RD SHARMA Solutions for Class 9 Maths Chapter 10 - Lines and Angles

Chapter 10 - Lines and Angles Exercise 10.51

Question 1

One angle is equal to three times its supplement. The measure of the angle is

(a) 130°

(b) 135°

(c) 90°

(d) 120°

Solution 1

Let the required angle be θ.

Then, measure of its supplement $= 180° - \theta$

According to question, we have

$\theta = 3(180 - \theta)$

$\Rightarrow \theta = 540^\circ - 3\theta$

$\Rightarrow 4\theta = 540^\circ$

$\Rightarrow \theta = 135^\circ$

Hence, correct option is (b).

Question 2

Two straight lines AB and CD intersect one another at the point O.

If $\angle AOC + \angle COB + \angle BOD = 274^\circ$, then $\angle AOD =$

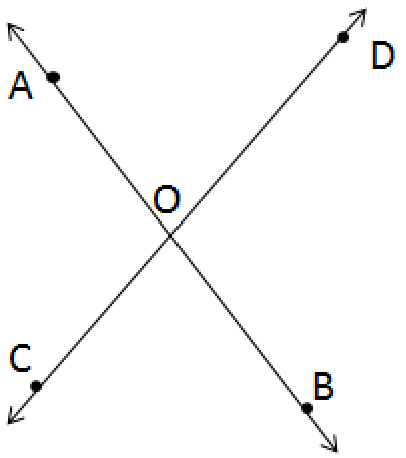
(a) 86°

(b) 90°

(c) 94°

(d) 137°

Solution 2



$\angle AOC + \angle COB + \angle BOD + \angle AOD = 360^\circ$ ....(1)

Now, $\angle AOC + \angle COB + \angle BOD = 274^\circ$ (Given) ....(2)

From (1) and (2),

$274^\circ + \angle AOD = 360^\circ$

$\Rightarrow \angle AOD = 360^\circ - 274^\circ$

$\Rightarrow \angle AOD = 86^\circ$

Hence, correct option is (a).

Question 3

Two straight lines AB and CD cut each other at 'O'.

If $\angle BOD = 63^\circ$, then $\angle BOC =$

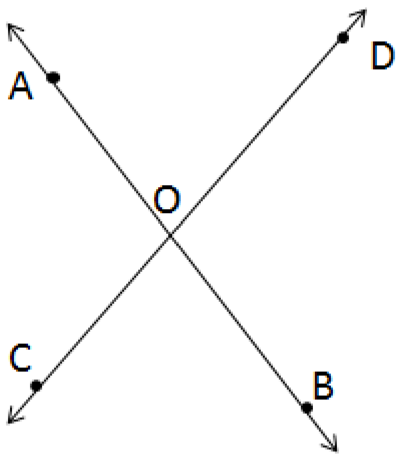
(a) 63°

(b) 117°

(c) 17°

(d) 153°

Solution 3



$\angle BOD$ and $\angle BOC$ form a linear pair.

$\therefore \angle BOD + \angle BOC = 180^\circ$

$\Rightarrow 63^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 117^\circ$

Hence, correct option is (b).

Question 4

Consider the following statements :

when two straight lines intersect :

(i) adjacent angles are complementary

(ii) adjacent angles are supplementary

(iii) opposite angles are equal

(iv) opposite angles are supplementary

Of these statements

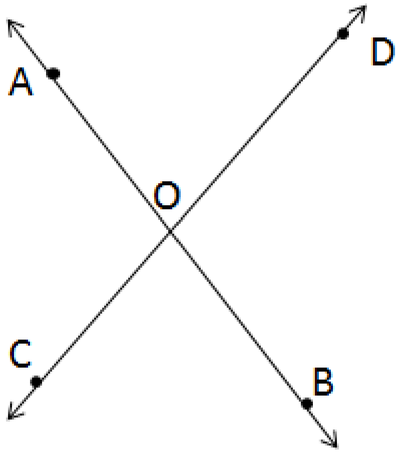
(a) (i) and (iii) are correct

(b) (ii) and (iii) are correct

(c) (i) and (iv) are correct

(d) (ii) and (iv) are correct

Solution 4



Let two lines AB and CD intersect each other at O.
Now we can see from figure any two adjacent angles,
$\angle AOD$ & $\angle DOB$, $\angle DOB$ & $\angle BOC$ etc are supplementary because their sum is $180°$.

$\angle AOD + \angle DOB = 180°$

$\angle DOB + \angle BOC = 180°$

So two adjacent angles are always supplementary.
Now two opposite angle like $\angle AOC$ & $\angle DOB$, $\angle AOD$ & $\angle COB$
are always equal to each other as they are vertically opposite angles

$\angle AOC = \angle DOB$

$\angle AOD = \angle COB$

Hence statement (ii) and (iii) are correct
Hence, correct option is (b).

Question 5

Given $\angle POR = 3x$ and $\angle QOR = 2x + 10°$ If POQ is a straight line then the value of x is

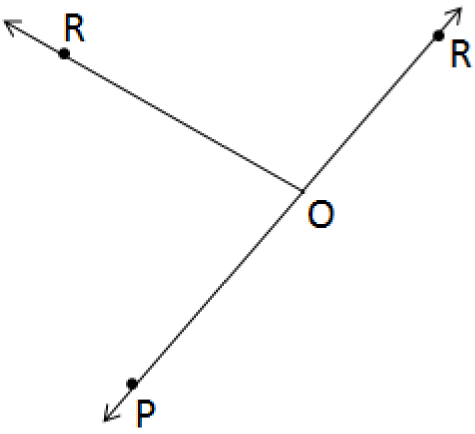
(a) $30°$

(b) $34°$

(c) $36°$

(d) None of these

Solution 5



$\angle POR = 3x$ and $\angle QOR = 2x + 10^\circ$

From figure, we can see that $\angle$ POR & $\angle$ QOR are two adjacent angles and are supplement.

$\Rightarrow \angle POR + \angle QOR = 180^\circ$

$\Rightarrow 3x + 2x + 10^\circ = 180^\circ$

$\Rightarrow 5x = 170^\circ$

$\Rightarrow x = 34^\circ$

Hence, correct option is (b).

Question 6

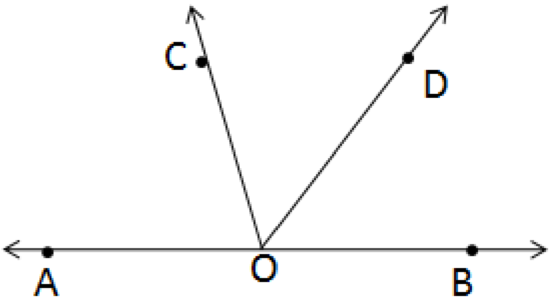
In *figure*, AOB is a straight line. If $\angle AOC + \angle BOD = 85^\circ$, then $\angle COD =$

(a) 85°

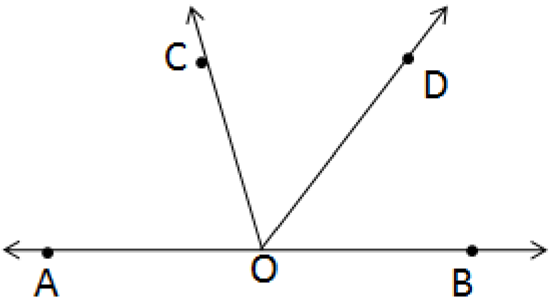
(b) 90°

(c) 95°

(d) 100°



Solution 6



From Figure, we can see

$\angle AOC + \angle COD + \angle BOD = 180^\circ$

But, $\angle AOC + \angle BOD = 85^\circ$ ....(Given)

$\Rightarrow 85^\circ + \angle COD = 180^\circ$

$\Rightarrow \angle COD = 95^\circ$

Hence, correct option is (c).

Question 7

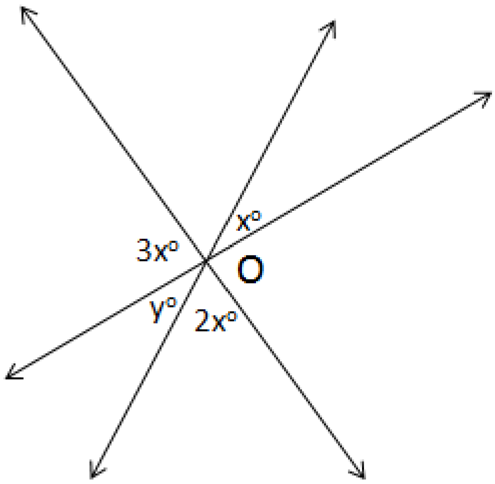
In figure, the value of y is

(a) 20°

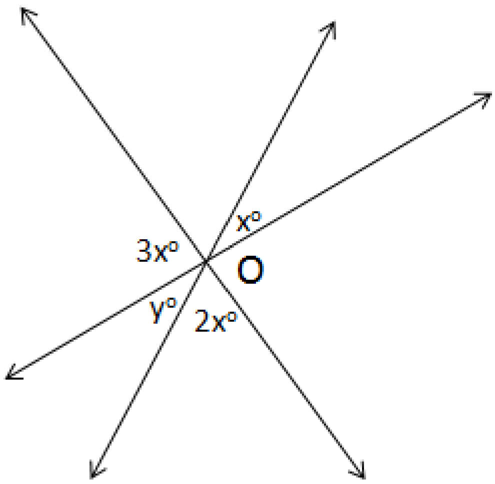
(b) 30°

(c) 45°

(d) 60°



Solution 7



From figure, we can see

$\angle x^\circ = \angle y^\circ$ (vertically opposite angles)

Also, $\angle 3x^\circ + \angle y^\circ + \angle 2x^\circ = 180^\circ$

Now, $\angle x^\circ = \angle y^\circ$

$\therefore \angle 3y^\circ + \angle y^\circ + \angle 2y^\circ = 180^\circ$

$\Rightarrow \angle 6y^\circ = 180^\circ$

$\Rightarrow \angle y^\circ = 30^\circ$

Hence, correct option is (b).

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Question 8

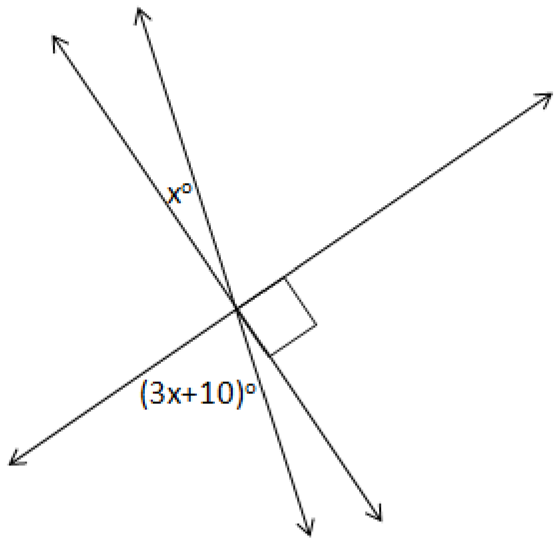
In figure, the value of x is

(a) 12

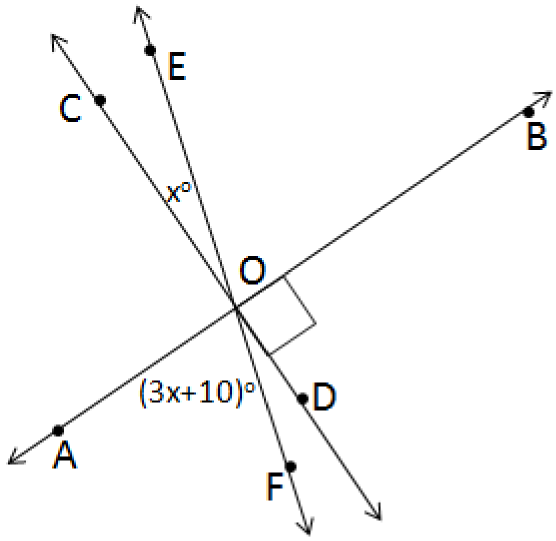
(b) 15

(c) 20

(d) 30



Solution 8



From figure, we can see that

∠BOD + ∠AOD = 180°

∠BOD = 90° (Given)

⇒ ∠AOD = 180° − 90° = 90°

Now, x° = ∠COE = ∠FOD (opposite angles are equal)

Now, ∠AOF + ∠FOD = 90° = ∠AOD

⇒ 3x° + 10° + x° = 90°

⇒ 4x° = 80°

⇒ x° = 20°

Hence, correct opiton is (c).

Question 9

In figure, which of the following statements must be true?

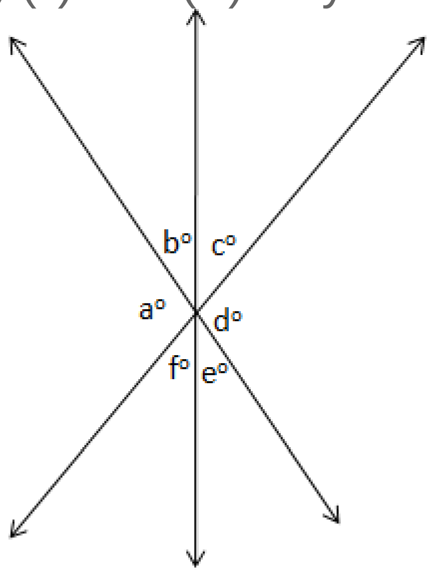
(i) a + b = d + c (ii) a + c + e = 180° (ii) b + f = c + e

(a) (i) only

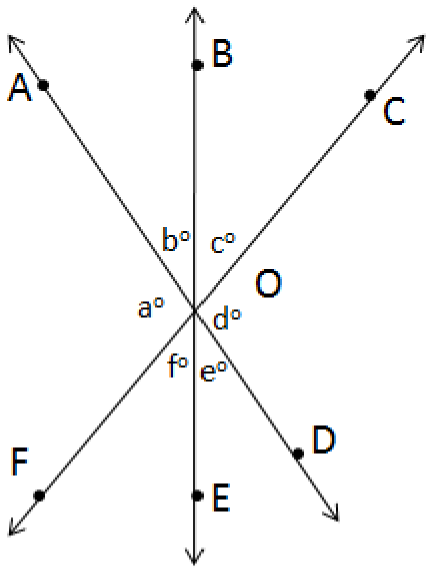
(b) (ii) only

(c) (iii) only

(d) (ii) and (iii) only



Solution 9



From figure, we can see that

$\angle a° + \angle b° + \angle c° = \angle FOC = 180°$

Also, $\angle b° = \angle e°$ (opposite angles)

So, $\angle a° + \angle e° + \angle c° = 180°$

$\Rightarrow$ (ii) is correct

Now, $\angle FOB \neq \angle DOB$

$\Rightarrow \angle a° + \angle b° \neq \angle d° + \angle c°$

$\Rightarrow$ (i) is incorrect

Now, $\angle b° = \angle e°$ and $\angle f° = \angle c°$ (opposite angles are equal)

Thus, $\angle b° + \angle f° = \angle e° + \angle c°$

$\Rightarrow$ (iii) is also correct

Hence, correct option is (d).

Chapter 10 - Lines and Angles Exercise 10.53

Question 10

If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3, then the measure of the larger angle is

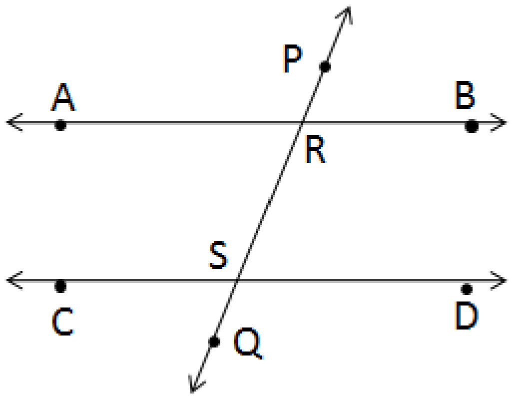
(a) 54°

(b) 120°

(c) 108°

(d) 136°

Solution 10



Let AB and CD are two Parallel lines and PQ is transverce to it.

According to Question,

$$\frac{\angle BRS}{\angle DSR} = \frac{2}{3}$$

$$\Rightarrow \angle BRS = \frac{2}{3}\angle DSR \quad ....(1)$$

Now, $\angle CSR = \angle BRS$ (Alternate angles)

Also, $\angle CSR + \angle DSR = 180°$

$$\Rightarrow \angle BRS + \angle DSR = 180°$$

$$\Rightarrow \frac{2}{3}\angle DSR + \angle DSR = 180°$$

$$\Rightarrow \angle DSR = \frac{\overset{36}{\cancel{180}} \times 3}{\cancel{5}} = 108°$$

$$\Rightarrow \angle BRS = \frac{2}{3} \times 108° = 72°$$

Thus, $\angle DSR = 108°$ and $\angle BRS = 72°$

$\Rightarrow$ Larger angle is $\angle DSR$.

Hence, correct option is (c).

Question 11

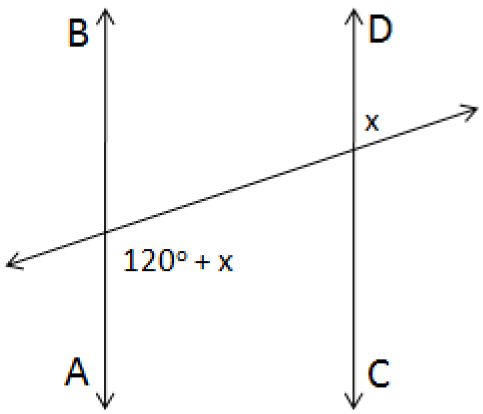
In figure, AB $\parallel$ CD, then the value of x is

(a) $20°$

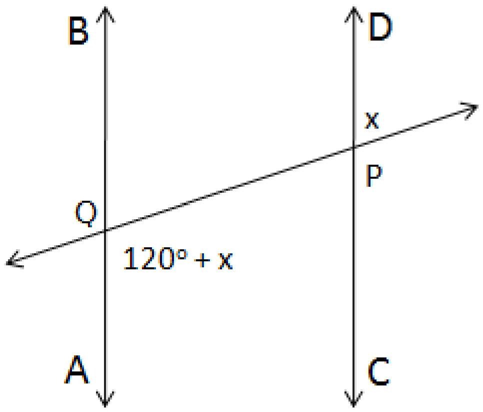
(b) $30°$

(c) $45°$

(d) $60°$



Solution 11



From figure,

$\angle DPQ + \angle x^\circ = 180^\circ$ (linear pair) ....(1)

Also, $\angle DPQ = \angle AQP$ (interior opposite angles)

$\Rightarrow \angle DPQ = 120^\circ + x$

From (1),

$120^\circ + x + x = 180^\circ$

$\Rightarrow 2x = 60^\circ$

$\Rightarrow x = 30^\circ$

Hence, option (b) is correct.

Question 12

Two lines AB and CD intersect at O. If $\angle AOC + \angle COB + \angle BOD = 270^\circ$, then $\angle AOC =$

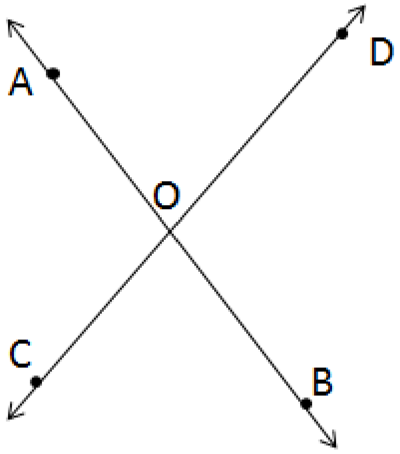
(a) 70°

(b) 80°

(c) 90°

(d) 180°

Solution 12



$\angle AOC + \angle COB + \angle BOD = 270°$ (Given)
From figure,
$\angle AOC + \angle COB + \angle BOD + \angle DOA = 360°$
$\Rightarrow 270° + \angle DOA = 360°$
$\Rightarrow \angle DOA = 360° - 270° = 90°$
Now $\angle DOA + \angle AOC = 180°$
$\Rightarrow \angle AOC = 180° - 90° = 90°$
Hence, correct option is (c).

Question 13

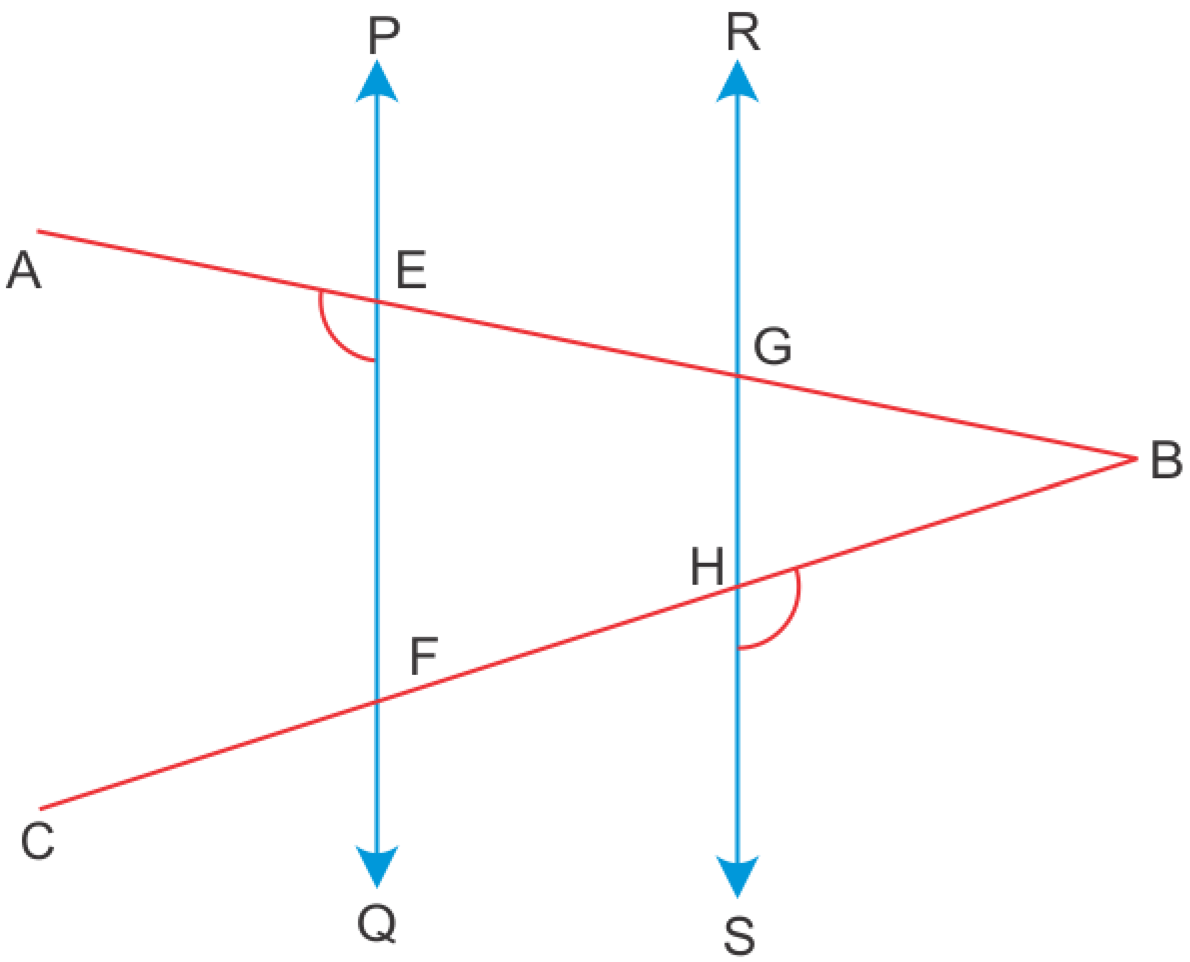
In figure, PQ $\parallel$ RS, $\angle AEF = 95°$, $\angle BHS = 110°$ and $\angle ABC = x°$. Then the value of x is

(a) $15°$

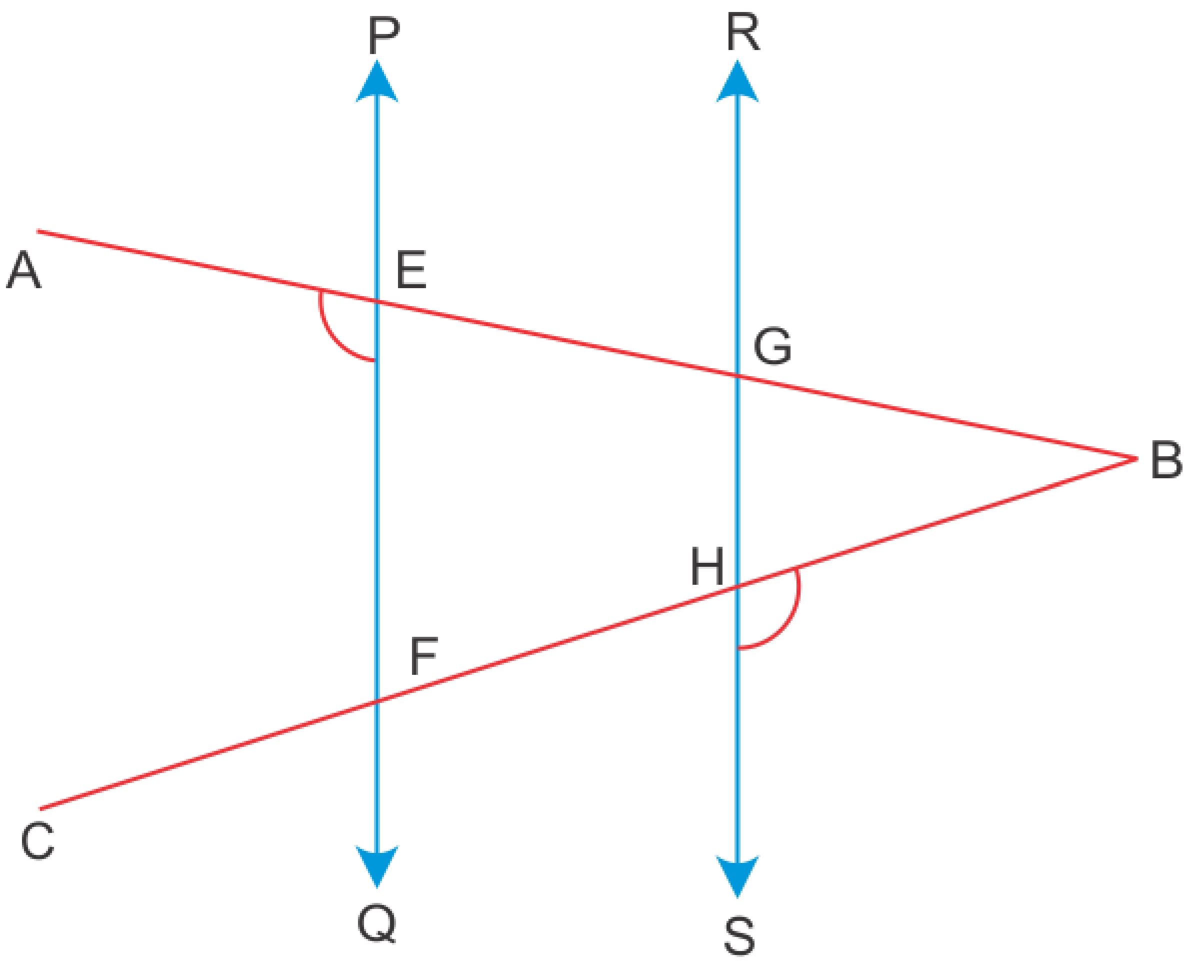
(b) $25°$

(c) $70°$

(d) $35°$



Solution 13



From figure,

$\angle AEF = \angle EGH$ (Corresponding angles)

$\Rightarrow \angle EGH = \angle AEF = 95°$

Also, $\angle BGH + \angle EGH = 180°$

$\Rightarrow \angle BGH = 180° - \angle EGH = 180° - 95° = 85°$

$\angle BHS = 110°$

Now, $\angle BHG + \angle BHS = 180°$

$\Rightarrow \angle BHG = 180° - \angle BHS = 180° - 110° = 70°$

Now, in $\triangle BHG$

$\angle BGH + \angle BHG + x = 180°$ (Sum of all angles of a $\triangle$ is $180°$)

$\Rightarrow 85° + 70° + x° = 180°$

$\Rightarrow x° = 180° - 155° = 25°$

Hence, correct option is (b).

Question 14

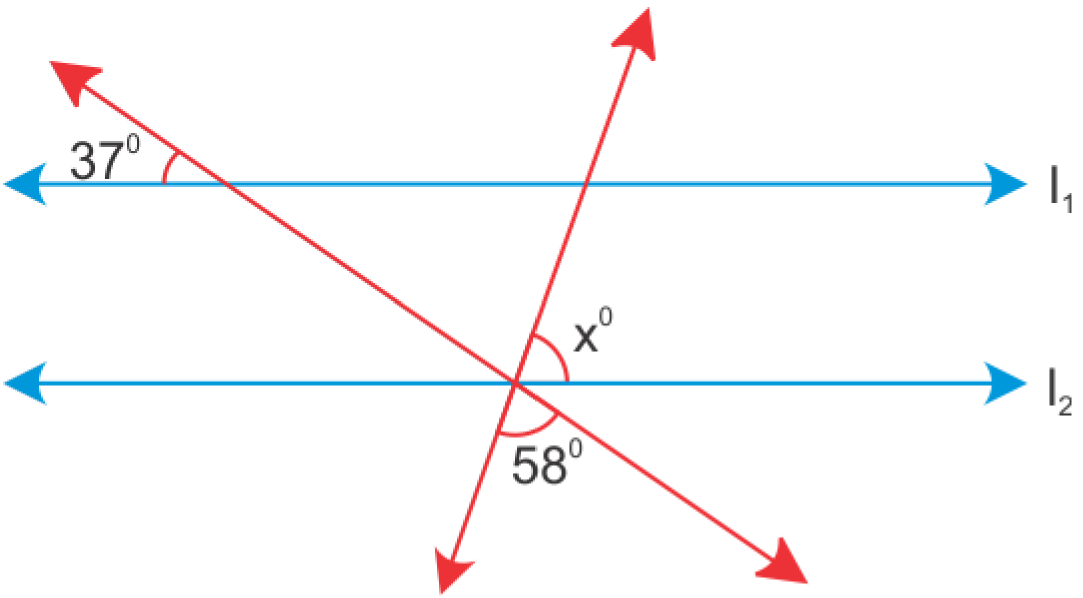
In figure, if $l_1 \parallel l_2$, what is the value of x?

(a) $90°$

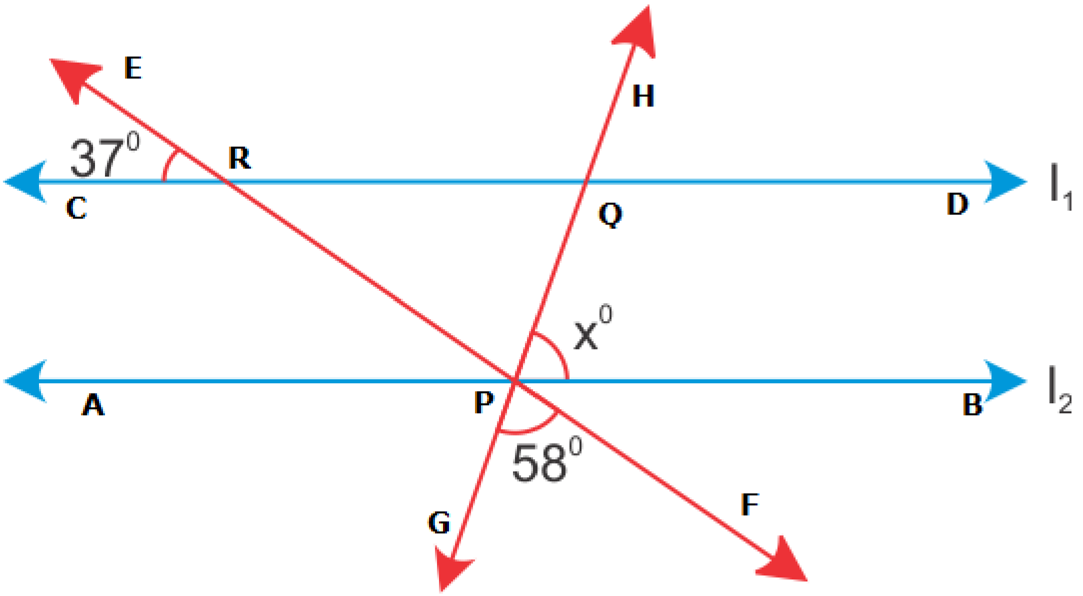
(b) $85°$

(c) $75°$

(d) $70°$



Solution 14



From figure,

$\angle ERC = \angle RPA$ (Correspondence angles are equal)

$\Rightarrow \angle ERC = 37° = \angle RPA$

Also, $\angle RPA = \angle BPF$ (opposite angles)

$\Rightarrow \angle RPA = 37° = \angle BPF$

Now, $\angle QPB + \angle BPF + \angle FPG = 180°$

$\Rightarrow x° + 37° + 58° = 180°$

$\Rightarrow x° = 85°$

Hence, correct option is (b).

Chapter 10 - Lines and Angles Exercise 10.54

Question 15

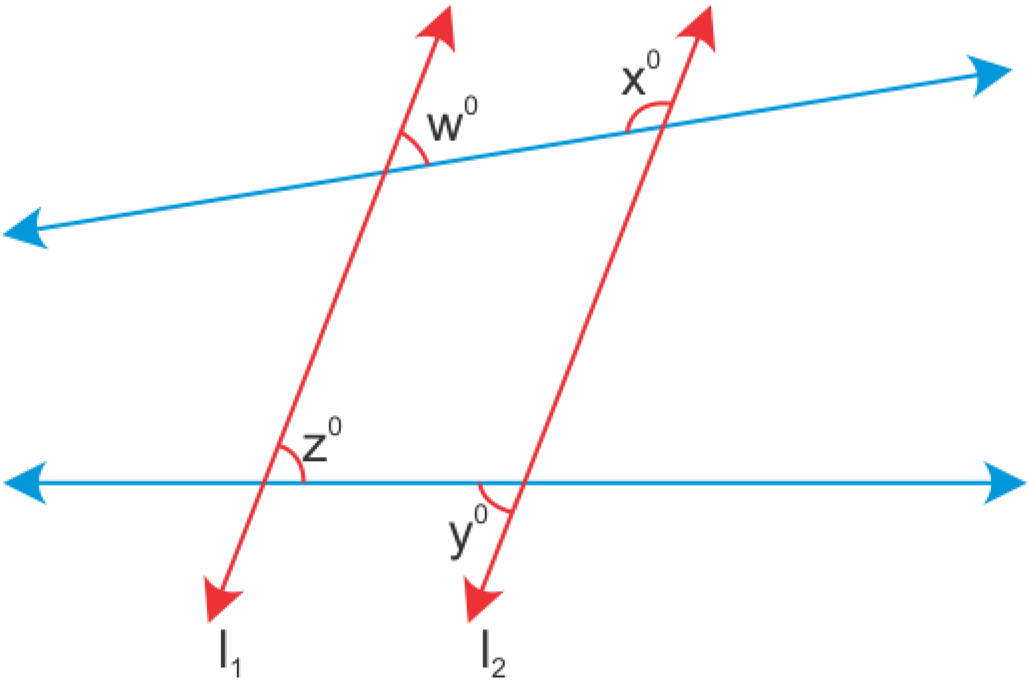
In figure, if $l_1 \parallel l_2$, what is x + y in terms of w and z?

(a) 180 − w + z

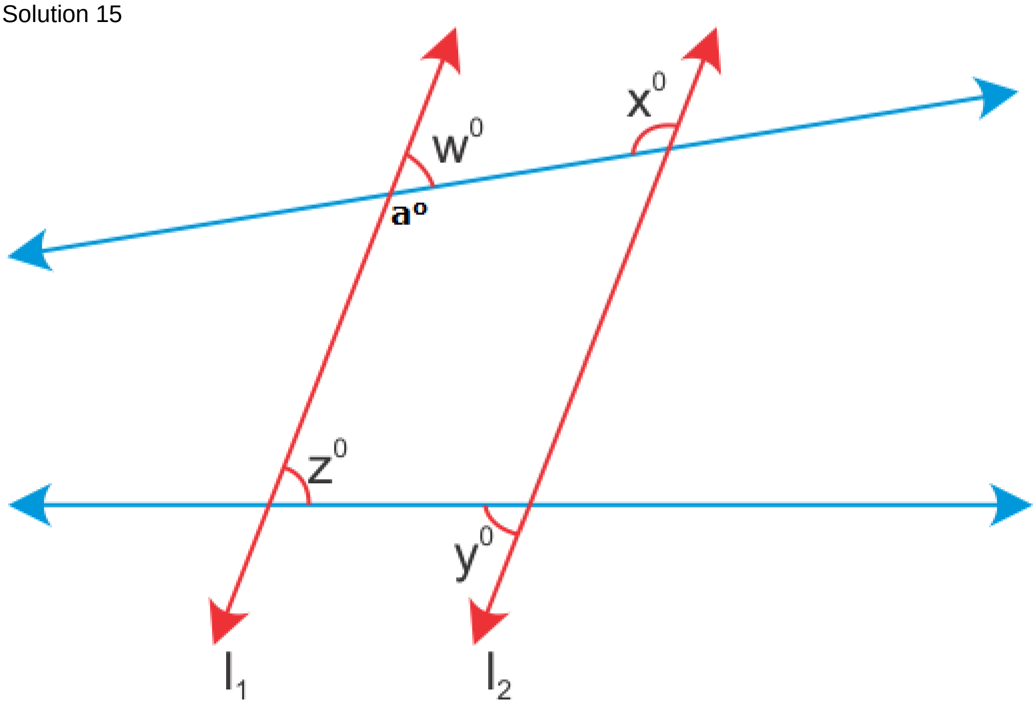
(b) 180 + w − z

(c) 180 − w − z

(d) 180 + w + z



Solution 15



Let angle supplement of w° be a°.

$\Rightarrow a° = 180° - w°$

Now, $a° = x°$ (Alternate opposite angles)

$\Rightarrow x° = 180° - w°$ ....(1)

Now, $y° = z°$ ....(2) (Alternate angles)

Adding (1) and (2), we get

$x° + y° = 180° - w° + z°$

Hence, correct option is (a).

Question 16

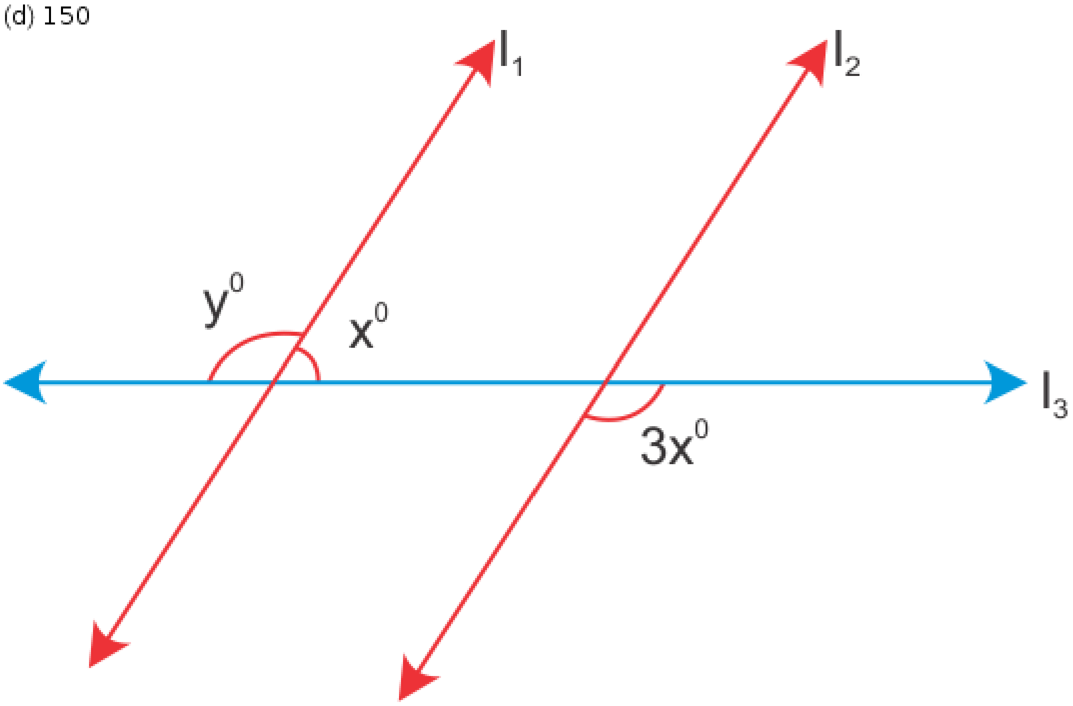
In figure, if $l_1 \parallel l_2$, what is the value of y?

(a) 100

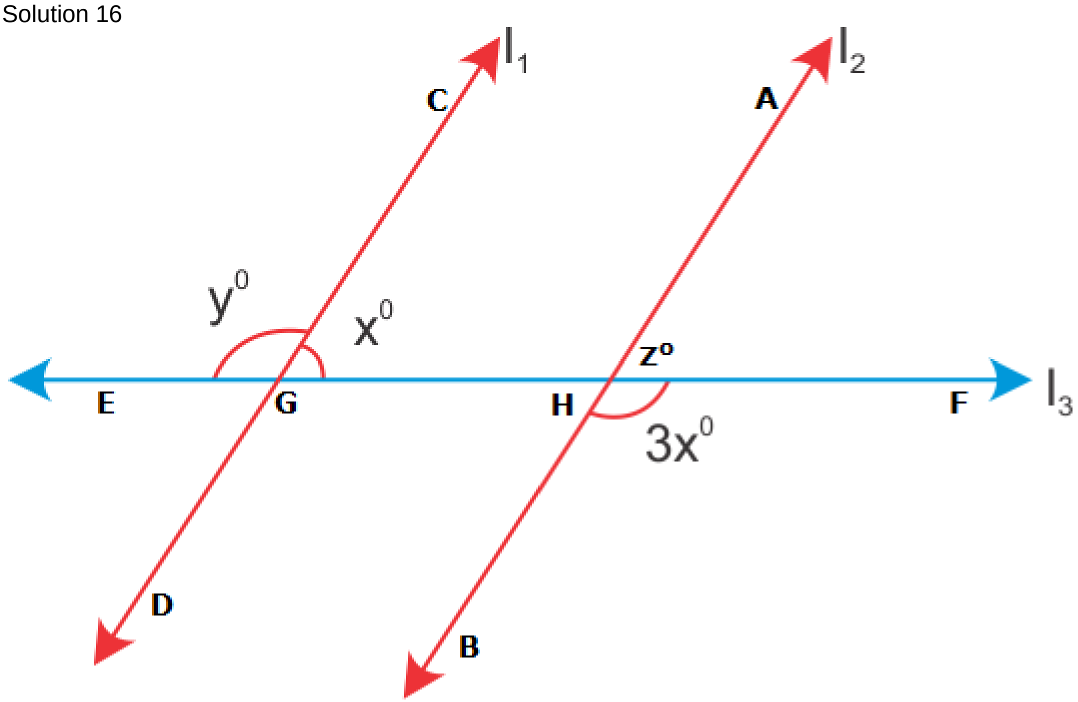
(b) 120

(c) 135

(d) 150



Solution 16



Let angle supplement of $3x°$ be $z°$

$\Rightarrow z° = 180° - 3x°$

$\Rightarrow \angle AHF + \angle FHB = 180°$

$\Rightarrow z° + 3x° = 180°$

$\Rightarrow z° = 180° - 3x°$

Now, $x° + y° = 180°$

Also, $x° = z°$ (correspondence angles)

$\Rightarrow x° = 180° - 3x°$

$\Rightarrow 4x° = 180°$

$\Rightarrow x° = 45°$

$x° + y° = 180°$

$\Rightarrow y° = 180° - x° = 180° - 45° = 135°$

Hence, correct option is (c).

Question 17

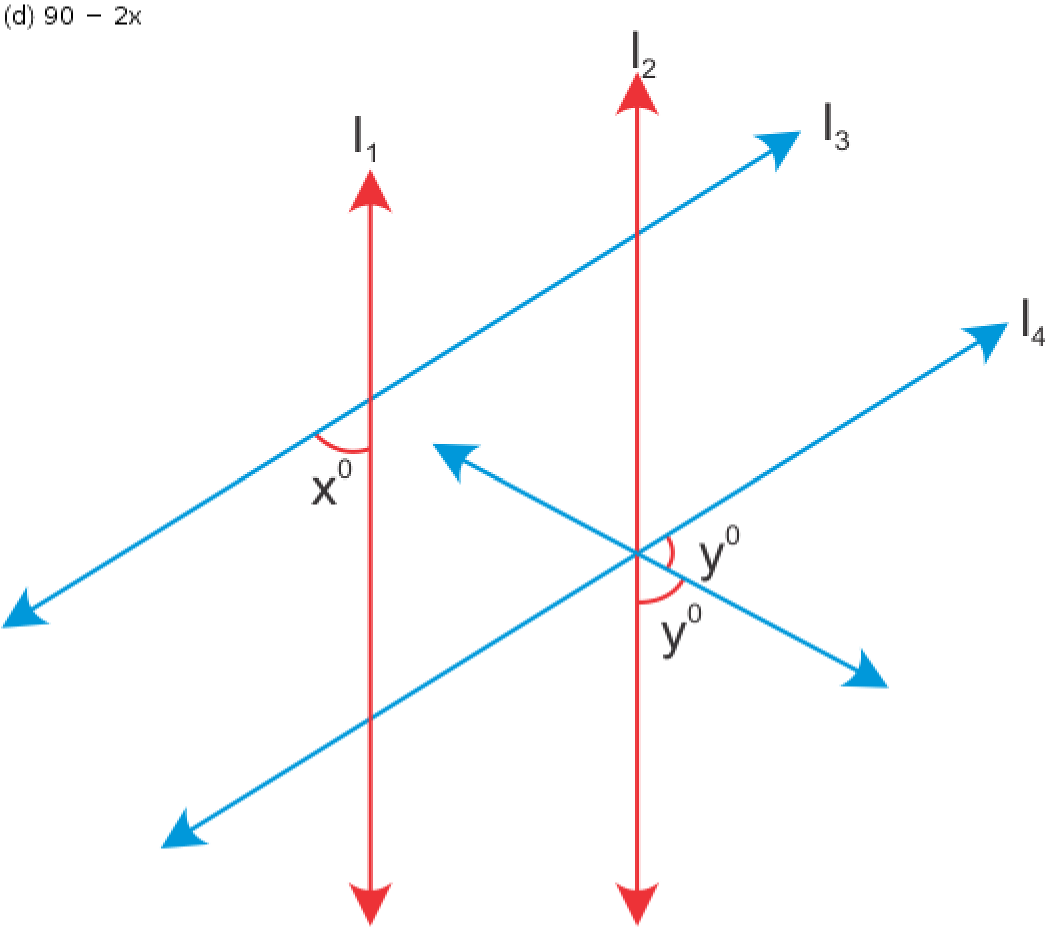
In *figure*, if $l_1 \parallel l_2$ and $l_3 \parallel l_4$, What is y in terms of x?

(a) $90 + x$

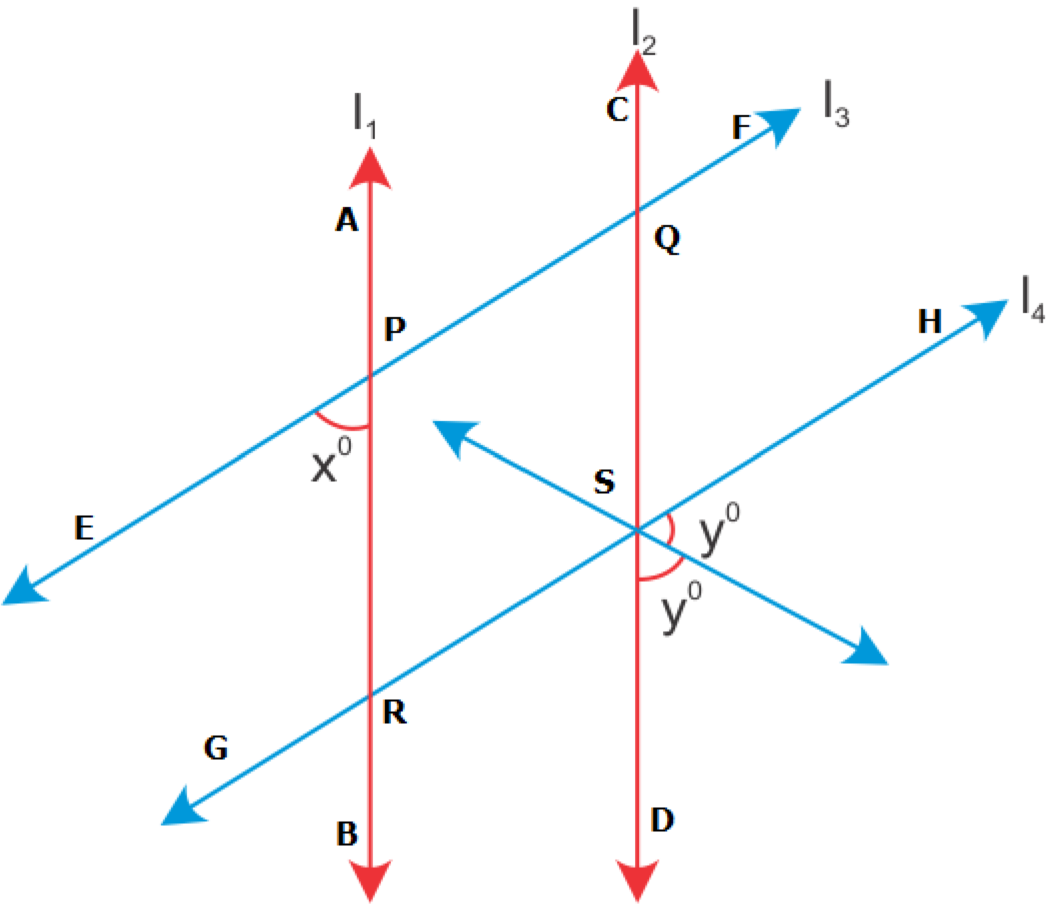
(b) $90 + 2x$

(c) $90 - \frac{x}{2}$

(d) $90 - 2x$



Solution 17



Fom figure,

$\angle EPR = \angle PQS$ (Correspondence angles are equal)

$\Rightarrow \angle PQS = x°$

Also, $\angle PQS = \angle RSD$ (Correspondence angles are equal)

$\Rightarrow \angle RSD = x°$

Now, $\angle RSD + y° + y° = 180°$

$\Rightarrow x° + 2y° = 180°$

$\Rightarrow y° = \dfrac{180° - x°}{2}$

$\Rightarrow y° = 90° - \dfrac{x°}{2}$

Hence, correct option is (c).

Chapter 10 - Lines and Angles Exercise 10.55

Question 18

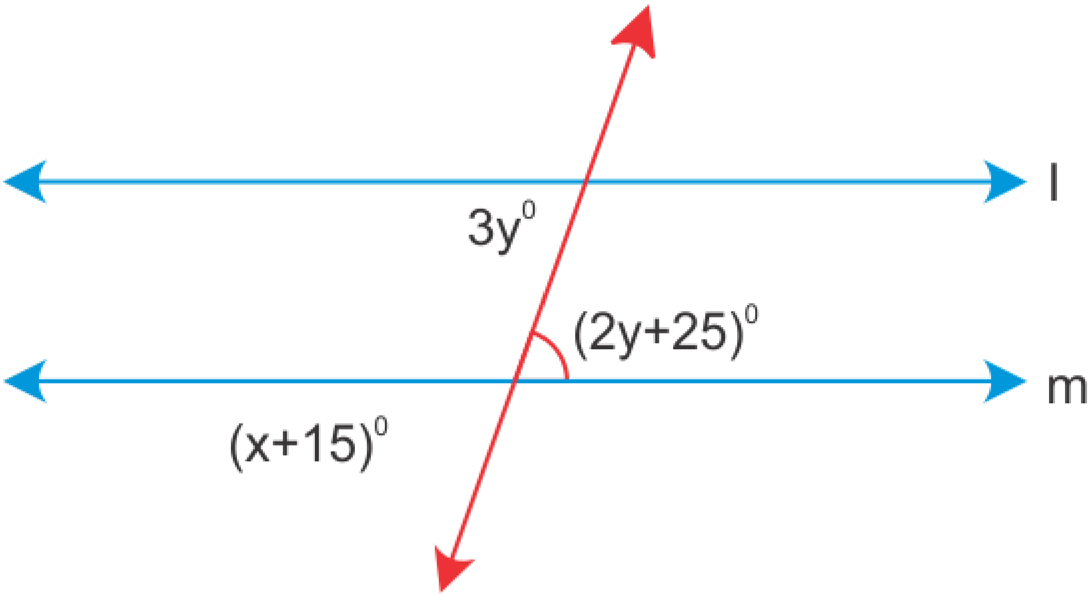
In figure, if l || m, What is the value of x?

(a) $60°$

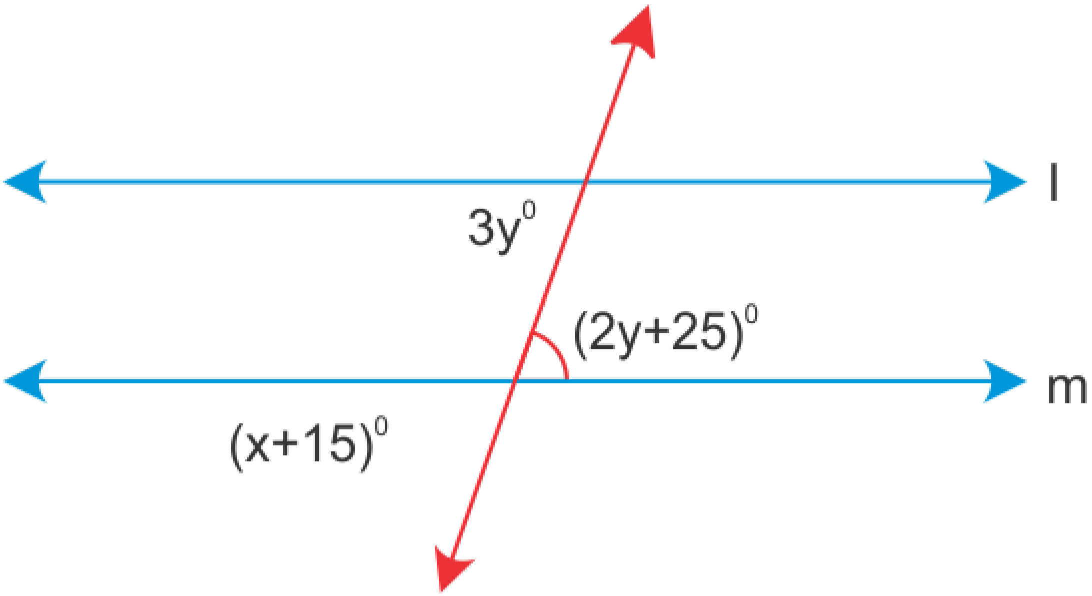
(b) $50°$

(c) $45°$

(d) $30°$



Solution 18



$3y° = 2y° + 25°$ (Alternate angles)

$\Rightarrow y° = 25°$

Now, $x° + 15° = 2y° + 25°$ (opposite angles)

$\Rightarrow x = 2y° + 25° - 15°$

$\Rightarrow x = 2y° + 10°$

$\Rightarrow x = 2 \times 25° + 10°$

$\Rightarrow x = 60°$

Hence, correct option is (a).

Question 19

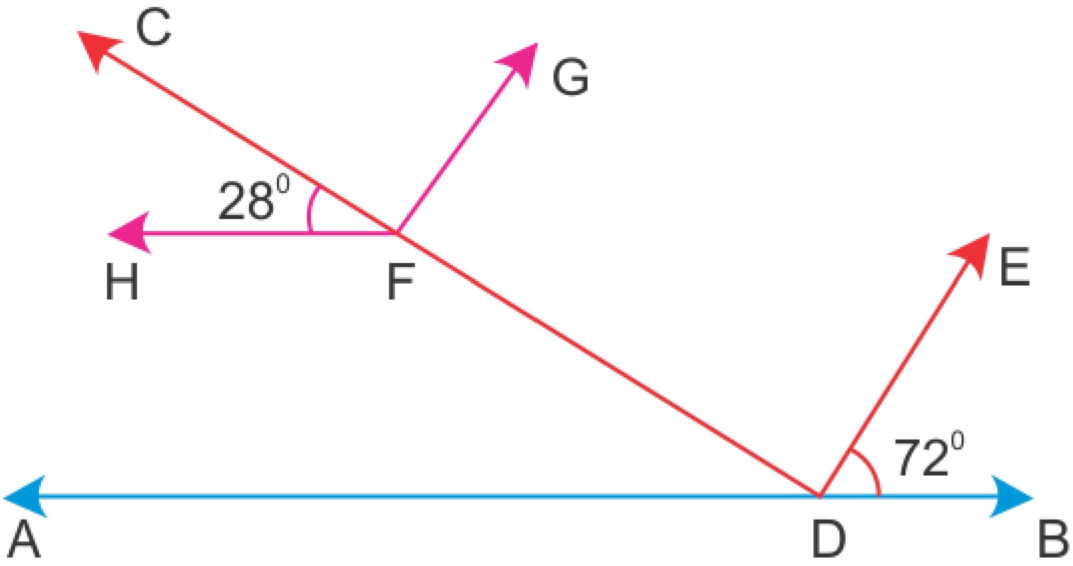
In figure, if AB || HF and DE || FG, then the measure of $\angle FDE$ is

(a) 108°

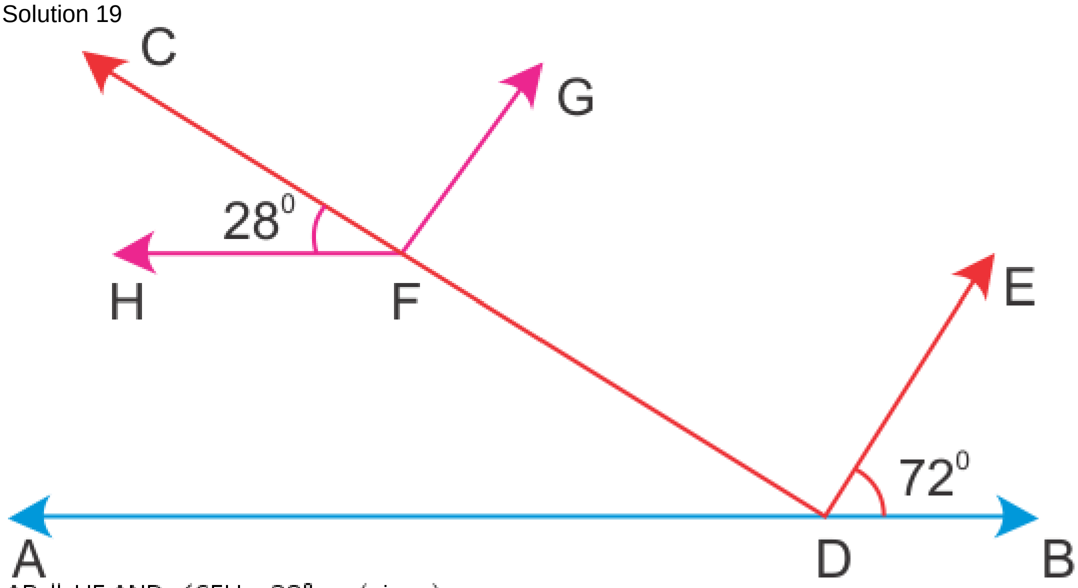
(b) 80°

(c) 100°

(d) 90°



Solution 19



AB || HF AND $\angle CFH = 28°$ (given)

$\angle CFH = \angle FDA$ (Correspondence Angles are equal)

$\Rightarrow \angle FDA = 28°$

Now, $\angle FDA + \angle FDE + \angle EDB = 180°$

$\Rightarrow 28° + \angle FDE + 72° = 180°$

$\Rightarrow \angle FDE = 80°$

Hence, correct option is (b).

Question 20

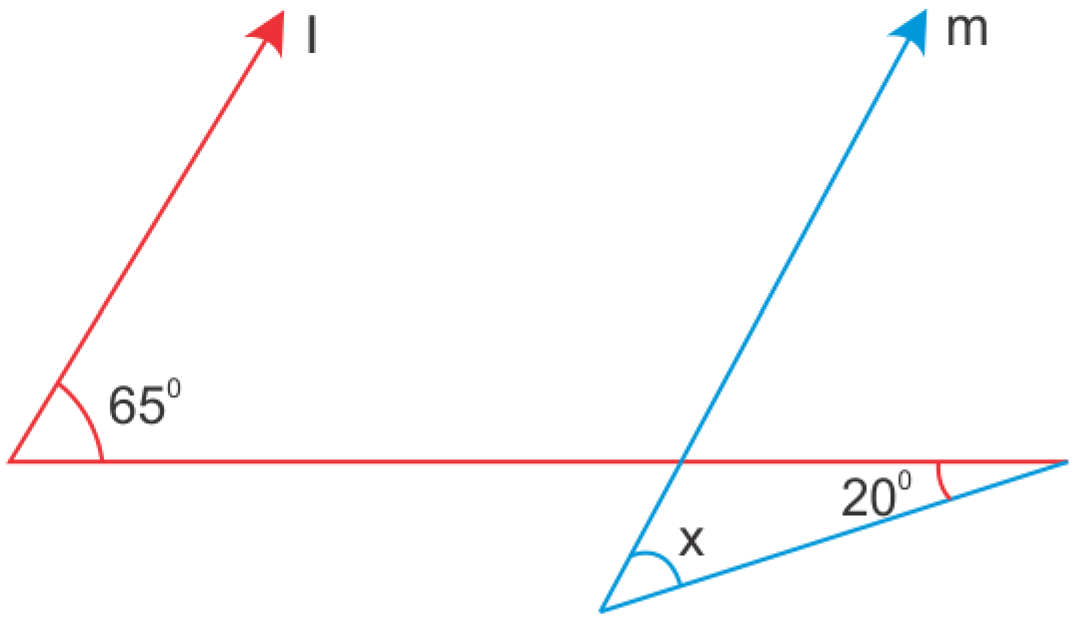
In figure, if lines l and m are parallel, then x =

(a) 20°

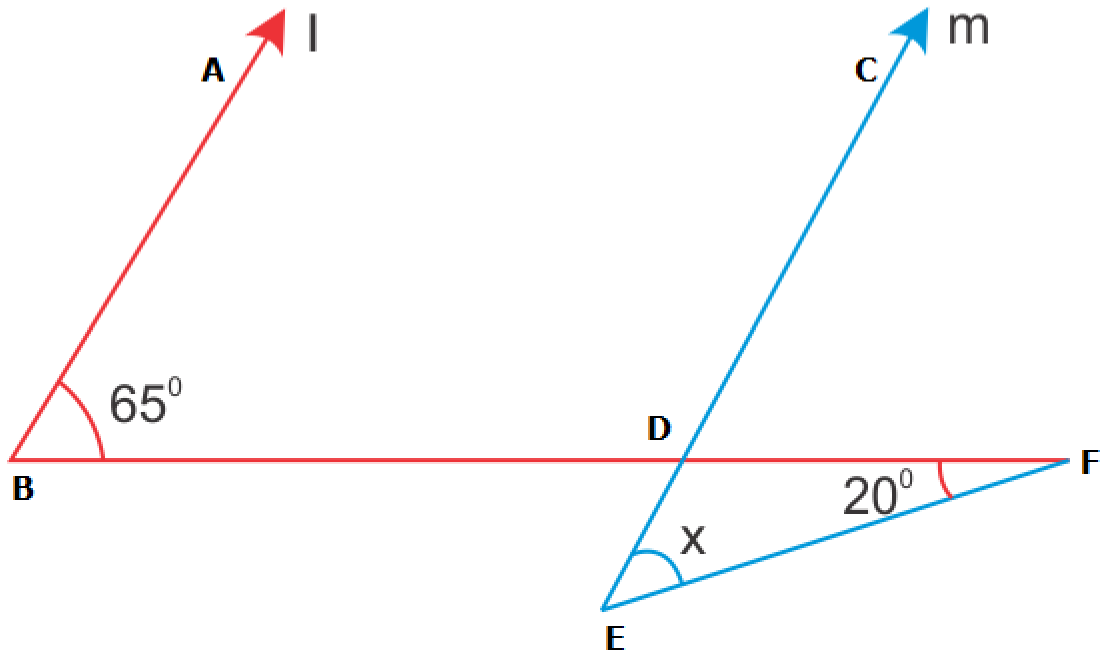
(b) 45°

(c) 65°

(d) 85°



Solution 20



From figure,

$\angle ABD = \angle CDF$ (Correspondence Angles)

$\Rightarrow \angle CDF = 65°$

Now, $\angle FDE = 180° - \angle CDF = 180° - 65°$

$\Rightarrow \angle FDE = 115°$

In $\triangle EDF$,

$\angle FDE + \angle DEF + \angle EFD = 180°$

$\Rightarrow 115° + x + 20° = 180°$ (Sum of all interior angles of a $\triangle$ as 180°)

$\Rightarrow x = 180° - 20° - 115° = 45°$

Hence, correct option is (b).

Question 21

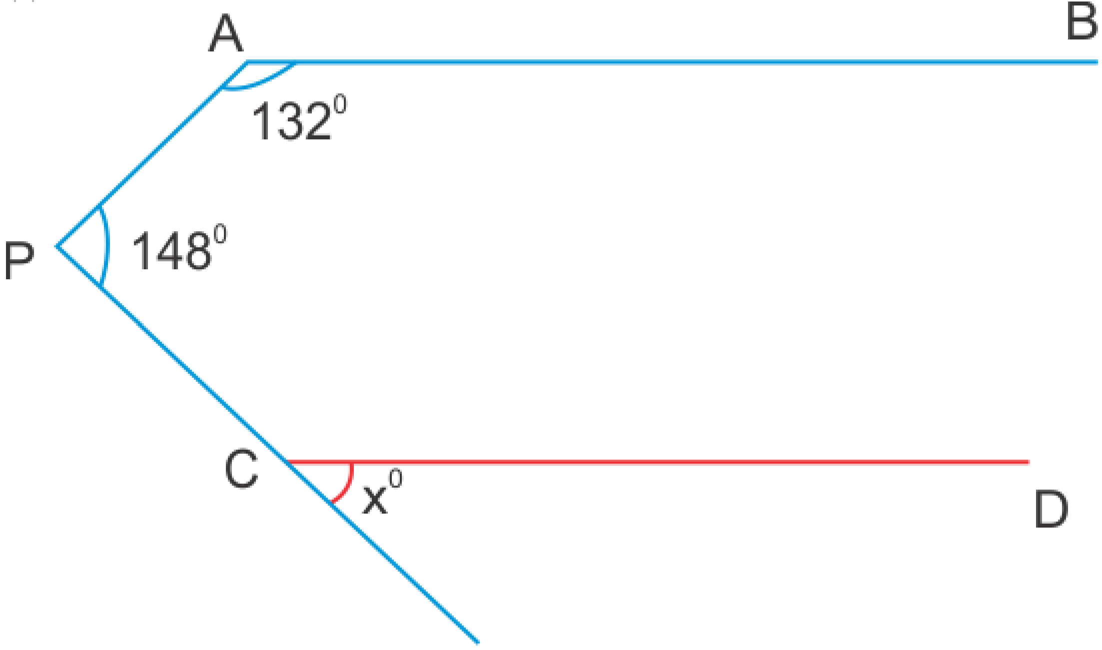
In figure, if AB || CD, then x =

(a) 100°

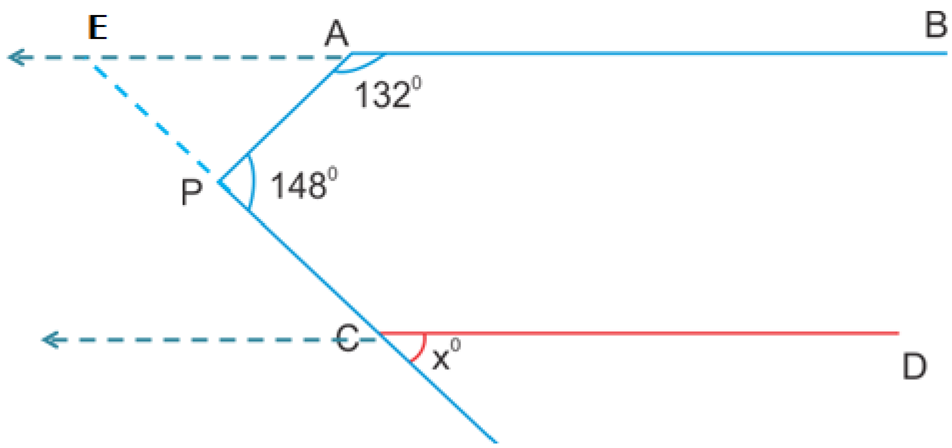
(b) 105°

(c) 110°

(d) 115°



Solution 21



Extending line BA and CP to meet at Point E.

$\angle APE = 180° - 148° = 32°$

$\angle EAP = 180° - 132° = 48°$

$\angle AEP = x°$ {(Correspondence angles) because AB || CD cut by transverse EC}

Now, in $\triangle APE$

$\angle APE + \angle EAP + \angle AEP = 180°$

$\Rightarrow 32° + 48° + x° = 180°$

$\Rightarrow x = 100°$

Hence, correct option is (a).

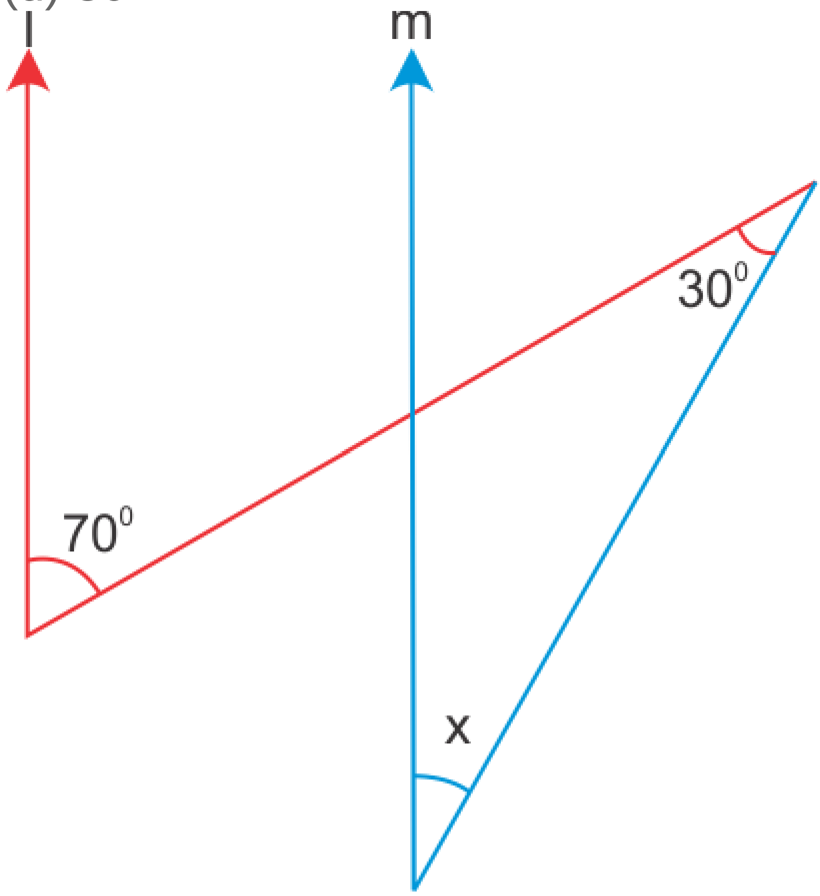
Chapter 10 - Lines and Angles Exercise 10.56

Question 22

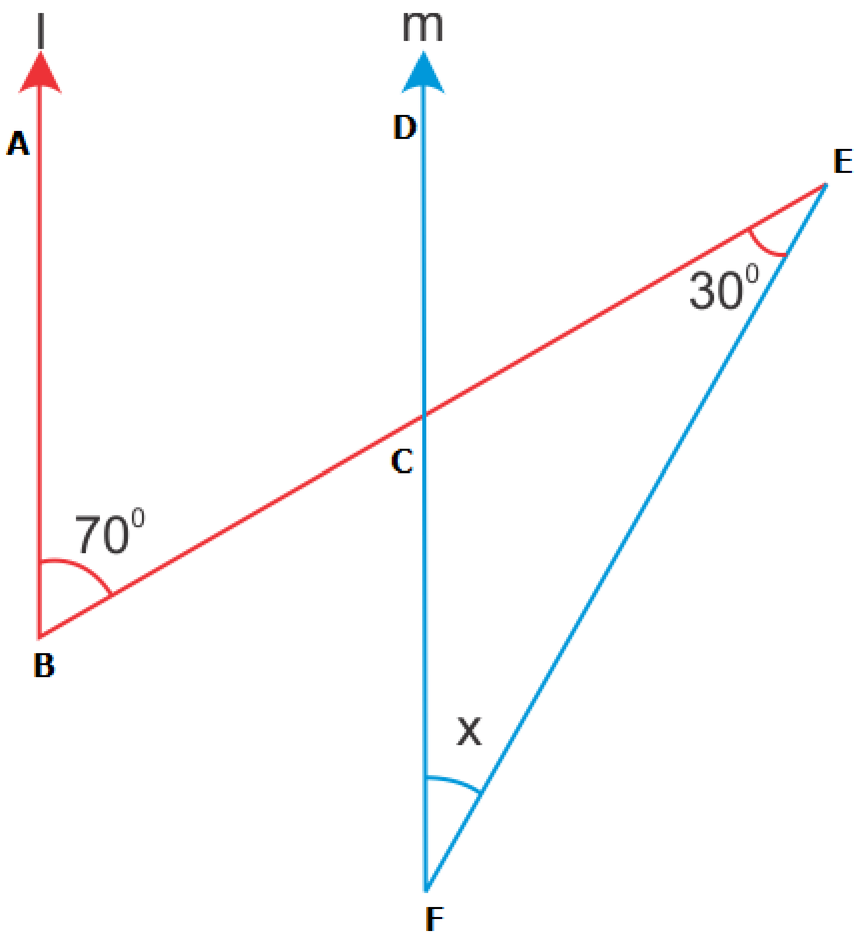
In figure, if lines l and m are parallel lines, then x =

(a) 70°

(b) 100°
(c) 40°
(d) 30°

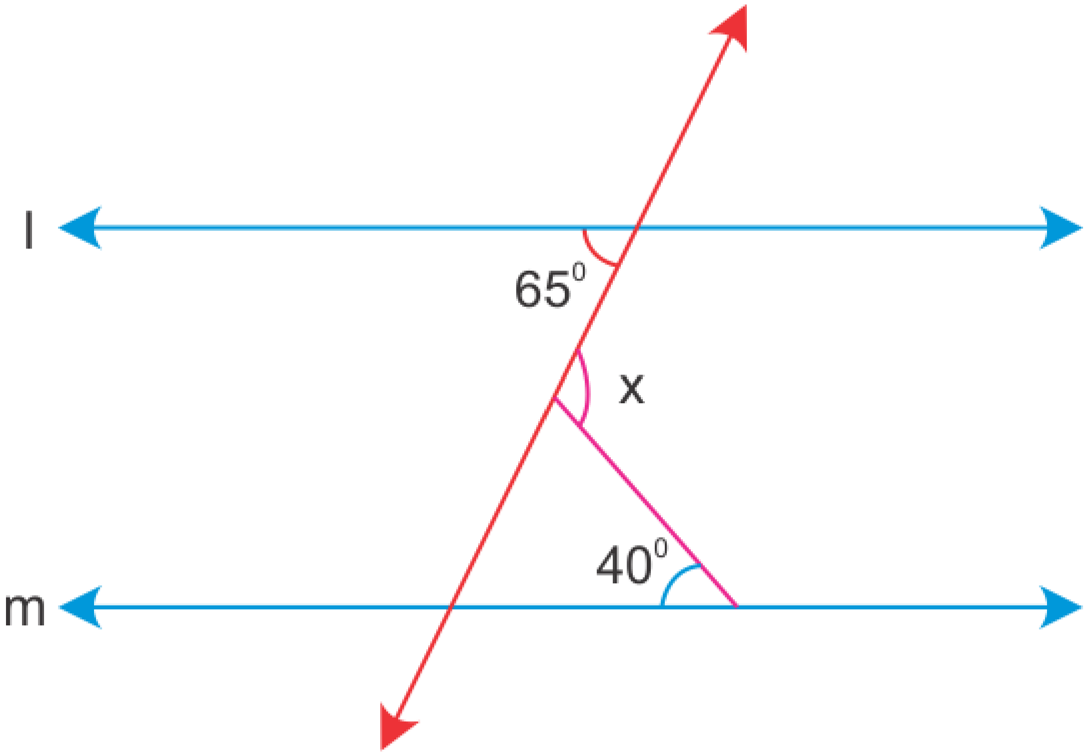


Solution 22

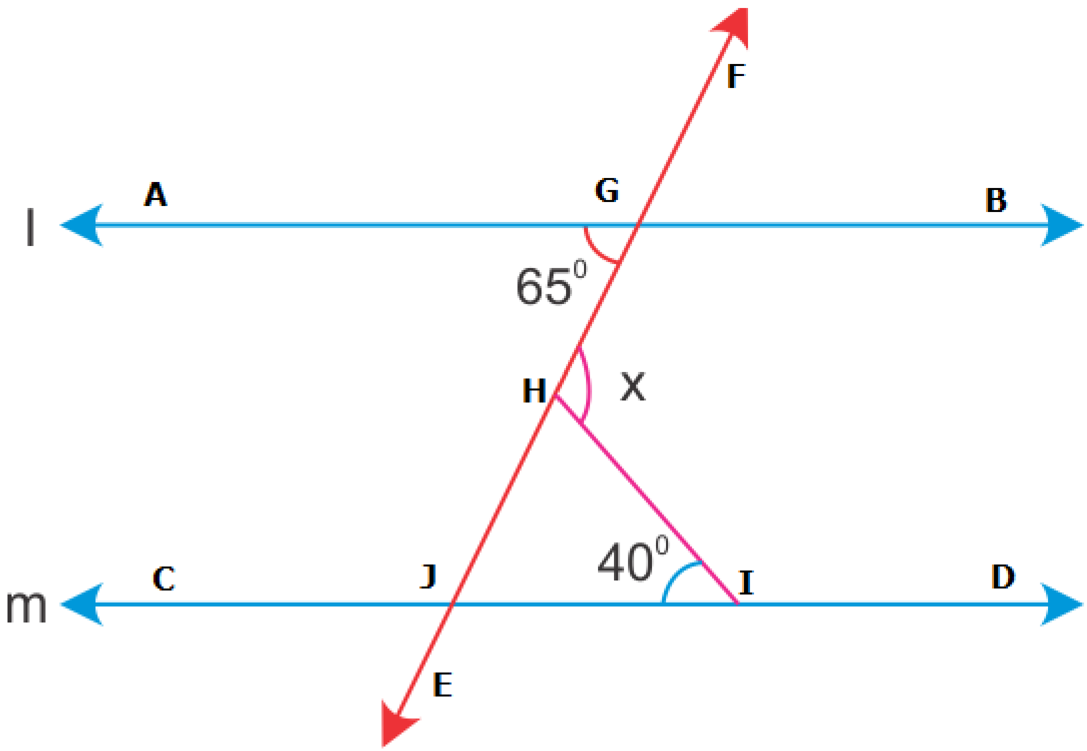


From figure,
$\angle ABC = \angle DCE$ .....(1) (Corresponding angles)
$\angle ECF = 180° - \angle DCE$ (Supplementry)
$= 180° - \angle ABC$ [From (1)]
$= 180° - 70°$
$\Rightarrow \angle ECF = 110°$
Now, in $\triangle CEF$
$\angle ECF + \angle CFE + \angle FEC = 180°$
$\Rightarrow 110° + x + 30° = 180°$
$\Rightarrow x = 40°$
Hence, correct option is (c).

Question 23
In figure, if l || m, then x =
(a) 105°
(b) 65°
(c) 40°
(d) 25°



Solution 23

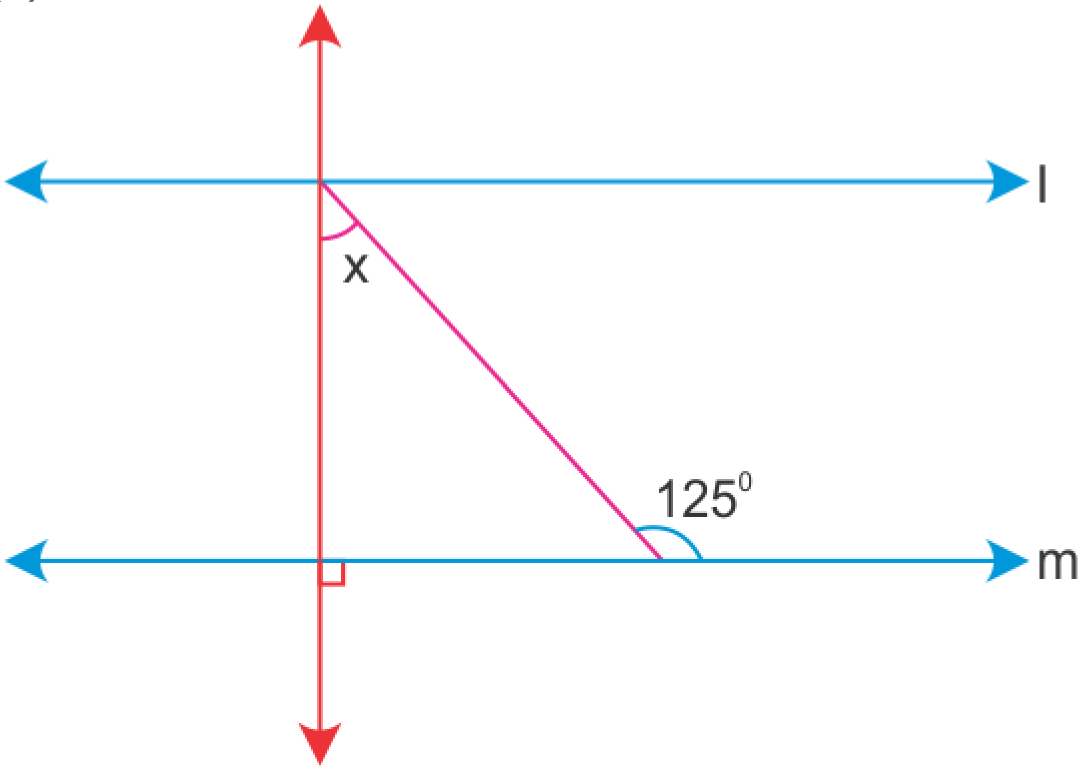


From figure,
$\angle AGE = \angle FGB$ (opposite angles)
$\Rightarrow \angle FGB = 65°$
Also, $\angle FGB = \angle HJI$ (Corresponding angle)
$\Rightarrow \angle HJI = 65°$
Now, in $\triangle HJI$,
$\angle HJI + \angle JIH + \angle IHJ = 180°$
$\Rightarrow 65° + 40° + \angle IHJ = 180°$
$\Rightarrow \angle IHJ = 180° - 65° - 40° = 75°$
Now, $x = 180° - \angle IHJ = 180° - 75° = 105°$
Hence, correct option is (a).

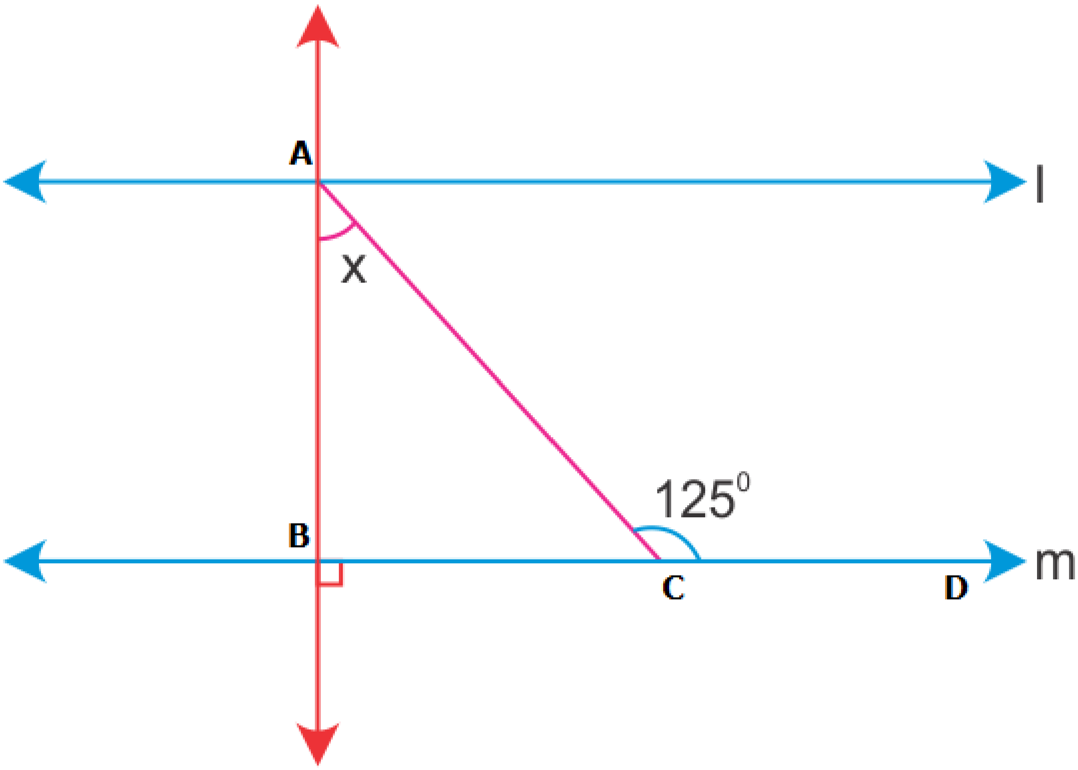
Question 24

In figure, if lines l and m are parallel, then the value of x is

(a) 35°
(b) 55°
(c) 65°
(d) 75°



Solution 24



From figure,
$\angle ACB = 180° - \angle ACD = 180° - 125° = 55°$
OR $\angle BCA = 55°$
In Right $\triangle$ ABC
$\angle ABC + \angle BCA + \angle CAB = 180°$
$\Rightarrow 90° + 55° + x = 180°$
$\Rightarrow x = 35°$
Hence, correct option is (a).

Question 25

Two complementary angles are such that two times the measure of one is equal to three times the measure of the other. The measure of the smaller angle is
(a) 45°
(b) 30°
(c) 36°
(d) none of these

Solution 25

Correct option (c)
Let one angle be θ
Then, its complementary = 90 - θ
According to question,
2θ = 3(90 - θ)

$5\theta = 270$

$\theta = 54°$

Then, $90 - \theta° = 36°$

Hence, the smaller angle is 36°.

Hence, correct option is (c).

Chapter 10 - Lines and Angles Exercise 10.57

Question 26

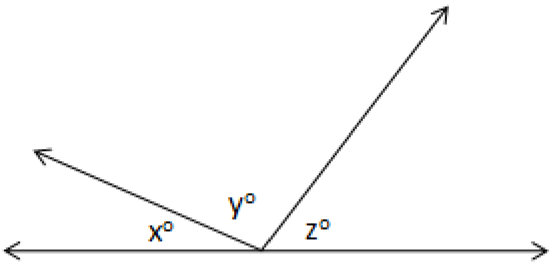
In figure, if $\frac{y}{x} = 5$ and $\frac{z}{x} = 4$, then the value of x is

(a) 8°

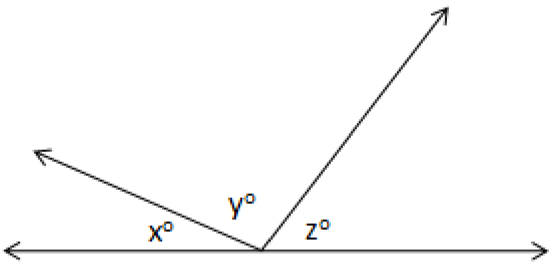
(b) 18°

(c) 12°

(d) 15°



Solution 26



From figure, we can see that

$\angle x° + \angle y° + \angle z° = 180°$ ...(1)

Now, $\frac{y}{x} = 5 \Rightarrow y = 5x$

And, $\frac{z}{x} = 4 \;\; z = 4x$

Substituting these value in equation (1), we have

$\angle x° + \angle 5x° + \angle 4x = 180°$

$\Rightarrow \angle 10x° = 180°$

$\Rightarrow \angle x° = 18°$

Hence, correct option is (b).

Question 27

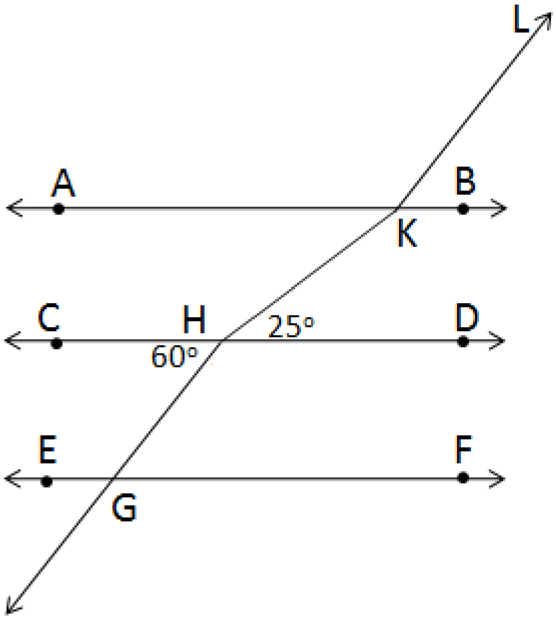
In figure, AB || CD || EF and GH || KL. The measure of ∠HKL is

(a) 85°

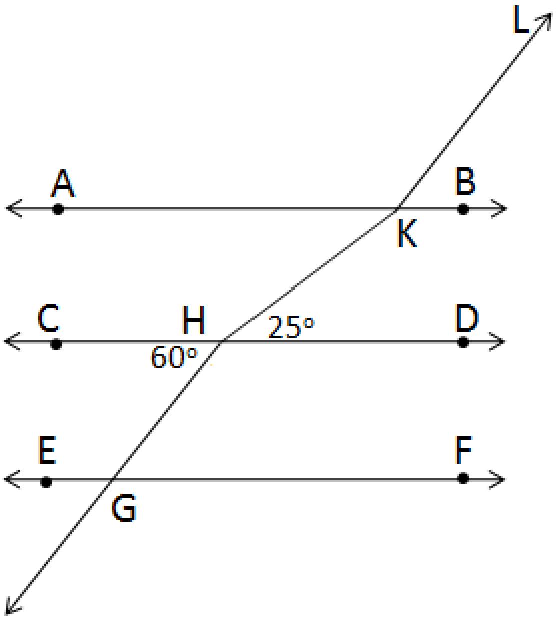
(b) 135°

(c) 145°

(d) 215°



Solution 27



GH || KL

$\Rightarrow \angle GHK = \angle HKL$ (interior opposite angles)

Now, $\angle GHK = \angle GHD + \angle DHR$

$= (180° - \angle GHC) + \angle DHK$ ($\angle GHC$ and $\angle GHD$ are supplementary)

$= 180° - 60° + 25°$

$\Rightarrow \angle GHK = 145°$

$\Rightarrow \angle HKL = \angle GHK = 145°$

Hence, correct option is (c).

Question 28

AB and CD are two parallel lines. PQ acuts AB and CD at E and F respectively. EL is bisector of $\angle FEB$. If $\angle LEB = 35°$, then $\angle CFQ$ will be

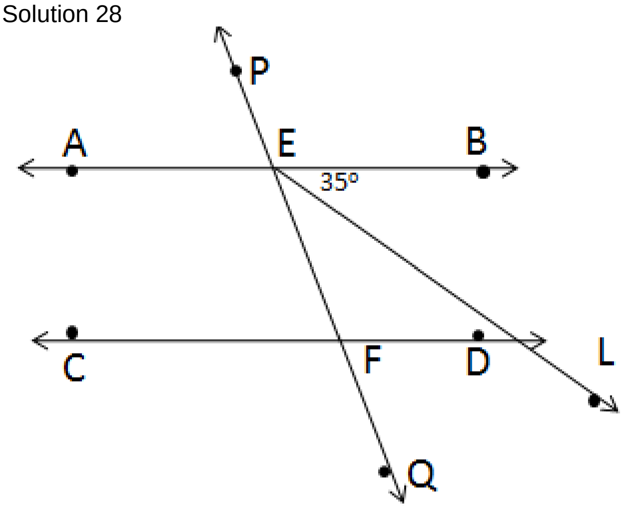
(a) 55°

(b) 70°

(c) 110°

(d) 130°

Solution 28



From figure,

$\angle LEB = \angle FEL$ (EL is bisector of $\angle FEB$)

Now, $\angle FEB = 2\angle LEB = 2 \times 35° = 70°$

Also, $\angle FEB = \angle CFE$ (Alternate interior angles)

$\Rightarrow \angle CFE = 70°$

Now, $\angle CFE + \angle CFQ = 180°$

$\Rightarrow 70° + \angle CFQ = 180°$

$\Rightarrow \angle CFQ = 180° - 70° = 110°$

Hence, correct option is (c).

Question 29

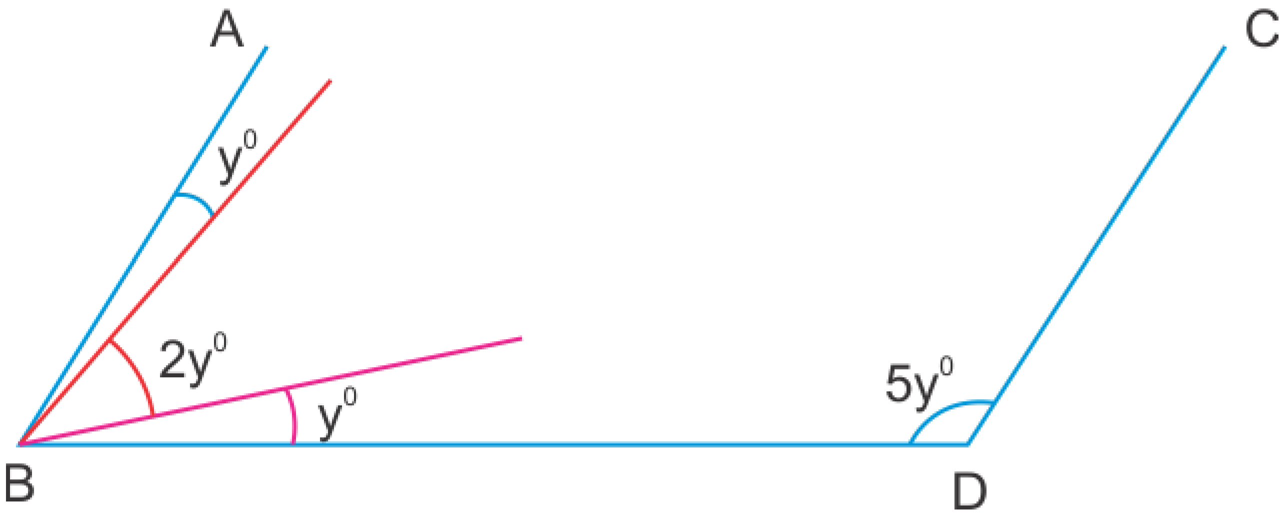
In figure, if line segment AB is parallel to the line segment CD, What is the value of y?

(a) 12

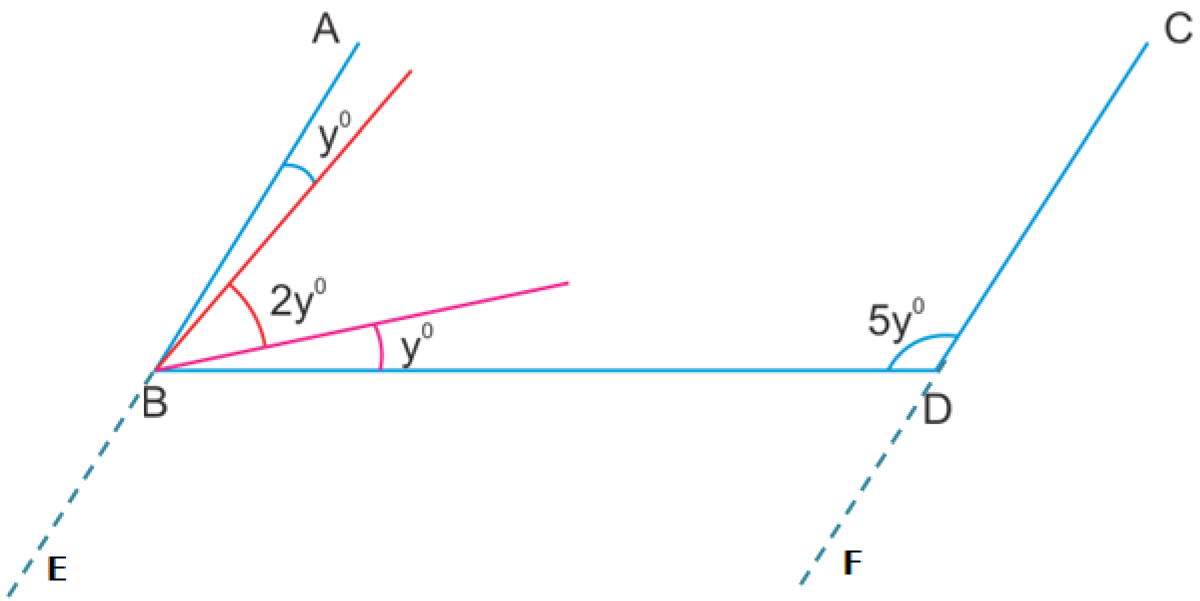
(b) 15

(c) 18

(d) 20



Solution 29



From figure,

$\angle ABD + \angle EBD = 180°$

$\Rightarrow \angle EBD = 180° - \angle ABD$ ....(1)

Now, $\angle ABD = y° + 2y° + y°$

$\Rightarrow \angle ABD = 4y°$ ....(2)

Substituting (2) in (1), we have

$\angle EBD = 180° - 4y°$

Now, $\angle EBD = \angle BDC$ (Alternate angles)

$\Rightarrow 180° - 4y° = 5y°$

$\Rightarrow 180° = 9y°$

$\Rightarrow y = 20°$

Hence, correct option is (d).

Question 30

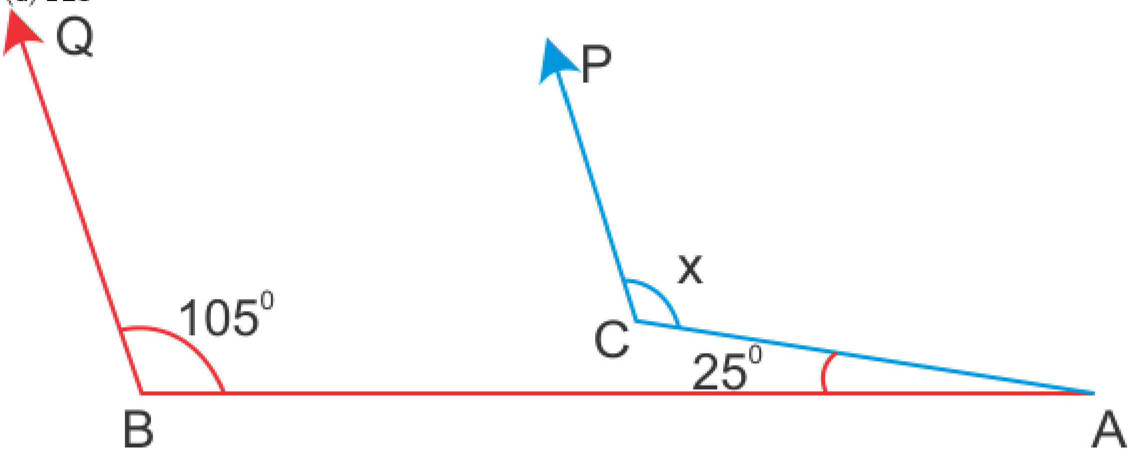
In figure, if CP || DQ, then the measure of x is

(a) 130°

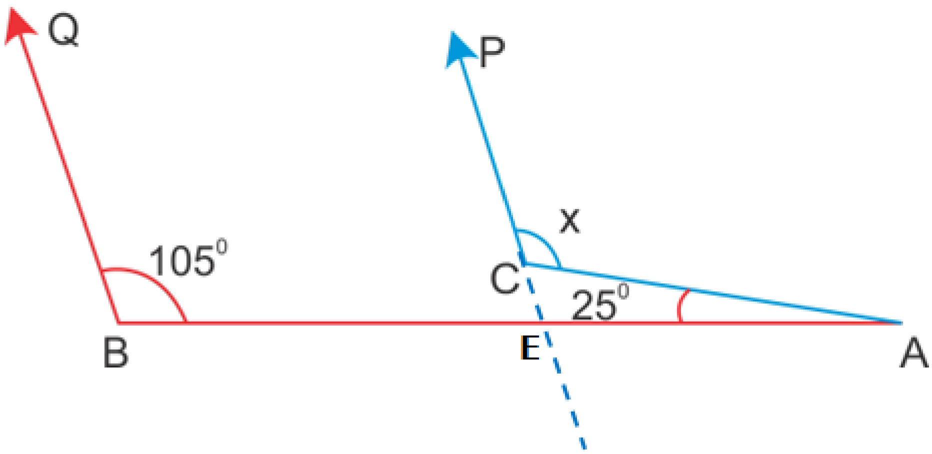
(b) 105°

(c) 175°

(d) 125°



Solution 30



From Figure,

$\angle QBA = \angle CEA$ (Correspondence angles)

$\Rightarrow \angle CEA = 105°$ ....(1)

In $\triangle ACE$,

$\angle CEA + \angle EAC + \angle ACE = 180°$

$\Rightarrow 105° + 25° + \angle ACE = 180°$ [from (1)]

$\Rightarrow 130° + \angle ACE = 180°$

$\Rightarrow \angle ACE = 50°$

Now, $x = \angle ACP = 180° - \angle ACE = 180° - 50° = 130°$

Hence, correct option is (a).

Chapter 10 - Lines and Angles Exercise Ex. 10.1

Question 1

Write the complement of each of the following angles:

(i) 20°

(ii) 35°

(iii) 90°

(iv) 77º

(v) 30º

Solution 1

(i) 20°

Since, the sum of an angle and its complement is 90°.

Therefore, its complement will be $(90 - 20 = 70^\circ)$.

(ii) 35°

Since, the sum of an angle and its complement is 90°.

Therefore, its complement will be $(90 - 35 = 55^\circ)$.

(iii) 90°

Since, the sum of an angle and its complement is 90°.

Therefore, its complement will be $(90 - 90 = 0^\circ)$.

(iv) 77°

Since, the sum of an angle and its complement is 90°.

Therefore, its complement will be $(90 - 77 = 13^\circ)$.

(v) 30°

Since, the sum of an angle and its complement is 90°.

Therefore, its complement will be $(90 - 30 = 60^\circ)$.

Question 2

Write the supplement of each of the following angles:

(i) 54º

(ii) 132º

(iii) 138º

Solution 2

(i) 54°
Since, the sum of an angle and its supplement is 180°

∴Its supplement will be 180° - 54° = 126°.

(ii) 132°
Since, the sum of an angle and its supplement is 180°
∴Its supplement will be 180° - 132° = 48°.

(iii) 138°
Since, the sum of an angle and its supplement is 180°
∴Its supplement will be 180° - 138° = 42°.

Question 3

If an angle is 28º less than its complement, find its measure.

Solution 3

Let the measure of the angle be x°.

$\therefore$ Its complement will be $(90-x)^\circ$

It is given that

Angle = complement – 28°.

$\therefore x^\circ = (90-x)^\circ - 28^\circ$

$\Rightarrow 2x^\circ = 62^\circ$

$\Rightarrow x = 31^\circ$

$\therefore$ Angle measured is 31°.

Question 4

If an angle is 30° more than one half of its complement, find the measure of the angle.

Solution 4

Let the measure of the angle be x°.

$\therefore$ Its complement will be $(90-x)^\circ$.

It is given that

Angle = $30^\circ + \frac{1}{2}$ complement

$\Rightarrow x^\circ = 30^\circ + \frac{1}{2}(90-x)^\circ$

$\Rightarrow x = \frac{60^\circ + 90 - x}{2}$

$\Rightarrow 2x = 60^\circ + 90 - x$

$\Rightarrow 3x = 150$

$\Rightarrow x = 50$

$\therefore$ Angle is 50°.

Question 5

Two supplementary angles are in the ratio 4 : 5. find the angles.

Solution 5

Let the angles be $4x$ and $5x$

It is given that they are supplementary angles.

$\therefore\ 4x + 5x = 180$

$\Rightarrow\ 9x = 180$

$\Rightarrow\ x = 20^\circ$

Hence $4x = 80, 5x = 100$

$\therefore$ Angles are 80° and 100°.

Question 6

Two supplementary angles differ by $48°$. Find the angles.

Solution 6

Let the measure of an angle be $x°$.

$\therefore$ Its supplementary will be $(180-x)°$.

It is given that

$(180-x)° - x° = 48°$

$\Rightarrow \quad 180-2x = 48$

$\Rightarrow \quad 132 = 2x$

$\Rightarrow \quad x = 66°$

Hence, $180 - x = 180 - 66 = 114°$.

Therefore, angles are $66°$ and $114°$.

Question 7

An angle is equal to 8 times its complement. Determine its measure.

Solution 7

Let the measure of the angle be $x°$.

Therefore its complement will be $(90-x)°$.

It is given that,

Angle = 8 complement

$\Rightarrow x° = 8(90-x)°$

$\Rightarrow x = 720 - 8x$

$\Rightarrow 9x = 720$

$\Rightarrow x = 80$

Therefore, the measure of the angle is $80°$.

Question 8

If the angle $(2x-10)°$ and $(x-5)°$ are complementary angles, find x.

Solution 8

Since the angles are complementary

Therefore their sum will be $90°$.

$\Rightarrow (2x-10)° + (x-5)° = 90°$

$\Rightarrow 2x - 10 + x - 5 = 90$

$\Rightarrow 3x = 105°$

$\Rightarrow x = 35°$

Question 9

If an angle differs from its complement by $10°$, find the angle.

Solution 9

Let the measure of the angle be $x°$.

$\therefore$ Its complement will be $(90 - x)°$.

It is given that

$x° - (90 - x)° = 10°$

$\Rightarrow x - 90 + x = 10$

$\Rightarrow 2x = 100$

$\Rightarrow x = 50$

$\therefore$ The measure of the angle will be 50°.

Question 10

If the supplement of an angle is two-third of itself. Determine the angle and its supplement.

Solution 10

Let the measure of the angle be $x°$.

$\therefore$ its supplement will be $(180 - x)°$.

It is given that

$(180 - x)° = \frac{2}{3}x°$

$\Rightarrow 180 - x = \frac{2}{3}x$

$\Rightarrow \frac{2}{3}x + x = 180$

$\Rightarrow \frac{5}{3}x = 180$

$\Rightarrow x = 108°$

Hence, supplement $= 180 - 108 = 72°$

$\therefore$ Angle will be 108° and its supplement will be 72°.

Question 11

An angle is 14° more than its complementary angle. What is its measure?

Solution 11

Let the angle be x°.

$\therefore$ The complementary of $x = (90 - x)^\circ$.

As per question,

$x - (90 - x) = 14$

$\Rightarrow x - 90 + x = 14$

$\Rightarrow 2x - 90 = 14$

$\Rightarrow 2x = 104$

$\Rightarrow x = \frac{104}{2} = 52^\circ$

$\therefore$ The angle measures 52°.

Question 12

The measure of an angle is twice the measure of its supplementary angle. Find its measure.

Solution 12

Let the measure of angle is x°.

$\therefore$ Its supplementary will be $(180 - x)^\circ$.

It is given that

$x^\circ = 2(180 - x)^\circ$

$\Rightarrow x = 360 - 2x$

$\Rightarrow 3x = 360$

$\Rightarrow x = 120^\circ$

$\therefore$ The measure of angle is 120°.

Question 13

If the complement of an angle is equal to the supplement of the thrice of it. Find the measure of angle.

Solution 13

Let the measure of the angle be x^o.

Its complement will be $(90^o - x^o)$ and its supplement will be $(180^o - x^o)$.

Supplement of thrice of the angle = $(180^o - 3x^o)$

According to the given information:

$(90^o - x^o) = (180^o - 3x^o)$

$3x - x = 180 - 90$

$2x = 90$

$x = 45$

Thus, the measure of the angle is 45^o.

The measure of the angle is 45^o

Question 14

If the supplement of an angle is three times its complement, find the angle.

Solution 14

Let the measure of the angle be $x°$.

$\therefore$ Its supplement will be $(180 - x)°$.

And its complement will be $(90 - x)°$.

It is given that

$(180 - x)° = 3(90 - x)°$

$\Rightarrow 180 - x = 270 - 3x$

$\Rightarrow 2x = 90$

$\Rightarrow x = 45$

$\therefore$ The measure of the angle will be $45°$.

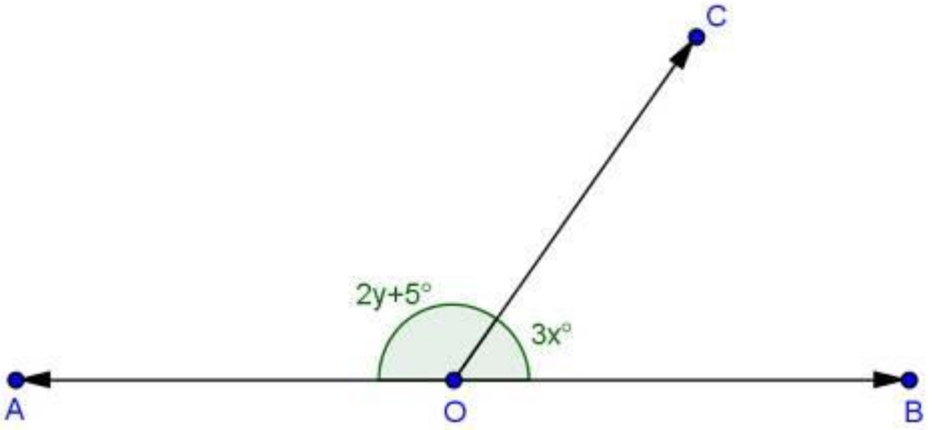
Chapter 10 - Lines and Angles Exercise Ex. 10.2

Question 1

In fig., OA and OB are opposite rays:

(i) If x = 25°, what is the value of y?

(ii) if y = 35°, what is the value of x?



Solution 1

(i) Since,

$\angle AOC + \angle BOC = 180^\circ$ [linear pair]

$\Rightarrow (2y + 5) + 3x = 180$

Given, $x = 25^\circ$

$\Rightarrow 2y + 5 + 75 = 180$

$\Rightarrow 2y = 180 - 80 = 100$

$\Rightarrow y = 50^\circ$

(ii) Since,

$\angle AOC + \angle BOC = 180^\circ$ [linear pair]

$\Rightarrow (2y + 5) + 3x = 180$

Given, $y = 35^\circ$

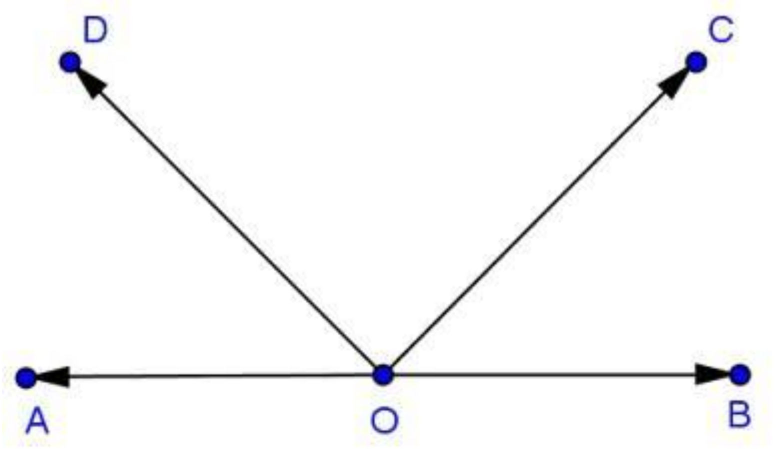
$\Rightarrow 70 + 5 + 3x = 180$

$\Rightarrow 3x = 105$

$\Rightarrow x = 35^\circ$

Question 2

In fig., write all pairs of adjacent angles and all the linear pairs.



Solution 2

Adjacent angles:

(i)$\angle AOD$, $\angle COD$;

(ii)$\angle BOC$, $\angle COD$;

(iii) $\angle AOD$, $\angle BOD$;

(iv) $\angle AOC$, $\angle BOC$

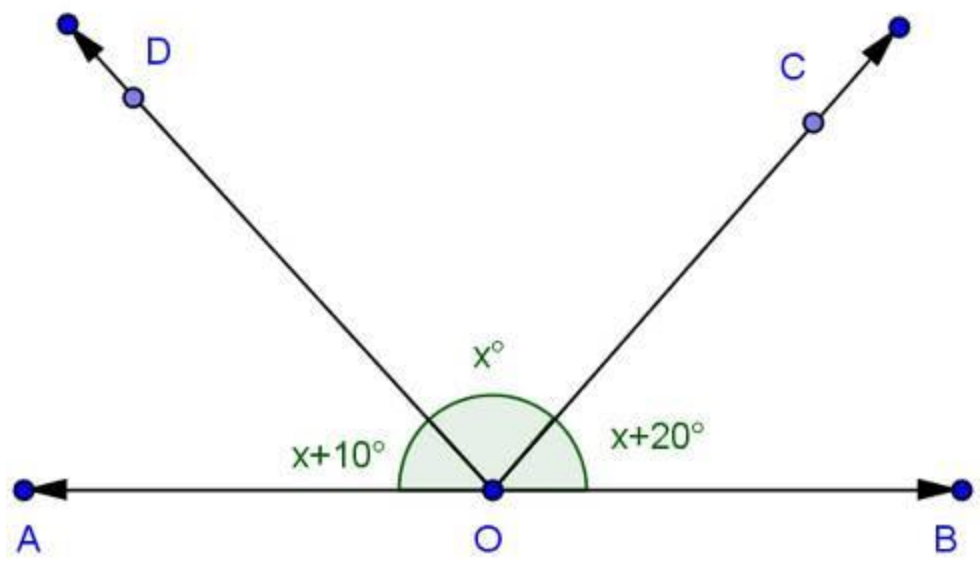
Linear angles:

(i) $\angle AOD$, $\angle BOD$

(ii) $\angle AOC$, $\angle BOC$

Question 3

In fig., find x. further find ∠BOC, ∠COD and ∠AOD



Solution 3

Since,

$\angle AOD + \angle BOD = 180°$ [linear pair]

$\angle AOD + \angle COD + \angle BOC = 180°$

$\Rightarrow (x + 10)° + (x)° + (x + 20)° = 180°$

$\Rightarrow 3x + 30 = 180$

$\Rightarrow x = \dfrac{150}{3} = 50$

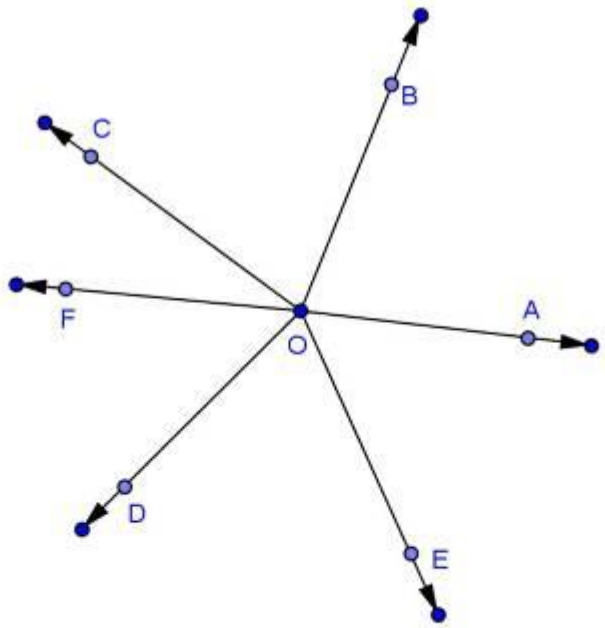
$\therefore x = 50$

$\therefore \angle AOD = (x + 10)° = (50 + 10)° = 60°$

$\Rightarrow \angle COD = (x)° = 50°$

$\Rightarrow \angle BOC = (x + 20)° = (50 + 20)° = 70°$

Question 4



In fig., rays OA, OB, OC, OD and OE have the common end point O. Show that $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$

Solution 4

Construction : A ray OF opposite to OA is drawn

Now,

$\angle AOB + \angle BOF = 180°$ [linear pair]

$\angle AOB + \angle BOC + \angle COF = 180° - - - (1)$

Also

$\angle AOE + \angle EOF = 180°$ [linear pair]

$\angle AOE + \angle DOF + \angle DOE = 180° - - - (2)$

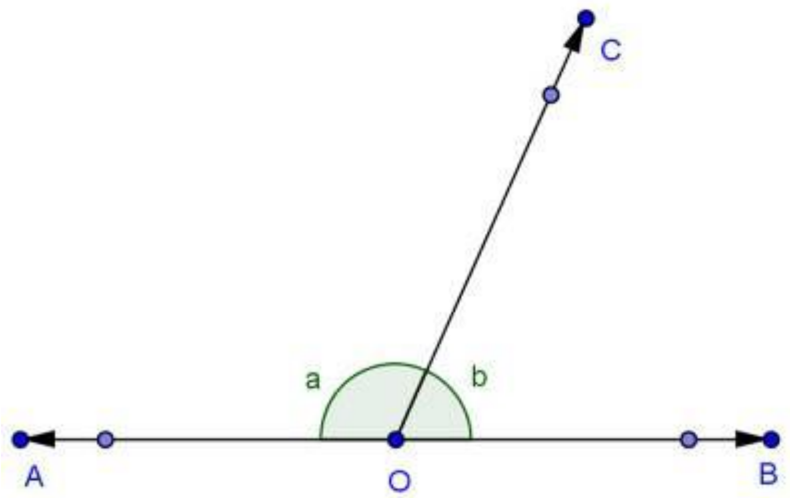
Adding (1) and (2) we get

$\angle AOB + \angle BOC + \angle COF + \angle DOF + \angle DOE + \angle AOE = 360°$

$\Rightarrow \angle AOB + \angle BOC + \angle COD + \angle DOE + \angle AOE = 360°$

Hence proved.

Question 5



In fig., $\angle AOC$ and $\angle BOC$ form a linear pair. If $a - 2b = 30°$, find a and b.

Solution 5

$\because a+b=180^\circ \quad ---(1) \qquad$ [linear pair]

$\text{And } a-2b = 30^\circ \quad ---(2) \qquad$ [given]

Subtracting (2) from (1)

$(a+b)-(a-2b) = 180-30$

$\Rightarrow a+b-a+2b = 150^\circ$

$\Rightarrow 3b = 150^\circ$

$\Rightarrow b = 50^\circ$

Hence, $a = 180^\circ - b$

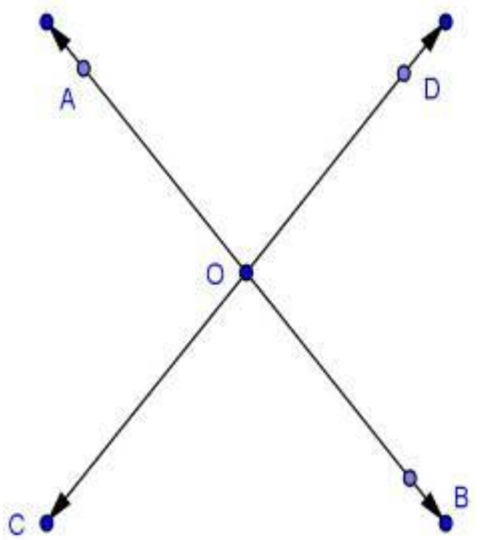
$= 180^\circ - 50^\circ \qquad$ [Substituting $b = 50^\circ$]

$= 130^\circ$

$\therefore a = 130^\circ \textit{ and } b = 50^\circ$

Question 6

How many pairs of adjacent angles are formed when two lines intersect in a point?



Solution 6

$\angle AOD, \angle DOB$

$\angle DOB, \angle BOC$

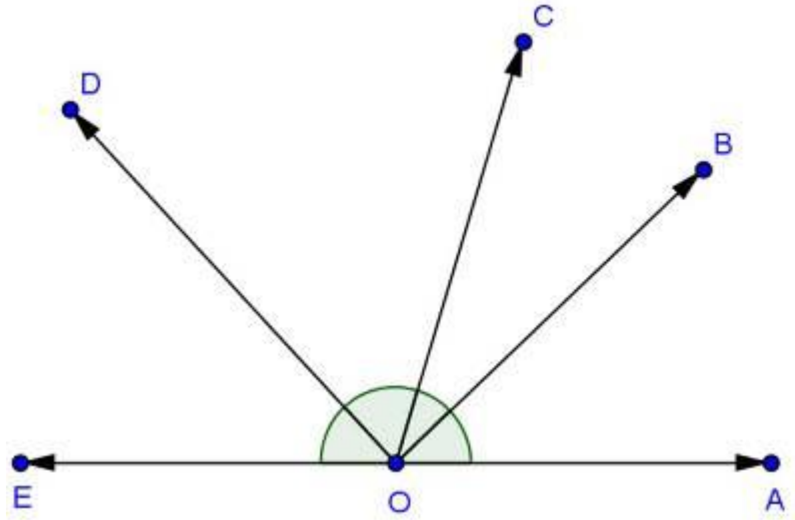
$\angle BOC, \angle COA$

$\angle COA, \angle AOD$

Hence 4 pairs.

Question 7

How many pairs of adjacent angles, in all, can you name in fig.



Solution 7

$\angle EOD, \angle DOC$

$\angle EOD, \angle DOB$

$\angle EOD, \angle DOA$

$\angle DOC, \angle COB$

$\angle DOC, \angle COA$

$\angle BOC, \angle BOA$

$\angle BOA, \angle BOD$

$\angle BOA, \angle BOE$

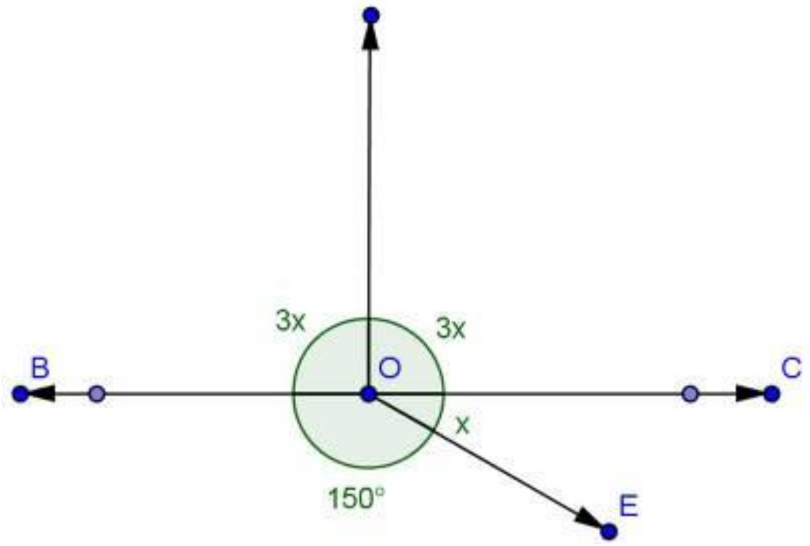
$\angle EOC, \angle COA$

$\angle EOC, \angle COB$

$\therefore$ 10 pairs.

Question 8

In fig., determine the value of x.



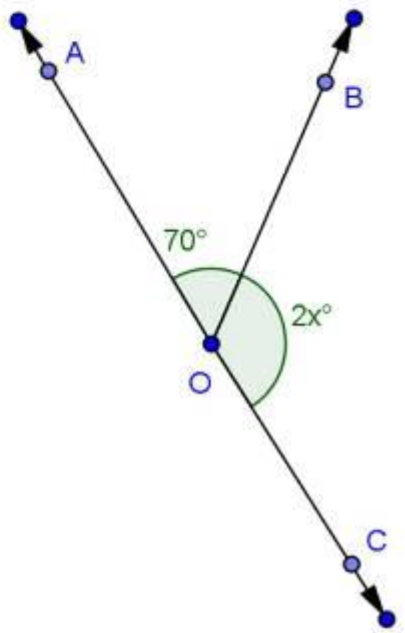
Solution 8

Since the sum of all the angles around a point is 360°.

$$\Rightarrow 3x + 3x + 150° + x = 360°$$
$$\Rightarrow 7x = 360° - 150° = 210°$$
$$\Rightarrow x = \frac{210}{7} = 30°$$

Question 9

In fig., AOC is a line, find x.



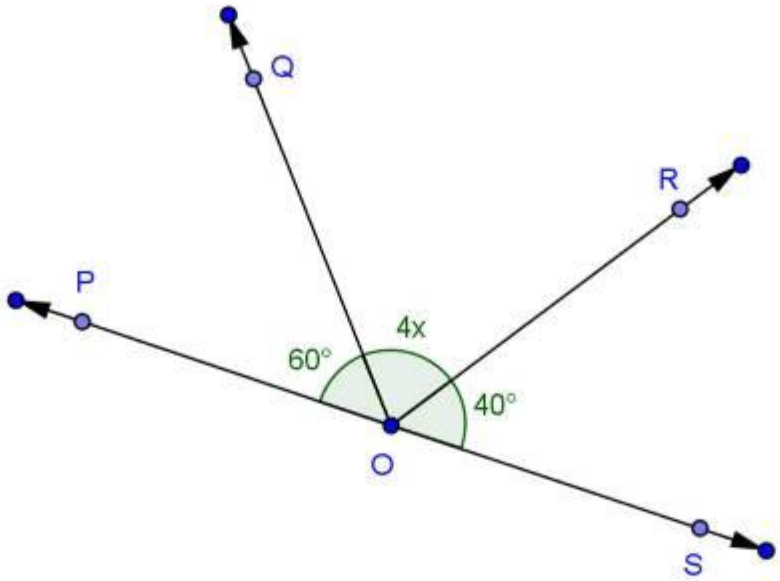
Solution 9

Since, $\angle AOB + \angle BOC = 180°$ [linear pair]

$$\Rightarrow 70° + 2x° = 180°$$
$$\Rightarrow 2x° = 180° - 70° = 110°$$
$$\Rightarrow x = 55°$$

Question 10

In Fig., POS is a line, find x.



Solution 10

Since, $\angle POQ + \angle QOS = 180^\circ$ [linear pair]

$\Rightarrow \angle POQ + \angle QOR + \angle SOR = 180^\circ$

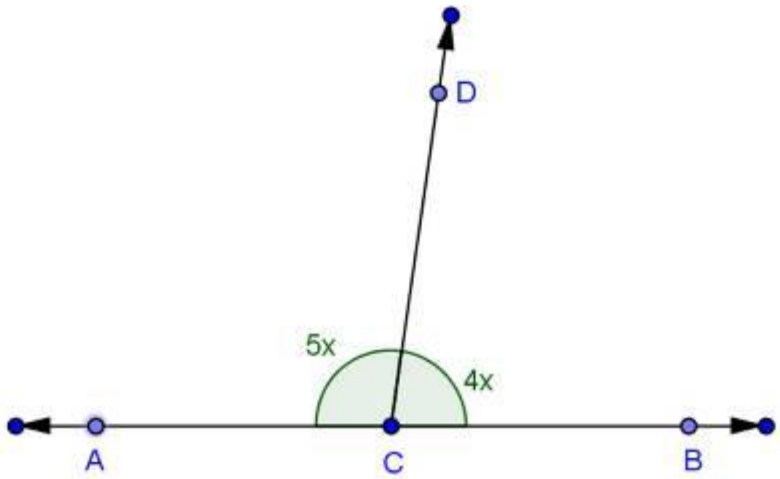
$\Rightarrow 60^\circ + 4x^\circ + 40^\circ = 180^\circ$

$\Rightarrow 4x = 80^\circ$

$\Rightarrow x = 20^\circ$

Question 11

In fig., ACB is a line such that ∠DCA = 5x and ∠DCB = 4x. Find the values of ∠DCA and ∠DCB



Solution 11

$\text{Here}, \angle ACD + \angle BCD = 180^\circ$ [linear pair]

$\Rightarrow 5x + 4x = 180^\circ$

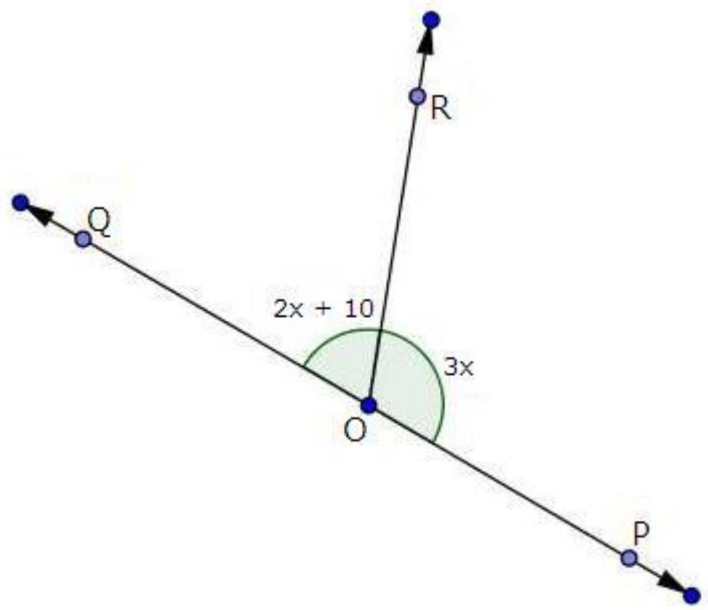
$\Rightarrow 9x = 180^\circ$

$\Rightarrow x = 20^\circ$

$\therefore x = 20^\circ$

Question 12

Give $\angle$POR = 3x and $\angle$QOR = 2x + 10, find the value of x for which POQ will be aline.



Solution 12

$\text{Since}, \angle QOR + \angle POR = 180^\circ$ [linear pair]

$\Rightarrow 2x + 10 + 3x = 180^\circ$

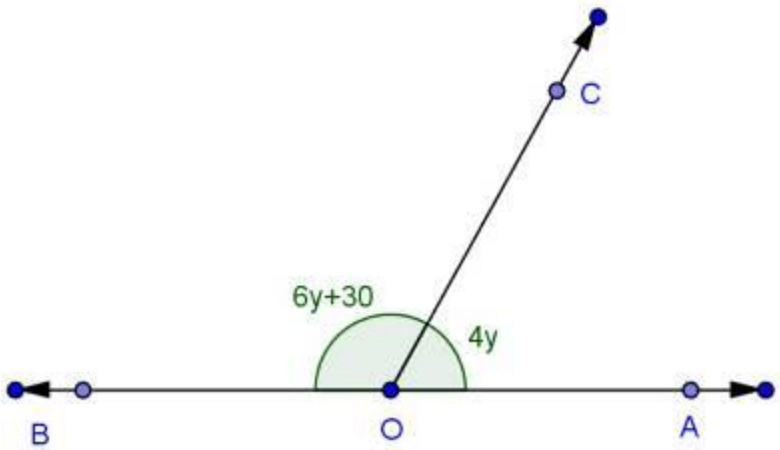
$\Rightarrow 5x + 10 = 180^\circ$

$\Rightarrow 5x = 170^\circ$

$\Rightarrow x = 34^\circ$

Question 13

What value of y would make AOB a line in fig., if $\angle$AOC = 4y and $\angle$BOC = (6y + 30)?



Solution 13

Since $\angle AOC + \angle BOC = 180^\circ$ [linear pair]

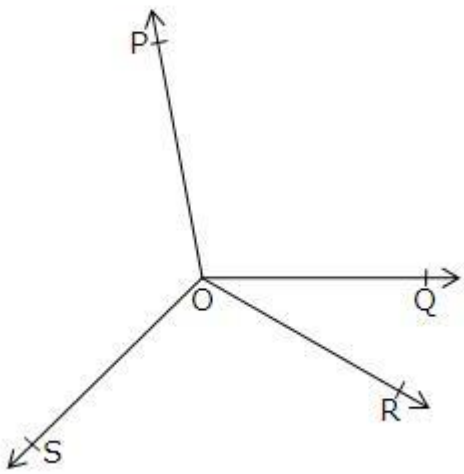
$\Rightarrow 6y + 30 + 4y = 180^\circ$

$\Rightarrow 10y = 150^\circ$

$\Rightarrow \quad y = 15^\circ$

Question 14

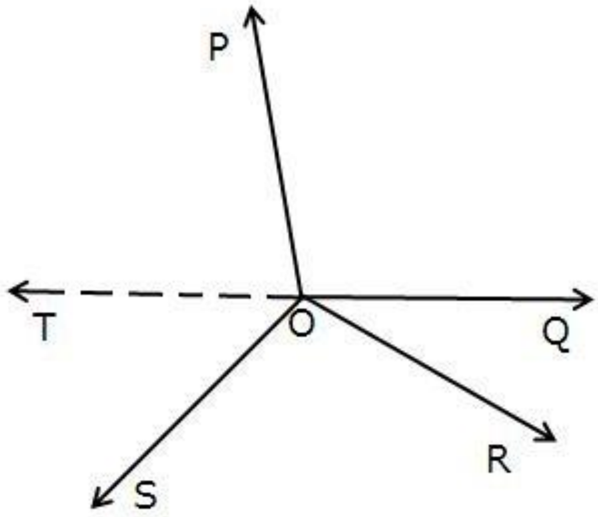
In fig., OP, OQ, OR and OS are four rays. Prove that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^0$



Solution 14

In fig., you need to produce any of the rays OP, OQ, OR or OS backwards to a point. Let us produce ray OQ backwards to a point T so that TOQ is a line.

Now, ray OP stands on line TOQ.



Therefore, $\angle TOP + \angle POQ = 180°$ (1)

(Linear pair axiom)

Similarly, ray OS stands on line TOQ.

Therefore, $\angle TOS + \angle SOQ = 180°$ (2)

But $\angle SOQ = \angle SOR + \angle QOR$

So, (2) becomes

$\angle TOS + \angle SOR + \angle OQR = 180°$

Now, adding (1) and (3), you get

$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360°$

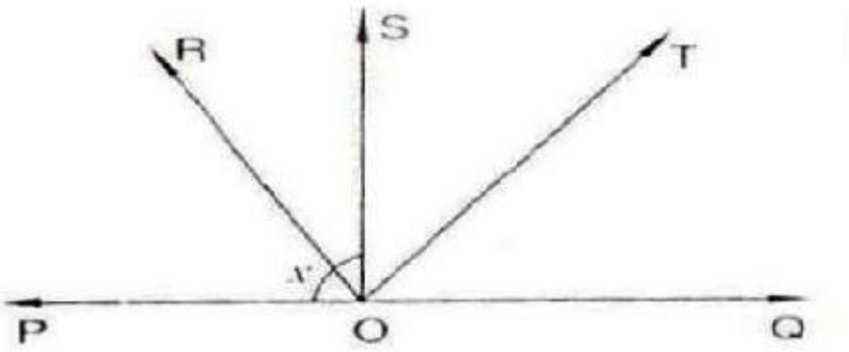
But $\angle TOP + \angle TOS = \angle POS$

Therefore, (4) becomes

$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$

Question 15

In fig., ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$, respectively. IF $\angle POS = x$, find $\angle ROT$.



Solution 15

Ray OS stands on the line POQ.

Therefore, $\angle POS + \angle SOQ, = 180°$

But, $\angle POS = x$

Therefore, $x + \angle SOQ = 180°$

So, $\angle SOQ = 180° - x$

Now, ray OR bisects $\angle POS$, therefore,

$$\angle ROS = \frac{1}{2} \times \angle POS$$
$$= \frac{1}{2} \times x = \frac{x}{2}$$

$$\text{Similarly, } \angle SOT = \frac{1}{2} \times \angle SOQ$$
$$= \frac{1}{2} \times \left(180^\circ - x\right)$$
$$= 90^\circ - \frac{x}{2}$$

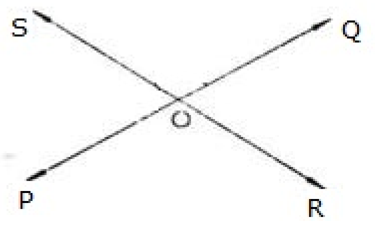
Now, $\angle ROT = \angle ROS + \angle SOT$

$$= \frac{x}{2} + 90^\circ - \frac{x}{2}$$

$$= 90°$$

Question 16

In Fig., Lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 5:7$, find all the angles.



Solution 16

$\angle POR + \angle ROQ = 180°$

(Linear pair of angles)

But $\angle POR : \angle ROQ = 5:7$

(Given)

Therefore, $\angle POR = \frac{5}{12} \times 180^o = 75^o$

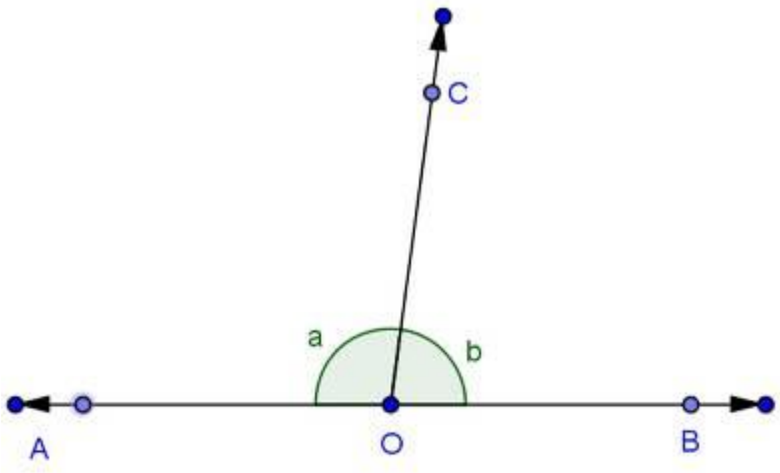
Similarly, $\angle ROQ = \frac{7}{12} \times 180^o = 105^o$

Now, $\angle POS = \angle ROQ = 105^o$ (vertically opposite angles)

and $\angle SOQ = \angle POR = 75^o$ (Vertically opposite angles)

Question 17

In Fig. If a greater than b by one third of a right-angle. find the values of a and b.



Solution 17

$\text{Since } a + b = 180^\circ$ [linear pair]

$\Rightarrow a = 180 - b - - - (1)$

Now,

$a = b + \frac{1}{3} \times 90^\circ$ [given]

$\Rightarrow a = b + 30^\circ - - - (2)$

Equating (1) and (2) we get

$b + 30^\circ = 180^\circ - b$

$\Rightarrow 2b = 150^\circ$

$\Rightarrow b = 75^\circ$

$\therefore b = 75^\circ$

Hence, $a = 180 - b = 180 - 75 = 105^\circ$ [putting $b = 75^\circ$ in 1]

Question 18

If fig , $\angle AOF$ and $\angle FOG$ form a linear pair.

$\angle EOB = \angle FOC = 90^\circ$ and $\angle DOC = \angle FOG = \angle AOB = 30^\circ$

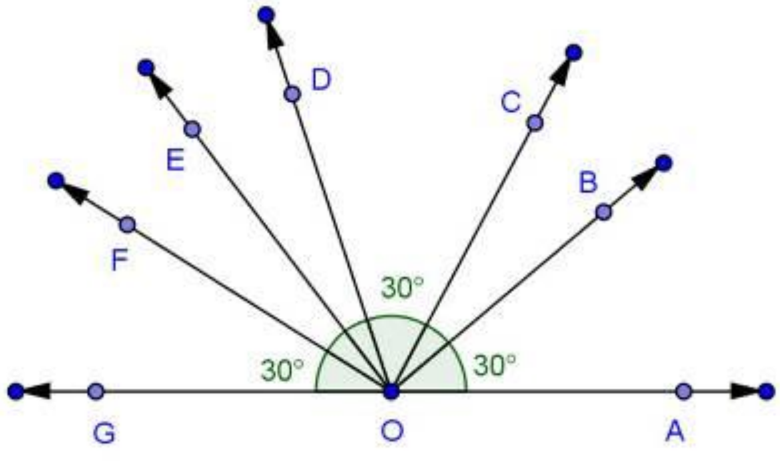
(i) Find the measure of $\angle FOE$, $\angle COB$ and $\angle DOE$.

(ii) Name all the right angles.

(iii) Name three pairs of adjacent complementary angles.

(iv) Name three pairs of adjacent supplementary angles.

(v) Name three pairs of adjacent angles.



Solution 18

(i)Let, $\angle FOE = x, \angle DOE = y$ & $\angle BOC = z$

$\because \angle AOF + \angle FOG = 180^\circ$ [linear pair]

$\Rightarrow \angle AOF + 30^\circ = 180^\circ$ $[\angle FOG = 30^\circ]$

$\Rightarrow \angle AOF = 150^\circ$

$\Rightarrow \angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOF = 150^\circ$

$\Rightarrow 30^\circ + z + 30^\circ + y + x = 150^\circ$

$\Rightarrow x + y + z = 90^\circ - - - (1)$

Now, $\angle FOC = 90^\circ$

$\Rightarrow \angle FOE + \angle EOD + \angle DOC = 90^\circ$

$\Rightarrow x + y + 30^\circ = 90^\circ$

$\Rightarrow x + y = 60^\circ - - - (2)$

$\therefore$ Substituitting (2) & (1)

$\Rightarrow 60 + z = 90^\circ$

$\Rightarrow z = 30^\circ$ i.e $\angle BOC = 30^\circ$

Now $\angle BOE = 90^\circ$ [given]

$\Rightarrow \angle BOC + \angle COD + \angle DOE = 90^\circ$

$\Rightarrow 30^\circ + 30^\circ + \angle DOE = 90^\circ$

$\angle DOE = x = 30^\circ$

Now, also we have, $x + y = 60^\circ$

$\Rightarrow y = 60^\circ - x = 60^\circ - 30^\circ = 30^\circ$

$\therefore \angle FOE = 30^\circ$

(ii) $\angle AOD, \angle BOE, \angle COF, \angle DOG.$

(iii) $\angle AOB, \angle BOD; \angle AOC, \angle COD; \angle BOC, \angle COE.$

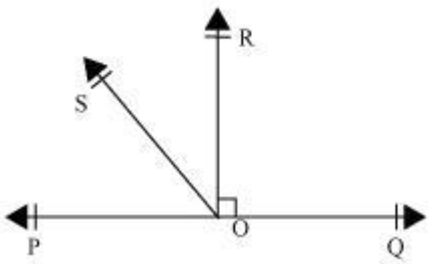
(iv) $\angle AOB, \angle BOG; \angle AOC, \angle COG; \angle AOD, \angle DOG.$

(v) $\angle BOC, \angle COD; \angle COD, \angle DOE; \angle DOE, \angle EOF.$

Question 19

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



Solution 19

Given that OR $\perp$ PQ

$\therefore \angle$POR = 90°

$\Rightarrow \angle$POS + $\angle$SOR = 90°

$\angle$ROS = 90° - $\angle$POS ... (1)

$\angle$QOR = 90° (As OR $\perp$ PQ)

$\angle$QOS - $\angle$ROS = 90°

$\angle$ROS = $\angle$QOS - 90° ... (2)

On adding equations (1) and (2), we have

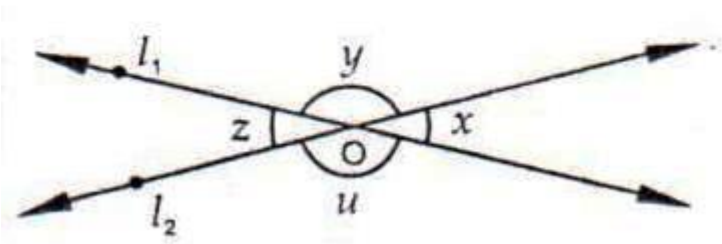
2 $\angle$ROS = $\angle$QOS - $\angle$POS

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Chapter 10 - Lines and Angles Exercise Ex. 10.3

Question 1

In fig., lines l_1 aans l_2 intersect at O, forming angles as shown in the figure. If x = 45, find the values of y, z and u.



Solution 1

Given,

$x = 45^\circ$

$\therefore z = x = 45^\circ$ [vertically opposite angle]

Now, $z + u = 180^\circ$ [linear pair]

$\Rightarrow 45^\circ + u = 180^\circ$

$\Rightarrow \quad u = 135^\circ$

Also $x + y = 180^\circ$ [linear pair]

$\Rightarrow y = 180^\circ - x$

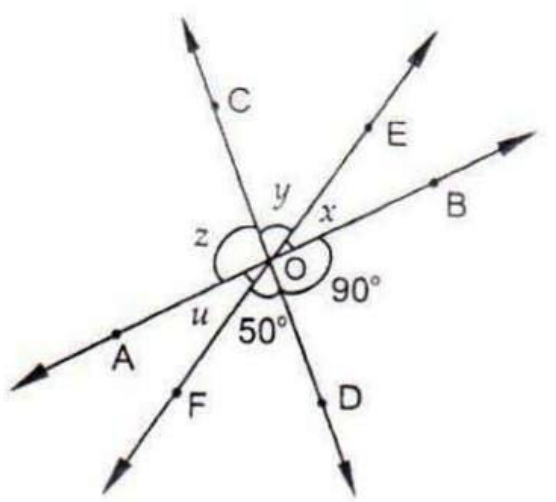
$= 180^\circ - 45^\circ$

$= 135^\circ$

$\therefore y = 135^\circ$

Question 2

In fig., three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x, y, z and u.



Solution 2

$z = \angle BOD = 90^\circ$ [vertically opposite angle]

$y = \angle DOF = 50^\circ$ [vertically opposite angle]

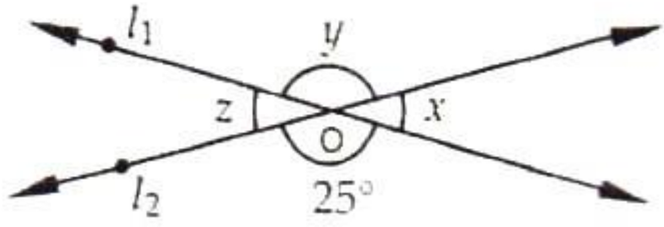
Now, $x + y + z = 180^\circ$ [linear pair]

$\Rightarrow x + 90^\circ + 50^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 140^\circ = 40^\circ$

Question 3

In fig. , find the values of x, y and z.



Solution 3

From the given figure:

$\angle y = 25°$ [vertically opposite angle]

Now,

$\angle x + \angle y = 180°$ [linear pair]

$\Rightarrow \angle x = 180° - 25°$

$\Rightarrow \angle x = 155°$

Also,

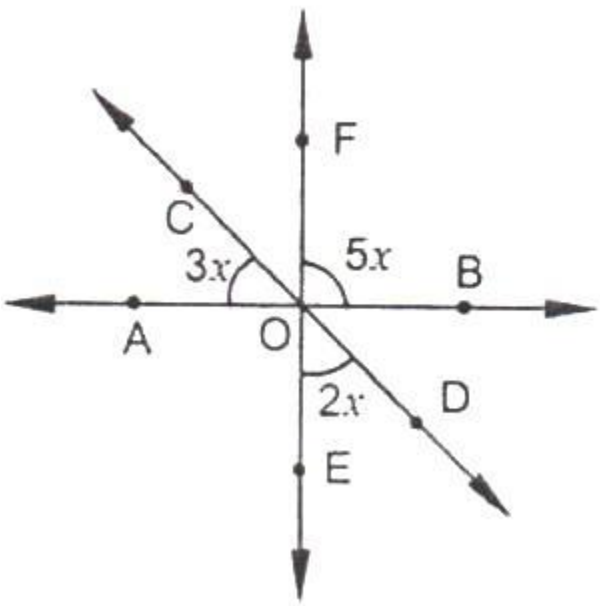
$\angle z = \angle x = 155°$ [vertically opposite angle]

$\therefore \angle y = 25°$

$\angle z = \angle x = 155°$

Question 4

In Fig., find the value of X.



Solution 4

$\angle AOE = \angle BOF = 5x$ [vertically opposite angle]

$\angle COA + \angle AOE + \angle EOD = 180^\circ$ [linear pair]

$\Rightarrow 3x + 5x + 2x = 180^\circ$

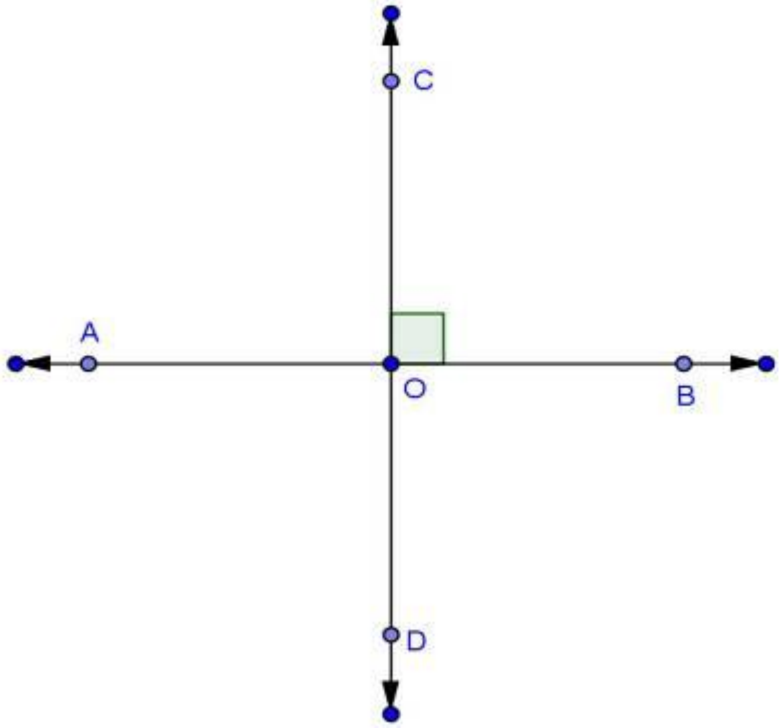
$\Rightarrow \quad 10x = 180^\circ$

$\Rightarrow \quad x = 18^\circ$

Question 5

If one of the four angles formed by two intersecting lines is a right angle, then show that each of the four angles is a right angle.

Solution 5



Given: AB and CD are two lines intersecting at O such that $\angle BOC = 90^\circ$.

R.T.P: $\angle AOC = 90^\circ, \angle AOD = 90^\circ$ & $\angle BOD = 90^\circ$

Proof:

We have, $\angle BOC = 90^\circ$ [given]

Also, $\angle BOC = \angle AOD = 90^\circ$ [vertically opposite angles]

$\angle AOC + \angle BOC = 180^\circ$ [linear pair]

$\Rightarrow \angle AOC + 90^\circ = 180^\circ$

$\Rightarrow \angle AOC = 90^\circ$

Now, $\angle AOC = \angle BOD = 90^\circ$ [vertically opposite angles]

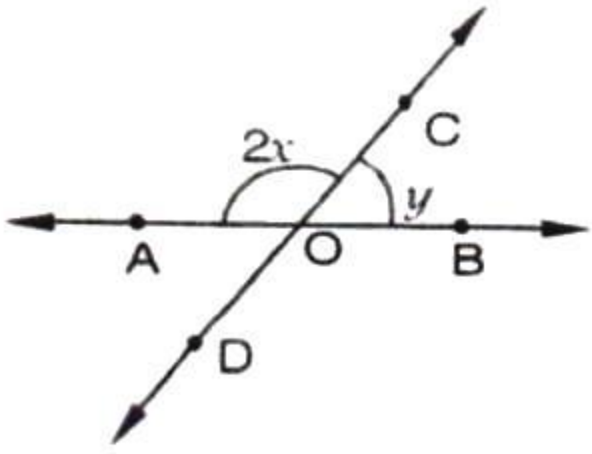
Hence, $\angle AOC = \angle BOC = \angle BOD = \angle AOD = 90^\circ$

Question 6

In fig., rays AB and CD intersect at O.

(i) Determine y when $x = 60^\circ$

(ii) Determine x when $y = 40$



Solution 6

(i)

Here, $\angle AOC + \angle BOC = 180^\circ$ [linear pair]

$\Rightarrow 2x + y = 180^\circ$ [linear pair]

$\Rightarrow 2 \times 60^\circ + y = 180^\circ$

$\Rightarrow y = 180^\circ - 120^\circ = 60^\circ$

$\therefore\ y = 60^\circ$

(ii)

Here, $\angle AOC + \angle BOC = 180^\circ$ [linear pair]

$\Rightarrow 2x + y = 180^\circ$

$\Rightarrow 2x + 40 = 180^\circ$

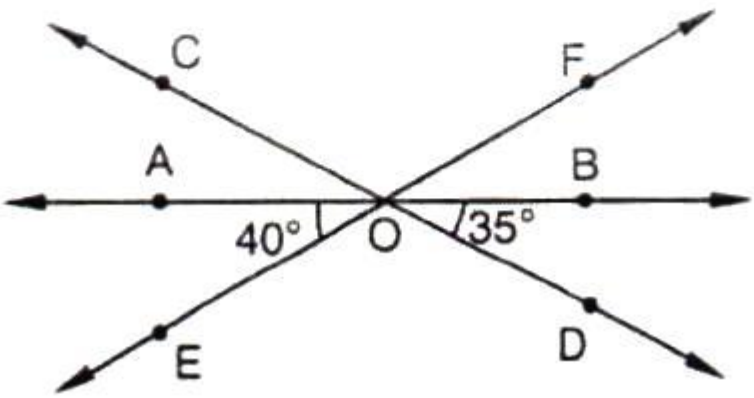
$\Rightarrow 2x = 180 - 40 = 140^\circ$

$\Rightarrow 2x = 140^\circ$

$\Rightarrow x = 70^\circ$

Question 7

In fig., lines AB, CD and EF intersect at O. Find the measures of ∠AOC, ∠COF, ∠DOE and ∠BOF.



Solution 7

$\angle AOE + \angle DOE + \angle BOD = 180^\circ$ [linear pair]

$\Rightarrow \angle DOE = 180^\circ - 40^\circ - 35^\circ = 105^\circ$

$\angle DOE = \angle COF = 105^\circ$ [vertically opposite angles]

Now, $\angle AOE + \angle AOF = 180^\circ$ [linear pair]

$\Rightarrow \angle AOE + \angle AOC + \angle COF = 180^\circ$

$\Rightarrow 40^\circ + \angle AOC + 105^\circ = 180^\circ$

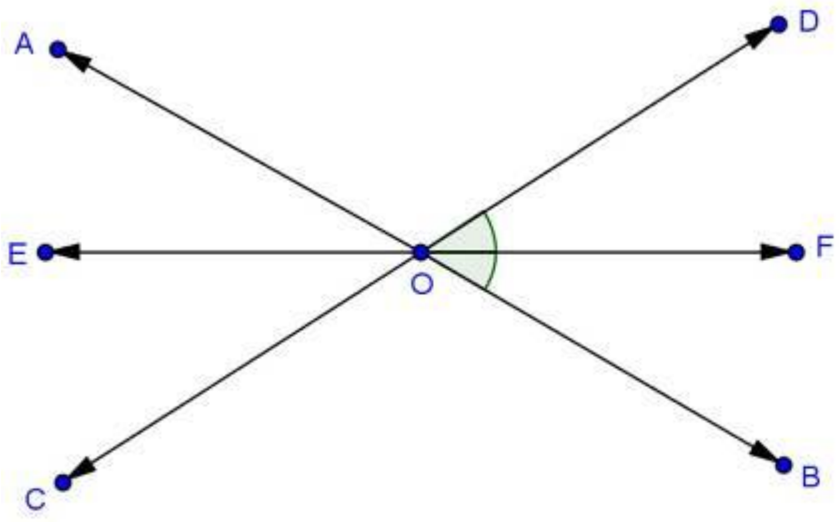
$\Rightarrow \angle AOC = 35^\circ$

Also, $\angle BOF = \angle AOE = 40^\circ$ [vertically opposite angles]

Question 8

AB, CD and EF are three concurrent lines passing through the point O such that OF bisects $\angle$BOD. If $\angle$BOF = 35°, find $\angle$BOC and $\angle$AOD.

Solution 8



$\angle BOF = 35^\circ$

$\therefore \angle BOD = 2\angle BOF = 70^\circ$ [$\because$ OF bisects $\angle BOD$]

$\angle BOD = \angle AOC = 70^\circ$ [vertically opposite angles]

Now, $\angle BOC + \angle AOC = 180^\circ$ [linear Pair]

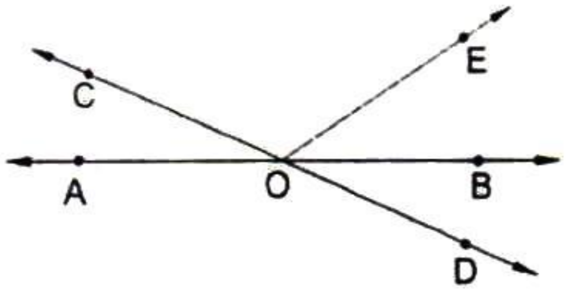
$\Rightarrow \angle BOC + 70^\circ = 180^\circ$

$\Rightarrow \angle BOC = 110^\circ$

$\therefore \angle AOD = \angle BOC = 110^\circ$ [vertically opposite angles]

Question 9

In fig., lines AB, and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Solution 9

$Here, \angle BOD = \angle AOC = 40^\circ$ [vertically opposite angles]

$Now \angle AOC + \angle BOE = 70^\circ$ [given]

$\Rightarrow \angle BOE = 70^\circ - 40^\circ = 30^\circ$

$Now \angle AOC + \angle BOC = 180^\circ$ [linear Pair]

$\Rightarrow \angle AOC + \angle COE + \angle BOE = 180^\circ$

$\Rightarrow \angle COE = 180^\circ - 30^\circ - 40^\circ = 110^\circ$

$\therefore$ reflex $\angle COE = 360^\circ - 110^\circ = 250^\circ$

Question 10

Which of the following statements are true (T) and which are false (F)?

(i) Angles forming a linear pair are supplementary.

(ii) If two adjacent angles are equal, then each angle measures 90^o.

(iii) Angles forming a linear pair can both be acute angles.

(iv) If angles forming a linear pair are equal, then each of these angles is of measure 90^o.

Solution 10

(i) True

(ii) False

(iii) False

(iv) True

Question 11

Fill in the blanks so as to make the following statements true:

(i) If one angle of a linear pair is acute, then its other angle will be ________.

(ii) A ray stands on a line, then the sum of the two adjacent angles so formed is _______.

(iii) If the sum of two adjacent angles is 180^o, then the ______ arms of the two angles are opposite rays.

Solution 11

(i) obtuse.

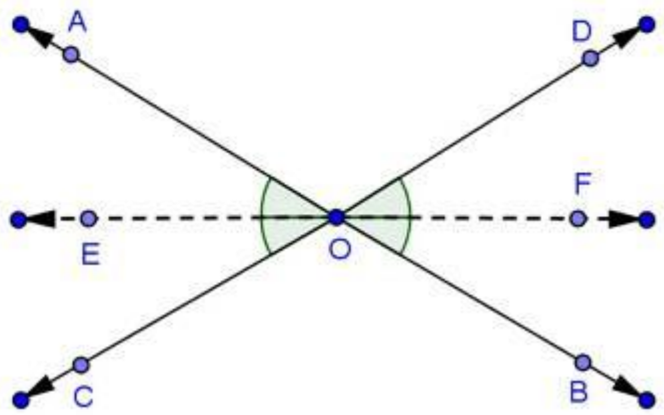
(ii) 180^o

(iii) uncommon

Question 12

Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

Solution 12



Given: lines AOB and COD intersect at point O such that
$\angle AOC = \angle BOD$. Also OE is the bisector $\angle AOC$ and OF is the bisector $\angle BOD$.

To Prove: EOF is a straight line.

$\angle AOD = \angle BOC = 5x$ $\quad$ [vertically opposite angle] $- - (1)$

Also $\angle AOC + \angle BOD$ $\quad$ [vertically opposite angle]

$\Rightarrow 2\angle AOE = 2\angle DOF$ $\quad$ $- - - (2)$

Now, $\angle AOD + \angle AOC + \angle BOC + \angle BOD = 360°$ [Sum of all angles around a point is 360°]

$\Rightarrow 2\angle AOD + 2\angle AOE + 2\angle DOF = 360°$

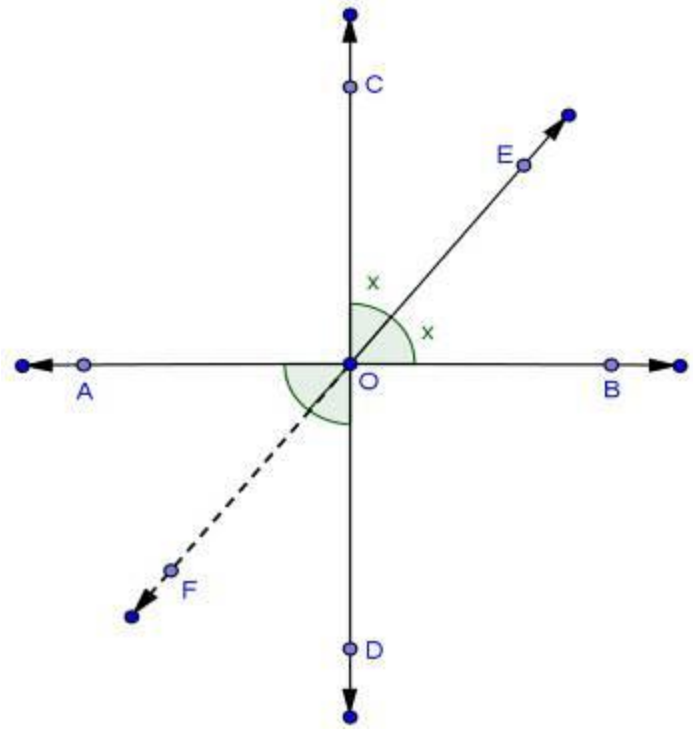
$\Rightarrow \angle AOD + \angle AOE + \angle DOF = 180°$

From this we conclude that EOF is a straight line.

Question 13

If two straight lines intersect each other, prove that the ray opposite to the bisector of one of the angles thus formed bisects the vertically opposite angle.

Solution 13



Given: AB & CD intersect each other at O.
OE bisects $\angle COB$

To prove: $\angle AOF = \angle DOF$

Proof: let $\angle COE = \angle EOB = x$ [$\because$ OE bisects $\angle COB$]

$\angle COE = \angle DOF = x$ [vertically opposite angles] --- (1)

$\angle BOE = \angle AOF = x$ [vertically opposite angles] --- (2)

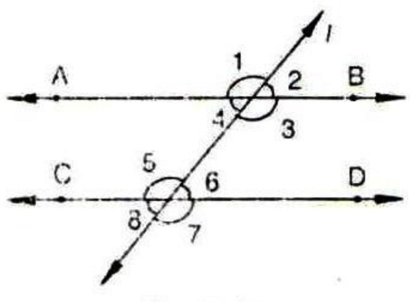
From (1) & (2)

$\angle AOF = \angle DOF = x$

Chapter 10 - Lines and Angles Exercise Ex. 10.4

Question 1

In fig., AB $\parallel$ CD and $\angle$1 and $\angle$2 are in the ratio 3:2. determine all angles from 1 to 8.



Solution 1

Let, $\angle 1 = 3x, \angle 2 = 2x$

$$\text{Now}, \angle 1 + \angle 2 = 180° \quad [\text{linear pair}]$$
$$\Rightarrow 3x + 2x = 180°$$
$$\Rightarrow \quad 5x = 180°$$
$$\Rightarrow \quad x = 36°$$

$\therefore \angle 1 = 3x = 108°, \angle 2 = 2x = 72°$

$\angle 1 = \angle 3 = 108°$ [vertically opposite angles]

$\angle 2 = \angle 4 = 72°$ [vertically opposite angles]

$\angle 1 = \angle 5 = 108°$ [corresponding angles]

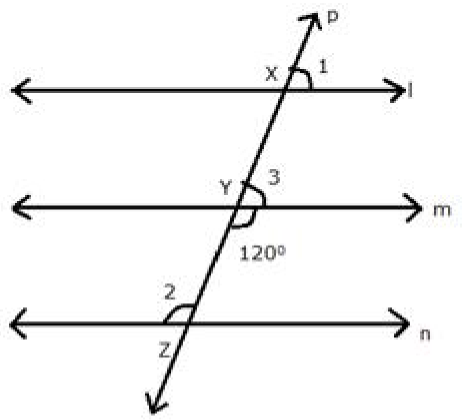
$\angle 2 = \angle 6 = 72°$ [corresponding angles]

$\angle 5 = \angle 7 = 108°$ [vertically opposite angles]

$\angle 6 = \angle 8 = 72°$ [vertically opposite angles]

Question 2

In fig., l, m and n are parallel lines intersected by transversal p at x, y and z respectively. find $\angle 1$, $\angle 2$, $\angle 3$.



Solution 2

From the given figure:

$\angle 3 + \angle m YZ = 180°$ [linear pair]

$\Rightarrow \angle 3 = 180° - 120°$

$\Rightarrow \angle 3 = 60°$

Now, $l \parallel m$

$\therefore \angle 1 = \angle 3$ [Corresponding angles]

$\Rightarrow \angle 1 = 60°$

Also, $m \parallel n$

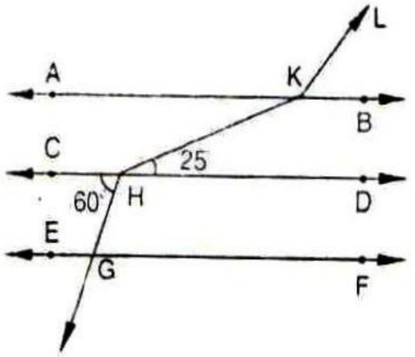
$\Rightarrow \angle 2 = 120°$ [alternate interior angles]

$\therefore \angle 1 = \angle 3 = 60°$

$\angle 2 = 120°$

Question 20

In fig., AB $\parallel$ CD $\parallel$ EF and GH $\parallel$ KL. Find $\angle$HKL



Solution 20

Produce LK to meet GF at N.

Now,

$\angle CHG = \angle HGN = 60°$ [alternate angles]

$\angle HGN = \angle KNF = 60°$ [corresponding angles]

$\therefore \angle KNG = 180° - 60° = 120°$ [linear pair]

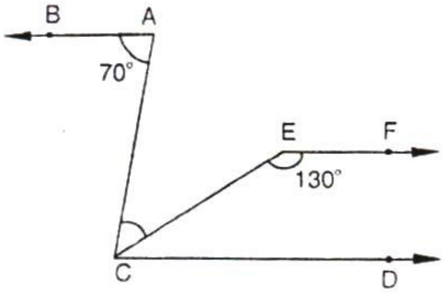
$\angle GNK = \angle AKL = 120°$ [corresponding angles]

$\angle AKH = \angle KHD = 25°$ [alternate angles]

$\therefore \angle HKL = \angle AKH + \angle AKL = 25° + 120° = 145°$

Question 3

In fig., if AB $\parallel$ CD and CD $\parallel$ EF, find $\angle$ACE.



Solution 3

Since $EF \parallel CD$

$\therefore \angle FEC + \angle ECD = 180^\circ$ [co-interior angles are supplementary]

$\Rightarrow \angle ECD = 180^\circ - 130^\circ = 50^\circ$

Also $BA \parallel CD$

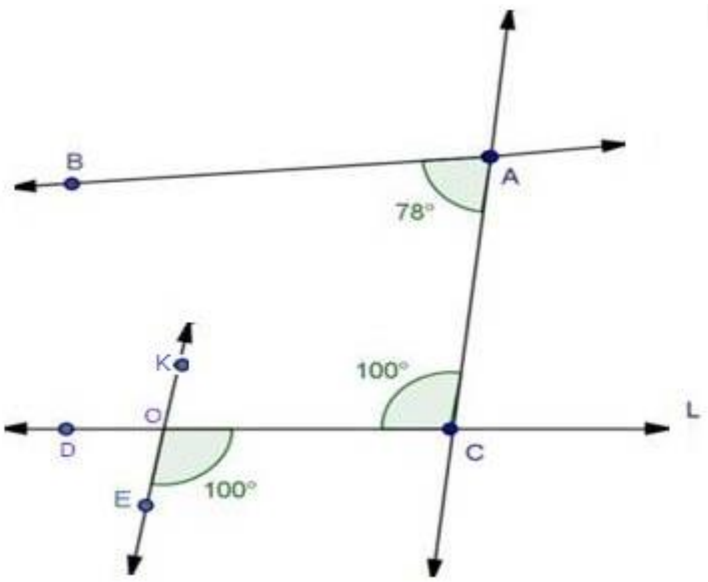
$\Rightarrow \angle BAC = \angle ACD = 70^\circ$ [alternate interior angles]

But $\angle ACE + \angle ECD = 70^\circ$

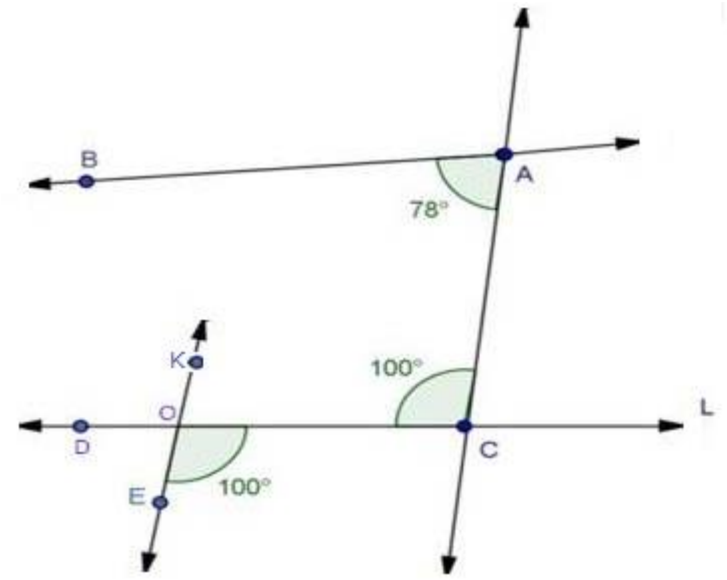
$\Rightarrow \angle ACE = 70^\circ - 50^\circ = 20^\circ$

Question 4

In fig., state which lines are parallel and why.



Solution 4



$\angle EOC = \angle DOK = 100^\circ$ [vertically opposite angles]

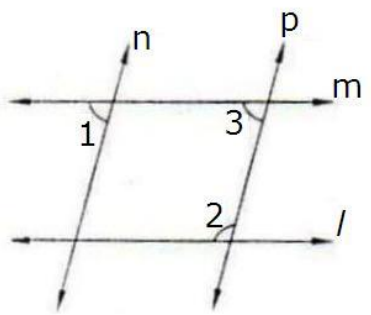
$and\ \angle DOK = \angle ACO = 100^\circ$

Here two lines EK and AC cut by a third line 'l' and the corresponding angles to it are equal.

$\therefore EK \parallel AC$

Question 5

In fig. if l $\parallel$ m, n $\parallel$ p and $\angle$1 = 85°, find $\angle$2.



Solution 5

$\angle 1 = \angle 3 = 85^\circ$ [corresponding angles]

$\angle 2 + \angle 3 = 180^\circ$ [co-interior angles are supplementary]

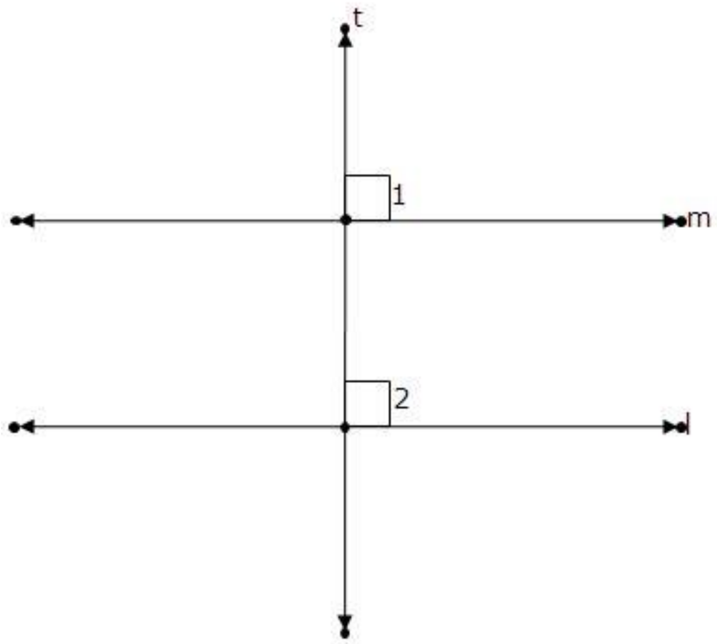
$\angle 2 = 180^\circ - 85^\circ = 95^\circ$

$\therefore \angle 2 = 95^\circ$

Question 6

If two straight lines are perpendicular to the same line, prove that they are parallel to each other.

Solution 6



$t \perp m$ [given]

$t \perp l$ [given]

$\angle 1 = \angle 2 = 90°$

Since l and m are two lines and t is transversal and the corresponding angles are equal.

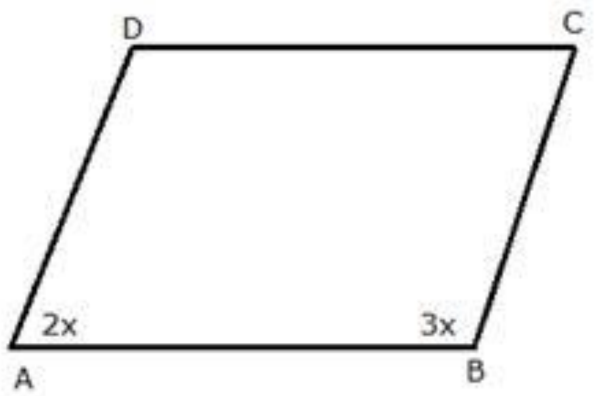
$\therefore l \parallel m$

Hence proved.

Question 7

Two unequal angles of a parallelogram are in the ratio 2 : 3. Find all its angles in degrees.

Solution 7



Let $\angle A = 2x$ and $\angle B = 3x$

Now,

$$\angle A + \angle B = 180° \quad \begin{bmatrix} \text{Co. interior angles are supplementary} \\ AD \parallel BC \text{ and } AB \text{ is the transversal} \end{bmatrix}$$

$$2x + 3x = 180°$$

$$5x = 180°$$

$$x = \frac{180°}{5} = 36°$$

$$\therefore \quad \angle A = 2 \times 36° = 72°$$

$$\angle b = 3 \times 36 = 108°$$

Now,

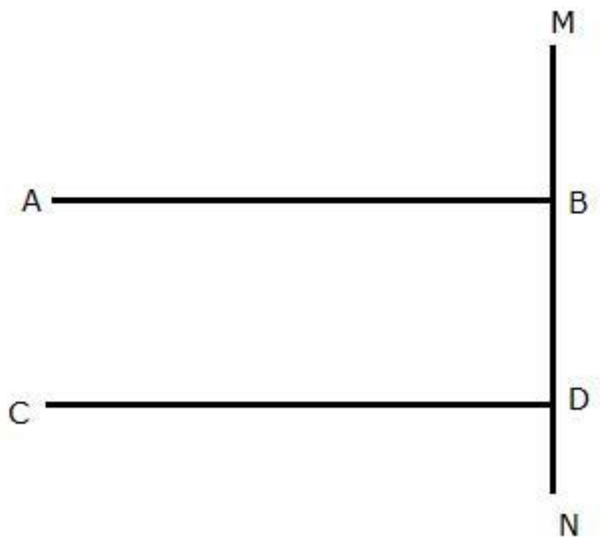
$$\angle A = \angle C = 72° \quad [\text{opposite angles of a parallelogram are equal}]$$

$$\angle B = \angle D = 108° \quad [\text{opposite angles of a parallelogram are equal}]$$

Question 8

If each of the two lines is perpendicular to the same line, what kind of lines are they to each other?

Solution 8



Let *AB* and *CD* be perpendicuar to line *MN*.

$\angle ABD = 90°$ [Since $AB \perp MN$] ---(i)
$\angle CDN = 90°$ [Since $CD \perp MN$] ---(ii)

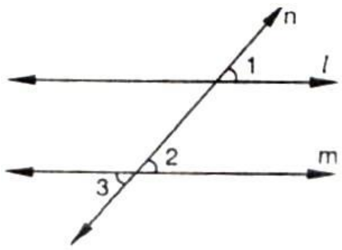
Now,

$$\angle ABD = \angle CDN = 90° \text{ [From (i) and (ii)]}$$

$\therefore$ $AB \parallel CD$, since corresponding angles are equal.

Question 9

In fig., $\angle 1 = 60°$ and $\angle 2 = (2/3)^{rd}$ of a right angle. prove that $l \parallel m$



Solution 9

Given: $\angle 1 = 60°$, $\angle 2 = \left[\frac{2}{3}\right]^{rd}$ of right angle

To prove: $l \parallel m$

Proof: $\angle 1 = 60°$

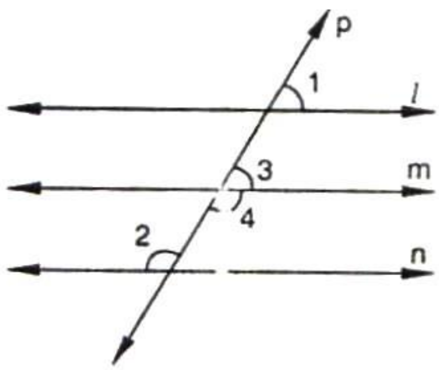
$$\angle 2 = \frac{2}{3} \times 90° = 60°$$

Since, $\angle 1 = \angle 2 = 60°$

$\therefore$ $l \parallel m$ as pair of corresponding angles are equal.

Question 10

In fig., if $l \parallel m \parallel n$ and $\angle 1 = 60°$, find $\angle 2$.



Solution 10

Since $l \parallel m$ and P is the transversal

$\therefore$ Given: $l \parallel m \parallel n$, $\angle 1 = 60°$

To find: $\angle 2$

$\angle 1 = \angle 3 = 60°$ [Corresponding angles]

Now,

$\angle 3 + \angle 4 = 180°$ [Linear pair]

$60° + \angle 4 = 180°$

$\angle 4 = 180° - 60° = 120°$

Also,

$m \parallel n$ and p is the transversal

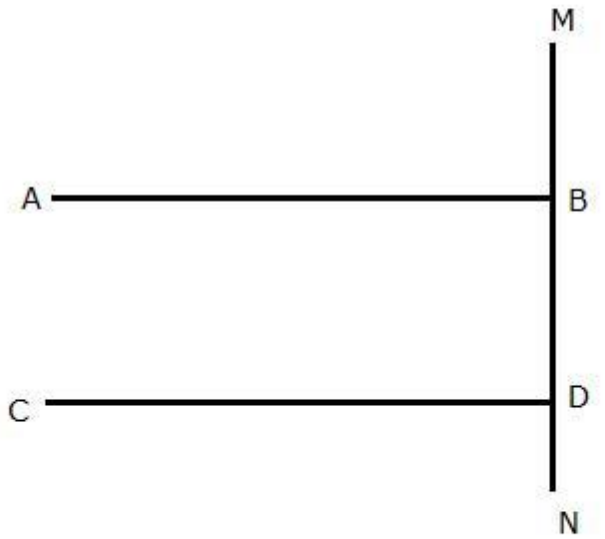
$\therefore$ $\angle 4 = \angle 2 = 120°$ [Alternate interior angle]

Hence, $\angle 2 = 120°$

Question 11

Prove that the straight lines perpendicular to the same straight line are parallel to one another.

Solution 11



Let *AB* and *CD* be perpendicuar to line *MN*.

$\angle ABD = 90°$ [Since $AB \perp MN$] ---(i)

$\angle CDN = 90°$ [Since $CD \perp MN$] ---(ii)

Now,

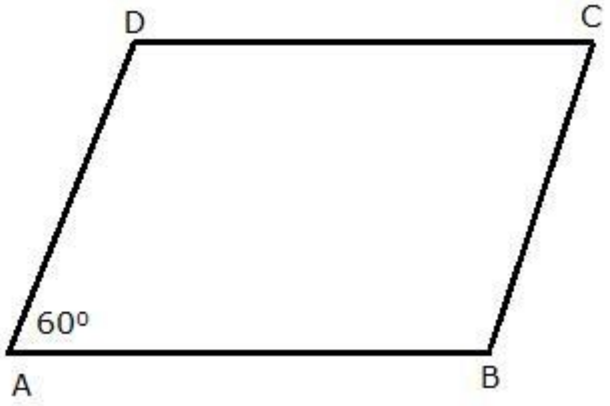
$\angle ABD = \angle CDN = 90°$ [From (i) and (ii)]

$\therefore$ $AB \parallel CD$, since corresponding angles are equal.

Question 12

The opposite sides of a quadrilateral are parallel. If one angle of the quadrilateral is 60°, find the other angles.

Solution 12



Given: $AB \parallel CD$

$AD \parallel BC$

Since $AB \parallel CD$ and AD is the transversal

$\therefore \quad \angle A + \angle D = 180°$ [Co. interior angles are supplementary]

$60° + \angle D = 180°$

$\angle D = 180° - 60° = 120°$

Now,

$AD \parallel BC$ and AB is the transversal

$\angle A + \angle B = 180°$ [Co. interior angles are supplementary]

$60° + \angle B = 180°$

$\angle B = 180° - 60° = 120°$

Also,

$\angle B + \angle C = 180°$ [Co. interior angles are supplementary]

$120° + \angle C = 180°$

$\angle C = 180° - 120° = 60°$

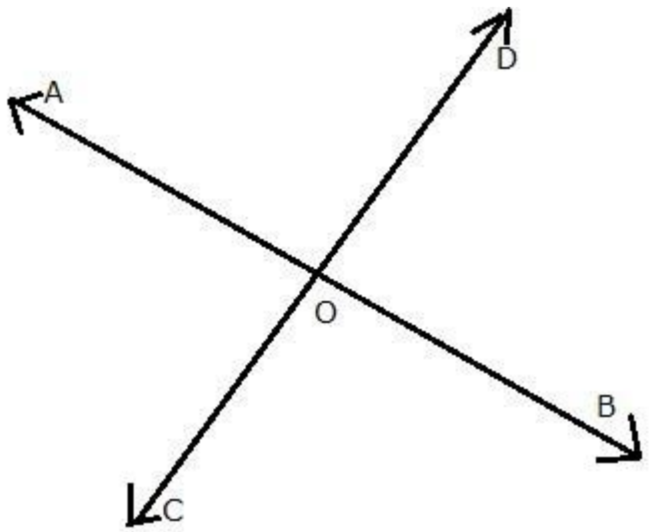
Hence, $\angle A = \angle C = 60°$

$\angle B = \angle D = 120°$

Question 13

Two lines AB and CD intersect at O. If $\angle AOC + \angle COB + \angle BOD = 270°$, find the measures of $\angle AOC$, $\angle COB$, $\angle BOD$ and $\angle DOA$.

Solution 13



Given: $\angle AOC + \angle COB + \angle BOD = 270°$

To find: $\angle AOC$, $\angle COB$, $\angle BOD$ and $\angle DOA$

Here, $\angle AOC + \angle COB + \angle BOD + \angle AOD = 360°$ [complete Angle]

$\Rightarrow \quad 270 + \angle AOD = 360°$ $[\because \angle AOC + \angle COB + \angle BOD = 270°]$

$\Rightarrow \quad \angle AOD = 360 - 270 = 90°$

Now, $\angle AOD + \angle BOD = 180°$ [linear pair]

$90 + \angle BOD = 180$

$\Rightarrow \quad \angle BOD = 180 - 90$

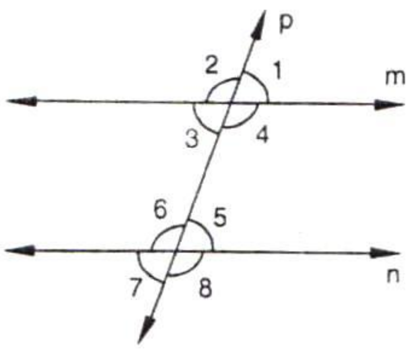
$\therefore \quad \angle BOD = 90°$

$\angle AOD = \angle BOC = 90°$ [vertically opposite angles]

$\angle BOD = \angle AOC = 90°$ [vertically opposite angles]

Question 14

In fig., p is transversal to lines m and n, $\angle 2 = 120°$ and $\angle 5 = 60°$. Prove that m $\parallel$ n.



Solution 14

Given: $\angle 2 = 120^\circ$, $\angle 5 = 60^\circ$
To prove: $m \parallel n$

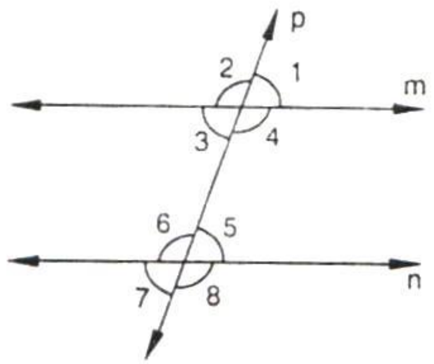
Proof: $\angle 2 + \angle 1 = 180^\circ$ [linear pair]

$120 + \angle 1 = 180^\circ$
$\angle 1 = 180 - 120$
$\angle 1 = 60$

Since $\angle 1 = \angle 5 = 60^\circ$
$\therefore m \parallel n$ [As pair of corresponding angles are equal]

Question 15

In fig., transceral l intersects two lines m and n, $\angle 4 = 110°$ and $\angle 7 = 65°$. is m $\parallel$ n ?

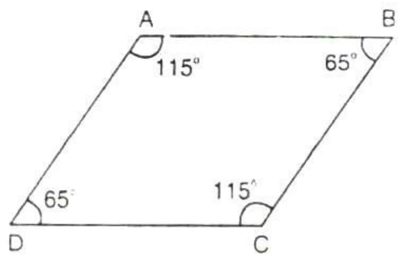


Solution 15

Given: $\angle 4 = 110^\circ, \angle 7 = 65^\circ$
To find: Is $m \parallel n$
Here, $\angle 7 = \angle 5 = 65^\circ$ [Vertically opposite angle]
Now, $\angle 4 + \angle 5 = 110 + 65 = 175^\circ$
$\therefore$ m is not parallel to n as the pair of co. interior angles is not supplementary.

Question 16

Which pair of lines in Fig., are parallel? give reasons.



Solution 16

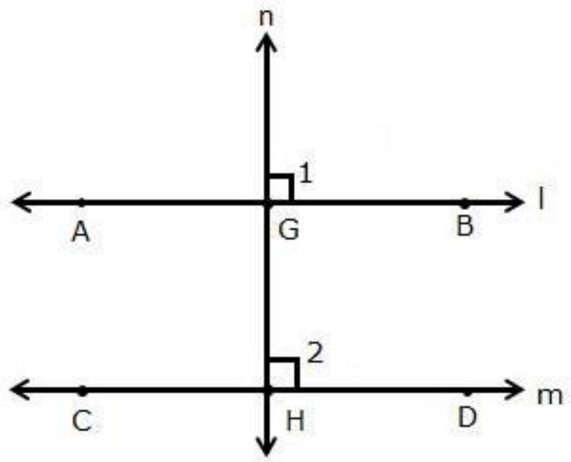
$\angle A + \angle B = 115 + 65 = 180^\circ$
$\therefore AD \parallel BC$ [As sum of co. interior angles we supplementary]
$\angle B + \angle C = 65 + 115 = 180^\circ$

$\therefore AB \parallel CD$ [As sum of co. interior angles are supplementary]

Question 17

If l, m, n are three lines such that $l \parallel m$ and $n \perp l$, prove that $n \perp m$.

Solution 17



Given: $l \parallel m$, $n \perp l$

To prove: $n \perp m$

Since l||m and n intersects them at G and H respectively

$\therefore \angle 1 = \angle 2$ [corresponding angles]

But, $\angle 1 = 90°$ [$n \perp l$]

$\Rightarrow \angle 2 = 90°$

hence, $n \perp m$

Question 18

Which of the following statements are true (T) and which are false (F)? Give reasons.

(i) If two lines are intersected by a transversal, then corresponding angles are equal.

(ii) If two parallel lines are intersected by a transversal, then alternate interior angles are equal.

(iii) Two lines perpendicular to the same line are perpendicular to each other.

(iv) Two lines parallel to the same line are parallel to each other.

(v) If two parallel lines are intersected by a transversal, then the interior angles on the same side of the transversal are equal.

Solution 18

(i) False

(ii) True

(iii) False

(iv) True

(v) False

Question 19

Fill in the blanks in each of the following to make the statement true:

(i) if two parallel lines are intersected by a transversal, then each pair of corresponding angles are ____.

(ii) If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are ____.

(iii) Two lines perpendicular to the same line are ____ to each other.

(iv) Two lines parallel to the same line are ____ to each other.

(v) If a transversal intersects a pair of lines in such away that a pair of alternate angles are equal, then the lines are ____.

(vi) If a transversal intersects a pair of lines in such away that the sum of interior angles on the same side of transversal is 180°, then the lines are ____.

Solution 19

(i) Equal

(ii) Supplementary

(iii) Parallel

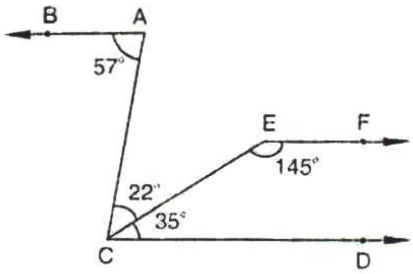
(iv) Parallel

(v) Parallel

(vi) Parallel

Question 21

In fig., show that AB $\parallel$ EF.



Solution 21

Produce EF to intersect AC at K.

Now, $\angle DCE + \angle CEF = 35^\circ + 145^\circ = 180^\circ$

$\therefore EF \parallel CD$ $\quad$ $[\because$ sum of co-interior angles is $180^\circ]$ --- (1)

Now $\angle BAC = \angle ACD = 57^\circ$

$\Rightarrow BA \parallel CD$ $\quad$ $[\because$ alternate interior angles are equal $]$ --- (2)

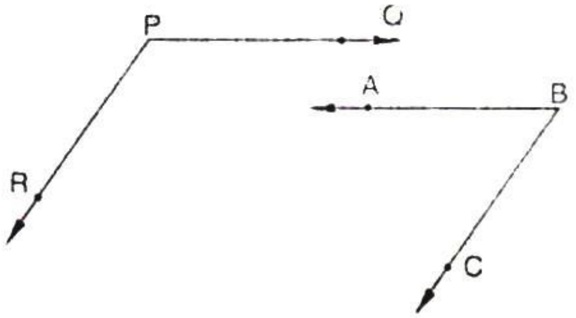
from (1) and (2)

$AB \parallel EF$ $\quad$ [lines parallel to the same line are parallel to each other]

Hence proved.

Question 22

In Fig., PQ $\parallel$ AB and PR BC. IF $\angle$QPR = 102º, determine $\angle$ABC. Give reasons.



Solution 22

AB isproduced to meet PR at K.

Since $PQ \parallel AB$

$\angle QPR = \angle BKR = 102^\circ$ [corresponding angles]

Since $PR \parallel BC$

$\therefore \angle RKB + \angle KBC = 180^\circ$ [co-interior angles are supplementary]

$\Rightarrow \angle KBC = 180^\circ - 102^\circ = 78^\circ$

$\therefore \angle KBC = \angle ABC = 78^\circ$

Question 23

Prove that if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

Solution 23

Consider the angles AOB and ACB.

Given: $CA \perp AO$
$CB \perp BO$

To prove:
$\angle AOB = \angle ACB$
or
$\angle AOB + \angle ACB = 180°$

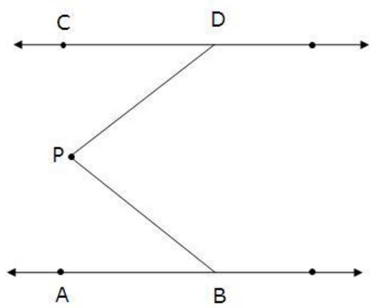
Proof: In quadrilateral $AOBC$
$\Rightarrow \angle A + \angle O + \angle B + \angle C = 360°$ [sum of angles of a quadrilateral]
$\Rightarrow 90 + \angle O + 90 + \angle C = 360$
$\Rightarrow 180 + \angle O + \angle C = 360$
$\Rightarrow \angle O + \angle C = 360 - 180 = 180°$

Hence, $\angle AOB + \angle ACB = 180°$. ....(i)
Also,
$\angle B + \angle ACB = 180°$
$\Rightarrow \angle ACB = 180° - 90°$
$\Rightarrow \angle ACB = 90°$.........(ii)
From (i) and (ii)
$\therefore \angle ACB = \angle AOB = 90°$

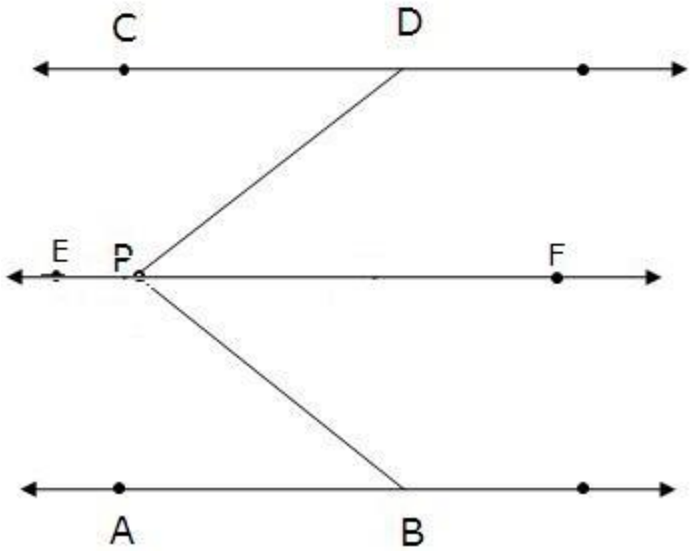
Hence, The angles are equal as well as supplementary.

Question 24

In fig., lines AB and CD are parallel and p is any point as shown in the figure. Show that $\angle ABP + \angle CDP = \angle DPB$.



Solution 24



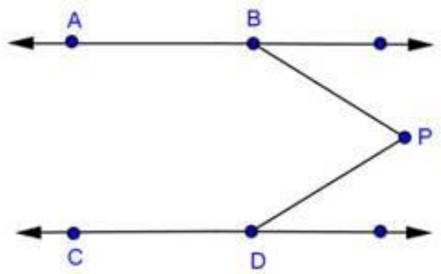
Given AB || CD.
Let EF be the parallel line to AB and CD which passes through P.

It can be seen from the figure
$\angle ABP = \angle BPF$ (Alternate int. angles)
$\angle CDP = \angle DPF$ (Alternate int. angles)
$\Rightarrow \angle ABP + \angle CDP = \angle BPF + \angle DPF$
$\Rightarrow \angle ABP + \angle CDP = \angle DPB$
Hence proved

Question 25

In fig., AB $\parallel$ CD and P is any point shown in the figure. Prove that:

$\angle ABP + \angle BPD + \angle CDP = 360°$



Solution 25

Given: $AB \parallel CD$, P is any point.

To prove: $\angle ABP + \angle BPD + \angle CDP = 360°$

Construction: Draw $EF \parallel AB$ passing through P

Proof:

Since $AB \parallel EF$ and $AB \parallel CD$

$\therefore \quad EF \parallel CD \quad$ [Lines parallel to the same line are parallel to each other]

$\angle ABP + \angle EPB = 180° \quad \begin{bmatrix}\text{Sum of co. interior angles is } 180° \\ AB \parallel EF \text{ and } BP \text{ is the transversal}\end{bmatrix} \quad$ ---(i)

$\angle EPD + \angle CDP = 180° \quad \begin{bmatrix}\text{Sum of co. interior angles is } 180° \\ EF \parallel CD \text{ and } DP \text{ is the transversal}\end{bmatrix} \quad$ ---(ii)

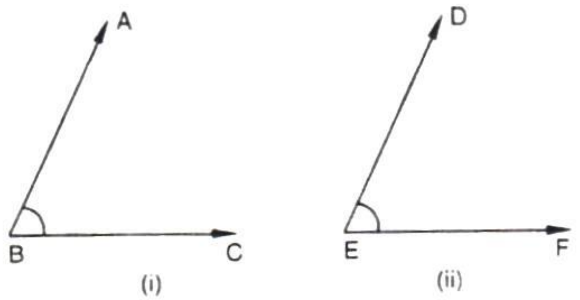
Adding (i) and (ii)

$$\angle ABP + \angle EPB + \angle EPD + \angle CDP = 180° + 180°$$

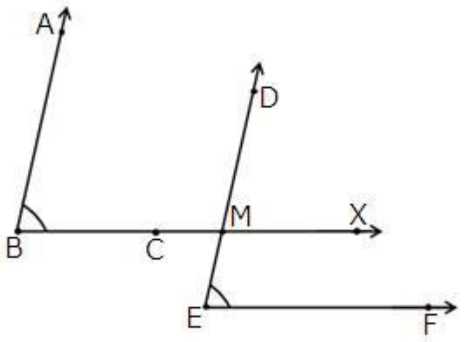
$$\angle ABP + \angle BPD + \angle CDP = 360°$$

Question 26

In fig., arms BA and BC of ∠ABC are respectively parallel to arms ED and EF of ∠DEF. Prove that ∠ABC = ∠DEF



Solution 26



Given: $AB \parallel DE$

$BC \parallel EF$

To prove: $\angle ABC = \angle DEF$

Construction: Produce BC to X such that it intersects DE at M.

Proof: Since $AB \parallel DE$ and BX is the transversal

$\therefore \quad \angle ABC = \angle DMX \quad$ [Corresponding angles] ---(i)

Also,

$BX \parallel EF$ and DE is the transversal

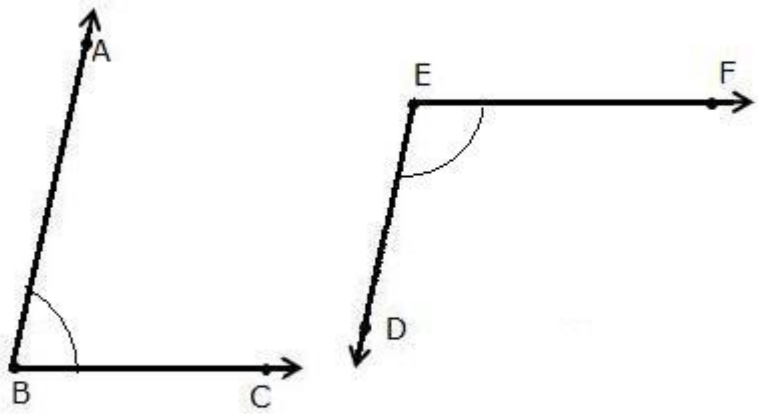
$\therefore \quad \angle DMX = \angle DEF \quad$ [Corresponding angles] ---(ii)

From (i) and (ii)

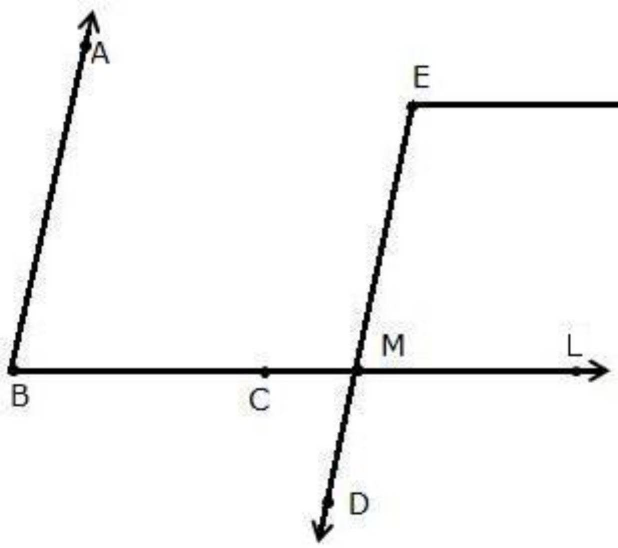
$\therefore \quad \angle ABC = \angle DEF$

Question 27

In fig., arms BA and BC of $\angle ABC$ are respectively parallel to arms ED and EF of $\angle DEF$. Prove that $\angle ABC + \angle DEF = 180°$.



Solution 27



Given: $AB \parallel DE$, $BC \parallel EF$

To prove: $\angle ABC + \angle DEF = 180°$

Construction: Produce BC to intersect DE at M

Proof: Since, $AB \parallel EM$ and BL is the transversal

$\therefore \quad \angle ABC = \angle EML \quad$ [Corresponting angles] --- (i)

Also,

$EF \parallel ML$ and EM is the transversal

Hence, $\angle DEF + \angle EML = 180°$ [Co-interior angles are supplementary] --- (ii)

From (i) and (ii)

$\therefore \quad \angle DEF + \angle ABC = 180°$