NCERT Solutions for Class 9 Maths Chapter 10 - Circles

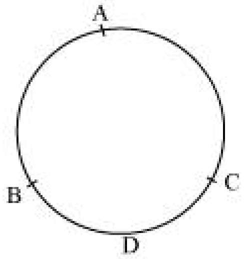
Chapter 10 - Circles Exercise Ex. 10.1

Solution 1

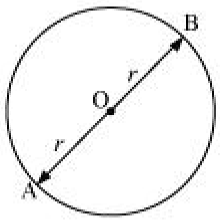
(i) The centre of a circle lies in interior of the circle. (exterior/interior)
(ii) A point, whose distance from the centre of a circle is greater than its radius lies in exterior of the circle. (exterior/interior)
(iii) The longest chord of a circle is a diameter of the circle.
(iv) An arc is a semicircle when its ends are the ends of a diameter.
(v) Segment of a circle is the region between an arc and chord of the circle.
(vi) A circle divides the plane, on which it lies, in three parts.

Solution 2

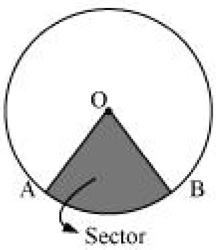
(i) True, all the points on circle are at equal distance from the centre of circle, and this equal distance it called as radius of circle.
(ii) False, on a circle there are infinite points. So, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.
(iii) False, consider three arcs of same length as AB, BC and CA. Now we may observe that for minor arc BDC. CAB is major arc. So AB, BC and CA are minor arcs of circle.



(iv) True, let AB be a chord which is twice as long as its radius. In this situation our chord will be passing through centre of circle. So it will be the diameter of circle.



(v) False, sector is the region between an arc and two radii joining the centre to the end points of the arc as in the given figure OAB is the sector of circle.



(vi) True, A circle is a two dimensional figure and it can also be referred as plane figure.

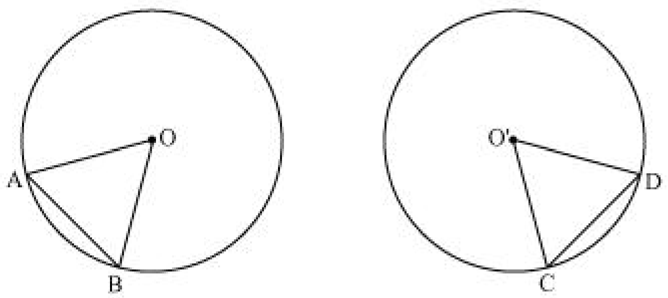
Chapter 10 - Circles Exercise Ex. 10.2

Solution 1

A circle is a collection of points which are equidistant from a fix point. This fix point is called as the centre of circle and this equal distance is called as radius of circle. And thus shape of a circle depends on the radius of the circle.

So, if we try to superimpose two circles of equal radius, one each other both circles will cover each other.
So, two circles are congruent if they have equal radius.

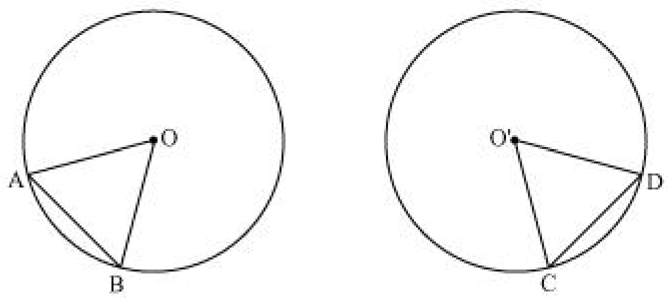
Now consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths



Now in $\triangle$AOB and $\triangle$CO'D
AB = CD (chords of same length)
OA = O'C (radii of congruent circles)
OB = O'D (radii of congruent circles)
$\therefore$ $\triangle$AOB $\cong$ $\triangle$CO'D (SSS congruence rule)

$\Rightarrow$ $\angle$AOB = $\angle$CO'D (by CPCT)
Hence equal chords of congruent circles subtend equal angles at their centres.

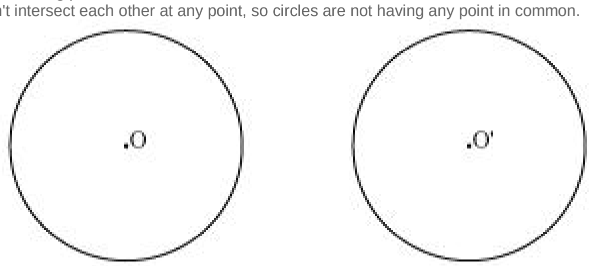
Solution 2
Let us consider two congruent circles (circles of same radius) with centres as O and O'.



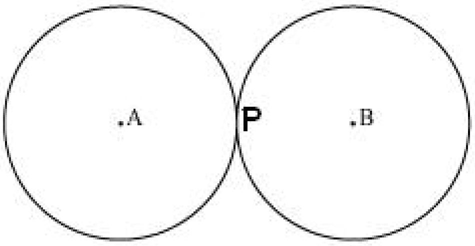
In $\triangle$AOB and $\triangle$CO'D

$\angle$AOB = $\angle$CO'D (given)
OA = O'C (radii of congruent circles)
OB = O'D (radii of congruent circles)
$\therefore$ $\triangle$AOB $\cong$ $\triangle$CO'D (SSS congruence rule)
$\Rightarrow$ AB = CD (by CPCT)
Hence, if chords of congruent circles subtend equal angles at their centres then chords are equal.

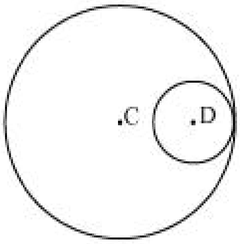
Chapter 10 - Circles Exercise Ex. 10.3
Solution 1
Consider the following pair of circles.
(i) circles don't intersect each other at any point, so circles are not having any point in common.



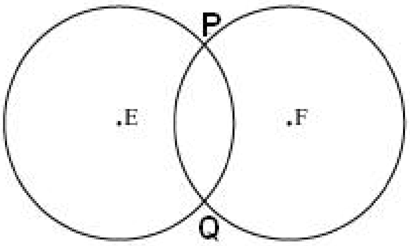
(ii) Circles touch each other only at one point P so there is only 1 point in common.



(iii) Circles touch each other at 1 point X only. So the circles have 1 point in common.



(iv) These circles intersect each other at two points P and Q. So the circles have two points in common. We may observe that there can be maximum 2 points in common.

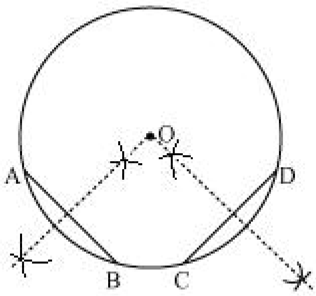


We can have a situation in which two congruent circles are superimposed on each other, this situation can be referred as if we are drawing circle two times.

Solution 2

Following are the steps of construction:

Step1. Take the given circle centered at point O.
Step2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these chords.
Step3. Let these perpendicular bisectors meet at point O. Now, O is the centre of given circle.

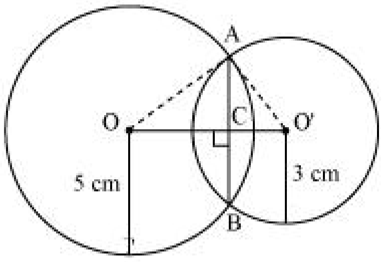


Solution 3

Consider two circles centered at point O and O' intersect each other at point A and B respectively.
Join AB. AB is the chord for circle centered at O, so perpendicular bisector of AB will pass through O.
Again AB is also chord of circle centered at O', so, perpendicular bisector of AB will also pass through O'.
Clearly centres of these circles lie on the perpendicular bisector of common chord.

Chapter 10 - Circles Exercise Ex. 10.4

Solution 1



Let radius of circle centered at O and O' be 5 cm and 3 cm respectively.
OA = OB = 5 cm
O'A = O'B = 3 cm
OO' will be the perpendicular bisector of chord AB.

$\therefore$ AC = CB

Given that OO' = 4 cm
Let OC be x. so, O'C will be 4 - x

In $\triangle$ OAC
$OA^2 = AC^2 + OC^2$

$\Rightarrow 5^2 = AC^2 + x^2$

$\Rightarrow 25 - x^2 = AC^2$ ... (1)

In $\triangle$ O'AC
$O'A^2 = AC^2 + O'C^2$

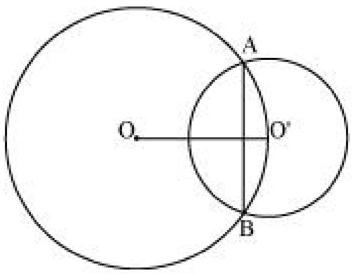
$\Rightarrow 3^2 = AC^2 + (4 - x)^2$

$\Rightarrow 9 = AC^2 + 16 + x^2 - 8x$

$\Rightarrow AC^2 = - x^2 - 7 + 8x$ ... (2)

From equations (1) and (2), we have
$25 - x^2 = - x^2 - 7 + 8x$
$8x = 32$
$x = 4$

So, the common chord will pass through the centre of smaller circle i.e. O'. and hence it will be diameter of smaller circle.



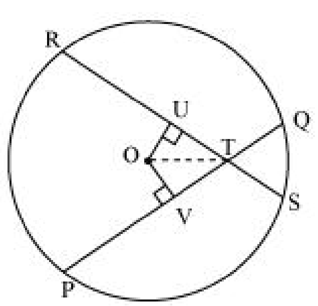
Now, $AC^2 = 25 - x^2 = 25 - 4^2 = 25 - 16 = 9$

$\therefore$ AC = 3 m

The length of the common chord AB = 2 AC = (2 $\times$ 3) m = 6 m

Solution 2

Let PQ and RS are two equal chords of a given circle and there are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT

OV = OU (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90^o)

OT = OT (common)

$\therefore \Delta OVT \cong \Delta OUT$ (RHS congruence rule)

$\therefore$ VT = UT (by CPCT) ... (1)

It is given that

PQ = RS ... ... (2)

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$

$\Rightarrow$ PV = RU ... ... (3)

On adding equations (1) and (3), we have

PV + VT = RU + UT

$\Rightarrow$ PT = RT ... ... (4)

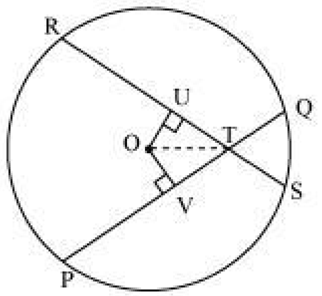
On subtracting equation (4) from equation (2), we have

PQ - PT = RS - RT

$\Rightarrow$ QT = ST ... ... (5)

Equations (4) and (5) shows that the corresponding segments of chords PQ and RS are congruent to each other.

Solution 3



Let PQ and RS are two equal chords of a given circle and there are intersecting each other at point T.

Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT

OV = OU (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90^o)

OT = OT (common)

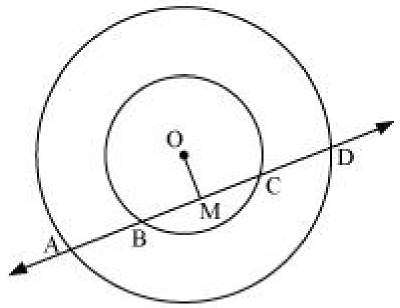
$\therefore \Delta OVT \cong \Delta OUT$ (RHS congruence rule)

$\therefore \angle OTV = \angle OTU$ (by CPCT)

Hence, the line joining the point of intersection to the centre makes equal angles with the chords.

Solution 4

Let us draw a perpendicular OM on line AD.



Here, BC is chord of smaller circle and AD is chord of bigger circle.
We know that the perpendicular drawn from centre of circle bisects the chord.

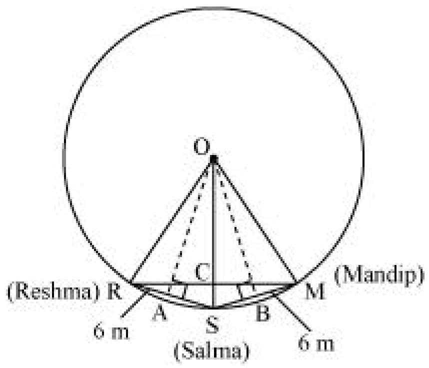
$\therefore$ BM = MC ... (1)

And AM = MD ... (2)
Subtracting equations (2) from (1), we have
AM - BM = MD - MC

$\Rightarrow$ AB = CD

Solution 5

Draw perpendiculars OA and OB on RS and SM respectively.
Let R, S and M be the position of Reshma, Salma and Mandip respectively.



AR = AS = $\frac{6}{2}$ = 3cm
OR = OS = OM = 5 m (radii of circle)
In OAR
$OA^2 + AR^2 = OR^2$
$OA^2 + (3\text{ m})^2 = (5\text{ m})^2$
$OA^2 = (25 - 9)\text{ m}^2 = 16\text{ m}^2$
OA = 4 m

We know that in an isosceles triangle altitude divides the base, so in $\triangle$RSM

$\angle$RCS will be of 90^o and RC = CM

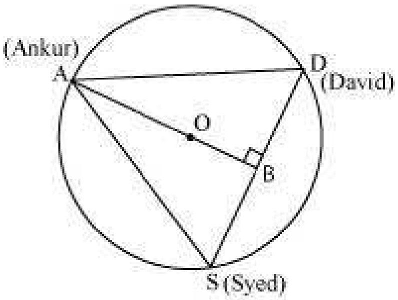
Area of $\triangle$ORS = $\frac{1}{2} \cdot$ OA $\cdot$ RS

$$\frac{1}{2} \cdot RC \cdot OS = \frac{1}{2} \cdot 4 \cdot 6$$

$$RC \cdot 5 = 24$$

RC = 4.8
RM = 2RC = 2(4.8)= 9.6
So, distance between Reshma and Mandip is 9.6 m.

Solution 6



Given that AS = SD = DA

So, ASD is a equilateral triangle

OA (radius) = 20 m.

Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ABC.

We also know that median intersect each other at the 2: 1. As AB is the median of equilateral triangle ABC, we can write

$$\Rightarrow \frac{OA}{OB} = \frac{2}{1}$$

$$\Rightarrow \frac{20\ m}{OB} = \frac{2}{1}$$

$$\Rightarrow OB = \left[\frac{20}{2}\right] m = 10\ m$$

$\therefore$ AB = OA + OB = (20 + 10) m = 30 m.

In $\triangle$ ABD

$AD^2 = AB^2 + BD^2$

$AD^2 = (30)^2 + \left[\frac{AD}{2}\right]^2$

$$AD^2 = 900 + \frac{1}{4}AD^2$$

$$\frac{3}{4}AD^2 = 900$$

$$AD^2 = 1200$$

$$AD = 20\sqrt{3}$$

So, length of string of each phone will be $20\sqrt{3}$ m.

Chapter 10 - Circles Exercise Ex. 10.5

Solution 1

We may observe that

$\angle$AOC = $\angle$AOB + $\angle$BOC

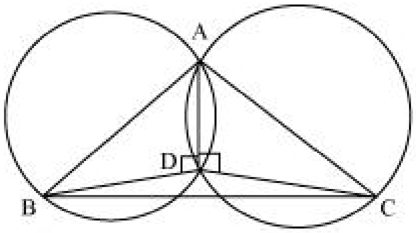
$= 60^o + 30^o$

$= 90^o$

We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

$$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \cdot 90° = 45°$$

Solution 2



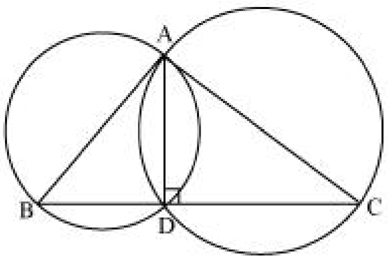
Consider a $\triangle$ABC
Two circles are drawn while taking AB and AC as diameter.
Let they intersect each other at D and let D does not lie on BC.
Join AD

$\angle ADB = 90^o$ (Angle subtend by semicircle)

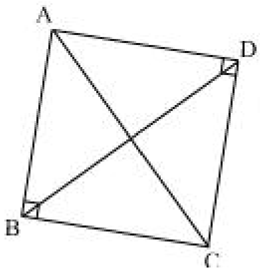
$\angle ADC = 90^o$ (Angle subtend by semicircle)

$\angle BDC = \angle ADB + \angle ADC = 90^o + 90^o = 180^o$
Hence BDC is straight line and our assumption was wrong.

Thus, Point D lies on third side BC of $\triangle$ABC



Solution 3



In $\triangle$ABC

$\angle ABC + \angle BCA + \angle CAB = 180^o$ (Angle sum property of a triangle)

$\Rightarrow 90^o + \angle BCA + \angle CAB = 180^o$

$\Rightarrow \angle BCA + \angle CAB = 90^o$ ... (1)

In $\triangle$ADC

$\angle CDA + \angle ACD + \angle DAC = 180^o$ (Angle sum property of a triangle)

$\Rightarrow 90^o + \angle ACD + \angle DAC = 180^o$

$\Rightarrow \angle ACD + \angle DAC = 90^o$ ... (2)

Adding equations (1) and (2), we have

$\angle BCA + \angle CAB + \angle ACD + \angle DAC = 180^o$

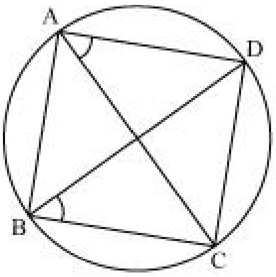
$\Rightarrow (\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) = 180^o \quad \angle BCD + \angle DAB = 180^o \quad ... (3)$

But it is given that

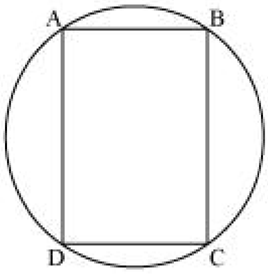
$\angle B + \angle D = 90^o + 90^o = 180^o \quad ... (4)$

From equations (3) and (4), we can see that quadrilateral ABCD is having sum of measures of opposite angles as 180^o.
So, it is a cyclic quadrilateral.
Consider chord CD.

Now, $\angle CAD = \angle CBD$ (Angles in same segment)



Solution 4



Let ABCD be a cyclic parallelogram.

$\angle A + \angle C = 180^o$ (Opposite angle of cyclic quadrilateral) ... (1)

We know that opposite angles of a parallelogram are equal

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

From equation (1)

$\angle A + \angle C = 180^o$

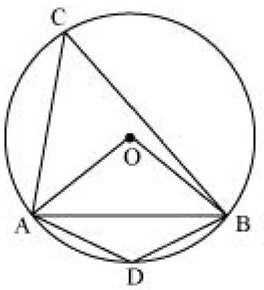
$\Rightarrow \angle A + \angle A = 180^o$

$\Rightarrow 2 \angle A = 180^o$

$\Rightarrow \angle A = 90^o$

Parallelogram ABCD is having its one of interior angles as 90^o, so, it is a rectangle.

Solution 5



In $\triangle OAB$

AB = OA = OB = radius

$\therefore \triangle OAB$ is an equilateral triangle.

So, each interior angle of this triangle will be of 60^o

$\therefore \angle AOB = 60^o$

$$\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}(60^\circ) = 30^\circ$$

Now,

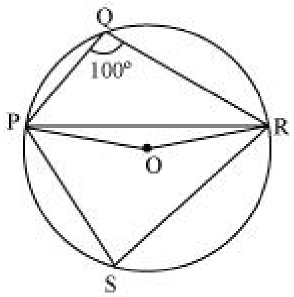
In cyclic quadrilateral ACBD

$\angle ACB + \angle ADB = 180^o$ (Opposite angle in cyclic quadrilateral)

$\Rightarrow \angle ADB = 180^o - 30^o = 150^o$

So, angle subtended by this chord at a point on major arc and minor arc are 30^o and 150^o respectively.

Solution 6



Consider PR as a chord of circle.
Take any point S on major arc of circle.
Now PQRS is a cyclic quadrilateral.

$\angle PQR + \angle PSR = 180^o$ (Opposite angles of cyclic quadrilateral)

$\Rightarrow \angle PSR = 180^o - 100^o = 80^o$

We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

$\therefore \angle POR = 2\angle PSR = 2\,(80^o) = 160^o$

In $\triangle POR$

OP = OR (radii of same circle)

$\therefore \angle OPR = \angle ORP$ (Angles opposite equal sides of a triangle)

$\angle OPR + \angle ORP + \angle POR = 180^o$ (Angle sum property of a triangle)

$2\angle OPR + 160^o = 180^o$

$2\angle OPR = 180^o - 160^o = 20^o$

$\angle OPR = 10^o$

Solution 7

In $\triangle ABC$

$\angle BAC + \angle ABC + \angle ACB = 180^o$ (Angle sum property of a triangle)

$\Rightarrow \angle BAC + 69^o + 31^o = 180^o$

$\Rightarrow \angle BAC = 180^o - 100^o$

$\Rightarrow \angle BAC = 80^o$

$\angle BDC = \angle BAC = 80^o$ (Angles in same segment of circle are equal)

Solution 8

In $\triangle$CDE

$\angle$CDE + $\angle$DCE = $\angle$CEB (Exterior angle)

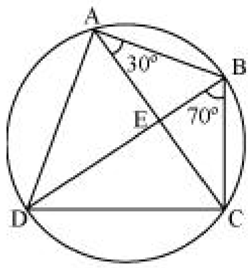
$\Rightarrow$ $\angle$CDE + 20^o = 130^o

$\Rightarrow$ $\angle$CDE = 110^o

But $\angle$BAC = $\angle$CDE (Angles in same segment of circle)

$\Rightarrow$ $\angle$BAC = 110^o

Solution 9



For chord CD

$\angle$CBD = $\angle$CAD (Angles in same segment)

$\angle$CAD = 70^o

$\angle$BAD = $\angle$BAC + $\angle$CAD = 30^o + 70^o = 100^o

$\angle$BCD + $\angle$BAD = 180^o (Opposite angles of a cyclic quadrilateral)

$\angle$BCD + 100^o = 180^o

$\angle$BCD = 80^o

In $\triangle$ABC
AB = BC (given)

$\therefore$ $\angle$BCA = $\angle$CAB (Angles opposite to equal sides of a triangle)

$\angle$BCA = 30^o

We have $\angle$BCD = 80^o

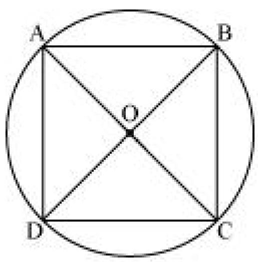
$\angle$BCA + $\angle$ACD = 80^o

30^o + $\angle$ACD = 80^o

$\angle$ACD = 50^o

$\angle$ECD = 50^o

Solution 10



Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.

$$\angle BAD = \frac{1}{2}\angle BOD = \frac{180^\circ}{2} = 90^\circ$$

(Consider BD as a chord)

$\angle BCD + \angle BAD = 180^o$ (Cyclic quadrilateral)

$\angle BCD = 180^o - 90^o = 90^o$

$$\angle ADC = \frac{1}{2}\angle AOC = \frac{180^\circ}{2} = 90^\circ$$

(Considering AC as a chord)

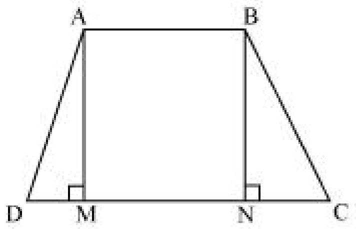
$\angle ADC + \angle ABC = 180^o$ (Cyclic quadrilateral)

$90^o + \angle ABC = 180^o$

$\angle ABC = 90^o$

Here, each interior angle of cyclic quadrilateral is of 90^o. Hence it is a rectangle.

Solution 11



Consider a trapezium ABCD with AB | |CD and BC = AD Draw AM $\perp$ CD and BN $\perp$ CD

In $\triangle AMD$ and $\triangle BNC$

AD = BC (Given)

$\angle AMD = \angle BNC$ (By construction each is 90^o)

AM = BM (Perpendicular distance between two parallel lines is same)

$\therefore \triangle AMD \cong \triangle BNC$ (RHS congruence rule)

$\therefore \angle ADC = \angle BCD$ (CPCT) ... (1)

$\angle BAD$ and $\angle ADC$ are on same side of transversal AD

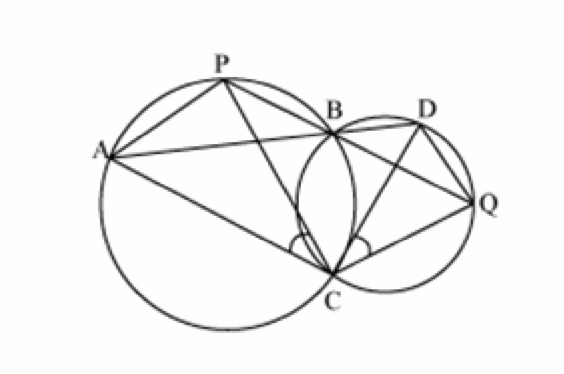
$\angle BAD + \angle ADC = 180^o$ ... (2)

$\angle BAD + \angle BCD = 180^o$ [Using equation (1)]

This equation shows that the opposite angles are supplementary.

So, ABCD is a cyclic quadrilateral.

Solution 12



Join chords AP and DQ
For chord AP

$\angle$PBA = $\angle$ACP (Angles in same segment) ... (1)
For chord DQ

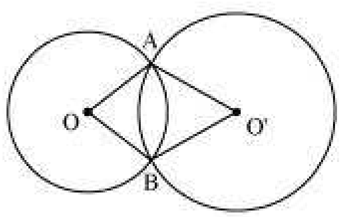
$\angle$DBQ = $\angle$QCD (Angles in same segment) ... (2)
ABD and PBQ are line segments intersecting at B.

$\therefore$ $\angle$PBA = $\angle$DBQ (Vertically opposite angles) ... (3)
From equations (1), (2) and (3), we have

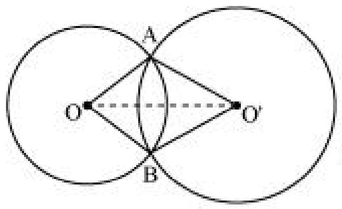
$\angle$ACP = $\angle$QCD

Chapter 10 - Circles Exercise Ex. 10.6

Solution 1



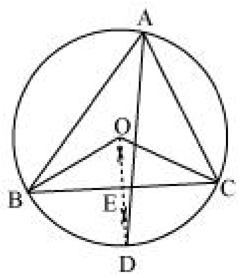
Let two circles having their centres as O and intersect each other at point A and B respectively.
Construction: Let us join OO',



Now in $\triangle$AOO' and $\triangle$BOO'
OA = OB (radius of circle 1)
O'A = O'B (radius of circle 2)
OO' = OO' (common)
$\triangle$AOO' $\cong$ $\triangle$BOO' (by SSS congruence rule)

$\angle$OAO' = $\angle$OBO' (by CPCT)
So, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution 2



Let perpendicular bisector of side BC and angle bisector of $\angle A$ meet at point D.
Let perpendicular bisector of side BC intersects it at E.

Perpendicular bisector of side BC will pass through circum centre O of circle. Now, $\angle BOC$ and $\angle BAC$ are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively.
We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\angle BOC = 2\angle BAC = 2\angle A$ ... (1)

In $\triangle BOE$ and $\triangle COE$
OE = OE (common)
OB = OC (radii of same circle)

$\angle OEB = \angle OEC$ (Each 90^{o} as $OD \perp BC$)

$\therefore \triangle BOE \cong \triangle COE$ (RHS congruence rule)

$\angle BOE = \angle COE$ (by CPCT) ... (2)

But $\angle BOE + \angle COE = \angle BOC$

$\Rightarrow \angle BOE + \angle BOE = 2\angle A$ [Using equations (1) and (2)]

$\Rightarrow 2\angle BOE = 2\angle A$

$\Rightarrow \angle BOE = \angle A$

$\therefore \angle BOE = \angle COE = \angle A$

The perpendicular bisector of side BC and angle bisector of $\angle A$ meet at point D.

$\therefore \angle BOD = \angle BOE = \angle A$ ... (3)

Since AD is the bisector of angle $\angle A$

$\angle BAD = \dfrac{\angle A}{2}$

$\Rightarrow 2\angle BAD = \angle A$ ... (4)

From equations (3) and (4), we have

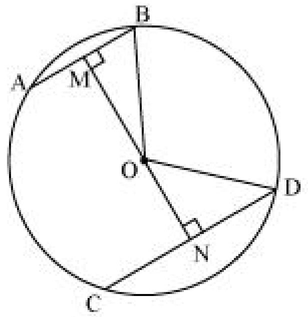
$\angle BOD = 2\angle BAD$

It is possible only if BD will be a chord of the circle. For this the point D lies on circum circle.

Therefore, the perpendicular bisector of side BC and angle bisector of $\angle A$ meet on the circum circle of triangle ABC.

Solution 3

Draw $OM \perp AB$ and $ON \perp CD$. Join OB and OD



$$BM = \frac{AB}{2} = \frac{5}{2}$$

(Perpendicular from centre bisects the chord)

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let ON be x, so OM will be 6 - x

In $\triangle$MOB

$$OM^2 + MB^2 = OB^2$$

$$(6-x)^2 + \left[\frac{5}{2}\right]^2 = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \qquad \ldots(1)$$

In $\triangle$NOD

$$ON^2 + ND^2 = OD^2$$

$$x^2 + \left[\frac{11}{2}\right]^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \qquad \ldots(2)$$

We have OB = OD (radii of same circle)
So, from equation (1) and (2)

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$$

$$x = 1$$

From equation (2)

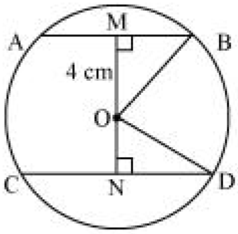
$$(1)^2 + \left[\frac{121}{4}\right] = OD^2$$

$$OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$$

$$OD = \frac{5}{2}\sqrt{5}$$

So, radius of circle is found to be $\frac{5}{2}\sqrt{5}$ cm.

Solution 4



Distance of smaller chord AB from centre of circle = 4 cm.
OM = 4 cm

$$MB = \frac{AB}{2} = \frac{6}{2} = 3 \text{ cm}$$

In $\triangle$ OMB

$$\begin{aligned} OM^2 + MB^2 &= OB^2 \\ (4)^2 + (3)^2 &= OB^2 \\ 16 + 9 &= OB^2 \\ OB &= \sqrt{25} \\ OB &= 5 \text{ cm} \end{aligned}$$

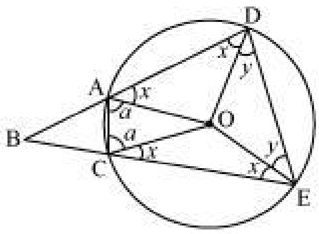
In $\triangle$ OND
OD=OB=5cm (radii of same circle)

$$ND = \frac{CD}{2} = \frac{8}{2} = 4\text{cm}$$

$$\begin{aligned} ON^2 + ND^2 &= OD^2 \\ ON^2 + (4)^2 &= (5)^2 \\ ON^2 &= 25 - 16 = 9 \\ ON &= 3 \end{aligned}$$

So, distance of bigger chord from centre is 3 cm.

Solution 5



In $\triangle$ AOD and $\triangle$ COE
OA = OC (radii of same circle)
OD = OE (radii of same circle)
AD = CE (given)

$\therefore \triangle AOD \cong \triangle COE$ (SSS congruence rule)

$\angle OAD = \angle OCE$ (by CPCT) ... (1)

$\angle ODA = \angle OEC$ (by CPCT) ... (2)

We also have

$\angle OAD = \angle ODA$ (As OA = OD) ... (3)
From equations (1), (2) and (3), we have

$\angle OAD = \angle OCE = \angle ODA = \angle OEC$

Let $\angle OAD = \angle OCE = \angle ODA = \angle OEC = x$

In $\triangle OAC$,
OA = OC

$\therefore \angle OCA = \angle OAC$ (let a)

In $\triangle ODE$,
OD = OE

$\angle OED = \angle ODE$ (let y)
ADEC is a cyclic quadrilateral

$\therefore \angle CAD + \angle DEC = 180^o$ (opposite angles are supplementary)
$x + a + x + y = 180^o$
$2x + a + y = 180^o$
$y = 180 - 2x - a$ ... (4)

But $\angle DOE = 180 - 2y$

And $\angle AOC = 180 - 2a$

Now, $\angle DOE - \angle AOC = 2a - 2y = 2a - 2(180 - 2x - a)$
$= 4a + 4x - 360^o$ ... (5)

Now, $\angle BAC + \angle CAD = 180$ (Linear pair)

$\Rightarrow \angle BAC = 180 - \angle CAD = 180 - (a + x)$

Similarly, $\angle ACB = 180 - (a + x)$

Now, in $\triangle ABC$

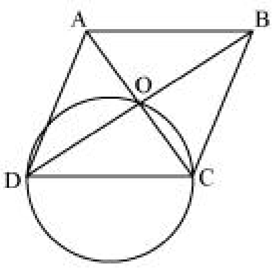
$\angle ABC + \angle BAC + \angle ACB = 180$ (Angle sum property of a triangle)

$\angle ABC = 180 - \angle BAC - \angle ACB$
$= 180 - (180 - a - x) - (180 - a - x)$
$= 2a + 2x - 180$

$= \frac{1}{2}[4a + 4x - 360^o]$

$\angle ABC = \frac{1}{2}[\angle DOE - \angle AOC]$ [Using equation (5)]

Solution 6



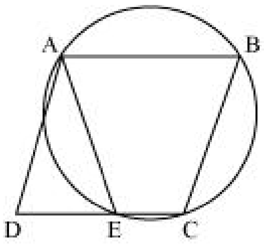
Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn taking side CD as its diameter.
We know that angle in a semicircle is of 90^o.

$\therefore \angle COD = 90^o$

Also in rhombus the diagonals intersect each other at 90^o

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^o$
So, point O has to lie on the circle.

Solution 7



We see that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral sum of opposite angles is 180^o

$\angle AEC + \angle CBA = 180^o$

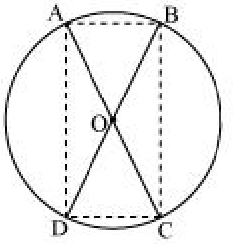
$\angle AEC + \angle AED = 180^o$ (linear pair)

$\angle AED = \angle CBA$ ... (1)
For a parallelogram opposite angles are equal.

$\angle ADE = \angle CBA$ ... (2)
From (1) and (2)

$\angle AED = \angle ADE$
AD = AE (angles opposite to equal sides of a triangle)

Solution 8



Let two chords AB and CD are intersecting each other at point O.

In $\triangle AOB$ and $\triangle COD$
OA = OC (given)
OB = OD (given)

$\angle AOB = \angle COD$ (vertically opposite angles)

$\triangle AOB \cong \triangle COD$ (SAS congruence rule)
AB = CD (by CPCT)

Similarly, we can prove $\triangle AOD \cong \triangle COB$

$\therefore$ AD = CB (by CPCT)

Since in quadrilateral ACBD opposite sides are equal in length.
Hence, ACBD is a parallelogram.
We know that opposite angles of a parallelogram are equal

$\therefore \angle A = \angle C$

But $\angle A + \angle C = 180^o$ (ABCD is a cyclic quadrilateral)

$\Rightarrow \angle A + \angle A = 180^o$

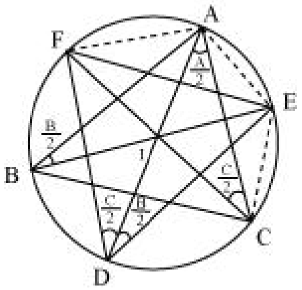
$\Rightarrow 2\angle A = 180^o$

$\Rightarrow \angle A = 90^o$

As ACBD is a parallelogram and one of its interior angles is 90^o, so it is a rectangle.

$\angle A$ is the angle subtended by chord BD. And as $\angle A = 90^o$, so BD should be diameter of circle. Similarly AC is diameter of circle.

Solution 9



It is given that BE is the bisector of $\angle B$

$\therefore \angle ABE = \frac{\angle B}{2}$

But $\angle ADE = \angle ABE$ (angles in same segment for chord AE)

$\Rightarrow \angle ADE = \frac{\angle B}{2}$

Similarly, $\angle ACF = \angle ADF = \frac{\angle C}{2}$ (angle in same segment for chord AF)

Now, $\angle D = \angle ADE + \angle ADF$

$$= \frac{\angle B}{2} + \frac{\angle C}{2}$$

$$= \frac{1}{2}(\angle B + \angle C)$$

$$= \frac{1}{2}(180^\circ - \angle A)$$

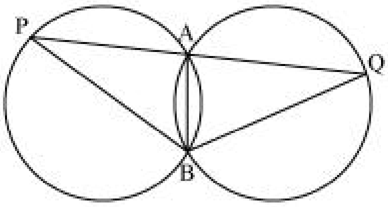
$$= 90^\circ - \frac{1}{2}\angle A$$

Similarly we can prove that

$$\angle E = 90^\circ - \frac{1}{2}\angle B$$

$$\angle F = 90^\circ - \frac{1}{2}\angle C$$

Solution 10



AB is common chord in both congruent circles.

$\therefore \angle APB = \angle AQB$

Now in $\triangle BPQ$

$\angle APB = \angle AQB$

$\therefore BP = BQ$ (angles opposite to equal sides of a triangle)