

FIT3139: Applied questions for week 8

Question 1

Design an algorithm to perform Monte Carlo integration on a positive and continuous function $f(x)$ over the interval $[a, b]$.

Question 2

Revise the Buffon's needle problem handled in Week 7 where we computed the axiomatic probability of a needle of length $r < 1$ will land crossing some line, given a floor with equally spaced parallel lines, where each pair of lines are a unit distance apart.

Derive the expression of the axiomatic probability if the needle had a length $r > 1$?

Implement a Monte Carlo method to find the probability of the needle crossing some line for various values of r (including values greater than 1) and plot the result.

Question 3

Review the Inverse Sampling method for sampling random variables. How does it work? In what contexts can it be used?

You want to sample a random variable X with density

$$f(x) = \frac{a}{x+1},$$

where a is some constant and $0 \leq x \leq 3$.

- Find the value of a .
- Implement a function to sample this distribution using Inverse Sampling.

Question 4

Review the Rejection Sampling method for sampling random variables. How does it work? In what contexts can it be used?

You want to generate a random variable X with density

$$f(x) = 140x^3(1-x)^3,$$

defined for $0 < x < 1$. Describe how to do this then implement a function to solve the problem. If you find the first two derivatives of the function, you can use your code from the week 5 applied session to find its maximum.

Question 5: Optional Extension

You want to sample a random variable X with density

$$f(x) = \lambda e^{-\lambda x},$$

with $\lambda = 2$ and $x \geq 0$. You wish to use Rejection Sampling.

- What problem occurs if you try and use a Continuous Uniform Distribution as your bounding probability distribution?
- What problem occurs if you try and use a Normal Distribution as your bounding probability distribution?
- Can you make an assumption that makes it possible for either of these distributions to be used? Under this assumption, write an algorithm using Rejection Sampling to obtain samples from this random variable.