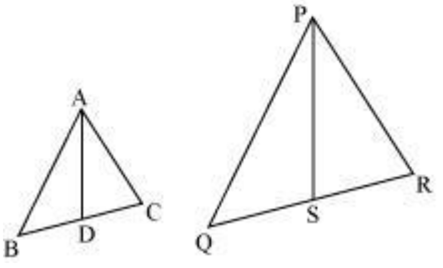
NCERT Solutions for Class 10 Maths Chapter 6 - Triangles

Chapter 6 - Triangles Exercise Ex. 6.4

Solution 6



Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$. Let AD and PS be the medians of these triangles.

So, $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$ (1)

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Since, AD and PS are medians

So, $BD = DC = \dfrac{BC}{2}$

And $QS = SR = \dfrac{QR}{2}$

So, equation (1) becomes

$\dfrac{AB}{PQ} = \dfrac{BD}{QS} = \dfrac{AC}{PR}$

Now in ΔABD and ΔPQS

$\angle B = \angle Q$

And, $\dfrac{AB}{PQ} = \dfrac{BD}{QS}$

So, $\Delta ABD \sim \Delta PQS$

So, we may say that

$\dfrac{AB}{PQ} = \dfrac{BD}{QS} = \dfrac{AD}{PS}$ (2)

Now $\dfrac{area(\Delta ABC)}{area(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{AC}{PR}\right)^2$

From equation (1) and (2) we may find that

$\dfrac{AB}{PQ} = \dfrac{AD}{PS}$

And hence,

$\dfrac{area(\Delta ABC)}{area(\Delta PQR)} = \left(\dfrac{AD}{PS}\right)^2$

Solution 1

$$\text{If } \Delta ABC \sim \Delta DEF$$

$$\text{Therefore } \frac{\text{area}(\Delta ABC)}{\text{area }(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

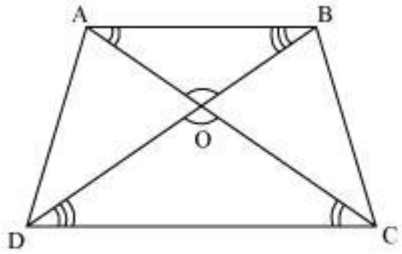
Since $EF = 15.4$, $\text{area}(\Delta ABC) = 64$; $\text{area}(\Delta DEF) = 121$

$$\text{Therefore } \frac{\text{area}(ABC)}{\text{area}(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{64}{121} = \frac{BC^2}{15.4^2} \Rightarrow \frac{BC}{15.4} = \frac{8}{11}$$

$$\Rightarrow BC = \frac{8 \times 15.4}{11} = 8 \times 1.4 = 11.2 \text{ cm}.$$

Solution 2



Since AB || CD

$\angle$OAB = $\angle$OCD (Alternate interior angles)

$\angle$OBA = $\angle$ODC (Alternate interior angles)

$\angle$AOB = $\angle$COD (Vertically opposite angles)

Therefore $\triangle$AOB ~ $\triangle$COD (By AAA rule)

$$\text{Therefore } \frac{\text{area}(\Delta AOB)}{\text{area }(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

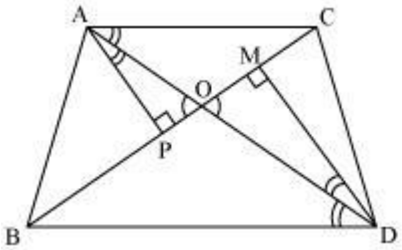
Since $AB = 2\ CD$

$$\text{Therefore } \frac{\text{area}(\Delta AOB)}{\text{area }(\Delta COD)} = \frac{4}{1}$$

Solution 3

We know that area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{height}$

Since $\triangle$ABC and $\triangle$DBC are one same base,
Therefore ratio between their areas will be as ratio of their heights.
Let us draw two perpendiculars AP and DM on line BC.



In $\triangle$APO and $\triangle$DMO,

$\angle APO = \angle DMO = 90$

$\angle AOP = \angle DOM$ (vertically opposite angles)

$\angle OAP = \angle ODM$ (remaining angle)

Therefore $\triangle APO \sim \triangle DMO$ (By AAA rule)

Therefore $\frac{AP}{DM} = \frac{AO}{DO}$

Therefore $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$

Solution 4

Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$

Now $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Since area (ΔABC) = area (ΔPQR)

Therefore $AB = PQ$

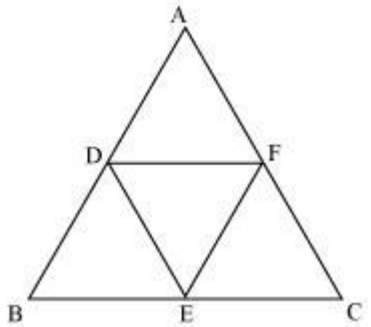
$BC = QR$

$AC = PR$

So, respective sides of two similiar triangles are also of same length

So, $\Delta ABC \cong \Delta PQR$ (by SSS rule)

Solution 5



Since D and E are mid points of $\triangle$ ABC

Therefore DE || AC and DE = $\frac{1}{2}$ AC

Now in $\Delta(BED)$ and $\Delta(BCA)$

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

Therefore $\Delta(BED) - \Delta(BCA)$

Therefore $\frac{area(BED)}{area(BCA)} = \left(\frac{DE}{AC}\right)^2$

Therefore $\frac{area(BED)}{area(BCA)} = \frac{1}{4}$

Similarly, $\frac{area(CFE)}{area(CAB)} = \frac{1}{4}$ and $\frac{area(ADF)}{area(ABC)} = \frac{1}{4}$

Now, area(ΔABC) = area(ΔBED) + area(ΔCFE) + area(ΔADF) + area(ΔDEF)

$\therefore$ area(ΔDEF) = area(ΔABC) – area(ΔBED) – area(ΔCFE) – area(ΔADF)

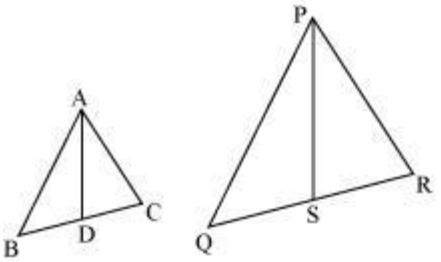
$= 1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{4}$ [considering area(ΔABC) = 1]

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

Therefore $\frac{area(DEF)}{area(ABC)} = \frac{\frac{1}{4}}{\frac{1}{1}} = \frac{1}{4}$

Solution 6



Let us assume two similar triangles as $\triangle$ ABC ~ $\triangle$ PQR. Let AD and PS be the medians of these triangles.

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ (1)

$\angle$A = $\angle$P, $\angle$B = $\angle$Q, $\angle$C = $\angle$R

Since, AD and PS are medians

So, $BD = DC = \frac{BC}{2}$

And $QS = SR = \frac{QR}{2}$

So, equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$

Now in ΔABD and ΔPQS

$\angle B = \angle Q$

And, $\frac{AB}{PQ} = \frac{BD}{QS}$

So, $\Delta ABD \sim \Delta PQS$

So, we may say that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \qquad (2)$

Now $\frac{area(\Delta ABC)}{area(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

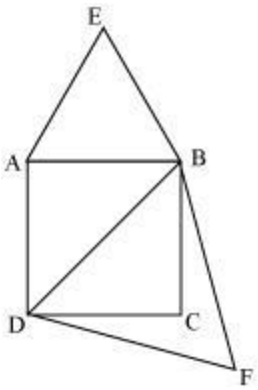
From equation (1) and (2) we may find that

$\frac{AB}{PQ} = \frac{AD}{PS}$

And hence,

$\frac{area(\Delta ABC)}{area(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$

Solution 7



Let ABCD be a square of side a.

Therefore its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as $\triangle$ ABE and $\triangle$ DBF

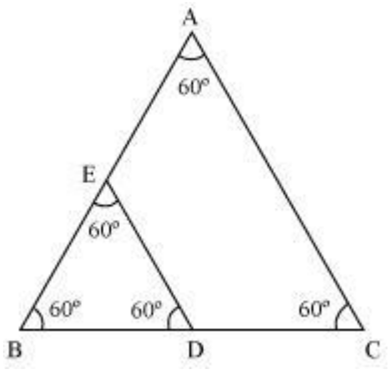
Side of an equilateral triangle $\triangle$ ABE described on one of its side = a

Side of an equilateral triangle $\triangle$ DBF described on one of its diagonal

We know that equilateral triangles are having all its angles as 60º and all its sides of same length. So, all equilateral triangles are similar to each other. So, ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.

$\frac{\text{area of } \Delta ABE}{\text{area of } \Delta DBF} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$

Solution 8



We know that equilateral triangles are having all its angles as 60º and all its sides of same length. So, all equilateral triangles are similar to each other. So, ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.

Let side of $\triangle$ ABC = x

Therefore side of $\Delta BDE = \frac{x}{2}$

So, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$ Hence, (c)

Solution 9

If, two triangles are similar to each other, ratio between areas of these triangles will be equal to the square of the ratio between sides of these triangles.

Given that sides are in the ratio 4:9.

So, ratio between areas of these triangles $= \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, (d).

Chapter 6 - Triangles Exercise Ex. 6.1

Solution 1

(i) All circles are SIMILAR.

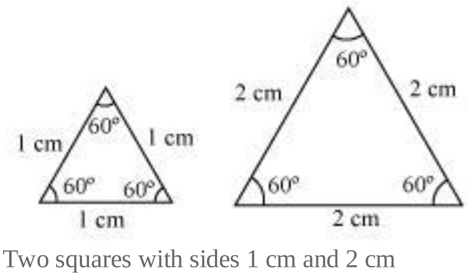
(ii) All squares are SIMILAR.

(iii) All EQUILATERAL triangles are similar.

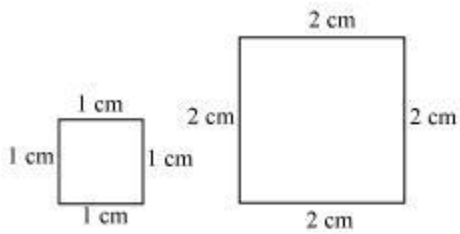
(iv) Two polygons of same number of sides are similar, if their corresponding angles are EQUAL and their corresponding sides are PROPORTIONAL.

Solution 2

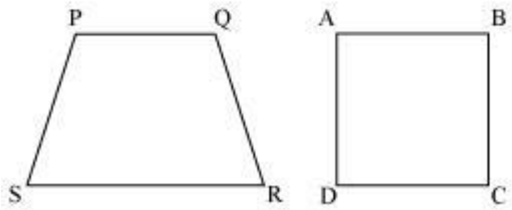
(i) Two equilateral triangles with sides 1 cm and 2 cm.



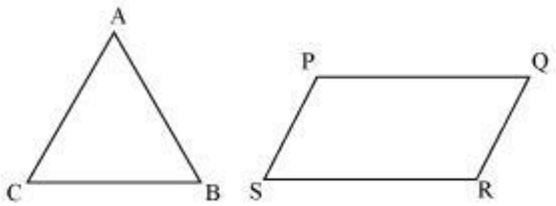
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and Square



Triangle and Parallelogram



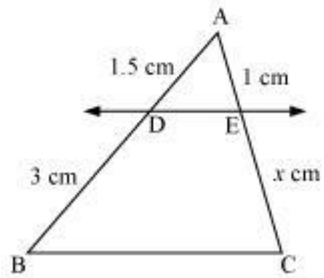
Solution 3

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional i.e. 1:2 but their corresponding angles are not equal.

Chapter 6 - Triangles Exercise Ex. 6.2

Solution 1

(i)



Let EC = x

Since DE || BC.

Therefore, by basic proportionality theorem,

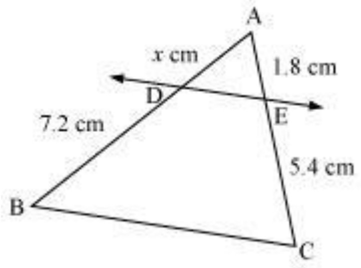
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

Therefore EC = 2 cm (ii)



Let AD = x
Since DE || BC,
Therefore by basic proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

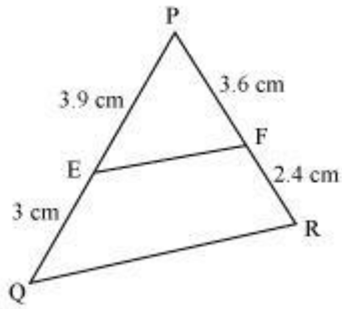
$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

Therefore AD = 2.4 cm

Solution 2
(i)



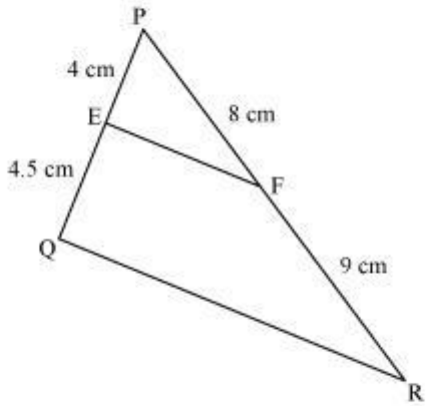
Given that PE = 3.9, EQ = 3, PF = 3.6, FR = 2.4
Now,

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Since $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore EF is not parallel with QR.
(ii)



PE = 4, QE = 4.5, PF = 8, RF = 9

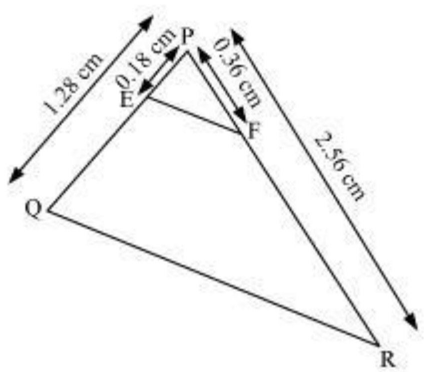
$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$

$\frac{PF}{FR} = \frac{8}{9}$

Since $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore EF is parellel with QR.

(iii)



PQ = 1.28, PR = 2.56, PE = 0.18, PF = 0.36

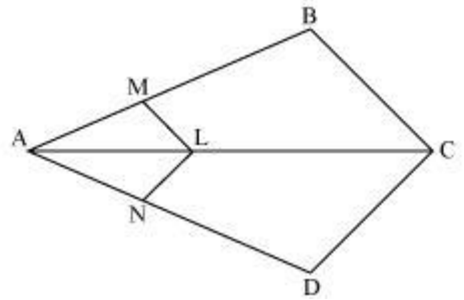
$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$

$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$

since $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore EF is parellel with QR.

Solution 3



In the given figure
Since LM || CB,
Therefore by basic proportionality theorem,

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad (i)$$

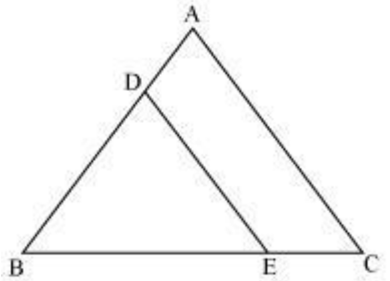
similarly since LN || CD

$$\frac{AN}{AD} = \frac{AL}{AC} \qquad (ii)$$

From (i) and (ii)
we can say,

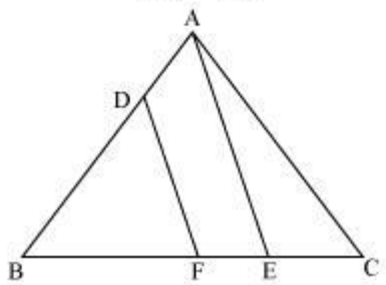
$$\frac{AM}{AB} = \frac{AN}{AD}$$

Solution 4



In $\triangle$ABC,
Since DE || AC

Therefore $\frac{BD}{DA} = \frac{BE}{EC}$ (i)



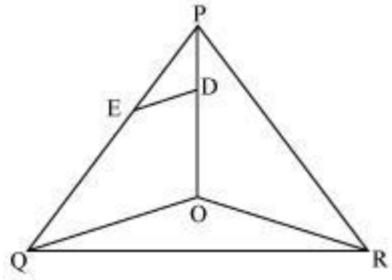
In ΔBAE,
Since $DF \parallel AE$

$\frac{BD}{DA} = \frac{BF}{FE}$ (ii)

from (i) and (ii)

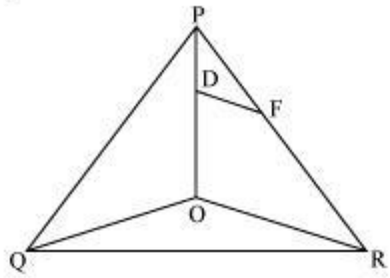
$\frac{BE}{EC} = \frac{BF}{FE}$

Solution 5



In $\triangle$ POQ
Since DE || OQ

$\frac{PE}{EQ} = \frac{PD}{DO}$ (i) [By basic proportionality theorem]



In Δ POR

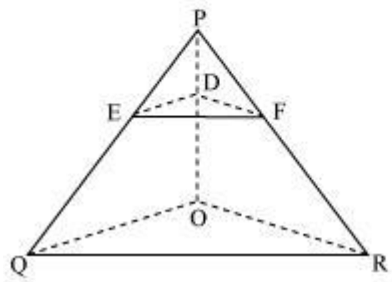
Since $DF \parallel OR$

$\frac{PF}{FR} = \frac{PD}{DO}$ (ii) [By basic proportionality theorem]

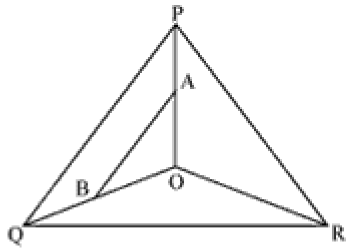
From (i) and (ii)

$\frac{PE}{EQ} = \frac{PF}{FR}$

By basic proportionality theorem
$EF \parallel QR$

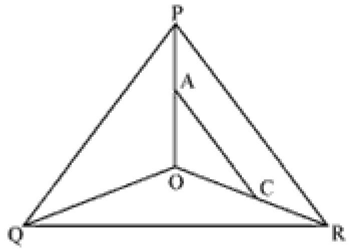


Solution 6



In Δ POQ
Since AB || PQ,

$$\frac{OA}{AP} = \frac{OB}{BQ} \quad \text{(i)} \quad [\text{By basic proportionality theorem}]$$



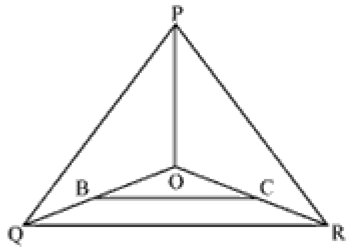
In Δ POR
Since AC || PR

Therefore $\frac{OA}{AP} = \frac{OC}{CR}$ (ii) [By basic proportionality theorem]

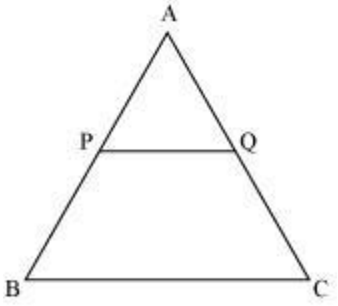
From (i) and (ii)

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore BC || QR
(By basic proportionality theorem)



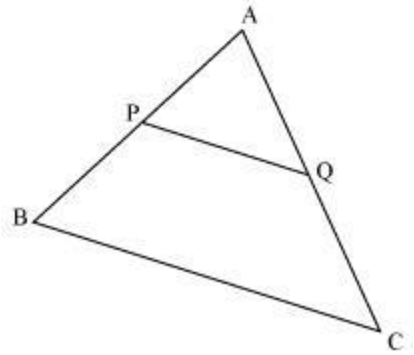
Solution 7



Consider the given figure
PQ is a line segment drawn through midpoint P of line AB such that PQ||BC
i.e. AP = PB
Now, by basic proportionality theorem

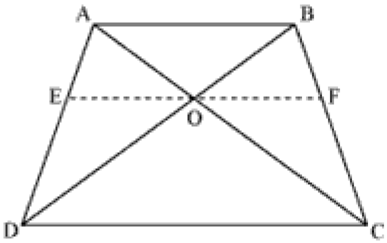
$$\frac{AQ}{QC} = \frac{AP}{PB}$$
$$\frac{AQ}{QC} = \frac{1}{1}$$
i.e. AQ = QC
Or, Q is midpoint of AC.

Solution 8



Consider the given figure
PQ is a line segment joining midpoints P and Q of line AB and AC respectively.
i.e. AP = PB and AQ = QC
Now, we may observe that
$$\frac{AP}{PB} = \frac{1}{1}$$
$$\text{and} \frac{AQ}{QC} = \frac{1}{1}$$
$$\text{Thus} \frac{AP}{PB} = \frac{AQ}{QC}$$
And hence basic proportionality theorem is verified
So, PQ||BC

Solution 9



Draw a line EF through point O such that EF||CD
In $\triangle ADC$
EO||CD
So, by basic proportionality theorem

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

Similarly in $\triangle BDC$
FO||CD
So, by basic proportionality theorem

$$\frac{BF}{FC} = \frac{BO}{OD} \qquad (2)$$

Now consider trapezium ABCD
As FE||CD

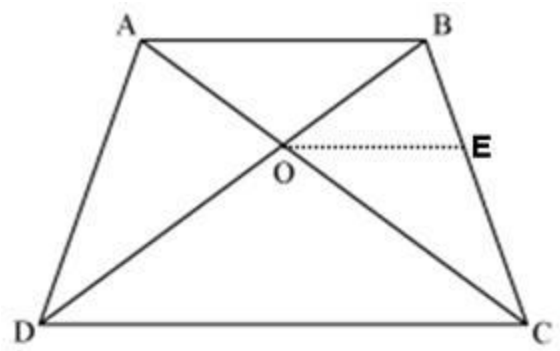
So, $\frac{AE}{ED} = \frac{BF}{FC} \qquad (3)$

Now from equation (1), (2), (3)

$$\frac{AO}{OC} = \frac{BO}{OD}$$

Or, $\frac{AO}{BO} = \frac{OC}{OD}$

Solution 10



Draw a line OE || AB

In ΔABC,

Since OE || AB

Therefore $\frac{AO}{OC} = \frac{BE}{EC}$

But by the given relation, we have:

$\frac{AO}{OC} = \frac{OB}{OD}$

Therefore $\frac{BE}{EC} = \frac{OB}{OD}$

Therefore EO || DC

Therefore AB || OE || DC

$\Rightarrow$ AB || CD

$\Rightarrow$ ABCD is a trapezium.

Chapter 6 - Triangles Exercise Ex. 6.3

Solution 1

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore $\Delta ABC \sim \Delta PQR$ [by AAA rule]

(ii)

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

Therefore $\Delta ABC \sim \Delta QRP$ [by SSS rule]

(iii) Triangles are not similar as the corresponding sides are not proportional.

(iv) Triangles are not similar as the corresponding sides are not proportional.

(v) Triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF

$\angle D + \angle E + \angle F = 180°$

(Sum of measures of angles of a triangle is 180)

$70° + 80° + \angle F = 180°$

$\angle F = 30°$

Similarly in $\angle$ PQR

$\angle P + \angle Q + \angle R = 180°$

(Sum of measures of angles of a triangle is 180)

$\angle P + 80° + 30° = 180°$

$\angle P = 70°$

Now In $\triangle$ DEF and $\triangle$ PQR

$\angle D = \angle P = 70°$

$\angle E = \angle Q = 80°$

$\angle F = \angle R = 30°$

Therefore $\triangle DEF \sim \triangle PQR$ [by AAA rule]

Solution 2

Since DOB is a straight line

Therefore $\angle DOC + \angle COB = 180°$

Therefore $\angle DOC = 180° - 125°$

$= 55°$

In $\angle$ DOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$

$\angle DCO + 70° + 55° = 180°$

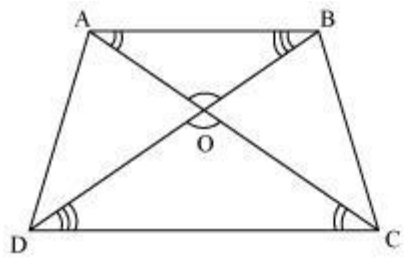
$\angle DCO = 55°$

Since $\triangle ODC \sim \triangle OBA$,

Therefore $\angle OCD = \angle OAB$ [corresponding angles equal in similar triangles]

Therefore $\angle OAB = 55°$

Solution 3



In $\triangle$ DOC and $\triangle$ BOA

AB || CD

Therefore $\angle CDO = \angle ABO$ [Alternate interior angles]

$\angle DCO = \angle BAO$ [Alternate interior angles]
$\angle DOC = \angle BOA$ [Vertically opposite angles]

Therefore $\triangle DOC \sim \triangle BOA$ [AAA rule)

Therefore $\frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

Therefore $\frac{OA}{OC} = \frac{OB}{OD}$

Solution 4

In $\triangle PQR$
$\angle PQR = \angle PRQ$
Therefore PQ = PR (i)
Given,

$\frac{QR}{QS} = \frac{QT}{PR}$

using (i)

$\frac{QR}{QS} = \frac{QT}{QP}$ (ii)

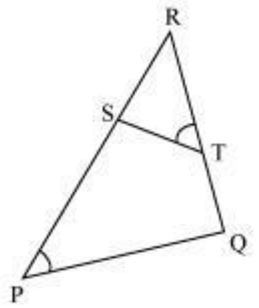
in ΔPQS and ΔTQR,

$\frac{QR}{QS} = \frac{QT}{QP}$ [using (ii)]

$\angle Q = \angle Q$

Therefore $\Delta PQS - \Delta TQR$ [SAS rule]

Solution 5



In $\triangle RPQ$ and $\triangle RST$
$\angle RTS = \angle QPS$ [given]
$\angle R = \angle R$ [common angle]
$\angle RST = \angle RQP$ [Remaining angles]
Therefore $\triangle RPQ \sim \triangle RTS$ [by AAA rule]

Solution 6

Since $\triangle ABE \cong \triangle ACD$
Therefore AB = AC (1)
AD = AE (2)

Now, in $\triangle$ ADE and $\triangle$ ABC,
Dividing equation (2) by (1)

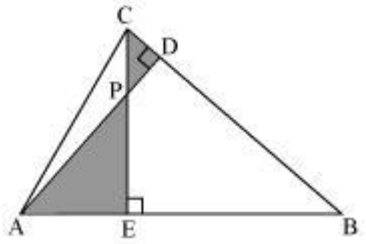
$$\frac{AD}{AB} = \frac{AE}{AC}$$

$\angle A = \angle A$ [common angle]

Therefore $\Delta ADE \sim \Delta ABC$ [by SAS rule]

Solution 7

(i)



In $\triangle$ AEP and $\triangle$ CDP

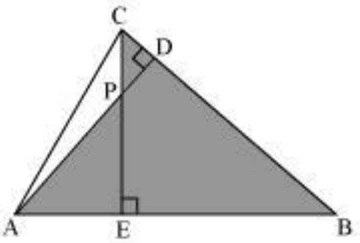
Since $\angle$ CDP = $\angle$ AEP = 90°

$\angle$ CPD = $\angle$ APE (vertically opposite angles)

$\angle$ PCD = $\angle$ PAE (remaining angle)

Therefore by AAA rule,

$\triangle$ AEP ~ $\triangle$ CDP



(ii)

In $\triangle$ ABD and $\triangle$ CBE

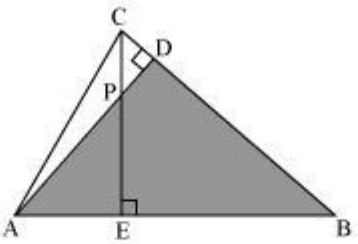
$\angle$ ADB = $\angle$ CEB = 90°

$\angle$ ABD = $\angle$ CBE (common angle)

$\angle$ DAB = $\angle$ ECB (remaining angle)

Therefore by AAA rule

$\triangle$ ABD ~ $\triangle$ CBE



(iii)

In $\triangle$AEP and $\triangle$ADB

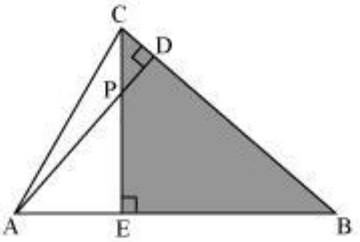
$\angle$AEP = $\angle$ADB = 90°

$\angle$PAE = $\angle$DAB (common angle)

$\angle$APE = $\angle$ABD (remaining angle)

Therefore by AAA rule

$\triangle$AEP ~ $\triangle$ADB

(iv) 

In $\triangle$PDC and $\triangle$BEC

$\angle$PDC = $\angle$BEC = 90°

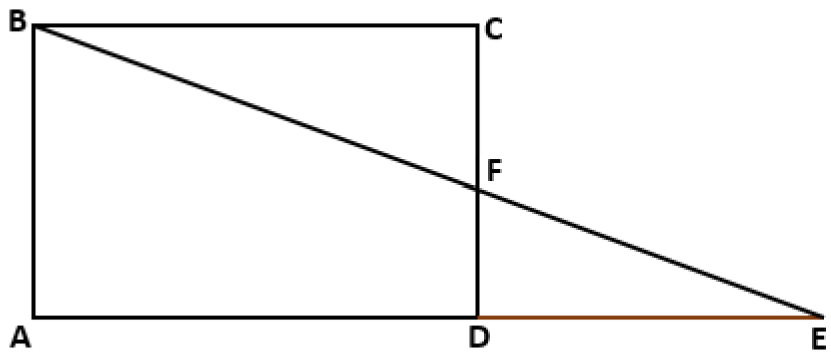
$\angle$PCD = $\angle$BCE (common angle)

$\angle$CPD = $\angle$CBE

Therefore by AAA rule

$\triangle$PDC ~ $\triangle$BEC

Solution 8



$\triangle$ABE and $\triangle$CFB

$\angle$A = $\angle$C (opposite angles of a parallelogram)

$\angle$AEB = $\angle$CBF (Alternate interior angles AE || BC)

$\angle$ABE = $\angle$CFB (remaining angle)

Therefore $\triangle$ABE ~ $\triangle$CFB (by AAA rule)

Solution 9

In $\triangle$ABC and $\triangle$AMP

$\angle$ABC = $\angle$AMP = 90

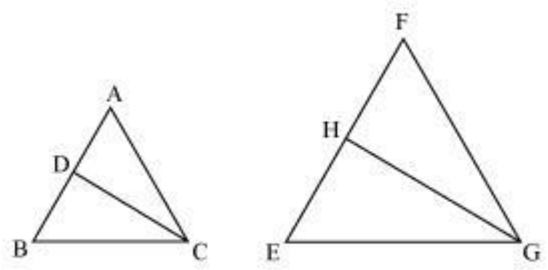
$\angle$A = $\angle$A (common angle)

$\angle$ACB = $\angle$APM (remaining angle)

Therefore $\triangle$ABC ~ $\triangle$AMP (by AAA rule)

Therefore $\frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides are proportional)

Solution 10



Since $\triangle$ABC ~ $\triangle$FEG

Therefore $\angle$A = $\angle$F

$\angle$B = $\angle$E

As, $\angle$ACB = $\angle$FGE

Therefore $\angle$ACD = $\angle$FGH (angle bisector)

And $\angle$DCB = $\angle$HGE (angle bisector)

Therefore $\triangle$ACD ~ $\triangle$FGH (by AAA rule)

And $\triangle$DCB ~ $\triangle$HGE (by AAA rule)

For $\triangle$ACD and $\triangle$FGH

$\frac{CD}{GH} = \frac{AC}{FG}$

and $\angle$ACD = $\angle$FGH

Therefore $\triangle$DCA ~ $\triangle$HGF (by SAS rule)

Solution 11

In $\triangle$ABD and $\triangle$ECF,

Given that AB = AC (isosceles triangles)

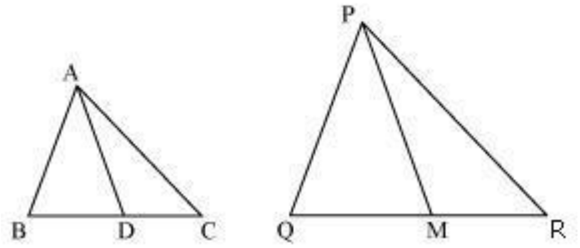
So, $\angle$ABD = $\angle$ECF

$\angle$ADB = $\angle$EFC = 90

$\angle$ BAD = $\angle$ CEF

Therefore $\triangle$ ABD ~ $\triangle$ ECF (by AAA rule)

Solution 12



Median divides opposite side.

So, $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

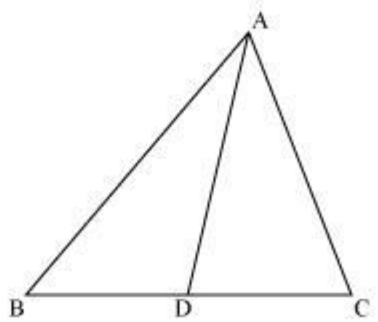
So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Therefore $\triangle$ ABD ~ $\triangle$ PQM (by SSS rule)

Therefore $\angle$ ABD = $\angle$ PQM (corresponding angles of similar triangles)

Therefore $\triangle$ ABC ~ $\triangle$ PQR (by SAS rule)

Solution 13



In $\triangle$ ADC and $\triangle$ BAC

Given that $\angle$ ADC = $\angle$ BAC

$\angle$ ACD = $\angle$ BCA (common angle)

$\angle$ CAD = $\angle$ CBA (remaining angle)

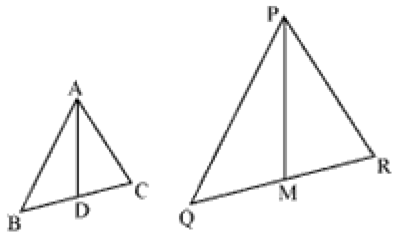
Hence, $\triangle$ ADC ~ $\triangle$ BAC [by AAA rule]

So, corresponding sides of similar triangles will be proportional to each other

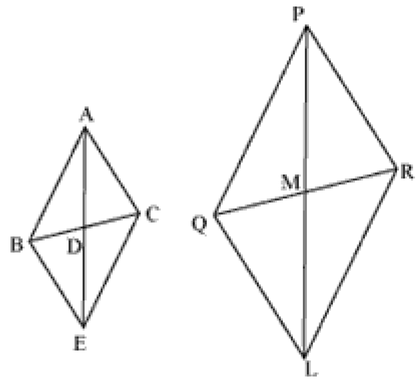
$\frac{CA}{CB} = \frac{CD}{CA}$

Hence $CA^2 = CB \times CD$

Solution 14



Given that
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

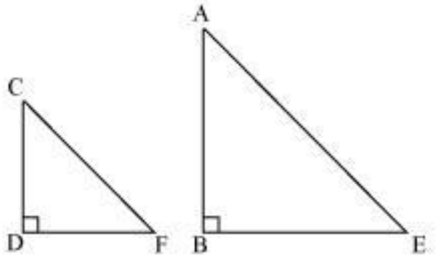


Let us extend AD and PM up to point E and L respectively such that AD = DE and PM = ML. Now join B to E, C to E, Q to L and R to L.
We know that medians divide opposite sides.
So, BD = DC and QM = MR
Also, AD = DE (By construction)
And PM = ML (By construction)
So, in quadrilateral ABEC, diagonals AE and BC bisects each other at point D.
So, quadrilateral ABEC is a parallelogram.
Similarly quadrilateral PQLR is a parallelogram
Now, consider quadrilateral ABEC
AC = BE (since it is a parallelogram opposite sides will be equal)
AB = EC (since it is a parallelogram opposite sides will be equal)
Similarly in quadrilateral PQLR,
PR = QL (since it is a parallelogram opposite sides will be equal)
PQ = LR (since it is a parallelogram opposite sides will be equal)
It was given that $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

So, we may say that $\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$
So, $\Delta ABE \sim \Delta PQL$ (by SSS rule)
Therefore $\angle BAE = \angle QPL$ (corresponding angles of similar triangles)
Similarly we may find that $\Delta AEC \sim \Delta PLR$
$\angle CAE = \angle RPL$ (corresponding angles of similar triangles)
Or $\angle CAB = \angle RPQ$
$\frac{AB}{PQ} = \frac{AC}{PR}$
So, we had observed that two respective sides are in same proportion in both triangles and also angle included between them is respectively equal
Hence, $\Delta ABC \sim \Delta PQR$ (by SAS rule)

Solution 15



Let AB be a tower
CD be a pole
Shadow of AB is BE
Shadow of CD is DF

The light rays from sun will fall on tower and pole at same angle and at the same time.

So, $\angle$DCF = $\angle$BAE

And $\angle$DFC = $\angle$BEA

$\angle$CDF = $\angle$ABE (tower and pole are vertical to ground)

Therefore $\triangle$ABE ~ $\triangle$CDF

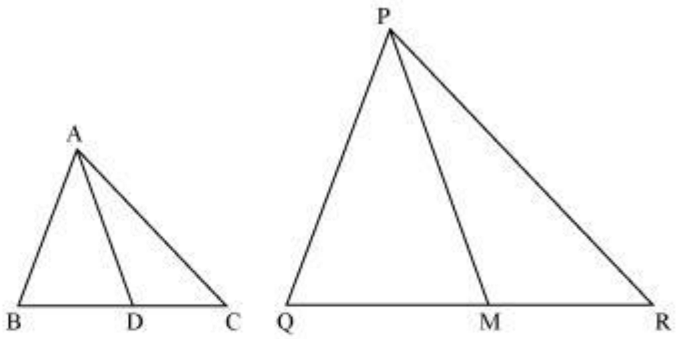
Therefore $\frac{AB}{CD} = \frac{BE}{DF}$

$\frac{AB}{6} = \frac{28}{4}$

$AB = 42$

So, height of tower will be 42 meters.

Solution 16



Since $\triangle$ABC ~ $\triangle$PQR

So, their respective sides will be in proportion

Or, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ (1)

Also, $\angle$A = $\angle$P, $\angle$B = $\angle$Q, $\angle$C = $\angle$R (2)

Since, AD and PM are medians so they will divide their opposite sides in equal halves.

Or, $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ (3)

From equation (1) and (3)

$\frac{AB}{PQ} = \frac{BD}{QM}$

$\angle B = \angle Q$ (from equation 2)

So, we had observed that two respective sides are in same proportion in both triangles and also angle included between them is respectively equal

Hence, $\triangle ABD \sim \triangle PQM$ (by SAS rule)

So, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Chapter 6 - Triangles Exercise Ex. 6.5

Solution 1

i.Given that sides are 7 cm, 24 cm, and 25 cm. Squaring the lengths of these sides we get 49, 576, and 625.
Clearly, 49 + 576 = 625 or $7^2 + 24^2 = 25^2$.
Therefore, given triangle is satisfying Pythagoras theorem. So, it is a right triangle. The longest side in a right angled triangle is the hypotenuse.
Therefore length of hypotenuse of this triangle = 25 cm.

ii.Given that sides are 3 cm, 8 cm, and 6 cm. Squaring the lengths of these sides we may get 9, 64, and 36. Clearly, sum of squares of lengths of two sides is not equal to square of length of third side. Therefore given triangle is not satisfying Pythagoras theorem. So, it is not a right triangle

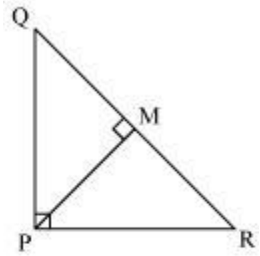
iii.Given that sides are 50 cm, 80 cm, and 100 cm. Squaring the lengths of these sides we may get 2500, 6400, and 10000. Clearly, sum of squares of lengths of two sides is not equal to square of length of third side. Therefore given triangle is not satisfying Pythagoras theorem. So, it is not a right triangle.

iv.Given that sides are 13 cm, 12 cm, and 5 cm. Squaring the lengths of these sides we may get 169, 144, and 25.
Clearly, 144 +25 = 169 Or, $12^2 + 5^2 = 13^2$.
Therefore given triangle is satisfying Pythagoras theorem. So, it is a right triangle.

The longest side in a right angled triangle is the hypotenuse.

Therefore length of hypotenuse of this triangle = 13 cm.

Solution 2



$\text{Let} \angle MPR = x$

$\text{In} \Delta MPR$

$\angle MRP = 180^o - 90^o - x$

$\angle MRP = 90^o - x$

$\text{Similarly in} \Delta MPQ$

$\angle MPQ = 90^o - \angle MPR$

$= 90^o - x$

$\angle MQP = 180^o - 90^o - (90^o - x)$

$\angle MQP = x$

Now in ΔMPQ and ΔMRP and we may observe that

$\angle MPQ = \angle MRP$

$\angle PMQ = \angle RMP$

$\angle MQP = \angle MPR$

Therefore $\Delta MPQ \sim \Delta MRP$ (by AAA rule)

Therefore $\frac{QM}{PM} = \frac{MP}{MR}$

$\Rightarrow PM^2 = QM \times MR$

Solution 3

(ii) Let $\angle CAB = x$

In ΔCBA

$\angle CBA = 180° - 90° - x$

$\angle CBA = 90° - x$

Similarly in ΔCAD

$\angle CAD = 90° - \angle CAB$

$= 90° - x$

$\angle CDA = 180° - 90° - (90° - x)$

$\angle CDA = x$

Now in ΔCBA and ΔCAD we may observe that

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA = 90°$

Therefore ΔCBA ▯ ΔCAD (by AAA rule)

Therefore $\dfrac{AC}{DC} = \dfrac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

iii. In ΔDCA & ΔDAB

$\angle$DCA = $\angle$DAB = 90Âº

$\angle$CDA = $\angle$ADB (common angle)

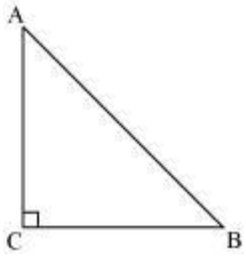
$\angle$DAC = $\angle$DBA (remaining angle)

ΔDCA ▯ ΔDAB (AAA property)

Therefore $\dfrac{DC}{DA} = \dfrac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

Solution 4

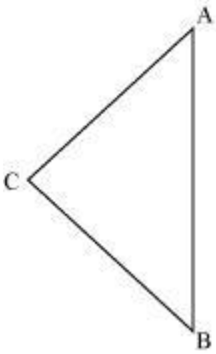


Given that $\triangle$ ABC is an isosceles triangle.
Therefore AC = CB
Applying Pythagoras theorem in ABC (i.e. right angled at point C)

$AC^2 + CB^2 = AB^2$

$2AC^2 = AB^2 \qquad (as\, AC = CB)$

Solution 5



Given that

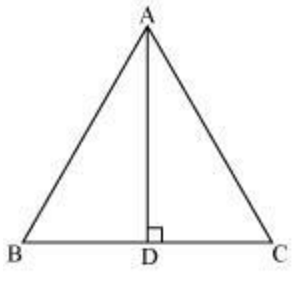
$AB^2 = 2AC^2$

$AB^2 = AC^2 + AC^2$

$AB^2 = AC^2 + BC^2 \quad (as\ AC = BC)$

Since triangle is satisfying the pythagoras theorem
Therefore, given triangle is a right angled triangle.

Solution 6



Let AD be the altitude in given equilateral triangle $\triangle$ ABC.
We know that altitude bisects the opposite side.
So, BD = DC = a

In ΔADB,

$\angle ADB = 90°$

Now applying pythagoras theorem

$AD^2 + DB^2 = AB^2$

$AD^2 + a^2 = (2a)^2$

$AD^2 + a^2 = 4a^2$

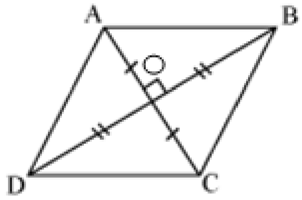
$AD^2 = 3a^2$

$AD = a\sqrt{3}$

Since in an equilateral triangle, all the altitudes are equal in length.

So, length of each altitude will be $\sqrt{3}a$

Solution 7



In $\triangle$ AOB, $\triangle$ BOC, $\triangle$ COD, $\triangle$ AOD

Applying Pythagoras theorem

$AB^2 = AO^2 + OB^2$

$BC^2 = BO^2 + OC^2$

$CD^2 = CO^2 + OD^2$

$AD^2 = AO^2 + OD^2$

Adding all these equations,

$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$

$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$ (diagonals bisect each other.)

$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$

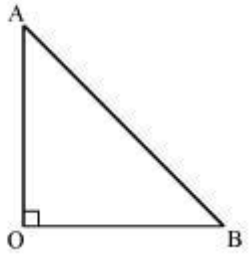
$= (AC)^2 + (BD)^2$

Solution 8

i. In ΔAOF
Applying Pythagoras theorem
$OA^2 = OF^2 + AF^2$
Similarly in ΔBOD
$OB^2 = OD^2 + BD^2$
Similarly in ΔCOE
$OC^2 = OE^2 + EC^2$
adding these equations
$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$
$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$

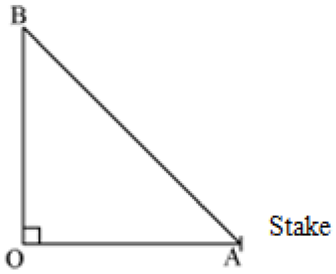
ii. As from above result
$AF^2 + BD^2 + EC^2 = (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$
Therefore $AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$

Solution 9



Let OA be the wall and AB be the ladder
Therefore by Pythagoras theorem,
$AB^2 = OA^2 + BO^2$
$10^2 = 8^2 + OB^2$
$100 = 64 + OB^2$
$OB^2 = 36$
$OB = 6$
Therefore distance of foot of ladder from base of the wall = 6 m

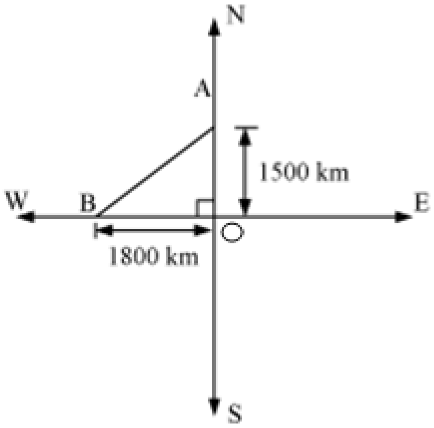
Solution 10



Let OB be the pole and AB be the wire.
Therefore by Pythagoras theorem,

$AB^2 = OB^2 + OA^2$
$24^2 = 18^2 + OA^2$
$OA^2 = 576 - 324$
$OA = \sqrt{252} = \sqrt{6 \times 6 \times 7} = 6\sqrt{7}$
Therefore distance from base = $6\sqrt{7}$ m

Solution 11



Distance traveled by the plane flying towards north in $1\frac{1}{2}$hrs

$= 1{,}000 \times 1\frac{1}{2} = 1{,}500$ km

Similarly, distance traveled by the plane flying towards west in $1\frac{1}{2}$hrs

$= 1{,}200 \times 1\frac{1}{2} = 1{,}800$ km

Let these distances are represented by OA and OB respectively.
Now applying Pythagoras theorem

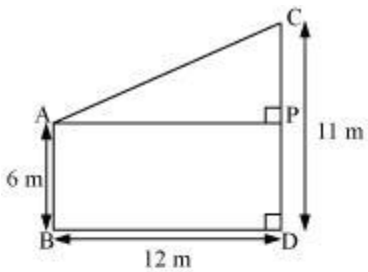
Distance between these planes after $1\frac{1}{2}$hrs AB $= \sqrt{OA^2 + OB^2}$

$= \sqrt{(1{,}500)^2 + (1{,}800)^2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9 \times 610000} = 300\sqrt{61}$

So, distance between these planes will be $300\sqrt{61}$ km. after $1\frac{1}{2}$hrs.

Solution 12



Let CD and AB be the poles of height 11 and 6 m.
Therefore CP = 11 - 6 = 5 m
From the figure we may observe that AP = 12m
In Δ APC, by applying Pythagoras theorem

$AP^2 + PC^2 = AC^2$

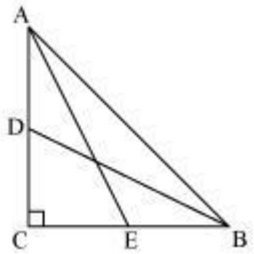
$12^2 + 5^2 = AC^2$

$AC^2 = 144 + 25 = 169$

$AC = 13$

Therefore distance between their tops = 13 m.

Solution 13



In $\triangle$ ACE,

$AC^2 + CE^2 = AE^2 \qquad (i)$

In ΔBCD,

$BC^2 + CD^2 = BD^2 \qquad (ii)$

Adding (1) and (2)

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \qquad (iii)$

or

$CD^2 + CE^2 + AC^2 + BC^2 = AE^2 + BD^2$

In ΔCDE

$DE^2 = CD^2 + CE^2$

In ΔABC

$AB^2 = AC^2 + CB^2$

Putting the values in equation (iii)

$DE^2 + AB^2 = AE^2 + BD^2$

Solution 14

Given that $3DC = DB$

$$DC = \frac{BC}{4} \qquad [DB:DC = 3:1] \qquad (1)$$

and

$$DB = \frac{3BC}{4} \qquad (2)$$

In ΔACD

$AC^2 = AD^2 + DC^2$

$AD^2 = AC^2 - DC^2 \qquad (3)$

In ΔABD

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2 \qquad (4)$

From equation (1) and (2)

Therefore $AC^2 - DC^2 = AB^2 - DB^2$

since given that $3DC = DB$

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2 \qquad \text{from (1) and (2)}$$

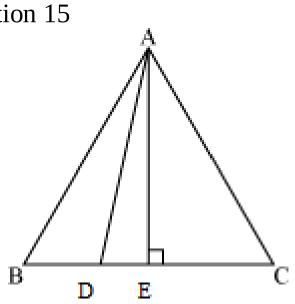
$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$16AC^2 - BC^2 = 16AB^2 - 9BC^2$

$16AB^2 - 16AC^2 = 8BC^2$

$2AB^2 = 2AC^2 + BC^2$

Solution 15



Let side of equilateral triangle be a. And AE be the altitude of ΔABC

So, $BE = EC = \frac{BC}{2} = \frac{a}{2}$

And, $AE = \frac{a\sqrt{3}}{2}$

Given that $BD = \frac{1}{3}BC = \frac{a}{3}$

So, $DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Now, in ΔADE by applying Pythagoras theorem

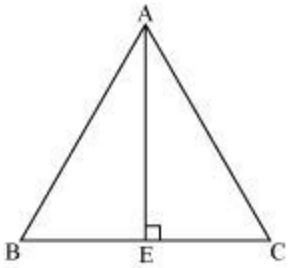
$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right) = \frac{28a^2}{36}$$

Or, $9\ AD^2 = 7\ AB^2$

Solution 16



Let side of equilateral triangle be a. And AE be the altitude of $\triangle$ ABC

So, $BE = EC = \frac{BC}{2} = \frac{a}{2}$

And, $AE = \frac{a\sqrt{3}}{2}$

Now, in $\triangle$ ABE by applying Pythagoras theorem

$AB^2 = AE^2 + BE^2$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

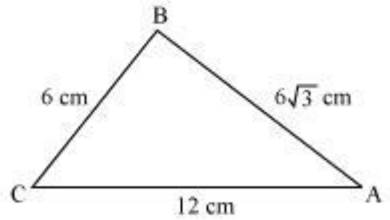
$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$4AE^2 = 3a^2$.

Or, $4AE^2 = 3 \times (\text{square of one side})^2$.

Solution 17



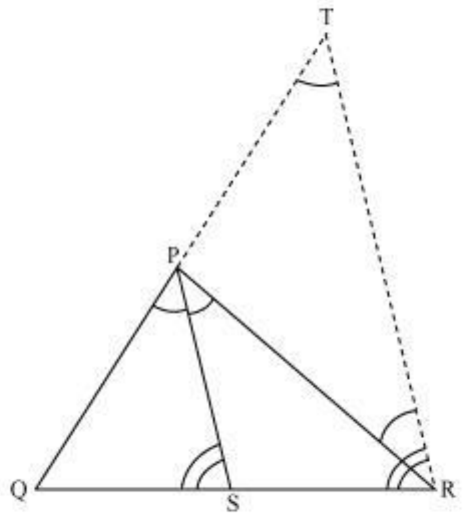
Given that AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm
We may observe that
$AB^2 = 108$
$AC^2 = 144$
And $BC^2 = 36$
$AB^2 + BC^2 = AC^2$

Thus the given triangle $\triangle$ ABC is satisfying Pythagoras theorem

Therefore triangle is a right angled triangle right angled at B
Therefore $\angle B = 90°$.

Hence, (c).

Chapter 6 - Triangles Exercise Ex. 6.6
Solution 1



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.
Given that PS is angle bisector of $\angle$ QPR.
$\angle$ QPS = $\angle$ SPR (1)
$\angle$ SPR = $\angle$ PRT (As PS || TR) (2)
$\angle$ QPS = $\angle$ QTR (As PS || TR) (3)

Using these equations we may find
$\angle$PRT = $\angle$ QTR from (2) and (3)
So, PT = PR (Since $\triangle$ PTR is isosceles triangle)
Now in $\triangle$ QPS and $\triangle$ QTR
$\angle$ QSP = $\angle$QRT (As PS || TR)
$\angle$ QPS = $\angle$ QTR (As PS || TR)
$\angle$ Q is common

$\triangle$QPS ~ $\triangle$QTR

So, $\frac{QS}{QR} = \frac{QP}{QT}$

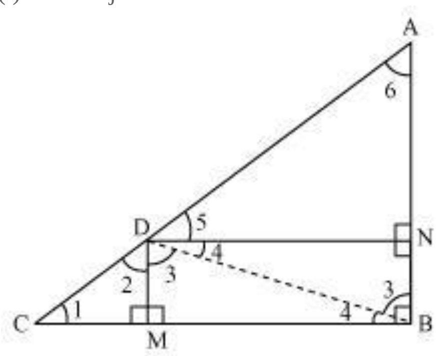
$\frac{QS}{QR - QS} = \frac{QP}{QT - QP}$

$\frac{QS}{SR} = \frac{QP}{PT}$

$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$

Solution 2

(i). Let us join DB.



DN || CB
DM || AB
So, DN = MB
DM = NB

The condition to be proved is the case when DNBM is a square or D is the midpoint of side AC.

Then $\angle$CDB = $\angle$ADB = 90°

$\angle$2 + $\angle$3 = 90° (1)

In $\triangle$CDM

$\angle$1 + $\angle$2 + $\angle$DMC = 180°

$\angle$1 + $\angle$2 = 90° (2)

In $\triangle$DMB

$\angle$3 + $\angle$DMB + $\angle$4 = 180°

$\angle$3 + $\angle$4 = 90° (3)

From equation (1) and (2)

$\angle$1 = $\angle$3

From equation (1) and (3)

$\angle$2 = $\angle$4

$\triangle$BDM ~ $\triangle$DCM

$\frac{BM}{DM} = \frac{DM}{MC}$

$\frac{DN}{DM} = \frac{DM}{MC}$

$DM^2 = DN \times MC$

(ii). Similarly in $\triangle$DBN

$\angle$4 + $\angle$3 = 90° (4)

In $\triangle$DAN

$\angle$5 + $\angle$6 = 90° (5)

In $\triangle$DAB

$\angle 4 + \angle 5 = 90°$ (6)
From equation (4) and (6)
$\angle 3 = \angle 5$
From equation (5) and (6)
$\angle 4 = \angle 6$
$\triangle DNA \sim \triangle BND$
$$\frac{AN}{DN} = \frac{DN}{NB}$$
$DN^2 = AN \times NB$
$DN^2 = AN \times DM$ (as NB = DM)

Solution 3
In $\triangle ADB$ applying Pythagoras theorem
$AB^2 = AD^2 + DB^2$ (1)
In $\triangle ACD$ applying Pythagoras theorem
$AC^2 = AD^2 + DC^2$
$AC^2 = AD^2 + (DB + BC)^2$
$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$
Now using equation (1)
$AC^2 = AB^2 + BC^2 + 2BC . BD$

Solution 4
In $\triangle ADB$ applying Pythagoras theorem
$AD^2 + DB^2 = AB^2$
$AD^2 = AB^2 - DB^2$ (1)
In $\triangle ADC$ applying Pythagoras theorem
$AD^2 + DC^2 = AC^2$ (2)
Now using equation (1)
$AB^2 - BD^2 + DC^2 = AC^2$
$AB^2 - BD^2 + (BC - BD)^2 = AC^2$
$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC. BD$
$= AB^2 + BC^2 - 2BC. BD$

Solution 5
(i). In $\triangle AMD$
$AM^2 + MD^2 = AD^2$ (1)
In $\triangle AMC$
$AM^2 + MC^2 = AC^2$ (2)
$AM^2 + (MD + DC)^2 = AC^2$
$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$
Using equation (1) we may get
$AD^2 + DC^2 + 2MD.DC = AC^2$
Now using the result $DC = \frac{BC}{2}$
$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$
$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii). In $\triangle ABM$ applying Pythagoras theorem
$AB^2 = AM^2 + MB^2$
$= (AD^2 - DM^2) + MB^2$
$= (AD^2 - DM^2) + (BD - MD)^2$
$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD.MD$
$= AD^2 + BD^2 - 2BD.MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii). In $\triangle$ AMB

$AM^2 + MB^2 = AB^2$ (1)

In $\triangle$ AMC

$AM^2 + MC^2 = AC^2$ (2)

Adding equation (1) and (2)

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

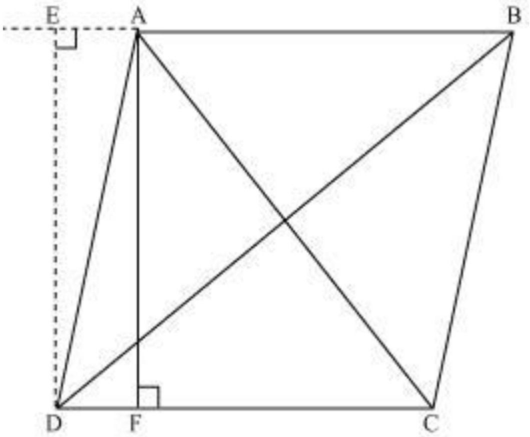
$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Solution 6



Let $\square$ ABCD be a parallelogram

Let us draw perpendicular DE on extended side AB and AF on side DC.

In $\triangle$ DEA

$DE^2 + EA^2 = DA^2$ (i)

In $\triangle$ DEB

$DE^2 + EB^2 = DB^2$

$DE^2 + (EA + AB)^2 = DB^2$

$(DE^2 + EA^2) + AB^2 + 2EA.\ AB = DB^2$

$DA^2 + AB^2 + 2EA.AB = DB^2$ (ii)

In $\triangle$ ADF

$AD^2 = AF^2 + FD^2$

In $\triangle$ AFC

$AC^2 = AF^2 + FC^2$

$= AF^2 + (DC - FD)^2$

$= AF^2 + DC^2 + FD^2 - 2DC - FD$

$= (AF^2 + FD^2) + DC^2 - 2DC\ .\ FD$

$AC^2 = AD^2 + DC^2 - 2DC\ \ FD$ (iii)

Since ABCD is a parallelogram

AB = CD (iii)

And BC = AD (iv)

In $\triangle$ DEA and $\triangle$ ADF

$\angle$ DEA = $\angle$ AFD

$\angle$ EAD = $\angle$ FDA (EA || DF)

$\angle$ EDA = $\angle$ FAD (AF || ED)

AD is common in both triangles.

Since respective angles are same and respective sides are same

$\triangle$ DEA $\cong$ $\triangle$ AFD

So EA = DF (v)

Adding equation (ii) and (iii)

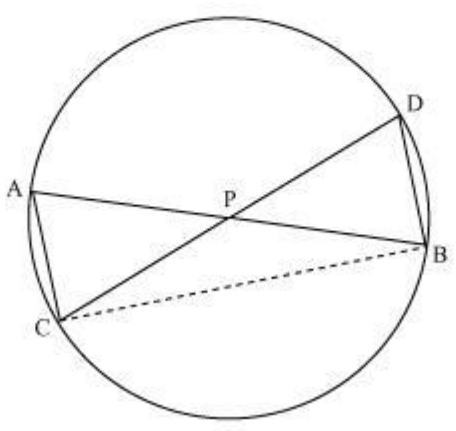
$DA^2 + AB^2 + 2EA.AB + AD^2 + DC^2 - 2DC.FD = DB^2 + AC^2$

$DA^2 + AB^2 + AD^2 + DC^2 + 2EA.AB - 2DC.FD = DB^2 + AC^2$

$BC^2 + AB^2 + AD^2 + DC^2 + 2EA.AB - 2AB.EA = DB^2 + AC^2$

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Solution 7



Let us join CB

(i) In $\triangle$ APC and $\triangle$ DPB

$\angle$ APC = $\angle$ DPB {Vertically opposite angles}

$\angle$ CAP = $\angle$ BDP {Angles in same segment for chord CB}

$\triangle$ APC ~ $\triangle$ DPB {BY AA similarly criterion}

(ii) We know that corresponding sides of similar triangles are proportional

$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$

$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$

$\therefore AP.\ PB = PC.\ DP$

Solution 8

(i) In $\triangle$ PAC and $\triangle$ PDB

$\angle$ P = $\angle$ P (common)

$\angle$ PAC = $\angle$ PDB (exterior angle of a cyclic quadrilateral is equal to opposite interior angle)

$\angle$ PCA = $\angle$ PBD

$\triangle$ PAC ~ $\triangle$ PDB

(ii) We know that corresponding sides of similar triangles are proportional.

$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB}$

$\Rightarrow \frac{PA}{PD} = \frac{PC}{PB}$

$\therefore PA.PB = PC.PD$

Solution 9

In ΔDBA and ΔDCA

$\frac{BD}{CD} = \frac{AB}{AC}$ (given)

AD = AD (common)

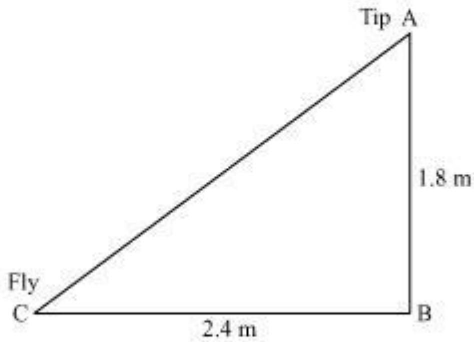
So, $\Delta DBA \sim \Delta DCA$ (By SSS)

Now, corresponding angles of similar triangles will be equal.

$\angle BAD = \angle CAD$

AD is angle bisector of $\angle BAC$

Solution 10



Let AB be the height of tip of fishing rod from water surface. Let BC be the horizontal distance of fly from the tip of fishing rod.

Then, AC is the length of string.

AC can be found by applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

$AC^2 = (1.8)^2 + (2.4)^2$

$AC^2 = 3.24 + 5.76$

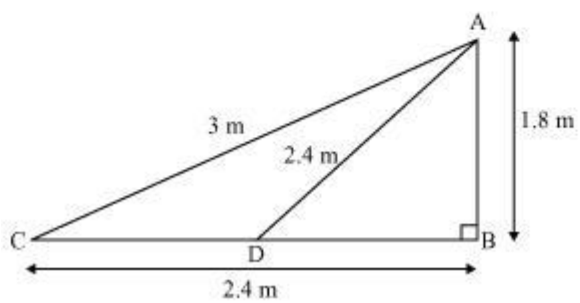
$AC^2 = 9.00$

$AC = \sqrt{9} = 3$

Thus, length of string out is 3 m.

Now, she pulls string at rate of 5 cm per second.

So, string pulled in 12 seconds = 12 x 5 = 60 cm = 0.6 m



Let after 12 second Fly be at point D.

Length of string out after 12 second is AD

AD = AC - string pulled by Nazima in 12 seconds

= 3.00 - 0.6

= 2.4

In ΔADB

$AB^2 + BD^2 = AD^2$
$(1.8)^2 + BD^2 = (2.4)^2$
$BD^2 = 5.76 - 3.24 = 2.52$
$BD = 1.587$
Horizontal distance of fly = BD + 1.2
= 1.587 + 1.2
= 2.787
= 2.79 m