RD SHARMA Solutions for Class 9 Maths Chapter 20 - Surface Areas and Volume of A Right Circular Cone

Chapter 20 - Surface Areas and Volume of A Right Circular Cone Exercise 20.24

Question 1

The number of surfaces of a cone has, is

(a) 1

(b) 2

(c) 3

(d) 4

Solution 1

A cone has two surfaces as follows: one curved surface and another bottom surface.

Hence, correct option is (b).

Question 2

The area of the curved surface of a cone of radius 2r and slant height $\frac{l}{2}$, is

(a) πrl

(b) $2\pi rl$

(c) $\frac{1}{2}\pi rl$

(d) $\pi(r + l)\, r$

Solution 2

Curved surface area of a cone of radius 'r' and slant height 'l' $= \pi rl$

Now, if $r' = 2r$ and $l' = \frac{l}{2}$, then

$$C.S.A. = \pi(2r)\left(\frac{l}{2}\right) = \pi rl$$

Hence, correct option is (a).

Question 3

The total surface area of a cone of radius $\frac{r}{2}$ and length 2l, is

(a) $2\pi r(l + r)$

(b) $\pi r\left(l + \frac{r}{4}\right)$

(c) $\pi r(l + r)$

(d) $2\pi rl$

Solution 3

Total surface Area of a cone $= \pi R(L + R)$

Where, R = Radius and L = Slant height

$$\therefore T.S.A. = \pi\left(\frac{r}{2}\right)\left(2l + \frac{r}{2}\right) = \pi r\left(l + \frac{r}{4}\right)$$

Hence, correct option is (b).

Question 4

A solid cylinder is melted and cast into a cone of same radius. The heights of the cone and cylinder are in the ratio

(a) 9 : 1
(b) 1 : 9
(c) 3 : 1
(d) 1 : 3

Solution 4

Volume of cylinder = Volume of cone

$\Rightarrow \pi r^2 h_1 = \frac{1}{3}\pi r^2 h_2$ (Let Radius be r for both)

$\Rightarrow \frac{h1}{h2} = \frac{1}{3}$

$\Rightarrow \frac{h_2(\text{cone})}{h_1(\text{cylinder})} = \frac{3}{1}$

Hence, correct option is (c).

Question 5

If the radius of the base of a right circular cone is 3r and its height is equal to the radius of the base, then its volume is

(a) $\frac{1}{3}\pi r^3$

(b) $\frac{2}{3}\pi r^3$

(c) $3\pi r^3$

(d) $9\pi r^3$

Solution 5

Volume of a cone $= \frac{1}{3}\pi R^2 H$

where, R = Radius of Base, H = height

If R = 3r, H = R = 3r,

Then $V = \frac{1}{3} \times \pi \times (3r)^2 \times 3r = 9\pi r^3$

Heans, correct option is (d).

Question 6

If the volumes of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, then the ratio of their heights, is

(a) 1 : 5
(b) 5 : 4
(c) 5 : 16
(d) 25 : 64

Solution 6

Let the volume of 1^{st} cone $= \frac{1}{3}\pi R_1^2 H_1 = V_1$

Let the volume of 2^{nd} cone $= \frac{1}{3}\pi R_2^2 H_2 = V_2$

$$\frac{V_1}{V_2} = \frac{R_1^2 H_1}{R_2^2 H_2} = \frac{1}{4} \quad \text{and} \quad \frac{d_1}{d_2} = \frac{4}{5} \Rightarrow \frac{2R_1}{2R_2} = \frac{4}{5} \Rightarrow \frac{R_1}{R_2} = \frac{4}{5}$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 \frac{H_1}{H_2} = \frac{1}{4}$$

$$\Rightarrow \frac{H_1}{H_2} = \frac{25}{64}$$

Hence, correct option is (d).

Question 7

The curved surface area of one cone is twice that of the other while the slant height of the latter is twice that of the former. The ratio of their radii is

(a) 2 : 1
(b) 4 : 1
(c) 8 : 1
(d) 1 : 1

Solution 7

Let the Curved Surface Area of one cone $= \pi R_1(L_1)$

where, L = Slant height, R = Radius

Curved Surface Area of other cone $= \pi R_2(L_2)$

Now, $\pi R_1(L_1) = 2\,\pi R_2(L_2)$ and $L_2 = 2L_1$

$\Rightarrow \pi R_1 L_1 = 2\pi R_2 (2L_1)$

$$\Rightarrow \frac{R_1}{R_2} = \frac{4}{1}$$

Hence, correct option is (b).

Chapter 20 - Surface Areas and Volume of A Right Circular Cone

Exercise 20.25

Question 8

If the height and radius of a cone of volume V are doubled, then the volume of the cone, is

(a) 3V
(b) 4V
(c) 6V
(d) 8V

Solution 8

$V = \frac{1}{3}\pi R^2 H$

If $R' = 2R$ and $H' = 2H$, then

$V' = \frac{1}{3}\pi(2R)^2(2H)$

$= 8\left(\frac{1}{3}\pi R^2 H\right)$

$= 8V$

$\Rightarrow V' = 8V$

Hence, correct option is (d).

Question 9

The ratio of the volume of a right circular cylinder and a right circular cone of the same base and height, is

(a) 1 : 3

(b) 3 : 1

(c) 4 : 3

(d) 3 : 4

Solution 9

Volume of a right circular cylinder of Height H and Radius R $= \pi R^2 H = V_1$

Volume of a cone of height H and Radius R $= \frac{1}{3}\pi R^2 H = V_2$

$\frac{V_1}{V_2} = \frac{\pi R^2 H}{\frac{1}{3}\pi R^2 H} = \frac{3}{1} = 3:1$

Hence, correct option is (b).

Question 10

A right circular cylinder cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is

(a) 3 : 5

(b) 2 : 5

(c) 3 : 1

(d) 1 : 3

Solution 10

Volume of a Right circular cylinder $= \pi R_1^2 H_1 = V_1$

Volume of a Right circular cone $= \frac{1}{3}\pi R_2^2 H_2 = V_2$

If $V_1 = V_2$ and $R_1 = R_2$, then

$\pi R_1^2 H_1 = \frac{1}{3}\pi (R_1)^2 H_2$

$\Rightarrow \frac{H_1}{H_2} = \frac{1}{3}$

Hence, correct option is (d).

Question 11

The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, the ratio of their curved surface areas, is

(a) 4 : 5

(b) 25 : 16

(c) 16 : 25
(d) 5 : 4

Solution 11

Curved Surface Area of cone $= \pi RL$
where, R = Radius and L = Slant height
Ratio of C.S.A. of two cones,
$C.S.A_1 : C.S.A_2 = \pi R_1 L_1 : \pi R_2 L_2$

If $\frac{L_1}{L_2} = \frac{5}{4}$, then $\frac{2R_1}{2R_2} = 1 \quad (\because R_1 = R_2)$

$$\Rightarrow \frac{C.S.A_1}{C.S.A_2} = \frac{\pi R_1 L_1}{\pi R_2 L_2}$$

$$\Rightarrow \frac{C.S.A_1}{C.S.A_2} = \frac{R_1}{R_1} \times \frac{5}{4}$$

$$\Rightarrow \frac{C.S.A_1}{C.S.A_2} = \frac{5}{4}$$

Hence, correct option is (d).

Question 12

If the heights of two cones are in the ratio of 1 : 4 and the radii of their bases are in the ratio 4 : 1, then the ratio of their volumes is
(a) 1 : 2
(b) 2 : 3
(c) 3 : 4
(d) 4 : 1

Solution 12

Let the volume of cone 1 $= \frac{1}{3}\pi R_1^2 H_1 = V_1$

Let the volume of cone 2 $= \frac{1}{3}\pi R_2^2 H_2 = V_2$

$$\frac{V_1}{V_2} = \frac{\cancel{\frac{1}{3}}\pi R_1^2 H_1}{\cancel{\frac{1}{3}}\pi R_2^2 H_2} = \frac{R_1^2}{R_2^2}\,\frac{H_1}{H_1} \quad \left\{\frac{H_1}{H_2} = \frac{1}{4},\ \frac{R_1}{R_2} = \frac{4}{1} \quad \text{(given)}\right\}$$

$$= \left(\frac{4}{1}\right)^2 \left(\frac{1}{4}\right)$$

$$= \frac{4}{1}$$

$$\Rightarrow V_1 : V_2 = 4 : 1$$

Hence, correct option is (d).

Question 13

The slant height of a cone is increased by 10%. If the radius remains the same, the curved surface area is increased by
(a) 10%
(b) 12.1%
(c) 20%
(d) 21%

Solution 13

C.S.A of a cone $= \pi rl$

If $l' = l + 10\%$ of $l = l + \frac{10}{100} \times l = l + \frac{l}{10}$

And, $r' = r$

C.S.A. $= \pi r\left(l + \frac{l}{10}\right) = \frac{11}{10}\pi rl$

So, increase in C.S.A. $= \frac{\frac{11}{10}\pi rl - \pi rl}{\pi rl} \times 100\% = 10\%$

Hence, correct option is (a).

Question 14

The height of a solid cone is 12 cm and the area of the circular base is $64\prod$ cm^2. A plane parallel to the base of the cone cuts through the cone 9 cm above the vertex of the cone, thee area of the base of the new cone so formed is

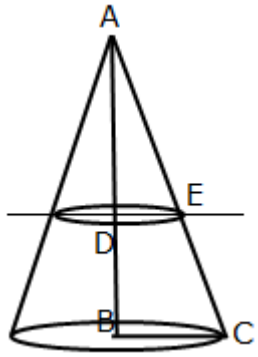
(a) $9\prod$ cm^2

(b) $16\prod$ cm^2

(c) $25\prod$ cm^2

(d) $36\prod$ cm^2

Solution 14



AB = 12 cm

Area of circular Base $= \pi r^2 = 64\pi$

$\Rightarrow r = 8$ cm

AD = 9 cm

Consider $\triangle ADE$ and $\triangle ABC$,

$\angle DAE = \angle BAC$ (common)

$\angle ADE = \angle ABC$ (each 90°)

$\angle AED = \angle ACB$ (third angle will also be same)

Hence $\triangle ADE \sim \triangle ABC$

So $\frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{9}{12} = \frac{DE}{8}$

$\Rightarrow DE = 6$ cm

Radius of Base of new cone = 6 cm

$\Rightarrow$ Area $= \pi(6)^2 = 36\pi$ cm^2

Hence, correct option is (d).

Question 15

If the base radius and the height of a right circular cone are increased by 20%, then the percentage increase in volume is approximately

(a) 60

(b) 68

(c) 73

(d) 78

Solution 15

Let the radius of the cone = R and height = H

Then, Volume $= \frac{1}{3}\pi R^2 H$

Now, $R' = R + 20\%$ of $R = R + \frac{R}{5} = \frac{6R}{5}$

$H' = H + 20\%$ of $H = H + \frac{H}{5} = \frac{6H}{5}$

New volume, $V' = \frac{1}{3}\pi R'^2 H'$

$$= \frac{1}{3}\pi\left(\frac{6R}{5}\right)^2\left(\frac{6H}{5}\right)$$

$$= \frac{216}{125}\left(\frac{1}{3}\pi R^2 H\right)$$

$$= \frac{216}{125}V$$

$$\%\text{ increase in Volume} = \frac{V' - V}{V} \times 100$$

$$= \frac{\frac{216v}{125} - V}{v} \times 100$$

$$= \frac{91}{125} \times 100$$

$$= 72.8\%$$

$$\approx 73\%$$

Hence, correct option is (c).

Question 16

If h, S and V denote respectively the height, curved surface area and volume of a right circular cone, then $3\prod Vh^3 - S^2h^2 + 9V^2$ is equal to

(a) 8

(b) 0

(c) $4\prod$

(d) $32\prod^2$

Solution 16

For a cone,

$V = \frac{1}{3}\pi R^2 h$

S = Curved Surface Area $= \pi RL$

$L = \sqrt{h^2 + R^2}$

$3\pi Vh^3 - S^2h^2 + 9V^2 = 3\pi\left(\frac{1}{3}\pi R^2 h\right)h^3 - \pi^2R^2(h^2 + R^2)\,h^2 + \cancel{9} \times \frac{1}{\cancel{9}}\pi^2R^4h^2$

$= \cancel{\pi^2R^2h^4} - \cancel{\pi^2R^2h^4} - \cancel{\pi^2R^4h^2} + \cancel{\pi^2R^4h^2}$

$= 0$

Hence, correct option is (b).

Question 17

If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of upper and lower part is

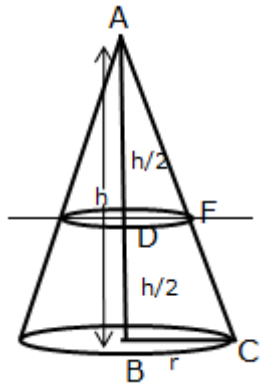
(a) 1 : 2

(b) 2 : 1

(c) 1 : 7

(d) 1 : 8

Solution 17



$\frac{AD}{AB} = \frac{DF}{BC}$

$\Rightarrow \frac{h/2}{h/2 + h/2} = \frac{DF}{BC} \Rightarrow \frac{DF}{BC} = \frac{1}{2} \Rightarrow DF = \frac{BC}{2} = \frac{r}{2}$

Volume of full cone $= \frac{1}{3}\pi r^2 h$

volume of small cone formed $= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2\frac{h}{2} = \frac{1}{3}\pi\frac{r^2}{4}\frac{h}{2} = \frac{1}{8}\left(\frac{\pi r^2 h}{3}\right)$

Ratio of volume of two parts

$= \frac{\text{Volume of small cone}}{\text{Volume of full cone} - \text{Volume of small cone}}$

$= \frac{\frac{1}{8}\left(\frac{\pi r^2 h}{3}\right)}{\frac{\pi r^2 h}{3} - \frac{\pi r^2 h}{8 \times 3}} = \frac{1}{7}$

Hence, correct option is (c).

Chapter 20 - Surface Areas and Volume of A Right Circular Cone

Exercise Ex. 20.1

Question 1

Find the curved surface area of a cone, if its slant height is 60 cm and the radius of its base is 21 cm.

Solution 1

Curved surface area of a cone $= \pi rl$

Given $r = 21\,cm$

$l = 60\,cm$

$\therefore$ Curved surface area $= \pi \times 21 \times 60$

$$= \frac{22}{7} \times 21 \times 60$$

$$= 3960\,cm^2$$

Question 2

The radius of a cone is 5cm and vertical height is 12 cm. Find the area of the curved surface.

Solution 2

Radius of a cone $= 5cm = r$

Height of a cone $= 12cm = h$

Slant height of cone

$$= \sqrt{r^2 + h^2} = \sqrt{5^2 + 12^2} = 13cm$$

$\therefore$ Curved surface area of cone $= \pi rl$

$$= \frac{22}{7} \times 5 \times 13$$

$$= 204.28cm^2$$

Question 3

The radius of a cone is 7cm and area of curved surface is $176cm^2$. Find the slant height.

Solution 3

Given radius 'r' of cone $= 7cm$

Let 'l' be the slant height of a cone

$\therefore$ Curved surface area $= \pi rl$

$\Rightarrow 176 = \pi \times 7 \times l$

$$\Rightarrow l = \frac{176}{7\pi} = \frac{176}{7 \times \frac{22}{7}} = 8cm$$

Question 4

The height of a cone is $21cm$. Find the area of the base if the slant height is $28cm$.

Solution 4

Since slant height 'l' = $28cm$
Height 'h' of cone = $21cm$

$\therefore$ Radius 'r' of cone = $\sqrt{28^2 - 21^2}$ [by pythagoras theorem]
$= 7\sqrt{7}cm$

$\therefore$ Area of base = πr^2
$= \frac{22}{7} \times \left(7\sqrt{7}\right)^2$
$= \frac{22}{7} \times 7 \times 7 \times 7 = 1078cm^2$

Question 5

Find the total surface area of a right circular cone with radius $6cm$ and height $8cm$.

Solution 5

Total surface area = $\pi rl + \pi r^2$

Now $l = \sqrt{h^2 + r^2}$ [by pythagoras theorem]

Here $r = 6cm$ & $h = 8cm$
$\Rightarrow l = \sqrt{6^2 + 8^2}$
$= 10cm$

$\therefore$ Total surface area = $\pi rl + \pi r^2$
$= \left(\frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 6 \times 6\right)$
$= \frac{1320}{7} + \frac{792}{7}$
$= 301.71cm^2$

Question 6

Find its curved surface area of a cone with base radius 5.25 cm and slant height 10 cm.

Solution 6

Radius of base of cone = 5.25 cm
Slant height of cone = 10 cm
CSA of cone = $\pi rl = \left(\frac{22}{7} \times 5.25 \times 10\right) cm^2 = (22 \times 0.75 \times 10)\, cm^2 = 165 cm^2$
Thus, the curved surface area of cone is 165 cm^2.

Question 7

Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Solution 7

Radius of base of cone $= \left(\frac{24}{2}\right)$ m $= 12$ cm

Slant height of cone $= 21$ m

Total surface area of cone $= \pi r^2 + \pi rl = \pi r(r+1)$

$$= \left[\frac{22}{7} \times 12 \times (12+21)\right] m^2$$
$$= \left[\frac{22}{7} \times 12 \times 33\right] m^2$$
$$= 1244.57\ m^2$$

Question 8

The area of the curved surface of a cone is $60\pi cm^2$. If the slant height of the cone be $8cm$, find the radius of the base.

Solution 8

Given curved surface area of cone $= 60\pi cm^2$

$\because$ Slant height 'l' of cone $= 8cm$

i.e., $\pi rl = 60\pi$

$\Rightarrow \pi \times r \times 8 = 60\pi$

$\Rightarrow r = \frac{60}{8} = \frac{15}{2} = 7.5$

$\therefore$ Radius of cone $= 7.5cm$.

Question 9

The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What is its slant height? (Use π = 22/7).

Solution 9

Given diameter = $70cm$

$\Rightarrow 2r = 70cm$

$\Rightarrow r = 35cm$

Now curved surface area = $4070cm^2$

$\Rightarrow \pi rl = 4070$

Where r = radius of the cone

l = slant height of the cone.

$\therefore \pi rl = 4070$

$\Rightarrow \frac{22}{7} \times 35 \times l = 4070$

$\Rightarrow l = \frac{4070 \times 7}{22 \times 35} = 37cm$

$\therefore$ Slant height of the cone = $37cm$

Question 10

The radius and slant height of a cone are in the ratio of 4 : 7. If its curved surface area is $792cm^2$, find its radius. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution 10

Let the radius $4x$ and slant height $7x$

Now, curved surface area = 792

$\Rightarrow \pi \times 4x \times 7x = 792$

$\Rightarrow \frac{22}{7} \times 4x \times 7x = 792$

$\Rightarrow 88x^2 = 792$

$\Rightarrow x^2 = \frac{792}{88} = 9$

$\Rightarrow x = 3$

$\therefore$ Radius = $4x = 4 \times 3 = 12cm$

Question 11

A joker's cap is in the form of right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Solution 11

Radius (r) of conical cap = 7 cm

Height (h) of conical cap = 24 cm

Slant height (l) of conical cap $= \sqrt{r^2+h^2} = \left[\sqrt{(7)^2+(24)^2}\right]\text{cm} = 25\text{cm}$

CSA of 1 conical cap = $\pi rl = \left(\frac{22}{7}\times 7\times 25\right)\text{cm}^2 = 550\ \text{cm}^2$

CSA of 10 such conical caps = $(10 \times 550)\ \text{cm}^2 = 5500\ \text{cm}^2$

Thus, 5500 cm^2 sheet will be required to make the 10 caps.

Question 12

Find the ratio of the curved surface area of two cones if their diameters of the bases are equal and slant heights are in the ratio 4:3.

Solution 12

Since diameter of two cones are equal

$\therefore$ Their radius are equal

$\therefore r_1 = r_2 = r\,(say)$

Let ratio be x.

$\therefore$ Slant height 'l_1' of 1st cone = $4x$

Similarly, slant height 'l_2' of 2nd cone = $3x$

$$\therefore \frac{C.S.A_1}{C.S.A_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{\pi \times r \times 4x}{\pi \times r \times 3x} = \frac{4}{3}$$

Question 13

There are two cones. The curved surface area of one is twice that of the other. The slant height of the later is twice that of the former. Find the ratio of their radii.

Solution 13

Let curved surface area of IInd cone = c

$\therefore$ curved surface area of Ist cone = $2c$.

and slant height of Ist cone = l

$\therefore$ Slant height of IInd cone = $2l$.

$$\therefore \frac{\text{curved surface area of I}^{st}\text{ cone}}{\text{curved surface area of II}^{nd}\text{ cone}} = \frac{2c}{c} = \frac{2}{1}$$

$$\Rightarrow \frac{\pi r_1 l}{\pi r_2 (2l)} = \frac{2}{1}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{4}{1}$$

i.e., Ratio of r_1 and r_2 is $(4:1)$

Question 14

The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, find the ratio of their curved surfaces.

Solution 14

Since diameters of two cones are equal.

$\therefore$ Their radius are also equal. *ie* $r_1 = r_2$

Let the ratio of slant height be x.

$\therefore l_1 = 5x$ and $l_2 = 4x$.

$\therefore$ Ratio of curved surface area $= \frac{C_1}{C_2}$

$\therefore \frac{C_1}{C_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2}$

$= \frac{\pi r_1 5x}{\pi r_2 4x} = \frac{5}{4}$

$\therefore$ Ratio of curved surface area = 5 : 4

Question 15

Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find the radius of the base and total surface area of the cone.

Solution 15

(i) Slant height of cone = 14 cm

Let radius of circular end of cone be r.

CSA of cone = πrl

$$308\ cm^2 = \left(\frac{22}{7} \times r \times 14\right)\ cm$$

$$\Rightarrow r = \left(\frac{308}{44}\right)\ cm = 7\ cm$$

Thus, the radius of circular end of the cone is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base

$= \pi rl + \pi r^2$

$$= \left[308 + \frac{22}{7} \times (7)^2\right]\ cm^2$$

$$= (308 + 154)\ cm^2$$

$$= 462\ cm^2$$

Thus, the total surface area of the cone is 462 cm^2.

Question 16

The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs 210 per 100 m^2.

Solution 16

Slant height (l) of conical tomb = 25 m

Base radius (r) of tomb = $\frac{14}{2}$ m = 7 m

CSA of conical tomb = $\pi rl = \left(\frac{22}{7} \times 7 \times 25\right) m^2 = 550 m^2$

Cost of white-washing 100 m^2 area = Rs 210

Cost of white-washing 550 m^2 area = Rs $\left(\frac{210 \times 500}{100}\right)$ = Rs 1155

Thus, the cost of white washing the conical tomb is Rs 1155.

Question 17

A conical tent is 10 m high and the radius of its base is 24 m. Find slant height of the tent. If the cost of 1 m^2 canvas is Rs 70, find the cost of the canvas required to make the tent,

Solution 17

(i) Height (h) of conical tent = 10 m

Radius (r) of conical tent = 24 m

Let slant height of conical tent be l.

$l^2 = h^2 + r^2 = (10\ m)^2 + (24\ m)^2 = 676\ m^2$

$\therefore l = 26$ m

Thus, the slant height of the conical tent is 26 m.

(ii) CSA of tent = $\pi rl = \left(\frac{22}{7} \times 24 \times 26\right) m^2 = \frac{13728}{7} m^2$

Cost of 1 m^2 canvas = Rs 70

Cost of $\frac{13728}{7}$ m^2 canvas = Rs $\left(\frac{13728}{7} \times 70\right)$

= Rs 137280

Thus, the cost of canvas required to make the tent is Rs 137280.

Question 18

The circumference of the base of a $10m$ height conical tent is 44 metres.

calculate the length of canvas used in making the tent if width of the canvas is $2m$. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution 18

Curved surface area of cone = πrl

Given circumference = $2\pi r$

$\Rightarrow 2\pi r = 44$

$\Rightarrow 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow r = 7m$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 10^2} = \sqrt{49 + 100} = \sqrt{149}m$

$\therefore$ curved surface area of tent = πrl

$= \frac{22}{7} \times 7 \times \sqrt{149}$

$= 22\sqrt{149}$

$\therefore$ The length of canvas used in making tent = $\frac{\text{Area of canvas}}{\text{width of canvas}}$

$= \frac{22\sqrt{149}}{2} = 11\sqrt{149}m = 134.2m$

Question 19

What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use π = 3.14).

Solution 19

Height (h) of conical tent = 8 m

Radius (r) of base of tent = 6 m

Slant height (l) of tent = $\sqrt{r^2 + h^2} = \left(\sqrt{6^2 + 8^2}\right)\text{m} = \left(\sqrt{100}\right)\text{m} = 10\text{m}$

CSA of conical tent = πrl = (3.14 $\times$ 6 $\times$ 10) m^2 = 188.4 m^2

Let length of tarpaulin sheet required be L.

As 20 cm will be wasted so, effective length will be (L - 0.2 m)

Breadth of tarpaulin = 3 m

Area of sheet = CSA of tent

[(L - 0.2 m) $\times$ 3] m = 188.4 m^2

L - 0.2 m = 62.8 m

L = 63 m

Thus, the length of the tarpaulin sheet will be 63 m.

Question 20

A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$).

Solution 20

Radius (r) of cone $= \frac{40}{2} = 20$ cm $= 0.2$ m

Height (h) of cone $= 1$ m

Slant height (l) of cone $= \sqrt{h^2+r^2} = [(1)^2 + (0.2)^2]\text{m} = (\sqrt{1.04})\text{m} = 1.02\text{m}$

CSA of each cone $= \pi rl = (3.14 \times 0.2 \times 1.02)\ m^2 = 0.64056\ m^2$
CSA of 50 such cones $= (50 \times 0.64056)\ m^2 = 32.028\ m^2$
Cost of painting 1 m^2 area = Rs 12
Cost of painting 32.028 m^2 area = Rs (32.028×12) = Rs 384.336
Thus, it will cost Rs 384.34 (approximately) in painting the 50 hollow cones.

Question 21

A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the radius of each to the height of each is 3:4.

Solution 21

Let radius of cone $= r$
Also, radius of cylinder $= r$
Height of cone $= h$
And, height of cylinder $= h$

Let c_1 be the curved surface area of cone

$\therefore c_1 = \pi r\sqrt{r^2+h^2}$

Similarly, c_2 be the curved surface area of cylinder

$\therefore c_2 = 2\pi rh$

According to question $\frac{C_2}{C_1} = \frac{8}{5}$

$\Rightarrow \frac{2\pi rh}{\pi r\sqrt{r^2+h^2}} = \frac{8}{5}$

$\Rightarrow 10h = 8\sqrt{r^2+h^2}$

$\Rightarrow 100h^2 = 64r^2 + 64h^2$

$\Rightarrow 36h^2 = 64r^2$

$\Rightarrow \frac{r^2}{h^2} = \frac{36}{64} \Rightarrow \frac{r}{h} = \frac{6}{8} = \frac{3}{4}$

Hence proved.

Question 22

A tent is in the form of a right circular cylinder surmounted by a cone.
The diameter of cylinder is 24m. The height of the cylindrical portion is 11m while the vertex of the cone is 16m above the ground. Find the area of the canvas required for the tent.

Solution 22

Since diameters of cylinder = $24m$

$\therefore$ Radius of cylinder = $12m$
Also radius of cone = $12m$
Height of cylinder = $11m$
Height of cone = $16 - 11 = 5m$

Slant height of cone = $\sqrt{5^2 + 12^2} = 13m$ $\quad \left[\because l = \sqrt{r^2 + h^2}\right]$

Therefore, area of the canvas required for the tent = $\pi rl + 2\pi rh$

$$= \left[\frac{22}{7} \times 12 \times 13 + 2 \times \frac{22}{7} \times 12 \times 11\right]$$
$$= 490.285 + 829.714$$
$$= 1320m^2$$

Question 23

A circus tent is cylindrical to a height of 3 metres and conical above it. If its diameter is $105m$ and the slant height of the conical portion is $53m$, calculate the length of the canvas $5m$ wide to make the required tent.

Solution 23

Since diameters = $105m$

$\therefore$ Radius of cone = radius of cylindrical portion = $\frac{105}{2}m = 52.5m$

$\therefore$ Curved surface area of circus tent = $\pi rl + 2\pi rh$

$$= \frac{22}{7} \times 52.5 \times 53 + 2 \times \frac{22}{7} \times 52.5 \times 3$$
$$= 8745 + 990$$
$$= 9735m^2$$

$\therefore$ Length of the canvas required for the tent = $\frac{\text{Area of Canvas}}{\text{Width of canvas}} = \frac{9735}{5} = 1947m$

Chapter 20 - Surface Areas and Volume of A Right Circular Cone
Exercise Ex. 20.2

Question 1

Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm

(iii) height 21 cm and slant height 28 cm.

Solution 1

(i) Radius (r) of cone = 6 cm
Height (h) of cone = 7 cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right] \text{ cm}^3 = 264 \text{ cm}^3$

(ii) Radius (r) of cone = 3.5 cm
Height (h) of cone = 12 cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] \text{ cm}^3 = 154 \text{ cm}^3$

(iii)

From $l^2 + r^2 + h^2$, we have

$r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2}\,\text{cm} = 7\sqrt{7}\,\text{cm}$

So, volume of the cone =

$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21 \text{cm}^3$

$= 7546 \text{ cm}^3$

Question 2

Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm
(ii) height 12 cm, slant height 13 cm

Solution 2

(i) Radius (r) of cone = 7 cm
Slant height (l) of cone = 25 cm

$= \sqrt{l^2 - r^2} = \left(\sqrt{25^2 - 7^2}\right)\text{cm} = 24 \text{ cm}$

Height (h) of cone

$= \frac{1}{3}\pi r^2 h = \left[\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right]\text{cm}^3 = 1232 \text{ cm}^3$

Volume of cone

Capacity of the conical vessel = $\left(\frac{1232}{1000}\right)$ litres= 1.232 litres

(ii) Height (h) of cone = 12 cm
Slant height (l) of cone = 13 cm

$= \sqrt{l^2 - r^2} = \left(\sqrt{13^2 - 12^2}\right)\text{cm} = 5 \text{ cm}$

Radius (r) of cone

Volume of cone $= \frac{1}{3}\pi r^2 h = \left[\frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12\right] cm^3 = \left(\frac{2200}{7}\right) cm^3$ = 314.28 cm^3

Capacity of the conical vessel = $\left(\frac{2200}{7000}\right)$ litres = $\frac{11}{35}$ litres.

Question 3

Two cones have their heights in the ratio 1:3 and the radii of their bases in the ratio 3:1. Find the ratio of their volumes.

Solution 3

Let the ratio of height be h.

$\therefore$ Height of I^{st} cone = h

Height of II^{nd} cone = $3h$.

Let the ratio of radii be r

$\therefore$ Radius of I^{st} cone = $3r$

And, radius of II^{nd} cone = r

$\therefore$ Ratio of volume = $\frac{V_1}{V_2}$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

$$= \frac{(3r)^2 \times h}{(r)^2 \times 3h}$$

$$= \frac{9r^2 h}{3r^2 h} = \frac{3}{1}$$

Question 4

The radius and the height of a right circular cone are in the ratio 5:12. If its volume is 314 cubic metre, find the slant height and the radius (Use $\pi = 3.14$).

Solution 4

Let the ratio be x

$\therefore$ Radius 'r' $= 5x$

Height 'h' $= 12x$

$\therefore$ Slant height $= \sqrt{r^2 + h^2} = \sqrt{(5x)^2 + (12x)^2} = 13x$

Now, volume $= 314m^3$ [given]

$\Rightarrow \frac{1}{3}\pi r^2 h = 314$

$\Rightarrow \frac{1}{3} \times 3.14 \times 25x^2 \times 12x = 314$

$\Rightarrow x^3 = \frac{314 \times 3}{3.14 \times 25 \times 12}$

$\Rightarrow x = 1$

$\therefore$ Slant height $= 13x = 13m.$

Radius $= 5x = 5m$

Question 5

The radius and the height of a right circular cone are in the ratio 5 : 12 and its volume is 2512 cubic cm. Find the slant height and radius of the cone. (Use $\pi = 3.14$).

Solution 5

Let the ratio be x

$\therefore$ Radius 'r' $= 5x$

Height 'h' $= 12x$

$\therefore$ Slant height 'l' $= \sqrt{r^2 + h^2} = \sqrt{(5x)^2 + (12x)^2} = 13x$

Now volume $= 2512cm^3$

$\Rightarrow \frac{1}{3} \times \pi \times (5x)^2 \times 12x = 2512$

$\Rightarrow \frac{1}{3} \times 3.14 \times 25x^2 \times 12x = 2512$

$\Rightarrow x^3 = \frac{2512 \times 3}{3.14 \times 25 \times 12}$

$\Rightarrow x = 2$

$\therefore$ Slant height $= 13x = 13 \times 2 = 26cm$

And, radius of the cone $= 5x = 5 \times 2 = 10cm$

Question 6

The ratio of volumes of two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. Find the ratio of their vertical heights.

Solution 6

Let the ratio of volume be 'v'.
Volume of first cone = $4v$

Similarly volume of second cone = $5v$

Let ratio of radius be 'r'
Radius of first cone = $2r$
Radius of second cone = $3r$

$$\therefore \frac{v_1}{v_2} = \frac{4v}{5v} = \frac{4}{5}$$

$$\Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2}\frac{(2r)^2}{(3r)^2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} \times \frac{4r^2}{9r^2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} \times \frac{36}{20} = \frac{18}{10} = \frac{9}{5}$$

$\therefore$ Ratio of their height is 9:5

Question 7

A cylinder and a cone have equal radii of their bases and equal heights. Show that their volumes are in the ratio 3:1.

Solution 7

Let, radius of cone = radius of cylinder = r

Let, height of cone = height of cylinder = h

Let v_1 = volume of cone
v_2 = volume of cylinder

$$\Rightarrow \frac{v_1}{v_2} = \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 h} = \frac{1}{3}$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{3}{1}$$

Hence their volumes are in the ratio 3:1

Question 8

If the radius of the base of a cone is halved, keeping the height same. What is the ratio of the volume of the reduced cone to that of the original cone?

Solution 8

Let radius of cone = r,

Height = h

$\therefore$ Volume 'v_1' $= \frac{1}{3}\pi r^2 h$

In another case,

Radius of cone $= \frac{1}{2}r$,

Height $= h$

$$\therefore \text{Volume } 'v_2' = \frac{1}{3}\pi\left(\frac{1}{2}r\right)^2 h$$

$$= \frac{1}{3}\pi \times \frac{r^2}{4} \times h$$

$$= \frac{1}{12}\pi r^2 h$$

$$\therefore \frac{v_2}{v_1} = \frac{\frac{1}{12}\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{12} = \frac{1}{4}$$

$\therefore$ Ratio will be $(1:4)$

Question 9

A heap of wheat is in the form of a cone of diameter 9 m and height is 3.5 m. Find its volume. How much canvas cloth is required to just cover the heap? (use π = 3.14).

Solution 9

Radius (r) of heap $= \left(\frac{9}{2}\right)$ m = 4.5 m

Height (h) of heap = 3.5 m

Volume of heap $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times 3.14 \times (4.5)^2 \times 3.5\right] \text{ m}^3 = 74.18 \text{ m}^3$$

Slant height (l) $= \sqrt{r^2 + h^2} = \sqrt{(4.5)^2 + (3.5)^2} = 5.70$ m

Area of canvas required = CSA of cone

$$= \pi r l = (3.14 \times 4.5 \times 5.7) \text{ m}^2 = 80.54 \text{ m}^2$$

Question 10

Find the weight of a solid cone whose base is of diameter $14cm$ and vertical height $51cm$, supposing the material of which it is made weighs 10 grams per cubic cm.

Solution 10

Given diameter of cone $= 14cm$

$\therefore$ Radius of cone $= 7cm$

Height of cone $= 51cm$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 51$

$= 2618cm^3$

Now it is given that $1cm^3$ weighs $10gm$.

$\therefore$ $2618cm^3$ weighs $(2618 \times 10)gm$

i.e., $26.180kg$

Question 11

A right angled triangle of which the sides containing the right angle are 6.3 cm and 10 cm in length, is made to turn round on the longer side. Find the volume of the solid, thus generated. Also, find its curved surface area.

Solution 11

It is given that

Radius of cone $= 6.3cm$

Height of cone $= 10cm$

$\therefore$ Slant height of cone $= \sqrt{r^2 + h^2} = \sqrt{(6.3)^2 + (10)^2} = 11.819cm$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (6.3)^2 \times 10$

$= 415.8cm^3$

And curved surface area of cone $= \pi rl$

$= \frac{22}{7} \times 6.3 \times 11.819$

$= 234.01cm^2$

Question 12

Find the volume of the largest right circular cone that can be fitted in a cube whose edge is $14cm$.

Solution 12

For largest right circular cone radius of the base of cone $= \frac{1}{2}$ edge of cube

$= \frac{1}{2} \times 14 = 7cm$

And height of the cone $= 14cm$.

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 14$

$= 718.666cm^3$

Question 13

The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

Solution 13

(i) Radius of cone = $\left(\frac{28}{2}\right)$ cm =14 cm

Let height of cone be h.

Volume of cone = 9856 cm^3

$\Rightarrow \frac{1}{3}\pi r^2 h = 9856 \text{ cm}^2$

$\Rightarrow \left[\frac{1}{3} \times \frac{22}{7} \times (14)^2 \times h\right] \text{cm}^2 = 9856 \text{ cm}^3$

h = 48 cm

Thus, the height of the cone is 48 cm.

(ii) Slant height (l) of cone $= \sqrt{r^2 + h^2}$

$= \left[\sqrt{(14)^2 + (48)^2}\right]$ cm

$= \left[\sqrt{196 + 2304}\right]$ cm

$= 50$ cm

Thus, the slant height of the cone is 50 cm.

(iii) CSA of cone = π rl = $\left(\frac{22}{7} \times 14 \times 50\right) \text{cm}^2 = 2200 \text{ cm}^2$

Question 14

A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Solution 14

Radius (r) of pit = $\left(\frac{3.5}{2}\right)\text{m} = 1.75\ \text{m}$

Depth (h) of pit = 12 m

Volume of pit = $\frac{1}{3}\pi r^2 h = \left[\frac{1}{3}\times\frac{22}{7}\times(1.75)^2\times 12\right] = 38.5\ \text{m}^3$

$\therefore$ Capacity of the pit = (38.5 $\times$ 1) kilolitres = 38.5 kilolitres

Question 15

Monica has a piece of canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m^2, find the volume of the tent that can be made with it.

Solution 15

Since the area of the canvas = 551 m^2 and area of the canvas lost in wastage is 1 m^2, therefore the area of canvas available for making the tent is (551 – 1) m^2 = 550 m^2.

Now, the surface area of the tent = 550 m^2 and the required base radius of the conical tent = 7 m

Therefore, curved surface area of tent = 550 m^2

That is, $\pi rl = 550$

or, $\frac{22}{7}\times 7\times l = 550$

$l = \frac{550}{22}\text{m} = 25\text{m}$

Now, $l^2 = r^2 + h^2$

Therefore, $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2}\text{m} = \sqrt{625-49}\text{m} = \sqrt{576}\text{m}$

$= 24$ m

So, the volume of the conical tent =

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24\text{m}^3 = 1232\text{m}^3$.