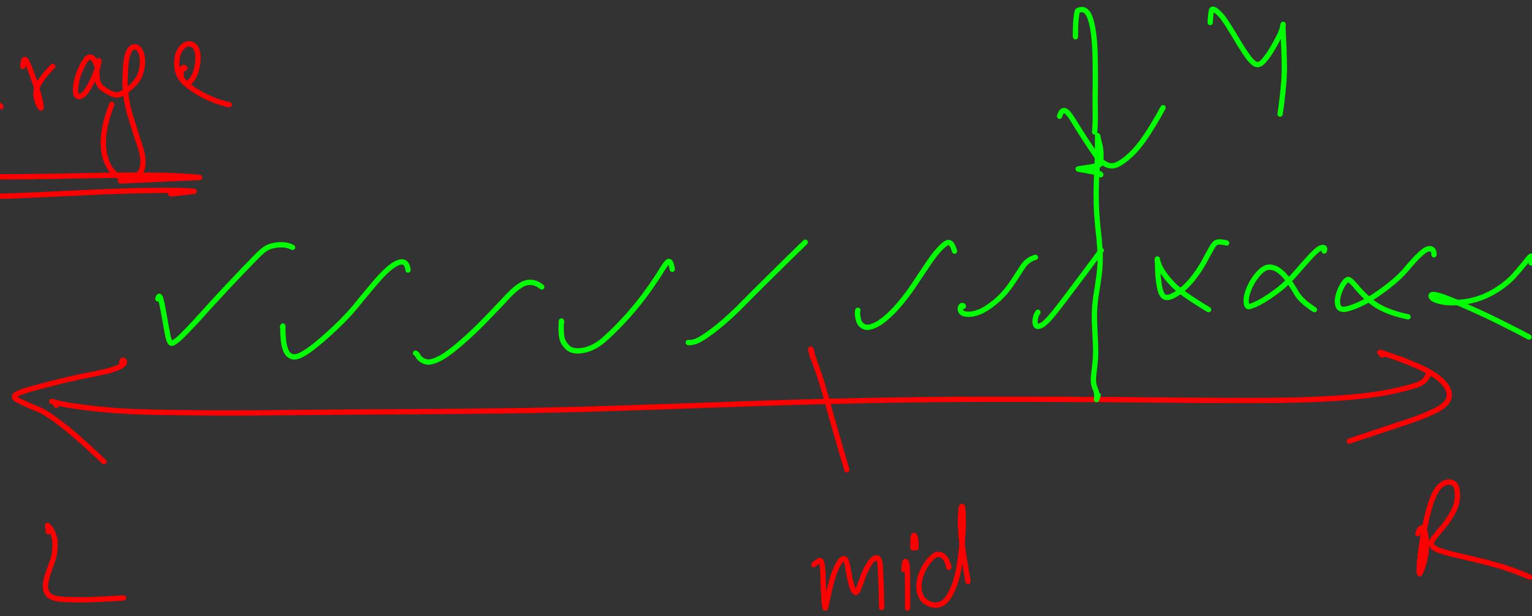
Advanced Binary Search ~~2~~ 3

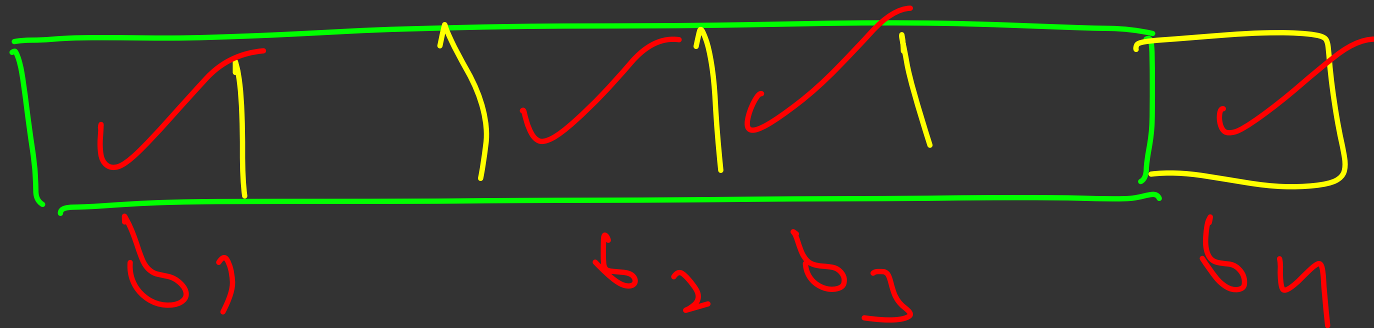
Problem 1: Average & Median

Harder

Average



if average can be $\geq$ mid or not



$f(x)$

suppose we chose k element

$$\frac{b_1 + b_2 + b_3 + b_4}{4} \geq x$$

$$b_1 + b_2 + b_3 + b_4 \geq 4x$$

$$\underline{(b_1 - x)} + \underline{(b_2 - x)} \ldots \underline{(b_4 - x)} \geq 0$$

$A =$

$B =$

$A_i - x \geq 0$

x

Average

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our search space for average = [L to R]

guess for Mid => Whether or not the average can be greater than or equal to Mid
If True -> The optimal answer might be on the right side
If False -> The optimal answer will be on the left side



F(x) = True or False based on whether or not the average of the chosen elements
can be >= x

A = [a1, a2, a3, ..... an]

(a1 + a2 + a4 + a6) / 4 >= x

(a1 + a2 + a4 + a6) >= 4x

(a1 - x) + (a2 - x) + (a4 - x) + (a6 - x) >= 0
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B = [a1 − x, a2 − x, a3 − x... an − x]


what is the maximum sum of elements that I can choose from B = M

if M >= 0 then answer = T, otherwise F



How to find out maximum sum


[2, −1, 4, −5]

C[i] = maximum sum of elements from 0 to i, such that you are choosing the ith
element

C[0] = B[0]
C[1] = B[1] + 2 = 1
C[2] = B[2] + max(C[1], C[0]) = 4 + max(1, 2) = 4 + 2 = 6
C[3] = B[3] + max(C[2], C[1]) = −5 + max(6, 1) = −5 + 6 = 1

max(C[n − 1], C[n − 2]) = maximum sum of elements that can be chosen from the
entire array such that no 2 consecutive elements are skipped
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Examples

[10, 20, 30, 40]

C[0] = 10
C[1] = 20 + 10 = 30
C[2] = 30 + max(C[0], C[1]) = 30 + max(30, 20) = 60
C[3] = 40 + max(C[2], C[1]) = 40 + max(60, 60) = 100

C[2] = arr[2] + C[1] = you are not skipping arr[1]
C[2] = arr[2] + C[0] = you are skipping arr[1] but not skipping arr[0]
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Median

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Median search space [L to R]

if Median can be >= Mid then search for a better median in the right half
otherwise search for the median in the left half

F(x) = True or False based on whether or not the median can be >= X


A = [a1, a2, a3, a4, a5, a6, a7]
original values = a1, a2, a4, a6, a7
sorted = b1, b2, b4, b6, b7
b4 >= X for median to be >= X


F(X) = Whether or not you can choose more elements that are >= X than the elements that
are < X

Choose more elements that are >= 5 than the elements that are < 5

5, 6, 7, 8, 9 good elements
1, 2, 3, 4 bad elements


Let's denote every element >= X with +1
Let's denote every element < X with -1


B[i] = +1 if A[i] >= X, and -1 if A[i] < X
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B[i] = +1 if A[i] >= X, and -1 if A[i] < X

Find the maximum sum of array B following the condition if that sum > 0, the answer = True, otherwise the answer = False

How to find out maximum sum - Same as what we did in the Average Case

Problem 2: Pair Selection

Binary Search on Answer

① Assume that answer $= x$

$\le x$

$\ge x$

② Come up with a checker function

$f(x) \longrightarrow$ ③ Plot the checker function

T T T T T f f f f f f f B.S.

f f f f T T T T T T B.S.

A a_1 | a_2 | a_3 | a_4 | a_5 | a_6

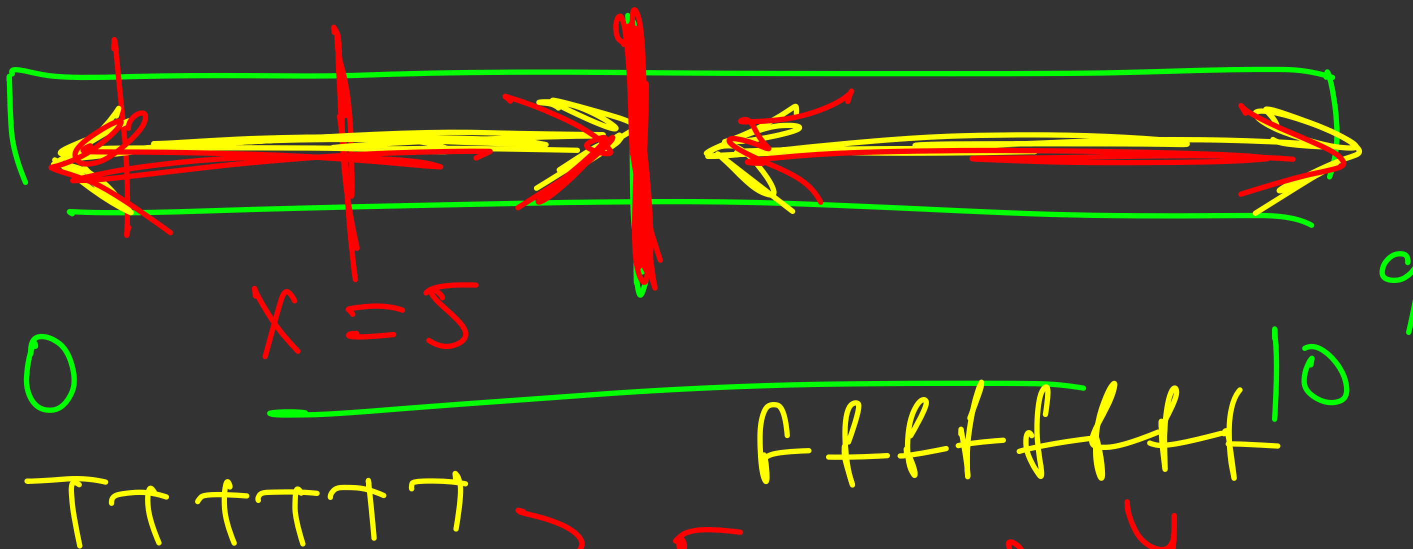
B b_1 | b_2 | b_3 | b_4 | b_5 | b_6

1 2 3 4 5 6

Can my answer be $\geq x$ or not

Optimal Answer = Y

Y = 10



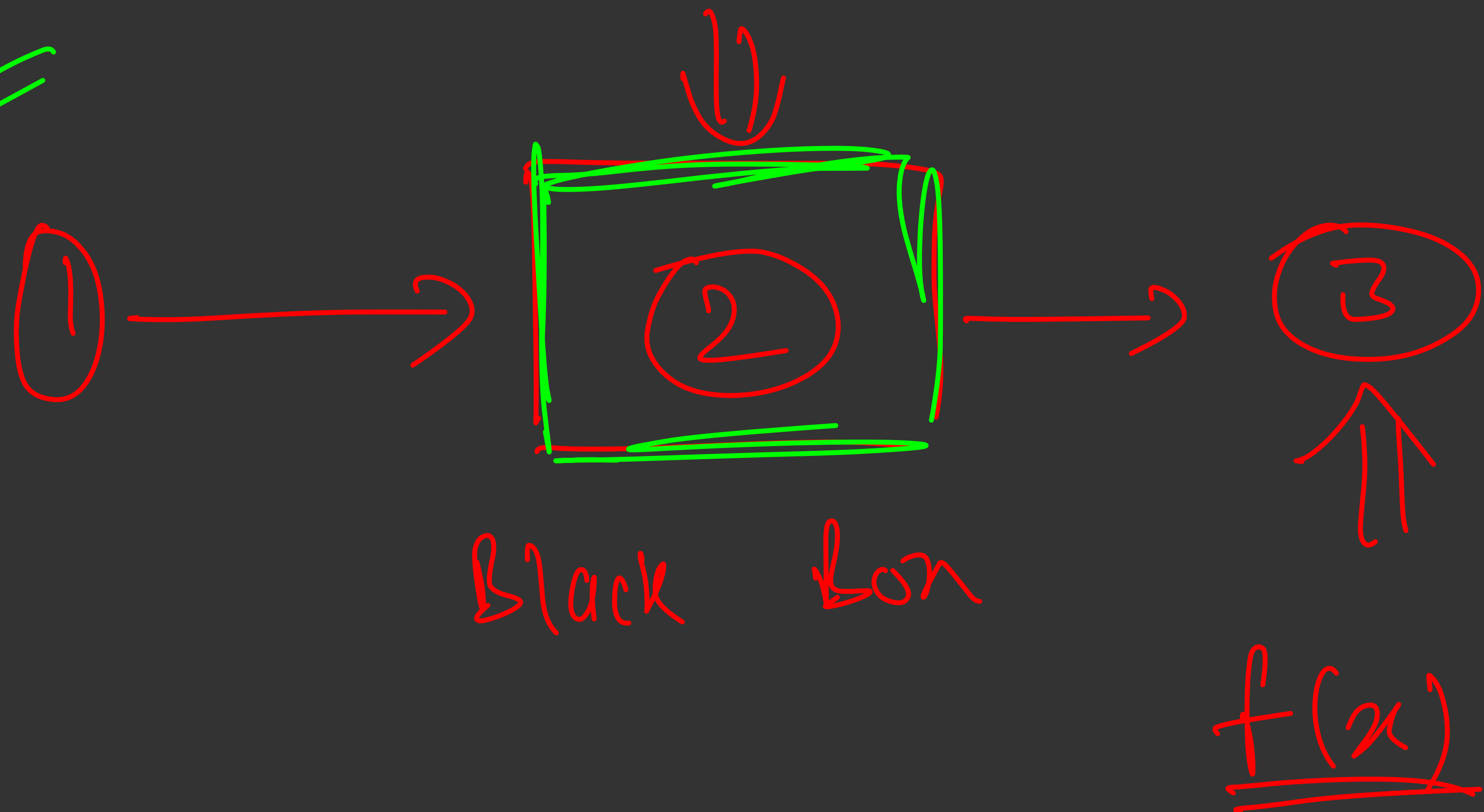
$\geq 5 \longrightarrow$ Yes

$\geq 4 \longrightarrow$ Yes

$\geq 11 \longrightarrow$ No

$\geq 12 \longrightarrow$ No

1. Assume can your ans $\geq x$
2. Writing a checker function $f(x)$
3. Plot the checker function



How to check if ans > x or not

$\boxed{k = 3}$

$$\frac{a_1 + a_3 + a_5}{b_1 + b_3 + b_5} > x$$

$$a_1 + a_3 + a_5 \geq x\,(b_1 + b_3 + b_5)$$

$$(a_1 - x b_1) + (a_3 - x b_3) + (a_5 - x b_5) \geq 0$$

A

B

$f(x)$

C

$A_i - x B_i$

$\cancel{a_i - b_i}$

$\boxed{k}$

$\boxed{\text{sum} \geq 0}$

Sort array C and choose last k elements and check if sum ≥ 0 or not

Binary Search $\rightarrow$ $\log_2(\text{search space})$

$1 \le a_i \le 10^5$

$1 \le s_i \le 10^5$

$\boxed{10 \text{ to } 10^6}$

1

$$\frac{10^5 \cdot 10^5}{10^5}$$

$\boxed{10^6}$

$T.C = (n \log n) \cdot (\log_2 10^{12})$

B.S $\rightarrow$ $O(\log_2 (\text{search space}))$

$O\left(\log_2\left(10^6 \cdot \frac{1}{p}\right)\right)$

Binary Search :

$$O(\log_2(\text{searchspace}))$$

$$\boxed{k \le n}$$

$f(x)$:

$$O(n\log n + O(k))$$

$$= O(n\log n + n)$$

$$= O(n\log n)$$

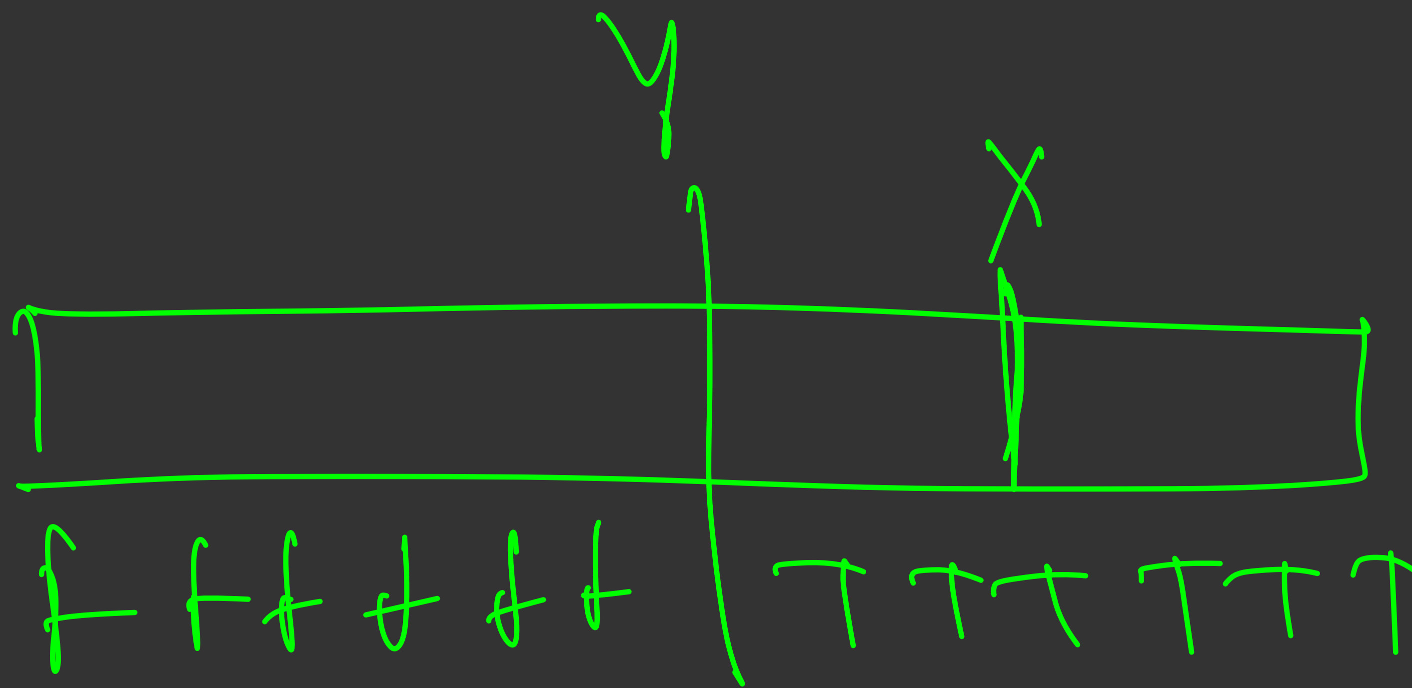
$1 \le a_i \le 10^5$

$1 \le d_i \le 10^5$ $\qquad \underline{\underline{10^6}}$

$$\frac{\sum a_i \to \max}{\sum d_i \to \min} = \frac{\cancel{x} \cdot 10^5}{\cancel{x}}$$

① Assume that answer $\geq x$

② Assume that answer $\leq x$

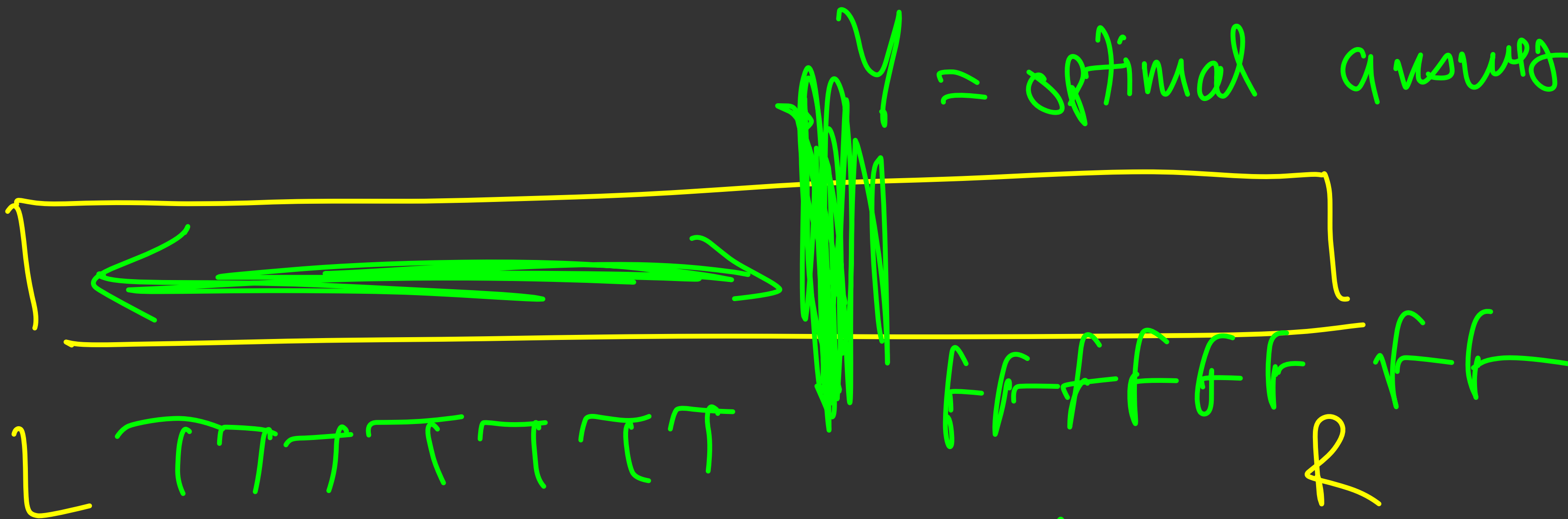


1. figuring out the answer is in a monotonic space

$O(\log_2(\text{search space}))$

2. writing the checker function

① maximum answer



$$f(x) \Rightarrow \begin{cases} T & \text{if } ans \geq x \\ F & \text{if } ans < x \end{cases}$$

Homework: Maximum Average Segment