Access the RD Sharma Solutions For Class 10 Chapter 6 – Trigonometric Identities

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Prove the following trigonometric identities:

1. $(1 - \cos^2 A) \operatorname{cosec}^2 A = 1$

Solution:

Taking the L.H.S,

$(1 - \cos^2 A) \operatorname{cosec}^2 A$

$= (\sin^2 A) \operatorname{cosec}^2 A$ [$\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A$]

$= 1^2$

$= 1$ = R.H.S

– Hence Proved

2. $(1 + \cot^2 A) \sin^2 A = 1$

Solution:

By using the identity,

$\operatorname{cosec}^2 A - \cot^2 A = 1 \Rightarrow \operatorname{cosec}^2 A = \cot^2 A + 1$

Taking,

L.H.S $= (1 + \cot^2 A) \sin^2 A$

$= \operatorname{cosec}^2 A \sin^2 A$

$= (\operatorname{cosec} A \sin A)^2$

$= ((1/\sin A) \times \sin A)^2$

$= (1)^2$

$= 1$

= R.H.S

– Hence Proved

3. $\tan^2\theta \cos^2\theta = 1 - \cos^2\theta$

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

Taking,

L.H.S $= \tan^2\theta \cos^2\theta$

$= (\tan\theta \times \cos\theta)^2$

$= (\sin\theta)^2$

$= \sin^2\theta$

$= 1 - \cos^2\theta$

= R.H.S

– Hence Proved

4. $\text{cosec}\ \theta\ \sqrt{(1 - \cos^2 \theta)} = 1$

Solution:

Using identity,

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

Taking L.H.S,

L.H.S = $\text{cosec}\ \theta\ \sqrt{(1 - \cos^2 \theta)}$

= $\text{cosec}\ \theta\ \sqrt{(\sin^2\theta)}$

= $\text{cosec}\ \theta \times \sin\theta$

= 1

= R.H.S

– Hence Proved

5. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1) = 1$

Solution:

Using identities,

$(\sec^2\theta - \tan^2\theta) = 1$ and $(\text{cosec}^2\theta - \cot^2\theta) = 1$

We have,

L.H.S = $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

= $\tan^2\theta \times \cot^2\theta$

= $(\tan \theta \times \cot \theta)^2$

= $(\tan \theta \times 1/\tan \theta)^2$

= 1^2

= 1

= R.H.S

– Hence Proved

6. $\tan \theta + 1/ \tan \theta = \sec \theta\ \text{cosec}\ \theta$

Solution:

We have,

L.H.S = $\tan \theta + 1/ \tan \theta$

= $(\tan^2 \theta + 1)/ \tan \theta$

= $\sec^2 \theta / \tan \theta$ [$\because \sec^2\theta - \tan^2\theta = 1$]

= $(1/\cos^2 \theta) \times 1/ (\sin \theta/\cos \theta)$ [$\because \tan \theta = \sin \theta / \cos \theta$]

= $\cos \theta/ (\sin \theta \times \cos^2 \theta)$

= $1/ \cos \theta \times 1/ \sin \theta$

= $\sec \theta \times \text{cosec}\ \theta$

= $\sec \theta\ \text{cosec}\ \theta$

= R.H.S

– Hence Proved

7. cos θ/ (1 – sin θ) = (1 + sin θ)/ cos θ

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1+ sin θ), we get

$$\frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta}{(1+\sin\theta)(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{(1+\sin\theta)}{\cos\theta}$$

L.H.S =

= R.H.S

– Hence Proved

8. **cos θ/ (1 + sin θ) = (1 – sin θ)/ cos θ**

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1- sin θ), we get

$$\frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(\cos^2\theta)}$$

$$= \frac{(1-\sin\theta)}{\cos\theta}$$

$$= \frac{(1-\sin\theta)}{\cos\theta}$$

L.H.S =

= R.H.S

– Hence Proved

9. $\cos^2\theta + 1/(1 + \cot^2\theta) = 1$

Solution:

We already know that,

$\text{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$

Taking L.H.S,

$$\text{L.H.S} = \cos^2 A + \frac{1}{1+\cot^2 A}$$

$$= \cos^2 A + \frac{1}{\text{cosec}^2 A}$$

$$= \cos^2 A + \left(\frac{1}{\text{cosec}\, A}\right)^2$$

$= \cos^2 A + \sin^2 A$

$= 1$

= R.H.S

– Hence Proved

10. $\sin^2 A + 1/(1 + \tan^2 A) = 1$

Solution:

We already know that,

$\sec^2\theta - \tan^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$

Taking L.H.S,

$$\text{L.H.S} = \sin A^2 + \frac{1}{1+\tan^2 A}$$

$$= \sin A^2 + \frac{1}{\sec^2 A}$$

$$= \sin A^2 + \left(\frac{1}{\sec A}\right)^2$$

= $\sin^2 A + \cos^2 A$

= 1

= R.H.S

– Hence Proved

11.

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

Solution:

We know that, $\sin^2\theta + \cos^2\theta = 1$

Taking the L.H.S,

$$\text{L.H.S} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

= R.H.S

– Hence Proved

12. 1 – cos θ/ sin θ = sin θ/ 1 + cos θ

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1+ cos θ), we get

$$\text{L.H.S} = \frac{1-\cos^2\theta}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin\theta)}{(1+\cos\theta)}$$

= R.H.S

– Hence Proved

13. sin θ/ (1 – cos θ) = cosec θ + cot θ

Solution:

Taking L.H.S,

$$\text{L. H. S} = \frac{\sin\theta}{1-\cos\theta}$$

On multiplying by its conjugates, we have

$$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

Since, $(1 - \cos^2\theta) = \sin^2\theta$

$$= \frac{\sin\theta + (\sin\theta \times \cos\theta)}{\sin^2\theta}$$

$$= \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta \times \cos\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

= cosec θ + cot θ

= R.H.S

– Hence Proved

14. (1 – sin θ) / (1 + sin θ) = (sec θ – tan θ)^2

Solution:

Taking the L.H.S,

$$\text{L.H.S} = \frac{1-\sin\theta}{1+\sin\theta}$$

On multiplying by its conjugate, we have

$$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

Since, $1 - \sin^2\theta = \cos^2\theta$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$= (\sec\theta - \tan\theta)^2$

= R.H.S

– Hence Proved

15. $$\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$$

Solution:

Taking L.H.S,

$$\text{L.H.S} = \frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}$$

Here, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$= \frac{\operatorname{cosec}^2\theta \times \tan\theta}{\sec^2\theta}$$

$$= \frac{1}{\sin^2\theta} \times \frac{\cos^2\theta}{1} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$

= R.H.S

– Hence Proved

16. $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

Solution:

Taking L.H.S,

L.H.S = $\tan^2\theta - \sin^2\theta$

$$\text{Since, } \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$

$$= \sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$$

= $\tan^2\theta \sin^2\theta$

= R.H.S

– Hence Proved

17. (cosec θ + sin θ)(cosec θ – sin θ) = $\cot^2\theta + \cos^2\theta$

Solution:

Taking L.H.S = (cosec θ + sin θ)(cosec θ – sin θ)

On multiplying we get,

= $\text{cosec}^2\theta - \sin^2\theta$

= $(1 + \cot^2\theta) - (1 - \cos^2\theta)$ [Using $\text{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]

= $1 + \cot^2\theta - 1 + \cos^2\theta$

= $\cot^2\theta + \cos^2\theta$

= R.H.S

– Hence Proved

18. (sec θ + cos θ) (sec θ – cos θ) = $\tan^2\theta + \sin^2\theta$

Solution:

Taking L.H.S = (sec θ + cos θ)(sec θ – cos θ)

On multiplying we get,

= $\sec^2\theta - \sin^2\theta$

= $(1 + \tan^2\theta) - (1 - \sin^2\theta)$ [Using $\sec^2\theta - \tan^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]

= $1 + \tan^2\theta - 1 + \sin^2\theta$

= $\tan^2\theta + \sin^2\theta$

= R.H.S

– Hence Proved

19. sec A(1- sin A) (sec A + tan A) = 1

Solution:

Taking L.H.S = sec A(1 – sin A)(sec A + tan A)

Substituting sec A = 1/cos A and tan A =sin A/cos A in the above we have,

L.H.S = 1/cos A (1 – sin A)(1/cos A + sin A/cos A)

= 1 – sin^2 A / cos^2 A [After taking L.C.M]

= cos^2 A / cos^2 A [∵ 1 – sin^2 A = cos^2 A]

= 1

= R.H.S

– Hence Proved

20. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1

Solution:

Taking L.H.S = (cosec A – sin A)(sec A – cos A)(tan A + cot A)

Putting, $\operatorname{cosec} A = \dfrac{1}{\sin A}, \sec A = \dfrac{1}{\cos A}, \tan A = \dfrac{\sin A}{\cos A}, \cot A = \dfrac{\cos A}{\sin A}$

Subsituting the above in the L.H.S, we get

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right) \left(\frac{1 - \cos^2 A}{\cos A}\right) \left(\frac{\sin^2 A + \cos^2 A}{\sin A \ \cos A}\right)$$

= (cos^2 A/ sin A) (sin^2 A/ cos A) (1/ sin A cos A) [∵ sin^{2}θ + cos^{2}θ = 1]

= sin A x cos A x (1/ cos A sin A)

= R.H.S

– Hence Proved

21. (1 + tan^{2}θ)(1 – sin θ)(1 + sin θ) = 1

Solution:

Taking L.H.S = (1 + tan^{2}θ)(1 – sin θ)(1 + sin θ)

And, we know sin^2 θ + cos^2 θ = 1 and sec^2 θ – tan^2 θ = 1

So,

L.H.S = (1 + tan^2 θ)(1 – sin θ)(1 + sin θ)

= (1 + tan^{2}θ){(1 – sin θ)(1 + sin θ)}

= (1 + tan^{2}θ)(1 – sin^{2}θ)

= sec^{2}θ (cos^2 θ)

= (1/ cos^2 θ) x cos^2 θ

= 1

= R.H.S

– Hence Proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

Solution:

We know that,

$\cot^2 A = \cos^2 A/ \sin^2 A$ and $\tan^2 A = \sin^2 A/\cos^2 A$

Substituting the above in L.H.S, we get

L.H.S = $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A$

= $\{\sin^2 A (\cos^2 A/ \sin^2 A)\} + \{\cos^2 A (\sin^2 A/\cos^2 A)\}$

= $\cos^2 A + \sin^2 A$

= 1 [$\because \sin^2\theta + \cos^2\theta = 1$]

= R.H.S

– Hence Proved

$$\text{(i)}\ \cot\theta - \tan\theta = \frac{2\cos 2\theta - 1}{\sin\theta * \cos\theta}$$

$$\text{(ii)}\ \tan\theta - \cot\theta = \left(\frac{2\sin^2\theta - 1}{\sin\theta * \cos\theta}\right)$$

23.

Solution:

(i) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S = $\cot\theta - \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \ x \cos\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta \ x \cos\theta}$$

$$= \frac{\cos^2\theta - 1 - \cos^2\theta}{\sin\theta \ x \cos\theta}$$

$$= \left(\frac{2\cos^2\theta - 1}{\sin\theta \ x \cos\theta}\right)$$

= R.H.S

– Hence Proved

(ii) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S = tan θ – cot θ

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 + \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \left(\frac{2\sin^2\theta - 1}{\sin\theta \quad \cos\theta}\right)$$

= R.H.S

– Hence Proved

24. ($\cos^2\theta$/ sin θ) – cosec θ + sin θ = 0

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta - 1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{-\sin^2\theta}{\sin\theta}\right) + \sin\theta$$

= – sin θ + sin θ

= 0

= R.H.S

- Hence proved

$$\frac{1}{1+\sin A}+\frac{1}{1-\sin A}=2\sec^2 A$$ **25.**

Solution:

Taking L.H.S,

$$\text{LHS}=\frac{1}{1+\sin A}+\frac{1}{1-\sin A}$$

$$=\frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)}$$

$$=\frac{1-\sin A+1+\sin A}{1-\sin^2 A} \quad \because (1+\sin A)(1-\sin A)=1-\sin^2 A$$

$$=\frac{2}{1-\sin^2 A}$$

$$=\frac{2}{\cos^2 A} \quad [\because 1-\sin^2 A=\cos A]$$

= 2 $\sec^2$ A

= R.H.S

- Hence proved

$$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

26.

Solution:

Taking the LHS and using $\sin^2\theta+\cos^2\theta=1$, we have

$$\text{LHS}=\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}$$

$$=\frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$=\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

= 2/ cos θ

= 2 sec θ

= R.H.S

- Hence proved

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$$

27.

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L.H.S}=\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}$$

$$=\frac{(1+2\sin\theta+\sin^2\theta)+(1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}$$

$$=\frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}$$

$$=\frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$=\frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$$

$$=\frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$$

= R.H.S

- Hence proved

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}=\left[\frac{1-\tan\theta}{\cot\theta}\right]^2=\tan^2\theta$$

28.

Solution:

Taking L.H.S,

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}$$

Using $\sec^2\theta - \tan^2\theta = 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

$$= \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\text{cosec}^2\theta}$$

$$= \frac{1}{\cos^2\theta \quad 1}\sin^2\theta = \tan^2\theta$$

= R.H.S

And, taking

$$\left[\frac{1-\tan\theta}{\cot\theta}\right]^2 =$$

$$\frac{1+tan^2\theta - 2tan\theta}{1+cot^2\theta - 2cot\theta}$$

=

$$\frac{sec^2\theta - 2tan\theta}{cosec^2\theta - 2cot\theta}$$ [Using $\sec^2\theta - \tan^2\theta = 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$]

=

$$\frac{\frac{1}{\cos^2\theta} - \frac{2\sin\theta}{\cos\theta}}{\frac{1}{\sin^2\theta} - \frac{2\cos\theta}{\sin\theta}} = \frac{\frac{1-2\sin\theta\,\cos\theta}{\cos^2\theta}}{\frac{1-2\sin\theta\,.\cos\theta}{\sin^2\theta}}$$

=

$$\frac{sin^2\theta}{cos^2\theta}$$

= $\tan^2\theta$ = R.H.S

- Hence proved

$$\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$$

29.

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

Multiplying by (1 - cos θ) to numerator and denominator

$$\text{LHS} = \frac{1+\sec\theta}{\sec\theta}$$

$$= \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\cos\theta+1}{\cos\theta}\cdot\cos\theta$$

$$= 1+\cos\theta$$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

= R.H.S

- Hence proved

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta$$

30.

Solution:

Taking LHS, we have

$$\text{LHS} = \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\cot\theta}{1 - \tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{\cot\theta}{1 - \tan\theta}$$

$$= \frac{1}{1 - \tan\theta}\left[\frac{1}{\tan\theta} - \tan^2\theta\right]$$

$$= \frac{1}{1 - \tan\theta}\left[\frac{1 - \tan^3\theta}{\tan\theta}\right]$$

$$= \frac{1}{1 - \tan\theta}\ \frac{(1 - \tan\theta)(1 + \tan\theta + \tan^2\theta)}{\tan\theta}\ \left[\text{Since, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$$

$$= \frac{1 + \tan\theta + \tan^2\theta}{\tan\theta}$$

$$= \frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{\tan^2\theta}{\tan\theta}$$

= 1 + tan θ + cot θ

= R.H.S

- Hence proved

31. $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Solution:

From trig. Identities we have,

$\sec^2\theta - \tan^2\theta = 1$

On cubing both sides,

$(\sec^2\theta - \tan^2\theta)^3 = 1$

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta) = 1$

[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]
$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta = 1$

$\Rightarrow \sec^6\theta = \tan^6\theta + 3\sec^2\theta\tan^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

32. $\text{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\text{cosec}^2\theta + 1$

Solution:

From trig. Identities we have,

$\text{cosec}^2\theta - \cot^2\theta = 1$

On cubing both sides,

$(\text{cosec}^2\theta - \cot^2\theta)^3 = 1$

$\text{cosec}^6\theta - \cot^6\theta - 3\text{cosec}^2\theta \cot^2\theta (\text{cosec}^2\theta - \cot^2\theta) = 1$

[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]
$\text{cosec}^6\theta - \cot^6\theta - 3\text{cosec}^2\theta \cot^2\theta = 1$

$\Rightarrow \text{cosec}^6\theta = \cot^6\theta + 3\ \text{cosec}^2\theta \cot^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

$$\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \tan\theta$$ **33.**

Solution:

Taking L.H.S and using $\sec^2\theta - \tan^2\theta = 1 \Rightarrow 1 + \tan^2\theta = \sec^2\theta$

$$\begin{aligned} \text{LHS} &= \frac{\sec^2\theta \cdot \cot\theta}{\text{cosec}^2\theta} \\ &= \frac{1 \cdot \sin^2\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin\theta}{\cos\theta} = \tan\theta \end{aligned}$$

= R.H.S

- Hence proved

$$\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$$ **34.**

Solution:

Taking L.H.S and using the identity $\sin^2A + \cos^2A = 1$, we get

$\sin^2A = 1 - \cos^2A$

$\Rightarrow \sin^2A = (1 - \cos A)(1 + \cos A)$

$$\text{LHS} = \frac{1+\cos A}{(1-\cos A)(1+\cos A)}$$

$$= \frac{1}{(1-\cos A)}$$

- Hence proved

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$$ **35.**

Solution:

We have,

$$\text{LHS} = \frac{\sec A - \tan A}{\sec A + \tan A}$$

Rationalizing the denominator and numerator with (sec A + tan A) and using $\sec^2\theta - \tan^2\theta = 1$ we get,

$$= \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec^2 A + \tan^2 A + 2\sec A\tan A)}$$

$$= \frac{1}{\left(\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos A}\right)}$$

$$\Rightarrow \frac{\cos^2 A}{1 + \sin^2 A + 2\sin A}$$

$$= \frac{\cos^2 A}{(1+\sin A)^2}$$

= R.H.S

- Hence proved

$$1 + \frac{\cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

36.

Solution:

We have,

$$\text{LHS} = \frac{1+\cos A}{\sin A}$$

On multiplying numerator and denominator by (1 – cos A), we get

$$= \frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin A}{1-\cos A}$$

= R.H.S

- Hence proved

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

37. (i)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with √(1 + sin A), we get

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1+\sin A)}{\cos A}}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

= R.H.S

- Hence proved

$$\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}} = 2\ \text{cosec}\ A$$

(ii)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(\sin^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(\sin^2 A)}}$$

$$= \frac{(1-\cos A)}{(\sin A)} + \frac{(1+\cos A)}{(\sin A)}$$

$$= \frac{(1-\cos A+1+\cos A)}{(\sin A)}$$

$$= \frac{(2)}{(\sin A)}$$

= 2 cosec A

= R.H.S

- Hence proved

38. Prove that:

$$\sqrt{\frac{(\sec\theta-1)}{(\sec\theta+1)}} + \sqrt{\frac{(\sec\theta+1)}{(\sec\theta-1)}} = 2\operatorname{cosec}\theta$$

(i)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(\sec\theta - 1)(\sec\theta - 1)}{(\sec\theta + 1)(\sec\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)(\sec\theta + 1)}{(\sec\theta - 1)(\sec\theta + 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{(\sec^2\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)^2}{(\sec^2\theta - 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$$

$$= \frac{(\sec\theta - 1)}{\tan\theta} + \frac{(\sec\theta + 1)}{\tan\theta}$$

$$= \frac{(\sec\theta - 1 + \sec\theta + 1)}{\tan\theta}$$

$$= \frac{(2\cos\theta)}{\cos\theta\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

= R.H.S

- Hence proved

$$\sqrt{\frac{(1 + \sin\theta)}{(1 - \sin\theta)}} + \sqrt{\frac{(1 - \sin\theta)}{(1 + \sin\theta)}} = 2\sec\theta$$

(ii)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{(1-\sin^2\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(\cos^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{(\cos^2\theta)}}$$

$$= \frac{(1+\sin\theta)}{(\cos\theta)} + \frac{(1-\sin\theta)}{(\cos\theta)}$$

$$= \sqrt{\frac{(1+\sin\theta+1-\sin\theta)}{(\cos\theta)}}$$

$$= \frac{(2)}{(\cos\theta)} = 2\sec\theta$$

= R.H.S

- Hence proved

$$\sqrt{\frac{(1+\cos\theta)}{(1-\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}} = 2\ \mathrm{cosec}\,\theta$$ **(iii)**

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} + \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(\sin^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(\sin^2\theta)}}$$

$$= \frac{(1-\cos\theta)}{(\sin\theta)} + \frac{(1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(1-\cos\theta+1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(2)}{(\sin\theta)}$$

= 2 cosec θ

= R.H.S

- Hence proved

$$\frac{\sec\theta-1}{\sec\theta+1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$$ **(iv)**

Solution:

Taking L.H.S, we have

$$= \frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

On multiplying numerator and denominator by $1 + \cos\theta$, we get

$$= \frac{(1 - \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 + \cos\theta)}$$

$$= \frac{(1 - \cos^2\theta)}{(1 + \cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1 + \cos\theta)^2}$$

$$= \left(\frac{\sin\theta}{1 + \cos\theta}\right)^2$$

= R.H.S

- Hence proved

$$(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$$

39.

Solution:

Taking LHS = (sec A – tan A)^2 , we have

$$= \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right]^2$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A}$$

$$= \frac{(1 - \sin A)^2}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{(1 - \sin A)}{(1 + \sin A)}$$

= R.H.S

- Hence proved

$$\frac{1-\cos A}{1+\cos A}=(\cot A-\operatorname{cosec} A)^2$$

40.

Solution:

Taking L.H.S and rationalizing the numerator and denominator with (1 – cos A), we get

$$=\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$=\frac{(1-\cos A)^2}{(1-\cos^2 A)}$$

$$=\frac{(1-\cos A)^2}{(\sin^2 A)}$$

$$=\left(\frac{1}{\sin A}-\frac{\cos A}{\sin A}\right)^2$$

= $(\operatorname{cosec} A - \cot A)^2$

= $(\cot A - \operatorname{cosec})^2$

= R.H.S

- Hence proved

$$\frac{1}{\sec A-1}+\frac{1}{\sec A+1}=2\operatorname{cosec} A\cot A$$

41.

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)}$$

$$= \frac{2 \sec A}{(\sec^2 A - 1)}$$

$$= \frac{2 \sec A}{(\tan^2 A)}$$

$$= \frac{2 \cos^2 A}{(\cos A \sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin A)(\sin A)}$$

= 2 cosec A cot A = RHS

- Hence proved

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

42.

Solution:

Taking LHS, we have

$$= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

= cos A + sin A

= RHS

- Hence proved

$$\frac{(\text{cosec A})}{(\text{cosec A} - 1)} + \frac{(\text{cosec A})}{(\text{cosec A} + 1)} = 2\sec^2 A$$ **43.**

Solution:

Considering L.H.S and taking L.C.M and on simplifying we have,

$$= \frac{(\text{cosec A})(\text{cosec A} + 1 + \text{cosec A} - 1)}{(\text{cosec}^2 A - 1)}$$

$$= \frac{(2\,\text{cosec}^2 A)}{\cot^2 A}$$

$$= \frac{2\sin^2 A}{\sin^2 A.\ \cos^2 A}$$

$$= \frac{2}{\cos^2 A}$$

= 2 $\sec^2$A

= RHS

- Hence proved

RD Sharma Class 10 Chapter 6 Exercise 6.2 Page No: 6.54

1. If cos θ = 4/5, find all other trigonometric ratios of angle θ.

Solution:

We have,

cos θ = 4/5

And we know that,

sin θ = √(1 – $\cos^2$θ)

⇒ sin θ = √(1 – $(4/5)^2$)

= √(1 – (16/25))

= √[(25 – 16)/25]

= √(9/25)

= 3/5

∴ sin θ = 3/5

Since, cosec θ = 1/ sin θ

= 1/ (3/5)

$\Rightarrow$ cosec θ = 5/3

And, sec θ = 1/ cos θ
= 1/ (4/5)

$\Rightarrow$ cosec θ = 5/4

Now,
tan θ = sin θ/ cos θ
= (3/5)/ (4/5)

$\Rightarrow$ tan θ = 3/4

And, cot θ = 1/ tan θ
= 1/ (3/4)

$\Rightarrow$ cot θ = 4/3

2. If sin θ = 1/√2, find all other trigonometric ratios of angle θ.
Solution:

We have,
sin θ = 1/√2
And we know that,
cos θ = $\sqrt{(1 - \sin^2 \theta)}$

$\Rightarrow$ cos θ = $\sqrt{(1 - (1/\sqrt{2})^2)}$

= √(1 – (1/2))
= √[(2 – 1)/2]
= √(1/2)
= 1/√2

∴ cos θ = 1/√2

Since, cosec θ = 1/ sin θ
= 1/ (1/√2)

⇒ cosec θ = √2

And, sec θ = 1/ cos θ

= 1/ (1/√2)

⇒ cosec θ = √2

Now,

tan θ = sin θ/ cos θ

= (1/√2)/ (1/√2)

⇒ tan θ = 1

And, cot θ = 1/ tan θ

= 1/ (1)

⇒ cot θ = 1

$$\text{If } \tan\theta = \frac{1}{\sqrt{2}}, \text{find the value of } \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta}.$$

3.

Solution:

Given,

tan θ = 1/√2

By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} \quad = \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2} \quad = \sqrt{1+\frac{1}{2}} \quad = \sqrt{\frac{3}{2}}$$

And,

$$\cot\theta = \frac{1}{\tan\theta} \quad = \frac{1}{\frac{1}{\sqrt{2}}} \quad = \sqrt{2}$$

From identity, we have

$$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} \ = \sqrt{1+2} \ = \sqrt{3}$$

Substituting the values, we get

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta} \quad = \frac{\left(\sqrt{3}\right)^2 - \left(\sqrt{\frac{3}{2}}\right)^2}{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5} = \frac{3}{10}$$

If $\tan\theta = \frac{3}{4}$, find the value of $\frac{1-\cos\theta}{1+\cos\theta}$

4.

Solution:

Given,

$\tan\theta = 3/4$

By using $\sec^2\theta - \tan^2\theta = 1$,

$$\sec\theta = \sqrt{1+\tan^2\theta} \quad = \sqrt{1+\left(\frac{3}{4}\right)^2} \quad = \sqrt{1+\frac{9}{16}} \quad = \sqrt{\frac{16+9}{16}} \quad = \sqrt{\frac{25}{16}}$$

sec $\theta = 5/4$

Since, $\sec\theta = 1/\cos\theta$

$\Rightarrow \cos\theta = 1/\sec\theta$

$= 1/(5/4)$

$= 4/5$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \cot^2\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}$$

So,

$$\text{If } \tan\theta = \frac{12}{5}, \text{find the value of } \frac{1+\sin\theta}{1-\sin\theta}$$

5.

Solution:

Given, tan θ = 12/5

Since, cot θ = 1/ tan θ = 1/ (12/5) = 5/12

Now, by using $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

cosec θ = $\sqrt{(1 + \cot^2\theta)}$

= $\sqrt{(1 + (5/12)^2)}$

= √(1 + 25/144)

= √(169/ 25)

⇒ cosec θ = 13/5

Now, we know that

sin θ = 1/ cosec θ

= 1/ (13/5)

⇒ sin θ = 5/13

Putting value of sin θ in the expression we have,

$$= \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}}$$

= 25/ 1

= 25

$$\text{If } \cot\theta = \frac{1}{\sqrt{3}}, \text{find the value of } \frac{1-\cos^2\theta}{2-\sin^2\theta}$$

6.

Solution:

Given,

cot θ = 1/√3

Using $\operatorname{cosec}^2\theta - \cot^2\theta = 1$, we can find cosec θ cosec θ = √(1 + cot² θ)

= √(1 + (1/√3)²)

= √(1 + (1/3)) = √((3 + 1)/3)

= √(4/3)

⇒ cosec θ = 2/√3

So, sin θ = 1/ cosec θ = 1/ (2/√3)

⇒ sin θ = √3/2

And, we know that

cos θ = √(1 – sin² θ)

= √(1 – (√3/2)²)

= √(1 – (3/4))

= √((4 – 3)/4)

= √(1/4)

⇒ cos θ = 1/2

Now, using cos θ and sin θ in the expression, we have

$$= \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1-\frac{1}{4}}{2-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}}$$

= 3/5

If cosec A $= \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$.

7.

Solution:

Given,

cosec A = √2

Using cosec² A − cot² A = 1, we find cot A

$$\cot A = \sqrt{\operatorname{cosec}^2 A - 1} \quad = \sqrt{(\sqrt{2})^2 - 1} \quad = \sqrt{2-1} \quad = 1$$

So, tan A = 1/ cot A
= 1/ 1 = 1

And, sin A = 1/ cosec A = 1/ $\sqrt{2}$

$$\cos A = \sqrt{1 - \sin^2 A} \quad = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

On substituting we get,

$$= \frac{2\left[\frac{1}{\sqrt{2}}\right]^2 + 3[1]^2}{4\left[[1] - \left[\frac{1}{\sqrt{2}}\right]^2\right]}$$

$$= \frac{2 \times \frac{1}{2} + 3}{4\left[1 - \frac{1}{2}\right]} \Rightarrow \frac{1+3}{4.\frac{1}{2}}$$