



Sliding Window Problem Solving

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Problem 1:

Given an array of N integers, find the ~~length of~~ ^{sum of} the subarray with largest sum.

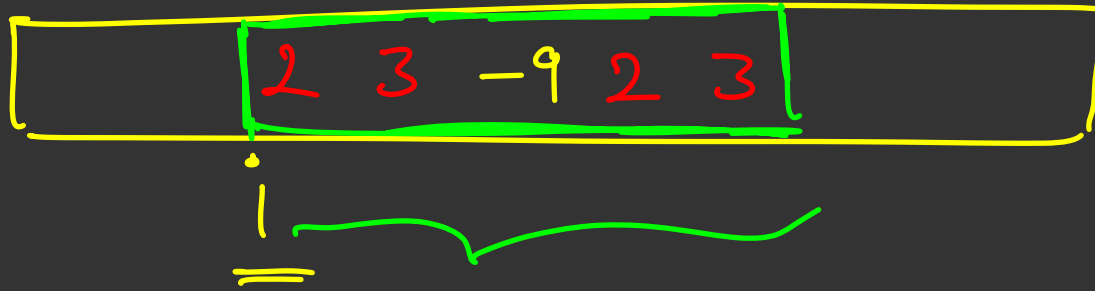
$$1 \leq N \leq 10^5$$

$$-10^9 \leq a[i] \leq 10^9$$

Example:

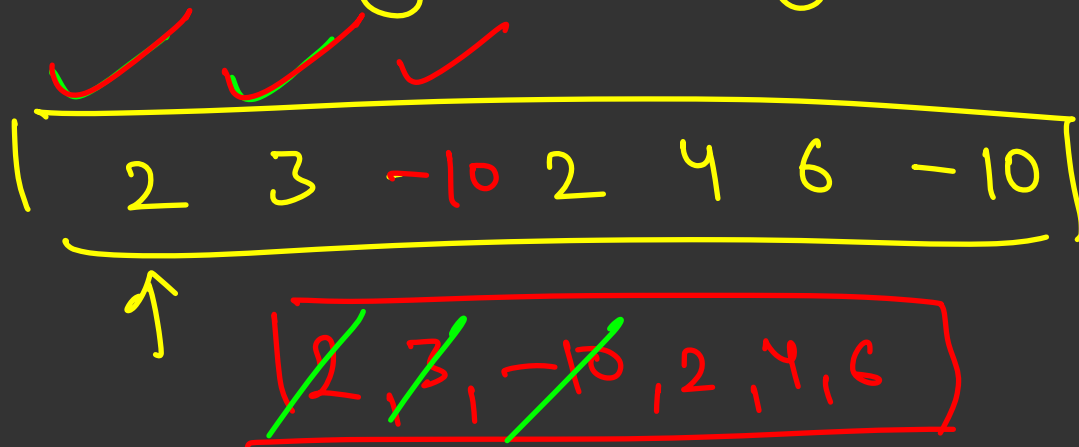
$$A = [1, 2, -1, 4, -10, 4]$$

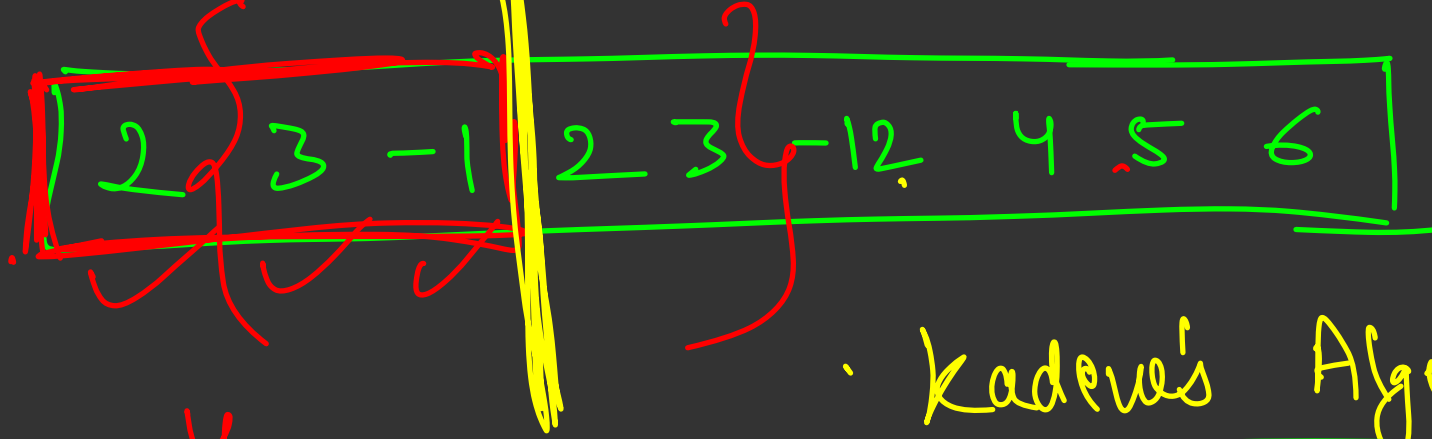
$$\text{Ans} = 6 \text{ by choosing } [1, 2, -1, 4]$$



sum is maximum

subarray is starting at index



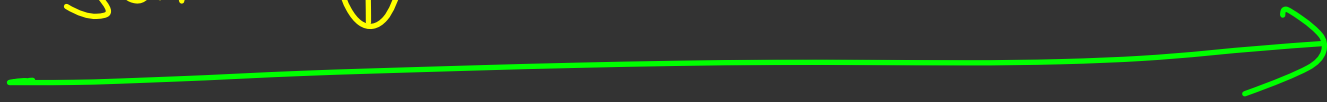


sum \rightarrow 4

Kadane's Algorithm

$O(n)$

← →
max suffix max prefix



0
= 0
< 0

Kadane's Algo

```
int sum = 0, best_sum = 0
```

```
for (int i = 0; i < n; i++) {
```

```
    sum += arr[i]
```

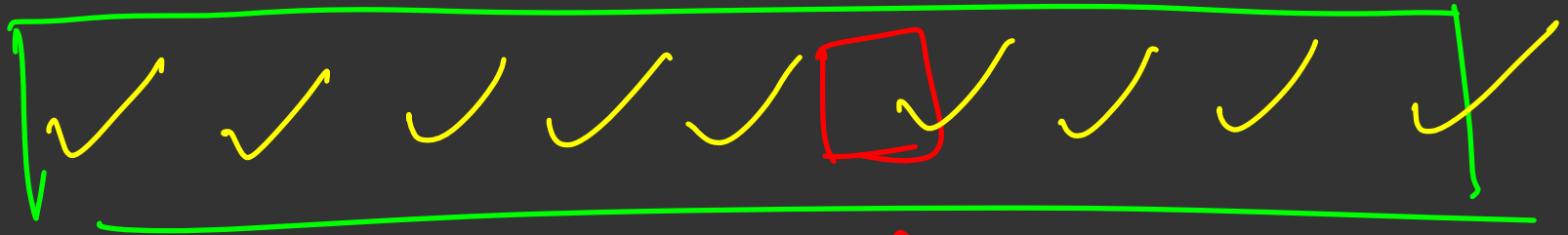
```
    if (sum < 0) {
```

```
        sum = 0
```

```
        best_sum = max(best_sum, sum)
```

```
}
```

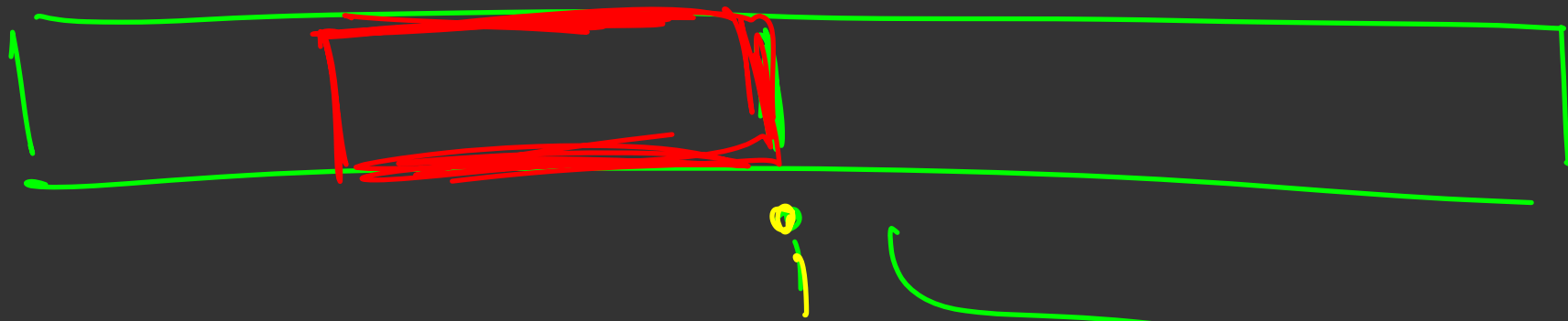
Approach (2)



find out best sum subarray
ending at i

① max sum subarray of entire array

② (max subarray ending at every index)
max

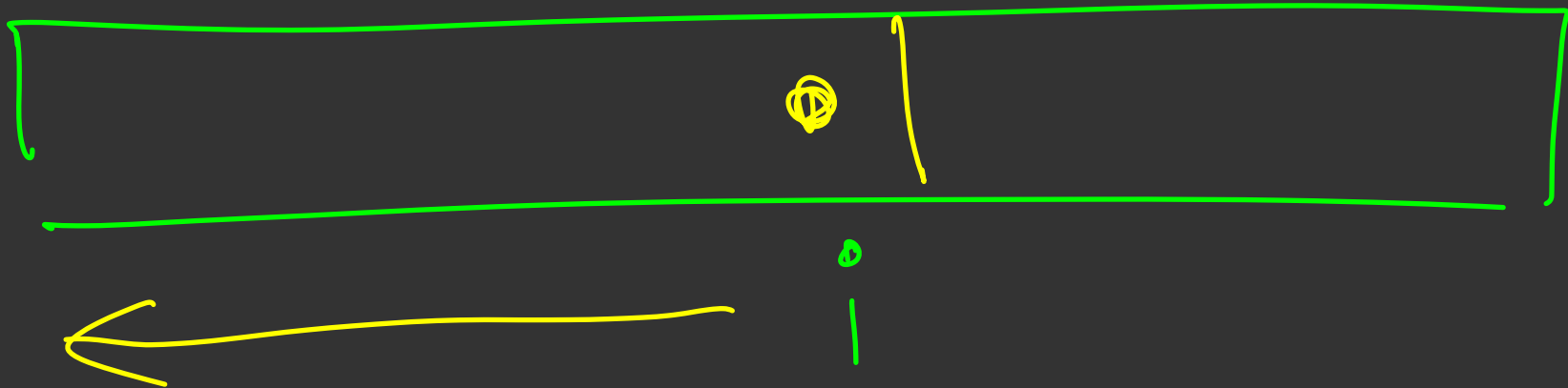


sum of subarray from $(l \text{ to } r)$

$$\text{prefix}[r] - \text{prefix}[l-1]$$

$$\text{prefix}[i] - \text{prefix}[j-1]$$

Search that $j \leq i$



max sum subarray ending at 'i'

$$= \max (\text{prefix}[i] - \text{prefix}[j-1])$$

$$0 \leq j \leq i$$

$$= \boxed{\text{prefix}[i]} - \min(\text{prefix}[j-1])$$

①

$$\min(\text{prefix}[j-1])$$

$$0 \leq j \leq i$$

2	2	-10	2	3	4	5	6	7
---	---	-----	---	---	---	---	---	---

i

0 1 2 3 4 5 6 7 8

max sum subarray ending at 4

subarray from l to r = $p[r] - p[l-1]$

$$\max(p[i] - p[j-1])$$

$0 \leq j \leq i$

$$\hookrightarrow p[i] = \min_{0 \leq j \leq i} (p[j-1])$$

✓ ✓ ✓ ✓ ✓ ;

$p(0)$ $p(1)$ $p(2)$ $p(3)$ $p(4)$ $p(5)$

$p(5)$ — min-so-far

vector<int> pratin(n+1)

pratin[0] = 0

1
1 2 3 4 5 6

int min_so_far = 0, max_ans = 0

for (int i = 1; i ≤ n; i++) {

pratin[i] = ~~pratin[i-1]~~ + arr[i]

min_so_far = min(min_so_far, pratin[i])

max_ans = max(max_ans, pratin[i] - min_so_far)

0 | 2 3 -10 2 2 6 -20 3 4

P | 0 2 5 -5 -3 -1 5 -15 -12 -8
— — — — —

M 0 0 0 -5 -5 -5 -5 -15 -15 -15
— — — — —

ans
0 2 5 0 2 4 10 0 3 7

10

Problem 1:

Given an array of N integers, find the length of the subarray with largest sum.

$$1 \leq N \leq 10^5$$

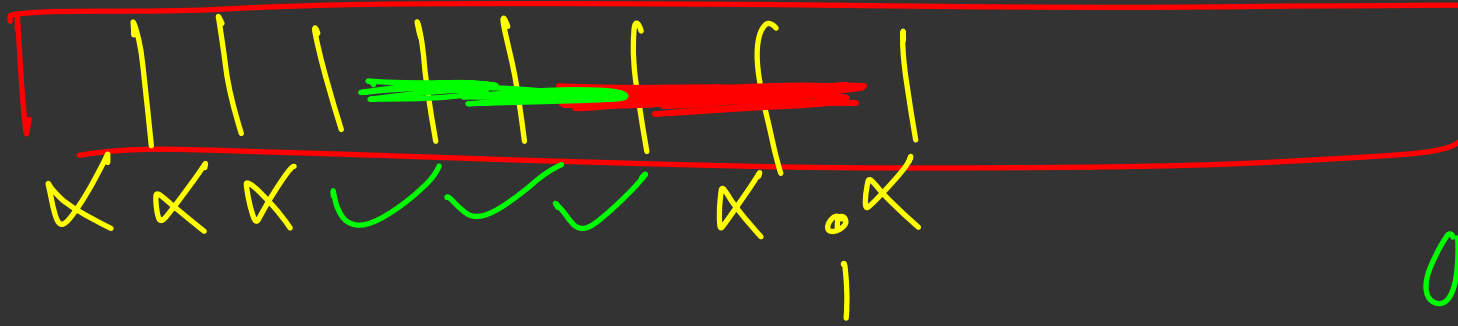
$$-10^9 \leq a[i] \leq 10^9$$

Example:

$$A = [1, 2, -1, 4, -10, 4]$$

Ans = 6 by choosing $[1, 2, -1, 4]$

Problem 2: [Link](#)



$$a = 3$$

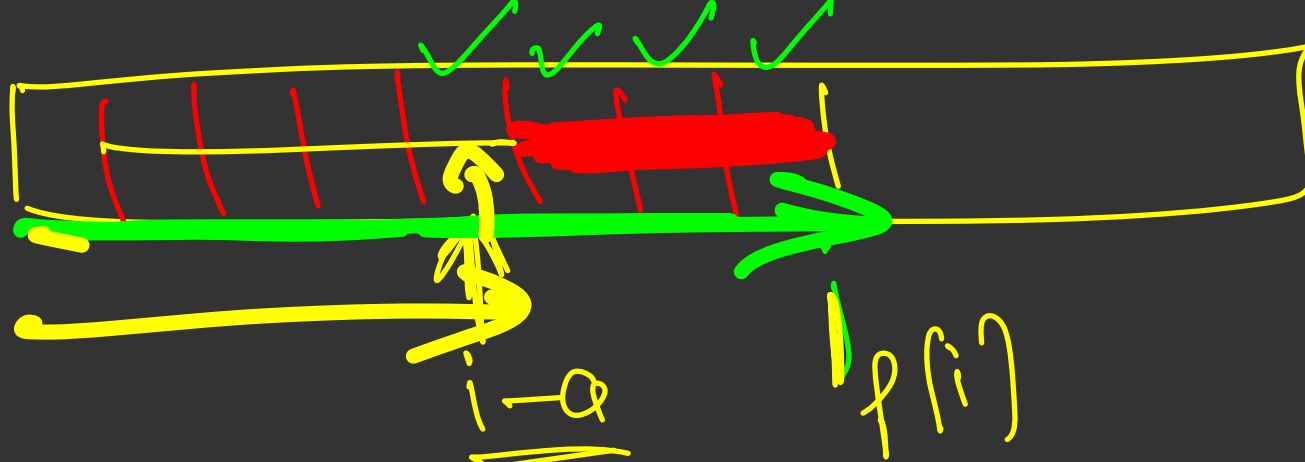
$$b = 5$$

max sum subarray ending at i

$$\max \left(\text{prefix}(i) - \min_{1 \leq j \leq i} \text{prefix}(j-1) \right)$$

$$1 \leq j \leq i$$

$$\text{prefix}(i) - \min_{1 \leq j \leq i} (\text{prefix}(j-1))$$



$$p[i] - p[i-a]$$

$$p[i] - p[i-a-1]$$

⋮

$$p[i] - p[i-1]$$

of length a

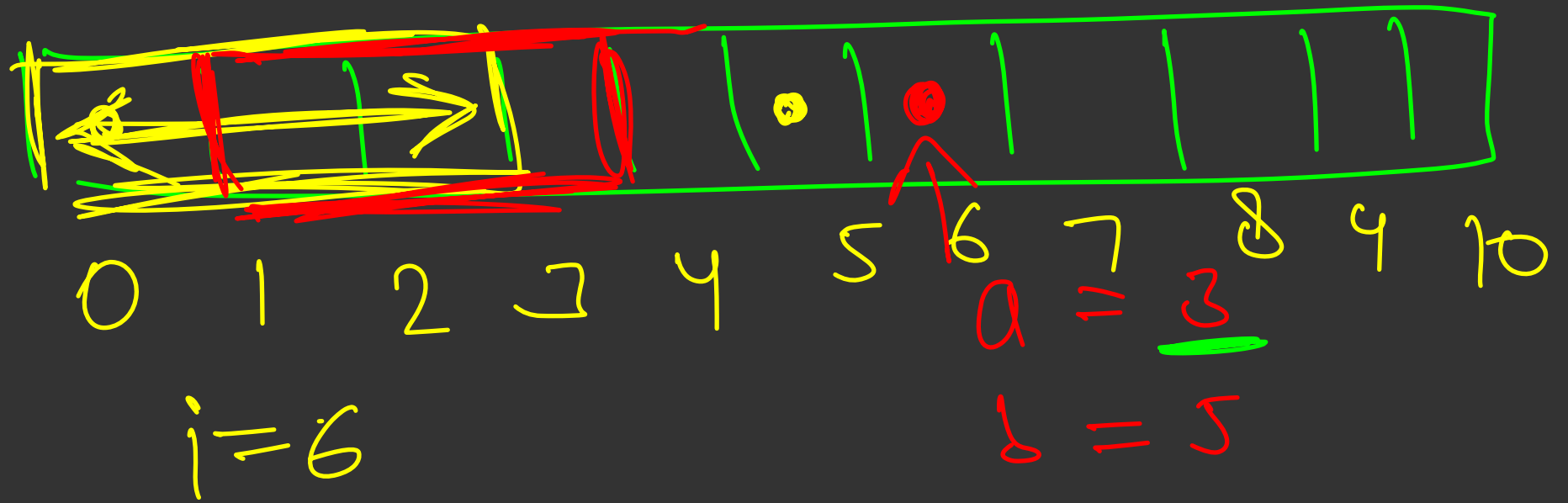
of length $a+1$

$$a = 3$$

max

$$\max \left(\begin{array}{c} f(i) - f(i-a) \\ \vdots \\ f(i) - f(i-a-1) \\ \vdots \\ f(i) - f(i-1) \end{array} \right)$$

$$f(i) - \underbrace{\min(f(i-a), f(i-a-1), \dots, f(i-1))}_{\text{min value in the range [i-a, i-1]}}$$



$$\underline{f[i]} = \min(f[i-a], f[i-a-1], \dots, f[i-b])$$

\downarrow 2 | 3 | -10 | 2 | 2 | 10 | 6 | -20 | 2 | 7 | 8 | 10

0 | 2 | 5 | -5 | -3 | -1 | 9 | 15 | -5 | -3 | 4 | 12 | 22

0 1 2 3 4 5 6 7 8 9 10 11 12

$$a = 3$$

$$b = 5$$

$$p[7] = \min(p[7-3], p[7-3-1], p[7-3-2])$$

$$p[7] = \min(p[4], p[3], p[2])$$

$$15 = \min(-3, -5, 5)$$

$$15 - (-5) = \underline{\underline{20}}$$

set

multiset

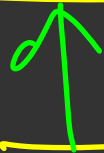
duplicates

values in sorted order well

insert
delete
search } $O(\log n)$

multiset

2 2 3 4 5 5 6



2 2 3 4 5 5 6

ordered set

find index of an element
find element given an index

multiset ^m → { 2, 2, 3, 4, 5 }

m.erase(2);

m.erase(m.lower_bound(2));
m.erase(m.find(2));

Problem 3:

Given an array of N integers, make all the elements equal to some value X , total cost of doing this is sum of $|a[i] - x|$ for all i from 0 to $N - 1$

$$1 \leq N \leq 10^5$$

$$10^9 \leq a[i] \leq 10^9$$

Example:

$$A = [1, 2, 3, 4, 5]$$

It is optimal to bring every number to 3.

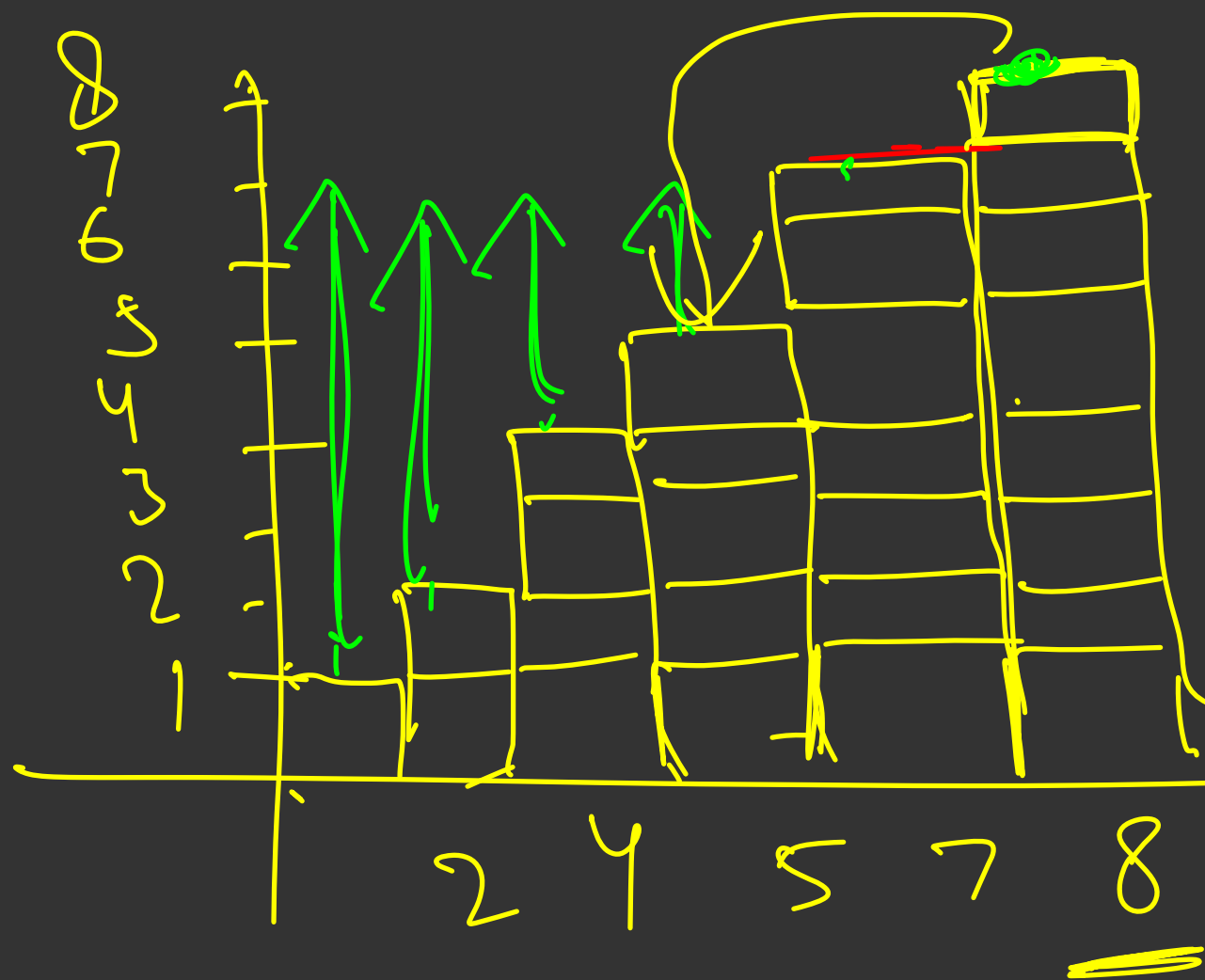
$$\text{So answer} = |1 - 3| + |2 - 3| + |3 - 3| + |4 - 3| + |5 - 3| = 6$$

1 2 3 4 5

X	2	2	2	2	2	X = 2
	1	0	1	2	3	→ 7
	1	1	1	1	1	X = 1
	0	1	2	3	4	→ 10
	3	3	3	3	3	X = 3
	2	1	0	1	2	→ 6

Bring all element to median to
minimize the overall distance
of every element from X

$$A = [1 \ 2 \ 4 \ 5 \ 7 \ 8]$$



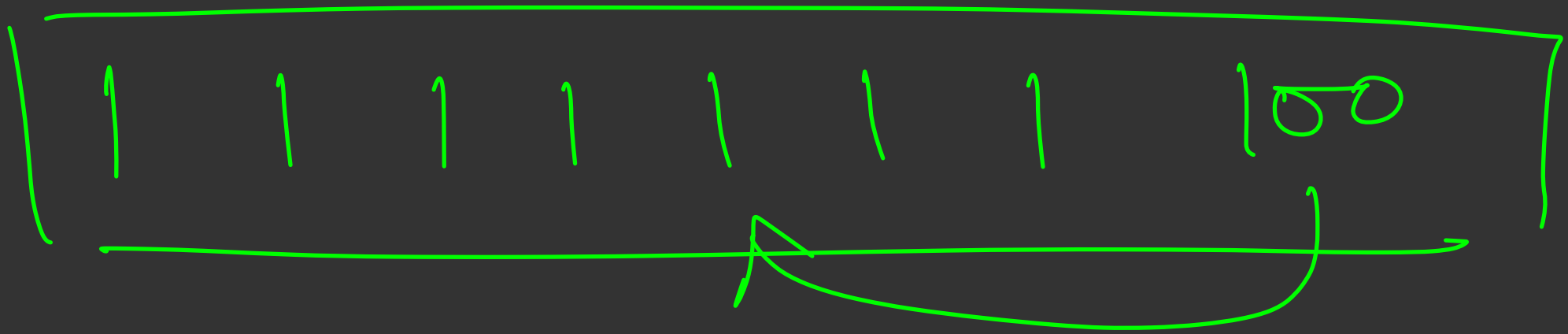
$$(8, 4)$$

$$(7, 4 - 5 + 1)$$

$$(7, 2)$$

$$(6, 2 - 4 + 2)$$





50

A →

1	2	4	6	9	14	16
---	---	---	---	---	----	----

X →

- 16 • 4
- 15 • $4 - 6 + 1$
- 14 • $(4 - 6 + 1) - 6 + 1$
- 13 • $[(4 - 6 + 1) - 6 + 1] - 5 + 2$
- ⋮
- 1

$$\boxed{-k + (n - k)}$$

$$k \geq n/2$$



$$X = 5$$

$$, \text{ moment} = 10$$

$$X = 4$$

$$11 = 7$$

$$X = 3$$

$$11 = 6$$

$$X = 2$$

$$11 = 7$$

$$X = 1$$

$$11 = 10$$

1 1 2 4 10

2

4

1

0

2

8

3

2

0

6

Problem 4: [Link](#)