Prepared by Julian García, based on material by
Arun Konagurthu

Workshop 17
Final Project

FIT 3139 Computational Modelling and Simulation

MONASH University
Information Technology

Due week 15

Outline

- Purpose of the final project
- Go through the final project specification
- How to select a project
- My own final project
- Marking rubric
- Other tips

Purpose of final project

- The main purpose of the final project is to **assess all learning outcomes** of the unit in a way that allows you to **integrate unit knowledge** into a single piece of work
- The learning outcomes of the unit are:

1. Explain and apply the process of computational scientific **model building, verification and interpretation**;
2. Analyse the **differences between core classes of modelling approaches** (Numerical versus Analytical; Linear versus Non-linear; Continuous versus Discrete; Deterministic versus Stochastic);
3. Evaluate the **implications of choosing different modelling approaches**;
4. Rationalise the role of **simulation and data visualisation** in science;
5. **Apply all of the above** to solving idealisations of real-world problems across various scientific disciplines.

Project specification

Task description

To demonstrate all learning outcomes, you will develop an **extension of a model discussed in the classroom**. An extension addresses the same problem, but adds or relaxes specific assumptions about the model. For example, taking a deterministic model and introducing assumptions to do a stochastic analysis, or providing stochastic analysis for a simulation.

Your extension should address the same problem, but contain some different assumptions that may or may not lead to different conclusions — an analysis should be presented comparing the results of the original model and the extended model. The model extension should be explained, interpreted an analysed, and it should allow you to showcase **at least <u>two</u> of the following techniques**:

- Markov chains
- Montecarlo simulation
- Heuristics
- Game theory

Your extension should address **two different modelling questions**, and use the algorithms, techniques and visualisations discussed in the clasroom to answer those questions.

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What to submit

Base model	One sentence description of the base model
Extension assumptions	One paragraph description on how assumptions are modified and the nature of the extension
Techniques showcased	Technique 1. Technique 2.
Modelling question 1	Questions being addressed.
Modelling question 2	

Report PDF!

- **Section 1: Specification table**
- **Section 2: Introduction**
- **Section 3: Model description**
- **Section 4: Results**
- **Section 5: List of algorithms and concepts**

What is your assignment about and what are your goals. What is your plan of attack (including techniques) around some questions you want to answer. <u>Get the reader excited about what's coming.</u>

Start by describing <u>base model</u> in detail. Then add: assumptions, mathematics, algorithms that define <u>your model</u>. Take time to discuss differences in approach.

Answer your questions, and give supporting evidence (including plots or visualisations). Discuss what you think is driving your results and how these compare to the results of the base model.

Tell us in bullet points what concepts and algorithms where part of your project (even if you used a library)
<u>What, Where and Why</u>

Video presentation

Source code

Video presentation

No longer than 10 minutes, but can be 5 minutes long.

Structured as the project is.

Tell us what you would like to have done if you have had more time.

Share your enthusiasm about the project.

Submit slides in PDF and Video in MP4 format

Source code

Code used for model and result analysis, marked as per each section of the report

Comment and make it easy to read.

Any of the standard libraries can be used, but you need to understand.

Submit all source code with appropriate filenames etc.

Report 85%

Section 1: Specification table

Section 2: Introduction (10 %)

Section 3: Model description (35 %)

Section 4: Results (35 %)

Section 5: List of algorithms and concepts (5 %)

Video presentation 15%

Source code (supporting model and results - checked for correctness)

How to pick?

Base model + Extension component

(Novel, not in the lectures or labs or assignments)

Technique requirement

<u>Two</u> out of:

Markov chain, Montecarlo simulation, Heuristics, Game Theory

Base model

- Schelling's segregation model
- Pipes and reactors (linear balance)
- Exponential Growth discrete time
- Exponential Growth continuous time
- Logistic growth discrete time
- Logistic growth continuous time
- Restricted population growth
- Pharmacokinetics
- SIR Discrete time
- SIR Continuous time
- SEIR and other variants
- Lotka Volterra
- Predator prey model (with humans)
- Guerrilla warfare
- Stochastic SIR and variants
- Land of oz with Markov Chains
- Snakes and ladders game
- Random web surfer
- Euclidian TSP
- Solving sudoku puzzles
- Rational commuter two roads
- Tennis game with game theory
- Tennis game with Markov chains
- Repeated prisoner's dilemma

Be creative!

Example 1

- **Base model:** Guerrilla warfare
- **Extension:** Stochastic Guerilla Warfare
- **Showcase:**
  - Markov chains: Discrete time, stochastic outcomes, tractable
  - Montecarlo simulation: Simulate larger versions of the above, relax assumptions
- **Questions**
  - What is the effect of randomness on warfare outcomes? Compare base vs stochastic version.
  - What factors drive the length of warfare? Absorption times, simulated and exact.

Example 2

- **Base model:** SIR deterministic time
- **Extension:** SIR and risk averse human behaviour
- **Showcase:**
  - Game theory: Agents can limit their exposure based on a game depending on infection numbers - two types of agents (extra compartments)
  - Montecarlo simulation: Simulate more types and non-rational agents
- **Questions**
  - How does human risk behaviour impact the spread of disease? risk vs no risk comparison.
  - Does risk diversity improve outcomes? More types via simulation

Example 3

- **Base model:** Iterated prisoner's dilemma
- **Extension:** Arbitrary memory and heuristic search for equilibria
- **Showcase:**
  - <u>Game theory</u>: Memory size determines a normal form game
  - <u>Heuristics</u>: For larger games, we use a heuristic to search for Nash equilibria
- **Questions**
  - How does the number of cooperative equilibria vary with memory size?
  - How effective can heuristics be in finding Nash equilibria?

Example 4

- **Base model:** Tennis with Markov Chains
- **Extension:** Simulate a complete tennis season based on skill distribution
- **Showcase:**
  - <u>Markov chains</u>: Simulate a tennis game with Markov chains, based on players disparity
  - <u>Montecarlo simulation</u>: Simulate a set, a match, a tournament a season (based on the game model)
- **Questions**
  - How does skill distribution affect rankings at the end of the season?
    Compare initial skill distribution, vs point distribution at end of season.
  - What is more effective in revealing quality? Longer seasons or longer matches?
    Restrict 3 set vs 5 format in simulation, inspect impact of season length

Final project

FIT 3139

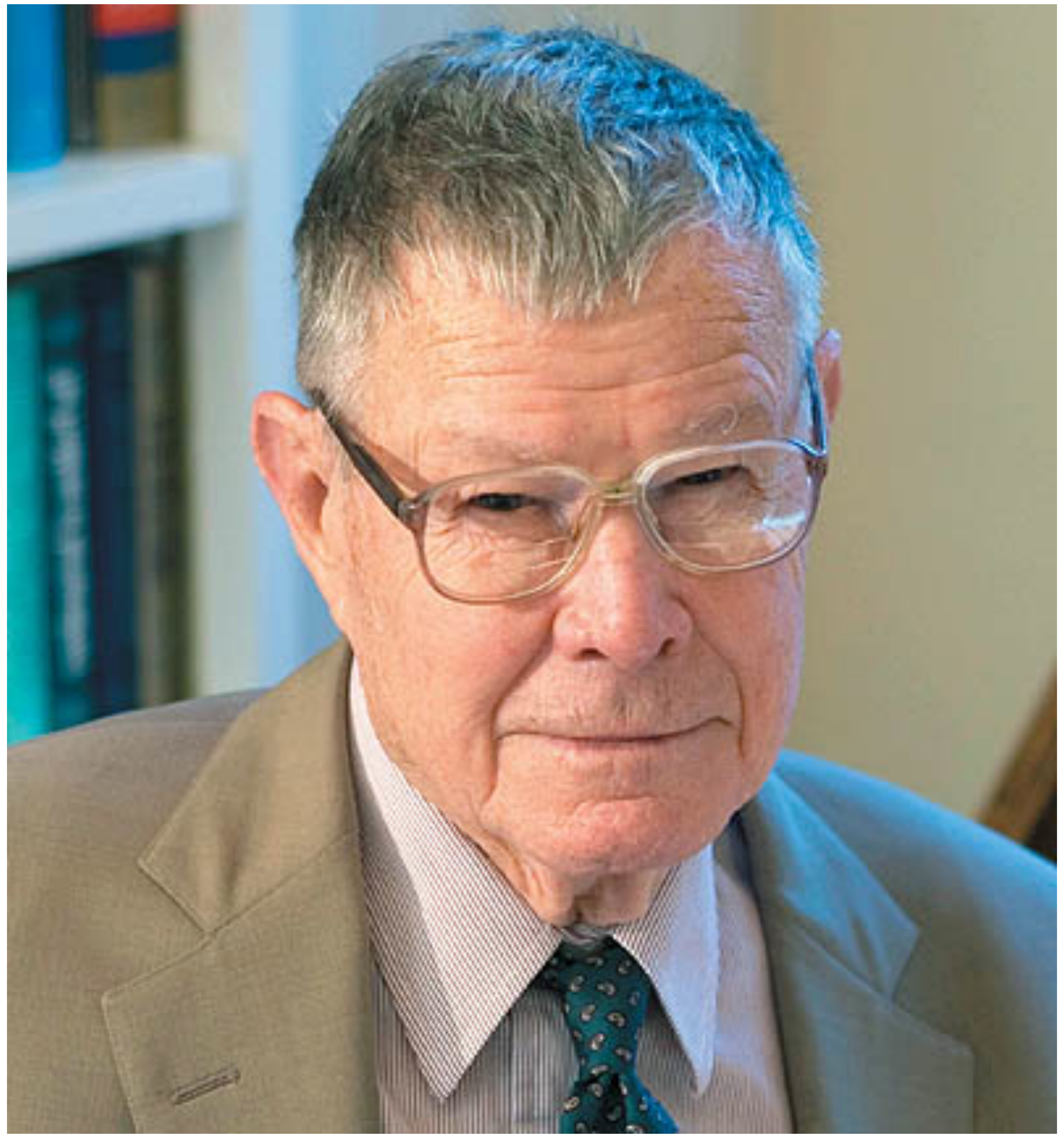
Computational Modelling and Simulation

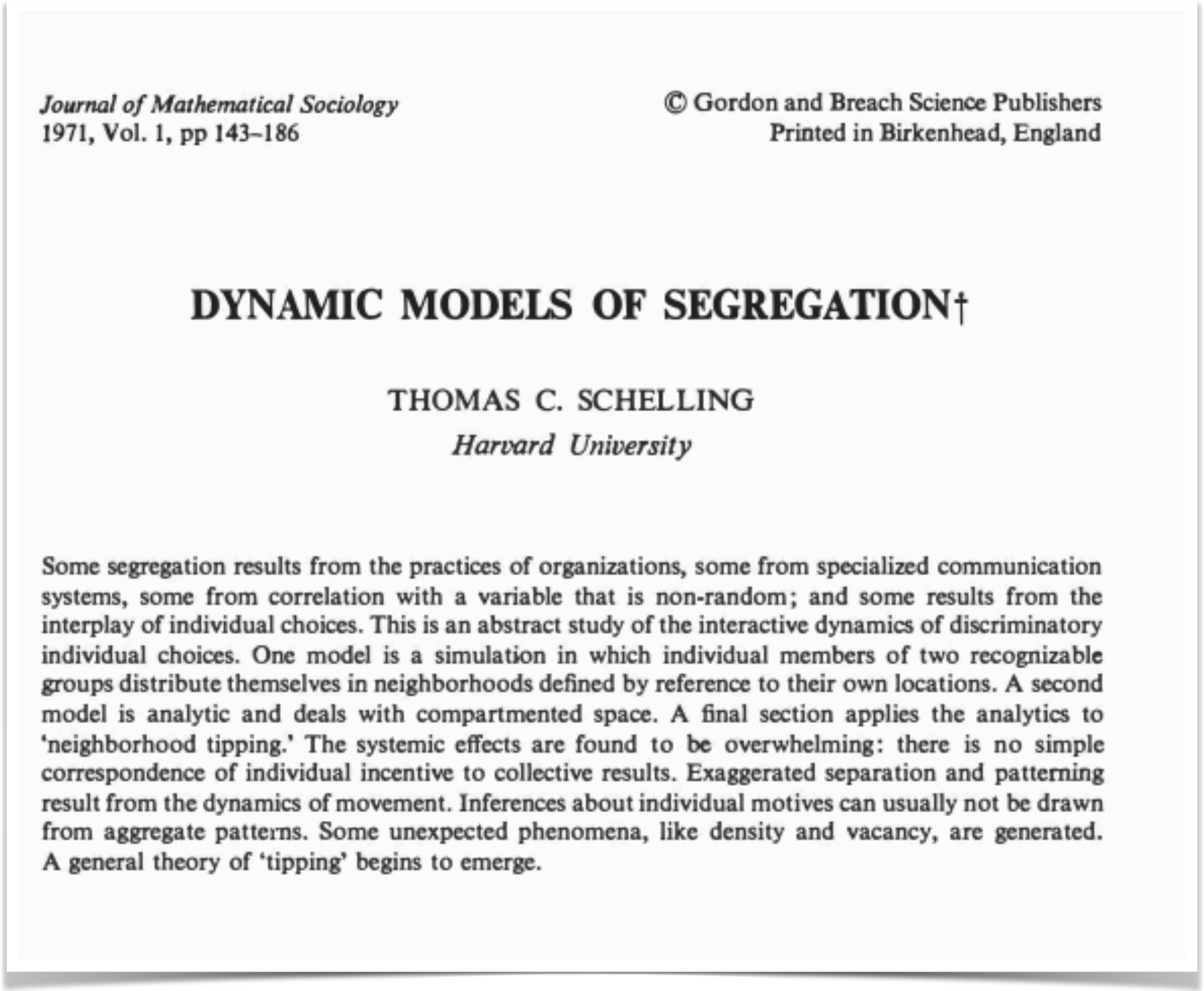
MONASH University
Information Technology

Example 5

- **Base model:** Schelling model
- **Extension:** Simplify Schelling model to provide exact calculations using Markov Chain Theory
- **Showcase:**
  - Markov chains: Exact calculations for a simple Schelling model
  - Montecarlo simulation: Simulate larger versions of the system, and agents misjudging happiness.
- **Questions**
  - Can a simple model reflect the qualitative outcomes of the Schelling model?
    Compare qualitatively the original model and the simplified model
  - Does randomness of choice affect the qualitative output of the model?
    Introduce misjudgement of happiness and analyse theory and simulation

Thomas Schelling





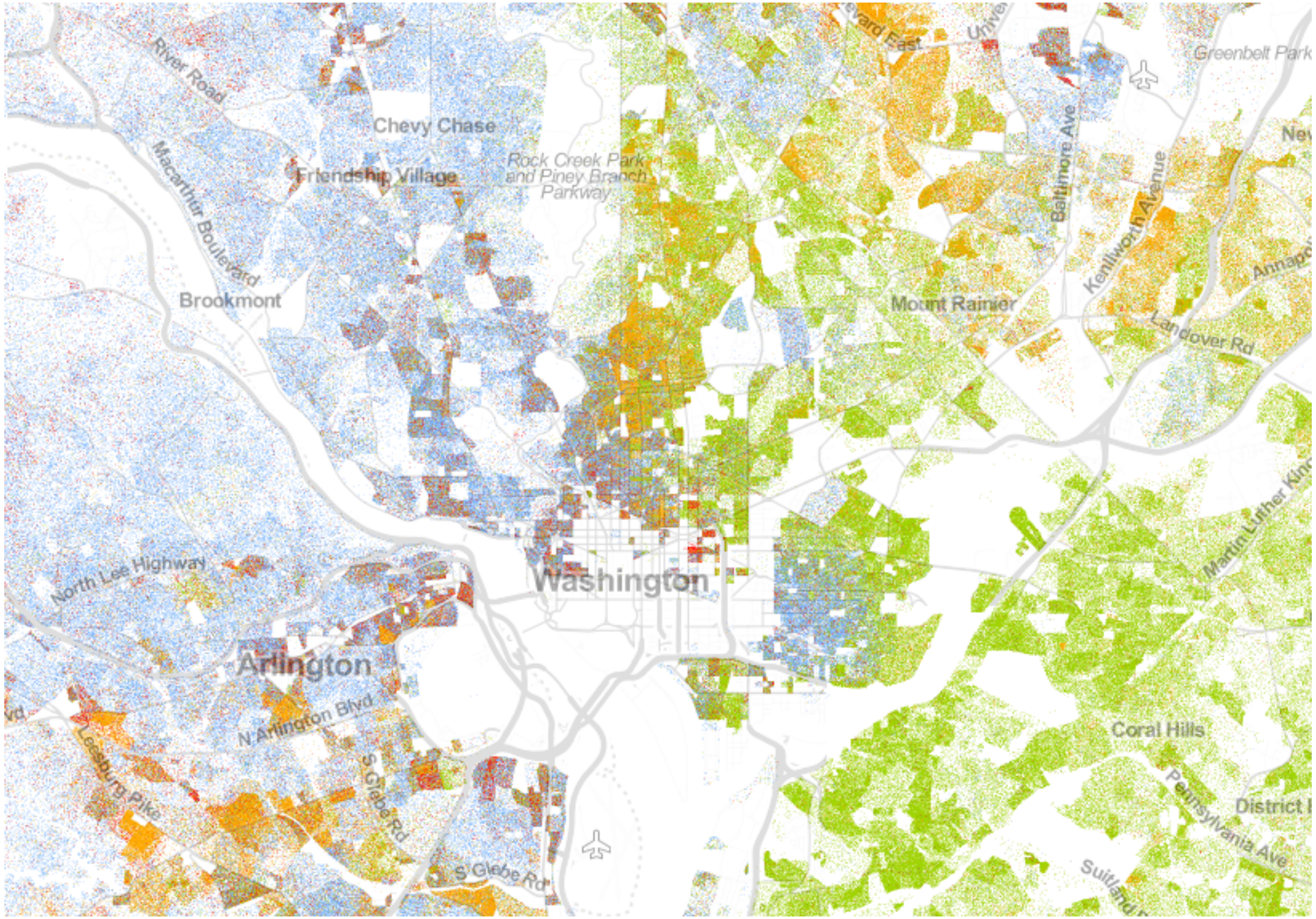
Journal of Mathematical Sociology
1971, Vol. 1, pp 143–186

DYNAMIC MODELS OF SEGREGATION†

THOMAS C. SCHELLING
Harvard University

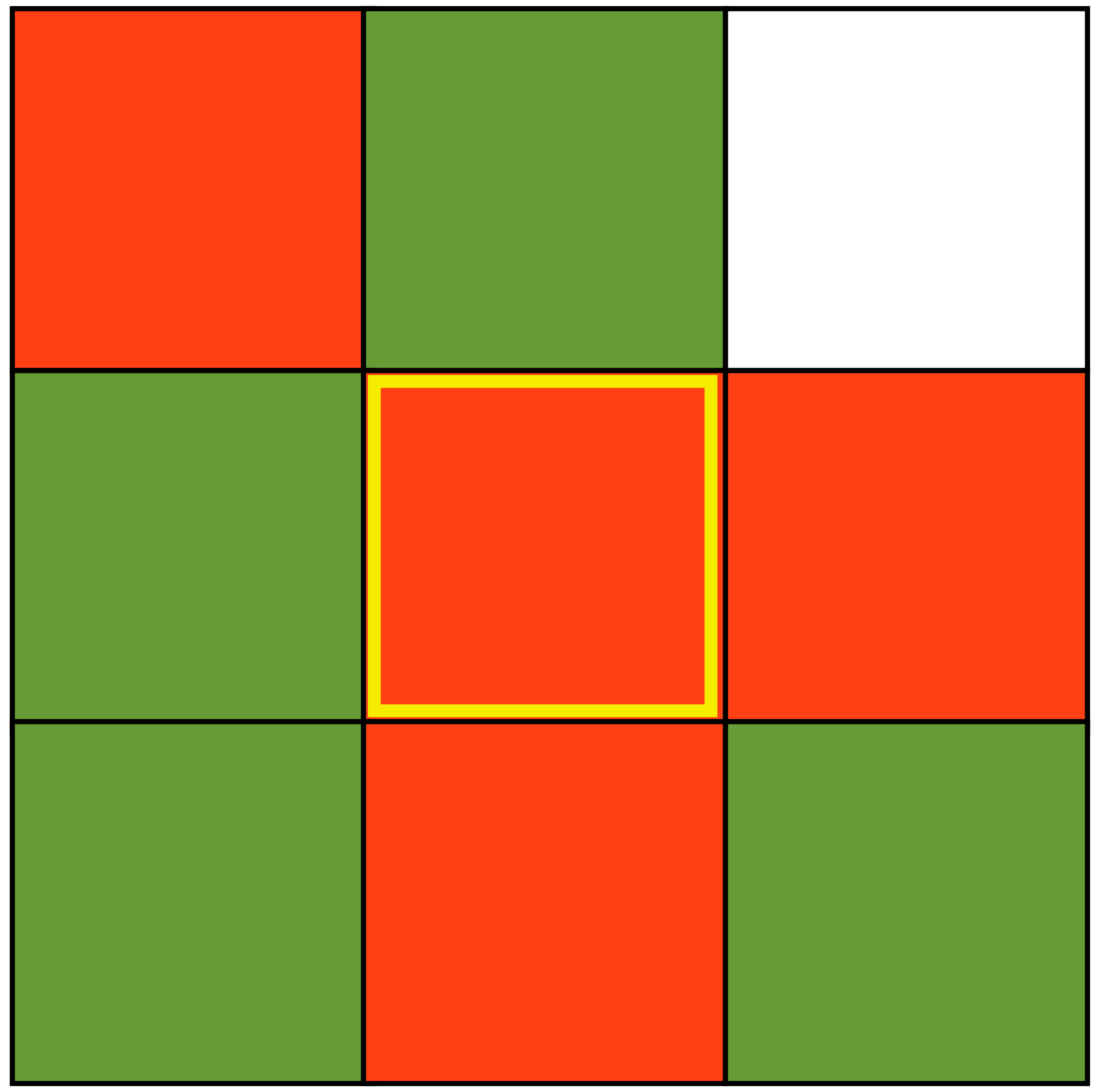
Some segregation results from the practices of organizations, some from specialized communication systems, some from correlation with a variable that is non-random; and some results from the interplay of individual choices. This is an abstract study of the interactive dynamics of discriminatory individual choices. One model is a simulation in which individual members of two recognizable groups distribute themselves in neighborhoods defined by reference to their own locations. A second model is analytic and deals with compartmented space. A final section applies the analytics to 'neighborhood tipping.' The systemic effects are found to be overwhelming: there is no simple correspondence of individual incentive to collective results. Exaggerated separation and patterning result from the dynamics of movement. Inferences about individual motives can usually not be drawn from aggregate patterns. Some unexpected phenomena, like density and vacancy, are generated. A general theory of 'tipping' begins to emerge.

Washington



River Road
Chevy Chase
Friendship Village
Rock Creek Park and Piney Branch Parkway
Macarthur Boulevard
Brookmont
Greenbelt Park
Baltimore Ave
Kenilworth Avenue
Mount Rainier
Landover Rd
North Lee Highway
Washington
Arlington
N Arlington Blvd
Leesburg Pike
S Glebe Rd
Martin Luther King
Coral Hills
Pennsylvania Ave

Should I stay or should I go



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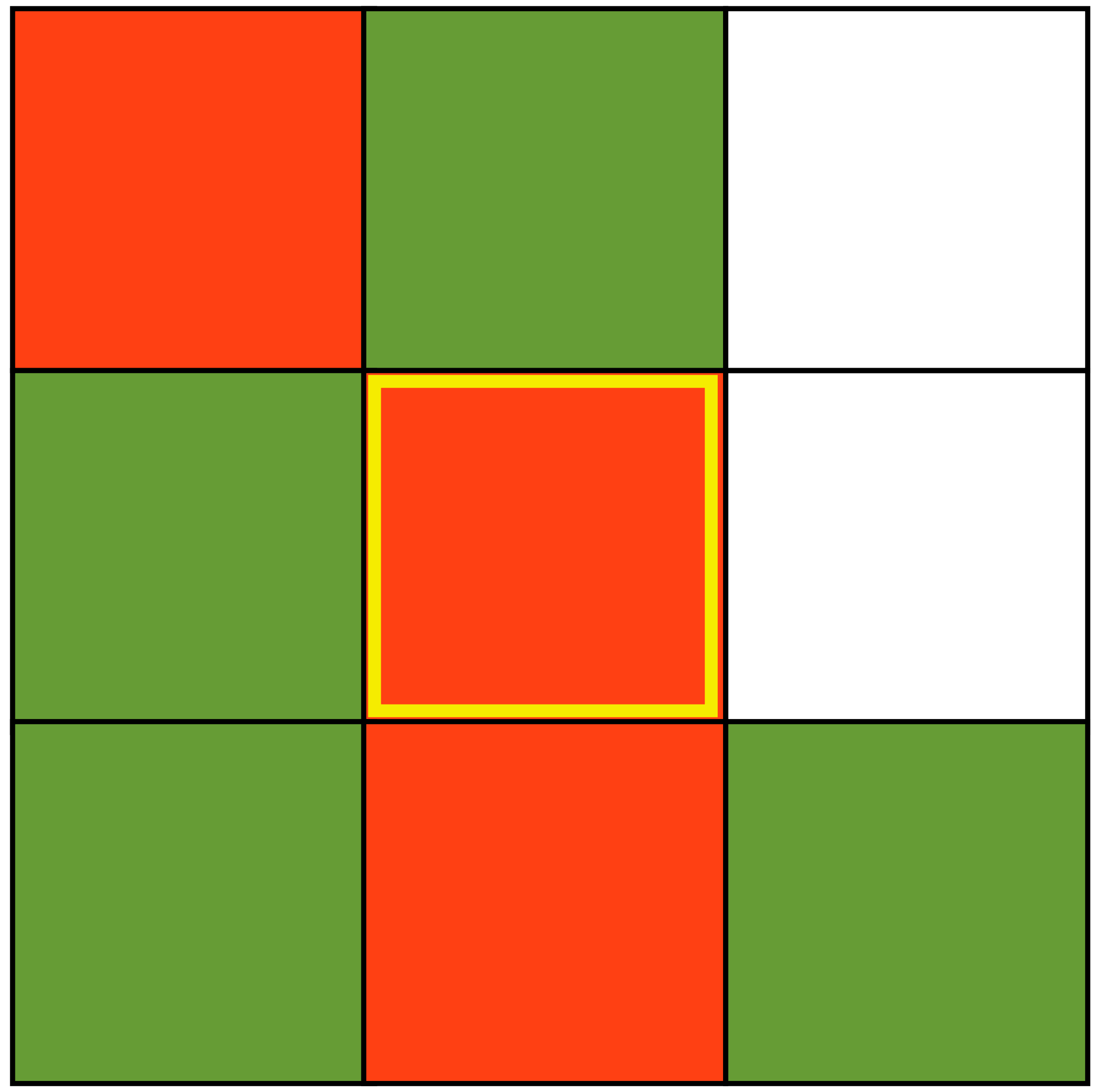
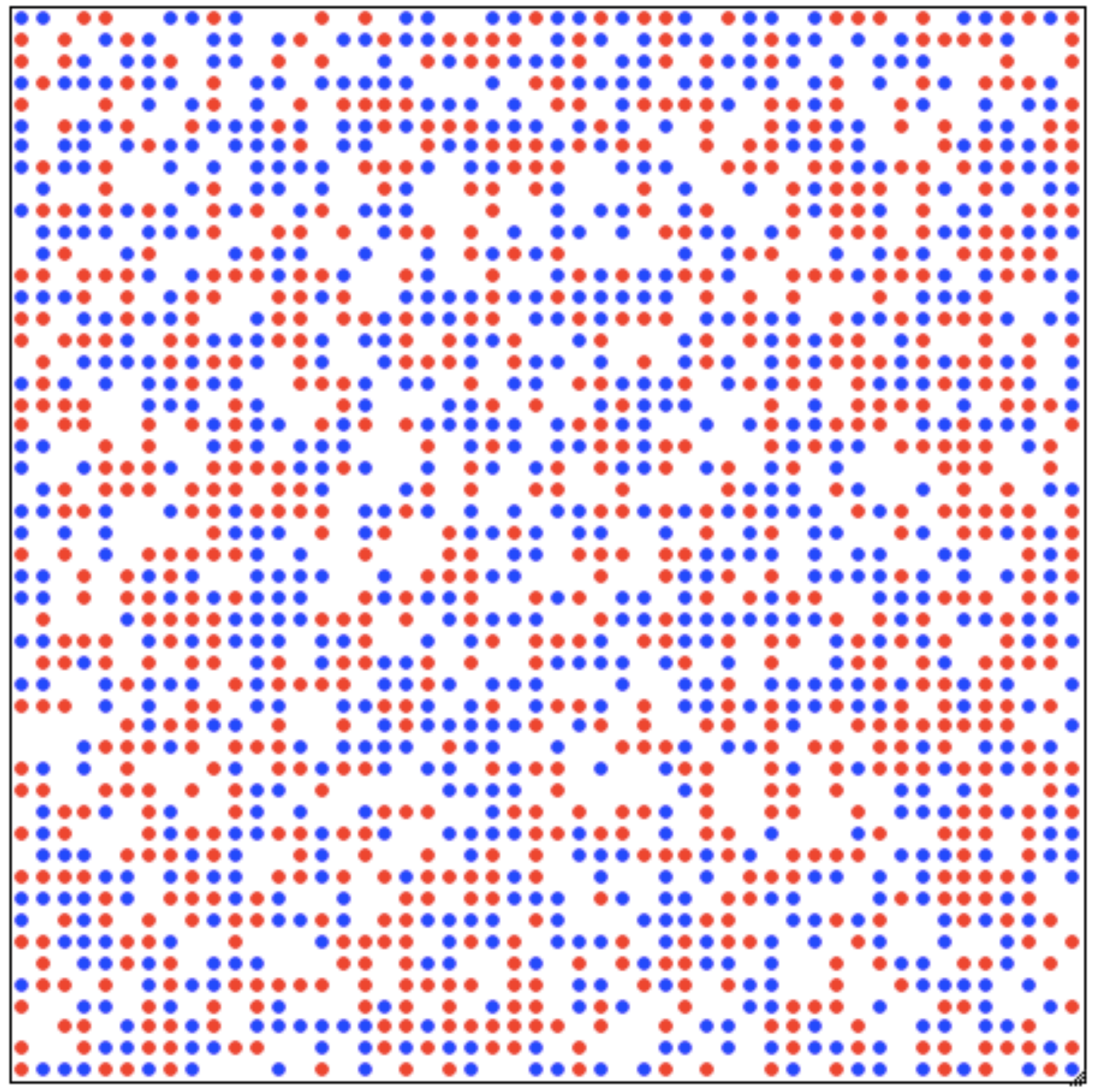


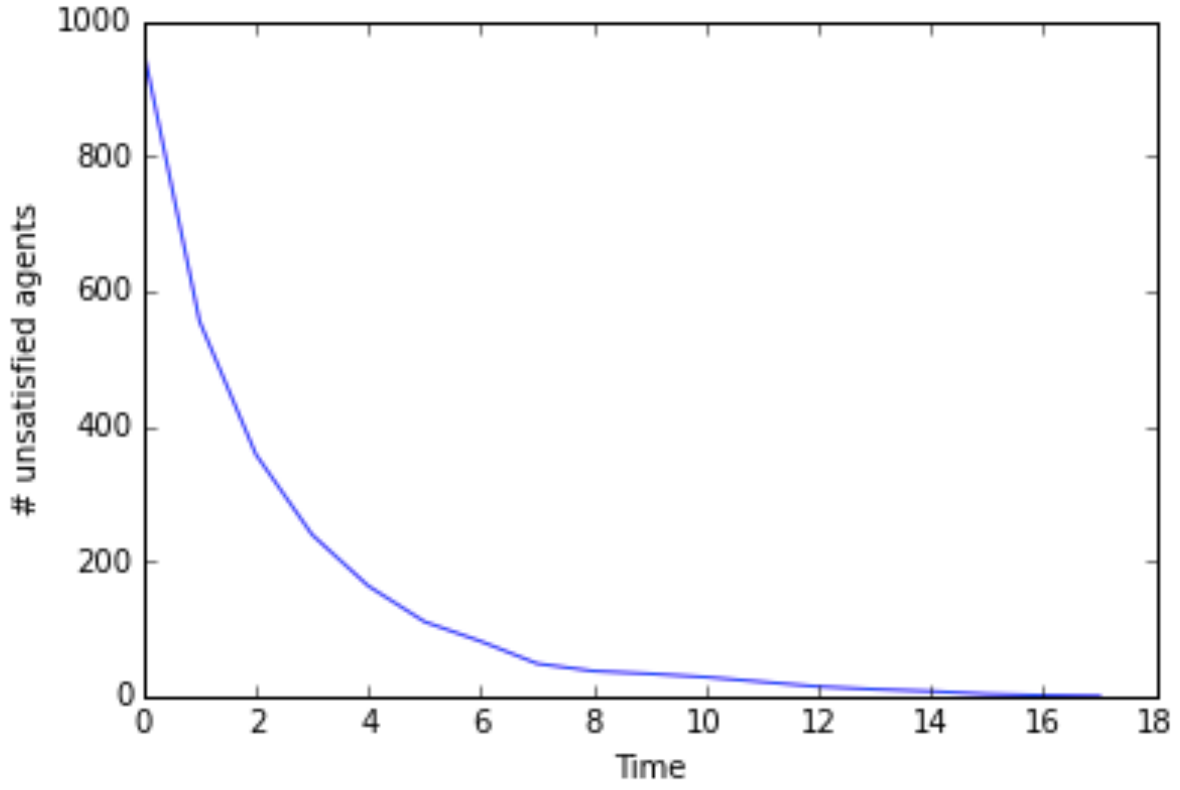
2/6

Schelling's segregation model.

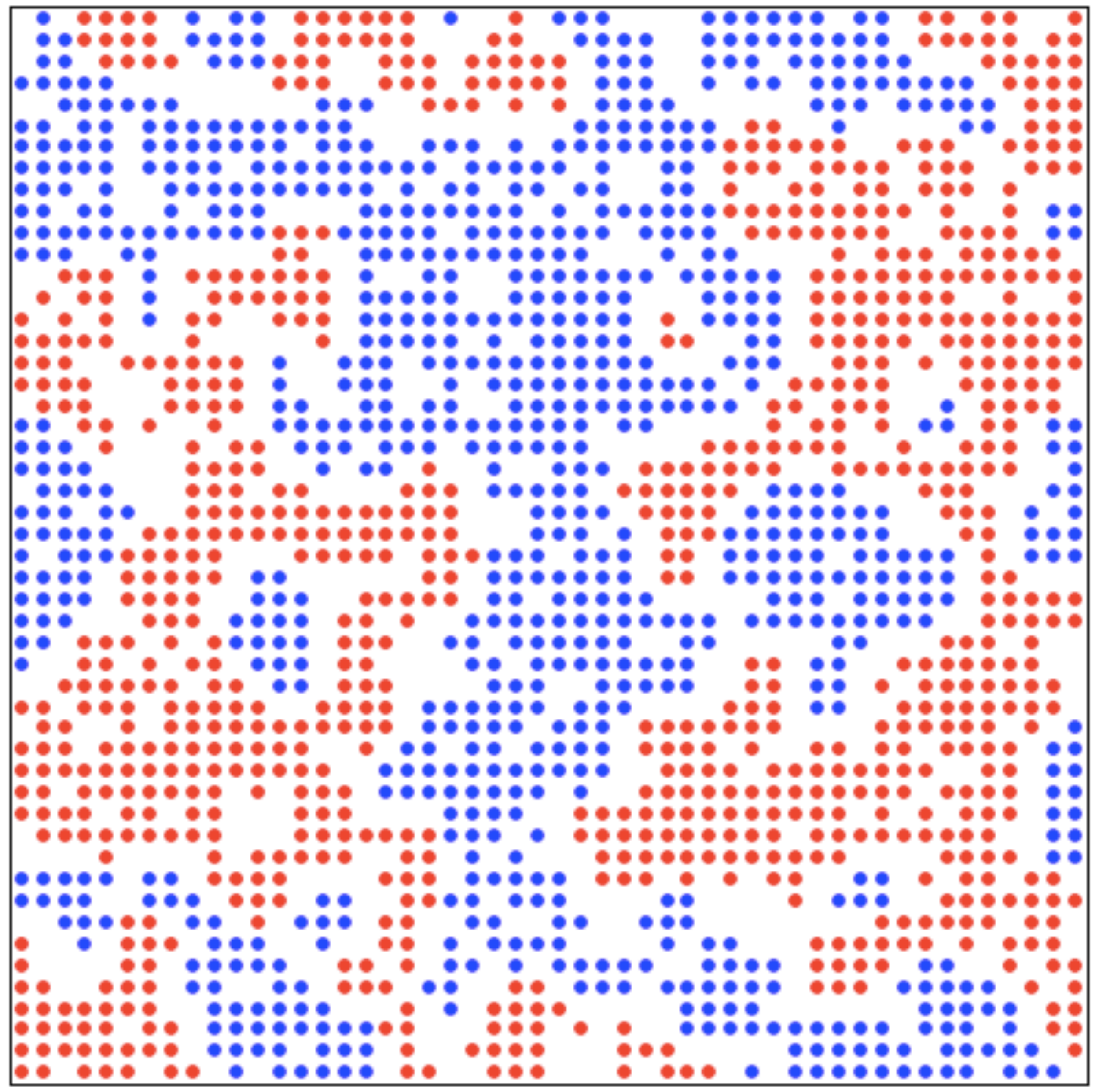
```
place agents on the grid.
for each agent, calculate happiness.
while (maximum number of iterations not
reached) do
    for each unhappy agent:
        find a new location randomly.
    for each agent:
        calculate happiness.
    if (everyone is happy):
        return
end
```

Note: To analyse the outcomes of this model I need to pick a quantity of interest and repeat many times, i.e., Montecarlo!





unsatisfied agents
Time
0
200
400
600
800
1000
0
2
4
6
8
10
12
14
16
18



Sketch of my report

Introduction

- The Schelling model is traditionally studied using Simulation
- The structure of the Schelling model is actually a Markov chain with a very large number of states
- The idea is to restrict the number of states drastically, and attempt an analysis using <u>Markov chain Theory</u>
- Then use <u>Montecarlo simulations</u> to expand on the theory
- <u>Questions</u>
  - Can a simple model reflect the qualitative outcomes of the Schelling model? Compare qualitatively the original model and the simplified model
  - Does randomness of choice affect the qualitative output of the model? Introduce misjudgement of happiness and analyse theory and simulation

Model description

- In the simplified Schelling model agents live on a cycle of finite size n. Agents can be of two types, say *0* and *1*. There are no empty positions, thus, a cycle of size n also implies n agents
- In this version there are no thresholds. Instead, an agent is "happy" if at least one of her neighbours is of the same type.
- Agents will agree to trade places if and only if at least one of the two agents benefits and no one is worse off after trading places.
- Since we obtain an absorbing Markov chain we can use the theory to understand certain quantities related to the questions we asked
- Further:
  - Agents can make mistakes when trading places (ergodic chain!)
  - Number of states is very restricted, for theory but we can estimate same quantities via montecarlo simulations in larger systems

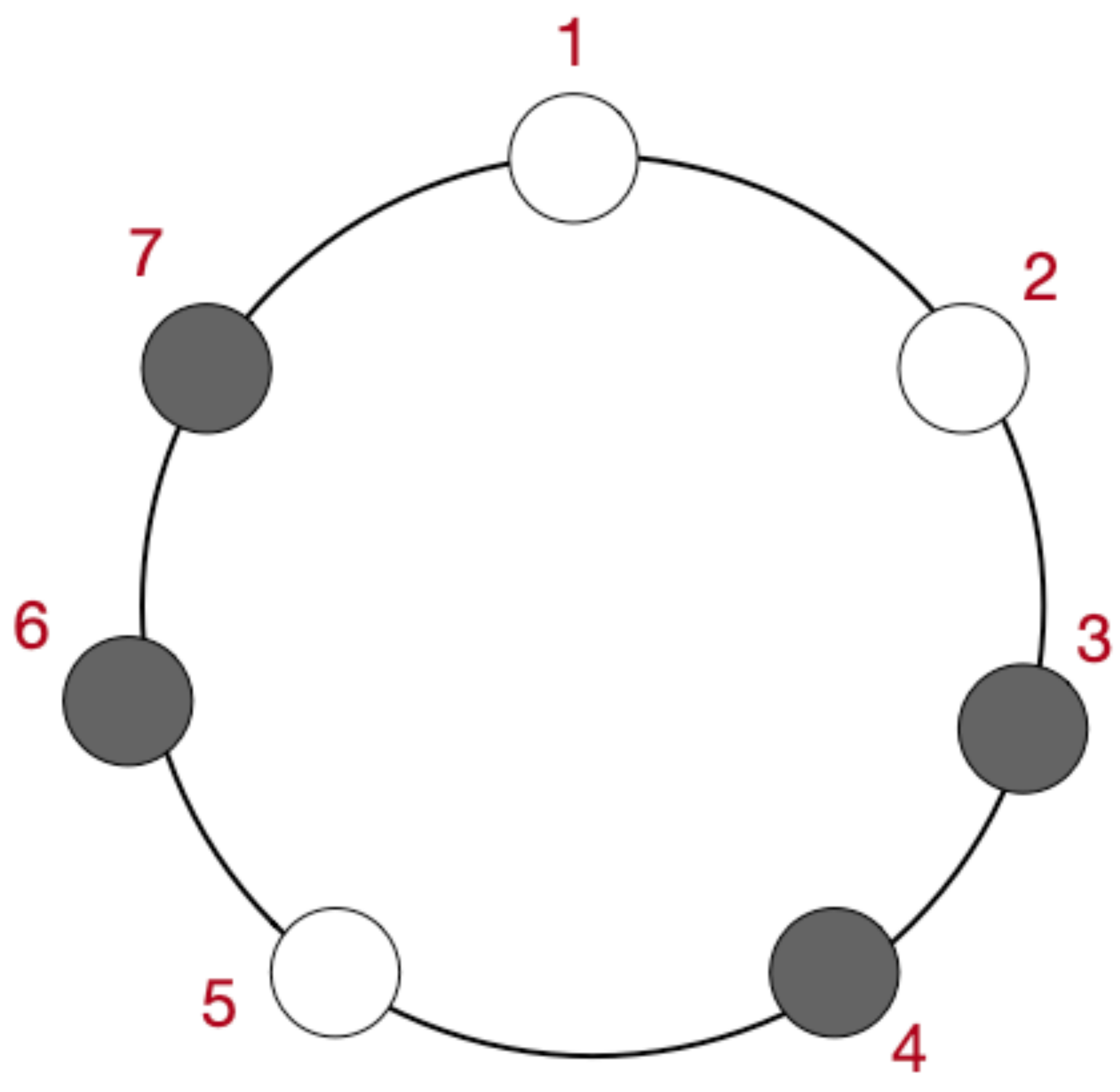


Figure 1: Example: A cycle of size 7. According to the rules above, all agents are "happy" except agent 5

Results

Two distinct individuals are chosen at each timestep. Each such pair has probability 1/6.
Trading towards absorbing states always happens.
Some transitions between transient states are not possible.

4 agents

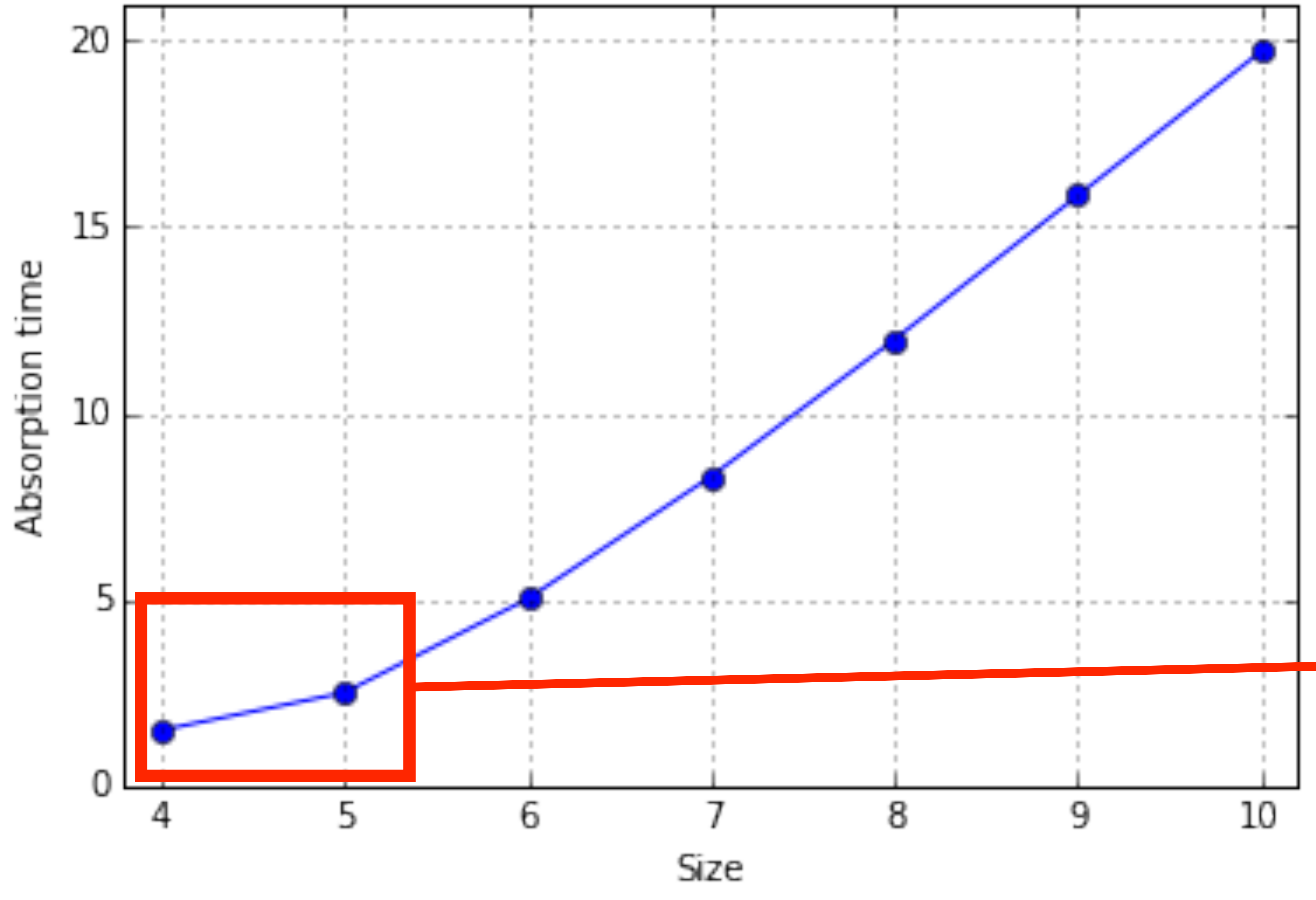
0	(0,	0,	1,	1)
1	(0,	1,	0,	1)
2	(0,	1,	1,	0)
3	(1,	0,	0,	1)
4	(1,	0,	1,	0)
5	(1,	1,	0,	0)

This chain has four absorbing states:
0, 2,3 and 5.
Other states are transient.

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & 0.0 & \frac{1}{6} \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ \frac{1}{6} & 0.0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

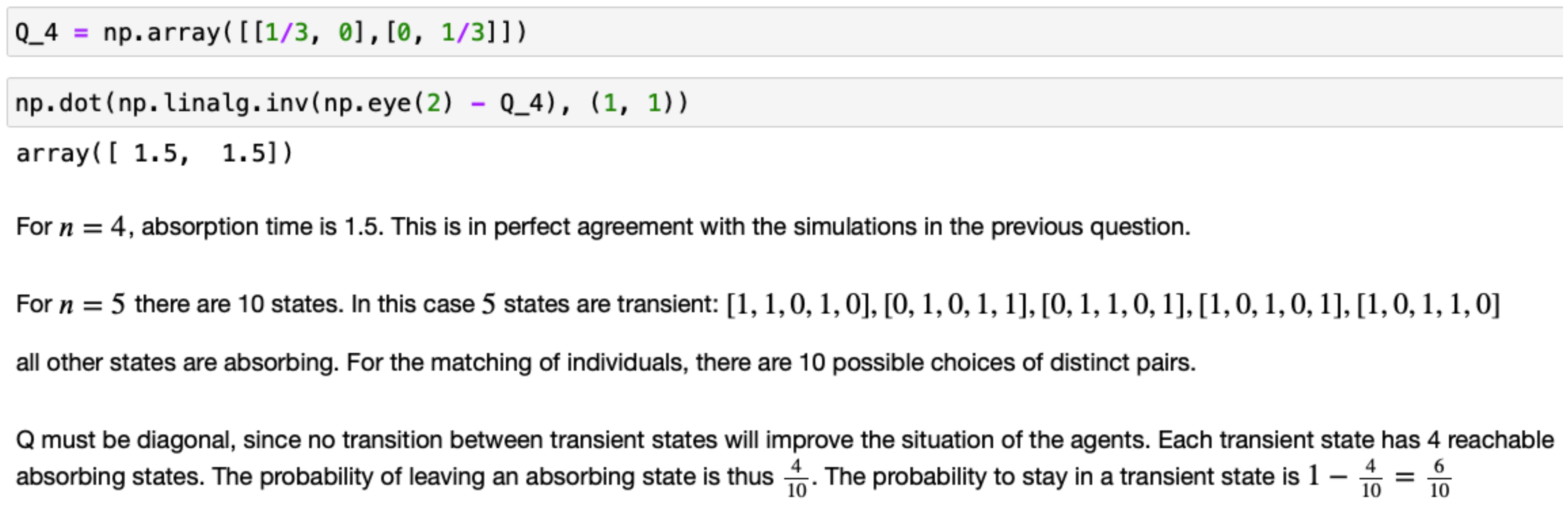
$$\begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Results



Absorption times via simulation

These two values match my theory



Results

$$\begin{bmatrix} -\frac{2\epsilon}{3}+1 & \frac{\epsilon}{6} & \frac{\epsilon}{6} & \frac{\epsilon}{6} & \frac{\epsilon}{6} & 0.0 \\ -\frac{\epsilon}{6}+\frac{1}{6} & \frac{2\epsilon}{3}+\frac{1}{3} & -\frac{\epsilon}{6}+\frac{1}{6} & -\frac{\epsilon}{6}+\frac{1}{6} & 0.0 & -\frac{\epsilon}{6}+\frac{1}{6} \\ \frac{\epsilon}{6} & \frac{\epsilon}{6} & -\frac{2\epsilon}{3}+1 & 0.0 & \frac{\epsilon}{6} & \frac{\epsilon}{6} \\ \frac{\epsilon}{6} & \frac{\epsilon}{6} & 0.0 & -\frac{2\epsilon}{3}+1 & \frac{\epsilon}{6} & \frac{\epsilon}{6} \\ -\frac{\epsilon}{6}+\frac{1}{6} & 0.0 & -\frac{\epsilon}{6}+\frac{1}{6} & -\frac{\epsilon}{6}+\frac{1}{6} & \frac{2\epsilon}{3}+\frac{1}{3} & -\frac{\epsilon}{6}+\frac{1}{6} \\ 0.0 & \frac{\epsilon}{6} & \frac{\epsilon}{6} & \frac{\epsilon}{6} & \frac{\epsilon}{6} & -\frac{2\epsilon}{3}+1 \end{bmatrix}$$

An ergodic markov chain!

```
stationary = np.linalg.matrix_power(P_N, 1000000)[0]
for i, state in enumerate(states_schelling):
    print("{}. {}; {}".format(i, state, stationary[i]))
```

```
0. (0, 0, 1, 1); 0.248743718581695
1. (0, 1, 0, 1); 0.00251256281395651
2. (0, 1, 1, 0); 0.248743718581695
3. (1, 0, 0, 1); 0.248743718581695
4. (1, 0, 1, 0); 0.00251256281395651
5. (1, 1, 0, 0); 0.248743718581692
```

The Montecarlo estimation of the stationary distribution is as follows:

```
observations = sum(list(ct.values()))
np.set_printoptions(suppress=True, precision=3)
for i in ct.keys():
    print(i, ct[i]/observations)
```

```
(False, True, True, False) 0.2513488
(True, False, True, False) 0.0025284
(True, True, False, False) 0.2475202
(False, True, False, True) 0.0025124
(False, False, True, True) 0.2474332
(True, False, False, True) 0.248657
```

- Questions
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Give me the nuance!

List of concepts

- The Schelling model is used as the basis for our model
- We formulate a Markov chain model, two versions: absorbing and ergodic
- Segregation follows absorption, so we inspect at absorption times derived from the canonical form of the Absorbing chain
- For a version with mistakes we inspect the stationary distribution of the ergodic Markov chain - and compute it using the power method.
- Larger versions of the system estimate the same quantities using montecarlo simulations.

Just a sketch!

What grade do I want?

PASS	Just what is asked with some minor correctness issues
CREDIT	Just what is asked in a format that is easy to read/ watch/interpret. (I know what you did)
DISTINCTION	What is asked with extensive analysis, more than two techniques implemented in a meaningful way around questions posed
HIGH DISTINCTION	Extensive analysis and more than two techniques implemented in a meaningful way around an interesting question showing deep understanding of the modelling exercise

Tips!

Good luck!

See you in the workshop…