

Geometry: JEE Maths

G V V Sharma*

1. In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is the length of the segment BD ?
2. In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on line segment CD so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E . What is the area of $\triangle AEB$?
3. $ABCD$ is a square and $AB = 1$. Equilateral triangles AYB and CXD are drawn such that X and Y are inside the square. What is the length of XY ?
4. Let O and I are the circumcentre and incentre of $\triangle ABC$ respectively. Suppose O lies in the interior of $\triangle ABC$ and I lies on the circle passing through B , O and C . What is the magnitude of $\angle BAC$ in degrees?
5. PS is a line segment of length 4 and O is the midpoint of PS . A semicircular arc is drawn with PS as diameter. Let X be the midpoint of this arc. Q and R are points on the arc PXS such that QR is parallel to PS and the semicircular arc drawn with QR as diameter is tangent to PS . What is the area of the region $QXROQ$ bounded by the two semicircular arcs?
6. Let AD and BC be the parallel sides of a trapezium $ABCD$. Let P and Q be the midpoints of the diagonals AC and BD . If $AD = 16$ and $BC = 20$, what is the length of PQ ?
7. In a triangle ABC , let H , I and O be the orthocentre, incentre and circumcentre respectively. If the points B , H , I , C lie on a circle, what is the magnitude of $\angle BOC$ in degrees?
8. Let ABC be an equilateral triangle. Let P and S be points on AB and AC , respectively, and let Q and R be points on BC such that $PQRS$ is a rectangle. If $PQ = \sqrt{3}PS$ and the area of $PQRS$ is $28\sqrt{3}$, what is the length of PC ?
9. Let A_1, B_1, C_1, D_1 be the midpoints of the sides of a convex quadrilateral $ABCD$ and let A_2, B_2, C_2, D_2 be the midpoints of the sides of the quadrilateral A_1, B_1, C_1, D_1 . If A_2, B_2, C_2, D_2 is a rectangle with sides 4 and 6, then what is the product of the lengths of the diagonals of $ABCD$?
10. Let S be a circle with centre O . A chord AB , not a diameter, divides S into two regions R_1 and R_2 such that O belongs to R_2 . Let S_1 be a circle with centre in R_1 , touching AB at X and S internally. Let S_2 be a circle with centre in R_2 , touching AB at Y , the circle S internally and passing through the centre of S . The point X lies on the diameter passing through the centre of S_2 and $\angle YXO = 30^\circ$. If the radius of S_2 is 100, then what is the radius of S_1 ?
11. In a triangle ABC with $\angle BCA = 90^\circ$, the perpendicular bisector of AB intersects segments AB and AC at X and Y , respectively. If the ratio of the area of quadrilateral $BXYC$ to the area of triangle ABC is $13 : 18$ and $BC = 12$ then what is the length of AC ?
12. Let $ABCD$ be a convex quadrilateral with perpendicular diagonals. If $AB = 20$, $BC = 70$ and $CD = 90$, then what is the value of DA ?
13. In a triangle ABC , X and Y are points on the segments AB and AC , respectively, such that $AX : XB = 1 : 2$ and $AY : YC = 2 : 1$. If the area triangle AXY is 10 then what is the area of triangle ABC ?
14. Let $ABCD$ be a convex quadrilateral with $\angle DAB = \angle BDC = 90^\circ$. Let the incircles of triangles ABD and BCD touch BD at P and Q , respectively, with P lying in between B and Q . If $AD = 999$ and $PQ = 200$ then what is the sum of the radii of the incircles of triangles ABD and BDC ?
15. Let XOY be a triangle with $\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY , respectively. Suppose that $XN = 19$ and $YM = 22$. What is XY ?
16. In a triangle ABC , let I denote the incenter. Let the lines AI , BI and CI intersect the incircle at

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

- P, Q and R, respectively. If $\angle BAC = 40^\circ$, what is the value of $\angle QPR$ in degrees?
17. In rectangle ABCD, $AB = 8$ and $BC = 20$. Let P be a point on AD such that $\angle BPC = 90^\circ$. If r_1, r_2, r_3 are the radii of the incircles of triangles APB, BPC and CPD, what is the value of $r_1 + r_2 + r_3$?
 18. In acute-angled triangle ABC, let D be the foot of the altitude from A, and E be the midpoint of BC. Let F be the midpoint of AC. Suppose $\angle BAE = 40^\circ$. If $\angle DAE = \angle DFE$, what is the magnitude of $\angle ADF$ in degrees?
 19. The circle ω touches the circle Ω internally at P. The centre O of Ω is outside ω . Let XY be a diameter of Ω which is also tangent to ω . Assume $PY > PX$. Let PY intersect ω at Z. If $YZ = 2PZ$, what is the magnitude of $\angle PYX$ in degrees?
 20. In a rectangle ABCD, E is the midpoint of AB; F is a point on AC such that BF is perpendicular to AC; and FE perpendicular to BD. Suppose $BC = \sqrt{3}$, find AB?
 21. Suppose in the plane 10 pairwise non-parallel lines intersect one another. What is the maximum possible number of polygons that can be formed?
 22. Let P be an interior point of a triangle ABC whose sidelengths are 26, 65, 78. The line through P parallel to BC meets the AB in K and AC in L. The line through P parallel to CA meets BC in M and BA in N. The line through P parallel to AB meets CA in S and CB in T. If KL, MN, ST are of equal lengths, find this common length?
 23. Let ABCD be a rectangle and let E and F be points on CD and BC respectively such that $\text{area}(\triangle ADE) = 16$, $\text{area}(\triangle CEF) = 9$ and $\text{area}(\triangle ABF) = 25$. What is the area of triangle AEF?
 24. Let AB and CD be two parallel chords in a circle with radius 5 such that the centre O lies between these chords. Suppose $AB = 6$, $CD = 8$. Suppose further that the area of the part of the circle lying between the chords AB and CD is $m\pi + n/k$, where m, n, k are positive integers with $\gcd(m, n, k) = 1$. What is the value of $m + n + k$?
 25. Let Ω_1 be a circle with centre O and let AB be a diameter of Ω_1 . Let P be a point on the segment OB different from O. Suppose another circle Ω_2 with centre P lies in the interior of Ω_1 . Tangents are drawn from A and B to the circle Ω_2 intersecting Ω_1 again at A_1 and B_1 respectively such that A_1 and B_1 are on the opposite sides of AB. Given that $A_1B = 5$, $AB_1 = 15$ and $OP = 10$, find the radius of Ω_1 .
 26. Consider the areas of the 4 triangles obtained by drawing the diagonals AC and BD of a trapezium ABCD. The product of these areas, taken two at a time, are computed. If among the 6 products so obtained, 2 products are 1296 and 576, determine the square root of the maximum possible area of the trapezium to the nearest integer.
 27. Let D be an interior point of the side BC of a triangle ABC. Let I_1 and I_2 be the incentres of triangles ABD and ACD respectively. Let AI_1 and AI_2 meet BC in E and F respectively. If $\angle BI_1E = 60^\circ$, what is the measure of $\angle CI_2F$ in degrees?
 28. Let ABC be an acute-angled triangle and let H be its orthocentre. Let G_1, G_2 and G_3 be the centroids of the triangles HBC, HCA and HAB respectively. If the area of the triangle $G_1G_2G_3$ is 7 units, what is the area of the triangle ABC?
 29. Triangles ABC and DEF are such that $\angle A = \angle D$, $AB = DE = 17$, $BC = EF = 10$ and $AC - DF = 12$. What is $AC + DF$?
 30. In a triangle ABC, right-angled at A, the altitude through A and the internal bisector of $\angle A$ have lengths 3 and 4 respectively. Find the length of the median through A?
 31. In a triangle ABC, the median from B to CA is perpendicular to the median from C to AB. If the median from A to BC is 30, determine $(BC^2 + CA^2 + AB^2)/100$.
 32. Let AB be a chord of a circle with centre O. Let C be a point on the circle such that $\angle ABC = 30^\circ$ and O lies inside the triangle ABC. Let D be a point on AB such that $\angle DCO = \angle OCB = 20^\circ$. Find the measure of $\angle CDO$ in degrees.
 33. Let ABCD be a trapezium in which $AB \parallel CD$ and $AD \perp AB$. Suppose ABCD has an incircle which touches AB at Q and CD at P. Given that $PC = 36$ and $QB = 49$, find PQ.
 34. In a quadrilateral ABCD, it is given that $AB = AD = 13$, $BC = CD = 20$, $BD = 24$. If r is the radius of the circle inscribable, then what is the integer closest to r?

35. Let ABC be a triangle and let Ω be its circum-circle. The internal bisectors of angles A , B and C intersect Ω at A_1, B_1 , and C_1 , respectively, and the internal bisectors of angle A_1, B_1, C_1 of the triangle $A_1B_1C_1$ intersect Ω at A_2, B_2 , and C_2 , respectively. If the smallest angle of triangle ABC is 40° , What is the magnitude of the smallest angle of triangle $A_2B_2C_2$.
36. How many distinct triangles ABC are there, upto similarity, such that the magnitudes of angles A , B , and C in degrees are positive integers and satisfy

$$\cos A \cos B + \sin A \sin B \sin kC = 1$$

for some positive integer k , where kC does not exceed 360° ?

37. In how many ways can a pair of parallel diagonals of a regular polygon of 10 sides be selected?
38. Let AB be a diameter of a circle and let C be a point on the segment AB such that $AC : CB = 6 : 7$. Let D be a point on the circle such that DC is perpendicular to AB . Let DE be the diameter through D . If $[XYZ]$ denote the area of the triangle XYZ , find $[ABD]/[CDE]$ to the nearest integer?
39. Let $ABCD$ be a convex cyclic quadrilateral. Suppose P is a point in the plane of the quadrilateral such that the sum of its distances from the vertices $ABCD$ is the least. If

$$\{PA, PB, PC, PD\} = \{3, 4, 6, 8\}$$

What is the maximum possible area of $ABCD$?

40. Let ABC be a triangle with sides 51, 52, 53. Let Ω denote the incircle of $\triangle ABC$. Draw tangents to Ω which are parallel to the sides of ABC . Let r_1, r_2, r_3 be the inradii of the three corner triangles so formed. Find the largest integer that does not exceed $r_1 + r_2 + r_3$?
41. In a triangle ABC , the median, the median AD (with D on BC) and the angle bisector BE (with E on AC) are perpendicular to each other. If $AD = 7$, and $BE = 9$, find the integer nearest to the area of triangle ABC .