

**RD SHARMA Solutions for Class 9 Maths Chapter 13 -  
Quadrilaterals**

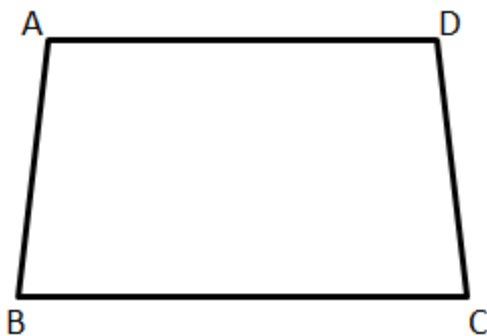
**Chapter 13 - Quadrilaterals Exercise 13.70**

**Question 1**

The opposite sides of a quadrilateral have

- (a) no common point
- (b) one common point
- (c) two common points
- (d) infinitely many common points

**Solution 1**



ABCD is a Quadrilateral.

The opposite sides AB and DC, AD and BC have no common point.

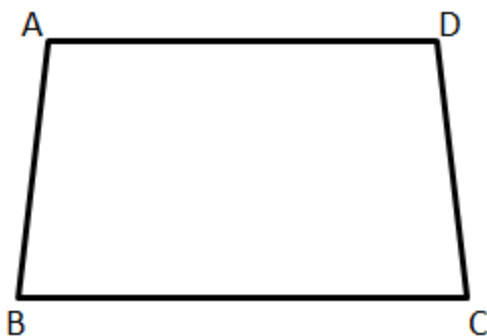
Hence, correct option is (a).

**Question 2**

The consecutive sides of a quadrilateral have

- (a) no common point
- (b) one common point
- (c) two common points
- (d) infinitely many common points

**Solution 2**



Consecutive sides of a Quadrilateral ABCD are

AB and BC,  
BC and CD,  
CD and AD,  
AD and AB,

which have only one point in common  
i.e the joint point of their ends.

Hence, correct option is (b).

### Chapter 13 - Quadrilaterals Exercise 13.71

#### Question 3

PQRS is a quadrilateral. PR and QS intersect each other at O. In which of the following cases, PQRS is parallelogram?

- (a)  $\angle P = 100^\circ$ ,  $\angle Q = 80^\circ$ ,  $\angle R = 100^\circ$
- (b)  $\angle P = 85^\circ$ ,  $\angle Q = 85^\circ$ ,  $\angle R = 95^\circ$
- (c)  $PQ = 7\text{ cm}$ ,  $QR = 7\text{ cm}$ ,  $RS = 8\text{ cm}$ ,  $SP = 8\text{ cm}$
- (d)  $OP = 6.5\text{ cm}$ ,  $OQ = 6.5\text{ cm}$ ,  $OR = 5.2\text{ cm}$ ,  $OS = 5.2\text{ cm}$

#### Solution 3

In a parallelogram, opposite corner angles are equal and sum of adjacent angles =  $180^\circ$

Hence, in quadrilateral PQRS,

$$\Rightarrow \angle P = \angle R \text{ and } \angle Q = \angle S$$

$$\text{Also, } \angle P + \angle Q = \angle Q + \angle R = 180^\circ$$

Hence, if  $\angle P = 100^\circ$  and  $\angle Q = 80^\circ$ , then

$$\angle P + \angle Q = 100^\circ + 80^\circ = 180^\circ$$

Also, if  $\angle Q = 80^\circ$  and  $\angle R = 100^\circ$ , then

$$\angle Q + \angle R = 80^\circ + 100^\circ = 180^\circ$$

Hence, correct option is (a).

#### Question 4

Which of the following quadrilateral is not a rhombus?

- (a) All four sides are equal
- (b) Diagonals bisect each other
- (c) Diagonals bisect opposite angles
- (d) One angle between the diagonals is  $60^\circ$

#### Solution 4

For a rhombus, the angle between the diagonals is  $90^\circ$  and not  $60^\circ$ .

Hence, correct option is (d).

#### Question 5

Diagonals necessarily bisect opposite angles in a

- (a) rectangle
- (b) parallelogram
- (c) isosceles trapezium
- (d) square

#### Solution 5

Diagonals necessarily bisect opposite angles in a square.

Hence, correct option is (d).

#### Question 6

The two diagonals are equal in a

- (a) parallelogram
- (b) rhombus
- (c) rectangle

(d) trapezium

**Solution 6**

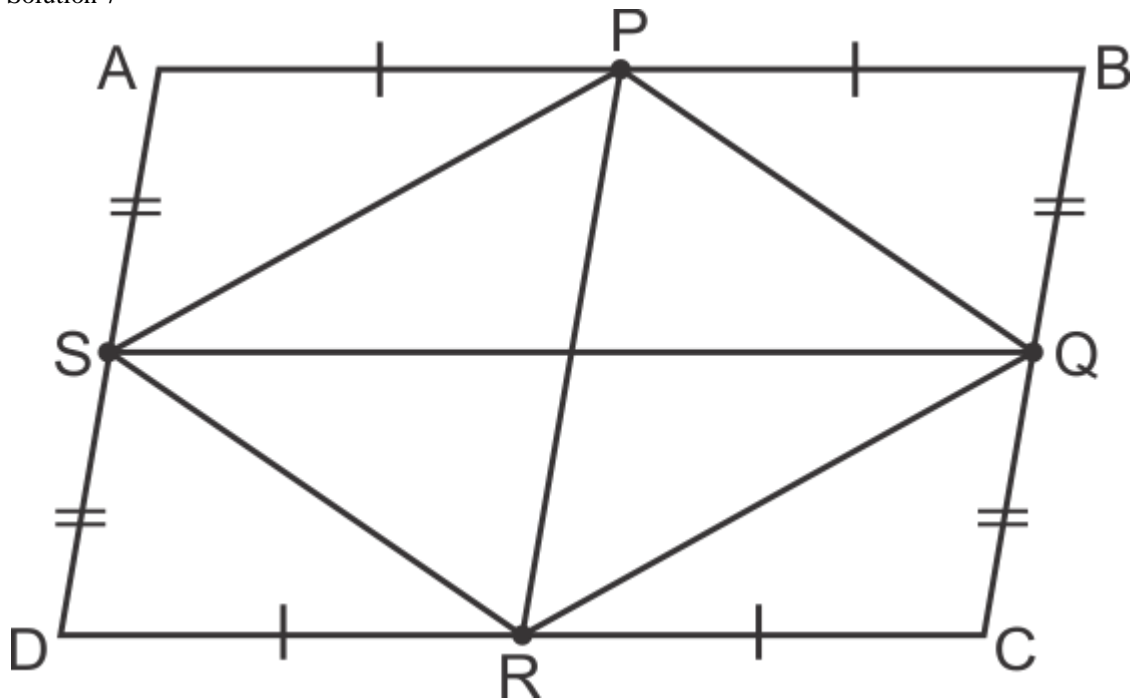
The two diagonals are equal in a rectangle (property).  
Hence, correct option is (c).

**Question 7**

We get a rhombus by joining the mid-points of the sides of a

- (a) parallelogram
- (b) rhombus
- (c) rectangle
- (d) triangle

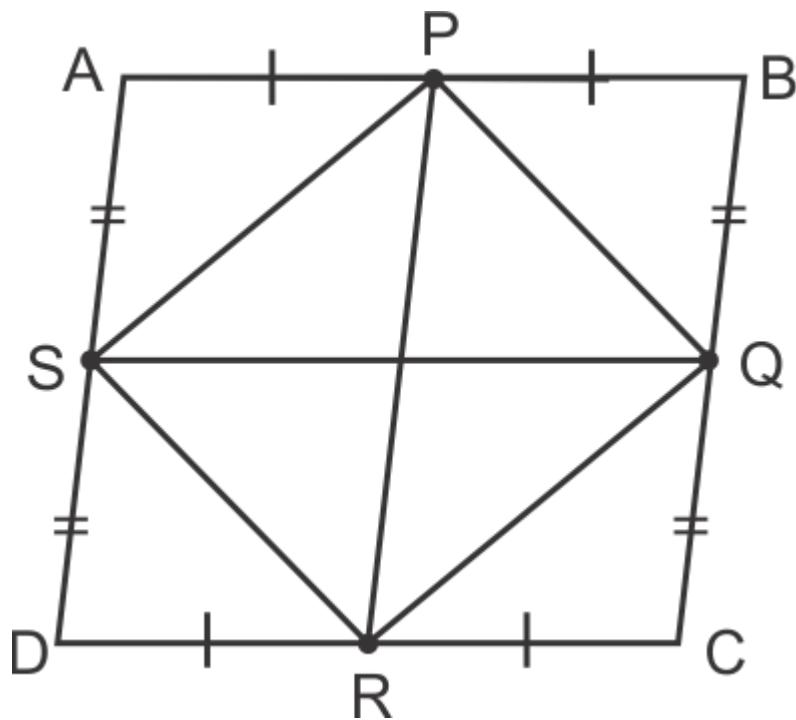
**Solution 7**



$PR \parallel AD \Rightarrow AB \nparallel AD$

$QS \parallel AB \Rightarrow PR \nparallel QS$

Since diagonals of PQRS are not making  $90^\circ$  between them,  
PQRS is not a Rhombus.



P, Q, R and S are the mid – points,

PR and QS are diagonals of quadrilateral PQRS.

$PR \parallel AD$ ,  $QS \parallel AB$

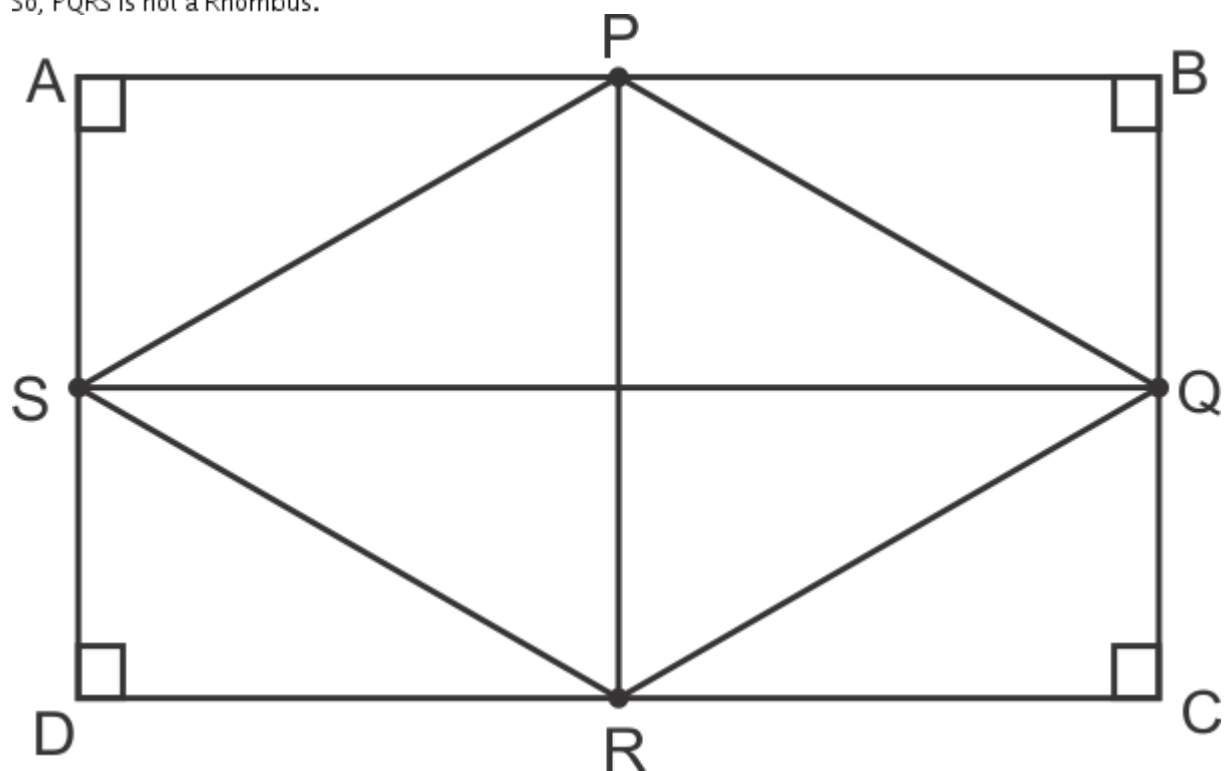
Because they are Formed by joning of mid – point of sides of Rhombus ABCD.

AD is not  $\perp$  to AB

$\Rightarrow$  PR will not be  $\perp$  to QS

i.e angle between diagonals PR & QS is not  $90^\circ$ .

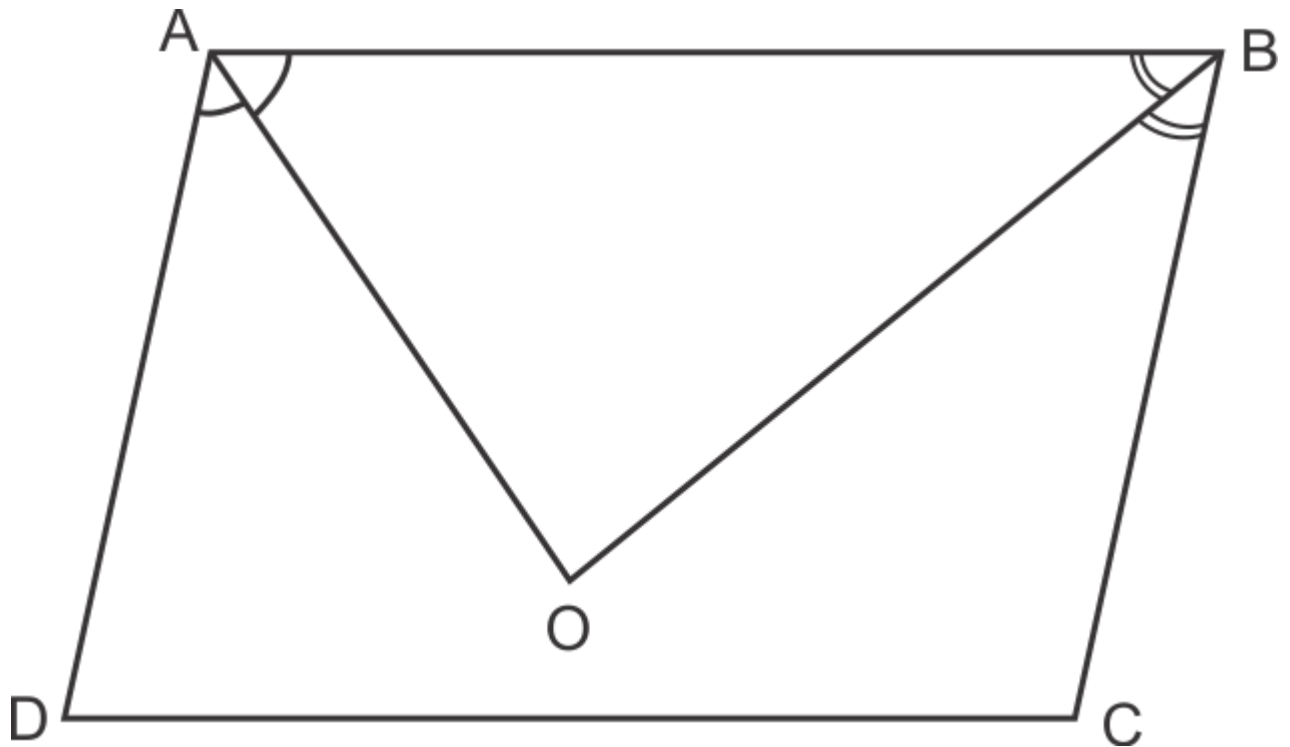
So, PQRS is not a Rhombus.



hence, correct option is (c).

The bisectors of any two adjacent angles of a parallelogram intersect at

- ### Solution 8



In a parallelogram, sum of adjacent angles =  $180^\circ$

$$\Rightarrow \angle A + \angle B = 180^\circ$$

$$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} = 90^\circ \quad \dots(1)$$

$$\Rightarrow \angle OAB = \frac{\angle A}{2} \text{ and } \angle OBA = \frac{\angle B}{2}$$

Thus,  $\angle OAB + \angle OBA = 90^\circ$  [From eq (1)]

$$\Rightarrow \angle AOB = 180^\circ - (\angle OAB + \angle OBA) = 180^\circ - 90^\circ$$

$$\Rightarrow \angle AOB = 90^\circ$$

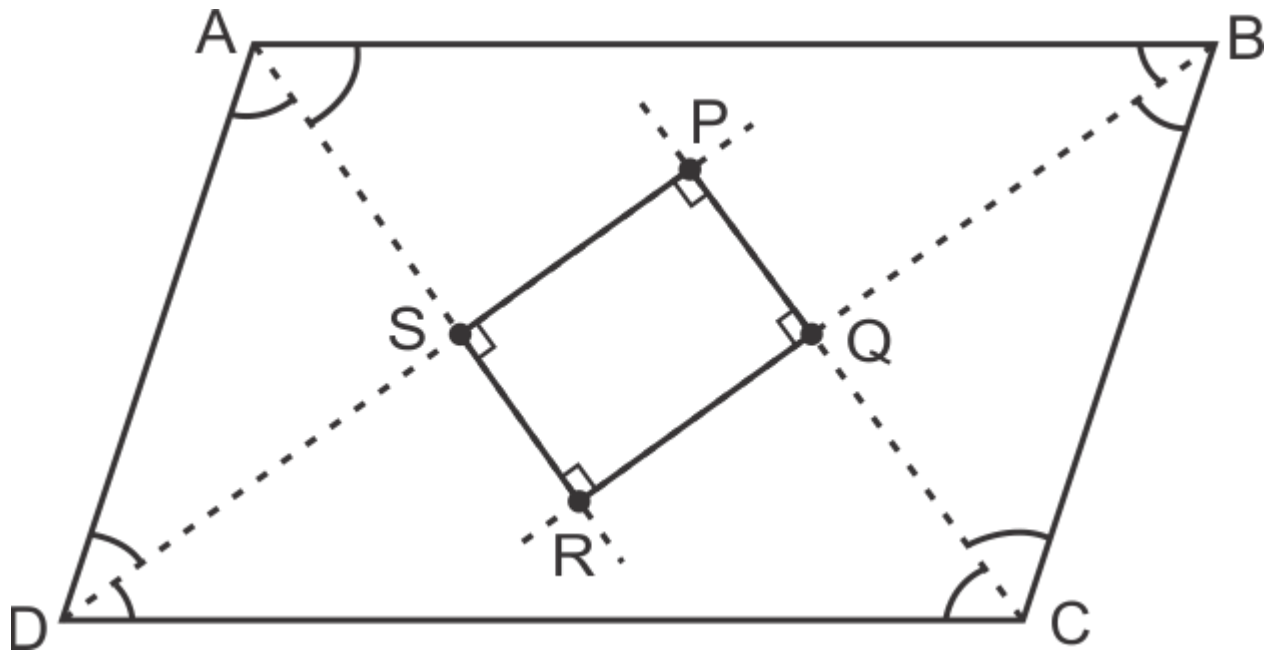
Hence, correct option is (d).

#### Question 9

The bisectors of the angle of a parallelogram enclose a

- (a) parallelogram
- (b) rhombus
- (c) rectangle
- (d) square

Solution 9



AR, BR, CP, DP are the bisectors of angles of parallelogram.  
 Because two bisectors of adjacent angles make  $90^\circ$  between them So  
 PQRS is a Rectangle

Because DP and BR are acute angle bisectors so the distance between  
 them  $PQ < PS$  (The distance between other two bisectors)

So  $PQ \neq PS$  (So PQRS is not a square, but only a rectangle)

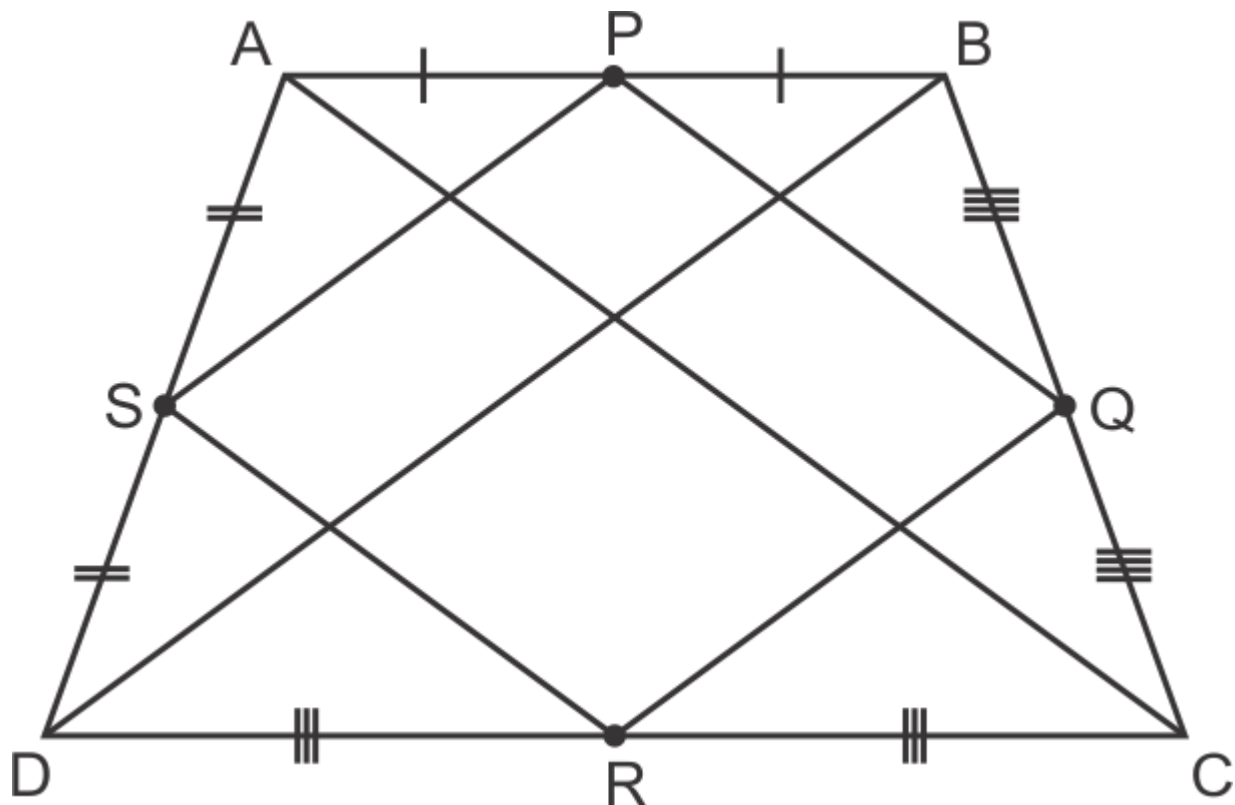
Hence, correct option is (c).

#### Question 10

The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a

- (a) parallelogram
- (b) rectangle
- (c) square
- (d) rhombus

Solution 10



P, Q, R & S are the mid – points of AB, BC, CD & AD respectively.

Consider  $\triangle ADB$ ,

If in a triangle, the mid – points of two sides are joined by a line then the line is parallel to the third side.

$\Rightarrow PS \parallel DB$  in  $\triangle ADB$

Similarly in  $\triangle CDB$ ,

$RQ \parallel DB$

Hence  $PS \parallel RQ$  ... (1)

Similarly in  $\triangle ABC$  and  $\triangle ADC$

$SR \parallel AC$ ,  $PQ \parallel AC$

$\Rightarrow SR \parallel PQ$  ... (2)

From eq. (1) and (2), PQRS is a parallelogram.

Hence, correct option is (a).

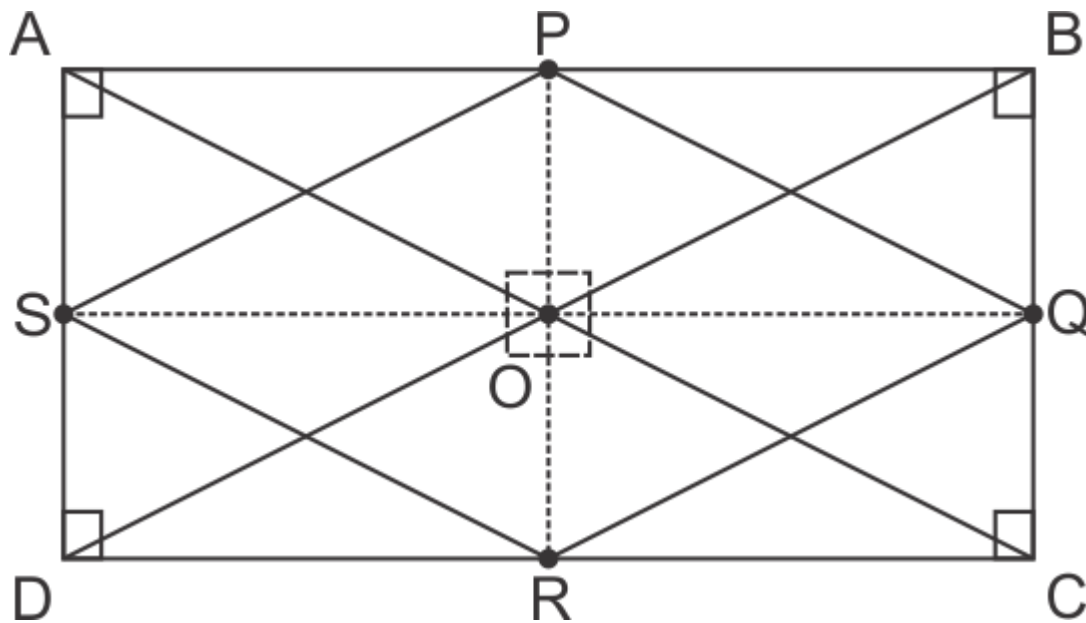
#### Question 11

The figure formed by joining the mid-points of the adjacent sides of a rectangle is a

- (a) square
- (b) rhombus
- (c) trapezium
- (d) none of these

Solution 11





$PQ \parallel AC$  (since in  $\triangle ABC$  mid – points of AB & BC are meeting by PQ)

Similarly,  $SR \parallel AC$

$\Rightarrow PQ \parallel SR$

Now in  $\triangle ABD$  and  $\triangle CBD$ ,

$PS \parallel BD$  and  $QR \parallel BD$

$\Rightarrow PS \parallel QR$

Hence, PQRS is a parallelogram.

But  $PR \perp QS$

$\Rightarrow$  Diagonals cut at  $90^\circ$

$\Rightarrow$  PQRS is a Rhomus

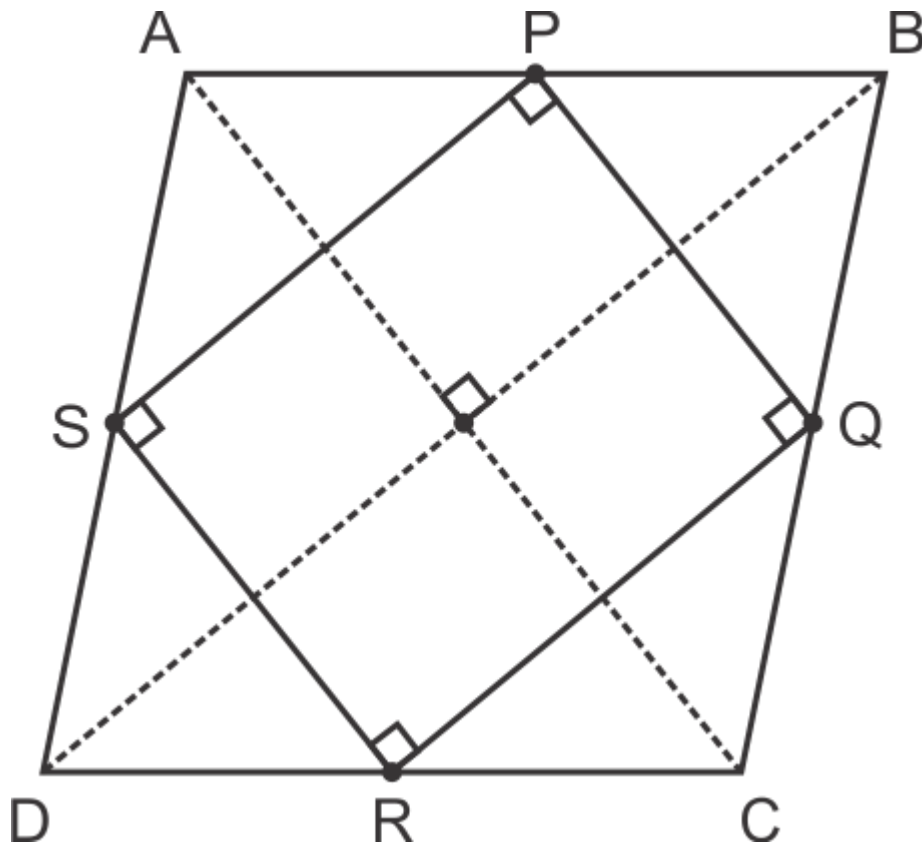
Hence, correct option is (b).

#### Question 12

The figure formed by joining the mid-points of the adjacent sides of a rhombus is a

- (a) square
- (b) rectangle
- (c) trapezium
- (d) none of these

Solution 12



In  $\triangle ABD$  and  $\triangle CBD$

$PS \parallel BD$  and  $QR \parallel BD$

{A line joining mid – points of two sides of  $\triangle$  is parallel to third side}

$\Rightarrow PS \parallel QR$

Similarly  $PQ \parallel SR$

Because  $SR \parallel AC$  and  $QR \parallel BD$ ,

And angle between the diagonals of a Rhombus  $AC$  and  $BD = 90^\circ$ ,

Angle between  $SR$  and  $QR = 90^\circ$

$\Rightarrow PQRS$  is a Rectangle.

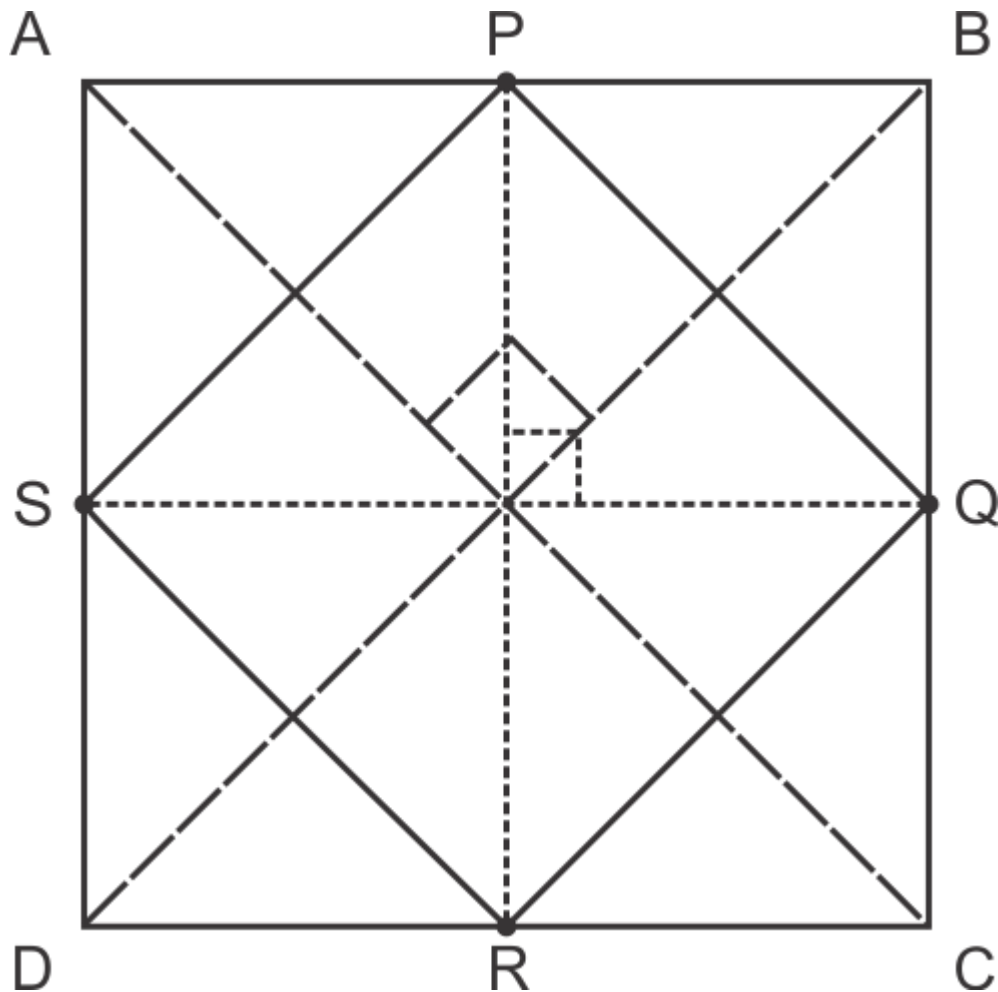
Hence, correct option is (b).

### Question 13

The figure formed by joining the mid-points of the adjacent sides of a square is a

- (a) rhombus
- (b) square
- (c) rectangle
- (d) parallelogram

**Solution 13**



$PS \parallel QR, PQ \parallel SR \dots(1)$

{Because lines joining the midpoint of any two sides of a  $\triangle$  are parallel to the third side}

$AC \perp BD$  &  $PR \perp QS$  (From Figure)

$SR \parallel AC$  and  $QR \parallel BD$

$AC \perp BD$

$\Rightarrow SR \perp QR$

Hence  $\angle SRQ = 90^\circ \dots(2)$

Also  $\triangle APS \cong \triangle DSR$

$\Rightarrow PS = SR \dots(3)$

From equations (1), (2), (3)

PQRS is a square.

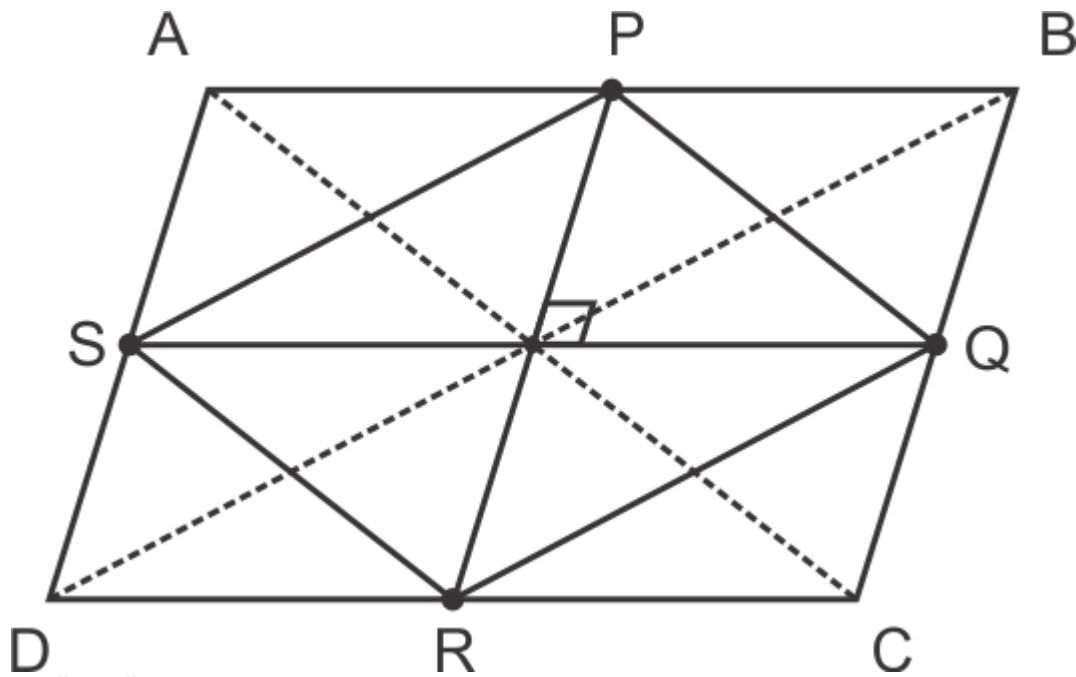
Hence, correct option is (b).

#### Question 14

The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a

- (a) rectangle
- (b) parallelogram
- (c) rhombus
- (d) square

Solution 14



D

$PQ \parallel SR \parallel AC$

$QR \parallel PS \parallel BD$

{Because line joining the mid – points of two sides of  $\triangle$  is  $\parallel$  to third side}

Now because AC is not perpendicular to BD in parallelogram,

$\Rightarrow$  SR is not perpendicular to QR

Also  $\triangle ASP \not\cong \triangle DRS$

$\Rightarrow PS \neq SR$

$\Rightarrow$  PQRS is just a parallelogram.

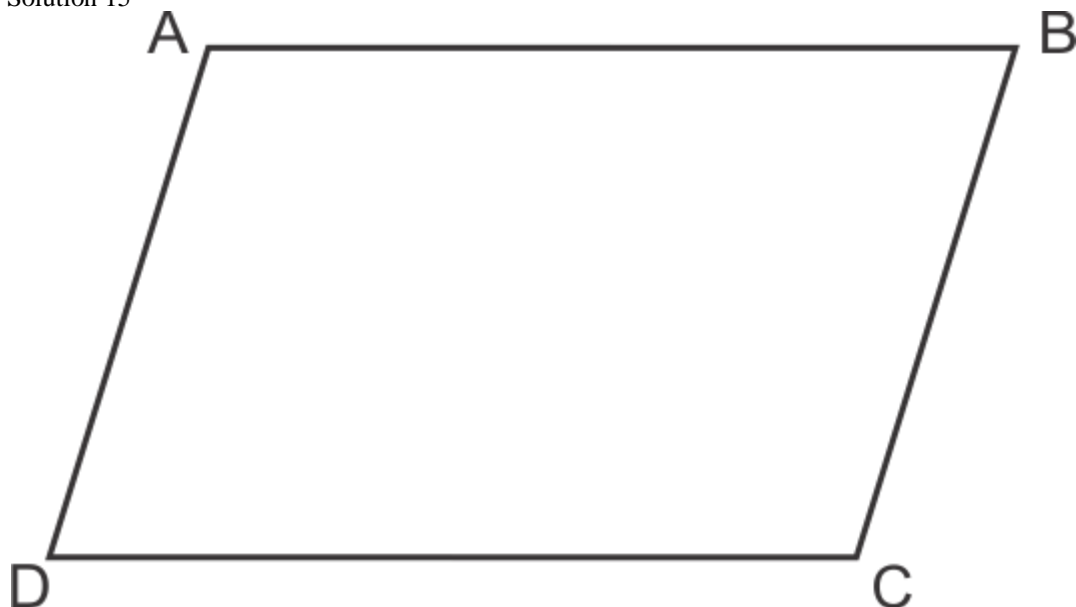
Hence, correct option is (b).

#### Question 15

If one angle of a parallelogram is  $24^\circ$  less than twice the smallest angle, then the measure of the largest angle of the parallelogram is

- (a)  $176^\circ$
- (b)  $68^\circ$
- (c)  $112^\circ$
- (d)  $102^\circ$

Solution 15



Let the smallest angle =  $\angle ADC = x^\circ$

Other angle =  $\angle BCD$

$$\Rightarrow \angle BCD = 2x^\circ - 24^\circ$$

Also,  $\angle ADC + \angle BCD = 180^\circ$  (Sum of adjacent angles in  $\parallel^{\text{gram}} = 180^\circ$ )

$$\Rightarrow x^\circ + 2x^\circ - 24^\circ = 180^\circ$$

$$\Rightarrow 3x^\circ = 204^\circ$$

$$\Rightarrow x = 68^\circ$$

$$\Rightarrow \text{Largest angle} = \angle BCD = 2 \times 68^\circ - 24^\circ = 112^\circ$$

Hence, correct option is (c).

#### Question 16

In a parallelogram ABCD, if  $\angle DAB = 75^\circ$  and  $\angle DBC = 60^\circ$ , then  $\angle BDC =$

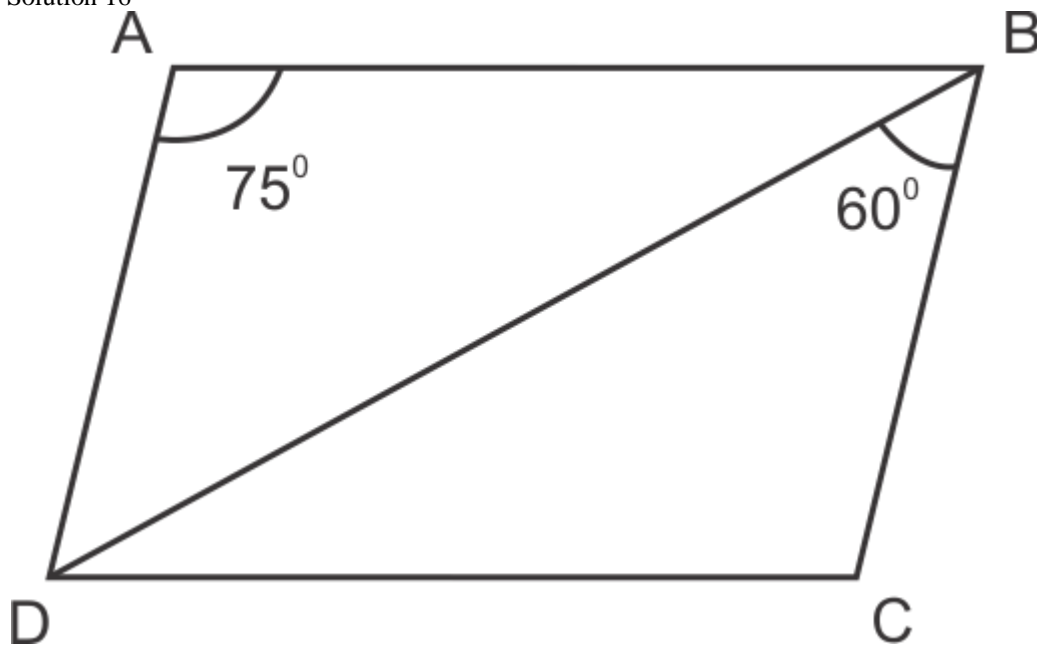
(a)  $75^\circ$

(b)  $60^\circ$

(c)  $45^\circ$

(d)  $55^\circ$

#### Solution 16



In parallelogram ABCD,

$$\angle A + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 75^\circ = 105^\circ$$

$$\angle ADB = \angle DBC \text{ (Alternate angles)}$$

$$\Rightarrow \angle ADB = 60^\circ$$

$$\angle BDC = \angle ADC - \angle ADB = 105^\circ - 60^\circ = 45^\circ$$

Hence, correct option is (c).

#### Question 17

ABCD is a parallelogram and E and F are the centroids of triangles ABD and BCD respectively, then EF =

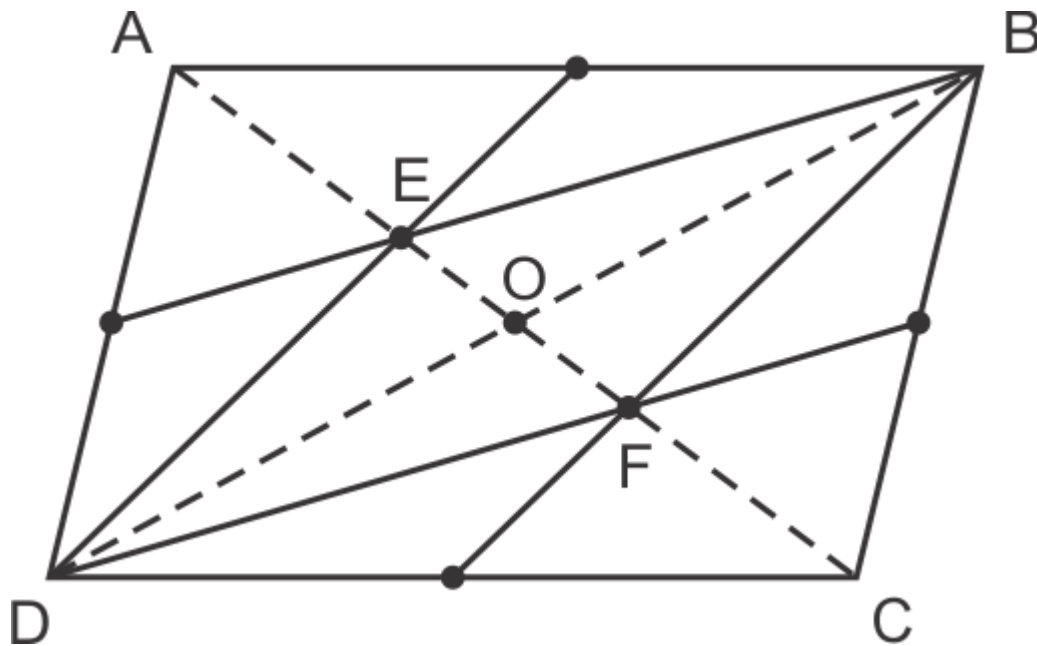
(a) AE

(b) BE

(c) CE

(d) DE

#### Solution 17



Centroid is the point where all medians of a  $\triangle$  meet.

In  $\triangle ABD$ , E is the centroid,

and in  $\triangle BCD$ , F is the centroid.

By the property of centroid, centroid divides a median in 2 : 1

So from figure,

$$\frac{AE}{EO} = \frac{2}{1} \Rightarrow EO = \frac{AE}{2} \dots(1)$$

$$\text{Also } \frac{CF}{FO} = \frac{2}{1} \Rightarrow FO = \frac{CF}{2} \dots(2)$$

Because AC is a diagonal of a parallelogram, O is its midpoint.

$$\Rightarrow OA = OC$$

$$\Rightarrow AE = CF$$

Adding equations (1) & (2),

$$EO + FO = \frac{AE + CF}{2} = \frac{2AE}{2}$$

$$\Rightarrow EF = \frac{2AE}{2}$$

$$\Rightarrow EF = AE$$

Hence, correct option is (a).

## Chapter 13 - Quadrilaterals Exercise 13.72

### Question 18

ABCD is a parallelogram, M is the mid – point of BD and BM bisects  $\angle B$ . Then,  $\angle AMB =$

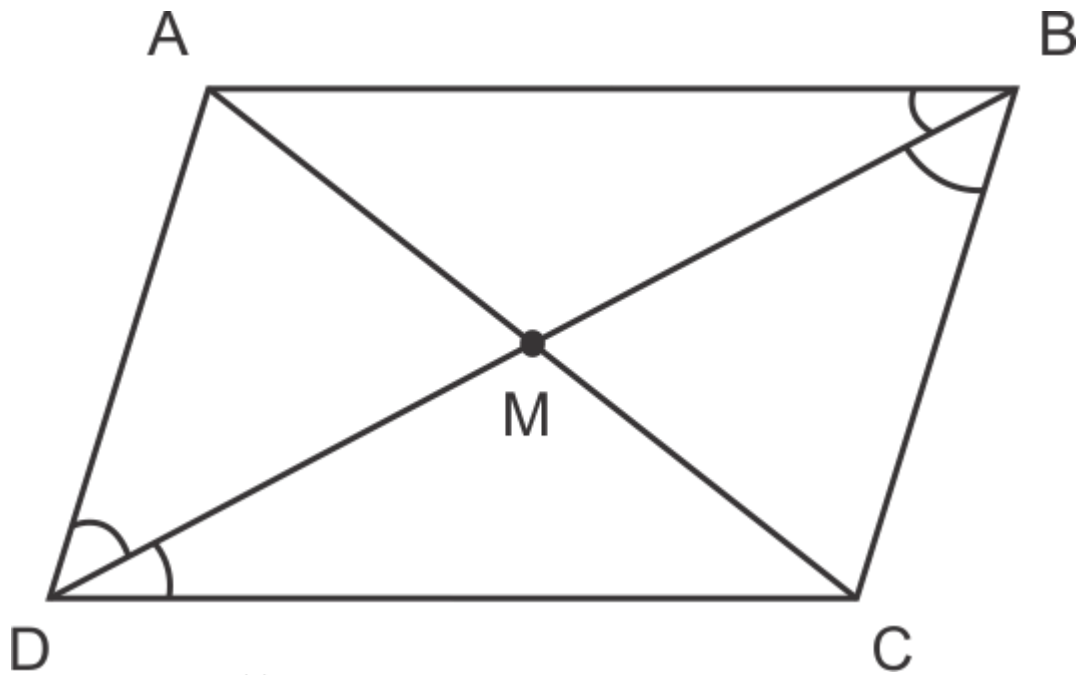
(a)  $45^\circ$

(b)  $60^\circ$

(c)  $90^\circ$

(d)  $75^\circ$

Solution 18



$\angle ABM = \angle CBM$  ... (1) (BM bisects  $\angle B$ )  
 $\angle ABM = \angle MDC$  ... (2) (Alternate angles)  
 $\angle CBM = \angle ADM$  ... (3) (Alternate angles)  
 From equations (1), (2) & (3)  
 $\angle MDC = \angle ADM$  ... (4)

Now, consider  $\triangle ABD$  &  $\triangle CBD$   
 $\angle CBD = \angle ABD$  {From eq (1)}  
 $DB = DB$  (Common)  
 $\angle ADB = \angle CDB$  {From eq (4)}  
 Hence, by ASA property,  
 $\triangle ABD \cong \triangle CBD$   
 $\Rightarrow AB = CB, AD = CD$

Hence, it becomes a Rhombus.

So, now diagonals of a Rhombus bisect each other at  $90^\circ$ .

$\Rightarrow \angle AMB = 90^\circ$

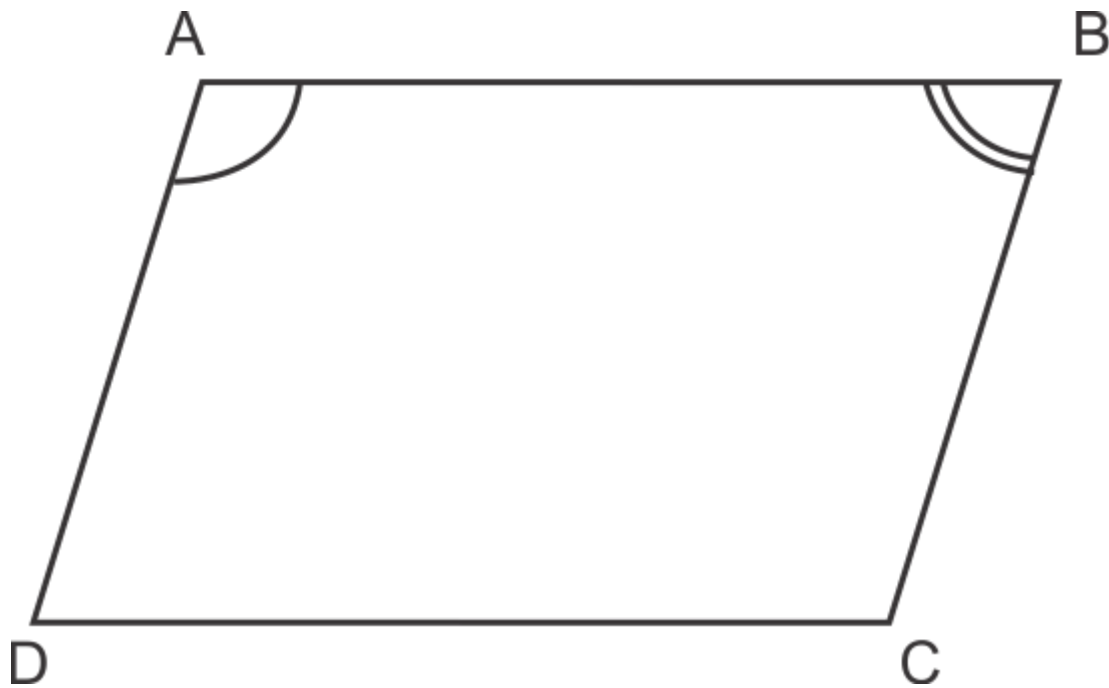
Hence, correct option is (c).

#### Question 19

If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is

- (a)  $108^\circ$
- (b)  $54^\circ$
- (c)  $72^\circ$
- (d)  $81^\circ$

Solution 19



Let ABCD be a parallelogram and  $\angle A = \frac{2}{3}\angle B$

Also,  $\angle A + \angle B = 180^\circ$  (adjacent angles in a Parallelogram are supplementary)

$$\Rightarrow \frac{2}{3}\angle B + \angle B = 180^\circ$$

$$\Rightarrow \angle B = 108^\circ \text{ and } \angle A = 72^\circ$$

$\Rightarrow$  Smallest angle is  $72^\circ$ .

Hence, correct option is (c).

#### Question 20

If the degree measures of the angles of quadrilateral are  $4x$ ,  $7x$ ,  $9x$  and  $10x$ , what is the sum of the measures of the smallest angle and largest angle?

- (a)  $140^\circ$
- (b)  $150^\circ$
- (c)  $168^\circ$
- (d)  $180^\circ$

#### Solution 20

Sum of all angles of a Quadrilateral =  $360^\circ$

$$4x + 7x + 9x + 10x = 360^\circ$$

$$30x = 360^\circ$$

$$x = 12^\circ$$

So, sum of smallest and largest angle,

$$\text{i.e. } 4x + 10x = 14x = 14 \times 12 = 168^\circ$$

Hence, correct option is (c).

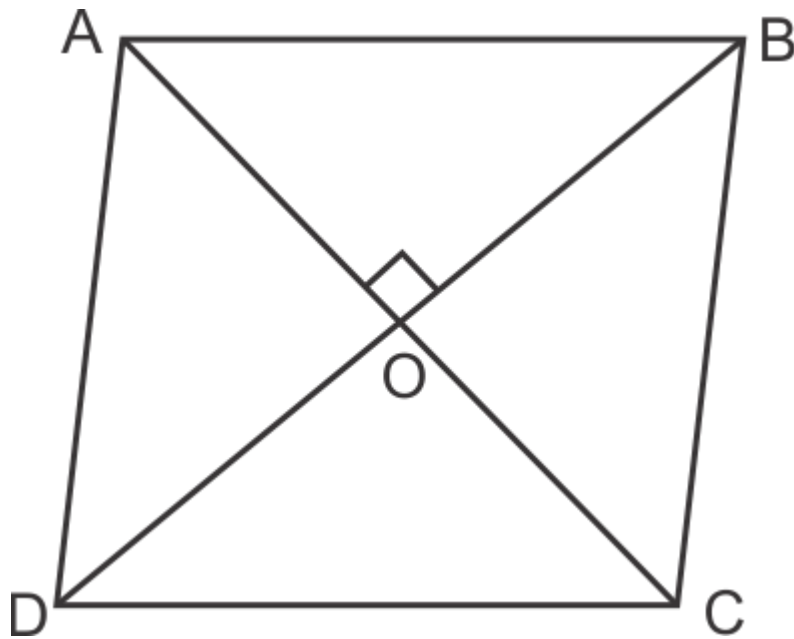
#### Question 21

If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to

- (a) 16 cm
- (b) 15 cm
- (c) 20 cm
- (d) 17 cm

#### Solution 21





Let  $BD = 24$  cm and  $AC = 18$  cm (Given)

Now,  $AO = \frac{AC}{2} = \frac{18}{2} = 9$  cm and  $BO = \frac{BD}{2} = \frac{24}{2} = 12$  cm

Now,  $AB = \sqrt{(AO)^2 + (BO)^2}$  (Diagonals make  $90^\circ$  between them)  
 $= \sqrt{9^2 + 12^2}$   
 $= \sqrt{81 + 144}$   
 $= \sqrt{225}$

$\Rightarrow AB = 15$  cm

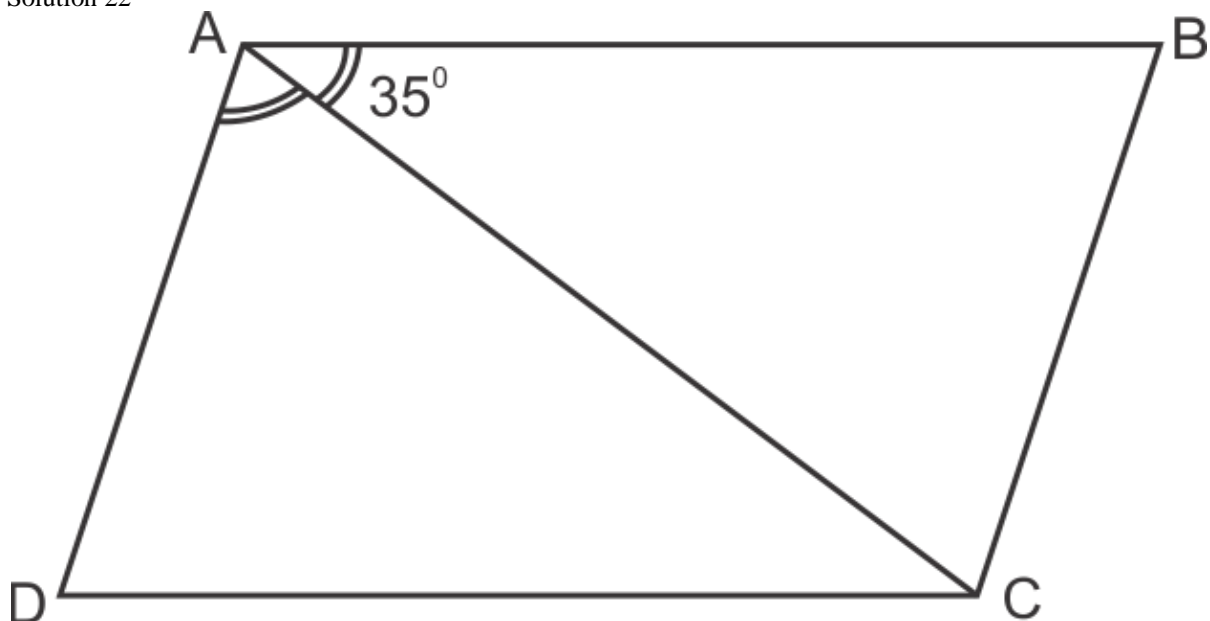
Hence, correct option is (b).

Question 22

ABCD is a parallelogram in which diagonal AC bisects  $\angle BAD$ . If  $\angle BAC = 35^\circ$ , then  $\angle ABC =$

- (a)  $70^\circ$
- (b)  $110^\circ$
- (c)  $90^\circ$
- (d)  $120^\circ$

Solution 22



AC bisects  $\angle DAB$ .

$$\Rightarrow \angle DAC = \angle BAC = 35^\circ$$

$$\Rightarrow \angle BAD = 2 \times 35^\circ = 70^\circ$$

$\angle A + \angle B = 180^\circ$  (Sum of any two adjacent angles in Parallelogram =  $180^\circ$ )

$$\Rightarrow \angle B = \angle ABC = 180^\circ - \angle BAD = 180^\circ - 70^\circ = 110^\circ$$

Hence, correct option is (b).

Question 23

In a rhombus ABCD, if  $\angle ACB = 40^\circ$ , then  $\angle ADB =$

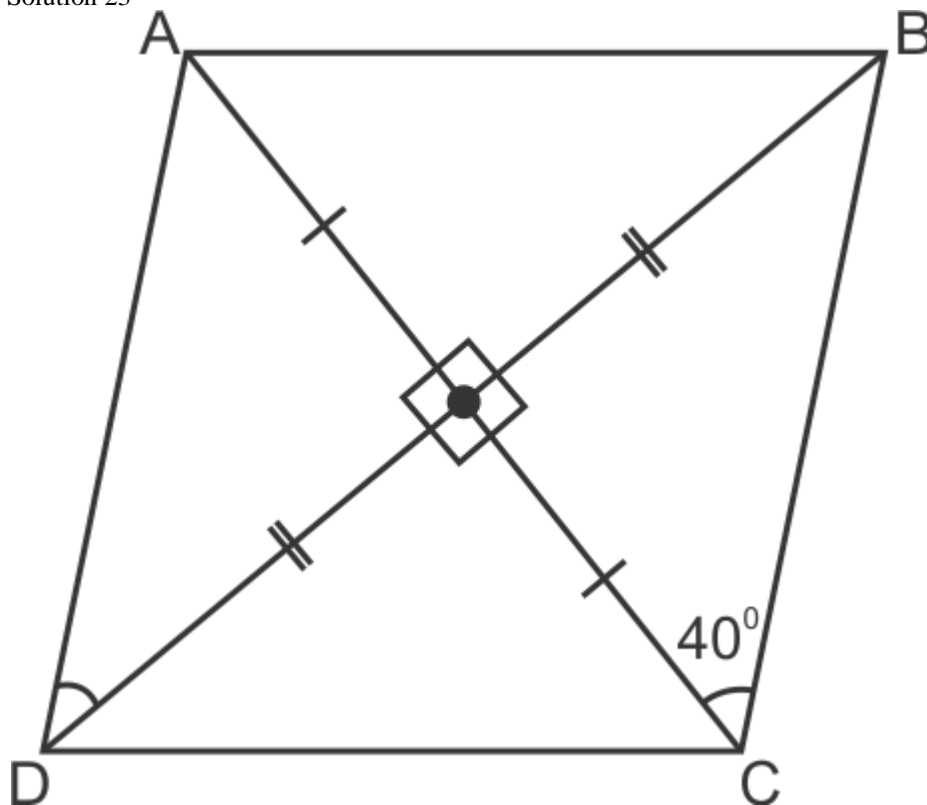
(a)  $70^\circ$

(b)  $45^\circ$

(c)  $50^\circ$

(d)  $60^\circ$

Solution 23



Consider  $\triangle AOD$  &  $\triangle COB$

$$\angle AOD = \angle COB = 90^\circ$$

$AD = BC$  (Sides of Rhombus)

$AO = CO$  (Diagonals bisect each other)

So by RHS property,  $\triangle AOD \cong \triangle COB$

$$\Rightarrow \angle OAD = \angle OCB = 40^\circ$$

In  $\triangle AOD$ ,

$$\angle ADB = \angle ADO = 180^\circ - 90^\circ - 40^\circ = 50^\circ$$

Hence, correct option is (c).

Question 24

In  $\triangle ABC$ ,  $\angle A = 30^\circ$ ,  $\angle B = 40^\circ$  and  $\angle C = 110^\circ$ . The angles of the triangle formed by joining the mid-points of the sides of this triangle are

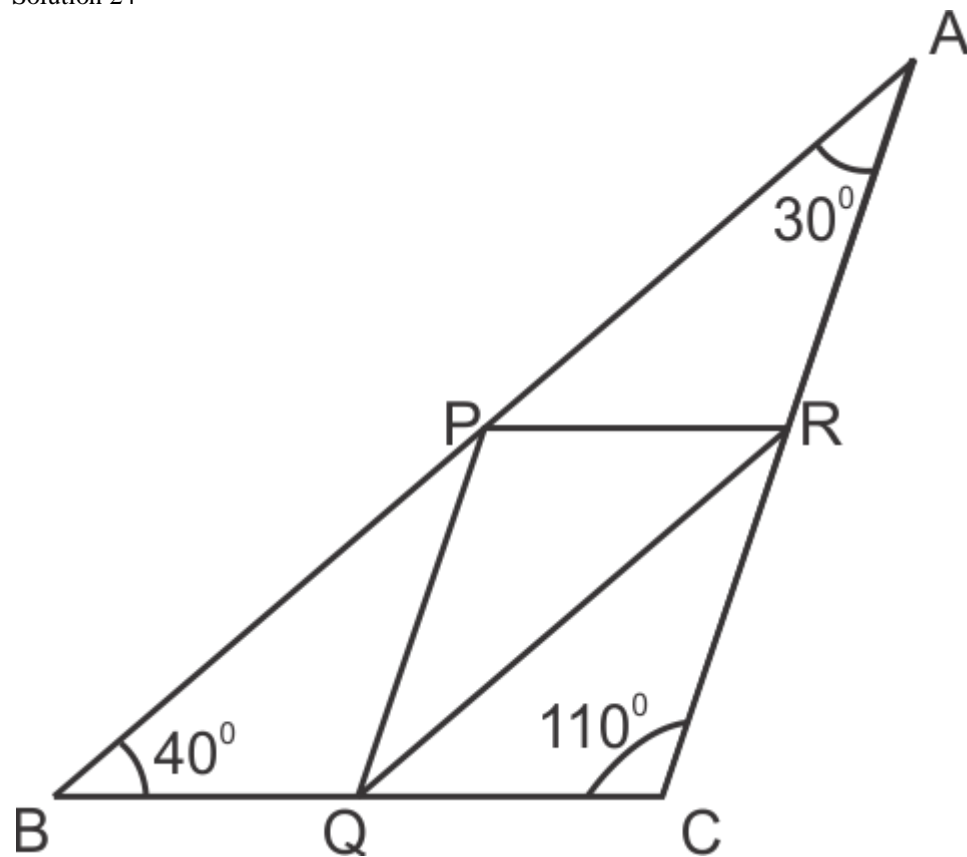
(a)  $70^\circ, 70^\circ, 40^\circ$

(b)  $60^\circ, 40^\circ, 80^\circ$

(c)  $30^\circ, 40^\circ, 110^\circ$

(d)  $60^\circ, 70^\circ, 50^\circ$

Solution 24



If in any triangle, all the mid – points (of each sides) are joined to form a triangle, then that triangle is similar to a parent triangle.

i.e.  $\triangle PQR \sim \triangle ABC$

So angles of  $\triangle PQR$  will be same as angles of  $\triangle ABC$ .

Thus, angles are  $30^\circ$ ,  $40^\circ$ ,  $110^\circ$ .

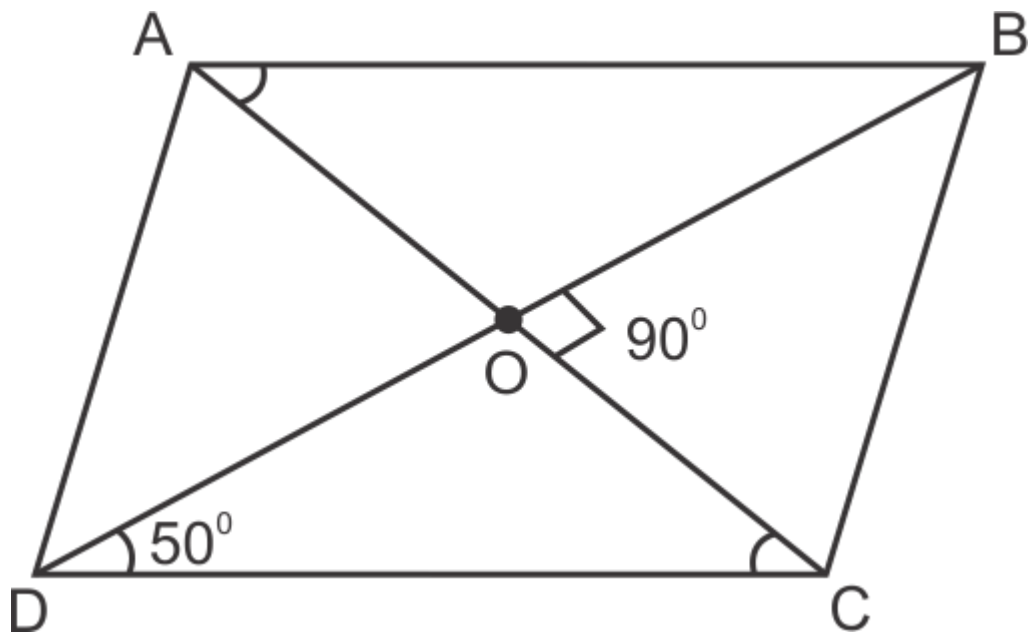
Hence, correct option is (c).

Question 25

The diagonals of a parallelogram ABCD intersect at O. If  $\angle BOC = 90^\circ$  and  $\angle BDC = 50^\circ$ , then  $\angle OAB =$

- (a)  $40^\circ$
- (b)  $50^\circ$
- (c)  $10^\circ$
- (d)  $90^\circ$

Solution 25



**D**

In a parallelogram ABCD,

$$\angle OAB = \angle OCD$$

In  $\triangle OCD$

$$\angle OCD + \angle COD + \angle ODC = 180^\circ$$

$$\angle COD = 90^\circ$$

$$\angle ODC = 50^\circ \text{ (given)}$$

$$\angle OCD = 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

$$\Rightarrow \angle OAB = \angle OCD = 40^\circ$$

Hence, correct option is (a).

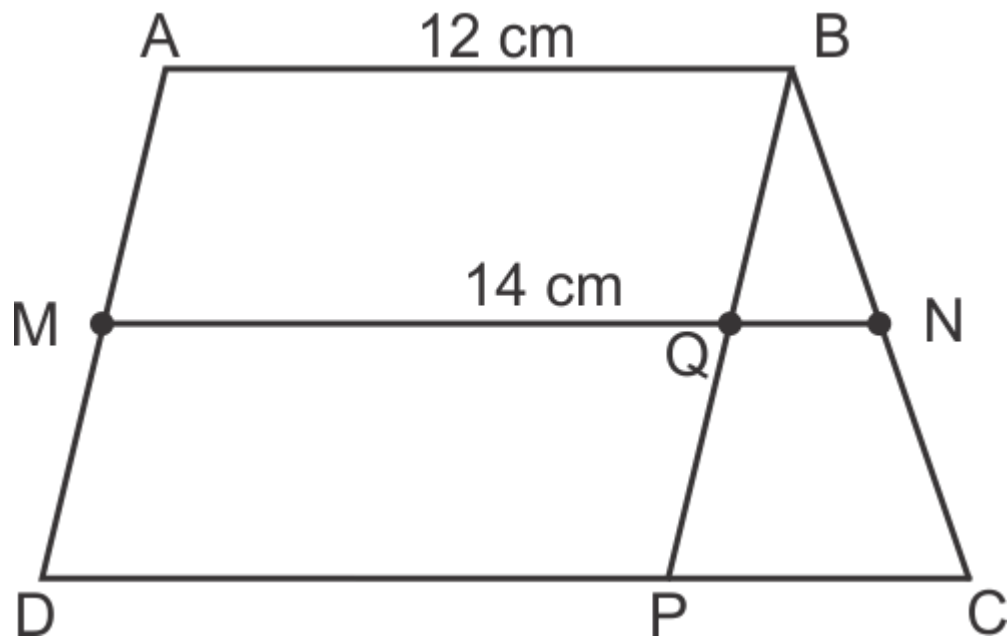
**Question 26**

ABCD is a trapezium in which  $AB \parallel DC$ . M and N are the mid – points of AD and BC respectively.

If  $AB = 12$  cm,  $MN = 14$  cm, then  $CD =$

- (a) 10 cm
- (b) 12 cm
- (c) 14 cm
- (d) 16 cm

**Solution 26**



Let a line BP is drawn  $\parallel$  to AD to meet DC at P.

ABPD is a parallelogram.

$AB \parallel PD$ ,  $AD \parallel BP$

So  $AB = DP$

Let BP cuts MN at Q.

MQ is also  $\parallel$  to  $AB \parallel PD$

So  $AB = MQ = DP = 12 \text{ cm} \dots(1)$

$QN = MN - MQ = 14 - 12 = 2 \text{ cm}$

Consider  $\triangle BPC$ .

Q and N are the mid – points of BP & BC, and the line joining them  $QN \parallel PC$ .

Then by property,  $\frac{QN}{PC} = \frac{1}{2}$

$\Rightarrow PC = 2QN = 2 \times 2 = 4 \text{ cm}$

Now,  $DC = DP + PC$

$DP = 12 \text{ cm}$  [From (1)]

$\Rightarrow DC = 12 + 4 = 16 \text{ cm}$

Hence, correct option is (d).

**Question 30**

ABCD is a parallelogram, and E is the mid – point of BC.

DC and AB when produced meet at F. Then,  $AF =$

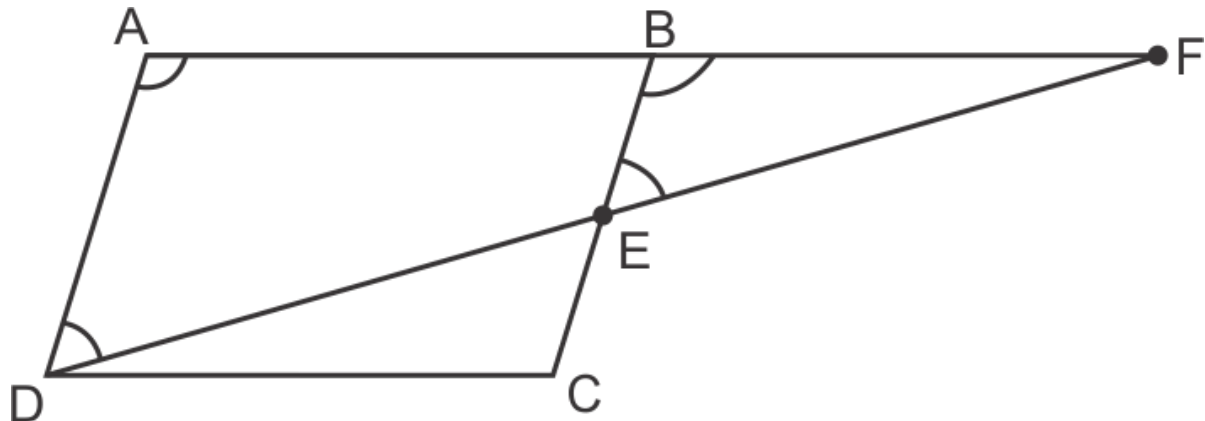
(a)  $\frac{3}{2}AB$

(b)  $2AB$

(c)  $3AB$

(d)  $\frac{5}{4}AB$

**Solution 30**



D

$BC \parallel AD$

$\Rightarrow BE \parallel AD$

Now, consider  $\triangle FAD$

$BE \parallel AD$

Also  $\frac{BE}{AD} = \frac{1}{2}$

In  $\triangle FBE$  and  $\triangle FAD$ ,

$\angle FAD = \angle FBE$  {Corresponding angles}

$\angle ADF = \angle BEF$  {Corresponding angles}

$\angle F = \angle F$  {common}

Hence,  $\triangle FBE \sim \triangle FAD$

$$\Rightarrow \frac{BF}{AF} = \frac{BE}{AD} = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{BF}{AF} = 1 - \frac{1}{2}$$

$$\Rightarrow \frac{AF - BF}{AF} = \frac{1}{2}$$

$$\Rightarrow \frac{AB}{AF} = \frac{1}{2}$$

$$\Rightarrow AF = 2AB$$

Hence, correct option is (b).

Question 27

Diagonals of a quadrilateral ABCD bisect each other. If  $\angle A = 45^\circ$ , then  $\angle B =$

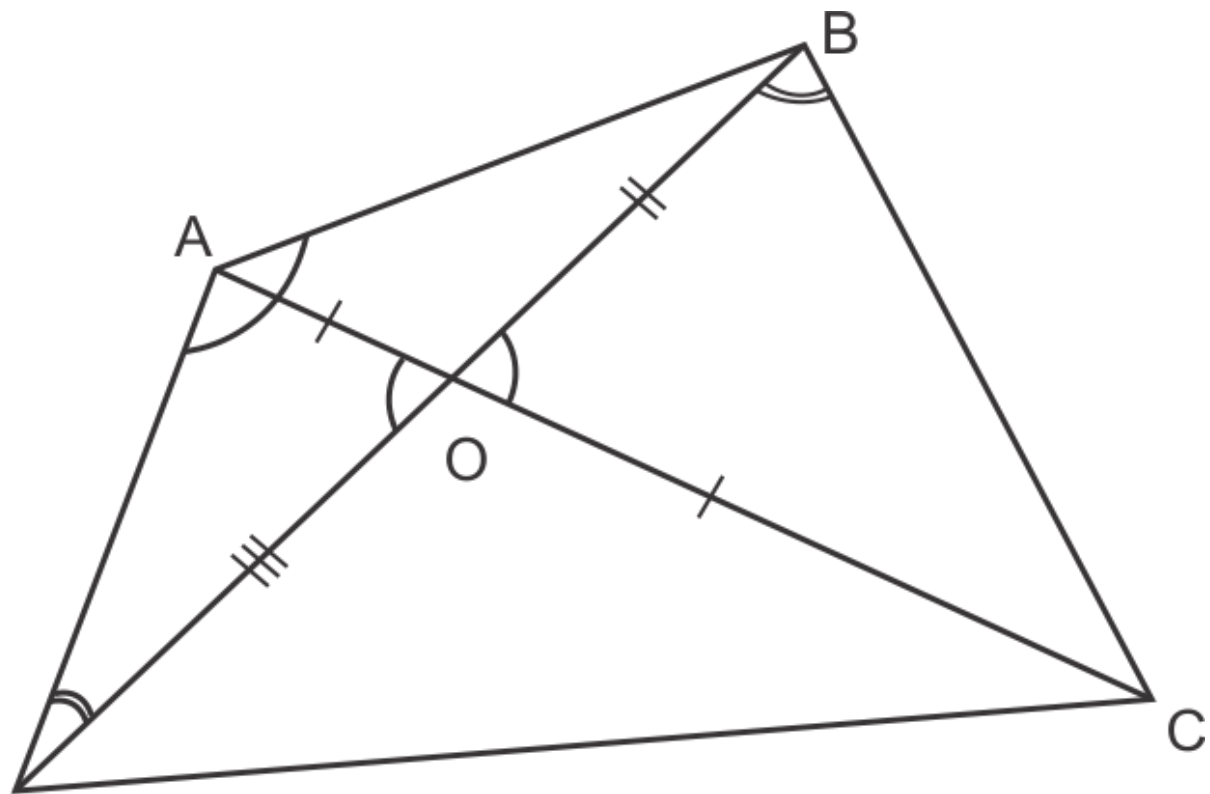
(a)  $115^\circ$

(b)  $120^\circ$

(c)  $125^\circ$

(d)  $135^\circ$

Solution 27



**D**

Consider  $\triangle AOD$  &  $\triangle COB$ ,

$AO = CO$  {Diagonals Bisect each other}

$OD = OB$  {Diagonals Bisect each other}

$\angle AOD = \angle COB$  (opposite angles)

So, by SAS property,  $\triangle AOD \cong \triangle COB$

$\Rightarrow \angle ADO = \angle CBO \dots(1)$

$\angle ABD = 180^\circ - \angle A - \angle ADO$  (in  $\triangle ADB$ )

$$= 180 - 45^\circ - \angle ADO$$

$\angle ABD = 135^\circ - \angle ADO \dots(2)$

$\angle B = \angle ABD + \angle CBO$

Putting values From eq (1) and (2)

$$\angle B = 135^\circ - \cancel{\angle ADO} + \cancel{\angle ADO}$$

$$\angle B = 135^\circ$$

Hence, correct option is (d).

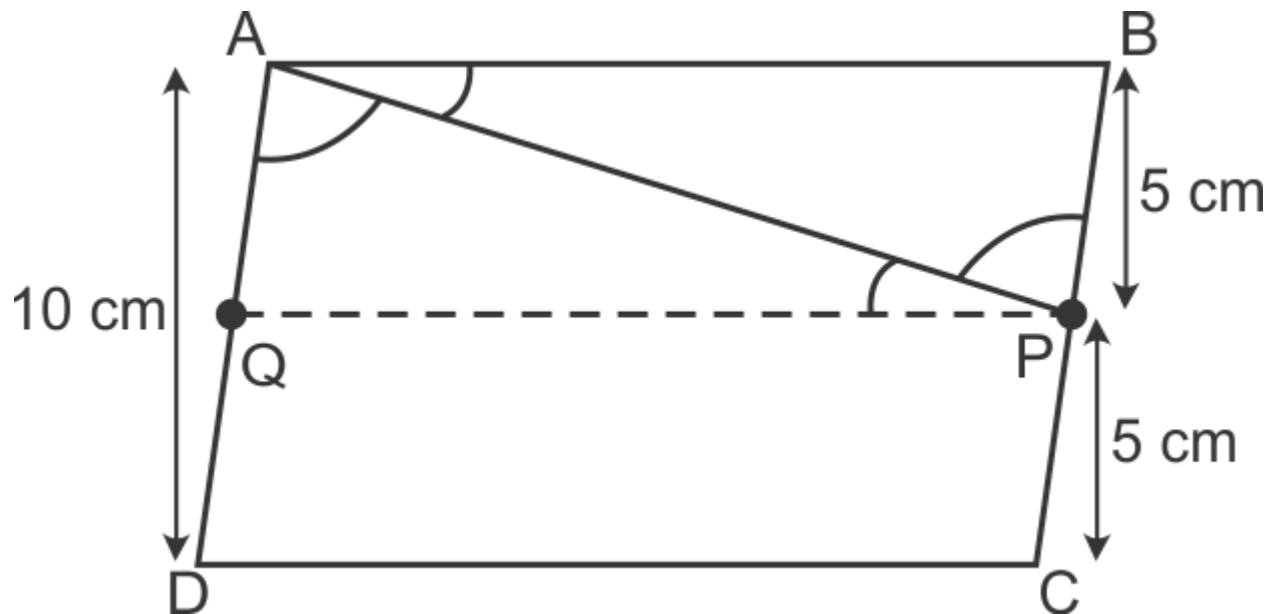
**Question 28**

P is the mid - point of side BC of a parallelogram ABCD such that

$\angle BAP = \angle DAP$ . If  $AD = 10$  cm, then  $CD =$

- (a) 5 cm
- (b) 6 cm
- (c) 8 cm
- (d) 10 cm

**Solution 28**



Let a line parallel to AB is drawn from P to meet AD at Q.

$PQ \parallel AB \parallel DC$

Q is also mid – point of AD.

Now, consider parallelogram ABPQ.

$\angle PAQ = \angle APB$  (Alternate angles)

Also  $\angle PAQ = \angle BAP$  (Given)

$\Rightarrow \angle APB = \angle BAP$

So  $\triangle ABP$  is isosceles  $\triangle$ .

$\Rightarrow BP = AB$

i.e.  $AB = \frac{10}{2} = 5 \text{ cm}$

Hence, correct option is (a).

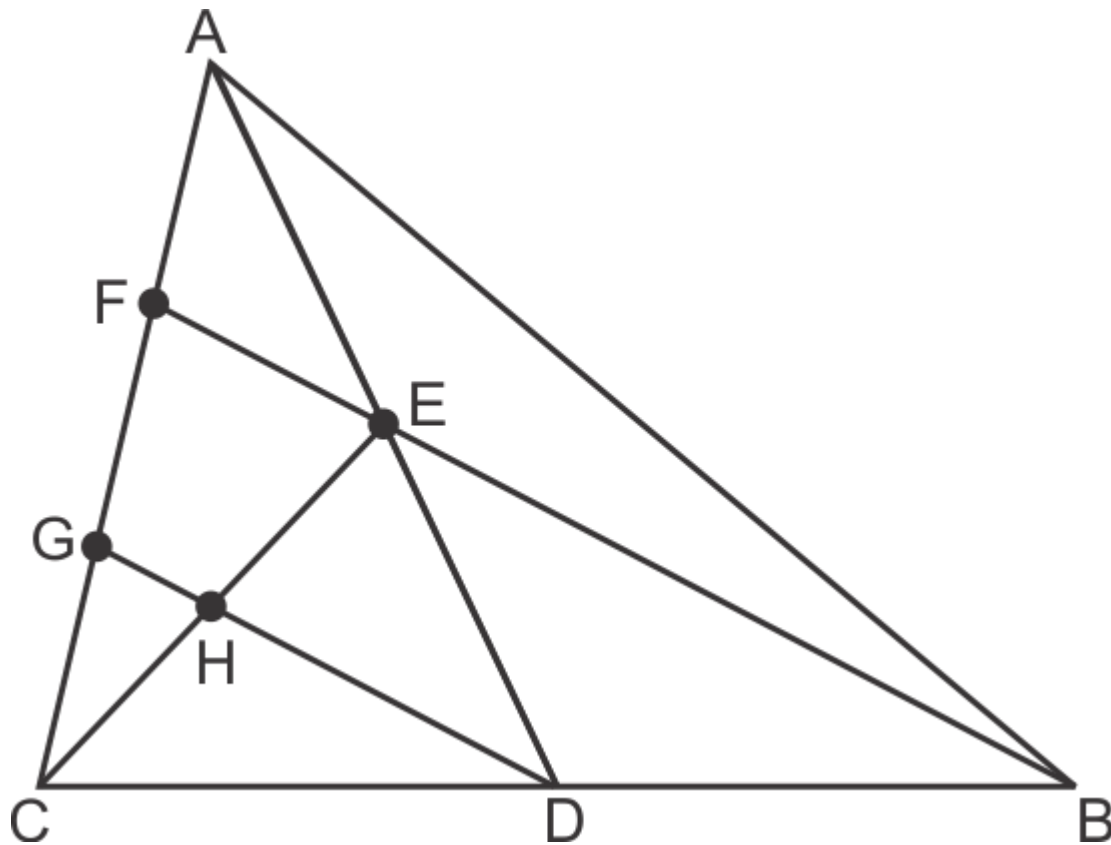
#### Question 29

In  $\triangle ABC$ , E is the mid-point of median AD such that BE produced meets AC at F. If  $AC = 10.5 \text{ cm}$ , then  $AF =$

- (a) 3 cm
- (b) 3.5 cm
- (c) 2.5 cm
- (d) 5 cm

#### Solution 29





A line DG is drawn parallel to FE to meet AC.

$FE \parallel DG$  and  $FE \parallel GH$

Now, consider  $\triangle ADG$ .

E is the mid – point of AD and EF is line From E  $\parallel$  to Base DG.

So by Property, it will meet AG at its midpoint

i.e. F is midpoint of AG.

$$\Rightarrow AF = FG \dots\dots(1)$$

Now, consider  $\triangle FBC$  &  $\triangle GDC$

$FE \parallel GH$  and  $FE \parallel GD$

D is mid – point of BC.

$$\Rightarrow \frac{DC}{BC} = \frac{1}{2} \dots\dots(2)$$

Because  $\triangle FBC \sim \triangle GDC$ ,

$$\Rightarrow \frac{GC}{FC} = \frac{1}{2} \Rightarrow FC = 2GC$$

OR  $FG = GC \dots\dots(3)$

From equations (1) and (3),

$$AF = FG = GC$$

$$\Rightarrow AF = \frac{AC}{3} = \frac{10.5}{3} = 3.5 \text{ cm}$$

Hence, correct option is (b).

## Chapter 13 - Quadrilaterals Exercise 13.73

### Question 31

In a quadrilateral ABCD,  $\angle A + \angle C$  is 2 times  $\angle B + \angle D$ . If  $\angle A = 140^\circ$  and  $\angle D = 60^\circ$ , then  $\angle B =$

(a)  $60^\circ$

(b)  $80^\circ$

(c)  $120^\circ$

(d) None of these

Solution 31

In a quadrilateral ABCD,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ \quad \dots(1)$$

$$\text{Now, } \angle A + \angle C = 2(\angle B + \angle D) \quad (\text{given}) \dots(2)$$

$$\text{Also, } \angle A = 140^\circ \quad \angle D = 60^\circ$$

Putting value of  $(\angle A + \angle C)$  from eq. (2) in eq. (1)

$$2(\angle B + \angle D) + \angle B + \angle D = 360^\circ$$

$$3(\angle B + \angle D) = 360^\circ$$

$$\Rightarrow \angle B + \angle D = 120^\circ$$

$$\Rightarrow \angle B + 60^\circ = 120^\circ$$

$$\Rightarrow \angle B = 60^\circ$$

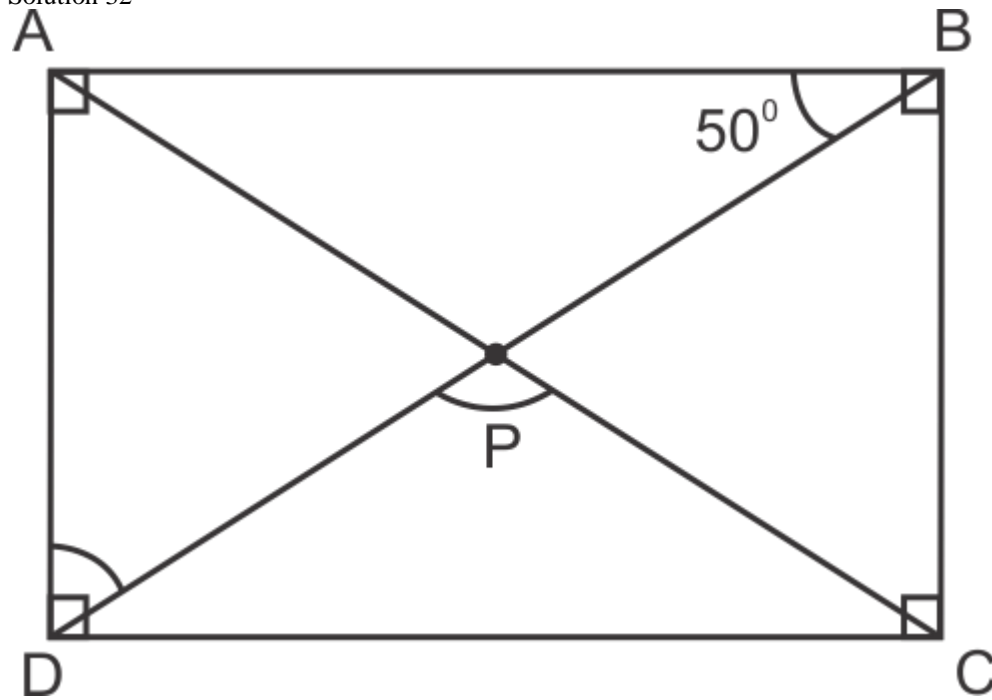
Hence, correct option is (a).

Question 32

The diagonals AC and BD of a rectangle ABCD intersect each other at P. If  $\angle ABD = 50^\circ$ , then  $\angle DPC =$

- (a)  $70^\circ$
- (b)  $90^\circ$
- (c)  $80^\circ$
- (d)  $100^\circ$

Solution 32



In  $\triangle ABD$ ,

$$\angle BDA + \angle ABD + \angle DAB = 180^\circ$$

$$\angle ABD = 50^\circ \text{ and } \angle DAB = 90^\circ$$

$$\Rightarrow \angle BDA = 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

Consider  $\triangle ABD$  &  $\triangle BAC$

$$AD = BC, \angle DAB = \angle ABC = 90^\circ, BD = AC$$

Hence, by RHS property  $\triangle ABD \cong \triangle BAC$

$$\Rightarrow \angle ABD = \angle BAC = 50^\circ$$

Now, consider  $\triangle ABP$

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$\angle PAB = \angle BAC = 50^\circ$$

$$\angle PBA = \angle ABD = 50^\circ$$

$$\Rightarrow \angle APB = 180^\circ - 50^\circ - 50^\circ = 80^\circ$$

Now,  $\angle APB = \angle DPC$  (Opposite angles)

$$\Rightarrow \angle DPC = 80^\circ$$

Hence, correct option is (c).

## Chapter 13 - Quadrilaterals Exercise Ex. 13.1

Question 1

**Three angles of a quadrilateral are respectively equal to  $110^\circ$ ,  $50^\circ$  and  $40^\circ$ . Find its fourth angle.**

Solution 1

**Let fourth angle be  $x$ .**

**We have,**

$$\text{Sum of all angles of a quadrilateral} = 360^\circ$$

$$\Rightarrow 110^\circ + 50^\circ + 40^\circ + x = 360^\circ$$

$$\Rightarrow 200^\circ + x = 360^\circ$$

$$\Rightarrow x = 160^\circ$$

Question 2

**In a quadrilateral  $ABCD$ , the angles  $A, B, C$  and  $D$  are in the ratio  $1 : 2 : 4 : 5$ . Find the measure of each angle of the quadrilateral.**

Solution 2

**Let the angles of the quadrilateral be  $A = x, B = 2x, C = 4x$  and  $D = 5x$ . Then,**

$$A + B + C + D = 360^\circ$$

$$\Rightarrow x + 2x + 4x + 5x = 360$$

$$\Rightarrow 12x = 360$$

$$\Rightarrow x = 30$$

$$\therefore A = 30^\circ, B = 60^\circ, C = 120^\circ \text{ and } D = 150^\circ$$

Question 3

The angles of quadrilateral are in the ratio 3: 5: 9: 13, Find all the angles of the quadrilateral.

Solution 3

**Let the common ratio between the angles is  $x$ . So, the angles will be  $3x, 5x, 9x$  and  $13x$  respectively.**

Since the sum of all interior angles of a quadrilateral is  $360^\circ$ .

$$\therefore 3x + 5x + 9x + 13x = 360^\circ$$

$$30x = 360^\circ$$

$$x = 12^\circ$$

Hence, the angles are

$$3x = 3 \times 12 = 36^\circ$$

$$5x = 5 \times 12 = 60^\circ$$

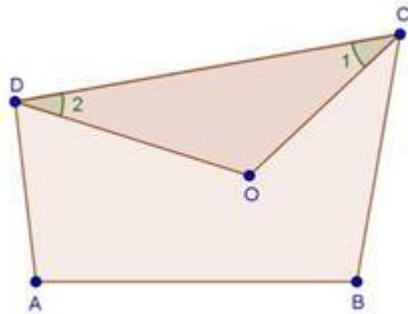
$$9x = 9 \times 12 = 108^\circ$$

$$13x = 13 \times 12 = 156^\circ$$

Question 4

In a quadrilateral  $ABCD$ ,  $CO$  and  $DO$  are the bisectors of  $\angle C$  and  $\angle D$  respectively. Prove that  $\angle COD = \frac{1}{2}(\angle A + \angle B)$ .

Solution 4



In  $\triangle DOC$

$$\angle 1 + \angle COD + \angle 2 = 180^\circ$$

[Angle sum property of a triangle]

$$\Rightarrow \angle COD = 180 - \angle 1 - \angle 2$$

$$\Rightarrow \angle COD = 180 - (\angle 1 + \angle 2)$$

$$\Rightarrow \angle COD = 180 - \left( \frac{1}{2}\angle C + \frac{1}{2}\angle D \right)$$

[ $\because OC$  &  $OD$  are bisectors of  $\angle C$  &  $\angle D$  respectively]

$$\Rightarrow \angle COD = 180 - \frac{1}{2}(\angle C + \angle D)$$

—(i)

In quadrilateral  $ABCD$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

[Angle sum property of a quadrilateral]

$$\angle C + \angle D = 360 - (\angle A + \angle B)$$

—(ii)

Substituting (ii) in (i)

$$\Rightarrow \angle COD = 180 - \frac{1}{2}(360 - (\angle A + \angle B))$$

$$\Rightarrow \angle COD = 180 - 180 + \frac{1}{2}(\angle A + \angle B)$$

$$\Rightarrow \angle COD = \frac{1}{2}(\angle A + \angle B)$$

#### Question 1

Two opposite angles of a parallelogram are  $(3x - 2)^\circ$  and  $(50 - x)^\circ$ . Find the measure of each of the parallelogram.

#### Solution 1

**Since opposite angles of a parallelogram are equal**

$$\therefore 3x - 2 = 50 - x$$

$$\Rightarrow 3x + x = 50 + 2$$

$$\Rightarrow 4x = 52$$

$$\Rightarrow x = 13$$

$$\therefore (3x - 2)^\circ = (3 \times 13 - 2)^\circ = 37^\circ$$

$$(50 - x)^\circ = (50 - 13)^\circ = 37^\circ$$

**Adjacent angles of a parallelogram are supplementary.**

$$\therefore x + 37^\circ = 180^\circ$$

$$\therefore x = 180^\circ - 37^\circ = 143^\circ$$

**Hence, four angles are :  $37^\circ, 143^\circ, 37^\circ, 143^\circ$**

#### Question 2

If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

#### Solution 2

**Let the measure of the angle be  $x$**

$$\therefore \text{The measure of the adjacent angle is } \frac{2x}{3}$$

**Since the adjacent angle of a parallelogram is supplementary**

$$\text{Hence, } x + \frac{2x}{3} = 180^\circ$$

$$\Rightarrow \frac{5x}{3} = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ \times 3}{5} = 108^\circ$$

**Adjacent angles are supplementary**

$$\therefore 108^\circ + x = 180^\circ$$

$$\Rightarrow x = 72^\circ$$

**Hence, four angles are :  $108^\circ, 72^\circ, 108^\circ, 72^\circ$**

#### Question 3

Find the measure of all the angles of a parallelogram, if one angle is  $24^\circ$  less than twice the smallest angle.

#### Solution 3

**Let the smallest angle be  $x$**

**$\therefore$  the other angle is  $(2x - 24)$**

**Now,**

**$x + 2x - 24 = 180$  [ $\because$  Sum of adjacent angle of a parallelogram is  $180^\circ$ ]**

$$\Rightarrow 3x - 24 = 180$$

$$\Rightarrow 3x = 180 + 24$$

$$\Rightarrow 3x = 204$$

$$\Rightarrow x = \frac{204}{3} = 68^\circ$$

$$x = 68^\circ$$

$$\Rightarrow 2x - 24^\circ = 2 \times 68^\circ - 24^\circ = 136^\circ - 24^\circ = 112^\circ$$

**Hence, four angles are :  $68^\circ$ ,  $112^\circ$ ,  $68^\circ$ ,  $112^\circ$**

Question 4

The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm what is the measure of the shorter side?

Solution 4

**Let the shorter side be  $x$**

$$\text{Perimeter} = x + 6.5 + x + 6.5 \quad [\text{Sum of all sides}]$$

$$22 = 2(x + 6.5)$$

$$11 = x + 6.5$$

$$x = 11 - 6.5 = 4.5 \text{ cm}$$

**$\therefore$  Shorter side = 4.5 cm.**

Question 5

In a parallelogram ABCD,  $\angle D = 135^\circ$ , determine the measures of  $\angle A$  and  $\angle B$ .

Solution 5

**In parallelogram ABCD**

$$\angle D + \angle C = 180^\circ \quad [\text{Adjacent angles are supplementary}]$$

$$135^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 135^\circ = 45^\circ$$

$$\angle A = \angle C = 45^\circ \quad [\text{Opposite angles of a parallelogram are equal}]$$

$$\angle B = \angle D = 135^\circ \quad [\text{Opposite angles of a parallelogram are equal}]$$

Question 6

ABCD is a parallelogram in which  $\angle A = 70$ . Compute  $\angle B$ ,  $\angle C$  and  $\angle D$ .

Solution 6

**In parallelogram ABCD**

$$\angle A = 70^\circ$$

$$\angle A + \angle B = 180^\circ \quad [\text{Adjacent angles are supplementary}]$$

$$70^\circ + \angle B = 180^\circ$$

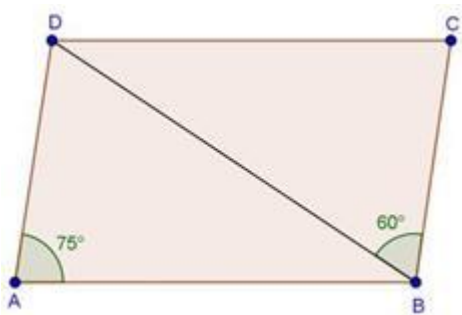
$$\angle B = 180^\circ - 70^\circ = 110^\circ$$

$$\angle A = \angle C = 70^\circ \quad [\text{Opposite angles of a parallelogram are equal}]$$

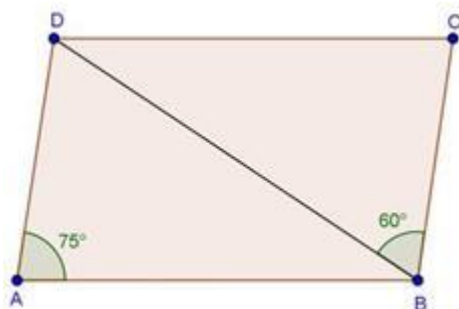
$$\angle B = \angle D = 110^\circ \quad [\text{Opposite angles of a parallelogram are equal}]$$

Question 7

In fig., ABCD is a parallelogram in which  $\angle DAB = 75^\circ$  and  $\angle DBC = 60^\circ$ . Compute  $\angle CDB$  and  $\angle ADB$ .



Solution 7



To find  $\angle CDB$  &  $\angle ADB$

$$\angle CBD = \angle ADB = 60^\circ$$

[Alternate interior angle  $AD \parallel BC$  and  $BD$  is the transversal]

In parallelogram  $ABCD$

$$\angle A = \angle C = 75^\circ$$

[opposite angles of a parallelogram]

In  $\triangle BDC$

$$\angle CBD + \angle C + \angle CDB = 180^\circ$$

[Angle sum property]

$$\Rightarrow 60^\circ + 75^\circ + \angle CDB = 180^\circ$$

$$\Rightarrow \angle CDB = 180^\circ - (60^\circ + 75^\circ)$$

$$\angle CDB = 180^\circ - 135^\circ = 45^\circ$$

Hence,  $\angle CDB = 45^\circ$ ,  $\angle ADB = 60^\circ$

Question 8

Which of the following statements are true (T) and which are false (F)?

- (i) In a parallelogram, the diagonals are equal.
- (ii) In a parallelogram, the diagonals bisect each other.
- (iii) In a parallelogram, the diagonals intersect each other at right angles.
- (iv) In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.
- (v) If all the angles of a quadrilateral are equal, it is a parallelogram.
- (vi) If three sides of a quadrilateral are equal, it is a parallelogram.
- (vii) If three angles of a quadrilateral are equal, it is a parallelogram.
- (viii) If all the sides of a quadrilateral are equal it is a parallelogram.

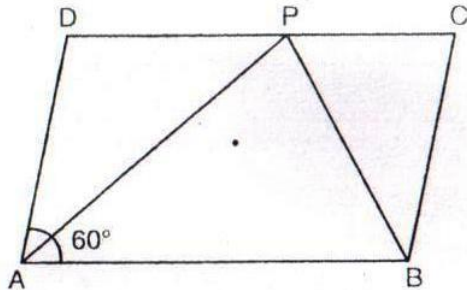
Solution 8

- i. F
- ii. T
- iii. F
- iv. F
- v. T
- vi. F
- vii. F
- viii. T

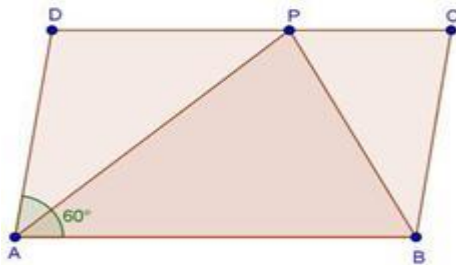
Question 9



In fig., ABCD is a parallelogram in which  $\angle A = 60^\circ$ . If the bisectors of  $\angle A$  and  $\angle B$  meet at P, prove that  $AD = DP$ ,  $PC = BC$  and  $DC = 2AD$ .



Solution 9



$$\angle DAP = \angle PAB = 30^\circ$$

[ $\because AP$  bisects  $\angle A$ ]

$$\angle A + \angle B = 180^\circ$$

[Adjacent angles are supplementary]

$$60^\circ + \angle B = 180^\circ$$

$$\angle B = 120^\circ$$

$$\angle PBA = \angle PBC = 30^\circ$$

[ $\because BP$  bisect  $\angle B$ ]

$$\angle PAB = \angle APD = 30^\circ$$

[Alternate interior angles]

$$\therefore AD = DP$$

[Sides opposite to equal angle are equal in length]

Similarly

$$\angle PBA = \angle BPC = 30^\circ$$

[Alternate interior angle]

$$\therefore PC = BC$$

$$DC = DP + PC$$

$$[\because DP = AD, PC = BC]$$

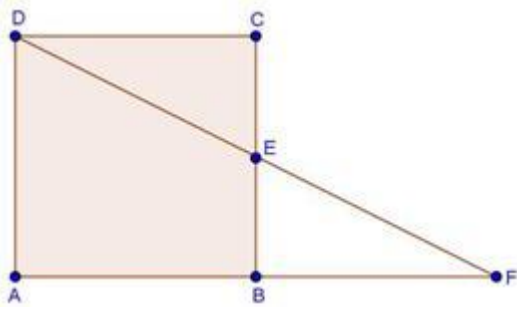
$$DC = AD + BC$$

[ $\because AD = BC$  opposite sides of a parallelogram are equal]

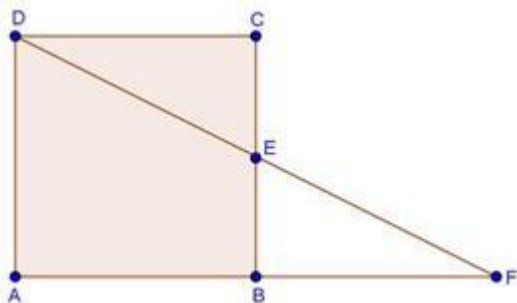
$$DC = 2AD$$

Question 10

In fig., ABCD is a parallelogram and E is the mid-point of side BC. If DE and AB when produced meet at F, prove that  $AF = 2AB$ .



Solution 10



**In  $\triangle BEF$  and  $\triangle CED$**

$$\angle BEF = \angle CED$$

$$BE = CE$$

$$\angle EBF = \angle ECD$$

$$\therefore \triangle BEF \cong \triangle CED$$

$$\therefore BF = CD$$

$$AF = AB + BF$$

$$AF = AB + AB$$

$$AF = 2AB$$

**[Vertically opposite angle]**

**[ $\therefore E$  is the mid-point of  $BC$ ]**

**[Alternate interior angles]**

**[A.S.A congruence]**

**[C.P.C.T]**

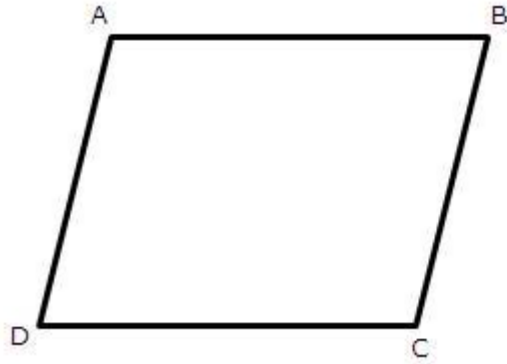
**[ $\therefore BF = CD, CD = AB$ ]**

## Chapter 13 - Quadrilaterals Exercise Ex. 13.3

### Question 1

In a parallelogram ABCD, determine sum of angles  $\angle C$  and  $\angle D$ .

Solution 1



$\angle C$  and  $\angle D$  are cosecutive interior angles on the same side of the transversal CD. Therefore,  
 $\angle C + \angle D = 180^\circ$

Question 2

In a parallelogram ABCD, if  $\angle B = 135^\circ$ , determine the measures of its other angles.

Solution 2

**We have,  $\angle B = 135^\circ$**

**Since ABCD is a parallelogram**

**$\therefore \angle A = \angle C, \angle B = \angle D$  and  $\angle A + \angle B = 180^\circ$**

**$\Rightarrow \angle A + 135^\circ = 180^\circ$**

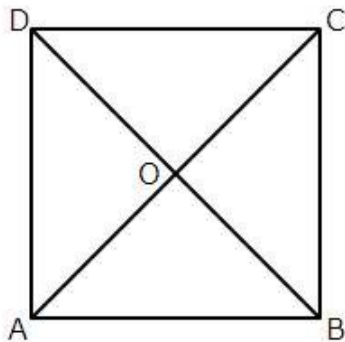
**$\Rightarrow \angle A = 45^\circ$**

**$\Rightarrow \angle A = \angle C = 45^\circ$  and  $\angle B = \angle D = 135^\circ$**

Question 3

ABCD is a square. AC and BD intersect at O. State the measure of  $\angle AOB$ .

Solution 3

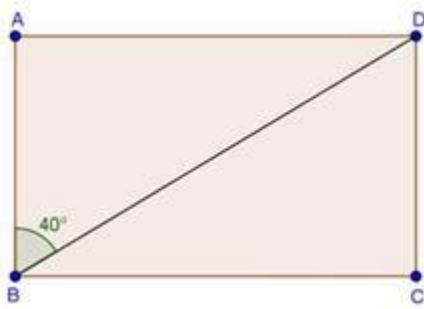


Since, diagonals of a square bisect each other at right angle. Therefore,  $\angle AOB = 90^\circ$

Question 4

**ABCD is a rectangle with  $\angle ABD = 40^\circ$ . Determine  $\angle DBC$ .**

Solution 4



**We have,**

$$\angle ABC = 90^\circ$$

$$\Rightarrow \angle ABD + \angle DBC = 90^\circ$$

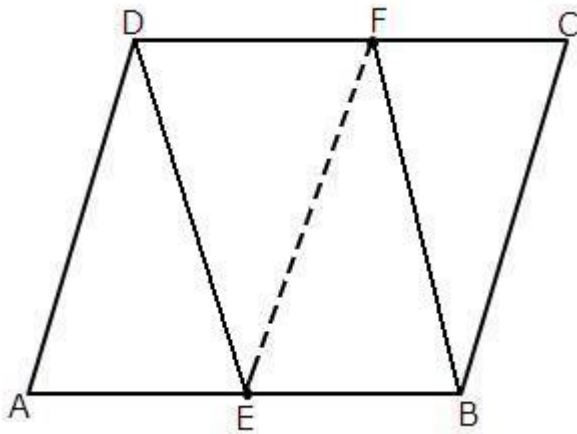
$$\Rightarrow 40^\circ + \angle DBC = 90^\circ$$

$$\Rightarrow \angle DBC = 50^\circ$$

**Question 5**

The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

**Solution 5**



**Since ABCD is a parallelogram. Therefore,**

$$AB \parallel DC \text{ and } AB = DC$$

$$\Rightarrow EB \parallel DF \text{ and } \frac{1}{2} AB = \frac{1}{2} DC$$

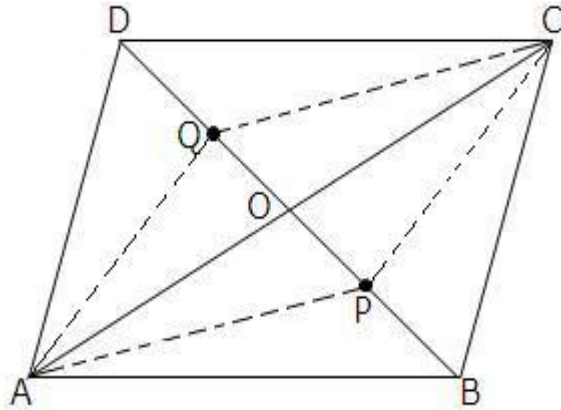
$$\Rightarrow EB \parallel DF \text{ and } EB = DF$$

$$\Rightarrow \text{EBFD is a parallelogram.}$$

**Question 6**

P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Prove also that AC bisects PQ.

**Solution 6**



Since, diagonals of a parallelogram bisect each other.

Therefore,  $OA = OC$  and  $OB = OD$ .

Since,  $P$  and  $Q$  are points of trisection of  $BD$ .

$$\therefore BP = PQ = QD$$

Now,  $OB = OD$  and  $BP = QD$

$$\Rightarrow OB - BP = OD - QD$$

$$\Rightarrow OP = OQ$$

Thus, in quadrilateral  $APCQ$ , we have

$$OA = OC \text{ and } OP = OQ$$

$\Rightarrow$  Diagonals of quadrilateral  $APCQ$  bisect each other

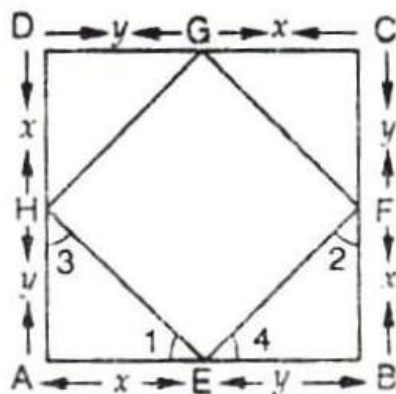
$\Rightarrow APCQ$  is a parallelogram.

Hence,  $AP \parallel CQ$ .

Question 7

ABCD is a square E, F, G and H are points on AB, BC, CD and DA respectively, such that  $AE = BF = CG = DH$ . Prove that EFGH is square.

Solution 7



We have,

$$AE = BF = CG = DH = x \text{ (say)}$$

$$\therefore BE = CF = DG = AH = y \text{ (say)}$$

In  $\Delta$ 's  $AEH$  and  $BEF$ , we have

$$AE = BF$$

$$\angle A = \angle B$$

$$\text{and, } AH = BE$$

So, by SAS congruence criterion, we have

$$\Delta AEH \cong \Delta BFE$$

$$\Rightarrow \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

$$\text{But, } \angle 1 + \angle 3 = 90^\circ \text{ and } \angle 2 + \angle 4 = 90^\circ$$

$$\Rightarrow \angle 1 + \angle 3 + \angle 2 + \angle 4 = 90^\circ + 90^\circ$$

$$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^\circ$$

$$\Rightarrow 2(\angle 1 + \angle 4) = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 4 = 90^\circ$$

$$\Rightarrow \angle HEF = 90^\circ$$

Similarly, we have

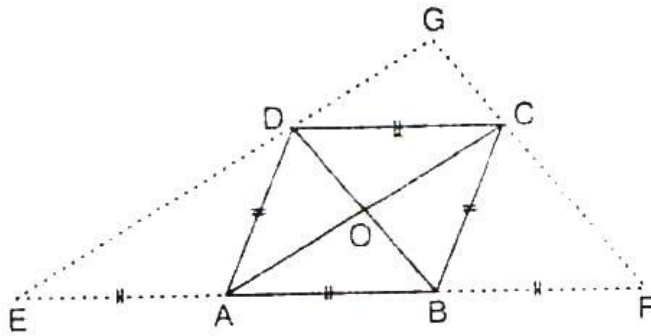
$$\angle F = \angle G = \angle H = 90^\circ$$

Hence,  $EFGH$  is a square.

Question 8

$ABCD$  is a rhombus,  $EABF$  is a straight line such that  $EA = AB = BF$ . Prove that  $ED$  and  $FC$  when produced meet at right angles.

Solution 8



We know that the diagonals of a rhombus are perpendicular bisector of each other.

$$\therefore OA = OC, OB = OD, \angle AOD = \angle COD = 90^\circ$$

$$\text{And, } \angle AOB = \angle COB = 90^\circ$$

In  $\triangle BDE$ , A and O are mid-points of BE and BD respectively.

$$\therefore OA \parallel DE$$

$$\Rightarrow OC \parallel DG$$

In  $\triangle CFA$ , B and O are mid-points of AF and AC respectively

$$\therefore OB \parallel CF$$

$$\Rightarrow OD \parallel GC$$

Thus, in quadrilateral DOCG, we have

$$OC \parallel DG \text{ and } OD \parallel GC$$

$$\Rightarrow DOCG \text{ is a parallelogram.}$$

$$\therefore \angle DGC = \angle DOC$$

$$\Rightarrow \angle DGC = 90^\circ$$

Question 9

ABCD is a parallelogram, AD is produced to E so that  $DE = DC$  and EC produced meets AB produced in F. Prove that  $BF = BC$ .

Solution 9

Draw a parallelogram ABCD with AC and BD intersecting at O.

Produce AD to E such that DE = DC.

Join EC and produce it to meet AB produced at F.

In  $\triangle DCE$ ,

$\therefore \angle DCE = \angle DEC$  ...(1) (In a triangle, equal sides have equal angles opposite to them)

$AB \parallel CD$  (Opposite sides of the parallelogram are parallel)

$\therefore AF \parallel CD$  (AB lies on AF)

$AF \parallel CD$  and EF is the transversal,

$\therefore \angle DCE = \angle BFC$  ...(2) (Pair of corresponding angles)

From (1) and (2), we get

$$\angle DEC = \angle BFC$$

In  $\triangle AFE$ ,

$$\angle AFE = \angle AEF \quad (\angle DEC = \angle BFC)$$

$\therefore AE = AF$  (In a triangle, equal angles have equal sides opposite to them)

$$\Rightarrow AD + DE = AB + BF$$

$$\Rightarrow BC + AB = AB + BF \quad (\because AD = BC, DE = CD \text{ and } CD = AB, AB = DE)$$

$$\Rightarrow BC = BF$$

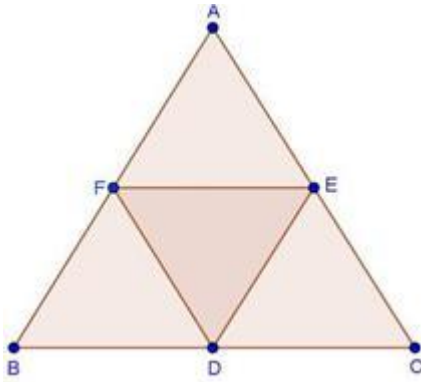
Chapter 13 - Quadrilaterals Exercise Ex. 13.4

Question 1

**In a  $\triangle ABC$ , D, E and F are, respectively, the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of  $\triangle DEF$ .**

Solution 1





$AB = 7 \text{ cm}, BC = 8 \text{ cm}, AC = 9 \text{ cm}$

In  $\triangle ABC$

$\therefore F \text{ \& } E$  are the mid-points of  $AB$  and  $AC$

$\therefore EF = \frac{1}{2} BC$  [Mid-points theorem]

Similarly

$$DF = \frac{1}{2} AC, DE = \frac{1}{2} AB$$

Perimeter of  $\triangle DEF = DE + EF + DF$

$$= \frac{1}{2} AB + \frac{1}{2} BC + \frac{1}{2} AC$$

$$= \frac{1}{2} \times 7 + \frac{1}{2} \times 8 + \frac{1}{2} \times 9$$

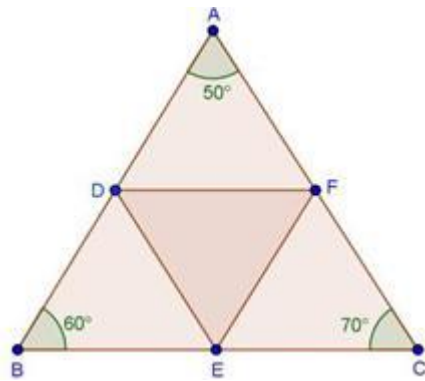
$$= 3.5 + 4 + 4.5 = 12 \text{ cm}$$

$\therefore P \text{ of } \triangle DEF = 12 \text{ cm}.$

Question 2

in a triangle  $\triangle ABC$ ,  $\angle A = 50^\circ$ ,  $\angle B = 60^\circ$  and  $\angle C = 70^\circ$ . Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution 2



**In  $\triangle ABC$**

**$D$  &  $E$  are mid-points of  $AB$  and  $BC$**

$$\therefore DE \parallel AC, DE = \frac{1}{2} AC$$

**[By mid-point theorem]**

$$DE = \frac{1}{2} AC = CF$$

**[ $\because F$  is the mid-point of  $AC$ ]**

**In quadrilateral  $DECF$**

$$DE \parallel AC, DE = CF$$

**Hence  $DECF$  is a parallelogram**

$$\therefore \angle C = \angle D = 70^\circ$$

**[Opposite angles of a parallelogram]**

**Similarly,**

$$BEFD \text{ is a parallelogram, } \angle B = \angle F = 60^\circ$$

$$ADEF \text{ is a parallelogram, } \angle A = \angle E = 50^\circ$$

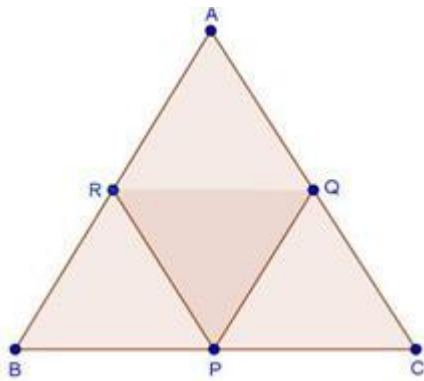
**$\therefore$  Angles of  $\triangle DEF$**

$$\angle D = 70^\circ, \angle E = 50^\circ, \angle F = 60^\circ$$

Question 3

**In a triangle,  $P, Q$  and  $R$  are the mid-points of sides  $BC, CA$  and  $AB$  respectively, If  $AC = 21$  cm,  $BC = 29$  cm and  $AB = 30$  cm, find the perimeter of the quadrilateral  $ARPQ$ .**

Solution 3



In  $\triangle ABC$

$R$  &  $P$  are mid-points of  $AB$  and  $BC$

$$\therefore RP \parallel AC, RP = \frac{1}{2} AC$$

[By mid-point theorem]

In quadrilateral  $ARPQ$

$$RP \parallel AQ, RP = AQ$$

$$\therefore ARPQ \text{ is a parallelogram}$$

[A pair of side is parallel and equal]

$$AR = \frac{1}{2} AB = \frac{1}{2} \times 30 = 15 \text{ cm}$$

$$\Rightarrow AR = QP = 15$$

[opposite sides are equal]

$$RP = \frac{1}{2} AC = \frac{1}{2} \times 21 = 10.5 \text{ cm}$$

$$\Rightarrow RP = AQ = 10.5$$

[Opposite sides are equal]

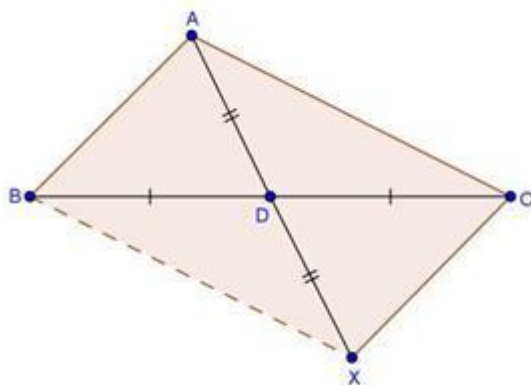
Now,

$$\begin{aligned} \text{Perimeter of } ARPQ &= AR + QP + RP + AQ \\ &= 15 + 15 + 10.5 + 10.5 \\ &= 51 \text{ cm} \end{aligned}$$

Question 4

In  $\triangle ABC$  median  $AD$  is produced to  $X$  such that  $AD = DX$ . Prove that  $ABXC$  is a parallelogram.

Solution 4



In quadrilateral  $ABXC$ , we have

$$AD = DX$$

[Given]

$$BD = DC$$

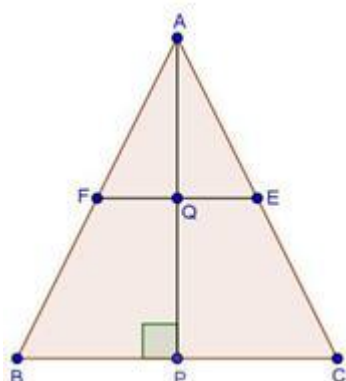
[Given]

So, diagonals  $AX$  and  $BC$  bisect each other. Therefore,  $ABXC$  is a parallelogram.

Question 5

In  $\triangle ABC$   $E$  and  $F$  are the mid-points of  $AC$  and  $AB$  respectively. The altitude  $AP$  to  $BC$  intersects  $FE$  at  $Q$ . Prove that  $AQ = QP$ .

Solution 5



In  $\triangle ABC$

$F$  and  $E$  are mid-points of  $AB$  &  $AC$

$$\therefore FE \parallel BC, FE = \frac{1}{2} BC$$

[By mid-point theorem]

In  $\triangle ABP$

$F$  is the mid-point of  $AB$  and  $FQ \parallel BP$

[ $\because FE \parallel BC$ ]

$$\therefore Q \text{ is the mid-point of } AP$$

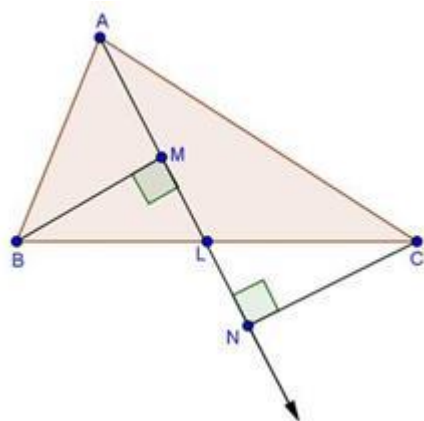
[By converse of mid-point theorem]

Hence,  $AQ = QP$

Question 6

In  $\triangle ABC$   $BM$  and  $CN$  are perpendiculars from  $B$  and  $C$  respectively on any line passing through  $A$ . If  $L$  is the mid-point of  $BC$ , prove that  $ML = NL$ .

Solution 6



In  $\triangle BLM$  and  $\triangle CLN$

$$\angle BML = \angle CNL = 90^\circ$$

[Given]

$$BL = CL$$

[ $L$  is the mid-point of  $BC$ ]

$$\angle MLB = \angle NLC$$

[Vertically opposite angle]

$$\therefore \triangle BLM \cong \triangle CLN$$

[A.A.S]

$$\therefore LM = LN$$

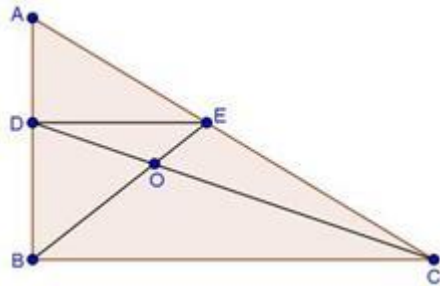
[Corresponding parts of congruent triangles]

Question 7

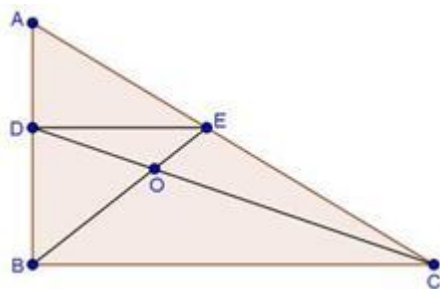
In fig., triangle ABC is right-angled at B. Give that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC

(ii) The area of  $\triangle ADE$ .



Solution 7



In right  $\triangle ABC$ ,  $\angle B = 90^\circ$

$$AC^2 = AB^2 + BC^2$$

[By pythagoras theorem]

$$\Rightarrow 15^2 = 9^2 + BC^2$$

$$\Rightarrow 225 - 81 = BC^2$$

$$\Rightarrow 144 = BC^2$$

$$\Rightarrow BC = \sqrt{144} = 12 \text{ cm}$$

In  $\triangle ABC$

D and E are mid-points of AB and AC

$$\therefore DE \parallel BC, DE = \frac{1}{2} BC$$

[By mid-point theorem]

$$AD = DB = \frac{AB}{2} = \frac{9}{2} = 4.5 \text{ cm}$$

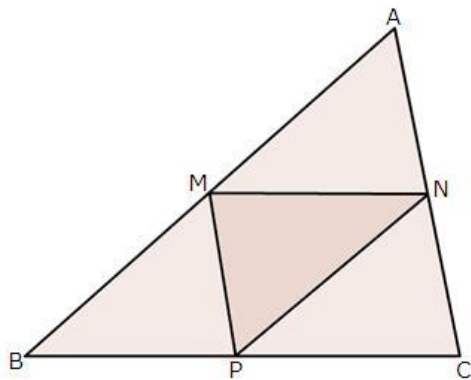
[ $\because$  D is the mid-point of AB]

$$DE = \frac{BC}{2} = \frac{12}{2} = 6 \text{ cm}$$

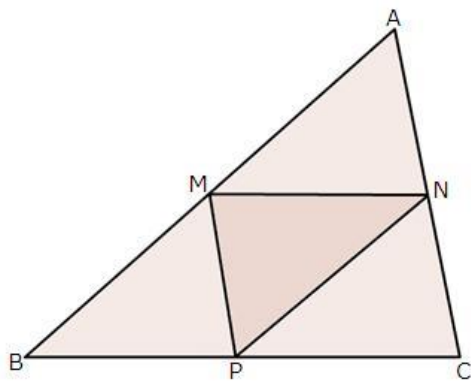
$$\begin{aligned} \text{Area of } \triangle ADE &= \frac{1}{2} \times AD \times DE \\ &= \frac{1}{2} \times 4.5 \times 6 = 4.5 \times 3 = 13.5 \text{ cm}^2. \end{aligned}$$

Question 8

In fig., M, N, and P are the mid-points of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, Calculate BC, AB and AC.



Solution 8



**Given,  $MN = 3 \text{ cm}$ ,  $NP = 3.5 \text{ cm}$  and  $MP = 2.5 \text{ cm}$**

**To find,  $BC$ ,  $AB$  and  $AC$**

**In  $\triangle ABC$**

**$M$  and  $N$  are mid-points of  $AB$  and  $AC$**

$$\therefore MN = \frac{1}{2} BC, MN \parallel BC$$

**[By mid-point theorem]**

$$\Rightarrow 3 = \frac{1}{2} BC$$

$$\Rightarrow 3 \times 2 = BC$$

$$\Rightarrow BC = 6 \text{ cm}$$

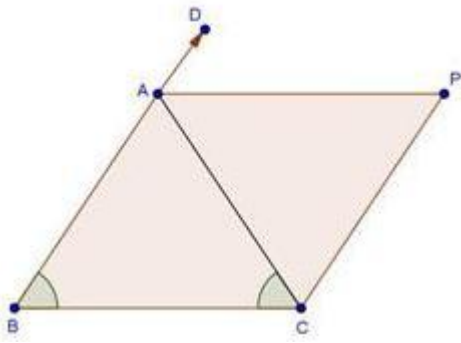
**Similarly,**

$$AC = 2MP = 2 \times 2.5 = 5 \text{ cm}$$

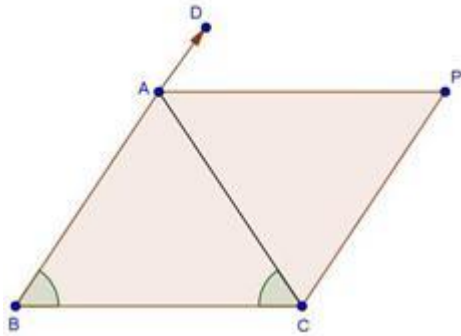
$$AB = 2NP = 2 \times 3.5 = 7 \text{ cm}$$

Question 9

In fig.,  $AB = AC$  and  $CP \parallel BA$  and  $AP$  is the bisector of exterior  $\angle CAD$  of  $\triangle ABC$ . Prove that (i)  $\angle PAC = \angle BCA$  (ii)  $ABCP$  is a parallelogram.



Solution 9



**Given,**

$AB = AC$  and  $CP \parallel BA$  and  $AP$  is the bisector of exterior  $\angle CAD$  of  $\triangle ABC$ .

**To prove:**

(i)  $\angle PAC = \angle BCA$

(ii)  $ABCP$  is a parallelogram

**Proof:**

(i) We have,

$$AB = AC$$

$$\Rightarrow \angle ACB = \angle ABC$$

[Opposite angles of equal sides of triangle are equal]

Now,

$$\angle CAD = \angle ABC + \angle ACB$$

$$\Rightarrow \angle PAC + \angle PAD = 2\angle ACB$$

$$\Rightarrow 2\angle PAC = 2\angle ACB$$

$$[\because \angle PAC = \angle PAD]$$

$$\Rightarrow \angle PAC = \angle ACB$$

$$\Rightarrow \angle PAC = \angle BCA$$

(ii) Now,

$$\angle PAC = \angle BCA$$

$$\Rightarrow AP \parallel BC$$

and,  $CP \parallel BA$

[Given]

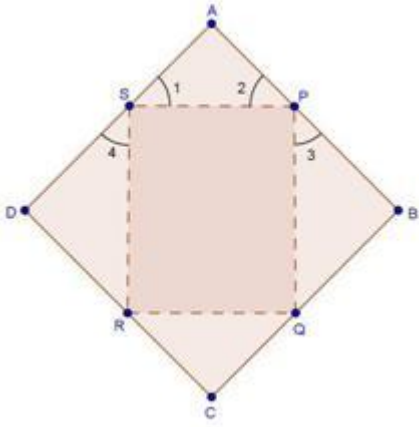
$\therefore ABCP$  is a parallelogram.

Question 10

$ABCD$  is a kite having  $AB = AD$  and  $BC = CD$ . Prove that the figure formed by joining the mid-points of the sides, in order, is a rectangle.

Solution 10





Given,

A kite  $ABCD$  having  $AB = AD$  and  $BC = CD$ .  $P, Q, R, S$  are the mid-points of sides  $AB, BC, CD$  and  $DA$  respectively.  $PQ, QR, RS$  and  $SP$  are joined.

To prove:

$PQRS$  is a rectangle.

Proof: In  $\triangle ABC$ ,  $P$  and  $Q$  are the mid-points of  $AB$  and  $BC$  respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \text{---(i)}$$

In  $\triangle ADC$ ,  $R$  and  $S$  are the mid-points of  $CD$  and  $AD$  respectively.

$$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2} AC \quad \text{---(ii)}$$

From (i) and (ii), we have

$$PQ \parallel RS \text{ and } PQ = RS.$$

Thus, in quadrilateral  $PQRS$ , a pair of opposite sides are equal and parallel.

So,  $PQRS$  is a parallelogram. Now, we shall prove that one angle of parallelogram  $PQRS$  is a right angle.

Since,  $AB = AD$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AD$$

$$\Rightarrow AP = AS$$

$$\Rightarrow \angle 1 = \angle 2$$

$$\text{---(iii)} \left[ \because P \text{ and } S \text{ are the mid-points of } B \text{ and } AD \text{ respectively} \right]$$

$$\text{---(iv)}$$

Now, in  $\triangle PBQ$  and  $\triangle SDR$ , we have

$$PB = SD$$

$$BQ = DR$$

$$\text{and, } PQ = SR$$

$$\left[ \begin{array}{l} \because AD = AB \\ \Rightarrow \frac{1}{2} AD = \frac{1}{2} AB \\ \therefore PB = SD \end{array} \right]$$

$$[\because BC = DC]$$

$$\left[ \begin{array}{l} \because PQRS \text{ is parallelogram} \\ \therefore PQ = SR \end{array} \right]$$

So, by SSS criterion of congruence, we have

$$\triangle PBQ \cong \triangle SDR$$

$$\Rightarrow \angle 3 = \angle 4$$

$$[C.P.C.T.]$$

Now,

$$\angle 3 + \angle SPQ + \angle 2 = 180^\circ$$

$$\text{and } \angle 1 + \angle PSR + \angle 4 = 180^\circ$$

$$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$$

$$\Rightarrow \angle SPQ = \angle PSR$$

$$[\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4]$$

Now, transversal  $PS$  cuts parallel lines  $SR$  and  $PQ$  at  $S$  and  $P$  respectively.

$$\therefore \angle SPQ + \angle PSR = 180^\circ$$

$$\Rightarrow 2\angle SPQ = 180^\circ$$

$$[\because \angle SPQ = \angle PSR]$$

$$\Rightarrow \angle SPQ = 90^\circ$$

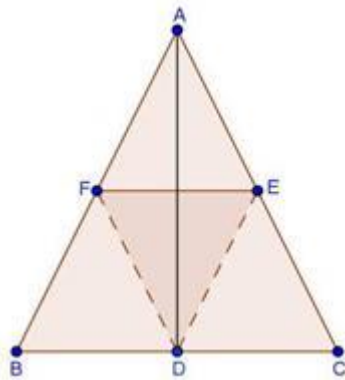
Thus,  $PQRS$  is a parallelogram such that  $\angle SPQ = 90^\circ$

Hence,  $PQRS$  is a rectangle.

Question 11

Let  $ABC$  be an isosceles triangle in which  $AB = AC$ . If  $D, E, F$  be the mid-points of the sides  $BC, CA$  and  $AB$  respectively, show that the segment  $AD$  and  $EF$  bisect each other at right angles.

Solution 11



Since  $D, E$  and  $F$  are the mid-points of sides  $BC, CA$  and  $AB$  respectively.

$$\therefore AB \parallel DE \text{ and } AC \parallel FD$$

$$\Rightarrow AF \parallel DE \text{ and } AE \parallel FD$$

$$\Rightarrow AFDE \text{ is a parallelogram}$$

$$\Rightarrow AF = DE \text{ and } AE = DF$$

$$\Rightarrow \frac{1}{2}AB = DE \text{ and } \frac{1}{2}AC = DF$$

$$\Rightarrow DE = DF$$

$$[\because AB = AC]$$

$$\Rightarrow AE = AF = DE = DF$$

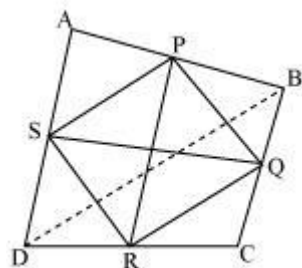
$$\Rightarrow AEDF \text{ is a rhombus}$$

$$\Rightarrow AD \text{ and } FE \text{ bisect each other at right angle.}$$

Question 12

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution 12



Let ABCD is a quadrilateral in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively. Join PQ, QR, RS, SP and BD.

In  $\triangle ABD$ , S and P are mid points of AD and AB respectively.

So, By using mid-point theorem, we can say that

$$SP \parallel BD \text{ and } SP = \frac{1}{2} BD \quad \dots (1)$$

Similarly in  $\triangle BCD$

$$QR \parallel BD \text{ and } QR = \frac{1}{2} BD \quad \dots (2)$$

From equations (1) and (2), we have

$$SP \parallel QR \text{ and } SP = QR$$

As in quadrilateral SPQR one pair of opposite sides are equal and parallel to each other.

So, SPQR is a parallelogram.

Since, diagonals of a parallelogram bisect each other.

Hence, PR and QS bisect each other.

Question 13

Fill in the blanks to make the following statements correct:

- (i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is \_\_\_\_\_.
- (ii) The triangle formed by joining the mid-points of the sides of a right triangle is \_\_\_\_\_.
- (iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is \_\_\_\_\_.

Solution 13

(i) isosceles

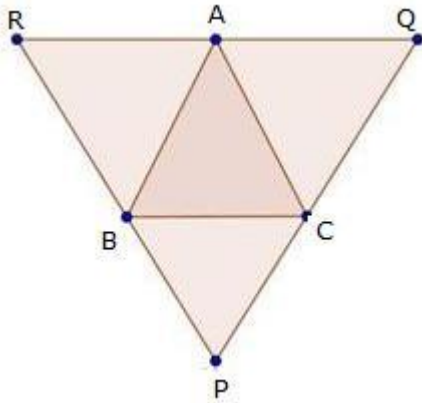
(ii) right triangle

(iii) parallelogram

Question 14

***ABC* is a triangle and through *A, B, C* lines are drawn parallel to *BC, CA* and *AB* respectively intersecting at *P, Q* and *R*. Prove that the perimeter of  $\triangle PQR$  is double the perimeter of  $\triangle ABC$ .**

Solution 14



Clearly,  $ABCQ$  and  $ARBC$  are parallelograms.

$$\therefore BC = AQ \text{ and } BC = AR$$

$$\Rightarrow AQ = AR$$

$\Rightarrow A$  is the mid-point of  $QR$ .

Similarly,  $B$  and  $C$  are the mid-points of  $PR$  and  $PQ$  respectively.

$$\therefore AB = \frac{1}{2}PQ, BC = \frac{1}{2}QR \text{ and } CA = \frac{1}{2}PR$$

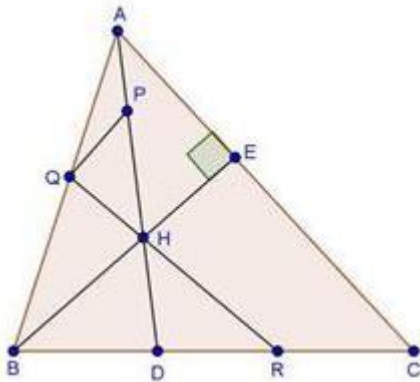
$$\Rightarrow PQ = 2AB, QR = 2BC \text{ and } PR = 2CA$$

$$\Rightarrow PQ + QR + RP = 2(AB + BC + CA)$$

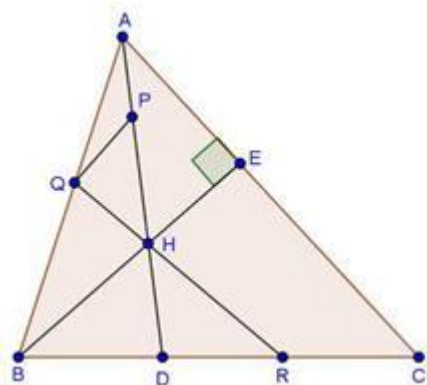
$$\Rightarrow \text{Perimeter of } \triangle PQR = 2(\text{Perimeter of } \triangle ABC)$$

Question 15

In fig.,  $BE \perp AC$ .  $AD$  is any line from  $A$  to  $BC$  intersecting  $BE$  in  $H$ .  $P$ ,  $Q$  and  $R$  are respectively the mid-points of  $AH$ ,  $AB$  and  $BC$ . Prove that  $\angle PQR = 90^\circ$ .



Solution 15



**Given,**

$BE \perp AC$  and  $P, Q$  and  $R$  are respectively mid-point of  $AH, AB$  and  $BC$ .

**To prove:**

$$\angle PQR = 90^\circ$$

**Proof:** In  $\triangle ABC$ ,  $Q$  and  $R$  are the mid-points of  $AB$  and  $BC$  respectively.

$$\therefore QR \parallel AC \quad \text{---(i)}$$

In  $\triangle ABH$ ,  $Q$  and  $P$  are the mid-points of  $AB$  and  $AH$  respectively.

$$\therefore QP \parallel BH$$

$$\Rightarrow QP \parallel BE \quad \text{---(ii)}$$

But,  $AC \perp BE$ . Therefore, from equation (i) and equation (ii) we have

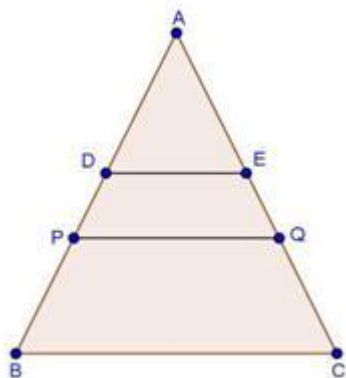
$$QP \perp QR$$

$$\Rightarrow \angle PQR = 90^\circ \quad \text{Hence proved.}$$

Question 16

$ABC$  is a triangle.  $D$  is a point on  $AB$  such that  $AD = \frac{1}{4} AB$  and  $E$  is a point on  $AC$  such that  $AE = \frac{1}{4} AC$ . Prove that  $DE = \frac{1}{4} BC$ .

Solution 16



Let  $P$  and  $Q$  be the mid-points of  $AB$  and  $AC$  respectively.

Then,  $PQ \parallel BC$  such that

$$PQ = \frac{1}{2} BC \quad \text{---(i)}$$

In  $\triangle APQ$ ,  $D$  and  $E$  are mid-points of  $AP$  and  $AQ$  are respectively.

$$\therefore DE \parallel PQ \text{ and } DE = \frac{1}{2} PQ \quad \text{---(ii)}$$

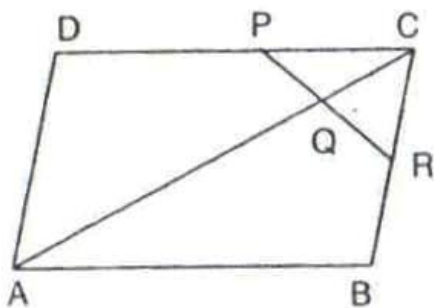
From equation (i) and equation (ii), we get

$$DE = \frac{1}{2} PQ = \frac{1}{2} \left[ \frac{1}{2} BC \right]$$

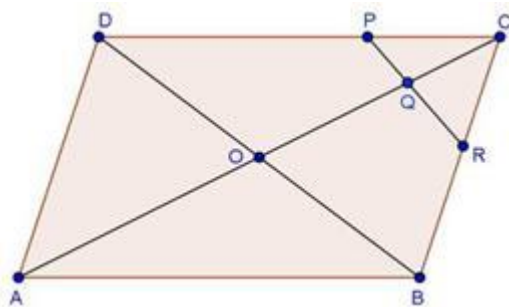
$$\Rightarrow DE = \frac{1}{4} BC \quad \text{Hence proved.}$$

#### Question 17

In fig.,  $ABCD$  is a parallelogram in which  $P$  is the mid-point of  $DC$  and  $Q$  is a point on  $AC$  such that  $CQ = (1/4)AC$ . If  $PQ$  produced meets  $BC$  at  $R$ , prove that  $R$  is a mid-point of  $BC$ .



#### Solution 17



Join  $B$  and  $D$ . Suppose  $AC$  and  $BD$  cut at  $O$ . Then,

$$OC = \frac{1}{2} AC$$

Now,

$$CQ = \frac{1}{4} AC$$

$$\Rightarrow CQ = \frac{1}{2} \left[ \frac{1}{2} AC \right]$$

$$= \frac{1}{2} \times OC$$

$$\left[ \because OC = \frac{1}{2} AC \right]$$

In  $\triangle DCO$ ,  $P$  and  $Q$  are mid-points of  $DC$  and  $OC$  respectively.

$\therefore PQ \parallel DO$

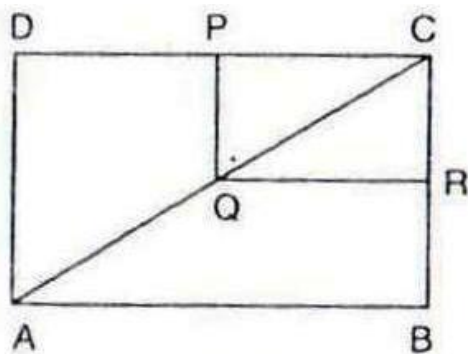
Also, in  $\triangle COB$ ,  $Q$  is mid-point of  $OC$  and  $QR \parallel OB$

$\therefore R$  is the mid-point of  $BC$ .

Question 18

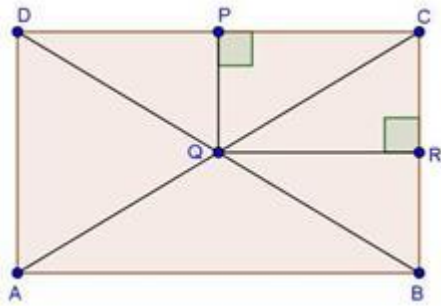
In fig.,  $ABCD$  and  $PQRC$  are rectangle and  $Q$  is the mid-point of  $AC$ . Prove that

i.  $DP = PC$  ii.  $PR = \frac{1}{2} AC$



Solution 18





(i) In  $\triangle ADC$ ,  $Q$  is the mid-point of  $AC$  such that  $PQ \parallel AD$

$\therefore P$  is the mid-point of  $DC$

$\Rightarrow DP = PC$

[Using converse of mid-point theorem]

(ii) Similarly,  $R$  is the mid-point of  $BC$ .

$\therefore PR = \frac{1}{2}BD$

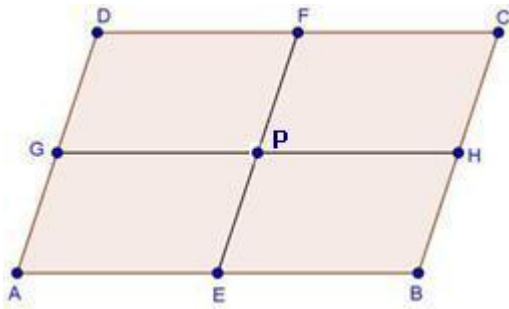
$\Rightarrow PR = \frac{1}{2}AC$

[Diagonals of rectangle are equal,  $\therefore BD = AC$ ]

Question 19

ABCD is a parallelogram, E and F are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC and G, P and H respectively. Prove that GP = PH.

Solution 19



Since  $E$  and  $F$  are mid-points  $AB$  and  $CD$  respectively.

$$\therefore AE = BE = \frac{1}{2} AB$$

$$\text{and } CF = DF = \frac{1}{2} CD$$

$$\text{But, } AB = CD$$

$$\therefore \frac{1}{2} AB = \frac{1}{2} CD$$

$$\Rightarrow BE = CF$$

$$\text{Also, } BE \parallel CF$$

$$[\because AB \parallel CD]$$

$$\therefore BEFC \text{ is a parallelogram.}$$

$$\Rightarrow BC \parallel EF \text{ and } BE = PH$$

$$\text{---(i)}$$

$$\text{Now, } BC \parallel EF$$

$$\Rightarrow AD \parallel EF$$

$$[\because BC \parallel AD \text{ as } ABCD \text{ is a parallelogram}]$$

$$\Rightarrow AEFD \text{ is a parallelogram}$$

$$\Rightarrow AE = GP$$

$$\text{---(ii)}$$

But,  $E$  is the mid-point of  $AB$ .

$$\therefore AE = BE$$

$$\Rightarrow GP = PH$$

$$[\text{Using (i) and (ii)}]$$

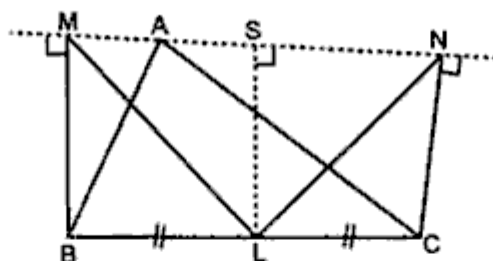
Question 20

BM and CN are perpendiculars to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Solution 20

To prove:  $LM = LN$

Draw  $LS$  perpendicular to line  $MN$ .



Therefore, the lines  $BM$ ,  $LS$  and  $CN$  being the same perpendiculars, on line  $MN$  are parallel to each other.

According to intercept theorem,

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

In the drawn figure,  $MB$  and  $LS$  and  $CN$  are three parallel lines and the two transversal lines are  $MN$  and  $BC$ .

We have,  $BL = LC$  (As  $L$  is the given mid-point of  $BC$ )

$\therefore$  Using intercept theorem, we get,

$$MS = SN \quad \dots(1)$$

Now, In  $\triangle MLS$  and  $\triangle LSN$

$$MS = SN \text{ using } \dots(1)$$

$$\angle LSM = \angle LSN = 90^\circ \text{ } LS \perp MN$$

and  $SL = LS$  common

$$\therefore \triangle MLS \cong \triangle LSN \quad (\text{SAS congruency criterion})$$

$$\therefore LM = LN \quad (\text{CPCT})$$