# FIT3139: Applied questions for week 5

## Question 1

Find the root of the following function $f(x) = x^2 - 2$ by implementing a function that uses Newton's method and another that uses the Seacant method to find the root. For Newton's method, start with $x_0 = 1$ and for the Seacant with $x_0 = 1$ and $x_1 = 2$. Terminate the iteration when the absolute difference between successive values, $|x_{k+1} - x_k|$, falls below a tolerance of $10^{-9}$.

Your `find_root_secant` should receive a function argument, and your `find_root_newton` should receive a function argument and another function which is the derivative.

Compare the convergence from the two algorithms. Is it aligned with what we learned in theory? You can approximate the convergence rate with the following equation:

$$r \approx \frac{\log(|x_{k+1} - x_k / x_k - x_{k-1}|)}{\log(|x_k - x_{k-1} / x_{k-1} - x_{k-2}|)}$$

## Question 2

Implement Newton's method in multiple dimensions to find a solution of a set of non-linear equations.

Your function should receive as input function arguments for the system being solved ($\boldsymbol{f}(\boldsymbol{x})$) and the corresponding Jacobian Matrix ($J_{\boldsymbol{f}}(\boldsymbol{x})$), as well as an initial starting point.

Terminate when the L2-norm[1] of the absolute difference between successive approximations, $||\boldsymbol{x}_{k+1} - \boldsymbol{x}_k||_2 = ||\boldsymbol{h}_k||_2$, falls below a tolerance of $10^{-9}$.

[1] `https://mathworld.wolfram.com/L2-Norm.html`

Use your implementation to find a solution to the set of equations

$$x_1^2 + x_2^2 = 4$$
$$e^{x_1} + e^{x_2} = 2$$

using the initial point $\boldsymbol{x}_0 = (0, 1)$.

## Question 3

A model of insect populations (in which all adults are assumed to die before next breeeding) leads to the difference equation

$$N_{k+1} = \frac{\lambda N_k}{1 + a N_k}$$

where $\lambda$ and $a$ are positive constants.

- Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the growth rate.
- What is the general shape of the graph of $R(N_k)$?
- Express the unrestricted growth rate $r$ and the carrying capacity $K$, for this model, in terms of the parameters $a$ and $\lambda$.

*Question 4*

Consider the discrete dynamical system given by the following difference equation:

$$x_{t+1} = x_t^2 + c$$

where, $c > 0$. Draw the cobweb model of the system for different values of $c$ for which the dynamics is qualitatively different. What conclusions can you draw about the system?

As a challenge, try to write a function which creates the cobwebbing plots for you.