Access answers to RD Sharma Solutions for Class 11 Maths Chapter 28 – Introduction to Three Dimensional Coordinate Geometry

ExERCISE 28.1 PAGE NO: 28.6

1. Name the octants in which the following points lie:
(i) (5, 2, 3)
(ii) (-5, 4, 3)
(iii) (4, -3, 5)
(iv) (7, 4, -3)
(v) (-5, -4, 7)
(vi) (-5, -3, -2)
(vii) (2, -5, -7)
(viii) (-7, 2, -5)

Solution:

(i) (5, 2, 3)

In this case, since x, y and z all three are positive then octant will be XOYZ

(ii) (-5, 4, 3)

In this case, since x is negative and y and z are positive then the octant will be X′OYZ

(iii) (4, -3, 5)

In this case, since y is negative and x and z are positive then the octant will be XOY′Z

(iv) (7, 4, -3)

In this case, since z is negative and x and y are positive then the octant will be XOYZ′

(v) (-5, -4, 7)

In this case, since x and y are negative and z is positive then the octant will be X′OY′Z

(vi) (-5, -3, -2)

In this case, since x, y and z all three are negative then octant will be X′OY′Z′

(vii) (2, -5, -7)

In this case, since z and y are negative and x is positive then the octant will be XOY′Z′

(viii) (-7, 2, -5)

In this case, since x and z are negative and x is positive then the octant will be X′OYZ′

2. Find the image of:
(i) (-2, 3, 4) in the yz-plane
(ii) (-5, 4, -3) in the xz-plane

(iii) (5, 2, -7) in the xy-plane
(iv) (-5, 0, 3) in the xz-plane
(v) (-4, 0, 0) in the xy-plane

Solution:

(i) (-2, 3, 4)

Since we need to find its image in yz-plane, a sign of its x-coordinate will change

So, Image of point (-2, 3, 4) is (2, 3, 4)

(ii)(-5, 4, -3)

Since we need to find its image in xz-plane, sign of its y-coordinate will change

So, Image of point (-5, 4, -3) is (-5, -4, -3)

(iii) (5, 2, -7)

Since we need to find its image in xy-plane, a sign of its z-coordinate will change

So, Image of point (5, 2, -7) is (5, 2, 7)

(iv) (-5, 0, 3)

Since we need to find its image in xz-plane, sign of its y-coordinate will change

So, Image of point (-5, 0, 3) is (-5, 0, 3)

(v) (-4, 0, 0)

Since we need to find its image in xy-plane, sign of its z-coordinate will change

So, Image of point (-4, 0, 0) is (-4, 0, 0)

3. A cube of side 5 has one vertex at the point (1, 0, 1), and the three edges from this vertex are, respectively, parallel to the negative x and y-axes and positive z-axis. Find the coordinates of the other vertices of the cube.

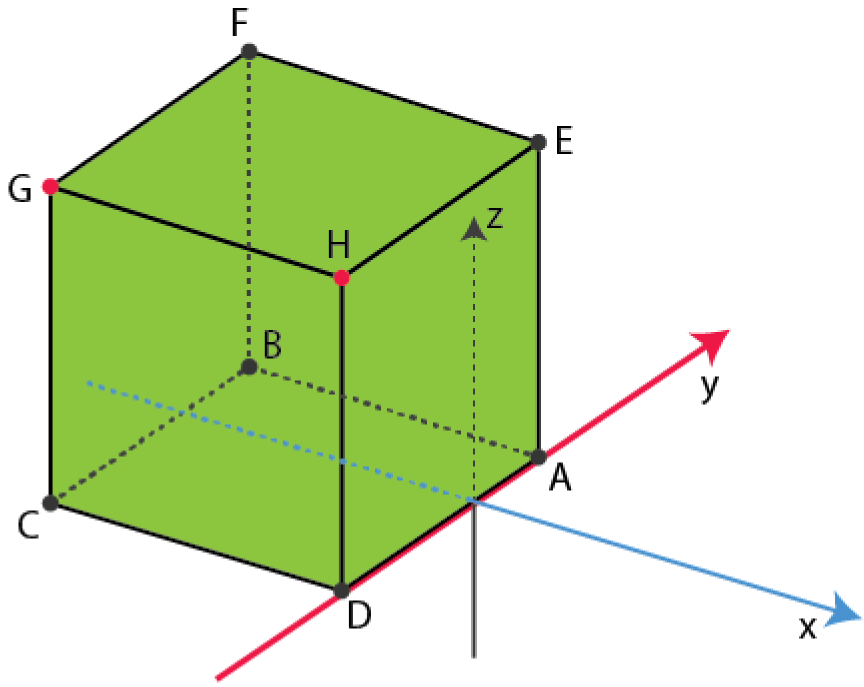
Solution:

Given: A cube has side 4 having one vertex at (1, 0, 1)

Side of cube = 5

We need to find the coordinates of the other vertices of the cube.

So let the Point A(1, 0, 1) and AB, AD and AE is parallel to –ve x-axis, -ve y-axis and +ve z-axis respectively.



Since side of cube = 5

Point B is (-4, 0, 1)

Point D is (1, -5, 1)

Point E is (1, 0, 6)

Now, EH is parallel to –ve y-axis

Point H is (1, -5, 6)

HG is parallel to –ve x-axis

Point G is (-4, -5, 6)

Now, again GC and GF is parallel to –ve z-axis and +ve y-axis respectively

Point C is (-4, -5, 1)

Point F is (-4, 0, 6)

4. Planes are drawn parallel to the coordinates planes through the points (3, 0, -1) and (-2, 5, 4). Find the lengths of the edges of the parallelepiped so formed.

Solution:

Given:

Points are (3, 0, -1) and (-2, 5, 4)

We need to find the lengths of the edges of the parallelepiped formed.

For point (3, 0, -1)

$x_1 = 3$, $y_1 = 0$ and $z_1 = -1$

For point (-2, 5, 4)

$x_2 = -2$, $y_2 = 5$ and $z_2 = 4$

Plane parallel to coordinate planes of x_1 and x_2 is yz-plane

Plane parallel to coordinate planes of y_1 and y_2 is xz-plane

Plane parallel to coordinate planes of z_1 and z_2 is xy-plane

Distance between planes $x_1 = 3$ and $x_2 = -2$ is $3 – (-2) = 3 + 2 = 5$

Distance between planes $x_1 = 0$ and $y_2 = 5$ is $5 – 0 = 5$

Distance between planes $z_1 = -1$ and $z_2 = 4$ is $4 – (-1) = 4 + 1 = 5$

∴Theedges of parallelepiped is 5, 5, 5

5. Planes are drawn through the points (5, 0, 2) and (3, -2, 5) parallel to the coordinate planes. Find the lengths of the edges of the rectangular parallelepiped so formed.

Solution:

Given:

Points are (5, 0, 2) and (3, -2, 5)

We need to find the lengths of the edges of the parallelepiped formed

For point (5, 0, 2)

$x_1 = 5$, $y_1 = 0$ and $z_1 = 2$

For point (3, -2, 5)

$x_2 = 3$, $y_2 = -2$ and $z_2 = 5$

Plane parallel to coordinate planes of x_1 and x_2 is yz-plane

Plane parallel to coordinate planes of y_1 and y_2 is xz-plane

Plane parallel to coordinate planes of z_1 and z_2 is xy-plane

Distance between planes $x_1 = 5$ and $x_2 = 3$ is $5 – 3 = 2$

Distance between planes $x_1 = 0$ and $y_2 = -2$ is $0 – (-2) = 0 + 2 = 2$

Distance between planes $z_1 = 2$ and $z_2 = 5$ is $5 – 2 = 3$

∴Theedges of parallelepiped is 2, 2, 3

6. Find the distances of the point P (-4, 3, 5) from the coordinate axes.

Solution:

Given:

The point P (-4, 3, 5)

The distance of the point from x-axis is given as:

$$
\begin{aligned}
\text{Distance} &= \sqrt{y^2 + z^2} \\
&= \sqrt{3^2 + 5^2} \\
&= \sqrt{9 + 25} \\
&= \sqrt{34}
\end{aligned}
$$

The distance of the point from y-axis is given as:

$$
\begin{aligned}
\text{Distance} &= \sqrt{x^2 + z^2} \\
&= \sqrt{(-4)^2 + 5^2} \\
&= \sqrt{16 + 25} \\
&= \sqrt{41}
\end{aligned}
$$

The distance of the point from z-axis is given as:

$$
\begin{aligned}
\text{Distance} &= \sqrt{x^2 + y^2} \\
&= \sqrt{(-4)^2 + 3^2} \\
&= \sqrt{16 + 9} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$

7. The coordinates of a point are (3, -2, 5). Write down the coordinates of seven points such that the absolute values of their coordinates are the same as those of the coordinates of the given point.

Solution:

Given:

Point (3, -2, 5)

The Absolute value of any point(x, y, z) is given by,

$\sqrt{(x^2 + y^2 + z^2)}$

We need to make sure that absolute value to be the same for all points.

So let the point A(3, -2, 5)

Remaining 7 points are:

Point B(3, 2, 5) (By changing the sign of y coordinate)

Point C(-3, -2, 5) (By changing the sign of x coordinate)

Point D(3, -2, -5) (By changing the sign of z coordinate)

Point E(-3, 2, 5) (By changing the sign of x and y coordinate)

Point F(3, 2, -5) (By changing the sign of y and z coordinate)

Point G(-3, -2, -5) (By changing the sign of x and z coordinate)

Point H(-3, 2, -5) (By changing the sign of x, y and z coordinate)

ExERCISE 28.2 PAGE NO: 28.9

1. Find the distance between the following pairs of points:
(i) P(1, -1, 0) and Q (2, 1, 2)

(ii) A(3, 2, -1) and B (-1, -1, -1)

Solution:

(i) P(1, -1, 0) and Q (2, 1, 2)

Given:

The points P(1, -1, 0) and Q (2, 1, 2)

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

Distance between the points (1, -1, 0) and (2, 1, 2) is given as

$= \sqrt{(1-2)^2+(-1-1)^2+(0-2)^2}$

$= \sqrt{(-1)^2+(-2)^2+(-2)^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

$= 3$

∴ The Distance between P and Q is 3 units.

(ii) A (3, 2, -1) and B (-1, -1, -1)

Given:

The points A (3, 2, -1) and B (-1, -1, -1)

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

Distance between the points (3, 2, -1) and (-1, -1, -1) is given as

$$= \sqrt{(3-(-1))^2+(2-(-1))^2+(-1-(-1))^2}$$

$$= \sqrt{(3+1)^2+(2+1)^2+(-1+1)^2}$$

$$= \sqrt{(4)^2+(3)^2+(0)^2}$$

$$= \sqrt{16+9+0}$$

$$= \sqrt{25}$$

$$= 5$$

= 5

∴ The Distance between A and B is 5 units.

2. Find the distance between the points P and Q having coordinates (-2, 3, 1) and (2, 1, 2).

Solution:

Given:

The points (-2, 3, 1) and (2, 1, 2)

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

Distance between the points (-2, 3, 1) and (2, 1, 2) is given as

$$= \sqrt{(-2-2)^2+(3-1)^2+(1-2)^2}$$

$$= \sqrt{(-4)^2+(2)^2+(-1)^2}$$

$$= \sqrt{16+4+1}$$

$$= \sqrt{21}$$

∴ The Distance between the given two points is $\sqrt{21}$ units.

3. Using distance formula prove that the following points are collinear:
(i) A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)

(ii) P(0, 7, -7), Q(1, 4, -5) and R(-1, 10, -9)

(iii) A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)
Solution:

(i) A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)

Given:

The points A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)

Points A, B and C are collinear if AB + BC = AC or AB + AC = BC or AC + BC = AB

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

Distance between the points A (4, -3, -1) and B (5, -7, 6) is AB,

$$= \sqrt{(4-5)^2+(-3-(-7))^2+(-1-6)^2}$$

$$= \sqrt{(-1)^2+(-4)^2+(-7)^2}$$

$$= \sqrt{1+16+49}$$

$$= \sqrt{66}$$

= √66

Distance between the points B (5, -7, 6) and C (3, 1, -8) is BC,

$$= \sqrt{(5-3)^2+(-7-1)^2+(6-(-8))^2}$$

$$= \sqrt{(-2)^2+(-8)^2+(14)^2}$$

$$= \sqrt{4+64+196}$$

$$= \sqrt{264}$$

$$= 2\sqrt{66}$$

Distance between the points A (4, -3, -1) and C (3, 1, -8) is AC,

$$= \sqrt{(4-3)^2+(-3-1)^2+(-1-(-8))^2}$$

$$= \sqrt{(1)^2+(-4)^2+(7)^2}$$

$$= \sqrt{1+16+49}$$

$$= \sqrt{66}$$

Clearly,

AB + AC

$$= \sqrt{66}+\sqrt{66}$$

$$= 2\sqrt{66}$$

= BC

∴The points A, B and C are collinear.

(ii) P (0, 7, -7), Q (1, 4, -5) and R (-1, 10, -9)

Given:

The points P (0, 7, -7), Q (1, 4, -5) and R (-1, 10, -9)

Points P, Q and R are collinear if PQ + QR = PR or PQ + PR = QR or PR + QR = PQ

By using the formula,

Distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

Distance between the points P (0, 7, -7) and Q (1, 4, -5) is PQ,

$$= \sqrt{(0-1)^2+(7-4)^2+(-7-(-5))^2}$$

$$= \sqrt{(-1)^2+(3)^2+(-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

= √14

Distance between the points Q (1, 4, -5) and R (-1, 10, -9) is QR,

$$= \sqrt{(1-(-1))^2+(4-10)^2+(-5-(-9))^2}$$

$$= \sqrt{(2)^2+(-6)^2+(4)^2}$$

$$= \sqrt{4+36+16}$$

$$= \sqrt{56}$$

$$= 2\sqrt{14}$$

Distance between the points P (0, 7, -7) and R (-1, 10, -9) is PR,

$$= \sqrt{(0-(-1))^2+(7-10)^2+(-7-(-9))^2}$$

$$= \sqrt{(1)^2+(-3)^2+(2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

It is clear that,

PQ + PR

$$= \sqrt{14}+\sqrt{14}$$

$$= 2\sqrt{14}$$

= QR

∴The points P, Q and R are collinear.

(iii) A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

Given:

The points A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

Points A, B and C are collinear if AB + BC = AC or AB + AC = BC or AC + BC = AB

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

Distance between the points A (3, -5, 1) and B (-1, 0, 8) is AB,

$$= \sqrt{(3-(-1))^2+(-5-0)^2+(1-8)^2}$$

$$= \sqrt{(4)^2+(-5)^2+(-7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10}$$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90}$

$= 3\sqrt{10}$

Distance between the points B (-1, 0, 8) and C (7, -10, -6) is BC,

$= \sqrt{(-1-7)^2 + (0-(-10))^2 + (8-(-6))^2}$

$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$

$= \sqrt{64 + 100 + 196}$

$= \sqrt{360}$

$= 6\sqrt{10}$

Distance between the points A (3, -5, 1) and C (7, -10, -6) is AC,

$= \sqrt{(3-7)^2 + (-5-(-10))^2 + (1-(-6))^2}$

$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90}$

$= 3\sqrt{10}$

It is clear that,

AB + AC

$= 3\sqrt{10} + 3\sqrt{10}$

$= 6\sqrt{10}$

= BC

∴The points A, B and C are collinear.

4. Determine the points in (i) xy-plane (ii) yz-plane and (iii) zx-plane which are equidistant from the points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1).

Solution:

Given:

The points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1)

(i) xy-plane

We know z = 0 in xy-plane.

So let P(x, y, 0) be any point in xy-plane

According to the question:

PA = PB = PC

$PA^2 = PB^2 = PC^2$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

So,

Distance between the points P (x, y, 0) and A (1, -1, 0) is PA,

$$= \sqrt{(x-1)^2 + (y-(-1))^2 + (0-0)^2}$$

$$= \sqrt{(x-1)^2 + (y+1)^2}$$

The distance between the points P (x, y, 0) and B (2, 1, 2) is PB,

$$= \sqrt{(x-2)^2 + (y-1)^2 + (0-2)^2}$$

$$= \sqrt{(x-2)^2 + (y-1)^2 + 4}$$

Distance between the points P (x, y, 0) and C (3, 2, -1) is PC,

$$= \sqrt{(x-3)^2 + (y-2)^2 + (0-(-1))^2}$$

$$= \sqrt{(x-3)^2 + (y-2)^2 + 1}$$

We know $PA^2 = PB^2$

So, $(x-1)^2 + (y+1)^2 = (x-2)^2 + (y-1)^2 + 4$

$x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$

$-2x + 2 + 2y = 9 - 4x - 2y$

$-2x + 2 + 2y - 9 + 4x + 2y = 0$

$2x + 4y - 7 = 0$

$2x = -4y + 7$……………………(1)

Since, $PA^2 = PC^2$

So, $(x-1)^2 + (y+1)^2 = (x-3)^2 + (y-2)^2 + 1$

$x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$

$-2x + 2 + 2y = 14 - 6x - 4y$

$-2x + 2 + 2y - 14 + 6x + 4y = 0$

$4x + 6y - 12 = 0$

$2(2x + 3y - 6) = 0$

Now substitute the value of 2x (obtained in equation (1)), we get

$7 – 4y + 3y – 6 = 0$

$– y + 1 = 0$

$y = 1$

By substituting the value of y back in equation (1) we get,

$2x = 7 – 4y$

$2x = 7 – 4(1)$

$2x = 3$

$x = 3/2$

∴The point P (3/2, 1, 0) in xy-plane is equidistant from A, B and C.

(ii) yz-plane

We know x = 0 in yz-plane.

Let Q(0, y, z) any point in yz-plane

According to the question:

$QA = QB = QC$

$QA^2 = QB^2 = QC^2$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

Distance between the points Q (0, y, z) and A (1, -1, 0) is QA,

$= \sqrt{(0-1)^2+(y-(-1))^2+(z-0)^2}$

$= \sqrt{1+(y+1)^2+z^2}$

The distance between the points Q (0, y, Z) and B (2, 1, 2) is QB,

$= \sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$

$= \sqrt{(z-2)^2+(y-1)^2+4}$

Distance between the points Q (0, y, z) and C (3, 2, -1) is QC,

$= \sqrt{(0-3)^2+(y-2)^2+(z-(-1))^2}$

$= \sqrt{(z+1)^2+(y-2)^2+9}$

We know, $QA^2 = QB^2$

So, $1 + z^2 + (y + 1)^2 = (z – 2)^2 + (y – 1)^2 + 4$

$z^2 + 1 + y^2 + 1 + 2y = z^2 + 4 – 4z + y^2 + 1 – 2y + 4$

$2 + 2y = 9 – 4z – 2y$

$2 + 2y – 9 + 4z + 2y = 0$

$4y + 4z – 7 = 0$

$4z = –4y + 7$

$z = [–4y + 7]/4$ …. (1)

Since, $QA^2 = QC^2$

So, $1 + z^2 + (y + 1)^2 = (z + 1)^2 + (y – 2)^2 + 9$

$^2 + 1 + y^2 + 1 + 2y = z^2 + 1 + 2z + y^2 + 4 – 4y + 9$

$2 + 2y = 14 + 2z – 4y$

$2 + 2y – 14 – 2z + 4y = 0$

$–2z + 6y – 12 = 0$

$2(–z + 3y – 6) = 0$

Now, substitute the value of z [obtained from (1)] we get

$$3y - \frac{(-4y + 7)}{4} - 6 = 0$$

$$\frac{12y - (-4y + 7) - 24}{4} = 0$$

12y + 4y – 7 – 24 = 0

16y – 31 = 0

y = 31/16

Substitute the value of y back in equation (1), we get

$$z = \frac{-4y + 7}{4}$$

$$= \frac{-4\left(\frac{31}{16}\right) + 7}{4}$$

$$= \frac{-\frac{124}{16} + 7}{4}$$

$$= \frac{\frac{-124 + 112}{16}}{4}$$

$$= \frac{-12}{4 \times 16}$$

$$= \frac{-3}{16}$$

= -3/16

∴The point Q (0, 31/16, -3/16) in yz-plane is equidistant from A, B and C.

(iii) zx-plane

We know y = 0 in xz-plane.

Let R(x, 0, z) any point in xz-plane

According to the question:

RA = RB = RC

$RA^2 = RB^2 = RC^2$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

Distance between the points R (x, 0, z) and A (1, -1, 0) is RA,

$= \sqrt{(x-1)^2+(0-(-1))^2+(z-0)^2}$

$= \sqrt{1+(x-1)^2+z^2}$

Distance between the points R (x, 0, z) and B (2, 1, 2) is RB,

$= \sqrt{(x-2)^2+(0-1)^2+(z-2)^2}$

$= \sqrt{(z-2)^2+(x-2)^2+1}$

Distance between the points R (x, 0, z) and C (3, 2, -1) is RC,

$= \sqrt{(x-3)^2+(0-2)^2+(z-(-1))^2}$

$= \sqrt{(z+1)^2+(x-3)^2+4}$

We know, $RA^2 = RB^2$

So, $1 + z^2 + (x – 1)^2 = (z – 2)^2 + (x – 2)^2 + 1$

$z^2 + 1 + x^2 + 1 – 2x = z^2 + 4 – 4z + x^2 + 4 – 4x + 1$

$2 – 2x = 9 – 4z – 4x$

$2 + 4z – 9 + 4x – 2x = 0$

$2x + 4z – 7 = 0$

$2x = –4z + 7$…………………………….(1)

Since, $RA^2 = RC^2$

So, $1 + z^2 + (x – 1)^2 = (z + 1)^2 + (x – 3)^2 + 4$

$z^2 + 1 + x^2 + 1 – 2x = z^2 + 1 + 2z + x^2 + 9 – 6x + 4$

$2 – 2x = 14 + 2z – 6x$

$2 – 2x – 14 – 2z + 6x = 0$

$–2z + 4x – 12 = 0$

$2(2x) = 12 + 2z$

Substitute the value of 2x [obtained from equation (1)] we get,

$2(–4z + 7) = 12 + 2z$

$–8z + 14 = 12 + 2z$

$14 – 12 = 8z + 2z$

10z = 2

z = 2/10

= 1/5

Now, substitute the value of z back in equation (1), we get

2x = -4z + 7

$$2x = -4\left(\frac{1}{5}\right) + 7$$

$$= -\frac{4}{5} + 7$$

$$= \frac{-4 + 35}{5}$$

$$= \frac{31}{5}$$

$$x = \frac{31}{10}$$

∴The point R (31/10, 0, 1/5) in xz-plane is equidistant from A, B and C.

5. Determine the point on z-axis which is equidistant from the points (1, 5, 7) and (5, 1, -4)

Solution:

Given:

The points (1, 5, 7) and (5, 1, -4)

We know x = 0 and y = 0 on z-axis

Let R(0, 0, z) any point on z-axis

According to the question:

RA = RB

$RA^2 = RB^2$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

Distance between R (0, 0, z) and A (1, 5, 7) is RA,

$= \sqrt{(0-1)^2+(0-5)^2+(z-7)^2}$

$= \sqrt{1+25+(z-7)^2}$

$= \sqrt{26+(z-7)^2}$

Distance between R (0, 0, z) and B (5, 1, -4) is RB,

$= \sqrt{(0-5)^2+(0-1)^2+(z-(-4))^2}$

$= \sqrt{(z+4)^2+25+1}$

$= \sqrt{(z+4)^2+26}$

We know, $RA^2 = RB^2$

$26+ (z – 7)^2 = (z + 4)^2 + 26$

$z^2+ 49 – 14z + 26 = z^2+ 16 + 8z + 26$

49 – 14z = 16 + 8z

49 – 16 = 14z + 8z

22z = 33

z = 33/22

= 3/2

∴The point R (0, 0, 3/2) on z-axis is equidistant from (1, 5, 7) and (5, 1, -4).

6. Find the point on y-axis which is equidistant from the points (3, 1, 2) and (5, 5, 2).

Solution:

Given:

The points (3, 1, 2) and (5, 5, 2)

We know x = 0 and z = 0 on y-axis

Let R(0, y, 0) any point on the y-axis

According to the question:

RA = RB

$RA^2 = RB^2$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

The distance between the points R (0, y, 0) and A (3, 1, 2) is RA,

$$= \sqrt{(0-3)^2+(y-1)^2+(0-2)^2}$$

$$= \sqrt{9+4+(y-1)^2}$$

$$= \sqrt{13+(y-1)^2}$$

Distance between the points R (0, y, 0) and B (5, 5, 2) is RB,

$$= \sqrt{(0-5)^2+(y-5)^2+(0-2)^2}$$

$$= \sqrt{(y-5)^2+25+4}$$

$$= \sqrt{(y-5)^2+29}$$

We know, $RA^2 = RB^2$

$13+ (y – 1)^2 = (y – 5)^2 + 29$

$y^2+ 1 – 2y + 13 = y^2+ 25 – 10y + 29$

10y – 2y = 54 – 14

8y = 40

y = 40/8

= 5

∴The point R (0, 5, 0) on y-axis is equidistant from (3, 1, 2) and (5, 5, 2).

7. Find the points on z-axis which are at a distance√21 from the point (1, 2, 3).

Solution:

Given:

The point (1, 2, 3)

Distance = √21

We know x = 0 and y = 0 on z-axis

Let R(0, 0, z) any point on z-axis

According to question:

RA = √21

$RA^2 = 21$

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

Distance between the points R (0, 0, z) and A (1, 2, 3) is RA,

$$= \sqrt{(0-1)^2+(0-2)^2+(z-3)^2}$$

$$= \sqrt{1+4+(z-3)^2}$$

$$= \sqrt{5+(z-3)^2}$$

$$= \sqrt{1+4+(z-3)^2}$$

$$= \sqrt{5+(z-3)^2}$$

We know, $RA^2 = 21$

$5 + (z – 3)^2 = 21$

$z^2 + 9 – 6z + 5 = 21$

$z^2 – 6z = 21 – 14$

$z^2 – 6z – 7 = 0$

$z^2 – 7z + z – 7 = 0$

z(z– 7) + 1(z – 7) = 0

(z– 7) (z + 1) = 0

(z– 7) = 0 or (z + 1) = 0

z= 7 or z = -1

∴The points (0, 0, 7) and (0, 0, -1) on z-axis is equidistant from (1, 2, 3).

8. Prove that the triangle formed by joining the three points whose coordinates are (1, 2, 3), (2, 3, 1) and (3, 1, 2) is an equilateral triangle.

Solution:

Given:

The points (1, 2, 3), (2, 3, 1) and (3, 1, 2)

An equilateral triangle is a triangle whose all sides are equal.

So let us prove AB = BC = AC

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

The distance between the points A (1, 2, 3) and B (2, 3, 1) is AB,

$= \sqrt{(1-2)^2+(2-3)^2+(3-1)^2}$

$= \sqrt{(-1)^2+(-1)^2+2^2}$

$= \sqrt{1+1+4}$

$= \sqrt{6}$

The distance between the points B (2, 3, 1) and C (3, 1, 2) is BC,

$= \sqrt{(2-3)^2+(3-1)^2+(1-2)^2}$

$= \sqrt{(-1)^2+2^2+(-1)^2}$

$= \sqrt{1+1+4}$

$= \sqrt{6}$

The distance between the points A (1, 2, 3) and C (3, 1, 2) is AC,

$= \sqrt{(1-3)^2+(2-1)^2+(3-2)^2}$

$= \sqrt{(-2)^2+1^2+1^2}$

$= \sqrt{1+1+4}$

$= \sqrt{6}$

= √6

The distance between the points A (1, 2, 3) and C (3, 1, 2) is AC,

$= \sqrt{(1-3)^2+(2-1)^2+(3-2)^2}$

$= \sqrt{(-2)^2+1^2+1^2}$

$= \sqrt{1+1+4}$

$= \sqrt{6}$

It is clear that,

AB = BC = AC

Δ ABC is a equilateral triangle

Hence Proved.

9. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of an isosceles right-angled triangle.

Solution:

Given:

The points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6)

Isosceles right-angled triangle is a triangle whose two sides are equal and also satisfies Pythagoras Theorem.

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

So,

The distance between the points A (0, 7, 10) and B (-1, 6, 6) is AB,

$$= \sqrt{(0-(-1))^2 + (7-6)^2 + (10-6)^2}$$

$$= \sqrt{1^2 + 1^2 + 4^2}$$

$$= \sqrt{1 + 1 + 16}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

The distance between the points B (-1, 6, 6) and C (-4, 9, 6) is BC,

$$= \sqrt{(-1-(-4))^2 + (6-9)^2 + (6-6)^2}$$

$$= \sqrt{3^2 + (-3)^2 + 0^2}$$

$$= \sqrt{9 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points A (0, 7, 10) and C (-4, 9, 6) is AC,

$= \sqrt{(0 - (-4))^2 + (7 - 9)^2 + (10 - 6)^2}$

$= \sqrt{4^2 + (-2)^2 + 4^2}$

$= \sqrt{16 + 4 + 16}$

$= \sqrt{36}$

$= 6$

Since, AB = BC

So, $AB^2 + BC^2$

$= (3\sqrt{2})^2 + (3\sqrt{2})^2$

= 18 + 18

= 36

$= AC^2$

We know that, AB = BC and $AB^2 + BC^2 = AC^2$

So, Δ ABC is an isosceles-right angled triangle

Hence Proved.

10. Show that the points A(3, 3, 3), B(0, 6, 3), C(1, 7, 7) and D(4, 4, 7) are the vertices of squares.

Solution:

Given:

The points A (3, 3, 3), B (0, 6, 3), C (1, 7, 7) and D (4, 4, 7)

We know that all sides of a square are equal.

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

The distance between the points A (3, 3, 3) and B (0, 6, 3) is AB,

$= \sqrt{(3-0)^2+(3-6)^2+(3-3)^2}$

$= \sqrt{3^2+3^2+0^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points B (0, 6, 3) and C (1, 7, 7) is BC,

$= \sqrt{(0-1)^2+(6-7)^2+(3-7)^2}$

$= \sqrt{1^2+1^2+4^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points C (1, 7, 7) and D (4, 4, 7) is CD,

$= \sqrt{(1-4)^2+(7-4)^2+(7-7)^2}$

$= \sqrt{3^2+3^2+0^2}$

$= \sqrt{9+9+0}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between the points A (3, 3, 3) and D (4, 4, 7) is AD,

$= \sqrt{(3-4)^2+(3-4)^2+(3-7)^2}$

$= \sqrt{1^2+1^2+4^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

It is clear that,

AB = BC = CD = AD

Quadrilateral formed by ABCD is a square. [Since all sides are equal]

Hence Proved.

11. Prove that the point A(1, 3, 0), B(-5, 5, 2), C(-9, -1, 2) and D(-3, -3, 0) taken in order are the vertices of a parallelogram. Also, show that ABCD is not a rectangle.

Solution:

Given:

The points A (1, 3, 0), B (-5, 5, 2), C (-9, -1, 2) and D (-3, -3, 0)

We know that, opposite sides of both parallelogram and rectangle are equal.

But diagonals of a parallelogram are not equal whereas they are equal for rectangle.

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

So,

The distance between the points A (1, 3, 0) and B (-5, 5, 2) is AB,

$= \sqrt{(1-(-5))^2+(3-5)^2+(0-2)^2}$

$= \sqrt{6^2+2^2+2^2}$

$= \sqrt{36+4+4}$

$= \sqrt{44}$

$= 2\sqrt{11}$

The distance between the points B (-5, 5, 2) and C (-9, -1, 2) is BC,

$= \sqrt{(-5-(-9))^2+(5-(-1))^2+(2-2)^2}$

$= \sqrt{4^2+6^2+0^2}$

$= \sqrt{16+36+0}$

$= \sqrt{52}$

$= 2\sqrt{13}$

The distance between the points C (-9, -1, 2) and D (-3, -3, 0) is CD,

$= \sqrt{(-9-(-3))^2+(-1-(-3))^2+(2-0)^2}$

$= \sqrt{6^2+2^2+2^2}$

$= \sqrt{36+4+4}$

$= \sqrt{44}$

$= 2\sqrt{11}$

The distance between the points A (1, 3, 0) and D (-3, -3, 0) is AD,

$= \sqrt{(1-(-3))^2+(3-(-3))^2+(0-0)^2}$

$= \sqrt{4^2+6^2+0^2}$

$= \sqrt{16+36+0}$

$= \sqrt{52}$

$= 2\sqrt{13}$

$= \sqrt{16+36+0}$

$= \sqrt{52}$

$= 2\sqrt{13}$

It is clear that,

AB = CD

BC = AD

Opposite sides are equal

Now, let us find the length of diagonals

By using the formula,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

The distance between the points A (1, 3, 0) and C (-9, -1, 2) is AC,

$$= \sqrt{(1-(-9))^2+(3-(-1))^2+(0-2)^2}$$

$$= \sqrt{10^2+4^2+2^2}$$

$$= \sqrt{100+16+4}$$

$$= \sqrt{120}$$

$$= 2\sqrt{30}$$

The distance between the points B (-5, 5, 2) and D (-3, -3, 0) is BD,

$$= \sqrt{(-5-(-3))^2+(5-(-3))^2+(2-0)^2}$$

$$= \sqrt{(-2)^2+8^2+2^2}$$

$$= \sqrt{4+64+4}$$

$$= \sqrt{72}$$

$$= 6\sqrt{2}$$

It is clear that,

AC ≠ BD

The diagonals are not equal, but opposite sides are equal.

So we can say that quadrilateral formed by ABCD is a parallelogram but not a rectangle.

Hence Proved.

12.Show that the points A(1, 3, 4), B(-1, 6, 10), C(-7, 4, 7) and D(-5, 1, 1) are the vertices of a rhombus.

Solution:

Given:

The points A (1, 3, 4), B (-1, 6, 10), C (-7, 4, 7) and D (-5, 1, 1)

We know that, all sides of both square and rhombus are equal.

But diagonals of a rhombus are not equal whereas they are equal for square.

By using the formula,

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

The distance between the points A (1, 3, 4) and B (-1, 6, 10) is AB,

$= \sqrt{(1-(-1))^2+(3-6)^2+(4-10)^2}$

$= \sqrt{2^2+(-3)^2+(-6)^2}$

$= \sqrt{4+9+36}$

$= \sqrt{49}$

$= 7$

The distance between the points B (-1, 6, 10) and C (-7, 4, 7) is BC,

$= \sqrt{(-1-(-7))^2+(6-4)^2+(10-7)^2}$

$= \sqrt{6^2+2^2+3^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

$= 7$

The distance between the points C (-7, 4, 7) and D (-5, 1, 1) is CD,

$= \sqrt{(-7-(-5))^2+(4-1)^2+(7-1)^2}$

$= \sqrt{(-2)^2+3^2+6^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

$= 7$

The distance between the points A (1, 3, 4) and D (-5, 1, 1) is AD,

$= \sqrt{(1-(-5))^2+(3-1)^2+(4-1)^2}$

$= \sqrt{6^2+2^2+3^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

$= 7$

$= \sqrt{6^2 + 2^2 + 3^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

$= 7$

It is clear that,

AB = BC = CD = AD

So, all sides are equal

Now, let us find the length of diagonals

By using the formula,

$\sqrt{(a - m)^2 + (b - n)^2 + (c - o)^2}$

So,

The distance between the points A (1, 3, 4) and C (-7, 4, 7) is AC,

$= \sqrt{(1 - (-7))^2 + (3 - 4)^2 + (4 - 7)^2}$

$= \sqrt{8^2 + (-1)^2 + (-3)^2}$

$= \sqrt{64 + 1 + 9}$

$= \sqrt{74}$

The distance between the points B (-1, 6, 10) and D (-5, 1, 1) is BD,

$= \sqrt{(-1 - (-5))^2 + (6 - 1)^2 + (10 - 1)^2}$

$= \sqrt{4^2 + 5^2 + 9^2}$

$= \sqrt{16 + 25 + 81}$

$= \sqrt{112}$

$= 4\sqrt{7}$

It is clear that,

AC ≠ BD

The diagonals are not equal but all sides are equal.

So we can say that quadrilateral formed by ABCD is a rhombus but not square.

Hence Proved.

ExERCISE 28.3 PAGE NO: 28.19

1. The vertices of the triangle are A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2). The internal bisector of angle A meets BC at D. Find the coordinates of D and the length AD.

Solution:

Given:

The vertices of the triangle are A (5, 4, 6), B (1, -1, 3) and C (4, 3, 2).

By using the formulas let us find the coordinates of D and the length of AD

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

The section formula is given as

$$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$$

The distance between the points A (5, 4, 6) and B (1, -1, 3) is AB,

$$= \sqrt{(5-1)^2+(4-(-1))^2+(6-3)^2}$$

$$= \sqrt{4^2+5^2+3^2}$$

$$= \sqrt{16+25+9}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

The distance between the points A (5, 4, 6) and C (4, 3, 2) is AC,

$$= \sqrt{(5-4)^2+(4-3)^2+(6-2)^2}$$

$$= \sqrt{1^2+1^2+4^2}$$

$$= \sqrt{1+1+16}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

So,

$$\frac{AB}{AC} = \frac{5\sqrt{2}}{3\sqrt{2}} = \frac{5}{3}$$

AB : AC = 5:3

BD: DC = 5:3

So, m = 5 and n = 3

B(1, -1, 3) and C(4, 3, 2)

Coordinates of D using section formula:

$$= \left(\frac{3(1)+5(4)}{5+3}, \frac{3(-1)+5(3)}{5+3}, \frac{3(3)+5(2)}{5+3}\right)$$

$$= \left(\frac{3+20}{8}, \frac{-3+15}{8}, \frac{9+10}{8}\right)$$

$$= \left(\frac{23}{8}, \frac{12}{8}, \frac{19}{8}\right)$$

$$= \left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8}\right)$$

The distance between the points A (5, 4, 6) and D (23/8, 3/2, 19/8) is AD,

$$= \sqrt{\left(5-\frac{23}{8}\right)^2 + \left(4-\frac{3}{2}\right)^2 + \left(6-\frac{19}{8}\right)^2}$$

$$= \sqrt{\left(\frac{40-23}{8}\right)^2 + \left(\frac{8-3}{2}\right)^2 + \left(\frac{48-19}{8}\right)^2}$$

$$= \sqrt{\left(\frac{17}{8}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{29}{8}\right)^2}$$

$$= \sqrt{\frac{289}{64} + \frac{25}{4} + \frac{361}{64}}$$

$$= \sqrt{\frac{289+400+841}{64}}$$

$$= \sqrt{\frac{1530}{64}}$$

$$= \frac{\sqrt{1530}}{8} \text{ Units}$$

∴ The Coordinates of D are $\left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8}\right)$ and the length of AD is $\frac{\sqrt{1530}}{8}$ units.

2. A point C with z-coordinate 8 lies on the line segment joining the points A(2, -3, 4) and B(8, 0, 10). Find the coordinates.

Solution:

Given:

The points A (2, -3, 4) and B (8, 0, 10)

By using the section formula,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let Point C(x, y, 8), and C divides AB in ratio k: 1

So, m = k and n = 1

A(2, -3, 4) and B(8, 0, 10)

Coordinates of C are:

$$(x, y, 8) = \left(\frac{k(8) + 1(2)}{k + 1}, \frac{k(0) + 1(-3)}{k + 1}, \frac{k(10) + 1(4)}{k + 1}\right)$$
$$= \left(\frac{8k + 2}{k + 1}, \frac{-3}{k + 1}, \frac{10k + 4}{k + 1}\right)$$

On comparing we get,

[10k + 4] / [k + 1] = 8
10k + 4 = 8(k + 1)

10k + 4 = 8k + 8

10k – 8k = 8 – 4

2k = 4

k = 4/2

= 2

Here C divides AB in ratio 2:1

x = [8k + 2] / [k + 1]

= [8(2) + 2] / [2 + 1]

= [16 + 2] / [3]

= 18/3

= 6

y = -3 / [k + 1]

= -3 / [2 + 1]

= -3 / 3

= -1

∴ The Coordinates of C are (6, -1, 8).

3. Show that the three points A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10) are collinear and find the ratio in which C divides AB.

Solution:

Given:

The points A (2, 3, 4), B (-1, 2, -3) and C (-4, 1, -10)

By using the section formula,

$$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$$

Let C divides AB in ratio k: 1

Three points are collinear if the value of k is the same for x, y and z coordinates.

So, m = k and n = 1

A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)

Coordinates of C are:

$$(-4, 1, -10) = \left(\frac{k(-1)+1(2)}{k+1}, \frac{k(2)+1(3)}{k+1}, \frac{k(-3)+1(4)}{k+1}\right)$$
$$= \left(\frac{-k+2}{k+1}, \frac{2k+3}{k+1}, \frac{-3k+4}{k+1}\right)$$

On comparing we get,

[-k + 2] / [k + 1] = -4
-k + 2 = -4(k + 1)

-k + 2 = -4k – 4

4k – k = – 2 – 4

3k = -6

k = -6/3

= -2

[2k + 3] / [k + 1] = 1
2k + 3 = k + 1

2k – k = 1 – 3

k = – 2

[-3k + 4] / [k + 1] = -10
-3k + 4 = -10(k + 1)

-3k + 4 = -10k – 10

-3k + 10k = -10 – 4

7k = -14

k = -14/7

= -2

The value of k is the same in all three cases.

So, A, B and C are collinear [as k = -2]

∴We can say that, C divides AB externally in ratio 2:1

4. Find the ratio in which the line joining (2, 4, 5) and (3, 5, 4) is divided by the yz-plane.

Solution:

Given:

The points (2, 4, 5) and (3, 5, 4)

By using the section formula,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

We know X coordinate is always 0 on yz-plane

So, let Point C(0, y, z), and let C divide AB in ratio k: 1

Then, m = k and n = 1

A(2, 4, 5) and B(3, 5, 4)

The coordinates of C are:

$$(0, y, z) = \left(\frac{k(3) + 1(2)}{k + 1}, \frac{k(5) + 1(4)}{k + 1}, \frac{k(4) + 1(5)}{k + 1}\right)$$

$$= \left(\frac{3k + 2}{k + 1}, \frac{5k + 4}{k + 1}, \frac{4k + 5}{k + 1}\right)$$

On comparing we get,

[3k + 2] / [k + 1] = 0
3k + 2 = 0(k + 1)

3k + 2 = 0

3k = – 2

k = -2/3

∴We can say that, C divides AB externally in ratio 2: 3

5. Find the ratio in which the line segment joining the points (2, -1, 3) and (-1, 2, 1) is divided by the plane x + y + z = 5.

Solution:

Given:

The points(2, -1, 3) and (-1, 2, 1)

By using the section formula,

$$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$$

Let C(x, y, z) be any point on the given plane and C divides AB in ratio k: 1

Then, m = k and n = 1

A(2, -1, 3) and B(-1, 2, 1)

Coordinates of C are:

$$(x,y,z) = \left(\frac{k(-1)+1(2)}{k+1}, \frac{k(2)+1(-1)}{k+1}, \frac{k(-1)+1(3)}{k+1}\right)$$
$$= \left(\frac{-k+2}{k+1}, \frac{2k-1}{k+1}, \frac{-k+3}{k+1}\right)$$

On comparing we get,

[-k + 2] / [k + 1] = x
[2k – 1] / [k + 1] = y
[-k + 3] / [k + 1] = z
We know that x + y + z = 5

$$\frac{-k+2}{k+1} + \frac{2k-1}{k+1} + \frac{-k+3}{k+1} = 5$$

$$\frac{-k+2+2k-1-k+3}{k+1} = 5$$

$$\frac{4}{k+1} = 5$$

5(k + 1) = 4

5k + 5 = 4

5k = 4 – 5

5k = – 1

k = -1/5

∴We can say that, the plane divides AB externally in the ratio 1:5

6. If the points A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6) are collinear, find the ratio in which C divided AB.

Solution:

Given:

The points A (3, 2, -4), B (9, 8, -10) and C (5, 4, -6)

By using the section formula,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let C divides AB in ratio k: 1

Three points are collinear if the value of k is the same for x, y and z coordinates.

Then, m = k and n = 1

A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6)

Coordinates of C are:

$$(5,4,-6) = \left(\frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1}\right)$$
$$= \left(\frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1}\right)$$

On comparing we get,

[9k + 3] / [k + 1] = 5
9k + 3 = 5(k + 1)

9k + 3 = 5k + 5

9k – 5k = 5 – 3

4k = 2

k = 2/4

= ½

[8k + 2] / [k + 1] = 4
8k + 2 = 4(k + 1)

8k + 2 = 4k + 4

8k – 4k = 4 – 2

4k = 2

k = 2/4

$= \frac{1}{2}$

$[-10k - 4] / [k + 1] = -6$
$-10k - 4 = -6(k + 1)$

$-10k - 4 = -6k - 6$

$-10k + 6k = 4 - 6$

$-4k = -2$

$k = -2/-4$

$= \frac{1}{2}$

The value of k is the same in all three cases.

So, A, B and C are collinear [as, $k = \frac{1}{2}$]

∴We can say that, C divides AB externally in ratio 1:2

7. The mid-points of the sides of a triangle ABC are given by (-2, 3, 5), (4, -1, 7) and (6, 5, 3). Find the coordinates of A, B and C.

Solution:

Given:

The mid-points of the sides of a triangle ABC is given as (-2, 3, 5), (4, -1, 7) and (6, 5, 3).

By using the section formula,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

We know the mid-point divides side in the ratio of 1:1.

The coordinates of C is given by,

$$\left(\frac{x + a}{2}, \frac{y + b}{2}, \frac{z + c}{2}\right)$$

P(-2, 3, 5) is mid-point of A(x_1, y_1, z_1) and B(x_2, y_2, z_2)

Then,

$$(-2, 3, 5) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$(-4, 6, 10) = (x_1 + x_2, y_1 + y_2, z_1 + z_2) \text{ …. } (1)$$

Q(4, -1, 7) is mid-point of B(x_2, y_2, z_2) and C(x_3, y_3, z_3)

Then,

$$(4,-1,7) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}, \frac{z_2 + z_3}{2}\right)$$

$(8, -2, 14) = (x_2 + x_3, y_3 + y_3, z_3 + z_3)$ …. (2)

R(6, 5, 3) is mid-point of A(x_1, y_1, z_1) and C(x_3, y_3, z_3)

Then,

$$(6,5,3) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}, \frac{z_1 + z_3}{2}\right)$$

$(12, 10, 6) = (x_1 + x_3, y_1 + y_3, z_1 + z_3)$ …. (3)

Now solving for 'x' terms

$x_1 + x_2 = -4$……………………(4)

$x_2 + x_3 = 8$………………………(5)

$x_1 + x_3 = 12$……………………(6)

By adding equation (4), (5), (6)

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 8 + 12 – 4$

$2x_1 + 2x_2 + 2x_3 = 16$

$2(x_1 + x_2 + x_3) = 16$

$x_1 + x_2 + x_3 = 8$………………………(7)

Now, subtract equation (4), (5) and (6) from equation (7) separately:

$x_1 + x_2 + x_3 – x_1 – x_2 = 8 – (-4)$

$x_3 = 12$

$x_1 + x_2 + x_3 – x_2 – x_3 = 8 – 8$

$x_1 = 0$

$x_1 + x_2 + x_3 – x_1 – x_3 = 8 – 12$

$x_2 = -4$

Now solving for 'y' terms

$y_1 + y_2 = 6$……………………(8)

$y_2 + y_3 = -2$……………………(9)

$y_1 + y_3 = 10$……………………(10)

By adding equation (8), (9) and (10) we get,

$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 10 + 6 - 2$

$2y_1 + 2y_2 + 2y_3 = 14$

$2(y_1 + y_2 + y_3) = 14$

$y_1 + y_2 + y_3 = 7$……………………….(11)

Now, subtract equation (8), (9) and (10) from equation (11) separately:

$y_1 + y_2 + y_3 - y_1 - y_2 = 7 - 6$

$y_3 = 1$

$y_1 + y_2 + y_3 - y_2 - y_3 = 7 - (-2)$

$y_1 = 9$

$y_1 + y_2 + y_3 - y_1 - y_3 = 7 - 10$

$y_2 = -3$

Now solving for 'z' terms

$z_1 + z_2 = 10$……………………(12)

$z_2 + z_3 = 14$……………………(13)

$z_1 + z_3 = 6$……………………(14)

By adding equation (12), (13) and (14) we get,

$z_1 + z_2 + z_2 + z_3 + z_1 + z_3 = 6 + 14 + 10$

$2z_1 + 2z_2 + 2z_3 = 30$

$2(z_1 + z_2 + z_3) = 30$

$z_1 + z_2 + z_3 = 15$……………………….(15)

Now, subtract equation (8), (9) and (10) from equation (11) separately:

$z_1 + z_2 + z_3 - z_1 - z_2 = 15 - 10$

$z_3 = 5$

$z_1 + z_2 + z_3 - z_2 - z_3 = 15 - 14$

$z_1 = 1$

$z_1 + z_2 + z_3 - z_1 - z_3 = 15 - 6$

$z_2 = 9$

∴Thevertices of sides of a triangle ABC are A(0, 9, 1) B(-4,-3, 9) and C(12, 1, 5).

8. A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3) are the vertices of a triangle ABC. Find the point in which the bisector of the angle ∠BAC meets BC.

Solution:

Given:

The vertices of a triangle are A (1, 2, 3), B (0, 4, 1), C (-1, -1, -3)

By using the distance formula,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

So,

The distance between the points A (1, 2, 3) and B (0, 4, 1) is AB,

$$= \sqrt{(1-0)^2+(2-4)^2+(3-1)^2}$$

$$= \sqrt{1^2+(-2)^2+2^2}$$

$$= \sqrt{1+4+4}$$

$$= \sqrt{9}$$

$$= 3$$

The distance between the points A (1, 2, 3) and C (-1, -1, -3) is AC,

$$= \sqrt{(1-(-1))^2+(2-(-1))^2+(3-(-3))^2}$$

$$= \sqrt{2^2+3^2+6^2}$$

$$= \sqrt{4+9+36}$$

$$= \sqrt{49}$$

$$= 7$$

$$= \sqrt{(1-(-1))^2+(2-(-1))^2+(3-(-3))^2}$$

$$= \sqrt{2^2+3^2+6^2}$$

$$= \sqrt{4+9+36}$$

$$= \sqrt{49}$$

$$= 7$$

So, AB/AC = 3/7

AB: AC = 3:7

BD: DC = 3:7

Then, m = 3 and n = 7

B(0, 4, 1) and C(-1, -1, -3)

Coordinates of D by using section formula is given as

$$= \left(\frac{7(0)+3(-1)}{7+3}, \frac{7(4)+3(-1)}{7+3}, \frac{7(1)+3(-3)}{7+3}\right)$$

$$= \left(\frac{0-3}{10}, \frac{28-3}{10}, \frac{7-9}{10}\right)$$

$$= \left(\frac{-3}{10}, \frac{25}{10}, \frac{-2}{10}\right)$$

$$= \left(\frac{-3}{10}, \frac{5}{2}, \frac{-1}{5}\right)$$

∴The coordinates of D are (-3/10, 5/2, -1/5).