    mid = (l+r)/2
    if( mid x mid ≤ x)
        ans = mid
        l = mid+1
    else
        r = mid-1 }
```