RD SHARMA Solutions for Class 9 Maths Chapter 4 - Algebraic Identities

Chapter 4 - Algebraic Identities Exercise 4.30

Question 1

If $x+\frac{1}{x}=5$, then $x^2+\frac{1}{x^2}=$

(a) 25
(b) 10
(c) 23
(d) 27

Solution 1

By using Identity $(a+b)^2 = a^2+b^2+2ab$, we have

$$\left(x+\frac{1}{x}\right)^2 = x^2+\left(\frac{1}{x}\right)^2 + 2\times\cancel{x}\times\frac{1}{\cancel{x}}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow (5)^2 = x^2+\frac{1}{x^2}+2 \qquad \left\{x+\frac{1}{x}=5 \text{ given}\right\}$$

$$\Rightarrow x^2+\frac{1}{x^2}=25-2$$

$$\Rightarrow x^2+\frac{1}{x^2} = 23$$

Hence, correct option is (c).

Question 2

If $x+\frac{1}{x}=2$, then $x^3+\frac{1}{x^3}=$

(a) 64
(b) 14
(c) 8
(d) 2

Solution 2

By using identity,

$(a+b)^3 = a^3+b^3+3ab(a + b)$

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3} + 3.\cancel{x}.\frac{1}{\cancel{x}}\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

Now $x+\frac{1}{x}=2$

$$\Rightarrow (2)^3 = x^3+\frac{1}{x^3}+3(2)$$

$$\Rightarrow x^3+\frac{1}{x^3}=(2)^3 - 3\times 2=8-6=2$$

Hence, correct option is (d).

Question 3

If $x+\frac{1}{x}=4$, then $x^4+\frac{1}{x^4}=$

(a) 196
(b) 194
(c) 192
(d) 190

Solution 3

$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$\left(x+\frac{1}{x}\right)=4$ (given)

$\Rightarrow x^2+\frac{1}{x^2} = (4)^2 - 2 = 16-2 = 14 \quad \ldots.(1)$

Squaring equation (1),

$\left(x^2+\frac{1}{x^2}\right)^2 = (14)^2$

$\Rightarrow (x^2)^2+\left(\frac{1}{x^2}\right)^2+2.x^2.\frac{1}{x^2} = 196$

$\Rightarrow x^4+\frac{1}{x^4} = 196-2$

$\Rightarrow x^4+\frac{1}{x^4} = 194$

Hence, correct option is (b).

Question 4

If $x+\frac{1}{x}=3$, then $x^6+\frac{1}{x^6}=$

(a) 927
(b) 414
(c) 364
(d) 322

Solution 4

$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$x+\frac{1}{x} = 3$ (*given*)

$\Rightarrow x^2+\frac{1}{x^2} = (3)^2 - 2$

$\Rightarrow x^2+\frac{1}{x^2} = 7 \quad \ldots.(1)$

Cubing both side of equation (1), *we have*

$\left(x^2+\frac{1}{x^2}\right)^3 = (7)^3$

$\Rightarrow (x^2)^3+\left(\frac{1}{x^2}\right)^3 + 3.x^2.\frac{1}{x^2}\left(x^2+\frac{1}{x^2}\right) = 7^3$

$\Rightarrow x^6+\frac{1}{x^6} + 3(7) = 7^3$

$\Rightarrow x^6+\frac{1}{x^6} = 343-21$

$\Rightarrow x^6+\frac{1}{x^6} = 322$

Hence, correct option is (d).

Question 5

If $x^2+\frac{1}{x^2}=102$, then $x-\frac{1}{x}=$

(a) 8
(b) 10
(c) 12
(d) 13

Solution 5

Correct option (b)

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2.x.\frac{1}{x}$$

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$$

$$=102-2 \qquad \left\{x^2+\frac{1}{x^2}=102\right\}$$

$$=100$$

$$\Rightarrow x-\frac{1}{x}=\sqrt{100}$$

$$\Rightarrow x-\frac{1}{x}=10$$

Hence, correct option is (b).

Question 6

If $x^3+\frac{1}{x^3}=110$, then $x+\frac{1}{x}=$

(a) 5
(b) 10
(c) 15
(d) none of these

Solution 6

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=110$$

Let $x+\frac{1}{x}=t$

$$\Rightarrow t^3-3t-110=0$$

$t=5$ is one of it's solution which is real,
other two solutions are imaginary

$$\Rightarrow x+\frac{1}{x}=5$$

Hence, correct option is (a).

Question 7

If $x^3-\frac{1}{x^3}=14$, then $x-\frac{1}{x}=$

(a) 5
(b) 4
(c) 3
(d) 2

Solution 7

$$\left(x-\frac{1}{x}\right)^3 = x^3-\frac{1}{x^3} - 3\cancel{x}\frac{1}{\cancel{x}}\left(x-\frac{1}{x}\right)$$

$$x^3-\frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right) - x^3 - \frac{1}{x^3} = 0$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right) - 14 = 0$$

Let $x-\frac{1}{x} = t$

$\Rightarrow t^3 + 3t - 14 = 0$

$\Rightarrow t^3 - 2t^2 + 2t^2 - 4t + 7t - 14 = 0$

$\Rightarrow t(t-2) + 2t(t-2) + 7(t-2) = 0$

$\Rightarrow (t-2)(t+2t+7) = 0$

$t^2 + 2t + 7 = 0$ has no real roots

so, $t = 2$ is a solution

$\Rightarrow x-\frac{1}{x} = 2$

Hence, correct option is (d).

Question 8

If a + b + c = 9 and ab + bc + ca = 23, then $a^2 + b^2 + c^2 =$

(a) 35

(b) 58

(c) 127

(d) none of these

Solution 8

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Here, a + b + c = 9, ab + bc + ca = 23

Thus, we have

$(9)^2 = a^2 + b^2 + c^2 + 2(23)$

$81 = a^2 + b^2 + c^2 + 46$

$a^2 + b^2 + c^2 = 81 - 46$

$a^2 + b^2 + c^2 = 35$

Hence, correct option is (a).

Question 9

$(a - b)^3 + (b - c)^3 + (c - a)^3 =$

(a) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(b) (a - b)(b - c)(c - a)

(c) 3(a - b)(b - c)(c - a)

(d) none of these

Solution 9

Let

$a - b = A$

$b - c = B$

$c - a = C$

Now $(A + B + C)^3 = A^3 + B^3 + C^3 + 3(A + B)(B + C)(C + A)$

$\Rightarrow A^3 + B^3 + C^3 = (A + B + C)^3 - 3(A + B)(B + C)(C + A)$

Now putting values of A, B and C, we get

$(a-b)^3 + (b-c)^3 + (c-a)^3 = (\cancel{a}-\cancel{b}+\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a})^3 - 3(a-\cancel{b}+\cancel{b}-c)(b-\cancel{c}+\cancel{c}-a)(c-\cancel{a}+\cancel{a}-b)$

$\Rightarrow (a-b)^3 + (b-c)^3 + (c-a)^3 = 0 - 3(a-c)(b-a)(c-b)$

$\Rightarrow (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$

Hence, correct option is (c).

Question 10

If $\frac{a}{b}+\frac{b}{a}=-1$, then $a^3-b^3=$
(a) 1
(b) −1
(c) $\frac{1}{2}$
(d) 0

Solution 10
$\frac{a}{b}+\frac{b}{a}=-1$
$\Rightarrow \frac{a^2+b^2}{ab}=-1$
$\Rightarrow a^2+b^2+ab=0$
Now using identity,
a^3-b^3
$=(a-b)(a^2+b^2+ab)$
$=(a-b)(0)\quad (\because a^2+b^2+ab=0)$
$=0$
Hence, correct option is (d).

Question 11
If a - b = -8 and ab = -12, then $a^3 - b^3 =$
(a) -244
(b) -240
(c) -224
(d) -260

Solution 11
a - b = -8
$(a - b)^2 = 64$
$a^2 + b^2 - 2ab = 64$
$a^2 + b^2 - 2ab + 3ab = 64 + 3ab$
$a^2 + b^2 + ab = 64 + 3(-12)$
$a^2 + b^2 + ab = 64 - 36$
$a^2 + b^2 + ab = 28$
Now $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
= (-8)(28)
= -224
Hence, correct option is (c).

Chapter 4 - Algebraic Identities Exercise 4.31
Question 1
If the volume of a cuboid is $3x^2 - 27$, then its possible dimensions are
(a) 3, x^2, -27x
(b) 3, x - 3, x + 3
(c) 3, x^2, 27x
(d) 3, 3, 3

Solution 1
Volume of a cuboid of side a, b and c = abc
Now, Volume = $3x^2 - 27$ (given)
abc = $3(x^2 - 9)$
abc = 3(x - 3)(x + 3)
So, possible dimensions are 3, x - 3 and x + 3
Hence, correct option is (b).

Question 2
75 × 75 + 2 × 75 × 25 + 25 × 25 is equal to
(a) 10000
(b) 6250
(c) 7500
(d) 3750

Solution 2
Given expression is 75 × 75 + 2 × 75 × 25 + 25 × 25
Let 75 = a and 25 = b
Then, we have
a × a + 2 × a × b + b × b
$= a^2 + 2ab + b^2$
$= (a + b)^2$
$= (75 + 25)^2$
$= (100)^2$
= 10000
Hence, Correct option is (a).

Question 3
$(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$ is equal to
(a) $x^{16} - y^{16}$
(b) $x^8 - y^8$
(c) $x^8 + y^8$
(d) $x^{16} + y^{16}$

Solution 3
$(x - y)(x + y) = x^2 - y^2$ [by identity $(a + b)(a - b) = a^2 - b^2$]
$(x^2 - y^2)(x^2 + y^2) = x^4 - y^4$
$(x^4 - y^4)(x^4 + y^4) = x^8 - y^8$
Now,
$(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$
$= (x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$
$= (x^4 - y^4)(x^4 + y^4)$
$= x^8 - y^8$
Hence, correct option is (b).

Question 4
If $x^4 + \frac{1}{x^4} = 623$, then $x + \frac{1}{x} =$
(a) 27
(b) 25
(c) $3\sqrt{3}$
(d) $-3\sqrt{3}$

Solution 4

$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2.x.\frac{1}{x} = x^2+\frac{1}{x^2}+2$

$\Rightarrow x^2+\frac{1}{x^2} = \left\{\left(x+\frac{1}{x}\right)^2-2\right\}$

Squaring both sides,

$\left(x^2+\frac{1}{x^2}\right)^2 = \left\{\left(x+\frac{1}{x}\right)^2-2\right\}^2$

$\Rightarrow x^4+\frac{1}{x^4}+2.x^2.\frac{1}{x^2} = \left\{\left(x+\frac{1}{x}\right)^2-2\right\}^2$

$\Rightarrow x^4+\frac{1}{x^4}+2 = \left\{\left(x+\frac{1}{x}\right)^2-2\right\}^2 = (623) + 2$

$\Rightarrow 623+2 = \left\{\left(x+\frac{1}{x}\right)^2-2\right\}^2 \quad \left\{x^4+\frac{1}{x^4} = 623\right\}$

$\Rightarrow 625 = \left\{\left(x+\frac{1}{x}\right)^2-2\right\}^2$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 - 2 = \sqrt{625} = 25$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 25+2 = 27$

$\Rightarrow \left(x+\frac{1}{x}\right) = \sqrt{27}$

$x+\frac{1}{x} = 3\sqrt{3}$

Hence, correct option is (c).

Question 5

If $x^4+\frac{1}{x^4} = 194$, then $x^3+\frac{1}{x^3} =$

(a) 76
(b) 52
(c) 64
(d) none of these

Solution 5

$x^4 + \frac{1}{x^4} = 194$

Now $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 194 + 2 = 196$

$\Rightarrow x^2 + \frac{1}{x^2} = 14 \quad ....(1)$

Now $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \qquad \left\{x^2 + \frac{1}{x^2} = 14\right\}$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 14 + 2 = 16 \quad$ [From (1)]

$\Rightarrow x + \frac{1}{x} = \sqrt{16}$

$\Rightarrow x + \frac{1}{x} = 4 \qquad ....3$

By identity $a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$$
$$= (4)(14-1)$$
$$= 4 \times 13$$
$$= 52$$

Hence, correct option is (b).

Question 6

If $x - \frac{1}{x} = \frac{15}{4}$, then $x + \frac{1}{x} =$

(a) 4

(b) $\frac{17}{4}$

(c) $\frac{13}{4}$

(d) $\frac{1}{4}$

Solution 6

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \quad ......(1)$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \quad .....(2)$

SUbtracting eq. (2) from eq. (1), we get

$\left(x - \frac{1}{x}\right)^2 - \left(x + \frac{1}{x}\right)^2 = -4$

$\Rightarrow \left(\frac{15}{4}\right)^2 - \left(x + \frac{1}{x}\right)^2 = -4$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = \left(\frac{15}{4}\right)^2 + 4 = \frac{225}{16} + 4$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = \frac{225 + 64}{16} = \frac{189}{16}$

$\Rightarrow \left(x + \frac{1}{x}\right) = \sqrt{\frac{289}{16}}$

$\Rightarrow x + \frac{1}{x} = \frac{17}{4}$

Hence, correct option is (b).

Question 7

If $3x+\frac{2}{x}=7$, then $\left(9x^2-\frac{4}{x^2}\right)=$

(a) 25
(b) 35
(c) 49
(d) 30

Solution 7

$\left(3x+\frac{2}{x}\right)^2 = 9x^2+\frac{4}{x^2}+12$ .....(1)

$\left(3x-\frac{2}{x}\right)^2 = 9x^2+\frac{4}{x^2}-12$ .....(2)

Subtracting equation (1) from eq. (2), we get

$\left(3x-\frac{2}{x}\right)^2 - \left(3x+\frac{2}{x}\right)^2 = -24$

$\Rightarrow \left(3x-\frac{2}{x}\right)^2 = (7)^2 - 24 = 25$

$\Rightarrow 3x-\frac{2}{x} = 5$

Now, $\left(3x+\frac{2}{x}\right)\left(3x-\frac{2}{x}\right) = 7 \times 5$

$\Rightarrow \left(9x^2-\frac{4}{x^2}\right)=35$

Hence, correct option is (b).

Question 8

If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then

(a) $a + b = c$
(b) $b + c = a$
(c) $c + a = b$
(d) $a = b = c$

Solution 8

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

Multiplying by 2 on both the sides, we have

$2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca = 0$

$(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ac) = 0$

$(a - b)^2 + (b - c)^2 + (a - c)^2 = 0$

$(a - b)^2 = 0, (b - c)^2 = 0, (a - c)^2 = 0$

$(a - b) = 0, (b - c) = 0, (a - c) = 0$

$a = b, b = c, a = c$

or we can say $a = b = c$

Hence, correct option is (d).

Question 9

If $a + b + c = 0$, then $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=$

(a) 0
(b) 1
(c) −1
(d) 3

Solution 9

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$, then

$a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc \quad ....(1)$

Now, consider $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$

Multiplying and dividng by a, b, and c in $\frac{a^2}{bc}$, $\frac{b^2}{ca}$ and $\frac{c^2}{ab}$ respectively, we get

$\frac{a^3}{abc} + \frac{b^3}{bca} + \frac{c^3}{cab}$

$= \frac{a^3+b^3+c^3}{abc}$

$= \frac{3abc}{abc} \quad ....[\text{from } (1)]$

$= 3$

Hence, correct option is (d).

Question 10

If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then

(a) $a + b + c = 0$

(b) $(a + b + c)^3 = 27abc$

(c) $a + b + c = 3abc$

(d) $a^3 + b^3 + c^3 = 0$

Solution 10

Let $a^{1/3} = A$, $b^{1/3} = B$ and $c^{1/3} = C$

Now, $A + B + C = 0$ (given)

If $A + B + C = 0$, then $A^3 + B^3 + C^3 - 3ABC = 0$

$\Rightarrow A^3 + B^3 + C^3 - 3ABC = 0$

$\Rightarrow A^3 + B^3 + C^3 = 3ABC \quad ....(1)$

$$\begin{Bmatrix} A = a^{1/3} & B = b^{1/3} & C = c^{1/3} \\ A^3 = a & B^3 = b & C^3 = c \end{Bmatrix}$$

Then, equation (1) becomes

$a + b + c = 3(abc)^{1/3}$

Cubing both sides of above equation, we get

$(a + b + c)^3 = 27abc$

Hence, correct option is (b).

Question 11

If $a + b + c = 9$ and $ab + bc + ca = 23$, then $a^3 + b^3 + c^3 - 3abc =$

(a) 108

(b) 207

(c) 669

(d) 729

Solution 11

Given, $a + b + c = 9$

Hence, $(a + b + c)^2 = 81$

So, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 81$

i.e. $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$

i.e. $a^2 + b^2 + c^2 + 2(23) = 81$

i.e. $a^2 + b^2 + c^2 = 81 - 46 = 35$

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$

$= (9)[35 - 23]$

$= 9 \times 12$
$= 108$
Hence, correct option is (a).

Question 12

$$\frac{(a^2-b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3} =$$

(a) $3(a + b)(b + c)(c + a)$
(b) $3(a - b)(b - c)(c - a)$
(c) $(a - b)(b - c)(c - a)$
(d) none of these

Solution 12

If $a + b + c = 0$ then, $a^3 + b^3 + c^3 = 3abc$

Now, $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

Again, $(a - b) + (b - c) + (c - a) = a - b + b - c + c - a = 0$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

Thus, we have

$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$$

$$= \frac{3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}{3(a - b)(b - c)(c - a)}$$

$$= \frac{(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)}{(a - b)(b - c)(c - a)}$$

$= (a + b)(b + c)(c + a)$

Hence, correct option is (d).

Chapter 4 - Algebraic Identities Exercise 4.32

Question 1

The product $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$ is equal to
(a) $a^6 + b^6$
(b) $a^6 - b^6$
(c) $a^3 - b^3$
(d) $a^3 + b^3$

Solution 1

$(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$
$= (a^2 - b^2)(a^2 + b^2 - ab)(a^2 + b^2 + ab)$
$= (a^2 - b^2)\{(a^2 + b^2)^2 - (ab)^2\}$
$= (a^2 - b^2)\{a^4 + b^4 + 2a^2b^2 - a^2b^2\}$
$= (a^2 - b^2)\{a^4 + b^4 + a^2b^2\}$
$= \{a^6 + a^2\cancel{b^4} + a^4\cancel{b^2} - b^2\cancel{a^4} - b^6 - b^4\cancel{a^2}\}$
$= a^6 - b^6$
Hence, correct option is (b).

Question 2

The product $(x^2 - 1)(x^4 + x^2 + 1)$ is equal to
(a) $x^8 - 1$
(b) $x^8 + 1$
(c) $x^6 - 1$
(d) $x^6 + 1$

Solution 2

Given expression is $(x^2 - 1)(x^4 + x^2 + 1)$

Let $x^2 = A$ and $1 = B$

Then, we have

$(A - B)(A^2 + AB + B^2)$

$= A^3 - B^3$

$= (x^2)^3 - (1)^3$

$= x^6 - 1$

Hence, correct option is (c).

Question 3

If $\frac{a}{b} + \frac{b}{a} = 1$, then $a^3 + b^3 =$

(a) 1

(b) −1

(c) $\frac{1}{2}$

(d) 0

Solution 3

$\frac{a}{b} + \frac{b}{a} = 1 \Rightarrow a^2 + b^2 - ab = 0$

Now by identity $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$,

if $a^2 + b^2 - ab = 0$,

then $a^3 + b^3 = 0$

Hence, correct option is (d).

Question 4

If $49a^2 - b = \left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right)$, then the value of b is

(a) 0

(b) $\frac{1}{4}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{2}$

Solution 4

$\left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right) = (7a)^2 - \left(\frac{1}{2}\right)^2$

[by using identity $(a+b)(a-b) = a^2 - b^2$]

$\Rightarrow \left(7a + \frac{1}{2}\right)\left(7a - \frac{1}{2}\right) = 49a^2 - \frac{1}{4}$

$\Rightarrow 49a^2 - b = 49a^2 - \frac{1}{4}$

$\Rightarrow b = \frac{1}{4}$

Hence, correct option is (b).

Chapter 4 - Algebraic Identities Exercise Ex. 4.1

Question 1

Evaluate $\left(2x - \frac{1}{x}\right)^2$ using an identity.

Solution 1

We have,

$$\left(2x-\frac{1}{x}\right)^2=(2x)^2+\left(\frac{1}{x}\right)^2-2\times 2x\times\frac{1}{x} \qquad \left[\because (a-b)^2=a^2-2ab+b^2\right]$$

$$=4x^2+\frac{1}{x^2}-4$$

$$\therefore \qquad \left(2x-\frac{1}{x}\right)^2=4x^2+\frac{1}{x^2}-4$$

Question 2

Evaluate $(2x+y)(2x-y)$ using an identity.

Solution 2

We have,

$$(2x+y)(2x-y)$$

$$=(2x)^2-(y)^2 \qquad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$=4x^2-y^2$$

$$\therefore \qquad (2x+y)(2x-y)=4x^2-y^2$$

Question 3

Evaluate $\left(a^2b-b^2a\right)^2$ using an identity.

Solution 3

We have,

$$\left(a^2b-b^2a\right)^2$$

$$=\left(a^2b\right)^2+\left(b^2a\right)^2-2\times a^2b\times b^2a \qquad \left[\because (a-b)^2=a^2-2ab+b^2\right]$$

$$=a^4b^2+b^4a^2-2a^3b^3$$

$$\therefore \qquad \left(a^2b-b^2a\right)^2=a^4b^2+b^4a^2-2a^3b^3$$

Question 4

Evaluate $(a-0.1)(a+0.1)$ using an identity.

Solution 4

We have,

$(a-0.1)(a+0.1) = (a)^2 - (0.1)^2 \qquad [\because (a-b)(a+b) = a^2 - b^2]$

$= a^2 - 0.01$

$\therefore \qquad (a-0.1)(a+0.1) = a^2 - 0.01$

Question 5

Evaluate $\left(1.5x^2 - 0.3y^2\right)\left(1.5x^2 + 0.3y^2\right)$ using an identity.

Solution 5

We have,

$\left(1.5x^2 - 0.3y^2\right)\left(1.5x^2 + 0.3y^2\right)$

$= \left(1.5x^2\right)^2 - \left(0.3y^2\right)^2 \qquad [\because (a-b)(a+b) = a^2 - b^2]$

$= 2.25x^4 - 0.09y^4$

$\therefore \qquad \left(1.5x^2 - 0.3y^2\right)\left(1.5x^2 + 0.3y^2\right) = 2.25x^4 - 0.09y^4$

Question 6

Evaluate $(399)^2$ using an identity.

Solution 6

We have,

$(399)^2 = (400-1)^2$

$= (400)^2 + (1)^2 - 2 \times 400 \times 1 \qquad [\because (a-b)^2 = a^2 + b^2 - 2ab]$

$= 160000 + 1 - 800$

$= 160001 - 800$

$= 159201$

$\therefore \qquad (399)^2 = 159201$

Question 7

Evaluate $(0.98)^2$ using an identity.

Solution 7

We have,

$(0.98)^2 = (1 - 0.02)^2$

$= (1)^2 + (0.02)^2 - 2 \times 1 \times 0.02$ $\quad\left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$= 1 + 0.0004 - 0.04$

$= 1.0004 - 0.04$

$= 0.9604$

$\therefore \quad (0.98)^2 = 0.9604$

Question 8

Evaluate 991×1009 using an identity.

Solution 8

We have,

991×1009

$= (1000 - 9)(1000 + 9)$

$= (1000)^2 - (9)^2$ $\quad\left[\because (a-b)(a+b) = a^2 - b^2\right]$

$= 1000000 - 81$

$= 999919$

$\therefore \quad 991 \times 1009 = 999919$

Question 9

Evaluate 117×83 using an identity.

Solution 9

We have,

$117 \times 83 = (100 + 17)(100 - 17)$

$= (100)^2 - (17)^2$ $\quad\left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= 10000 - 289$

$= 9711$

$\therefore \quad 117 \times 83 = 9711$

Question 10

Simplify $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

Solution 10

We have,

$175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

$= (175 + 25)^2 \qquad [\because a^2 + 2ab + b^2 = (a + b)^2]$

$= (200)^2$

$= 40000$

$\therefore \quad 175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = 40000$

Question 11

Simplify $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

Solution 11

We have,

$322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

$= (322 - 22)^2 \qquad [\because a^2 - 2ab + b^2 = (a - b)^2]$

$= (300)^2$

$= 90000$

$\therefore \quad 322 \times 322 - 2 \times 322 \times 22 + 22 \times 22 = 90000$

Question 12

Simplify $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

Solution 12

We have,

$0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

$= (0.76 + 0.24)^2 \qquad [\because a^2 + b^2 + 2ab = (a + b)^2]$

$= (1.00)^2$

$= 1$

$\therefore \quad 0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24 = 1$

Question 13

Simplify $\dfrac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$

Solution 13

We have,

$\dfrac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$

$= \dfrac{(7.83 + 1.17)(7.83 - 1.17)}{6.66} \qquad [\because a^2 - b^2 = (a + b)(a - b)]$

$= \dfrac{(9.00)(6.66)}{6.66}$

$= 9$

$\therefore \quad \dfrac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66} = 9$

Question 14

If $x + \frac{1}{x} = 11$, find the value of $x^2 + \frac{1}{x^2}$

Solution 14

We have,

$$x + \frac{1}{x} = 11$$

Now, $\left(x + \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x}\right)^2 + 2 \times x \times \frac{1}{x}$

$$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \quad (11)^2 = x^2 + \frac{1}{x^2} + 2 \qquad \left[\because x + \frac{1}{x} = 11\right]$$

$$\Rightarrow \quad 121 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 121 - 2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 119$$

$$\therefore \quad x^2 + \frac{1}{x^2} = 119$$

Question 15

If $x - \frac{1}{x} = -1$, find the value of $x^2 + \frac{1}{x^2}$

Solution 15

We have,

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$$

$$\Rightarrow \quad (-1)^2 = x^2+\frac{1}{x^2}-2 \qquad \left[\because x-\frac{1}{x}=-1\right]$$

$$\Rightarrow \quad 1 = x^2+\frac{1}{x^2}-2$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 1+2$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 3$$

Question 16

If $x+\frac{1}{x}=\sqrt{5}$, find the value of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$

Solution 16

We have,

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$\Rightarrow \quad \left(\sqrt{5}\right)^2 = x^2+\frac{1}{x^2}+2 \qquad \left[\because x+\frac{1}{x}=\sqrt{5}\right]$

$\Rightarrow \quad 5 = x^2+\frac{1}{x^2}+2$

$\Rightarrow \quad x^2+\frac{1}{x^2} = 5-2$

$\Rightarrow \quad x^2+\frac{1}{x^2} = 3 \qquad ---(1)$

Now, $\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}$

$\Rightarrow \quad \left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2$

$\Rightarrow \quad (3)^2 = x^4+\frac{1}{x^4}+2 \qquad$ [Using equation 1]

$\Rightarrow \quad 9 = x^4+\frac{1}{x^4}+2$

$\Rightarrow \quad x^4+\frac{1}{x^4} = 9-2$

$\Rightarrow \quad x^4+\frac{1}{x^4} = 7$

Hence, $x^2+\frac{1}{x^2}=3$ and $x^4+\frac{1}{x^4}=7$

Question 17

If $9x^2+25y^2=181$ and $xy=-6$, find the value of $3x+5y$

Solution 17

We have,

$$(3x+5y)^2 = (3x)^2 + (5y)^2 + 2\times 3x \times 5y$$

$\Rightarrow \quad (3x+5y)^2 = 9x^2 + 25y^2 + 30xy$

$= 181 + 30(-6) \quad \left[\because 9x^2 + 25y^2 = 181 \text{ and } xy = -6\right]$

$= 181 - 180$

$= 1$

$\Rightarrow \quad (3x+5y)^2 = 1$

$\Rightarrow \quad (3x+5y)^2 = (\pm 1)^2$

$\Rightarrow \quad 3x + 5y = \pm 1$

Question 18

If $2x + 3y = 8$ and $xy = 2$, find the value of $4x^2 + 9y^2$

Solution 18

We have,

$$(2x+3y)^2 = (2x)^2 + (3y)^2 + 2\times 2x \times 3y$$

$\Rightarrow \quad (2x+3y)^2 = 4x^2 + 9y^2 + 12xy$

$\Rightarrow \quad (8)^2 = 4x^2 + 9y^2 + 12\times 2 \quad \left[\because 2x + 3y = 8 \text{ and } xy = 2\right]$

$\Rightarrow \quad 64 = 4x^2 + 9y^2 + 24$

$\Rightarrow \quad 4x^2 + 9y^2 = 64 - 24$

$\Rightarrow \quad 4x^2 + 9y^2 = 40$

Question 19

If $3x - 7y = 10$ and $xy = -1$, find the value of $9x^2 + 49y^2$

Solution 19

We have,

$$(3x-7y)^2 = (3x)^2 + (7y)^2 - 2\times 3x \times 7y$$

$\Rightarrow \quad (3x-7y)^2 = 9x^2 + 49y^2 - 42xy$

$\Rightarrow \quad (10)^2 = 9x^2 + 49y^2 - 42(-1) \quad \left[\because 3x - 7y = 10 \text{ and } xy = -1\right]$

$\Rightarrow \quad 100 = 9x^2 + 49y^2 + 42$

$\Rightarrow \quad 9x^2 + 49y^2 = 100 - 42$

$\Rightarrow \quad 9x^2 + 49y^2 = 58$

Question 20

Simplify $\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)$

Solution 20

We have,

$$\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)$$

$$=\left(\frac{1}{2}a-3b\right)\left(\frac{1}{2}a+3b\right)\left(\frac{1}{4}a^2+9b^2\right)$$

$$=\left[\left(\frac{1}{2}a\right)^2-(3b)^2\right]\left[\frac{1}{4}a^2+9b^2\right] \qquad \left[\because (a-b)(a+b)=a^2-b^2\right]$$

$$=\left[\frac{1}{4}a^2-9b^2\right]\left[\frac{1}{4}a^2+9b^2\right]$$

$$=\left(\frac{1}{4}a^2\right)^2-\left(9b^2\right)^2 \qquad \left[(a-b)(a+b)=a^2-b^2\right]$$

$$=\frac{1}{16}a^4-81b^4$$

$$\therefore \left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)=\frac{1}{16}a^4-81b^4$$

Question 21

Simplify $\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$

Solution 21

We have,

$$\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$$

$$=\left(m+\frac{n}{7}\right)^2\left(m+\frac{n}{7}\right)\left(m-\frac{n}{7}\right)$$

$$=\left(m+\frac{n}{7}\right)^2\left[m^2-\left(\frac{n}{7}\right)^2\right] \qquad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$=\left(m+\frac{n}{7}\right)^2\left[m^2-\frac{n^2}{49}\right]$$

Question 22

If $x^2+\frac{1}{x^2}=66$, find the value of $x-\frac{1}{x}$

Solution 22
We have,

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = 66-2 \qquad \left[\because x^2+\frac{1}{x^2}=66\right]$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = 64$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = (\pm 8)^2$$

$$\Rightarrow \quad x-\frac{1}{x} = \pm 8$$

Question 23
If $x^2+\frac{1}{x^2}=79$, find the value of $x+\frac{1}{x}$

Solution 23
We have,

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = 79+2 \qquad \left[\because x^2+\frac{1}{x^2}=79\right]$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = 81$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = (\pm 9)^2$$

$$\Rightarrow \quad x+\frac{1}{x} = \pm 9$$

Question 24
Simplify $\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$

Solution 24

We have,

$$\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$$

$$= -\left(\frac{2}{5}-\frac{x}{2}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$$

$$= -\left(\frac{2}{5}-\frac{x}{2}\right)^2-x^2+2x$$

$$= -\left[\left(\frac{2}{5}\right)^2+\left(\frac{x}{2}\right)^2-2\times\frac{2}{5}\times\frac{x}{2}\right]-x^2+2x$$

$$= -\left[\frac{4}{25}+\frac{x^2}{4}-\frac{2x}{5}\right]-x^2+2x$$

$$= -\frac{4}{25}-\frac{x^2}{4}+\frac{2x}{5}-x^2+2x$$

$$= -\frac{x^2}{4}-x^2+\frac{2x}{5}+2x-\frac{4}{25}$$

$$= \frac{-x^2-4x^2}{4}+\frac{2x+10x}{5}-\frac{4}{25}$$

$$= -\frac{5x^2}{4}+\frac{12x}{5}-\frac{4}{25}$$

$$\therefore \left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x = -\frac{5x^2}{4}+\frac{12x}{5}-\frac{4}{25}$$

Question 25

Simplify $(x^2+x-2)(x^2-x+2)$

Solution 25

We have,

$\left(x^2 + x - 2\right)\left(x^2 - x + 2\right)$

$= \left[(x)^2 + (x-2)\right]\left[(x)^2 - (x-2)\right]$

$= \left(x^2\right)^2 - (x-2)^2 \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$

$= x^4 - \left(x^2 + 4 - 4x\right)$

$= x^4 - x^2 - 4 + 4x$

$= x^4 - x^2 + 4x - 4$

$\Rightarrow \quad \left(x^2 + x - 2\right)\left(x^2 - x + 2\right) = x^4 - x^2 + 4x - 4$

Question 26

Simplify $\left(x^3 - 3x^2 - x\right)\left(x^2 - 3x + 1\right)$

Solution 26

We have,

$\left(x^3 - 3x^2 - x\right)\left(x^2 - 3x + 1\right)$

$= x\left[x^2 - 3x - 1\right]\left[x^2 - 3x + 1\right]$

$= x\left[\left(x^2 - 3x\right) - 1\right]\left[\left(x^2 - 3x\right) + 1\right]$

$= x\left[\left(x^2 - 3x\right)^2 - (1)^2\right] \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$

$= x\left[\left(x^2\right)^2 + (3x)^2 - 2 \times x^2 \times 3x - 1\right]$

$= x\left[x^4 + 9x^2 - 6x^3 - 1\right]$

$= x^5 + 9x^3 - 6x^4 - x$

$= x^5 - 6x^4 + 9x^3 - x$

$\therefore \left(x^3 - 3x^2 - x\right)\left(x^2 - 3x + 1\right) = x^5 - 6x^4 + 9x^3 - x$

Question 27

Simplify $\left(2x^4 - 4x^2 + 1\right)\left(2x^4 - 4x^2 - 1\right)$

Solution 27

We have,

$\left(2x^4 - 4x^2 + 1\right)\left(2x^4 - 4x^2 - 1\right)$

$= \left[\left(2x^4 - 4x^2\right) + 1\right]\left[\left(2x^4 - 4x^2\right) - 1\right]$

$= \left[\left(2x^4 - 4x^2\right)^2 - (1)^2\right]$ $\left[\because\ (a+b)(a-b) = a^2 - b^2\right]$

$= \left[\left(2x^4\right)^2 + \left(4x^2\right)^2 - 2 \times 2x^4 \times 4x^2 - 1\right]$

$= \left[4x^8 + 16x^4 - 16x^6 - 1\right]$

$= 4x^8 - 16x^6 + 16x^4 - 1$

$\therefore\ \left(2x^4 - 4x^2 + 1\right)\left(2x^4 - 4x^2 - 1\right) = 4x^8 - 16x^6 + 16x^4 - 1$

Question 28

Prove that $a^2 + b^2 + c^2 - ab - bc - ca$ is always non-negative for all values of a, b, c.

Solution 28

We have,

$a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{2}{2}\left[a^2 + b^2 + c^2 - ab - bc - ca\right]$ [Multiplying and dividing by 2]

$= \frac{1}{2}\left[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca\right]$

$= \frac{1}{2}\left[a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca\right]$

$= \frac{1}{2}\left[\left(a^2 + b^2 - 2ab\right) + \left(b^2 + c^2 - 2bc\right) + \left(c^2 + a^2 - 2ca\right)\right]$

$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$ $\left[\because\ (a-b)^2 = a^2 + b^2 - 2ab\right]$

$= \dfrac{(a-b)^2 + (b-c)^2 + (c-a)^2}{2} \geq 0$

$\therefore\ a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

Hence, $a^2 + b^2 + c^2 - ab - bc - ca$ is always non-negative for all values of a, b and c.

Chapter 4 - Algebraic Identities Exercise Ex. 4.2

Question 1

Write $(a + 2b + c)^2$ in the expanded form.

Solution 1

We have,

$(a + 2b + c)^2$

using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (a)^2 + (2b)^2 + (c)^2 + 2 \times a \times 2b + 2 \times 2b \times c + 2 \times c \times a$

$= a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ca$

$\therefore\ (a + 2b + c)^2 = a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ca$

Question 2

Write $(2a - 3b - c)^2$ in the expanded form.

Solution 2

We have,

$(2a - 3b - c)^2$

using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (2a)^2 + (-3b)^2 + (-c)^2 + 2 \times 2a \times (-3b) + 2 \times (-3b) \times (-c) + 2 \times (-c) \times 2a$

$= 4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca$

$\therefore\ (2a - 3b - c)^2 = 4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca$

Question 3

Write $(-3x + y + z)^2$ in the expanded form:

Solution 3

We have,

$(-3x + y + z)^2$

using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= (-3x)^2 + y^2 + z^2 + 2 \times (-3x) \times y + 2 \times y \times z + 2 \times z \times (-3x)$

$= 9x^2 + y^2 + z^2 - 6xy + 2yz - 6zx$

$\therefore\ (-3x + y + z)^2 = 9x^2 + y^2 + z^2 - 6xy + 2yz - 6zx$

Question 4

Write $(m + 2n - 5p)^2$ in the expanded form:

Solution 4

We have,

$\left(m+2n-5p\right)^2$

using identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= \left(m\right)^2+\left(2n\right)^2+\left(-5p\right)^2+2\times m\times 2n+2\times 2n\times\left(-5p\right)+2\times\left(-5p\right)\times m$

$= m^2+4n^2+25p^2+4mn-20np-10pm$

$\therefore\ \left(m+2n-5p\right)^2 = m^2+4n^2+25p^2+4mn-20np-10pm$

Question 5

Write $\left(2+x-2y\right)^2$ in the expanded form:

Solution 5

We have,

$\left(2+x-2y\right)^2$

using identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= \left(2\right)^2+x^2+\left(-2y\right)^2+2\times 2\times x+2\times x\times\left(-2y\right)+2\times\left(-2y\right)\times 2$

$= 4+x^2+4y^2+4x-4xy-8y$

$\therefore\ \left(2+x-2y\right)^2 = 4+x^2+4y^2+4x-4xy-8y$

Question 6

Write $\left(a^2+b^2+c^2\right)^2$ in the expanded form:

Solution 6

We have,

$\left(a^2+b^2+c^2\right)^2$

using identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= \left(a^2\right)^2+\left(b^2\right)^2+\left(c^2\right)^2+2\times a^2\times b^2+2\times b^2\times c^2+2\times c^2\times a^2$

$= a^4+b^4+c^4+2a^2b^2+2b^2c^2+2c^2a^2$

$\therefore\ \left(a^2+b^2+c^2\right)^2 = a^4+b^4+c^4+2a^2b^2+2b^2c^2+2c^2a^2$

Question 7

Write $\left(ab+bc+ca\right)^2$ in the expanded form:

Solution 7

We have,

$\left(ab+bc+ca\right)^2$

using identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= \left(ab\right)^2+\left(bc\right)^2+\left(ca\right)^2+2\times ab\times bc+2\times ca\times ab+2\times bc\times ca$

$= a^2b^2+b^2c^2+c^2a^2+2ab^2c+2ca^2b+2bc^2a$

$\therefore\ \left(ab+bc+ca\right)^2 = a^2b^2+b^2c^2+c^2a^2+2ab^2c+2ca^2b+2bc^2a$

Question 8

Write $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$ in the expanded form:

Solution 8

We have,

$\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$

using identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$= \left(\frac{x}{y}\right)^2+\left(\frac{y}{z}\right)^2+\left(\frac{z}{x}\right)^2+2\times\frac{x}{y}\times\frac{y}{z}+2\times\frac{y}{z}\times\frac{z}{x}+2\times\frac{z}{x}\times\frac{x}{y}$

$= \frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+\frac{2x}{z}+\frac{2y}{x}+\frac{2z}{y}$

$\therefore\ \left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2 = \frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+\frac{2x}{z}+\frac{2y}{x}+\frac{2z}{y}$

Question 9

Write $\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$ in the expanded form:

Solution 9

We have,

$$\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$$

using identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$$= \left(\frac{a}{bc}\right)^2+\left(\frac{b}{ca}\right)^2+\left(\frac{c}{ab}\right)^2+2\times\frac{a}{bc}\times\frac{b}{ca}+2\times\frac{b}{ca}\times\frac{c}{ab}+2\times\frac{c}{ab}\times\frac{a}{bc}$$

$$= \frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+\frac{2}{c^2}+\frac{2}{a^2}+\frac{2}{b^2}$$

$$\therefore \left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2 = \frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+\frac{2}{a^2}+\frac{2}{b^2}+\frac{2}{c^2}$$

Question 10

Write the following in the expanded form:

$(x + 2y + 4z)^2$

Solution 10

$$\begin{aligned}(x+2y+4z)^2 &= x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)\\ &= x^2+4y^2+16z^2+4xy+16yz+8xz\end{aligned}$$

Question 11

Write the following in the expanded form:

$(2x - y + z)^2$

Solution 11

$$\begin{aligned}(2x-y+z)^2 &= (2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(z)(2x)\\ &= 4x^2+y^2+z^2-4xy-2yz+4xz\end{aligned}$$

Question 12

Write the following in the expanded form:

$(-2x + 3y + 2z)^2$

Solution 12

$$\begin{aligned}(-2x+3y+2z)^2 &= (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)\\ &= 4x^2+9y^2+4z^2-12xy+12yz-8xz\end{aligned}$$

Question 13

If $a+b+c=0$ *and* $a^2+b^2+c^2=16$, find the value of $ab+bc+ca$.

Solution 13

We know that,

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$\Rightarrow \quad (0)^2 = 16+2(ab+bc+ca) \qquad [\because a+b+c=0 \text{ and } a^2+b^2+c^2=16]$

$\Rightarrow \quad 2(ab+bc+ca) = -16$

$\Rightarrow \quad ab+bc+ca = -\dfrac{16}{2} = -8$

$\therefore \quad ab+bc+ca = -8$

Question 14

If $a^2+b^2+c^2=16$ and $ab+bc+ca=10$, find the value of $a+b+c$.

Solution 14

We know that,

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$\Rightarrow \quad (a+b+c)^2 = 16+2(10) \qquad [\because a^2+b^2+c^2=16 \text{ and } ab+bc+ca=10]$

$\Rightarrow \quad (a+b+c)^2 = 16+20$

$\Rightarrow \quad (a+b+c)^2 = 36$

$\Rightarrow \quad (a+b+c)^2 = (\pm 6)^2$

$\Rightarrow \quad a+b+c = \pm 6$

Question 15

If $a+b+c=9$ and $ab+bc+ca=23$, find the value of $a^2+b^2+c^2$.

Solution 15

We know that,

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$\Rightarrow \quad (9)^2 = a^2+b^2+c^2+2(23) \qquad [\because a+b+c=9 \text{ and } ab+bc+ca=23]$

$\Rightarrow \quad 81 = a^2+b^2+c^2+46$

$\Rightarrow \quad a^2+b^2+c^2 = 81-46$

$\Rightarrow \quad a^2+b^2+c^2 = 35$

Question 16

Find the value of $4x^2+y^2+25z^2+4xy-10yz-20zx$ when $x=4, y=3$ and $z=2$.

Solution 16

We have,

$4x^2 + y^2 + 25z^2 + 4xy - 10yz - 20zx$

$= (2x)^2 + (y)^2 + (-5z)^2 + 2 \times 2x \times y + 2 \times y \times (-5z) + 2 \times (-5z) \times 2x$

$= [2x + y - 5z]^2$

$= [2 \times 4 + 3 - 5 \times 2]^2$ $\quad [\because x = 4, y = 3 \text{ and } z = 2]$

$= [8 + 3 - 10]^2$

$= [1]^2$

$= 1$

$\therefore \quad 4x^2 + y^2 + 25z^2 + 4xy - 10yz - 20zx = 1$

Question 17

Simplify:

$(a + b + c)^2 + (a - b + c)^2$

Solution 17

We have,

$(a + b + c)^2 + (a - b + c)^2$

$= (a + b + c)^2 + (a + (-b) + c)^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$ $\quad$ [using identity $(a + b + c)^2 = a^2 + b^2 + c$

$= 2a^2 + 2b^2 + 2c^2 + 4ca$

$\therefore \; (a + b + c)^2 + (a - b + c)^2 = 2a^2 + 2b^2 + 2c^2 + 4ca$

Question 18

Simplify:

$(a + b + c)^2 - (a - b + c)^2$

Solution 18

We have,

$$(a+b+c)^2-(a-b+c)^2$$

$$= \left[a+b+c\right]^2-\left[a+(-b)+c\right]^2$$

$$= [a^2+b^2+c^2+2ab+2bc+2ca]-[a^2+b^2+c^2-2ab-2bc+2ca]$$

$$= a^2+b^2+c^2+2ab+2bc+2ca-a^2-b^2-c^2+2ab+2bc-2ca$$

$$= 4ab+4bc$$

$$\therefore\ (a+b+c)^2-(a-b+c)^2 = 4ab+4bc = 4(ab+bc)$$

Question 19

Simplify:

$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$$

Solution 19

We have,

$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$$

$$= \left[a+b+c\right]^2+\left[a+(-b)+c\right]^2+\left[a+b+(-c)\right]^2$$

$$= \left[a^2+b^2+c^2+2ab+2bc+2ca\right]+\left[a^2+b^2+c^2-2ab-2bc+2ca\right]+\left[a^2+b^2+c^2+2ab-2bc-2ca\right]$$

$$= 3a^2+3b^2+3c^2+2ab-2bc+2ca$$

$$= 3\left(a^2+b^2+c^2\right)+2ab-2bc+2ca$$

Question 20

Simplify:

$$(2x+p-c)^2-(2x-p+c)^2$$

Solution 20

We have,

$(2x+p-c)^2-(2x-p+c)^2$

$= [2x+p+(-c)]^2 - [2x+(-p)+c]^2$

$= \left[(2x)^2+p^2+(-c)^2+2\times 2x\times p+2\times p\times(-c)+2\times(-c)\times 2x\right]$
$-\left[(2x)^2+(-p)^2+c^2+2\times 2x\times(-p)+2\times(-p)\times c+2\times c\times 2x\right]$
$= \left[4x^2+p^2+c^2+4xp-2pc-4cx\right]-\left[4x^2+p^2+c^2-4xp-2pc+4cx\right]$
$= 4x^2+p^2+c^2+4xp-2pc-4cx-4x^2-p^2-c^2+4xp+2pc-4cx$
$= 8xp-8cx$

$= 8x(p-c)$

$\therefore\ (2x+p-c)^2-(2x-p+c)^2 = 8x(p-c)$

Question 21
Simplify:

$\left(x^2+y^2-z^2\right)^2-\left(x^2-y^2+z^2\right)^2$

Solution 21
We have,

$\left(x^2+y^2-z^2\right)^2-\left(x^2-y^2+z^2\right)^2$

$= \left[x^2+y^2+\left(-z^2\right)\right]^2-\left[x^2+\left(-y^2\right)+z^2\right]^2$

$= \left[\left(x^2\right)^2+\left(y^2\right)^2+\left(-z^2\right)^2+2\left(x^2\right)\left(y^2\right)+2\left(y^2\right)\left(-z^2\right)+2\left(-z^2\right)\left(x^2\right)\right]$
$-\left[\left(x^2\right)^2+\left(-y^2\right)^2+\left(z^2\right)^2+2\left(x^2\right)\left(-y^2\right)+2\left(-y^2\right)\left(z^2\right)+2\left(z^2\right)\left(x^2\right)\right]$
$= \left[x^4+y^4+z^4+2x^2y^2-2y^2z^2-2z^2x^2\right]-\left[x^4+y^4+z^4-2x^2y^2-2y^2z^2+2z^2x^2\right]$
$= x^4+y^4+z^4+2x^2y^2-2y^2z^2-2z^2x^2-x^4-y^4-z^4+2x^2y^2+2y^2z^2-2z^2x^2$
$= 4x^2y^2-4z^2x^2$

$= 4x^2\left(y^2-z^2\right)$

$\therefore\ \left(x^2+y^2-z^2\right)^2-\left(x^2-y^2+z^2\right)^2 = 4x^2\left(y^2-z^2\right)$

Question 22

Simplify $(x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$

Solution 22

We have,

$$(x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$$

$$=\left[x^2+y^2+z^2+2(xy+yz+zx)\right]+\left[x^2+\frac{y^2}{4}+\frac{z^2}{9}+2\left(\frac{xy}{2}+\frac{yz}{6}+\frac{zx}{3}\right)\right]$$
$$-\left[\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}+2\left(\frac{xy}{6}+\frac{yz}{12}+\frac{zx}{8}\right)\right]$$
$$=x^2+y^2+z^2+2xy+2yz+2zx+x^2+\frac{y^2}{4}+\frac{z^2}{9}+\frac{2xy}{2}+\frac{2yz}{6}+\frac{2zx}{3}-\frac{x^2}{4}-\frac{y^2}{9}-\frac{z^2}{16}-\frac{2xy}{6}-\frac{2yz}{12}-\frac{2zx}{8}$$
$$=2x^2-\frac{x^2}{4}+y^2+\frac{y^2}{4}-\frac{y^2}{9}+z^2+\frac{z^2}{9}-\frac{z^2}{16}+2xy+xy-\frac{xy}{3}+2yz+\frac{yz}{3}-\frac{yz}{6}+2zx+\frac{2zx}{3}-\frac{zx}{4}$$
$$=\frac{8x^2-x^2}{4}+\frac{36y^2+9y^2-4y^2}{36}+\frac{144z^2+16z^2-9z^2}{144}+\frac{6xy+3xy-xy}{3}+\frac{12yz+2yz-yz}{6}+\frac{24zx+8zx-3z}{12}$$

$$=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

$$\therefore\ (x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

Question 23

Simplify $(x+y-2z)^2-x^2-y^2-3z^2+4xy$

Solution 23

We have,

$$(x+y-2z)^2-x^2-y^2-3z^2+4xy$$

$$=\left[x^2+y^2+(-2z)^2+2\times x\times y+2\times y\times(-2z)+2\times(-2z)\times x\right]-x^2-y^2-3z^2+4xy$$
$$=x^2+y^2+4z^2+2xy-4yz-4zx-x^2-y^2-3z^2+4xy$$

$$=z^2+6xy-4yz-4zx$$

$$\therefore\ (x+y-2z)^2-x^2-y^2-3z^2+4xy=z^2+6xy-4yz-4zx$$

Question 24

Simplify $(x^2-x+1)^2-(x^2+x+1)^2$

Solution 24

We have,

$$\left(x^2 - x + 1\right)^2 - \left(x^2 + x + 1\right)^2$$

$= [(x^2)^2 + (-x)^2 + (1)^2 + 2 \times x^2 \times (-x) + 2 \times (-x) \times 1 + 2 \times 1 \times x^2]$
$-[(x^2)^2 + x^2 + (1)^2 + 2 \times x^2 \times x + 2 \times x \times 1 + 2 \times 1 \times x^2]$

$= [x^4 + x^2 + 1 - 2x^3 - 2x + 2x^2] - [x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2]$

$= x^4 + x^2 + 1 - 2x^3 - 2x + 2x^2 - x^4 - x^2 - 1 - 2x^3 - 2x - 2x^2$

$= -4x^3 - 4x$

$= -4x\left[x^2 + 1\right]$

$\therefore \quad \left(x^2 - x + 1\right)^2 - \left(x^2 + x + 1\right)^2 = -4x\left[x^2 + 1\right]$

Chapter 4 - Algebraic Identities Exercise Ex. 4.3

Question 1

Find the cube of $\frac{1}{x} + \frac{y}{3}$.

Solution 1

We know that,

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Replacing a by $\frac{1}{x}$ and b by $\frac{y}{3}$,

We have

$$\left(\frac{1}{x} + \frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3 \times \frac{1}{x} \times \frac{y}{3}\left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x}\left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

$$\therefore \quad \left(\frac{1}{x} + \frac{y}{3}\right)^3 = \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

Question 2

Find the cube of $\frac{3}{x} - \frac{2}{x^2}$.

Solution 2

We know that,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Replacing a by $\frac{3}{x}$ and b by $\frac{2}{x^2}$,

We get,

$$\begin{aligned}\left(\frac{3}{x}-\frac{2}{x^2}\right)^3 &= \left(\frac{3}{x}\right)^3 - \left(\frac{2}{x^2}\right)^3 - 3\times\frac{3}{x}\times\frac{2}{x^2}\left(\frac{3}{x}-\frac{2}{x^2}\right)\\ &= \frac{27}{x^3}-\frac{8}{x^6}-\frac{18}{x^3}\left(\frac{3}{x}-\frac{2}{x^2}\right)\\ &= \frac{27}{x^3}-\frac{8}{x^6}-\frac{54}{x^4}+\frac{36}{x^5}\end{aligned}$$

$$\therefore \left(\frac{3}{x}-\frac{2}{x^2}\right)^3 = \frac{27}{x^3}-\frac{8}{x^6}-\frac{54}{x^4}+\frac{36}{x^5}$$

Question 3

Find the cube of $2x+\frac{3}{x}$.

Solution 3

We know that,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

Replacing a by $2x$ and b by $\frac{3}{x}$,

We have,

$$\begin{aligned}\left(2x+\frac{3}{x}\right)^3 &= (2x)^3 + \left(\frac{3}{x}\right)^3 + 3\times 2x\times\frac{3}{x}\left(2x+\frac{3}{x}\right)\\ &= 8x^3+\frac{27}{x^3}+18\left(2x+\frac{3}{x}\right)\\ &= 8x^3+\frac{27}{x^3}+36x+\frac{54}{x}\end{aligned}$$

$$\therefore \left(2x+\frac{3}{x}\right)^3 = 8x^3+\frac{27}{x^3}+36x+\frac{54}{x}$$

Question 4

Find the cube of $4-\frac{1}{3x}$.

Solution 4

We know that,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

Replacing a by 4 and b by $\frac{1}{3x}$,

We have,

$$\left(4-\frac{1}{3x}\right)^3 = (4)^3 - \left(\frac{1}{3x}\right)^3 - 3\times4\times\frac{1}{3x}\left(4-\frac{1}{3x}\right)$$

$$= 64 - \frac{1}{27x^3} - \frac{4}{x}\left(4-\frac{1}{3x}\right)$$

$$= 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2}$$

$$\therefore \left(4-\frac{1}{3x}\right)^3 = 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2}$$

Question 5

If $a+b=10$ and $ab=21$, find the value of a^3+b^3.

Solution 5

We know that,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$\Rightarrow \quad (10)^3 = a^3 + b^3 + 3\times21\times10 \qquad [\because a+b=10 \text{ and } ab=21]$

$\Rightarrow \quad 1000 = a^3 + b^3 + 630$

$\Rightarrow \quad a^3 + b^3 = 1000 - 630$

$\Rightarrow \quad a^3 + b^3 = 370$

Question 6

If $a-b=4$ and $ab=21$, find the value of a^3-b^3.

Solution 6

We know that,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\Rightarrow (4)^3 = a^3 - b^3 - 3 \times 21 \times 4 \qquad [\because a - b = 4 \text{ and } ab = 21]$$
$$\Rightarrow \quad 64 = a^3 - b^3 - 252$$
$$\Rightarrow \quad a^3 - b^3 = 252 + 64$$

$$\Rightarrow \quad a^3 - b^3 = 316$$

Question 7

If $x + \frac{1}{x} = 5$, find the value of $x^3 + \frac{1}{x^3}$.

Solution 7

We know that,

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \quad (5)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) \qquad \left[\because x + \frac{1}{x} = 5\right]$$
$$\Rightarrow \quad (5)^3 = x^3 + \frac{1}{x^3} + 3 \times 5$$
$$\Rightarrow \quad 125 = x^3 + \frac{1}{x^3} + 15$$
$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 125 - 15$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 110$$

Question 8

If $x - \frac{1}{x} = 7$, find the value of $x^3 - \frac{1}{x^3}$.

Solution 8

We know that,

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right)$$

$\Rightarrow \quad (7)^3 = x^3 - \frac{1}{x^3} - 3(7) \qquad \left[\because x - \frac{1}{x} = 7\right]$

$\Rightarrow \quad 343 = x^3 - \frac{1}{x^3} - 21$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 343 + 21$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 364$

Question 9

If $x - \frac{1}{x} = 5$, find the value of $x^3 - \frac{1}{x^3}$.

Solution 9

We know that,

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right)$$

$\Rightarrow \quad (5)^3 = x^3 - \frac{1}{x^3} - 3 \times 5 \qquad \left[\because x - \frac{1}{x} = 5\right]$

$\Rightarrow \quad 125 = x^3 - \frac{1}{x^3} - 15$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 125 + 15$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 140$

Question 10

If $x^2 + \frac{1}{x^2} = 51$, find the value of $x^3 - \frac{1}{x^3}$.

Solution 10

We know that,

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right)$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) \qquad --(1)$$

Now,

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = 51 - 2 \qquad \left[\because x^2 + \frac{1}{x^2} = 51\right]$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = 49$$

$$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = (7)^2$$

$$\Rightarrow \quad x - \frac{1}{x} = 7 \qquad --(2)$$

Using equation (1) and equation (2)
We get,

$$(7)^3 = x^3 - \frac{1}{x^3} - 3 \times 7$$

$$\Rightarrow \quad 343 = x^3 - \frac{1}{x^3} - 21$$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} = 343 + 21$$

$$\Rightarrow \quad x^3 - \frac{1}{x^3} = 364$$

Question 11

If $x^2 + \frac{1}{x^2} = 98$, find the value of $x^3 + \frac{1}{x^3}$.

Solution 11

We know that,

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) \qquad --(1)$$

Now,

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = 98+2 \qquad \left[\because x^2+\frac{1}{x^2}=98\right]$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = 100$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = (10)^2$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right) = 10 \qquad --(2)$$

Using equation (1) and equation (2) We get,

$$(10)^3 = x^3+\frac{1}{x^3}+3\times 10$$

$$\Rightarrow \quad 1000 = x^3+\frac{1}{x^3}+30$$

$$\Rightarrow \quad x^3+\frac{1}{x^3} = 1000-30$$

$$\Rightarrow \quad x^3+\frac{1}{x^3} = 970$$

Question 12

If $2x+3y=13$ and $xy=6$, find the value of $8x^3+27y^3$.

Solution 12

We know that,

$$(2x+3y)^3=(2x)^3+(3y)^3+3\times 2x\times 3y\,(2x+3y)$$

$\Rightarrow \quad (13)^3=8x^3+27y^3+18xy\times 13 \qquad [\because 2x+3y=13]$

$\Rightarrow \quad 2197=8x^3+27y^3+18\times 6\times 13 \qquad [\because xy=6]$

$\Rightarrow \quad 2197=8x^3+27y^3+1404$

$\Rightarrow \quad 8x^3+27y^3=2197-1404$

$\Rightarrow \quad 8x^3+27y^3=793$

Question 13

If $3x-2y=11$ and $xy=12$, find the value of $27x^3-8y^3$.

Solution 13

We know that,

$$(3x-2y)^3=(3x)^3-(2y)^3-3\times 3x\times 2y\,(3x-2y)$$

$\Rightarrow \quad (11)^3=27x^3-8y^3-18\times 12\times 11 \qquad [\because 3x-2y=11 \text{ and } xy=12]$

$\Rightarrow \quad 1331=27x^3-8y^3-2376$

$\Rightarrow \quad 27x^3-8y^3=2376+1331$

$\Rightarrow \quad 27x^3-8y^3=3707$

Question 14

Evaluate $(103)^3$

Solution 14

We have,

$$\begin{aligned}(103)^3&=(100+3)^3\\&=(100)^3+(3)^3+3\times 100\times 3(100+3)\\&=1000000+27+900\times 103\\&=1000027+92700\\&=1092727\end{aligned}$$

$\Rightarrow \quad (103)^3=1092727$

Question 15

Evaluate $(98)^3$

Solution 15

We have,

$$(98)^3 = (100-2)^3$$
$$= (100)^3 - (2)^3 - 3\times100\times2\,(100-2)$$
$$= 1000000 - 8 - 600\times98$$
$$= 999992 - 58800$$

$$= 941192$$

$$\Rightarrow \quad (98)^3 = 941192$$

Question 16

Evaluate $(9.9)^3$

Solution 16

We have,

$$(9.9)^3 = (10-0.1)^3$$
$$= (10)^3 - (0.1)^3 - 3\times10\times0.1\,(10-0.1)$$
$$= 1000 - 0.001 - 3\times9.9$$
$$= 999.999 - 29.7$$

$$= 970.299$$

$$\therefore \quad (9.9)^3 = 970.299$$

Question 17

Evaluate $(10.4)^3$

Solution 17

We have,

$$(10.4)^3 = (10+0.4)^3$$
$$= (10)^3 + (0.4)^3 + 3\times10\times0.4\,(10+0.4)$$
$$= 1000 + 0.064 + 12\times10.4$$
$$= 1000.064 + 124.8$$

$$= 1124.864$$

$$\therefore \quad (10.4)^3 = 1124.864$$

Question 18

Evaluate $(598)^3$

Solution 18

We have,

$$(598)^3 = (600-2)^3$$
$$= (600)^3 - (2)^3 - 3 \times 600 \times 2\,(600-2)$$
$$= 216000000 - 8 - 3600 \times 598$$
$$= 215999992 - 2152800$$
$$= 213847192$$
$$\therefore\ (598)^3 = 213847192$$

Question 19

Evaluate $(99)^3$

Solution 19

We have,

$$(99)^3 = (100-1)^3$$
$$= (100)^3 - (1)^3 - 3 \times 100 \times 1\,(100-1)$$
$$= 1000000 - 1 - 300 \times 99$$
$$= 999999 - 29700$$
$$= 970299$$
$$\therefore\ (99)^3 = 970299$$

Question 20

Evaluate $111^3 - 89^3$

Solution 20

We have,

$$111^3 - 89^3 = (100+11)^3 - (100-11)^3$$
$$= 2\left(11^3 + 3 \times 100^2 \times 11\right) \qquad \left[\because (a+b)^3 - (a-b)^3 = 2\left(b^3 + 3a^2b\right)\right]$$
$$= 2\left(1331 + 3 \times 10000 \times 11\right)$$
$$= 2\left(1331 + 330000\right)$$
$$= 2\left(331331\right)$$
$$= 662662$$
$$\therefore\ 111^3 - 89^3 = 662662$$

Question 21

Evaluate $46^3 + 34^3$

Solution 21

We have,

$46^3 + 34^3$

$= (40+6)^3 + (40-6)^3$

$= 2\left((40)^3 + 3 \times 40 \times 6^2\right)$ $\qquad \left[\because (a+b)^3 + (a-b)^3 = 2\left(a^3 + 3ab^2\right)\right]$

$= 2\left[64000 + 3 \times 40 \times 36\right]$

$= 2\left(64000 + 120 \times 36\right)$

$= 2\left(64000 + 4320\right)$

$= 2\left(68320\right)$

$= 136640$

$\therefore \quad 46^3 + 34^3 = 136640$

Question 22

Evaluate $104^3 + 96^3$

Solution 22

We have,

$104^3 + 96^3 = (100+4)^3 + (100-4)^3$

$= 2\left(100^3 + 3 \times 100 \times 4^2\right)$ $\qquad \left[\because (a+b)^3 + (a-b)^3 = 2\left[a^3 + 3ab^2\right]\right]$

$= 2\left[1000000 + 300 \times 16\right]$

$= 2\left(1000000 + 4800\right)$

$= 2\left(1004800\right)$

$= 2009600$

$\therefore \quad 104^3 + 96^3 = 2009600$

Question 23

Evaluate $93^3 - 107^3$

Solution 23

We have,

$93^3 - 107^3$

$= (100-7)^3 - (100+7)^3$

$= -\left[(100+7)^3 - (100-7)^3\right]$

$= -\left[2\left(7^3 + 3\times 100^2 \times 7\right)\right]$ $\quad\left[\because\ (a+b)^3 - (a-b)^3 = 2\left[b^3 + 3a^2b\right]\right]$

$= -2\left(343 + 21\times 10000\right)$

$= -2\left(343 + 210000\right)$

$= -2\left(210343\right)$

$= -420686$

$\therefore\ 93^3 - 107^3 = -420686$

Question 24

If $x + \frac{1}{x} = 3$, calculate $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$

Solution 24

We know that,

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$\Rightarrow \quad (3)^2 = x^2+\frac{1}{x^2}+2 \qquad \left[\because \left(x+\frac{1}{x}\right)=3\right]$

$\Rightarrow \quad 9 = x^2+\frac{1}{x^2}+2$

$\Rightarrow \quad x^2+\frac{1}{x^2} = 7 \qquad \ldots(1)$

Now,

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$(3)^3 = x^3+\frac{1}{x^3}+3\times 3 \qquad \left[\because \left(x+\frac{1}{x}\right)=3\right]$

$\Rightarrow \quad 27 = x^3+\frac{1}{x^3}+9$

$\Rightarrow \quad x^3+\frac{1}{x^3} = 27-9$

$\Rightarrow \quad x^3+\frac{1}{x^3} = 18 \qquad \ldots(2)$

Now,

$$\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2$$

$\Rightarrow \quad (7)^2 = x^4+\frac{1}{x^4}+2 \qquad \left[\because x^2+\frac{1}{x^2}=7\right]$

$\Rightarrow \quad 49 = x^4+\frac{1}{x^4}+2$

$\Rightarrow \quad x^4+\frac{1}{x^4} = 49-2$

$\Rightarrow \quad x^4+\frac{1}{x^4} = 47 \qquad \ldots(3)$

From (1), (2) and (3) we have,

$x^2+\frac{1}{x^2}=7, \quad x^3+\frac{1}{x^3}=18, \qquad$ and $x^4+\frac{1}{x^4}=47$

Question 25

Find the value of $27x^3+8y^3$, if

$3x+2y=14$ and $xy=8$

Solution 25

We know that,

$$(3x+2y)^3 = (3x)^3 + (2y)^3 + 3 \times 3x \times 2y\,(3x+2y)$$

$$\Rightarrow \quad (14)^3 = 27x^3 + 8y^3 + 18xy \times 14 \qquad [\because 3x+2y=8]$$
$$\Rightarrow \quad 2744 = 27x^3 + 8y^3 + 18 \times 8 \times 14$$
$$\Rightarrow \quad 2744 = 27x^3 + 8y^3 + 2016$$

$$\Rightarrow \quad 27x^3 + 8y^3 = 2744 - 2016$$
$$= 728$$

$$\therefore \; 27x^3 + 8y^3 = 728$$

Question 26

Find the value of $27x^3 + 8y^3$, if

$3x + 2y = 20$ and $xy = \frac{14}{9}$

Solution 26

We know that,

$$(3x+2y)^3 = (3x)^3 + (2y)^3 + 3 \times 3x \times 2y\,(3x+2y)$$

$$(20)^3 = 27x^3 + 8y^3 + 18 \times \frac{14}{9} \times 20 \qquad \left[\because 3x+2y=20 \text{ and } xy = \frac{14}{9}\right]$$
$$\Rightarrow \quad 8000 = 27x^3 + 8y^3 + 2 \times 14 \times 20$$
$$\Rightarrow \quad 8000 = 27x^3 + 8y^3 + 560$$

$$\Rightarrow \quad 27x^3 + 8y^3 = 8000 - 560$$

$$\Rightarrow \quad 27x^3 + 8y^3 = 7440$$

Question 27

Find the value of $64x^3 - 125z^3$, if $4x - 5z = 16$ and $xz = 12$

Solution 27

We know that,

$$(4x - 5z)^3 = (4x)^3 - (5z)^3 - 3 \times 4x \times 5z(4x - 5z)$$

$\Rightarrow \quad (4x - 5z)^3 = 64x^3 - 125z^3 - 60xz(4x - 5z)$

$\Rightarrow \quad (16)^3 = 64x^3 - 125z^3 - 60 \times 12 \times 16$

$\Rightarrow \quad 4096 = 64x^3 - 125z^3 - 11520$

$\Rightarrow \quad 64x^3 - 125z^3 = 4096 + 11520$

$= 15616$

$\therefore \; 64x^3 - 125z^3 = 15616$

Question 28

If $x - \frac{1}{x} = 3 + 2\sqrt{2}$, find the value of $x^3 - \frac{1}{x^3}$

Solution 28

We know that,

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \times \frac{1}{x}\left(x - \frac{1}{x}\right)$$

$\Rightarrow \quad \left(3 + 2\sqrt{2}\right)^3 = x^3 - \frac{1}{x^3} - 3 \times \left(3 + 2\sqrt{2}\right)$

$\Rightarrow \quad (3)^3 + \left(2\sqrt{2}\right)^3 + 3 \times 3 \times 2\sqrt{2}\left(3 + 2\sqrt{2}\right) = x^3 - \frac{1}{x^3} - 9 - 6\sqrt{2}$

$\Rightarrow \quad 27 + 16\sqrt{2} + 18\sqrt{2}\left(3 + 2\sqrt{2}\right) = x^3 - \frac{1}{x^3} - 9 - 6\sqrt{2}$

$\Rightarrow \quad 27 + 16\sqrt{2} + 54\sqrt{2} + 72 = x^3 - \frac{1}{x^3} - 9 - 6\sqrt{2}$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 72 + 27 + 9 + 16\sqrt{2} + 54\sqrt{2} + 6\sqrt{2}$

$= 108 + 76\sqrt{2}$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 108 + 76\sqrt{2}$

Question 29

Simplify $\left(\frac{x}{2} + \frac{y}{3}\right)^3 - \left(\frac{x}{2} - \frac{y}{3}\right)^3$

Solution 29

We have,

$$\left(\frac{x}{2}+\frac{y}{3}\right)^3-\left(\frac{x}{2}-\frac{y}{3}\right)^3$$

$$=\left[\left(\frac{x}{2}\right)^3+\left(\frac{y}{3}\right)^3+3\times\frac{x}{2}\times\frac{y}{3}\left(\frac{x}{2}+\frac{y}{3}\right)\right]-\left[\left(\frac{x}{2}\right)^3-\left(\frac{y}{3}\right)^3-3\times\frac{x}{2}\times\frac{y}{3}\left(\frac{x}{2}-\frac{y}{3}\right)\right]$$
$$=\left[\frac{x^3}{8}+\frac{y^3}{27}+\frac{xy}{2}\left(\frac{x}{2}+\frac{y}{3}\right)\right]-\left[\frac{x^3}{8}-\frac{y^3}{27}-\frac{xy}{2}\left(\frac{x}{2}-\frac{y}{3}\right)\right]$$
$$=\left[\frac{x^3}{8}+\frac{y^3}{27}+\frac{x^2y}{4}+\frac{xy^2}{6}\right]-\left[\frac{x^3}{8}-\frac{y^3}{27}-\frac{x^2y}{4}+\frac{xy^2}{6}\right]$$
$$=\frac{x^3}{8}+\frac{y^3}{27}+\frac{x^2y}{4}+\frac{xy^2}{6}-\frac{x^3}{8}+\frac{y^3}{27}+\frac{x^2y}{4}-\frac{xy^2}{6}$$
$$=\frac{2y^3}{27}+\frac{2x^2y}{4}$$

$$=\frac{2y^3}{27}+\frac{x^2y}{2}$$

Question 30

Simplify $\left(x+\frac{2}{x}\right)^3+\left(x-\frac{2}{x}\right)^3$

Solution 30

We have,

$$\left(x+\frac{2}{x}\right)^3+\left(x-\frac{2}{x}\right)^3$$

$$=\left[x^3+\left(\frac{2}{x}\right)^3+3\times x\times\frac{2}{x}\left(x+\frac{2}{x}\right)\right]+\left[x^3-\left(\frac{2}{x}\right)^3-3\times x\times\frac{2}{x}\left(x-\frac{2}{x}\right)\right]$$
$$=\left[x^3+\frac{8}{x^3}+6\left(x+\frac{2}{x}\right)\right]+\left[x^3-\frac{8}{x^3}-6\left(x-\frac{2}{x}\right)\right]$$
$$=\left[x^3+\frac{8}{x^3}+6x+\frac{12}{x}\right]+\left[x^3-\frac{8}{x^3}-6x+\frac{12}{x}\right]$$
$$=x^3+\frac{8}{x^3}+6x+\frac{12}{x}+x^3-\frac{8}{x^3}-6x+\frac{12}{x}$$

$$=2x^3+\frac{24}{x}$$

$$\therefore\quad\left(x+\frac{2}{x}\right)^3+\left(x-\frac{2}{x}\right)^3=2x^3+\frac{24}{x}$$

Question 31

Simplify $(2x-5y)^3-(2x+5y)^3$

Solution 31

We have,

$(2x-5y)^3-(2x+5y)^3$

$= \left[(2x)^3-(5y)^3-3(2x)(5y)(2x-5y)\right]-\left[(2x)^3+(5y)^3+3(2x)(5y)(2x+5y)\right]$
$= \left[8x^3-125y^3-30xy(2x-5y)\right]-\left[8x^3+125y^3+30xy(2x+5y)\right]$
$= \left[8x^3-125y^3-60x^2y+150xy^2\right]-\left[8x^3+125y^3+60x^2y+150xy^2\right]$
$= 8x^3-125y^3-60x^2y+150xy^2-8x^3-125y^3-60x^2y-150xy^2$

$= -250y^3-120x^2y$

$\therefore\ (2x-5y)^3-(2x+5y)^3 = -250y^3-120x^2y$

Question 32

Simplify $(x+3)^3+(x-3)^3$

Solution 32

We have,

$(x+3)^3+(x-3)^3$

$= \left[x^3+(3)^3+3\times x\times 3(x+3)\right]+\left[x^3-(3)^3-3\times x\times 3(x-3)\right]$
$= \left[x^3+27+9x(x+3)\right]+\left[x^3-27-9x(x-3)\right]$
$= x^3+27+9x^2+27x+x^3-27-9x^2+27x$

$= 2x^3+54x$

$\therefore\ (x+3)^3+(x-3)^3 = 2x^3+54x$

Question 33

If $x^4+\frac{1}{x^4}=194$, find $x^3+\frac{1}{x^3}$, $x^2+\frac{1}{x^2}$ and $x+\frac{1}{x}$

Solution 33

We know that,

$$\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2$$

$$\Rightarrow \quad \left(x^2+\frac{1}{x^2}\right)^2 = 194+2 \qquad \left[\because\ x^4+\frac{1}{x^4}=194\right]$$

$$\Rightarrow \quad \left(x^2+\frac{1}{x^2}\right)^2 = 196$$

$$\Rightarrow \quad \left(x^2+\frac{1}{x^2}\right)^2 = (14)^2$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 14 \qquad \ldots(1)$$

Now,

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = 14+2 \qquad \left[\because\ x^2+\frac{1}{x^2}=14\right]$$

$$= 16$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2 = (4)^2$$

$$\Rightarrow \quad x+\frac{1}{x} = 4 \qquad \ldots(2)$$

Now,

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \quad (4)^3 = x^3+\frac{1}{x^3}+3\times 4 \qquad \left[\because\ x+\frac{1}{x}=4\right]$$

$$\Rightarrow \quad 64 = x^3+\frac{1}{x^3}+12$$

$$\Rightarrow \quad x^3+\frac{1}{x^3} = 52 \qquad \ldots(3)$$

From (1), (2) and (3) we have,

$x^2+\frac{1}{x^2}=14$, $x+\frac{1}{x}=4$ and $x^3+\frac{1}{x^3}=52$

Question 34

If $x^4+\frac{1}{x^4}=119$, find the value of $x^3-\frac{1}{x^3}$.

Solution 34

We know that,

$$\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2 = 119+2 = 121 \qquad \left[\because x^4+\frac{1}{x^4}=119\right]$$

$$\Rightarrow \quad \left(x^2+\frac{1}{x^2}\right)^2 = (11)^2$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 11 \qquad ---(1)$$

Now,

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$$

$$= 11-2 \qquad [\text{Using equation 1}]$$

$$= 9$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2 = (3)^2$$

$$\Rightarrow \quad x-\frac{1}{x} = 3$$

We know that,

$$\left(x-\frac{1}{x}\right)^3 = x^3-\frac{1}{x^3}-3\times x\times\frac{1}{x}\left(x-\frac{1}{x}\right)$$

$$\Rightarrow \quad (3)^3 = x^3-\frac{1}{x^3}-3(3) \qquad \left[\because x-\frac{1}{x}=3\right]$$

$$\Rightarrow \quad 27 = x^3-\frac{1}{x^3}-9$$

$$\Rightarrow \quad x^3-\frac{1}{x^3} = 27+9$$

$$= 36$$

$$\therefore \quad x^3-\frac{1}{x^3} = 36$$

Chapter 4 - Algebraic Identities Exercise Ex. 4.4

Question 1

Find the following product:

$$(3x+2y)(9x^2-6xy+4y^2)$$

Solution 1

We have,

$(3x+2y)(9x^2-6xy+4y^2)$

$= (3x+2y)\left[(3x)^2 - 3x \times 2y + (2y)^2\right]$

$= (3x)^3 + (2y)^3 \qquad \left[\because a^3+b^3 = (a+b)(a^2-ab+b^2)\right]$

$= 27x^3 + 8y^3$

$\therefore (3x+2y)(9x^2-6xy+4y^2) = 27x^3+8y^3$

Question 2

Find the following product:

$(4x-5y)(16x^2+20xy+25y^2)$

Solution 2

We have,

$(4x-5y)(16x^2+20xy+25y^2)$

$= (4x-5y)\left[(4x)^2 + 4x \times 5y + (5y)^2\right]$

$= (4x)^3 - (5y)^3 \qquad \left[\because a^3-b^3 = (a-b)(a^2+ab+b^2)\right]$

$= 64x^3 - 125y^3$

$\therefore (4x-5y)(16x^2+20xy+25y^2) = 64x^3-125y^3$

Question 3

Find the following product:

$(7p^4+q)(49p^8-7p^4q+q^2)$

Solution 3

We have,

$(7p^4+q)(49p^8-7p^4q+q^2)$

$= (7p^4+q)\left[(7p^4)^2 - 7p^4 \times q + (q)^2\right]$

$= (7p^4)^3 + (q)^3 \qquad \left[\because a^3+b^3 = (a+b)(a^2-ab+b^2)\right]$

$= 343p^{12} + q^3$

$\therefore (7p^4+q)(49p^8-7p^4q+q^2) = 343p^{12}+q^3$

Question 4

Find the following product:

$\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)$

Solution 4

We have,

$$\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)$$

$$=\left(\frac{x}{2}+2y\right)\left[\left(\frac{x}{2}\right)^2-xy-(2y)^2\right]$$

$$=\left(\frac{x}{2}\right)^3+(2y)^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$=\frac{x^3}{8}+8y^3$$

$$\therefore \left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)=\frac{x^3}{8}+8y^3$$

Question 5

Find the following product:

$$\left(\frac{3}{x}-\frac{5}{y}\right)\left(\frac{9}{x^2}+\frac{25}{y^2}+\frac{15}{xy}\right)$$

Solution 5

We have,

$$\left(\frac{3}{x}-\frac{5}{y}\right)\left(\frac{9}{x^2}+\frac{25}{y^2}+\frac{15}{xy}\right)$$

$$=\left(\frac{3}{x}-\frac{5}{y}\right)\left[\left(\frac{3}{x}\right)^2+\left(\frac{5}{y}\right)^2+\frac{3}{x}\times\frac{5}{y}\right]$$

$$=\left(\frac{3}{x}\right)^3-\left(\frac{5}{y}\right)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$$

$$=\frac{27}{x^3}-\frac{125}{y^3}$$

$$\therefore \left(\frac{3}{x}-\frac{5}{y}\right)\left(\frac{9}{x^2}+\frac{25}{y^2}+\frac{15}{xy}\right)=\frac{27}{x^3}-\frac{125}{y^3}$$

Question 6

Find the following product:

$$\left(3+\frac{5}{x}\right)\left(9-\frac{15}{x}+\frac{25}{x^2}\right)$$

Solution 6

We have,

$$\left(3+\frac{5}{x}\right)\left(9-\frac{15}{x}+\frac{25}{x^2}\right)$$

$$=\left(3+\frac{5}{x}\right)\left[(3)^2-3\times\frac{5}{x}+\left(\frac{5}{x}\right)^2\right]$$

$$=(3)^3+\left(\frac{5}{x}\right)^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$=27+\frac{125}{x^3}$$

$$\therefore\left(3+\frac{5}{x}\right)\left(9-\frac{15}{x}+\frac{25}{x^2}\right)=27+\frac{125}{x^3}$$

Question 7

Find the following product:

$$\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right)$$

Solution 7

We have,

$$\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right)$$

$$=\left(\frac{2}{x}+3x\right)\left[\left(\frac{2}{x}\right)^2+(3x)^2-\frac{2}{x}\times 3x\right]$$

$$=\left(\frac{2}{x}\right)^3+(3x)^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$=\frac{8}{x^3}+27x^3$$

$$\therefore\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right)=\frac{8}{x^3}+27x^3$$

Question 8

Find the following product:

$$\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4+6x\right)$$

Solution 8

We have,

$$\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4+6x\right)$$

$$=\left(\frac{3}{x}-2x^2\right)\left[\left(\frac{3}{x}\right)^2+(2x)^2+\frac{3}{x}\times 2x^2\right]$$

$$=\left(\frac{3}{x}\right)^3-\left(2x^2\right)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$$

$$=\frac{27}{x^3}-8x^6$$

$$\therefore\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4+6x\right)=\frac{27}{x^3}-8x^6$$

Question 9

Find the following product:

$(1-x)\left(1+x+x^2\right)$

Solution 9

We have,

$$(1-x)\left(1+x+x^2\right)$$

$$=(1-x)\left[(1)^2+1\times x+(x)^2\right]$$

$$=1^3-x^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$$

$$=1-x^3$$

$$\therefore(1-x)\left(1+x+x^2\right)=1-x^3$$

Question 10

Find the following product:

$(1+x)\left(1-x+x^2\right)$

Solution 10

We have,

$$(1+x)\left(1-x+x^2\right)$$

$$=(1+x)\left[(1)^2-1\times x+(x)^2\right]$$

$$=1^3+x^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$=1+x^3$$

$$\therefore(1+x)\left(1-x+x^2\right)=1+x^3$$

Question 11

Find the following product:

$\left(x^2-1\right)\left(x^4+x^2+1\right)$

Solution 11

We have,

$\left(x^2-1\right)\left(x^4+x^2+1\right)$

$= \left(x^2-1\right)\left[\left(x^2\right)^2+1\times x^2+1^2\right]$

$= \left(x^2\right)^3-(1)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$= x^6-1$

$\therefore \left(x^2-1\right)\left(x^4+x^2+1\right)=x^6-1$

Question 12

Find the following product:

$\left(x^3+1\right)\left(x^6-x^3+1\right)$

Solution 12

We have,

$\left(x^3+1\right)\left(x^6-x^3+1\right)$

$= \left(x^3+1\right)\left[\left(x^3\right)^2-1\times x^3+(1)^2\right]$

$= \left(x^3\right)^3+(1)^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$= x^9+1$

$\therefore \left(x^3+1\right)\left(x^6-x^3+1\right)=x^9+1$

Question 13

If $x = 3$ and $y = -1$, find the value of the following using in identity:

$\left(9y^2-4x^2\right)\left(81y^4+36x^2y^2+16x^4\right)$

Solution 13

We have,

$\left(9y^2-4x^2\right)\left(81y^4+36x^2y^2+16x^4\right)$

$= \left(9y^2-4x^2\right)\left[\left(9y^2\right)^2+9y^2\times 4x^2+\left(4x^2\right)^2\right]$

$= \left(9y^2\right)^3-\left(4x^2\right)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$= 729y^6-64x^6$

$= 729\times(-1)^6-64(3)^6 \qquad [\because x = 3 \text{ and } y = -1]$

$= 729-64\times 729$

$= 729-46656$

$= -45927$

$\left(9y^2-4x^2\right)\left(81y^4+36x^2y^2+16x^4\right)=-45927$

Question 14

If $x = 3$ and $y = -1$, find the value of the following using in identity:

$$\left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right)$$

Solution 14

We have,

$$\left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right)$$

$$= \left(\frac{3}{x} - \frac{x}{3}\right)\left[\left(\frac{x}{3}\right)^2 + \left(\frac{3}{x}\right)^2 + \frac{3}{x} \times \frac{x}{3}\right]$$

$$= \left(\frac{3}{x}\right)^3 - \left(\frac{x}{3}\right)^3 \qquad \left[\because x^3 - y^3 = (x - y)\left(x^2 + y^2 + xy\right)\right]$$

$$= \frac{27}{x^3} - \frac{x^3}{27}$$

$$= \frac{27}{(3)^3} - \frac{(3)^3}{27} \qquad [\because x = 3]$$

$$= \frac{27}{27} - \frac{27}{27}$$

$$= 1 - 1$$

$$= 0$$

$$\therefore \left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right) = 0$$

Question 15

If $x = 3$ and $y = -1$, find the value of the following using in identity:

$$\left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right)$$

Solution 15

We have,

$$\left(\frac{x}{7}+\frac{y}{3}\right)\left(\frac{x^2}{49}+\frac{y^2}{9}-\frac{xy}{21}\right)$$

$$=\left(\frac{x}{7}+\frac{y}{3}\right)\left[\left(\frac{x}{7}\right)^2+\left(\frac{y}{3}\right)^2-\frac{x}{7}\times\frac{y}{3}\right]$$

$$=\left(\frac{x}{7}\right)^3+\left(\frac{y}{3}\right)^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$=\frac{x^3}{343}+\frac{y^3}{27}$$

$$=\frac{(3)^3}{343}+\frac{(-1)^3}{27} \qquad \left[\because x=3 \text{ and } y=-1\right]$$

$$=\frac{27}{343}+\frac{-1}{27}$$

$$=\frac{729-343}{9261}=\frac{386}{9261}$$

$$\therefore \left(\frac{x}{7}+\frac{y}{3}\right)\left(\frac{x^2}{49}+\frac{y^2}{9}-\frac{xy}{21}\right)=\frac{386}{9261}$$

Question 16

If $x = 3$ and $y = -1$, find the value of the following using in identity:

$$\left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{xy}{12}+\frac{y^2}{9}\right)$$

Solution 16

We have,

$$\left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{xy}{12}+\frac{y^2}{9}\right)$$

$$=\left(\frac{x}{4}-\frac{y}{3}\right)\left[\left(\frac{x}{4}\right)^2+\frac{x}{4}\times\frac{y}{3}+\left(\frac{y}{3}\right)^2\right]$$

$$=\left(\frac{x}{4}\right)^3-\left(\frac{y}{3}\right)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$$

$$=\frac{x^3}{64}-\frac{y^3}{27}$$

$$=\frac{(3)^3}{64}-\frac{(-1)^3}{27} \qquad \left[\because x=3,\ y=-1\right]$$

$$=\frac{27}{64}+\frac{1}{27}$$

$$=\frac{729+64}{1728}=\frac{793}{1728}$$

$$\therefore \left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{xy}{12}+\frac{y^2}{9}\right)=\frac{793}{1728}$$

Question 17

If $x = 3$ and $y = -1$, find the value of the following using in identity:

$$\left(\frac{5}{x}+5x\right)\left(\frac{25}{x^2}-25+25x^2\right)$$

Solution 17

We have,

$$\left(\frac{5}{x}+5x\right)\left(\frac{25}{x^2}-25+25x^2\right)$$

$$=\left(\frac{5}{x}+5x\right)\left[\left(\frac{5}{x}\right)^2-\frac{5}{x}\times 5x+(5x)^2\right]$$

$$=\left(\frac{5}{x}\right)^3+(5x)^3 \qquad \left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$=\frac{125}{x^3}+125x^3$$

$$=\frac{125}{(3)^3}+125(3)^3 \qquad [\because x=3]$$

$$=\frac{125}{27}+125\times 27$$

$$=\frac{125}{27}+3375$$

$$=\frac{125+3375\times 27}{27}=\frac{125+91125}{27}$$

$$=\frac{91250}{27}$$

Question 18

If $a+b=10$ and $ab=16$, find the value of a^2-ab+b^2 and a^2+ab+b^2.

Solution 18

We have,

$a^2 - ab + b^2 = a^2 + b^2 - ab$

$= a^2 + b^2 - ab + 2ab - 2ab$ $\qquad$ [Adding and subtracting $2ab$]

$= \left(a^2 + b^2 + 2ab\right) - 3ab$

$= \left(a + b\right)^2 - 3ab$ $\qquad$ $\left[\because \left(a + b\right)^2 = a^2 + b^2 - 2ab\right]$

$= \left(10\right)^2 - 3 \times 16$ $\qquad$ $[\because a + b = 10 \text{ and } ab = 16]$

$= 100 - 48$

$= 52$

$\Rightarrow a^2 - ab + b^2 = 52$

We have,

$a^2 + ab + b^2 = a^2 + ab + b^2 + ab - ab$ $\qquad$ [Adding and subtracting ab]

$= \left(a^2 + b^2 + 2ab\right) - ab$

$= \left(a + b\right)^2 - ab$ $\qquad$ $\left[\because \left(a + b\right)^2 = a^2 + b^2 + 2ab\right]$

$= \left(10\right)^2 - 16$ $\qquad$ $[\because a + b = 10 \text{ and } ab = 16]$

$= 100 - 16$

$\Rightarrow a^2 + ab + b^2 = 84$

Hence, $a^2 - ab + b^2 = 52$, and $a^2 + ab + b^2 = 84$

Question 19

If $a + b = 8$ and $ab = 6$, find the value of $a^3 + b^3$.

Solution 19

We have,

$a^3 + b^3 = \left(a + b\right)\left(a^2 - ab + b^2\right)$

$= \left(a + b\right)\left(a^2 + b^2 - ab\right)$

$= \left(a + b\right)\left(a^2 + b^2 - ab + 2ab - 2ab\right)$ $\qquad$ [Adding and subtracting $2ab$ in the second bracket]

$= \left(a + b\right)\left[\left(a^2 + b^2 + 2ab\right) - 3ab\right]$

$= \left(a + b\right)\left[\left(a + b\right)^2 - 3ab\right]$ $\qquad$ $\left[\because \left(a + b\right)^2 = a^2 + b^2 + 2ab\right]$

$= 8 \times \left[\left(8\right)^2 - 3 \times 6\right]$ $\qquad$ $[\because a + b = 8 \text{ and } ab = 6]$

$= 8 \times [64 - 18]$

$= 8 \times 46$

$= 368$

$\therefore a^3 + b^3 = 368$

Question 20

If $a - b = 6$ and $ab = 20$, find the value of $a^3 - b^3$.

Solution 20

We have,

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (a-b)(a^2 + ab + b^2 - 2ab + 2ab)$ [Adding and subtracting $2ab$ in the second bracket]

$= (a-b)\left[(a^2 + b^2 - 2ab) + 3ab\right]$

$= (a-b)\left[(a-b)^2 + 3ab\right]$ $\left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$= 6 \times \left[(6)^2 + 3 \times 20\right]$ $[\because a - b = 6 \text{ and } ab = 20]$

$= 6 \times [36 + 60]$

$= 6 \times 96$

$= 576$

$\therefore a^3 - b^3 = 576$

Question 21

If $x = -2$ and $y = 1$, by using an identity find the value of the following:

$$\left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right)$$

Solution 21

We have,

$$\left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right)$$

$$= \left(5y + \frac{15}{y}\right)\left[(5y)^2 - 5y \times \frac{15}{y} + \left(\frac{15}{y}\right)^2\right]$$

$$= (5y)^3 + \left(\frac{15}{y}\right)^3 \qquad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$

$$= 125y^3 + \frac{3375}{y^3}$$

$$= 125(1)^3 + \frac{3375}{(1)^3} \qquad [\because y = 1]$$

$= 125 + 3375$

$= 3500$

$$\therefore \left(5y + \frac{15}{y}\right)\left(25y^2 - 75 + \frac{225}{y^2}\right) = 3500$$

Question 22

If $x = -2$ and $y = 1$, by using an identity find the value of the following:

$$\left(4y^2 - 9x^2\right)\left(16y^4 + 36x^2y^2 + 81x^4\right)$$

Solution 22

We have,

$$\left(4y^2-9x^2\right)\left(16y^4+36x^2y^2+81x^4\right)$$

$$=\left(4y^2-9x^2\right)\left[\left(4y^2\right)^2+4y^2\times 9x^2+\left(9x^2\right)^2\right]$$

$$=\left(4y^2\right)^3-\left(9x^2\right)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$$

$$=64y^6-729x^6$$

$$=64(1)^6-729(-2)^6 \qquad \left[\because x=-2 \text{ and } y=1\right]$$

$$=64-729\times 64$$

$$=64-46656$$

$$=-46592$$

$$\therefore \left(4y^2-9x^2\right)\left(16y^4+36x^2y^2+81x^4\right)=-46592$$

Question 23

If $x=-2$ and $y=1$, by using an identity find the value of the following:

$$\left(\frac{2}{x}-\frac{x}{2}\right)\left(\frac{4}{x^2}+\frac{x^2}{4}+1\right)$$

Solution 23

We have,

$$\left(\frac{2}{x}-\frac{x}{2}\right)\left(\frac{4}{x^2}+\frac{x^2}{4}+1\right)$$

$$=\left(\frac{2}{x}-\frac{x}{2}\right)\left[\left(\frac{2}{x}\right)^2+\left(\frac{x}{2}\right)^2+\frac{2}{x}\times\frac{x}{2}\right]$$

$$=\left(\frac{2}{x}\right)^3-\left(\frac{x}{2}\right)^3 \qquad \left[\because a^3-b^3=(a-b)\left(a^2+b^2+2ab\right)\right]$$

$$=\frac{8}{x^3}-\frac{x^3}{8}$$

$$=\frac{8}{(-2)^3}-\frac{(-2)^3}{8} \qquad \left[\because x=-2\right]$$

$$=\frac{8}{-8}+\frac{8}{8}$$

$$=-1+1=0$$

$$\therefore \left(\frac{2}{x}-\frac{x}{2}\right)\left(\frac{4}{x^2}+\frac{x^2}{4}+1\right)=0$$

Chapter 4 - Algebraic Identities Exercise Ex. 4.5

Question 1

Find the following product:

$$(3x+2y+2z)\left(9x^2+4y^2+4z^2-6xy-4yz-6zx\right)$$

Solution 1

We have,

$(3x+2y+2z)\left(9x^2+4y^2+4z^2-6xy-4yz-6zx\right)$

$=(3x+2y+2z)\left((3x)^2+(2y)^2+(2z)^2-3x\times 2y-2y\times 2z-2z\times 3x\right)$

$=(3x)^3+(2y)^3+(2z)^3-3\times 3x\times 2y\times 2z$ $\quad\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right.$

$=27x^3+8y^3+8z^3-36xyz$

$\therefore (3x+2y+2z)\left(9x^2+4y^2+4z^2-6xy-4yz-6zx\right)=27x^3+8y^3+8z^3-36xyz$

Question 2

Find the following product:

$(4x-3y+2z)\left(16x^2+9y^2+4z^2+12xy+6yz-8zx\right)$

Solution 2

We have,

$(4x-3y+2z)\left(16x^2+9y^2+4z^2+12xy+6yz-8zx\right)$

$=(4x+(-3y)+2z)\left[(4x)^2+(-3y)^2+(2z)^2-(4x)(-3y)-(-3y)(2z)-(2z)(4x)\right]$

$=(4x)^3+(-3y)^3+(2z)^3-3(4x)(-3y)(2z)$ $\quad\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-\right.\right.$

$=64x^3-27y^3+8z^3+72xyz$

$\therefore (4x-3y+2z)\left(16x^2+9y^2+4z^2+12xy+6yz-8zx\right)=64x^3-27y^3+8z^3+72xyz$

Question 3

Find the following product:

$(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

Solution 3

We have,

$(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

$=(2a+(-3b)+(-2c))\left((2a)^2+(-3b)^2+(-2c)^2-(2a)(-3b)-(-3b)(-2c)-(-2c)(2a)\right)$

$=(2a)^3+(-3b)^3+(-2c)^3-3(2a)(-3b)(-2c)$

$\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$

$=8a^3-27b^3-8c^3-36abc$

$\therefore (2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)=8a^3-27b^3-8c^3-36abc$

Question 4

Find the following product:

$(3x-4y+5z)\left(9x^2+16y^2+25z^2+12xy-15zx+20yz\right)$

Solution 4

We have,

$\left(3x - 4y + 5z\right)\left(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz\right)$

$= \left(3x + \left(-4y\right) + 5z\right)\left(\left(3x\right)^2 + \left(-4y\right)^2 + \left(5z\right)^2 - \left(3x\right)\left(-4y\right) - \left(-4y\right)\left(5z\right) - \left(5z\right)\left(3x\right)\right)$

$= \left(3x\right)^3 + \left(-4y\right)^3 + \left(5z\right)^3 - 3\left(3x\right)\left(-4y\right)\left(5z\right)$

$\left[\because a^3 + b^3 + c^3 - 3abc = \left(a + b + c\right)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)\right]$

$= 27x^3 - 64y^3 + 125z^3 + 180xyz$

$\therefore \left(3x - 4y + 5z\right)\left(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz\right) = 27x^3 - 64y^3 + 125z^3 + 180xyz$

Question 5

Evaluate:

$25^3 - 75^3 + 50^3$

Solution 5

Let $a = 25, b = -75$ and $c = 50$

Then,

$a + b + c = 25 - 75 + 50$

$= 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \left(25\right)^3 + \left(-75\right)^3 + \left(50\right)^3 = 3 \times 25 \times \left(-75\right) \times 50$

$= -75 \times 75 \times 50$

$= -5625 \times 50$

$= -281250$

$\therefore 25^3 - 75^3 + 50^3 = -281250$

Question 6

Evaluate:

$48^3 - 30^3 - 18^3$

Solution 6

Let $a = 48$, $b = -30$ and $c = -18$

Then,

$a + b + c = 48 - 30 - 18$

$= 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (48)^3 + (-30)^3 + (-18)^3 = 3 \times (48) \times (-30) \times (-18)$

$= 144 \times 540$

$= 77760$

$\therefore 48^3 - 30^3 - 18^3 = 77760$

Question 7

Evaluate:

$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$

Solution 7

Let $a = \frac{1}{2}$, $b = \frac{1}{3}$ and $c = \frac{-5}{6}$

Then,

$a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6}$

$= \frac{3+2}{6} - \frac{5}{6}$

$\Rightarrow a + b + c = \frac{5}{6} - \frac{5}{6} = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{-5}{6}\right)^3 = 3 \times \left(\frac{1}{2}\right) \times \left(\frac{1}{3}\right) \times \left(\frac{-5}{6}\right)$

$= \frac{-5}{12}$

$\therefore \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \frac{-5}{12}$

Question 8

Evaluate:

$(0.2)^3 - (0.3)^3 + (0.1)^3$

Solution 8

Let $a = 0.2, b = -0.3,$ and $c = 0.1$

Then,

$a + b + c = 0.2 - 0.3 + 0.1$

$= 0.3 - 0.3$

$\Rightarrow a + b + c = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (0.2)^3 + (-0.3)^3 + (0.1)^3 = 3 \times (0.2) \times (-0.3) \times (0.1)$

$= -0.018$

Question 9

If $x + y + z = 8$ and $xy + yz + zx = 20$ find the value of $x^3 + y^3 + z^3 - 3xyz$.

Solution 9

We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = (x + y + z)\left[(x^2 + y^2 + z^2) - (xy + yz + zx)\right]$ --- (1)

It follow from the above identity that we require the values of $x + y + z, x^2 + y^2 + z^2,$ and $xy + yz + zx$ to get the value of $x^3 + y^3 + z^3 - 3xyz$. The values of $x + y + z$ and $xy + yz + zx$ are known to us. So we require the value of $x^2 + y^2 + z^2$.

Now,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow (8)^2 = x^2 + y^2 + z^2 + 2(20)$ $\quad [\because x + y + z = 8 \text{ and } xy + yz + zx = 20]$

$\Rightarrow 64 = x^2 + y^2 + z^2 + 40$

$\Rightarrow x^2 + y^2 + z^2 = 64 - 40 = 24$

Substituting the values of $x^2 + y^2 + z^2, x + y + z$ and $xy + yz + zx$ in equation (1), we get,

$x^3 + y^3 + z^3 - 3xyz = 8 \times (24 - 20)$

$= 8 \times 4$

$= 32$

$\therefore x^3 + y^3 + z^3 - 3xyz = 32$

Question 10

If $a + b + c = 9$ and $ab + bc + ca = 26$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Solution 10

We know that

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$\Rightarrow a^3+b^3+c^3-3abc=(a+b+c)\left[(a^2+b^2+c^2)-(ab+bc+ca)\right]\ ---(1)$$

It follows from the above identity that we require the values of $a+b+c$, $a^2+b^2+c^2$, and $ab+bc+ca$ to get the value of $a^3+b^3+c^3-3abc$. The values of $a+b+c$ and $ab+bc+ca$ are known to us. So we require the value of $a^2+b^2+c^2$.

Now,

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

$$\Rightarrow (9)^2=a^2+b^2+c^2+2\times 26 \qquad [\because a+b+c=9 \text{ and } ab+bc+ca=26]$$

$$\Rightarrow 81=a^2+b^2+c^2+52$$

$$\Rightarrow a^2+b^2+c^2=81-52=29$$

Substituting the values of $a^2+b^2+c^2$ in (1), we get,

$$a^3+b^3+c^3-3abc=9(29-26) \qquad (\because a+b+c=9 \text{ and } ab+bc+ca=26)$$

$$=9\times 3$$

$$=27$$

$$\therefore a^3+b^3+c^3-3abc=27$$

Question 11

If $a+b+c=9$ and $a^2+b^2+c^2=35$, find the value of $a^3+b^3+c^3-3abc$.

Solution 11

We know that,

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow a^3+b^3+c^3-3abc=[a+b+c]\left[(a^2+b^2+c^2)-(ab+bc+ca)\right]\ ---(1)$

It follows from the above identity that we require the values of $a+b+c$, $a^2+b^2+c^2$, and $ab+bc+ca$ to get the value of $a^3+b^3+c^3-3abc$. The values of $a+b+c$ and $a^2+b^2+c^2$ are known to us. So we require the value of $ab+bc+ca$.

Now,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (9)^2=35+2(ab+bc+ca)\qquad [\because a+b+c=9 \text{ and } a^2+b^2+c^2=35]$

$\Rightarrow 81=35+2(ab+bc+ca)$

$\Rightarrow 2(ab+bc+ca)=81-35=46$

$\Rightarrow ab+bc+ca=\dfrac{46}{2}=23$

Substituting the values of $ab+bc+ca$ in (1), we get,

$a^3+b^3+c^3-3abc=9(35-23)\qquad [\because a+b+c=9\ and\ a^2+b^2+c^2=35]$

$=9\times 12$

$=108$

$\therefore a^3+b^3+c^3-3abc=108$