Access answers to RD Sharma Solutions for Class 11 Maths Chapter 8 – Transformation Formulae

EXERCISE 8.1 PAGE NO: 8.6

1. Express each of the following as the sum or difference of sines and cosines:
(i) 2 sin 3x cos x
(ii) 2 cos 3x sin 2x
(iii) 2 sin 4x sin 3x
(iv) 2 cos 7x cos 3x

Solution:

(i) 2 sin 3x cos x

By using the formula,

2 sin A cos B = sin (A + B) + sin (A – B)

2 sin 3x cos x = sin (3x + x) + sin (3x – x)

= sin (4x) + sin (2x)

= sin 4x + sin 2x

(ii) 2 cos 3x sin 2x

By using the formula,

2 cos A sin B = sin (A + B) – sin (A – B)

2 cos 3x sin 2x = sin (3x + 2x) – sin (3x – 2x)

= sin (5x) – sin (x)

= sin 5x – sin x

(iii) 2 sin 4x sin 3x

By using the formula,

2 sin A sin B = cos (A – B) – cos (A + B)

2 sin 4x sin 3x = cos (4x – 3x) – cos (4x + 3x)

= cos (x) – cos (7x)

= cos x – cos 7x

(iv) 2 cos 7x cos 3x

By using the formula,

2 cos A cos B = cos (A + B) + cos (A – B)

2 sin 3x cos x = cos (7x + 3x) + cos (7x – 3x)

= cos (10x) + cos (4x)

= cos 10x + cos 4x

2. Prove that:
(i) 2 sin 5π/12 sin π/12 = 1/2

(ii) 2 cos 5π/12 cos π/12 = 1/2

(iii) 2 sin 5π/12 cos π/12 = (√3 + 2)/2

Solution:

(i) 2 sin 5π/12 sin π/12 = 1/2

By using the formula,

2 sin A sin B = cos (A – B) – cos (A + B)

2 sin 5π/12 sin π/12 = cos (5π/12 – π/12) – cos (5π/12 + π/12)

= cos (4π/12) – cos (6π/12)

= cos (π/3) – cos (π/2)

= cos (180°/3) – cos (180°/2)

= cos 60° – cos 90°

= 1/2 – 0

= 1/2

Hence Proved.

(ii) 2 cos 5π/12 cos π/12 = 1/2

By using the formula,

2 cos A cos B = cos (A + B) + cos (A – B)

2 cos 5π/12 cos π/12 = cos (5π/12 + π/12) + cos (5π/12 – π/12)

= cos (6π/12) + cos (4π/12)

= cos (π/2) + cos (π/3)

= cos (180°/2) + cos (180°/3)

= cos 90° + cos 60°

= 0 + 1/2

= 1/2

Hence Proved.

(iii) 2 sin 5π/12 cos π/12 = (√3 + 2)/2

By using the formula,

2 sin A cos B = sin (A + B) + sin (A – B)

2 sin 5π/12 cos π/12 = sin (5π/12 + π/12) + sin (5π/12 – π/12)

= sin (6π/12) + sin (4π/12)

= sin (π/2) + sin (π/3)

= sin (180°/2) + sin (180°/3)

= sin 90° + sin 60°

= 1 + √3

= (2 + √3)/2

= (√3 + 2)/2

Hence Proved.

3. show that:

(i) sin 50° cos 85° = (1 – √2sin 35°)/2√2

(ii) sin 25° cos 115° = 1/2 {sin 140° – 1}

Solution:

(i) sin 50° cos 85° = (1 – √2sin 35°)/2√2

By using the formula,

2 sin A cos B = sin (A + B) + sin (A – B)

sin A cos B = [sin (A + B) + sin (A – B)] / 2

sin 50° cos 85° = [sin(50° + 85°) + sin (50° – 85°)] / 2

= [sin (135°) + sin (-35°)] / 2

= [sin (135°) – sin (35°)] / 2 (since, sin (-x) = -sin x)

$= [\sin (180^\circ – 45^\circ) – \sin 35^\circ] / 2$

$= [\sin 45^\circ – \sin 35^\circ] / 2$

$= [(1/\sqrt{2}) – \sin 35^\circ] / 2$

$= [(1 – \sin 35^\circ)/\sqrt{2}] / 2$

$= (1 – \sin 35^\circ) / 2\sqrt{2}$

Hence proved.

(ii) $\sin 25^\circ \cos 115^\circ = 1/2 \{\sin 140^\circ – 1\}$

By using the formula,

$2 \sin A \cos B = \sin (A + B) + \sin (A – B)$

$\sin A \cos B = [\sin (A + B) + \sin (A – B)] / 2$

$\sin 20^\circ \cos 115^\circ = [\sin(25^\circ + 115^\circ) + \sin (25^\circ – 115^\circ)] / 2$

$= [\sin (140^\circ) + \sin (-90^\circ)] / 2$

$= [\sin (140^\circ) – \sin (90^\circ)] / 2$ (since, $\sin (-x) = -\sin x$)

$= 1/2 \{\sin 140^\circ – 1\}$

Hence proved.

4. Prove that:

$4 \cos x \cos (\pi/3 + x) \cos (\pi/3 – x) = \cos 3x$

Solution:

Let us consider LHS:

$4 \cos x \cos (\pi/3 + x) \cos (\pi/3 – x) = 2 \cos x (2 \cos (\pi/3 + x) \cos (\pi/3 – x))$

By using the formula,

$2 \cos A \cos B = \cos (A + B) + \cos (A – B)$

$2 \cos x (2 \cos (\pi/3+x) \cos (\pi/3 – x)) = 2 \cos x (\cos (\pi/3+x + \pi/3-x) + \cos (\pi/3+x – \pi/3+ x))$

$= 2 \cos x (\cos (2\pi/3) + \cos (2x))$

$= 2 \cos x \{\cos 120° + \cos 2x\}$

$= 2 \cos x \{\cos (180° – 60°) + \cos 2x\}$

$= 2 \cos x (\cos 2x – \cos 60°)$ (since, $\{\cos (180° – A) = – \cos A\}$)

$= 2 \cos 2x \cos x – 2 \cos x \cos 60°$

$= (\cos (x + 2x) + \cos (2x – x)) – (2\cos x)/2$

$= \cos 3x + \cos x – \cos x$

$= \cos 3x$

= RHS

Hence Proved.

EXERCISE 8.2 PAGE NO: 8.17

1. Express each of the following as the product of sines and cosines:
(i) $\sin 12x + \sin 4x$
(ii) $\sin 5x – \sin x$
(iii) $\cos 12x + \cos 8x$
(iv) $\cos 12x – \cos 4x$

(v) $\sin 2x + \cos 4x$

Solution:

(i) $\sin 12x + \sin 4x$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

$\sin 12x + \sin 4x = 2 \sin (12x + 4x)/2 \cos (12x – 4x)/2$

$= 2 \sin 16x/2 \cos 8x/2$

$= 2 \sin 8x \cos 4x$

(ii) $\sin 5x – \sin x$

By using the formula,

$\sin A – \sin B = 2 \cos (A+B)/2 \sin (A-B)/2$

$\sin 5x – \sin x = 2 \cos (5x + x)/2 \sin (5x – x)/2$

$= 2 \cos 6x/2 \sin 4x/2$

$= 2 \cos 3x \sin 2x$

(iii) $\cos 12x + \cos 8x$

By using the formula,

$\cos A + \cos B = 2 \cos (A+B)/2 \cos (A-B)/2$

$\cos 12x + \cos 8x = 2 \cos (12x + 8x)/2 \cos (12x – 8x)/2$

$= 2 \cos 20x/2 \cos 4x/2$

$= 2 \cos 10x \cos 2x$

(iv) $\cos 12x – \cos 4x$

By using the formula,

$\cos A – \cos B = -2 \sin (A+B)/2 \sin (A-B)/2$

$\cos 12x – \cos 4x = -2 \sin (12x + 4x)/2 \sin (12x – 4x)/2$

$= -2 \sin 16x/2 \sin 8x/2$

$= -2 \sin 8x \sin 4x$

(v) $\sin 2x + \cos 4x$

$\sin 2x + \cos 4x = \sin 2x + \sin (90^\circ – 4x)$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

$\sin 2x + \sin (90^\circ – 4x) = 2 \sin (2x + 90^\circ – 4x)/2 \cos (2x – 90^\circ + 4x)/2$

$= 2 \sin (90^\circ – 2x)/2 \cos (6x – 90^\circ)/2$

$= 2 \sin (45° – x) \cos (3x – 45°)$

$= 2 \sin (45° – x) \cos \{-(45° – 3x)\}$ (since, $\{\cos (-x) = \cos x\}$)

$= 2 \sin (45° – x) \cos (45° – 3x)$

$= 2 \sin (\pi/4 – x) \cos (\pi/4 – 3x)$

2. Prove that :
(i) sin 38° + sin 22° = sin 82°

(ii) cos 100° + cos 20° = cos 40°

(iii) sin 50° + sin 10° = cos 20°

(iv) sin 23° + sin 37° = cos 7°

(v) sin 105° + cos 105° = cos 45°

(vi) sin 40° + sin 20° = cos 10°

Solution:

(i) $\sin 38° + \sin 22° = \sin 82°$

Let us consider LHS:

sin 38° + sin 22°

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

sin 38° + sin 22° = 2 sin (38° + 22°)/2 cos (38° – 22°)/2

= 2 sin 60°/2 cos 16°/2

= 2 sin 30° cos 8°

= 2 × 1/2 × cos 8°

= cos 8°

= cos (90° – 82°)

= sin 82° (since, {cos (90° – A) = sin A})

= RHS

Hence Proved.

(ii) cos 100° + cos 20° = cos 40°

Let us consider LHS:

cos 100° + cos 20°

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos 100° + cos 20° = 2 cos (100° + 20°)/2 cos (100° – 20°)/2

= 2 cos 120°/2 cos 80°/2

= 2 cos 60° cos 4°

= 2 × 1/2 × cos 40°

= cos 40°

= RHS

Hence Proved.

(iii) sin 50° + sin 10° = cos 20°

Let us consider LHS:

sin 50° + sin 10°

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

sin 50° + sin 10° = 2 sin (50° + 10°)/2 cos (50° – 10°)/2

= 2 sin 60°/2 cos 40°/2

= 2 sin 30° cos 20°

= 2 × 1/2 × cos 20°

= cos 20°

= RHS

Hence Proved.

(iv) sin 23° + sin 37° = cos 7°

Let us consider LHS:

sin 23° + sin 37°

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

sin 23° + sin 37° = 2 sin (23° + 37°)/2 cos (23° – 37°)/2

$= 2 \sin 60^{\circ}/2 \cos -14^{\circ}/2$

$= 2 \sin 30^{\circ} \cos -7^{\circ}$

$= 2 \times 1/2 \times \cos -7^{\circ}$

$= \cos 7^{\circ}$ (since, {cos (-A) = cos A})

= RHS

Hence Proved.

(v) sin 105° + cos 105° = cos 45°

Let us consider LHS: sin 105° + cos 105°

$\sin 105° + \cos 105° = \sin 105^{\circ} + \sin (90^{\circ} – 105^{\circ})$ [since, {sin (90° – A) = cos A}]

$= \sin 105^{\circ} + \sin (-15^{\circ})$

$= \sin 105^{\circ} – \sin 15^{\circ}$ [{sin(-A) = – sin A}]

By using the formula,

Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

$\sin 105^{\circ} – \sin 15^{\circ} = 2 \cos (105^{\circ} + 15^{\circ})/2 \sin (105^{\circ} – 15^{\circ})/2$

$= 2 \cos 120^{\circ}/2 \sin 90^{\circ}/2$

$= 2 \cos 60^{\circ} \sin 45^{\circ}$

$= 2 \times 1/2 \times 1/\sqrt{2}$

$= 1/\sqrt{2}$

$= \cos 45^{\circ}$

= RHS

Hence proved.

(vi) sin 40° + sin 20° = cos 10°

Let us consider LHS:

sin 40° + sin 20°

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

$\sin 40° + \sin 20° = 2 \sin (40^{\circ} + 20^{\circ})/2 \cos (40^{\circ} – 20^{\circ})/2$

$= 2 \sin 60^{\circ}/2 \cos 20^{\circ}/2$

$= 2 \sin 30^{\circ} \cos 10^{\circ}$

$= 2 \times 1/2 \times \cos 10^{\circ}$

$= \cos 10^{\circ}$

= RHS

Hence Proved.

3. Prove that:

(i) cos 55° + cos 65° + cos 175° = 0

(ii) sin 50° – sin 70° + sin 10° = 0

(iii) cos 80° + cos 40° – cos 20° = 0

(iv) cos 20° + cos 100° + cos 140° = 0

(v) sin 5π/18 – cos 4π/9 = √3 sin π/9

(vi) cos π/12 – sin π/12 = 1/√2

(vii) sin 80° – cos 70° = cos 50°

(viii) sin 51° + cos 81° = cos 21°

Solution:

(i) cos 55° + cos 65° + cos 175° = 0

Let us consider LHS:

cos 55° + cos 65° + cos 175°

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos 55° + cos 65° + cos 175° = 2 cos (55° + 65°)/2 cos (55° – 65°) + cos (180° – 5°)

= 2 cos 120°/2 cos (-10°)/2 – cos 5° (since, {cos (180° – A) = – cos A})

= 2 cos 60° cos (-5°) – cos 5° (since, {cos (-A) = cos A})

= 2 × 1/2 × cos 5° – cos 5°

= 0

= RHS

Hence Proved.

(ii) sin 50° – sin 70° + sin 10° = 0

Let us consider LHS:

sin 50° – sin 70° + sin 10°

By using the formula,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

sin 50° – sin 70° + sin 10° = 2 cos (50° + 70°)/2 sin (50° – 70°) + sin 10°

= 2 cos 120°/2 sin (-20°)/2 + sin 10°

= 2 cos 60° (- sin 10°) + sin 10° [since,{sin (-A) = -sin (A)}]

= 2 × 1/2 × – sin 10° + sin 10°

= 0

= RHS

Hence proved.

(iii) cos 80° + cos 40° – cos 20° = 0

Let us consider LHS:

cos 80° + cos 40° – cos 20°

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos 80° + cos 40° – cos 20° = 2 cos (80° + 40°)/2 cos (80° – 40°) – cos 20°

= 2 cos 120°/2 cos 40°/2 – cos 20°

= 2 cos 60° cos 20° – cos 20°

= 2 × 1/2 × cos 20° – cos 20°

= 0

= RHS

Hence Proved.

(iv) cos 20° + cos 100° + cos 140° = 0

Let us consider LHS:

cos 20° + cos 100° + cos 140°

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos 20° + cos 100° + cos 140° = 2 cos (20° + 100°)/2 cos (20° – 100°) + cos (180° – 40°)

= 2 cos 120°/2 cos (-80°)/2 – cos 40° (since, {cos (180° – A) = – cos A})

= 2 cos 60° cos (-40°) – cos 40° (since, {cos (-A) = cos A})

= 2 × 1/2 × cos 40° – cos 40°

= 0

= RHS

Hence Proved.

(v) sin 5π/18 – cos 4π/9 = √3 sin π/9

Let us consider LHS:

sin 5π/18 – cos 4π/9 = sin 5π/18 – sin (π/2 – 4π/9) (since, cos A = sin (90° – A))

= sin 5π/18 – sin (9π – 8π)/18

= sin 5π/18 – sin π/18

By using the formula,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

$$sin\frac{5\pi}{18} - sin\frac{\pi}{18} = 2\,cos\left(\frac{\frac{5\pi}{18}+\frac{\pi}{18}}{2}\right) sin\left(\frac{\frac{5\pi}{18}-\frac{\pi}{18}}{2}\right)$$

= 2 cos (6π/36) sin (4π/36)

= 2 cos π/6 sin π/9

= 2 cos 30° sin π/9

= 2 × √3/2 × sin π/9

= √3 sin π/9

= RHS

Hence proved.

(vi) cos π/12 – sin π/12 = 1/√2

Let us consider LHS:

cos π/12 – sin π/12 = sin (π/2 – π/12) – sin π/12 (since, cos A = sin(90° – A))

= sin (6π – 5π)/12 – sin π/12

= sin 5π/12 – sin π/12

By using the formula,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

$$sin\frac{5\pi}{12} - sin\frac{\pi}{12} = 2\,cos\left(\frac{\frac{5\pi}{12}+\frac{\pi}{12}}{2}\right) sin\left(\frac{\frac{5\pi}{12}-\frac{\pi}{12}}{2}\right)$$

= 2 cos (6π/24) sin (4π/24)

= 2 cos π/4 sin π/6

= 2 cos 45° sin 30°

= 2 × 1/√2 × 1/2

= 1/√2

= RHS

Hence proved.

(vii) sin 80° – cos 70° = cos 50°

sin 80° = cos 50° + cos 70°

So, now let us consider RHS

cos 50° + cos 70°

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos 50° + cos 70° = 2 cos (50° + 70°)/2 cos (50° – 70°)/2

= 2 cos 120°/2 cos (-20°)/2

= 2 cos 60° cos (-10°)

= 2 × 1/2 × cos 10° (since, cos (-A) = cos A)

= cos 10°

= cos (90° – 80°)

= sin 80° (since, cos (90° – A) = sin A)

= LHS

Hence Proved.

(viii) sin 51° + cos 81° = cos 21°

Let us consider LHS:

sin 51° + cos 81° = sin 51° + sin (90° – 81°)

= sin 51° + sin 9° (since, sin (90° – A) = cos A)

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

sin 51° + sin 9° = 2 sin (51° + 9°)/2 cos (51° – 9°)/2

= 2 sin 60°/2 cos 42°/2

= 2 sin 30° cos 21°

= 2 × 1/2 × cos 21°

= cos 21°

= RHS

Hence proved.

4. Prove that:

(i) cos (3π/4 + x) – cos (3π/4 – x) = -√2 sin x

(ii) cos (π/4 + x) + cos (π/4 – x) = √2 cos x

Solution:

(i) cos (3π/4 + x) – cos (3π/4 – x) = -√2 sin x

Let us consider LHS:

cos (3π/4 + x) – cos (3π/4 – x)

By using the formula,

cos A – cos B = -2 sin (A+B)/2 sin (A-B)/2

cos (3π/4 + x) – cos (3π/4 – x) = -2 sin (3π/4 + x + 3π/4 – x)/2 sin (3π/4 + x – 3π/4 + x)/2

= -2 sin (6π/4)/2 sin 2x/2

= -2 sin 6π/8 sin x

= -2 sin 3π/4 sin x

= -2 sin (π – π/4) sin x

$= -2 \sin \pi/4 \sin x$ (since, $(\pi-A) = \sin A$)

$= -2 \times 1/\sqrt{2} \times \sin x$

$= -\sqrt{2} \sin x$

= RHS

Hence proved.

(ii) $\cos (\pi/4 + x) + \cos (\pi/4 - x) = \sqrt{2} \cos x$

Let us consider LHS:

$\cos (\pi/4 + x) + \cos (\pi/4 - x)$

By using the formula,

$\cos A + \cos B = 2 \cos (A+B)/2 \cos (A-B)/2$

$\cos (\pi/4 + x) + \cos (\pi/4 - x) = 2 \cos (\pi/4 + x + \pi/4 - x)/2 \cos (\pi/4 + x - \pi/4 + x)/2$

$= 2 \cos (2\pi/4)/2 \cos 2x/2$

$= 2 \cos 2\pi/8 \cos x$

$= 2 \sin \pi/4 \cos x$

$= 2 \times 1/\sqrt{2} \times \cos x$

$= \sqrt{2} \cos x$

= RHS

Hence proved.

5. Prove that:

(i) $\sin 65^\circ + \cos 65^\circ = \sqrt{2} \cos 20^\circ$

(ii) $\sin 47^\circ + \cos 77^\circ = \cos 17^\circ$

Solution:

(i) $\sin 65^\circ + \cos 65^\circ = \sqrt{2} \cos 20^\circ$

Let us consider LHS:

$\sin 65^\circ + \cos 65^\circ = \sin 65^\circ + \sin (90^\circ - 65^\circ)$

$= \sin 65^\circ + \sin 25^\circ$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

$\sin 65^\circ + \sin 25^\circ = 2 \sin (65^\circ + 25^\circ)/2 \cos (65^\circ - 25^\circ)/2$

$= 2 \sin 90^\circ/2 \cos 40^\circ/2$

$= 2 \sin 45^\circ \cos 20^\circ$

$= 2 \times 1/\sqrt{2} \times \cos 20^\circ$

$= \sqrt{2} \cos 20^\circ$

= RHS

Hence proved.

(ii) $\sin 47^\circ + \cos 77^\circ = \cos 17^\circ$

Let us consider LHS:

$\sin 47^\circ + \cos 77^\circ = \sin 47^\circ + \sin (90^\circ - 77^\circ)$

$= \sin 47^\circ + \sin 13^\circ$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

$\sin 47^\circ + \sin 13^\circ = 2 \sin (47^\circ + 13^\circ)/2 \cos (47^\circ - 13^\circ)/2$

$= 2 \sin 60^\circ/2 \cos 34^\circ/2$

$= 2 \sin 30^\circ \cos 17^\circ$

$= 2 \times 1/2 \times \cos 17^\circ$

$= \cos 17^\circ$

= RHS

Hence proved.

6. Prove that:
(i) cos 3A + cos 5A + cos 7A + cos 15A = 4 cos 4A cos 5A cos 6A

(ii) cos A + cos 3A + cos 5A + cos 7A = 4 cos A cos 2A cos 4A

(iii) sin A + sin 2A + sin 4A + sin 5A = 4 cos A/2 cos 3A/2 sin 3A

(iv) sin 3A + sin 2A – sin A = 4 sin A cos A/2 cos 3A/2

(v) cos 20° cos 100° + cos 100° cos 140° – cos 140° cos 200° = – 3/4

(vi) sin x/2 sin 7x/2 + sin 3x/2 sin 11x/2 = sin 2x sin 5x

(vii) cos x cos x/2 – cos 3x cos 9x/2 = sin 4x sin 7x/2

Solution:

(i) cos 3A + cos 5A + cos 7A + cos 15A = 4 cos 4A cos 5A cos 6A

Let us consider LHS:

cos 3A + cos 5A + cos 7A + cos 15A

So now,

(cos 5A + cos 3A) + (cos 15A + cos 7A)

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

(cos 5A + cos 3A) + (cos 15A + cos 7A)

= [2 cos (5A+3A)/2 cos (5A-3A)/2] + [2 cos (15A+7A)/2 cos (15A-7A)/2]

= [2 cos 8A/2 cos 2A/2] + [2 cos 22A/2 cos 8A/2]

= [2 cos 4A cos A] + [2 cos 11A cos 4A]

= 2 cos 4A (cos 11A + cos A)

Again by using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

2 cos 4A (cos 11A + cos A) = 2 cos 4A [2 cos (11A+A)/2 cos (11A-A)/2]

= 2 cos 4A [2 cos 12A/2 cos 10A/2]

= 2 cos 4A [2 cos 6A cos 5A]

= 4 cos 4A cos 5A cos 6A

= RHS

Hence proved.

(ii) cos A + cos 3A + cos 5A + cos 7A = 4 cos A cos 2A cos 4A

Let us consider LHS:

cos A + cos 3A + cos 5A + cos 7A

So now,

(cos 3A + cos A) + (cos 7A + cos 5A)

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

(cos 3A + cos A) + (cos 7A + cos 5A)

= [2 cos (3A+A)/2 cos (3A-A)/2] + [2 cos (7A+5A)/2 cos (7A-5A)/2]

= [2 cos 4A/2 cos 2A/2] + [2 cos 12A/2 cos 2A/2]

= [2 cos 2A cos A] + [2 cos 6A cos A]

= 2 cos A (cos 6A + cos 2A)

Again by using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

2 cos A (cos 6A + cos 2A) = 2 cos A [2 cos (6A+2A)/2 cos (6A-2A)/2]

= 2 cos A [2 cos 8A/2 cos 4A/2]

= 2 cos A [2 cos 4A cos 2A]

= 4 cos A cos 2A cos 4A

= RHS

Hence proved.

(iii) sin A + sin 2A + sin 4A + sin 5A = 4 cos A/2 cos 3A/2 sin 3A

Let us consider LHS:

sin A + sin 2A + sin 4A + sin 5A

So now,

(sin 2A + sin A) + (sin 5A + sin 4A)

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

(sin 2A + sin A) + (sin 5A + sin 4A) =

= [2 sin (2A+A)/2 cos (2A-A)/2] + [2 sin (5A+4A)/2 cos (5A-4A)/2]

= [2 sin 3A/2 cos A/2] + [2 sin 9A/2 cos A/2]

= 2 cos A/2 (sin 9A/2 + sin 3A/2)

Again by using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

2 cos A/2 (sin 9A/2 + sin 3A/2) = 2 cos A/2 [2 sin (9A/2 + 3A/2)/2 cos (9A/2 – 3A/2)/2]

= 2 cos A/2 [2 sin ((9A+3A)/2)/2 cos ((9A-3A)/2)/2]

= 2 cos A/2 [2 sin 12A/4 cos 6A/4]

= 2 cos A/2 [2 sin 3A cos 3A/2]

= 4 cos A/2 cos 3A/2 sin 3A

= RHS

Hence proved.

(iv) sin 3A + sin 2A – sin A = 4 sin A cos A/2 cos 3A/2

Let us consider LHS:

sin 3A + sin 2A – sin A

So now,

(sin 3A – sin A) + sin 2A

By using the formula,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

(sin 3A – sin A) + sin 2A = 2 cos (3A + A)/2 sin (3A – A)/2 + sin 2A

= 2 cos 4A/2 sin 2A/2 + sin 2A

We know that, $\sin 2A = 2 \sin A \cos A$

$= 2 \cos 2A \operatorname{Sin} A + 2 \sin A \cos A$

$= 2 \sin A (\cos 2A + \cos A)$

Again by using the formula,

$\cos A + \cos B = 2 \cos (A+B)/2 \cos (A-B)/2$

$2 \sin A (\cos 2A + \cos A) = 2 \sin A [2 \cos (2A+A)/2 \cos (2A-A)/2]$

$= 2 \sin A [2 \cos 3A/2 \cos A/2]$

$= 4 \sin A \cos A/2 \cos 3A/2$

= RHS

Hence proved.

(v) $\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ – \cos 140^\circ \cos 200^\circ = – 3/4$

Let us consider LHS:

$\cos 20^\circ \cos 100^\circ + \cos 100^\circ \cos 140^\circ – \cos 140^\circ \cos 200^\circ =$

We shall multiply and divide by 2 we get,

$= 1/2 [2 \cos 100^\circ \cos 20^\circ + 2 \cos 140^\circ \cos 100^\circ – 2 \cos 200^\circ \cos 140^\circ]$

We know that $2 \cos A \cos B = \cos (A+B) + \cos (A-B)$

So,

$= 1/2 [\cos (100^\circ + 20^\circ) + \cos (100^\circ – 20^\circ) + \cos (140^\circ + 100^\circ) + \cos (140^\circ – 100^\circ) – \cos (200^\circ + 140^\circ) – \cos (200^\circ – 140^\circ)]]$

$= 1/2 [\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ – \cos 340^\circ – \cos 60^\circ]$

$= 1/2 [\cos (90^\circ + 30^\circ) + \cos 80^\circ + \cos (180^\circ + 60^\circ) + \cos 40^\circ – \cos (360^\circ – 20^\circ) – \cos 60^\circ]$

We know, $\cos (180^\circ + A) = – \cos A$, $\cos (90^\circ + A) = – \sin A$, $\cos (360^\circ – A) = \cos A$

So,

$= 1/2 [- \sin 30^\circ + \cos 80^\circ – \cos 60^\circ + \cos 40^\circ – \cos 20^\circ – \cos 60^\circ]$

$= 1/2 [- \sin 30^\circ + \cos 80^\circ + \cos 40^\circ – \cos 20^\circ – 2 \cos 60^\circ]$

Again by using the formula,

$\cos A + \cos B = 2 \cos (A+B)/2 \cos (A-B)/2$

$= 1/2 [- \sin 30^\circ + 2 \cos (80^\circ+40^\circ)/2 \cos (80^\circ-40^\circ)/2 – \cos 20^\circ – 2 \times 1/2]$

$= 1/2 [- \sin 30^\circ + 2 \cos 120^\circ/2 \cos 40^\circ/2 – \cos 20^\circ – 1]$

$= 1/2 [- \sin 30^\circ + 2 \cos 60^\circ \cos 20^\circ – \cos 20^\circ – 1]$

$= 1/2 [- 1/2 + 2\times1/2\times\cos 20^\circ – \cos 20^\circ – 1]$

$= 1/2 [-1/2 + \cos 20^\circ – \cos 20^\circ – 1]$

$= 1/2 [-1/2 -1]$

$= 1/2 [-3/2]$

$= -3/4$

= RHS

Hence proved.

(vi) $\sin x/2 \sin 7x/2 + \sin 3x/2 \sin 11x/2 = \sin 2x \sin 5x$

Let us consider LHS:

$\sin x/2 \sin 7x/2 + \sin 3x/2 \sin 11x/2 =$

We shall multiply and divide by 2 we get,

$= 1/2 [2 \sin 7x/2 \sin x/2 + 2 \sin 11x/2 \sin 3x/2]$

We know that 2 sin A sin B = cos (A-B) – cos (A+B)

So,

= 1/2 [cos (7x/2 – x/2) – cos (7x/2 + x/2) + cos (11x/2 – 3x/2) – cos (11x/2 + 3x/2)]

= 1/2 [cos (7x-x)/2 – cos (7x+x)/2 + cos (11x-3x)/2 – cos (11x+3x)/2]

= 1/2 [cos 6x/2 – cos 8x/2 + cos 8x/2 – cos 14x/2]

= 1/2 [cos 3x – cos 7x]

= – 1/2 [cos 7x – cos 3x]

Again by using the formula,

cos A – cos B = -2 sin (A+B)/2 sin (A-B)/2

= -1/2 [-2 sin (7x+3x)/2 sin (7x-3x)/2]

= -1/2 [-2 sin 10x/2 sin 4x/2]

= -1/2 [-2 sin 5x sin 2x]

= -2/-2 sin 5x sin 2x

= sin 2x sin 5x

= RHS

Hence proved.

(vii) cos x cos x/2 – cos 3x cos 9x/2 = sin 4x sin 7x/2

Let us consider LHS:

cos x cos x/2 – cos 3x cos 9x/2 =

We shall multiply and divide by 2 we get,

= 1/2 [2 cos x cos x/2 – 2 cos 9x/2 cos 3x]

We know that 2 cos A cos B = cos (A+B) + cos (A-B)

So,

= 1/2 [cos (x + x/2) + cos (x – x/2) – cos (9x/2 + 3x) – cos (9x/2 – 3x)]

= 1/2 [cos (2x+x)/2 + cos (2x-x)/2 – cos (9x+6x)/2 – cos (9x-6x)/2]

= 1/2 [cos 3x/2 + cos x/2 – cos 15x/2 – cos 3x/2]

= 1/2 [cos x/2 – cos 15x/2]

= – 1/2 [cos 15x/2 – cos x/2]

Again by using the formula,

cos A – cos B = -2 sin (A+B)/2 sin (A-B)/2

= – 1/2 [-2 sin (15x/2 + x/2)/2 sin (15x/2 – x/2)/2]

= -1/2 [-2 sin (16x/2)/2 sin (14x/2)/2]

= -1/2 [-2 sin 16x/4 sin 7x/2]

= – 1/2 [-2 sin 4x sin 7x/2]

= -2/-2 [sin 4x sin 7x/2]

= sin 4x sin 7x/2

= RHS

Hence proved.

7. Prove that:

$$(i)\ \frac{sin\,A + sin\,3A}{cos\,A - cos\,3A} = cot\,A$$

$$(ii) \frac{sin\,9A - sin\,7A}{cos\,7A - cos\,9A} = cot\,8A$$

$$(iii) \frac{sinA - sinB}{cosA + cosB} = tan\frac{A-B}{2}$$

$$(iv) \frac{sinA + sinB}{sinA - sinB} = tan(\frac{A+B}{2})cot(\frac{A-B}{2})$$

$$(v) \frac{cosA + cosB}{cosB - cosA} = cot(\frac{A+B}{2})cot(\frac{A-B}{2})$$

Solution:

$$(i)\ \frac{sin\,A + sin\,3A}{cos\,A - cos\,3A} = cot\,A$$

Solution:

Let us consider LHS:

$$\frac{sin\,A + sin\,3A}{cos\,A - cos\,3A}$$

By using the formulas,

Sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

Cos A – cos B = - 2 sin (A+B)/2 sin (A-B)/2

So now,

$$\frac{sin\,3A + sin\,A}{cos\,A - cos\,3A} = \frac{2(sin\frac{A+3A}{2}\ cos\frac{3A-A}{2})}{-2(sin\frac{A+3A}{2}\ sin\frac{A-3A}{2})}$$

$$= -\frac{sin\,\frac{4A}{2}\ cos\frac{2A}{2}}{sin\frac{4A}{2}\ sin\frac{-2A}{2}}$$

$$= -\frac{cos\,A}{-sinA}\ (since, sin(-A) = -sinA)$$

$$= \frac{cos\,A}{sin\,A}$$

= cot A

= RHS

Hence proved.

$(ii) \dfrac{sin\,9A - sin\,7A}{cos 7A - cos\,9A} = cot\,8A$

Solution:

Let us consider LHS:

$\dfrac{sin\,9A - sin\,7A}{cos 7A - cos\,9A}$

By using the formulas,

Sin A - sin B = 2 cos (A+B)/2 sin (A-B)/2

Cos A – cos B = - 2 sin (A+B)/2 sin (A-B)/2

So now,

$$\frac{sin9A - sin7A}{cos7A - cos9A} = \frac{2(cos\frac{9A+7A}{2}sin\frac{9A-7A}{2})}{-2(sin\frac{7A+9A}{2}sin\frac{7A-9A}{2})}$$

$$= -\frac{(cos\,\frac{16A}{2}sin\,\frac{2A}{2})}{(sin\,\frac{16A}{2}\;sin\frac{(-2A)}{2})}$$

$$= -\frac{(cos 8A\;sin\;A)}{(-sin\;8A\;sin\;A)}(since, sin(-A) = -sinA)$$

$$= \frac{cos\;8A}{sin\;8A}$$

= cot A

= RHS

Hence proved.

$$(iii) \frac{sinA - sinB}{cosA + cosB} = tan\frac{A-B}{2}$$

Solution:

Let us consider LHS:

$$\frac{sinA - sinB}{cosA + cosB}$$

By using the formulas,

Sin A - sin B = 2 cos (A+B)/2 sin (A-B)/2

Cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

So now,

$$\frac{sin\,A - sinB}{cos\,A + cosB} = \frac{2(cos\,\frac{A+B}{2}\,sin\,\frac{A-B}{2})}{2(cos\,\frac{A+B}{2}\,cos\,\frac{A-B}{2})}$$

$$= \frac{sin(\frac{A-B}{2})}{cos(\frac{A-B}{2})}$$

$$= tan\frac{A-B}{2}$$

= RHS

Hence proved.

$$(iv) \frac{sinA + sinB}{sinA - sinB} = tan(\frac{A+B}{2})cot(\frac{A-B}{2})$$

Solution:

Let us consider LHS:

$$\frac{sinA + sinB}{sinA - sinB}$$

By using the formulas,

Sin A - sin B = 2 cos (A+B)/2 sin (A-B)/2

Sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

So now,

$$\frac{sinA + sinB}{sinA - sinB} = \frac{2(sin\,\frac{A+B}{2}\,cos\,\frac{A-B}{2})}{2(cos\,\frac{A+B}{2}\,sin\,\frac{A-B}{2})}$$

$$= \frac{sin\frac{A+B}{2}\,cos\frac{A-B}{2}}{cos\frac{A+B}{2}\,sin\frac{A-B}{2}}$$

$$= tan(\frac{A+B}{2})cot(\frac{A-B}{2})$$

= RHS

Hence proved.

$$(v)\frac{cosA+cosB}{cosB-cosA}=cot(\frac{A+B}{2})cot(\frac{A-B}{2})$$

Solution:

Let us consider LHS:

$$\frac{cosA+cosB}{cosB-cosA}$$

By using the formulas,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos A - cos B = -2 sin (A+B)/2 sin (A-B)/2

So now,

$$\frac{cosA+cosB}{cosB-cosA}=\frac{2(cos\,\frac{A+B}{2}\,cos\,\frac{A-B}{2})}{-2(sin\,\frac{A+B}{2}\,sin\,\frac{B-A}{2})}$$

$$=-\frac{cos\,\frac{A+B}{2}\,cos\,\frac{A-B}{2}}{sin\,\frac{A+B}{2}\,sin(-\frac{A-B}{2})}$$

$$=-\frac{cos\,\frac{A+B}{2}\,cos\,\frac{A-B}{2}}{-sin\,\frac{A+B}{2}\,sin(\frac{A-B}{2})}(since, sin(-x)=-sinx)$$

$$=cot(\frac{A+B}{2})cot(\frac{A-B}{2})$$

= RHS

Hence proved.

8. Prove that:

$$\text{(i)} \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$$

$$\text{(ii)} \frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \frac{\cos 5A}{\cos 3A}$$

$$\text{(iii)} \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \cot 3A$$

$$\text{(iv)} \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$$

$$\text{(v)} \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$$

$$\text{(vi)} \frac{\sin 5A \cos 2A - \sin 6A \cos A}{\sin A \sin 2A - \cos 2A \cos 3A} = \tan A$$

$$\text{(vii)} \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$

$$\text{(viii)} \frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A} = \tan 2A$$

$$\text{(ix)} \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A$$

$$\text{(x)} \frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A}$$

$$\text{(xi)} \frac{\sin(\theta + \phi) - 2\sin\theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2\cos\theta + \cos(\theta - \phi)} = \tan\theta$$

Solution:

(i) $\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$

Let us consider LHS:

$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$

By using the formulas,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

So now,

$$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A}$$

$$= \frac{\left(2\sin\frac{5A + A}{2}\cos\frac{5A - A}{2}\right) + \sin 3A}{\left(2\cos\frac{5A + A}{2}\cos\frac{5A - A}{2}\right) + \cos 3A}$$

$$= \frac{\left(2\sin\frac{6A}{2}\cos\frac{4A}{2}\right) + \sin 3A}{\left(2\cos\frac{6A}{2}\cos\frac{4A}{2}\right) + \cos 3A}$$

$$= \frac{(2\sin 3A\cos 2A) + \sin 3A}{(2\cos 3A\cos 2A) + \cos 3A}$$

$$= \frac{\sin 3A\,(2\cos 2A + 1)}{\cos 3A\,(2\cos 2A + 1)}$$

= tan 3A

= RHS

Hence proved.

(ii) $\frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \frac{\cos 5A}{\cos 3A}$

Let us consider LHS:

$\frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A}$

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

So now,

$$\frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \frac{(\cos 7A + \cos 3A) + 2\cos 5A}{(\cos 5A + \cos A) + 2\cos 3A}$$

$$= \frac{\left(2\cos\frac{7A + 3A}{2}\cos\frac{7A - 3A}{2}\right) + 2\cos 5A}{\left(2\cos\frac{5A + A}{2}\cos\frac{5A - A}{2}\right) + 2\cos 3A}$$

$$= \frac{\left(2\cos\frac{10A}{2}\cos\frac{4A}{2}\right) + 2\cos 5A}{\left(2\cos\frac{6A}{2}\cos\frac{4A}{2}\right) + 2\cos 3A}$$

$$= \frac{(2\cos 5A\cos 2A) + 2\cos 5A}{(2\cos 3A\cos 2A) + 2\cos 3A}$$

$$= \frac{2\cos 5A(\cos 2A + 1)}{2\cos 3A(\cos 2A + 1)}$$

$$= \frac{\cos 5A}{\cos 3A}$$

= RHS

Hence proved.

(iii) $\frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \cot 3A$

Let us consider LHS:

$$\frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A}$$

By using the formula,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

So now,

$$\frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \frac{(\cos 4A + \cos 2A) + \cos 3A}{(\sin 4A + \sin 2A) + \sin 3A}$$

$$= \frac{\left(2\cos\frac{4A + 2A}{2}\cos\frac{4A - 2A}{2}\right) + \cos 3A}{\left(2\sin\frac{4A + 2A}{2}\cos\frac{4A - 2A}{2}\right) + \sin 3A}$$

$$= \frac{\left(2\cos\frac{6A}{2}\cos\frac{2A}{2}\right) + \cos 3A}{\left(2\sin\frac{6A}{2}\cos\frac{2A}{2}\right) + \sin 3A}$$

$$= \frac{(2\cos 3A\cos A) + \cos 3A}{(2\sin 3A\cos A) + \sin 3A}$$

$$= \frac{\cos 3A\,(2\cos A + 1)}{\sin 3A\,(2\cos A + 1)}$$

= cot 3A

= RHS

Hence proved.

(iv) $\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$

Let us consider LHS:

$$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$

By using the formulas,

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

So now,

$$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$

When we rearrange we get,

$$= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)}$$

$$= \frac{\left(2\sin\frac{9A+3A}{2}\cos\frac{9A-3A}{2}\right) + \left(2\sin\frac{7A+5A}{2}\cos\frac{7A-5A}{2}\right)}{\left(2\cos\frac{9A+3A}{2}\cos\frac{9A-3A}{2}\right) + \left(2\cos\frac{7A+5A}{2}\cos\frac{7A-5A}{2}\right)}$$

$$= \frac{\left(2\sin\frac{12A}{2}\cos\frac{6A}{2}\right) + \left(2\sin\frac{12A}{2}\cos\frac{2A}{2}\right)}{\left(2\cos\frac{12A}{2}\cos\frac{6A}{2}\right) + \left(2\cos\frac{12A}{2}\cos\frac{2A}{2}\right)}$$

$$= \frac{(2\sin 6A\cos 3A) + (2\sin 6A\cos A)}{(2\cos 6A\cos 3A) + (2\cos 6A\cos A)}$$

$$= \frac{2\sin 6A\,(\cos 3A + \cos A)}{2\cos 6A\,(\cos 3A + \cos A)}$$

$$= \frac{\sin 6A}{\cos 6A}$$

= tan 6A

= RHS

Hence proved.

(v) $\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$

Let us consider LHS:

$$\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A}$$

By using the formulas,

cos A - cos B = -2 sin (A+B)/2 sin (A-B)/2

sin A - sin B = 2 cos (A+B)/2 sin (A-B)/2

So now,

$$\frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \frac{-(\sin 7A - \sin 5A) + (\sin 8A - \sin 4A)}{(\cos 7A - \cos 5A) - (\cos 8A - \cos 4A)}$$

When we rearrange we get,

$$= \frac{-\left(2\cos\frac{7A + 5A}{2}\sin\frac{7A - 5A}{2}\right) + \left(2\cos\frac{8A + 4A}{2}\sin\frac{8A - 4A}{2}\right)}{\left(-2\sin\frac{7A + 5A}{2}\sin\frac{7A - 5A}{2}\right) - \left(-2\sin\frac{8A + 4A}{2}\sin\frac{8A - 4A}{2}\right)}$$

$$= \frac{-\left(2\cos\frac{12A}{2}\sin\frac{2A}{2}\right) + \left(2\cos\frac{12A}{2}\sin\frac{4A}{2}\right)}{\left(-2\sin\frac{12A}{2}\sin\frac{2A}{2}\right) - \left(-2\sin\frac{12A}{2}\sin\frac{4A}{2}\right)}$$

$$= \frac{-(2\cos 6A \sin A) + (2\cos 6A \sin 2A)}{-(2\sin 6A \sin A) + (2\sin 6A \sin 2A)}$$

$$= \frac{2\cos 6A\,(-\sin A + \sin 2A)}{2\sin 6A\,(-\sin A + \sin 2A)}$$

$$= \frac{\cos 6A}{\sin 6A}$$

= cot 6A

= RHS

Hence proved.

(vi) $\frac{\sin 5A \cos 2A - \sin 6A \cos A}{\sin A \sin 2A - \cos 2A \cos 3A} = \tan A$

Let us consider LHS:

$\frac{\sin 5A \cos 2A - \sin 6A \cos A}{\sin A \sin 2A - \cos 2A \cos 3A}$

Let us multiply and divide the above expression by 2 we get,

$$\frac{\sin 5A \cos 2A - \sin 6A \cos A}{\sin A \sin 2A - \cos 2A \cos 3A} = \frac{(2\sin 5A \ \cos 2A) - (2\sin 6A \cos A)}{(2\sin 2A \sin A) - (2\cos 3A \cos 3A)}$$

We know that, 2 sin A sin B = cos (A – B) – cos (A + B),

2 sin A cos B = sin (A + B) + sin (A – B) and

2 cos A cos B = cos (A + B) + cos (A – B)

So now,

$$= \frac{\{\sin(5A + 2A) + \sin(5A - 2A)\} - \{\sin(6A + A) + \sin(6A - A)\}}{\{\cos(2A - A) - \cos(2A + A)\} - \{\cos(3A + 2A) + \cos(3A - 2A)\}}$$

$$= \frac{\{\sin 7A + \sin 3A\} - \{\sin 7A + \sin 5A\}}{\{\cos A - \cos 3A\} - \{\cos 5A + \cos A\}}$$

$$= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos A - \cos 3A - \cos 5A - \cos A}$$

$$= \frac{\sin 3A - \sin 5A}{-(\cos 5A + \cos 3A)}$$

$$= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A + \cos 3A)}$$

$$= \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$$

By using the formulas,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

so,

$$= \frac{2\cos\frac{5A + 3A}{2}\sin\frac{5A - 3A}{2}}{2\cos\frac{5A + 3A}{2}\cos\frac{5A - 3A}{2}}$$

$$= \frac{2\cos\frac{5A+3A}{2}\sin\frac{5A-3A}{2}}{2\cos\frac{5A+3A}{2}\cos\frac{5A-3A}{2}}$$

$$= \frac{2\cos\frac{8A}{2}\sin\frac{2A}{2}}{2\cos\frac{8A}{2}\cos\frac{2A}{2}}$$

$$= \frac{\sin A}{\cos A}$$

= tan A

= RHS

Hence proved.

$$\text{(vii)}\ \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$

Let us consider LHS:

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A}$$

Let us multiply and divide the above expression by 2 we get,

$$\frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \frac{(2\sin 11A\ \sin A) + (2\sin 7A \sin 3A)}{(2\cos 11A\ \sin A) + (2\cos 7A\ \sin 3A)}$$

We know that, 2 sin A sin B = cos (A – B) – cos (A + B),

2 cos A sin B = sin (A + B) - sin (A – B)

So now,

$$= \frac{\{\cos(11A - A) - \cos(11A + A)\} + \{\cos(7A - 3A) - \cos(7A + 3A)\}}{\{\sin(11A + A) - \sin(11A - A)\} + \{\sin(7A + 3A) - \sin(7A - 3A)\}}$$

$$= \frac{\{\cos 10A - \cos 12A\} + \{\cos 4A - \cos 10A\}}{\{\sin 12A - \sin 10A\} + \{\sin 10A - \sin 4A\}}$$

$$= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A}$$

$$= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A}$$

By using the formulas,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

cos A – cos B = -2 sin (A+B)/2 sin (A-B)/2 so,

$$= -\frac{-2\sin\frac{12A + 4A}{2}\sin\frac{12A - 4A}{2}}{2\cos\frac{12A + 4A}{2}\sin\frac{12A - 4A}{2}}$$

$$= \frac{\sin\frac{16A}{2}\sin\frac{8A}{2}}{\cos\frac{16A}{2}\sin\frac{8A}{2}}$$

$$= \frac{\sin 8A}{\cos 8A}$$

= tan 8A

= RHS

Hence proved.

$$\text{(viii)}\ \frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A} = \tan 2A$$

Let us consider LHS:

$$\frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A}$$

Let us multiply and divide the above expression by 2 we get,

$$\frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A} = \frac{(2\sin 3A\ \cos 4A) - (2\sin A\cos 2A)}{(2\sin 4A\sin A\ ) + (2\cos 6A\cos A)}$$

We know that, 2 sin A sin B = cos (A – B) – cos (A + B),

2 cos A cos B = cos (A + B) + cos (A – B)

2 sin A cos B = sin (A + B) + sin (A – B)

So now,

$$= \frac{\{\sin(3A + 4A) + \sin(3A - 4A)\} - \{\sin(A + 2A) + \sin(A - 2A)\}}{\{\cos(4A - A) - \cos(4A + A)\} + \{\cos(6A + A) + \cos(6A - A)\}}$$

$$= \frac{\{\sin 7A + \sin(-A)\} - \{\sin 3A + \sin(-A)\}}{\{\cos 3A - \cos 5A\} + \{\cos 7A + \cos 5A\}}$$

$$= \frac{\{\sin 7A - \sin A\} - \{\sin 3A - \sin A\}}{\{\cos 3A - \cos 5A\} + \{\cos 7A + \cos 5A\}}$$ (since, (-A) = -sin A)

$$= \frac{\sin 7A - \sin A - \sin 3A + \sin A}{\cos 3A - \cos 5A + \cos 7A + \cos 5A}$$

$$= \frac{\sin 7A - \sin 3A}{\cos 7A + \cos 3A}$$

By using the formulas,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

so,

$$= \frac{2\cos\frac{7A + 3A}{2}\sin\frac{7A - 3A}{2}}{2\cos\frac{7A + 3A}{2}\cos\frac{7A - 3A}{2}}$$

$$= \frac{\cos\frac{10A}{2}\sin\frac{4A}{2}}{\cos\frac{10A}{2}\cos\frac{4A}{2}}$$

$$= \frac{\sin 2A}{\cos 2A}$$

= tan 2A

= RHS

Hence proved.

(ix) $\frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A$

Let us consider LHS:

$$\frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A}$$

Let us multiply and divide the above expression by 2 we get,

$$\frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \frac{(2\sin 2A\ \sin A) + (2\sin 6A \sin 3A)}{(2\sin A\cos 2A\ ) + (2\sin 3A\cos 6A)}$$

We know that, 2 sin A sin B = cos (A – B) – cos (A + B),

2 sin A cos B = sin (A + B) + sin (A – B)

So now,

$$= \frac{\{\cos(2A - A) - \cos(2A + A)\} + \{\cos(6A - 3A) - \cos(6A + 3A)\}}{\{\sin(A + 2A) + \sin(A - 2A)\} + \{\sin(3A + 6A) + \sin(3A - 6A)\}}$$

$$= \frac{\{\cos A - \cos 3A\} + \{\cos 3A - \cos 9A\}}{\{\sin 3A + \sin(-A)\} + \{\sin 9A + \sin(-3A)\}}$$

$$= \frac{\{\cos A - \cos 3A\} + \{\cos 3A - \cos 9A\}}{\{\sin 3A - \sin A\} + \{\sin 9A - \sin 3A\}}$$ (since, sin (-A) = -sin A)

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A}$$

$$= \frac{-(\cos 9A - \cos A)}{\sin 9A - \sin A}$$

By using the formulas,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

cos A – cos B = -2 sin (A+B)/2 sin (A-B)/2

so,

$$= -\frac{-2\sin\frac{9A + A}{2}\sin\frac{9A - A}{2}}{2\cos\frac{9A + A}{2}\sin\frac{9A - A}{2}}$$

$$= \frac{\sin\frac{10A}{2}\sin\frac{8A}{2}}{\cos\frac{10A}{2}\sin\frac{8A}{2}}$$

$$= \frac{\sin 5A}{\cos 5A}$$

= tan 5A

= RHS

Hence proved.

(x) $\frac{\sin \mathbf{A} + 2\sin 3\mathbf{A} + \sin 5\mathbf{A}}{\sin 3\mathbf{A} + 2\sin 5\mathbf{A} + \sin 7\mathbf{A}} = \frac{\sin 3\mathbf{A}}{\sin 5\mathbf{A}}$

Let us consider LHS:

$$\frac{\sin \mathbf{A} + 2\sin 3\mathbf{A} + \sin 5\mathbf{A}}{\sin 3\mathbf{A} + 2\sin 5\mathbf{A} + \sin 7\mathbf{A}}$$

$$\frac{\sin \mathbf{A} + 2\sin 3\mathbf{A} + \sin 5\mathbf{A}}{\sin 3\mathbf{A} + 2\sin 5\mathbf{A} + \sin 7\mathbf{A}} = \frac{(\sin 5A + \sin A) + 2\sin 3A}{(\sin 7A + \sin 3A) + 2\sin 5A}$$

We know that,

Sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

So now,

$$= \frac{\left(2\sin\frac{5A + A}{2}\cos\frac{5A - A}{2}\right) + 2\sin 3A}{\left(2\sin\frac{7A + 3A}{2}\cos\frac{7A - 3A}{2}\right) + 2\sin 5A}$$

$$= \frac{\left(2\sin\frac{6A}{2}\cos\frac{4A}{2}\right) + 2\sin 3A}{\left(2\sin\frac{10A}{2}\cos\frac{4A}{2}\right) + 2\sin 5A}$$

$$= \frac{(2\sin 3A\cos 2A) + 2\sin 3A}{(2\sin 5A\cos 2A) + 2\sin 5A}$$

$$= \frac{2\sin 3A\,(\cos 2A + 1)}{2\sin 5A\,(\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\sin 5A}$$

= RHS

Hence proved.

(xi) $\frac{\sin(\theta + \phi) - 2\sin\theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2\cos\theta + \cos(\theta - \phi)} = \tan\theta$

Let us consider LHS:

$$\frac{\sin(\theta + \phi) - 2\sin\theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2\cos\theta + \cos(\theta - \phi)}$$

$$\frac{\sin(\theta + \phi) - 2\sin\theta + \sin(\theta - \phi)}{\cos(\theta + \phi) - 2\cos\theta + \cos(\theta - \phi)} = \frac{\{\sin(\theta + \Phi) + \sin(\theta - \Phi)\} - 2\sin\theta}{\{\cos(\theta + \Phi) + \cos(\theta - \Phi)\} - 2\cos\theta}$$

We know that,

Sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

Cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

So now,

$$= \frac{\left(2\sin\frac{\theta + \Phi + \theta - \Phi}{2}\cos\frac{\theta + \Phi - \theta + \Phi}{2}\right) - 2\sin\theta}{\left(2\cos\frac{\theta + \Phi + \theta - \Phi}{2}\cos\frac{\theta + \Phi - \theta + \Phi}{2}\right) - 2\cos\theta}$$

$$= \frac{\left(2\sin\frac{2\theta}{2}\cos\frac{2\Phi}{2}\right) - 2\sin\theta}{\left(2\cos\frac{2\theta}{2}\cos\frac{2\Phi}{2}\right) - 2\cos\theta}$$

$$= \frac{(2\sin\theta\cos\Phi) - 2\sin\theta}{(2\cos\theta\cos\Phi) - 2\cos\theta}$$

$$= \frac{2\sin\theta(\cos\Phi - 1)}{2\cos\theta(\cos\Phi - 1)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

= tan θ

= RHS

Hence proved.

9. Prove that:

(i) sin α + sin β + sin γ – sin (α + β + γ) = 4 sin (α + β)/2 sin (β + γ)/2 sin (α + γ)/2

(ii) cos (A + B + C) + cos (A – B + C) + cos (A + B – C) + cos (-A + B + C) = 4 cos A cos B cos C

Solution:

(i) sin α + sin β + sin γ – sin (α + β + γ) = 4 sin (α + β)/2 sin (β + γ)/2 sin (α + γ)/2

Let us consider LHS:

sin α + sin β + sin γ – sin (α + β + γ)

By using the formulas,

Sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

$$\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha + \beta + \gamma)$$

$$= (\sin\alpha + \sin\beta) + \{\sin\gamma - \sin(\alpha + \beta + \gamma)\}$$

$$= \left(2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}\right) + \left(2\cos\frac{\gamma + \alpha + \beta + \gamma}{2}\sin\frac{\gamma - \alpha - \beta - \gamma}{2}\right)$$

$$= \left(2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}\right) + \left(2\cos\frac{\alpha + \beta + 2\gamma}{2}\sin\frac{-(\alpha + \beta)}{2}\right)$$

$$= \left(2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}\right) - \left(2\cos\frac{\alpha + \beta + 2\gamma}{2}\sin\frac{\alpha + \beta}{2}\right)$$ (since,sin (-A) =-sin A)

$$= 2\sin\frac{\alpha + \beta}{2}\left(\cos\frac{\alpha - \beta}{2} - \cos\frac{\alpha + \beta + 2\gamma}{2}\right)$$

Again by using the formula,

Cos A – cos B = -2 sin (A+B)/2 sin (A-B)/2

$$= 2\sin\frac{\alpha + \beta}{2}\left(-2\sin\frac{\frac{\alpha - \beta}{2} + \frac{\alpha + \beta + 2\gamma}{2}}{2}\sin\frac{\frac{\alpha - \beta}{2} - \frac{\alpha + \beta + 2\gamma}{2}}{2}\right)$$

$$= 2\sin\frac{\alpha + \beta}{2}\left(-2\sin\frac{\frac{\alpha - \beta + \alpha + \beta + 2\gamma}{2}}{2}\sin\frac{\frac{\alpha - \beta - (\alpha + \beta + 2\gamma)}{2}}{2}\right)$$

$$= 2\sin\frac{\alpha + \beta}{2}\left(-2\sin\frac{\frac{2\alpha + 2\gamma}{2}}{2}\sin\frac{\frac{\alpha - \beta - \alpha - \beta - 2\gamma)}{2}}{2}\right)$$

$$= 2\sin\frac{\alpha + \beta}{2}\left(-2\sin\frac{\frac{2\alpha + 2\gamma}{2}}{2}\sin\frac{\frac{-2\beta - 2\gamma)}{2}}{2}\right)$$

$$= 2\sin\frac{\alpha + \beta}{2}\left(-2\sin\frac{\frac{2(\alpha + \gamma)}{2}}{2}\sin\frac{\frac{-2(\beta + \gamma)}{2}}{2}\right)$$

$$= 2\sin\frac{\alpha + \beta}{2}\left(-2\sin\frac{\alpha + \gamma}{2}\sin\frac{-(\beta + \gamma)}{2}\right)$$

$$= 2\sin\frac{\alpha + \beta}{2}\left(2\sin\frac{\alpha + \gamma}{2}\sin\frac{\beta + \gamma}{2}\right)$$

$$= 4\sin\frac{\alpha + \beta}{2}\sin\frac{\beta + \gamma}{2}\sin\frac{\alpha + \gamma}{2}$$

= RHS

Hence proved.

(ii) cos (A + B + C) + cos (A – B + C) + cos (A + B – C) + cos (-A + B + C) = 4 cos A cos B cos C

Let us consider LHS:

cos (A + B + C) + cos (A – B + C) + cos (A + B – C) + cos (-A + B + C)

so,

cos (A + B + C) + cos (A – B + C) + cos (A + B – C) + cos (-A + B + C) =

={cos (A + B + C) + cos (A – B + C)} + {cos (A + B – C) + cos (-A + B + C)}

By using the formula,

Cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

$$= \left\{2\cos\frac{(A+B+C)+(A-B+C)}{2}\cos\frac{(A+B+C)-(A-B+C)}{2}\right\}$$
$$+\left\{2\cos\frac{(A+B-C)+(-A+B+C)}{2}\cos\frac{(A+B-C)-(-A+B+C)}{2}\right\}$$

$$= \left\{2\cos\frac{A+B+C+A-B+C}{2}\cos\frac{A+B+C-A+B-C}{2}\right\}$$
$$+\left\{2\cos\frac{A+B-C-A+B+C}{2}\cos\frac{A+B-C+A-B-C}{2}\right\}$$

$$= \left\{2\cos\frac{2A+2C}{2}\cos\frac{2B}{2}\right\}+\left\{2\cos\frac{2B}{2}\cos\frac{2A-2C}{2}\right\}$$

$$= \left\{2\cos\frac{2(A+C)}{2}\cos\frac{2B}{2}\right\}+\left\{2\cos\frac{2B}{2}\cos\frac{2(A-C)}{2}\right\}$$

$= 2\cos(A+C)\cos B + 2\cos B\cos(A-C)$

$= 2\cos B\,\{\cos(A+C)+\cos(A-C)\}$

By using the formula,

$\cos A + \cos B = 2\cos(A+B)/2\cos(A-B)/2$

$$= 2\cos B\left\{2\cos\frac{A+C+(A-C)}{2}\cos\frac{A+C-(A-C)}{2}\right\}$$

$$= 2\cos B\left\{2\cos\frac{A+C+A-C}{2}\cos\frac{A+C-A+C}{2}\right\}$$

$$= 2\cos B\left\{2\cos\frac{2A}{2}\cos\frac{2C}{2}\right\}$$

$= 4\cos A\cos B\cos C$

= RHS

Hence proved.