FIT3139: Applied exercises for Week 6

Question 1

Modify the Lotka-Volterra equations to account for the fish population growing logistically in isolation.

What is the steady state of this modified system?

Solve this equation using an existing differential equation solver (**scipy.integrate.odeint** or **scipy.integrate.solve_ivp** for Python or MATLAB function **ode45**) and plot your results, along with horizontal lines at the steady state.

What do you observe?

Question 2

Find the phase-plane trajectories for the coupled system given by:

$$\frac{dx}{dt} = -y$$

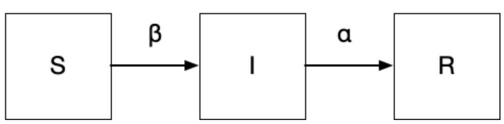
$$\frac{dy}{dt} = x$$

Sketch the trajectories. What do they look like?

Question 3

You want to come up with a differential equation model for a mysterious virus that shall not be named. Your model is based on a standard SIR model, similar to the one discussed in the lectures, but using differential equations instead of difference equations.

The flowchart diagram of this model is as follows:



where the constant β is the transmission rate, and $alpha$ is the rate of recovery. The system of ODE describing the dynamics is then:

$$\dot{S} = -\beta S I$$

$$\dot{I} = \beta S I - \alpha I$$

$$\dot{R} = \alpha I$$

You want to extend the model to account for the following aspects. Develop your model incrementally, and specify the equations and diagrams for each model.

- Adding an extra compartment of exposed individuals who have acquired the virus but are not symptomatic or contagious. How do things change when you assume that individuals in this compartment are contagious?
- Assume there are different levels of clinical progression. That is, some patients will get mild symptoms and recover, while some others may progress to show severe symptoms. Account for mild, for progression from mild to strong and then to severe symptoms. Assume transmission rates differ depending on how strong the symptoms are.
- How could you account for individuals that die from the disease.
- How would you model interventions like social distancing?

Discuss the models, their parameters and the assumptions behind them.

Question 4

You want to model and Zombie apocalypse scenario. To do so you consider a model with 3 compartments: Humans (variable S for susceptible), Zombies (variable Z) and Removed (variable R). The assumptions are as follows: Humans are born at a constant rate Π, and die of natural death at a rate δ. They get infected by other zombies (i.e., turning into zombies) at a *transmission rate* β. Zombies can be defeated by humans – removed – at a rate α. Removed individual can resurrect and become zombies at a rate γ.

Formulate a system of ODE describing the dynamics.
Consider different ways you could extend this model to be more interesting. For example, you could add a compartment for people who have been permanently cured.

Optional: Use a computer to explore the dynamics.