make observations

Mindset

make guesses

Greedy Algorithms

n

n=100

- Priyansh Agarwal

10	20	30	4	15

a_2 $\uparrow m$ a_1

$$a_1 + a_2 + a_3$$

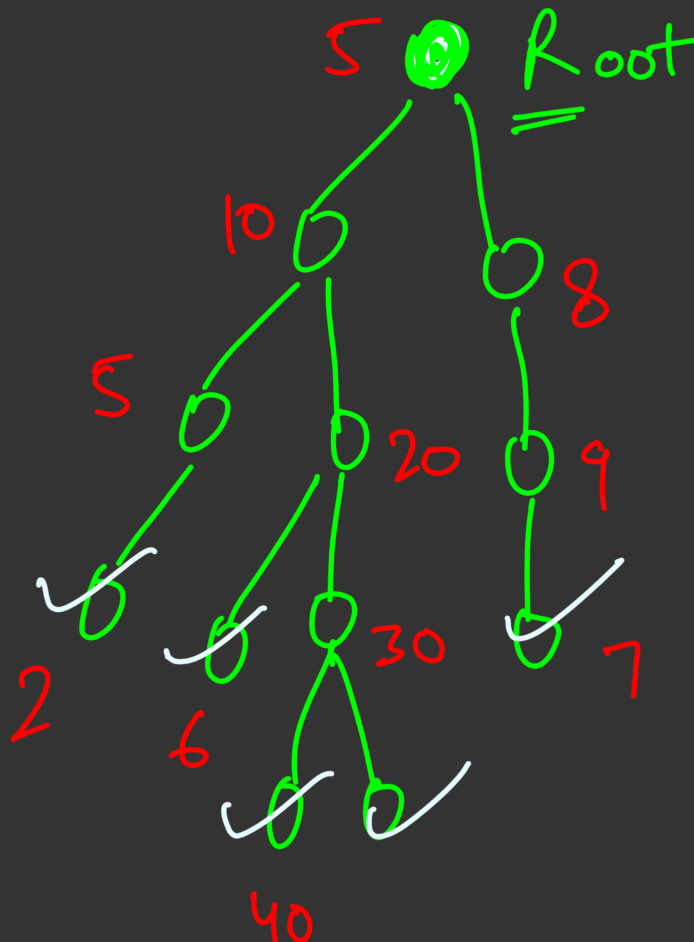
$$\boxed{\boxed{a_1 + a_2} + m} > \boxed{a_1 + a_2 + a_3}$$

What is a Greedy Strategy?

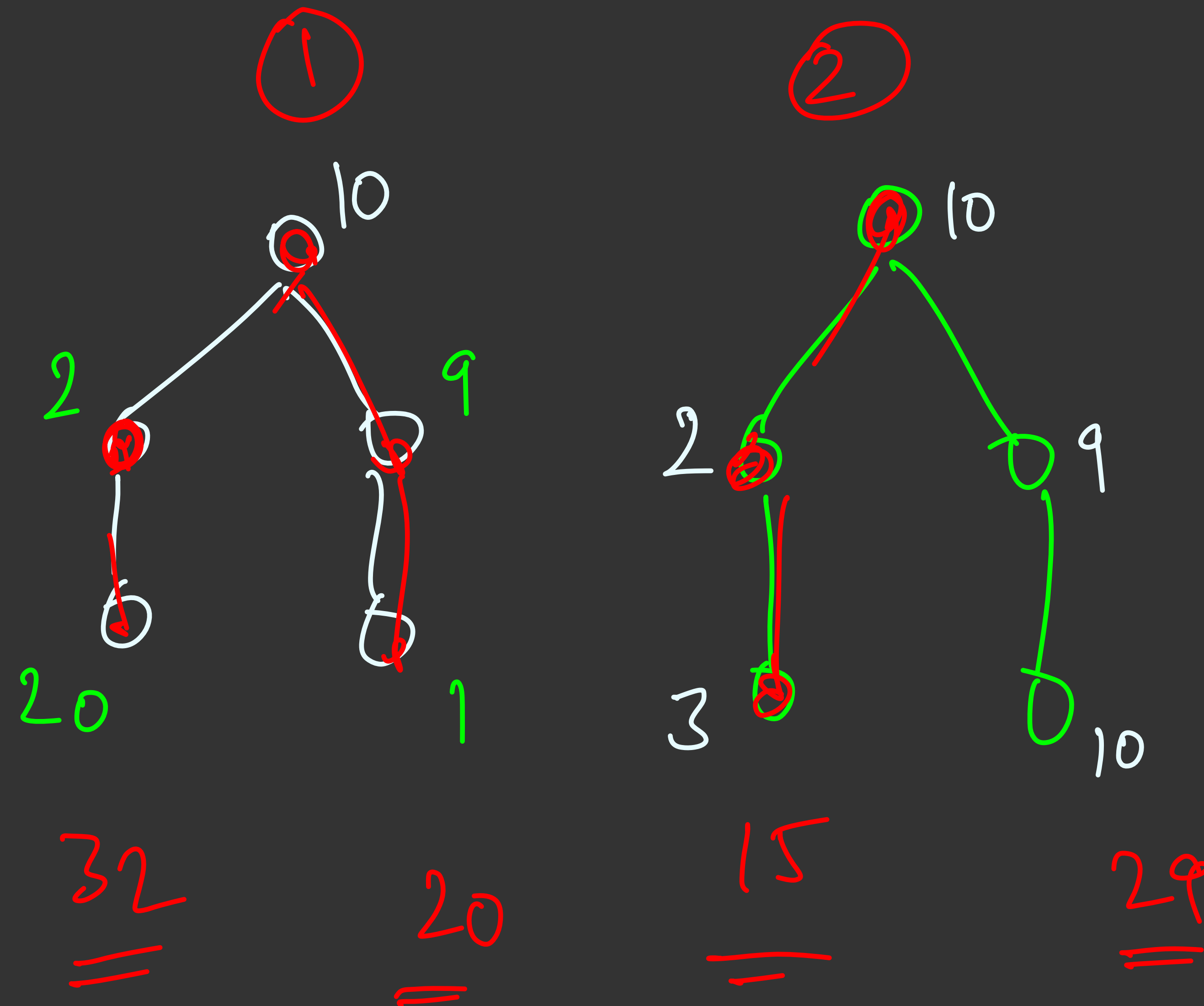
- A strategy that assumes that the best answer can be found using $\boxed{\text{some}}$ (not all) possibilities and only tries out those limited possibilities
- It involves coming up with a claim (greedy) and then proving it
- How to prove a Greedy Strategy?
  - Formal or intuitive proof
  - Trying out too many cases and failing to disprove

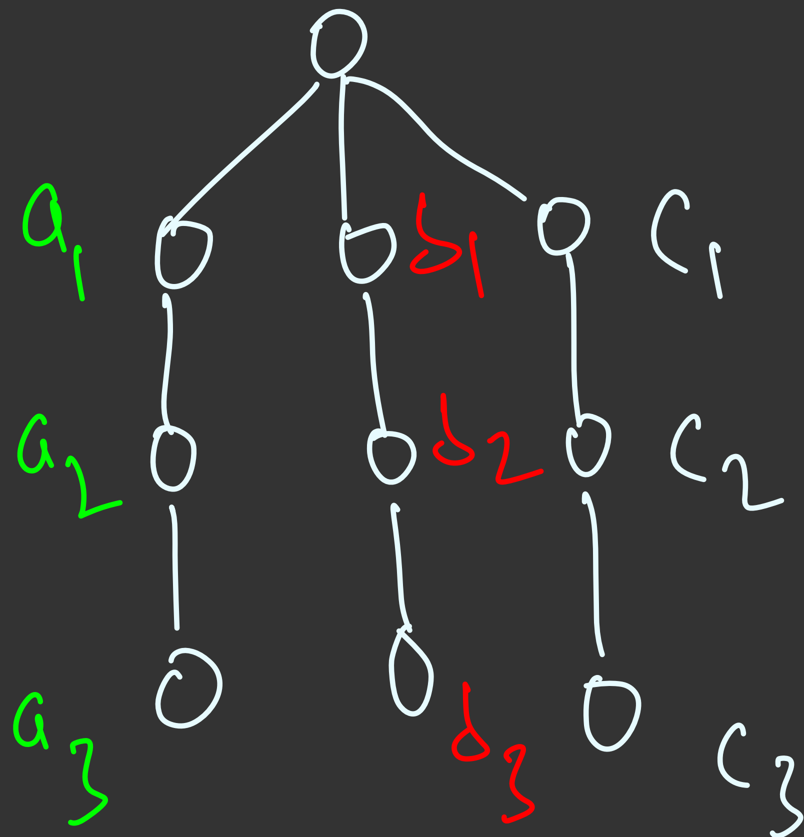
1

2



Root
5
10
8
5
20
9
2
6
30
7
40


1
10
2
9
20
1
32
20
2
10
2
9
3
10
15
29

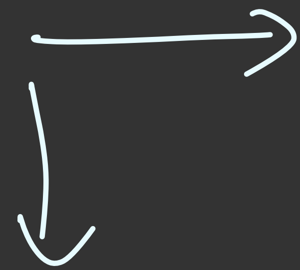


$$\boxed{a_2 \le b_2 \le c_2}$$

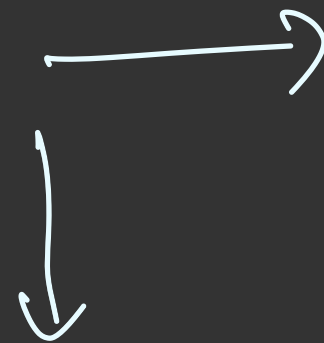
$$\boxed{a_3 \le b_3 \le c_3}$$

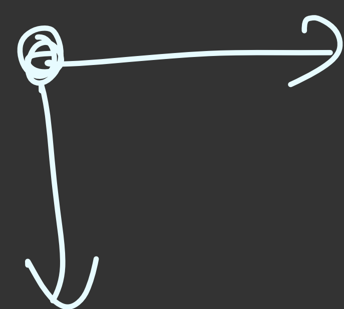
$$\boxed{a_1 \le b_1 \le c_1}$$

0	10	-	-
-	9	-	-
2	8	-	-
-	3	-	-
-	-	-	0



5	10	4
20	2	3
1	100	2





5	10	100
20	100	100
1	1	1

✗

fails

10

100

finite → calculatoru

1000

Let’s take some examples

1. Given an array, find a subset of K elements with maximum sum
2. Given an array, find the maximum difference between 2 elements
3. Given an array, find the minimum difference between 2 elements
4. Given an array reorder the array so that sum of a[i]*i is maximized

$a_1 \; a_2 \; a_3 \; a_4$

$1 < n \leq 10^5$

$C(n,k) \rightarrow$ exponential $< 2^n$

$$\boxed{a_1 \quad a_2 \quad - \, \ldots \quad a_n}$$

$$a_i \leq a_{i+1}$$

$|a_n - a_1|$ is the biggest possible diff b/w 2 elements

$a_1 \quad a_2 \quad \ldots \ldots \quad a_n$

$a_i \qquad a_j \qquad j > i$

$(a_j - a_i) \quad B \qquad a_n$

$\boxed{a_j \le a_n}$

$(\underbrace{a_n - a_i}_{A})$

$(A - B)$

$(a_n - a_i) - (a_j - a_i) \geq 0$

$X \quad \boxed{a_n - a_i}$

$i \mathrel{!}= 1$

a_1

$Y \quad \boxed{a_n - a_1}$

$X - Y \leq 0$

$\boxed{Y \geq X}$

$(X - Y) \longrightarrow (a_n - a_i) - (a_n - a_1)$

$\longrightarrow a_1 - a_i$

$\longrightarrow \boxed{\leq 0}$

$$A \rightarrow a_1 \quad a_2 \quad a_2 \; \ldots \ldots \; a_n$$

$$a_1 \le a_2 \le a_3 \ldots \ldots \le a_n$$

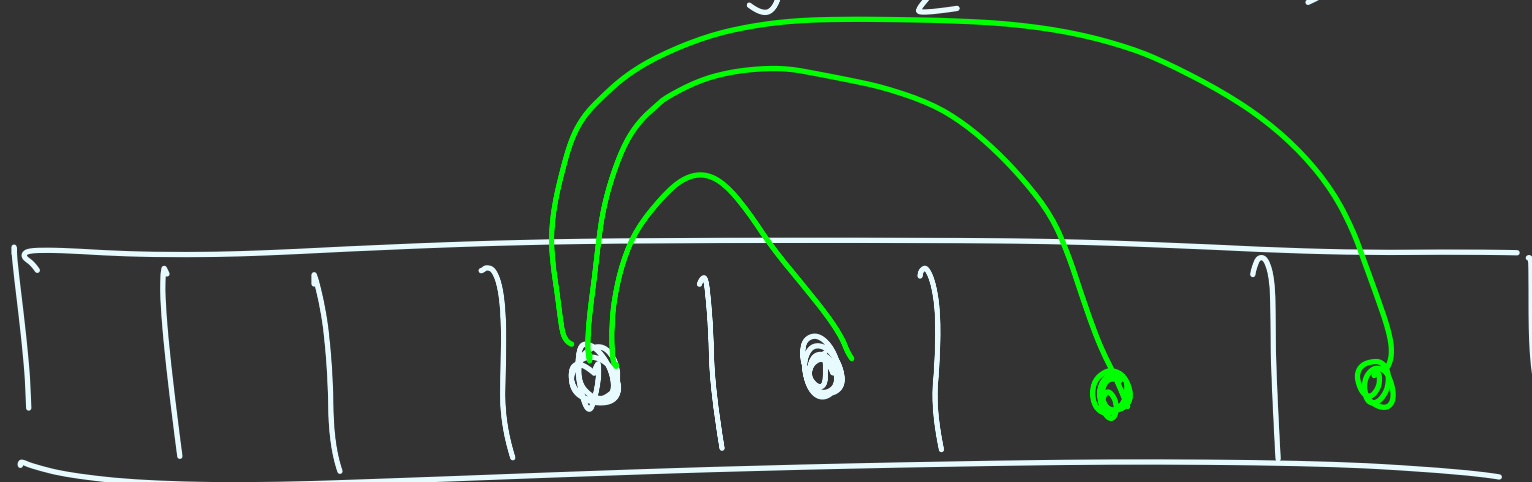
$$\boxed{a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7}$$

$$(a_6 - a_2) \ge (a_5 - a_2) \ge (a_4 - a_2) \ge (a_3 - a_2)$$

$a_i \qquad a_j$

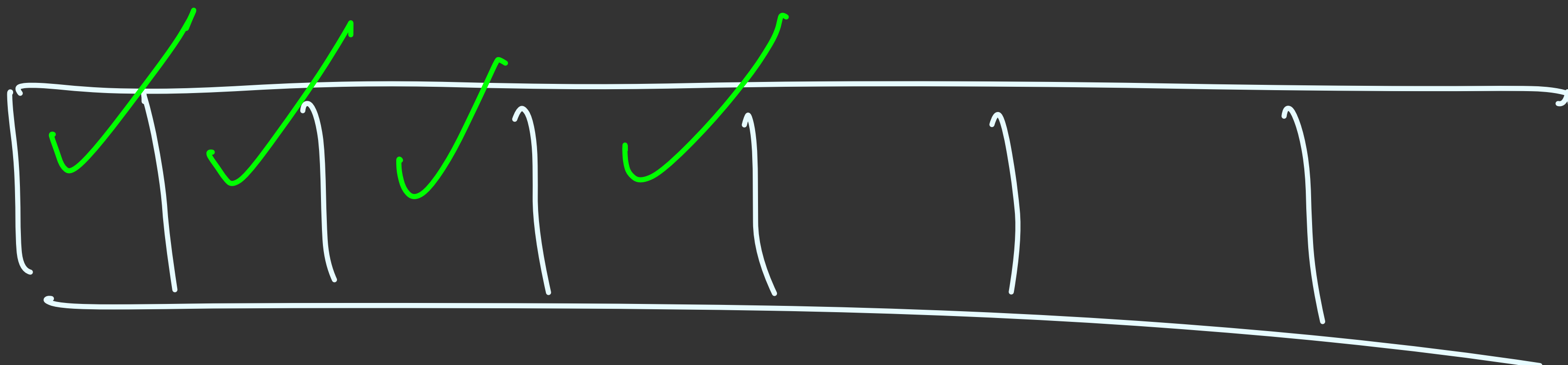
$$(a_j - a_i) \geq (a_{j-1} - a_i)$$

$$\geq (a_{j-2} - a_i) \ldots\ldots$$



$a_i \quad a_{i+1}$

a_i ✓✓ $\qquad \underline{\underline{a_{i+1}}}$



$(a_2 - a_1) \quad O(n) \quad (a_3 - a_2)$

$\geq$

$(a_4 - a_3) \quad \cdots\cdots \quad (a_n - a_{n-1})$

$$\boxed{a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7}$$

$$a_1 \leq a_2 \leq a_3 \ldots \leq a_7$$

$$(a_1 \times 1) + (a_2 \times 2) + (a_3 \times 3) \ldots$$

$$+ (a_n \times n)$$

$a_1 \quad a_2 \quad a_3 \quad \ldots\ldots \quad a_n$

$$\sum_{k=1}^{n} a_k \times k$$

$a_i \le a_{i+1}$

$$a_1 \quad a_2 \quad a_3 \quad \text{———} \cdots \quad a_n$$

$$\leq \quad \leq \quad \leq$$

$$a_i \qquad a_j$$

$$a_j \geq a_i \qquad j > i$$

$$Sum1 = \underline{a_1 x_1 + a_2 x_2 \cdots} a_i x_i \underline{\cdots} a_j x_j \cdots \underline{a_n x_n}$$

$$\downarrow$$

$$Sum2 = \underline{a_1 x_1 + a_2 x_2 \cdots} a_j x_i \underline{\cdots} a_i x_j \cdots \underline{a_n x_n}$$

Sum1 − Sum2

$$(a_i \times i + a_j \times j) - (a_i \times j + a_j \times i)$$

$$i(a_i - a_j) - j(a_i - a_j)$$

$$= \underline{\underline{(i-j)}}(a_i - a_j) \qquad a_j > a_i$$

$$= \underbrace{\boxed{<0} \quad \boxed{<0}}_{} \rightarrow \underline{\underline{\geq 0}}$$

a_1	b_1
a_2	b_2
a_3	b_3
a_4	b_4
a_5	b_5
Sort A	Sort B

$$\sum_{i=1}^{n} a_i \times b_i$$

-10 $\quad$ -2 $\quad$ 1 $\quad$ 2

-6 $\quad$ 5 $\quad$ 10 $\quad$ 20

-10^9 $\quad$ 1 $\quad$ 2 $\quad$ 3 $\quad$ 10^5

$\boxed{10^9}$ $\quad$ 2 $\quad$ 3 $\quad$ 5 $\quad$ 6

Coin Change Problem: Link

Given the following denominations of coins [1, 5, 10, 50, 100] with unlimited supply, find the minimum number of coins required to make a sum of X (1 <= X <= 1e9)

$$(a_{i+1} \% a_i) = 0$$

Examples:

X = 125 can be made with 100 + 10 + 10 + 5

X = 256 can be made with 100 + 100 + 50 + 5 + 1

1

5

10

50

100

$\boxed{x}$

$\boxed{\text{3 coins of 5}}$ 4 coins

and $\boxed{\text{1 coin of 1}}$

$\boxed{\text{1 coin of 10}}$

+ $\boxed{\text{1 coin of 5}}$ + $\boxed{\text{1 coin of 1}}$

x

1

5

10

50

100

$$\boxed{d_1 \quad d_2 \quad d_3 \quad \ldots\ldots \quad d_n}$$

$$d_1 \le d_2 \le d_3 \ldots\ldots \le d_n$$

$$\boxed{d_1 = 1}$$

$$(d_{i+1}) \% \, d_i = 0$$

$$\boxed{\times}$$

1

5

10

50

100

$\boxed{60}$

10 + 50

10 + 10 + 10 + 10 + 10 + 10

6 denominations = 10

= 1 denomination of 50

+ 1 den of 10

$$d_{i+1} \% d_i = 0$$

$$d_i \times \underline{\underline{k}} = d_{i+1}$$

$\underline{\underline{m}}$ dlmen of d_i

$\boxed{m > k}$

$\underline{(m-k) \text{ dm of } d_i}$

$\underline{1 \text{ dm of } d_{i+1}}$

$d_i \qquad d_{i+1}$

$$\boxed{d_i \times k = d_{i+1}}$$

$$X = \underline{m \cdot d_i}$$

$\underline{\underline{m > k}}$

$\underline{\underline{m}}$

$\underline{(m-k)d_i} + \underline{1\, k d_i}$

$\underline{m-k+1}$

$$\underline{(m-k)\, d_i + 1\, d_{i+1}}$$

260

50, 10, 100

$10 \times 5 = 50$

$26(10) \rightarrow 21(10) + 1(50)$

$50 \times 2 = 100$

$16(10) + 2(50)$

$11(10) + 3(50)$

$6(10) + 4(50)$

$1(10) + 5(50)$

$$1(10) + 3(50) + 1(100)$$

$$1(10) + 1(50) + 2(100)$$

Coin Change Problem

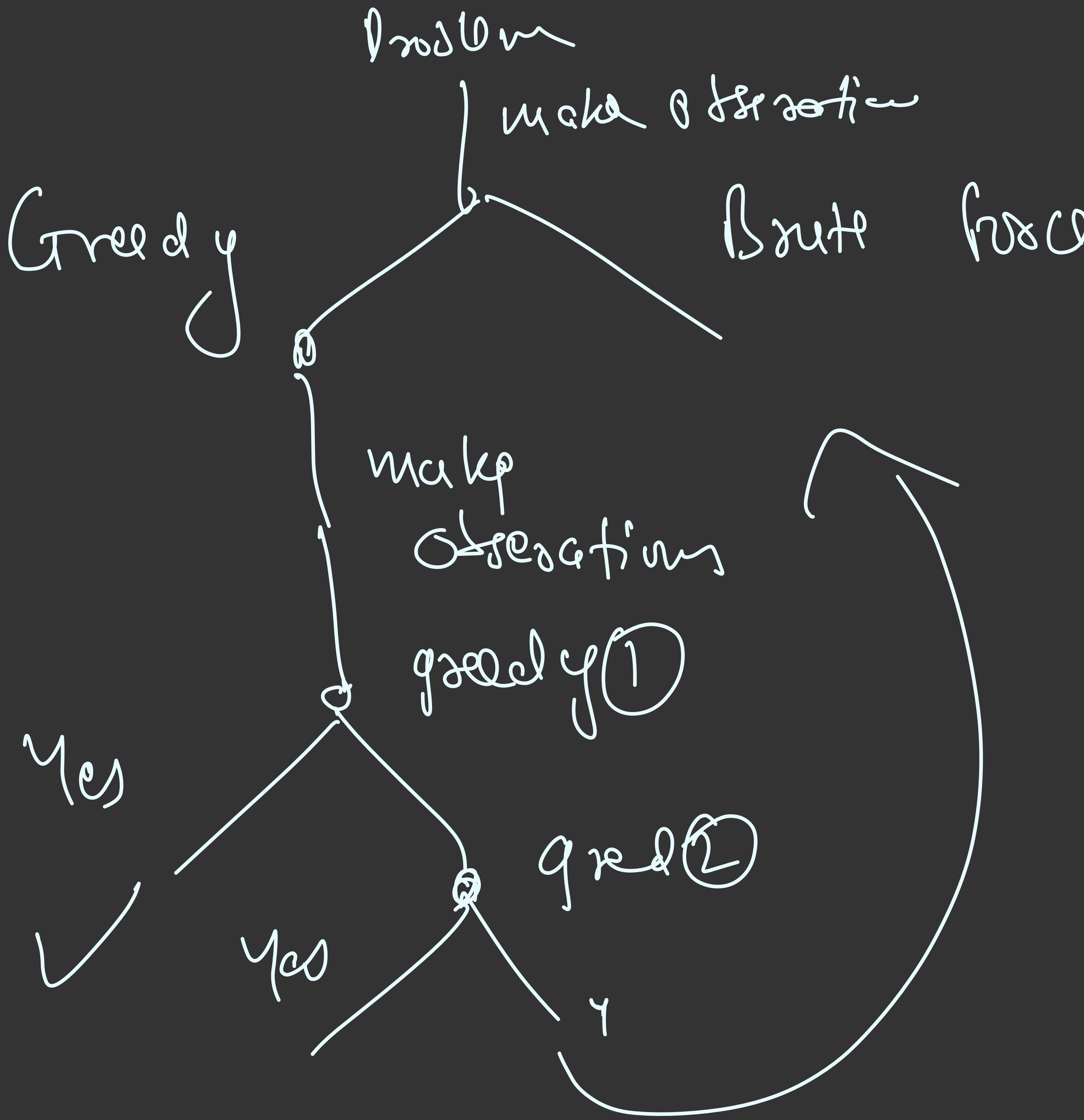
DP

Does the greedy strategy always work?

Let's change the denominations to [1, 8, 10]. Construct X = 16

Greedy strategy -> [10, 1, 1, 1, 1, 1, 1]

Optimal strategy -> [8, 8]



Problem
make observation
Greedy
Brute force
make observations
greedy 1
Yes
Yes
greed 2
Y

Coin Change Problem

When does the greedy strategy work then?

If we can make a[i + 1] with a[i], we should always first try to use a[i + 1] as many times as possible and only then use a[i], when required coin change is lesser than a[i + 1]

5 10 15 20 25 30

$x \rightarrow \underline{\underline{5k}}$

1 2 5 6

$x \rightarrow x/5$

4 2

5 1 5 is not divisible by 2

5 10 25 30

$x = 11$

$x \to 65$

1 2 5 6

$x \to 15$

$6 \to 2$ time

$5 \to 3$ time

$2 \to 1$ time

$1 \leftarrow 1$ time

when $a_{i+1} \% a_i == 0 \longrightarrow$ Greedy ✓✓

$\downarrow$

✗

Maximum Product Problem

Given N (1 <= N <= 1e9), find two numbers A and B such that A + B = N and A * B is maximized.

$A \rightarrow x$

$B \rightarrow N - x$

Example:

N = 5, (1 * 4 = 4), (2 * 3 = 6), (3 * 2 = 6), (4 * 1 = 4)

$(x \times (N-x))$

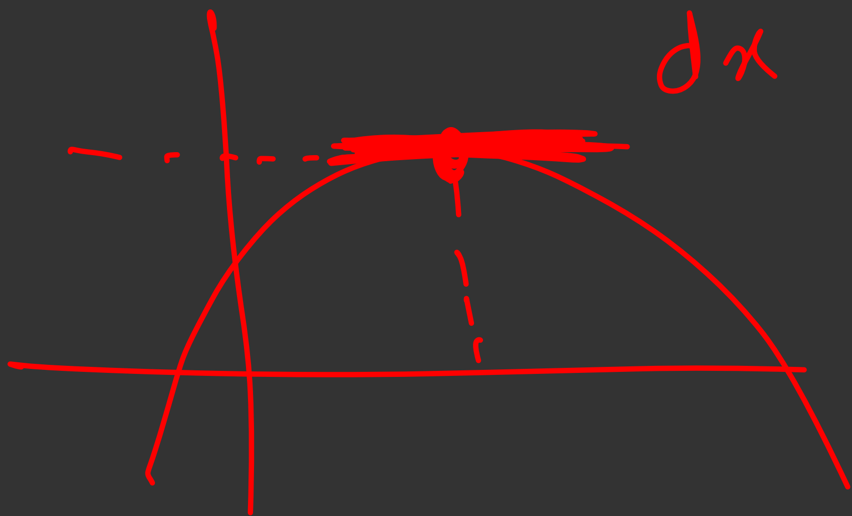
N = 8, (1 * 7 = 7), (2 * 6 = 12), (3 * 5 = 15), (4 * 4 = 16), (5 * 3 = 15)...

$(Nx - x^2)$

$$\frac{d\,(Nx - x^2)}{dx} = N - 2x$$

$$= 0$$

$$\underline{\underline{x = N/2}}$$



$$N = \underline{\underline{26}}$$

$A = 8, \quad B = 8 \qquad \underline{\underline{A \cdot B}}$

$A = 8 - 1 \qquad B = 8 + 1 \qquad \underline{\underline{8^2}}$

$A = 8 - 2 \qquad B = 8 + 2$

$$\underline{\underline{N}}$$

$$A = \underline{\underline{N/2}} \qquad B = \underline{\underline{N/2}}$$

$$A = \underline{(N/2 - x)} \qquad B = \underline{(N/2 + x)}$$

$$(N/2 - x)(N/2 + x) \to \boxed{(N/2)^2 - x^2}$$

$$\to (p-q)(p+q) \to p^2 - q^2$$

$$\to p^2 - pq + pq - q^2 \to p^2 - q^2$$

$N = 19$

$N \to (N-1)/2$

$N \to (N+1)/2$

$A = 9$ $B = 10$

$A = 9.5$ $B = 9.5$

$A = 8$ $B = 11$

$A = 9$ $B = 10$

$$\underline{\underline{N}}$$

$N/2 \qquad N/2$

$(N/2 - x) \times (N/2 + x)$

$$\left[N/2\right]^2 - \underline{\underline{x^2}}$$

$x\uparrow \qquad \downarrow$

$\underline{\underline{19}}$

$9.5 \quad 9.5$

$0.5 \quad 0.5$

$1.5 \quad 1.5$

Maximum Product Problem

11 20 27 32 35 36 35 32 27 20 11

Can we observe a pattern?

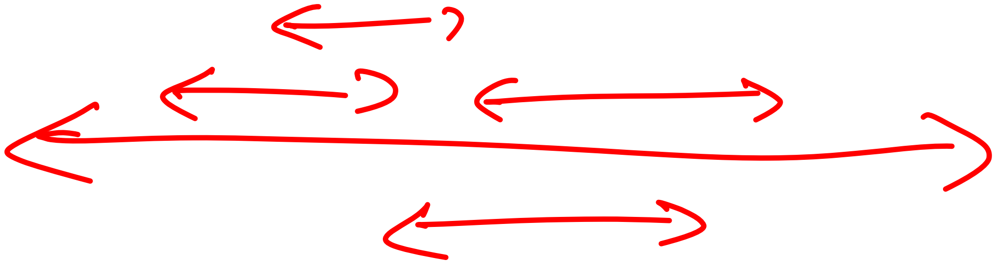
N = 12

(1 * 11 = 11), (2 * 10 = 20), (3 * 9 = 27), (4 * 8 = 32), (5 * 7 = 35), (6 * 6 = 36),
(7 * 5 = 35), (8 * 4 = 32), (9 * 3 = 27), (10 * 2 = 20), (11 * 1 = 11)

How to formally prove this?

Activity Selection Problem: [Link]

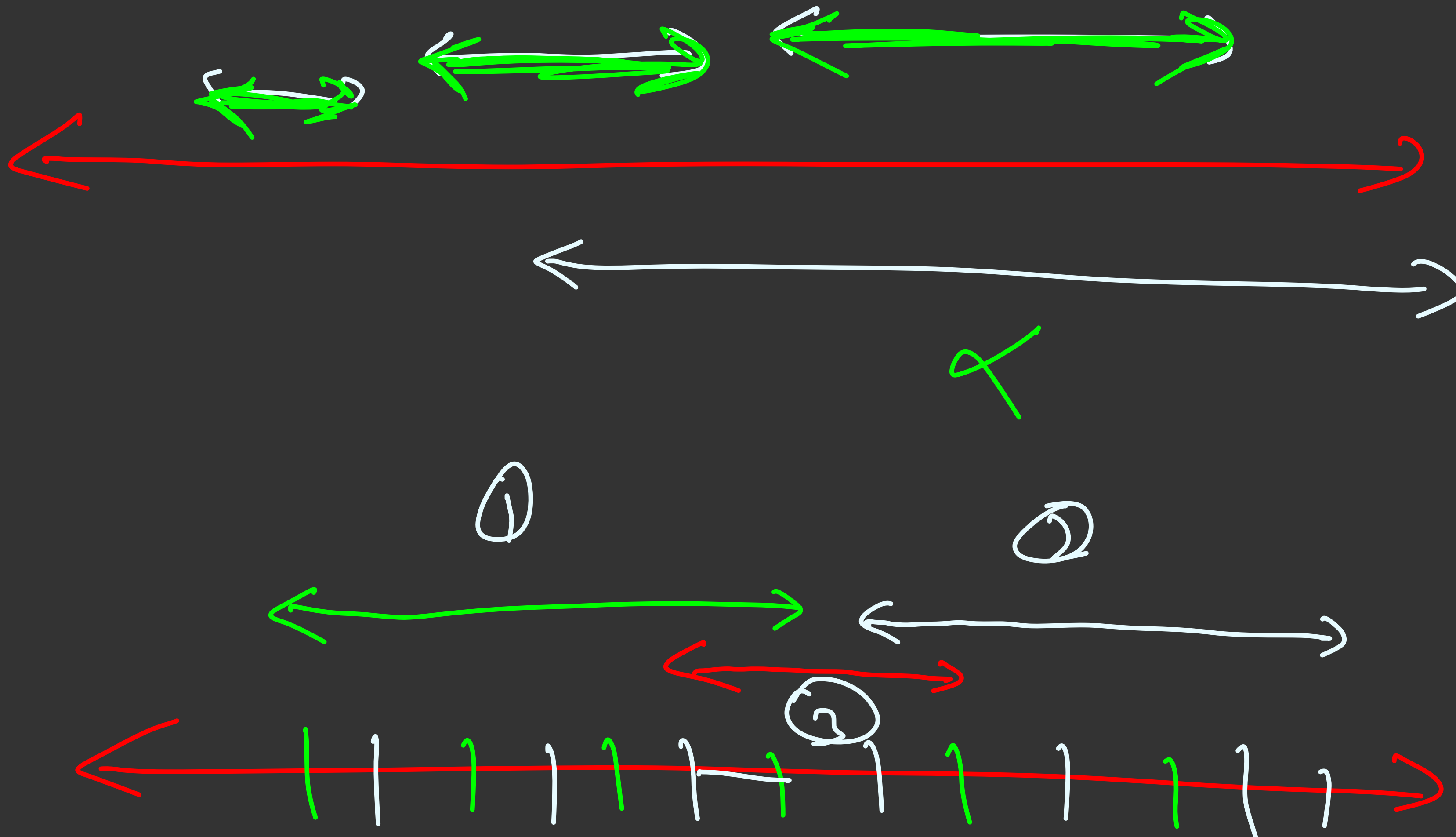
Given N activities with their start and finish day. Select the maximum number of activities that can be performed, assuming that you can only work on a single activity at a given day.

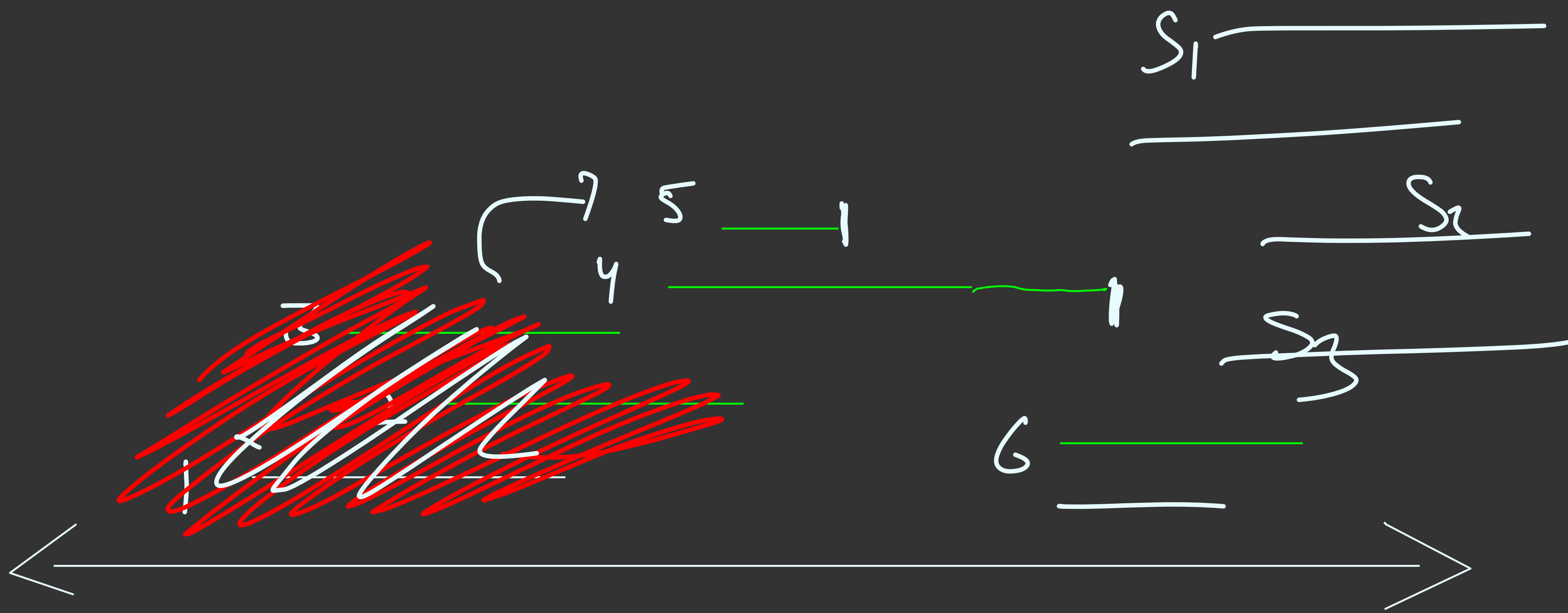


Example:

Activities = [[1, 5], [2, 3], [4, 6]]

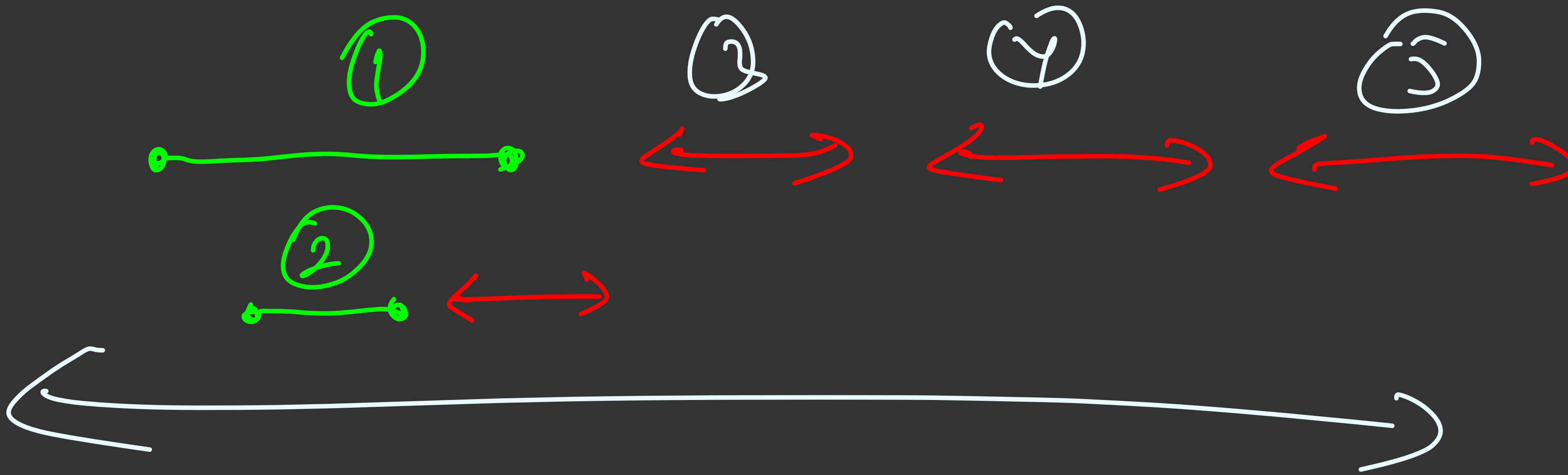
Ans = 2, selected activities: [[2, 3], [4, 6]]





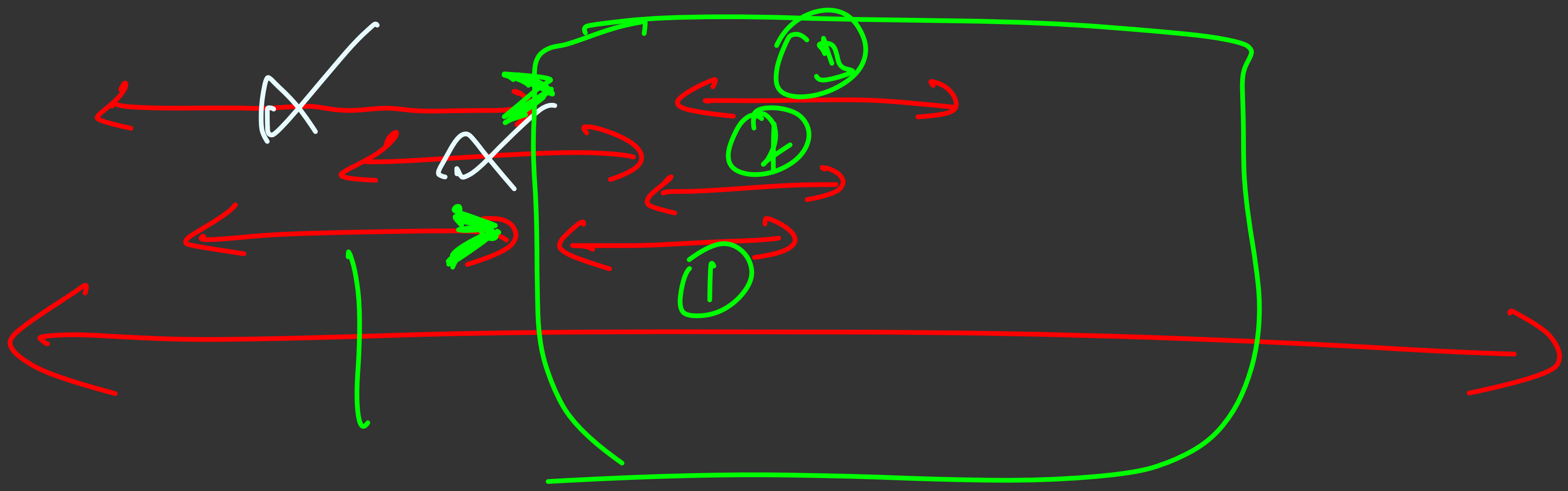
5 ✓ ans 1 no. of chosen segments after 5

4 ✓ ans 2 no. of chosen segments after 4



Any answer from 1 can also be constructed from 2

Any ans from 2 might not be possible from 1


1
2
3

Activity Selection Problem

If we want to choose the very first activity, which one should it be?

Choosing that activity which has the lowest ending time allows us to choose more activities in future.

We know this works for first activity, we can argue the same for 2nd activity, the 3rd activity and so on….

Let’s just sort all the activities by their ending time and then pick them up in order.