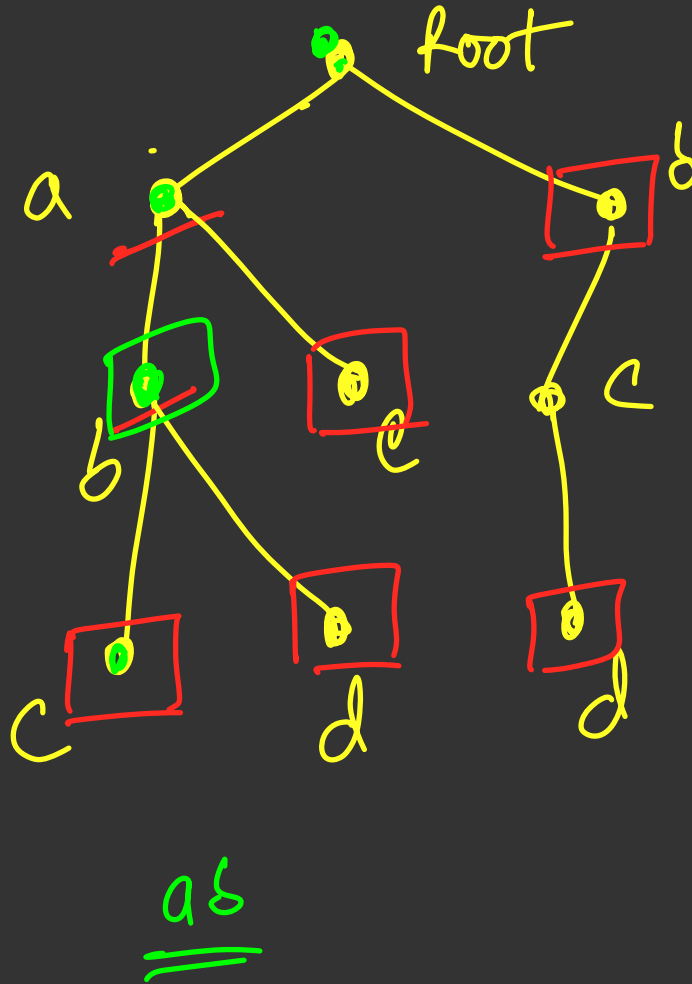


(Binary Tries)

Tries 2

- Priyansh Agarwal

$\frac{abc}{abd}$ ✓
 $\frac{ae}{bcd}$
 $\frac{b}{}$



$\frac{abc}{x}$
 $\frac{2}{2}$

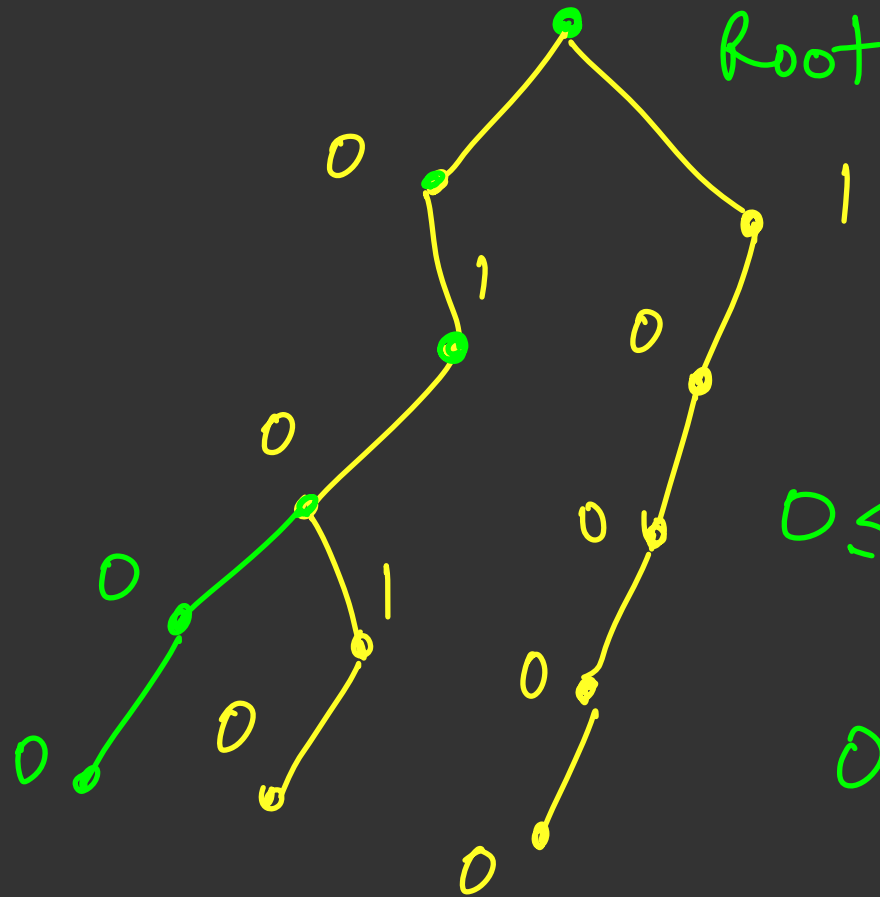
$$16 \overline{) 10000}$$

10 01010

8 01000

7 00111

6 0 0 1 1 0



$$0 \leq Q_i \leq 2^{63}$$

$$0 \leq a_i \leq 10^9$$

$$0 \leq a_i \leq \underline{2^3}$$

Problem 1

$$2 \leq N \leq 10^5$$

- Given an array A of N integers ($0 \leq a_i \leq 1e9$), find the maximum XOR of 2 numbers.

$$0 \leq a_i \leq 10^9$$

10 2 3 9 11

$$9 \wedge 2$$

$$1001 \wedge 0010$$

$$1011 \rightarrow (11)$$

$$10 \wedge 3$$

$$1010 \wedge 0011$$

$$\rightarrow 1001 (9)$$

$$10 \wedge 11$$

$$1010 \wedge 1011 = (1)$$

$$2^i > 2^{i-1} + 2^{i-2} + 2^{i-3} \dots 2^0$$

/ - - - - ↓ 4 3 2 1 0 /

A 1 0 0 0 0

B 0 ← →

A > B

A 0 0 0 0 0 | 0 0 0 0 0 0 0 0
 → / ✓

B 0 0 0 0 0 0 1 1 1 1 1 1 1
 →

10 — 1010(3)

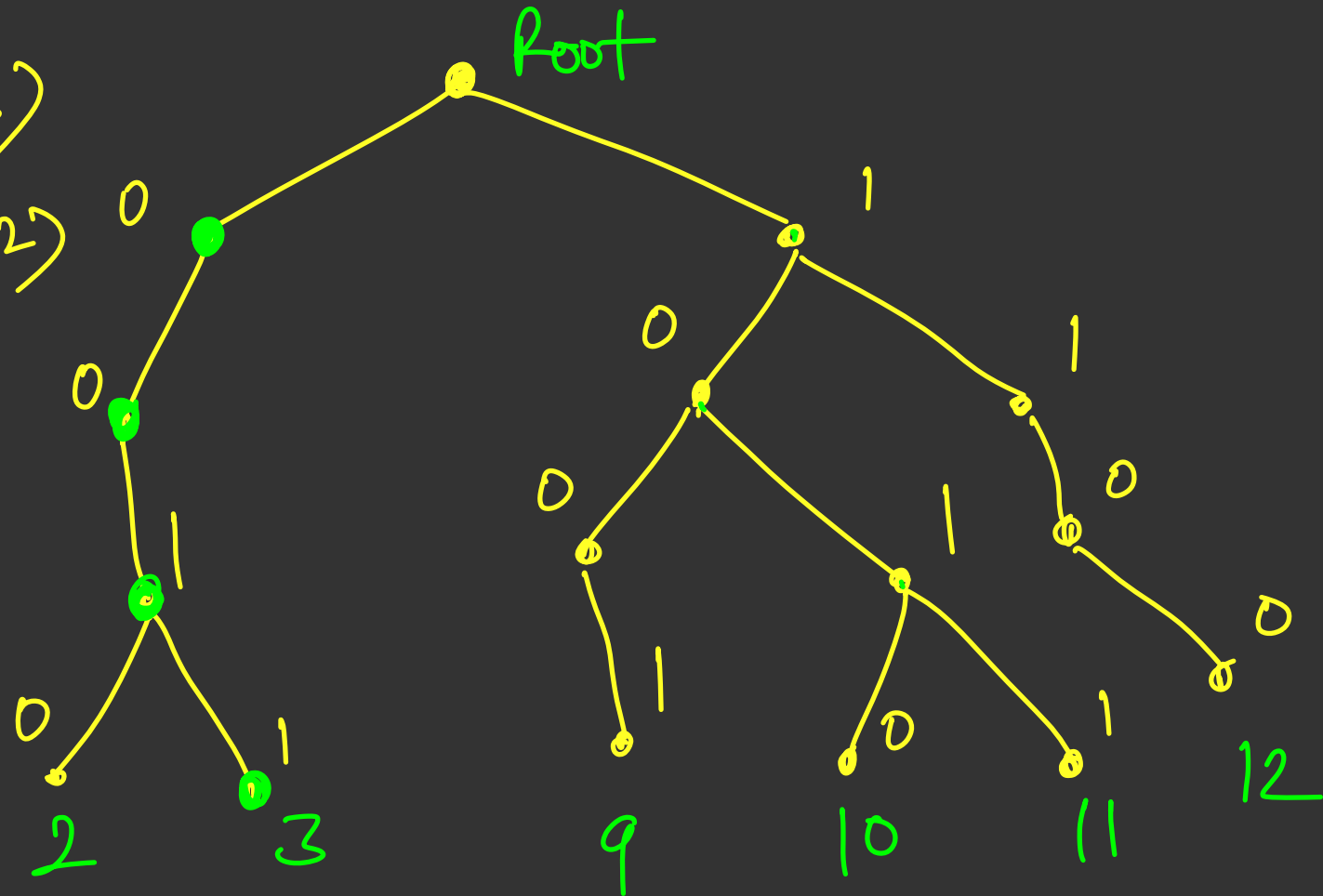
2 — 0010(12)

3 — 0011

9 — 1001

11 — 1011

12 — 1100



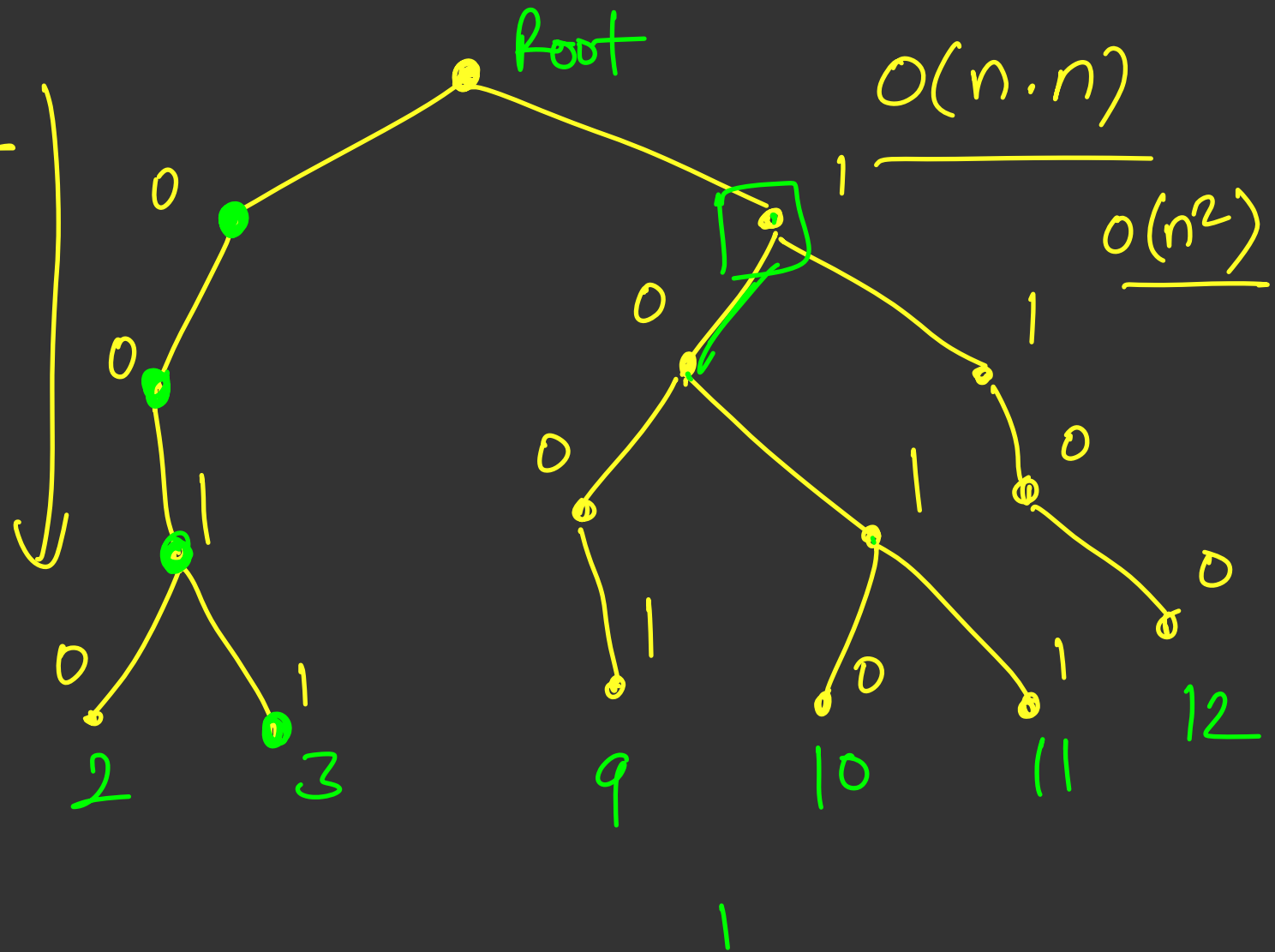
N $0 \leq q_i \leq 10^9$ $0 \leq q_i \leq 2^{31}$

Problem 2



Given an array A of N integers ($0 \leq a_i \leq 1e9$), find the maximum AND of 2 numbers.

10 — $\downarrow\downarrow$ 1010
 2 — 0010
 3 — 0011
 9 — 1001
 11 — 1011
 12 — 1100
 0 — 00000



10

0

11

1

1010

↑↑↑

We want maximum AND & w

two no.s

a_1 — 1001

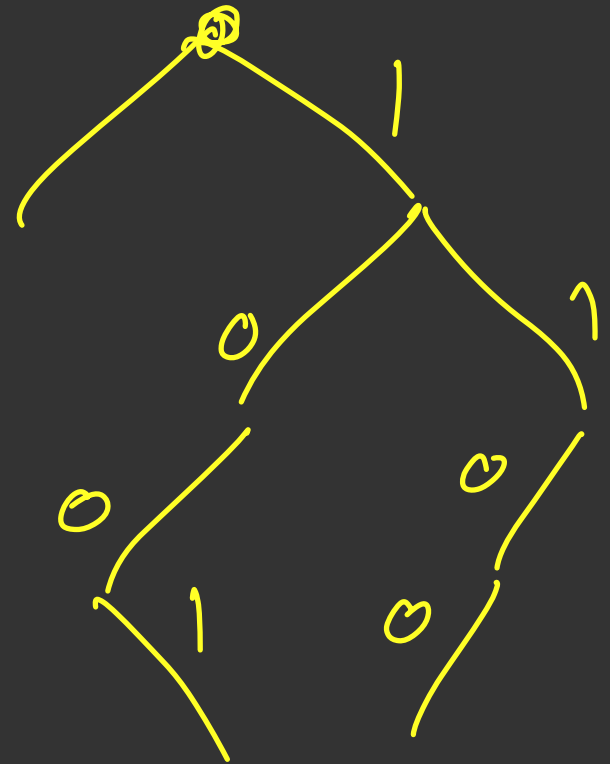
a_2

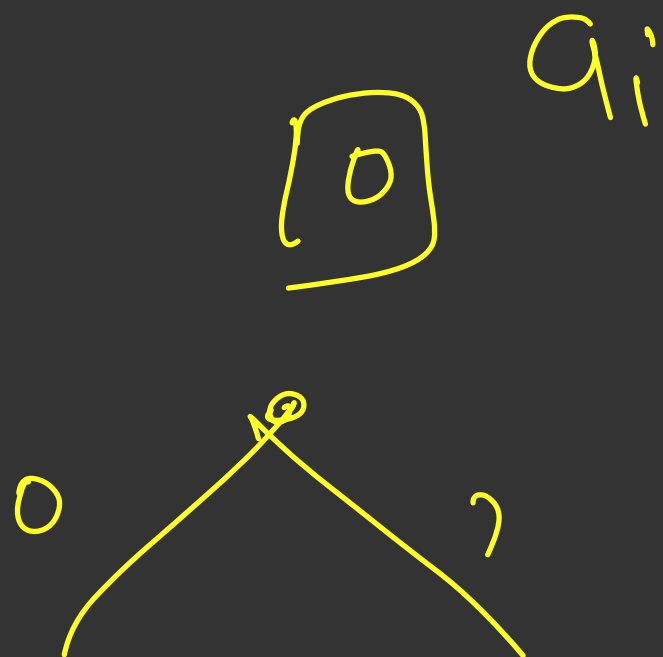
a_3

⋮

a_n

$[a_i] \& [a_j]$

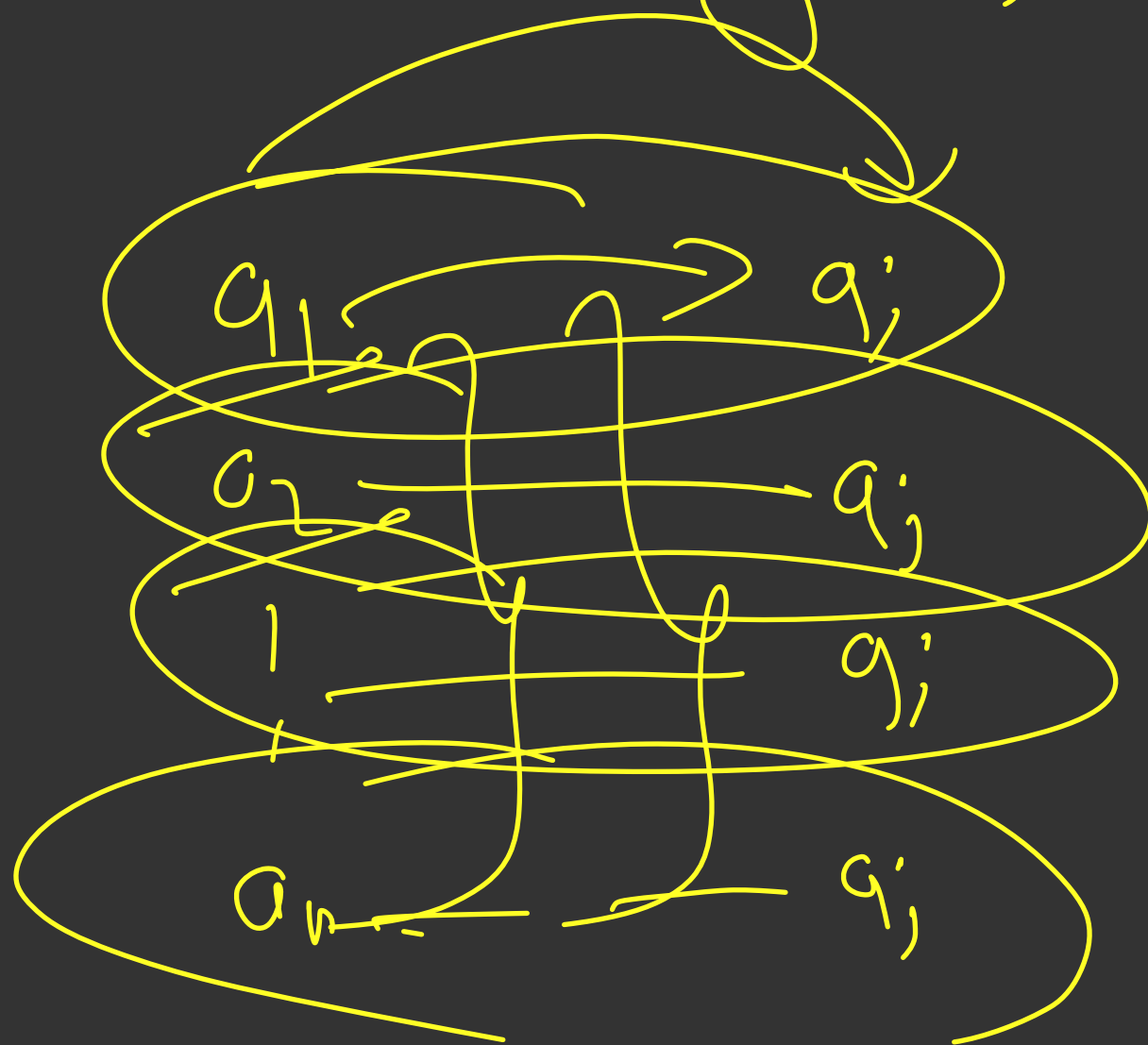


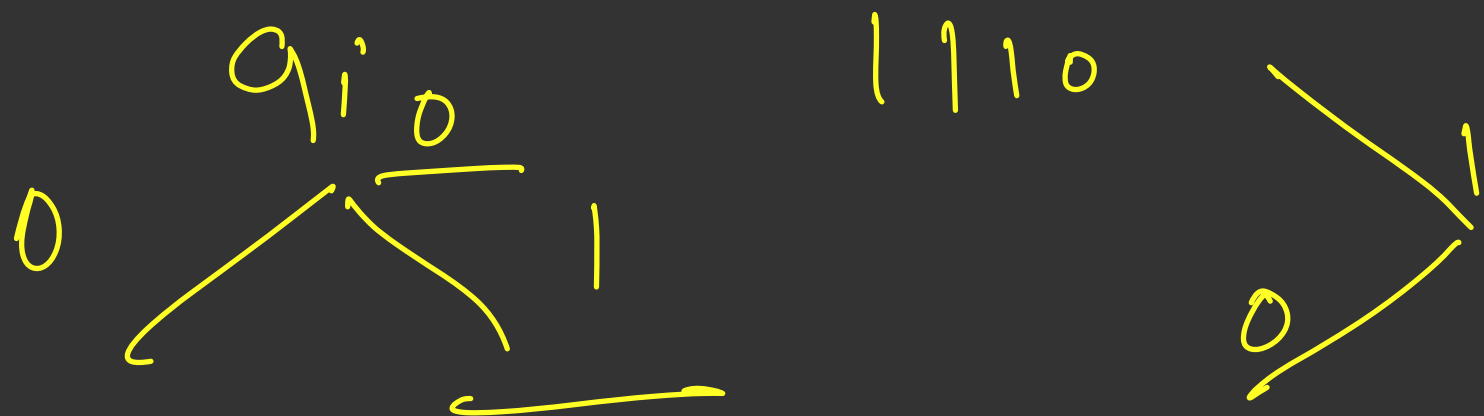


$$q_j < q_i$$

An arrow pointing upwards from below the horizontal line.

$q_i \rightarrow$ any q_j such that
 $q_i \leq q_j$ is
maximum





$q_i \rightarrow$ that q_j such that
 $q_j < q_i$ and
 $q_i \& q_j$
 is maximum

Problem 3

$$1 \leq N \leq 10^5$$

$$(0 \leq a_i \leq 10^9)$$

- Given an array `arr` of size `N` consisting of non-negative integers. You are also given `Q` queries represented by 2D integer array queries, where queries[i] = [xi, mi].

$$1 \leq Q \leq 10^5 \quad (0 \leq x_i, m_i \leq 10^9)$$

- The answer to the *i*th query is the maximum bitwise XOR value of `xi` and any element of `arr` that does not exceed `mi`. In other words, the answer is $\max(arr[j] \text{ XOR } x_i)$ for all j such that $arr[j] \leq mi$. If all elements in `nums` are larger than `mi`, then answer is -1.

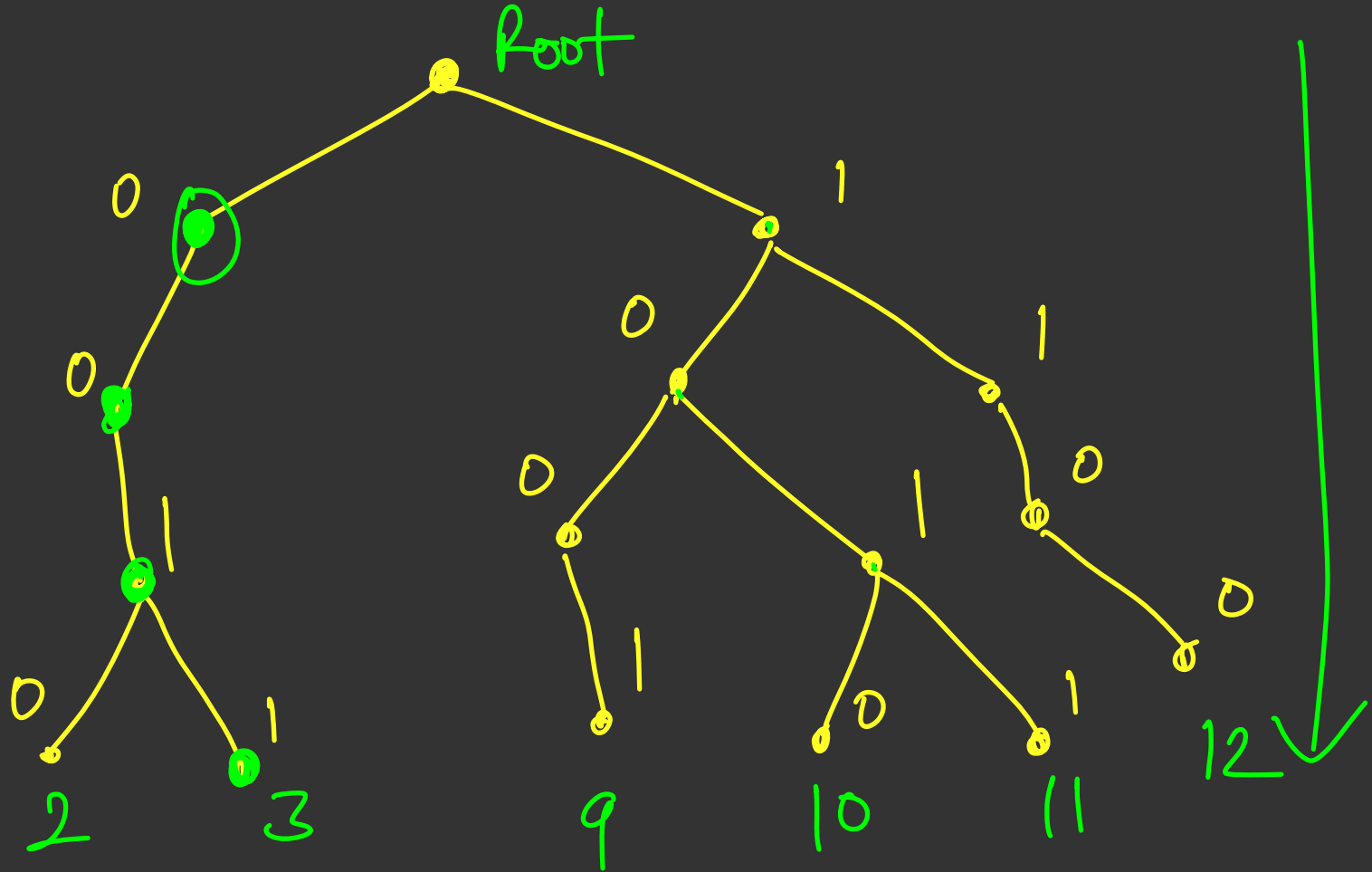
$$10 - 1010$$

2 — 0010

3 — 0011

9 — 1001

11 — 1011

$$12 - 1/100$$

$$X = 8, M = 6$$

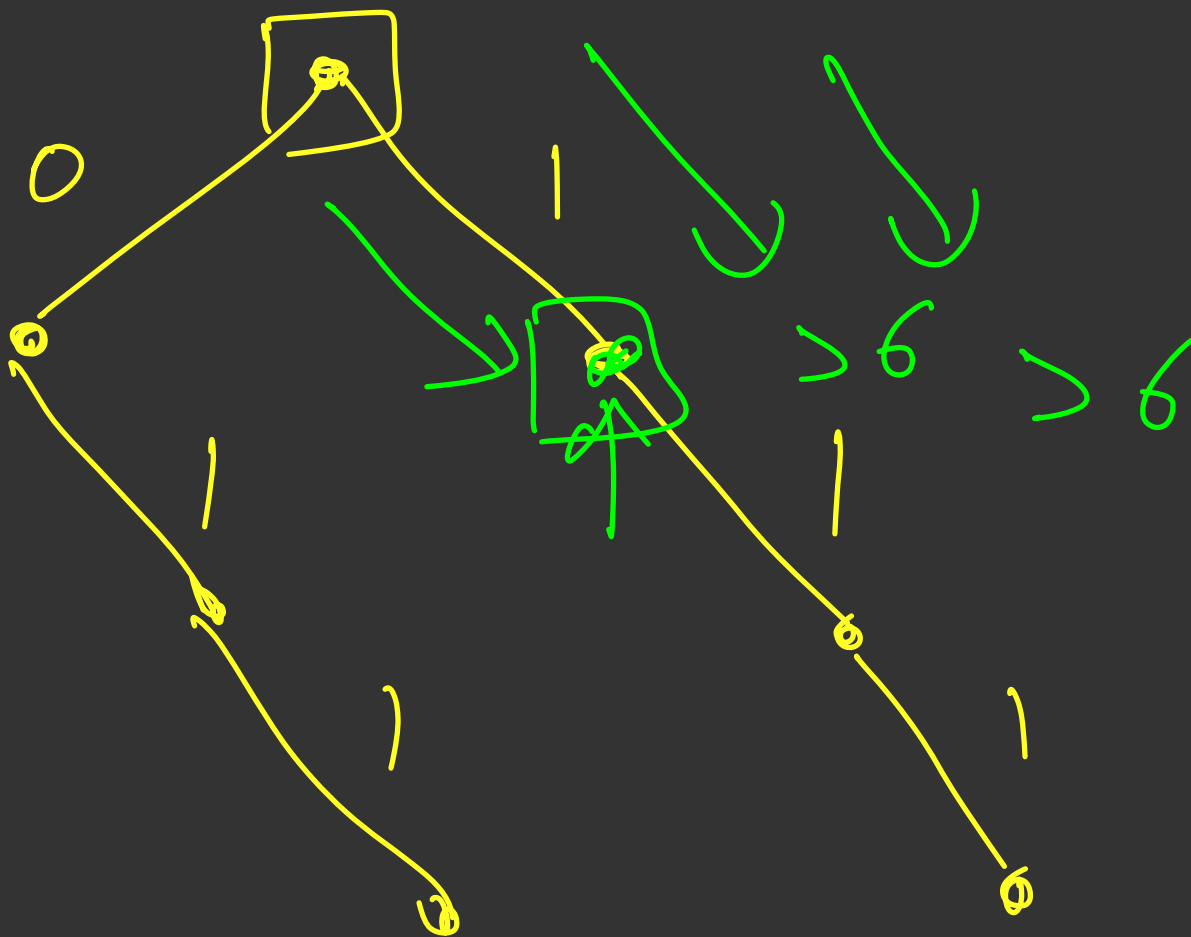
1000
↑↑

0110

4 → 001
↑

$$X = 0$$

$$M = 6$$



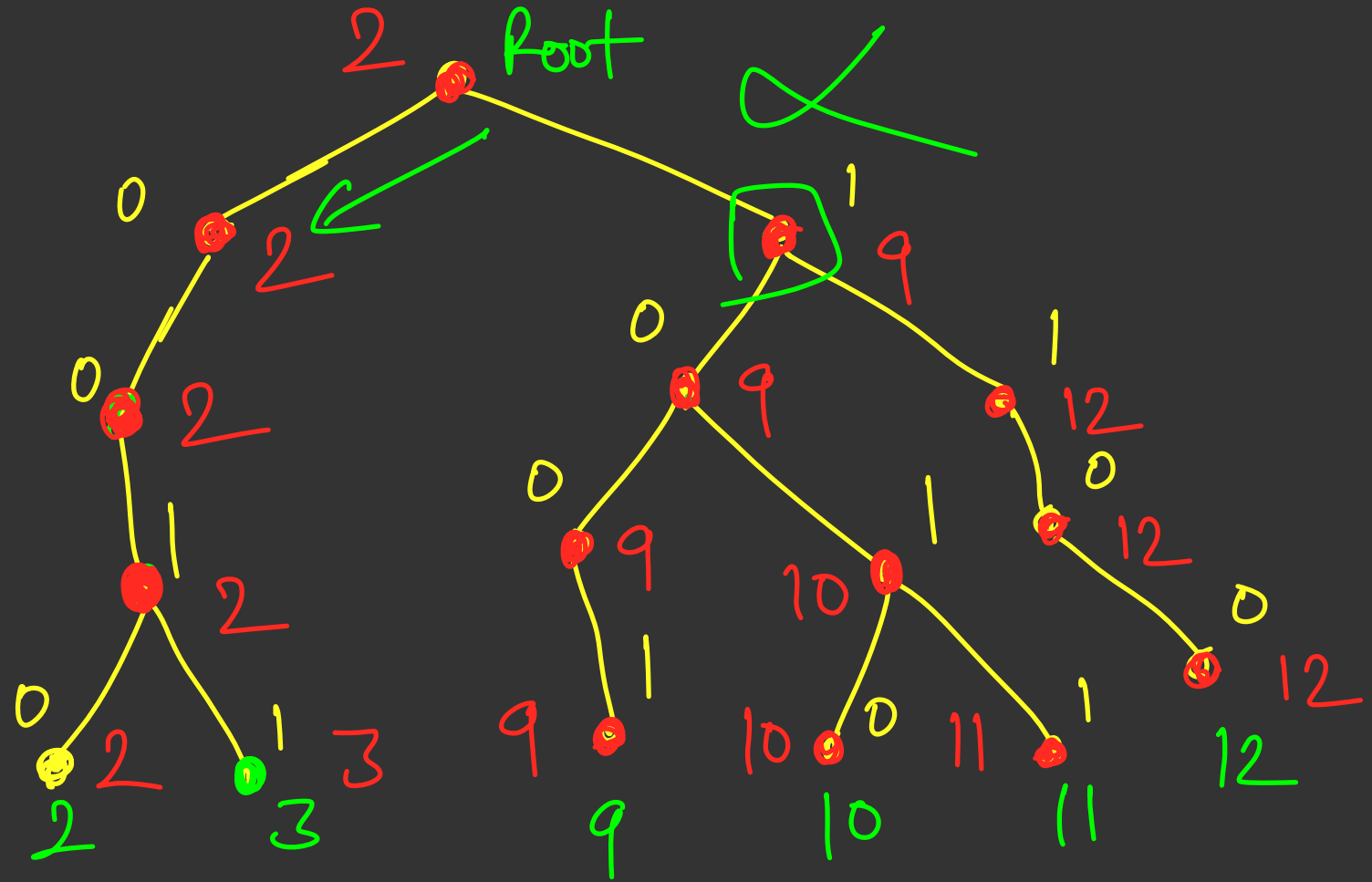
We are standing on i th bit if
the i th bit in x is

0 \rightarrow we want to go towards 1

1 \rightarrow we want to go towards 0



10 — 1010
 2 — 0010
 3 — 0011
 9 — 1001
 11 — 1011
 12 — 1100



$X = 7 \rightarrow 0111$

$M = \underline{8}$