RD SHARMA Solutions for Class 12-science Maths Chapter 32 - Mean and variance of a random variable

Chapter 32 - Mean and variance of a random variable

Exercise Ex. 32.1

Question 1

Which of the following distributions of probabilities of a random variable X are the probability distributions?

(i)

X :	3	2	1	0	−1
$P(x)$:	0.3	0.2	0.4	0.1	0.05

(ii)

X :	0	1	2
$P(x)$:	0.6	0.4	0.2

(iii)

X :	0	1	2	3	4
$P(x)$:	0.1	0.5	0.2	0.1	0.1

(iv)

X :	0	1	2	3
$P(x)$:	0.3	0.2	0.4	0.1

Solution 1

(i) Here

$$X: \quad 3 \quad 2 \quad 1 \quad 0 \quad -1$$
$$p(x): \quad 0.3 \quad 0.2 \quad 0.4 \quad 0.1 \quad 0.05$$

$$p(x=3)+p(x=2)+p(x=1)+p(x=0)+p(x=-1)$$
$$=0.3+0.2+0.4+0.1+0.05$$
$$=1.05 \neq 1$$

So, the given distribution of probabilities is not a probability distribution.

(ii) Here

$$X: \quad 0 \quad 1 \quad 2$$
$$p(x): \quad 0.6 \quad 0.4 \quad 0.2$$

$$p(x=0)+p(x=1)+p(x=2)$$
$$=0.6+0.4+0.2$$
$$=1.2 \neq 1$$

So, the given distribution of probabilities is not a probability distribution.

(iii) Here

$$X: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$
$$p(x): \quad 0.1 \quad 0.5 \quad 0.2 \quad 0.1 \quad 0.1$$

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)$$
$$=0.1+0.5+0.2+0.1+0.1$$
$$=1$$

So, the given distribution of probabilities is a probability distribution.

(iv) Here

$$X: \quad 0 \quad 1 \quad 2 \quad 3$$
$$p(x): \quad 0.3 \quad 0.2 \quad 0.4 \quad 0.1$$

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)$$
$$=0.3+0.2+0.4+0.1$$
$$=1$$

So, the given distribution of probabilities is a probability distribution.

Question 2

A random variable X has the following probability distribution

Values of x :	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	k

Find the value of k.

Solution 2

Here

x :	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	k

We know that,

$$p(-2)+p(-1)+p(0)+p(1)+p(2)+p(3)=1$$

$\Rightarrow \quad 0.1+k+0.2+2k+0.3+k=1$

$\Rightarrow \quad 4k+0.6=1$

$\Rightarrow \quad 4k=1-0.6$

$\Rightarrow \quad 4k=0.4$

$\Rightarrow \quad k=\frac{0.4}{4}$

$\Rightarrow \quad k=\frac{1}{10}$

$\Rightarrow \quad k=0.1$

Question 3

A random variable X has the following probability distribution

Values of x :	0	1	2	3	4	5	6	7	8
$P(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Determine:

(i) The value of a

(ii) $P(X<3)$, $P(x \geq 3)$, $P(0<X<5)$.

Solution 3

Here

X:	0	1	2	3	4	5	6	7	8
$P(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Since $\sum p(x) = 1$

$$P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) = 1$$

$\Rightarrow \quad = a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$\Rightarrow \quad 81a = 1$

$\Rightarrow \quad a = \frac{1}{81}$

(ii) $P(x < 3) = P(0) + P(1) + P(2)$

$= a + 3a + 5a$

$= 9a$

$= 9\left(\frac{1}{81}\right)$

$\therefore P(x < 3) = \frac{1}{9}$

$P(x \geq 3) = 1 - P(x < 3) = 1 - \frac{1}{9} = \frac{8}{9}$

$P(0 < x < 5) = P(1) + P(2) + P(3) + P(4)$

$= 3a + 5a + 7a + 9a$

$= 24a$

$= 24\left(\frac{1}{81}\right)$

$\therefore P(0 < x < 5) = \frac{8}{27}$

Question 4

The probability distribution function of a random variable X is given by

x_i:	0	1	2
p_i:	$3c^2$	$4c - 10c^2$	$5c - 1$

where $c > 0$

Find: (i) c (ii) $P(X < 2)$ (iii) $P(1 < X \leq 2)$

Solution 4

Here:-

x:	0	1	2
$P(x)$:	$3c^2$	$4c-10c^2$	$5c-1$

Where $c > 0$

(i) since $\sum P(x) = 1$

$\Rightarrow \quad P(0)+P(1)+P(2) = 1$

$\Rightarrow \quad 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$\Rightarrow \quad 3c^3 - 10c^2 + 9c - 2 = 0$

$\Rightarrow \quad 3c^3 - 3c^2 - 7c^2 + 7c + 2c - 2 = 0$

$\Rightarrow \quad 3c^2(c-1) - 7c(c-1) + 2(c-1) = 0$

$\Rightarrow \quad (c-1)(3c^2 - 7c + 2) = 0$

$\Rightarrow \quad (c-1)(3c^2 - 6c - c + 2) = 0$

$\Rightarrow \quad (c-1)(3c(c-2) - 1(c-2)) = 0$

$(c-1)(3c-1)(c-2) = 0$

$c = 1, \quad c = 2, \quad c = \frac{1}{3}$

Only $c = \frac{1}{3}$ is possible. Because if c = 1, or c = 2 then P(2) will become negative.

(ii) $P(x<2) = P(0)+P(1)$

$= 3c^3 + 4c - 10c^2$

$= 3\left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2$

$= \frac{3}{27} + \frac{4}{3} - \frac{10}{9}$

$= \frac{1}{9} + \frac{4}{3} - \frac{10}{9}$

$= \frac{3}{9}$

$\therefore P(x<2) = \frac{1}{3}$

(iii) $P(1 < x \leq 2) = P(2)$

$= 5c - 1$

$= 5\left(\frac{1}{3}\right) - 1$

$\therefore P(1 < x \leq 2) = \frac{2}{3}$

Question 5

Let X be a random variable which assumes value x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.Find the probability distribution of X.

Solution 5

Here,

$$2P(x_1) = 3P(x_2) = P(x_3) = 5P(x_4)$$

Let $P(x_3) = a$

$$2P(x_1) = P(x_3) \quad \Rightarrow \quad P(x_1) = \frac{a}{2}$$

$$3P(x_2) = P(x_3) \quad \Rightarrow \quad P(x_2) = \frac{a}{3}$$

$$5P(x_4) = P(x_3) \quad \Rightarrow \quad P(x_4) = \frac{a}{5}$$

Since $\quad P(x_1) + P(x_2) + P(x_3) + P(x_4) = 1$

$$\Rightarrow \quad \frac{a}{2} + \frac{a}{3} + \frac{a}{1} + \frac{a}{5} = 1$$

$$\Rightarrow \quad \frac{15a + 10a + 30a + 6a}{30} = 1$$

$$\Rightarrow \quad 61a = 30$$

$$\Rightarrow \quad a = \frac{30}{61}$$

so,

X :	x_1	x_2	x_3	x_4
$P(x)$:	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

Question 6

A random variable X takes the values 0, 1, 2 and 3 such that:

$P(X = 0) = P(X > 0) = P(X < 0)$; $P(X = -3) = P(X = -2) = P(X = -1)$;

$P(X = 1) = P(X = 2) = P(X = 3)$.Obtain the probability distirbution of X.

Solution 6

Here,

$$P(x=0)=P(x>0)=P(x<0)$$

Let $P(x=0)=k$

$\Rightarrow \quad P(x>0)=k=P(x<0)$

Since $\sum P(x)=1$

$\Rightarrow \quad P(x<0)+P(x=0)+P(x>0)=1$

$\Rightarrow \quad k+k+k=1$

$\Rightarrow \quad 3k=1$

$\Rightarrow \quad k=\frac{1}{3}$

So, $P(x<0)=$

$P(x=-1)+P(x=-2)+P(x=-3)=\frac{1}{3}$

$3P(x=-1)=\frac{1}{3}, \qquad [\because \quad P(x=-1)=P(x=-2)=P(x=-3)]$

$P(x=-1)=\frac{1}{9}$

$\Rightarrow \quad P(x=-1)=P(x=-2)=P(x=-3)=\frac{1}{9} ----(i)$

$\Rightarrow \quad P(x=0)=\frac{1}{3} ---------(ii)$

and

$P(x>0)=k$

$P(x=1)+P(x=2)+P(x=3)=\frac{1}{3}$

$3P(x=1)=\frac{1}{3}, \qquad [\because \quad P(x=1)=P(x=2)=P(x=3)]$

$\Rightarrow \quad P(x=1)=\frac{1}{9}$

$\Rightarrow \quad P(x=1)=P(x=2)=P(x=3)=\frac{1}{9} ---(iii)$

From equation (i), (ii), (iii),

x :	-3	-2	-1	0	1	2	3
$P(x)$:	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Question 7

Two cards are drawn from a well shuffled pack of 52 cards.Find the probability distribution of the number of aces.

Solution 7

Let X denote number of aces in a sample of 2 cards drawn.
There are four aces in a pack of 52 cards.
So, X can have values 0, 1, 2

Now,

$$P(x=0)=\frac{48C_2}{52C_2}=\frac{48\times 47}{2}\times\frac{2}{52\times 51}=\frac{188}{221}$$

$$P(x=1)=\frac{48C_1\times 4C_1}{52C_2}=\frac{48\times 4\times 2}{52\times 51}=\frac{32}{221}$$

$$P(x=2)=\frac{4C_2}{52C_2}=\frac{4\times 3}{2}\times\frac{2}{52\times 51}=\frac{1}{221}$$

So,

x	0	1	2
$P(x)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Question 8

Find the probability distribution of the number of heads, when three coins are tossed.

Solution 8

Probability of getting a Head in one throw of a coin $= \frac{1}{2}$

$$P(H) = \frac{1}{2}$$

$$\Rightarrow \quad P(T) = 1 - \frac{1}{2}$$

$$\Rightarrow \quad P(T) = \frac{1}{2}$$

Let X denote the number of heads obtained is 3 throws of a coin .

Then $X = 0, 1, 2, 3$

Now,

$$P(x=0) = P(T)P(T)P(T) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(x=1) = P(H)P(T)P(T) + P(T)P(H)P(T) + P(T)P(T)P(H)$$

$$= \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$P(x=1) = \frac{3}{8}$$

$$P(x=2) = P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H)$$

$$= \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$P(x=2) = \frac{3}{8}$$

$$P(x=3) = P(H)P(H)P(H)$$

$$= \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$= \frac{1}{8}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Question 9

Four cards are drawn simultaneously from a well shuffled pack of 52 playing cards. Find the probability distribution of the number of aces.

Solution 9

Let x denote number of aces drawn out of 4 cards drawn.
There are four ace aces in a pack of 52.

So, $X = 0, 1, 2, 3, 4$

Now,

$$P(x = 0) = \frac{48C_4}{52C_4}$$

$$P(x = 1) = \frac{48C_3 \times 4C_1}{52C_4}$$

$$P(x = 2) = \frac{48C_2 \times 4C_2}{52C_4}$$

$$P(x = 3) = \frac{48C_1 \times 4C_3}{52C_4}$$

$$P(x = 4) = \frac{4C_4}{52C_4}$$

So,

Required probability distribution is

x :	0	1	2	3	4
$P(x)$:	$\frac{48C_4}{52C_4}$	$\frac{48C_3 \times 4C_1}{52C_4}$	$\frac{48C_2 \times 4C_2}{52C_4}$	$\frac{48C_1 \times 4C_3}{52C_4}$	$\frac{4C_4}{52C_4}$

Question 10

A bag contains 4 red and 6 black balls. Three balls are drawn at random. Find the probability distribution of the number of red balls.

Solution 10

A bag has 4 red and 6 black balls. Three balls are drawn.

Let X denote number of red balls out of 3 drawn.

Then X = 0,1,2,3.

So,

$$P\left(\text{no red balls}\right) = P\left(X = 0\right) = \frac{6C_3}{10C_3} = \frac{6\times5\times4}{3\times2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{6}$$

$$P\left(\text{one red balls}\right) = P\left(X = 1\right) = \frac{4C_1\times6C_2}{10C_3} = \frac{4\times6\times5}{2}\times\frac{3\times2}{10\times9\times8} = \frac{1}{2}$$

$$P\left(\text{two red balls}\right) = P\left(X = 1\right) = \frac{4C_2\times6C_1}{10C_3} = \frac{4\times3\times6}{2}\times\frac{3\times2}{10\times9\times8} = \frac{3}{10}$$

$$P\left(\text{all three red}\right) = P\left(X = 3\right) = \frac{4C_3}{10C_3} = \frac{4\times3\times2}{10\times9\times8} = \frac{1}{30}$$

The required probability distribution is

X :	0	1	2	3
$P(x)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Question 11

Five defective mangoes are accidently mixed with 15 good ones.Four mangoes are drawn at random from this lot. Find the probability distribution of the number of defective mangoes.

Solution 11

Here 5 defective and 15 non-defective mangoes.Let X denote the defective mangoes drawn out of 4 mangoes drawn.

So, $X = 0,1,2,3,4$.

$$P\left(X = 0\right) = \frac{15C_4}{20C_4}$$
$$= \frac{15\times14\times13\times12}{4\times3\times2\times1}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{91}{323}$$

$$P\left(X = 1\right) = \frac{5C_1\times15C_3}{20C_4}$$
$$= \frac{5\times15\times14\times13}{3\times2\times1}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{455}{969}$$

$$P\left(X = 2\right) = \frac{5C_2\times15C_2}{20C_4}$$
$$= \frac{5\times4}{2}\times\frac{15\times14}{2}\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{70}{323}$$

$$P\left(X = 3\right) = \frac{5C_3\times15C_1}{20C_4}$$
$$= \frac{5\times4}{2}\times15\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{10}{323}$$

$$P\left(X = 4\right) = \frac{5C_4}{20C_4}$$
$$= 5\times\frac{4\times3\times2\times1}{20\times19\times18\times17}$$
$$= \frac{1}{969}$$

So, required probability distribution is

x :	0	1	2	3	4
$P\left(x\right)$:	$\frac{91}{323}$	$\frac{455}{969}$	$\frac{70}{323}$	$\frac{10}{323}$	$\frac{1}{969}$

Question 12

Two dice are thrown together and the number appearing on them noted. X denotes the sum of the two numbers. Assuming that all the 36 outcomes are equally likely, what is the probability distribution of X.

Solution 12

Here, X denote the number of sum of two number or two dice thrown together

So, $X = 2,3,4,5,6,7,8,9,10,11,12.$

So,

$P(x=2)=\frac{1}{36}$ [Possible pairs: $(1,1)$]

$P(x=3)=\frac{2}{36}=\frac{1}{18}$ [Possible pairs: $(1,2),(2,1)$]

$P(x=4)=\frac{3}{36}=\frac{1}{12}$ [Possible pairs: $(1,3),(2,2),(3,1)$]

$P(x=5)=\frac{4}{36}=\frac{1}{9}$ [Possible pairs: $(1,4),(2,3),(3,2),(4,1)$]

$P(x=6)=\frac{5}{36}$ [Possible pairs: $(1,5),(2,4),(3,3),(4,2),(5,1)$]

$P(x=7)=\frac{6}{36}=\frac{1}{6}$ [Possible pairs: $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$]

$P(x=8)=\frac{5}{36}$ [Possible pairs: $(2,6),(3,5),(4,4),(5,3),(6,2)$]

$P(x=9)=\frac{4}{36}=\frac{1}{9}$ [Possible pairs: $(3,6),(4,5),(5,4),(6,3)$]

$P(x=10)=\frac{3}{36}=\frac{1}{12}$ [Possible pairs: $(4,6),(5,5),(6,4)$]

$P(x=11)=\frac{2}{36}=\frac{1}{18}$ [Possible pairs: $(5,6),(6,5)$]

$P(x=12)=\frac{1}{36}$ [Possible pairs: $(6,6)$]

So, required probability distribution is

x :	2	3	4	5	6	7	8	9	10	11	12
$P(x)$:	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Question 13

A class has 15 students whose ages are 14, 17, 15, 14, 21, 19, 20, 16, 18, 17,20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being selected and the age X of the selected student is recorded. What is the probability distribution of the random variable X?

Solution 13

There are 15 students in the class. Each student has the same chance to be chosen. Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X = 14) = \frac{2}{15}$, $P(X = 15) = \frac{1}{15}$, $P(X = 16) = \frac{2}{15}$, $P(X = 16) = \frac{3}{15}$,

$P(X = 18) = \frac{1}{15}$, $P(X = 19) = \frac{2}{15}$, $P(X = 20) = \frac{3}{15}$, $P(X = 21) = \frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Question 14

Five defective bolts are accidently mixed with twenty good ones.If four bolts are drawn at the random from this lot,find the probability of the number of defective bolts.

Solution 14

Here 5 defective and 20 non-defective bolts. Let X denote the number of defective bolts drawn out of 4 bolts drawn.So, X can have values 0,1,2,3,4.

$$P(X=0)=\frac{20C_4}{25C_4}$$
$$=\frac{20\times19\times18\times17}{25\times24\times23\times22}$$
$$=\frac{969}{2530}$$

$$P(X=1)=\frac{5C_1\times20C_3}{25C_4}$$
$$=\frac{5\times20\times19\times18}{3\times2\times1}\times\frac{4\times3\times2}{25\times24\times23\times22}$$
$$=\frac{114}{253}$$

$$P(X=2)=\frac{5C_2\times20C_2}{25C_4}$$
$$=\frac{5\times4}{2}\times\frac{20\times19}{2}\times\frac{4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{38}{253}$$

$$P(X=3)=\frac{5C_3\times20C_1}{25C_4}$$
$$=\frac{5\times4}{2}\times\frac{20\times4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{4}{253}$$

$$P(X=4)=\frac{5C_4}{25C_4}$$
$$=5\times\frac{4\times3\times2\times1}{25\times24\times23\times22}$$
$$=\frac{1}{2530}$$

So, required probability distribution is

X :	0	1	2	3	4
$P(X)$:	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$

Question 15

Two cards are drawn successively with replacement from well shuffled pack of 52 cards.Find the probability distribution of number of aces.

Solution 15

Two cards are drawn successively with replacement from a pack of 52 cards.

Let X be the number of aces obtained. Then X = 0,1,2.

$$P\left(X=0\right) = P\left(\overline{A}_1\right) \times P\left(\overline{A}_2\right)$$
$$= \frac{48}{52} \times \frac{48}{52}$$
$$= \frac{144}{169}$$

$$P\left(X=1\right) = P\left(A_1\right) P\left(\overline{A}_2\right) + P\left(\overline{A}_1\right) P\left(A_2\right)$$
$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}$$
$$= \frac{24}{169}$$

$$P\left(X=2\right) = P\left(A_1\right) P\left(A_2\right)$$
$$= \frac{4}{52} \times \frac{4}{52}$$
$$= \frac{1}{169}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Question 16

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards.Find the probability distribution of number of kings.

Solution 16

Two cards are drawn successively with replcement from a pack of 52 cards.Let X denote the number of kings drawn out of 2 cards.
So, $X = 0,1,2$.

$$P(X = 0) = P(\overline{K}_1) \times P(\overline{K}_2)$$
$$= \frac{48}{52} \times \frac{48}{52}$$
$$= \frac{144}{169}$$

$$P(X = 1) = P(K_1)P(\overline{K}_2) + P(\overline{K}_1)P(K_2)$$
$$= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}$$
$$= \frac{24}{169}$$

$$P(X = 2) = P(K_1)P(K_2)$$
$$= \frac{4}{52} \times \frac{4}{52}$$
$$= \frac{1}{169}$$

So, required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Question 17

Two cards are drawn successively without replacement from a well shuffled pack of 52 cards.Find the probability distribution of number of aces.

Solution 17

Two cards are drawn without replcement from a pack of 52 cards. Let X denote the number of aces drawn from pack out of 2 cards.
So, $X = 0,1,2.$

$$P(X = 0) = \frac{48C_2}{52C_2}$$
$$= \frac{48 \times 47}{2} \times \frac{2 \times 1}{52 \times 51}$$
$$= \frac{188}{221}$$

$$P(X = 1) = \frac{4C_1 \times 48C_1}{52C_2}$$
$$= \frac{4 \times 48 \times 2}{52 \times 51}$$
$$= \frac{32}{221}$$

$$P(X = 2) = \frac{4C_2}{52C_2}$$
$$= \frac{4 \times 3}{2} \times \frac{2}{52 \times 51}$$
$$= \frac{1}{221}$$

So, required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Question 18

Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement, from a bag containing 4 white and 6 red balls.

Solution 18

Given bag have 4 white and 6 red balls.Let X denote the number of white balls out of 3 balls drawn without replcement,
So, $X = 0,1,2,3.$

$$P\left(\text{No white ball}\right) = \frac{6C_3}{10C_3}$$
$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{3 \times 2}{10 \times 9 \times 8}$$
$$= \frac{5}{30}$$
$$P\left(\text{One white ball}\right) = \frac{4C_1 \times 6C_2}{10C_3}$$
$$= \frac{4 \times 6 \times 5}{2} \times \frac{3 \times 2}{10 \times 9 \times 8}$$
$$= \frac{15}{30}$$
$$P\left(\text{Two white balls}\right) = \frac{4C_2 \times 6C_1}{10C_3}$$
$$= \frac{4 \times 3}{2} \times \frac{6 \times 3 \times 2}{10 \times 9 \times 8}$$
$$= \frac{9}{30}$$
$$P\left(\text{Three white balls}\right) = \frac{4C_3}{10C_3}$$
$$= \frac{4 \times 3 \times 2 \times 1}{10 \times 9 \times 8}$$
$$= \frac{1}{30}$$
So,
Required probability distribution is

x :	0	1	2	3
$P\left(x\right)$:	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

Question 19

Find the probability distribution of Y in two throws of two dice, where Y represents the numbers of time a total of 9 appears.

Solution 19

Since total is 9 when dice has $(3,6)(4,5)(5,4)(6,3)$

$\therefore P(\text{A total of 9 appears}) = p(A) = \frac{4}{36}$

Two dice are thrown 2 times.

Here, Y denotes the numbers of times a total of 9 appears.

So, $Y = 0,1,2$

$$P(Y=0) = P(\overline{A}_1) \times P(\overline{A}_2)$$
$$= \frac{32}{36} \times \frac{32}{36}$$
$$= \frac{64}{81}$$
$$P(Y=1) = P(A_1)P(\overline{A}_2) + P(\overline{A}_1)P(A_2)$$
$$= \frac{4}{36} \times \frac{32}{36} + \frac{32}{36} \times \frac{4}{36}$$
$$= \frac{16}{81}$$
$$P(Y=2) = P(A_1)P(A_2)$$
$$= \frac{4}{36} \times \frac{4}{36}$$
$$= \frac{1}{81}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

Question 20

From a lot containing 25 items, 5 of which are defective, 4 are chosen at random. Let X be the number of defectives found. Obtain the probability distribution of X if the items are chosen without replacement.

Solution 20

Given 25 items in the lot. 5 are defective. Good items are 20.
4 items are chosen at random.
Let X be the random variable that denotes the number of defective items in the selected lot.

P (X = 0) = P (4 non-defective and 0 defective) = ${}^{5}C_{0}.{}^{20}C_{4}/{}^{25}C_{4}$

= 4845/12650

P (X = 1) = P (3 non-defective and 1 defective) = ${}^{5}C_{1}.\ {}^{20}C_{3}/{}^{25}C_{4}$

=5x1140/12650

P (X = 2) = P (2 non-defective and 2 defective) = ${}^{5}C_{2}.\ {}^{20}C_{2}/{}^{25}C_{4}$

=10x190/12650

P (X = 3) = P (1 non-defective and 3 defective) = ${}^{5}C_{3}.\ {}^{20}C_{1}/{}^{25}C_{4}$

=10x20/12650

P (X = 4) = P (0 non-defective and 4 defective) = ${}^{5}C_{4}.\ {}^{20}C_{0}/{}^{25}C_{4}$

=5/12650

The probability distribution of X is

X	0	1	2	3	4
P(X)	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$

Question 21

Three cards are drawn successively with replacement from a well-shuffled deck of 52 cards. A random variable X denotes the number of hearts in the three cards drawn. Determine the probability distribution of X.

.

Solution 21

Three cards are thrown with replacement.Let X denote the numbers of hearts if three cards are drawn.

So, X has values 0,1,2,3

$$P(X=0)=P(\overline{H_1})\times P(\overline{H_2})\times P(\overline{H_3})$$
$$=\frac{39}{52}\times\frac{39}{52}\times\frac{39}{52}$$
$$=\frac{27}{26}$$
$$P(X=1)=P(H_1)P(\overline{H_2})P(\overline{H_3})+P(\overline{H_1})P(H_2)P(\overline{H_3})+P(\overline{H_1})P(\overline{H_2})P(H_3)$$
$$=\frac{13}{52}\times\frac{39}{52}\times\frac{39}{52}+\frac{39}{52}\times\frac{13}{52}\times\frac{39}{52}+\frac{39}{52}\times\frac{39}{52}\times\frac{13}{52}$$
$$=\frac{27}{64}$$
$$P(X=2)=P(H_1)P(H_2)P(\overline{H_3})+P(H_1)P(\overline{H_2})P(H_3)+P(\overline{H_1})P(H_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{13}{52}\times\frac{39}{52}+\frac{13}{52}\times\frac{39}{52}\times\frac{13}{52}+\frac{39}{52}\times\frac{13}{52}\times\frac{13}{52}$$
$$=\frac{9}{64}$$
$$P(X=3)=P(H_1)P(H_2)P(H_3)$$
$$=\frac{13}{52}\times\frac{13}{52}\times\frac{13}{52}$$
$$=\frac{1}{64}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Question 22

An urn contains 4 red and 3 blue balls.Find the probability distribution of the number of blue balls in a random draw of 3 balls with replacement.

Solution 22

Urn has 4 red and 3 blue balls.3 balls are drawn with replacement.

Let X denote numbers of blue balls drawn out of 3 drawn.

So, X has values 0,1,2,3

$$P(X=0)=P(\bar{B}_1)\times P(\bar{B}_2)\times P(\bar{B}_3)$$
$$=\frac{4}{7}\times\frac{4}{7}\times\frac{4}{7}$$
$$=\frac{64}{343}$$

$$P(X=1)=P(B_1)P(\bar{B}_2)P(\bar{B}_3)+P(\bar{B}_1)P(B_2)P(\bar{B}_3)+P(\bar{B}_1)P(\bar{B}_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{4}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{4}{7}\times\frac{4}{7}\times\frac{3}{7}$$
$$=\frac{144}{343}$$

$$P(X=2)=P(B_1)P(B_2)P(\bar{B}_3)+P(B_1)P(\bar{B}_2)P(B_3)+P(\overline{B_1})P(B_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{3}{7}\times\frac{4}{7}\times\frac{3}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{3}{7}$$
$$=\frac{108}{343}$$

$$P(X=3)=P(B_1)P(B_2)P(B_3)$$
$$=\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}$$
$$=\frac{27}{343}$$

So,

Required probability distribution is

x :	0	1	2	3
$P(x)$:	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Question 23

Two cards are drawn simultaneously from a well-shuffled deck of 52 cards. Find the probability distribution of the number of successes,when getting a spade is considered a success.

Solution 23

Two cards are drawn simultaneously .Let X denote the number of spades obtained.

So, X can have values 0,1,2.

$$P(X=0)=\frac{39C_2}{52C_2}$$
$$=\frac{39\times 38}{52\times 51}$$
$$=\frac{19}{34}$$
$$P(X=1)=\frac{39C_1\times 13C_1}{52C_2}$$
$$=\frac{13\times 39\times 2}{52\times 51}$$
$$=\frac{13}{34}$$
$$P(X=2)=\frac{13C_2}{52C_2}$$
$$=\frac{13\times 12}{52\times 51}$$
$$=\frac{2}{34}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{2}{34}$

Question 24

A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success.Find the probability distribution of number of successs.

Solution 24

Let A be the event of occurence of a number less than 3.

$P(A) = \frac{2}{6}$ $\quad\quad\quad\quad [\because 1, 2 \text{ are less than } 3.]$

$P(A) = \frac{1}{3}$

Let X denote the number of success is 2 throws of die.
So, X has value 0,1,2.

$$P(X = 0) = P(\overline{A}_1) \times P(\overline{A}_2)$$
$$= \frac{2}{3} \times \frac{2}{3}$$
$$= \frac{4}{9}$$
$$P(X = 1) = P(A_1) P(\overline{A}_2) + P(\overline{A}_1) P(A_2)$$
$$= \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3}$$
$$= \frac{4}{9}$$
$$P(X = 2) = P(A_1) P(A_2)$$
$$= \frac{1}{3} \times \frac{1}{3}$$
$$= \frac{1}{9}$$

So,

Required probability distribution is

x :	0	1	2
$P(x)$:	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Question 25

An urn contains 5 red and 2 black balls.Two balls are randomly selected.
Let X represent the number of black balls.What are possible value of X.
Is X a random variable?

Solution 25

Urn has 5 red and 2 black balls. 2 balls are randomly selected.
Here, X denote the numbers of black balls.
So, possible values of $X = 0, 1, 2$

$$P(X=0) = P(\bar{B}_1) \times P(\bar{B}_2)$$
$$= \frac{5}{7} \times \frac{5}{7}$$
$$= \frac{25}{49}$$
$$P(X=1) = P(B_1)P(\bar{B}_2) + P(\bar{B}_1)P(B_2)$$
$$= \frac{2}{7} \times \frac{5}{7} + \frac{5}{7} \times \frac{2}{7}$$
$$= \frac{20}{49}$$
$$P(X=2) = P(B_1)P(B_2)$$
$$= \frac{2}{7} \times \frac{2}{7}$$
$$= \frac{4}{49}$$

Now,
$$P(x=0) + P(x=1) + P(x=2)$$
$$= \frac{25}{49} + \frac{20}{49} + \frac{4}{49}$$
$$= \frac{49}{49}$$
$$= 1$$

So, $\sum P(x) = 1$

Therefore

X is a random variable

Question 26

Let X represent the difference between the number of heads and the number of tails when a coin is tossed 6 times. What are possible value of X?

Solution 26

Here, coin is tossed 6 times.
So, there can have
$1H\,5T$ or $2H\,4T$ or $3H\,3T$ or
$4H\,2T$ or $5H\,1T$ or $6H$ or
$6T$
Here, X denote the difference between the number of head and number of tails.
So,

$$X = 6, 4, 2, 0, -2, -4, -6$$

Question 27

From a lot of 10 bulbs which include 3 defectives, a sample of 2 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution 27

It is given that out of 10 bulbs, 3 are defective.

Number of non-defective bulbs $= 10 - 3 = 7$

2 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$$\therefore P(X = 0) = \frac{{}^{7}C_2}{{}^{10}C_2} = \frac{7}{15}$$

$$\therefore P(X = 1) = \frac{{}^{3}C_1 \times {}^{7}C_1}{{}^{10}C_2} = \frac{7}{15}$$

$$\therefore P(X = 2) = \frac{{}^{3}C_2}{{}^{10}C_2} = \frac{1}{15}$$

Therefore, the required probability distribution is

X	0	1	2
$P(X)$	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

Question 28

Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If X denotes the number of red balls drawn, find the probability distribution of X.

Solution 28

Clearly, X can assume values 0, 1, 2, 3, 4 such that

$$P(X = 0) = (\text{Probablity of getting no red ball}) = \frac{{}^{8}C_0 \times {}^{4}C_4}{{}^{12}C_4} = \frac{1 \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{1}{495}$$

$$P(X = 1) = (\text{Probablity of getting one red ball}) = \frac{{}^{8}C_1 \times {}^{4}C_3}{{}^{12}C_4} = \frac{8 \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{32}{495}$$

$$P(X = 2) = (\text{Probablity of getting two red balls}) = \frac{{}^{8}C_2 \times {}^{4}C_2}{{}^{12}C_4} = \frac{\frac{8 \times 7}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{168}{495}$$

$$P(X = 3) = (\text{Probablity of getting three red balls}) = \frac{{}^{8}C_3 \times {}^{4}C_1}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{224}{495}$$

$$P(X = 4) = (\text{Probablity of getting four red balls}) = \frac{{}^{8}C_4 \times {}^{4}C_0}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{70}{495}$$

Thus, probablity distribution of random veriable X is,

X	0	1	2	3	4
P(X)	$\frac{1}{495}$	$\frac{32}{495}$	$\frac{168}{495}$	$\frac{224}{495}$	$\frac{70}{495}$

Question 29

The probability distribution of a random variable X is given below:

X :	0	1	2	3
P(x) :	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

(i) Determine the value of k

(ii) Determine P (X $\le$ 2) and P b(X > 2)

(iii) Find P (X $\le$ 2) + P(X > 2)

Solution 29

(i) We know that,

$$P(0)+P(1)+P(2)+P(3)=1$$

$$k+\frac{k}{2}+\frac{k}{4}+\frac{k}{8}=1$$

$$15k=8$$

$$k=\frac{8}{15}$$

(ii) $P(X \le 2)$

$$=P(0)+P(1)+P(2)$$

$$=k+\frac{k}{2}+\frac{k}{4}$$

$$=\frac{8}{15}+\frac{8}{30}+\frac{8}{60}$$

$$=\frac{14}{15}$$

$$P(X>2)=P(3)=\frac{k}{8}=\frac{1}{15}$$

(iii) $P(X \le 2)+P(X>2)$

$$=P(0)+P(1)+P(2)+P(3)$$

$$=1$$

Chapter 32 - Mean and variance of a random variable

Exercise Ex. 32.2

Question 1(i)

Find the mean and standard deviation of each of the following probability distributions:

x_i : 2 3 4

p_i : 2.2 0.5 0.3

Solution 1(i)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
		$\sum p_i x_i = 3.1$	$\sum p_i x_i^2 = 10.1$

$$\text{Mean} = \sum p_i x_i = 3.1$$

$$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 = 10.1 - (3.1)^2 = 0.49$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = 0.7$$

Question 1(ii)

Find the mean and standard deviation of the following probability distributions:

x_i :	1	3	4	5
p_i :	0.4	0.1	0.2	0.3

Solution 1(ii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
3	0.1	0.3	0.9
4	0.2	0.8	3.2
5	0.3	1.5	7.5
		$\sum xp = 3$	$\sum x^2 p = 12$

$\text{Mean} = \sum xp$

mean = 3

$$\text{Standard deviation} = \sqrt{\sum x^2 p - (\text{mean})^2}$$

$$= \sqrt{12 - (3)^2}$$

$$= \sqrt{3}$$

Standard Deviation = 1.732

Question 1(iii)

Find the mean and standard deviation of the following probability distributions:

$$\begin{array}{lcccc} x_i: & -5 & -4 & 1 & 2 \\ p_i: & \frac{1}{4} & \frac{1}{8} & \frac{1}{2} & \frac{1}{8} \end{array}$$

Solution 1(iii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-5	$\frac{1}{4}$	$-\frac{5}{4}$	$\frac{25}{4}$
-4	$\frac{1}{8}$	$-\frac{1}{2}$	2
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
		$\Sigma xp = -1$	$\Sigma x^2p = \frac{37}{4}$

Mean $= \Sigma xp$

mean $= -1$

Standard deviation $= \sqrt{\Sigma x^2p - (\text{mean})^2}$

$= \sqrt{\frac{37}{4} - (-1)^2}$

$= \sqrt{\frac{33}{4}}$

$= \sqrt{8.25}$

Standard Deviation = 2.9

Question 1(iv)

Find the mean and standard deviation of the following probability distributions:

$$\begin{array}{lccccc} x_i: & -1 & 0 & 1 & 2 & 3 \\ p_i: & 0.3 & 0.1 & 0.1 & 0.3 & 0.2 \end{array}$$

Solution 1(iv)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-1	0.3	-0.3	0.3
0	0.1	0	0
1	0.1	0.1	0.1
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$\sum xp = 1$	$\sum x^2p = 3.4$

Mean = $\sum xp$
mean = 1

Standard deviation = $\sqrt{\sum x^2p - (\text{mean})^2}$

$= \sqrt{(3.4) - (1)^2}$

$= \sqrt{2.4}$

Standard Deviation = 1.5

Question 1(v)

Find the mean and standard deviation of the following probability distributions:

x_i :	1	2	3	4
p_i :	0.4	0.3	0.2	0.1

Solution 1(v)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1 2 3 4	0.4 0.3 0.2 0.1	0.4 0.6 0.6 0.4	0.4 1.2 1.8 1.6
		$\sum xp = 2$	$\sum x^2p = 5$

Mean = $\sum xp$
mean = 2

Standard deviation = $\sqrt{\sum x^2p - (\text{mean})^2}$
$= \sqrt{5 - (2)^2}$

Standard Deviation = 1

Question 1(vi)

Find the mean and standard deviation of the following probability distributions:

x_i :	0	1	3	5
p_i :	0.2	0.5	0.2	0.1

Solution 1(vi)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	0.2	0	0
1	0.5	0.5	0.5
3	0.2	0.6	1.8
5	0.1	0.5	2.5
		$\sum xp = 1.6$	$\sum x^2p = 4.8$

Mean = $\sum xp$

mean = 1.6

Standard deviation = $\sqrt{\sum x^2p - (\text{mean})^2}$

$= \sqrt{4.8 - (1.6)^2}$

$= \sqrt{4.8 - 2.56}$

$= \sqrt{2.24}$

Standard Deviation = 1.497

Question 1(vii)

Find the mean and standard deviation of the following probability distributions:

x_i :	− 2	−1	0	1	2
p_i :	0.1	0.2	0.4	0.2	0.1

Solution 1(vii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.4	0	0
1	0.2	0.2	0.2
2	0.1	0.2	0.4
		$\sum xp = 0$	$\sum x^2 p = 1.2$

Mean = $\sum xp$

mean = 0

Standard deviation = $\sqrt{\sum x^2 p - (\text{mean})^2}$

$= \sqrt{(1.2)^2 - (0)^2}$

Standard Deviation = 1.2

Question 1(viii)

Find the mean and standard deviation of the following probability distributions:

x_i : − 3 − 1 0 1 3

p_i : 0.05 0.45 0.20 0.25 0.05

Solution 1(viii)

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
-3	0.05	-0.15	0.45
-1	0.45	-0.45	0.45
0	0.20	0	0
1	0.25	0.25	0.25
3	0.05	0.15	0.45
		$\sum xp = -0.2$	$\sum x^2p = 1.6$

Mean $= \sum xp$

mean $= -0.2$

$$\text{Standard deviation} = \sqrt{\sum x^2p - (\text{mean})^2}$$

$$= \sqrt{1.6 - (-0.2)^2}$$

$$= \sqrt{1.6 - 0.04}$$

$$= \sqrt{1.56}$$

Standard Deviation = 1.249

Question 1(ix)

Find the mean and standard deviation of each of the following probability distributions:

x_i :	0	1	2	3	4	5
p_i :	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

Solution 1(ix)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{5}{18}$	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
3	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$
4	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{16}{9}$
5	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{25}{18}$
		$\sum p_i x_i = \frac{35}{18}$	$\sum p_i x_i^2 = \frac{35}{6}$

$$\text{Mean} = \sum p_i x_i = \frac{35}{18}$$

$$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 = \frac{35}{6} - \left(\frac{35}{18}\right)^2 = \frac{665}{324}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \frac{\sqrt{665}}{18}$$

Question 2

A discrete random variable X has the probability distribution given below:

X : 0.5 1 1.5 2

P(X) : k k^2 $2k^2$ k

(i) Find the value of k.

(ii) Determine the mean of the distribution.

Solution 2

(i) We know that,

$P(0.5)+P(1)+P(1.5)+P(2) = 1$

$k + k^2 + 2k^2 + k = 1$

$3k^2 + 2k - 1 = 0$

$3k^2 + 3k - k - 1 = 0$

$(3k-1)(k+1) = 0$

$k = \frac{1}{3}$ or $k = -1$

We know that $0 \le P(X) \le 1$

$\therefore k = \frac{1}{3}$

(ii)

x_i	p_i	$p_i x_i$
0.5	$\frac{1}{3}$	$\frac{1}{6}$
1	$\frac{1}{9}$	$\frac{1}{9}$
1.5	$\frac{2}{9}$	$\frac{1}{3}$
2	$\frac{1}{3}$	$\frac{2}{3}$
		$\sum p_i x_i = \frac{23}{18}$

$\text{Mean} = \sum p_i x_i = \frac{23}{18}$

Question 3

Find the mean variance and standard deviation of the following probability distribution

x_i :	a	b
p_i :	p	q

where $p + q = 1$.

Solution 3

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
a	p	ap	a^2p
b	q	bq	b^2q

Mean = $\sum xp$

mean = $ap + bq$

$$\begin{aligned}
\text{Variance} &= \sum x^2p - (\text{mean})^2 \\
&= \left(a^2p + b^2q\right) - (ap + bq)^2 \\
&= a^2p + b^2q - a^2p^2 - b^2q^2 - 2abpq \\
&= a^2pq + b^2pq - 2abpq \qquad [\because \quad p + q = 1] \\
&= pq\left(a^2 + b^2 - 2ab\right)
\end{aligned}$$

Variance = $pq\,(a - b)^2$

Standard deviation = $|a - b|\sqrt{pq}$

Question 4

Find the mean and variance of the number of heads in three tosses of a coin.

Solution 4

We know that in a throw of coin.

$P(H)=\frac{1}{2}, \qquad P(T)=\frac{1}{2}$

Let X denote the number of heads in three tosses of coin.

So, $X = 0,1,2,3$

$$
\begin{aligned}
P(x=0) &= P(T)P(T)P(T) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{1}{8} \\
P(x=1) &= P(H)P(T)P(T)+P(T)P(H)P(T)+P(T)P(T)P(H) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{3}{8} \\
P(x=2) &= P(H)P(H)P(T)+P(H)P(T)P(H)+P(T)P(H)P(H) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{3}{8} \\
P(x=3) &= P(H)P(H)P(H) \\
&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \\
&= \frac{1}{8}
\end{aligned}
$$

So,

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\Sigma xp = \frac{3}{2}$	$\Sigma x^2 p = 3$

$$\text{Mean} = \sum xp$$

$$\text{mean} = \frac{3}{2}$$

$$\text{Variance} = \sum x^2 p - (\text{mean})^2$$

$$= 3 - \frac{9}{4}$$

$$\text{Variance} = \frac{3}{4}$$

Question 5

Two cards are drawn simultaneously from a pack of 52 cards.Compute the mean and standard deviation of the number of kings.

Solution 5

Two cards are drawn simultneously from a pack of 52 cards.
Let X denotes the number of kings drawn.

So, $X = 0,1,2$

$$P(x=0) = \frac{48C_2}{52C_2}$$
$$= \frac{48 \times 47}{52 \times 51}$$
$$= \frac{188}{221}$$
$$P(x=1) = \frac{4C_1 \times 48C_1}{52C_2}$$
$$= \frac{4 \times 48 \times 2}{52 \times 51}$$
$$= \frac{32}{221}$$
$$P(x=2) = \frac{4C_2}{52C_2}$$
$$= \frac{4 \times 3}{52 \times 51}$$
$$= \frac{1}{221}$$

So,

x_i	p_i	$x_i\,p_i$	$x_i^2 p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$	$\frac{4}{221}$
		$\Sigma xp = \frac{34}{221}$	$\Sigma x^2 p = \frac{36}{221}$

Mean = $\sum xp$

mean = $\frac{34}{221}$

$$\text{Variance} = \sum x^2 p - (\text{mean})^2$$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2$$

$$= \frac{7956 - 1156}{48841}$$

$$= \frac{6800}{48841}$$

$$\text{Variance} = \frac{400}{2873}$$

Question 6

Find the mean, variance and standard deviation of the number of tails in three tosses of a coin.

Solution 6

We know that ,in a throw of coin.

$P(T) = \frac{1}{2}, \qquad P(H) = \frac{1}{2}$

Let X denote the number of tails in three throws of coins.

So, X can take values from 0,1,2,3

$$P(x=0) = P(H)P(H)P(H)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{8}$$
$$P(x=1) = P(T)P(H)P(H) + P(H)P(T)P(H) + P(H)P(H)P(T)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{3}{8}$$
$$P(x=2) = P(T)P(T)P(H) + P(T)P(H)P(T) + P(H)P(T)P(T)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{3}{8}$$
$$P(x=3) = P(T)P(T)P(T)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{8}$$

So,

Mean $= \sum xp$

mean $= \frac{3}{2}$

$$\text{Variance} = \sum x^2 p - (\text{mean})^2$$
$$= 3 - \left(\frac{3}{2}\right)^2$$
$$= 3 - \frac{9}{4}$$

Variance $= \frac{3}{4}$

Standard Deviation $= \sqrt{\text{Variance}}$

$$= \sqrt{\frac{3}{4}}$$

Standard Deviation = 0.87

Question 7

Two bad eggs are accidently mixed up with ten good ones. Three eggs are drawn at random with replacement from this lot. Cumpute the mean for the number of bad eggs drawn.

Solution 7

Total 12 good and bad eggs. 10 are good and 2 are bad.
3 eggs are drawn from this lot
Let X be the random variable that denotes the number of bad eggs in the lot.

P (X = 0) = P (3good and 0 bad) = $^{3}C_{0}.^{10}C_{3}/^{12}C_{3}$

= 1 x 120/220 = 6/11

P (X = 1) = P (2good and 1 bad) = $^{2}C_{1}.^{10}C_{2}/^{12}C_{3}$

=2x 45/220 = 9/22

P (X = 2) = P (1good and 2 bad) = $^{2}C_{2}.^{10}C_{1}/^{12}C_{3}$

= 1x10/220 = 1/22

The probability distribution of X is

X	0	1	2
P(X)	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

The mean = $0 \times \frac{6}{11} + 1 \times \frac{9}{22} + 2 \times \frac{1}{22} = \frac{11}{22}$ = 1/2

Question 8

A pair of fair dice is thrown.Let X be the random variable which denotes the minimum of the two numbers which appear. Find the probability distribution, mean and variance of X.

Solution 8

A pair of dice is thrown. And X denote minimum of the two number appeared.
So, X can have values 2,3,4,5,6.

$P(x=1)=\frac{11}{36}$ $\left[\text{Possible pairs: } (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\right]$

$P(x=2)=\frac{9}{36}$ $\left[\text{Possible pairs: } (2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\right]$

$P(x=3)=\frac{7}{36}$ $\left[\text{Possible pairs: } (3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\right]$

$P(x=4)=\frac{5}{36}$ $\left[\text{Possible pairs: } (4,4),(4,5),(4,6),(5,4),(6,4)\right]$

$P(x=5)=\frac{3}{36}$ $\left[\text{Possible pairs: } (5,5),(5,6),(6,5)\right]$

$P(x=6)=\frac{1}{36}$ $\left[\text{Possible pairs: } (6,6)\right]$

x_i	p_i	$x_i\,p_i$	$x_i^2 p_i$
1	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{11}{36}$
2	$\frac{9}{36}$	$\frac{18}{36}$	$\frac{36}{36}$
3	$\frac{7}{36}$	$\frac{21}{36}$	$\frac{63}{36}$
4	$\frac{5}{36}$	$\frac{20}{36}$	$\frac{80}{36}$
5	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{75}{36}$
6	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{36}{36}$
		$\sum xp=\frac{91}{36}$	$\sum x^2p=\frac{301}{36}$

Mean = $\sum xp$

Mean = $\frac{91}{36}$

Variance = $\sum x^2 p - (\text{mean})^2$

$$= \frac{301}{36} - \left(\frac{91}{36}\right)^2$$

$$= \frac{10836 - 8281}{1296}$$

$$= \frac{2555}{1296}$$

Variance = 1.97

Probability distribution is

x :	1	2	3	4	5	6
$P(x)$:	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

Question 9

A fair coin is tossed four times.Let X denote the nomber of head occurring. Find the probability distribution, mean and variance of X.

Solution 9

We know that ,In a toss of coin.

$P(T) = \frac{1}{2}$, $\quad P(H) = \frac{1}{2}$

Let X denote the number of occuring head in 4 throws of coins.

So, X can take values from $X = 0,1,2,3,4$

$$P(x=0) = P(T)P(T)P(T)P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{16}$$
$$P(x=1) = P(H)P(T)P(T)P(T)\times {}^4C_1$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 4$$
$$= \frac{4}{16}$$
$$P(x=2) = P(H)P(H)P(T)P(T)\times {}^4C_2$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 6$$
$$= \frac{6}{16}$$
$$P(x=3) = P(H)P(H)P(H)P(T)\times {}^4C_3$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 4$$
$$= \frac{4}{16}$$
$$P(x=4) = P(H)P(H)P(H)P(H)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{16}$$

So,

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{1}{16}$	0	0
1	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	$\frac{24}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{36}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{16}{16}$
		$\sum xp = 2$	$\sum x^2 p = 5$

Mean $= \sum xp$
mean $= 2$

Variance $= \sum x^2 p - (\text{mean})^2$
$= 5 - (2)^2$
Variance $= 1$

Probability distribution is

$$\begin{array}{cccccc} x & : & 0 & 1 & 2 & 3 & 4 \\ P(x) & : & \frac{1}{16} & \frac{4}{16} & \frac{6}{16} & \frac{4}{16} & \frac{1}{16} \end{array}$$

Question 10

A fair die is tossed .Let X denote twice the number appearing.Find the probability distribution, mean and variance of X.

Solution 10

X denotes twice the number appearing on the die.
So, $X = 2,4,6,8,10,12$.

Probability distribution is

X :	2	4	6	8	10	12
$P(x)$:	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
2	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{16}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$	$\frac{36}{6}$
8	$\frac{1}{6}$	$\frac{8}{6}$	$\frac{64}{6}$
10	$\frac{1}{6}$	$\frac{10}{6}$	$\frac{100}{6}$
12	$\frac{1}{6}$	$\frac{12}{6}$	$\frac{144}{6}$
		$\sum xp = 7$	$\sum x^2p = \frac{364}{6}$

Mean $= \sum xp$
mean $= 7$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$
$$= \left(\frac{364}{6}\right) - (7)^2$$
$$= \frac{364 - 294}{6}$$
$$= \frac{70}{6}$$

Variance $= 11.7$

Question 11

A fair die is tossed .Let X denote 1 or 3 according as an odd or an even number appears.Find the probability distribution, mean and variance of X.

Solution 11

Probability of even number $= P(E) = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \quad P(O) = \frac{1}{2}$

Here, X have values 1 or 3 according as an odd or even number.

So,

$$\begin{array}{lll} X: & 1 & 3 \\ P(x): & \frac{1}{2} & \frac{1}{2} \end{array}$$

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$
		$\sum xp = 2$	$\sum x^2p = 5$

Mean $= \sum xp$
mean $= 2$

Variance $= \sum x^2p - (\text{mean})^2$
$= 5 - 4$

Variance $= 1$

Question 12

A fair coin is tossed four times .Let X denote longest string of heads occurring. Find the probability distribution, mean and variance of X.

Solution 12

Let the event of getting a head = H and getting a tail = T
Let X denote the variable longest consecutive heads occurring in 4 tosses. The possible values are

X = 0 (no head) {T, T, T, T}
X = 1 (1 heads) {H, T, T, T}
X = 2 (2 heads) {H, H, T, T}
X = 3 (3 heads) {H, H, H, T}
X = 4 (4 heads) {H, H, H, H}

n(S) = {(HHHH),(HHHT),(HHTT),(HTHH),(HTHT),(HTTH),(HTTT),
(THHH),(THTH),(THHT),(THTT),(THTT),(TTHH),(TTHT)
(TTTH),(TTTT)}

$P(X=0) = \frac{1}{16}$

$P(X=1) = \frac{7}{16}$

$P(X=2) = \frac{5}{16}$

$P(X=3) = \frac{2}{16}$

$P(X=4) = \frac{1}{16}$

Probability distribution is

X	0	1	2	3	4
$p_i=P(X)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
$p_i x_i^2$	0	$\frac{7}{16}$	$\frac{20}{16}$	$\frac{18}{16}$	1

Mean = $\sum_{i=1\ to\ n} X_i \times P(X_i)$

Mean, $\mu = 0\times\frac{1}{16}+1\times\frac{7}{16}+2\times\frac{5}{16}+3\times\frac{2}{16}+4\times\frac{1}{16}$

$= 0+\frac{7}{16}+\frac{10}{16}+\frac{6}{16}+\frac{4}{16}$

$=\frac{27}{16} = 1.7$

Variance $Var(X) = \sum p_i x_i^2 - (\sum p_i x_i)^2$

$= \frac{61}{16} - 1.7^2$

$=3.825-2.89$

$= 0.935$

Question 13

Two card are selected at random from a box which contains five cards numbered 1,1,2,2 and 3.Let X denote the sum and Y the maximum of the two numbers drawn. Find the probability distribution, mean and variance of X and Y.

Solution 13

Box contains five cards 1,1,2,2,3.

Here,

X denotes the sum of two number on cards drawn.

Y denotes the maximum of the two number.

So, $X = 2,3,4,5$

$Y = 1,2,3$

$$P(X=2) = P(1)P(1)$$
$$= \frac{2}{5} \times \frac{1}{4}$$
$$= 0.1$$
$$P(X=3) = P(1)P(2) + P(2)P(1)$$
$$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4}$$
$$= 0.4$$
$$P(X=4) = P(2)P(2) + P(1)P(3) + P(3)P(1)$$
$$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$
$$= 0.3$$
$$P(X=5) = P(2)P(3) + P(3)P(2)$$
$$= \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$
$$= 0.2$$

Probability Distribution for X

X :	2	3	4	5
$P(x)$:	0.1	0.4	0.3	0.2

Now,

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
2	0.1	0.1	0.4
3	0.4	1.2	3.6
4	0.3	1.2	4.8
5	0.2	1.0	5.0
		$\sum xp = 3.6$	$\sum x^2 p = 13.8$

Mean = $\sum xp$

mean = 3.6

Variance = $\sum x^2p - (\text{mean})^2$

$= 13.8 - (3.6)^2$

$= 13.8 - 12.96$

Variance = 0.84

$P(Y = 1) = P(1)P(1)$

$= \frac{2}{5} \times \frac{1}{4}$

$= \frac{2}{20}$

$= 0.1$

$P(Y = 2) = P(1)P(2) + P(2)P(1) + P(2)P(2)$

$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$

$P(Y = 2) = 0.5$

$P(Y = 3) = P(1)P(3) + P(2)P(3) + P(3)P(1) + P(3)P(2)$

$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4}$

$= 0.4$

Probability distribution for Y is

X :	1	2	3
$P(x)$:	0.1	0.5	0.4

Y_i	p_i	$Y_i\, p_i$	$Y_i^2 p_i$
1	0.1	0.1	0.1
2	0.5	1.0	2.0
3	0.4	1.2	3.6
		$\sum xp = 2.6$	$\sum x^2p = 5.7$

Mean = $\sum xp$

mean = 2.3

Variance = $\sum x^2 p - (\text{mean})^2$

$= 5.7 - (2.3)^2$

Variance = 0.41

Question 14

A die is tossed twice. A 'success' is getting an odd number on a toss. Find the variance of the number of successes.

Solution 14

Probability of getting an odd number = $P(O) = \frac{1}{2}$

$\Rightarrow \quad P(E) = \frac{1}{2}$

Die is tossed twice. Let X denote the number of times an odd number occurs.

So, $X = 0,1,2$.

$$P(X = 0) = P(E)P(E)$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(X = 1) = P(O)P(E) + P(E)P(O)$$
$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{2}$$
$$P(X = 2) = P(O)P(O)$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$P(X = 2) = \frac{1}{4}$$

x_i	p_i	$x_i\ p_i$	$x_i^2 p_i$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
		$\sum xp = 1$	$\sum x^2 p = \frac{3}{2}$

Mean = $\sum xp = 1$

Variance = $\sum x^2 p - (\text{mean})^2$

$$= \frac{3}{2} - 1$$

Variance = $\frac{1}{2}$

Question 15

A box contains 13 bulbs, out of which 5 are defective.3 bulbs are randomly drawn, one by one without replacement, from the box. Find the probability distribution of the number of defective bulbs.

Solution 15

Out of 13 bulbs 5 are defective $\Rightarrow$ 8 bulbs are good.

3 bulbs are drawn without replacement ,

Let X denote number of defective bulbs,

So, X can have values 0,1,2,3

$$P(X=0)=p(\text{No defective})$$
$$=\frac{8C_3}{13C_3}$$
$$=\frac{8\times7\times6}{13\times12\times11}$$
$$=\frac{28}{143}$$
$$P(X=1)=p(\text{Only one defective})$$
$$=\frac{8C_2\times5C_1}{13C_3}$$
$$=\frac{8\times7\times5}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$=\frac{70}{143}$$
$$P(X=2)=p(\text{Only two defective})$$
$$=\frac{8C_1\times5C_2}{13C_3}$$
$$=\frac{8\times5\times4}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$=\frac{40}{143}$$
$$P(X=3)=p(\text{all three are defective})$$
$$=\frac{5C_3}{13C_3}$$
$$=\frac{4\times5}{2}\times\frac{3\times2\times1}{13\times12\times11}$$
$$=\frac{5}{143}$$

So, Probability distribution is

X :	0	1	2	3
$P(x)$:	$\frac{28}{143}$	$\frac{70}{143}$	$\frac{40}{143}$	$\frac{5}{143}$

Question 16

In roulette, the wheel has 13 numbers 0,1,2,.......,12 marked on equally spaced slots. A player sets Rs 10 on a given number. He receives Rs 100 from the organiser of the game if the ball comes to rest in this slot; otherwise get nothing. If X denotes the player's net gain/loss, find $E(X)$.

Solution 16

$P(\text{win}) = \frac{1}{13} \Rightarrow \qquad P(\text{lose}) = \frac{12}{13}$

He gains Rs 90 if he wins and loses Rs 10 if his number does not appear.

Let X denote total loss or gain, so,

X :	90	-10
$P(x)$:	$\frac{1}{13}$	$\frac{12}{13}$
XP :	$\frac{90}{13}$	$\frac{-120}{13}$

$$E(X) = \Sigma XP$$
$$= \frac{90}{13} - \frac{120}{13}$$
$$E(X) = -\frac{30}{13}$$

Question 17

Three cards are drawn at random (without replacement) from a well shuffled pack of 52 cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution.

Solution 17

Let 'X' be the random variable which can assume values from 0 to 3.

$$P(X=0)=\frac{^{26}C_3}{^{52}C_3}=\frac{2600}{22100}=\frac{2}{17}$$

$$P(X=1)=\frac{^{26}C_1\times{}^{26}C_2}{^{52}C_3}=\frac{8450}{22100}=\frac{13}{34}$$

$$P(X=2)=\frac{^{26}C_2\times{}^{26}C_1}{^{52}C_3}=\frac{8450}{22100}=\frac{13}{34}$$

$$P(X=3)=\frac{^{26}C_3}{^{52}C_3}=\frac{2600}{22100}=\frac{2}{17}$$

Probability distribution of X:

$X=x_i$	0	1	2	3
$p(X=x_i)$	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

$$\text{Mean}=\sum_{i=0}^{3}(x_i\times p_i)$$

$$=x_0p_0+x_1p_1+x_2p_2+x_3p_3$$

$$=0\times\frac{2}{17}+1\times\frac{13}{34}+2\times\frac{13}{34}+3\times\frac{2}{17}$$

$$=\frac{13+26+12}{34}$$

$$=\frac{51}{34}$$

$$=\frac{3}{2}$$

$$=1.5$$

Question 18

An urn contains 5 are 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes, find the mean and variance of X.

Solution 18

X can assume values 0, 1, 2.
Yes X is a random variable.

$$P(X = 0) = (\text{Probablity of getting no black ball}) = \frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{1 \times \frac{5 \times 4}{2 \times 1}}{\frac{7 \times 6}{2 \times 1}} = \frac{20}{42}$$

$$P(X = 1) = (\text{Probablity of getting one black ball}) = \frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{2 \times 5}{\frac{7 \times 6}{2 \times 1}} = \frac{20}{42}$$

$$P(X = 2) = (\text{Probablity of getting two black balls}) = \frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{1 \times 1}{\frac{7 \times 6}{2 \times 1}} = \frac{2}{42}$$

Thus, probablity distribution of random veriable X is,

X	0	1	2
P(X)	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{2}{42}$

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{20}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$
		$\Sigma p_i x_i = \frac{4}{7}$	$\Sigma p_i x_i^2 = \frac{2}{3}$

$$\text{Mean} = \Sigma p_i x_i = \frac{4}{7}$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{2}{3} - \left(\frac{4}{7}\right)^2 = \frac{50}{147}$$

Question 19

Two numbers are selected at random (without replacement) from positive integers 2,3,4,5, 6 and 7. Let X denote the larger of the two number obtained. Find the mean and variance of the probability distribution of X.

Solution 19

We can select two positive in $6 \times 5 = 30$ different ways.
X denotes tha larger number so, X can assume values 3, 4, 5, 6 and 7.
Yes X is a random variable.

$$P(X = 3) = P(\text{larger number is } 3) = \{(2,3),(3,2)\} = \frac{2}{30}$$

$$P(X = 4) = P(\text{larger number is } 4) = \{(2,4),(4,2),(3,4),(4,3)\} = \frac{4}{30}$$

$$P(X = 5) = P(\text{larger number is } 5) = \{(2,5),(5,2),(3,5),(5,3),(4,5),(5,4)\} = \frac{6}{30}$$

$$P(X = 6) = P(\text{larger number is } 6) = \{(2,6),(6,2),(3,6),(6,3),(4,6),(6,4),(5,6),(6,5)\} = \frac{8}{30}$$

$$P(X = 7) = P(\text{larger number is } 7) = \{(2,7),(7,2),(3,7),(7,3),(4,7),(7,4),(5,7),(7,5),(6,7),(7,6)\} = \frac{10}{30}$$

Thus, probablity distribution of random veriable X is,

x_i	p_i	$p_i x_i$	$p_i x_i^2$
3	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{18}{30}$
4	$\frac{4}{30}$	$\frac{16}{30}$	$\frac{64}{30}$
5	$\frac{6}{30}$	$\frac{30}{30}$	$\frac{150}{30}$
6	$\frac{8}{30}$	$\frac{48}{30}$	$\frac{288}{30}$
7	$\frac{10}{30}$	$\frac{70}{30}$	$\frac{490}{30}$
		$\Sigma p_i x_i = \frac{17}{3}$	$\Sigma p_i x_i^2 = \frac{101}{3}$

$$\text{Mean} = \Sigma p_i x_i = \frac{17}{3}$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{101}{3} - \left(\frac{17}{3}\right)^2 = \frac{14}{9}$$

Chapter 32 - Mean and variance of a random variable Exercise MCQ

Question 1

If a random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7	8
P(X):	a	3a	5a	7a	9a	11a	13a	15a	17a

then the value of a is

a. $\frac{7}{81}$

b. $\frac{5}{80}$

c. $\frac{2}{81}$

d. $\frac{1}{81}$

Solution 1

Correct option: (d)

$$\sum_{x=0}^{8} P(x) = 1$$

$$\Rightarrow a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$\Rightarrow 81a = 1$$

$$\Rightarrow a = \frac{1}{81}$$

Question 2

A random variable X has the following probability distribution:

X:	1	2	3	4	5	6	7	8
P(X):	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event E = {X:X is a prime number}, F = {X:X∪ F) is

a. 0.50

b. 0.77

c. 0.35

d. 0.87

Solution 2

Correct option: (b)

$P(E) = P(2) + P(3) + P(5) + P(7)$

$P(E) = 0.23 + 0.12 + 0.20 + 0.07$

$P(E) = 0.62$

And

$P(F) = P(1) + P(2) + P(3)$

$P(F) = 0.15 + 0.23 + 0.12$

$P(F) = 0.5$

Also,

$P(E \cap F) = P(2) + P(3)$

$P(E \cap F) = 0.23 + 0.12$

$P(E \cap F) = 0.35$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$P(E \cup F) = 0.62 + 0.5 - 0.35$

$P(E \cup F) = 0.77$

Question 3

A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If P(X=3)=2 P(X=1) and P (X=2)=0.3, then P(X=0) is

a. 0.1
b. 0.2
c. 0.3
d. 0.4

Solution 3

Correct option: (d)

Let $P(0) = x$ and $P(1) = y$

Given that $E(x) = 1.3$

$\sum_{0}^{3} xP(x) = 1.3$

$0 + P(1) + 2P(2) + 3P(3) = 1.3$

$y + 2 \times 0.3 + 6y = 1.3$

$7y = 0.7$

$y = 0.1$

Also,

$\sum_{0}^{3} P(x) = 1$

$x + 0.1 + 0.3 + 0.2 = 1$

$x = 0.4$

Question 4

A random variable has the following probability distribution:

$X=x_i$:	0	1	2	3	4	5	6	7
$P(X=x_i)$:	0	2p	2p	3p	P^2	$2p^2$	$7p^2$	2p

The value of P is
a. 1/10
b. -1
c. -1/10
d. 1/5

Solution 4
Correct option: (a)

$$\sum_{0}^{7} P(x) = 1$$

$$2p + 2p + 3p + p^2 + 2p^2 + 7p^2 + 2p = 1$$

$$9p + 10p^2 = 1$$

$$10p^2 + 9p - 1 = 0$$

$$(10p - 1)(p + 1) = 0$$

$$\Rightarrow p = \frac{1}{10}$$

Question 5
If X is a random -variable with probability distribution as given below:

$X=x_i$:	0	1	2	3
$P(X=x_i)$:	k	3k	3k	k

The value of k and its variance are
a. 1/8, 22/27
b. 1/8, 23/27
c. 1/8, 24/27
d. 1/8, 3/4

Solution 5
Correct option: (d)

$$\sum_{0}^{3} P(x) = 1$$

$$k + 3k + 3k + k = 1$$

$$k = \frac{1}{8}$$

x	$P(x)$	$xP(x)$	$x^2P(x)$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
Total		$E(x) = \frac{12}{8} = 1.5$	$E(x^2) = 3$

$$V(x) = E(x^2) - [E(x)]^2$$

$$V(x) = 3 - (1.5)^2$$

$$V(x) = 0.75 = \frac{3}{4}$$

Question 6

The probability distribution of a discrete random variable X is given below:

X:	2	3	4	5
P(X):	5/k	7/k	9/k	11/k

The value of E(x) is

a. 8

b. 16

c. 32

d. 48

Solution 6

Correct option: (c)

$$\sum_{2}^{5} P(x) = 1$$

$$\frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1$$

$$k = 32$$

NOTE: Question is modified.

Question 7

For the following probability distribution:

X:	-4	-3	-2	-1	0
P(X):	0.1	0.2	0.3	0.2	0.2

The value of E(X) is

a. 0

b. -1

c. -2

d. -1.8

Solution 7

Correct option: (d)

x	P(x)	xP(x)
−4	0.1	−0.4
−3	0.2	−0.6
−2	0.3	−0.6
−1	0.2	−0.2
0	0.2	0
		$E(x) = -1.8$

Question 8

For the following probability distribution:

X:	1	2	3	4
P(X):	1/10	1/5	3/10	2/5

The value of $E(X^2)$ is

a. 3

b. 5

c. 7

d. 10

Solution 8

Correct option: (d)

x	P(x)	$x^2P(x)$
1	$\frac{1}{10}$	$\frac{1}{10}$
2	$\frac{1}{5}$	$\frac{4}{5}$
3	$\frac{3}{10}$	$\frac{27}{10}$
4	$\frac{2}{5}$	$\frac{32}{5}$
		$E(X^2) = 10$

Question 9

Let X be a discrete random variable. Then the variance of X is

a. $E(X^2)$

b. $E(X^2) + (E(X))^2$

c. $E(X^2) - (E(X))^2$

d. $\sqrt{E(X^2) - (E(X))^2}$

Solution 9

Correct option: (c)

Variance of discrete random variable is always $E(X^2) - (E(X))^2$

Chapter 32 - Mean and variance of a random variable Exercise Ex. 32VSAQ

Question 1

Write the values of 'a' for which the following distribution of probabilities becomes a probability distribution:

$X = x_i$:	-2	-1	0	1
$P(X = x_i)$:	$\frac{1-a}{4}$	$\frac{1+2a}{4}$	$\frac{1-2a}{4}$	$\frac{1+a}{4}$

Solution 1

Here,

X :	-2	-1	0	1
$P(x)$:	$\frac{1-a}{4}$	$\frac{1+2a}{4}$	$\frac{1-2a}{4}$	$\frac{1+a}{4}$

Now, $\sum P(x) = P(-2) + P(-1) + P(0) + P(1)$

$$= \frac{1-a}{4} + \frac{1+2a}{4} + \frac{1-2a}{4} + \frac{1+a}{4}$$

$$= \frac{1-a+1+2a+1-2a+1+a}{4}$$

$$= 1$$

So, the sum of all probablities is equal to 1.

Now, each value of the probability must be positive and less than or equal to 1.

Now,

$0 \le \frac{1-a}{4} \le 1 \Rightarrow 0 \le 1-a \le 4 \Rightarrow -1 \le -a \le 3 \Rightarrow 1 \ge a \ge -3$

$0 \le \frac{1+2a}{4} \le 1 \Rightarrow 0 \le 1+2a \le 4 \Rightarrow -1 \le 2a \le 3 \Rightarrow -\frac{1}{2} \le a \le \frac{3}{2}$

$0 \le \frac{1-2a}{4} \le 1 \Rightarrow 0 \le 1-2a \le 4 \Rightarrow -1 \le -2a \le 3 \Rightarrow \frac{1}{2} \ge a \ge \frac{-3}{2}$

$0 \le \frac{1+a}{4} \le 1 \Rightarrow 0 \le 1+a \le 4 \Rightarrow -1 \le a \le 3$

Therefore, $-\frac{1}{2} \le a \le \frac{1}{2}$

Question 2

For what value of k the following distribution is a probability distribution?

$X = x_i$:	0	1	2	3
$P(X = x_i)$:	$2k^4$	$3k^2 - 5k^3$	$2k - 3k^2$	$3k - 1$

Solution 2

Here,

$$X = x_i : \quad 0 \qquad 1 \qquad 2 \qquad 3$$

$$P(X = x_i) : \quad 2k^4 \qquad 3k^2 - 5k^3 \qquad 2k - 3k^2 \qquad 3k - 1$$

Since $\sum P(x) = 1$

$\Rightarrow \quad P(0) + P(1) + P(2) + P(3) = 1$

$\Rightarrow \quad 2k^4 + 3k^2 - 5k^3 + 2k - 3k^2 + 3k - 1 = 1$

$\Rightarrow \quad 2k^4 - 5k^3 + 5k - 2 = 0$

$\Rightarrow \quad 2k^4 - 2k^3 - 3k^3 + 3k^2 - 3k^2 + 3k + 2k - 2 = 0$

$\Rightarrow \quad 2k^3(k-1) - 3k^2(k-1) - 3k(k-1) + 2(k-1) = 0$

$\Rightarrow \quad (k-1)(2k^3 - 3k^2 - 3k + 2) = 0$

$\Rightarrow \quad (k-1)\left[2k^3 - k^2 - 2k^2 + k - 4k + 2\right] = 0$

$\Rightarrow \quad (k-1)\left[k^2(2k-1) - k(2k-1) - 2(2k-1)\right] = 0$

$\Rightarrow \quad (k-1)(2k-1)(k^2 - k - 2) = 0$

$\Rightarrow \quad (k-1)(2k-1)(k-2)(k+1) = 0$

$\Rightarrow \quad k = 1, \frac{1}{2}, 2, -1$

only $k = \frac{1}{2}$ is possible

Question 3

If X denotes the number on the upper face of a cubical die when it is thrown, find the mean of X.

Solution 3

Here, X denote upper face of a die ,So

X can have values 1,2,3,4,5,6

x_i	p_i	$x_i \, p_i$
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$
		$\Sigma xp = \frac{21}{6}$

Mean = $\sum XP$

$= \frac{21}{6}$

Mean = 3.5

Question 4

If the probability distribution of a random variable X is given by

$X = x_i$:	1	2	3	4
$P(X = x_i)$:	$2k$	$4k$	$3k$	k

Write the value of k.

Solution 4

Here,

$X = x_i$:	1	2	3	4
$P(X = x_i)$:	$2k$	$4k$	$3k$	k

Since $\sum P(x) = 1$

$\Rightarrow \quad P(1) + P(2) + P(3) + P(4) = 1$

$\Rightarrow \quad 2k + 4k + 3k + k = 1$

$\Rightarrow \quad 10k = 1$

$\Rightarrow \quad k = \frac{1}{10}$

$\Rightarrow \quad k = 0.1$

Question 5

Find the mean of the following probability distribution:

$X = x_i$:	1	2	3
$P(X = x_i)$:	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{8}$

Solution 5

Here,

X :	1	2	3
$P(X)$:	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{8}$
XP :	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{15}{8}$

$\Sigma P(x) = \frac{1}{4} + \frac{2}{8} + \frac{15}{8}$

Mean = $\frac{19}{8}$

Question 6

If the probability distribution of a random variable X is as given below:

$X = x_i$:	1	2	3	4
$P(X = x_i)$:	c	$2c$	$4c$	$4c$

Write the value of $P(X \le 2)$.

Solution 6

Here,

$X = x_i$:	1	2	3	4
$P(X = x_i)$:	c	$2c$	$4c$	$4c$

Since $\sum P(x) = 1$

$\Rightarrow \quad c + 2c + 4c + 4c = 1$

$\Rightarrow \quad 11c = 1$

$\Rightarrow \quad c = \frac{1}{11}$

$P(X \le 2)$

$= P(X = 1) + P(X = 2)$

$= c + 2c$

$= 3c$

$= \frac{3}{11}$

$P(X \le 2) = \frac{3}{11}$

$\Rightarrow P(x \le 2) = 0.3$

Question 7

A random variable has the following probability distribution:

$X = x_i$:	1	2	3	4
$P(X = x_i)$:	k	$2k$	$3k$	$4k$

Write the value of $P(X \ge 3)$.

Solution 7

Here,

$X = x_i$:	1	2	3	4
$P(X = x_i)$:	k	$2k$	$3k$	$4k$

Since $\sum P(x) = 1$

$\Rightarrow \quad P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) = 1$

$\Rightarrow \quad k + 2k + 3k + 4k = 1$

$\Rightarrow \quad 10k = 1$

$\Rightarrow \quad k = \frac{1}{10}$

$P(X \geq 3)$

$= P(X = 3) + P(X = 4)$

$= 3k + 4k$

$= 7k$

$= \frac{7}{10}$

$P(X \geq 3) = \frac{7}{10}$