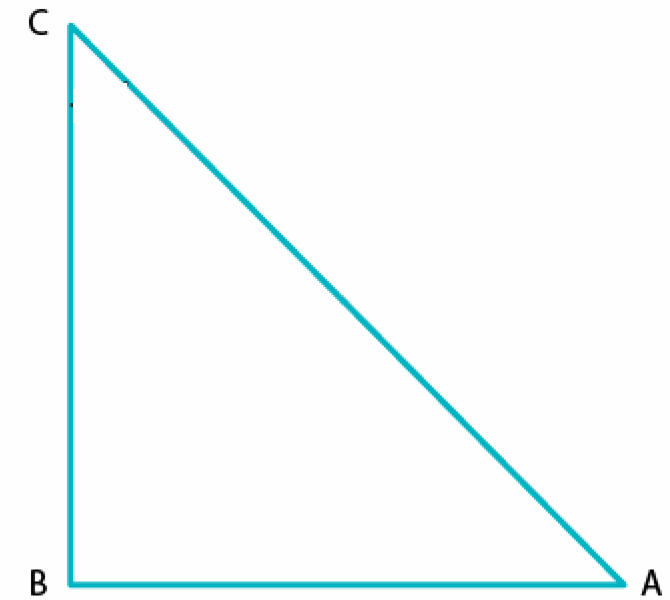
Chapter 5 – Trigonometric Ratios of RD Sharma contains three exercises for which the RD Sharma Solutions for Class 10 provides answers to the problems with full explanations. Trigonometry is the science of measuring triangles. Further, in this chapter, you will learn about the trigonometric ratios and their relations between them. In addition, the trigonometric ratios of some specific angles are also discussed.

Access the RD Sharma Solutions For Class 10 Chapter 5 – Trigonometric Ratios

RD Sharma Class 10 Chapter 5 Exercise 5.1 Page No: 5.23

1. In each of the following, one of the six trigonometric ratios s given. Find the values of the other trigonometric ratios.



(i) sin A = 2/3

Solution:

We have,

sin A = 2/3 ………… (1)

As we know, by sin definition;

sin A = Perpendicular/ Hypotenuse = 2/3 ….(2)

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

$\Rightarrow 3^2 = AB^2 + 2^2$

$AB^2 = 3^2 - 2^2$

$AB^2 = 9 - 4$

$AB^2 = 5$

$AB = \sqrt{5}$

Hence, Base = $\sqrt{5}$

By definition,

$\cos A$ = Base/Hypotenuse

$\Rightarrow \cos A = \sqrt{5}/3$

Since, $\operatorname{cosec} A = 1/\sin A$ = Hypotenuse/Perpendicular

$\Rightarrow \operatorname{cosec} A = 3/2$

And, $\sec A$ = Hypotenuse/Base

$\Rightarrow \sec A = 3/\sqrt{5}$

And, $\tan A$ = Perpendicular/Base

$\Rightarrow \tan A = 2/\sqrt{5}$

And, $\cot A = 1/\tan A$ = Base/Perpendicular

$\Rightarrow \cot A = \sqrt{5}/2$

(ii) cos A = 4/5

Solution:

We have,

$\cos A = 4/5$ ………. (1)

As we know, by cos defination

$\cos A$ = Base/Hypotenuse …. (2)

By comparing eq. (1) and (2), we get

Base = 4 and Hypotenuse = 5

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get

$5^2 = 4^2 + BC^2$

$BC^2 = 5^2 – 4^2$

$BC^2 = 25 – 16$

$BC^2 = 9$

$BC = 3$

Hence, Perpendicular = 3

By definition,

$\sin A$ = Perpendicular/Hypotenuse

⇒ sin A = 3/5

Then, cosec A = 1/sin A

⇒ cosec A= 1/ (3/5) = 5/3 = Hypotenuse/Perependicular

And, sec A = 1/cos A

⇒ sec A =Hypotenuse/Base

sec A = 5/4

And, tan A = Perpendicular/Base

⇒ tan A = 3/4

Next, cot A = 1/tan A = Base/Perpendicular

∴ cot A = 4/3

(iii) tan θ = 11/1

Solution:

We have, tan θ = 11……… (1)

By definition,

tan θ = Perpendicular/ Base…. (2)

On Comparing eq. (1) and (2), we get;

Base = 1 and Perpendicular = 5

Now, using Pythagoras theorem in Δ ABC.

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get;

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

AC= √122

Hence, hypotenuse = √122

By definition,

sin = Perpendicular/Hypotenuse

⇒ sin θ = 11/√122

And, cosec θ = 1/sin θ

⇒ cosec θ = √122/11

Next, cos θ = Base/ Hypotenuse

⇒ cos θ = 1/√122

And, sec θ = 1/cos θ

⇒ sec θ = √122/1 = √122

And, cot θ = 1/tan θ

∴ cot θ = 1/11

(iv) sin θ = 11/15

Solution:

We have, sin θ = 11/15 ………. (1)

By definition,

sin θ = Perpendicular/ Hypotenuse …. (2)

On Comparing eq. (1) and (2), we get;

Perpendicular = 11 and Hypotenuse= 15

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$15^2 = AB^2 + 11^2$

$AB^2 = 15^2 – 11^2$

$AB^2 = 225 – 121$

$AB^2 = 104$

AB = √104

AB= √ (2×2×2×13)

AB= 2√(2×13)

AB= 2√26

Hence, Base = 2√26

By definition,

cos θ = Base/Hypotenuse

∴ cosθ = 2√26/ 15

And, cosec θ = 1/sin θ

∴ cosec θ = 15/11

And, secθ = Hypotenuse/Base

∴ secθ =15/ 2√26

And, tan θ = Perpendicular/Base

∴ tanθ =11/ 2√26

And, cot θ = Base/Perpendicular

∴ cotθ =2√26/ 11

(v) tan α = 5/12

Solution:

We have, tan α = 5/12 …. (1)

By definition,

tan α = Perpendicular/Base…. (2)

On Comparing eq. (1) and (2), we get

Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2= 169$

AC = 13 [After taking sq root on both sides]

Hence, Hypotenuse = 13

By definition,

sin α = Perpendicular/Hypotenuse

∴ sin α = 5/13

And, cosec α = Hypotenuse/Perpendicular

∴ cosec α = 13/5

And, cos α = Base/Hypotenuse

∴ cos α = 12/13

And, sec α =1/cos α

∴ sec α = 13/12

And, tan α = sin α/cos α

∴ tan α=5/12

Since, cot α = 1/tan α

∴ cot α =12/5

(vi) sin θ = √3/2

Solution:

We have, sin θ = √3/2 …………. (1)

By definition,

sin θ = Perpendicular/ Hypotenuse….(2)

On Comparing eq. (1) and (2), we get;

Perpendicular = √3 and Hypotenuse = 2

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get;

$2^2 = AB^2 + (\sqrt{3})^2$

$AB^2 = 2^2 – (\sqrt{3})^2$

$AB^2 = 4 – 3$

$AB^2 = 1$

AB = 1

Thus, Base = 1

By definition,

cos θ = Base/Hypotenuse

∴ cos θ = 1/2

And, cosec θ = 1/sin θ

Or cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ =2/√3

And, sec θ = Hypotenuse/Base

∴ sec θ = 2/1

And, tan θ = Perpendicula/Base

∴ tan θ = √3/1

And, cot θ = Base/Perpendicular

∴ cot θ = 1/√3

(vii) cos θ = 7/25

Solution:

We have, cos θ = 7/25 ……….. (1)

By definition,

cos θ = Base/Hypotenuse

On Comparing eq. (1) and (2), we get;

Base = 7 and Hypotenuse = 25

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC)

$25^2 = 7^2 + BC^2$

$BC^2 = 25^2 – 7^2$

$BC^2 = 625 – 49$

$BC^2 = 576$

BC= √576

BC= 24

Hence, Perpendicular side = 24

By definition,

sin θ = perpendicular/Hypotenuse

∴ sin θ = 24/25

Since, cosec θ = 1/sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 25/24

Since, sec θ = 1/cosec θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 25/7

Since, tan θ = Perpendicular/Base

∴ tan θ = 24/7

Now, cot = 1/tan θ

So, cot θ = Base/Perpendicular

∴ cot θ = 7/24

(viii) tan θ = 8/15

Solution:

We have, tan θ = 8/15 …………. (1)

By definition,

tan θ = Perpendicular/Base …. (2)

On Comparing eq. (1) and (2), we get;

Base = 15 and Perpendicular = 8

Now, using Pythagoras theorem in Δ ABC

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

AC = √289

AC = 17

Hence, Hypotenuse = 17

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ = 8/17

Since, cosec θ = 1/sin θ

Also, cosec θ = Hypotenuse/Perpendicualar

∴ cosec θ = 17/8

Since, cos θ = Base/Hypotenuse

∴ cos θ = 15/17

Since, sec θ = 1/cos θ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 17/15

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular

∴ cot θ = 15/8

(ix) cot θ = 12/5

Solution:

We have, cot θ = 12/5 …………. (1)

By definition,

cot θ = 1/tan θ

cot θ = Base/Perpendicular ……. (2)

On Comparing eq. (1) and (2), we have

Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC);

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

AC = √169

AC = 13

Hence, Hypotenuse = 13

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ= 5/13

Since, cosec θ = 1/sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 13/5

Since, cos θ = Base/Hypotenuse

∴ cos θ = 12/13

Since, sec θ = 1/cosθ

Also, sec θ = Hypotenuse/Base

∴ sec θ = 13/12

Since, tanθ = 1/cot θ

Also, tan θ = Perpendicular/Base

∴ tan θ = 5/12

(x) sec θ = 13/5

Solution:

We have, sec θ = 13/5………. (1)

By definition,

sec θ = Hypotenuse/Base…………. (2)

On Comparing eq. (1) and (2), we get

Base = 5 and Hypotenuse = 13

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

And. putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC)

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 – 5^2$

$BC^2=169 – 25$

$BC^2= 144$

BC= √144

BC = 12

Hence, Perpendicular = 12

By definition,

Since, sin θ = perpendicular/Hypotenuse

∴ sin θ= 12/13

Since, cosec θ= 1/ sin θ

Also, cosec θ= Hypotenuse/Perpendicualar

∴ cosec θ = 13/12

Since, cos θ= 1/sec θ

Also, cos θ = Base/Hypotenuse

∴ cos θ = 5/13

Since, tan θ = Perpendicular/Base

∴ tan θ = 12/5

Since, cot θ = 1/tan θ

Also, cot θ = Base/Perpendicular

∴ cot θ = 5/12

(xi) cosec θ = √10

Solution:

We have, cosec θ = √10/1   ……..… (1)

By definition,

cosec θ = Hypotenuse/ Perpendicualar ……….(2)

And, cosecθ = 1/sin θ

On comparing eq.(1) and(2), we get

Perpendicular side = 1 and Hypotenuse = √10

Now, using Pythagoras theorem in Δ ABC

$AC^2 = AB^2 + BC^2$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB)

$(\sqrt{10})^2 = AB^2 + 1^2$

$AB^2 = (\sqrt{10})^2 – 1^2$

$AB^2 = 10 – 1$

AB = √9

AB = 3

So, Base side = 3

By definition,

Since, sin θ = Perpendicular/Hypotenuse

∴ $\sin\theta = 1/\sqrt{10}$

Since, $\cos\theta$ = Base/Hypotenuse

∴ $\cos\theta = 3/\sqrt{10}$

Since, $\sec\theta = 1/\cos\theta$

Also, $\sec\theta$ = Hypotenuse/Base

∴ $\sec\theta = \sqrt{10}/3$

Since, $\tan\theta$ = Perpendicular/Base

∴ $\tan\theta = 1/3$

Since, $\cot\theta = 1/\tan\theta$

∴ $\cot\theta = 3/1$

(xii) $\cos\theta$ =12/15

Solution:

We have; $\cos\theta = 12/15$ ………. (1)

By definition,

$\cos\theta$ = Base/Hypotenuse……… (2)

By comparing eq. (1) and (2), we get;

Base =12 and Hypotenuse = 15

Now, using Pythagoras theorem in Δ ABC, we get

$AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC);

$15^2 = 12^2 + BC^2$

$BC^2 = 15^2 - 12^2$

$BC^2 = 225 - 144$

$BC^2 = 81$

$BC = \sqrt{81}$

BC = 9

So, Perpendicular = 9

By definition,

Since, $\sin\theta$ = perpendicular/Hypotenuse

∴ $\sin\theta = 9/15 = 3/5$

Since, $\operatorname{cosec}\theta = 1/\sin\theta$

Also, $\operatorname{cosec}\theta$ = Hypotenuse/Perpendicualar

∴ $\operatorname{cosec}\theta = 15/9 = 5/3$

Since, $\sec\theta = 1/\cos\theta$

Also, $\sec\theta$ = Hypotenuse/Base

∴ $\sec\theta = 15/12 = 5/4$

Since, $\tan\theta$ = Perpendicular/Base

∴ $\tan\theta = 9/12 = 3/4$

Since, $\cot\theta = 1/\tan\theta$

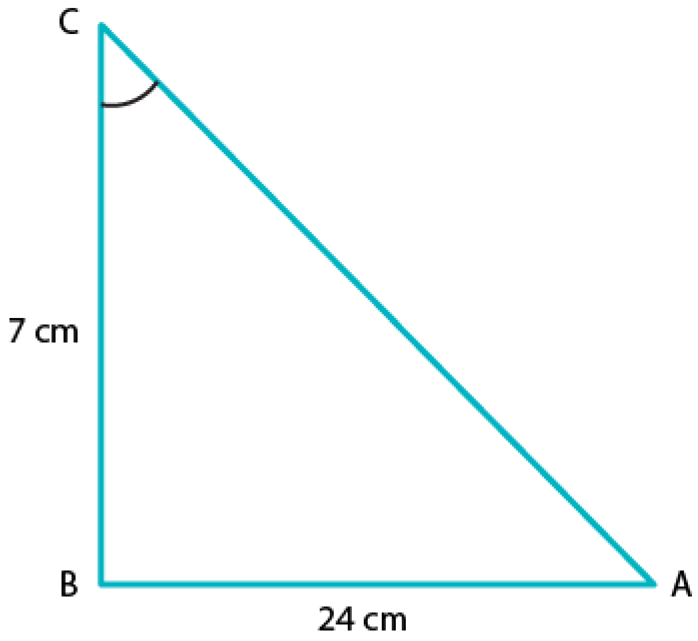
Also, $\cot\theta$ = Base/Perpendicular

∴ $\cot\theta = 12/9 = 4/3$

2. In a △ ABC, right angled at B, AB = 24 cm , BC = 7 cm. Determine

(i) sin A , cos A (ii) sin C, cos C

Solution:



(i) Given: In △ABC, AB = 24 cm, BC = 7cm and ∠ABC = 90°

To find: sin A, cos A

By using Pythagoras theorem in △ABC we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

$AC = \sqrt{625}$

AC= 25

Hence, Hypotenuse = 25

By definition,

sin A = Perpendicular side opposite to angle A/ Hypotenuse

sin A = BC/ AC

sin A = 7/ 25

And,

cos A = Base side adjacent to angle A/Hypotenuse

cos A = AB/ AC

cos A = 24/ 25

(ii) Given: In △ABC , AB = 24 cm and BC = 7cm and ∠ABC = 90°

To find: sin C, cos C

By using Pythagoras theorem in △ABC we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625

AC= 25

Hence, Hypotenuse = 25

By definition,

sin C = Perpendicular side opposite to angle C/Hypotenuse

sin C = AB/ AC

sin C = 24/ 25

And,

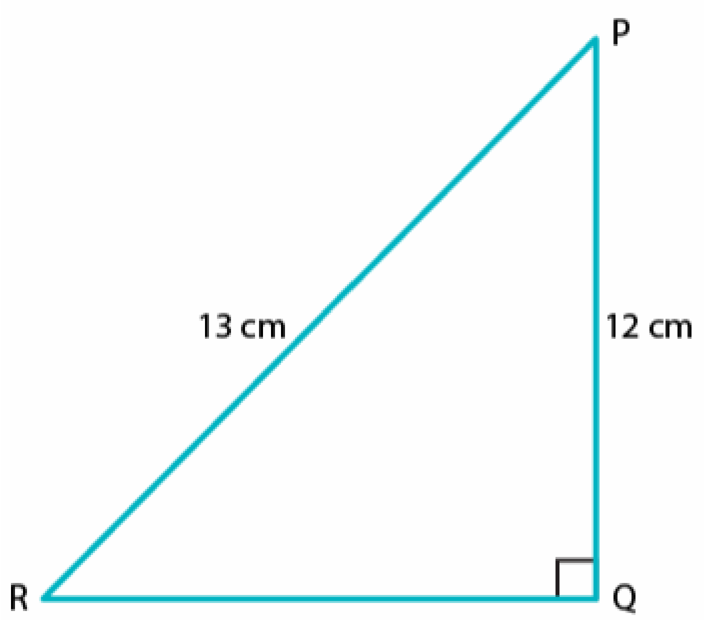
cos C = Base side adjacent to angle C/Hypotenuse

cos A = BC/AC

cos A = 7/25

3. In fig. 5.37, find tan P and cot R. Is tan P = cot R?

Solution:



By using Pythagoras theorem in △PQR, we have

$PR^2 = PQ^2 + QR^2$

Putting the length of given side PR and PQ in the above equation

$13^2 = 12^2 + QR^2$

$QR^2 = 13^2 – 12^2$

$QR^2 = 169 – 144$

$QR^2 = 25$

$QR = \sqrt{25} = 5$

By definition,

tan P = Perpendicular side opposite to P/ Base side adjacent to angle P

tan P = QR/PQ

tan P = 5/12 ………. (1)

And,

cot R= Base/Perpendicular

cot R= QR/PQ

cot R= 5/12 …. (2)

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

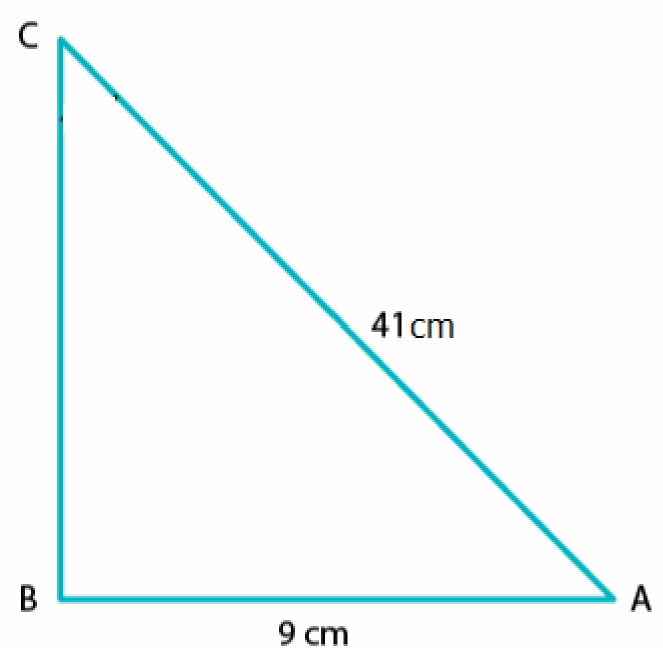
Therefore, L.H.S of both equations should also be equal.

∴ tan P = cot R

Yes, tan P = cot R = 5/12

4. If sin A = 9/41, compute cos A and tan A.

Solution:



Given that, sin A = 9/41 …………. (1)

Required to find: cos A, tan A

By definition, we know that

sin A = Perpendicular/ Hypotenuse……………(2)

On Comparing eq. (1) and (2), we get;

Perpendicular side = 9 and Hypotenuse = 41

Let's construct △ABC as shown below,

And, here the length of base AB is unknown.

Thus, by using Pythagoras theorem in △ABC, we get;

$AC^2 = AB^2 + BC^2$

$41^2 = AB^2 + 9^2$

$AB^2 = 41^2 – 9^2$

$AB^2 = 168 – 81$

AB= 1600

AB = √1600

AB = 40

⇒ Base of triangle ABC, AB = 40

We know that,

cos A = Base/ Hypotenuse

cos A =AB/AC

cos A =40/41

And,

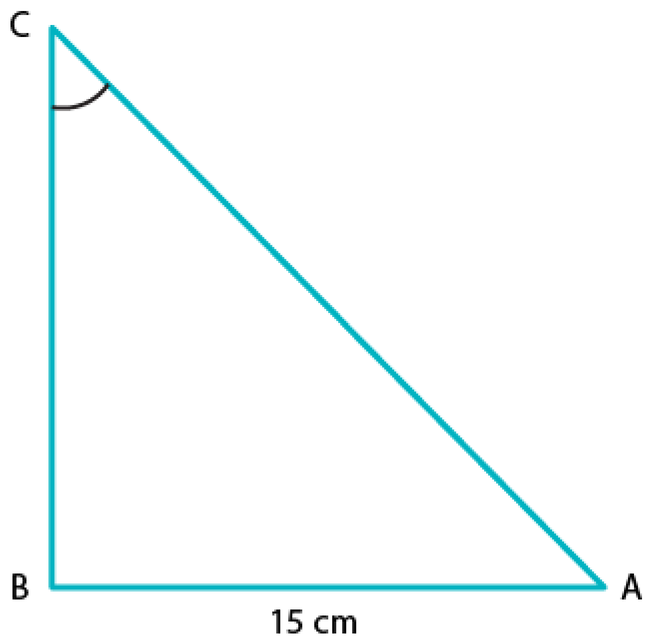
tan A = Perpendicular/ Base

tan A = BC/AB

tan A = 9/40

5. Given 15cot A= 8, find sin A and sec A.

Solution



We have, 15cot A = 8

Required to find: sin A and sec A

As, 15 cot A = 8

⇒ cot A = 8/15 …….(1)

And we know,

cot A = 1/tan A

Also by definition,

Cot A = Base side adjacent to ∠A/ Perpendicular side opposite to ∠A …. (2)

On comparing equation (1) and (2), we get;

Base side adjacent to ∠A = 8

Perpendicular side opposite to ∠A = 15

So, by using Pythagoras theorem to △ABC, we have

$AC^2 = AB^2 + BC^2$

Substituting values for sides from the figure

$AC^2 = 8^2 + 15^2$

$AC^2 = 64 + 225$

$AC^2 = 289$

$AC = \sqrt{289}$

AC = 17

Therefore, hypotenuse =17

By definition,

sin A = Perpendicular/Hypotenuse

⇒ sin A= BC/AC

sin A= 15/17 (using values from the above)

Also,

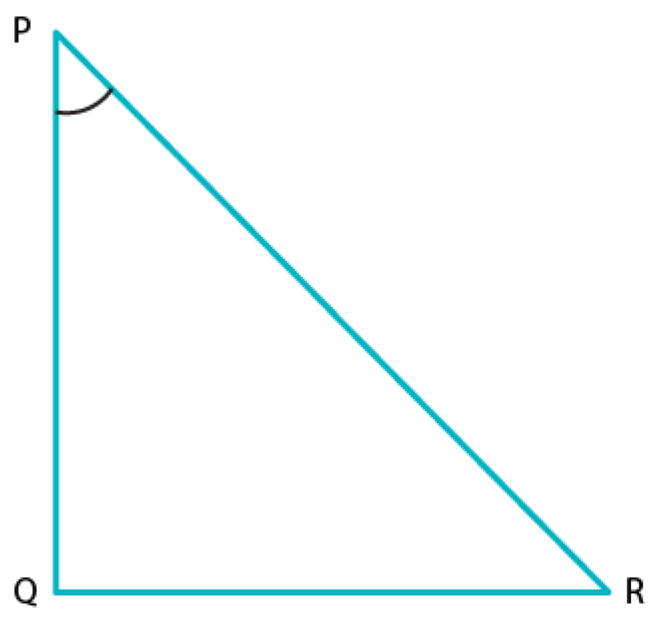
sec A= 1/ cos A

⇒ secA = Hypotenuse/ Base side adjacent to ∠A

∴ sec A= 17/8

6. In △PQR, right-angled at Q, PQ = 4cm and RQ = 3 cm. Find the value of sin P, sin R, sec P and sec R.

Solution:



Given:

△PQR is right-angled at Q.

PQ = 4cm

RQ = 3cm

Required to find: sin P, sin R, sec P, sec R

Given △PQR,

By using Pythagoras theorem to △PQR, we get

$PR^2 = PQ^2 + RQ^2$

Substituting the respective values,

$PR^2 = 4^2 + 3^2$

$PR^2 = 16 + 9$

$PR^2 = 25$

$PR = \sqrt{25}$

PR = 5

⇒ Hypotenuse =5

By definition,

sin P = Perpendicular side opposite to angle P/ Hypotenuse

sin P = RQ/ PR

⇒ sin P = 3/5

And,

sin R = Perpendicular side opposite to angle R/ Hypotenuse

sin R = PQ/ PR

⇒ sin R = 4/5

And,

sec P=1/cos P

secP = Hypotenuse/ Base side adjacent to ∠P

sec P = PR/ PQ

⇒ sec P = 5/4

Now,

sec R = 1/cos R

secR = Hypotenuse/ Base side adjacent to ∠R

sec R = PR/ RQ

⇒ sec R = 5/3

7. If cot θ = 7/8, evaluate

(i) (1+sin θ)(1–sin θ)/ (1+cos θ)(1–cos θ)

(ii) $\cot^2 \theta$

Solution:

(i) Required to evaluate:

$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$, given = cot θ = 7/8

Taking the numerator, we have

(1+sin θ)(1–sin θ) = 1 – $\sin^2$ θ [Since, (a+b)(a-b) = $a^2 - b^2$]

Similarly,

(1+cos θ)(1–cos θ) = 1 – $\cos^2$ θ

We know that,

$\sin^2$ θ + $\cos^2$ θ = 1

⇒ 1 – $\cos^2$ θ = $\sin^2$ θ

And,

1 – $\sin^2$ θ = $\cos^2$ θ

Thus,

(1+sin θ)(1 –sin θ) = 1 – $\sin^2$ θ = $\cos^2$ θ

(1+cos θ)(1–cos θ) = 1 – $\cos^2$ θ = $\sin^2$ θ

⇒

$$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$$

= $\cos^2\theta / \sin^2\theta$

= $(\cos\theta/\sin\theta)^2$

And, we know that (cos θ/sin θ) = cot θ

⇒

$$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$$

= $(\cot\theta)^2$

= $(7/8)^2$

= 49/ 64

(ii) Given,

cot θ = 7/8

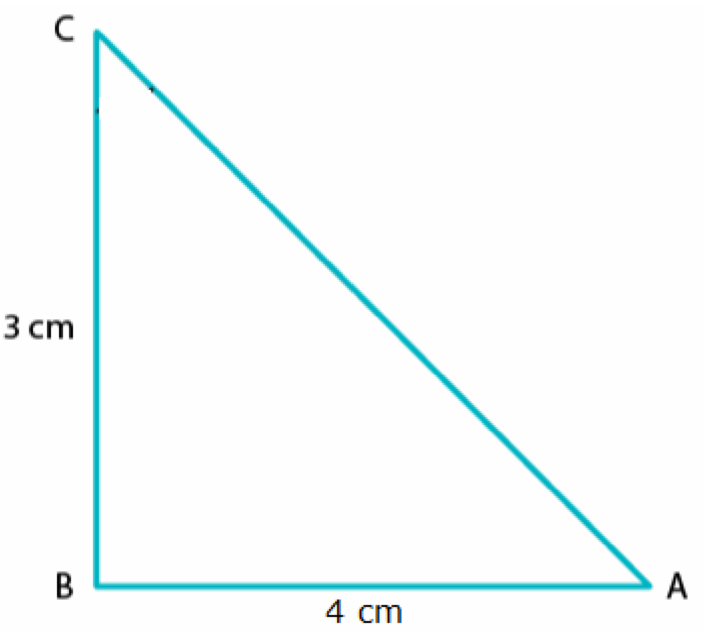
So, by squaring on both sides we get

$(\cot\theta)^2 = (7/8)^2$

∴ $\cot\theta^2 = 49/64$

8. If 3cot A = 4, check whether $(1–\tan^2A)/(1+\tan^2A) = (\cos^2A – \sin^2A)$ or not.

Solution:



Given,

3cot A = 4

⇒ cot A = 4/3

By definition,

tan A = 1/ Cot A = 1/ (4/3)

⇒ tan A = 3/4

Thus,

Base side adjacent to ∠A = 4

Perpendicular side opposite to ∠A = 3

In ΔABC, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in ΔABC

We get

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

AC = √25

AC = 5

Hence, hypotenuse = 5

Now, we can find that

sin A = opposite side to ∠A/ Hypotenuse = 3/5

And,

cos A = adjacent side to ∠A/ Hypotenuse = 4/5

Taking the LHS,

L.H.S = $\frac{1-\tan^2 A}{1+\tan^2 A}$

Putting value of tan A

We get,

L.H.S= $\frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

Take L.C.M of both numerator and denominator;

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}$

Thus, LHS = 7/25

Now, taking RHS

R.H.S = $\cos^2 A - \sin^2 A$

Putting value of sin A and cos A

R.H.S= $(\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = (\frac{4}{5})^2 - (\frac{3}{5}^2)$

$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$

$\cos^2 A - \sin^2 A = \frac{16-9}{25}$

$\cos^2 A - \sin^2 A = \frac{7}{25}$

Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. If tan θ = a/b, find the value of (cos θ + sin θ)/ (cos θ – sin θ)

Solution:

Given,

tan θ = a/b

And, we know by definition that

tan θ = opposite side/ adjacent side

Thus, by comparison

Opposite side = a and adjacent side = b

To find the hypotenuse, we know that by Pythagoras theorem that

Hypotenuse^2 = opposite side^2 + adjacent side^2

⇒ Hypotenuse = $\sqrt{(a^2 + b^2)}$

So, by definition

sin θ = opposite side/ Hypotenuse

sin θ = a/ $\sqrt{(a^2 + b^2)}$

And,

cos θ = adjacent side/ Hypotenuse

cos θ = b/ $\sqrt{(a^2 + b^2)}$

Now,

After substituting for cos θ and sin θ, we have

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)/\sqrt{(a^2+b^2)}}{(a-b)/\sqrt{(a^2+b^2)}}$$

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{(a+b)}{(a-b)}$$

∴

Hence Proved.

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

10. If 3 tan θ = 4, find the value of

Solution:

Given, 3 tan θ = 4

⇒ tan θ = 4/3

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

From, let's divide the numerator and denominator by cos θ.

We get,

(4 – tan θ) / (2 + tan θ)

⇒ (4 – (4/3)) / (2 + (4/3)) [using the value of tan θ]

⇒ (12 – 4) / (6 + 4) [After taking LCM and cancelling it]

⇒ 8/10 = 4/5

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

∴ = 4/5

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$$

11. If 3 cot θ = 2, find the value of

Solution:

Given, 3 cot θ = 2

⇒ cot θ = 2/3

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$$

From, let's divide the numerator and denominator by sin θ.

We get,

(4 –3 cot θ) / (2 + 6 cot θ)

⇒ (4 – 3(2/3)) / (2 + 6(2/3)) [using the value of tan θ]

⇒ (4 – 2) / (2 + 4) [After taking LCM and simplifying it]

⇒ 2/6 = 1/3

$$\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$$

∴ = 1/3

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

12. If tan θ = a/b, prove that

Solution:

Given, tan θ = a/b

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$$

From LHS, let's divide the numerator and denominator by cos θ.

And we get,

(a tan θ – b) / (a tan θ + b)

$\Rightarrow$ (a(a/b) – b) / (a(a/b) + b) [using the value of tan θ]

$\Rightarrow$ $(a^2 - b^2)/b^2 / (a^2 + b^2)/b^2$ [After taking LCM and simplifying it]

$\Rightarrow$ $(a^2 - b^2)/ (a^2 + b^2)$

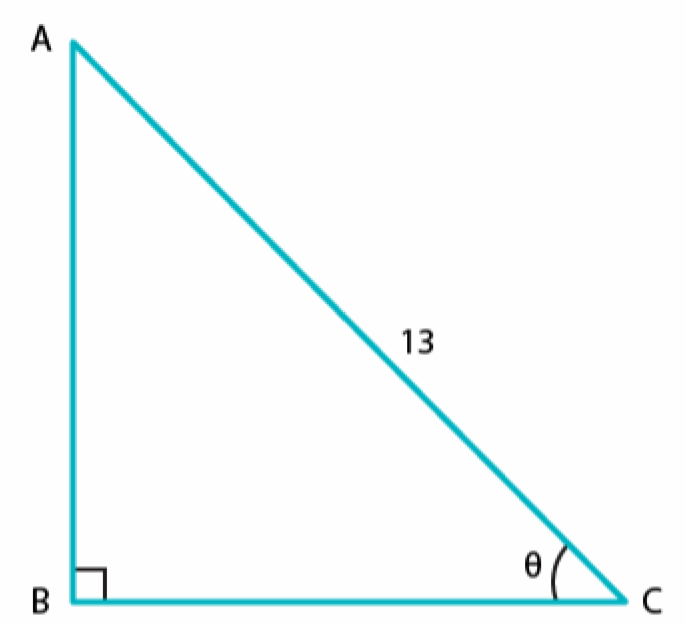
= RHS

– Hence Proved

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$$

13. If sec θ = 13/5, show that

Solution:



Given,

sec θ = 13/5

We know that,

sec θ = 1/ cos θ

$\Rightarrow$ cos θ = 1/ sec θ = 1/ (13/5)

∴ cos θ = 5/13 ……. (1)

By definition,

cos θ = adjacent side/ hypotenuse ….. (2)

Comparing (1) and (2), we have

Adjacent side = 5 and hypotenuse = 13

By Pythagoras theorem,

Opposite side = $\sqrt{((\text{hypotenuse})^2 - (\text{adjacent side})^2)}$

= $\sqrt{(13^2 - 5^2)}$

= √(169 – 25)

= √(144)

= 12

Thus, opposite side = 12

By definition,

tan θ = opposite side/ adjacent side

∴ tan θ = 12/ 5

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$$

From, let’s divide the numerator and denominator by cos θ.

We get,

(2 tan θ – 3) / (4 tan θ – 9)

⇒ (2(12/5) – 3) / (4(12/5) – 9) [using the value of tan θ]

⇒ (24 – 15) / (48 – 45) [After taking LCM and cancelling it]

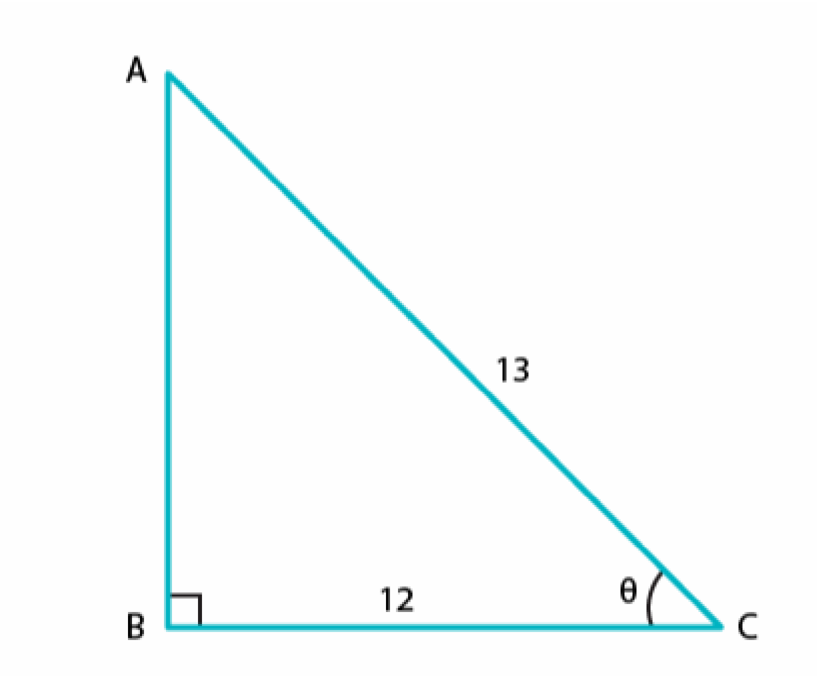
⇒ 9/3 = 3

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$$

∴ = 3

14. If cos θ = 12/13, show that sin θ(1 – tan θ) = 35/156

Solution:



Given, cos θ = 12/13…… (1)

By definition we know that,

cos θ = Base side adjacent to ∠θ / Hypotenuse……. (2)

When comparing equation (1) and (2), we get

Base side adjacent to ∠θ = 12 and Hypotenuse = 13

From the figure,

Base side BC = 12

Hypotenuse AC = 13

Side AB is unknown here and it can be found by using Pythagoras theorem

Thus by applying Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 12^2$

Therefore,

$AB^2 = 13^2 – 12^2$

$AB^2 = 169 – 144$

$AB^2 = 25$

AB = √25

AB = 5 …. (3)

Now, we know that

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse

Thus, sin θ = AB/AC [from figure]

⇒ sin θ = 5/13… (4)

And, tan θ = sin θ / cos θ = (5/13) / (12/13)

⇒ tan θ = 12/13… (5)

Taking L.H.S we have

L.H.S = sin θ (1 – tan θ)

Substituting the value of sin θ and tan θ from equation (4) and (5)

We get,

$$\Rightarrow \quad \text{L.H.S} = \frac{5}{13}\left(1-\frac{5}{12}\right)$$

$$\text{L.H.S} = \frac{5}{13}\left(\frac{1\times 12}{1\times 12}-\frac{5}{12}\right) \quad \text{[Taking LCM]}$$

$$\text{L.H.S} = \frac{5}{13}\left(\frac{12-5}{12}\right)$$

$$\text{L.H.S} = \frac{5}{13}\left(\frac{7}{12}\right)$$

$$\text{L.H.S} = \frac{5\times 7}{13\times 12}$$

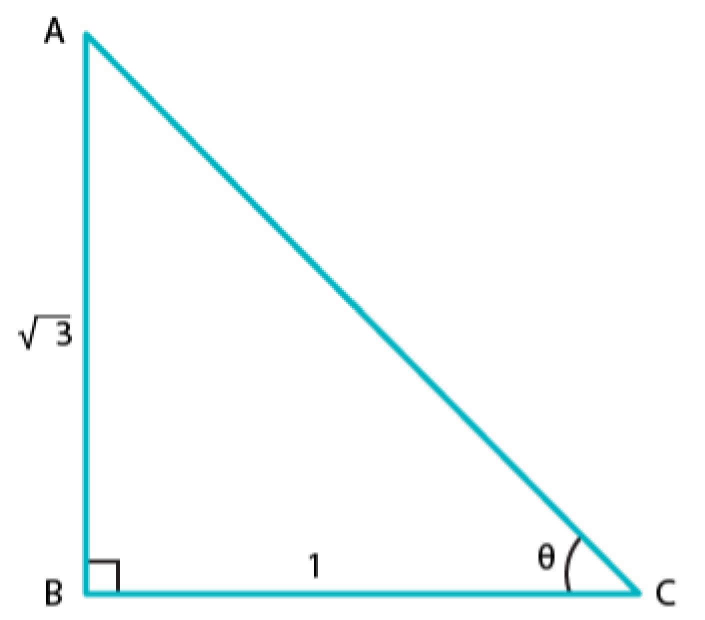
$$\text{L.H.S} = \frac{35}{136}$$

Therefore it's shown that $\sin\theta\,(1-\tan\theta) = \frac{35}{136}$

If $\cot\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

15.

Solution:



Given, cot θ = 1/3……. (1)

By definition we know that,

cot θ = 1/ tan θ

And, since tan θ = perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ cot θ = Base side adjacent to ∠θ / perpendicular side opposite to ∠θ …… (2)

[Since they are reciprocal to each other]
On comparing equation (1) and (2), we get

Base side adjacent to ∠θ = 1 and Perpendicular side opposite to ∠θ = √3

Therefore, the triangle formed is,

On substituting the values of known sides as AB = √3 and BC = 1

$AC^2 = (\sqrt{3}) + 1$

$AC^2 = 3 + 1$

$AC^2 = 4$

AC = √4

Therefore, AC = 2 … (3)

Now, by definition

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = √3/ 2 ……(4)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = 1/ 2 ….. (5)

Now, taking L.H.S we have

$$\text{L. H. S} = \frac{1-\cos^2\theta}{2-\sin^2\theta}$$

Substituting the values from equation (4) and (5), we have

$$\text{L. H. S} = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\text{L. H. S} = \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

Now by taking L.C.M in numerator and denominator, we get

$$\text{L. H. S} = \frac{\frac{(4\times 1)-1}{4}}{\frac{(4\times 2)-3}{4}}$$

$$\text{L. H. S} = \frac{\frac{4-1}{4}}{\frac{8-3}{4}}$$

$$\text{L. H. S} = \frac{3}{4}\times\frac{4}{5}$$

$$\text{L. H. S} = \frac{3}{5} = \text{R. H. S}$$

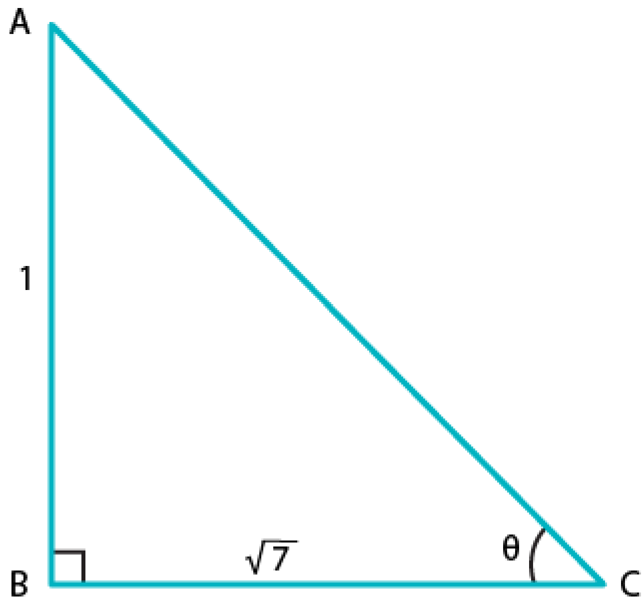
Therefore, $\dfrac{1-\cos^2\theta}{2-\sin^2\theta} = \dfrac{3}{5}$

If $\tan\theta = \frac{1}{\sqrt{7}}$, then show that $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

16.

Solution:

Given, $\tan\theta = 1/\sqrt{7}$ …..(1)



By definition, we know that

$\tan\theta$ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ ……(2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to ∠θ = 1

Base side adjacent to ∠θ = $\sqrt{7}$

Thus, the triangle representing ∠ θ is,

Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + (\sqrt{7})^2$

$AC^2 = 1 + 7$

$AC^2 = 8$

$AC = \sqrt{8}$

⇒ $AC = 2\sqrt{2}$

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ $\sin\theta = 1/ 2\sqrt{2}$

And, since $\text{cosec}\,\theta = 1/\sin\theta$

⇒ $\text{cosec}\,\theta = 2\sqrt{2}$ …….. (3)

Now,

cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ $\cos\theta = \sqrt{7}/ 2\sqrt{2}$

And, since $\sec\theta = 1/ \sin\theta$

⇒ $\sec\theta = 2\sqrt{2}/ \sqrt{7}$ ……. (4)

Taking the L.H.S of the equation,

$$\text{L.H.S} = \frac{\text{cosec}^2\,\theta - \sec^2\,\theta}{\text{cosec}^2\theta + \sec^2\theta}$$

Substituting the value of cosec θ and sec θ from equation (3) and (4), we get

$$\text{L.H.S} = \frac{[(2\sqrt{2})]^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{[(2\sqrt{2})]^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$\text{L.H.S} = \frac{(8) - \left(\frac{8}{7}\right)}{(8) + \left(\frac{8}{7}\right)} \qquad = \frac{\frac{56-8}{7}}{\frac{56+8}{7}} \quad \text{[Taking L.C.M and simplifying]}$$

$$\text{L.H.S} = \frac{\frac{48}{7}}{\frac{64}{7}}$$

Therefore,

L.H.S = 48/64 = 3/4 = R.H.S

Hence proved that $\dfrac{\text{cosec}^2\,\theta - \sec^2\theta}{\text{cosec}^2\,\theta + \sec^2\theta} = \dfrac{3}{4}$

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$$

17. If sec θ = 5/4, find the value of

Solution:

Given,

sec θ = 5/4

We know that,

sec θ = 1/ cos θ

⇒ cos θ = 1/ (5/4) = 4/5 …… (1)

By definition,

cos θ = Base side adjacent to ∠θ / Hypotenuse …. (2)

On comparing equation (1) and (2), we have

Hypotenuse = 5

Base side adjacent to ∠θ = 4

Thus, the triangle representing ∠ θ is ABC.

Perpendicular side opposite to ∠θ, AB is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 + BC^2$

$AB^2 = 5^2 – 4^2$

$AB^2 = 25 – 16$

$AB = \sqrt{9}$

⇒ AB = 3

By definition,

sin θ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ sin θ = 3/ 5 …..(3)

Now, tan θ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ

⇒ tan θ = 3/ 4 ……(4)

And, since cot θ = 1/ tan θ

⇒ cot θ = 4/ 3 ……(5)

Now,

Substituting the value of sin θ, cos θ, cot θ and tan θ from the equations (1), (3), (4) and (5) we have,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$$

= 12/7

Therefore,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{12}{7}$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

18. If tan θ = 12/13, find the value of

Solution:

Given,

$\tan\theta = 12/13$ …….. (1)

We know that by definition,

$\tan\theta$ = Perpendicular side opposite to ∠θ / Base side adjacent to ∠θ …… (2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to ∠θ = 12

Base side adjacent to ∠θ = 13

Thus, in the triangle representing ∠ θ we have,

Hypotenuse AC is the unknown and it can be found by using Pythagoras theorem

So by applying Pythagoras theorem, we have

$AC^2 = 12^2 + 13^2$

$AC^2 = 144 + 169$

$AC^2 = 313$

⇒ $AC = \sqrt{313}$

By definition,

$\sin\theta$ = Perpendicular side opposite to ∠θ / Hypotenuse = AB / AC

⇒ $\sin\theta = 12/\sqrt{313}$…..(3)

And, cos θ = Base side adjacent to ∠θ / Hypotenuse = BC / AC

⇒ cos θ = 13/ √313 …..(4)

Now, substituting the value of sin θ and cos θ from equation (3) and (4) respectively in the equation below

$$\frac{2\sin\theta\cos\theta}{\cos^2\Theta\ -\sin^2\theta} = \frac{2\times\frac{13}{\sqrt{313}}\times\frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2-\left(\frac{12}{\sqrt{313}}\right)^2}$$

$$=\frac{\frac{312}{313}}{\frac{25}{313}}$$

$$=\frac{312}{25}$$

Therefore,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}=\frac{312}{25}$$

RD Sharma Class 10 Chapter 5 Exercise 5.2 Page No: 5.41

Evaluate each of the following:

1. sin 45· sin 30· + cos 45· cos 30·

Solution:

Sin 45° sin 30° + cos 45° cos 30°

Value of trigonometric ratios are:

$sin45^\circ = \frac{1}{\sqrt{2}}$ $sin30^\circ = \frac{1}{2}$

$cos45^\circ = \frac{1}{\sqrt{2}}$ $cos30^\circ = \frac{\sqrt{3}}{2}$

Substituting in the given equation, we get

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

= $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{2}}$

= $\frac{\sqrt{3}+1}{2\sqrt{2}}$

2. sin 60· cos 30· + cos 60· sin 30·

Solution:

sin 60° cos 30° + cos 60° sin 30°

By trigonometric ratios we have ,

$sin60^\circ = \frac{\sqrt{3}}{2}$ $sin30^\circ = \frac{1}{2}$

$cos30^\circ = \frac{\sqrt{3}}{2}$ $cos60^\circ = \frac{1}{2}$

Substituting the values in given equation

= $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

= $\frac{3}{4} + \frac{1}{4}$ = $\frac{4}{4}$ = 1

3. cos 60· cos 45· – sin 60· sin 45·

Solution:

cos 60° cos 45° – sin 60° sin 45°

We know that by trigonometric ratios

$cos60^\circ = \frac{1}{2}$ $cos45^\circ = \frac{1}{\sqrt{2}}$

$sin60^\circ = \frac{\sqrt{3}}{2}$ $sin45^\circ = \frac{1}{\sqrt{2}}$

Substituting the values in given equation

$\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

= $\frac{1-\sqrt{3}}{2\sqrt{2}}$

4. $sin^2 30^\circ + sin^2 45^\circ + sin^2 60^\circ + sin^2 90^\circ$

Solution:

$sin^2 30^\circ + sin^2 45^\circ + sin^2 60^\circ + sin^2 90^\circ$

We know that by trigonometric ratios

$sin30^\circ = \frac{1}{2}$ $sin45^\circ = \frac{1}{\sqrt{2}}$

$sin60^\circ = \frac{\sqrt{3}}{2}$ $sin90^\circ$ = 1

Substituting the values in given equation, we get

= $\left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + 1$

= $\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

= $\frac{5}{2}$

5. $cos^2 30^\circ + cos^2 45^\circ + cos^2 60^\circ + cos^2 90^\circ$

Solution:

$$cos^2 30^\circ + cos^2 45^\circ + cos^2 60^\circ + cos^2 90^\circ$$

We know that by trigonometric ratios

$cos30^\circ = \frac{\sqrt{3}}{2}$ $cos45^\circ = \frac{1}{\sqrt{2}}$

$cos60^\circ = \frac{1}{2}$ $cos90^\circ$ = 0

Substituting the values in given equation

$$\left[\frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{1}{2}\right]^2 + 0$$

= $\frac{3}{4} + \frac{1}{2} + \frac{1}{4}$

= $\frac{3}{2}$

6. tan² 30° + tan² 45° + tan² 60°

Solution:

$$tan^2 30^\circ + tan^2 45^\circ + tan^2 60^\circ$$

We know that by trigonometric ratios

$tan30^\circ = \frac{1}{\sqrt{3}}$ $tan60^\circ = \sqrt{3}$

$tan45^\circ = 1$

Substituting the values in given equation

$$\left[\frac{1}{\sqrt{3}}\right]^2 + [\sqrt{3}]^2 + 1$$

= $\frac{1}{3} + 3 + 1$

= $\frac{13}{3}$

7. 2sin² 30° − 3cos² 45° + tan² 60°

Solution:

$$2sin^2 30^\circ - 3cos^2 45^\circ + tan^2 60^\circ$$

We know that by trigonometric ratios

$$sin30^\circ = \frac{1}{2} \; cos45^\circ = \frac{1}{\sqrt{2}}$$

$$tan60^\circ = \sqrt{3}$$

Substituting the values in given equation

= $2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2$

= $2\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$

= $\frac{1-3+6}{2}$

= 2

8. sin² 30· cos²45· + 4tan² 30· + (1/2) sin² 90· − 2cos² 90· + (1/24) cos20·

Solution:

$sin^2 30^\circ cos^2 45^\circ + 4tan^2 30^\circ + \frac{1}{2} sin^2 90^\circ - 2cos^2 90^\circ + \frac{1}{24} cos^2 0^\circ$

We know that by trigonometric ratios

$sin30^\circ = \frac{1}{2}$

$cos45^\circ = \frac{1}{\sqrt{2}}$

$tan30^\circ = \frac{1}{\sqrt{3}}$

$sin90^\circ$ = 1

$cos90^\circ$ = 0

$cos0^\circ$ = 1

Substituting the values in given equation

$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$

= $\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

= $\frac{48}{24}$

= 2

$$sin^2 30^\circ cos^2 45^\circ + 4tan^2 30^\circ + \frac{1}{2} sin^2 90^\circ - 2cos^2 90^\circ + \frac{1}{24} cos^2 0^\circ$$

We know that by trigonometric ratios

$sin 30^\circ = \frac{1}{2}$

$cos 45^\circ = \frac{1}{\sqrt{2}}$

$tan 30^\circ = \frac{1}{\sqrt{3}}$

$sin 90^\circ$ = 1

$cos 90^\circ$ = 0

$cos 0^\circ$ = 1

Substituting the values in given equation

$$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$$

= $\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

= $\frac{48}{24}$

= 2

9. $4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$

Solution:

$$4\left(sin^4 60^\circ + cos^4 30^\circ\right) - 3\left(tan^2 60^\circ - tan^2 45^\circ\right) + 5cos^2 45^\circ$$

We know that by trigonometric ratios we have ,

$sin60^\circ = \frac{\sqrt{3}}{2}$ $cos45^\circ = \frac{1}{\sqrt{2}}$

$tan60^\circ = \sqrt{3}$ $cos30^\circ = \frac{\sqrt{3}}{2}$

Substituting the values in given equation

$= 4 \cdot \frac{18}{16} - 6 + \frac{5}{2}$

$= \frac{1}{4} - 6 + \frac{5}{2}$

$= \frac{14}{2} - 6 = 7 - 6 = 1$

10. (cosec² 45· sec² 30·)(sin² 30· + 4cot² 45· − sec² 60·)

Solution:

$$\left(cosec^2 45^\circ sec^2 30^\circ\right)\left(sin^2 30^\circ + 4cot^2 45^\circ - sec^2 60^\circ\right)$$

We know that by trigonometric ratios,

$cosec45^\circ = \sqrt{2}$ $sec30^\circ = \frac{2}{\sqrt{3}}$

$sin30^\circ = \frac{1}{2}$ $cot45^\circ = 1$

$sec60^\circ = 2$

Substituting the values in given equation

$$\left(\left[\sqrt{2}\right]^2 . \left[\frac{2}{\sqrt{3}}\right]^2\right)\left(\left[\frac{1}{2}\right]^2 + 4\,(1)\,(2)^2\right)$$

$= 3 \cdot \frac{4}{3} \cdot \frac{1}{4}$

$= \frac{2}{3}$

11. cosec³ 30· cos60· tan³ 45· sin² 90· sec² 45· cot30·

Solution:

$cosec^3 30^\circ cos60^\circ tan^3 45^\circ sin^2 90^\circ sec^2 45^\circ cot30^\circ$

Using trigonometric values, we have

$= (2)^3 \times (\frac{1}{2}) \times (1^3) \times (1^2) \times (\sqrt{2}^2) \times (\sqrt{3})$

$= 8 \times (\frac{1}{2}) \times (1) \times (1) \times (2) \times (\sqrt{3})$

$= 8\sqrt{3}$

12. $cot^2 30^\circ - 2cos^2 60^\circ - (3/4)sec^2 45^\circ - 4sec^2 30^\circ$

Solution:

Using trigonometric values, we have

$cot^2 30^\circ - 2cos^2 60^\circ - \frac{3}{4} sec^2 45^\circ - 4sec^2 30^\circ$

$= (\sqrt{3}^2) \times 2(\frac{1}{2})^2 \times (\frac{3}{4} \times \sqrt{2}^2) \times (4 \times (\frac{2}{\sqrt{3}})^2)$

$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$

$= \frac{-13}{3}$

13. $(cos0^\circ + sin45^\circ + sin30^\circ)(sin90^\circ - cos45^\circ + cos60^\circ)$

Solution:

$(cos0^\circ + sin45^\circ + sin30^\circ)(sin90^\circ - cos45^\circ + cos60^\circ)$

Using trigonometric values, we have

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$$

$$= \left(\left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right)$$

$$= \frac{9}{4} - \frac{1}{2}$$

$$= \frac{7}{4}$$

14. $\dfrac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \tan 60°}$

Solution:

Given,

$$\frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \tan 60°}$$

Using trigonometric values, we have

$$= \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

$$= \frac{3}{2}$$

15. 4/cot² 30° + 1/sin² 60° − cos² 45°

Solution:

$$\frac{4}{cot^2 30^\circ} + \frac{1}{sin^2 60^\circ} - cos^2 45^\circ$$
$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{(\frac{\sqrt{3}}{2})^2} - (\frac{1}{\sqrt{2}})^2$$
$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2}$$
$$= \frac{16–3}{6}$$
$$= \frac{13}{6}$$

16. 4(sin⁴30∘ + cos² 60∘) − 3(cos² 45∘ − sin² 90∘) − sin² 60∘

Solution:

Using trigonometric values, we have

$$4(sin^4 30^\circ + cos^2 60^\circ) - 3(cos^2 45^\circ - sin^2 90^\circ) - sin^2 60^\circ$$
$$= 4((\tfrac{1}{2})^4 + (\tfrac{1}{2})^2) - 3((\tfrac{1}{\sqrt{2}})^2 - 1) - (\tfrac{\sqrt{3}}{2})^2$$
$$= 4(\tfrac{1}{16} + \tfrac{1}{4}) + \tfrac{3}{2} - \tfrac{3}{4}$$
$$= \tfrac{8}{4} = 2$$

$$\frac{tan^2 60^\circ + 4cos^2 45^\circ + 3sec^2 30^\circ + 5cos^2 90^\circ}{cosec 30^\circ + sec 60^\circ - cot^2 30^\circ}$$

17.

Solution:

Using trigonometric values, we have

$$\frac{tan^2 60^\circ + 4cos^2 45^\circ + 3sec^2 30^\circ + 5cos^2 90^\circ}{cosec 30^\circ + sec 60^\circ - cot^2 30^\circ}$$
$$= \frac{(\sqrt{3})^2 + 4(\frac{1}{\sqrt{2}})^2 + 3(\frac{2}{\sqrt{3}})^2 + 5(0)}{2 + 2 – (\sqrt{3})^2}$$
$$= 3 + 2 + 4$$
$$= 9$$

$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$ **18.**

Solution:

Using trigonometric values, we have

$$\frac{\sin 30°}{\sin 45°}+\frac{\tan 45°}{\sec 60°}-\frac{\sin 60°}{\cot 45°}-\frac{\cos 30°}{\sin 90°}$$

$$=\frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}+\frac{1}{2}-\frac{\frac{\sqrt{3}}{2}}{1}-\frac{\frac{\sqrt{3}}{2}}{1}$$

$$=\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$$

$$=\frac{\sqrt{2}+1-2\sqrt{3}}{2}$$

$$\frac{\tan 45°}{\operatorname{cosec} 30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$$

19.

Solution:

Using trigonometric values, we have

$$\frac{\tan 45°}{\operatorname{cosec} 30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$$

$$=\frac{1}{2}+\frac{2}{1}-\frac{5(1)}{2(1)}$$

$$=\frac{5}{2}-\frac{5}{2}$$

$$=0$$

Find the value of x in each of the following: (20-25)

20. 2sin 3x = √3

Solution:

Given,

2 sin 3x = √3

sin 3x = √3/2

sin 3x = sin 60°

3x = 60°

x = 20°

RD Sharma Class 10 Chapter 5 Exercise 5.3 Page No: 5.52

1. Evalute the following:

(i) sin 20°/ cos 70°

(ii) cos 19°/ sin 71°

(iii) sin 21°/ cos 69°

(iv) tan 10°/ cot 80°

(v) sec 11°/ cosec 79°

Solution:

(i) We have,

sin 20°/ cos 70° = sin (90° – 70°)/ cos 70° = cos 70°/ cos70° = 1 [∵ sin (90 – θ) = cos θ]

(ii) We have,

cos 19°/ sin 71° = cos (90° – 71°)/ sin 71° = sin 71°/ sin 71° = 1 [∵ cos (90 – θ) = sin θ]

(iii) We have,

sin 21°/ cos 69° = sin (90° – 69°)/ cos 69° = cos 69°/ cos69° = 1 [∵ sin (90 – θ) = cos θ]

(iv) We have,

tan 10°/ cot 80° = tan (90° – 10°) / cot 80° = cot 80°/ cos80° = 1 [∵ tan (90 – θ) = cot θ]

(v) We have,

sec 11°/ cosec 79° = sec (90° – 79°)/ cosec 79° = cosec 79°/ cosec 79° = 1

[∵ sec (90 – θ) = cosec θ]

2. Evaluate the following:

(i) $\left(\frac{\sin 49°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\sin 49°}\right)^2$

Solution:

We have, [∵ sin (90 – θ) = cos θ and cos (90 – θ) = sin θ]

$$\left(\frac{\sin 49°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\sin 49°}\right)^2$$

$$= \left(\frac{\sin(90° - 41°)}{\cos 41°}\right)^2 + \left(\frac{\cos(90° - 49°)}{\sin 49°}\right)^2$$

$$= \left(\frac{\cos 41°}{\cos 41°}\right)^2 + \left(\frac{\sin 49°}{\sin 49°}\right)^2$$

= $1^2 + 1^2$ = 1 + 1

= 2

(ii) cos 48°- sin 42°

Solution:

We know that, cos (90° − θ) = sin θ.

So,

cos 48° – sin 42° = cos (90° − 42°) – sin 42° = sin 42° – sin 42° = 0

Thus the value of cos 48° – sin 42° is 0.

$$\text{(iii) } \frac{\cot 40°}{\tan 50°} - \frac{1}{2}\left(\frac{\cos 35°}{\sin 55°}\right)$$

Solution:

We have, [∵ cot (90 – θ) = tan θ and cos (90 – θ) = sin θ]

$$\frac{\cot 40°}{\tan 50°} - \frac{1}{2}\left(\frac{\cos 35°}{\sin 55°}\right)$$

$$= \frac{\cot(90° - 50°)}{\tan 50°} - \frac{1}{2}\left(\frac{\cos(90° - 55°)}{\sin 55°}\right)$$

$$= \frac{\tan 50°}{\tan 50°} - \frac{1}{2}\left(\frac{\sin 55°}{\sin 55°}\right)$$

= 1 – 1/2(1)

= 1/2

$$\text{(iv) } \left(\frac{\sin 27°}{\cos 63°}\right)^2 - \left(\frac{\cos 63°}{\sin 27°}\right)^2$$

Solution:

We have, [∵ sin (90 – θ) = cos θ and cos (90 – θ) = sin θ]

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$$

$$=\left(\frac{\sin(90^\circ-63^\circ)}{\cos 63^\circ}\right)^2-\left(\frac{\cos(90^\circ-27^\circ)}{\sin 27^\circ}\right)^2$$

$$=\left(\frac{\cos 63^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\sin 27^\circ}{\sin 27^\circ}\right)^2$$

= 1 – 1

= 0

$$\text{(v)}\,\frac{\tan 35^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan 12^\circ}-1$$

Solution:

We have, [∵ cot (90 – θ) = tan θ and tan (90 – θ) = cot θ]

$$\frac{\tan 35^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan 12^\circ}-1$$

= tan (90° – 35°)/ cot 55° + cot (90° – 12°)/ tan 12° – 1

= cot 55°/ cot 55° + tan 12°/ tan 12° – 1

= 1 + 1 – 1

= 1

$$\text{(vi)}\,\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}$$

Solution:

We have , [∵ sin (90 – θ) = cos θ and sec (90 – θ) = cosec θ]

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}$$

= sec (90° – 20°)/ cosec 20° + sin (90° – 31°)/ cos 31°

= cosec 20°/ cosec 20° + cos 12°/ cos 12°

= 1 + 1

= 2

(vii) cosec 31° – sec 59°

Solution:

We have,

cosec 31° – sec 59°

Since, cosec (90 – θ) = cos θ

So,

cosec 31° – sec 59° = cosec (90° – 59°) – sec 59° = sec 59° – sec 59° = 0

Thus,

cosec 31° – sec 59° = 0

(viii) (sin 72° + cos 18°) (sin 72° – cos 18°)

Solution:

We know that,

sin (90 – θ) = cos θ

So, the given can be expressed as

(sin 72° + cos 18°) (sin (90 – 18)° – cos 18°)

= (sin 72° + cos 18°) (cos 18° – cos 18°)

= (sin 72° + cos 18°) x 0

= 0

(ix) sin 35° sin 55° – cos 35° cos 55°

Solution:

We know that,

sin (90 – θ) = cos θ

So, the given can be expressed as

sin (90 – 55)° sin (90 – 35)° – cos 35° cos 55°

= cos 55° cos 35° – cos 35° cos 55°

= 0

(x) tan 48° tan 23° tan 42° tan 67°

Solution:

We know that,

tan (90 – θ) = cot θ

So, the given can be expressed as

tan (90 – 42)° tan (90 – 67)° tan 42° tan 67°

= cot 42° cot 67° tan 42° tan 67°

= (cot 42° tan 42°)(cot 67° tan 67°)

= 1 x 1 [∵ tan θ x cot θ = 1]

= 1

(xi) sec 50° sin 40° + cos 40° cosec 50°

Solution:

We know that,

sin (90 – θ) = cos θ and cos (90 – θ) = sin θ

So, the given can be expressed as

sec 50° sin (90 – 50)° + cos (90 – 50)° cosec 50°

= sec 50° cos 50° + sin 50° cosec 50°

= 1 + 1 [∵ sin θ x cosec θ = 1 and cos θ x sec θ = 1]

= 2

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) sin 59° + cos 56° (ii) tan 65° + cot 49° (iii) sec 76° + cosec 52°

(iv) cos 78° + sec 78° (v) cosec 54° + sin 72° (vi) cot 85° + cos 75°

(vii) sin 67° + cos 75°

Solution:

Using the below trigonometric ratios of complementary angles, we find the required

sin (90 – θ) = cos θ cosec (90 – θ) = sec θ

cos (90 – θ) = sin θ sec (90 – θ) = cosec θ

tan (90 – θ) = cot θ cot (90 – θ) = tan θ

(i) sin 59° + cos 56° = sin (90 – 31)° + cos (90 – 34)° = cos 31° + sin 34°

(ii) tan 65° + cot 49° = tan (90 – 25)° + cot (90 -31)° = cot 25° + tan 31°

(iii) sec 76° + cosec 52° = sec (90 – 14)° + cosec (90 – 38)° = cosec 14° + sec 38°

(iv) cos 78° + sec 78° = cos (90 – 12)° + sec (90 – 12)° = sin 12° + cosec 12°

(v) cosec 54° + sin 72° = cosec (90 – 36)° + sin (90 – 18)° = sec 36° + cos 18°

(vi) cot 85° + cos 75° = cot (90 – 5)° + cos (90 – 15)° = tan 5° + sin 15°

4. Express cos 75° + cot 75° in terms of angles between 0° and 30°.

Solution:

Given,

cos 75° + cot 75°

Since, cos (90 – θ) = sin θ and cot (90 – θ) = tan θ

cos 75° + cot 75° = cos (90 – 15)° + cot (90 – 15)° = sin 15° + tan 15°

Hence, cos 75° + cot 75° can be expressed as sin 15° + tan 15°

5. If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

Solution:

Given,

sin 3A = cos (A – 26°)

Using cos (90 – θ) = sin θ, we have

sin 3A = sin (90° – (A – 26°))

Now, comparing both L.H.S and R.H.S

3A = 90° – (A – 26°)

3A + (A – 26°) = 90°

4A – 26° = 90°

4A = 116°

A = 116°/4

∴ $A = 29^\circ$

6. If A, B, C are the interior angles of a triangle ABC, prove that

(i) tan ((C + A)/ 2) = cot (B/2) (ii) sin ((B + C)/ 2) = cos (A/2)

Solution:

We know that, in triangle ABC the sum of the angles i.e $A + B + C = 180^\circ$

So, $C + A = 180^\circ - B \Rightarrow (C + A)/2 = 90^\circ - B/2$ …… (i)

And, $B + C = 180^\circ - A \Rightarrow (B + C)/2 = 90^\circ - A/2$ ……. (ii)

(i) L.H.S = tan ((C + A)/ 2)

⇒ tan ((C + A)/ 2) = tan (90° – B/2) [From (i)]

= cot (B/2) [∵ tan (90 – θ) = cot θ]

= R.H.S

- Hence Proved

(ii) L.H.S = sin ((B + C)/2)

⇒ tan ((B + C)/ 2) = tan (90° – A/2) [From (ii)]

= cot (B/2) [∵ tan (90 – θ) = cot θ]

= R.H.S

- Hence Proved

7. Prove that:

(i) tan 20°tan 35°tan 45°tan 55°tan 70°= 1

(ii) sin 48°sec 48°+ cos 48°cosec 42°= 2

$$\text{(iii)}\ \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \quad \operatorname{cosec} 42^\circ = 0$$

$$\text{(iv)}\ \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \ \ \operatorname{cosec} 31^\circ = 2$$

Solution:

(i) Taking L.H.S = tan 20° tan 35° tan 45° tan 55° t an 70°

= tan (90° – 70°) tan (90° – 55°) tan 45° tan 55° ta n70°

= cot 70° cot 55° tan 45° tan 55° tan 70° [∵ tan (90 – θ) = cot θ]

= (tan 70° cot 70°)(tan 55° cot 55°) tan 45° [∵ tan θ x cot θ = 1]

= 1 × 1 × 1 = 1

- Hence proved

(ii) Taking L.H.S = sin 48° sec 48° + cos 48° cosec 42°

= sin 48° sec (90° – 48°) cos 48° cosec (90° – 48°)

[∵sec (90 – θ) = cosec θ and cosec (90 – θ) = sec θ]

= sin 48° cosec 48° + cos 48° sec 48° [∵ cosec θ x sin θ = 1 and cos θ x sec θ = 1]

= 1 + 1 = 2

- Hence proved

(iii) Taking the L.H.S,

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \ \operatorname{cosec} 20^\circ$$

$$= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\cos 70^\circ}{\sin 20^\circ} - 2\cos 70^\circ \ \operatorname{cosec}(90^\circ - 70^\circ)$$

$$= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} - 2\cos 70^\circ \ \operatorname{cosec}(90^\circ - 70^\circ)$$

$$= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sin 20^\circ}{\sin 20^\circ} - 2 \times 1$$

= 1 + 1 – 2

= 2 – 2

= 0

- Hence proved

(iv) Taking L.H.S,

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \ \mathrm{cosec}\ 31^\circ$$

$$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \ \mathrm{cosec}(90^\circ - 59^\circ)$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \ \sec 59^\circ$$

= 1 + 1

= 2

- Hence proved

8. Prove the following:

(i) sinθ sin (90° – θ) – cos θ cos (90° – θ) = 0

Solution:

Taking the L.H.S,

sinθ sin (90° – θ) – cos θ cos (90° – θ)

= sin θ cos θ – cos θ sin θ [∵ sin (90 – θ) = cos θ and cos (90 – θ) = sin θ]

= 0

$$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\mathrm{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta} = 2$$ **(ii)**

Solution:

Taking the L.H.S,

$$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\tan\theta}{\mathrm{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta}$$

$$= \frac{\sin\theta\ \mathrm{cosec}\,\theta\ \tan\theta}{\sec\theta\ \cos\theta\ \tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1\times\tan\theta}{1\times\tan\theta} + 1 = \frac{\tan\theta}{\tan\theta} + 1$$

[∵ cosec θ x sin θ = 1 and cos θ x sec θ = 1]

= 1 + 1

= 2 = R.H.S

- Hence Proved

$$\frac{\tan(90^\circ - A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$$ **(iii)**

Solution:

Taking the L.H.S, [∵ tan (90° – θ) = cot θ]

$$\text{L.H.S.} = \frac{\tan(90^\circ - A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A$$

$$= \frac{\cot A \cot A}{\operatorname{cosec}^2 A} - \cos^2 A$$

$$= \frac{\cot^2 A}{\operatorname{cosec}^2 A} - \cos^2 A = \frac{\frac{\cos^2 A}{\sin^2 \theta}}{\frac{1}{\sin^2 A}} - \cos^2 A$$

$$= \frac{\cos^2 A \times \sin^2 A}{\sin^2 A \times 1} - \cos^2 A = \cos^2 A - \cos^2 A$$

= 0 = R.H.S

- Hence Proved

$$\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$$ **(iv)**

Solution:

Taking L.H.S, [∵ sin (90 – θ) = cos θ and cos (90 – θ) = sin θ]

$$\text{L.H.S.} = \frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)}$$

$$= \frac{\sin A\ \cos A}{\cot A} = \frac{\sin A\ \cos A}{\frac{\cos A}{\sin A}}$$

$$= \frac{\sin A\ \cos A \times \sin A}{\cos A} = \sin A \times \sin A$$

= $\sin^2 A$ = R.H.S

- Hence Proved

(v) sin (50° + θ) – cos (40° – θ) + tan 1° tan 10° tan 70° tan 80° tan 89° = 1

Solution:

Taking the L.H.S,

= sin (50° + θ) – cos (40° – θ) + tan 1° tan 10° tan 20° tan 70° tan 80° tan 89°

= sin (50° + θ) – sin (90° – (40° – θ)) + tan (90 – 89)° tan (90 – 80)° tan (90 – 70)° tan 70° tan 80° tan 89°[∵ sin (90 –

θ) = cos θ]

= sin (50° + θ) – sin (50° + θ) + cot 89° cot 80° cot 70° tan 70° tan 80° tan 89°

[∵ tan (90° – θ) = cot θ]

= 0 + (cot 89° x tan 89°) (cot 80° x tan 80°) (cot 70° x tan 70°)

= 0 + 1 x 1 x 1 [∵ tan θ x cot θ = 1]

= 1= R.H.S

- Hence Proved