RD SHARMA Solutions for Class 12-science Maths Chapter 29 - The plane

Chapter 29 - The plane Exercise Ex. 29.1

Question 1(i)

Find the equation of the plane passing through the points:

$(2,1,0),(3,-2,-2)$ and $(3,1,7)$

Solution 1(i)

Given three points are,

$(2,1,0),(3,-2,-2)$ and $(3,1,7)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = (x-2)(-21-0)-(y-1)(7+2)+z(0+3)=0$$

$$= -21x + 42 - 9y + 9 + 3z = 0$$
$$= -21x - 9y + 3z + 51 = 0$$

Dividing by -3, we get

Equation of plane, $7x + 3y - z - 17 = 0$

Question 1(ii)

Find the equation of the plane passing through the points:

$(-5,0,-6),(-3,10,-9)$ and $(-2,6,-6)$

Solution 1(ii)

Given points are,
$(-5,0,-6),(-3,10,-9)$ and $(-2,6,-6)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x+5 & y-0 & z+6 \\ -3+5 & 10-0 & -9+6 \\ -2+5 & 6-0 & -6+6 \end{vmatrix}=0$$

$$\begin{vmatrix} x+5 & y & z+6 \\ 2 & 10 & -3 \\ 3 & 6 & 0 \end{vmatrix}=0$$

$(x+5)(0+18)-y(0+9)+(z+6)(12-30)=0$
$(x+5)(18)-y(9)+(z+6)(-18)=0$
$18x+90-9y-18z-108=0$

Dividing by 9, we get

Equation of plane, $2x-y-2z-2=0$

Question 1(iii)
Find the equation of the plane passing through the points:

$(1,1,1),(1,-1,2)$ and $(-2,-2,2)$

Solution 1(iii)

Given three points are,
$(1,1,1), (1,-1,2)$ and $(-2,-2,2)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 2-1 \\ -2-1 & -2-1 & 2-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 1 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$$(x-1)(-2+3) - (y-1)(0+3) + (z-1)(0-6) = 0$$
$$(x-1)(1) - (y-1)(3) + (z-1)(-6) = 0$$
$$x - 1 - 3y + 3 - 6z + 6 = 0$$

$$x - 3y - 6z + 8 = 0$$

Equation of plane is, $x - 3y - 6z + 8 = 0$

Question 1(iv)

Find the equation of the plane passing through the points:

$(2,3,4), (-3,5,1)$ and $(4,-1,2)$

Solution 1(iv)

Given points are,
$(2,3,4),(-3,5,1)$ and $(4,-1,2)$

We know that, equation of plane passing through three points are given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -3-2 & 5-3 & 1-4 \\ 4-2 & -1-3 & 2-4 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -5 & 2 & -3 \\ 2 & -4 & -2 \end{vmatrix} = 0$$

$(x-2)(-4-12)-(y-3)(10+6)+(z-4)(20-4)=0$
$(x-2)(-16)-(y-3)(16)+(z-4)(16)=0$
$-16x+32-16y+48+16z-64=0$

$-16x-16y+16z+16=0$

Dividing by (-16), we get,

Equation of plane, $x+y-z-1=0$

Question 1(v)
Find the equation of the plane passing through the points:

$(0,-1,0),(3,3,0)$ and $(1,1,1)$

Solution 1(v)

Given points are,
$(0,-1,0),(3,3,0)$ and $(1,1,1)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 3-0 & 3+1 & 0-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 3 & 4 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$x(4-0)-(y+1)(3-0)+z(6-4)=0$
$4x-(y+1)(3)+z(2)=0$
$4x-3y-3+2z=0$

$4x-3y+2z-3=0$

Equation of plane is, $4x-3y+2z-3=0$

Question 2

Show that the four point $(0,-1,-1),(4,5,1),(3,9,4)$ and $(-4,4,4)$ are coplanar and find the equation of the common plane.

Solution 2

We have to prove that points

$(0,-1,-1),(4,5,1),(3,9,4)$ and $(-4,4,4)$ are coplanar.

First we shall find the equation of plane passing through three points:

$(0,-1,1),(4,5,1)$ and $(3,9,4)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$x(30-20)-(y+1)(20-6)+(z+1)(40-18)=0$

$10x-(y+1)(14)+(z+1)(22)=0$

$10x-14y-14+22z+22=0$

$10x-14y+22z+8=0$

Dividing by 2, we get

$5x-7y+11z+4=0 \qquad ---(i)$

Now, for the fourth point $(-4,4,4)$ put $x=-4,\ y=4,\ z=4$ in equation (i),

$5(-4)-7(4)+11(4)+4=0$

$-20-28+44+4=0$

$-48+48=0$

$0=0$

$LHS = RHS$

Since, fourth point satisfies the equation of plane passing through three points

So, all four points are collinear

Equation of common plane is, $5x-7y+11z+4=0$

Question 3(i)

Show that the following points are coplanar:

$(0,-1,0),(2,1,-1),(1,1,1)$ and $(3,3,0)$

Solution 3(i)

Given, four points are

$(0,-1,0),(2,1,-1),(1,1,1)$ and $(3,3,0)$.

Now, first we find the equation of plane passing through three points:

$(0,-1,0),(2,1,-1)$, $(1,1,1)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 2-0 & 1+1 & -1-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y+1 & z \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$x(2+2)-(y+1)(2+1)+z(4-2)=0$

$x(4)-(y+1)(3)+z(2)=0$

$4x-3y-3+2z=0$

$4x-3y+2z-3=0 \qquad ---(i)$

Put, $x=3$, $y=3$, $z=0$ in equation (i), we get

$4x-3y+2z-3=0$

$4(3)-3(3)+2(0)-3=0$

$12-9+0-3=0$

$12-12=0$

$0=0$

$LHS=RHS$

Since, fourth point satisfies the equation of plane passing through three points,

Hence, four points are coplanar

Question 3(ii)

Show that the following points are coplanar:

$(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$

Solution 3(ii)

Given, four points are

$(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$

First we shall find the equation of plane passing through three points:

$(0,4,3),(-1,-5,-3),(-2,-2,1)$

We know that, equation of plane passing through three given points is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-0 & y-4 & z-3 \\ -1-0 & -5-4 & -3-3 \\ -2-0 & -2-4 & 1-3 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y-4 & z-3 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix}=0$$

$x(18-36)-(y-4)(2-12)+(z-3)(6-18)=0$

$x(-18)-(y-4)(-10)+(z-3)(-12)=0$

$-18x+10y-40-12z+36=0$

$-18x+10y-12z-4=0$ --- (i)

Put, $x=1$, $y=1$, $z=-1$ in equation (i),

$-18(1)+10(1)-12(-1)-4=0$

$-18+10+12-4=0$

$-22+22=0$

$0=0$

$LHS=RHS$

So, fourth point $(1,1,-1)$ satisfies the equation of plane passing through three points,

Hence, four points are coplanar

Chapter 29 - The plane Exercise Ex. 29.2

Question 1

Write the equation of the plane whose intercepts on the coordinate axes are 2, – 3 and 4.

Solution 1

Given, intercepts on the coordinate axes are 2, – 3 and 4

We know that,
The equation of a plane whose intercepts on the coordinate axes are a, b and c respectively, is given by

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(\text{i})$$

Here, $a = 2$, $b = -3$, $c = 4$

So,
Equation of required plane is

$$\frac{x}{2}+\frac{y}{-3}+\frac{z}{4}=1$$

$$\frac{6x-4y+3z}{12}=1$$

$$6x-4y+3z=12$$

Question 2(i)

Reduce the equations of the following plane in intercept form and find its intercepts on the coordinate axes:

$$4x+3y-6z-12=0$$

Solution 2(i)

Reduce the equation $4x + 3y - 6z - 12 = 0$ in intercept form:

$$4x + 3y - 6z - 12 = 0$$
$$4x + 3y - 6z = 12$$

Divide by 12,

$$\frac{4x}{12} + \frac{3y}{12} - \frac{6z}{12} = \frac{12}{12}$$
$$\frac{x}{3} + \frac{y}{4} - \frac{z}{2} = 1$$

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{(-2)} = 1 \qquad ---(i)$$

This is of the form,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a = 3$, $b = 4$, $c = -2$

Intercepts on the coordinate axes are $3, 4, -2$

Question 2(ii)

Reduce the equations of the following plane in intercept form and find its intercepts on the coordinate axes:

$$2x + 3y - z = 6$$

Solution 2(ii)

Reduce $2x + 3y - z = 6$ in the intercept form:

$2x + 3y - z = 6$

Divide by 6,

$$\frac{2x}{6} + \frac{3y}{6} - \frac{z}{6} = \frac{6}{6}$$
$$\frac{x}{3} + \frac{y}{2} - \frac{z}{6} = 1$$

$$\frac{x}{3} + \frac{y}{2} + \frac{z}{(-6)} = 1 \qquad ---(i)$$

We know intercept form of plane with a, b, c as intercepts on coordinate axes is,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a = 3$, $b = 2$, $c = -6$

So, intercepts on coordinate axes by the given plane are $3, 2, -6$

Question 2(iii)

Reduce the equations of the following plane in intercept form and find its intercepts on the coordinate axes:

$2x - y + z = 5$

Solution 2(iii)

We have to find intercepts on coordinate axes by plane $2x - y + z = 5$

$2x - y + z = 5$

Divide by 5,

$$\frac{2x}{5} - \frac{y}{5} + \frac{z}{5} = \frac{5}{5}$$
$$\frac{x}{\left(\frac{5}{2}\right)} + \frac{y}{(-5)} + \frac{z}{5} = 1 \qquad ---(i)$$

We know that if a, b, c are intercepts on coordinate axes by the plane,
then equation of such plane is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$$a = \frac{5}{2},\ b = -5,\ c = 5$$

So, intercepts on coordinate axes by the plane are $\frac{5}{2}, -5, 5$.

Question 3

Find the equation of a plane which meets the axes in A, B and C, given that the centroid of the triangle ABC is the point (α, β, γ).

Solution 3

Here, it is given that the plane meets axes in A, B and C

Let, $A = (a,0,0)$, $B = (0,b,0)$, $C = (0,0,c)$

We have centroid of $\triangle ABC$ is (α, β, γ) we know that, centroid of $\triangle ABC$ is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(\alpha, \beta, \gamma) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$(\alpha, \beta, \gamma) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

So,

$$\frac{a}{3} = \alpha \quad \Rightarrow \quad a = 3\alpha \qquad ---(\text{i})$$

$$\frac{b}{3} = \beta \quad \Rightarrow \quad b = 3\beta \qquad ---(\text{ii})$$

$$\frac{c}{3} = \gamma \quad \Rightarrow \quad c = 3\gamma \qquad ---(\text{iii})$$

We know that, if a, b, c are intercepts by plane on coordinate axes, then equation of the plane is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Put a, b, c from equation (i), (ii) and (iii),

$$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$$

Multiplying by 3 on both the sides,

$$\frac{3x}{3\alpha} + \frac{3y}{3\beta} + \frac{3z}{3\gamma} = 3$$

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

Question 4

Find the equation of the plane passing through the point $(2, 4, 6)$ and making equal intercepts on the coordinate axes.

Solution 4

Intercepts on the coordinate axes are equal.

We know that, if a, b, c are intercepts on coordinate axes by a plane, then equation of the plane is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Here, it is givin that $a = b = c = p$ (Say)

$$\frac{x}{p} + \frac{y}{p} + \frac{z}{p} = 1$$
$$\frac{x + y + z}{p} = 1$$

$$x + y + z = p \qquad ---(i)$$

It is given that plane is passing through the point $(2,4,6)$, so, using equation (i)

$$x + y + z = p$$
$$2 + 4 + 6 = p$$

$$12 = p$$

Put, value of p in equation (i)

$$x + y + z = 12$$

So, the required equation of the plane is given by,

$$x + y + z = 12$$

Question 5

A plane meets the coordinate axes at A, B and C respectively such that the centroid of triangle ABC is $(1,-2,3)$. Find the equation of the plane.

Solution 5

Here, it is given that plane meets the coordinate axes at A, B and C with centroid of □ABC is $(1, -2, 3)$

The equation of plane with intercepts a, b and c on the coordinate axes is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(\text{i})$$

We know that, centroid of a triangle is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(1, -2, 3) = \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$$

$$(1, -2, 3) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

Comparing LHS and RHS,

$$\frac{a}{3} = 1 \Rightarrow \qquad a = 3 \qquad ---(\text{i})$$

$$\frac{b}{3} = -2 \Rightarrow \qquad b = -6 \qquad ---(\text{ii})$$

$$\frac{c}{3} = 3 \Rightarrow \qquad c = 9 \qquad ---(\text{iii})$$

Put, a, b, c is equation (i), we get the equation of required plane

$$\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$$

$$\frac{6x - 3x + 2z}{18} = 1$$

$$6x - 3x + 2z = 18$$

Chapter 29 - The plane Exercise Ex. 29.3

Question 1

Find the vector equation of a plane passing through a point having position vector $2\hat{i} - \hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 2\hat{j} - 3\hat{k}$

Solution 1

We know that, vector equation of a plane passing through a point $\vec{a}$ and normal to $\vec{n}$ is given by,

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0 \qquad ---(i)$$

Here,
$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{n} = 4\hat{i} + 2\hat{j} - 3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i)

$$\left[\vec{r} - \left(2\hat{i} - \hat{j} + \hat{k}\right)\right].\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$$
$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left(2\hat{i} - \hat{j} + \hat{k}\right).\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$$
$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[(2)(4) + (-1)(2) + (1)(-3)\right] = 0$$
$$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[8 - 2 - 3\right] = 0$$

$$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - 3 = 0$$

So, equation of required plane is given by,

$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 3$$

Question 2(i)

Find the cartesian form of equation of a plane whose vector equation is

$$\vec{r}.\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$

Solution 2(i)

Given the vector equation of a plane,

$$\vec{r}.\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$

let, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$
$$(x)(12) + (y)(-3) + (z)(4) + 5 = 0$$

$$12x - 3y + 4z + 5 = 0$$

Cartesian form of the equation of the plane is given by

$$12x - 3y + 4z + 5 = 0$$

Question 2(ii)
Find the cartesian form of equation of a plane whose vector equation is

$$\vec{r}.\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$

Solution 2(ii)
Here, equation of the plane is,

$$\vec{r}.\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$

let, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$
$$(x)(-1) + (y)(1) + (z)(2) = 9$$

$$-x + y + 2z = 9$$

Cartesian form of the equation of plane is,

$$-x + y + 2z = 9$$

Question 3
Find the vector equations of the coordinates planes.

Solution 3

We have to find vector equation of coordinate planes.
For xy-plane.

It passes through origin and is perpendicular to z-axis, so
Put $\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{k}$ in the vector equation of plane passing through point $\vec{a}$ and perpendicular to vector $\vec{n}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{k} = 0$$

$$\vec{r}.\hat{k} = 0 \qquad ---(\text{i})$$

For xz-plane,
It passes through origin and perpendicular to y-axis, so

$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{j}$

Equation of xz-plane is given by
$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{j} = 0$$

$$\vec{r}.\hat{j} = 0$$

For yz-plane.

It passes through origin and is perpendicular to x-axis, so
$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$, $\vec{n} = \hat{i}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{i} = 0$$

$$\vec{r}.\hat{i} = 0$$

Hence, equation of xy, yz, zx-plane are given by

$$\vec{r}.\hat{k} = 0$$
$$\vec{r}.\hat{i} = 0$$
$$\vec{r}.\hat{j} = 0$$

Question 4(i)

Find the vector equation of each one of following plane

$2x - y + 2z = 8$

Solution 4(i)

Given, equation of plane is,

$2x - y + 2z = 8$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

So,

Vector equation of the plane is $\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

Question 4(ii)

Find the vector equation of each one of following plane

$x + y - z = 5$

Solution 4(ii)

Given, cartesian equation of the plane is,

$x + y - z = 5$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

$\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

So,

Vector equation of the plane is $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

Question 4(iii)

Find the vector equation of each one of following plane

$x + y = 3$

Solution 4(iii)

Given, cartesian equation of plane is,

$x + y = 3$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j}\right) = 3$$
$$\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$$

So,
Vector equation of the plane is $\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

Question 5

Find the vector and cartesian equations of a plane passing through the point $(1, -1, 1)$ and normal to the line joining the points $(1, 2, 5)$ and $(-1, 3, 1)$.

Solution 5

We know that, vector equation of a plane passing through point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n} = 0 \qquad ---(\text{i})$$

The given plane is passing through the point $(1,-1,1)$ and normal to the line joining $A(1,2,5)$ and $B(-1,3,1)$. So,

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{n} = \overrightarrow{AB}$
$=$ Position vector of $\vec{B}$ $-$ Position vector of A
$= \left(-\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$
$= -\hat{i} + 3\hat{j} + \hat{k} - \hat{i} - 2\hat{j} - 5\hat{k}$

$= -2\hat{i} + \hat{j} - 4\hat{k}$

Put, $\vec{n}$ and $\vec{a}$ in equation (i),

$\left[\vec{r} - \left(\hat{i} - \hat{j} + \hat{k}\right)\right].\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$
$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left(\hat{i} - \hat{j} + \hat{k}\right)\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$
$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[(1)(-2) + (-1)(1) + (1)(-4)\right] = 0$
$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-2 - 1 - 4\right] = 0$
$\vec{r}\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-7\right] = 0$
$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) + 7 = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = -7$
Multiplying by (-1) on both the sides

$\vec{r}\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$
Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7$
$(x)(2) + (y)(-1) + (2)(4) = 7$
$2x - y + 4z = 7$

So, vector and cartesian equation the plane is

$\vec{r}.\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 7,\ 2x - y + 4z = 7$

Question 6

If $\bar{n}$ is a vector of magnitude $\sqrt{3}$ and is equally inclined with an acute angle with the coordinate axes. Find the vector and cartesian forms of equations of a plane which passes through $(2,1,-1)$ and is normal to $\bar{n}$.

Solution 6

Here, it is given that $\bar{n} = \sqrt{3}$ and $\bar{n}$ makes equal angle with coordinate axes.

Let, $\bar{n}$ has direction cosine as l, m and n and it makes angle of α, β and γ with the coordinate axes. So

Here, $\alpha = \beta = \gamma$

$\Rightarrow \quad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \quad l = m = n = p \text{ (Say)}$

We know that,

$l^2 + m^2 + n^2 = 1$

$p^2 + p^2 + p^2 = 1$

$3p^2 = 1$

$p^2 = \frac{1}{3}$

$p = \pm\frac{1}{\sqrt{3}}$

So,

$l = \pm\frac{1}{\sqrt{3}}$

$\cos\alpha = \pm\frac{1}{\sqrt{3}}$

Now, $\alpha = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

It gives, α is an obtuse angle so, neglect it.

Again, $\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

It gives, α is an acute angle, so

$\cos\alpha = \frac{1}{\sqrt{3}}$

$\therefore \quad l = m = n = \frac{1}{\sqrt{3}}$

So,

$\vec{n} = |\vec{n}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$= \sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

And, $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$

We know that, vector equation of a plane passing through the point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$

$\left[\vec{r} - \left(2\hat{i} + \hat{j} - \hat{k}\right)\right].\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[(2)(1) + (1)(1) + (-1)(1)\right] = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[2 + 1 - 1\right] = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - 2 = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

Put, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

$(x)(1) + (y)(1) + (z)(1) = 2$

$x + y + z = 2$

So, vector and cartesian equation of the plane is,

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$, $x + y + z = 2$

Question 7

The foot of the perpendicular drawn from the origin to a plane is $(12, -4, 3)$. Find the equation of the plane.

Solution 7

Here, it is given that foot of the perpendicular drawn from origin O to the plane is $P(12,-4,3)$

It means, the required plane is passing through $P(12,-4,3)$ and perpendicular to OP.

We know that, equation of a plane passing through $\bar{a}$ and perpendicular to $\bar{n}$ is given by,

$(\vec{r}-\bar{a}).\bar{n}=0$ --- (i)

Here, $\bar{a}=12\hat{i}-4\hat{j}+3\hat{k}$

And, $\bar{n}=\overline{OP}$

= Position vector of P – Position vector of O

$=\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$

$\bar{n}=12\hat{i}-4\hat{j}+3\hat{k}$

Put, value of $\bar{a}$ and $\bar{n}$ in equation (i),

$\left[\vec{r}-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\right].\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$

$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$

$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[(12)(12)+(-4)(-4)+(3)(3)\right]=0$

$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[144+16+9\right]=0$

$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$

$(x)(12)+(y)(-4)+(z)(3)=169$

$12x-4y+3z=169$

So, the vector and cartesian equation of the required plane is,

$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=169,\ 12x-4y+3z=169$

Question 8

Find the equation of the plane passing through the point $(2,3,1)$ having 5,3,2 as direction ratios of normal to the plane.

Solution 8

Given that, the plane is passing through $P(2,3,1)$ having 5,3,2 as the direction ratios of the normal to the plane.

We know that,
Equation of a plane passing through a point $\vec{a}$ and $\vec{n}$ is a vector normal to the plane, is given by,

$(\vec{r}-\vec{a}).\vec{n}=0$ --- (i)

So, $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$
$\vec{n}=5\hat{i}+3\hat{j}+2\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\right]\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[(2)(5)+(3)(3)+(1)(2)\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[10+9+2\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$
$$(x)(5)+(y)(3)+(z)(2)=21$$

$$5x+3y+2z=21$$

Question 9

If the axes are rectangular and P is the point $(2,3,-1)$, find the equation of the plane through P at right angles to OP.

Solution 9

Here, given that P is the point $(2,3,-1)$ and required plane is passing through P at right angles to OP

It means, the plane is passing through P and OP is the vector normal to the plane.

We know that, equation of a plane, passing through a point $\vec{a}$ and $\vec{n}$ is vector normal to the plane, is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---\text{(i)}$$

Here, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$

$\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P $-$ Position vector of O

$$=\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$$

$$\vec{n}=2\hat{i}+3\hat{j}-\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right].\left(2\hat{i}+3\hat{j}-\hat{k}\right)=0$$

$$\vec{r}\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}-\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right]=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[(2)(2)+(3)(3)+(-1)(-1)\right]=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[4+9+1\right]=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-14=0$$

$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$

$$(x)(2)+(y)(3)+(z)(-1)=14$$

$$2x+3y-z=14$$

Equation of required plane is,

$$2x+3y-z=14$$

Question 10

Find the intercepts made on the coordinate axes by the plane $2x+y-2z=3$ and find also the direction cosines of the normal to the plane.

Solution 10

Here, given equation of plane is,

$2x + y - 2z = 3$

Dividing by 3 on both the sides,

$$\frac{2x}{3} + \frac{y}{3} - \frac{2z}{3} = \frac{3}{3}$$

$$\frac{x}{\frac{3}{2}} + \frac{y}{3} + \frac{z}{-\frac{3}{2}} = 1 \qquad ---(i)$$

We know that, if a, b, c are the intercepts by a plane on the coordinate axes, new equation of the plane is,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$a = \frac{3}{2}$, $b = 3$, $c = -\frac{3}{2}$

Again, given equation of plane is,

$$2x + y - 2z = 3$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

$$\vec{r}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

So, vector normal to the plane is given by

$$\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\left|\vec{n}\right| = \sqrt{(2)^2 + (1)^2 + (-2)^2}$$

$$= \sqrt{4 + 1 + 4}$$

$$= \sqrt{9}$$

$$\left|\vec{n}\right| = 3$$

Direction vector of $\bar{n} = 2, 1, -2$

Direction vector of $\bar{n} = \frac{2}{|\bar{n}|}, \frac{1}{|\bar{n}|}, \frac{-2}{|\bar{n}|}$

$$= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$$

So,

Intercepts by the plane on coordinate axes are $= \frac{3}{2}, 3, -\frac{3}{2}$

Direction cosine of normal to the plane are $= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$

Question 11

A plane passes through the point $(1, -2, 5)$ and is perpendicular to the line joining the origin to the point $3\hat{i} + \hat{j} - \hat{k}$. Find the vector and catresian forms of the equation of the plane.

Solution 11

Here, given that, the required plane passes through the point $(1,-2,5)$ and is perpendicular to the line joining origin O to the point $P\left(3\hat{i}+\hat{j}-\hat{k}\right)$.

We know that, equation of a plane passing through a point $\vec{a}$ and perpendicular to a vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(\text{i})$$

Here, $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$

$\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P – Position vector of O

$=\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$

$\vec{n}=3\hat{i}+\hat{j}-\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\right]\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[(1)(3)+(-2)(1)+(5)(-1)\right]=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[3-2-5\right]=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[-4\right]=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)+4=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)=-4$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(3\hat{i}+\hat{j}-\hat{k}\right)=-4$$
$$(x)(3)+(y)(1)+(z)(-1)=-4$$

$$3x+y-z=-4$$

Hence, equation of required plane is,

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)=-4$$

Or, $3x+y-z=-4$

Question 12

Find the equation of the plane that bisects the line joining $(1,2,3)$ and $(3,4,5)$ and is at right angle to the line.

Solution 12

We have to find the equation of plane that bisects $A(1,2,3)$ and $B(3,4,5)$ perpendicularly

We know that, equation of a plane passing through the point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n} = 0 \qquad ---(i)$$

Here, $\vec{a}$ = mid-point of AB

$$= \frac{\text{Position vector of } A + \text{Position vector of } B}{2}$$

$$= \frac{\hat{i}+2\hat{j}+3\hat{k} + 3\hat{i}+4\hat{j}+5\hat{k}}{2}$$

$$\vec{a} = \frac{4\hat{i}+6\hat{j}+8\hat{k}}{2}$$

$$\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$$

And, $\vec{n} = \overrightarrow{AB}$

$$= \text{Position vector of } B - \text{Position vector of } A$$

$$= \left(3\hat{i}+4\hat{j}+5\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$= 3\hat{i}+4\hat{j}+5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$$

$$\vec{n} = 2\hat{i}+2\hat{j}+2\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}-\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right) = 0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)\right] = 0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[(2)(2)+(3)(2)+(4)(2)\right] = 0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[4+6+8\right] = 0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-18 = 0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right) = 18$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 18$$

$$(x)(2) + (y)(2) + (z)(2) = 18$$

$$2x + 2y + 2z = 18$$

Dividing by 2 on both the sides,

$$\frac{2x}{2} + \frac{2y}{2} + \frac{2z}{2} = \frac{18}{2}$$

$$x + y + z = 9$$

The required equation of the plane is,

$$x + y + z = 9$$

Question 13(i)

Show that the normals to the following pairs of planes are perpendicular to each other:

$x - y + z - 2 = 0$ and $3x + 2y - z + 4 = 0$

Solution 13(i)

Given, two equation of plane are,

$x - y + z - 2 = 0$ and
$3x + 2y - z + 4 = 0$

$x - y + z = 2$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} - \hat{j} + \hat{k}\right) = 2$
$\vec{r}.\overrightarrow{n_1} = 2 \qquad ---(i)$

$3x + 2y - z = -4$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}.\overrightarrow{n_2} = -4 \qquad ---(i)$

From equation (i) and (ii), we get that
$\overrightarrow{n_1}$ is normal to equation (i) and
$\overrightarrow{n_2}$ is normal to equation (ii).

Now,
$\overrightarrow{n_1}.\overrightarrow{n_2} = \left(\hat{i} - \hat{j} + \hat{k}\right).\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$
$= (1)(3) + (-1)(2) + (1)(-1)$
$= 3 - 2 - 1$
$= 3 - 3$

$= 0$

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$

So, $\overrightarrow{n_1}$ is perpendicular to $\overrightarrow{n_2}$

Question 13(ii)
Show that the normals to the following pairs of planes are perpendicular to each other:

$\vec{r}.\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 5$ and $\vec{r}.\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$

Solution 13(ii)

Given, two vector equation of plane are,

$\vec{r}.\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 5$

$\vec{r}.\vec{n_1} = 5$

So, $\vec{n_1} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

And, $\vec{r}.\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$

$\vec{r}.\vec{n_2} = 5$

So, $\vec{n_2} = 2\hat{i} - 2\hat{j} - 2\hat{k}$

Now, $\vec{n_1}.\vec{n_2}$

$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right).\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right)$

$= (2)(2) + (-1)(-2) + (3)(-2)$

$= 4 + 2 - 6$

$= 6 - 6$

$= 0$

$\vec{n_1}.\vec{n_2} = 0$

Hence, normals to planes $\vec{n_1}$ and $\vec{n_2}$ are perpendicular.

Question 14

Show that the normal vector to the plane $2x + 2y + 2z = 3$ is equally inclined with the coordinate axes.

Solution 14

Given, equation of plane is,

$$2x + 2y + 2z = 3$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 3$$
$$\vec{r}.\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right) = 3$$

$$\vec{r}.\vec{n} = d$$

Normal to the plane $\vec{n} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

Direction ratio of $\vec{n} = 2,2,2$

Direction cosine of $\vec{n} = \frac{2}{\left|\vec{n}\right|}, \frac{2}{\left|\vec{n}\right|}, \frac{2}{\left|\vec{n}\right|}$

$$\left|\vec{n}\right| = \sqrt{(2)^2 + (2)^2 + (2)^2}$$
$$= \sqrt{4 + 4 + 4}$$
$$= \sqrt{12}$$

$$\left|\vec{n}\right| = 2\sqrt{3}$$

Direction cosine of $\left|\vec{n}\right| = \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}$

$$= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

So, $l = \frac{1}{\sqrt{3}}$, $m = \frac{1}{\sqrt{3}}$, $n = \frac{1}{\sqrt{3}}$

Let, α, β, γ be the angle that normal $\vec{n}$ makes with the coordinate axes respectively.

$l = \cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad ---(i)$

$m = \cos\beta = \frac{1}{\sqrt{3}}$

$\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad ---(ii)$

$n = \cos\gamma = \frac{1}{\sqrt{3}}$

$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad ---(iii)$

From equation (i), (ii) and (iii),

$\alpha = \beta = \gamma$

So, normal to the plane, $\bar{n}$ is equally inclined with the coordinate axes.

Question 15

Find the vector of magnitude 26 units normal to the plane $12x - 3y + 4z = 1$.

Solution 15

Given, equation of plane is,

$12x - 3y + 4z = 1$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$

$\vec{r}.\vec{n} = 1$

So, normal to the plane is

$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$

$= \sqrt{144 + 9 + 16}$

$= \sqrt{144 + 25}$

$= 169 = 13$

Unit vector $\hat{n} = \dfrac{12\hat{i} - 3\hat{j} + 4\hat{k}}{13}$

$= \dfrac{12\hat{i}}{13} - \dfrac{3}{13}\hat{j} + 4\hat{k}$

A vector normal to the plane with magnitude

$26 = 26\hat{n}$

$= 26\left(\dfrac{12\hat{i}}{13} - 3\hat{j} + 4\hat{k}\right)$

Required vector $= 24\hat{i} - 6\hat{j} + 8\hat{k}$

Question 16

If the line drawn from $(4, -1, 2)$ meets a plane at right angles at the point $(-10, 5, 4)$ find the equation of the plane.

Solution 16

Given that, line drawn from $A(4,-1,2)$ meets a plane at right angle, at the point $B(-10,5,4)$.

We know that,
Equation of a plane passing through the point $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(\text{i})$$

Here, $\vec{a}$ = Position vector B

$$\vec{a}=-10\hat{i}+5\hat{j}+4\hat{k}$$

$$\begin{aligned}\vec{n} &= \overrightarrow{AB}\\ &= \text{Position vector of } B - \text{Position vector of } A\\ &= \left(-10\hat{i}+5\hat{j}+4\hat{k}\right)-\left(4\hat{i}-\hat{j}+2\hat{k}\right)\\ &= -10\hat{i}+5\hat{j}+4\hat{k}-4\hat{i}+\hat{j}-2\hat{k}\end{aligned}$$

$$\vec{n}=-14\hat{i}+6\hat{j}+2\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(-10\hat{i}+5\hat{j}+4\hat{k}\right)\right].\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)=0$$

$$\vec{r}.\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-\left(-10\hat{i}+5\hat{j}+4\hat{k}\right)\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)=0$$

$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-\left[(-10)(-14)+(5)(6)+(4)(2)\right]=0$$

$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-\left[140+30+8\right]=0$$

$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)-178=0$$

$$\vec{r}\left(-14\hat{i}+6\hat{j}+2\hat{k}\right)=178$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 178$$

$$(x)(-14) + (y)(6) + (z)(2) = 178$$

$$-14x + 6y + 2z = 178$$

Dividing both the sides by (-2), we get

$$\frac{-14x}{-2} + \frac{6y}{-2} + \frac{2z}{-2} = \frac{178}{-2}$$

$$7x - 3y - z = -89$$

The equation of required plane is,

$$7x - 3y - z + 89 = 0$$

Question 17

Find the equation of the plane which bisects the line joining the points $(-1, 2, 3)$ and $(3, -5, 6)$ at right angles.

Solution 17

We have to find the equation of plane which bisects the line joining the points $A(-1,2,3)$ and $B(3,-5,6)$ at right angles.

Let, C be the mid-point of AB

We know that, equation of a plane passing through a point $\bar{a}$ and perpendicular to a vector $\bar{n}$ is given by,

$$(\bar{r}-\bar{a}).\bar{n}=0 \qquad ---(i)$$

Here, $\bar{a}$ = Position vector of C

= Mid-point of A and B

$$= \frac{\text{Position vector of } A + \text{Position vector of } B}{2}$$

$$\bar{a} = \frac{-\hat{i}+2\hat{j}+3\hat{k} + 3\hat{i}-5\hat{j}+6\hat{k}}{2}$$

$$= \frac{2\hat{i}}{2} - \frac{3\hat{j}}{2} + \frac{9\hat{k}}{2}$$

$$\bar{a} = \hat{i} - \frac{3}{2}\hat{j} + \frac{9}{2}\hat{k}$$

$\bar{n} = \overline{AB}$

= Position vector of B – Position vector of A

$$= \frac{\left(3\hat{i}-5\hat{j}+6\hat{k}\right)-\left(-\hat{i}+2\hat{j}+3\hat{k}\right)}{2}$$

$$= \frac{3\hat{i}-5\hat{j}+6\hat{k}+\hat{i}-2\hat{j}-3\hat{k}}{2}$$

$$= \frac{4\hat{i}-7\hat{j}+3\hat{k}}{2}$$

$$= \frac{4}{2}\hat{i} - \frac{7}{2}\hat{j} + \frac{3}{2}\hat{k}$$

$$\bar{n} = 2\hat{i} - \frac{7}{2}\hat{j} + \frac{3}{2}\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\right]\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[(1)(2)+\left(-\frac{3}{2}\right)\left(-\frac{7}{2}\right)+\left(\frac{9}{2}\right)\left(+\frac{3}{2}\right)\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[2+\frac{21}{4}+\frac{27}{4}\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[\frac{29+27}{4}\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\frac{56}{4}=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-14=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-14=0$$

$$(x)(2)+(y)\left(-\frac{7}{2}\right)+(z)\left(+\frac{3}{2}\right)-14=0$$

$$2x-\frac{7y}{2}+\frac{3z}{2}-14=0$$

$$\frac{4x-7y+3z-28}{2}=0$$

$$4x-7y+3z=28$$

Equation of required plane is,

$$4x-7y+3z=28$$

Question 18

Find the vector and Cartesian equations of the plane which passes through the point (5, 2, -4) and perpendicular to the line with direction ratios 2, 3, -1.

Solution 18

Vector equation of the plane:

Given that the required plane passes through the point $(5, 2, -4)$ having the position vector

$\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$

Also given that the required plane is perpendicular to the line with direction ratios 2, 3 and -1.

Thus the vector equation of the normal vector to the plane is $\vec{n} = 2\hat{i} + 3\hat{j} - \hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = (5\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 10 + 6 + 4$

$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$

The Cartesian equation of the plane is

$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 2$

$\Rightarrow 2x + 3y - z = 20$

Question 19

If *O* be the origin and the coordinates of *P* be (1, 2, -3), then find the equation of the plane passing through *P* and perpendicular to *OP*.

Solution 19

Consider the point $P(1, 2, -3)$.

Thus the position vector of the point P is

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

Direction ratios of the line OP, where O is the origin, are 1,2 and -3

Thus the vector equation of the normal vector, OP, to the plane is $\vec{n} = \hat{i} + 2\hat{j} - 3\hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$\vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = (\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$

$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 1 + 4 + 9$

$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$

$\Rightarrow x + 2y - 3z = 14$

Question 20

If O is the origin and the coordinates of A are (a, b, c). Find the direction cosines of OA and the equation of the plane through A at right angles to OA.

Solution 20

O is the origin and the coordinates of A are (a, b, c).

$\overrightarrow{OA} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore$ The direction the direction ratios of OA are proportional to, a, b, c.

$\therefore$ Direction cosines are,

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

The equation of the line passing through A(a, b, c) and perpendicular to $\overrightarrow{OA}$ is,

$$\left\{x\hat{i} + y\hat{j} + z\hat{k} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \bullet a\hat{i} + b\hat{j} + c\hat{k} = 0$$

$$ax + by + cz = a^2 + b^2 + c^2$$

Chapter 29 - The plane Exercise Ex. 29.4

Question 1

Find the vector equation of a plane which is at a distance of 3 units from the origin and has $\hat{k}$ as the unit vector normal to it.

Solution 1

Here, it is given that, the required plane is at a distanceof 3 unit from origin and k is unit vector normal to it. We know that, vector equation of a plane normal to unit vector $\hat{n}$ and at distance d from origin, is

$$\vec{r}.\hat{n} = d$$

So, here d = 3 unit

$\hat{n} = \hat{k}$

The equation of the required plane is,

$$\vec{r}.\hat{k} = 3$$

Question 2

Find the vector equation of a plane which is at a distance of 5 units from the origin and which is normal to the vector $\hat{i} - 2\hat{j} - 2\hat{k}$.

Solution 2

We know that, vector equation of a plane which is at a distance d unit from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(i)$$

Here, d = 5 unit

$\vec{n} = \hat{i} - 2\hat{j} - 2\hat{k}$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$\hat{n} = \frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)$$

Put, value of d and $\hat{n}$ in equation (i),

The equation of required plane is,

$$\vec{r}.\frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$$

Question 3

Reduce the equation $2x - 3y - 6z = 14$ to the normal form and hence find the length of perpendicular from the origin to the plane. Also, find the direction cosines of the normal to the plane.

Solution 3

Given equation of plane is,

$$2x - 3y - 6z = 14$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right) = 14$$

Dividing the equation by $\sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$$\vec{r}.\frac{\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)}{\sqrt{4+9+36}} = \frac{14}{\sqrt{4+9+36}}$$
$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = \frac{14}{7}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2 \qquad ---(\text{i})$$

We know that the vector equation of a plane with distance d from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(\text{ii})$$

Comparing (i) and (ii),
$d = 2$ and
$$\hat{n} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

So, distance of plane from origin = 2 unit
Direction cosine of normal to plane = $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

Question 4

Reduce the equation $\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) + 6 = 0$ to normal form and hence find the length of perpendicular from the origin to the plane.

Solution 4

Given equation of plane is,

$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) + 6 = 0$$
$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = -6$$

Multiplying both the sides by (-1),

$$\vec{r}.\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = 6$$
$$\vec{r}.\vec{n} = 6 \qquad ---(i)$$

Here, $\vec{n} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$|\vec{n}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2}$$
$$= \sqrt{1+4+4}$$
$$= \sqrt{9}$$
$$= 3$$

Dividing equation (i) by $|\vec{n}| = 3$ both the sides,

$$\vec{r}.\frac{\vec{n}}{|\vec{n}|} = \frac{6}{|\vec{n}|}$$
$$\vec{r}.\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = \frac{6}{3}$$
$$\vec{r}\left(-\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2 \qquad ---(ii)$$

We know that, equation of a plane at distance d from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing equation (ii) and (iii),

$d = 2$

$$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

So, normal form of equation of plane is,

$$\vec{r}.\left(-\frac{1}{3}\hat{i}+\frac{2}{3}\hat{j}-\frac{2}{3}\hat{k}\right)=2$$

Length of perpendicular from origin to plane
d = 2 unit

Question 5
Write the normal form of the equation of the plane $2x - 3y + 6z + 14 = 0$.

Solution 5
Given equation of plane is,

$$2x - 3y + 6z + 14 = 0$$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}-3\hat{j}+6\hat{k}\right)=-14$$
$$\vec{r}.\left(2\hat{i}-3\hat{j}+6\hat{k}\right)=-14$$

Multiplying by (-1) both the sides,

$$\vec{r}.\left(-2\hat{i}+3\hat{j}-6\hat{k}\right)=14 \qquad ---(\text{i})$$

So, $\vec{r}.\vec{n} = 14$

$$\vec{n}=-2\hat{i}+3\hat{j}-6\hat{k}$$
$$\left|\vec{n}\right|=\sqrt{(-2)^2+(3)^2+(-6)^2}$$
$$=\sqrt{4+9+36}$$
$$=\sqrt{49}$$

$$\left|\vec{n}\right|=7$$

Dividing equation (i) by $\left|\vec{n}\right| \Rightarrow$ both the sides,

$$\vec{r}.\frac{\left(-2\hat{i}+3\hat{j}-6\hat{k}\right)}{7}=\frac{14}{7}$$

$$\vec{r}.\left(-\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=2$$

$$-\frac{2}{7}x+\frac{3}{7}y-\frac{6}{7}z=2$$

Question 6

The direction ratios of the perpendicular from the origin to a plane are 12, –3, 4 and the length of the perpendicular is 5. find the equation on the plane.

Solution 6

Given, direction ratios of perpendicular from origin to a plane is 12, – 3, 4

So,

Normal vector = $12\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$$

$$= \sqrt{144 + 9 + 16}$$

$$= \sqrt{169}$$

$$\left|\vec{n}\right| = 13$$

Normal unit vector $\hat{n} = \dfrac{\vec{n}}{\left|\vec{n}\right|}$

$$= \frac{1}{13}\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

Given that, perpendicular distance of plane from origin is 5 unit.

$\Rightarrow \quad d = 5$ unit

We know that, equation of a plane at a distance d from origin and normal unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d$$

So, vector equation of required plane is

$$\vec{r}.\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$

$$(x)\left(\frac{12}{13}\right) + (y)\left(-\frac{3}{13}\right) + (z)\left(\frac{4}{13}\right) = 5$$

$$\frac{12}{13}x - \frac{3}{13}y + \frac{4}{13}z = 5$$

Question 7

Find the unit normal vector to the plane $x + 2y + 3z - 6 = 0$.

Solution 7

Given equation of plane is

$$x + 2y + 3z - 6 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 6 = 0$$
$$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6 \qquad ---(i)$$

$$\vec{r}.\vec{n} = 6$$

So, $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$
$$\left|\vec{n}\right| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{1 + 4 + 9}$$

$$\left|\vec{n}\right| = \sqrt{14}$$

Dividing equation (i) by $\sqrt{14}$, we get

$$\vec{r}.\left(\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}\right) = \frac{6}{\sqrt{14}} \qquad ---(ii)$$

We know that, vector equation of a plane at distance d unit from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing (ii) and (iii), we get

Normal unit vector $= \hat{n} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

Question 8

Find the equation of a plane which is at a distance of $3\sqrt{3}$ units from the origin and the normal to which is equally inclined with the coordinate axes.

Solution 8

We know that, vector equation of a plane which is at a distance d from origin and normal to unit vector $\hat{n}$ is given by

$\vec{r}.\hat{n} = d$ $\quad ---(\text{i})$

Here, given $\vec{d} = 3\sqrt{3}$ unit.

Let, $\vec{a} = \left(p\hat{i} + q\hat{j} + r\hat{k}\right)$

Where $\vec{a}$ is normal vector.

Given that, $\vec{a}$ is equally inclined to the coordinate axes

If l, m, n are direction cosines of $\vec{n}$,

Here, $l = m = n$ $\quad ---(\text{ii})$

We know that,

$l^2 + m^2 + n^2 = 1$

$l^2 + l^2 + l^2 = 1$ $\quad \left[\text{Using (ii)}\right]$

$3l^2 = 1$

$l = \dfrac{1}{\sqrt{3}}$

So, $l = m = n = \dfrac{1}{\sqrt{3}}$

Here,

$$l = \frac{p}{|\bar{a}|} = \frac{1}{\sqrt{3}}$$

$$m = \frac{q}{|\bar{a}|} = \frac{1}{\sqrt{3}}$$

$$n = \frac{r}{|\bar{a}|} = \frac{1}{\sqrt{3}}$$

Now,

$$\bar{a} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

$$= \frac{p}{|\bar{a}|}\hat{i} + \frac{q}{|\bar{a}|}\hat{j} + \frac{r}{|\bar{a}|}\hat{k}$$

$$\hat{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Put the value of $d = 3\sqrt{3}$ unit and $\hat{n} = \hat{a} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ in equation (i), vector equation of the required plane is

$$\vec{r}.\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) = 3\sqrt{3}$$

$$\vec{r}\left(\hat{i} + \hat{j} + \hat{k}\right) = 9$$

$$x + y + z = 9$$

Question 9

Find the equation of the plane passing through the point $(1,2,1)$ and perpendicular to the line joining the points $(1,4,2)$ and $(2,3,5)$. Find also the perpendicular distance of the origin from this plane.

Solution 9

Here, we have to find equating a plane passing through $A(1,2,1)$ and perpendicular to line joining $B(1,4,2)$ and $C(2,3,5)$.

We know that, the vector equation of a plane passing through a point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$

$\vec{n}=\overrightarrow{BC}$

$= \text{Position vector of } C - \text{Position vector of } B$

$=\left(2\hat{i}+3\hat{j}+5\hat{k}\right)-\left(\hat{i}+4\hat{j}+2\hat{k}\right)$

$=2\hat{i}+3\hat{j}+5\hat{k}-\hat{i}-4\hat{j}-2\hat{k}$

$\vec{n}=\hat{i}-\hat{j}+3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

Vector equation of plane is

$$\left[\vec{r}-\left(\hat{i}+2\hat{j}+\hat{k}\right)\right].\left(\hat{i}-\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left(\hat{i}+2\hat{j}+\hat{k}\right)\left(\hat{i}-\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left[(1)(1)+(2)(-1)+(1)(3)\right]=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left[1-2+3\right]=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-(4-2)=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-2=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)=2 \qquad ---(ii)$$

$$|\vec{n}| = \sqrt{(1)^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{1+1+9}$$
$$= \sqrt{11}$$

Dividing equation (i) by $\sqrt{11}$,

$$\vec{r}.\left(\frac{1}{\sqrt{11}}\hat{i} - \frac{1}{\sqrt{11}}\hat{j} + \frac{3}{\sqrt{11}}\hat{k}\right) = \frac{2}{\sqrt{11}}$$

$$\vec{r}.\hat{n} = d$$

So, perpendicular distance of plane from origin = $\frac{2}{\sqrt{11}}$ units

Equation of plane, $\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 2$

Equation of plane, $x - y + 3z - 2 = 0$

Question 10

Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also, find its cartesian form.

Solution 10

We know that the vector equation of a plane at a distance

'p' from the origin and normal to the unit vector $\hat{n}$ is $\vec{r}\cdot\hat{n} = p$

Vector normal to the plane is $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

The unit vector normal to the plane is

$$\hat{n} = \frac{2}{\sqrt{2^2+(-3)^2+4^2}}\hat{i} - \frac{3}{\sqrt{2^2+(-3)^2+4^2}}\hat{j} + \frac{4}{\sqrt{2^2+(-3)^2+4^2}}\hat{k}$$

$$\Rightarrow \hat{n} = \frac{2}{\sqrt{4+9+16}}\hat{i} - \frac{3}{\sqrt{4+9+16}}\hat{j} + \frac{4}{\sqrt{4+9+16}}\hat{k}$$

$$\Rightarrow \hat{n} = \frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$$

Here, given that $p = \frac{6}{\sqrt{29}}$

Thus, the vector equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

The Cartesian equation of the plane is

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow \left(\frac{2x}{\sqrt{29}} - \frac{3y}{\sqrt{29}} + \frac{4z}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow \left(\frac{2x-3y+4z}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow 2x - 3y + 4z = 6$$

Question 11

Find the distance of the plane 2x - 3y + 4z - 6 = 0 from the origin.

Solution 11

The Cartesian equation of the given plane is

$2x - 3y + 4z - 6 = 0.$

The above equation can be rewritten as

$2x - 3y + 4z = 6$

Therefore, the vector equation of the plane is

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 6$$

$$\Rightarrow \vec{r}\cdot\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 6 \ldots.(1)$$

We know that the vector equation of a plane at a distance

'p' from the origin and normal to unit vector $\hat{n}$ is $\vec{r}\cdot\hat{n} = p$

We have, $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}.$

Thus $|\vec{n}| = \sqrt{2^2+(-3)^2+4^2} = \sqrt{29}$

Dividing the equation (1) by $|\vec{n}| = \sqrt{29}$, we have,

$$\vec{r}\cdot\left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

Hence the normal form of the equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

Hence the perpendicular distance of the

origin from the plane is $p = \frac{6}{\sqrt{29}}$.

Chapter 29 - The plane Exercise Ex. 29.5

Question 1

Find the vector equation of the plane passing through the points $(1,1,1)$, $(1,-1,1)$ and $(-7,-3,-5)$.

Solution 1

Given that, plane is passing through

$(1,1,1)$, $(1,-1,1)$ and $(-7,-3,-5)$

We know that, equation of plane passing through 3 points,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 1-1 \\ -7-1 & -3-1 & -5-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 0 \\ -8 & -4 & -6 \end{vmatrix} = 0$$

$$(x-1)(12-0)-(y-1)(0-0)+(z-1)(0-16)=0$$
$$(x-1)(12)-(y-1)(0)+(z-1)(-16)=0$$
$$12x-12-0-16z+16=0$$

$$12x-16z+4=0$$

Dividing by 4,

$$3x-4z+1=0$$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(3\hat{i}+0\hat{j}-4\hat{k}\right)+1=0$$

$$\vec{r}.\left(3\hat{i}-4\hat{k}\right)+1=0$$

Equation of the required plane,

$$\vec{r}.\left(3\hat{i}-4\hat{k}\right)+1=0$$

Question 2

Find the vector equation of the plane passing through the points *P*(2, 5, -3), *Q*(-2, -3, 5) and *R*(5, 3, -3).

Solution 2

Let $P(2,5,-3)$, $Q(-2,-3,5)$ and $R(5,3,-3)$ be the three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.
Therefore, $\overrightarrow{PQ} \times \overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$

$\overrightarrow{PQ} = (-2-2)\hat{i} + (-3-5)\hat{j} + (5-(-3))\hat{k}$
$\Rightarrow \overrightarrow{PQ} = -4\hat{i} - 8\hat{j} + 8\hat{k}$
Similarly,
$\overrightarrow{PR} = (5-2)\hat{i} + (3-5)\hat{j} + (-3-(-3))\hat{k}$
$\Rightarrow \overrightarrow{PR} = 3\hat{i} - 2\hat{j} + 0\hat{k}$
Thus

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 16\hat{i} + 24\hat{j} + 32\hat{k}$$

The plane passes through the point P with position vector $\vec{p} = 2\hat{i} + 5\hat{j} - 3\hat{k}$
Thus, its vector equation is

$$\{\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})\} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) = 0$$
$$\Rightarrow \vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) - (32 + 120 - 96) = 0$$
$$\Rightarrow \vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) - 56 = 0$$
$$\Rightarrow \vec{r} \cdot (16\hat{i} + 24\hat{j} + 32\hat{k}) = 56$$
$$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7$$

Question 3

Find the vector equation of the plane passing through the points $A(a,0,0), B(0,b,0)$ and $C(0,0,c)$.
Reduce it to normal form.
If plane ABC is at a distance p from the origin, prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

Solution 3

Let $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,c)$ be three points on a plane having their position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are in the same plane.

Therefore, $\overrightarrow{AB} \times \overrightarrow{AC}$ is a vector perpendicular to the plane.

Let $\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$

$$\overrightarrow{AB} = (0-a)\hat{i} + (b-0)\hat{j} + (0-0)\hat{k}$$

$$\Rightarrow \overrightarrow{AB} = -a\hat{i} + b\hat{j} + 0\hat{k}$$

Similarly,

$$\overrightarrow{AC} = (0-a)\hat{i} + (0-0)\hat{j} + (c-0)\hat{k}$$

$$\Rightarrow \overrightarrow{AC} = -a\hat{i} + 0\hat{j} + c\hat{k}$$

Thus

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}$$

$$\vec{n} = bc\hat{i} + ac\hat{j} + ab\hat{k}$$

$$\Rightarrow \hat{n} = \frac{bc\hat{i} + ac\hat{j} + ab\hat{k}}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}$$

The plane passes through the point P with position vector $\vec{a} = a\hat{i} + 0\hat{j} + 0\hat{k}$

Thus, the vector equation in the normal form is

$$\{\vec{r} - \left((a\hat{i} + 0\hat{j} + 0\hat{k}\right)\} \cdot \left(\frac{bc\hat{i} + ac\hat{j} + ab\hat{k}}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}\right) = 0$$

$$\Rightarrow \vec{r} \cdot \frac{(bc\hat{i} + ac\hat{j} + ab\hat{k})}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}} = \frac{abc}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}}$$

$$\Rightarrow \vec{r} \cdot \frac{(bc\hat{i} + ac\hat{j} + ab\hat{k})}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}} = \frac{1}{\sqrt{\frac{b^2c^2 + a^2c^2 + a^2b^2}{a^2b^2c^2}}}$$

$$\Rightarrow \vec{r} \cdot \frac{(bc\hat{i} + ac\hat{j} + ab\hat{k})}{\sqrt{b^2c^2 + a^2c^2 + a^2b^2}} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \ldots(1)$$

The vector equation of a plane normal to the unit vector $\hat{n}$ and at a distance 'd' from the origin is $\vec{r} \cdot \hat{n} = d \ldots(2)$

Given that the plane is at a distance 'p' from the origin.

Comparing equations (1) and (2), we have,

$$d = p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Question 4

Find the vector equation of the plane passing through the points (1, 1, -1), (6, 4, -5) and (-4, -2, 3).

Solution 4

Let $P(1, 1, -1)$, $Q(6, 4, -5)$ and $R(-4, -2, 3)$ be three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.
Therefore, $\overrightarrow{PQ} \times \overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$

$\overrightarrow{PQ} = (6-1)\hat{i} + (4-1)\hat{j} + (-5-(-1))\hat{k}$
$\Rightarrow \overrightarrow{PQ} = 5\hat{i} + 3\hat{j} - 4\hat{k}$
Similarly,
$\overrightarrow{PR} = (-4-1)\hat{i} + (-2-1)\hat{j} + (3-(-1))\hat{k}$
$\Rightarrow \overrightarrow{PR} = -5\hat{i} - 3\hat{j} + 4\hat{k}$
Thus
Here, $\overrightarrow{PQ} = -\overrightarrow{PR}$
Therefore, the given points are collinear.
Thus, $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$ where, $5a + 3b - 4c = 0$
The plane passes through the point P with position vector $\vec{p} = \hat{i} + \hat{j} - \hat{k}$
Thus, its vector equation is
$\{\vec{r} - (\hat{i} + \hat{j} - \hat{k})\} \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = 0$, where, $5a + 3b - 4c = 0$

Question 5

Find the vector equation of the plane passing through the points

$3\hat{i} + 4\hat{j} + 2\hat{k}$, $2\hat{i} - 2\hat{j} - \hat{k}$ and $7\hat{i} + 6\hat{k}$.

Solution 5

Let, A, B, C be the points with position vector $\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$, $\left(2\hat{i}-2\hat{j}-\hat{k}\right)$ and $\left(7\hat{i}+6\hat{k}\right)$ respectively. Then

$\overrightarrow{AB}$ = Position vector of B – Poosition vector of A

$$= \left(2\hat{i}-2\hat{j}-\hat{k}\right)-\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$$
$$= 2\hat{i}-2\hat{j}-\hat{k}-3\hat{i}-4\hat{j}-2\hat{k}$$
$$= -\hat{i}-6\hat{j}-3\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Poosition vector of B

$$= \left(7\hat{i}+6\hat{k}\right)-\left(2\hat{i}-2\hat{j}-\hat{k}\right)$$
$$= 7\hat{i}+6\hat{k}-2\hat{i}+2\hat{j}+\hat{k}$$

$$\overrightarrow{BC} = 5\hat{i}+2\hat{j}+7\hat{k}$$

A vector normal to A, B, C is a vector perpendicular to $\overrightarrow{AB}\times\overrightarrow{BC}$

$$\vec{n} = \overrightarrow{AB}\times\overrightarrow{BC}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -6 & -3 \\ 5 & 2 & 7 \end{vmatrix}$$

$$\vec{n} = \hat{i}\left(-42+6\right)-\hat{j}\left(-7+15\right)+\hat{k}\left(-2+30\right)$$

$$= -36\hat{i}-8\hat{j}+28\hat{k}$$

We know that, equation of a plane passing through vector $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right) = \left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right)$$

$$= (3)(-36) + (4)(-8) + (2)(28)$$
$$= -108 - 32 + 56$$
$$= -140 + 56$$

$$\vec{r}.\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right) = -84$$

Dividing by (-4), we get

$$\vec{r}.\left(9\hat{i} + 2\hat{j} - 7\hat{k}\right) = 21$$

Equation of required plane is,

$$\vec{r}.\left(9\hat{i} + 2\hat{j} - 7\hat{k}\right) = 21$$

Chapter 29 - The plane Exercise Ex. 29.6

Question 1(i)

Find the angle between the planes:

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$ and $\vec{r}.\left(-\hat{i} + \hat{j}\right) = 4$

Solution 1(i)

Given equation of two planes are

$$\vec{r}.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1 \qquad ---(i)$$

$$\vec{r}.\left(-\hat{i} + \hat{j}\right) = 4 \qquad ---(ii)$$

We know that, angle between two planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \qquad ---(iii)$$

Here, $\vec{n_1} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{n_2} = -\hat{i} + \hat{j}$

$$\cos\theta = \frac{\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)\left(-\hat{i} + \hat{j}\right)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2}}$$

$$= \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{4+9+16}\sqrt{1+1}}$$

$$= \frac{-2-3+0}{\sqrt{29}\sqrt{2}}$$

$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

Question 1(ii)

Find the angle between the planes:

$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 6$ and $\vec{r}.\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right) = 9$

Solution 1(ii)

Given equation of planes are

$$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 6 \qquad ---(i)$$

$$\vec{r}.\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right) = 9 \qquad ---(ii)$$

We know that, angle between the planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \qquad ---(iii)$$

Here, from equation (i) and (ii),

$\vec{n_1} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{n_2} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta = \frac{\left(2\hat{i} - \hat{j} + 2\hat{k}\right)\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)}{\sqrt{(2)^2 + (-1)^2 + (2)^2}\sqrt{(3)^2 + (6)^2 + (-2)^2}}$$

$$= \frac{(2)(3) + (-1)(6) + (2)(-2)}{\sqrt{4 + 1 + 4}\sqrt{9 + 36 + 4}}$$

$$\cos\theta = \frac{6 - 6 + 4}{\sqrt{9}\sqrt{49}}$$

$$= \frac{-4}{3.7}$$

$$= \frac{-4}{21}$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

Question 1(iii)

Find the angle between the planes:

$\vec{r}.\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right) = 5$ and $\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = 9$

Solution 1(iii)

Given equation of planes are

$$\vec{r}.\left(2\hat{i}+3\hat{j}-6\hat{k}\right)=5 \qquad ---(i)$$
$$\vec{r}.\left(\hat{i}-2\hat{j}+2\hat{k}\right)=9 \qquad ---(ii)$$

We know that, angle between equation of planes

$\vec{r}.\overrightarrow{n_1}=d_1$ and $\vec{r}.\overrightarrow{n_2}=d_2$ is given by,

$$\cos\theta=\frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{\left|\overrightarrow{n_1}\right|\left|\overrightarrow{n_2}\right|} \qquad ---(iii)$$

From equation (i) and (ii),

$\overrightarrow{n_1}=2\hat{i}+3\hat{j}-6\hat{k}$

$\overrightarrow{n_2}=\hat{i}-2\hat{j}+2\hat{k}$

Put $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ in equation (iii),

$$\cos\theta=\frac{\left(2\hat{i}+3\hat{j}-6\hat{k}\right)\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{\sqrt{(2)^2+(3)^2+(-6)^2}\sqrt{(1)^2+(-2)^2+(2)^2}}$$
$$=\frac{(2)(1)+(3)(-2)+(-6)(2)}{\sqrt{4+9+36}\sqrt{1+4+4}}$$
$$=\frac{2-6-12}{\sqrt{49}\sqrt{9}}$$
$$=\frac{-16}{7.3}$$
$$=\frac{-16}{21}$$

$$\theta=\cos^{-1}\left(\frac{-16}{21}\right)$$

Question 2(i)

Find the angle between the planes:

$2x-y+z=4$ and $x+y+2z=3$

Solution 2(i)

Given, equation of planes are,

$$2x - y + z = 4 \qquad ---(i)$$
$$x + y + 2z = 3 \qquad ---(ii)$$

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(iii)$$

Here, from equation (i) and (ii),
$a_1 = 2,\ b_1 = -1,\ c_1 = 1$
$a_2 = 1,\ b_2 = 1,\ c_2 = 2$

Put then values in equation (iii),

$$\cos\theta = \frac{(2)(1) + (-1)(1) + (1)(2)}{\sqrt{(2)^2 + (-1)^2 + (1)^2}\sqrt{(1)^2 + (1)^2 + (2)^2}}$$
$$= \frac{2 - 1 + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}$$
$$\cos\theta = \frac{4 - 1}{\sqrt{6}\sqrt{6}}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$
$$\theta = \frac{\pi}{3}$$

Question 2(ii)

Find the angle between the planes:

$x + y - 2z = 3$ and $2x - 2y + z = 5$

Solution 2(ii)

Given, equation of two planes are,

$$x + y - 2z = 3 \qquad ---(i)$$
$$2x - 2y + z = 5 \qquad ---(ii)$$

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),
$a_1 = 1, b_1 = 1, c_1 = -2$
$a_2 = 2, b_2 = -2, c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(2) + (1)(-2) + (-2)(1)}{\sqrt{(1)^2 + (1)^2 + (-2)^2}\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$
$$\cos\theta = \frac{2 - 2 - 2}{\sqrt{1 + 1 + 4}\sqrt{4 + 4 + 1}}$$
$$= \frac{-2}{\sqrt{6}\sqrt{9}}$$
$$= \frac{-2}{3\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{-2}{3\sqrt{6}}\right)$$

Question 2(iii)

Find the angle between the planes:

$x - y + z = 5$ and $x + 2y + z = 9$

Solution 2(iii)

Given, equation of planes are,

$$x - y + z = 5 \qquad ---(i)$$
$$x + 2y + z = 9 \qquad ---(ii)$$

We know that, angle between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(iii)$$

From equation (i) and (ii),
$a_1 = 1,\ b_1 = -1,\ c_1 = 1$
$a_2 = 1,\ b_2 = 2,\ c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(1) + (-1)(2) + (1)(1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(1)^2 + (2)^2 + (1)^2}}$$
$$\cos\theta = \frac{1 - 2 + 1}{\sqrt{1+1+1}\sqrt{1+4+1}}$$
$$= \frac{0}{\sqrt{3}\sqrt{6}}$$
$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$
$$\theta = \frac{\pi}{2}$$

Question 2(iv)

Find the angle between the planes:

$2x - 3y + 4z = 1$ and $-x + y = 4$

Solution 2(iv)

Given, equation of planes are,

$$2x - 3y + 4z = 1 \qquad ---(i)$$
$$-x + y = 4 \qquad ---(ii)$$

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),
$a_1 = 2,\ b_1 = -3,\ c_1 = 4$
$a_2 = -1,\ b_2 = 1,\ c_2 = 0$

Put these values in equation (iii),

$$\cos\theta = \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2 + (0)^2}}$$
$$\cos\theta = \frac{-2 - 3 + 0}{\sqrt{4 + 9 + 16}\sqrt{1 + 1 + 0}}$$
$$= \frac{-5}{\sqrt{29}\sqrt{2}}$$
$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

Question 2(v)

Find the angle between the planes:
2x + y - 2z = 5 and 3x - 6y - 2z = 7

Solution 2(v)

We know that the angle between the planes
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Therefore, the angle between $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$

$$\cos\theta = \frac{2 \times 3 + 1 \times (-6) + (-2) \times (-2)}{\sqrt{2^2 + 1^2 + (-2)^2} \cdot \sqrt{3^2 + (-6)^2 + (-2)^2}}$$

$$\Rightarrow \cos\theta = \frac{6 - 6 + 4}{\sqrt{9} \cdot \sqrt{9 + 36 + 4}}$$

$$\Rightarrow \cos\theta = \frac{4}{3 \times 7}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

Question 3(i)

Show that the following planes are at right angles:

$\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 5$ and $\vec{r}.\left(-\hat{i} - \hat{j} + \hat{k}\right) = 3$

Solution 3(i)

Given equation of planes are

$$\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 5 \qquad --- (i)$$
$$\vec{r}.\left(-\hat{i} - \hat{j} + \hat{k}\right) = 3 \qquad --- (ii)$$

We know that, planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ are perpendicular

if $\vec{n_1}.\vec{n_2} = 0 \qquad --- (iii)$

From equation (i) and (ii),

$$\vec{n_1} = 2\hat{i} - \hat{j} + \hat{k}$$
$$\vec{n_2} = -\hat{i} - \hat{j} + \hat{k}$$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\vec{n_1}.\vec{n_2} = 0$$
$$\left(2\hat{i} - \hat{j} + \hat{k}\right)\left(-\hat{i} - \hat{j} + \hat{k}\right) = 0$$
$$(2)(-1) + (-1)(-1) + (1)(1) = 0$$
$$-2 + 1 + 1 = 0$$

$$0 = 0$$
$$LHS = RHS$$

Hence, planes are at right angle

Question 3(ii)

Show that the following planes are at right angles:

$x - 2y + 4z = 10$ and $18x + 17y + 4z = 49$

Solution 3(ii)

Given, equation of planes are,

$x - 2y + 4z = 10$
$18x + 17y + 4z = 49$

$\Rightarrow \quad x - 2y + 4z - 10 = 0 \qquad ---(i)$
$\quad\quad 18x + 17y + 4z - 49 = 0 \qquad ---(ii)$

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are at right angles if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

From (i) and (ii),
$a_1 = 1,\ b_1 = -2,\ c_1 = 4$
$a_2 = 18,\ b_2 = 17,\ c_2 = 4$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(1)(18) + (-2)(17) + (4)(4) = 0$
$18 - 34 + 16 = 0$

$0 = 0$
$LHS = RHS$

Hence, planes are at right angles

Question 4(i)

Determine the value of λ for which the following planes are perpendicular to each other.

$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 7$ and $\vec{r}.\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 26$

Solution 4(i)

Here, given equation of planes are

$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 7$ --- (i)

$\vec{r}.\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 26$ --- (ii)

We know that, planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ are perpendicular if

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$ --- (iii)

From equation (i) and (ii), we get

$\overrightarrow{n_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{n_2} = \lambda\hat{i} + 2\hat{j} - 7\hat{k}$

Since, (i) and (ii) are perpendicular, so from (iii),

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$

$\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$

$(1)(\lambda) + (2)(2) + (3)(-7) = 0$

$\lambda + 4 - 21 = 0$

$\lambda - 17 = 0$

$\lambda = 17$

Question 4(ii)

Determine the value of λ for which the following planes are perpendicular to each other.

$2x - 4y + 3z = 5$ and $x + 2y + \lambda z = 5$

Solution 4(ii)

Given, that plane $2x - 4y + 3z - 5 = 0$ ---(i)
and $x + 2y + \lambda z - 5 = 0$ are ---(ii) perpendicular.

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

From equation (i) and (ii),
$a_1 = 2,\ b_1 = -4,\ c_1 = 3$
$a_2 = 1,\ b_2 = 2,\ c_2 = \lambda$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(2)(1) + (-4)(2) + (3)(\lambda) = 0$
$2 - 8 + 3\lambda = 0$
$-6 + 3\lambda = 0$
$3\lambda = 6$
$\lambda = \frac{6}{3}$

$\lambda = 2$

Question 4(iii)
Determine the value of λ for which the following planes are perpendicular to each other.

$3x - 6y - 2z = 7$ and $2x + y - \lambda z = 5$

Solution 4(iii)

Given, that planes

$3x - 6y - 2z - 7 = 0 \quad ---(i)$

and $2x + y - \lambda z - 5 = 0 \quad ---(ii)$

are perpendicular.

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \quad ---(iii)$

From (i) and (ii),

$a_1 = 3,\ b_1 = -6,\ c_1 = -2$

$a_2 = 2,\ b_2 = 1,\ c_2 = -\lambda$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(3)(2) + (-6)(1) + (-2)(-\lambda) = 0$

$6 - 6 + 2\lambda = 0$

$0 + 2\lambda = 0$

$2\lambda = 0$

$\lambda = 0$

Question 5

Find the equation of a plane passing through the point $(-1,-1,2)$ and perpendicular to the planes $3x + 2y - 3z = 1$ and $5x - 4y + z = 5$.

Solution 5

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given, plane is passing through $(-1, -1, 2)$,

$$a(x + 1) + b(y + 1) + c(z - 2) = 0 \qquad ---(ii)$$

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given, plane (ii) is perpendicular to plane

$$3x + 2y - 3z = 1 \qquad ---(iv)$$

So, using (ii),(iv) in (iii),

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(a)(3) + (b)(2) + (c)(-3) = 0$$

$$3a + 2b - 3c = 0 \qquad ---(v)$$

Also, plane (ii) is perpendicular to plane

$$5x - 4y + z = 5 \qquad ---(vi)$$

So, using (ii),(vi) in (iii),

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(a)(5) + (b)(-4) + (c)(1) = 0$$

$$5a - 4b + c = 0 \qquad ---(vii)$$

On solving (v) and (vii),

$$\frac{a}{(2)(1)-(-3)(-4)}=\frac{b}{(5)(-3)-(3)(1)}=\frac{c}{(3)(-4)-(2)(5)}$$

$$\frac{a}{2-12}=\frac{b}{-15-3}=\frac{c}{-12-10}$$

$$\frac{a}{-10}=\frac{b}{-18}=\frac{c}{-22}=\lambda\ (\text{Say})$$

$a=-10\lambda,\ b=-18\lambda,\ c=-22\lambda$

Put the value of a,b,c in equation (ii)

$$a(x+1)+b(y+1)+c(z-2)=0$$

$$(-10\lambda)(x+1)+(-18\lambda)(y+1)+(-22\lambda)(z-2)=0$$

$$-10\lambda x-10\lambda-18\lambda y-18\lambda-22\lambda z+44\lambda=0$$

$$-10\lambda x-18\lambda y-22\lambda z+16\lambda=0$$

Dividing by -2λ,

$5x+9y+11z-8=0$

So, equation of required plane is,

$5x+9y+11z-8=0$

Question 6

Obtain the equation of the plane passing through the point $(1,-3,-2)$ and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$.

Solution 6

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Now, equation of plane passing through $(1, -3, -2)$,

$$a(x - 1) + b(y + 3) + c(z + 2) = 0 \qquad ---(ii)$$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given, plane (ii) is perpendicular to plane

$$x + 2y + 2z = 5 \qquad ---(iv)$$

Using equation $(ii), (iv)$ in (iii),

$$(a)(1) + (b)(2) + (c)(2) = 0$$

$$a + 2b + 2c = 0 \qquad ---(v)$$

Also, plane (ii) is perpendicular to plane

$$3x + 3y + 2z = 8 \qquad ---(vi)$$

Using equation $(ii), (vi)$ in (iii),

$$(a)(3) + (b)(3) + (c)(2) = 0$$

$$3a + 3b + 2c = 0 \qquad ---(vii)$$

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(2)-(3)(2)}=\frac{b}{(3)(2)-(1)(2)}=\frac{c}{(1)(3)-(2)(3)}$$

$$\frac{a}{4-6}=\frac{b}{6-2}=\frac{c}{3-6}$$

$$\frac{a}{-2}=\frac{b}{4}=\frac{c}{-3}=\lambda\,(\text{Say})$$

$a=-2\lambda,\ b=4\lambda,\ c=-3\lambda$

Put a,b,c in equation (ii)

$$a(x-1)+b(y+3)+c(z+2)=0$$

$$(-2\lambda)(x-1)+(4\lambda)(y+3)+(-3\lambda)(z+2)=0$$

$$-2\lambda x+2\lambda+4\lambda y+12\lambda-3\lambda z-6\lambda=0$$

$$-2\lambda x+4\lambda y-3\lambda z+8\lambda=0$$

Dividing by $(-\lambda)$,

$$2x-4y+3z-8=0$$

Equation of required plane is,

$$2x-4y+3z-8=0$$

Question 7

Find the equation of the plane passing through the origin and perpendicular to each of the planes, $x+2y-z=1$ and $3x-4y+z=5$.

Solution 7

We know that equation of a plane passing through a point (x_1, y_1, z_1) is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ --- (i)

Given that, plane is passing through origin, so

$a(x - 0) + b(y - 0) + c(z - 0) = 0$

$ax + by + cz = 0$ --- (ii)

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

Given that, plane (ii) is perpendicular to plane

$x + 2y - z = 1$ --- (iv)

Using (ii), (iv) in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(2) + (c)(-1) = 0$

$a + 2b - c = 0$ --- (v)

Given, plane (ii) is perpendicular to plane

$3x - 4y + z = 5$ --- (vi)

Using equation (ii), (vi) in (iii),

$(a)(3) + (b)(-4) + (c)(1) = 0$

$3a - 4b + c = 0$ --- (vii)

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(1)-(-4)(-1)}=\frac{b}{(3)(-1)-(1)(1)}=\frac{c}{(1)(-4)-(2)(3)}$$
$$\frac{a}{2-4}=\frac{b}{-3-1}=\frac{c}{-4-6}$$
$$\frac{a}{-2}=\frac{b}{-4}=\frac{c}{-10}=\lambda\ (\text{Say})$$
$$a=-2\lambda,\ b=-4\lambda,\ c=-10\lambda$$

Put a,b,c in equation (ii)

$$ax+by+cz=0$$
$$-2\lambda x-4\lambda y-10\lambda z=0$$

Dividing by -2λ,

$$x+2y+5z=0$$

Equation of required plane is,

$$x+2y+5z=0$$

Question 8

Find the equation of the plane passing through the points $(1,-1,2)$ and $(2,-2,2)$ and which is perpendicular to the plane $6x-2y+2z=9$.

Solution 8

We know that equation of a plane passing through (x_1, y_1, z_1) is given by
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given that, plane is passing through $(1, -1, 2)$, so

$$a(x - 1) + b(y + 1) + c(z - 2) = 0 \qquad ---(i)$$

Plane (i) is also passing through $(2, -2, 2)$, so $(2, -2, 2)$ must satisfy the equation (i),

$a(2 - 1) + b(-2 + 1) + c(2 - 2) = 0$

$$a - b = 0 \qquad ---(ii)$$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given that, plane (i) is perpendicular to plane

$$6x - 2y + 2z - 9 = 0 \qquad ---(iv)$$

Using plane (i), (iv) in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(6) + (b)(-2) + (c)(2) = 0$

$$6a - 2b + 2c = 0 \qquad ---(v)$$

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-1)(2) - (-2)(0)} = \frac{b}{(6)(0) - (1)(2)} = \frac{c}{(1)(-2) - (6)(-1)}$$

$$\frac{a}{-2 + 0} = \frac{b}{0 - 2} = \frac{c}{-2 + 6}$$

$$\frac{a}{-2} = \frac{b}{-2} = \frac{c}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \qquad a = -2\lambda,\ b = -2\lambda,\ c = 4\lambda$

Put a, b, c in equation (i)

$$a(x-1)+b(y+1)+c(z-2)=0$$
$$(-2\lambda)(x-1)+(-2\lambda)(y+1)+(4\lambda)(z-2)=0$$
$$-2\lambda x+2\lambda-2\lambda y-2\lambda+4\lambda z-8\lambda=0$$

$$-2\lambda x-2\lambda y+4\lambda z-8\lambda=0$$

Dividing by (-2λ),
$$x+y-2z+4=0$$

Equation of required plane is,

$$x+y-2z+4=0$$

Question 9

Find the equation of the plane passing through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=1$.

Solution 9

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here, the plane is passing through $(2, 2, 1)$

$$a(x - 2) + b(y - 2) + c(z - 1) = 0 \qquad ---(\text{i})$$

It is also passing through $(9, 3, 6)$, so it must satisfy the equation (i),
$a(9 - 2) + b(3 - 2) + c(6 - 1) = 0$
$$7a + b + 5c = 0 \qquad ---(\text{ii})$$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(\text{iii})$

Given that, plane (i) is perpendicular to plane
$$2x + 6y + 6z = 1 \qquad ---(\text{iv})$$

Using plane $(\text{i}), (\text{iv})$ in equation (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(2) + (b)(6) + (c)(6) = 0$

$$2a + 6b + 6c = 0 \qquad ---(\text{v})$$
Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(1)(6) - (5)(6)} = \frac{b}{(2)(5) - (7)(6)} = \frac{c}{(7)(6) - (2)(1)}$$
$$\frac{a}{6 - 30} = \frac{b}{10 - 42} = \frac{c}{42 - 2}$$
$$\frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} = \lambda \text{ (Say)}$$

$\Rightarrow \qquad a = -24\lambda, b = -32\lambda, c = 40\lambda$

Put a, b, c in equation (i),

$$a(x-2)+b(y-2)+c(z-1)=0$$
$$(-24\lambda)(x-2)+(-32\lambda)(y-2)+(40\lambda)(z-1)=0$$
$$-24\lambda x+48\lambda-32\lambda y+64\lambda+40\lambda z-40\lambda=0$$

$$-24\lambda x-32\lambda y+40\lambda z+72\lambda=0$$

Dividing by (-8λ),
$$3x+4y-5z-9=0$$

Equation of required plane is,

$$3x+4y-5z=9$$

Question 10

Find the equation of the plane passing through the points whose coordinates are $(-1,1,1)$ and $(1,-1,1)$ and perpendicular to the plane $x+2y+2z=5$.

Solution 10

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given, the required plane is passing through $(-1,1,1)$,

$a(x + 1) + b(y - 1) + c(z - 1) = 0$ $\qquad ---(\text{i})$

It is also passing through $(1,-1,1)$, so it must satisfy the equation (i),

$a(1 + 1) + b(-1 - 1) + c(1 - 1) = 0$

$2a - 2b = 0$ $\qquad ---(\text{ii})$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(\text{iii})$

Given, plane (i) is perpendicular to plane

$x + 2y + 2z = 5$ $\qquad ---(\text{iv})$

Using plane $(\text{i}),(\text{iv})$ in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(2) + (c)(2) = 0$

$a + 2b + 2c = 0$ $\qquad ---(\text{v})$

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-2)(2)-(2)(0)} = \frac{b}{(1)(0)-(2)(2)} = \frac{c}{(2)(2)-(1)(-2)}$$

$$\frac{a}{-4-0} = \frac{b}{0-4} = \frac{c}{4+2}$$

$$\frac{a}{-4} = \frac{b}{-4} = \frac{c}{6} = \lambda \text{ (Say)}$$

$\Rightarrow \qquad a = -4\lambda,\ b = -4\lambda,\ c = 6\lambda$

Put the value of a, b, c in equation (i),

$a(x+1)+b(y-1)+c(z-1)=0$
$(-4\lambda)(x+1)+(-4\lambda)(y-1)+(6\lambda)(z-1)=0$
$-4\lambda x+4\lambda-4\lambda y+4\lambda+6\lambda z-6\lambda=0$

$-4\lambda x-4\lambda y+6\lambda z-6\lambda=0$

Dividing by (-2λ), we get
$2x+2y-3z+3=0$

The equation of required plane is,

$2x+2y-3z+3=0$

Question 11
Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

Solution 11
The equation of the plane parallel to ZOX is y = constant.
Given that the $y-$intercept is 3.
Thus the equation of the plane is $y = 3$.

Question 12
Find the equation of the plane that contains the point (1, -1, 2) and is perpendicular to each of the planes 2x + 3y - 2z = 5 and x + 2y - 3z = 8.

Solution 12

The equation of any plane passing through $(1, -1, 2)$ is $a(x-1)+b(y+1)+c(z-2)=0$....(1)
Given that, plane (1) is perpendicular to the planes
$2x+3y-2z=5$
and
$x+2y-3z=8$
Therefore, we have,
$2a+3b-2c=0$...(2)
and
$a+2b-3c=0$...(3)
Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{3\times(-3)-2\times(-2)}=\frac{b}{1\times(-2)-2\times(-3)}=\frac{c}{2\times2-1\times3}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{-9+4}=\frac{b}{-2+6}=\frac{c}{4-3}=\lambda$$

$$\Rightarrow \frac{a}{-5}=\frac{b}{4}=\frac{c}{1}=\lambda$$

Thus, we have,
$a=-5\lambda, b=4\lambda$ and $c=\lambda$
Substituting the above values in equation (1), we have,
$-5\lambda(x-1)+4\lambda(y+1)+\lambda(z-2)=0$
Since $\lambda \neq 0$, we have,
$-5(x-1)+4(y+1)+(z-2)=0$
$\Rightarrow -5x+5+4y+4+z-2=0$
$\Rightarrow -5x+4y+z+7=0$
$\Rightarrow 5x-4y-z-7=0$
$\Rightarrow 5x-4y-z=7$
Thus the required equation of the plane is $5x-4y-z=7$

Question 13

Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$.

Solution 13

Given that the equation of the required plane is parallel to the plane
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$...(1)
$\therefore$ Vector equation of any plane parallel to (1) is
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=k$...(2)
Since the given plane passes through (a, b, c), then
$(a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=k$
$\Rightarrow a+b+c=k$...(3)

Substituting the above value of k in equation (2), we have,
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$
Thus the required equation of the plane is $x+y+z=a+b+c$

Question 14

Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.

Solution 14

The equation of any plane passing through $(-1,3,2)$

is $(x+1)+b(y-3)+c(z-2)=0....(1)$

Given that, Plane (1) is perpendicular to the planes

$x+2y+3z=5$

and

$3x+3y+z=0$

Therefore, we have,

$a+2b+3c=0...(2)$

and

$3a+3b+c=0...(3)$

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times1-3\times3}=\frac{b}{3\times3-1\times1}=\frac{c}{1\times3-3\times2}=\lambda(\text{say})$$

$$\Rightarrow\frac{a}{2-9}=\frac{b}{9-1}=\frac{c}{3-6}=\lambda$$

$$\Rightarrow\frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=\lambda$$

Thus, we have,

$a=-7\lambda, b=8\lambda$ and $c=-3\lambda$

Substituting the above values in equation (1), we have,

$-7\lambda(x+1)+8\lambda(y-3)-3\lambda(z-2)=0$

Since $\lambda\neq0$, we have,

$-7(x+1)+8(y-3)-3(z-2)=0$

$\Rightarrow -7x-7+8y-24-3z+6=0$

$\Rightarrow -7x+8y-3z-25=0$

$\Rightarrow 7x-8y+3z+25=0$

Thus the required equation of the plane is $7x-8y+3z+25=0$

Question 15

Find the vector equation of the plane through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane x - 2y + 4z = 10.

Solution 15

The equation of any plane passing through $(2, 1, -1)$
is $a(x-2)+b(y-1)+c(z+1)=0$....(1)
Also, the above plane passes through the point $(-1, 3, 4)$.
Thus, equation (1), becomes,
$a(-1-2)+b(3-1)+c(4+1)=0$
$\Rightarrow -3a+2b+5c=0$...(2)
Given that, Plane (1) is perpendicular to the plane
$x-2y+4z=10$
Therefore, we have,
$a-2b+4c=0$...(3)

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times 4-5\times(-2)}=\frac{b}{1\times 5-(-3)\times 4}=\frac{c}{(-3)\times(-2)-1\times 2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{8+10}=\frac{b}{5+12}=\frac{c}{6-2}=\lambda$$

$$\Rightarrow \frac{a}{18}=\frac{b}{17}=\frac{c}{4}=\lambda$$

Thus, we have,
$a=18\lambda, b=17\lambda$ and $c=4\lambda$
Substituting the above values in equation (1), we have,
$18\lambda(x-2)+17\lambda(y-1)+4\lambda(z+1)=0$
Since $\lambda \neq 0$, we have,
$18(x-2)+17(y-1)+4(z+1)=0$
$\Rightarrow 18x-36+17y-17+4z+4=0$
$\Rightarrow 18x+17y+4z-49=0$

Thus the required equation of the plane is $18x+17y+4z-49=0$

Chapter 29 - The plane Exercise Ex. 29.7

Question 1(i)

Find the vector equation of the following planes in scalar product form $\left(\vec{r}.\vec{n}=d\right)$:

$$\vec{r}=\left(2\hat{i}-\hat{k}\right)+\lambda\hat{i}+\mu\left(\hat{i}-2\hat{j}-\hat{k}\right)$$

Solution 1(i)

Here, $\vec{r} = \left(2\hat{i} - \hat{k}\right) + \lambda\hat{i} + \mu\left(\hat{i} - 2\hat{j} - \hat{k}\right)$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} - \hat{k}$, $\vec{b} = \hat{i}$, $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}\,(0 - 0) - \hat{j}\,(-1 - 0) + \hat{k}\,(-2 - 0)$$
$$= 0\,\hat{i} + \hat{j} - 2\hat{k}$$

$\vec{n} = \hat{j} - 2\hat{k}.$

We know that vector equation of plane in scalar product form is,
$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put $\vec{n}$ and $\vec{a}$ in equation (i),

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = \left(2\hat{i} - \hat{k}\right)\left(\hat{j} - 2\hat{k}\right)$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = (2)(0) + (0)(1) + (-1)(-2)$$
$$= 0 + 0 + 2$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The equation in required form is,

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

Question 1(ii)

Find the vector equation of the following planes in scalar product form $\left(\vec{r}.\vec{n} = d\right)$:

$$\vec{r} = (1 + s - t)\,\hat{i} + (2 - s)\,\hat{j} + (3 - 2s + 2t)\,\hat{k}$$

Solution 1(ii)

Here, $\vec{r} = (1 + s - t)\hat{i} + (2 - s)\hat{j} + (3 - 2s + 2t)\hat{k}$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + s\left(\hat{i} - \hat{j} - 2\hat{k}\right) + t\left(-\hat{i} + 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= \hat{i}(-2 - 0) - \hat{j}(2 - 2) + \hat{k}(0 - 1)$$

$$\vec{n} = -2\hat{i} - \hat{k}$$

We know that, vector equation of a plane in scalar product form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(\text{i})$$

Put value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = \left(-2\hat{i} - \hat{k}\right)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = (-2)(1) + (0)(2) + (-1)(3)$$

$$= -2 + 0 - 3$$

$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = -5$$

Multiplying both the sides by (-1),

$$\vec{r}.\left(2\hat{i} + \hat{k}\right) = 5$$

The equation in the required form,

$$\vec{r}.\left(2\hat{i} + \hat{k}\right) = 5$$

Question 1(iii)

Find the vector equation of the following planes in scalar product form $\left(\vec{r}.\vec{n} = d\right)$:

$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

Solution 1(iii)

Given, equation of plane,

$$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ is the equation of a plane passing through point $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = -\hat{i} + \hat{j} - 2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}\left(-4 + 1\right) - \hat{j}\left(-2 - 1\right) + \hat{k}\left(1 + 2\right)$$

$$= -3\hat{i} + 3\hat{j} + 3\hat{k}$$

We know that, the equation of plane in scalar product form is given by,
$\vec{r}.\vec{n} = \vec{a}\vec{n}$

$$\vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = \left(\hat{i} + \hat{j}\right)\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right)$$
$$= (1)(-3) + (1)(3) + (0)(3)$$
$$= -3 + 3$$

$$\vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = 0$$

Dividing by 3, we get

$$\vec{r}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = 0$$

Equation in required form is,

$$\vec{r}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = 0$$

Question 1(iv)

Find the vector equation of the following planes in scalar product form $\left(\vec{r}.\vec{n} = d\right)$:

$\vec{r} = \hat{i} - \hat{j} + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(4\hat{i} - 2\hat{j} + 3\hat{k}\right)$

Solution 1(iv)

$\bar{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4i - 2\hat{j} + 3k)$

Plane is passing through $(\hat{i} - \hat{j})$and parallel to

$b(\hat{i} + \hat{j} + \hat{k})$ *and* $c\,(4i - 2\hat{j} + 3k)$

$$n = b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix}$$

$n = 5i + j - 6k$

$\bar{r}.n = (\hat{i} - \hat{j}).(5i + j - 6k)$=5-1=4

$\bar{r}.(5i + j - 6k) = 4$

Question 2(i)

Find the cartesian form of the equation of the following planes:

$\vec{r} = \left(\hat{i} - \hat{j}\right) + s\left(-\hat{i} + \hat{j} + 2\hat{k}\right) + t\left(\hat{i} + 2\hat{j} + \hat{k}\right)$

Solution 2(i)

Here, given equation of plane is,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + s\left(-\hat{i} + \hat{j} + 2\hat{k}\right) + t\left(\hat{i} + 2\hat{j} + \hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

Given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}\left(1 - 4\right) - \hat{j}\left(-1 - 2\right) + \hat{k}\left(-2 - 1\right)$$

$$\vec{n} = -3\hat{i} + 3\hat{j} - 3\hat{k}$$

We know that, equation of plane in the scalar product form,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(\hat{i} - \hat{j}\right) = \left(\hat{i} - \hat{j}\right)\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right)$$

$$\vec{r}.\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = (1)(-3) + (-1)(3) + (0)(-3)$$
$$= -3 - 3 + 0$$

$$\vec{r}.\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = -6$$

Put $\vec{r}. = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right) = -6$$
$$(x)(-3) + (y)(3) + (z)(-3) = -6$$

$$-3x + 3y - 3z = -6$$

Dividing by (-3), we get

$$x - y + z = 2$$

Equation in required form is,

$$x - y + z = 2$$

Question 2(ii)

Find the cartesian form of the equation of the following planes:

$$\vec{r} = (1 + s + t)\hat{i} + (2 - s + t)\hat{j} + (3 - 2s + 2t)\hat{k}$$

Solution 2(ii)

Given, equation of plane,

$$\vec{r} = (1+s+t)\hat{i} + (2-s+t)\hat{j} + (3-2s+2t)\hat{k}$$
$$= \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + s\left(\hat{i} - \hat{j} - 2\hat{k}\right) + t\left(\hat{i} + \hat{j} + 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through the vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$
$\vec{c} = \hat{i} + \hat{j} + 2\hat{k}$

The given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(-2+2) - \hat{j}(2+2) + \hat{k}(1+1)$$
$$= 0.\left(\hat{i}\right) - 4\hat{j} + 2\hat{k}$$
$$\vec{n} = -4\hat{j} + 2\hat{k}$$

We know that, equation of plane in scalar product form is given by,
$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put, the value of $\vec{a}$ and $\vec{n}$ in (i),
$$\vec{r}.\left(-4\hat{j} + 2\hat{k}\right) = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\left(-4\hat{j} + 2\hat{k}\right)$$

$$\vec{r}.\left(-4\hat{j} + 2\hat{k}\right) = (1)(0) + (2)(-4) + (3)(2)$$
$$= 0 - 8 + 6$$

$$\vec{r}.\left(-4\hat{j} + 2\hat{k}\right) = -2$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-4\hat{j} + 2\hat{k}\right) = -2$$
$$(x)(0) + (y)(-4) + (z)(2) = -2$$

$$-4y + 2z = -2$$

Dividing by (-2), we get

$$2y - z = 1$$

The equation in required form is,

$$2y - z = 1$$

Question 3(i)

Find the vector equation of the following planes in non-parametric form:

$$\vec{r} = (\lambda - 2\mu)\hat{i} + (3 - \mu)\hat{j} + (2\lambda + \mu)\hat{k}$$

Solution 3(i)

Given, equation of plane is,

$$\vec{r} = (\lambda - 2\mu)\hat{i} + (3 - \mu)\hat{j} + (2\lambda + \mu)\hat{k}$$
$$\vec{r} = (3\hat{j}) + \lambda(\hat{i} + 2\hat{k}) + \mu(-2\hat{i} - \hat{j} + \hat{k})$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a point $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Given, $\vec{a} = 3\hat{j}$

$\vec{b} = \hat{i} + 2\hat{k}$

$\vec{c} = -2\hat{i} - \hat{j} + \hat{k}$

The given plane is perpendicular to

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(0 + 2) - \hat{j}(1 + 4) + \hat{k}(-1 - 0)$$

$$\vec{n} = 2\hat{i} - 5\hat{j} - \hat{k}$$

Vector equation of plane in non-parametric form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$
$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = (3\hat{j})(2\hat{i} - 5\hat{j} - \hat{k})$$

$$= (0)(2) + (3)(-5) + (0)(-1)$$
$$= 0 - 15 + 0$$

$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = -15$$
$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) + 15 = 0$$

The required form of equation is,

$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) + 15 = 0$$

Question 3(ii)

Find the vector equation of the following planes in non-parametric form:

$$\vec{r} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \mu\left(5\hat{i} - 2\hat{j} + 7\hat{k}\right)$$

Solution 3(ii)

Given, equation of plane is,

$$\vec{r} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \mu\left(5\hat{i} - 2\hat{j} + 7\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{c} = 5\hat{i} - 2\hat{j} + 7\hat{k}$

The given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= \hat{i}\left(14 + 6\right) - \hat{j}\left(7 - 15\right) + \hat{k}\left(-2 - 10\right)$$

$$\vec{n} = 20\hat{i} + 8\hat{j} - 12\hat{k}$$

We know that, equation of a plane in non-parametric form is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = \left(2\hat{i} + 2\hat{j} - \hat{k}\right)\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right)$$

$$= \left(2\right)\left(20\right) + \left(2\right)\left(8\right) + \left(-1\right)\left(-12\right)$$

$$= 40 + 16 + 12$$

$$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = 68$$

Dividing by 4,

$$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$

Equation of plane in required form is,

$$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$

Chapter 29 - The plane Exercise Ex. 29.8

Question 1

Find the equation of the plane which is parallel to $2x - 3y + z = 0$ and which passes through $(1, -1, 2)$.

Solution 1

Given, equation of plane is

$$2x - 3y + z = 0 \qquad ---(i)$$

We know that equation of a plane parallel the plane (i) is given by

$$2x - 3y + z + \lambda = 0 \qquad ---(ii)$$

Given that, plane (ii) is passing through the point $(1, -1, 2)$ so it must satisfy the equation (ii),

$$2(1) - 3(-1) + (2) + \lambda = 0$$
$$2 + 3 + 2 + \lambda = 0$$
$$7 + \lambda = 0$$

$$\lambda = -7$$

Put the value of λ in equation (ii),

$$2x - 3y + z - 7 = 0$$

So, equation of the required plane is,

$$2x - 3y + z = 7$$

Question 2

Find the equation of the plane through $(3, 4, -1)$ which is parallel to the plane $\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0$.

Solution 2

Given, equation of plane is

$$\vec{r}.\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+2=0 \qquad ---(i)$$

We know that equation of a plane parallel to the plane (i) is given by

$$\vec{r}.\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+\lambda=0 \qquad ---(ii)$$

Given that, plane (ii) is passing through vector $\left(3\hat{i}+4\hat{j}-\hat{k}\right)$ so it must satisfy equation (ii),

$\left(3\hat{i}+4\hat{j}-\hat{k}\right)\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+\lambda=0$
$(3)(2)+(4)(-3)+(-1)(5)+\lambda=0$
$6-12-5+\lambda=0$
$-11+\lambda=0$

$\lambda=11$

Put the value of λ in equation (ii),

$$\vec{r}.\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+11=0$$

Equation of required plane is,

$$\vec{r}.\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+11=0$$

Question 3

Find the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0$
$3x-5y+4z+11=0$ and the point $(-2,1,3)$.

Solution 3

We know that, equation of a plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Given, equations of plane is,

$2x - 7y + 4z - 3 = 0$ and
$3x - 5y + 4z + 11 = 0$

So, equation of plane passing through the line of intersection of given two planes is

$$\left(2x - 7y + 4z - 3\right) + \lambda\left(3x - 5y + 4z + 11\right) = 0$$
$$2x - 7y + 4z - 3 + 3\lambda x - 5\lambda y + 4\lambda z + 11\lambda = 0$$
$$x\left(2 + 3\lambda\right) + y\left(-7 - 5\lambda\right) + z\left(4 + 4\lambda\right) - 3 + 11\lambda = 0 \qquad ---(\text{i})$$

Plane (1) is passing through the points $(-2, 1, 3)$, so it satisfies the equation (i),

$$(-2)\left(2 + 3\lambda\right) + (1)\left(-7 - 5\lambda\right) + (3)\left(4 + 4\lambda\right) - 3 + 11\lambda = 0$$
$$-4 - 6\lambda - 7 - 5\lambda + 12 + 12\lambda - 3 + 11\lambda = 0$$
$$-2 + 12\lambda = 0$$
$$12\lambda = 2$$
$$\lambda = \frac{2}{12}$$

$$\lambda = \frac{1}{6}$$

Put λ in equation (i),

$$x(2+3\lambda)+y(-7-5\lambda)+z(4+4\lambda)-3+11\lambda=0$$
$$x\left(2+\frac{3}{6}\right)+y\left(-7-\frac{5}{6}\right)+z\left(4+\frac{4}{6}\right)-3+\frac{11}{6}=0$$
$$x\left(\frac{12+3}{6}\right)+y\left(\frac{-42-5}{6}\right)+z\left(\frac{24+4}{6}\right)-\frac{18+11}{6}=0$$

$$\frac{15}{6}x-\frac{47}{6}y+\frac{28}{6}z-\frac{7}{6}=0$$

Multiplying by 6, we get

$$15x-47y+28z-7=0$$

Therefore, equation of required plane is,

$$15x-47y+28z=7$$

Question 4

Find the equation of the plane through the point $2\hat{i}+\hat{j}-\hat{k}$ and passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$ and $\vec{r}.\left(\hat{j}+2\hat{k}\right)=0$.

Solution 4

We know that, equation of a plane passing the line of intersection of planes

$\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is given by

$\vec{r}.\left(\overrightarrow{n_1} + \lambda \overrightarrow{n_2}\right) = d_1 + \lambda d_2$

So, equation of plane through the line of intersection of planes $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$ and $\vec{r}.\left(\hat{j} + 2\hat{k}\right) = 0$ is given by

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0 \qquad ---(i)$$

Given that plane (i) is passing through the point $\left(2\hat{i} + \hat{j} - \hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{j} + 2\hat{k}\right) = 0$$
$$(2)(1) + (1)(3) + (-1)(-1) + \lambda\left[(2)(0) + (1)(1) + (-1)(2)\right] = 0$$
$$(2 + 3 + 1) + \lambda(1 - 2) = 0$$
$$6 - \lambda = 0$$

$$\lambda = 6$$

Put λ in equation (i),

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\hat{j} + 12\hat{k}\right] = 0$$

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

So, equation of required plane is,

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

Question 5

Find the equation of the plane passing through the line of intersection of the planes $2x - y = 0$, and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$.

Solution 5

We know that, equation of a plane passing through the line of intersection of $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of plane $2x - y = 0$ and $3z - y = 0$ is

$$\left(2x - y\right) + \lambda\left(3z - y\right) = 0$$

$$2x - y + 3\lambda z - \lambda y = 0$$

$$x\left(2\right) + y\left(-1 - \lambda\right) + z\left(3\lambda\right) = 0 \qquad ---\text{(i)}$$

We know that, two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---\text{(ii)}$$

Given, plane (i) is perpendicular to plane

$$4x + 5y - 3z = 8 \qquad ---\text{(iii)}$$

Using (i) and (iii) in equation (ii),

$$\left(2\right)\left(4\right) + \left(-1 - \lambda\right)\left(5\right) + \left(3\lambda\right)\left(-3\right) = 0$$

$$8 - 5 - 5\lambda - 9\lambda = 0$$

$$3 - 14\lambda = 0$$

$$-14\lambda = -3$$

$$\lambda = \frac{3}{14}$$

Put the value of λ in equation (i),

$$2x + y\left(-1 - \lambda\right) + z\left(3\lambda\right) = 0$$

$$2x + y\left(-1 - \frac{3}{14}\right) + z.3\left(\frac{3}{14}\right) = 0$$

$$2x + y\left(\frac{-14 - 3}{14}\right) + \frac{9z}{14} = 0$$

$$2x + y\left(-\frac{17}{14}\right) + \frac{9z}{14} = 0$$

Multiplying with 14, we get

$$28x - 17y + 9z = 0$$

Equation of required plane is,

$$28x - 17y + 9z = 0$$

Question 6

Find the equation of the plane which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and which is perpendicular to the plane $5x + 3y - 6z + 8 = 0$.

Solution 6

We know that, the equation plane passing through the line of intersection of plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Here, equation of plane passing through the intersection of plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x + 2y + 3z - 4 + 2\lambda x + \lambda y - \lambda z + 5\lambda = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(i)$$

We know, that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y - 6z + 8 = 0 \qquad ---(iii)$$

Using plane (i) and (iii) in equation (ii),

$$(5)\left(1 + 2\lambda\right) + (3)\left(2 + \lambda\right) + (-6)\left(3 - \lambda\right) = 0$$
$$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$
$$-7 + 19\lambda = 0$$
$$19\lambda = 7$$

$$\lambda = \frac{7}{19}$$

Put value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 + \frac{14}{19}\right) + y\left(2 + \frac{7}{19}\right) + z\left(3 - \frac{7}{19}\right) - 4 + \frac{35}{19} = 0$$
$$x\left(\frac{19 + 14}{19}\right) + y\left(\frac{38 + 7}{19}\right) + z\left(\frac{57 - 7}{19}\right)\frac{-76 + 35}{19} = 0$$

$$x\left(\frac{33}{19}\right) + y\left(\frac{45}{19}\right) + z\left(\frac{50}{19}\right) - \frac{41}{19} = 0$$

Multiplying by 19, we get

$$33x + 45y + 50z - 41 = 0$$

Equation of required plane is,

$$33x + 45y + 50z - 41 = 0$$

Question 7

Find the equation of the plane through the line of intersection of the planes $x + 2y + 3z + 4 = 0$ and $x - y + z + 3 = 0$ and passing through the origin.

Solution 7

We know that, equation of a plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x + 2y + 3z + 4 = 0$ and $x - y + z + 3 = 0$ is

$$\left(x + 2y + 3z + 4\right) + \lambda\left(x - y + z + 3\right) = 0$$

$$x\left(1 + \lambda\right) + y\left(2 - \lambda\right) + z\left(3 + \lambda\right) + 4 + 3\lambda = 0 \qquad ---(i)$$

Equation (i) is passing through origin, so

$$(0)(1 + \lambda) + (0)(2 - \lambda) + (0)(3 + \lambda) + 4 + 3(\lambda) = 0$$

$$0 + 0 + 0 + 4 + 3\lambda = 0$$

$$3\lambda = -4$$

$$\lambda = -\frac{4}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + \lambda\right) + y\left(2 - \lambda\right) + z\left(3 + \lambda\right) + 4 + 3\lambda = 0$$

$$x\left(1 - \frac{4}{3}\right) + y\left(2 + \frac{4}{3}\right) + z\left(3 - \frac{4}{3}\right) + 4 - \frac{12}{3} = 0$$

$$x\left(\frac{3 - 4}{3}\right) + y\left(\frac{6 + 4}{3}\right) + z\left(\frac{9 - 4}{3}\right) + 4 - 4 = 0$$

$$-\frac{x}{3} + \frac{10y}{3} + \frac{5z}{3} = 0$$

Multiplying by 3, we get

$$-x + 10y + 5z = 0$$

$$x - 10y - 5z = 0$$

The equation of required plane is,

$$x - 10y - 5z = 0$$

Question 8

Find the vector equation (in scalar product form) of the plane containing the line of intersection of the planes $x - 3y + 2z - 5 = 0$ and $2x - y + 3z - 1 = 0$ and passing through $(1, -2, 3)$.

Solution 8

We know that equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x - 3y + 2z - 5 = 0$ and $2x - y + 3z - 1 = 0$ is given by

$$\left(x - 3y + 2z - 5\right) + \lambda\left(2x - y + 3z - 1\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0 \qquad ---\left(\text{i}\right)$$

Plane (i) is passing through the point $(1, -2, 3)$ so,

$$\left(1\right)\left(1 + 2\lambda\right) + \left(-2\right)\left(-3 - \lambda\right) + \left(3\right)\left(2 + 3\lambda\right) - 5 - \lambda = 0$$
$$1 + 2\lambda + 6 + 2\lambda + 6 + 9\lambda - 5 - \lambda = 0$$
$$8 + 12\lambda = 0$$
$$12\lambda = -8$$
$$\lambda = -\frac{8}{12}$$
$$\lambda = -\frac{2}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0$$
$$x\left(1 - \frac{4}{3}\right) + y\left(-3 + \frac{2}{3}\right) + z\left(2 - \frac{6}{3}\right) - 5 + \frac{2}{3} = 0$$
$$x\left(\frac{3 - 4}{3}\right) + y\left(\frac{-9 + 2}{3}\right) + z\left(\frac{6 - 6}{3}\right)\frac{-15 + 2}{3} = 0$$

$$-\frac{1}{3}x - \frac{7}{3}y + z\left(0\right) - \frac{13}{3} = 0$$

Multiplying by (-3),

$$x + 7y + 13 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

Equation of required plane is,

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

Question 9

Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$.

Solution 9

We know that, equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes is $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y + 6z + 8 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$(5)\left(1 + 2\lambda\right) + (3)\left(2 + \lambda\right) + (6)\left(3 - \lambda\right) = 0$$
$$5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$
$$29 + 7\lambda = 0$$
$$7\lambda = -29$$

$$\lambda = -\frac{29}{7}$$

Put the value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 - \frac{58}{7}\right) + y\left(2 - \frac{29}{7}\right) + z\left(3 + \frac{29}{7}\right) - 4 - \frac{145}{7} = 0$$
$$x\left(\frac{7 - 58}{7}\right) + y\left(\frac{14 - 29}{7}\right) + z\left(\frac{21 + 29}{7}\right)\frac{-28 - 145}{7} = 0$$

$$x\left(-\frac{51}{7}\right) + y\left(-\frac{15}{7}\right) + z\left(\frac{50}{7}\right) - \frac{173}{7} = 0$$

Multiplying by (-7), we get

$51x + 15y - 50z + 173 = 0$

So, equation of required plane is,

$51x + 15y - 50z + 173 = 0$

Question 10

Find the equation of the plane through the line of intersection of the planes $\vec{r}.(\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which is at a unit distance from the origin.

Solution 10

$\vec{r}.(\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$

$x + 3y + 6 = 0;\ 3x - y - 4z = 0$

$x + 3y + 6 + \lambda(3x - y - 4z) = 0$

$x(1 + 3\lambda) + y(3 - \lambda) + -4z\lambda + 6 = 0$

$$\text{Distance from origin to plane} = \left| \frac{6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (4\lambda)^2}} \right| = 1$$

$36 = (1+3\lambda)^2 + (3-\lambda)^2 + (4\lambda)^2$

$36 = 1 + 6\lambda + 9\lambda^2 + 9 - 6\lambda + \lambda^2 + 16\lambda^2$

$26 = 26\lambda^2$

$\lambda^2 = 1$

$\lambda = \pm 1$

Case : 1 $\lambda = 1$

$x + 3y + 6 + 1(3x - y - 4z) = 0$

$4x + 2y - 4z + 6 = 0$

Case : 2 $\lambda = -1$

$x + 3y + 6 - 1(3x - y - 4z) = 0$

$2x - 4y - 4z - 6 = 0$

Question 11

Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z - 4 = 0$

Solution 11

We know that equation of a plane passing through the line of intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ is

$$\left(2x + 3y - z + 1\right) + \lambda\left(x + y - 2z + 3\right) = 0$$
$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0 \qquad ---\text{(i)}$$

We know that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---\text{(ii)}$$

Given, plane (i) is perpendicular to the plane,

$$3x - y - 2z - 4 = 0 \qquad ---\text{(iii)}$$

Using (i) and (iii) in equation (ii),

$$\left(3\right)\left(2 + \lambda\right) + \left(-1\right)\left(3 + \lambda\right) + \left(-2\right)\left(-1 - 2\lambda\right) = 0$$
$$6 + 3\lambda - 3 - \lambda + 2 + 4\lambda = 0$$
$$6\lambda + 5 = 0$$
$$6\lambda = -5$$

$$\lambda = -\frac{5}{6}$$

Put the value of λ in equation (i),

$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0$$
$$x\left(2 - \frac{5}{6}\right) + y\left(3 - \frac{5}{6}\right) + z\left(-1 + \frac{10}{6}\right) + 1 - \frac{15}{6} = 0$$
$$x\left(\frac{12 - 5}{6}\right) + y\left(\frac{18 - 5}{6}\right) + z\left(\frac{-6 + 10}{6}\right) + \frac{6 - 15}{6} = 0$$

$$\frac{7x}{6} + \frac{13y}{6} + \frac{4z}{6} - \frac{9}{6} = 0$$

Multiplying by 6, we get

$$7x + 13y + 4z - 9 = 0$$

Equation of the required plane is,

$$7x + 13y + 4z - 9 = 0$$

Question 12

Find the equation of the plane that contains the line of intersection of the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.

Solution 12

We know that, equation of a plane passing through the line of intersection of plane

$\vec{r}.\vec{n_1}-d_1=0$ and $\vec{r}.\vec{n_2}-d_2=0$ is

$\left(\vec{r}.\vec{n_1}-d_1\right)+\lambda\left(\vec{r}.\vec{n_2}-d_2\right)=0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ is given by

$\left[\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4\right]+\lambda\left[\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5\right]=0$

$\vec{r}.\left[\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}-\hat{k}\right)\right]-4+5\lambda=0 \qquad ---(i)$

We know that two planes are perpendicular if

$\vec{n_1}.\vec{n_2}=0 \qquad ---(ii)$

Given that plane (i) is perpendicular to plane

$\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0 \qquad ---(iii)$

Using (i) and (iii) in equation (ii),

$\left[\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}-\hat{k}\right)\right]\left(5\hat{i}+3\hat{j}-6\hat{k}\right)=0$

$\left[\hat{i}\left(1+2\lambda\right)+\hat{j}\left(2+\lambda\right)+\hat{k}\left(3-\lambda\right)\right]\left(5\hat{i}+3\hat{j}-6\hat{k}\right)=0$

$\left(1+2\lambda\right)\left(5\right)+\left(2+\lambda\right)\left(3\right)+\left(3-\lambda\right)\left(-6\right)=0$

$5+10\lambda+6+3\lambda-18+6\lambda=0$

$19\lambda-7=0$

$\lambda=\frac{7}{19}$

Put value of λ in equation (i),

$$\vec{r}.\left[\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}-\hat{k}\right)\right]-4+5\lambda=0$$

$$\vec{r}\left[\hat{i}+2\hat{j}+3\hat{k}+\frac{14}{19}\hat{i}+\frac{7}{19}\hat{j}-\frac{7}{19}\hat{k}\right]-4+5\left(\frac{7}{19}\right)=0$$

$$\vec{r}\left[\frac{33\hat{i}}{19}+\frac{45\hat{j}}{19}-\frac{50\hat{k}}{19}\right]\frac{-76+35}{19}=0$$

$$\vec{r}\left(\frac{33\hat{i}+45\hat{j}+50\hat{k}}{19}\right)-\frac{41}{19}=0$$

Multiplying by 19,

$$\vec{r}\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0$$

Equation of required plane is,

$$\vec{r}\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0$$
$$33x+45y+50z-41=0$$

Question 13

Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$ and the point (1, 1, 1).

Solution 13

The equation of a plane passing through the intersection of $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=-5$ is

$$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-6]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})+5]=0$$
$$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=(6-5\lambda)\ldots(1)$$
$$\Rightarrow [x\hat{i}+y\hat{j}+z\hat{k}]\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=(6-5\lambda)$$
$$\Rightarrow [x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda)]=(6-5\lambda)\ldots(2)$$

The requried plane also passes through the point(1, 1, 1).

Substiuting $x=1, y=1, z=1$ in equation (2), we have,

$$1\times(1+2\lambda)+1\times(1+3\lambda)+1\times(1+4\lambda)=(6-5\lambda)$$
$$\Rightarrow 1+2\lambda+1+3\lambda+1+4\lambda=6-5\lambda$$
$$\Rightarrow 3+9\lambda=6-5\lambda$$
$$\Rightarrow 14\lambda=6-3$$
$$\Rightarrow 14\lambda=3$$
$$\Rightarrow \lambda=\frac{3}{14}$$

Substituting the value $\lambda = \frac{3}{14}$ in equation (1), we have,

$$\vec{r}\cdot\left[\left(1+2\left(\frac{3}{14}\right)\right)\hat{i}+\left(1+3\left(\frac{3}{14}\right)\right)\hat{j}+\left(1+4\left(\frac{3}{14}\right)\right)\hat{k}\right]=\left(6-5\left(\frac{3}{14}\right)\right)$$

$$\Rightarrow \vec{r}\cdot\left[\frac{20}{14}\hat{i}+\frac{23}{14}\hat{j}+\frac{26}{14}\hat{k}\right]=\frac{69}{14}$$

$$\Rightarrow \vec{r}\cdot\left[20\hat{i}+23\hat{j}+26\hat{k}\right]=69$$

Question 14

Find the equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}+\hat{j}+3\hat{k}\right)=7$, $\vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)=9$ and the point $(2,1,3)$.

Solution 14

We know that, equation of the plane passing through the line of intersection of planes

$\vec{r}.\vec{n_1}-d_1=0$ and $\vec{r}.\vec{n_2}-d_2=0$ is

$$\left(\vec{r}.\vec{n_1}-d_1\right)+\lambda\left(\vec{r}.\vec{n_2}-d_2\right)=0$$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(2\hat{i}+\hat{j}+3\hat{k}\right)-7=0$ and $\vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9=0$ is given by

$$\left[\vec{r}.\left(2\hat{i}+\hat{j}+3\hat{k}\right)-7\right]+\lambda\left[\vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9\right]=0$$

$$\vec{r}\left[\left(2\hat{i}+\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+5\hat{j}+3\hat{k}\right)\right]-7-9\lambda=0$$

$$\vec{r}\left[(2+2\lambda)\hat{i}+(1+5\lambda)\hat{j}+(3+3\lambda)\hat{k}\right]-7-9\lambda=0 \qquad ---(i)$$

Given that plane (i) is passing through

$\left(2\hat{i}+\hat{j}+3\hat{k}\right)$, so

$$\left(2\hat{i}+\hat{j}+3\hat{k}\right)\left[(2+2\lambda)\hat{i}+(1+5\lambda)\hat{j}+(3+3\lambda)\hat{k}\right]-7-9\lambda=0$$

$$(2)(2+2\lambda)+(1)(1+5\lambda)+(3)(3+3\lambda)-7-9\lambda=0$$

$$4+4\lambda+1+5\lambda+9+9\lambda-7-9\lambda=0$$

$$9\lambda+7=0$$

$$9\lambda=-7$$

$$\lambda=-\frac{7}{9}$$

Put value of λ in equation (i),

$$\vec{r}.\left[(2+2\lambda)\hat{i}+(1+5\lambda)\hat{j}+(3+3\lambda)\hat{k}\right]-7-9\lambda=0$$

$$\vec{r}.\left[\left(2+\frac{14}{9}\right)\hat{i}+\left(1-\frac{35}{9}\right)\hat{j}+\left(3-\frac{21}{9}\right)\hat{k}\right]-7+\frac{63}{9}=0$$

$$\vec{r}.\left[\left(\frac{18-14}{9}\right)\hat{i}+\left(\frac{9-35}{9}\right)\hat{j}+\left(\frac{27-21}{9}\right)\hat{k}\right]-7+7=0$$

$$\vec{r}.\left[\left(\frac{4}{9}\right)\hat{i}-\frac{26}{9}\hat{j}+\frac{6\hat{k}}{9}\right]+0=0$$

$$\vec{r}.\left[\frac{4}{9}\hat{i}-\frac{26}{9}\hat{j}+\frac{6}{9}\hat{k}\right]=0$$

Multiplying by $\left(\frac{9}{2}\right)$, we get

$$\vec{r}\left[2\hat{i}-13\hat{j}+3\hat{k}\right]=0$$

Equation of required plane is,

$$\vec{r}.\left(2\hat{i}-13\hat{j}+3\hat{k}\right)=0$$

Question 15

Find the equation of the plane through the intersection of the planes 3x - y 2z = 4 and x + y + z = 2 and the point (2, 2, 1).

Solution 15

The equation of the family of planes through the intersection of planes

$3x-y+2z=4$ and $x+y+z=2$ is,

$(3x-y+2z-4)+\lambda(x+y+z-2)=0$.........(i)

If it passes through $(2,\ 2,\ 1)$, then

$(6-2+2-4)+\lambda(2+2+1-2)=0$

$\Rightarrow \lambda=-\frac{2}{3}$

Substituting $\lambda=-\frac{2}{3}$ in (i) we get, $7x-5y+4z=0$ as the equation of the required plane.

Question 16

Find the vector equation of the plane through the line of intersection of the plane x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x - y + z = 0.

Solution 16

The equation of the family of planes through the line of intersection of planes $x+y+z=1$ and $2x+3y+4z=5$ is,

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0\ldots\ldots\ldots(i)$$
$$(2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z=5\lambda+1$$

It is perpendicular to the plane $x-y+z=0$.

$$\therefore\ (2\lambda+1)(1)+(3\lambda+1)(-1)+(4\lambda+1)(1)=5\lambda+1$$
$$\Rightarrow 2\lambda+1-3\lambda-1+4\lambda+1=5\lambda+1$$
$$\Rightarrow \lambda=-\frac{1}{3}$$

Substituting $\lambda=-\frac{1}{3}$ in (i), we get, $x-z+2=0$ as the equation of the required plane and its vector equation is $\vec{r}\cdot(\hat{i}-\hat{k})+2=0$.

Question 17

Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$

Solution 17

The equation of the family of planes parallel to $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$ is,

$$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=d\ldots\ldots\ldots(i)$$

If it passes through (a, b, c) then

$$(a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=d$$
$$\Rightarrow a+b+c=d$$

Substituting $a+b+c=d$ in (i), we get,

$$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=a+b+c$$

$x+y+z=a+b+c$ as the equation of the required plane.

Chapter 29 - The plane Exercise Ex. 29.9

Question 1

Find the distance of the point $2\hat{i}-\hat{j}-4\hat{k}$ from the plane $\vec{r}.\left(3\hat{i}-4\hat{j}+12\hat{k}\right)-9=0$.

Solution 1

We know that distance of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right| \text{ unit}$$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and
plane $\vec{r}.\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9 = 0$

$\vec{r}.\vec{n} - d = 0$

So, required distance

$$D = \left|\frac{\left(2\hat{i} - \hat{j} - 4\hat{k}\right)\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}\right|$$

$$= \left|\frac{(2)(3) + (-1)(-4) + (-4)(12) - 9}{\sqrt{9 + 16 + 144}}\right|$$

$$= \left|\frac{6 + 4 - 48 - 9}{\sqrt{169}}\right|$$

$$= \left|-\frac{47}{13}\right|$$

$$= \frac{47}{13} \text{ units}$$

Required distance is $\frac{47}{13}$ units

Question 2

Show that the points $\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 3\hat{j} + 3\hat{k}$ are equidistant from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$.

Solution 2

We know that, distance of a point $\vec{a}$ to a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right| \qquad ---(i)$$

Let D_1 be the distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$

from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then

$$D_1 = \left|\frac{\left(\hat{i} - \hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right| \qquad \left[\text{Using equation } (i)\right]$$

$$= \left|\frac{(1)(5) + (-1)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{5 - 2 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|-\frac{9}{\sqrt{78}}\right|$$

$$D_1 = \frac{9}{\sqrt{78}} \text{ units} \qquad ---(ii)$$

Again, let D_2 be the distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then, using equation (i), we get

$$D_2 = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$

$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|\frac{9}{\sqrt{78}}\right|$$

$$= \frac{9}{\sqrt{78}} \text{ units} \qquad ---(iii)$$

From equation (ii) and (iii)

$D_1 = D_2$

Distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

= Distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

Question 3

Find the distance of the point $(2, 3, -5)$ from the plane $x + 2y - 2z - 9 = 0$.

Solution 3

We know that, distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(i)$$

So, distance of point $(2, 3, -5)$ from the plane $x + 2y - 2z - 9 = 0$ is given by

$$D = \left|\frac{2 + (2)(3) - 2(-5) - 9}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right| \qquad \left[\text{Using equation (i)}\right]$$

$$= \left|\frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}}\right|$$

$$= \left|\frac{9}{\sqrt{9}}\right|$$

$$= \left|\frac{9}{3}\right|$$

$D = 3$ units

Question 4

Find the equations of the planes parallel to the plane $x + 2y - 2z + 8 = 0$ which are at distance of 2 units from the point $(2, 1, 1)$.

Solution 4

Given equation of plane is

$$x + 2y - 2z + 8 = 0 \qquad ---(\text{i})$$

We know that, equation of the plane parallel to plane (i) is given by

$$x + 2y - 2z + \lambda = 0 \qquad ---(\text{ii})$$

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(\text{iii})$$

Given, $D = 2$ unit is the distance of the plane (ii) from the point $(2,1,1)$, so
Using (i),

$$2 = \left|\frac{2 + (2)(1) - 2(1) + \lambda}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right|$$

$$2 = \left|\frac{2 + 2 - 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$2 = \left|\frac{2 + \lambda}{\sqrt{9}}\right|$$

Squaring both the sides, we get

$$4 = \frac{(2 + \lambda)^2}{9}$$

$$36 = (2 + \lambda)^2$$

$$2 + \lambda = \pm 6$$

$$\Rightarrow \quad 2 + \lambda = 6 \quad \textit{or} \quad 2 + \lambda = -6$$

$$\Rightarrow \quad \lambda = 4 \quad \textit{or} \quad \lambda = -8$$

Put $\lambda = 4$ in equation (ii),

$$x + 2y - 2z + 4 = 0$$

Put $\lambda = -8$ in equation (ii),

$$x + 2y - 2z - 8 = 0$$

Hence, equation of the required plane are

$$x + 2y - 2z + 4 = 0$$

$$x + 2y - 2z - 8 = 0$$

Question 5

Show that the points $(1,1,1)$ and $(-3,0,1)$ are equidistant from the plane $3x+4y-12z+13=0$.

Solution 5

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax+by+cz+d=0$ is given by

$$D = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{(a)^2+(b)^2+(c)^2}}\right| \qquad ---(i)$$

Let D_1 be the distance of the point $(1,1,1)$ from plane $3x+4y-12z+13=0$, so using (i), we get

$$D_1 = \left|\frac{(3)(1)+(4)(1)-12(1)+13}{\sqrt{(3)^2+(4)^2+(-12)^2}}\right|$$

$$= \left|\frac{3+4-12+13}{\sqrt{9+16+144}}\right|$$

$$= \left|\frac{8}{\sqrt{169}}\right|$$

$$D = \frac{8}{13} \text{ units} \qquad ---(ii)$$

Let D_2 be the distane of a point $(-3,0,1)$ from the plane $3x+4y-12z+13=0$, so using equation (i),

$$D_2 = \left|\frac{(3)(-3)+(4)(0)-12(1)+13}{\sqrt{(3)^2+(4)^2+(-12)^2}}\right|$$

$$= \left|\frac{-9+0-12+13}{\sqrt{9+4+144}}\right|$$

$$= \left|-\frac{8}{\sqrt{169}}\right|$$

$$D_2 = \frac{8}{13} \text{ units} \qquad ---(iii)$$

Hence, from equation (ii) and (iii)

$$D_1 = D_2$$

Question 6

find the equation of the planes parallel to the plane $x-2y+2z-3=0$ and which are at a unit distance from the point $(1,1,1)$.

Solution 6

Given equation of plane is

$$x - 2y + 2z - 3 = 0 \qquad ---(i)$$

We know that, equation of a plane parallel to plane (i) is given by,

$$x - 2y + 2z + \lambda = 0 \qquad ---(ii)$$

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + b^2 + c^2}}\right| \qquad ---(iii)$$

Given that, distance of plane (ii) from a point $(1,1,1)$ is one unit, so using (iii),

$$1 = \left|\frac{(1) - 2(1) + 2(1) + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}}\right|$$

$$= \left|\frac{1 - 2 + 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$1 = \left|\frac{1 + \lambda}{\sqrt{9}}\right|$$

$$1 = \left|\frac{1 + \lambda}{3}\right|$$

Squaring both the sides,

$$1 = \frac{(1 + \lambda)^2}{9}$$

$$9 = (1 + \lambda)^2$$

$$1 + \lambda = \pm 3$$

$$\Rightarrow \quad 1 + \lambda = 3 \quad \text{or} \quad 1 + \lambda = -3$$

$$\Rightarrow \quad \lambda = 2 \quad \text{or} \quad \lambda = -4$$

Put the value of λ in equation (ii) to get the equations of required planes,

$$x - 2y + 2z + 2 = 0$$

$$x - 2y + 2z - 4 = 0$$

Question 7

find the distance of the point $(2,3,5)$ from the xy-plane.

Solution 7

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(i)$$

So, distance of point $(2,3,5)$ from xy-plane (we know that equation of xy-plane is $z = 0$) is

$$= \left|\frac{(2)(0) + (3)(0) + (5)(1) + 0}{\sqrt{(0)^2 + (0)^2 + (1)^2}}\right| \qquad [\text{Using } (i)]$$

$$= \left|\frac{0 + 0 + 5}{\sqrt{0 + 0 + 1}}\right|$$

= 5 unit

Distance of the point $(2, 3, 5)$ from xy-plane = 5 unit

Question 8

find the distance of the point $(3,3,3)$ from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9 = 0$

Solution 8

We know that, distance (D) of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(i)$$

So, distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9 = 0$ is

$$D = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$

$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|\frac{9}{\sqrt{78}}\right|$$

Therefore, required distance is

$= \frac{9}{\sqrt{78}}$ units

Question 9

If the product of distances of the point $(1,1,1)$ from the origin and the plane $x - y + z + \lambda = 0$ be 5, find the value of λ.

Solution 9

Distance of point $(1,1,1)$ from origin is $\sqrt{3}$

Distance of point $(1,1,1)$ from plane is $\left|\frac{1+\lambda}{\sqrt{3}}\right|$

Product $= \left|\frac{1+\lambda}{\sqrt{3}}\right| \times \sqrt{3} = 5$

$|1+\lambda| = 5$

so λ=4 or -6

Question 10

Find an equation for the set of all points that are equidistant from the planes $3x - 4y + 12z = 6$ and $4x + 3z = 7$

Solution 10

Consider

$$3x-4y+12z-6=0 \qquad \text{...... (1)}$$

$$4x+3z-7=0 \qquad \text{...... (2)}$$

The distance of a point (x_1, y_1, z_1) from the plane $3x-4y+12z-6=0$ is

$$D_1=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$=\left|\frac{3x_1-4y_1+12z_1-6}{\sqrt{3^2+(-4)^2+12^2}}\right|$$

$$=\left|\frac{3x_1-4y_1+12z_1-6}{\sqrt{169}}\right|$$

$$=\left|\frac{3x_1-4y_1+12z_1-6}{13}\right|$$

The distance of the point (x_1, y_1, z_1) from the plane $4x+3z-7=0$ is

$$D_2=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$=\left|\frac{4x_1+3z_1-7}{\sqrt{4^2+3^2}}\right|$$

$$=\left|\frac{4x_1+3z_1-7}{\sqrt{25}}\right|$$

$$=\left|\frac{4x_1+3z_1-7}{5}\right|$$

Since the point (x_1, y_1, z_1) are equidistant from the planes $3x-4y+12z-6=0$ and $4x+3z-7=0$

So

$$D_1=D_2$$

$$\left|\frac{3x_1-4y_1+12z_1-6}{13}\right|=\left|\frac{4x_1+3z_1-7}{5}\right|$$

$$\frac{3x_1-4y_1+12z_1-6}{13}=\pm\frac{4x_1+3z_1-7}{5}$$

Taking positive sign

$$\frac{3x_1-4y_1+12z_1-6}{13}=\frac{4x_1+3z_1-7}{5}$$

$$15x_1-20y_1+60z_1-30=52x_1+39z_1-91$$

$$37x_1+20y_1-21z_1-61=0$$

Taking negative sign

$$\frac{3x_1-4y_1+12z_1-6}{13}=-\frac{4x_1+3z_1-7}{5}$$

$$15x_1-20y_1+60z_1-30=-52x_1-39z_1+91$$

$$67x_1-20y_1+99z_1-121=0$$

Question 11

Find the distance between the point (7, 2, 4) and the plane determined by the points *A* (2, 5, -3), *B* (-2, -3, 5) and *C* (5, 3, -3)

Solution 11

The equation of any plane passing through A(2,5, −3)

is $a(x-2)+b(y-5)+c(z+3)=0$....(1)

The above plane passes through the point B(−2, −3,5)

and hence, we have,

$a(-2-2)+b(-3-5)+c(5+3)=0$

$\Rightarrow -4a-8b+8c=0$...(2)

Again the required plane passes through the point C(5,3, −3)

and hence, we have,

$a(5-2)+b(3-5)+c(-3+3)=0$

$\Rightarrow 3a-2b+0c=0$...(3)

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{(-8)\times 0-(-2)\times 8}=\frac{b}{3\times 8-(-4)\times 0}=\frac{c}{(-4)\times(-2)-3\times(-8)}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{0+16}=\frac{b}{24+0}=\frac{c}{8+24}=\lambda$$

$$\Rightarrow \frac{a}{16}=\frac{b}{24}=\frac{c}{32}=\lambda$$

$$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\lambda$$

$\Rightarrow a=2\lambda, b=3\lambda$ and $c=4\lambda$

Substituting the above values in equation (1), we have,

$2\lambda(x-2)+3\lambda(y-5)+4\lambda(z+3)=0$

Since $\lambda \neq 0$, we have,

$2(x-2)+3(y-5)+4(z+3)=0$

$\Rightarrow 2x-4+3y-15+4z+12=0$

$\Rightarrow 2x+3y+4z-7=0$

Thus the equation of the plane is

$2x+3y+4z-7=0$

The distance from the point P(7,2,4) to the plane is

$$d=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$\therefore \text{Distance, } d=\left|\frac{2x+3y+4z-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{2\times 7+3\times 2+4\times 4-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{29}{\sqrt{29}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\sqrt{29}\text{ units}$$

Question 12

A plane makes intercepts -6, 3, 4 respectively on the coordinate axes. Find the length of the perpendicular from the origin on it.

Solution 12

Given that a plane is making intercepts $-6, 3$ and 4 respectively on the coordinate axes.
Thus the equation of the plane is

$$\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}=1 \quad \dots(1)$$

We need to find the length of the perpendicular from the origin on the plane.

If the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is at a distance 'p', then

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \quad \dots(2)$$

Comparing equation (1) with the general equation, we get,
$a=-6, b=3$ and $c=4$
Thus, equation (2) becomes,

$$\frac{1}{p^2}=\frac{1}{(-6)^2}+\frac{1}{3^2}+\frac{1}{4^2}$$

$$\Rightarrow \frac{1}{p^2}=\frac{4+16+9}{144}$$

$$\Rightarrow \frac{1}{p^2}=\frac{29}{144}$$

$$\Rightarrow p^2=\frac{144}{29}$$

$$\Rightarrow p=\frac{12}{\sqrt{29}} \text{ units}$$

Chapter 29 - The plane Exercise Ex. 29.10

Question 1

Find the distance between the parallel planes, $2x-y+3z-4=0$ and $6x-3y+9z+13=0$.

Solution 1

Let $p\left(x_1, y_1, z_1\right)$ be any point as the plane $2x - y + 3z - 4 = 0$, then

$$2x_1 - y_1 + 3z_1 - 4 = 0 \qquad ---(i)$$

We know that distance (D) of a point $\left(x_1, y_1, z_1\right)$ from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + (b)^2 + (c)^2}}\right| \qquad ---(ii)$$

Length of perpendicular from $P\left(x_1, y_1, z_1\right)$ to the plane $6x - 3y + 9z + 13 = 0$ is

$$= \left|\frac{6x_1 - 3y_1 + 9z_1 + 13}{\sqrt{(6)^2 + (-3)^2 + (9)^2}}\right| \qquad [\text{Using } (ii)]$$

$$= \left|\frac{3\left(2x_1 - y_1 + 3z_1\right) + 13}{\sqrt{36 + 9 + 81}}\right| \qquad [\text{Using } (i)]$$

$$= \left|\frac{3(4) + 13}{\sqrt{126}}\right|$$

$$= \left|\frac{12 + 13}{\sqrt{126}}\right|$$

$$= \left|\frac{25}{\sqrt{126}}\right|$$

$$= \frac{25}{3\sqrt{14}} \qquad \text{units}$$

Since P is the point on plane (i) and $\frac{25}{3\sqrt{14}}$ is the distance of P from plane

$6x - 3y + 9z + 13 = 0$, so

the distance between the parallel planes is $\frac{25}{3\sqrt{14}}$ units

Question 2

find the equation of the plane which passes through the point $(3, 4, -1)$ and is parallel to the plane $2x - 3y + 5z + 7 = 0$. also find the distance between the two planes.

Solution 2

Equation of plane which is parallel to 2x-3y+5z+7=0
is of the form 2x-3y+5z=d
Above plane is passing through (3, 4, -1)
So, substitute above point in the equation, we get
6-12-5=d
d=-11
So plane equation is 2x-3y+5z=-11
Distance between planes is given by

$$\left|\frac{-7+11}{\sqrt{4+9+25}}\right| = \frac{4}{\sqrt{38}}$$

Question 3

find the equation of the plane mid-parallel to the planes $2x - 2y + z + 3 = 0$ and $2x - 2y + z + 9 = 0$.

Solution 3

Given equations of planes are

$$2x - 2y + z + 3 = 0 \qquad ---(i)$$

$$2x - 2y + z + 9 = 0 \qquad ---(ii)$$

Let equation of the mid parallel plane is

$$2x - 2y + z + \lambda = 0 \qquad ---(iii)$$

Let $P(x_1, y_1, z_1)$ be any point on plane (iii),

$$2x_1 - 2y_1 + z_1 + \lambda = 0 \qquad ---(iv)$$

We know that, distance of a plane $ax + by + cz + d = 0$ from a point (x_1, y_1, z_1) is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(v)$$

Let D_1 be the distance of $P(x_1, y_1, z_1)$ from plane (i), so using equation (v),

$$D_1 = \left|\frac{2x_1 - 2y_1 + z_1 + 3}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}\right|$$

$$D_1 = \left|\frac{-\lambda + 3}{\sqrt{4 + 4 + 1}}\right| \qquad \text{[Using equation (iv)]}$$

$$D_1 = \left|\frac{3 - \lambda}{3}\right| \qquad ---(vi)$$

Again, let D_2 be the distance of point (x_1, y_1, z_1) from plane (ii), so using equation (v),

$$D_2 = \left|\frac{2x_1 - 2y_1 + z_1 + 9}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}\right|$$

$$D_2 = \left|\frac{-\lambda + 9}{\sqrt{9}}\right| \qquad \text{[Using equation (iv)]}$$

$$D_2 = \left|\frac{9 - \lambda}{3}\right| \qquad \text{---(vii)}$$

Since, $P\ (x_1, y_1, z_1)$ is a point on mid parallel plane, so

$$D_1 = D_2$$

$$\left|\frac{3 - \lambda}{3}\right| = \left|\frac{9 - \lambda}{3}\right| \qquad \text{[Using (vi),(vii)]}$$

Squaring both the sides,

$$\frac{(3 - \lambda)^2}{9} = \frac{(9 - \lambda)^2}{9}$$

$$9 - 6\lambda + \lambda^2 = 81 - 18\lambda + \lambda^2$$

$$-6\lambda + 18\lambda = 81 - 9$$

$$12\lambda = 72$$

$$\lambda = \frac{72}{12}$$

$$\lambda = 6$$

Put the value of λ in equation (iii),

$$2x - 2y + z + 6 = 0$$

So, equation of required plane is

$$2x - 2y + z + 6 = 0$$

Question 4

find the distance between the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7 = 0$ and $\vec{r}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7 = 0$.

Solution 4

Let the position vector of any point P on a plane $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)+7$ is $\vec{a}$, so

$$\vec{a}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)+7=0 \qquad ---(i)$$

We know that distance (D) of a point $\vec{a}$ from a plane $\vec{r}.\vec{n}-d=0$ is given by

$$D=\left|\frac{\vec{a}.\vec{n}-d}{\vec{n}}\right| \qquad ---(ii)$$

Length of perpendicular from $P\left(\vec{a}\right)$ to plane $\vec{r}.\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+7=0$ is given by

$$=\left|\frac{\vec{a}.\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+7}{\sqrt{(2)^2+(4)^2+(6)^2}}\right|$$

$$=\left|\frac{2\vec{a}\left(\hat{i}+2\hat{j}+3\hat{k}\right)+7}{\sqrt{4+16+36}}\right|$$

$$=\left|\frac{2.(-7)+7}{\sqrt{56}}\right| \qquad \left[\text{Using equation (i)}\right]$$

$$=\left|\frac{-14+7}{\sqrt{56}}\right|$$

$$=\frac{7}{\sqrt{56}}$$

Distance between two given plane

= Distance of $P\left(\vec{a}\right)$ from plane $\vec{r}.\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+7=0$

$$=\frac{7}{\sqrt{56}}$$

So, required distance $=\dfrac{7}{\sqrt{56}}$ units

Chapter 29 - The plane Exercise Ex. 29.11

Question 1

find the angle between the line $\vec{r}=\left(2\hat{i}+3\hat{j}+9\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=5$.

Solution 1

$$\vec{r} = (2\hat{i} + 3\hat{j} + 9\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$
$$\vec{r}.(\hat{i} + \hat{j} + k) = 5$$

Angle between line and plane is given by

$$\cos\theta = \left|\frac{2+3+4}{\sqrt{(1+1+1)(4+9+16)}}\right| = \frac{9}{\sqrt{87}}$$

Question 2

find the angle between the line $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$ and the plane $2x + y - z = 4$.

Solution 2

We know that the angle (θ) between the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Given, equation of line is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$$

So, $a_1 = 1$, $b_1 = -1$, $c_1 = 1$

Given equation of plane is $2x + y - z - 4 = 0$
So, $a_2 = 2$, $b_2 = 1$, $c_2 = -1$

Put these value in equation (i),

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$= \frac{(1)(2) + (-1)(1) + (1)(-1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(2)^2 + (1)^2 + (-1)^2}}$$
$$\sin\theta = \frac{2-1-1}{\sqrt{1+1+1}\sqrt{4+1+1}}$$
$$= \frac{0}{\sqrt{3}\sqrt{6}}$$
$$\sin\theta = 0$$

$$\theta = 0^\circ$$

angle between plane and line = 0°

Question 3

find the angle between the line joining the points $(3,-4,-2)$ and $(12,2,0)$ and the plane $3x-y+z=1$.

Solution 3

We know that angle (θ) between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given that, line is passing through

$A(3,-4,-2)$ and $B(12,2,0)$, so direction ratios of line AB

$=(12-3,\ 2+4,\ 0+2)$

$=(9,\ 6,\ +2)$

So, $a_1=9,\ b_1=6,\ c_1=2$ $\qquad ---(ii)$

Given equation of plane is $3x-y+z=1$

$a_2=3,\ b_2=-1,\ c_2=1$ $\qquad ---(iii)$

Using (ii) and (iii) in equation (i),

Angle (θ) between plane and line is

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{(9)(3)+(6)(-1)+(2)(1)}{\sqrt{(9)^2+(6)^2+(2)^2}\sqrt{(3)^2+(-1)^2+(1)^2}}$$

$$=\frac{27-6+2}{\sqrt{81+36+4}\sqrt{9+1+1}}$$

$$=\frac{23}{\sqrt{121}\sqrt{11}}$$

$$=\frac{23}{11\sqrt{11}}$$

$$\theta=\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

so, required angle between plane and line is given by

$$\theta=\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

Question 4

The line $\vec{r} = \hat{i} + \lambda\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$ is parallel to the plane $\vec{r}.\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 4$. Find m.

Solution 4

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to plane $\vec{r}.\vec{n} = d$ if

$\vec{b}.\vec{n} = 0$ $\qquad$ ---(i)

Given, equation of line is $\vec{r} = \hat{i} + \lambda\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$ and equation of plane

$\vec{r}.\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 4$

So $\quad \vec{b} = \left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$

$\vec{n} = \left(m\hat{i} + 3\hat{j} + \hat{k}\right)$

Put $\vec{b}$ and $\vec{n}$ in equation (i),

$\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 0$

$(2)(m) + (-m)(3) + (-3)(1) = 0$

$2m - 3m - 3 = 0$

$-m - 3 = 0$

$-m = 3$

$m = -3$

Question 5

Show that the line whose vector equation is $\vec{r} = 2\hat{i} + 5\hat{j} + 7\hat{k} + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ is parallel to the plane whose vector equation is $\vec{r}.\left(\hat{i} + \hat{j} - \hat{k}\right) = 7$. Also, find the distance between them.

Solution 5

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} = d$ is parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, equation of line $\vec{r} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and equation of plane $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 7$, so

$\vec{b} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + \hat{j} - \hat{k}$

Now,

$$\begin{aligned}\vec{b}.\vec{n} &= \left(\hat{i} + 3\hat{j} + 4\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right)\\ &= (1)(1) + (3)(1) + (4)(-1)\\ &= 1 + 3 - 4\\ &= 0\end{aligned}$$

Since $\vec{b}.\vec{n} = 0$ so using (i), we get

Given line and plane are parallel

We know that, distance (D) of a plane $\vec{r}\vec{n} - d = 0$ from a point $\vec{a}$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(ii)$$

We have to find distance between line and plane which is equal to the distance between point $\vec{a} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)$ from plane, so

$$\begin{aligned}D &= \left|\frac{\left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) - 7}{\sqrt{(1)^2 + (1)^2 + (-1)^2}}\right|\\ &= \left|\frac{(2)(1) + (5)(1) + (7)(-1) - 7}{\sqrt{1 + 1 + 1}}\right|\\ &= \left|\frac{2 + 5 - 7 - 7}{\sqrt{3}}\right|\\ &= \left|\frac{-7}{\sqrt{3}}\right|\end{aligned}$$

$$D = \frac{7}{\sqrt{3}}$$

So, required distance between plane and line is $D = \frac{7}{\sqrt{3}}$ unit

Question 6

Find the vector equation of the line through the origin which is perpendicular to the plane $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)=3$.

Solution 6

Required line is perpendicular to plane $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)=3$, so line is parallel to the normal vector $\vec{n}=\hat{i}+2\hat{j}+3\hat{k}$ of plane.

And it is passing through point $\vec{a}=0\hat{i}+0\hat{j}+0\hat{k}$.

We know that equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r}=\vec{a}+\lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a}=0\hat{i}+0\hat{j}+0.\hat{k}$ and $\vec{b}=\vec{n}=\hat{i}+2\hat{j}+3\hat{k}$

So, $\vec{r}=\left(0\hat{i}+0\hat{j}+0.\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

Hence, equation required line is

$$\vec{r}=\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

Question 7

Find the equation of the plane through $(2,3,-4)$ and $(1,-1,3)$ and parallel to x-axis.

Solution 7

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

So, equation of plane passing through $(2, 3, -4)$ is

$$a(x - 2) + b(y - 3) + c(z + 4) = 0 \qquad ---(ii) \qquad [\text{Using (i)}]$$

It is also passing through $(1, -1, 3)$, so,

$$a(1 - 2) + b(-1 - 3) + c(3 + 4) = 0$$
$$-a - 4b + 7c = 0$$
$$a + 4b - 7c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if
$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Here, equation (ii) is parallel to x-axis

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0} \qquad ---(v)$$

Using (ii) and (v) in equation (iv),

$$(a)(1) + (b)(0) + c(0) = 0$$
$$a = 0 \qquad ---(vi)$$

Put the value of a in equation (iii),

$$a - 4b + 7c = 0$$
$$0 - 4b + 7c = 0$$
$$-4b = -7c$$
$$4b = 7c$$

$$b = \frac{7}{4}c$$

Put the value of a and b in equation (ii),

$$a(x-2)+b(y-3)+c(z+4)=0$$
$$0(x-2)+\left(\frac{7}{4}c\right)(y-3)+c(z+4)=0$$
$$0+\frac{7cy}{4}-\frac{21c}{4}+\frac{cz}{1}+\frac{4c}{1}=0$$

$$7cy-21c+4cz+16c=0$$

Dividing by c,

$$7y+4z-5=0$$

Equation of required plane is

$$7y+4z-5=0$$

Question 8

Find the equation of a plane passing through the points $(0,0,0)$ and $(3,-1,2)$ and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

Solution 8

We know that equation a plane passing through the point (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given the required plane is passing through $(0,0,0)$, so using (i),

$$a(x - 0) + b(y - 0) + c(z - 0) = 0$$
$$ax + by + cz = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(3,-1,2)$,

$$3a - b + 2c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Given that, plane (ii) is parallel to line

$$\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}, \text{ so}$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(7) = 0$$
$$a - 4b + 7c = 0 \qquad ---(ii)$$

Solving equation (iii) and (v) by cross-multiplication, we get

$$\frac{a}{(-1)(7) - (-4)(2)} = \frac{b}{(1)(2) - (3)(7)} = \frac{c}{(3)(-4) - (1)(-1)}$$
$$\frac{a}{-7 + 8} = \frac{b}{2 - 21} = \frac{c}{-12 + 1}$$
$$\frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = \lambda \text{ (Say)}$$

$$\Rightarrow \qquad a = \lambda,\ b = -19\lambda,\ c = -11\lambda$$

Put the value of a, b, c in equation (ii),

$$ax + by + cz = 0$$
$$\lambda x - 19\lambda y - 11\lambda z = 0$$

Dividing by λ, we get

$$x - 19y - 11z = 0$$

Equation of required plane is

$$x - 19y - 11z = 0$$

Question 9

Find the equation of the line passing through $(1, 2, 3)$ and parallel to the planes $x - y + 2z = 5$ and $3x + y + z = 6$.

Solution 9

We know that equation of a line passing through (x_1, y_1, z_1) is given by

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \qquad ---(i)$$

Here, required line is passing through $(1,2,3)$, is given by, $\quad$ [Using (i)]

$$\frac{x-1}{a_1} = \frac{y-2}{b_1} = \frac{z-3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$$

Given, line (ii) is parallel to plane $x - y + 2z = 5$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(1) + (b_1)(-1) + (c_1)(2) = 0$

$$a_1 - b_1 + 2c_1 = 0 \qquad ---(iv)$$

Also, given line (ii) is parallel to plane $3x + y + z = 6$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$

$$3a_1 + b_1 + c_1 = 0 \qquad ---(v)$$

Solving (iv) and (v) by cross-multiplication,

$$\frac{a_1}{(-1)(1)-(1)(2)} = \frac{b_1}{(3)(2)-(1)(1)} = \frac{c_1}{(1)(1)-(3)(-1)}$$

$$\frac{a_1}{-1-2} = \frac{b_1}{6-1} = \frac{c_1}{1+3}$$

$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$

Put a_1, b_1, c_1 in equation (ii),

$$\frac{x-1}{-3\lambda} = \frac{y-2}{5\lambda} = \frac{z-3}{4\lambda}$$

Multiplying by λ,

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

Equation of required line is

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

The vector equation of the line is

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

Question 10

Prove that the line of section of the planes $5x + 2y - 4z + 2 = 0$ and $2x + 8y + 2z - 1 = 0$ is parallel to the plane $4x - 2y - 5z - 2 = 0$.

Solution 10

Firstly we have to find the line of section of planes $5x + 2y - 4z + 2 = 0$ and $2x + 8y + 2z - 1 = 0$
Let a_1, b_1, c_1 be the direction ratios of the line $5x + 2y - 4z + 2 = 0$ and $2x + 8y + 2z - 1 = 0$

Since, line lies in both the planes, so it is perpendicular to both planes, so

$5a_1 + 2b_1 - 4c_1 = 0 \qquad ---(i)$

$2a_1 + 8b_1 + 2c_1 = 0 \qquad ---(ii)$

Solving equation (i) and (ii), by cross-multiplication

$$\frac{a_1}{(2)(2) - (-4)(8)} = \frac{b_1}{(2)(-4) - (5)(2)} = \frac{c_1}{(5)(8) - (2)(2)}$$
$$\frac{a_1}{4 + 32} = \frac{b_1}{-8 - 10} = \frac{c_1}{40 - 4}$$
$$\frac{a_1}{36} = \frac{b_1}{-18} = \frac{c_1}{36}$$
$$\frac{a_1}{2} = \frac{b_1}{-1} = \frac{c_1}{2} = \lambda \text{ (Say)}$$

$\Rightarrow \qquad a_1 = 2\lambda,\ b_1 = -\lambda,\ c_1 = 2\lambda$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if
$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

Here line with direction ratio a_1, b_1, c_1 is parallel to plane $4x - 2y - 5z - 2 = 0$,

$a_1a_2 + b_1b_2 + c_1c_2$
$= (2)(4) + (-1)(-2) + (2)(-5)$
$= 8 + 2 - 10$

$= 0$

Therefore, line of section is parallel to the plane.

Question 11

Find the vector equation of the line passing through the point $(1, -1, 2)$ and perpendicular to the plane $2x - y + 3z - 5 = 0$.

Solution 11

Equation of line passing through $\vec{a}$ and parallel to $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(\text{i})$$

Given that, required line is passing through $(1, -1, 2)$ is,

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\vec{b} \qquad ---(\text{ii})$$

Since, line (i) is perpendicular to plane $2x - y + 3z - 5 = 0$, so normal to plane is parallel to the line.

In vector form,
$\vec{b}$ is parallel to $\vec{n} = 2\hat{i} - \hat{j} + 3\hat{k}$
So, $\vec{b} = \mu\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$ as μ is any scaler
Thus, equation of required line is,

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\mu\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right)$$

$$\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \delta\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

Question 12

Find the equation of the plane passing through the points $(2, 2, -1)$ and $(3, 4, 2)$ and parallel to the line whose direction ratios are $7, 0, 6$.

Solution 12

We know that, equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given that, required plane is passing through $(2, 2, -1)$, so using (i),

$$a(x - 2) + b(y - 2) + c(z + 1) = 0 \qquad ---(ii)$$

Given, plane (ii) is passing through $(3, 4, 2)$,

$$a(3 - 2) + b(4 - 2) + c(2 + 1) = 0$$
$$a + 2b + 3c = 0 \qquad ---(iii)$$

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_2} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_2}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$

Given that, plane (ii) is parallel to a line whose direction ratios are $7, 0, 6$
so using (iv), we get

$$(a)(7) + (b)(0) + (c)(6) = 0$$
$$7a + 0 + 6c = 0$$
$$7a + 6c = 0$$

$$a = -\frac{6c}{7}$$

Put the value of a in equation (iii),

$$a + 2b + 3c = 0$$
$$-\frac{6c}{7} + 2b + 3c = 0$$
$$-6c + 14b + 21c = 0$$
$$14b + 15c = 0$$

$$b = -\frac{15c}{14}$$

Put the value of a and b in equation (ii),

$$a(x-2)+b(y-2)+c(z+1)=0$$

$$\left(-\frac{6c}{7}\right)(x-2)+\left(-\frac{15c}{14}\right)(y-2)+c(z+1)=0$$

$$-\frac{6cx}{7}+\frac{12c}{7}-\frac{15cy}{14}+\frac{30c}{14}+cz+c=0$$

Multiplying by $\left(\frac{14}{c}\right)$, we get

$$-12x+24-15y+30+14z+14=0$$

$$-12x+15y+14z+68=0$$

Multiplying by (-1),

Equation of required plane is,

$$12x+15y-14z-68=0$$

Question 13

find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x+4y+z+5=0$.

Solution 13

We know that angle (θ) between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given line is $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and equation of plane is $3x+4y+z+5=0$, so angle between plane and line is,

$$\sin\theta=\frac{(3)(3)+(-1)(4)+(2)(1)}{\sqrt{(3)^2+(-1)^2+(2)^2}\sqrt{(3)^2+(4)^2+(1)^2}}$$

$$=\frac{9-4+2}{\sqrt{9+1+4}\sqrt{9+16+1}}$$

$$=\frac{7\times\sqrt{7}}{\sqrt{14}\sqrt{26}\times\sqrt{7}}$$

$$=\frac{7\sqrt{7}}{7\sqrt{52}}$$

$$\theta=\sin^{-1}\left(\sqrt{\frac{7}{52}}\right)$$

Question 14

Find the equation of the plane passing through the intersection of the planes $x-2y+z=1$ and $2x+y+z=8$ and parallel to the line with direction ratios proportional to 1,2,1. Find also the perpendicular distance of $(1,1,1)$ from this plane.

Solution 14

We know that equation of plane passing through the intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the intersection of two planes $x - 2y + z - 1 = 0$ and $2x + y + z - 8 = 0$ is given by

$$\left(x - 2y + z - 1\right) + \lambda\left(2x + y + z - 8\right) = 0$$
$$x - 2y + z - 1 + 2\lambda x + \lambda y + \lambda z - 8\lambda = 0$$
$$x\left(1 + 2\lambda\right) + y\left(-2 + \lambda\right) + z\left(1 + \lambda\right) - 1 - 8\lambda = 0 \qquad ---(\text{i})$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(\text{ii})$

Given that plane (i) is parallel to line with direction ratio 1,2,1, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(1)\left(1 + 2\lambda\right) + (2)\left(-2 + \lambda\right) + (1)\left(1 + \lambda\right) = 0$$
$$1 + 2\lambda - 4 + 2\lambda + 1 + \lambda = 0$$
$$5\lambda - 2 = 0$$

$$\lambda = \frac{2}{5}$$

Put the value of λ in equation (i),

$$x\left(1 + \frac{4}{5}\right) + y\left(-2 + \frac{2}{5}\right) + z\left(1 + \frac{2}{5}\right) - 1 - \frac{16}{5} = 0$$

Multiplying by 5,

$$x\left(5 + 4\right) + y\left(-10 + 2\right) + z\left(5 + 2\right) - 5 - 16 = 0$$
$$9x - 8y + 7z - 21 = 0$$

So, equation of required plane is

$$9x - 8y + 7z - 21 = 0 \qquad ---(\text{iii})$$

We know that distance (D) of a point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is given by

$$D = \frac{ax_1 + by_1 + cz_1 + d_1}{\sqrt{a^2 + b^2 + c^2}}$$

So, distance of point $(1,1,1)$ from plane (i) is given by

$$D = \left|\frac{(9)(1) + (-8)(1) + (7)(1) - 21}{\sqrt{(9)^2 + (-8)^2 + (7)^2}}\right|$$

$$= \left|\frac{9 - 8 + 7 - 21}{\sqrt{81 + 64 + 49}}\right|$$

$$= \left|\frac{16 - 29}{\sqrt{194}}\right|$$

$$= \left|\frac{-13}{\sqrt{194}}\right|$$

$$D = \frac{13}{\sqrt{194}} \text{ units}$$

Question 15

State when the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to the plane $\vec{r}.\vec{n} = d$. Show that the line $\vec{r} = \hat{i} + \hat{j} + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ is paralle to the plane $\vec{r}.\left(2\hat{j} + \hat{k}\right) = 3$. Also, find the distance between the line and the plane.

Solution 15

We know that line $\vec{r} = \vec{a} + \lambda\vec{b}$ is paralle to plane $\vec{r}.\vec{n} = d$ if

$\vec{b}.\vec{n} = 0$

Given, line is $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and plane is $\vec{r}.\left(2\hat{j} + \hat{k}\right) = 3$, so

$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{a} = \left(\hat{i} + \hat{j}\right)$ and $\vec{n} = \left(2\hat{j} + \hat{k}\right)$

$$\begin{aligned}\text{Now, } \vec{b}\vec{n} &= \left(3\hat{i} - \hat{j} + 2\hat{k}\right)\left(2\hat{j} + \hat{k}\right)\\ &= (3)(0) + (-1)(2) + (2)(1)\\ &= 0 - 2 + 2\\ &= 0\end{aligned}$$

Since, $\vec{b}.\vec{n} = 0$, so line is parallel to plane

Distance between point $\vec{a}$ and plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right| \qquad ---(i)$$

$\vec{a}$ is a point on the line. So distance between line and plane is equal to the distance between $\vec{a} = \left(\hat{i} + \hat{j}\right)$ and plane $\vec{r}.\left(2\hat{j} + \hat{k}\right) = 3$, so using (i),

$$\begin{aligned}D &= \left|\frac{\left(\hat{i} + \hat{j}\right)\left(2\hat{j} + \hat{k}\right) - 3}{\sqrt{(2)^2 + (1)^2}}\right|\\ &= \left|\frac{(1)(0) + (1)(2) + (0)(1) - 3}{\sqrt{4 + 1}}\right|\\ &= \left|\frac{0 + 2 + 0 - 3}{\sqrt{5}}\right|\\ &= \left|\frac{-1}{\sqrt{5}}\right|\\ &= \frac{1}{\sqrt{5}} \text{ unit}\end{aligned}$$

So, required distance $= \frac{1}{\sqrt{5}}$ unit

Question 16

Show that the plane whose vector equation is $\vec{r}.\left(\hat{i}+2\hat{j}-\hat{k}\right)=1$ and the line whose vector equation is $\vec{r}=\left(-\hat{i}+\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+4\hat{k}\right)$ are parallel. Also find the distance between them.

Solution 16

We know that line $\vec{r} = \vec{a} + \lambda \vec{b}$ and plane $\vec{r}.\vec{n} - d = 0$ are parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, line $\vec{r} = \left(-\hat{i} + \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$ and plane is $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1 = 0$

So, $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$

Now, $\vec{b}.\vec{n} = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$

$= (2)(1) + (1)(2) + (4)(-1)$

$= 2 + 2 - 4$

$= 0$

Since, $\vec{b}.\vec{n} = 0$, so by equation (i), line is parallel to plane

Distance (D) between point $\vec{a}$ and plane $\vec{r}\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right| \qquad ---(ii)$$

Distance between given line and plane

= Distance of point $\vec{a} = \left(-\hat{i} + \hat{j} + \hat{k}\right)$ from $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1 = 0$

$D = \left|\frac{\vec{a}\vec{n} - d}{\left|\vec{n}\right|}\right|$

$= \left|\frac{\left(-\hat{i} + \hat{j} + \hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right) - 1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}}\right|$

$= \left|\frac{(-1)(1) + (1)(2) + (1)(-1) - 1}{\sqrt{1 + 4 + 1}}\right|$

$= \left|\frac{-1 + 2 - 1 - 1}{\sqrt{6}}\right|$

$= \left|\frac{-1}{\sqrt{6}}\right|$

$= \frac{1}{\sqrt{6}}$

So, required distance $= \frac{1}{\sqrt{6}}$ units

Question 17

Find the equation of the plane through the intersection of the planes $3x - 4y + 5z = 10$ and $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

Solution 17

We know that equation of plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0 \qquad ---(i)$$

So, equation of plane passing through the line of intersection of planes $3x - 4y + 5z - 10 = 0$ and $2x + 2y - 3z - 4 = 0$ is,

$$(3x - 4y + 5z - 10) + \lambda(2x + 2y - 3z - 4) = 0$$
$$(3 + 2\lambda)x + (-4 + 2\lambda)y + (5 - 3\lambda)z - 10 - 4\lambda = 0 \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ parallel to plane

$a_2x + b_2y + c_2z + d_2 = 0$ if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

Given that, plane (ii) is parallel to line $x = 2y = 3z$ or $\frac{x}{6} = \frac{y}{3} = \frac{z}{2}$

So,

$$(6)(3 + 2\lambda) + (3)(-4 + 2\lambda) + (2)(5 - 3\lambda) = 0$$
$$18 + 12\lambda - 12 + 6\lambda + 10 - 6\lambda = 0$$
$$12\lambda + 16 = 0$$
$$\lambda = -\frac{16}{12}$$

$$\lambda = -\frac{4}{3}$$

Put λ in equation (ii),

$$x(3 + 2\lambda) + y(-4 + 2\lambda) + z(5 - 3\lambda) - 10 - 4\lambda = 0$$
$$x\left(3 - \frac{8}{3}\right) + y\left(-4 - \frac{8}{3}\right) + z\left(5 + \frac{12}{3}\right) - 10 + \frac{16}{3} = 0$$

Multiplying by 3,

$$x(9 - 8) + y(-12 - 8) + z(15 + 12) - 30 + 16 = 0$$
$$x - 20y + 27z - 14 = 0$$

Equation of required plane is given by

$$x - 20y + 27z - 14 = 0$$

Question 18

Find the vector and cartesian forms of the equation of the plane passing through the point (1, 2, −4) and parallel to the lines $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$. Also, find the distance of the point (9, −8, −10) from the plane thus obtained.

Solution 18

The plane passes through the point $\vec{a}(1, 2, -4)$

A vector in a direction perpendicular to

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

is $\vec{n} = (2\hat{i} + 3\hat{j} + 6\hat{k}) \times (\hat{i} + \hat{j} - \hat{k})$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = -9\hat{i} + 8\hat{j} - \hat{k}$$

Equation of the plane is $(\vec{r} - \vec{a}).\vec{n} = 0$

$(\vec{r} - (\hat{i} + 2\hat{j} - 4\hat{k})).(-9\hat{i} + 8\hat{j} - \hat{k}) = 0$

$\Rightarrow \vec{r}.(-9\hat{i} + 8\hat{j} - \hat{k}) = 11$

Substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get the Cartesian form as

$-9x + 8y - z = 11$

The distance of the point (9, −8, −10) from the plane

$$= \left| \frac{-9(9) + 8(-8) - (-10) - 11}{\sqrt{9^2 + 8^2 + 1^2}} \right| = \frac{146}{\sqrt{146}} = \sqrt{146}$$

Question 19

Find the equation of the plane passing through the points $(3, 4, 1)$ and $(0, 1, 0)$ and parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$.

Solution 19

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) \qquad ---(i)$$

Given that, required equation of plane is passing through $(3,4,1)$, so

$$a(x - 3) + b(y - 4) + c(z - 1) = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(0,1,0)$, so

$$a(0 - 3) + b(1 - 4) + c(0 - 1) = 0$$
$$-3a - 3b - c = 0$$

$$3a + 3b + c = 0 \qquad ---(iii)$$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, line $\frac{x + 3}{2} = \frac{y - 3}{7} = \frac{z - 2}{5}$ is parallel to plane (ii), so

$$(2)(a) + (7)(b) + (c)(5) = 0$$
$$2a + 7b + 5c = 0 \qquad ---(iv)$$

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(3)(5) - (7)(1)} = \frac{b}{(2)(1) - (3)(5)} = \frac{c}{(3)(7) - (2)(3)}$$
$$\frac{a}{15 - 7} = \frac{b}{2 - 15} = \frac{c}{21 - 6}$$

$$\frac{a}{8} = \frac{b}{-13} = \frac{c}{15} = \lambda\,(\text{Say})$$

$\Rightarrow \quad a = 8\lambda,\ b = -13\lambda,\ c = 15\lambda$

Put a, b, c in equation (ii),

$$a(x-3)+b(y-4)+c(z-1)=0$$
$$8\lambda(x-3)+(-13\lambda)(y-4)+(15\lambda)(z-1)=0$$
$$8\lambda x-24\lambda-13\lambda y+52\lambda+15\lambda z-15\lambda=0$$

$$8\lambda x-13\lambda y+15\lambda z+13\lambda=0$$

Dividing by λ, equation of required plane is,

$$8x-13y+15z+13=0$$

Question 20

Find the coordinates of the point where the line

$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane $x-y+z-5=0$.

Also, find the angle between the line and the plane.

Solution 20

Find the coordinates of the point where the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=r$$

$\Rightarrow x=3r+2, y=4r-1, z=2r+2$

Substituting in the equation of the plane $x-y+z-5=0$,

we get $(3r+2)-(4r-1)+(2r+2)-5=0$

$\Rightarrow r=0$

$\Rightarrow x=2, y=-1, z=2$

Direction ratios of the line are $3,4,2$

Direction ratios of a line perpendicular to the plane are $1,-1,1$

$$\sin\theta=\frac{3\times1+4\times-1+2\times1}{\sqrt{9+16+4}\sqrt{1+1+1}}=\frac{1}{\sqrt{87}}$$

$$\theta=\sin^{-1}\frac{1}{\sqrt{87}}$$

Question 21

Find the vector equation of the straight line passing through $(1,2,3)$ and perpendicular to the plane $\vec{r}.\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9=0$.

Solution 21

We know that equation of line passing through point $\vec{a}$ and parallel to vector $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda \vec{b} \qquad ---(i)$$

Given that, line is passing through $(1,2,3)$.

So, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

It is given that line is perpendicular to plane $\vec{r}.\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) + 9 = 0$

So, normal to plane $\left(\vec{n}\right)$ is parallel to $\vec{b}$.

So, let $\vec{b} = \mu\vec{n} = \mu\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$

Put $\vec{a}$ and $\vec{b}$ in (i), equation of line is,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left[\mu\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)\right]$$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \delta\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) \qquad [\text{As } \delta = \lambda\mu]$$

Equation of required line is,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \delta\left(\hat{i} + 2\hat{j} - 5\hat{k}\right)$$

Question 22

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

and the plane $10x + 2y - 11z = 3$

Solution 22

Direction ratios of the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

are $(2,3,6)$

Direction ratio of a line perpendicular to the plane

$10x + 2y - 11z = 3$ are $10, 2, -11$

If θ is the angle between the line and the plane, then

$$\sin\theta = \frac{2 \times 10 + 3 \times 2 + 6 \times -11}{\sqrt{2^2 + 3^2 + 6^2}\sqrt{10^2 + 2^2 + 11^2}} = -\frac{40}{\sqrt{49}\sqrt{225}} = -\frac{40}{7 \times 15} = -\frac{8}{21}$$

$$\Rightarrow \theta = \sin^{-1}\left(-\frac{8}{21}\right)$$

Question 23

Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes $\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$.

Solution 23

We know that, equation of line passing through (x_1, y_1, z_1) is given by

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \qquad ---(i)$$

Given that, required line is passing through $(1, 2, 3)$, so

$$\frac{x - 1}{a_1} = \frac{y - 2}{b_1} = \frac{z - 3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given line (ii) is parallel to

$$\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5$$
$$\Rightarrow \quad x - y + 2z - 5 = 0, \text{ so}$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a_1)(1) + (b_1)(-1) + (c_1)(2) = 0$$

$$a_1 - b_1 + 2c_1 = 0 \qquad ---(iii)$$

Line (ii) is also parallel to plane

$$\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$$
$$\Rightarrow \quad 3x + y + z - 6 = 0, \text{ so}$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$$

$$3a_1 + b_1 + c_1 = 0 \qquad ---(iv)$$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a_1}{(-1)(1) - (2)(1)} = \frac{b_1}{(3)(2) - (1)(1)} = \frac{c_1}{(1)(1) - (3)(-1)}$$
$$\frac{a_1}{-1 - 2} = \frac{b_1}{6 - 1} = \frac{c_1}{1 + 3}$$
$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$

Put a_1, b_1, c_1 in equation (ii), so, equation line is given by

$$\frac{x-1}{-3\lambda} = \frac{y-2}{5\lambda} = \frac{z-3}{4\lambda}$$
$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

So, vector equation of required line is

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

Question 24

Find the value of λ such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane $3x - y - 2z = 7$.

Solution 24

Here, given mid line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$

So, normal vector of plane is parallel to line so,

Direction ratios of normal to plane are proparional to the direction ratios of line

Here,

$$\frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$$

cross multiplying the last two

$$-2\lambda = 4$$
$$\lambda = \frac{4}{-2}$$

$$\lambda = -2$$

Question 25

Find the equation of the plane passing through the points (-1, 2, 0),(2, 2, -1) and parallel to the line

$$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{1}.$$

Solution 25

The equation of a plane passing through $(-1, 2, 0)$ is

$a(x+1)+b(y-2)+c(z-0)=0$.............(i)

This passes through $(2, 2, -1)$

$\therefore\ a(2+1)+b(2-2)+c(-1-0)=0$

$3a-c=0$............(ii)

The plane in (i) is parallel to $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{1}$.

Theefore normal to the plane is perpendicular to the line.

$\therefore\ a(1)+b(2)+c(1)=0$.........(iii)

Solving (ii) and (iii) by cross multiplication we get,

$$\frac{a}{0-(-1)(2)}=\frac{b}{(1)(-1)-(3)(1)}=\frac{c}{(3)(2)-0}$$

$$\Rightarrow \frac{a}{2}=-\frac{b}{4}=\frac{c}{6}$$

$$\Rightarrow a=-\frac{b}{2}=\frac{c}{3}=\lambda(\text{say})$$

$$\Rightarrow a=\lambda, b=-2\lambda, c=3\lambda$$

Substituting $a=\lambda, b=-2\lambda, c=3\lambda$ in (i) we get,

$\lambda(x+1)-2\lambda(y-2)+3\lambda(z-0)=0$

$x-2y+3z+5=0$

$\therefore$ The required equation of the plane is $x-2y+3z+5=0$.

Chapter 29 - The plane Exercise Ex. 29.12

Question 1(i)

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the yz-plane.

Solution 1(i)

Direction ratios of the given line are

$(5-3, 1-4, 6-1)=(2,-3,5)$

Hence, equation of the line is

$$\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-6}{5}=r$$

$\Rightarrow x=2r+5, y=-3r+1, z=5r+6$

For any point on the yz $-$ plane $x=0$

$\Rightarrow 2r+5=0 \Rightarrow r=-\frac{5}{2}$

$y=-3(-\frac{5}{2})+1=\frac{17}{2}$

$z=5(-\frac{5}{2})+6=-\frac{13}{2}$

Hence, the coordinates of the point are $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$.

Question 1(ii)

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the zx-plane.

Solution 1(ii)

Direction ratios of the given line are

$(5-3, 1-4, 6-1) = (2, -3, 5)$

Hence, equation of the line is

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$$

$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$

For any point on $zx-$plane $y = 0$

$\Rightarrow -3r+1 = 0 \Rightarrow r = \frac{1}{3}$

$$x = 2\left(\frac{1}{3}\right)+5 = \frac{17}{3}$$

$$z = 5\left(\frac{1}{3}\right)+6 = \frac{23}{3}$$

Hence, the coordinates of the point are $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

Question 2

Find the coordinates of the point where the line through $(3, -4, -5)$ *and* $(2, -3, 1)$ *crosses the plane* $2x+y+z=7$

Solution 2

Let the coordinates of the points A and B be
$(3, -4, -5)$ *and* $(2, -3, 1)$ *repectively.*

Equation of the line joining the points (x_1, y_1, z_1) *and* (x_2, y_2, z_2) *is*

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = r, \text{ where } r \text{ is some constant.}$$

Thus equation of AB is

$$\frac{x-3}{2-3} = \frac{y-(-4)}{(-3)-(-4)} = \frac{z-(-5)}{1-(-5)} = r$$

$$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = r$$

Any point on the line AB is of the form

$-r+3,\ r-4,\ 6r-5$

Let P be the point of intersection of the line AB and the plane $2x+y+z=7$

Thus, we have,

$2(-r+3)+\ r-4+6r-5=7$

$\Rightarrow -2r+6+\ r-4+6r-5=7$

$\Rightarrow 5r=10$

$\Rightarrow r=2$

Substituting the value of r in $-r+3,\ r-4,\ 6r-5$*, the coordinates of P are:*

$(-2+3,\ 2-4,\ 6\times 2-5) = (1, -2, 7)$

Question 3

Find the distance of the point $(-1, -5, -10)$ *from the point of intersection of the line* $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ *and the plane* $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

Solution 3

The equation of the given line is

$$\vec{r} \cdot = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad ...(1)$$

The equation of the given plane is

$$\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5 \quad ...(2)$$

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})\right] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$\Rightarrow \left[(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}\right] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$\Rightarrow \lambda = 0$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates, (2, −1, 2). The point is (−1, −5, −10).

The distance d between the points, (2, −1, 2) and (−1, −5, −10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

Question 4

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$.

Solution 4

To find the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$, we substitute $\vec{r}$ of line in the plane.

$$[2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow [(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$$

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + 4(3\hat{i} + 4\hat{j} + 2\hat{k}) = 14\hat{i} + 12\hat{j} + 10\hat{k}$$

Hence, the distance of the point $2\hat{i} + 12\hat{j} + 5\hat{k}$ from $14\hat{i} + 12\hat{j} + 10\hat{k}$ is

$$\sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

Question 5

Find the distance of the point *P* (-1, -5, -10) from the point of intersection of the line joining the points *A* (2, -1, 2) and *B* (5, 3, 4) with the plane x - y + z = 5.

Solution 5

Equation of the line through the points A(2, − 1,2)

and B(5,3,4) is $\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2} = r$

$\Rightarrow \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$

$\Rightarrow x = 3r + 2, y = 4r - 1, z = 2r + 2$

Substituting these in the plane equation we get

$(3r + 2) - (4r - 1) + (2r + 2) = 5$

$\Rightarrow r = 0$

$\Rightarrow x = 2, y = -1, z = 2$

Distance of (2, − 1,2) from (− 1, − 5, − 10) is

$= \sqrt{(2-(-1))^2 + (-1-(-5))^2 + (2-(-10))^2} = \sqrt{3^2 + 4^2 + 12^2}$

$= \sqrt{169} = 13$

Question 6

Find the distance of the point P(3, 4,4) from the point, where the line joining the points A(3, -4, -5) and B (2, -3, 1) intersects the plane 2x + y + z =7.

Solution 6

The equation of a line joining the points A(3, -4, -5) and B(2, -3, 1)is

$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} = r$

$\Rightarrow x = 3 - r, y = -4 + r, z = -5 + 6r$

Substituting this into the given plane equation we get,

$2(3-r) + (-4 + r) + (-5 + 6r) = 7$

$\Rightarrow r = 2$

$\Rightarrow x = 1,\ y = -2,\ z = 7$

Distance of (1, -2, 7) from (3, 4, 4) is

$= \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2}$

$= \sqrt{4 + 36 + 9}$

$= \sqrt{49}$

$= 7$

Chapter 29 - The plane Exercise Ex. 29.13

Question 1

Show that the line $\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ are coplanar. Also, find the equation of the plane containing them.

Solution 1

$\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

We know that the lines,

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are coplanar if

$\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$

and the equation of the plane containing them is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

Here

$\vec{a_1} = 2\hat{j} - 3\hat{k}, \vec{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Therefore, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 0 + 4 + 3 = 7$

and

$\vec{a_2} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -2 + 12 - 3 = 7$

Since $\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$, the lines are coplanar.

Now the equation of the plane containing the given lines is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

$\vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = -7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + 7 = 0$

Question 2

Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar.
Also, find the equation of the plane containing them.

Solution 2

We know that lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

And equation of plane containing them is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

Here, equation of lines are

$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$

So, $x_1=-1$, $y_1=3$, $z_1=-2$, $l_1=-3$, $m_1=2$, $n_1=1$
$x_2=0$, $y_2=7$, $z_2=-7$, $l_2=1$, $m_2=-3$, $n_2=2$

So,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 0+1 & 7-3 & -7+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$= 1(4+3) - 4(-6-1) - 5(9-2)$

$= 7 + 28 - 35$

$= 0$

So, lines are coplanar.

Equation of plane containing line is

$$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$(x+1)(4+3) - (y-3)(-6-1) + (z+2)(9-2) = 0$

$7x + 7 + 7y - 21 + 7z + 14 = 0$

$7x + 7y + 7z = 0$

Question 3

Find the equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point $(0, 7, -7)$ and show that the line $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ also lies in the same plane.

Solution 3

We know that the plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Required plane is passing through $(0, 7, -7)$, so

$$a(x - 0) + b(y - 7) + c(z + 7) = 0$$
$$ax + b(y - 7) + c(z + 7) = 0 \qquad ---(ii)$$

Plane (ii) also contains line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ so, it passes through point $(-1, 3, -2)$,

$$a(-1) + b(3 - 7) + c(-2 + 7) = 0$$
$$-a - 4b + 5c = 0 \qquad ---(iii)$$

Also, plane (ii) will be parallel to line
so, $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$$(a)(-3) + (b)(2) + (c)(1) = 0$$
$$-3a + 2b + c = 0 \qquad ---(iv)$$

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(-4)(1)-(5)(2)} = \frac{b}{(-3)(5)-(-1)(1)} = \frac{c}{(-1)(2)-(-4)(-3)}$$
$$\frac{a}{-4-10} = \frac{b}{-15+1} = \frac{c}{-2-12}$$

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{14} = \lambda\,(\text{Say})$$

$$\Rightarrow \quad a = -14\lambda,\ b = -14\lambda,\ c = -14\lambda$$

Put a, b, c in equation (ii),

$$ax + b(y-7) + c(z+7) = 0$$
$$(-14\lambda)x + (-14\lambda)(y-7) + (-14\lambda)(z+7) = 0$$

Dividing by (-14λ), we get

$$x + y - 7 + z + 7 = 0$$
$$x + y + z = 0$$

So, equation of plane containing the given point and line is $x + y + z = 0$

The other line is $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$$(1)(1) + (1)(-3) + (1)(2) = 0$$
$$1 - 3 + 2 = 0$$
$$0 = 0$$

$$LHS = RHS$$

So, $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ lie on plane $x + y + z = 0$

Question 4

Find the equation of the plane which contains two parallel lines $\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$ and $\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$.

Solution 4

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Since, required plane contain lines

$$\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5} \text{ and } \frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$$

So, required plane passes through $(4, 3, 2)$ and $(3, -2, 0)$, so, equation of required plane is,

$$a(x - 4) + b(y - 3) + c(z - 2) = 0 \qquad ---(ii)$$

Plane (ii) also passes through $(3, -2, 0)$, so

$$a(3 - 4) + b(-2 - 3) + c(0 - 2) = 0$$
$$-a - 5b - 2c = 0$$
$$a + 5b + 2c = 0 \qquad ---(iii)$$

Now plane (ii) is also parallel to line with direction ratios $1, -4, 5$, so,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(5) = 0$$
$$a - 4b + 5c = 0 \qquad ---(iv)$$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a}{(5)(5)-(-4)(2)}=\frac{b}{(1)(2)-(1)(5)}=\frac{c}{(1)(-4)-(1)(5)}$$
$$\frac{a}{25+8}=\frac{b}{2-5}=\frac{c}{-4-5}$$

$$\frac{a}{33}=\frac{b}{-3}=\frac{c}{-9}$$

Multiplying by 3,

$$\frac{a}{11}=\frac{b}{-1}=\frac{c}{-3}=\lambda\left(\text{Say}\right)$$

$$\Rightarrow \quad a=11\lambda,\ b=-\lambda,\ c=-3\lambda$$

Put a, b, c in equation (ii),

$$a(x-4)+b(y-3)+c(z-2)=0$$
$$(11\lambda)(x-4)+(-\lambda)(y-3)+(-3\lambda)(z-2)=0$$
$$11\lambda x-44\lambda-\lambda y+3\lambda-3\lambda z+6\lambda=0$$

$$11\lambda x-\lambda y-3\lambda z-35\lambda=0$$

Dividing by λ,

$$11x-y-3z-35=0$$

So, equation of required plane is,

$$11x-y-3z-35=0$$

Question 5

Show that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and $3x-2y+z+5=0=2x+3y+4z-4$ intersect. Find the equation of the plane in which they lie and also their point of intersection.

Solution 5

We have, equation of line is

$$\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2} = \lambda \text{ (Say)}$$

General point on this line is given by

$$(3\lambda - 4,\ 5\lambda - 6,\ -2\lambda + 1) \qquad ---(i)$$

Another equation of line is

$$3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$$

Let a, b, c be the direction ratio of the line so, it will be perpendicular to normal of $3x - 2y + z + 5 = 0$ and $2x + 3y + 4z - 4 = 0$, so

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(3)(a) + (-2)(b) + (1)(c) = 0$$

$$3a - 2b + c = 0 \qquad ---(ii)$$

Again,

$$(2)(a) + (3)(b) + (4)(c) = 0$$

$$2a + 3b + 4c = 0 \qquad ---(iii)$$

Solving (ii) and (iii) by cross-multiplication,

$$\frac{a}{(-2)(4) - (3)(1)} = \frac{b}{(2)(1) - (3)(4)} = \frac{c}{(3)(3) - (-2)(2)}$$

$$\frac{a}{-8-3}=\frac{b}{2-12}=\frac{c}{9+4}$$

$$\frac{a}{-11}=\frac{b}{-10}=\frac{c}{13}$$

Direction ratios are proportional to $-11,-10,13$
Let $z=0$, so

$$3x-2y=-5 \qquad ---(A)$$
$$2x+3y=4 \qquad ---(B)$$

Solving (A) and (B),

$$\begin{array}{rrcr} & 6x-4y & = & -10 \\ - & \underset{-}{6x}+9y & = & \underset{-}{12} \\ \hline & -13y & = & -12 \end{array}$$

$$y=\frac{22}{13}$$

Put y in equation (A),

$$3x-2\left(\frac{22}{13}\right)=-5$$
$$3x-\frac{44}{13}=-5$$
$$3x=-5+\frac{44}{13}$$

$$3x = -\frac{21}{13}$$
$$x = -\frac{7}{13}$$

So, equation of line (2) in symmetrical form,

$$\frac{x+\frac{7}{13}}{-11} = \frac{y-\frac{22}{13}}{-10} = \frac{z-0}{13}$$

Put the general point of line (1) from equation (1)

$$\frac{3\lambda - 4 + \frac{7}{13}}{-11} = \frac{5\lambda - 6 - \frac{22}{13}}{-10} = \frac{-2\lambda + 1}{13}$$

$$\frac{39\lambda - 52 + 7}{-11 \times 13} = \frac{65\lambda - 78 - 22}{-10 \times 13} = \frac{-2\lambda + 1}{13}$$

$$\frac{39\lambda - 45}{-11} = \frac{65\lambda - 100}{-10} = \frac{-2\lambda + 1}{1}$$

The equation of the plane is $45x - 17y + 25z + 53 = 0$

Their point of intersection is $(2, 4, -3)$

Question 6

Show that the plane whose vector equation is $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) = 3$ contains the line whose vector equation is $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$.

Solution 6

We know that plane $\vec{r}.\vec{n} = d$ contains the line $\vec{r} = \vec{a} + \lambda\vec{b}$ if

(i) $\vec{b}.\vec{n} = 0$ (ii) $\vec{a}.\vec{n} = d$ --- (i)

Given, equation of plane $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) = 3$ and equation of line $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$

so, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{a} = \hat{i} + \hat{j}$

$d = 3$ $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$

$$\vec{b}.\vec{n} = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (2)(1) + (1)(2) + (4)(-1)$$
$$= 2 + 2 - 4$$

$$\vec{b}.\vec{n} = 0$$

$$\vec{a}.\vec{n} = \left(\hat{i} + \hat{j}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (1)(1) + (1)(2) + (0)(-1)$$
$$= 1 + 2 - 0$$
$$= 3$$

$$= d$$

since, $\vec{b}.\vec{n} = 0$ and $\vec{a}.\vec{n} = d$, so, from (i)

Given line lie on the given plane.

Question 7

Find the equation of the plane determined by the intersection of the lines $\frac{x+3}{3} = \frac{y}{-2} = \frac{z-7}{6}$ and $\frac{x+6}{1} = \frac{y+5}{-3} = \frac{z-1}{2}$

Solution 7

Let $L_1 : \frac{x+3}{3} = \frac{y}{-2} = \frac{z-7}{6}$ and

$L_2 : \frac{x+6}{1} = \frac{y+5}{-3} = \frac{z-1}{2}$ be the equations of two lines

Let the plane be $ax + by + cz + d = 0$...(1)

Given that the required plane passes through the intersection of the lines L_1 and L_2.

Hence the normal to the plane is perpendicular to the lines L_1 and L_2.

$\therefore 3a - 2b + 6c = 0$

$a - 3b + 2c = 0$

Using cross-multiplication, we get,

$$\frac{a}{-4+18} = \frac{b}{6-6} = \frac{c}{-9+2}$$

$$\Rightarrow \frac{a}{14} = \frac{b}{0} = \frac{c}{-7}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{0} = \frac{c}{-1}$$

Question 8

Find the vector equation of the plane passing through the points $(3,4,2)$ and $(7,0,6)$ and perpendicular to the plane $2x-5y-15=0$. Also show that the plane thus obtained contains the line $\bar{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$

Solution 8

Let the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$........(i)

Plane is passing through (3,4,2) and (7,0,6)

$$\frac{3}{a}+\frac{4}{b}+\frac{2}{c}=1$$

$$\frac{7}{a}+\frac{0}{b}+\frac{6}{c}=1$$

Required plane is perpendicular to $2x-5y-15=0$

$$\frac{2}{a}+\frac{-5}{b}+\frac{0}{c}=0$$

$$\Rightarrow 2b=5a$$

$$\therefore b=2.5a$$

$$\frac{3}{a}+\frac{4}{2.5a}+\frac{2}{c}=1$$

$$\frac{7}{a}+\frac{6}{c}=1$$

Solving the above 2 equations,

$a=3.4=\frac{17}{5}$, $b=\ 8.5=\frac{17}{2}$ and $c=\frac{-34}{6}=-\frac{17}{3}$

Substituting the values in (i)

$$\frac{x}{\frac{17}{5}}+\frac{y}{\frac{17}{2}}+\frac{z}{-\frac{17}{3}}=1$$

$$\Rightarrow \frac{5x}{17}+\frac{2y}{17}-\frac{3z}{17}=1$$

$$\Rightarrow 5x+2y-3z=17$$

$$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(5\hat{i}+2\hat{j}-3\hat{k})=17$$

$$\Rightarrow \vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$$

Vector equation of the plane is $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The line passes through $B(1,3,-2)$.

$5(1)+2(3)-3(-2)=17$

The point B lies on the plane.

$\therefore$ The line $\vec{r}=\hat{i}+3\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$ lies on the plane $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

Question 9

If the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of the plane containing these lines.

Solution 9

The direction ratio of the line $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ is

$r_1=(-3,-2k,2)$

The direction ratio of the line $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$r_2=(k,1,5)$

Since the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular so

$$\begin{aligned}
r_1\cdot r_2&=0\\
(-3,-2k,2)\cdot(k,1,5)&=0\\
-3k-2k+10&=0\\
-5k&=-10\\
k&=2
\end{aligned}$$

Therefore the equation of the lines are $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$

The equation of the plane containing the perpendicular lines $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$(-20-2)x-y(-15-4)+z(-3+8)+d=0$$

$$-22x+19y+5z+d=0$$

The line $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ pass through the point $(1,2,3)$ so putting $x=1, y=2, z=3$ in the equation $-22x+19y+5z+d=0$ we get

$$\begin{aligned}
-22(1)+19(2)+5(3)+d&=0\\
d&=22-38-15\\
d&=-31
\end{aligned}$$

Therefore the equation of the plane containing the lines is

$-22x+19y+5z=31$

Question 10

Find the coordinates of the point where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ *intersects the plane* $x-y+z-5=0$. *Also, find the angle between the line and the plane.*

Solution 10

Any point on the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=k$$

is of the form, $(3k+2,\ 4k-1,\ 2k+2)$.

If the point $P(3k+2,\ 4k-1,\ 2k+2)$ *lies in the plane* $x-y+z-5=0$, *we have,*

$(3k+2)-(4k-1)+(2k+2)-5=0$

$\Rightarrow 3k+2-4k+1+2k+2-5=0$

$\Rightarrow k=0$

Thus, the coordinates of the point of intersection of the line and the plane are: $P(3\times0+2,\ 4\times0-1,\ 2\times0+2)=P(2,-1,2)$

Let θ *be the angle between the line and the plane.*

Thus,

$\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$, *where*, l, m *and* n *are the direction ratios of the line and* a, b *and* c *are the direction ratios of the normal to the plane.*

Here, $l=3,\ m=4,\ n=2,\ a=1,\ b=-1$, *and* $c=1$

Hence,

$$\sin\theta=\frac{1\times3+(-1)\times4+1\times2}{\sqrt{1^2+(-1)^2+1^2}\sqrt{3^2+4^2+2^2}}$$

$$\Rightarrow \sin\theta=\frac{1}{\sqrt{3}\sqrt{29}}$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}\sqrt{29}}\right)$$

Question 11

Find the vector equation of the plane passing through three points with position vectors $\hat{i}+\hat{j}-2\hat{k},\ 2\hat{i}-\hat{j}+\hat{k}$ *and* $\hat{i}+2\hat{j}+\hat{k}$. *Also, find the coordinates of the point of intersection of this plane and the line* $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$.

Solution 11

Let A, B and C be three points with position vectors
$\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ *and* $\hat{i}+2\hat{j}+\hat{k}$.

Thus, $\overrightarrow{AB}=\vec{b}-\vec{a}=\left(2\hat{i}-\hat{j}+\hat{k}\right)-\left(\hat{i}+\hat{j}-2\hat{k}\right)=\hat{i}-2\hat{j}+3\hat{k}$

$\overrightarrow{AC}=\vec{c}-\vec{a}=\left(\hat{i}+2\hat{j}+\hat{k}\right)-\left(\hat{i}+\hat{j}-2\hat{k}\right)=\hat{j}+3\hat{k}$

Now consider $\overrightarrow{AB}\times\overrightarrow{AC}$:

$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}$$

$\vec{n}=\hat{i}(-6-3)-3\hat{j}+\hat{k}=-9\hat{i}-3\hat{j}+\hat{k}$

So, the equation of the required plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

$\Rightarrow(\vec{r}\cdot\vec{n})=(\vec{a}\cdot\vec{n})$

$\Rightarrow\left(\vec{r}\cdot\left(-9\hat{i}-3\hat{j}+\hat{k}\right)\right)=\left(\hat{i}+\hat{j}-2\hat{k}\right)\cdot\left(-9\hat{i}-3\hat{j}+\hat{k}\right)$

$\Rightarrow\vec{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$

Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$

Any point on the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ *is of the form,*

$(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

If the point $P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$ *lies in the plane,*

$\vec{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$, *we have,*

$9(3+2\lambda)-3(1+2\lambda)-(-1+\lambda)=14$

$\Rightarrow 27+18\lambda-3-6\lambda+1-\lambda=14$

$\Rightarrow 11\lambda=-11$

$\Rightarrow\lambda=-1$

Thus, the required point of intersection is

$P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

$\Rightarrow P(3+2(-1),\ -1-2(-1),\ -1+(-1))$

$\Rightarrow P(1,\ 1,\ -2)$

Question 12

Show that the lines $\dfrac{5-x}{-4}=\dfrac{y-7}{4}=\dfrac{z+3}{-5}$ and $\dfrac{x-8}{7}=\dfrac{2y-8}{2}=\dfrac{z-5}{3}$ are coplanar.

Solution 12

$$\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$$
$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}\text{.......(i)}$$

$$\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$$
$$\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}\text{.....(ii)}$$

Here,
$a_1 = 4, b_1 = 4, c_1 = -5$
$a_2 = 7, b_2 = 1, c_2 = 3$
$x_1 = 5, y_1 = 7, z_1 = -3$
$x_2 = 8, y_2 = 4, z_2 = 5$
Condition for two lines to be coplanar,
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$
$$\therefore \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$
$$= \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$
$= 3(12+5) + 3(12+35) + 8(4-28)$
$= 3 \times 17 + 3 \times 47 + 8 \times (-24)$
$= 51 + 141 - 192$
$= 192 - 192$
$= 0$

$\therefore$ The lines are coplanar to each other.

Question 13
Find the equation of a plane which passes through the point (3, 2, 0) and contains the line
$$\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}.$$

Solution 13

Required equation of plane is passing through the point $(3, 2, 0)$,

$\therefore\ a(x-3)+b(y-2)+c(z-0)=0$

$\Rightarrow a(x-3)+b(y-2)+cz=0$.........(i)

Required equation of plane also contains the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$,

so it passes through the point $(3, 2, 0)$

$\Rightarrow a(3-3)+b(6-2)+c4=0$

$\Rightarrow 4b+4c=0$...............................(ii)

Also plane will be parallel to,

$a(1)+b(5)+c(4)=0$

$a+5b+4c=0$.............(iii)

Solving (ii) and (iii) by cross multiplication,

$\frac{a}{16\text{-}20}=\frac{b}{4-0}=\frac{c}{0-4}=\lambda(\text{say})$

$-\frac{a}{4}=\frac{b}{4}=-\frac{c}{4}=\lambda(\text{say})$

$\Rightarrow a=-\lambda, b=\lambda,\ c=-\lambda$

Put $a=-\lambda, b=\lambda,\ c=-\lambda$ in equation (i) we get

$(-\lambda)(x-3)+(\lambda)(y-2)+(-\lambda)z=0$

$\Rightarrow -x+3+y-2-z=0$

$\Rightarrow x-y+z-1=0$

Chapter 29 - The plane Exercise Ex. 29.14

Question 1

Find the shortest distance between the lines $\frac{x-2}{-1}=\frac{y-5}{2}=\frac{z-0}{3}$ and $\frac{x-0}{2}=\frac{y+1}{-1}=\frac{z-1}{2}$.

Solution 1

Consider

$$l_1 : \frac{x-2}{-1} = \frac{y-5}{2} = \frac{z-0}{3}$$

$$l_2 : \frac{x-0}{2} = \frac{y+1}{-1} = \frac{z-1}{2}$$

Clearly line l_1 passes through the point $P(2,5,0)$

The equation of a plane containing line l_2 is

$$a(x-0)+b(y+1)+c(z-1)=0 \qquad \ldots\ldots (1)$$

Where $2a-b+2c=0$

If it is parallel to line l_1 then

$$-a+2b+3c=0$$

Therefore

$$\frac{a}{-7} = \frac{b}{-8} = \frac{c}{3}$$

Substituting values of a, b, c in the equation (1) we obtain

$$a(x-0)+b(y+1)+c(z-1)=0$$

$$-7(x-0)-8(y+1)+3(z-1)=0$$

$$-7x-8y-8+3z-3=0$$

$$7x+8y-3z+11=0 \qquad \ldots\ldots (2)$$

This is the equation of the plane containing line l_2 and parallel to line l_1

Shortest distance between l_1 and l_2 = Distance between point $P(2,5,0)$ and plane (2)

$$= \left| \frac{14+40+11}{\sqrt{7^2+8^2+(-3)^2}} \right| = \frac{65}{\sqrt{122}}$$

Question 2

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

Solution 2

$l_1: \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

$l_2: \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Let the equation of the plane containing l_1 be $a(x+1)+b(y+1)+c(z+1)=0$

Plane is parallel to l_1: $7a-6b+c=0$......(i)

Plane is parallel to l_2: $a-2b+c=0$........(ii)

Solving (i) and (ii),

$$\frac{a}{-6+2} = \frac{b}{1-7} = \frac{c}{-14+6}$$

$$\frac{a}{-4} = \frac{b}{-6} = \frac{c}{-8}$$

$\therefore$ Equation of the plane is $-4(x+1)-6(y+1)-8(z+1)=0$

$4(x+1)+6(y+1)+8(z+1)=0$ is the equation of the plane.

Question 3

Find the shortest distance between the lines

$\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ and $3x - y - 2z + 4 = 0 = 2x + y + z + 1$.

Solution 3

The equation of a plane containing the line $3x - y - 2z + 4 = 0 = 2x + y + z + 1$ is

$x(2\lambda+3)+y(\lambda-1)+z(\lambda-2)+\lambda+4=0$............(i)

If it is parallel to the line then $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ then,

$2(2\lambda+3)+4(\lambda-1)+(\lambda-2)=0$

$\Rightarrow \lambda = 0$

Putting $\lambda = 0$ in (i) we get,

$3x - y - 2z + 4 = 0$............(ii)

As this equation of the plane containing the second line and paralle to the first line.

Clearly the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z+2}{1}$ passes through the point $(1, 3, -2)$

So, the shortest distance 'd' between the given lines is equal to the length of perpendicular from $(1, 3, -2)$ on the plane (ii).

$$d = \left|\frac{3 - 3 + 4 + 4}{\sqrt{1+9+4}}\right| = \frac{8}{\sqrt{14}}$$

Chapter 29 - The plane Exercise Ex. 29.15

Question 1

Find the image of the point $(0,0,0)$ in the plane $3x+4y-6z+1=0$.

Solution 1

$3x+4y-6z+1=0$

Line passing through orgin and perpendicular to plane is given by

$$\frac{x}{3}=\frac{y}{4}=\frac{z}{-6}=r(say)$$

So let the image of (0,0,0) is (3r, 4r, -6r)

Midpoint of (0,0,0) and (3r, 4r, -6r) lies on plane.

$$3\left(\frac{3r}{2}\right)+2(4r)-3(-6r)+1=0$$

$$30.5r=-1$$

$$r=\frac{-2}{61}$$

So image is $\left(\frac{-6}{61},\frac{-8}{61},\frac{12}{61}\right)$

Question 2

Find the reflection of the point $(1,2,-1)$ in the plane $3x-5y+4z=5$.

Solution 2

Here, we have to find reflection of the point $P(1,2,-1)$ in the plane $3x-5y+4z=5$

Let Q be the reflection of the point P and R be the mid-point of PQ.
Then, R lies on the plane $3x-5y+4z=5$.

Direction ratios of PQ are proportional to $3,-5,4$ and PQ is passing through $(1,2,-1)$.
So, equation of PQ is given by,

$$\frac{x-1}{3}=\frac{y-2}{-5}=\frac{z+1}{4}=\lambda\ (\text{Say})$$

Let Q be $(3\lambda+1,\ -5\lambda+2,\ 4\lambda-1)$

The coordinates of R are $\left(\frac{3\lambda+1+1}{2},\ \frac{-5\lambda+2+2}{2},\ \frac{4\lambda-1-1}{2}\right)=\left(\frac{3\lambda+2}{2},\ \frac{-5\lambda+4}{2},\ \frac{4\lambda-2}{2}\right)$

Since, R lies on the given plane $3x-5y+4z=5$

$$\therefore \quad 3\left(\frac{3\lambda+2}{2}\right)-5\left(-\frac{5\lambda+4}{2}\right)+4\left(\frac{4\lambda-2}{2}\right)=5$$

$$\begin{aligned}
&\Rightarrow \quad 9\lambda+6+25\lambda-20+16\lambda-8=10\\
&\Rightarrow \quad 50\lambda-22=10\\
&\Rightarrow \quad 50\lambda=10+22\\
&\Rightarrow \quad 50\lambda=32\\
&\Rightarrow \quad \lambda=\frac{16}{25}
\end{aligned}$$

So, $Q=(3\lambda+1,\ -5\lambda+2,\ 4\lambda-1)$

$$=\left(3\left(\frac{16}{25}\right)+1,\ -5\left(\frac{16}{25}\right)+2,\ 4\left(\frac{16}{25}\right)-1\right)$$

$$=\left(\frac{48}{25}+1,\ -\frac{16}{5}+2,\ \frac{64}{25}-1\right)$$

$$=\left(\frac{73}{25},\ -\frac{6}{5},\ \frac{39}{25}\right)$$

So, reflection of $P(1,2,-1)=\left(\frac{73}{25},\ -\frac{6}{5},\ \frac{39}{25}\right)$

Question 3

Find the coordinates of the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$.

Hence or otherwise deduce the length of the perpendicular.

Solution 3

We have to find foot of the perpendicular, say Q, drawn from $P(5,4,2)$ to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda \quad (\text{say})$$

Let Q be $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

Direction ratios of line PQ are $(2\lambda - 1 - 5,\ 3\lambda + 3 - 4, -\lambda + 1 - 2)$ or $(2\lambda - 6,\ 3\lambda - 1, -\lambda - 1)$

Here, line PQ is perpendicular to line given(AB).

So,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(2\lambda - 6)(2) + (3\lambda - 1)(3) + (-\lambda - 1)(-1) = 0$$
$$4\lambda - 12 + 9\lambda - 3 + \lambda + 1 = 0$$
$$14\lambda - 14 = 0$$
$$\lambda = \frac{14}{14}$$
$$\lambda = 1$$

So, $Q = (2\lambda - 1, 3\lambda + 3, -\lambda + 1)$

$$= (2(1) - 1, 3(1) + 3, -(1) + 1)$$
$$= (2 - 1, 3 + 3, -1 + 1)$$
$$= (1, 6, 0)$$

Length of perpendicular PQ

$$= \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2} \qquad [\text{Using Distance formula}]$$
$$= \sqrt{16 + 4 + 4}$$
$$= \sqrt{24}$$
$$= 2\sqrt{6}$$

So,

Foot of perpendicular is $(1,6,0)$

Length of the perpendicular is $2\sqrt{6}$ units

Question 4

Find the image of the point with position vector $3\hat{i} + \hat{j} + 2\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$.

Also find the position vector of the foot of the perpendicular and the equation of the perpendicular line through $3\hat{i} + \hat{j} + 2\hat{k}$

Solution 4

Here, we have to find image of the point $P(3,1,2)$ in the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=4$ or $2x-y+z=4$.

Let Q be the image of the point P.

So,
Direction ratios of normal to the point are $2,-1,1$

Direction ratios of line PQ perpendicular to $2,-1,1$ and PQ is passing through $(3,1,2)$.

So equation of PQ is

$$\frac{x-3}{2}=\frac{y-1}{-1}=\frac{z-2}{1}=\lambda \quad \text{(say)} \qquad \left[\begin{array}{l}\text{Using equation of line passing through } (x_1,y_1,z_1) \text{ is} \\ \dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}\end{array}\right]$$

General point on the line PQ is $(2\lambda+3,\ -\lambda+1,\ \lambda+2)$

Let Q be $(2\lambda+3,\ -\lambda+1,\ \lambda+2)$

Let R be the mid point of PQ. Then,

coordinates of R are $\left(\dfrac{2\lambda+3+3}{2},\dfrac{-\lambda+1+1}{2},\dfrac{\lambda+2+2}{2}\right)=\left(\dfrac{2\lambda+6}{2},\dfrac{-\lambda+2}{2},\dfrac{\lambda+4}{2}\right)$

Since, R lies on the plane $2x-y+z=4$, we have,

$$2\left(\frac{2\lambda+6}{2}\right)-\left(\frac{-\lambda+2}{2}\right)+\left(\frac{\lambda+4}{2}\right)=4$$

$\Rightarrow \quad 4\lambda+12+\lambda-2+\lambda+4=8$

$\Rightarrow \quad 6\lambda=8-14$

$\Rightarrow \quad \lambda=\dfrac{-6}{6}$

$\Rightarrow \quad \lambda=-1$

So,

$$\text{Image of } P = Q\left(2(-1)+3, -(-1)+1, -1+2\right)$$

$$\text{Image of } P = (1, 2, 1)$$

The equation of the perpendicular line through $3\hat{i}+\hat{j}+2\hat{k}$ is

$$\bar{r} = 3\hat{i}+\hat{j}+2\hat{k}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$$

The position vector of the image point is

$$3\hat{i}+\hat{j}+2\hat{k}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)=(3+2\lambda)\hat{i}+(1-\lambda)\hat{j}+(2+k)\hat{k}$$

The position vector of the foot of the perpendicular is

$$\frac{\left[(3+2\lambda)\hat{i}+(1-\lambda)\hat{j}+(2+\lambda)\hat{k}\right]+\left[3\hat{i}+\hat{j}+2\hat{k}\right]}{2}$$

$$=(3+\lambda)\hat{i}+\left(1-\frac{\lambda}{2}\right)\hat{j}+\left(2+\frac{\lambda}{2}\right)\hat{k}$$

Putting $\lambda=-1$ the position vector of the foot of the perpendicular is

$$2\hat{i}+\frac{3}{2}\hat{j}+\frac{3}{2}\hat{k}$$

Question 5

Find the coordinates of the foot of the perpendicular from the point $(1,1,2)$ to the plane $2x-2y+4z+5=0$. Also, find the length of the perpendicular.

Solution 5

$$2x-2y+4z+5=0$$
$$(1,1,2)$$
$$=\left|\frac{2-2+8+5}{\sqrt{1+1+4}}\right|=\frac{13}{\sqrt{6}}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z-2}{4}=k$$
$$x=2k+1$$
$$y=-2k+1$$
$$z=4k+2$$

Substitute (x,y,z)=$(2k+1,\ -2k+1, 4k+2)$ in plane equation

$$2x-2y+4z+5=0$$
$$4k+2+4k-2+16k+8+5=0$$
$$24k=-13$$
$$k=\frac{-13}{24}$$
$$(x,y,z)=(\frac{-1}{12},\frac{5}{3},\frac{-1}{6})$$

Question 6

Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured along a line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.

Solution 6

Here, we have to find distance of the point $P(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to line AB, $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

Let Q be the mid point of the line joining P to plane.

Here, PQ is parallel to line AB

$\Rightarrow$ Direction ratios of line PQ are proportional to direction ratios of line AB

$\Rightarrow$ Direction ratios of line PQ are $2, 3, -6$ and PQ is passing through $P(1,-2,3)$.

So equation of PQ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda \quad \text{(say)}$$

General point on line PQ is $(2\lambda+1,\ 3\lambda-2, -6\lambda+3)$

Suppose coordinates of Q be $(2\lambda+1, 3\lambda-2, -6\lambda+3)$

Since Q lies on the plane $x-y+z=5$

$$(2\lambda+1)-(3\lambda-2)+(-6\lambda+3)=5$$

$$2\lambda+1-3\lambda+2-6\lambda+3=5$$

$$-7\lambda=5-6$$

$$-7\lambda=-1$$

$$\lambda=\frac{1}{7}$$

Coordinate of $Q=\ (2\lambda+1, 3\lambda-2, -6\lambda+3)=\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

Distance between $(1,-2,3)$ and plane $=PQ$

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}$$

$$=\sqrt{\frac{49}{49}}$$

$$=1$$

Required distance = 1 unit

Question 7

Find the coordinates of the foot the perpendicular from the point $(2,3,7)$ to the plane $3x-y-z=7$. Also, find the length of the perpendicular.

Solution 7

Let Q be the foot of the perpendicular.

Here, Direction ratios of normal to plane is $3, -1, -1$

$\Rightarrow$ Line PQ is parallel to normal to plane

$\Rightarrow$ Direction ratios of PQ are proportional to $3, -1, -1$ and PQ is passing through $P(2,3,7)$.

So,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 2}{3} = \frac{y - 3}{-1} = \frac{z - 7}{-1} = \lambda \quad \text{(say)}$$

General point on line PQ

$$= (3\lambda + 2, -\lambda + 3, -\lambda + 7)$$

Coordinates of Q be $(3\lambda + 2, -\lambda + 3, -\lambda + 7)$

Point Q lies on the plane $3x - y - z = 7$.

So,

$$3(3\lambda + 2) - (-\lambda + 3) - (-\lambda + 7) = 7$$
$$9\lambda + 6 + \lambda - 3 + \lambda - 7 = 7$$
$$11\lambda = 7 + 4$$
$$11\lambda = 11$$
$$\lambda = \frac{11}{11}$$
$$\lambda = 1$$

$\therefore$ Coordinate of Q $= (3\lambda + 2, -\lambda + 3, -\lambda + 7)$

$$= (3(1) + 2, -(1) + 3, -(1) + 7)$$
$$= (5, 2, 6)$$

Length of the perpendicular PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$
$$= \sqrt{(2 - 5)^2 + (3 - 2)^2 + (7 - 6)^2}$$
$$= \sqrt{9 + 1 + 1}$$
$$= \sqrt{11}$$

Question 8

Find the image of the point $(1,3,4)$ in the plane $2x - y + z + 3 = 0$.

Solution 8

Here, we have to find image of point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$

Let Q be the image of the point.

Here, Direction ratios of normal to plane are $2, -1, 1$

$\Rightarrow$ Direction ratios of PQ which is parallel to normal to the plane is proportional to $2, -1, 1$ and line PQ is passing through $P(1,3,4)$.

So, equation of line PQ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = \lambda \quad \text{(say)}$$

General point on line PQ

$$= (2\lambda + 1, \; -\lambda + 3, \lambda + 4)$$

Let Q be $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

Q is image of P, so R is the mid point of PQ

Coordinates of R $\left(\frac{2\lambda + 1 + 1}{2}, \frac{-\lambda + 3 + 3}{2}, \frac{\lambda + 4 + 4}{2}\right)$

$$= \left(\frac{2\lambda + 2}{2}, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

$$= \left(\lambda + 1, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

Point R is on th plane $2x - y + z + 3 = 0$

$$= 2(\lambda + 1) - \left(\frac{-\lambda + 6}{2}\right) + \left(\frac{\lambda + 8}{2}\right) = 0$$

$$4\lambda + 4 + \lambda - 6 + \lambda + 8 + 6 = 0$$

$$6\lambda = -12$$

$$\lambda = -2$$

So,

$$\text{Image } Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$$

$$= (-4 + 1, 2 + 3, -2 + 4)$$

$$= (-3, 5, 2)$$

Image of P $(1,3,4)$ is $(-3,5,2)$

Question 9

Find the distance of the point with position vector $-\hat{i} - 5\hat{j} - 10\hat{k}$ from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 12\hat{k})$ with the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

Solution 9

Here, we have to find distance of a point A with position vector $\left(-\hat{i}-5\hat{j}-10\hat{k}\right)$ from the point of intersection of line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$ with plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Let the point of intersection of line and plane be $B\left(\vec{b}\right)$

The line and the plane will intersect when,

$$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\left[\left(2+3\lambda\right)\hat{i}+\left(-1+4\lambda\right)\hat{j}+\left(2+12\lambda\right)\hat{k}\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\left(2+3\lambda\right)\left(1\right)+\left(-1+4\lambda\right)\left(-1\right)+\left(2+12\lambda\right)\left(1\right)=5$$
$$2+3\lambda+1-4\lambda+2+12\lambda=5$$
$$11\lambda=5-5$$
$$\lambda=0$$

So, the point B is given by

$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\left(0\right)\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)$$

$$\overrightarrow{AB}=\vec{b}-\vec{a}$$
$$=\left(2\hat{i}-\hat{j}+2\hat{k}\right)-\left(-\hat{i}-5\hat{j}-10\hat{k}\right) \quad =\left(2\hat{i}-\hat{j}+2\hat{k}+\hat{i}+5\hat{j}+10\hat{k}\right)=\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\left|\overrightarrow{AB}\right| \quad =\sqrt{(3)^2+(4)^2+(12)^2}=\sqrt{9+16+144}=\sqrt{169}=13$$

Required distance = 13 units

Question 10

Find the length and the foot of the perpendicular from the point $(1,1,2)$ to the plane $\vec{r}\cdot\left(\hat{i}-2\hat{j}+4\hat{k}\right)+5=0$.

Solution 10

$$x - 2y + 4z + 5 = 0$$

$$(1, 1, 2)$$

$$= \left|\frac{1 - 2 + 4 + 5}{\sqrt{1 + 4 + 16}}\right| = \frac{8}{\sqrt{21}}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-2}{4} = k$$

$$x = k + 1$$

$$y = -2k + 1$$

$$z = 4k + 2$$

Substitute (x,y,z)=(k+1, -2k+1, 4k+2) in plane equation

$$x - 2y + 4z + 5 = 0$$

$$k + 1 + 4k - 2 + 16k + 8 + 5 = 0$$

$$21k = -12$$

$$k = \frac{-12}{21} = \frac{-4}{7}$$

$$(x, y, z) = (\frac{3}{7}, \frac{15}{7}, \frac{-2}{7})$$

Question 11

Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point $P\,(3,2,1)$ from the plane $2x - y + z + 1 = 0$.Find also the image of the point in the plane.

Solution 11

$2x - y + z + 1 = 0$

$(3, 2, 1)$

$$= \left|\frac{6-2+1+1}{\sqrt{4+1+1}}\right| = \frac{6}{\sqrt{6}} = \sqrt{6}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = k$$

$x = 2k + 3$

$y = -k + 2$

$z = k - 1$

Substitute $(x,y,z) = (2k+3,\ -k+2,\ k-1)$ in plane equation

$2x - y + z + 1 = 0$

$4k + 6 + k - 2 + k - 1 + 1 = 0$

$6k = -4$

$$k = \frac{-4}{6} = \frac{-2}{3}$$

$$(x, y, z) = \left(\frac{5}{3}, \frac{8}{3}, \frac{-5}{3}\right)$$

Question 12

Find the direction cosines of the unit vector perpendicular to the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$ *passing through the origin.*

Solution 12

Given equation of the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$

Thus, the direction ratios normal to the plane are $6,\ -3$ *and* -2

Hence the direction cosines to the normal to the plane are

$$\frac{6}{\sqrt{6^2 + (-3)^2 + (-2)^2}}, \frac{-3}{\sqrt{6^2 + (-3)^2 + (-2)^2}}, \frac{-2}{\sqrt{6^2 + (-3)^2 + (-2)^2}}$$

$$= \frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}$$

$$= \frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$$

The direction cosines of the unit vector perpendicular to the plane are same as the direction cosines of the normal to the plane.

Thus, the direction cosines of the unit vector perpendicular to the plane are: $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

Question 13

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x-3y+4z-6=0$

Solution 13

Consider the given equation of the plane $2x-3y+4z-6=0$

The direction ratios of the normal to the plane are $2,\ -3$ *and* 4

Thus, the directio ratios of the line perpendicular to the plane are $2,\ -3$ *and* $4.$

The equation of the line passing (x_1,y_1,z_1) *having direction ratios* a, b *and* c *is*

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Thus, the equation of the line passing through the origin with direction ratios $2,\ -3$ *and* 4 *is*

$$\frac{x-0}{2}=\frac{y-0}{-3}=\frac{z-0}{4}$$

$$\Rightarrow \frac{x}{2}=\frac{y}{-3}=\frac{z}{4}=r, \text{ where } r \text{ is some constant}$$

Any point on the line is of the form $2r,\ -3r$ *and* $4r$

If the point $P(2r,\ -3r,\ 4r)$ *lies on the plane* $2x-3y+4z-6=0,$

it should satisfies the equation, $2x-3y+4z-6=0$

Thus, we have,

$$2(2r)-3(-3r)+4(4r)-6=0$$

$$\Rightarrow 4r+9r+16r-6=0$$

$$\Rightarrow 29r=6$$

$$\Rightarrow r=\frac{6}{29}$$

Thus, the coordinates of the point of intersection of the perpendicular from the origin and the plane are:

$$P\left(2\times\frac{6}{29},\ -3\times\frac{6}{29},\ 4\times\frac{6}{29}\right)=P\left(\frac{12}{29},\ \frac{18}{29},\ \frac{24}{29}\right)$$

Question 14

Find the length and the foot of perpendicular from the point (1, 3/2, 2) to the plane 2x - 2y + 4z + 5 = 0.

Solution 14

The length of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

$$d = \left|\frac{2 - 3 + 8 + 5}{\sqrt{4 + 4 + 16}}\right| = \frac{12}{2\sqrt{6}} = \sqrt{6}$$

Let the foot of perpendicular be (x, y, z). So DR's are in proportional

$$\frac{x - 1}{2} = \frac{y - \frac{3}{2}}{-2} = \frac{z - 2}{4} = k$$

$$x = 2k + 1$$
$$y = -2k + \frac{3}{2}$$
$$z = 4k + 2$$

So using values of x, y, z in equation of the plane we have,

$$2(2k + 1) - 2\left(-2k + \frac{3}{2}\right) + 4(4k + 2) + 5 = 0$$
$$4k + 2 + 4k - 3 + 16k + 8 + 5 = 0$$
$$24k = -12$$
$$k = -\frac{1}{2}$$

$$(x, y, z) = \left(0, \frac{5}{2}, 0\right)$$

Chapter 29 - The plane Exercise MCQ

Question 1

The plane $2x - (1 + \lambda)y + 3\lambda z = 0$ passes through the intersection of the planes

a. $2x - y = 0$ and $y - 3z = 0$
b. $2x + 3z = 0$ and $y = 0$
c. $2x - y + 3z = 0$ and $y - 3z = 0$
d. none of these

Solution 1

Correct option: (a)

$$2x - (1 + \lambda)y + 3\lambda z = 0$$
$$\Rightarrow 2x - y - \lambda y + 3\lambda z = 0$$
$$\Rightarrow 2x - y + \lambda(3z - y) = 0$$

The given plane passes through intersection of

$2x - y = 0$ and $-y + 3z = 0 \Rightarrow y - 3z = 0$

Question 2

The acute angle between the planes 2x - y + z = 6 and x + y + 2z = 3 is

a. 45°
b. 60°
c. 30°
d. 75°

Solution 2

Correct option: (b)

Directions of a plane $2x - y + z = 6$ are $2, -1, 1$.

Directions of a plane $x + y + 2z = 3$ are $1, 1, 2$.

Let angle between the planes be θ.

$$\Rightarrow \cos\theta = \left|\frac{2\times 1 - 1\times 1 + 1\times 2}{\sqrt{2^2+1^2+1^2}\sqrt{1^2+1^2+2^2}}\right|$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60°$$

Question 3

The equation of the plane through the intersection of the planes x + 2y + 3z = 4 and 2x + y - z = -5 and perpendicular to the plane 5x + 3y + 6z + 8 = 0 is

a. 7x - 2y + 3z + 81 = 0
b. 23x + 14y - 9z + 48 = 0
c. 51x - 15y - 50z + 173 = 0
d. none of these

Solution 3

Correct option: (d)

The equation of the plane passing through the line of intersection of the $x + 2y + 3z = 4$ and $2x + y - z = -5$ is

$$(x + 2y + 3x - 4) + \lambda(2x + y - z + 5) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (2 + \lambda)y + (3 - \lambda)z - 4 + 5\lambda = 0$$

Direction ratios of the above plane are $1 + 2\lambda, 2 + \lambda, 3 - \lambda$

This plane is perpendicular to the plane whose direction ratios are $5, 3, 6$.

$$\Rightarrow 5(1 + 2\lambda) + 3(2 + \lambda) + 6(3 - \lambda) = 0$$

$$\Rightarrow \lambda = \frac{-29}{7}$$

Put it in the equation

$$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$$

$$\Rightarrow (x + 2y + 3z - 4) - \frac{29}{7}(2x + y - z + 5) = 0$$

$$\Rightarrow -51x - 15y + 50z - 173 = 0$$

$$\Rightarrow 51x + 15y - 50z + 173 = 0$$

Question 4
The distance between the planes 2x + 2y - z + 2 = 0 and 4x + 4y - 2z + 5 = 0 is

a. $\frac{1}{2}$
b. $\frac{1}{4}$
c. $\frac{1}{6}$
d. none of these

Solution 4
Correct option: (c)
Given planes
$2x + 2y - z + 2 = 0$ and
$4x + 4y - 2z + 5 = 0 \Rightarrow 2x + 2y - z + \frac{5}{2} = 0$
Comparing with $ax + by + cz + d_1 = 0$
and $ax + by + cz + d_2 = 0$
$d_1 = 2$ and $d_2 = \frac{5}{2}$
Distance between the planes $= \left|\frac{2 - \frac{5}{2}}{\sqrt{4 + 4 + 1}}\right|$
Distance between the planes $= \frac{1}{6}$

Question 5
The image of the point (1, 3, 4) in the plane 2x - y + z + 3 = 0 is

a. (3, 5, 2)
b. (-3, 5, 2)
c. (3, 5, -2)
d. (3, -5, 2)

Solution 5
Correct option: (b)

Let P$(1,3,4)$ be a point.

LEt M be the point on the plane.

Equation of the plane is $2x - y + z + 3 = 0$.....(i)

Thus equation of a plane is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$$

$\Rightarrow x = 1 + 2\lambda,\ y = 3 - \lambda,\ z = 4 + \lambda$

Put in (i)

$\Rightarrow \lambda = -1$

$M = (-1, 4, 3)$

Let Q (x', y', z') be the image of P.

$\Rightarrow$ M is mid point of PQ

$$\frac{1+x'}{2} = -1, \frac{3+y'}{2} = 4, \frac{4+z'}{2} = 2$$

$\Rightarrow x' = -3, y' = 5, z' = 2$

Question 6

The equation of the plane containing the two lines

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-0}{3} \text{ and } \frac{x}{-2} = \frac{y-2}{-3} = \frac{z+1}{-1} \text{ is}$$

a. $8x + y - 5z - 7 = 0$
b. $8x + y + 5z - 7 = 0$
c. $8x - y - 5z - 7 = 0$
d. none of these

Solution 6

Correct option: (d)

The cartesian equation of a plane $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-0}{3}$

$\vec{r} = \hat{i} - \hat{j} + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$\vec{r} = 2\hat{j} - \hat{k} + \mu\left(-2\hat{i} - 3\hat{j} - \hat{k}\right)$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & 3\hat{k} \\ 2 & -1 & 3 \\ -2 & -3 & -1 \end{vmatrix}$$

$\vec{n} = 10\hat{i} - 4\hat{j} - 8\hat{k}$

$\vec{r} \cdot \left(10\hat{i} - 4\hat{j} - 8\hat{k}\right) = \left(10\hat{i} - 4\hat{j} - 8\hat{k}\right) \cdot \left(\hat{i} - \hat{j}\right)$

$\vec{r} \cdot \left(10\hat{i} - 4\hat{j} - 8\hat{k}\right) = 14$

$\Rightarrow 10x - 4y - 8k = 14$

$5x - 2y - 4z = 7$

NOTE: Answer not matching with back answer.

Question 7

The equation of the plane $\vec{r} = \hat{i} - \hat{j} + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ in scalar product form is

a. $\vec{r} \cdot \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right) = 7$

b. $\vec{r} \cdot \left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7$

c. $\vec{r} \cdot \left(5\hat{i} - 2\hat{j} + 3\hat{k}\right) = 7$

d. none of these

Solution 7

Correct option: (a)

Given $\vec{r} = \hat{i} - \hat{j} + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$

Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$\vec{b} \times \vec{c} = (3+2)\hat{i} - (3-1)\hat{j} + (-2-1)\hat{k}$

$\vec{b} \times \vec{c} = 5\hat{i} - 2\hat{j} - 3\hat{k}$

The equation of the plane in vector form

$\vec{r} \cdot \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right) = \left(\hat{i} - \hat{j}\right) \cdot \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right)$

$\vec{r} \cdot \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right) = 7$

Question 8

The distance of the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ from the plane $\vec{r} \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5,$ is

a. $\frac{5}{3\sqrt{3}}$

b. $\frac{10}{3\sqrt{3}}$

c. $\frac{25}{3\sqrt{3}}$

d. none of these

Solution 8

Correct option: (b)

$$\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$$

Here, $\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 4\hat{k}$, $d = 5$

Perpendicular distance of a point from the plane

$$= \frac{\left|\left(2\hat{i} - 2\hat{j} + 3\hat{k}\right) \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) - 5\right|}{\sqrt{1 + 5^2 + 1}}$$

$$= \frac{10}{3\sqrt{3}}$$

Question 9

The equation of the plane through the line $x+y+z+3 = 0$, $2x - y + 3z + 1=0$ and parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is

a. $x - 5y + 3z = 7$
b. $x - 5y + 3z = -7$
c. $x + 5y + 3z = 7$
d. $x + 5y + 3z = -7$

Solution 9

Correct option: (a)

The equation of the plane through the line

$x + y + z + 3 = 0$ and $2x - y + 3z + 1 = 0$

$\Rightarrow (x + y + z + 3) + \lambda(2x - y + 3z + 1) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 - \lambda)y + (1 + 2\lambda)z + 3 + \lambda = 0$

This plane is parallel to given line

$\Rightarrow$ The given line is perpendicular to the nomral of the intersection of the plane.

$(1 + 2\lambda) + 2(1 - \lambda) + 3(1 + 2\lambda) = 0$

$\Rightarrow \lambda = \frac{-2}{3}$

Equation of the plane is

$(x + y + z + 3) - \frac{2}{3}(2x - y + 3z + 1) = 0$

$\Rightarrow x - 5y + 3z = 7$

Question 10

The vector equation of the plane containing the line $\vec{r} = \left(-2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} - 2\hat{j} - \hat{k}\right)$ and the point $\hat{i} + 2\hat{j} + 3\hat{k}$ is

a. $\vec{r} \cdot \left(\hat{i} + 3\hat{k}\right) = 10$

b. $\vec{r} \cdot \left(\hat{i} - 3\hat{k}\right) = 10$

c. $\vec{r} \cdot \left(3\hat{i} + \hat{k}\right) = 10$

d. none of these

Solution 10

Correct option: (a)

The vector equation of the plane containing the line

$\vec{r} = \left(-2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} - 2\hat{j} - \hat{k}\right)$

$\Rightarrow$ Equation of a plane $3x - 2y - z = 0$

Plane is passing through the point $(-2, -3, 4)$

$x\left(x_1 + 2\right) + y\left(y_1 + 3\right) + z\left(z_1 - 4\right) = 0$

This plane is passing through the point $(1, 2, 3)$

$\Rightarrow 3a + 5b - c = 0$

Consider, these equations

$x\left(x_1 + 2\right) + y\left(y_1 + 3\right) + z\left(z_1 - 4\right) = 0$

$3x - 2y - z = 0$

$3a + 5b - c = 0$

$$\Rightarrow \begin{vmatrix} x_1 + 2 & y_1 + 3 & z_1 - 4 \\ 3 & -2 & -1 \\ 3 & 5 & -1 \end{vmatrix} = 0$$

$x_1 + 3z_1 = 10$

$\Rightarrow \vec{r} \cdot \left(\hat{i} + 3\hat{k}\right) = 10$

Question 11

A plane meets the coordinate axes at A, B, C such that the centroid of ΔABC is the point (a, b, c). If the equation of the plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = k$, then $k =$

a. 1
b. 2
c. 3
d. none of these

Solution 11

Correct option: (c)

A plane meets the coordinate axes at A, B, C then Intercepts made by these points on the coordinate axes are u, v and w respectively.

$\Rightarrow A(u,0,0)$, $B(0,v,0)$ and $C(0,0,w)$

$\Rightarrow$ Centroid of triangle $= (a,b,c)$

$$\Rightarrow \left(\frac{u}{3}, \frac{v}{3}, \frac{w}{3}\right) = (a,b,c)$$

$$\Rightarrow u = 3a,\ v = 3b,\ w = 3c$$

$$\frac{x}{u} + \frac{y}{v} + \frac{z}{w} = 1$$

$$\Rightarrow \frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$$

Question 12

The distance between the point $(3,4,5)$ and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane $x+y+z=17$, is

a. 1
b. 2
c. 3
d. none of these

Solution 12

Correct option: (c)

$$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$$

$$\Rightarrow x = 3+\lambda,\ y = 4+2\lambda,\ z = 5+2\lambda$$

$\Rightarrow (3+\lambda, 4+2\lambda, 5+2\lambda)$ point is on the $x+y+z=17$ plane.

$$3+\lambda+4+2\lambda+5+2\lambda = 17$$

$$\Rightarrow \lambda = 1$$

$$\Rightarrow (3+\lambda, 4+2\lambda, 5+2\lambda) = (4,6,7)$$

Distance between $(4,6,7)$ and $(3,4,5)$

$$= \sqrt{1+4+4} = 3$$

Question 13

A vector parallel to the line of intersection of the planes $\vec{r}\cdot\left(3\hat{i}-\hat{j}+\hat{k}\right) = 1$ and $\vec{r}\cdot\left(\hat{i}+4\hat{j}-2\hat{k}\right) = 2$ is

a. $-2\hat{i}+7\hat{j}+13\hat{k}$

b. $2\hat{i}+7\hat{j}-13\hat{k}$

c. $-2\hat{i}-7\hat{j}+13\hat{k}$

d. $2\hat{i}+7\hat{j}+13\hat{k}$

Solution 13

Correct option: (a)

Given equations of the planes in cartesian form

$3x - y + z = 1$

$x + 4y - 2z = 2$

$\Rightarrow \frac{x}{2-4} = \frac{y}{4+3} = \frac{z}{12+1}$

$\Rightarrow \frac{x}{-2} = \frac{y}{7} = \frac{z}{13}$

The equation of the vector $= -2\hat{i} + 7\hat{j} + 13\hat{k}$

Question 14

If a plane passes through the point $(1, 1, 1)$ and is perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ then its perpendicular distance from the origin is

a. 3/4

b. 4/3

c. 7/5

d. 1

Solution 14

Correct option: (c)

If the plane is perpendicular to the line then its direction ratios are proportional to 3, 0, 4.

Equation of the plane

$3x + 4z + d = 0$

Plane is passing through $(1, 1, 1)$

$\Rightarrow d = -7$

Perpendicular distance of a plane $3x + 4x - 7 = 0$ from origin

$= \frac{|-7|}{\sqrt{25}} = \frac{7}{5}$

Question 15

The equation of the plane parallel to the lines x - 1 = 2y - 5 = 2z and 3x = 4y - 11 = 3z - 4 and passing through the point (2, 3, 3) is

a. x - 4y + 2z + 4 = 0
b. x + 4y + 2z + 4 = 0
c. x - 4y + 2z - 4 = 0
d. none of these

Solution 15

Correct option: (a)

$$x-1=2y-5=2z$$

$$\Rightarrow \frac{x-1}{1}=\frac{y-\frac{5}{2}}{\frac{1}{2}}=\frac{z}{\frac{1}{2}}$$

also,

$$3x=4y-11=3z-4$$

$$\frac{x}{\frac{1}{3}}=\frac{y-\frac{11}{4}}{\frac{1}{4}}=\frac{z-\frac{4}{3}}{\frac{1}{3}}$$

$\Rightarrow$ Equation of plane

$$x+\frac{y}{2}+\frac{z}{2}=0 \Rightarrow 2x+y+z=0$$

$$\frac{x}{3}+\frac{y}{4}+\frac{z}{3}=0 \Rightarrow 4x+3y+4z=0$$

$$\Rightarrow \frac{x}{1}=\frac{y}{-4}=\frac{z}{2}=\lambda$$

equation of a plane is passing through $(2,3,3)$

$$x-4y+2z=1\times 2-4\times 3+2\times 3$$

$$x-4y+2z=-4$$

$$x-4y+2z+4=0$$

Question 16

The distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$ is

a. 9
b. 13
c. 17
d. none of these

Solution 16

Correct option: (b)

$$\vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 12\hat{k}\right)$$
$$\vec{r} = (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 12\lambda)\hat{k}$$
This point is on the plane $\vec{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$
$$\Rightarrow (2 + 3\lambda) - (-1 + 4\lambda) + (2 + 12\lambda) = 5$$
$$\lambda = 0$$
$$\Rightarrow \vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$
Distance between $(2, -1, 2)$ and $(-1, -5, -10)$
$$= \sqrt{(2 + 1)^2 + (-1 + 4)^2 + (2 + 10)^2}$$
$= 13$ units.

Question 17
The equation of the plane through the intersection of the planes ax + by + cz + d = 0 and lx + my + nz + p = 0 and parallel to the line y = 0, z = 0

a. (bl - am)y + (cl - an)z + dl - ap = 0
b. (am - bl)x + (mc - bn)z + md - bp = 0
c. (na - cl)x + (bn - cm)y + nd - cp = 0
d. none of these

Solution 17
Correct option: (a)

The equation of the plane through the intersection of the planes $ax + by + cz + d = 0$ and $lx + my + nz + p = 0$
$$\Rightarrow (ax + by + cz + d) + \lambda(lx + my + nz + p) = 0$$
$$\Rightarrow (a + \lambda l)x + (b + \lambda m)y + (c + \lambda n)z + d + p = 0$$
This plane is parallel to the line $y = 0, z = 0$
$$\Rightarrow a + \lambda l = 0$$
$$\Rightarrow \lambda = \frac{-a}{l}$$
Put in equation
$$(a + \lambda l)x + (b + \lambda m)y + (c + \lambda n)z + d + \lambda p = 0$$
$$\Rightarrow \left(a - \frac{a}{l}l\right)x + \left(b - \frac{a}{l}m\right)y + \left(c - \frac{a}{l}n\right)z + d - \frac{a}{l}p = 0$$
$$\Rightarrow (bl - am)y + (cl - an)z + dl - ap = 0$$

Question 18
The equation of the plane which cuts equal intercepts of unit length on the coordinate axes is

a. x + y + z = 1
b. x + y + z = 0
c. x + y - z = 1
d. x + y + z = 2

Solution 18
Correct option: (a)

Equation of the plane having intercepts a, b, c is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Given that intercepts are of unit length.

$$\frac{x}{1}+\frac{y}{1}+\frac{z}{1}=1$$

$$x+y+z=1$$

Chapter 29 - The plane Exercise Ex. 29VSAQ

Question 1

Write the equation of the plane parallel XOY -plane and passing through the point $(2,-3,5)$.

Solution 1

We know that, equation of XOY plane is $z=0$

so, any plane parallel to the XOY plane is given by

$$z=\lambda -----(i)$$

it is passing through $(2,-3,5)$ so,

$$5=\lambda$$

Put $\lambda=5$ is equation (i)

Equation of required plane is

$$z=5$$

Question 2

Write the equation of the plane parallel YOZ -plane and passing through the $(-4,1,0)$.

Solution 2

We know that, equation of YOZ plane is $x=0$

so, equation of a plane parallel to YOZ is given by

$$x=\lambda -----(i)$$

Plane (i) passing through $(-4,1,0)$ so,

$$-4=\lambda$$

Put $\lambda=-4$ is equation (i)

Equation of required plane is

$$x=-4$$

Question 3

Write the equation of the plane passing through points $(a,0,0)(0,b.0)$ and $(0,0,c)$.

Solution 3

Given a plane passing $(a,0,0)\,(0,b,0)\,(0,0,c)$ so
intercepts of the plane on coordinate axes are a,b,c respectively

So, equation fo the plane with intercepts a,b,c is given by

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Question 4

Write the general equation of a plane parallel to X-axis.

Solution 4

We know that general equation of a plane is

$$ax+by+cz+d=0$$

we know that direction vector of a normal to the plane parallel to X-axis are propartional to 0, 1, 1 so

Equation of plane parallel to X-axis is

$$(0)(x)+by+cz+d=0$$
$$by+cz+d=0$$

So, equation of required plane is

$$by+cz+d=0$$

Question 5

Write the value of k for which the planes $x-2y+kz=4$ and $2x+5y-z=9$ are perpendicular.

Solution 5

We know that the plane $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are perpendicular of $a_1a_2+b_1b_2+c_1c_2=0----(i)$

Here equation of planes that are perpendicular are

$$x-2y+kz=4$$
$$2x+5y-z=9$$

So, $a_1a_2+b_1b_2+c_1c_2=0$

$$(1)(2)+(-2)(5)+(k)(-1)=0$$
$$2-10-k=0$$
$$-8-k=0$$
$$k=-8$$

Question 6

Write the intercepts made by the plane $2x-3y+4z=12$ on the coordinate axes.

Solution 6

Given plane is $2x - 3y + 4z = 12$
dividing by 12,

$$\frac{2x}{12} - \frac{3y}{12} + \frac{4z}{12} = \frac{12}{12}$$

$$\frac{x}{6} + \frac{y}{(-4)} + \frac{z}{3} = 1$$

Camparing it with intercept from a plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$\Rightarrow \quad a = 6, b = -4, c = 3$

So,

Intercepts by plane on coordinate axes are 6,-4,3 respectively

Question 7

Write the ratio in which the plane $4x + 5y - 3z = 8$ divides the line segment joining points $(-2,1,3)$ and $(3,3,2)$.

Solution 7

Let the plane $4x + 5y - 3z = 8$ divide the line joining $A(-2,1,5)$ and $(3,3,2)$ in the ratio $k:1$.

Here, C is a point of internal section of AB,

So, coordinates of C is given by

$$= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + nyL_1}{m+n}, \frac{mz_2 + mz_1}{m+n}\right)$$

$$= \left(\frac{3k-2}{k+1}, \frac{3k+1}{k+1}, \frac{2k+5}{k+1}\right)$$

Point C lies as plane $4x - 5y - 3z = 8$ so,

$$4\left(\frac{3k-2}{k+1}\right) + 5\left(\frac{3k+1}{k+1}\right) - 3\left(\frac{2k+5}{k+1}\right) = 8$$

$$12k - 8 + 15k + 5 - 6k - 15 = 8k + 8$$

$$13k = 26$$

$$k = 2$$

Required ratio $= 2:1$

Question 8

Write the distance between the parallel planes $2x - y + 3z = 4$ and $2x - y + 3z = 18$

Solution 8

Let (x_1, y_1, z_1) be any point on the plane

$$2x - y + 3z = 4$$
$$2x_1 - y_1 + 3z_1 = 4 ------ (i)$$

Distance of (x_1, y_1, z_1) from plane $2x - y + 3z = 18$

$$= \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$
$$= \left|\frac{(2)(x_1) + (-1)(y_1) + (3)(z_1) + (-18)}{\sqrt{(2)^2 + (-1)^2 + (3)^2}}\right|$$
$$= \left|\frac{2x_1 - y_1 + 3z_1 + -18}{\sqrt{4+1+9}}\right|$$
$$= \left|\frac{4 - 18}{\sqrt{14}}\right| \qquad [\text{Using (i)}]$$
$$= \frac{14}{\sqrt{14}}$$
$$= \sqrt{14}$$

Required distance $= \sqrt{14}$ units

Question 9

Write the plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) = 14$ in normal form.

Solution 9

Given plane is $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) = 14$

$\Rightarrow \qquad \vec{r} \cdot \vec{n} = d$

where, $\vec{n} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

Hence, $|\vec{n}| = \sqrt{(2)^2 + (3)^2 + (-6)^2}$

$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$
$$= 7$$

$$\vec{r} \cdot \frac{\vec{n}}{|\vec{n}|} = \frac{d}{|\vec{n}|}$$
$$\vec{r} \cdot \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k}) = \frac{14}{7}$$
$$\vec{r} \cdot \left(\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}\right) = 2$$

Question 10

Write the equation of the plane $\bar{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)=12$ from the origin.

Solution 10

The equation of the plane $\bar{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)=12$ from the origin is

$$d=\frac{\left|\left(2\hat{i}-\hat{j}+2\hat{k}\right)\cdot\left(0\hat{i}+0\hat{j}+0\hat{k}\right)-12\right|}{\sqrt{2^2+(-1)^2+2^2}}$$

$$=\frac{|-12|}{\sqrt{9}}$$

$$=\frac{12}{3}$$

$$=4$$

Question 11

Write the equation of the plane $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ in scalar product form.

Solution 11

Given equation of plane is $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$

Normal vector to this plane is a vector perpendicular to $\vec{b}$ and $\vec{c}$ both,so

$\vec{n}=\vec{b}\times\vec{c}$ & Given that it is passing through $\vec{a}$, so,

vector equation is scalar product form is

$$\left(\vec{r}-\vec{a}\right)\times\vec{n}=0$$

$$\left(\vec{r}-\vec{a}\right)\times\left(\vec{b}\times\vec{c}\right)=0$$

Question 12

Write a vector normal to the plane $\vec{r}=l\vec{a}+m\vec{c}$.

Solution 12

Given plane is $\vec{r}=l\vec{a}+m\vec{c}$

this plane is parallel to $\vec{b}$ and $\vec{c}$,so

normal vector of the plane will be perpendicular to both $\vec{b}$ and $\vec{c}$

$$\Rightarrow \quad \vec{n}=\vec{b}\times\vec{c}$$

Question 13

Write the equation of the plane passing through $(2,-1,1)$ and parallel to the plane $3x+2y-z=7$.

Solution 13

Given equation of plane is

$$3x + 2y - z = 7 \text{------} (i)$$

any plane parallel to plane (i) is given by

$$3x + 2y - z = \lambda \text{------} (ii)$$

Given plane (ii) is passing through (2,-1,1),

so,

$$(3)(2) + (2)(-1) - 1 = \lambda$$

$$6 - 2 - 1 = \lambda$$

$$\lambda = 3$$

Put $\lambda = 3$ in equation (ii)

Equation of required plane is

$$3x + 2y - z = 3$$

Question 14

Write the equation of the plane containing the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{a} + \mu\vec{c}$.

Solution 14

We have to write the equation of plane containing the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{a} + \mu\vec{c}$

Since required plane contains the lines, so it will pass through $\vec{a}$ and normal to this plane will be perpendicular to both $\vec{b}$ and $\vec{c}$ so,

$$\vec{n} = \vec{b} \times \vec{c}$$

we know that equation of plane passing through $\vec{a}$ and perpendicular to vector $\vec{n}$ is given

$$(\vec{r} - \vec{a}) \times \vec{n} = 0$$

so, equation of required plane is

$$(\vec{r} - \vec{a})(\vec{b} \times \vec{c}) = 0$$

Question 15

Write the position vector of the point where the line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r} \cdot \vec{n} = 0$.

Solution 15

Line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets plane $\vec{r} \cdot \vec{n} = 0$ so,

so,

$$\vec{r} \cdot \vec{n} = 0$$
$$\left(\vec{a} + \lambda\vec{b}\right) \times \vec{n} = 0$$
$$\overleftrightarrow{an} + \lambda\overleftrightarrow{bn} = 0$$
$$\lambda = \frac{-\overleftrightarrow{an}}{\overleftrightarrow{bn}}$$

Put λ in equation of line that give, the point of intersection of line and plane

$$= \vec{a} + \left(\frac{-\overleftrightarrow{an}}{\overleftrightarrow{bn}}\right)\vec{b}$$

so,

Required point is

$$\vec{a} - \left(\frac{\overleftrightarrow{an}}{\overleftrightarrow{bn}}\right)\vec{b}$$

Question 16

Write the value of k for which the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{k}$ is perpendicular to the normal to the plane $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = 4$.

Solution 16

Given that line $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z-1}{k}$

is perpendicular to plane $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = 4$

$\Rightarrow \qquad 2x+3y+4z = 4$

Direction ratios of normal to the plane as 2,3,4

Since, line is perpendicular to normal to the plane, is

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(2)(2) + (3)(3) + (4)(k) = 0$$
$$4 + 9 + 4k = 0$$
$$4k = -13$$

$$k = \frac{-13}{4}$$

Question 17

Write the angle between the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane $x + y + 4 = 0$.

Solution 17

We know that, angle (θ) between line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and plane $a_2x + b_2y + c_2z = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1)^2 + (b_1)^2 + (c_1)^2}\sqrt{(a_2)^2 + (b_2)^2 + (c_2)^2}}$$

so, angle (θ) between line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$

and plane $x + y + z = 0$ is given by

$$\sin\theta = \frac{(2)(1) + (1)(1) + (-2)(0)}{\sqrt{(2)^2 + (1)^2 + (-2)^2}\sqrt{(1)^2 + (1)^2}}$$

$$= \frac{2+1+0}{\sqrt{4+1+4}\sqrt{2}}$$

$$= \frac{3}{3\sqrt{2}}$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta = 45^\circ$$

Question 18

Write the intercept cut off by the plane $2x + y - z = 5$ on x-axis.

Solution 18

At x-axis $y = 0, z = 0$.

Putting $y = 0, z = 0$ in the equation $2x + y - z = 5$ is

$$2x + 0 - 0 = 5$$

$$x = \frac{5}{2}$$

The intercept cut off by the plane $2x + y - z = 5$ on x-axis is $\frac{5}{2}$

Question 19

Find the length of the perpendicular drawn from the origin to the plane 2x - 3y + 6z + 21 = 0.

Solution 19

The length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$ is

$$\frac{2(0) - 3(0) + 6(0) + 21}{\sqrt{2^2 + 3^2 + 6^2}} = \frac{21}{\sqrt{49}} = \frac{21}{7} = 3$$

Question 20

Write the vector equation of the line passing through the point $(1, -2, -3)$ and normal to the plane $\vec{r}.(2\hat{i} + \hat{j} + 2\hat{k}) = 5$.

Solution 20

Position vector of the point is $\hat{i} - 2\hat{j} - 3\hat{k}$

A vector perpendicular to the plane is $2\hat{i} + \hat{j} + 2\hat{k}$

Hence, equation of the line is

$\hat{i} - 2\hat{j} - 3\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$

Question 21

Write the vector equation of the plane, passing through the point (a,b,c)and parallel to the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$.

Solution 21

The vector equation of a plane passing through a point having position vector $\vec{A}$ and normal to $\vec{n}$

is $[\vec{r} - \vec{A}] \cdot \vec{n} = 0$

Here, $\vec{n} = \hat{i} + \hat{j} + \hat{k}$ $(\because$ It is parallel to the plane $\vec{r} \cdot = (\hat{i} + \hat{j} + \hat{k})2)$

$\therefore$ The required equation of the plane,

is $[\vec{r} - (a\hat{i} + b\hat{j} + c\hat{k})] \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$