Chapter 3 – Pair Of Linear Equations In Two Variables contains eleven exercises and corresponding RD Sharma Solutions for Class 10 has all the answers to the problems in various exercise. This chapter is an extension of what you have learnt from the middle school about linear equation in one variable. Let's see some of the concepts discussed in this chapter:

- Systems of linear equations in two variables
- The solution of a system of linear equations in two variables
- Graphical and algebraic methods of solving a system of linear equations in two variables like substitution, elimination and cross-multiplication methods.
- Consistent and inconsistent system of equations.
- Applications of linear equations in two variables in solving simple problems from different areas.

Access the RD Sharma Solutions For Class 10 Chapter 3 – Pair Of Linear Equations In Two Variables

Exercise 3.1 Page No: 3.12

1. **Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a rig on the items in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the giant wheel. Each ride costs ₹3 and a game of hoopla costs ₹4. If she spent ₹20 in the fair, represent this situation algebraically and graphically.**

Solution:

Let 'x' be the number of rides Akhila had on the giant wheel.

And, let 'y' be the number of times she played Hoopla.

From the question we can write the below pair of equations.

$y = (1/2)x$ $\Rightarrow x - 2y = 0$……. (i)

$3x + 4y = 20$……. (ii)

To represent these equations graphically we need at least two solutions for each (i) and (ii).

And let's put them in a table for each:

For equation (i),

x	0	2
y = (1/2)x	0	1

For equation (ii),

x	0	20/3	4
y = (20 – 3x)/4	5	0	2

When:

The solution of the variable is zero; the equation can be solved easily. Putting x =0 in equation (ii) we get

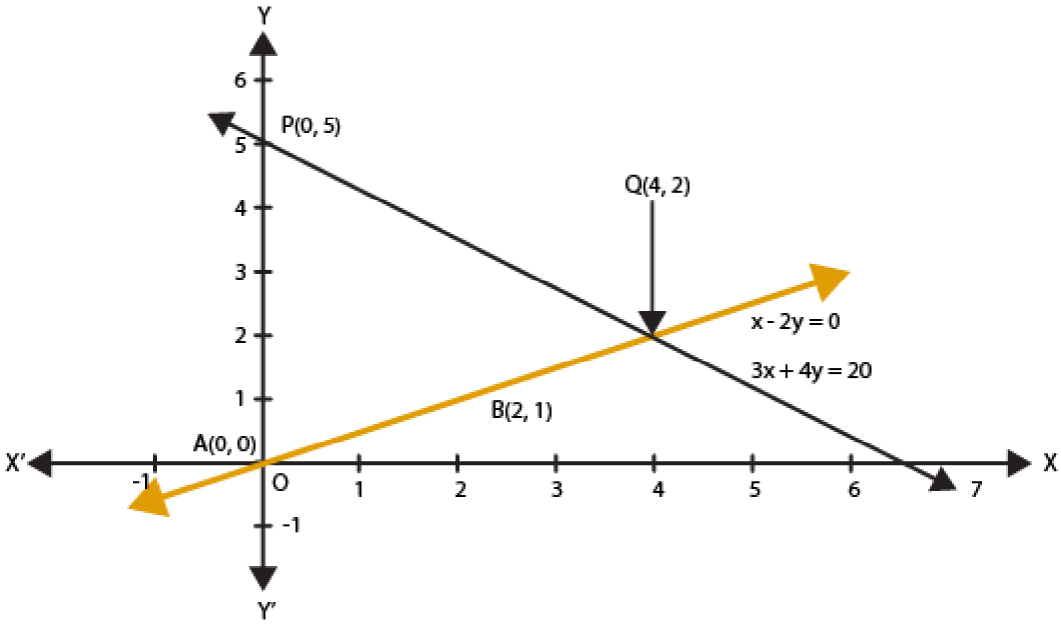
$4y = 20 \Rightarrow y = 5$

Similarly putting y = 0 in equation (ii) we get

$3x = 20 \Rightarrow x = 20/3$ but it is not an integer so it is not easy to plot on graph paper.

So, we chose $y=2$ which gives $x =4$ as an integer value.

The above can be plotted in a graph as below:



We can observe that the two lines represents the equations (i) and (ii) intersect at a single point.

2. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Is not this interesting? Represent this situation algebraically and graphically.

Solution:

Let the present age of Aftab and his daughter be x and y respectively.

Hence, seven years ago,

Age of Aftab = $x – 7$ and Age of his daughter = $y – 7$

According to the given condition,

$x – 7 = 7 (y – 7) \Rightarrow x – 7y = -42$……… (i)

Three years from the present age,

$x + 3 = 3 (y + 3) \Rightarrow x – 3y = 6$………..(ii)

Therefore, equations (i) and (ii) represents the situation algebraically.

To represent these equations graphically we need at least two solutions for each (i) and (ii).

And let's put them in a table for each:

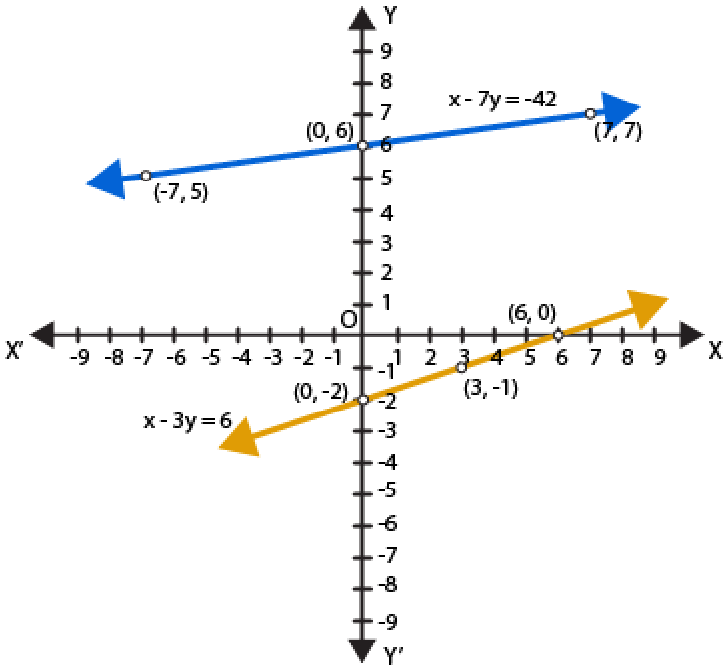
For equation (i),

x	-7	0	7
y = (x + 42)/7	5	6	7

For equation (ii),

x	6	3	0
y = (x – 6)/3	0	-1	-2

The above can be plotted in a graph as below:



3. The path of the train A is given by the equation 3x+4y-12 =0 and the path of another train B is given by the equation 6x+8y-48 =0. Represent this situation graphically.

Solution:

Given pair of linear equations which represents the paths of train A and train B,

3x + 4y – 12 = 0………………………….. (i)

6x + 8y – 48 = 0 ………………………….. (ii)

To represent these equations graphically we need at least two solutions for each (i) and (ii).

And let's put them in a table for each:

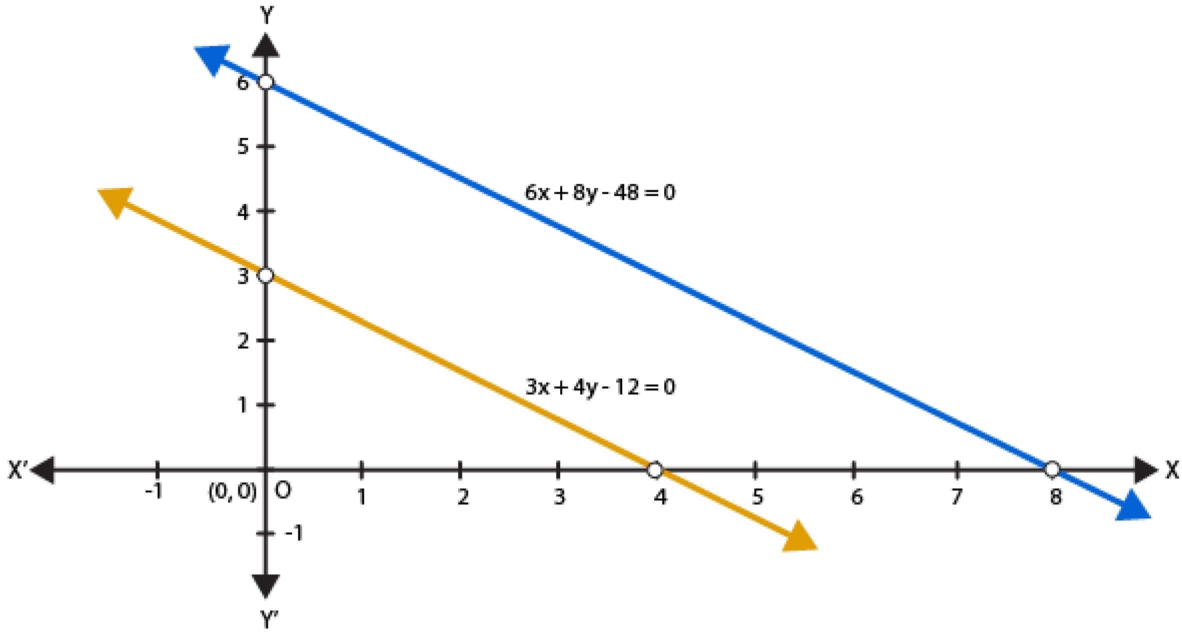
For equation (i),

x	0	4
y = (12 – 3x)/4	3	0

For equation (i),

x	0	8
y = (48 – 6x)/8	6	0

The above can be plotted in a graph as below:



Exercise 3.2 Page No: 3.29

Solve the following system of equations graphically:

1. x + y = 3

2x + 5y = 12

Solution:

Given,

x + y = 3……. (i)

2x + 5y = 12……. (ii)

For equation (i),

When y = 0, we have x =3

When x= 0, we have y =3

Thus we have the following table giving points on the line x + y = 3

x	0	3
y	3	0

For equation (ii),

We solve for y:

⇒ y = (12 – 2x)/5

So, when x = 1

y = (12 – 2(1))/5 = 2

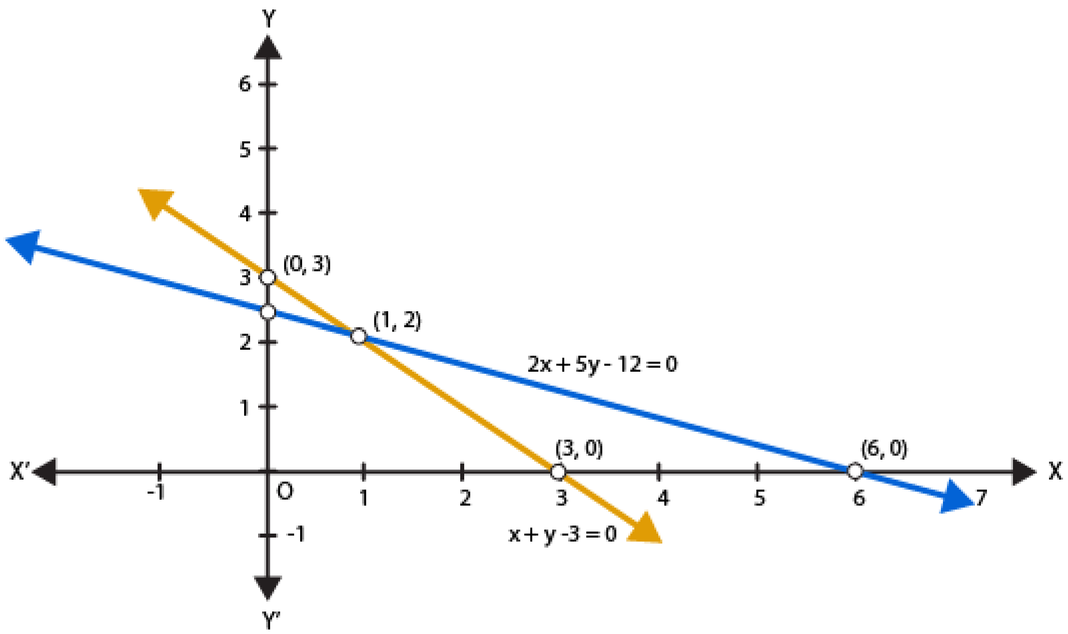
And, when x = 6

⇒ y = (12 – 2(6))/5 = 0

Thus we have the following table giving points on the line 2x + 5y = 12

x	1	6
y	2	0

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (1, 2)

Hence, x= 1 and y = 2

2. x – 2y = 5

2x + 3y = 10

Solution:

Given,

x – 2y = 5……. (i)

2x + 3y = 10……. (ii)

For equation (i),

⇒ y = (x – 5)/2

When y = 0, we have x = 5

When x = -1, we have y = -2

Thus we have the following table giving points on the line **x – 2y = 5**

x	5	-1

y	0	-2

For equation (ii),

We solve for y:

⇒ $y = (10 – 2x)/3$

So, when $x = 5$

$y = (10 – 2(5))/3 = 0$

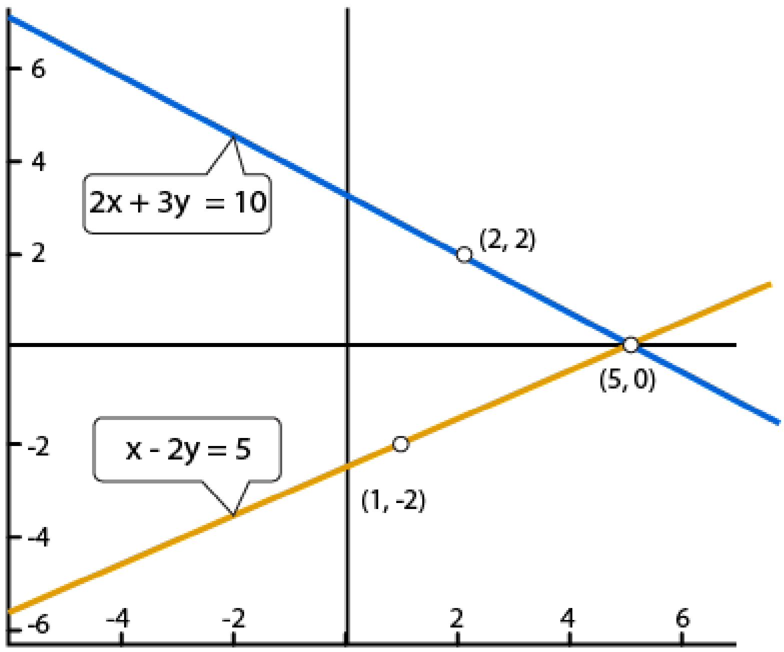
And, when $x = 2$

⇒ $y = (10 – 2(2))/3 =2$

Thus we have the following table giving points on the line **2x + 3y = 10**

x	5	2
y	0	2

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (5, 0)

Hence, x= 5 and y = 0

3. 3x+ y + 1 = 0

2x – 3y + 8 = 0

Solution:

Given,

3x+ y + 1 = 0 ……. (i)

2x – 3y + 8 = 0……. (ii)

For equation (i),

⇒ y = -(1 + 3x)

When x = 0, we have y = -1

When x = -1, we have y = 2

Thus we have the following table giving points on the line **3x+ y + 1 = 0**

x	-1	0
y	2	-1

For equation (ii),

We solve for y:

⇒ y = (2x + 8)/ 3

So, when x = -4

y = (2(-4) + 8)/3 = 0

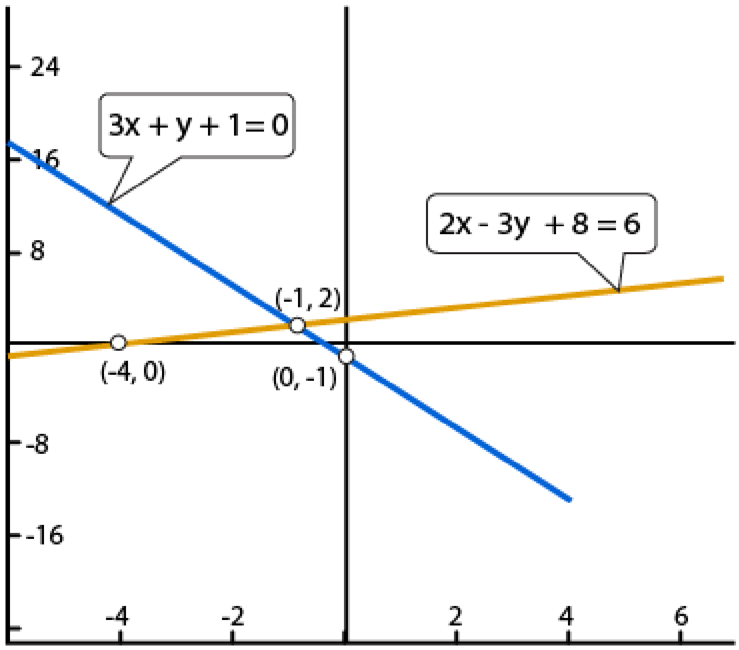
And, when x = -1

⇒ y = (2(-1) + 8)/3 = 2

Thus we have the following table giving points on the line **2x – 3y + 8 = 0**

x	-4	-1
y	0	2

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (-1, 2)

Hence, x= -4 and y = 0

4. 2x + y – 3 = 0

2x – 3y – 7 = 0

Solution:

Given,

2x + y – 3 = 0……. (i)

2x – 3y – 7 = 0……. (ii)

For equation (i),

⇒ y = (3 – 2x)

When x = 0, we have y = (3 – 2(0)) = 3

When x = 1, we have y = (3 – 2(1)) = 1

Thus we have the following table giving points on the line **2x + y – 3 = 0**

x	0	1
y	3	1

For equation (ii),

We solve for y:

⇒ y = (2x – 7)/ 3

So, when x = 2

y = (2(2) – 7)/3 = -1

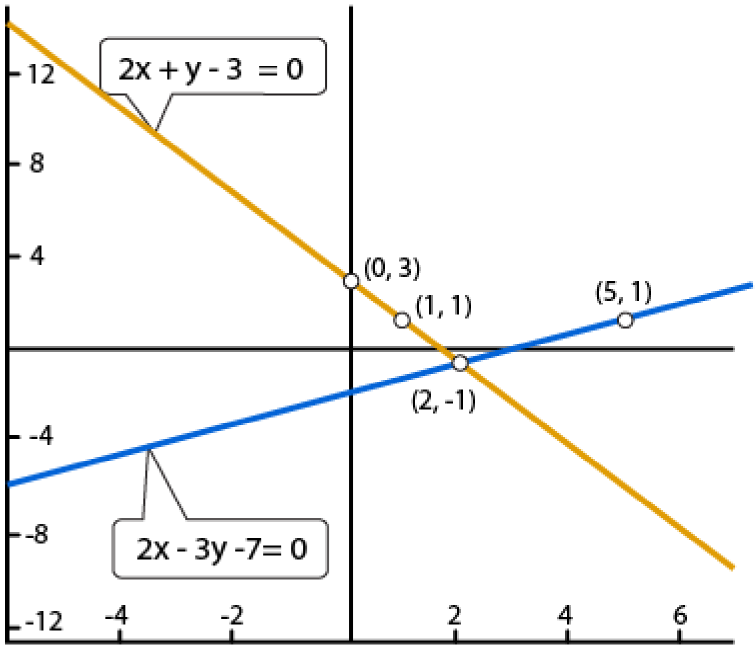
And, when x = 5

⇒ y = (2(5) – 7)/3 = 1

Thus we have the following table giving points on the line **2x – 3y – 7 = 0**

x	2	5
y	-1	1

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (2, -1)

Hence, x= 2 and y = -1

5. x + y = 6

x – y = 2

Solution:

Given,

x + y = 6……. (i)

x – y = 2……. (ii)

For equation (i),

⇒ y = (6 – x)

When x = 2, we have y = (6 – 2)) = 4

When x = 3, we have y = (6 – 3) = 3

Thus we have the following table giving points on the line **x + y = 6**

x	2	3
y	4	3

For equation (ii),

We solve for y:

⇒ y = (x – 2)

So, when x = 2

y = (0 – 2) = -2

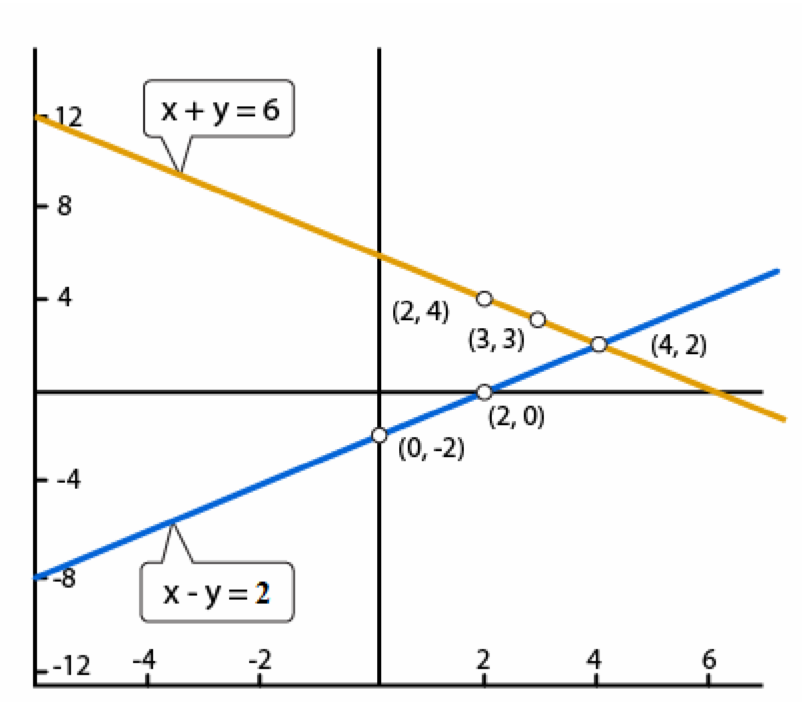
And, when x = 5

⇒ y = (2 – 2) = 0

Thus we have the following table giving points on the line **x – y = 2**

x	0	2
y	-2	0

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (4, 2)

Hence, x= 4 and y = 2

6. x – 2y = 6

3x – 6y = 0

Solution:

Given,

$x - 2y = 6$**……. (i)**

$3x - 6y = 0$**……. (ii)**

For equation (i),

$\Rightarrow y = (x - 6)/2$

When $x = 2$, we have $y = (2 - 6)/2 = -2$

When $x = 0$, we have $y = (0 - 6)/2 = -3$

Thus we have the following table giving points on the line $x - 2y = 6$

x	2	0
y	-2	-3

For equation (ii),

We solve for y:

$\Rightarrow y = x/ 2$

So, when $x = 0$

$y = 0/2 = 0$

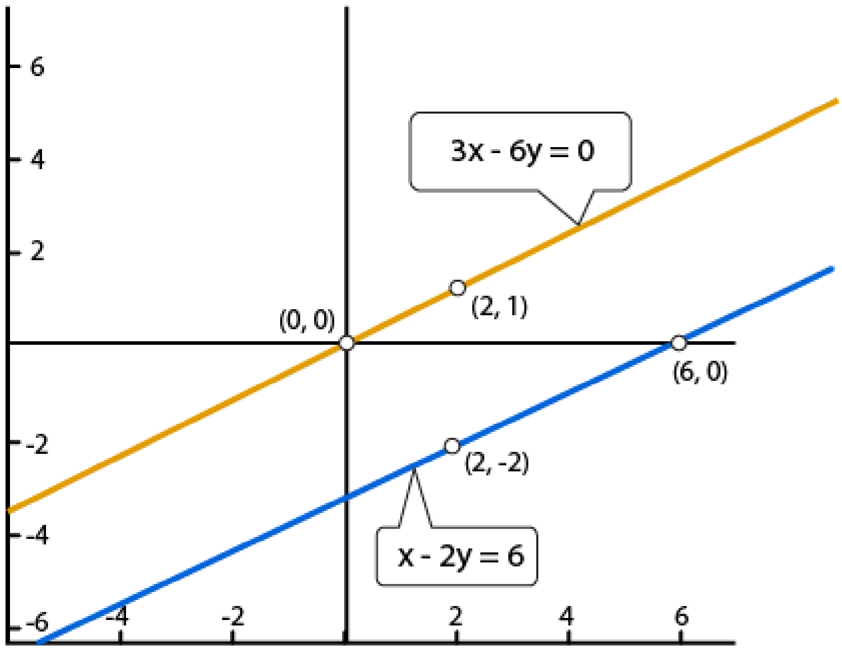
And, when $x = 2$

$\Rightarrow y = 2/2 = 1$

Thus we have the following table giving points on the line $3x - 6y = 0$

x	0	2
y	0	1

Graph of the equations (i) and (ii) is as below:



Clearly the two lines are parallel to each other. So, the two lines do not intersect.

Hence, the given system has no solutions.

7. x + y = 4

2x – 3y = 3

Solution:

Given,

x + y = 4……. (i)

2x – 3y = 3……. (ii)

For equation (i),

⇒ y = (4 – x)

When x = 4, we have y = (4 – 4) = 0

When x = 2, we have y = (4 – 2) = 2

Thus we have the following table giving points on the line **x + y = 4**

x	4	2
y	0	2

For equation (ii),

We solve for y:

⇒ y = (2x – 3)/3

So, when x = 3

y = (2(3) – 3)/3 = 1

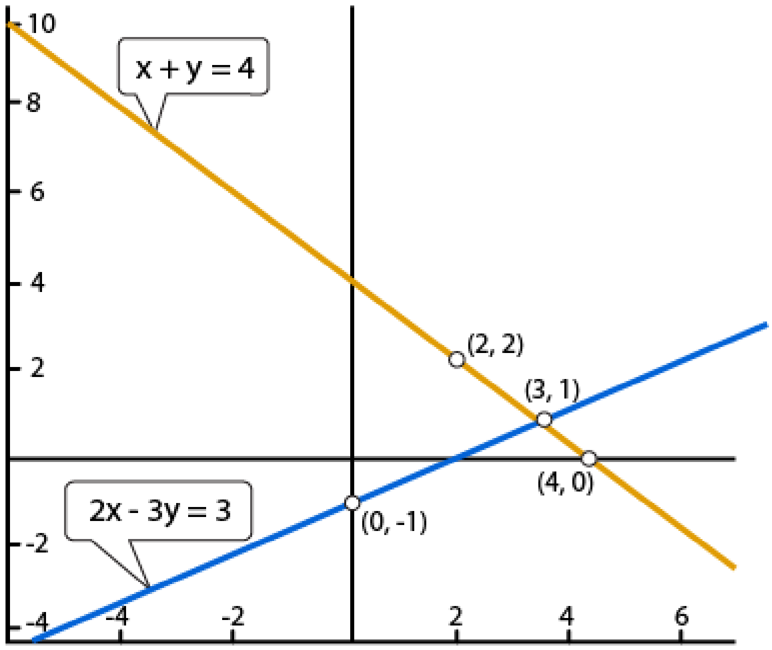
And, when x = 0

⇒ y = (2(0) – 3)/3 = -1

Thus we have the following table giving points on the line **2x – 3y = 3**

x	3	0
y	1	-1

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (3, 1)

Hence, x= 3 and y = 1

8. 2x + 3y = 4

x – y + 3 = 0

Solution:

Given,

2x + 3y = 4……. (i)

x – y + 3 = 0……. (ii)

For equation (i),

⇒ y = (4 – 2x) /3

When x = -1, we have y = (4 – 2(-1))/3 = 2

When x = 2, we have y = (4 – 2(2))/3 = 0

Thus we have the following table giving points on the line **2x + 3y = 4**

x	-1	2
y	2	0

For equation (ii),

We solve for y:

⇒ y = (x + 3)

So, when x = 0

y = (0 + 3) = 3

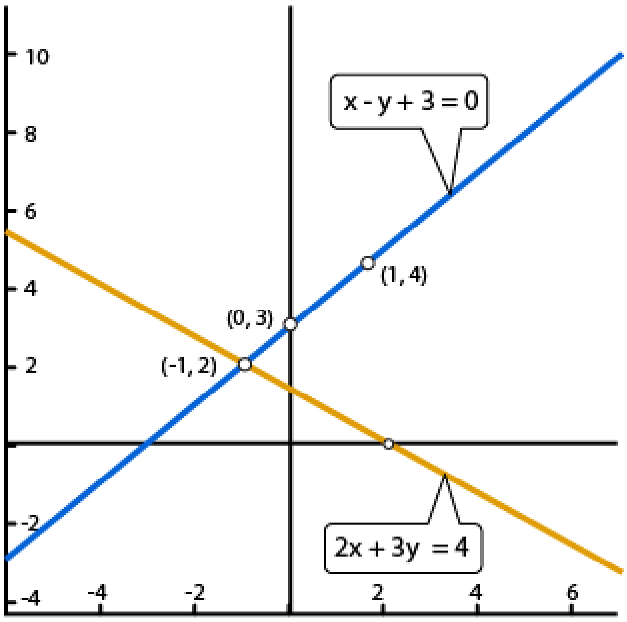
And, when x = 1

⇒ y = (1 + 3) = 4

Thus we have the following table giving points on the line **x – y + 3 = 0**

x	0	1
y	3	4

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (-1, 2)

Hence, x= -1 and y = 2

9. 2x – 3y + 13 = 0

3x – 2y + 12 = 0

Solution:

Given,

$2x – 3y + 13 = 0$……. (i)

$3x – 2y + 12 = 0$……. (ii)

For equation (i),

$\Rightarrow y = (2x + 13) /3$

When $x = -5$, we have $y = (2(-5) + 13))/3 = 1$

When $x = -2$, we have $y = (2(-2) + 13))/3 = 3$

Thus we have the following table giving points on the line **$2x – 3y + 13 = 0$**

x	-5	-2
y	1	3

For equation (ii),

We solve for y:

$\Rightarrow y = (3x + 12)/2$

So, when $x = -4$

$y = (3(-4) + 12)/2 = 0$

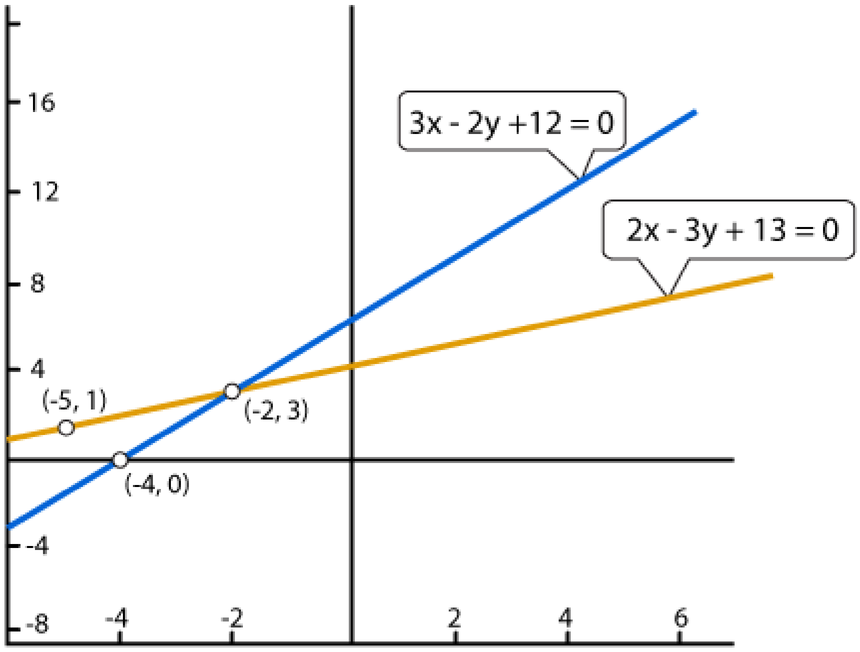
And, when $x = -2$

$\Rightarrow y = (3(-2) + 12)/2 = 3$

Thus we have the following table giving points on the line **$3x – 2y + 12 = 0$**

x	-4	-2
y	0	3

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (-2, 3)

Hence, x= -2 and y = 3

10. 2x + 3y + 5 = 0

3x + 2y – 12 = 0

Solution:

Given,

2x + 3y + 5 = 0……. (i)

3x – 2y – 12 = 0……. (ii)

For equation (i),

⇒ y = -(2x + 5) /3

When x = -4, we have y = -(2(-4) + 5))/3 = 1

When x = -2, we have y = -(2(-2) + 5))/3 = -1

Thus we have the following table giving points on the line **2x + 3y + 5 = 0**

x	-4	-1
y	1	-1

For equation (ii),

We solve for y:

⇒ y = (3x – 12)/2

So, when x = 4

y = (3(4) – 12)/2 = 0

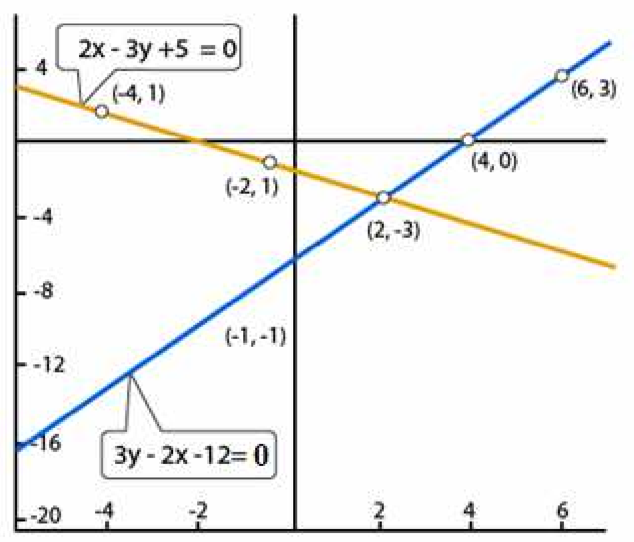
And, when x = 6

⇒ y = (3(6) – 12)/2 = 3

Thus we have the following table giving points on the line **3x – 2y – 12 = 0**

x	4	6
y	0	3

Graph of the equations (i) and (ii) is as below:



Clearly the two lines intersect at a single point P (2, -3)

Hence, x= 2 and y = -3

Show graphically that each one of the following systems of equation has infinitely many solution:

11. 2x + 3y = 6

4x + 6y = 12

Solution:

Given,

2x + 3y = 6……. (i)

4x + 6y = 12……. (ii)

For equation (i),

⇒ y = (6 – 2x) /3

When x = 0, we have y = (6 – 2(0))/3 = 2

When x = 3, we have y = (6 – 2(3))/3 = 0

Thus we have the following table giving points on the line **2x + 3y = 6**

x	0	3
y	2	0

For equation (ii),

We solve for y:

⇒ y = (12 – 4x)/6

So, when x = 0

y = (12 – 4(0))/6 = 2

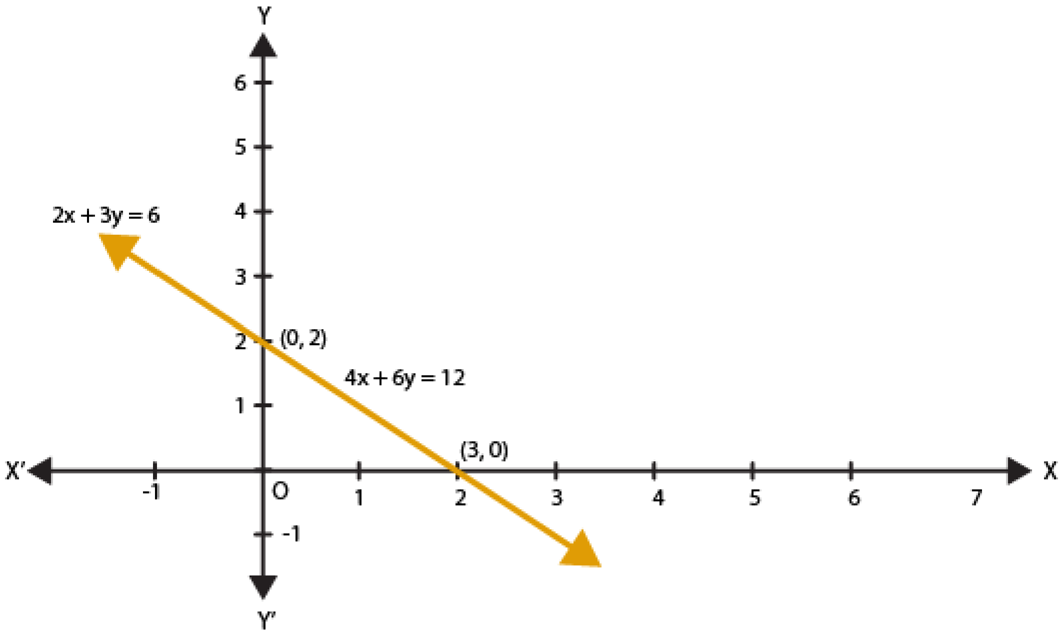
And, when x = 3

⇒ y = (12 – 4(3))/6 = 0

Thus we have the following table giving points on the line **4x + 6y = 12**

x	0	3
y	2	0

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

12. x – 2y = 5

3x – 6y = 15

Solution:

Given,

x – 2y = 5……. (i)

3x – 6y = 15……. (ii)

For equation (i),

$\Rightarrow y = (x – 5) /2$

When x = 3, we have y = (3 – 5) /2 = -1

When x = 5, we have y = (5 – 5) /2 = 0

Thus we have the following table giving points on the line **x – 2y = 5**

x	3	5
y	-1	0

For equation (ii),

We solve for y:

$\Rightarrow y = (3x – 15)/6$

So, when x = 3

y = (3(3) – 15)/6= -1

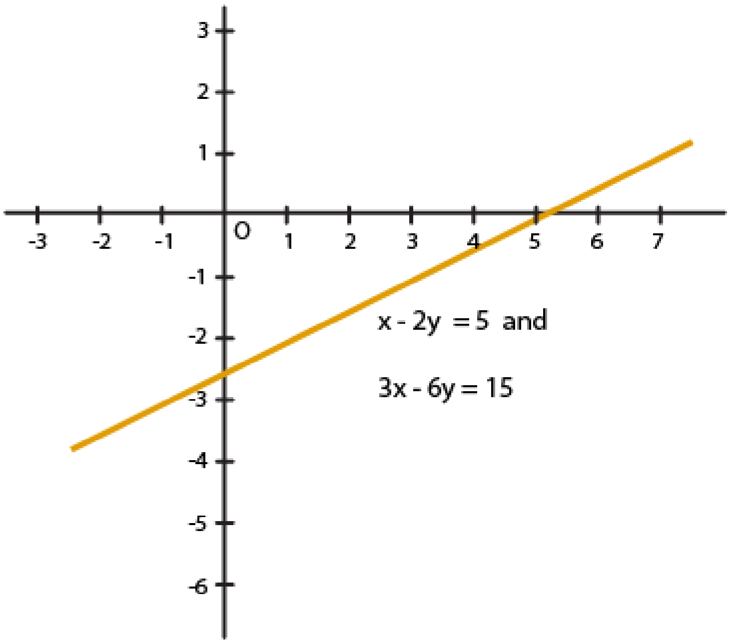
And, when x = 3

$\Rightarrow y = (3(5) – 15)/6= 0$

Thus we have the following table giving points on the line **3x – 6y = 15**

x	3	5
y	-1	0

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

13. 3x + y = 8

6x + 2y = 16

Solution:

Given,

3x + y = 8……. (i)

6x + 2y = 16……. (ii)

For equation (i),

⇒ y = (8 – 3x)

When x = 2, we have y = (8 – 3(2)) = 2

When x = 3, we have y = (8 – 3(3)) = -1

Thus we have the following table giving points on the line **3x + y = 8**

x	2	3
y	2	-1

For equation (ii),

We solve for y:

⇒ y = (16 – 6x)/2

So, when x = 3

y = (16 – 6(3))/2= -1

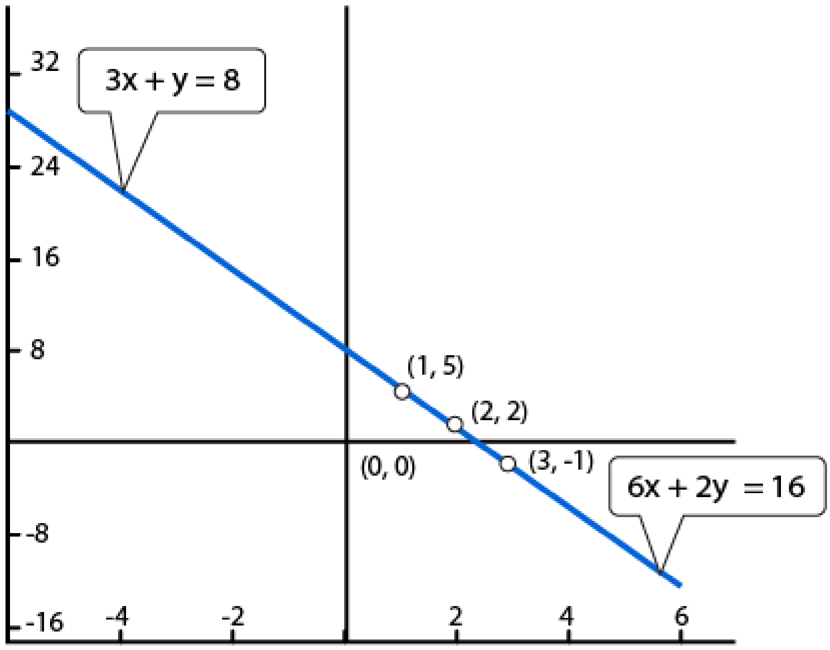
And, when x = 1

⇒ y = (16 – 6(1))/2= 5

Thus we have the following table giving points on the line **6x + 2y = 16**

x	3	1
y	-1	5

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

14. x – 2y + 11 = 0

3x + 6y + 33 = 0

Solution:

Given,

x – 2y + 11 = 0……. (i)

3x – 6y + 33 = 0……. (ii)

For equation (i),

⇒ y = (x + 11)/2

When x = -1, we have y = (-1 + 11)/2 = 5

When x = -3, we have y = (-3 + 11)/2 = 4

Thus we have the following table giving points on the line **x – 2y + 11 = 0**

x	-1	-3
y	5	4

For equation (ii),

We solve for y:

⇒ $y = (3x + 33)/6$

So, when $x = 1$

$y = (3(1) + 33)/6 = 6$

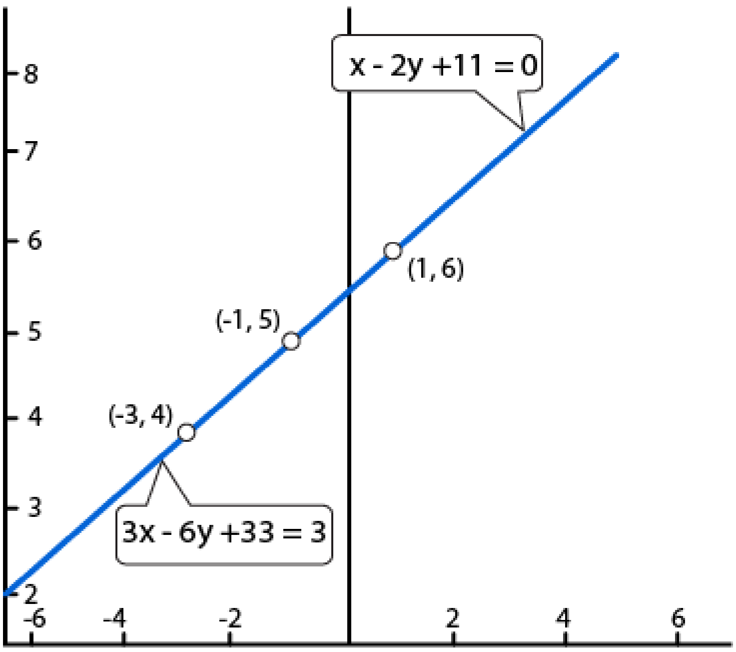
And, when $x = -1$

⇒ $y = (3(-1) + 33)/6 = 5$

Thus we have the following table giving points on the line **3x – 6y + 33 = 0**

x	1	-1
y	6	5

Graph of the equations (i) and (ii) is as below:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions.

Show graphically that each one of the following systems of equations is in-consistent (i.e has no solution):

15. 3x – 5y = 20

6x – 10y = – 40

Solution:

Given,

$3x - 5y = 20$……. (i)

$6x - 10y = -40$……. (ii)

For equation (i),

$\Rightarrow y = (3x - 20)/5$

When $x = 5$, we have $y = (3(5) - 20)/5 = -1$

When $x = 0$, we have $y = (3(0) - 20)/5 = -4$

Thus we have the following table giving points on the line **$3x - 5y = 20$**

x	5	0
y	-1	-4

For equation (ii),

We solve for y:

$\Rightarrow y = (6x + 40)/10$

So, when $x = 0$

$y = (6(0) + 40)/10 = 4$

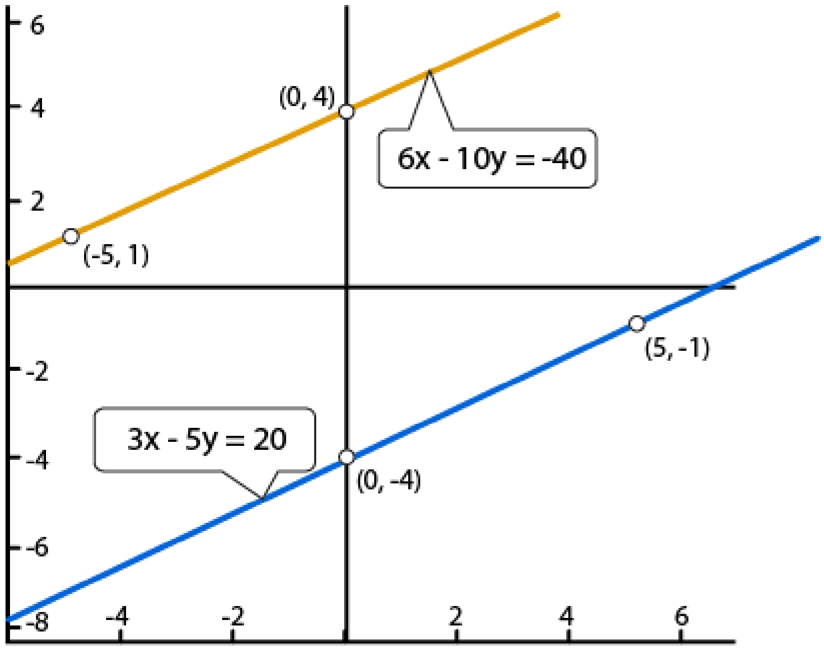
And, when $x = -5$

$\Rightarrow y = (6(-5) + 40)/10 = 1$

Thus we have the following table giving points on the line **$6x - 10y = -40$**

x	0	-5
y	4	1

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.

Hence, the given systems of equations is in-consistent.

16. x – 2y = 6

3x – 6y = 0

Solution:

Given,

x – 2y = 6……. (i)

3x – 6y = 0……. (ii)

For equation (i),

⇒ y = (x – 6)/2

When x = 6, we have y = (6 – 6)/2 = 0

When x = 2 we have y = (2 – 6)/2 = -2

Thus we have the following table giving points on the line **x – 2y = 6**

x	6	2
y	0	-2

For equation (ii),

We solve for y:

⇒ y = x/2

So, when x = 0

y = 0/2 = 0

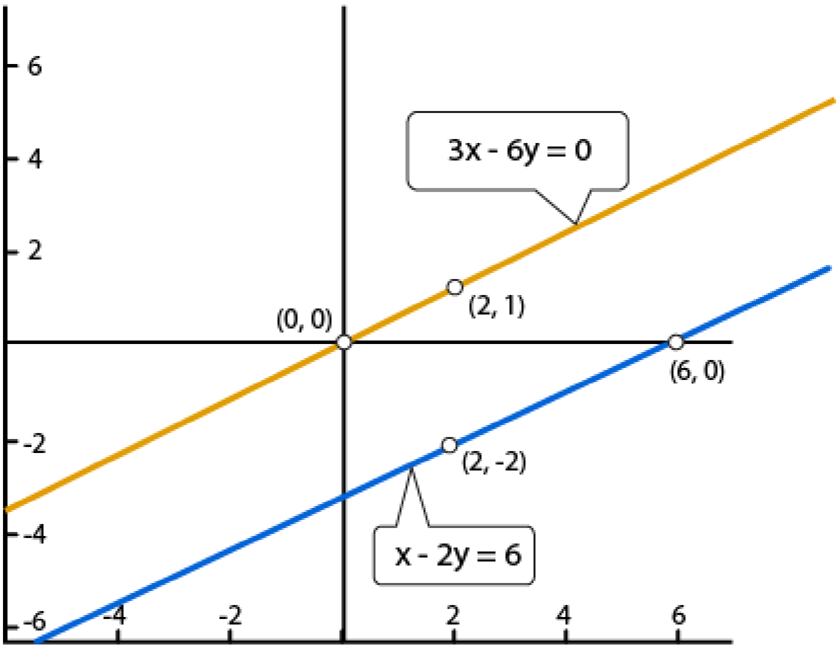
And, when x = 2

⇒ y = 2/2 = 1

Thus we have the following table giving points on the line **3x – 6y = 0**

x	0	2
y	0	1

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.

Hence, the given systems of equations is in-consistent.

17. 2y – x = 9

6y – 3x = 21

Solution:

Given,

2y – x = 9**……. (i)**

6y – 3x = 21……. (ii)

For equation (i),

⇒ y = (x + 9)/2

When x = -3, we have y = (-3 + 9)/2= 3

When x = -1, we have y = (-1 + 9)/2= 4

Thus we have the following table giving points on the line 2y – x = 9

x	-3	-1
y	3	4

For equation (ii),

We solve for y:

⇒ $y = (21 + 3x)/6$

So, when $x = -3$

$y = (21 + 3(-3))/6 = 2$

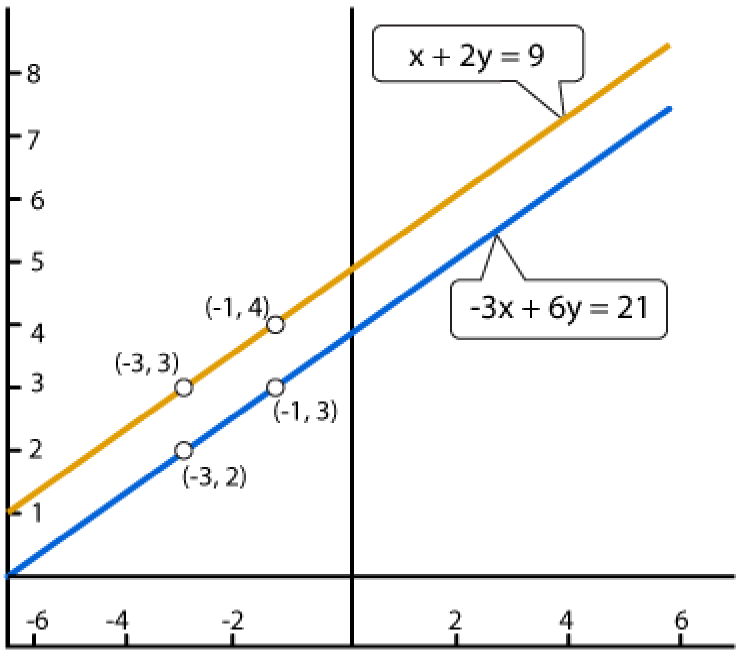
And, when $x = -1$

⇒ $y = (21 + 3(-1))/6 = 3$

Thus we have the following table giving points on the line **6y – 3x = 21**

x	-3	-1
y	2	3

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.

Hence, the given systems of equations is in-consistent.

18. 3x – 4y – 1 = 0

2x – (8/3)y + 5 = 0

Solution:

Given,

$3x – 4y – 1 = 0$……. (i)

$2x – (8/3)y + 5 = 0$**……. (ii)**

For equation (i),

$\Rightarrow y = (3x – 1)/4$

When $x = -1$, we have $y = (3(-1) – 1)/4= -1$

When $x = 3$, we have $y = (3(3) – 1)/4= 2$

Thus we have the following table giving points on the line **$3x – 4y – 1 = 0$**

x	-1	3
y	-1	2

For equation (ii),

We solve for y:

$\Rightarrow y = (6x + 15)/8$

So, when $x = -2.5$

$y = (6(-2.5) + 15)/8 = 0$

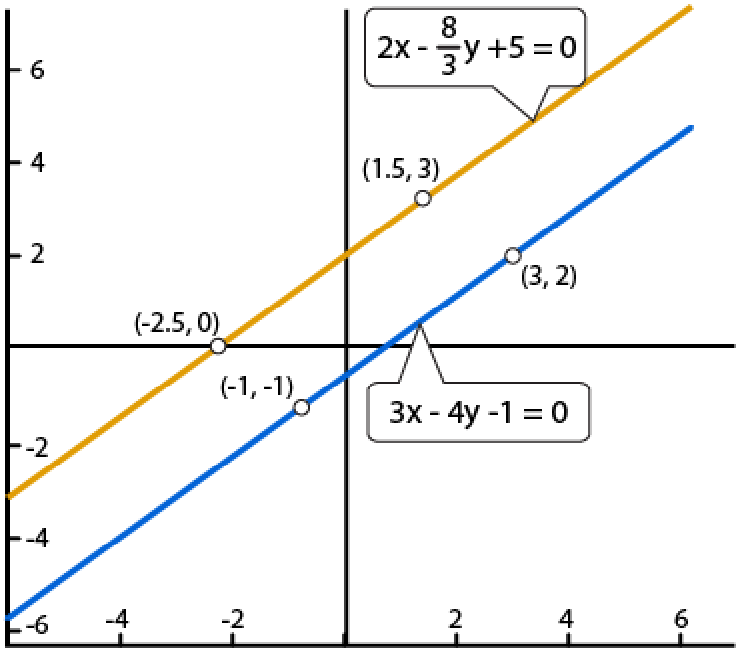
And, when $x = 1.5$

$\Rightarrow y = (6(1.5) + 15)/8 = 3$

Thus we have the following table giving points on the line $2x – (8/3)y + 5 = 0$

x	-2.5	1.5
y	0	3

Graph of the equations (i) and (ii) is as below:



It is clearly seen that, there is no common point between these two lines.

Hence, the given systems of equations is in-consistent.

19. Determine graphically the vertices of the triangle, the equations of whose sides are given below:

(i) 2y – x = 8, 5y – x = 14 and y – 2x = 1

Solution:

Given,

2y – x = 8……. (i)

5y – x = 14……. (ii)

y – 2x = 1……… (iii)

For equation (i),

⇒ y = (x + 8)/2

When x = -4, we have y = (-4 + 8)/2 = 2

When x = 0, we have y = (0 + 8)/2 = 4

Thus we have the following table giving points on the line **2y – x = 8**

x	-4	0
y	2	4

For equation (ii),

We solve for y:

⇒ y = (x + 14)/5

So, when x = -4

$y = ((-4) + 14)/5 = 2$

And, when $x = 1$

$\Rightarrow y = (1 + 14)/5 = 3$

Thus we have the following table giving points on the line **5y – x = 14**

x	-4	1
y	2	3

Finally, for equation (iii),

$\Rightarrow y = (2x + 1)$

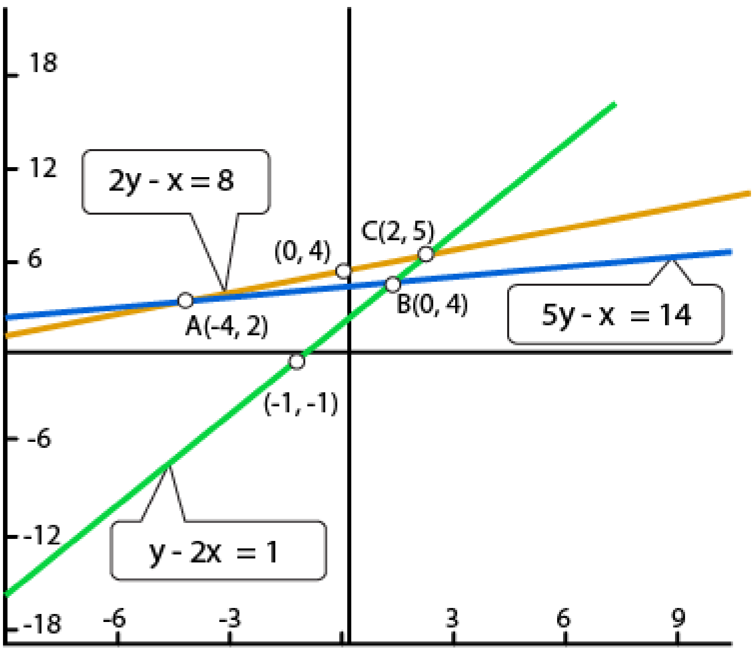
When $x = -1$, we have $y = (2(-1) + 1) = -1$

When $x = 1$, we have $y = (2(1) + 1) = 3$

Thus we have the following table giving points on the line **y – 2x = 1**

x	-1	1
y	1	3

Graph of the equations (i), (ii) and (iii) is as below:



From the above graph, we observe that the lines taken in pairs intersect at points A(-4,2), B(1,3) and C(2,5)

Hence the vertices of the triangle are A(-4, 2), B(1, 3) and C(2,5)

(ii) y = x, y = 0 and 3x + 3y = 10

Solution:

Given,

y = x ……. (i)

y = 0 ……. (ii)

3x + 3y = 10……… (iii)

For equation (i),

When x = 1, we have y = 1

When x = -2, we have y = -2

Thus we have the following table giving points on the line **y = x**

x	1	-2
y	1	-2

For equation (ii),

When x = 0

y = 0

And, when x = 10/3

⇒ y = 0

Thus we have the following table giving points on the line **y = 0**

x	0	10/3
y	0	10/3

Finally, for equation (iii),

⇒ y = (10 – 3x)/3

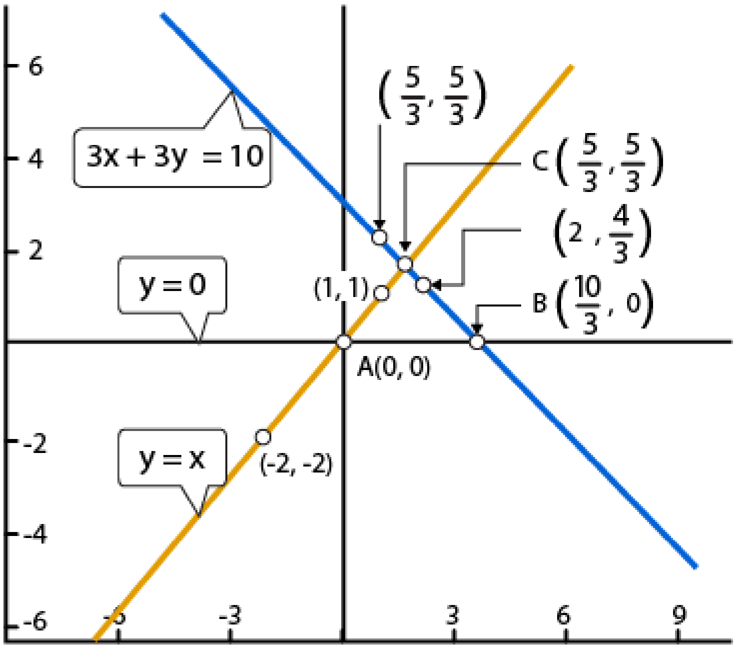
When x = 1, we have y = (10 – 3(1))/3) = 7/3

When x = 2, we have y = (10 – 3(2))/3 = 4/3

Thus we have the following table giving points on the line **3x + 3y = 10**

x	1	2
y	7/3	4/3

Graph of the equations (i), (ii) and (iii) is as below:



From the above graph, we observe that the lines taken in pairs intersect at points A(0,0) B(10/3,0) and C(5/3, 5/3)

Hence the vertices of the triangle are A(0,0) B(10/3,0) and C(5/3, 5/3).

20. Determine graphically whether the system of equations x – 2y = 2, 4x – 2y = 5 is consistent or in-consistent.

Solution:

Given,

x – 2y = 2……. (i)

4x – 2y = 5……. (ii)

For equation (i),

⇒ y = (x – 2)/2

When x = 2, we have y = (2 – 2)/2 = 0

When x = 0, we have y = (0 – 2)/2 = -1

Thus we have the following table giving points on the line **x – 2y = 2**

x	2	0
y	0	-1

For equation (ii),

We solve for x:

⇒ x = (5 + 2y)/4

So, when y = 0

x = (5 + 2(0))/4 = 5/4

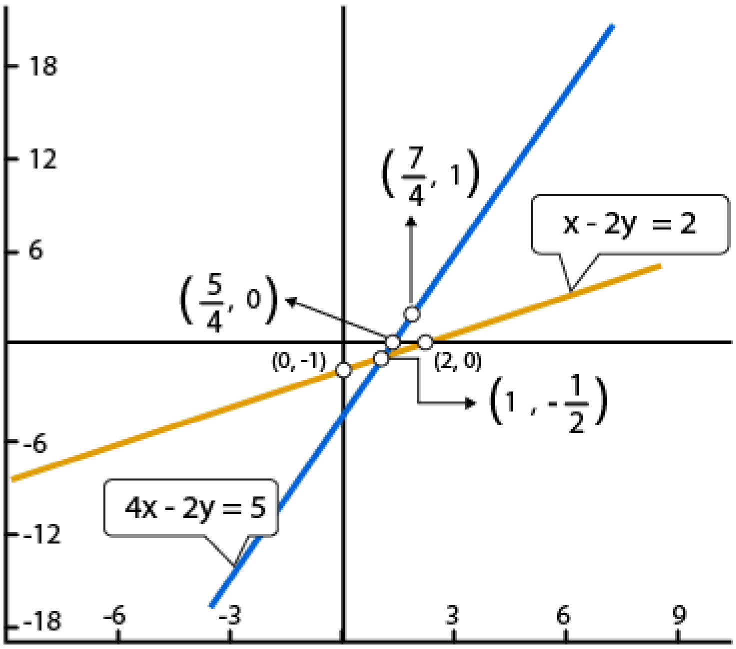
And, when y = 1.5

⇒ x = (5 + 2(1))/4 = 7/4

Thus we have the following table giving points on the line **4x – 2y = 5**

x	5/4	7/4
y	0	1

Graph of the equations (i) and (ii) is as below:



It is clearly seen that the two lines intersect at (1,0)

Hence, the system of equations is consistent.

21. Determine by drawing graphs, whether the following system of linear equation has a unique solution or not:

(i) 2x – 3y = 6 and x + y = 1

Solution:

Given,

2x – 3y = 6 ……. (i)

x + y = 1……. (ii)

For equation (i),

⇒ y = (2x – 6)/3

When x = 3, we have y = (2(3) – 6)/3= 0

When x = 0, we have y = (2(0) – 6)/3= -2

Thus we have the following table giving points on the line **2x – 3y = 6**

x	3	0
y	0	-2

For equation (ii),

We solve for y:

⇒ y = (1 – x)

So, when x = 0

y = (1 – 0) = 1

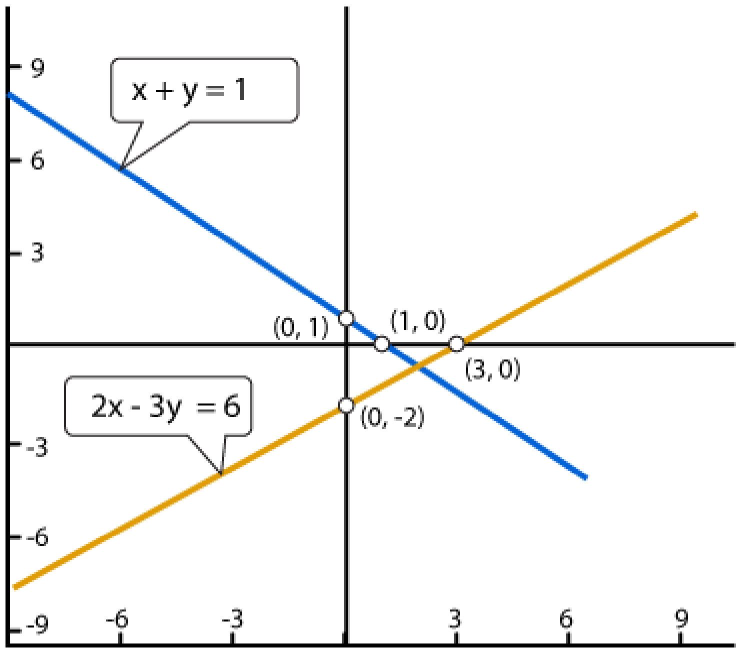
And, when x = 1

⇒ y = (1 – 1) = 0

Thus we have the following table giving points on the line **x + y = 1**

x	0	1
y	1	0

Graph of the equations (i) and (ii) is as below:



It's seen clearly that the two lines intersect at one.

Thus, we can conclude that the system of equations has a unique solution.

(ii) 2y = 4x – 6 and 2x = y + 3

Solution:

Given,

2y = 4x – 6……. (i)

$2x = y + 3$……. (ii)

For equation (i),

$\Rightarrow y = (4x – 6)/2$

When $x = 1$, we have $y = (4(1) – 6)/2 = -1$

When $x = 4$, we have $y = (4(4) – 6)/2= 5$

Thus we have the following table giving points on the line **$2y = 4x – 6$**

x	1	4
y	-1	5

For equation (ii),

We solve for y:

$\Rightarrow y = 2x – 3$

So, when $x = 2$

$y = 2(2) – 3 = 1$

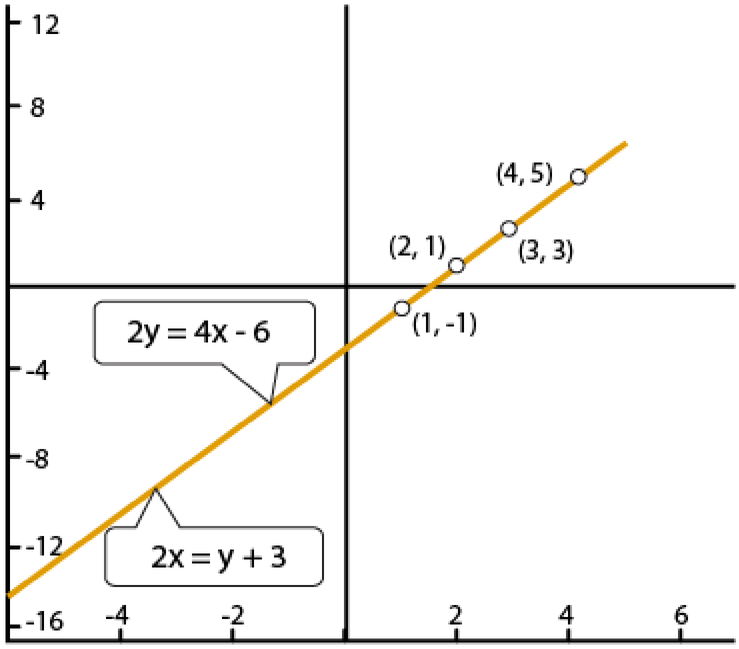
And, when $x = 3$

$\Rightarrow y = 2(3) – 3 = 3$

Thus we have the following table giving points on the line **$2x = y + 3$**

x	2	3
y	1	3

Graph of the equations (i) and (ii) is as below:



We see that the two lines are coincident. And, hence it has infinitely many solutions.

Therefore, the system of equations does not have a unique solution.

Exercise 3.3 Page No: 3.44

Solve the following system of equations:

1. $11x + 15y + 23 = 0$

$7x – 2y – 20 = 0$

Solution:

The given pair of equations are:

$11x + 15y + 23 = 0$ ………………………… (i)

$7x – 2y – 20 = 0$ …………………………….. (ii)

From (ii)

$2y = 7x – 20$

$\Rightarrow y = (7x - 20)/2$ ……………………………… (iii)

Now, substituting y in equation (i) we get,

$\Rightarrow 11x + 15((7x-20)/2) + 23 = 0$

$\Rightarrow 11x + (105x - 300)/2 + 23 = 0$

$\Rightarrow (22x + 105x – 300 + 46) = 0$

$\Rightarrow 127x – 254 = 0$

$\Rightarrow x = 2$

Next, putting the value of x in the equation (iii) we get,

$\Rightarrow y = (7(2) − 20)/2$

$\therefore y= -3$

Thus, the value of x and y is found to be 2 and -3 respectively.

2. $3x – 7y + 10 = 0$

$y – 2x – 3 = 0$

Solution:

The given pair of equations are:

$3x – 7y + 10 = 0$ …………………………. (i)

$y – 2x – 3 = 0$ ……………………………….. (ii)

From (ii)

$y – 2x – 3 = 0$

$y = 2x+3$ ……………………………… (iii)

Now, substituting y in equation (i) we get,

$\Rightarrow 3x – 7(2x+3) + 10 = 0$

$\Rightarrow 3x – 14x – 21 + 10 = 0$

$\Rightarrow -11x = 11$

$\Rightarrow x = -1$

Next, putting the value of x in the equation (iii) we get,

$\Rightarrow y = 2(-1) + 3$

$\therefore y = 1$

Thus, the value of x and y is found to be -1 and 1 respectively.

3. 0.4x + 0.3y = 1.7

0.7x – 0.2y = 0.8

Solution:

The given pair of equations are:

$0.4x + 0.3y = 1.7$

$0.7x – 0.2y = 0.8$

Let's, multiply LHS and RHS by 10 to make the coefficients as an integer

$4x + 3y = 17$ ……………………….. (i)

$7x – 2y = 8$ …………………………… (ii)

From (ii)

$7x – 2y = 8$

$x = (8 + 2y)/7$……………………………… (iii)

Now, substituting x in equation (i) we get,

$\Rightarrow 4[(8 + 2y)/7] + 3y = 17$

$\Rightarrow 32 + 8y + 21y = (17 \times 7)$

$\Rightarrow 29y = 87$

$\Rightarrow y = 3$

Next, putting the value of y in the equation (iii) we get,

$\Rightarrow x = (8 + 2(3))/ 7$

$\Rightarrow x = 14/7$

$\therefore x = 2$

Thus, the value of x and y is found to be 2 and 3 respectively.

4. $x/2 + y = 0.8$

$7/(x+ y/2) = 10$

Solution:

The given pair of equations are:

$x/2 + y = 0.8 \Rightarrow x + 2y = 1.6$…… (a)

$7/(x + y/2) = 10 \Rightarrow 7 = 10(x + y/2) \Rightarrow 7 = 10x + 5y$

Let's, multiply LHS and RHS of equation (a) by 10 for easy calculation

So, we finally get

$10x + 20y = 16$ ……………………….. (i) And,

$10x + 5y = 7$ …………………………… (ii)

Now, subtracting two equations we get,

$\Rightarrow$ (i) – (ii)

$15y = 9$

$\Rightarrow y = 3/5$

Next, putting the value of y in the equation (i) we get,

$x = [16 - 20(3/5)]/10$

$\Rightarrow (16 - 12)/10 = 4/10$

∴ $x = 2/5$

Thus, the value of x and y obtained are 2/5 and 3/5 respectively.

5. 7(y + 3) – 2(x + 2) = 14

4(y – 2) + 3(x – 3) = 2

Solution:

The given pair of equations are:

$7(y+3) - 2(x+2) = 14$………………………… (i)

$4(y-2) + 3(x-3) = 2$……………………………….. (ii)

From (i), we get

$7y + 21 - 2x - 4 = 14$

$7y = 14 + 4 - 21 + 2x$

⇒ $y = (2x - 3)/7$

From (ii), we get

$4y - 8 + 3x - 9 = 2$

$4y + 3x - 17 - 2 = 0$

⇒ $4y + 3x - 19 = 0$ …………….. (iii)

Now, substituting y in equation (iii)

$4[(2x - 3)/7] + 3x - 19=0$

$8x - 12 + 21x - (19 \times 17) = 0$ [after taking LCM]

$29x = 145$

⇒ $x = 5$

Now, putting the value of x and in the equation (ii)

$4(y-2) + 3(5-3) = 2$

⇒ $4y -8 + 6 = 2$

⇒ $4y = 4$

∴ $y = 1$

Thus, the value of x and y obtained are 5 and 1 respectively.

6. $x/7 + y/3 = 5$

$x/2 – y/9 = 6$

Solution:

The given pair of equations are:

$x/7 + y/3 = 5$…………………………. (i)

$x/2 – y/9 = 6$………………………………..(ii)

From (i), we get

$x/7 + y/3 = 5$

$\Rightarrow 3x + 7y = (5\times21)$ [After taking LCM]

$\Rightarrow 3x = 105 – 7y$

$\Rightarrow x = (105 – 7y)/3$……. (iv)

From (ii), we get

$x/2 – y/9 = 6$

$\Rightarrow 9x – 2y = 108$ ……………………… (iii) [After taking LCM]

Now, substituting x in equation (iii) we get,

$9[(105 − 7y)/3] – 2y = 108$

$\Rightarrow 945 – 63y – 6y = 324$ [After taking LCM]

$\Rightarrow 945 – 324 = 69y$

$\Rightarrow 69y = 621$

⇒ $y = 9$

Now, putting the value of y in the equation (iv)

$x = (105 - 7(9))/3$

⇒ $x = (105 - 63)/3 = 42/3$

∴ $x = 14$

Thus, the value of x and y obtained are 14 and 9 respectively.

7. x/3 + y/4 = 11

5x/6 − y/3 = −7

Solution:

The given pair of equations are:

$x/3 + y/4 = 11$…………………………. (i)

$5x/6 - y/3 = -7$……………………………….. (ii)

From (i), we get

$x/3 + y/4 = 11$

⇒$4x + 3y = (11\times12)$ [After taking LCM]

⇒ $4x = 132 - 3y$

⇒ $x = (132 - 3y)/4$……. (iv)

From (ii), we get

$5x/6 - y/3 = -7$

⇒ $5x - 2y = -42$ ……………………… (iii) [After taking LCM]

Now, substituting x in equation (iii) we get,

$5[(132 - 3y)/4] - 2y = -42$

⇒ $660 - 15y - 8y = -42 \times 4$ [After taking LCM]

⇒ 660 + 168 = 23y

⇒ 23y = 828

⇒ y = 36

Now, putting the value of y in the equation (iv)

x = (132 – 3(36))/4

⇒ x = (132 − 108)/4 = 24/4

∴ x = 6

Thus, the value of x and y obtained are 6 and 36 respectively.

8. 4/x + 3y = 8

6/x −4y = −5

Solution:

Taking 1/x = u

Then the two equation becomes,

4u + 3y = 8…………………… (i)

6u – 4y = -5……………………. (ii)

From (i), we get

4u = 8 – 3y

⇒ u = (8 − 3y)/4 …….. (iii)

Substituting u in (ii)

[6(8 − 3y)/4] – 4y = -5

⇒ [3(8−3y)/2] − 4y = −5

⇒ 24 − 9y −8y = −5 x 2 [After taking LCM]

$\Rightarrow 24 - 17y = -10$

$\Rightarrow -17y = -34$

$\Rightarrow y = 2$

Putting y=2 in (iii) we get,

$u = (8 - 3(2))/4$

$\Rightarrow u = (8 - 6)/4$

$\Rightarrow u = 2/4 = 1/2$

$\Rightarrow x = 1/u = 2$

$\therefore x = 2$

So, the solution of the pair of equations given is x=2 and y =2.

9. x + y/2 = 4

2y + x/3 = 5

Solution:

The given pair of equations are:

$x + y/2 = 4$ ……………………. (i)

$2y + x/3 = 5$……………………. (ii)

From (i) we get,

$x + y/2 = 4$

$\Rightarrow 2x + y = 8$ [After taking LCM]

$\Rightarrow y = 8 - 2x$ …..(iv)

From (ii) we get,

$x + 6y = 15$ ……………… (iii) [After taking LCM]

Substituting y in (iii), we get

$x + 6(8 - 2x) = 15$

$\Rightarrow x + 48 - 12x = 15$

$\Rightarrow -11x = 15 - 48$

$\Rightarrow -11x = -33$

$\Rightarrow x = 3$

Putting $x = 3$ in (iv), we get

$y = 8 - (2\times3)$

$\therefore y = 8 - 6 = 2$

Hence, the solution of the given system of equation are $x = 3$ and $y = 2$ respectively.

10. $x + 2y = 3/2$

$2x + y = 3/2$

Solution:

The given pair of equations are:

$x + 2y = 3/2$ …………………. (i)

$2x + y = 3/2$…………………… (ii)

Let us eliminate y from the given equations. The coefficients of y in the given equations are 2 and 1 respectively. The L.C.M of 2 and 1 is 2. So, we make the coefficient of y equal to 2 in the two equations.

Multiplying equation (i)x1 and (ii)x2 $\Rightarrow$

$x + 2y = 3/2$ ……………………… (iii)

$4x + 2y = 3$ ……………………………………………………. (iv)

Subtracting equation (iii) from (iv)

$(4x - x) + (2y-2y) = 3x = 3 - (3/2)$

$\Rightarrow 3x = 3/2$

$\Rightarrow x = 1/2$

Putting $x = 1/2$ in equation (iv)

$4(1/2) + 2y = 3$

$\Rightarrow 2 + 2y = 3$

$\therefore y = 1/2$

The solution of the system of equation is $x = 1/2$ and $y = 1/2$

11. $\sqrt{2}x - \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

Solution:

The given pair of equations are:

$\sqrt{2}x - \sqrt{3}y = 0$………………………. (i)

$\sqrt{3}x - \sqrt{8}y = 0$………………………. (ii)

From equation (i)

$x = \sqrt{(3/2)}y$ …………….(iii)

Substituting this value in equation (ii) we obtain

$\sqrt{3}x - \sqrt{8}y = 0$

$\Rightarrow \sqrt{3}(\sqrt{(3/2)}y) - \sqrt{8}y = 0$

$\Rightarrow (3/\sqrt{2})y - \sqrt{8}y = 0$

$\Rightarrow 3y - 4y = 0$

$\Rightarrow y = 0$

Now, substituting y in equation (iii) we obtain

⇒ $x=0$

Thus, the value of x and y obtained are 0 and 0 respectively.

12. 3x – (y + 7)/11 + 2 = 10

2y + (x + 11)/7 = 10

Solution:

The given pair of equations are:

$3x – (y + 7)/11 + 2 = 10$……………….. (i)

$2y + (x + 11)/7 = 10$…………………….. (ii)

From equation (i)

$33x – y – 7 + 22 = (10 \times 11)$ [After taking LCM]

⇒ $33x – y + 15 = 110$

⇒ $33x + 15 – 110 = y$

⇒ $y = 33x – 95$………. (iv)

From equation (ii)

$14 + x + 11 = (10 \times 7)$ [After taking LCM]

⇒ $14y + x + 11 = 70$

⇒ $14y + x = 70 – 11$

⇒ $14y + x = 59$ …………………….. (iii)

Substituting (iv) in (iii) we get,

$14 (33x – 95) + x = 59$

⇒ $462x – 1330 + x = 59$

$\Rightarrow 463x = 1389$

$\Rightarrow x = 3$

Putting $x = 3$ in (iii) we get,

$\Rightarrow y = 33(3) - 95$

$\therefore y = 4$

The solution for the given pair of equations is $x = 3$ and $y = 4$ respectively.

13. $2x - (3/y) = 9$

$3x + (7/y) = 2$,$y \neq 0$

Solution:

The given pair of equations are:

$2x - (3/y) = 9$……………………………. (i)

$3x + (7/y) = 2$…………………………… (ii)

Substituting $1/y = u$ the above equations becomes,

$2x - 3u = 9$ ………………………..(iii)

$3x + 7u = 2$………………………..(iv)

From (iii)

$2x = 9 + 3u$

$\Rightarrow x = (9+3u)/2$

Substituting the value of x from above in the equation (iv) we get,

$3[(9+3u)/2] + 7u = 2$

$\Rightarrow 27 + 9u + 14u = (2 \times 2)$

$\Rightarrow 27 + 23u = 4$

$\Rightarrow 23u = -23$

$\Rightarrow u = -1$

So, $y = 1/u = -1$

And putting $u = -1$ in $x = (9 + 3u)/2$ we get,

$\Rightarrow x = [9 + 3(-1)]/2 = 6/2$

$\therefore x = 3$

The solution of the pair of equations given are $y = 3$ and $x = -1$ respectively.

14. 0.5x + 0.7y = 0.74

0.3x + 0.5y = 0.5

Solution:

The given pair of equations are:

$0.5x + 0.7y = 0.74$………………………. (i)

$0.3x – 0.5y = 0.5$ ………………………….. (ii)

Now, let's multiply LHS and RHS by 100 for both (i) and (ii) for making integral coefficients and constants.

(i) x100 $\Rightarrow$

$50x +70y = 74$ ……………………….. (iii)

(ii) x100 $\Rightarrow$

$30x + 50y = 50$ …………………………… (iv)

From (iii)

$50x = 74 – 70y$

$x = (74-70y)/ 50$ ……………………………… (v)

Now, substituting x in equation (iv) we get,

$30[(74-70y)/ 50] + 50y = 50$

$\Rightarrow 222 – 210y + 250y = 250$ [After taking LCM]

$\Rightarrow 40y = 28$

⇒ $y = 0.7$

Now, by putting the value of y in the equation (v), we get

⇒ $x = [74 - 70(0.7)]/ 50=0$

⇒ $x =25/ 50 = 1/2$

∴ $x = 0.5$

Thus, the value of x and y so obtained are 0.5 and 0.7 respectively.

15. 1/(7x) + 1/(6y) = 3

1/(2x) – 1/(3y) = 5

Solution:

The given pair of equations are:

$1/(7x) + 1/(6y) = 3$………………………….. (i)

$1/(2x) - 1/(3y) = 5$……………………………. (ii)

Multiplying (ii) by 1/2 we get,

$1/(4x) - 1/(6y) = 52$……………………………. (iii)

Now, solving equations (i) and (iii)

$1/(7x) + 1/(6y) = 3$………………………….. (i)

$1/(4x) - 1/(6y) = 5/2$……………………………. (iii)

Adding (i) + (iii) we get,

$1/x(1/7 + 1/4) = 3 + 5/2$

⇒ $1/x(11/28) = 11/2$

⇒ $x = 1/14$

Using $x =1/ 14$ we get, in (i)

$1/[7(1/14)] + 1/(6y) = 3$

⇒ $2 + 1/(6y)=3$

⇒ $1/(6y) = 1$

⇒ $y = 1/6$

The solution for the given pair of equations is $x=1/14$ and $y=1/6$ respectively.

16. 1/(2x) + 1/(3y) = 2

1/(3x) + 1/(2y) = 13/6

Solution:

Let $1/x = u$ and $1/y = v$,

So the given equations becomes,

$u/2 + v/3 = 2$ ………………(i)

$u/3 + v/2 = 13/6$ ……………(ii)

From (i), we get

$u/2 + v/3 = 2$

⇒ $3u + 2v = 12$

⇒ $u = (12 – 2v)/3$ …………(iii)

Using (iii) in (ii)

$[(12 – 2v)/3]/3 + v/2 = 13/6$

⇒ $(12 – 2v)/9 + v/2 = 13/6$

⇒ $24 – 4v + 9v = (13/6) \times 18$ [after taking LCM]

⇒ $24 + 5v = 39$

⇒ $5v = 15$

⇒ $v = 3$

Substituting v in (iii)

u = (12 – 2(3))/3

⇒ u = 2

Thus, x = 1/u ⇒ x = 1/2 and

y = 1/v ⇒ y = 1/3

The solution for the given pair of equations is x = 1/2 and y = 1/3 respectively.

17. 15/u + 2/v = 17

1/u + 1/v = 36/5

Solution:

Let 1/x = u and 1/y = v

So, the given equations becomes

15x + 2y = 17 ………………………….. (i)

x + y = 36/5………………………. (ii)

From equation (i) we get,

2y = 17 – 15x

=y = (17 − 15x)/ 2 …………………. (iii)

Substituting (iii) in equation (ii) we get,

= x + (17 − 15x)/ 2 = 36/5

2x + 17 – 15x = (36 x 2)/ 5 [after taking LCM]

-13x = 72/5 – 17

= -13x = -13/5

⇒ x = 1/5

⇒ u = 1/x = 5

Putting x = 1/5in equation (ii) , we get

1/5 + y = 36/5

⇒ y = 7

⇒ v = 1/y = 1/7

The solution of the pair of equations given are u = 5 and v = 1/7 respectively.

18. 3/x – 1/y = −9

2/x + 3/y = 5

Solution:

Let 1/x = u and 1/y = v

So, the given equations becomes

3u – v = -9…………………..(i)

2u + 3v = 5 ……………………….(ii)

Multiplying equation (i) x 3 and (ii) x 1 we get,

9u – 3v = -27 ………………………….. (iii)

2u + 3v = 5 ……………………………… (iv)

Adding equation (iii) and (iv) we get ,

9u + 2u – 3v + 3v = -27 + 5

⇒ 11u = -22

⇒ u = -2

Now putting u =-2 in equation (iv) we get,

2(-2) + 3v = 5

⇒ 3v = 9

⇒ v = 3

Hence, x = 1/u = −1/2 and,

y = 1/v = 1/3

19. 2/x + 5/y = 1

60/x + 40/y = 19

Solution:

Let 1/x = u and 1/y = v

So, the given equations becomes

2u + 5v = 1…………………..(i)

60u + 40v = 19 ……………………….(ii)

Multiplying equation (i) x 8 and (ii) x 1 we get,

$16u + 40v = 8$ ………………………….. (iii)

$60u + 40v = 19$ …………………………… (iv)

Subtracting equation (iii) from (iv) we get,

$60u – 16u + 40v – 40v = 19 – 8$

$\Rightarrow 44u = 11$

$\Rightarrow u = 1/4$

Now putting $u = 1/4$ in equation (iv) we get,

$60(1/4) + 40v = 19$

$\Rightarrow 15 + 40v = 19$

$\Rightarrow v = 4/ 40 = 1/10$

Hence, $x = 1/u = 4$ and,

$y = 1/v = 10$

20. $1/(5x) + 1/(6y) = 12$

$1/(3x) – 3/(7y) = 8$

Solution:

Let $1/x = u$ and $1/y = v$

So, the given equations becomes

$u/5 + v/6 = 12$…………………..(i)

$u/3 – 3v/7 = 8$……………………….(ii)

Taking LCM for both equations, we get

$6u + 5v = 360$………. (iii)

$7u – 9v = 168$……….. (iv)

Subtracting (iii) from (iv)

$7u – 9v – (6u + 5v) = 168 – 360$

$\Rightarrow u – 14v = -192$

$\Rightarrow u = (14v – 192)$………. (v)

Using (v) in equation (iii), we get

$6(14v – 192) + 5v = 360$

$\Rightarrow 84v - 1152 + 5v = 360$

$\Rightarrow 89v = 1512$

$\Rightarrow v = 1512/89$

$\Rightarrow y = 1/v = 89/1512$

Now, substituting v in equation (v), we find u

$u = 14 \times (1512/89) – 192$

$\Rightarrow u = 4080/89$

$\Rightarrow x = 1/u = 89/ 4080$

Hence, the solution for the given system of equations is $x = 89/4080$ and $y = 89/ 1512$

21. $4/x + 3y = 14$

$3/x – 4y = 23$

Solution:

Taking $1/x = u$, the given equation becomes

$4u + 3y = 14$…………………….. (i)

$3u – 4y = 23$…………………….. (ii)

Adding (i) and (ii), we get

$4u + 3y + 3u – 4y = 14 + 23$

$\Rightarrow 7u – y = 37$

$\Rightarrow y = 7u – 37$……………………… (iii)

Using (iii) in (i),

$4u + 3(7u – 37) = 14$

⇒ $4u + 21u – 111 = 14$

⇒ $25u = 125$

⇒ $u = 5$

⇒ $x = 1/u = 1/5$

Putting u= 5 in (iii), we find y

$y = 7(5) – 37$

⇒ $y = -2$

Hence, the solution for the given system of equations is $x = 1/5$ and $y = -2$

22. 4/x + 5y = 7

3/x + 4y = 5

Solution:

Taking $1/x = u$, the given equation becomes

$4u + 5y = 7$…………………….. (i)

$3u + 4y = 5$…………………….. (ii)

Subtracting (ii) from (i), we get

$4u + 5y – (3u + 4y) = 7 – 5$

⇒ $u + y = 2$

⇒ $u = 2 – y$……………………… (iii)

Using (iii) in (i),

$4(2 – y) + 5y = 7$

⇒ $8 – 4y + 5y = 7$

⇒ y = -1

Putting y = -1 in (iii), we find u

u = 2 – (-1)

⇒ u = 3

⇒ x = 1/u = 1/3

Hence, the solution for the given system of equations is x = 1/3 and y = -1

23. 2/x + 3/y = 13

5/x – 4/y = -2

Solution:

Let 1/x = u and 1/y = v

So, the given equations becomes

2u + 3v = 13………………….. (i)

5u – 4v = -2 ……………………… (ii)

Adding equation (i) and (ii) we get,

2u + 3v + 5u – 4v = 13 – 2

⇒ 7u – v = 11

⇒ v = 7u – 11…….. (iii)

Using (iii) in (i), we get

2u + 3(7u – 11) = 13

⇒ 2u + 21u – 33 = 13

⇒ 23u = 46

⇒ u = 2

Substituting u = 2 in (iii), we find v

$v = 7(2) – 11$

$\Rightarrow v = 3$

Hence, $x = 1/u = 1/2$ and,

$y = 1/v = 1/3$

24. 2/x + 3/y = 2

4/x – 9/y = -1

Solution:

Let $1/\sqrt{x} = u$ and $1/\sqrt{y} = v$,

So, the given equations becomes

$2u + 3v = 2$………………….. (i)

$4u – 9v = -1$ ……………………… (ii)

Multiplying (ii) by 3 and

Adding equation (i) and (ii)x3 we get,

$6u + 9v + 4u – 9v = 6 – 1$

$\Rightarrow 10u = 5$

$\Rightarrow u = 1/2$

Substituting $u = 1/2$ in (i), we find v

$2(1/2) + 3v = 2$

$\Rightarrow 3v = 2 – 1$

$\Rightarrow v = 1/3$

Since, $1/\sqrt{x} = u$ we get $x = 1/u^2$

$\Rightarrow x = 1/(1/2)^2 = 4$

And,

$1/\sqrt{y} = v$ $y = 1/v^2$

$\Rightarrow y = 1/(1/3)^2 = 9$

Hence, the solution is $x = 4$ and $y = 9$.

25. $(x + y)/xy = 2$

$(x – y)/xy = 6$

Solution:

The given pair of equations are:

$(x + y)/xy = 2 \Rightarrow 1/y + 1/x = 2$……. (i)

$(x – y)/xy = 6 \Rightarrow 1/y – 1/x = 6$………(ii)

Let $1/x = u$ and $1/y = v$, so the equation (i) and (ii) becomes

$v + u = 2$……. (iii)

$v – u = 6$……..(iv)

Adding (iii) and (iv), we get

$2v = 8$

$\Rightarrow v = 4$

$\Rightarrow y = 1/v = 1/4$

Substituting $v = 4$ in (iii) to find x,

$4 + u = 2$

$\Rightarrow u = -2$

$\Rightarrow x = 1/u = -1/2$

Hence, the solution is $x = -1/2$ and $y = 1/4$.

26. $2/x + 3/y = 9/xy$

$4/x + 9/y = 21/xy$

Solution:

Taking LCM for both the given equations, we have

$(2y + 3x)/ xy = 9/xy \Rightarrow 3x + 2y = 9$………. (i)

(4y + 9x)/ xy = 21/xy ⇒ 9x + 4y = 21………(ii)

Performing (ii) – (i)x2⇒

9x + 4y – 2(3x + 2y) = 21 – (9×2)

⇒ 3x = 3

⇒ x = 1

Using x = 1 in (i), we find y

3(1) + 2y = 9

⇒ y = 6/2

⇒ y = 3

Thus, the solution for the given set of equations is x = 1 and y = 3.

Exercise 3.4 Page No: 3.57

Solve each of the following systems of equations by the method of cross-multiplication:

1. x + 2y + 1 = 0

2x – 3y – 12 = 0

Solution:

The given system of equations is

x + 2y + 1 = 0

2x – 3y – 12 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 1$, $b_1 = 2$, $c_1 = 1$

$a_2 = 2$, $b_2 = -3$, $c_2 = -12$

By cross multiplication method,

$$\frac{x}{-24+3} = \frac{-y}{-12-2} = \frac{1}{-3-4}$$

$$\frac{x}{-21} = \frac{-y}{-14} = \frac{1}{-7}$$

Now,

$$\frac{x}{-21} = \frac{1}{-7}$$

=x= 3

And,

$$\frac{-y}{-14} = \frac{1}{-7}$$

=y=-2

Hence, the solution for the given system of equations is x = 3 and y = -2.

2. 3x + 2y + 25 = 0

2x + y + 10 = 0

Solution:

The given system of equations is

3x + 2y + 25 = 0

2x + y + 10 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 3$, $b_1 = 2$, $c_1 = 25$

$a_2 = 2$, $b_2 = 1$, $c_2 = 10$

By cross multiplication method,

$$\frac{x}{20-25} = \frac{-y}{30-50} = \frac{1}{3-4}$$

$$\frac{x}{-5} = \frac{-y}{-20} = \frac{1}{-1}$$

Now,

$$\frac{x}{-5} = \frac{1}{-1}$$

=x= 5

And,

$$\frac{-y}{-20} = \frac{1}{-1}$$

=y=-20

Hence, the solution for the given system of equations is x = 5 and y = -20.

3. 2x + y = 35, 3x + 4y = 65

Solution:

The given system of equations can be written as

2x + y – 35 = 0

3x + 4y – 65 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 2$, $b_1 = 1$, $c_1 = -35$

$a_2 = 3$, $b_2 = 4$, $c_2 = -65$

By cross multiplication method,

$$\frac{x}{-65+140} = \frac{-y}{-130+105} = \frac{1}{8-3}$$

$$\frac{x}{75} = \frac{-y}{-25} = \frac{1}{5}$$

Now,

$$\frac{x}{75} = \frac{1}{5}$$

=x= 15

And,

$$\frac{-y}{-25} = \frac{1}{5}$$

=y=5

Hence, the solution for the given system of equations is x = 15 and y = 5.

4. 2x – y = 6, x – y = 2

Solution:

The given system of equations can be written as

2x – y – 6 = 0

x – y – 2 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 2$, $b_1 = -1$, $c_1 = -6$

$a_2 = 1$, $b_2 = -1$, $c_2 = -2$

By cross multiplication method,

$$\frac{x}{2-6} = \frac{-y}{-4+6} = \frac{1}{-2+1}$$

$$\frac{x}{-4} = \frac{-y}{2} = \frac{1}{-1}$$

Now,

$$\frac{x}{-4} = \frac{1}{-1}$$

=x= 4

And,

$$\frac{-y}{2} = \frac{1}{-1}$$

=y=2

Hence, the solution for the given system of equations is x = 4 and y = 2.

5. (x + y)/ xy = 2

(x – y)/ xy = 6

Solution:

The given system of equations is

(x + y)/ xy = 2 ⇒ 1/y + 1/x = 2…….. (i)

(x – y)/ xy = 6 ⇒ 1/y – 1/x = 6……… (ii)

Let 1/x = u and 1/y = v, so the equation becomes

u + y = 2….. (iii)

u – y = 6……(iv)

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations (iii) & (iv) with the general form, we get

$a_1 = 1$, $b_1 = 1$, $c_1 = -2$

$a_2 = 1$, $b_2 = -1$, $c_2 = -6$

By cross multiplication method,

$$\frac{u}{6-2} = \frac{-v}{6+2} = \frac{1}{-1-1}$$

$$\frac{u}{4} = \frac{-v}{8} = \frac{1}{-2}$$

Now,

$$\frac{u}{4} = \frac{1}{-2}$$

$= u = -2$

And,

$$\frac{-v}{-8} = \frac{1}{-2}$$

$= v = 4$

$\frac{1}{u} = x = \frac{-1}{2}$

$\frac{1}{v} = y = \frac{1}{4}$

Hence, the solution for the given system of equations is x = -1/2 and y = 1/4.

6. ax + by = a-b

bx – ay = a+b

Solution:

The given system of equations can be written as

ax + by – (a-b) = 0

bx – ay – (a+b) = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = a$, $b_1 = b$, $c_1 = -(a-b)$

$a_2 = b$, $b_2 = -a$, $c_2 = -(a+b)$

By cross multiplication method,

$$\frac{x}{-ab-b^2+ab-a^2} = \frac{-y}{-a^2-ab-b^2+ab} = \frac{1}{-a^2-b^2}$$

$$\frac{x}{-b^2-a^2} = \frac{-y}{-a^2-b^2} = \frac{1}{-a^2-b^2}$$

Now,

$$\frac{x}{-ab-b^2+ab-a^2} = \frac{1}{-a^2-b^2}$$

$=x= 1$

And,

$$\frac{-y}{-a^2-ab-b^2+ab} = \frac{1}{-a^2-b^2}$$

$=y=-1$

Hence, the solution for the given system of equations is x = 1 and y = -1.

7. x + ay = b

ax + by = c

Solution:

The given system of equations can be written as

x + ay – b = 0

ax + by – c = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 1$, $b_1 = a$, $c_1 = -b$

$a_2 = a$, $b_2 = -b$, $c_2 = -c$

By cross multiplication method,

$$\frac{x}{-ac-b^2} = \frac{-y}{-c+ab} = \frac{1}{-a^2-b}$$

Now,

$$\frac{x}{-ac-b^2} = \frac{1}{-a^2-b}$$

$$=x= \frac{b^2+ac}{a^2+b}$$

And,

$$\frac{-y}{-c+ab} = \frac{1}{-a^2-b}$$

$$=y= \frac{-c+ab}{a^2+b}$$

Hence, the solution for the given system of equations is $x = (b^2 + ac)/(a^2 + b^2)$ and $y = (-c^2 + ab)/(a^2 + b^2)$.

8. $ax + by = a^2$

$bx + ay = b^2$

Solution:

The given system of equations can be written as

$ax + by - (a^2) = 0$

$bx + ay - (b^2) = 0$

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

$$\Rightarrow \frac{x}{b_1c_2-b_2c_1} = \frac{-y}{a_1c_2-a_2c_1} = \frac{1}{a_1b_2-a_2b_1}$$

Here, According to the question,

$a_1= a$, $b_1= b$, $c_1= a^2$

$a_2= b$, $b_2= a$, $c_2= b^2$

By cross multiplication method,

$$\frac{x}{-b^2+a^2} = \frac{-y}{-ab^2-a^2b} = \frac{1}{a^2-b^2}$$

Now,

$$\frac{x}{-b^2+a^2} = \frac{1}{a^2-b^2}$$

$$=x= \frac{a^2+ab+b^2}{a+b}$$

And,

$$\frac{-y}{-ab^2-a^2b} = \frac{1}{a^2-b^2}$$

$$=y=-\frac{-ab(a-b)}{(a-b)(a+b)}$$

Hence, the solution for the given system of equations is $x = (a^2 + ab + b^2)/(a + b)$ and $y = -ab / (a+ b)$.

9. 5/(x + y) – 2/(x -y) = -1

15/(x + y) + 7/(x – y) = 10

Solution:

Let's substitute 1/(x + y) = u and 1/(x – y) = v, so the given equations becomes

5u – 2v = -1

15u + 7v = 10

For cross multiplication we use,

$$\frac{x}{b_1c_2-b_2c_1} = \frac{-y}{a_1c_2-a_2c_1} = \frac{1}{a_1b_2-a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 5$, $b_1 = -2$, $c_1 = 1$

$a_2 = 15$, $b_2 = 7$, $c_2 = -10$

By cross multiplication method,

$\frac{u}{20-7} = \frac{-v}{-50-15} = \frac{1}{35+30}$

$\frac{u}{13} = \frac{-v}{-65} = \frac{1}{65}$

Now,

$\frac{u}{13} = \frac{1}{-65}$

$=u= \frac{1}{5}$

$\frac{1}{u} =$ =x+y

=x+y=5 ……………………….(i)

And,

$\frac{-v}{-65} = \frac{1}{-65}$

=v=1

$\frac{1}{v} =$ =x-y

=x-y =1 ………………………… (ii)

Adding equation (i) and (ii)

2x= 6

=x=3

Substituting the value of x in equation (i)

3+y=5

=y=2

Hence, the solution for the given system of equations is x = 3 and y = 2.

10. 2/x + 3/y = 13

5/x – 4/y = -2

Solution:

Let 1/x = u and 1/y = v, so the equation becomes

2u + 3y = 13 ⇒ 2u + 3y – 13 = 0

5u – 4y = -2 ⇒ 5u – 4y + 2 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = 2$, $b_1 = 3$, $c_1 = -13$

$a_2 = 5$, $b_2 = -4$, $c_2 = 2$

By cross multiplication method,

$$\frac{u}{6-52} = \frac{-v}{4+65} = \frac{1}{-8-15}$$

$$\frac{u}{-46} = \frac{-v}{69} = \frac{1}{-23}$$

Now,

$$\frac{u}{-46} = \frac{1}{-23}$$

=u= 2

$$\frac{1}{u} = x$$

=x= $\frac{1}{2}$

And,

$$\frac{-v}{69} = \frac{1}{-23}$$

=v=3

$$\frac{1}{v} = y$$

=y = $\frac{1}{3}$

Hence, the solution for the given system of equations is x = 1/2 and y = 1/3.

11. 57/(x + y) + 6/(x – y) = 5

38/(x + y) + 21/(x – y) = 9

Solution:

Let's substitute 1/(x + y) = u and 1/(x – y) = v, so the given equations becomes

57u + 6v = 5 ⇒ 57u + 6v – 5 = 0

38u + 21v = 9 ⇒ 38u + 21v – 9 = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

a_1= 57, b_1= 6 , c_1= -5

a_2= 38 , b_2=21 , c_2= -9

By cross multiplication method,

$$\frac{u}{-54+105} = \frac{-v}{-513+190} = \frac{1}{1193-228}$$

$$\frac{u}{51} = \frac{-v}{-323} = \frac{1}{969}$$

Now,

$$\frac{u}{51} = \frac{1}{969}$$

=u= $\frac{1}{19}$

$\frac{1}{u}$ =x+y

=x+y=19 ....(i)

And,

$$\frac{-v}{-323} = \frac{1}{969}$$

=v= $\frac{1}{3}$

$\frac{1}{v}$ = =x-y

=x-y =3 .... (ii)

Adding equation (i) and (ii)

2x= 22

=x=11

Substituting the value of x in equation (i)

11+y=19

=y=8

Hence, the solution for the given system of equations is x = 11 and y = 8.

12. xa – yb = 2

ax – by = a^2-b^2

Solution:

The given system of equations can be written as

xa – yb – 2 **= 0**

ax – by – (a^2-b^2) = 0

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = \frac{1}{a}$, Let $b_1 = \frac{1}{b}$, Let $c_1 = -2$

$a_2 = a$, $b_2 = -b$, $c_2 = b^2 - a^2$

By cross multiplication method

$$= \frac{x}{\frac{b^2-a^2}{b}-2b} = \frac{-y}{\frac{b^2-a^2}{b}+2b} = \frac{1}{\frac{-b}{a}-\frac{a}{b}}$$

$$= \frac{x}{\frac{b^2-a^2-2b^2}{b}} = \frac{-y}{\frac{b^2-a^2+2b^2}{b}} = \frac{1}{\frac{-b^2-a^2}{ab}}$$

Now, $\frac{x}{\frac{b^2-a^2-2b^2}{b}} = \frac{1}{\frac{-b^2-a^2}{ab}}$

x= a

and, $\frac{-y}{\frac{b^2-a^2+2b^2}{b}} = \frac{1}{\frac{-b^2-a^2}{ab}}$

=y=b

Hence, the solution for the given system of equations is x = a and y = b.

13. x/a + y/b = a + b

$x/a^2 + y/b^2 = 2$

Solution:

The given system of equations can be written as

x/a + y/b – (a + b) **= 0**

$x/a^2 + y/b^2 - 2$ **= 0**

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = \frac{1}{a}$, Let $b_1 = \frac{1}{b}$, Let $c_1 = -(a+b)$

$a_2 = \frac{1}{a^2}$, $b_2 = \frac{1}{b^2}$, $c_2 = -2$

By cross multiplication method

$$= \frac{x}{\frac{-2}{b} + \frac{a}{b^2} + \frac{1}{b}} = \frac{-y}{\frac{-2}{a} + \frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{-1}{ab^2} - \frac{-1}{a^2b}}$$

$$= \frac{x}{\frac{a-b}{b^2}} = \frac{-y}{\frac{-a-b}{a^2} + \frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{-1}{ab^2} - \frac{-1}{a^2b}}$$

Now, $\frac{x}{\frac{a-b}{b^2}} = \frac{1}{\frac{-1}{ab^2} - \frac{-1}{a^2b}}$

$= x = a^2$

$$\frac{-y}{\frac{-a-b}{a^2} + \frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{-1}{ab^2} - \frac{-1}{a^2b}}$$

$= y = b^2$

Hence, the solution for the given system of equations is $x = a^2$ and $y = b^2$.

14. x/a = y/b

$ax + by = a^2 + b^2$

Solution:

The given system of equations can be written as

x/a – y/b **= 0**

$ax + by – (a^2 + b^2)$ **= 0**

For cross multiplication we use,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Comparing the above two equations with the general form, we get

$a_1 = \frac{1}{a}$, Let $b_1 = \frac{1}{b}$, $c_1 = 0$

Hence, $a_1 = a$, $b_2 = b$, Let $c_1 = -(a^2+b^2)$

By cross multiplication method

$$\frac{x}{\frac{a^2+b^2}{b}} = \frac{y}{\frac{a^2+b^2}{a}} = \frac{1}{\frac{a}{b} + \frac{b}{a}}$$

Now, $\frac{x}{\frac{a^2+b^2}{b}} = \frac{1}{\frac{a}{b} + \frac{b}{a}}$

$= x = a$

And $\frac{y}{\frac{a^2+b^2}{a}} = \frac{1}{\frac{a}{b} + \frac{b}{a}}$

$= y = b$

Hence, the solution for the given system of equations is x = a and y = b.

Exercise 3.5 Page No: 3.73

In each of the following systems of equation determine whether the system has a unique solution, no solution or infinite solutions. In case there is a unique solution, find it from 1 to 4:

1. $x - 3y = 3$

$3x - 9y = 2$

Solution:

The given system of equations is:

$x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$

$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1$, $b_1 = -3$, $c_1 = -3$

$a_2 = 3$, $b_2 = -9$, $c_2 = -2$

So according to the question, we get

$a_1 / a_2 = 1/3$

$b_1 / b_2 = -3/ -9 = 1/3$ and,

$c_1 / c_2 = -3/ -2 = 3/2$

$\Rightarrow a_1 / a_2 = b_1 / b2 \neq c_1 / c_2$

Hence, we can conclude that the given system of equation has no solution.

2. $2x + y = 5$

$4x + 2y = 10$

Solution:

The given system of equations is:

$2x + y - 5 = 0$

$4x + 2y - 10 = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$

$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = 1$, $c_1 = -5$

$a_2 = 4$, $b_2 = 2$, $c_2 = -10$

So according to the question, we get

$a_1 / a_2 = 2/4 = 1/2$

$b_1 / b_2 = 1/ 2$ and,

$c_1 / c_2 = -5/ -10 = 1/2$

$\Rightarrow a_1 / a_2 = b_1 / b2 = c_1 / c_2$

Hence, we can conclude that the given system of equation has infinity many solutions.

3. 3x – 5y = 20

6x – 10y = 40

Solution:

The given system of equations is:

$3x - 5y - 20 = 0$

$6x - 10y - 40 = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$

$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 3$, $b_1 = -5$, $c_1 = -20$

$a_2 = 6$, $b_2 = -10$, $c_2 = -40$

So according to the question, we get

$a_1 / a_2 = 3/6 = 1/2$

$b_1 / b_2 = -5/ -10 = 1/2$ and,

$c_1 / c_2 = -20/ -40 = 1/2$

$\Rightarrow a_1 / a_2 = b_1 / b2 = c_1 / c_2$

Hence, we can conclude that the given system of equation has infinity many solutions.

4. x – 2y = 8

5x – 10y = 10

Solution:

The given system of equations is:

$x - 2y - 8 = 0$

$5x - 10y - 10 = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$

$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -8$

$a_2 = 5$, $b_2 = -10$, $c_2 = -10$

So according to the question, we get

$a_1 / a_2 = 1/5$

$b_1 / b_2 = -2/ -10 = 1/5$ and,

$c_1 / c_2 = -8/ -10 = 4/5$

$\Rightarrow a_1 / a_2 = b_1 / b2 \neq c_1 / c_2$

Hence, we can conclude that the given system of equation has no solution.

Find the value of k for which the following system of equations has a unique solution: (5-8)

5. kx + 2y = 5

3x + y = 1

Solution:

The given system of equations is:

$kx + 2y – 5 = 0$

$3x + y – 1 = 0$

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = k$, $b_1 = 2$, $c_1 = -5$

$a_2 = 3$, $b_2 = 1$, $c_2 = -1$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 \neq b_1 / b_2$

$k/3 \neq 2/1$

$\Rightarrow k \neq 6$

Hence, the given system of equations will have unique solution for all real values of k other than 6.

6. 4x + ky + 8 = 0

2x + 2y + 2 = 0

Solution:

The given system of equations is:

$4x + ky + 8 = 0$

$2x + 2y + 2 = 0$

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = 4$, $b_1 = k$, $c_1 = 8$

$a_2 = 2$, $b_2 = 2$, $c_2 = 2$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 \neq b_1 / b_2$

$4/2 \neq k/2$

$\Rightarrow k \neq 4$

Hence, the given system of equations will have unique solution for all real values of k other than 4.

7. 4x – 5y = k

2x – 3y = 12

Solution

The given system of equations is:

4x – 5y – k = 0

2x – 3y – 12 = 0

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = 4$, $b_1 = 5$, $c_1 = -k$

$a_2 = 2$, $b_2 = 3$, $c_2 = 12$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 \neq b_1 / b_2$

4/2 ≠ 5/3

⇒k can have any real values.

Hence, the given system of equations will have unique solution for all real values of k.

8. x + 2y = 3

5x + ky + 7 = 0

Solution:

The given system of equations is:

x + 2y – 3 = 0

5x + ky + 7 = 0

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$

$a_2 = 5$, $b_2 = k$, $c_2 = 7$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 \neq b_1 / b_2$

1/5 ≠ 2/k

⇒ k ≠ 10

Hence, the given system of equations will have unique solution for all real values of k other than 10.

Find the value of k for which each of the following system of equations having infinitely many solution: (9-19)

9. 2x + 3y – 5 = 0

6x + ky – 15 = 0

Solution:

The given system of equations is:

$2x + 3y – 5 = 0$

$6x + ky – 15 = 0$

The above equations are of the form

$a_1 x + b_1 y − c_1 = 0$

$a_2 x + b_2 y − c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$

$a_2 = 6$, $b_2 = k$, $c_2 = -15$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/6 = 3/k$

$\Rightarrow k = 9$

Hence, the given system of equations will have infinitely many solutions, if k = 9.

10. 4x + 5y = 3

kx + 15y = 9

Solution:

The given system of equations is:

$4x + 5y – 3= 0$

$kx + 15y – 9 = 0$

The above equations are of the form

$a_1 x + b_1 y − c_1 = 0$

$a_2 x + b_2 y − c_2 = 0$

Here, $a_1 = 4$, $b_1 = 5$, $c_1 = -3$

$a_2 = k$, $b_2 = 15$, $c_2 = -9$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$4/ k = 5/ 15 = −3/ −9$

$4/ k = 1/ 3$

$\Rightarrow k = 12$

Hence, the given system of equations will have infinitely many solutions, if k = 12.

11. kx – 2y + 6 = 0

4x – 3y + 9 = 0

Solution:

The given system of equations is:

$kx – 2y + 6 = 0$

$4x – 3y + 9 = 0$

The above equations are of the form

$a_1 x + b_1 y − c_1 = 0$

$a_2 x + b_2 y − c_2 = 0$

Here, $a_1 = k$, $b_1 = -2$, $c_1 = 6$

$a_2 = 4$, $b_2 = -3$, $c_2 = 9$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$k/ 4 = −2/ −3 = 2/ 3$

$\Rightarrow k = 8/ 3$

Hence, the given system of equations will have infinitely many solutions, if k = 8/3.

12. 8x + 5y = 9

kx + 10y = 18

Solution:

The given system of equations is:

$8x + 5y – 9 = 0$

$kx + 10y – 18 = 0$

The above equations are of the form

$a_1 x + b_1 y − c_1 = 0$

$a_2 x + b_2 y − c_2 = 0$

Here, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$

$a_2 = k$, $b_2 = 10$, $c_2 = -18$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$8/k = 5/10 = −9/ −18 = 1/2$

$\Rightarrow k=16$

Hence, the given system of equations will have infinitely many solutions, if k = 16.

13. 2x – 3y = 7

(k+2)x – (2k+1)y = 3(2k-1)

Solution:

The given system of equations is:

$2x – 3y – 7 = 0$

$(k+2)x – (2k+1)y – 3(2k-1) = 0$

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = -3$, $c_1 = -7$

$a_2 = (k+2)$, $b_2 = -(2k+1)$, $c_2 = -3(2k-1)$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/ (k+2) = -3/ -(2k+1) = -7/ -3(2k-1)$

$2/(k+2) = -3/ -(2k+1)$ and $-3/ -(2k+1) = -7/ -3(2k-1$

$\Rightarrow 2(2k+1) = 3(k+2)$ and $3\times3(2k-1) = 7(2k+1)$

$\Rightarrow 4k+2 = 3k+6$ and $18k - 9 = 14k + 7$

$\Rightarrow k=4$ and $4k = 16 \Rightarrow k=4$

Hence, the given system of equations will have infinitely many solutions, if $k = 4$.

14. 2x + 3y = 2

(k+2)x + (2k+1)y = 2(k-1)

Solution:

The given system of equations is:

$2x + 3y - 2 = 0$

$(k+2)x + (2k+1)y - 2(k-1) = 0$

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$

$a_2 = (k+2)$, $b_2 = (2k+1)$, $c_2 = -2(k-1)$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/ (k+2) = 3/ (2k+1) = -2/ -2(k-1)$

$2/ (k+2) = 3/ (2k+1)$ and $3(2k+1) = 22(k-1$

$\Rightarrow 2(2k+1) = 3(k+2)$ and $3(k-1) = (2k+1)$

$\Rightarrow 4k+2 = 3k+6$ and $3k-3 = 2k+1$

$\Rightarrow k = 4$ and $k = 4$

Hence, the given system of equations will have infinitely many solutions, if k = 4.

15. x + (k+1)y = 4

(k+1)x + 9y = 5k + 2

Solution:

The given system of equations is:

$x + (k+1)y – 4= 0$

$(k+1)x + 9y – (5k + 2) = 0$

The above equations are of the form

$a_1 x + b_1 y - c_1 = 0$

$a_2 x + b_2 y - c_2 = 0$

Here, $a_1 = 1$, $b_1 = (k+1)$, $c_1 = -4$

$a_2 = (k+1)$, $b_2 = 9$, $c_2 = -(5k+2)$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$1/ k+1 = (k+1)/ 9 = -4/ -(5k+2)$

$1/ k+1 = k+1/ 9$ and $k+1/ 9 = 4/ 5k+2$

$\Rightarrow 9 = (k+1)^2$ and $(k+1)(5k+2) = 36$

$\Rightarrow 9 = k^2 + 2k + 1$ and $5k^2+2k+5k+2 = 36$

$\Rightarrow k^2+2k-8 = 0$ and $5k^2+7k-34 = 0$

$\Rightarrow k^2+4k-2k-8 = 0$ and $5k^2+17k-10k-34 = 0$

$\Rightarrow k(k+4)-2(k+4) = 0$ and $(5k+17)-2(5k+17) = 0$

$\Rightarrow(k+4)(k-2) = 0$ and $(5k+17)(k-2) = 0$

$\Rightarrow k = -4$ or $k = 2$ and $k = -17/5$ or $k = 2$

Its seen that k=2 satisfies both the condition.

Hence, the given system of equations will have infinitely many solutions, if k = 9.

16. kx + 3y = 2k + 1

2(k+1)x + 9y = 7k + 1

Solution:

The given system of equations is:

$kx + 3y - (2k + 1) = 0$

$2(k+1)x + 9y - (7k + 1) = 0$

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = k$, $b_1 = 3$, $c_1 = -(2k+1)$

$a_2 = 2(k+1)$, $b_2 = 9$, $c_2 = -(7k+1)$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$k/ 2(k+1) = 3/ 9$ and $3/ 9 = -(2k+1)/ -(7k+1)$

$3k = 2k + 2$ and $7k+1 = 3(2k+1) = 6k + 3$

$k = 2$ and $k = 2$

Hence, the given system of equations will have infinitely many solutions, if k = 2.

17. 2x + (k-2)y = k

6x + (2k-1)y = 2k + 5

Solution:

The given system of equations is:

$2x + (k-2)y - k = 0$

$6x + (2k-1)y - (2k+5) = 0$

The above equations are of the form

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

Here, $a_1 = 2$, $b_1 = k-2$, $c_1 = -k$

$a_2 = 6$, $b_2 = 2k-1$, $c_2 = -2k-5$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/6 = (k-2)/ (2k-1)$ and $(k-2)/ (2k-1) = -k/ -2k-5$

$4k - 2 = 6k - 12$ and $(k-2)(2k+5) = k(2k-1)$

$2k = 10$ and $2k^2 – 4k + 5k – 10 = 2k^2 – k$

$\Rightarrow k = 5$ and $2k = 10 \Rightarrow k = 5$

Hence, the given system of equations will have infinitely many solutions, if k = 5.

18. 2x + 3y = 7

(k+1)x + (2k-1)y = 4k+1

Solution:

The given system of equations is:

$2x + 3y – 7= 0$

$(k+1)x + (2k-1)y – (4k+1) = 0$

The above equations are of the form

$a_1 x + b_1 y − c_1 = 0$

$a_2 x + b_2 y − c_2 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = – 7$

$a_2 = (k+1)$, $b_2 = 2k-1$, $c_2 = – (4k+1)$

So according to the question,

For unique solution, the condition is

$a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

$2/ (k+1) = 3/ (2k−1) = −7/ −(4k+1)$

$2/ (k+1) = 3/(2k−1)$ and $3/ (2k−1) = 7/(4k+1)$

$2(2k−1) = 3(k+1)$ and $3(4k+1) = 7(2k−1)$

$\Rightarrow 4k−2 = 3k+3$ and $12k + 3 = 14k − 7$

$\Rightarrow k = 5$ and $2k = 10$

$\Rightarrow k = 5$ and $k = 5$

Hence, the given system of equations will have infinitely many solutions, if k = 5.

Exercise 3.6 Page No: 3.73

1. 5 pens and 6 pencils together cost 9 and 3 pens and 2 pencils cost 5. Find the cost of 1 pen and 1 pencil.

Solution:

Let’s assume the cost of a pen and pencil be x and y respectively.

Then, forming equations according to the question

$5x + 6y = 9$ … (i)

$3x + 2y = 5$ … (ii)

On multiplying equation (i) by 2 and equation (ii) by 6, we get

$10x + 12y = 18$ … (iii)

$18x + 12y = 30$ … (iv)

Now on subtracting equation (iii) from equation (iv), we get

$18x – 10x + 12y – 12y = 30 – 18$

$8x = 12$

$x = 3/2 = 1.5$

Putting $x = 1.5$ in equation (i), we find y

$5(1.5) + 6y = 9$

$6y = 9 – 7.5$

$y = (1.5)/ 6 = 0.25$

Therefore, the cost of one pen = □ 1.50 and so the cost of one pencil = □ 0.25

2. 7 audio cassettes and 3 videocassettes cost □ 1110, while 5 audio cassettes and 4 videocassettes cost □ 1350. Find the cost of audio cassettes and a video cassette.

Solution:

Let’s assume the cost of an audio cassette and that of a video cassette be □ x and □ y, respectively. Then forming equations according to the question, we have

$7x + 3y = 1110$ … (i)

$5x + 4y = 1350$ … (ii)

On multiplying equation (i) by 4 and equation (ii) by 3,

We get,

$28x + 12y = 4440$ … (iii)

$15x + 4y = 4050$ … (iv)

Subtracting equation (iv) from equation (iii),

$28x – 13x + 12y – 12y = 4440 – 4050$

$13x = 390$

$\Rightarrow x = 30$

On substituting $x = 30$ in equation (i)

$7(30) + 3y = 1110$

$3y = 1110 – 210$

$y = 900/ 3$

$\Rightarrow y = 300$

Therefore, it’s found that the cost of one audio cassette = □ 30

And the cost of one video cassette = □ 300

3. Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Solution:

Let's assume the number of pens and pencils be x and y, respectively.

Forming equations according to the question, we have

$x + y = 40$ … (i)

$(y+5) = 4(x-5)$

$y + 5 = 4x - 20$

$5 + 20 = 4x - y$

$4x - y = 25$ … (ii)

Adding equation (i) and (ii),

We get,

$x + 4x = 40 + 25$

$5x = 65$

$\Rightarrow x = 13$

Putting x=13 in equation (i), we get

$13 + y = 40$

$\Rightarrow y = 40 - 13 = 27$

Therefore, it's found that the number of pens Reena has is 13

And, the number of pencils Reena has is 27.

4. 4 tables and 3 chairs, together, cost □ 2250 and 3 tables and 4 chairs cost □ 1950. Find the cost of 2 chairs and 1 table.

Solution:

Let's assume the cost of 1 table is □ x and cost of 1 chair is □ y.

Then, according to the question

$4x + 3y = 2250$ … (i)

$3x + 4y = 1950$ … (ii)

On multiplying (i) with 3 and (ii) with 4,

We get,

$12x + 9y = 6750$ … (iii)

$12x + 16y = 7800$ … (iv)

Now, subtracting equation (iv) from (iii),

We get,

$-7y = -1050$

$y = 150$

Using y = 150 in (i), we find x

$4x + 3(150) = 2250$

$4x = 2250 - 450$

$x = 1800/ 4$

$\Rightarrow x = 450$

From the question, it's required to find the value of $(x + 2y) \Rightarrow 450 + 2(150) = 750$

Therefore, the total cost of 2 chairs and 1 table is □ 750.

5. 3 bags and 4 pens together cost □ 257 whereas 4 bags and 3 pens together cost □324. Find the total cost of 1 bag and 10 pens.

Solution:

Let the cost of a bag and a pen be □ x and □ y, respectively.

Then, according to the question

$3x + 4y = 257$ … (i)

$4x + 3y = 324$ … (ii)

On multiplying equation (i) by 3 and (ii) by 4,

We get,

$9x + 12y = 770$ … (iii)

$16x + 12y = 1296$ … (iv)

Subtracting equation (iii) from (iv), we get

$16x – 9x = 1296 – 771$

$7x = 525$

$x = 525/7 = 75$

Hence, the cost of a bag = □ 75

Substituting $x = 75$ in equation (i),

We get,

$3 \times 75 + 4y = 257$

$225 + 4y = 257$

$4y = 257 – 225$

$4y = 32$

$y = 32/4 = 8$

Hence, the cost of a pen = □ 8

From the question, it's required to find the value of $(x + 10y) \Rightarrow 75 +10(8) = 20$

Therefore, the total cost of 1 bag and 10 pens = 75 + 80 = □ 155.

6. 5 books and 7 pens together cost □ 79 whereas 7 books and 5 pens together cost □ 77. Find the total cost of 1 book and 2 pens.

Solution:

Let's assume the cost of a book a pen be □ x and □ y, respectively.

Then, according to the question

$5x + 7y = 79$ … (i)

$7x + 5y = 77$ … (ii)

On multiplying equation (i) by 5 and (ii) by 7,

We get,

$25x + 35y = 395$ … (iii)

$49x + 35y = 539$ … (iv)

Subtracting equation (iii) from (iv),

We have,

$49x – 25x = 539 – 395$

$24x = 144$

$x = 144/24 = 6$

Hence, the cost of a book = ₹ 6

Substituting, x= 6 in equation (i),

We get,

$5 (6) + 7y = 79$

$30 + 7y = 79$

$7y = 79 – 30$

$7y = 49$

$y = 49/ 7 = 7$

Hence, the cost of a pen = ₹ 7

From the question, it’s required to find the value of $(x + 2y) \Rightarrow 6 + 2(7) = 20$

Therefore, the total cost of 1 book and 2 pens = 6 + 14= ₹ 20

7. Jamila sold a table and a chair for ₹ 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got ₹ 1065. Find the cost price of each.

Solution:

Let the cost price of one table and one chair be ₹ x and ₹ y, respectively.

So,

The selling price of the table, when it’s sold at a profit of 10% = ₹ x + 10x/100 = ₹ 110x / 100

The selling price of the chair, when it’s sold at a profit of 25% = ₹ y + 25y/100 = ₹ 125y / 100

Hence, according to the question

$110x / 100 + 125y / 100 = 1050$ … (i)

Similarly,

The selling price of the table, when it’s sold at a profit of 25% = ₹ (x + 25x/100) = ₹ 125x/ 100

The selling price of the chair, when it’s sold at a profit of 10% = ₹ (y + 10y/100) = ₹ 110y / 100

Hence, again from the question

$125x / 100 + 110y / 100 = 1065$ … (ii)

Re- written (i) and (ii) with their simplest coefficients,

$11x/10 + 5y/4 = 1050$…….. (iii)

$5x/4 + 11y/10 = 1065$…….. (iv)

Adding (iii) and (iv), we get

$(11/10 + 5/4)x + (5/4 + 11/10)y = 2115$

$47/20x + 47/20y = 2115$

$x + y = 2115(20/47) = 900$

$\Rightarrow x = 900 – y$ ……. (v)

Using (v) in (iii),

$11(900 – y)/10 + 5y/4 = 1050$

$2(9900 - 11y) + 25y = 1050 \times 20$ [After taking LCM]

$19800 – 22y + 25y = 21000$

$3y = 1200$

$\Rightarrow y = 400$

Putting $y = 400$ in (v), we get

$x = 900 – 400 = 500$

Therefore, the cost price of the table is □ 500 and that of the chair is □ 400.

Exercise 3.7 Page No: 3.85

1. The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

Solution:

Let's assume the two numbers to be x and y.

Also let's consider that, x is greater than or equal to y.

Now, according to the question

The sum of the two numbers, $x + y = 8$………… (i)

Also given that, their sum is four times their difference. So, we can write;

$x + y = 4(x – y)$

$\Rightarrow x + y = 4x - 4y$

$\Rightarrow 4x – 4y – x – y = 0$

$\Rightarrow 3x – 5y = 0$………………. (ii)

Solving (i) and (ii), we can find x and y, so the required two numbers.

On multiplying equation (i) by 5 and then add with equation (ii), we get here;

$5 (x + y) + (3x – 5y) = 5 \times 8 + 0$

$\Rightarrow 5x + 5y + 3x – 5y = 40$

$\Rightarrow 8x = 40$

$\Rightarrow x = 5$

Putting the value of x in (i), we find y

$5 + y = 8$

$\Rightarrow y = 8 – 5$

$\Rightarrow y = 3$

Therefore, the two numbers are 5 and 3.

2. The sum of digits of a two-digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

Solution:

Let's assume the digit at the unit's place as x and at ten's place as y. Then the required number is 10y + x.

Also it's given that, the sum of the digits of the number is 13,

So, x + y = 13………… (i)

On interchanging the position of digits, the new number so formed will be 10x+y.

Again is it's given that, the difference between the new number so formed upon interchanging the digits and the original number is equal to 45. Therefore, this can be expressed as;

$(10x + y) – (10y + x) = 45$

$\Rightarrow 110x + y – 10y – x = 45$

$\Rightarrow 9x – 9y = 45$

$\Rightarrow 9(x – y) = 45$

$\Rightarrow x – y = 5$………..(ii)

Solving (i) and (ii) we can find x and y,

Now, adding (i) and (ii), we get;

$(x + y) + (x – y) = 13 + 5$

$\Rightarrow x + y + x – y = 18$

$\Rightarrow 2x = 18$

$\Rightarrow x = 9$

Putting the value of x in the equation (i), we find y;

$9 + y = 13$

$\Rightarrow y = 13 – 9$

$\Rightarrow y = 4$

Hence, the required number is, $10 \times 4 + 9 = 49$.

3. A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.

Solution:

Let's assume the digit at unit's place as x and ten's place as y. Thus, the number to be found is 10y + x.

From the question it's given as, the sum of the digits of the number is equal to 5.

Thus we can write, $x + y = 5$ ………….. (i)

On interchange the place of digits, the new number so formed will be 10x+ y.

Again from the question it's given as, the new number so obtained after interchanging the digits is greater by 9 from the original number. Therefore, this can be written as;

$10x + y = 10y + x +9$

$\Rightarrow 10x + y – 10y – x = 9$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow 9(x - y) = 9$

$\Rightarrow x - y = 1$………………. (ii)

Solving (i) and (ii), we can find x and y

Adding the eq. 1 and 2, we get;

$(x + y) + (x - y) = 5+1$

$\Rightarrow x + y + x - y = 5+1$

$\Rightarrow 2x = 6$

$\Rightarrow x = 6/2$

$\Rightarrow x = 3$

Putting the value of x in the equation 1, we get;

$3 + y = 5$

$\Rightarrow y = 5-3$

$\Rightarrow y = 2$

Hence, the required number is $10 \times 2 + 3 = 23$

4. The sum of digits of a two-digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Solution:

Let the digits at unit's place be x and ten's place be y, respectively. Thus, the number we need to find is 10y + x.

As per the given statement, the sum of the digits of the number is 15. Thus, we have;

$x+ y = 15$ ……………(i)

Upon interchanging the digit's place, the new number will so be 10x + y.

Also it's given from the question that, the new number obtained exceeds from the original number by 9. Therefore, we can write this as;

$10x + y = 10y + x + 9$

$\Rightarrow 10x + y - 10y - x = 9$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow 9(x - y) = 9$

$\Rightarrow x - y = 9/9$

$\Rightarrow x - y = 1$ ………………….. (ii)

Solving (i) and (ii), we can find x and y

Now, adding the equations (i) and (ii), we get;

$(x + y) + (x - y) = 15 + 1$

$\Rightarrow x + y + x - y = 16$

$\Rightarrow 2x = 16$

$\Rightarrow x = 16/2$

$\Rightarrow x = 8$

Putting the value of x in the equation (i), to get y

$8 + y = 5$

$\Rightarrow y = 15 - 8$

$\Rightarrow y = 7$

Hence, the required number is, $10 \times 7 + 8 = 78$

5. The sum of a two-digit number and the number formed by reversing the order of digits is 66. If the two digits differ by 2, find the number. How many such numbers are there?

Solution:

Let’s assume the digit at unit’s place as x and ten’s place as y. Thus from the question, the number needed to be found is 10y + x.

From the question it’s told as, the two digits of the number are differing by 2. Thus, we can write

$x - y = \pm 2$………….. (i)

Now after reversing the order of the digits, the number becomes 10x + y.

Again from the question it’s given that, the sum of the numbers obtained by reversing the digits and the original number is 66. Thus, this can be written as;

$(10x + y) + (10y + x) = 66$

$\Rightarrow 10x + y + 10y + x = 66$

$\Rightarrow 11x + 11y = 66$

$\Rightarrow 11(x + y) = 66$

$\Rightarrow x + y = 66/11$

$\Rightarrow x + y = 6$………….. (ii)

Now, we have two sets of systems of simultaneous equations

$x - y = 2$ and $x + y = 6$

$x - y = -2$ and $x + y = 6$

Let’s first solve the first set of system of equations;

$x - y = 2$ …………. (iii)

$x + y = 6$ ………….. (iv)

On adding the equations (iii) and (iv), we get;

$(x – y) + (x + y) = 2+6$

$\Rightarrow x – y + x + y = 8$

$\Rightarrow 2x =8$

$\Rightarrow x = 8/2$

$\Rightarrow x = 4$

Putting the value of x in equation (iii), we get

$4 – y = 2$

$\Rightarrow y = 4 – 2$

$\Rightarrow y = 2$

Hence, the required number is $10 \times 2 +4 = 24$

Now, let’s solve the second set of system of equations,

$x – y = -2$ …………. (v)

$x + y = 6$ ………….. (vi)

On adding the equations (v) and (vi), we get

$(x – y)+(x + y)= -2 + 6$

$\Rightarrow x – y + x + y = 4$

$\Rightarrow 2x = 4$

$\Rightarrow x = 4/2$

$\Rightarrow x = 2$

Putting the value of x in equation 5, we get;

$2 – y = -2$

$\Rightarrow y = 2+2$

$\Rightarrow y = 4$

Hence, the required number is $10\times4+ 2 = 42$

Therefore, there are two such possible numbers i.e, 24 and 42.

6. The sum of two numbers is 1000 and the difference between their square is 256000. Find the numbers.

Solution:

Let's assume the two numbers be x and y. And also assume that x is greater than or equal to y.

So as per the question, we can write the sum of the two numbers as

$x + y = 1000$ ……….. (i)

Again it's given that, the difference between the squares of the two numbers, thus writing it

$x^2 – y^2 = 256000$

$\Rightarrow (x + y) (x – y) = 256000$

$\Rightarrow 1000(x-y) = 256000$

$\Rightarrow x – y = 256000/1000$

$\Rightarrow x – y = 256$ ………….. (ii)

By solving (i) and (ii), we can find the two numbers

On adding the equations (i) and (ii), we get;

$(x+ y) + (x- y) = 1000 + 256$

$\Rightarrow x + y + x – y =1256$

$\Rightarrow 2x = 1256$

$\Rightarrow x = 1256/ 2$

$\Rightarrow x = 628$

Now, putting the value of x in equation (i), we get

$628 + y = 1000$

$\Rightarrow y = 1000 – 628$

$\Rightarrow y = 372$

Hence, the two required numbers are 628 and 372.

7. The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Solution:

Let's assume the digit at unit's place is x and ten's place is y. Thus from the question, the number we need to find is 10y + x.

From the question since the two digits of the number are differing by 3. Therefore,

$x – y = \pm 3$ …………. (i)

And, after reversing the digits, the number so obtained is 10x + y.

Again it's given from the question that, the sum of the numbers obtained by reversing the digit's places and the original number is 99. Thus, this can be written as;

(10x + y) + (I0y + x) = 99

$\Rightarrow 10x + y + 10y + x = 99$

$\Rightarrow 11x + 11y = 99$

$\Rightarrow 11(x + y) = 99$

$\Rightarrow x + y = 99/11$

$\Rightarrow x + y = 9$ …………… (ii)

So, finally we have two sets of systems of equations to solve. Those are,

$x – y = 3$ and $x + y = 9$

$x – y = -3$ and $x + y = 9$

Now, let's solve the first set of system of equations;

$x – y = 3$ ……….. (iii)

$x + y = 9$ ………. (iv)

Adding the equations (iii) and (iv), we get;

$(x – y) + (x + y) = 3 + 9$

$\Rightarrow x – y + x + y =12$

$\Rightarrow 2x = 12$

$\Rightarrow x = 12/2$

$\Rightarrow x = 6$

Putting the value of x in equation (iii), we find y

$6 – y = 3$

$\Rightarrow y = 6 – 3$

$\Rightarrow y = 3$

Hence, when considering this set the required number should be $10\times3 + 6 =36$

Now, when solving the second set of system of equations,

$x – y = –3$ ……….(v)

$x + y = 9$ ………….. (vi)

Adding the equations (v) and (vi), we get;

$(x – y) + (x + y) = –3 + 9$

$x – y + x + y = 6$

$2x = 6$

$x = 3$

Putting the value of x in equation 5, we get;

$3 – y = -3$

$\Rightarrow y = 3 + 3$

$\Rightarrow y = 6$

Hence, when considering this set the required number should be $10×6+3=63$

Therefore, there are two such numbers for the given question.

8. A two- digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Solution:

Let's assume the digit at unit's place is x and at ten's place is y. Thus from the question, the number we need to find is 10y + x.

From the question since the number is 4 times the sum of the two digits. We can write,

$10y + x = 4(x + y)$

$\Rightarrow 10y + x = 4x+ 4y$

$\Rightarrow 4x + 4y – 10y -x = 0$

$\Rightarrow 3x – 6y = 0$

$\Rightarrow 3(x – 2y) = 0$

$\Rightarrow x – 2y = 0$ ……………… (i)

Secondly, after reversing the digits, the new number formed is 10x + y.

Again it's given from the question that if 18 is added to the original number, the digits are reversed. Thus, we have

$(10y+x) + 18 = 10x+y$

$\Rightarrow 10x + y - 10y - x = 18$

$\Rightarrow 9x - 9y = 18$

$\Rightarrow 9(x - y) = 18$

$\Rightarrow x - y = 18/9$

$\Rightarrow x - y = 2$ ………… (ii)

Now by solving equation (i) and (ii) we can find the value of x and y and thus the number.

On subtracting the equation (i) from equation (ii), we get;

$(x - y) - (x - 2y) = 2 - 0$

$\Rightarrow x - y - x + 2y = 2$

$\Rightarrow y = 2$

Putting the value of y in the equation (i) to find x, we get

$x - 2 \times 2 = 0$

$\Rightarrow x - 4 = 0$

$\Rightarrow x = 4$

Hence, the required number is $10 \times 2 + 4 = 24$

Exercise 3.8 Page No: 3.88

1. The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Solution:

Let’s assume the numerator of the fraction to be x and the denominator of the fraction to be y.

So, the required fraction is x/y.

From the question it’s given as,

The numerator of the fraction is 4 less the denominator.

Thus, the equation so formed is,

$x = y – 4$

$\Rightarrow x – y = −4$ …… (i)

And also it’s given in the question as,

If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is 8 times the numerator.

Putting the above condition in an equation, we get

$y + 1 = 8(x-2)$

$\Rightarrow y + 1 = 8x–16$

$\Rightarrow 8x – y = 1 + 16$

$\Rightarrow 8x – y = 17$ …… (ii)

Solving (i) and (ii),

Subtracting the equation (ii) from (i), we get

$(x – y) – (8x – y) = – 4 – 17$

$\Rightarrow x – y − 8x + y = −21$

$\Rightarrow −7x = −21$

$\Rightarrow x = 21/7$

$\Rightarrow x = 3$

Substituting the value of x =3 in the equation (i), we find y

$3 - y = -4$

$\Rightarrow y = 3+4$

$\Rightarrow y = 7$

Therefore, the fraction is 3/7.

2. A fraction becomes 9/ 11 if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes 5/ 6. Find the fraction.

Solution:

Let’s assume the numerator of the fraction to be x and the denominator of the fraction to be y.

So, the required fraction is x/y.

From the question it’s given as,

If 2 is added to both numerator and the denominator, the fraction becomes 9/ 11 .

Thus, the equation so formed is,

$x + 2y + 2 = 9/ 11$

$\Rightarrow 11(x+2) = 9(y+2)$

$\Rightarrow 11x + 22 = 9y+18$

$\Rightarrow 11x - 9y = 18 - 22$

$\Rightarrow 11x - 9y + 4 = 0$ ……. (i)

And also it’s given in the question as,

If 3 is added to both numerator and the denominator, the fraction becomes 5/ 6,

Expressing the above condition in an equation, we have

$x + 3y + 3 = 56$

$\Rightarrow 6(x+3) = 5(y+3)$

$\Rightarrow 6x + 18 = 5y + 15$

⇒ 6x – 5y = 15 – 18

⇒ 6x – 5y + 3 = 0…….. (ii)

Solving (i) and (ii), to find the fraction

By using cross-multiplication method, we have

$$\frac{x}{-9*3-(-5)*4} = \frac{-y}{11*3-6*4} = \frac{1}{11*(-5)-6*(-9)}$$

$$\Rightarrow \frac{x}{-27+20} = \Rightarrow \frac{-y}{33-24} = \frac{1}{-55+54}$$

$$\Rightarrow \frac{x}{-7} = \frac{-y}{9} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{7} = \frac{y}{9} = 1$$

x = 7, y = 9

Hence, the required fraction is 7/ 9.

3. A fraction becomes 1/ 3 if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes 1/ 2. Find the fraction.

Solution:

Let’s assume the numerator of the fraction to be x and the denominator of the fraction to be y.

So, the required fraction is x/y.

From the question it’s given as,

If 1 is subtracted from both numerator and the denominator, the fraction becomes 1/ 3.

Thus, the equation so formed is,

(x – 1)/ (y −1) = 1/ 3

⇒ 3(x–1) = (y–1)

⇒ 3x – 3 = y – 1

⇒ 3x – y – 2 = 0…. (i)

And also it’s given in the question as,

If 1 is added to both numerator and the denominator, the fraction becomes 12. Expressing the above condition in an equation, we have

(x+1)/ (y+1) = 1/ 2

⇒ 2(x+1) = (y+1)

⇒ 2x + 2 = y + 1

⇒ 2x – y + 1 = 0 …….. (ii)

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\frac{x}{(-1)*1-(-1)*(-2)} = \frac{-y}{3*1-2*(-2)} = \frac{1}{3*(-1)-2*(-1)}$$

$$\Rightarrow \frac{x}{-1-2} = \Rightarrow \frac{-y}{3+4} = \frac{1}{-3+2}$$

$$\Rightarrow \frac{x}{-3} = \frac{-y}{7} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{7} = 1$$

⇒ x = 3, y = 7

Hence, the required fraction is 3/7.

4. If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction?

Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.

So, the required fraction is x/y.

From the question it's given as,

If 1 is added to the numerator and 1 is subtracted from the denominator, the fraction becomes 1.

Thus, the equation so formed is,

(x +1)/ (y−1) = 1

⇒ (x+1) = (y–1)

⇒ x + 1 – y + 1 = 0

⇒ x – y + 2 = 0 …….. (i)

And also it's given in the question as,

If 1 is added to the denominator, the fraction becomes 12.

Expressing the above condition in an equation, we have

x/ (y+1) = 1/ 2

⇒ 2x = (y+1)

⇒ 2x – y – 1 = 0 …… (ii)

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\frac{x}{(-1)*(-1)-(-1)*2} = \frac{-y}{1*(-1)-2*2} = \frac{1}{1*(-1)-2*(-1)}$$

$$\Rightarrow \frac{x}{1+2} = \Rightarrow \frac{-y}{-1-4} = \frac{1}{-1+2}$$

$$\Rightarrow \frac{x}{3} = \frac{-y}{-5} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{5} = 1$$

⇒x = 3, y = 5

Hence, the required fraction is 3/ 5.

5. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes 12. Find the fraction.

Solution:

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.

So, the required fraction is x/y.

From the question it's given as,

The sum of the numerator and denominator of the fraction is 12.

Thus, the equation so formed is,

x + y = 12

⇒ x + y – 12 = 0

And also it's given in the question as,

If the denominator is increased by 3, the fraction becomes 1/2.

Putting this as an equation, we get

$x/ (y+3) = 1/2$

$\Rightarrow 2x = (y+3)$

$\Rightarrow 2x - y - 3 = 0$

The two equations are,

$x + y - 12 = 0$…… (i)

$2x - y - 3 = 0$…….. (ii)

Adding (i) and (ii), we get

$x + y - 12 + (2x - y - 3) = 0$

$\Rightarrow 3x - 15 = 0$

$\Rightarrow x = 5$

Using $x = 5$ in (i), we find y

$5 + y - 12 = 0$

$\Rightarrow y = 7$

Therefore, the required fraction is 5/7.

Exercise 3.9 Page No: 3.92

1. A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.

Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.

From the question it's given that,

Father is 3 times as old as his son. (Present)

So, the equation formed is

$x = 3y$

$\Rightarrow x - 3y = 0$……. (i)

Also again from the question it's given as, After 12 years, father's age will be (x+12) years and son's age will be (y+12) years.

Furthermore, the relation between their ages after 12 years is given below

$x + 12 = 2(y + 12)$

$\Rightarrow x + 12 = 2y + 24$

$\Rightarrow x - 2y - 12 = 0$…… (ii)

Solving (i) and (ii), we get the solution

By using cross-multiplication, we have

$$\frac{x}{(-3)\times(-12)-(-2)\times 0} = \frac{-y}{1\times(-12)-1\times 0} = \frac{1}{1\times(-2)-1\times(-3)}$$

$$\Rightarrow \frac{x}{36-0} = \frac{-y}{-12-0}\frac{1}{-2+3}$$

$$\frac{x}{36} = \frac{-y}{-12} = \frac{1}{1}$$

$$\frac{x}{36} = \frac{y}{12} = 1$$

x = 36, y = 12

Hence, the present age of father is 36 years and the present age of son is 12 years.

2. Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. What are the present ages of A and B.

Solution:

Let the present ages of A be x years and that of B be y years

From the question it's given that,

After 10 years, A's age will be (x +10) years and B's age will be (y + 10) years.

Furthermore, the relation between their ages after 10 years is given below

$x + 10 = 2(y + 10)$

$\Rightarrow x + 10 = 2y + 20$

$\Rightarrow x - 2y - 10 = 0$…….. (i)

Also again from the question it's given as,

Before 5 years, the age of A was (x – 5) years and the age of B was (y – 5) years.

So, the equation formed is

$x - 5 = 3(y-5)$

⇒ x – 5 = 3y – 15

⇒ x – 3y + 10 = 0…….. (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we get,

$$\frac{x}{(-2)\times 10-(-3)\times(-10)}=\frac{-y}{1\times 10-1\times(-10)}=\frac{1}{1\times(-3)-1\times(-2)}$$

$$\Rightarrow \frac{x}{-20-3-}=\frac{-y}{10+10}=\frac{1}{-3+2}$$

$$\Rightarrow \frac{x}{-50}=\frac{-y}{20}=\frac{1}{-1}$$

$$\Rightarrow \frac{x}{50}=\frac{y}{20}=1$$

⇒x = 50, y = 20

Hence, the present age of A is 50 years and the present age if B is 20 years.

3. A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old as his sister S. If the age of the father and sister differ by 40 years, find the age of A.

Solution:

Assuming that, the present age of A = x

the present age of B = y

the present age of F = z

the present age of S = t

It's understood from the question that,

A is elder to b by 2 years. ⇒ x = y + 2

F is twice as old as A. ⇒ z = 2x

B is twice as old as S. ⇒ y = 2t

Also given that the ages of F and S is differing by 40 years. ⇒ z – t = 40.

So, the four equations are:

$x = y + 2$ … (i)

$z = 2x$ … (ii)

$y = 2t$ … (iii)

$z – t = 40$ …(iv)

It’s clearly seen from the equations obtained that x, y, z and t are unknowns.

And we have to find the value of x.

So, by using equation (iii) in (i),

(i) Becomes $x = 2t + 2$

From (iv), we have $t = z – 40$

Hence, we get

$x = 2(z – 40) + 2$

$= 2z – 80 + 2$

$= 2z – 78$

Using the equation (ii), we have

$x = 2\times2x – 78$

$\Rightarrow x = 4x - 78$

$\Rightarrow 4x – x = 78$

$\Rightarrow 3x = 78$

$\Rightarrow x = 783$

$\Rightarrow x = 26$

Hence, the age of A is 26 years.

4. Six year hence a man’s age will be three times age of his son and three years ago, he was nine times as old as his son. Find their present ages.

Solution:

Let’s assume the present ages of the father as x years and that of his son’s age as y years.

From the question it’s given that,

After 6 years, the man’s age will be (x + 6) years and son’s age will be (y + 6) years.

So, the equation formed is

$x + 6 = 3(y + 6)$

$x + 6 = 3y + 18$

x – 3y – 12 = 0……. (i)

Also again from the question it's given as,

Before 3 years, the age of the man was (x – 3) years and the age of son's was (y – 3) years.

Furthermore, the relation between their 3 years ago is given below

x – 3 = 9(y – 3)

x – 3 = 9y – 27

x – 9y + 24 = 0……. (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we get

$$\frac{x}{(-3)\times24-(-9)\times(-12)} = \frac{-y}{1\times24-1\times(-12)} = \frac{1}{1\times(-9)-1\times(-3)}$$

$$\Rightarrow \frac{x}{-72-108} = \frac{-y}{24+12} = \frac{1}{9+3}$$

$$\Rightarrow \frac{x}{-180} = \frac{-y}{36} = \frac{1}{-9+3}$$

$$\Rightarrow \frac{x}{180} = \frac{y}{36} = \frac{1}{6}$$

$$\Rightarrow x = \frac{180}{6}, y = \frac{36}{6}$$

⇒x = 30, y = 6

Hence, the present age of the man is 30 years and the present age of son is 6 years.

5. Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then. Find their present ages.

Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.

From the question it's given that,

After 10 years, father's age will be (x+10) years and son's age will be (y + 10) years.

So, the equation formed is

x + 10 = 2(y + 10)

x – 10 = 2y + 20

x – 2y – 10 = 0……… (i)

Also again from the question it's given as,

Before 10 years, the age of father was (x – 10) years and the age of son was (y – 10) years.

Furthermore, the relation between their 10 years ago is given below

x – 10 = 12(y – 10)

x – 10 = 12y – 120

x – 12y + 110 = 0……… (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we have

$$\frac{x}{(-2)\times110-(-12)\times(-10)} = \frac{-y}{1\times110-1\times(-10)} = \frac{1}{1\times(-12)-1\times(-12)}$$

$$\Rightarrow \frac{x}{-220-120} = \frac{-y}{110+10} = \frac{1}{-12+2}$$

$$\Rightarrow \frac{x}{-340} = \frac{-y}{120} = \frac{1}{-10}$$

$$\Rightarrow \frac{x}{340} = \frac{y}{120} = \frac{1}{10}$$

$$\Rightarrow x = \frac{340}{10}, y = \frac{120}{10}$$

⇒x = 34, y = 12

Hence, the present age of father is 34 years and the present age of the son is 12 years.

6. The present age of father is 3 years more than three times of the age of the son. Three years hence, father's age will be 10 years more than twice the age of the son. Determine their present age.

Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.

From the question it's given that,

The present age of father is three years more than three times the age of the son.

So, the equation formed is

x = 3y + 3

x – 3y -3 = 0 …….. (i)

Also again from the question it's given as,

After 3 years, father's age will be (x + 3) years and son's age will be (y + 3) years.

Furthermore, the relation between their ages after 3 years is given below

x + 3 = 2(y + 3) + 10

x – 2y – 13 = 0 …….. (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we have

$$\frac{x}{(-3)\times(-13)-(-2)\times(-3)} = \frac{-y}{1\times(-13)-1\times(-3)} = \frac{1}{1\times(-2)-1\times(-3)}$$

$$\Rightarrow \frac{x}{39-6} = \frac{-y}{-13+3} = \frac{1}{-2+3}$$

$$\Rightarrow \frac{x}{33} = \frac{-y}{-10} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{33} = \frac{y}{10} = 1$$

⇒x = 33, y = 10

Hence,

The present age of father = 33 years and the present age of his son = 10 years.

Exercise 3.10 Page No: 3.101

1. Points A and B are 70km. apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7hrs, but if they travel towards each other, they meet in one hour. Find the speed of two cars.

Solution:

Let's consider the car starting from point A as X and its speed as x km/hr.

And, the car starting from point B as Y and its speed as y km/hr.

It's seen that there are two cases in the question:

Case 1: Car X and Y are moving in the same direction

Case 2: Car X and Y are moving in the opposite direction

Let's assume that the meeting point in case 1 as P and in case 2 as Q.

Now, solving for case 1:

The distance travelled by car X = AP

And, the distance travelled by car Y = BP

As the time taken for both the cars to meet is 7 hours,

The distance travelled by car X in 7 hours = 7x km [∵ distance = speed x time]

⇒ AP = 7x

Similarly,

The distance travelled by car Y in 7 hours = 7y km

⇒ BP = 7Y

As the cars are moving in the same direction (i.e. away from each other), we can write

AP – BP = AB

So, 7x – 7y = 70

x – y = 10 ……………………… (i) [after taking 7 common out]

Now, solving for case 2:

In this case as it's clearly seen that,

The distance travelled by car X = AQ

And,

The distance travelled by car Y = BQ

As the time taken for both the cars to meet is 1 hour,

The distance travelled by car x in 1 hour = 1x km

⇒ AQ = 1x

Similarly,

The distance travelled by car y in 1 hour = 1y km

⇒ BQ = 1y

Now, since the cars are moving in the opposite direction (i.e. towards each other), we can write

AQ + BQ = AB

⇒ x + y = 70 …………… (ii)

Hence, by solving (i) and (ii) we get the required solution

From (i), we have x = 10 + y……. (iii)

Substituting this value of x in (ii).

⇒ (10 + y) + y = 70

⇒ y = 30

Now, using y = 30 in (iii), we get

⇒ x = 40

Therefore,

– Speed of car X = 40km/hr.

– Speed of car Y = 30 km/hr.

2. A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and the speed of the current.

Solution:

Let’s assume,

The speed of the sailor in still water as x km/hr

And,

The speed of the current as y km/hr

We know that,

Speed of the sailor in upstream = (x – y) km/hr

Speed of the sailor in downstream = (x + y) km/hr

So, time taken to cover 8 km upstream = 8/ (x – y) hr [∵ time = distance/ speed]

And, time taken to cover 8 km downstream = 8/ (x + y hr [∵ time = distance/ speed]

It's given that time taken to cover 8 km downstream in 40 minutes or, 40/ 60 hour or 2/3 hr.

8/ (x + y) = 2/3

8 × 3 = 2(x + y)

24 = 2x + 2y

x + y = 12 …………………… (i) [After taking 2 common out and rearranging]

Similarly, time taken to cover 8 km upstream in 1hour can be written as,

8/ (x – y) = 1

8 = 1(x – y)

⇒ x – y = 8 …………….. (ii)

Hence, by solving (i) and (ii) we get the required solution

On adding (i) and (ii) we get,

2x = 20

⇒ x = 10

Now, putting the value of x in (i), we find y

10 + y = 12

⇒ y = 2

Therefore, the speed of sailor is 10km/hr and the speed of the current is 2km/hr.

3. The boat goes 30km upstream and 44km downstream in 10 hours. In 13 hours, it can go 40km upstream and 55km downstream. Determine the speed of stream and that of the boat in still water.

Solution:

Let's assume,

The speed of the boat in still water as x km/hr

And,

The speed of the stream as y km/hr

We know that,

Speed of the boat in upstream = (x – y) km/hr

Speed of the boat in downstream = (x + y) km/hr

So,

Time taken to cover 30 km upstream = 30/ (x − y) hr [∵ time = distance/ speed]

Time taken to cover 44 km downstream =44/ (x + y) hr [∵ time = distance/ speed]

It's given that the total time of journey is 10 hours. So, this can expressed as

30/ (x – y) + 44/ (x + y) = 10 …….. (i)

Similarly,

Time taken to cover 40 km upstream = 40/ (x – y) hr [∵ time = distance/ speed]

Time taken to cover 55 km downstream = 55/ (x + y) hr [∵ time = distance/ speed]

And for this case the total time of the journey is given as 13 hours.

Hence, we can write

40/ (x – y) + 55/ (x + y) = 13 ……. (ii)

Hence, by solving (i) and (ii) we get the required solution

Taking, 1/ (x – y) = u and 1/ (x + y) = v in equations (i) and (ii) we have

30u + 44v = 10

40u + 55v = 10

Which may be re- written as,

30u + 44v – 10 = 0 ……. (iii)

40u + 55v – 13 = 0……… (iv)

Solving these equations by cross multiplication we get,

$$\frac{u}{44x-13-55x-10} = \frac{-v}{30x-13-40x-10} = \frac{1}{30\times55-40\times44}$$

$$u = \frac{-22}{-110}$$

$$v = \frac{10}{110}$$

$$u = \frac{2}{10}$$

$$v = \frac{1}{11}$$

Now,

1/ (x – y) = 2/10

⇒ 1 x 10 = 2(x – y)

⇒ $10 = 2x – 2y$

⇒ $x – y = 5$ ……. (v)

And,

$1/ (x + y) = 1/11$

⇒ $x + y = 11$ ……. (vi)

Again, solving (v) and (vi)

Adding (v) and (vi), we get

$2x = 16$

⇒ $x = 8$

Using x in (v), we find y

$8 – y = 5$

⇒ $y = 3$

Therefore, the speed of the boat in still water is 8 km/hr and the speed of the stream is 3 km/hr.

4. A boat goes 24km upstream and 28km downstream in 6hrs. It goes 30km upstream and 21km downstream in 6.5 hours. Find the speed of the boat in still water and also speed of the stream.

Solution:

Let’s assume,

The speed of the boat in still water as x km/hr

And,

The speed of the stream as y km/hr

We know that,

Speed of the boat in upstream = $(x – y)$ km/hr

Speed of the boat in downstream = $(x + y)$ km/hr

So, time taken to cover 28 km downstream = $28/ (x+y)$ hr [∵ time = distance/ speed]

Time taken to cover 24 km upstream =24/ (x – y) hr [∵ time = distance/ speed]

It's given that the total time of journey is 6 hours. So, this can expressed as

24/ (x – y) + 28/ (x + y) = 6…… (i)

Similarly,

Time taken to cover 30 km upstream = 30/ (x − y) [∵ time = distance/ speed]

Time taken to cover 21km downstream =21/ (x + y) [∵ time = distance/ speed]

And for this case the total time of the journey is given as 6.5 i.e 13/2 hours.

Hence, we can write

30/ (x – y) + 21/ (x + y) = 13/2 ….. (ii)

Hence, by solving (i) and (ii) we get the required solution

Taking, 1/ (x – y) = u and 1/ (x + y) = v in equations (i) and (ii) we have (after rearranging)

24u + 28v – 6 = 0 …… (iii)

30u + 21v – 13/2 = 0 ……. (iv)

Solving these equations by cross multiplication we get,

$$\frac{u}{28x-6.5-21x-6} = \frac{-v}{24x-6.5-30x-6} = \frac{1}{24\times21-30\times28}$$

u = 1/6 and v = 1/14

Now,

u = 1/ (x − y) = 1/ 6

x – y = 6 …. (v)

v = 1/ (x + y) = 1/ 14

x + y = 14……. (vi)

On Solving (v) and (vi)

Adding (v) and (vi), we get

2x = 20

⇒ x = 10

Using x = 10 in (v), we find y

10 + y = 14

⇒ y = 4

Therefore,

Speed of the stream = 4km/hr.

Speed of boat = 10km/hr.

5. A man walks a certain distance with a certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Solution:

Let the actual speed of the man be x km/hr and y be the actual time taken by him in hours.

So, we know that

Distance covered = speed × distance

⇒ Distance = x × y = xy ………………………….. (i)

First condition from the question says that,

If the speed of the man increase by 1/2 km/hr, the journey time will reduce by 1 hour.

Showing this using variables, we have

⇒ When speed is (x + 1/2) km/hr, time of journey = y – 1 hours

Now,

Distance covered = (x + 1/2) x (y – 1) km

Since the distance is the same i.e xy we can equate it, [from (i)]

xy = (x + 1/2) x (y – 1)

And we finally get,

-2x + y – 1 = 0 ………………….. (ii)

From the second condition of the question, we have

If the speed reduces by 1 km/hr then the time of journey increases by 3 hours.

⇒ When speed is (x-1) km/hr, time of journey is (y+3) hours

Since, the distance covered = xy [from (i)]

xy = (x-1)(y+3)

⇒ xy = xy – 1y + 3x – 3

⇒ xy = xy + 3x – 1y – 3

⇒ 3x – y – 3 = 0 ……………… (iii)

From (ii) and (iii), the value of x can be calculated by

(ii) + (iii) ⇒

$x - 4 = 0$

$x = 4$

Now, y can be obtained by using $x = 4$ in (ii)

$-2(4) + y - 1 = 0$

$\Rightarrow y = 1 + 8 = 9$

Hence, putting the value of x and y in equation (i), we find the distance

Distance covered = xy

$= 4 \times 9$

= 36 km

Thus, the distance is 36 km and the speed of walking is 4 km/hr.

6. A person rowing at the rate of 5km/h in still water, takes thrice as much as time in going 40 km upstream as in going 40km downstream. Find the speed of the stream.

Solution:

Let's assume x to be the speed of the stream.

So, we know that

Speed of boat in downstream = $(5 + x)$ and,

Speed of boat in upstream = $(5 - x)$

It is given that,

The distance in one way is 40km.

And,

Time taken during upstream = 3 × time taken during the downstream

Expressing it by equations, we have

$40/ (5 - x) = 3 \times 40/ (5 + x)$ [∵ time = distance/ speed]

By cross multiplication, we get

$(5+x) = 3(5-x)$

$\Rightarrow 5 + x = 3(5 - x)$

$\Rightarrow x + 3x = 15 - 5$

$\Rightarrow x = 10/4 = 2.5$

Therefore, the speed of the stream is 2.5 km/hr.

7. Ramesh travels 760km to his home partly by train and partly by car. He takes 8 hours if he travels 160km by train and the rest by car. He takes 12 minutes more if he travels 240km by train and the rest by car. Find the speed of the train and car respectively.

Solution:

Let's assume,

The speed of the train be x km/hr

The speed of the car = y km/hr

From the question, it's understood that there are two parts

Part 1: When Ramesh travels 160 Km by train and the rest by car.

Part 2: When Ramesh travels 240Km by train and the rest by car.

Part 1,

Time taken by Ramesh to travel 160 km by train = 160/x hrs [∵ time = distance/ speed]

Time taken by Ramesh to travel the remaining (760 – 160) km i.e., 600 km by car =600/y hrs

So, the total time taken by Ramesh to cover 760Km = 160/x hrs + 600/y hrs

It's given that,

Total time taken for this journey = 8 hours

So, by equations its

$160/x + 600/y = 8$

$20/x + 75/y = 1$ [on dividing previous equation by 8] …………………… (i)

Part 2,

Time taken by Ramesh to travel 240 km by train = 240/x hrs

Time taken by Ramesh to travel (760 – 240) = 520km by car = 520/y hrs

For this journey, it's given that Ramesh will take a total of is 8 hours and 12 minutes to finish.

$240/x + 520/y$ = 8hrs 12mins = 8 + (12/60) = 41/5 hr

$240/x + 520/y = 41/5$

$6/x + 13/y = 41/200$ ………. (ii)

Solving (i) and (ii), we get the required solution

Let's take $1/x = u$ and $1/y = v$,

So, (i) and (ii) becomes,

$20u + 75v = 1$ ……….. (iii)

$6u + 13v = 41/200$ ……. (iv)

On multiplying (iii) by 3 and (iv) by 10,

$60u + 225v = 3$

$60u + 130v = 41/20$

Subtracting the above two equations, we get

$(225 – 130)v = 3 – 41/20$

$95v = 19/ 20$

$\Rightarrow v = 19/ (20 \times 95) = 1/100$

$\Rightarrow y = 1/v = 100$

Using $v = 1/100$ in (iii) to find v,

$20u + 75(1/100) = 1$

$20u = 1 – 75/100$

$\Rightarrow 20u = 25/100 = 1/4$

$\Rightarrow u = 1/80$

$\Rightarrow x = 1/u = 80$

So, the speed of the train is 80km/hr and the speed of car is100km/hr.

8) A man travels 600 km partly by train and partly by car. If he covers 400km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if he travels 200km by train and the rest by car, he takes half an hour longer. Find the speed of the train and the speed of the car.

Solution:

Let's assume,

The speed of the train be x km/hr

The speed of the car = y km/hr

From the question, it's understood that there are two parts

Part 1: When the man travels 400 km by train and the rest by car.

Part 2: When Ramesh travels 200 km by train and the rest by car.

Part 1,

Time taken by the man to travel 400km by train = 400/x hrs [∵ time = distance/ speed]

Time taken by the man to travel (600 – 400) = 200km by car = 200/y hrs

Time taken by a man to cover 600km = 400/x hrs + 200/y hrs

Total time taken for this journey = 6 hours + 30 mins = 6 + 1/2 = 13/2

So, by equations its

$400/x + 200/y = 13/2$

$400/x + 200/y = 13/2$

$400/x + 200/y = 13/2$

$200 (2/x + 1/y) = 13/2$

$2/x + 1/y = 13/400$ ….(i)

Part 2,

Time taken by the man to travel 200 km by train = $200/x$ hrs. [∵ time = distance/ speed]

Time taken by the man to travel (600 – 200) = 400km by car = $200/y$ hrs

For the part, the total time of the journey is given as 6hours 30 mins + 30 mins that is 7hrs,

$200/x + 400/y = 7$

$200 (1/x + 2/y) = 7$

$1/x + 2/y = 7/200$ …..(ii)

Taking $1/x = u$, and $1/y = v$,

So, the equations (i) and (ii) becomes,

$2u + v = 13/400$ ….. (iii)

$u + 2v = 7/200$ ……. (iv)

Solving (iii) and (iv), by

(iv) x 2 – (iii) ⇒

$3v = 14/200 – 13/400$

$3v = 1/400 \times (28 – 13)$

$3v = 15/400$

$v = 1/80$

$\Rightarrow y = 1/v = 80$

Now, using v in (iii) we find u,

$2u + (1/80) = 13/400$

$2u = 13/400 – 1/80$

$2u = 8/400$

$u = 1/100$

$\Rightarrow x = 1/u = 100$

Hence, the speed of the train is 100km/hr and the speed of the car is 80km/hr.

9. Places A and B are 80km apart from each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite direction, they meet in 1hour and 20 minutes. Find the speeds of the cars.

Solution:

Let’s consider the car starting from point A as X and its speed as x km/hr.

And, the car starting from point B as Y and its speed as y km/hr.

It's seen that there are two cases in the question:

Case 1: Car X and Y are moving in the same direction

Case 2: Car X and Y are moving in the opposite direction

Let's assume that the meeting point in case 1 as P and in case 2 as Q.

Now, solving for case 1:

The distance travelled by car X = AP

And, the distance travelled by car Y = BP

As the time taken for both the cars to meet is 8 hours,

The distance travelled by car X in 7 hours = 8x km [∵ distance = speed x time]

⇒ AP = 8x

Similarly,

The distance travelled by car Y in 8 hours = 8y km

⇒ BP = 8Y

As the cars are moving in the same direction (i.e. away from each other), we can write

AP – BP = AB

So, 8x – 8y = 80

⇒ x – y = 10 ………………………. (i) [After taking 8 common out]

Now, solving for case 2:

In this case as it's clearly seen that,

The distance travelled by car X = AQ

And,

The distance travelled by car Y = BQ

As the time taken for both the cars to meet is 1 hour and 20 min, ⇒1 + (20/60) = 4/3 hr

The distance travelled by car x in 4/3 hour = 4x/3 km

⇒ AQ = 4x/3

Similarly,

The distance travelled by car y in 4/3 hour = 4y/3 km

⇒ $BQ = 4y/3$

Now, since the cars are moving in the opposite direction (i.e. towards each other), we can write

$AQ + BQ = AB$

⇒ $4x/3 + 4y/3 = 80$

⇒ $4x + 4y = 240$

⇒ $x + y = 60$ …………… (ii) [After taking LCM]

Hence, by solving (i) and (ii) we get the required solution

From (i), we have $x = 10 + y$……. (iii)

Substituting this value of x in (ii).

⇒ $(10 + y) + y = 60$

⇒ $2y = 50$

⇒ $y = 25$

Now, using $y = 30$ in (iii), we get

⇒ $x = 35$

Therefore,

– Speed of car X = 35 km/hr.

– Speed of car Y = 25 km/hr.

Exercise 3.11 Page No: 3.111

1. If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.

Solution:

Let’s assume the length and breadth of the rectangle be x units and y units respectively.

Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

Case 1:

Length is increased by 2 units ⇒ now, the new length is x+2 units

Breadth is reduced by 2 units ⇒ now, the new breadth is y-2 units

And it’s given that the area is reduced by 28 square units i.e. = xy – 28

So, the equation becomes

$(x+2)(y-2) = xy - 28$

$\Rightarrow xy - 2x + 2y - 4 = xy - 28$

$\Rightarrow -2x + 2y - 4 + 28 = 0$

$\Rightarrow -2x + 2y + 24 = 0$

$\Rightarrow 2x - 2y - 24 = 0$ ……… (i)

Case 2:

Length is reduced by 1 unit ⇒ now, the new length is x-1 units

Breadth is increased by 2 units ⇒ now, the new breadth is y+2 units

And, it’s given that the area is increased by 33 square units i.e. = i.e. = xy + 33

So, the equation becomes

$(x-1)(y+2) = xy + 33$

⇒ xy + 2x – y – 2 = x + 33

⇒ 2x – y − 2 − 33 = 0

⇒ 2x – y −35 = 0 ……….. (ii)

Solving (i) and (ii),

By using cross multiplication, we get

$$\frac{x}{(-2*-35)-(-1*-24)} = \frac{y}{(2*-35)-(2*-24)} = \frac{1}{(2*-1)-(2*-2)}$$

$$\frac{x}{70-24} = \frac{-y}{-70+48} = \frac{1}{-2+4}$$

$$\frac{x}{46} = \frac{-y}{-22} = \frac{1}{2}$$

x = 46/2

x = 23

And,

y = 22/2

y = 11

Hence,

The length of the rectangle is 23 units.

The breadth of the rectangle is 11 units.

So, the area of the actual rectangle = length x breadth,

= x×y

= 23 x 11

= 253 sq. units

Therefore, the area of rectangle is 253 sq. units.

2. The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and the breadth is increased by 5 metres. Find the dimensions of the rectangle.

Solution:

Let’s assume the length and breadth of the rectangle be x units and y units respectively.

Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

Case 1

Length is increased by 7 metres ⇒ now, the new length is x+7

Breadth is decreased by 3 metres ⇒ now, the new breadth is y-3

And it's given, the area of the rectangle remains same i.e. = xy.

So, the equation becomes

xy = (x+7)(y−3)

xy = xy + 7y − 3x − 21

3x – 7y + 21 = 0 ………. (i)

Case 2:

Length is decreased by 7 metres ⇒ now, the new length is x-7

Breadth is increased by 5 metres ⇒ now, the new breadth is y+5

And it's given that, the area of the rectangle still remains same i.e. = xy.

So, the equation becomes

xy = (x−7)(y+5)

xy = xy − 7y + 5x − 35

5x – 7y – 35 = 0 ………. (ii)

Solving (i) and (ii),

By using cross-multiplication, we get,

$$\frac{x}{(-7\times-5)-(-7\times21)} = \frac{y}{(3\times-35)-(5\times21)} = \frac{1}{(3\times-7)-(5\times-7)}$$

$$\frac{x}{245+147} = \frac{-y}{-105-105} = \frac{1}{-21+35}$$

$$\frac{x}{392} = \frac{-y}{-210} = \frac{1}{14}$$

x = 392/14

x = 28

And,

y = 210/14

y = 15

Therefore, the length of the rectangle is 28 m. and the breadth of the actual rectangle is 15 m.

3. In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the triangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimension of the rectangle.

Solution:

Let's assume the length and breadth of the rectangle be x units and y units respectively.

Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

According to the question,

Case 1:

Length is increased by 3 metres ⇒ now, the new length is x+3

Breadth is reduced by 4 metres ⇒ now, the new breadth is y-4

And it’s given, the area of the rectangle is reduced by 67 m^2 = xy – 67.

So, the equation becomes

xy – 67 = (x + 3)(y – 4)

xy – 67 = xy + 3y – 4x – 12

4xy – 3y – 67 + 12 = 0

4x – 3y – 55 = 0 —— (i)

Case 2:

Length is reduced by 1 m ⇒ now, the new length is x-1

Breadth is increased by 4 metre ⇒ now, the new breadth is y+4

And it’s given, the area of the rectangle is increased by 89 m^2 = xy + 89.

Then, the equation becomes

xy + 89 = (x -1)(y + 4)

4x – y – 93 = 0 —— (ii)

Solving (i) and (ii),

Using cross multiplication, we get

$$\frac{x}{(-3\times-93)-(-1\times-55)} = \frac{-y}{(4\times-93)-(4\times-55)} = \frac{1}{(4\times-1)-(4\times-3)}$$

$$\frac{x}{279-55} = \frac{-y}{-372+220} = \frac{1}{-4+12}$$

$$\frac{x}{224} = \frac{-y}{-152} = \frac{1}{8}$$

x = 224/8

x = 28

And,

y = 152/8

y = 19

Therefore, the length of rectangle is 28 m and the breadth of rectangle is 19 m.

4. The income of X and Y are in the ratio of 8: 7 and their expenditures are in the ratio 19: 16. If each saves ₹ 1250, find their incomes.

Solution:

Let the income be denoted by x and the expenditure be denoted by y.

Then, from the question we have

The income of X is □ 8x and the expenditure of X is 19y.

The income of Y is □ 7x and the expenditure of Y is 16y.

So, on calculating the savings, we get

Saving of X = 8x – 19y = 1250

Saving of Y = 7x – 16y = 1250

Hence, the system of equations formed are

8x – 19y – 1250 = 0 —– (i)

7x – 16y – 1250 = 0 —– (ii)

Using cross-multiplication method, we have

$$\frac{x}{(-19\times-1250)-(-16\times-1250)} = \frac{-y}{(8\times-1250)-(7\times-1250)} = \frac{1}{(8\times-16)-(7\times-19)}$$

$$\frac{x}{23750-20000} = \frac{-y}{-10000+8750} = \frac{1}{-128+133}$$

$$\frac{x}{3750} = \frac{y}{1250} = \frac{1}{5}$$

x = 3750/5

x = 750

If, x = 750, then

The income of X = 8x

= 8 x 750

= 6000

The income of Y = 7x

= 7 x 750

= 5250

Therefore, the income of X is □ 6000 and the income of Y is □ 5250

5. A and B each has some money. If A gives □ 30 to B, then B will have twice the money left with A. But, if B gives □ 10 to A, then A will have thrice as much as is left with B. How much money does each have?

Solution:

Let's assume the money with A be □ x and the money with B be □ y.

Then, from the question we have the following cases

Case 1: If A gives □ 30 to B, then B will have twice the money left with A.

So, the equation becomes

y + 30 = 2(x – 30)

y + 30 = 2x – 60

2x – y – 60 – 30 = 0

2x – y – 90 = 0 —— (i)

Case 2: If B gives □ 10 to A, then A will have thrice as much as is left with B.

x + 10 = 3(y – 10)

x + 10 = 3y – 10

x – 3y + 10 + 30 = 0

x – 3y + 40 = 0 —— (ii)

Solving (i) and (ii),

On multiplying equation (ii) with 2, we get,

$2x - 6y + 80 = 0$

Subtract equation (ii) from (i), we get,

$2x - y - 90 - (2x - 6y + 80) = 0$

$5y - 170 = 0$

$y = 34$

Now, on using $y = 34$ in equation (i), we find,

$x = 62$

Hence, the money with A is □ 62 and the money with B be □ 34

7. 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?

Solution:

Assuming that the time required for a man alone to finish the work be x days and also the time required for a boy alone to finish the work be y days.

Then, we know

The work done by a man in one day = $1/x$

The work done by a boy in one day = $1/y$

Similarly,

The work done by 2 men in one day = $2/x$

The work done by 7 boys in one day = $7/y$

So, the condition given in the question states that,

2 men and 7 boys together can finish the work in 4 days

$4(2/x + 7/y) = 1$

$8/x + 2/8y = 1$ ———(i)

And, the second condition from the question states that,

4 men and 4 boys can finish the work in 3 days

For this, the equation so formed is

$3(4/x + 4/y) = 1$

$12/x + 12/y = 1$ ———(ii)

Hence, solving (i) and (ii) ⇒

Taking, $1/x = u$ and $1/y = v$

So, the equations (i) and (ii) becomes,

$8u + 28v = 1$

$12u + 12v = 1$

$8u + 28v - 1 = 0$ —— (iii)

$12u + 12v - 1 = 0$ —— (iv)

By using cross multiplication, we get,

$u = 1/15$

$1/x = 1/15$

$x = 15$

And,

$v = 1/60$

$1/y = 1/60$

$y = 60$

Therefore,

The time required for a man alone to finish the work is 15 days and the time required for a boy alone to finish the work is 60 days.

8. In a ΔABC, ∠A = x°, ∠B = (3x – 2)°, ∠C = y°. Also, ∠C – ∠B = 9°. Find the three angles.

Solution:

It's given that,

$\angle A = x^\circ$,

$\angle B = (3x - 2)^\circ$,

$\angle C = y^\circ$

Also given that,

$\angle C - \angle B = 9^\circ$

$\Rightarrow \angle C = 9^\circ + \angle B$

$\Rightarrow \angle C = 9^\circ + 3x^\circ - 2^\circ$

$\Rightarrow \angle C = 7^\circ + 3x^\circ$

Substituting the value for

$\angle C = y^\circ$ in above equation we get,

$y^\circ = 7^\circ + 3x^\circ$

We know that, $\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow x^\circ + (3x^\circ - 2^\circ) + (7^\circ + 3x^\circ) = 180^\circ$

$\Rightarrow 7x^\circ + 5^\circ = 180^\circ$

$\Rightarrow 7x^\circ = 175^\circ$

$\Rightarrow x^\circ = 25^\circ$

Hence, calculating for the individual angles we get,

$\angle A = x^\circ = 25^\circ$

$\angle B = (3x - 2)^\circ = 73^\circ$

$\angle C = (7 + 3x)^\circ = 82^\circ$

Therefore,

$\angle A = 25^\circ$, $\angle B = 73^\circ$ and $\angle C = 82^\circ$.

9. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^\circ$, $\angle B = (y + 3)^\circ$, $\angle C = (2y + 10)^\circ$, $\angle D = (4x - 5)^\circ$. Find the four angles.

Solution:

We know that,
The sum of the opposite angles of cyclic quadrilateral should be 180°.
And, in the cyclic quadrilateral ABCD,

Angles $\angle A$ and $\angle C$ & angles $\angle B$ and $\angle D$ are the pairs of opposite angles.

So,

$\angle A + \angle C = 180^\circ$ and

$\angle B + \angle D = 180^\circ$

Substituting the values given to the above two equations, we have

For $\angle A + \angle C = 180^\circ$

$\Rightarrow \angle A = (2x + 4)^\circ$ and $\angle C = (2y + 10)^\circ$

$2x + 4 + 2y + 10 = 180^\circ$
$2x + 2y + 14 = 180^\circ$
$2x + 2y = 180^\circ – 14^\circ$
$2x + 2y = 166$ —— (i)

And for, $\angle B + \angle D = 180^\circ$, we have

$\Rightarrow \angle B = (y+3)^\circ$ and $\angle D = (4x – 5)^\circ$

$y + 3 + 4x – 5 = 180^\circ$
$4x + y – 5 + 3 = 180^\circ$
$4x + y – 2 = 180^\circ$
$4x + y = 180^\circ + 2^\circ$
$4x + y = 182^\circ$ ——- (ii)
Now for solving (i) and (ii), we perform
Multiplying equation (ii) by 2 to get,
$8x + 2y = 364$ —— (iii)
And now, subtract equation (iii) from (i) to get

$-6x = -198$

$x = -198 / -6$

$\Rightarrow x = 33^\circ$

Now, substituting the value of $x = 33^\circ$ in equation (ii) to find y

$4x + y = 182$

$132 + y = 182$

$y = 182 - 132$

$\Rightarrow y = 50$

Thus, calculating the angles of a cyclic quadrilateral we get:

$\angle A = 2x + 4$

$= 66 + 4$

$= 70^\circ$

$\angle B = y + 3$

$= 50 + 3$

$= 53^\circ$

$\angle C = 2y + 10$

$= 100 + 10$

$= 110^\circ$

$\angle D = 4x - 5$

$= 132 - 5$

$= 127^\circ$

Therefore, the angles of the cyclic quadrilateral ABCD are

$\angle A = 70^\circ$, $\angle B = 53^\circ$, $\angle C = 110^\circ$ and $\angle D = 127^\circ$

10. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Solution:

Let's assume that the total number of correct answers be x and the total number of incorrect answers be y.

Hence, their sum will give the total number of questions in the test i.e. $x + y$

Further from the question, we have two type of marking scheme:

1) When 3 marks is awarded for every right answer and 1 mark deducted for every wrong answer.

According to this type, the total marks scored by Yash is 40. (Given)

So, the equation formed will be

$3x – 1y = 40$ ….. (i)

Next,

2) When 4 marks is awarded for every right answer and 2 marks deducted for every wrong answer.

According to this type, the total marks scored by Yash is 50. (Given)

So, the equation formed will be

$4x – 2y = 50$ …… (ii)

Thus, by solving (i) and (ii) we obtained the values of x and y.

From (i), we get

$y = 3x – 40$ …….. (iii)

Using (iii) in (ii) we get,

$4x – 2(3x – 40) = 50$

$4x – 6x + 80 = 50$

$2x = 30$

$x = 15$

Putting $x = 14$ in (iii) we get,

$y = 3(15) – 40$

$y = 5$

So, $x + y = 15 + 5 = 20$

Therefore, the number of questions in the test were 20.

11. In a ΔABC, ∠A = x°, ∠B = 3x°, ∠C = y°. If 3y – 5x = 30, prove that the triangle is right angled.

Solution:

We need to prove that ΔABC is right angled.

Given:

$\angle A = x^\circ$, $\angle B = 3x^\circ$ and $\angle C = y^\circ$

Sum of the three angles in a triangle is 180° (Angle sum property of a triangle) i.e., $\angle A + \angle B + \angle C = 180^\circ$

$x + 3x + y = 180^\circ$

$4x + y = 180$ —— (i)

From question it's given that, $3y - 5x = 30$ —– (ii)

To solve (i) and (ii), we perform

Multiplying equation (i) by 3 to get,

$12x + 36y = 540$ —– (iii)

Now, subtracting equation (ii) from equation (iii) we get

$17x = 510$

$x = 510/17$

$\Rightarrow x = 30^\circ$

Substituting the value of $x = 30^\circ$ in equation (i) to find y

$4x + y = 180$

$120 + y = 180$

$y = 180 - 120$

$\Rightarrow y = 60^\circ$

Thus the angles $\angle A$, $\angle B$ and $\angle C$ are calculated to be

$\angle A = x^\circ = 30^\circ$

$\angle B = 3x^\circ = 90^\circ$

$\angle C = y^\circ = 60^\circ$

A right angled triangle is a triangle with any one side right angled to other, i.e., 90° to other.

And here we have,

$\angle B = 90^\circ$.

Therefore, the triangle ABC is right angled. Hence proved.

12. The car hire charges in a city comprise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is □ 89 and for a journey of 20 km, the charge paid is □ 145. What will a person have to pay for travelling a distance of 30 km?

Solution:

Let the fixed charge of the car be □ x and,

Let the variable charges of the car be □ y per km.

So according to the question, we get 2 equations

x + 12y = 89 —— (i) and,

x + 20y = 145 —— (ii)

Now, by solving (i) and (ii) we can find the charges.

On subtraction of (i) from (ii), we get,

-8y = -56

y = −56 − 8

⇒ y = 7

So, substituting the value of y = 7 in equation (i) we get

x + 12y = 89

x + 84 = 89

x = 89 – 84

⇒ x = 5

Thus, the total charges for travelling a distance of 30 km can be calculated as: x + 30y

⇒ x + 30y = 5 + 210 = □ 215

Therefore, a person has to pay □ 215 for travelling a distance of 30 km by the car.

Frequently Asked Questions

Find the value of k for which the system of equations kx + 2y = 5 and 3x + y = 1 has a unique solution.

The given system of equations is:
kx + 2y – 5 = 0
3x + y – 1 = 0
The above equations are of the form
a1x + b1y − c1 = 0
a2x + b2y − c2 = 0
Here, a1 = k, b1 = 2, c1 = −5
a2 = 3, b2 = 1, c2 = -1

So according to the question,
For unique solution, the condition is
$a_1 / a_2 \neq b_1 / b_2$
$k/3 \neq 2/1$

$\Rightarrow k \neq 6$

Hence, the given system of equations will have unique solution for all real values of k other than 6.

5 pens and 6 pencils together cost ▪ 9 and 3 pens and 2 pencils cost ▪ 5. Find the cost of 1 pen and 1 pencil.

Let’s assume the cost of a pen and pencil be ▪ x and ▪ y respectively.
Then, forming equations according to the question
$5x + 6y = 9$ … (i)
$3x + 2y = 5$ … (ii)
On multiplying equation (i) by 2 and equation (ii) by 6, we get
$10x + 12y = 18$ … (iii)
$18x + 12y = 30$ … (iv)
Now on subtracting equation (iii) from equation (iv), we get
$18x – 10x + 12y – 12y = 30 – 18$
$8x = 12$
$x = 3/2 = 1.5$
Putting $x = 1.5$ in equation (i), we find y
$5(1.5) + 6y = 9$
$6y = 9 – 7.5$
$y = (1.5)/ 6 = 0.25$
Therefore, the cost of one pen = ▪ 1.50 and so the cost of one pencil = ▪ 0.25

7 audio cassettes and 3 videocassettes cost ▪ 1110, while 5 audio cassettes and 4 videocassettes cost ▪ 1350. Find the cost of audio cassettes and a video cassette.

Let’s assume the cost of an audio cassette and that of a video cassette be ▪ x and ▪ y, respectively. Then forming equations according to the question, we have
$7x + 3y = 1110$ … (i)
$5x + 4y = 1350$ … (ii)
On multiplying equation (i) by 4 and equation (ii) by 3,
We get,
$28x + 12y = 4440$ … (iii)
$15x + 4y = 4050$ … (iv)
Subtracting equation (iv) from equation (iii),
$28x – 13x + 12y – 12y = 4440 – 4050$
$13x = 390$

$\Rightarrow x = 30$

On substituting $x = 30$ in equation (i)
$7(30) + 3y = 1110$
$3y = 1110 – 210$
$y = 900/ 3$

$\Rightarrow y = 300$

Therefore, it’s found that the cost of one audio cassette = ▪ 30
And the cost of one video cassette = ▪ 300

Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Let’s assume the number of pens and pencils be x and y, respectively.
Forming equations according to the question, we have
$x + y = 40$ … (i)
$(y+5) = 4(x-5)$
$y + 5 = 4x – 20$
$5 + 20 = 4x – y$
$4x – y = 25$ … (ii)
Adding equation (i) and (ii),
We get,
$x + 4x = 40 + 25$
$5x = 65$

$\Rightarrow x = 13$

Putting x=13 in equation (i), we get
$13 + y = 40$

$\Rightarrow y = 40 – 13 = 27$

Therefore, it’s found that the number of pens Reena has is 13
And, the number of pencils Reena has is 27.

4 tables and 3 chairs, together, cost ▪ 2250 and 3 tables and 4 chairs cost ▪ 1950. Find the cost of 2 chairs and 1 table.

Let’s assume the cost of 1 table is ▪ x and cost of 1 chair is ▪ y.
Then, according to the question
$4x + 3y = 2250$ … (i)
$3x + 4y = 1950$ … (ii)
On multiplying (i) with 3 and (ii) with 4,
We get,
$12x + 9y = 6750$ … (iii)
$12x + 16y = 7800$ … (iv)
Now, subtracting equation (iv) from (iii),
We get,
$-7y = -1050$
$y = 150$
Using y = 150 in (i), we find x
$4x + 3(150) = 2250$
$4x = 2250 – 450$
$x = 1800/ 4$

$\Rightarrow x = 450$

From the question, it’s required to find the value of $(x + 2y) \Rightarrow 450 + 2(150) = 750$

Therefore, the total cost of 2 chairs and 1 table is ▪ 750.

3 bags and 4 pens together cost ▪ 257 whereas 4 bags and 3 pens together cost ▪ 324. Find the total cost of 1 bag and 10 pens.

Let the cost of a bag and a pen be ▪ x and ▪ y, respectively.
Then, according to the question
$3x + 4y = 257$ … (i)
$4x + 3y = 324$ … (ii)
On multiplying equation (i) by 3 and (ii) by 4,
We get,

9x + 12y = 770 … (iii)
16x + 12y = 1296 … (iv)
Subtracting equation (iii) from (iv), we get
16x – 9x = 1296 – 771
7x = 525
x = 525/7 = 75
Hence, the cost of a bag = 75
Substituting x = 75 in equation (i),
We get,
3 x 75 + 4y = 257
225 + 4y = 257
4y = 257 – 225
4y = 32
y = 32/4 = 8
Hence, the cost of a pen = 8

From the question, it’s required to find the value of (x + 10y) ⇒ 75 +10(8) = 20

Therefore, the total cost of 1 bag and 10 pens = 75 + 80 = 155.