FIT3139: Applied questions for week 9

Question 1

Describe how you can sample a discrete random variable with states $S = \{1, 2, \dots n\}$, and probability mass $P\{X = i\} = p_i$.

Question 2

Use the Gillespie algorithm to write an algorithm for generating a stochastic trajectory of the Lotka-Volterra model. Implement this algorithm as a function and solve the system using the parameters $r = 0.8, s = -0.8, f = 0.1, \alpha = 0.045, \beta = 0.03$ and plot your results.

Question 3

Consider the absorbing Markov chain whose transition matrix is given below:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Provide the canonical form of the matrix. Use the canonical form to compute the absorbtion probabilities and absorbtion times for this Markov Chain.

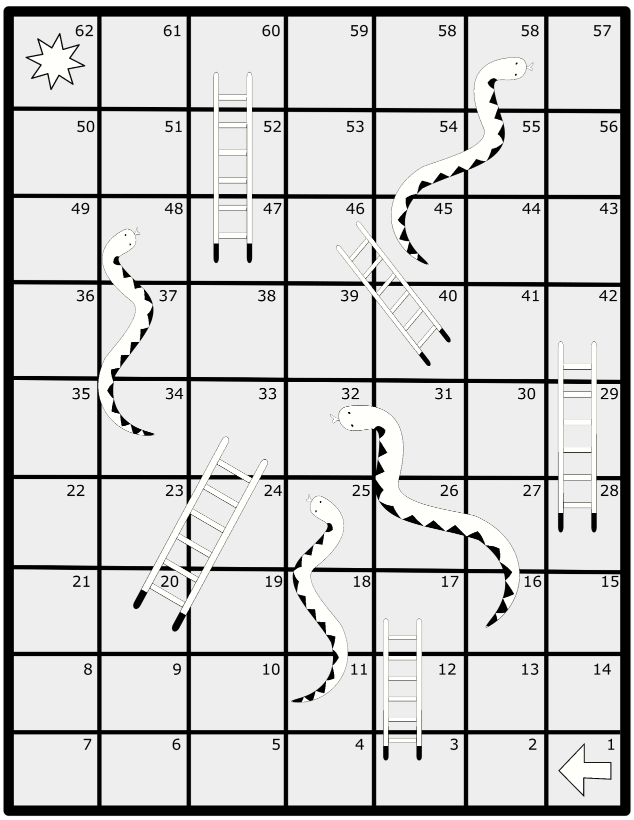
Question 4

Show that the fundamental matrix for an absorbing Markov chain is defined as $N = I + Q + Q^2 + Q^3 + \cdots$ if and only if, $N = (I - Q)^{-1}$. Optional: Show that if $N = I + Q + Q^2 + Q^3 + \dots$ then n_{ij} is the expected number of times the chain visits state j, starting in state i before absorption.

Question 5: Snakes and Ladders

Consider the game of snakes and ladders given by the following figure:

How would you represent this game with a Markov Chain? How would you use this Markov Chain to compute which states were visited most often, and how long it takes on average for the game to end? How about simulating a game?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