# RD SHARMA Solutions for Class 9 Maths Chapter 2 - Exponents of Real Numbers

## Chapter 2 - Exponents of Real Numbers Exercise 2.29

Question 1

The value of $\{2 - 3(2 - 3)^3\}^3$ is

(a) 5
(b) 125
(c) 1/5
(d) -125

Solution 1

$\{2 - 3(2 - 3)^3\}^3$
$= \{2 - 3(-1)^3\}^3$
$= \{2 - 3(-1)\}^3$
$= \{2 - (-3)\}^3$
$= \{2 + 3\}^3$
$= \{5)^3$
$= 5^3$
$= 125$

So, correct option is (b).

Question 2

The value of $x - y^{x-y}$ when $x = 2$ and $y = -2$ is

(a) 18
(b) -18
(c) 14
(d) -14

Solution 2

$x = 2, y = -2$
$x - y = 2 - (-2) = 2 + 2 = 4$
Now $x - y^{x-y} = 2 - (-2)^4 = 2 - 16 = -14$
So, correct option is (d).

Question 3

The product of the square root of x with the cube root of x is

(a) cube root of the square root of x
(b) sixth root of the fifth power of x
(c) fifth root of the sixth power of x
(d) sixth root of x

Solution 3

Square root of x $= \sqrt{x} = x^{\frac{1}{2}}$

Cube root of x $= \sqrt[3]{x} = x^{\frac{1}{3}}$

Thus,

$x^{\frac{1}{2}} \times x^{\frac{1}{3}}$

$= (x)^{\frac{1}{2}+\frac{1}{3}}$

$= x^{\frac{5}{6}}$

$= (x^5)^{\frac{1}{6}}$

$= \sqrt[6]{x^5}$

= sixth root of the fifth power of x

Hence, correct option is (b).

Question 4

The seventh root of x divided by the eighth root of x is

(a) x

(b) $\sqrt{x}$

(c) $\sqrt[56]{x}$

(d) $\frac{1}{\sqrt[56]{x}}$

Solution 4

Seventh root of x $= x^{\frac{1}{7}}$

Eighth root of x $= x^{\frac{1}{8}}$

Now, $\frac{x^{\frac{1}{7}}}{x^{\frac{1}{8}}} = (x)^{\frac{1}{7}-\frac{1}{8}} = x^{\frac{1}{56}} = \sqrt[56]{x}$

Hence, correct option is (c).

Question 5

The square root of 64 divided by the cube root of 64 is

(a) 64

(b) 2

(c) $\frac{1}{2}$

(d) $64^{2/3}$

Solution 5

Square root of 64 $= \sqrt{64} = 8$

Cube root of 64 $= \sqrt[3]{64} = 4$

Now $\dfrac{\sqrt{64}}{\sqrt[3]{64}} = \dfrac{8}{4} = 2$

Hence, correct option is (b).

## Chapter 2 - Exponents of Real Numbers Exercise 2.30

Question 1

Which of the following is (are) not equal to $\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6}$ ?

(a) $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}$

(b) $\dfrac{1}{\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{1/6}}$

(c) $\left(\frac{6}{5}\right)^{1/30}$

(d) $\left(\frac{5}{6}\right)^{-1/30}$

Solution 1

$$\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6} = \frac{1}{\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{1/6}} \quad \text{(option b)}$$

$$\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6} = \frac{1}{\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{1/6}} = \frac{1}{\left(\frac{5}{6}\right)^{\frac{1}{5}\times\frac{1}{6}}} = \frac{1}{\left(\frac{5}{6}\right)^{\frac{1}{30}}} \quad \text{(option c)}$$

$$\left(\frac{5}{6}\right)^{\frac{1}{5}\times\left(\frac{-1}{6}\right)} = \left(\frac{5}{6}\right)^{\frac{-1}{30}} \quad (option\ d)$$

$$\text{But, } \left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6} \neq \left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}} \quad \text{(option a)}$$

Hence, correct option is (a).

Question 2

When simplified $\left(x^{-1}+y^{-1}\right)^{-1}$ is equal to

(a) $xy$

(b) $x+y$

(c) $\frac{xy}{x+y}$

(d) $\frac{x+y}{xy}$

Solution 2

$\left(x^{-1}+y^{-1}\right)^{-1}$

$= \left(\frac{1}{x}+\frac{1}{y}\right)^{-1}$

$= \left(\frac{x+y}{xy}\right)^{-1}$

$= \frac{xy}{x+y}$

Hence, correct option is (c).

Question 3

If $8^{x+1} = 64$, what is the value of $3^{2x+1}$?

(a) 1
(b) 3
(c) 9
(d) 27

Solution 3

$8^{x+1} = 64 = (8)^2$

so, $x + 1 = 2$

Hence, $x = 1$

Now, $3^{2x+1} = 3^{2(1)+1} = 3^3 = 27$

Hence, correct option is (d).

Question 4

If $(2^3)^2 = 4^x$, then $3^x =$

(a) 3
(b) 6
(c) 9
(d) 27

Solution 4

$(2^3)^2 = (2^2)^3 \quad \left[\because (a^b)^c = (a^b)^c \text{ by properties}\right]$

$\therefore (2^3)^2 = 4^3 = 4^x$

$\Rightarrow x = 3$

Now $3^x = 3^3 = 27$

Hence, correct option is (d).

Question 5

If $x^{-2} = 64$, then $x^{1/3} + x^0 =$

(a) 2
(b) 3

(c) 3/2
(d) 2/3

Solution 5

$x^{-2} = 64$

$\Rightarrow x^2 = \frac{1}{64}$

$\Rightarrow x = \frac{1}{8}$

Now, $x^{1/3} + x^0$

$= \left(\frac{1}{8}\right)^{1/3} + \left(\frac{1}{8}\right)^0$

$= \left(\frac{1}{8}\right)^{1/3} + 1$

$= \left\{\left(\frac{1}{2}\right)^{\cancel{3}}\right\}^{1/\cancel{3}} + 1$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$

Hence, correct option is (c).

Question 6

When simplified $\left(-\frac{1}{27}\right)^{-2/3}$ is

(a) 9
(b) $-9$
(c) $\frac{1}{9}$
(d) $-\frac{1}{9}$

Solution 6

$\left(\frac{-1}{27}\right)^{-2/3}$

$= (-27)^{2/3}$

$= \left[(-3)^3\right]^{2/3}$

$= (-3)^2$

$= 9$

Hence, correct option is (a).

Question 7

Which one of the following is not equal to $\left(\sqrt[3]{8}\right)^{-1/2}$?

(a) $\left(\sqrt[3]{2}\right)^{-1/2}$

(b) $8^{-1/6}$

(c) $\dfrac{1}{\left(\sqrt[3]{8}\right)^{1/2}}$

(d) $\dfrac{1}{\sqrt{2}}$

Solution 7

Option (a):

$\sqrt[3]{2} \neq \sqrt[3]{8}$

$\Rightarrow \left(\sqrt[3]{2}\right)^{-1/2} \neq \left(\sqrt[3]{8}\right)^{-1/2}$

Option (b):

$\sqrt[3]{8} = (8)^{1/3}$

$\Rightarrow \left(\sqrt[3]{8}\right)^{-1/2} = 8^{-1/6}$

Option (c):

$$\left(\sqrt[3]{8}\right)^{-1/2} = \frac{1}{\left(\sqrt[3]{8}\right)^{1/2}}$$

Option (d):

$\sqrt[3]{8} = 8^{1/3} = \left(2^{\cancel{3}}\right)^{1/\cancel{3}} = 2$

$$\Rightarrow \left(\sqrt[3]{8}\right)^{-1/2} = (2)^{-1/2} = \frac{1}{(2)^{1/2}} = \frac{1}{\sqrt{2}}$$

Hence, correct option is (a).

Question 8

Which one of the following is not equal to $\left(\dfrac{100}{9}\right)^{-3/2}$?

(a) $\left(\dfrac{9}{100}\right)^{3/2}$

(b) $\dfrac{1}{\left(\dfrac{100}{9}\right)^{3/2}}$

(c) $\dfrac{3}{10} \times \dfrac{3}{10} \times \dfrac{3}{10}$

(d) $\sqrt{\dfrac{100}{9} \times \dfrac{100}{9} \times \dfrac{100}{9}}$

Solution 8

Option (a):

$$\left(\frac{100}{9}\right)^{-3/2} = \frac{1}{\left(\frac{100}{9}\right)^{3/2}} = \left(\frac{9}{100}\right)^{3/2} \qquad \left\{\because a^{-x} = \frac{1}{a^{x}}\right\}$$

Option (b):

$$\left(\frac{100}{9}\right)^{-3/2} = \frac{1}{\left(\frac{100}{9}\right)^{3/2}} \qquad \left\{\because a^{-x} = \frac{1}{a^{x}}\right\}$$

Option (c):

$$\left(\frac{100}{9}\right)^{-3/2} = \left(\frac{9}{100}\right)^{3/2} = \left\{\left(\frac{3}{10}\right)^{\cancel{2}}\right\}^{3/\cancel{2}} = \left(\frac{3}{10}\right)^{3} = \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$$

Option (d):

$$\left(\frac{100}{9}\right)^{-3/2} = \left(\frac{9}{100}\right)^{3/2} = \left\{\left(\frac{9}{100}\right)^{3}\right\}^{1/2} = \sqrt{\frac{9}{100} \times \frac{9}{100} \times \frac{9}{100}} \neq \sqrt{\frac{100}{9} \times \frac{100}{9} \times \frac{100}{9}$$

Hence, correct option is (d).

Question 9

If a, b, c are positive real numbers, then $\sqrt{a^{-1}b} \times \sqrt{b^{-1}c} \times \sqrt{c^{-1}a}$ is equal to

(a) 1

(b) abc

(c) $\sqrt{abc}$

(d) $\frac{1}{abc}$

Solution 9

$$a^{-1} = \frac{1}{a},\ b^{-1} = \frac{1}{b},\ c^{-1} = \frac{1}{c}$$

$$\Rightarrow \sqrt{a^{-1}b} = \sqrt{\frac{b}{a}},\ \sqrt{b^{-1}c} = \sqrt{\frac{c}{b}},\ \sqrt{c^{-1}a} = \sqrt{\frac{a}{c}}$$

Now,

$$\sqrt{a^{-1}b} \times \sqrt{b^{-1}c} \times \sqrt{c^{-1}a}$$

$$= \sqrt{\frac{b}{a}} \times \sqrt{\frac{c}{b}} \times \sqrt{\frac{a}{c}}$$

$$= \frac{\sqrt{b}}{\sqrt{a}} \times \frac{\sqrt{c}}{\sqrt{b}} \times \frac{\sqrt{a}}{\sqrt{c}}$$

$$= 1$$

Hence, correct option is (a).

Question 10

If $\left(\frac{2}{3}\right)^{x}\left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$, then $x =$

(a) 2

(b) 3

(c) 4

(d) 1

Solution 10

$$\left(\frac{2}{3}\right)^{x} = \left(\frac{3}{2}\right)^{-x} \qquad \left\{\because a^{x} = \frac{1}{a^{-x}}\right\}$$

$$\therefore \left(\frac{2}{3}\right)^{x} \times \left(\frac{3}{2}\right)^{2x}$$

$$= \left(\frac{3}{2}\right)^{-x}\left(\frac{3}{2}\right)^{2x}$$

$$= \left(\frac{3}{2}\right)^{2x-x}$$

$$= \left(\frac{3}{2}\right)^{x}$$

Thus,

$$\left(\frac{3}{2}\right)^{x} = \frac{81}{16}$$

$$\Rightarrow \left(\frac{3}{2}\right)^{x} = \frac{3^4}{2^4} = \left(\frac{3}{2}\right)^{4}$$

$$\Rightarrow x = 4$$

Hence, correct option is (c).

## Chapter 2 - Exponents of Real Numbers Exercise 2.31

Question 1

$(256)^{0.16} \times (256)^{0.09} =$

(a) 4

(b) 16

(c) 64

(d) 256.25

Solution 1

$a^p \times a^n = a^{p+n}$

$\therefore (256)^{0.16} \times (256)^{0.09}$

$= (256)^{0.16+0.09}$

$= (256)^{0.25}$

$= (256)^{\frac{1}{4}}$

$= \left(4^4\right)^{\frac{1}{4}}$

$= 4$

So, correct option is (a).

Question 2

If $10^{2y} = 25$, then $10^{-y}$ equals

(a) $\frac{1}{-5}$

(b) $\frac{1}{50}$

(c) $\frac{1}{625}$

(d) $\frac{1}{5}$

Solution 2

$10^{2y} = \left(10^2\right)^y = 100^y = 10^y.10^y = \left(10^y\right)^2$

But, $10^{2y} = 25$

$\Rightarrow \left(10^y\right)^2 = 25$

$\Rightarrow \left(10^y\right)^2 = (5)^2$

$\Rightarrow 10^y = 5$

$\Rightarrow \frac{1}{10y} = \frac{1}{5}$

$\Rightarrow 10^{-y} = \frac{1}{5}$

So, correct option is (d).

Question 3

If $9^{x+2} = 240 + 9^x$, then x =

(a) 0.5

(b) 0.2

(c) 0.4

(d) 0.1

Solution 3

$9^{x+2} = 240 + 9^x$

$\Rightarrow 9^x \times 9^2 = 240 + 9^x$

$\Rightarrow 9^x \times 81 = 240 + 9^x$

$\Rightarrow 9^x(81) - 9^x = 240$

$\Rightarrow 9^x(81 - 1) = 240$

$\Rightarrow 9^x \times 80 = 240$

$\Rightarrow 9^x = \frac{240}{80} = 3 = (3^2)^{\frac{1}{2}} = (9)^{\frac{1}{2}}$

$\Rightarrow x = \frac{1}{2} = 0.5$

Hence, correct option is (a).

Question 4

If x is a positive real number and $x^2 = 2$, then $x^3 =$

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) $3\sqrt{2}$

(d) 4

Solution 4

$x^2 = 2$

$\therefore x = \sqrt{2}$

Now,

$x^3 = x^2 . x = 2\sqrt{2}$

Hence, correct option is (b).

Question 5

If $\frac{x}{x^{1.5}} = 8x^{-1}$ and $x > 0$, then $x =$

(a) $\frac{\sqrt{2}}{4}$

(b) $2\sqrt{2}$

(c) 4

(d) 64

Solution 5

$$\frac{x}{x^{1.5}} = 8x^{-1}$$

$$\Rightarrow (x)^{1-1.5} = \frac{8}{x}$$

$$\Rightarrow x^{-0.5} = \frac{8}{x}$$

$$\Rightarrow x^{-1/2} = \frac{8}{x}$$

$$\Rightarrow x.x^{-1/2} = 8$$

$$\Rightarrow x^{1-\frac{1}{2}} = 8$$

$$\Rightarrow x^{1/2} = 8$$

$$\Rightarrow \left(x^{1/2}\right)^2 = (8)^2$$

$$\Rightarrow x = 64$$

Hence, correct option is (d).

Question 6

The value of $\{8^{-4/3} \div 2^{-2}\}^{1/2}$ is

(a) $\frac{1}{2}$

(b) 2

(c) $\frac{1}{4}$

(d) 4

Solution 6

$$8^{-4/3} = \left(2^{\cancel{3}}\right)^{-4/\cancel{3}} = 2^{-4}$$

So, $8^{-4/3} \div 2^{-2} = \dfrac{2^{-4}}{2^{-2}} = 2^{-2}$

Now,

$$\{8^{-4/3} \div 2^{-2}\}^{1/2} = \{2^{-\cancel{2}}\}^{1/\cancel{2}} = 2^{-1} = \frac{1}{2}$$

Hence, correct option is (a).

Question 7

If a, b, c are positive real numbers, then $\sqrt[5]{3125\ a^{10}b^5c^{10}}$ is equal to

(a) $5a^2bc^2$

(b) $25ab^2c$

(c) $5a^3bc^3$

(d) $125a^2bc^2$

Solution 7

$3125\ a^{10}b^5c^{10} = 5^5(a^2)^5\ (b)^5\ (c^2)^5$

Now,

$\sqrt[5]{3125\ a^{10}\ b^5\ c^{10}}$

$= (3125 \times a^{10} \times b^5 \times c^{10})^{1/5}$

$= \left[5^5 \times \left(a^2\right)^5 \times (b)^5 \times \left(c^2\right)^5\right]^{1/5}$

$= \left[\left(5a^2bc^2\right)^5\right]^{1/5}$

$= 5a^2bc^2$

Hence, correct option is (a).

Question 8

If a, m, n are positive integers, then $\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn}$ is equal to

(a) $a^{mn}$

(b) a

(c) $a^{m/n}$

(d) 1

Solution 8

$\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn}$

$= \left\{\sqrt[m]{(a)^{1/n}}\right\}^{mn}$

$= \left\{\left[(a)^{1/n}\right]^{1/m}\right\}^{mn}$

$= \{(a)^{1/mn}\}^{mn}$

$= a$

Hence, correct option is (b).

Question 9

If x = 2 and y = 4, then $\left(\frac{x}{y}\right)^{x-y} + \left(\frac{y}{x}\right)^{y-x} =$

Solution 9

$x = 2$ and $y = 4$

Now,

$$\left(\frac{x}{y}\right)^{x-y} + \left(\frac{y}{x}\right)^{y-x}$$

$$= \left(\frac{2}{4}\right)^{2-4} + \left(\frac{4}{2}\right)^{4-2}$$

$$= \left(\frac{1}{2}\right)^{-2} + (2)^2$$

$$= (2)^2 + (2)^2$$

$$= 4 + 4$$

$$= 8$$

Hence, correct option is (b).

Question 10

The value of m for which $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4} = 7^m$, is

(a) $\frac{-1}{3}$

(b) $\frac{1}{4}$

(c) $-3$

(d) 2

Solution 10

$$\left\{\left[\left(\frac{1}{7^2}\right)^{-2}\right]^{-1/3}\right\}^{1/4} = 7^m$$

$$\Rightarrow \left\{\left[\left(7^{-2}\right)^{-2}\right]^{-1/3}\right\}^{1/4} = 7^m$$

$$\Rightarrow \left\{\left[(7)^4\right]^{-1/3}\right\}^{1/4} = 7^m$$

$$\Rightarrow \{(7)^{-4/3}\}^{1/4} = 7^m$$

$$\Rightarrow \{(7)^{-4/3}\}^{1/4 =} = 7^m$$

$$\Rightarrow \{(7)^{-1/3}\} = 7^m$$

$$\Rightarrow -\frac{1}{3} = m$$

Hence, correct option is (a).

Question 11

The value of $\left\{\left(23+2^2\right)^{2/3} + (140-19)^{1/2}\right\}^2$, is

(a) 196

(b) 289

(c) 324

(d) 400

Solution 11

$$\left\{\left(23+2^2\right)^{2/3} + (140-19)^{1/2}\right\}^2$$
$$=\{(23+4)^{2/3} + (121)^{1/2}\}^2$$
$$=\{(27)^{2/3} + (121)^{1/2}\}^2$$
$$=\left\{\left(3^3\right)^{2/3} + \left(11^2\right)^{1/2}\right\}^2$$
$$=\{3^2+11\}^2$$
$$=\{9+11\}^2$$
$$=\{20\}^2$$
$$=400$$

Hence, correct option is (d).

## Chapter 2 - Exponents of Real Numbers Exercise 2.32

Question 1

If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equal

(a) $5\sqrt{5}$

(b) $\sqrt{5}$

(c) $25\sqrt{5}$

(d) 125

Solution 1

$4^{x} - 4^{x-1} = 24$

$\Rightarrow 4^{x-1}(4-1) = 24$

$\Rightarrow 4^{x-1}(3) = 24$

$\Rightarrow 4^{x-1} = 8$

$\Rightarrow 4^{x-1} = 8$

$\Rightarrow 2^{2(x-1)} = 2^{3}$

$\Rightarrow 2(x-1) = 3$

$\Rightarrow x = \frac{3}{2} + 1$

$\Rightarrow x = \frac{5}{2}$

Now, $(2x)^{x} = \left(\cancel{2} \times \frac{5}{\cancel{2}}\right)^{5/2} = 5^{5/2} = \left(5^{5}\right)^{1/2} = \sqrt{5^{5}} = 5^{2}\sqrt{5} = 25\sqrt{5}$

Hence, correct option is (c).

Question 2

If $g = t^{2/3} + 4t^{-1/2}$, what is the value of g when $t = 64$?

(a) $\frac{31}{2}$

(b) $\frac{33}{2}$

(c) 16

(d) $\frac{257}{16}$

Solution 2

$g = t^{2/3} + 4t^{-1/2}$

Substituting $t = 64$, we get

$g = (64)^{2/3} + 4(64)^{-1/2}$

$= (4^{3})^{2/3} + 4(8^{2})^{-1/2}$

$= 4^{2} + 4(8)^{-1}$

$= 16 + \frac{4}{8}$

$= 16 + \frac{1}{2}$

$= \frac{33}{2}$

Hence, correct option is (b).

Question 3

When simplified $(256)^{-\left(4^{-3/2}\right)}$ is

(a) 8

(b) $\frac{1}{8}$

(c) 2

(d) $\frac{1}{2}$

Solution 3

$(256)^{-\left(4^{-3/2}\right)}$

$= \left(2^8\right)^{-\left(2^2\right)^{-3/2}}$

$= \left(2^8\right)^{-(2)^{-3}}$

$= \left(2^8\right)^{-1/8}$

$= 2^{-1}$

$= \frac{1}{2}$

Hence, correct option is (d).

Question 4

If $\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$, then $x =$

(a) 2

(b) 3

(c) 5

(d) 4

Solution 4

$$\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$$

$$\Rightarrow \frac{3^{2x-8}}{25 \times 9} = \frac{5^3}{5^x}$$

$$\Rightarrow \frac{3^{2x-8}}{5^2 \times 3^2} = \frac{5^3}{5^x}$$

$$\Rightarrow 3^{2x-8} = \frac{3^2.5^2.5^3}{5^x}$$

$$\Rightarrow 3^{2x-8} = \left(3^2\right)\left(5^{5-x}\right)$$

$$\Rightarrow 5^{5-x} = \frac{3^{2x-8}}{3^2}$$

$$\Rightarrow \frac{1}{5^{(x-5)}} = 3^{2x-10}$$

$$\Rightarrow \frac{1}{5^{(x-5)}} = \left(3^2\right)^{(x-5)}$$

$$\Rightarrow \frac{1}{5^{(x-5)}} = (9)^{x-5}$$

$$\Rightarrow 1 = (5 \times 9)^{x-5}$$

$$\Rightarrow x - 5 = 0 \qquad ....\left(\text{If } a^p = 1, \text{ then } p = 0\right)$$

$$\Rightarrow x = 5$$

Hence, correct option is (c).

Question 5

The value of $64^{-1/3}\left(64^{1/3} - 64^{2/3}\right)$, is

(a) 1

(b) $\frac{1}{3}$

(c) $-3$

(d) $-2$

Solution 5

Correct option : (c)

$64^{-1/3}\left(64^{1/3} - 64^{2/3}\right)$

$= \left(4^{\cancel{3}}\right)^{-1/\cancel{3}} \left\{\left(4^{\cancel{3}}\right)^{1/\cancel{3}} - \left(4^{\cancel{3}}\right)^{2/\cancel{3}}\right\}$

$= 4^{-1}\left(4^{1} - 4^{2}\right)$

$= \frac{1}{4}(4 - 16)$

$= \frac{-12}{4}$

$= -3$

Hence, correct option is (c).

Question 6

If $\sqrt{5^n} = 125$, then $5^{\sqrt[n]{64}} =$

(a) 25

(b) $\frac{1}{125}$

(c) 625

(d) $\frac{1}{5}$

Solution 6

$\sqrt{5^n} = 125$

$\Rightarrow \left(5^n\right)^{1/2} = 5^3$

$\Rightarrow 5^{n/2} = 5^3$

$\Rightarrow \frac{n}{2} = 3$

$\Rightarrow n = 6$

Now,

$5^{\sqrt[n]{64}} = (5)^{(64)^{1/n}} = (5)^{(64)^{1/6}} = (5)^{\left(2^6\right)^{1/6}} = 5^2 = 25$

Hence, correct option is (a).

Question 7

If $(16)^{2x+3} = (64)^{x+3}$, then $4^{2x-2} =$

(a) 64

(b) 256

(c) 32

(d) 512

Solution 7

$(16)^{2x+3} = (64)^{x+3}$

$\Rightarrow \left(4^2\right)^{2x+3} = \left(4^3\right)^{x+3}$

$\Rightarrow (4)^{2(2x+3)} = \left(4^{3(x+3)}\right)$

$\Rightarrow 2(2x+3) = 3(x+3)$

$\Rightarrow 4x+6 = 3x+9$

$\Rightarrow x = 3$

Now, $4^{2x-2} = 4^{2\times3-2} = 4^4 = 256$

Hence, correct option is (b).

Question 8

If $2^{-m} \times \frac{1}{2^m} = \frac{1}{4}$, then $\frac{1}{14}\left\{\left(4^m\right)^{1/2} + \left(\frac{1}{5^m}\right)^{-1}\right\}$ is equal to

(a) $\frac{1}{2}$

(b) 2

(c) 4

(d) $-\frac{1}{4}$

Solution 8

$$2^{-m} \times \frac{1}{2^m} = \frac{1}{4}$$

$$\Rightarrow 2^{-m} \times 2^{-m} = \frac{1}{2^2}$$

$$\Rightarrow 2^{-2m} = 2^{-2}$$

$$\Rightarrow -2m = -2$$

$$\Rightarrow m = 1$$

Now,

$$\frac{1}{14}\left\{(4^m)^{1/2} + \left(\frac{1}{5^m}\right)^{-1}\right\}$$

$$= \frac{1}{14}\left\{(4)^{1/2} + \left(\frac{1}{5}\right)^{-1}\right\}$$

$$= \frac{1}{14}\left\{\left(2^2\right)^{1/2} + 5\right\}$$

$$= \frac{1}{14}(2+5)$$

$$= \frac{7}{14}$$

$$= \frac{1}{2}$$

Hence, correct option is (a).

Question 9

If $\frac{2^{m+n}}{2^{n-m}} = 16$, $\frac{3^p}{3^n} = 81$ and $a = 2^{1/10}$, then $\frac{a^{2m+n-p}}{\left(a^{m-2n+2p}\right)^{-1}} =$

(a) 2

(b) $\frac{1}{4}$

(c) 9

(d) $\frac{1}{8}$

Solution 9

$$\frac{2^{m+n}}{2^{n-m}} = 16$$

$\Rightarrow 2^{m+n-(n-m)} = 2^4$

$\Rightarrow 2^{2m} = 2^4$

$\Rightarrow 2m = 4$

$\Rightarrow m = 2$ ......(1)

And, $\frac{3^p}{3^n} = 81$

$\Rightarrow 3^{p-n} = 3^4$

$\Rightarrow p - n = 4$

$\Rightarrow n - p = -4$ .....(2)

Now, $\frac{a^{2m+n-p}}{(a^{m-2n+2p})^{-1}}$

$= \frac{a^{2m+(n-p)}}{(a^{m-2(n-p)})^{-1}}$

$= \frac{a^{2(2)+(-4)}}{(a^{2-2(-4)})^{-1}}$ ....[From (1) and (2)]

$= \frac{a^{4-4}}{(a^{2+8})^{-1}}$

$= \frac{a^0}{a^{-10}}$

$= a^{10}$

$= (2^{1/10})^{10}$

$= 2$

Hence, correct option is (a).

Question 10

If $\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$, then x =

(a) 3

(b) $-3$

(c) $\frac{1}{3}$

(d) $-\frac{1}{3}$

Solution 10

$$\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$$

$$\Rightarrow \frac{3^{5x}}{3^{2x}} \times 81^2 \times 6561 = 3^7$$

$$\Rightarrow 3^{5x-2x} \times \left(3^4\right)^2 \times (81)^2 = 3^7$$

$$\Rightarrow 3^{3x} \times 3^8 \times 3^8 = 3^7$$

$$\Rightarrow 3^{(3x+8+8)} = 3^7$$

$$\Rightarrow 3x + 8 + 8 = 7$$

$$\Rightarrow 3x + 16 = 7$$

$$\Rightarrow 3x = -9$$

$$\Rightarrow x = -3$$

Hence, correct option is (b).

## Chapter 2 - Exponents of Real Numbers Exercise 2.33

Question 1

If $0 < y < x$, which statement must be true?

(a) $\sqrt{x} - \sqrt{y} = \sqrt{x-y}$

(b) $\sqrt{x} + \sqrt{x} = \sqrt{2x}$

(c) $x\sqrt{y} = y\sqrt{x}$

(d) $\sqrt{xy} = \sqrt{x}\sqrt{y}$

Solution 1

Option (a) is incorrect because $\sqrt{x} - \sqrt{y} \neq \sqrt{x-y}$

In square root operation, we can not take square root in common.

Option (b) is incorrect because $\sqrt{x} + \sqrt{x} = 2\sqrt{x} \neq \sqrt{2x}$

Option (c) is incorrect because in $x\sqrt{y}$, $\sqrt{y}$ is irrational and

in $y\sqrt{x}$, $\sqrt{x}$ is irrational.

Two numbers $x\sqrt{y}$ and $y\sqrt{x}$ will be equal if and only if $x = y$.

But $x > y$.

Now, option (d) $\sqrt{xy} = \sqrt{x}.\sqrt{y}$ is true because $x, y > 0$ and

this is a properly of square root operations.

Hence, correct option is (d).

Question 2

If $10^x = 64$, what is the value of $10^{\frac{x}{2}+1}$?

(a) 18

(b) 42

(c) 80

(d) 81

Solution 2

$10^x = 64$

Taking square root on both sides, we have

$\left(10^x\right)^{1/2} = (64)^{1/2}$

$\Rightarrow 10^{\frac{x}{2}} = \left(8^2\right)^{1/2}$

$\Rightarrow 10^{\frac{x}{2}} = 8$

Multiplying by 10 on both sides, we have

$10^{\frac{x}{2}} \times \ 10\ 8 \times 10$

$\Rightarrow 10^{\frac{x}{2}} \times \ 10 = 80$

$\Rightarrow 10^{\frac{x}{2}+1} = \ 80$

Hence, correct option is (c).

Question 3

$\dfrac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}}$ is equal to

(a) $\dfrac{5}{3}$

(b) $-\dfrac{5}{3}$

(c) $\dfrac{3}{5}$

(d) $-\dfrac{3}{5}$

Solution 3

$$\frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^{n} - 2 \times 5^{n+1}}$$

Taking $5^n$ common from Numerotor and denominator, we have

$$\frac{5^n(5^2 - 6 \times 5^1)}{5^n(13 - 2 \times 5^1)}$$

$$= \frac{25 - 30}{13 - 10}$$

$$= \frac{-5}{3}$$

Hence, correct option is (b).

Question 4

If $\sqrt{2^n} = 1024$, then $3^{2\left(\frac{n}{4} - 4\right)} =$

(a) 3

(b) 9

(c) 27

(d) 81

Solution 4

$\sqrt{2^n} = 1024$

$\Rightarrow (2^n)^{1/2} = 2^{10}$

$= 2^{n/2} = 2^{10}$

$\Rightarrow \frac{n}{2} = 10$

$\Rightarrow n = 20$

Now, $3^{2\left(\frac{n}{2} - 4\right)} = 3^{2\left(\frac{20}{4} - 4\right)} = 3^{2(5-4)} = 3^2 = 9$

Hence, correct option is (b).

## Chapter 2 - Exponents of Real Numbers Exercise Ex. 2.1

Question 1

Simplify:

$3(a^4b^3)^{10} \times 5(a^2b^2)^3$

Solution 1

$3\left(a^4b^3\right)^{10} \times 5\left(a^2b^2\right)^3$

$= 3 \times a^{4\times10} \times b^{3\times10} \times 5 \times a^{2\times3} \times b^{2\times3}$

$= 3 \times a^{40} \times b^{30} \times 5 \times a^6 \times b^6$

$= 3 \times 5 \times a^{40} \times a^6 \times b^{30} \times b^6$

$= 15 \times a^{40+6} \times b^{30+6}$

$= 15 \times a^{46} \times b^{36}$

$= 15a^{46}b^{36}$

Question 2

Simplify:

$(2x^{-2}y^3)^3$

Solution 2

$\left(2x^{-2}y^3\right)^3$

$= 2^3 \times x^{-2\times3} \times y^{3\times3}$

$= 8 \times x^{-6} \times y^9$

$= 8x^{-6}y^9$

Question 3

Simplify:

$$\frac{\left(4 \times 10^7\right)\left(6 \times 10^{-5}\right)}{8 \times 10^4}$$

Solution 3

$$\frac{\left(4 \times 10^7\right)\left(6 \times 10^{-5}\right)}{8 \times 10^4}$$

$$= \frac{4 \times 10^7 \times 6 \times 10^{-5}}{8 \times 10^4}$$

$$= \frac{4 \times 6 \times 10^{7-5}}{8 \times 10^4}$$

$$= \frac{24 \times 10^2}{8 \times 10^4}$$

$$= \frac{24}{8 \times 10^{4-2}}$$

$$= \frac{3}{10^2}$$

$$= \frac{3}{100}$$

Question 4

Simplify:

$$\frac{4ab^2\left(-5ab^3\right)}{10a^2b^2}$$

Solution 4

$$\frac{4ab^2\left(-5ab^3\right)}{10a^2b^2}$$
$$= \frac{4 \times (-5) \times a \times a \times b^2 \times b^3}{10 \times a^2 \times b^2}$$
$$= \frac{-20 \times a^{1+1} \times b^2 \times b^3}{10 \times a^2 \times b^2}$$
$$= \frac{-20 \times a^2 \times b^2 \times b^3}{10 \times a^2 \times b^2}$$
$$= -2 \times b^3$$
$$= -2b^3$$

Question 5

Simplify:

$$\left(\frac{x^2y^2}{a^2b^3}\right)^n$$

Solution 5

$$\left(\frac{x^2y^2}{a^2b^3}\right)^n$$
$$= \frac{x^{2n} \times y^{2n}}{a^{2n} \times b^{3n}}$$
$$= \frac{x^{2n}y^{2n}}{a^{2n}b^{3n}}$$

Question 6

Simplify:

$$\frac{\left(a^{3n-9}\right)^6}{a^{2n-4}}$$

Solution 6

$$\frac{\left(a^{3n-9}\right)^6}{a^{2n-4}}$$

$$= \frac{\left(\frac{a^{3n}}{a^9}\right)^6}{\frac{a^{2n}}{a^4}}$$

$$= \frac{\frac{a^{18n}}{a^{54}}}{\frac{a^{2n}}{a^4}}$$

$$= \frac{a^{18n}}{a^{54}} \times \frac{a^4}{a^{2n}}$$

$$= a^{18n-2n} \times a^{4-54}$$

$$= a^{16n} \times a^{-50}$$

$$= a^{16n-50}$$

Question 7

If a = 3 and b = -2, find the value of:

$a^a + b^b$

Solution 7

Given, $a = 3$ and $b = -2$

$$\therefore\ a^a + b^b = 3^3 + (-2)^{-2}$$

$$= 27 + \frac{1}{(-2)^2}$$

$$= 27 + \frac{1}{4}$$

$$= \frac{108+1}{4}$$

$$= \frac{109}{4}$$

Question 8

If a = 3 and b = -2, find the value of:

$a^b + b^a$

Solution 8

Given, $a = 3$ and $b = -2$

$$\therefore\ a^b + b^a = 3^{-2} + (-2)^3$$

$$= \frac{1}{3^2} + (-8)$$

$$= \frac{1}{9} - 8$$

$$= \frac{1-72}{9}$$

$$= -\frac{71}{9}$$

Question 9

If a = 3 and b = -2, find the value of:

$(a + b)^{ab}$

Solution 9

$$\text{Given, } a = 3 \text{ and } b = -2$$

$$\begin{aligned}\therefore (a+b)^{ab} &= [3+(-2)]^{3\times(-2)} \\ &= [3-2]^{-6} \\ &= [1]^{-6} \\ &= \frac{1}{1^6} \\ &= 1\end{aligned}$$

Question 10

Prove that:

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$$

Solution 10

$$\begin{aligned}\text{L.H.S.} &= \left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} \\ &= \left(x^{a-b}\right)^{a^2+ab+b^2} \times \left(x^{b-c}\right)^{b^2+bc+c^2} \times \left(x^{c-a}\right)^{c^2+ca+a^2} \\ &= x^{(a-b)(a^2+ab+b^2)} \times x^{(b-c)(b^2+bc+c^2)} \times x^{(c-a)(c^2+ca+a^2)} \\ &= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3} \\ &= x^{a^3-b^3+b^3-c^3+c^3-a^3} \\ &= x^0 \\ &= 1 \\ &= \text{R.H.S.}\end{aligned}$$

Question 11

Prove that:

$$\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = 1$$

Solution 11

$$\text{L.H.S.} = \left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b$$
$$= \left(x^{a-b}\right)^c \times \left(x^{b-c}\right)^a \times \left(x^{c-a}\right)^b$$
$$= x^{(a-b)c} \times x^{(b-c)a} \times x^{(c-a)b}$$
$$= x^{ac-bc} \times x^{ba-ca} \times x^{cb-ab}$$
$$= x^{ac-bc+ba-ca+cb-ab}$$
$$= x^0$$
$$= 1$$
$$= \text{R.H.S.}$$

Question 12

Prove that:

$$\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

Solution 12

Multiplying the numerators and denominators of two terms on L.H.S. by $x^b$ and $x^a$ respectively, we obtain

$$\text{L.H.S.} = \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$$
$$= \frac{x^b}{x^b + x^{a-b+b}} + \frac{x^a}{x^a + x^{b-a+a}}$$
$$= \frac{x^b}{x^b + x^a} + \frac{x^a}{x^a + x^b}$$
$$= \frac{x^b + x^a}{x^b + x^a}$$
$$= 1$$
$$= \text{R.H.S.}$$

Question 13

Prove that:

$$\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$$

Solution 13

Multiplying the numerators and denominators of three terms on L.H.S. by $x^a$, $x^b$ and $x^c$ respectively, we obtain

$$\text{L.H.S.} = \frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}}$$
$$= \frac{x^a}{x^a + x^{b-a+a} + x^{c-a+a}} + \frac{x^b}{x^b + x^{a-b+b} + x^{c-b+b}} + \frac{x^c}{x^c + x^{b-c+c} + x^{a-c+c}}$$
$$= \frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^b + x^a + x^c} + \frac{x^c}{x^c + x^b + x^a}$$
$$= \frac{x^a + x^b + x^c}{x^a + x^b + x^c}$$
$$= 1$$
$$= \text{R.H.S.}$$

Question 14

Prove that:

$$\frac{a + b + c}{a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}} = abc$$

Solution 14

$$\text{L.H.S.} = \frac{a + b + c}{a^{-1}b^{-1} + b^{-1}c^{-1} + c^{-1}a^{-1}}$$
$$= \frac{a + b + c}{\frac{1}{a} \times \frac{1}{b} + \frac{1}{b} \times \frac{1}{c} + \frac{1}{c} \times \frac{1}{a}}$$
$$= \frac{a + b + c}{\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}}$$
$$= \frac{a + b + c}{\frac{c + a + b}{abc}}$$
$$= \frac{abc(a + b + c)}{a + b + c}$$
$$= abc$$
$$= \text{R.H.S.}$$

Question 15

Prove that:

$$\left(a^{-1} + b^{-1}\right)^{-1} = \frac{ab}{a + b}$$

Solution 15

$$\text{L.H.S.} = \left(a^{-1} + b^{-1}\right)^{-1}$$
$$= \left(\frac{1}{a} + \frac{1}{b}\right)^{-1}$$
$$= \left(\frac{b+a}{ab}\right)^{-1}$$
$$= \frac{ab}{a+b}$$
$$= \text{R.H.S.}$$

Question 16

If $abc = 1$, show that $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$

Solution 16

$abc = 1$

$$\text{L.H.S.} = \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$$
$$= \frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$$
$$= \frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{1}{1+\frac{1}{ab}+\frac{1}{a}} \quad \ldots\left[\because abc = 1 \Rightarrow \frac{1}{c} = ab \text{ and } c = \frac{1}{ab}\right]$$
$$= \frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{ab}{ab+1+b}$$
$$= \frac{b+1+ab}{b+ab+1}$$
$$= 1$$
$$= \text{R.H.S.}$$

Question 17

Simplify:

$$\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

Solution 17

$$\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$
$$= 3^{n-(n-1)} \times 9^{n+1-(n-1)}$$
$$= 3^{n-n+1} \times 9^{n+1-n+1}$$
$$= 3^1 \times 9^2$$
$$= 3 \times 81$$
$$= 243$$

Question 18

Simplify:

$$\frac{5 \times 25^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (25)^{n+1}}$$

Solution 18

$$\frac{5 \times 25^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (25)^{n+1}}$$

$$= \frac{5 \times (5^2)^{n+1} - 5^2 \times 5^{2n}}{5 \times 5^{2n+3} - (5^2)^{n+1}}$$

$$= \frac{5 \times 5^{2n+2} - 5^2 \times 5^{2n}}{5 \times 5^{2n+3} - 5^{2n+2}}$$

$$= \frac{5 \times 5^{2n+2} - 5^{2+2n}}{5 \times 5^{2n+2+1} - 5^{2n+2}}$$

$$= \frac{5 \times 5^{2n+2} - 5^{2n+2}}{5 \times 5^{2n+2} \times 5 - 5^{2n+2}}$$

$$= \frac{5^{2n+2}(5-1)}{5^{2n+2}(25-1)}$$

$$= \frac{4}{24}$$

$$= \frac{1}{6}$$

Question 19

Simplify:

$$\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^{x} - 2^2 \times 5^{n}}$$

Solution 19

$$\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^{n} - 2^2 \times 5^{n}}$$

$$= \frac{5^n \times 5^3 - 6 \times 5^n \times 5}{9 \times 5^n - 2^2 \times 5^n}$$

$$= \frac{5^n (5^3 - 6 \times 5)}{5^n (9 - 2^2)}$$

$$= \frac{125 - 30}{9 - 4}$$

$$= \frac{95}{5}$$

$$= 19$$

Question 20

Simplify:

$$\frac{6(8)^{n+1} + 16(2)^{3n-2}}{10(2)^{3n+1} - 7(8)^{n}}$$

Solution 20

$$\frac{6(8)^{n+1}+16(2)^{3n-2}}{10(2)^{3n+1}-7(8)^{n}}$$

$$=\frac{6(2^3)^{n+1}+16(2)^{3n-2}}{10(2)^{3n+1}-7(2^3)^{n}}$$

$$=\frac{6\times 2^{3n+3}+16\times 2^{3n-2}}{10\times 2^{3n+1}-7\times 2^{3n}}$$

$$=\frac{6\times 2^{3n}\times 2^{3}+16\times 2^{3n}\times 2^{-2}}{10\times 2^{3n}\times 2-7\times 2^{3n}}$$

$$=\frac{2^{3n}\left(6\times 2^{3}+16\times 2^{-2}\right)}{2^{3n}\left(10\times 2-7\right)}$$

$$=\frac{6\times 8+16\times \frac{1}{2^2}}{20-7}$$

$$=\frac{48+4}{13}$$

$$=\frac{52}{13}$$

$$=4$$

Question 21

Solve the equation for x:

$7^{2x+3}=1$

Solution 21

$$7^{2x+3}=1$$

$$\Rightarrow 7^{2x}\times 7^{3}=1$$

$$\Rightarrow 7^{2x}=\frac{1}{7^3}$$

$$\Rightarrow 7^{2x}=7^{-3}$$

$$\Rightarrow 2x=-3$$

$$\Rightarrow x=-\frac{3}{2}$$

Question 22

Solve the equation for x:

$2^{x+1}=4^{x-3}$

Solution 22

$2^{x+1} = 4^{x-3}$

$\Rightarrow 2^{x+1} = \left(2^2\right)^{x-3}$

$\Rightarrow 2^{x+1} = 2^{2x-6}$

$\Rightarrow x + 1 = 2x - 6$

$\Rightarrow x - 2x = -6 - 1$

$\Rightarrow -x = -7$

$\Rightarrow x = 7$

Question 23

Solve the equation for x:

$2^{5x+3} = 8^{x+3}$

Solution 23

$2^{5x+3} = 8^{x+3}$

$\Rightarrow 2^{5x+3} = \left(2^3\right)^{x+3}$

$\Rightarrow 2^{5x+3} = 2^{3x+9}$

$\Rightarrow 5x + 3 = 3x + 9$

$\Rightarrow 5x - 3x = 9 - 3$

$\Rightarrow 2x = 6$

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow x = 3$

Question 24

Solve the equation for x:

$4^{2x} = \frac{1}{32}$

Solution 24

$4^{2x} = \frac{1}{32}$

$\Rightarrow \left(2^2\right)^{2x} = \frac{1}{2^5}$

$\Rightarrow 2^{4x} = 2^{-5}$

$\Rightarrow 4x = -5$

$\Rightarrow x = -\frac{5}{4}$

Question 25

Solve the equation for x:

$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$

Solution 25

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$
$$\Rightarrow (2^2)^{x-1} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^x$$
$$\Rightarrow 2^{2x-2} \times \frac{1}{2^{3-2x}} = \frac{1}{2^{3x}}$$
$$\Rightarrow 2^{2x} \times 2^{-2} \times \frac{1}{2^3 \times 2^{-2x}} = \frac{1}{2^{3x}}$$
$$\Rightarrow 2^{2x} \times \frac{1}{2^2} \times \frac{2^{2x}}{2^3} = \frac{1}{2^{3x}}$$
$$\Rightarrow \frac{2^{2x+2x}}{4 \times 8} = \frac{1}{2^{3x}}$$
$$\Rightarrow \frac{2^{4x}}{32} = \frac{1}{2^{3x}}$$
$$\Rightarrow 2^{4x} \times 2^{3x} = 32$$
$$\Rightarrow 2^{7x} = 2^5$$
$$\Rightarrow 7x = 5$$
$$\Rightarrow x = \frac{5}{7}$$

Question 26
Solve the equation for x:
$2^{3x-7} = 256$

Solution 26
$$2^{3x-7} = 256$$
$$\Rightarrow 2^{3x} \times 2^{-7} = 2^8$$
$$\Rightarrow 2^{3x} = \frac{2^8}{2^{-7}}$$
$$\Rightarrow 2^{3x} = 2^{8+7}$$
$$\Rightarrow 2^{3x} = 2^{15}$$
$$\Rightarrow 3x = 15$$
$$\Rightarrow x = 5$$

Question 27
Solve the equation for x:
$2^{2x} - 2^{x+3} + 2^4 = 0$

Solution 27

$2^{2x} - 2^{x+3} + 2^4 = 0$

$\Rightarrow \left(2^x\right)^2 - 2^x \times 2^3 + 2^4 = 0$

$\Rightarrow \left(2^x\right)^2 - 2^x \times 8 + 16$

Let $2^x = a$

Thus, we have

$a^2 - 8a + 16$

$\Rightarrow a^2 - 4a - 4a + 16$

$\Rightarrow a\left(a - 4\right) - 4\left(a - 4\right)$

$\Rightarrow \left(a - 4\right)\left(a - 4\right) = 0$

$\Rightarrow \left(a - 4\right)^2 = 0$

$\Rightarrow a - 4 = 0$

$\Rightarrow a = 4$

$\Rightarrow 2^x = 4$

$\Rightarrow 2^x = 2^2$

$\Rightarrow x = 2$

Question 28

Solve the equation for x:

$3^{2x + 4} + 1 = 2.3^{x + 2}$

Solution 28

$3^{2x+4} + 1 = 2.3^{x+2}$

$\Rightarrow 3^4.3^{2x} + 1 = 2.3^x.3^2$

$\Rightarrow 81\left(3^x\right)^2 + 1 = 18.3^x$

$\Rightarrow 81\left(3^x\right)^2 - 18.3^x + 1 = 0$

Let $3^x = a$

Thus, we have

$81a^2 - 18a + 1 = 0$

$\Rightarrow 81a^2 - 9a - 9a + 1 = 0$

$\Rightarrow 9a\left(9a - 1\right) - 1\left(9a - 1\right) = 0$

$\Rightarrow \left(9a - 1\right)\left(9a - 1\right) = 0$

$\Rightarrow \left(9a - 1\right)^2 = 0$

$\Rightarrow 9a - 1 = 0$

$\Rightarrow 9a = 1$

$\Rightarrow a = \frac{1}{9}$

$\Rightarrow 3^x = \frac{1}{9}$

$\Rightarrow 3^x = \frac{1}{3^2} = 3^{-2}$

$\Rightarrow x = -2$

Question 29

If $49392 = a^4b^2c^3$, find the values of a, b and c, where a, b and c are different positive primes.

Solution 29

By prime factorisation, we have

$$49392 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 7 = 2^4 \times 3^2 \times 7^3$$

Now, $a^4b^2c^3 = 49392$

$$\Rightarrow a^4b^2c^3 = 2^4 \times 3^2 \times 7^3$$

$$\Rightarrow a = 2,\ b = 3 \text{ and } c = 7$$

Question 30

If $1176 = 2^a \times 3^b \times 7^c$, find a, b and c.

Solution 30

By prime factorisation, we have

$$1176 = 2 \times 2 \times 2 \times 3 \times 7 \times 7 = 2^3 \times 3 \times 7^2$$

Now, $2^a \times 3^b \times 7^c = 1176$

$$\Rightarrow 2^a \times 3^b \times 7^c = 2^3 \times 3 \times 7^2$$

$$\Rightarrow a = 3,\ b = 1 \text{ and } c = 2$$

Question 31

Given $4725 = 3^a\, 5^b\, 7^c$ , find

i. the integral values of a, b and c
ii. the values of $2^{-a}3^b7^c$

Solution 31

i By prime factorisation, we have

$$4725 = 3 \times 3 \times 3 \times 5 \times 5 \times 7 = 3^3 \times 5^2 \times 7$$

Now, $3^a \times 5^b \times 7^c = 4725$

$$\Rightarrow 3^a \times 5^b \times 7^c = 3^3 \times 5^2 \times 7$$

$$\Rightarrow a = 3,\ b = 2 \text{ and } c = 1$$

ii $$2^{-a} \times 3^b \times 7^c = 2^{-3} \times 3^2 \times 7^1$$

$$= \frac{1}{2^3} \times 9 \times 7$$

$$= \frac{1}{8} \times 9 \times 7$$

$$= \frac{63}{8}$$

Question 32

If $a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$, prove that $a^{q-r}b^{r-p}\, c^{p-q} = 1$.

Solution 32

$a = xy^{p-1}$, $b = xy^{q-1}$ and $c = xy^{r-1}$

$$\therefore\ a^{q-r} \times b^{r-p} \times c^{p-q} = \left(xy^{p-1}\right)^{q-r} \times \left(xy^{q-1}\right)^{r-p} \times \left(xy^{r-1}\right)^{p-q}$$
$$= x^{q-r} \times y^{(p-1)(q-r)} \times x^{r-p} \times y^{(q-1)(r-p)} \times x^{p-q} \times y^{(r-1)(p-q)}$$
$$= x^{q-r} \times y^{pq-pr-q+r} \times x^{r-p} \times y^{qr-pq-r+p} \times x^{p-q} \times y^{pr-qr-p+q}$$
$$= x^{q-r+r-p+p-q} \times y^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$$
$$= x^0 \times y^0$$
$$= 1 \times 1$$
$$= 1$$

# Chapter 2 - Exponents of Real Numbers Exercise Ex. 2.2

Question 1

Assuming that $x$ is positive real number, simplify $\left(\sqrt{x^{-3}}\right)^5$.

Solution 1

We have,

$$\left(\sqrt{x^{-3}}\right)^5 = \left(\sqrt{\frac{1}{x^3}}\right)^5$$

$$= \left(\frac{1}{x^{\frac{3}{2}}}\right)^5$$

$$= \frac{1}{x^{\frac{3}{2}\times 5}}$$

$$= \frac{1}{x^{\frac{15}{2}}}$$

$$\Rightarrow \quad \left(\sqrt{x^{-3}}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$$

Question 2

Assuming that $x, y$ are positive real numbers, simplify $\sqrt{x^3 y^{-2}}$.

Solution 2

We have,

$$\sqrt{x^3 y^{-2}} = \sqrt{\frac{x^3}{y^2}}$$

$$= \left(\frac{x^3}{y^2}\right)^{\frac{1}{2}}$$

$$= \frac{x^{3\times\frac{1}{2}}}{y^{2\times\frac{1}{2}}}$$

$$= \frac{x^{\frac{3}{2}}}{y}$$

$$\Rightarrow \quad \sqrt{x^3 y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$$

Question 3

Assuming that $x, y$ are positive real numbers, simplify $\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2$.

Solution 3

We have,

$$\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2 = \left(\frac{1}{x^{\frac{2}{3}} y^{\frac{1}{2}}}\right)^2$$

$$= \left(\frac{1}{x^{\frac{2}{3}\times 2} y^{2\times\frac{1}{2}}}\right)$$

$$= \frac{1}{x^{\frac{4}{3}} y^1}$$

$$= \frac{1}{x^{\frac{4}{3}} y}$$

$$\Rightarrow \quad \left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2 = \frac{1}{x^{\frac{4}{3}} y}$$

Question 4

Assuming that $x, y$ are positive real numbers, simplify $\left(\sqrt{x}\right)^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$.

Solution 4

We have,

$$\left(\sqrt{x}\right)^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(y^2\right) \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{2}\times-\frac{2}{3}}y^2}{\left(xy^{-\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$= \frac{x^{-\frac{1}{3}}y^2}{x^{\frac{1}{2}}y^{-\frac{1}{2}\times\frac{1}{2}}}$$

$$= \left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}\right) \times \left(y^2 \times y^{\frac{1}{4}}\right)$$

$$= \left(x^{-\frac{1}{3}-\frac{1}{2}}\right)\left(y^{2+\frac{1}{4}}\right)$$

$$= \left(x^{\frac{-2-3}{6}}\right)\left(y^{\frac{8+1}{4}}\right)$$

$$= \left(x^{-\frac{5}{6}}\right)\left(y^{\frac{9}{4}}\right)$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

$$\Rightarrow \quad \left(\sqrt{x}\right)^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}} = \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

Question 5

Assuming that $x, y, z$ are positive real numbers, simplify $\sqrt[5]{243x^{10}y^{5}z^{10}}$.

Solution 5

We have,

$$\sqrt[5]{243x^{10}y^{5}z^{10}} = \left(243x^{10}y^{5}z^{10}\right)^{\frac{1}{5}}$$

$$= (243)^{\frac{1}{5}}x^{\frac{10}{5}}y^{\frac{5}{5}}z^{\frac{10}{5}}$$

$$= \left(3^{5}\right)^{\frac{1}{5}}x^{2}y^{1}z^{2}$$

$$= 3^{5\times\frac{1}{5}}x^{2}yz^{2}$$

$$= 3x^{2}yz^{2}$$

$$\Rightarrow \quad \sqrt[5]{243x^{10}y^{5}z^{10}} = 3x^{2}yz^{2}$$

Question 6

Assuming that $x, y$ are positive real numbers, simplify $\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$.

Solution 6

We have,

$$\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}} = \left(\frac{y^{10}}{x^{4}}\right)^{\frac{5}{4}}$$

$$= \frac{y^{10\times\frac{5}{4}}}{x^{4\times\frac{5}{4}}}$$

$$= \frac{y^{\frac{25}{2}}}{x^{5}}$$

$$\Rightarrow \quad \left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}} = \frac{y^{\frac{25}{2}}}{x^{5}}$$

Question 7

Assuming that x, y, z are positive real numbers, simplify each of the following:

$$\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{5}\left(\frac{6}{7}\right)^{2}$$

Solution 7

$$\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5\left(\frac{6}{7}\right)^2 = \frac{\left(\sqrt{2}\right)^5}{\left(\sqrt{3}\right)^5}\times\frac{6^2}{7^2}$$

$$= \frac{4\sqrt{2}}{9\sqrt{3}}\times\frac{36}{49}$$

$$= \frac{4\sqrt{2}\times 4}{\sqrt{3}\times 49}$$

$$= \frac{\sqrt{256}\times\sqrt{2}}{\sqrt{3}\times\sqrt{2401}}$$

$$= \frac{\sqrt{512}}{\sqrt{7203}}$$

$$= \left(\sqrt{\frac{512}{7203}}\right)$$

$$= \left(\frac{512}{7203}\right)^{\frac{1}{2}}$$

Question 8

Simplify:

$$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$$

Solution 8

We have,

$$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} = 16^{-\frac{1}{5}\times\frac{5}{2}}$$

$$= 16^{-\frac{1}{2}}$$

$$= \frac{1}{16^{\frac{1}{2}}}$$

$$= \frac{1}{\left(4^2\right)^{\frac{1}{2}}}$$

$$= \frac{1}{4^{2\times\frac{1}{2}}}$$

$$= \frac{1}{4}$$

$$\Rightarrow \quad \left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} = \frac{1}{4}$$

Question 9

Simplify:

$$\sqrt[5]{(32)^{-3}}$$

Solution 9

$$\sqrt[5]{(32)^{-3}} = \left[(32)^{-3}\right]^{\frac{1}{5}}$$

$$= \left[\left(2^5\right)^{-3}\right]^{\frac{1}{5}}$$

$$= 2^{5\times(-3)\times\frac{1}{5}}$$

$$= 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

Question 10

Simplify:

$$\sqrt[3]{(343)^{-2}}$$

Solution 10

We have,

$$\sqrt[3]{(343)^{-2}} = \sqrt[3]{\frac{1}{(343)^2}}$$

$$= \frac{1}{(343)^{\frac{2}{3}}}$$

$$= \frac{1}{\left(7^3\right)^{\frac{2}{3}}}$$

$$= \frac{1}{7^{3\times\frac{2}{3}}}$$

$$= \frac{1}{7^2}$$

$$= \frac{1}{49}$$

$$\Rightarrow \quad \sqrt[3]{(343)^{-2}} = \frac{1}{49}$$

Question 11

Simplify:

$$(0.001)^{\frac{1}{3}}$$

Solution 11

We have,

$$(0.001)^{\frac{1}{3}} = \left(\frac{1}{1000}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1}{10^3}\right)^{\frac{1}{3}}$$

$$= \frac{1}{10^{3\times\frac{1}{3}}}$$

$$= \frac{1}{10}$$

$$= 0.1$$

$$\Rightarrow \quad (0.001)^{\frac{1}{3}} = 0.1$$

Question 12

Simplify:

$$\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$$

Solution 12

We have,

$$\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$$

$$= \frac{5^{2\times\frac{3}{2}} \times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}} \times 2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$= \frac{3375}{512}$$

$$\Rightarrow \quad \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \frac{3375}{512}$$

Question 13

Simplify:

$$\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$$

Solution 13

We have,

$$\left(\frac{\sqrt{2}}{5}\right)^{8} \div \left(\frac{\sqrt{2}}{5}\right)^{13}$$

$$= \left(\frac{\sqrt{2}}{5}\right)^{8} \times \left(\frac{5}{\sqrt{2}}\right)^{13}$$

$$= \frac{\left(\sqrt{2}\right)^{8}}{5^{8}} \times \frac{5^{13}}{\left(\sqrt{2}\right)^{13}}$$

$$= \frac{5^{13} \times 5^{-8}}{\left(\sqrt{2}\right)^{13} \times \left(\sqrt{2}\right)^{-8}}$$

$$= \frac{5^{13-8}}{\left(\sqrt{2}\right)^{13-8}}$$

$$= \frac{5^{5}}{\left(\sqrt{2}\right)^{5}} = \frac{3125}{4\sqrt{2}}$$

$$\Rightarrow \qquad \left(\frac{\sqrt{2}}{5}\right)^{8} \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \frac{3125}{4\sqrt{2}}$$

Question 14

Simplify:

$$\left(\frac{5^{-1} \times 7^{2}}{5^{2} \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^{3}}{5^{3} \times 7^{-5}}\right)^{-\frac{5}{2}}$$

Solution 14

We have,

$$\left(\frac{5^{-1}\times 7^2}{5^2\times 7^{-4}}\right)^{\frac{7}{2}}\times\left(\frac{5^{-2}\times 7^3}{5^3\times 7^{-5}}\right)^{-\frac{5}{2}}$$

$$=\left(\frac{7^{2+4}}{5^{2+1}}\right)^{\frac{7}{2}}\times\left(\frac{7^{3+5}}{5^{3+2}}\right)^{-\frac{5}{2}}$$

$$=\left(\frac{7^6}{5^3}\right)^{\frac{7}{2}}\times\left(\frac{7^8}{5^5}\right)^{-\frac{5}{2}}$$

$$=\frac{7^{6\times\frac{7}{2}}}{5^{3\times\frac{7}{2}}}\times\frac{7^{8\times-\frac{5}{2}}}{5^{5\times-\frac{5}{2}}}$$

$$=\frac{7^{21}}{5^{\frac{21}{2}}}\times\frac{7^{-20}}{5^{-\frac{25}{2}}}$$

$$=\frac{7^{21-20}}{5^{\frac{21}{2}-\frac{25}{2}}}=\frac{7}{5^{-\frac{4}{2}}}$$

$$=7\times 5^{\frac{4}{2}}=7\times 5^2$$

$$=7\times 25=175$$

$$\Rightarrow\quad\left(\frac{5^{-1}\times 7^2}{5^2\times 7^{-4}}\right)^{\frac{7}{2}}\times\left(\frac{5^{-2}\times 7^3}{5^3\times 7^{-5}}\right)^{-\frac{5}{2}}=175$$

Question 15

Prove that:

$$\sqrt{3\times 5^{-3}}\div\sqrt[3]{3^{-1}}\sqrt{5}\times\sqrt[6]{3\times 5^6}=\frac{3}{5}$$

Solution 15

We have,

$$\sqrt{3\times5^{-3}} \div \sqrt[3]{3^{-1}}\sqrt{5} \times \sqrt[6]{3\times5^{6}} = \frac{3}{5}$$

$$= \frac{\left(3\times5^{-3}\right)^{\frac{1}{2}}}{\sqrt[3]{3^{-1}}\sqrt{5}} \times \sqrt[6]{3\times5^{6}}$$

$$= \frac{3^{\frac{1}{2}}\times5^{-\frac{3}{2}}\times\left(3\times5^{6}\right)^{\frac{1}{6}}}{\left(3^{-1}\right)^{\frac{1}{3}}\times(5)^{\frac{1}{2}}}$$

$$= \frac{3^{\frac{1}{2}}\times3^{\frac{1}{6}}\times5^{-\frac{3}{2}}\times5^{6\times\frac{1}{6}}}{3^{-\frac{1}{3}}\times5^{\frac{1}{2}}}$$

$$= \left(3^{\frac{1}{2}}\times3^{\frac{1}{6}}\times3^{\frac{1}{3}}\right)\times\left(5^{-\frac{3}{2}}\times5^{1}\times5^{-\frac{1}{2}}\right)$$

$$= \left(3^{\frac{1}{2}+\frac{1}{6}+\frac{1}{3}}\right)\times\left(5^{-\frac{3}{2}-\frac{1}{2}+1}\right)$$

$$= 3^{\frac{3+1+2}{6}}\times5^{\frac{-3-1}{2}+1}$$

$$= 3^{\frac{6}{6}}\times5^{-\frac{4}{2}+1}$$

$$= 3^{1}\times5^{-2+1}$$

$$= 3\times5^{-1} = \frac{3}{5}$$

Question 16

Prove that:

$$9^{\frac{3}{2}} - 3\times5^{0} - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$$

Solution 16

We have,

$$9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$$

$$= \left(3^2\right)^{\frac{3}{2}} - 3 \times 1 - \left(\frac{1}{9^2}\right)^{-\frac{1}{2}}$$

$$= 3^{2 \times \frac{3}{2}} - 3 - \left(9^{-2}\right)^{-\frac{1}{2}}$$

$$= 3^3 - 3 - 9^{-2 \times \left(-\frac{1}{2}\right)}$$

$$= 3^3 - 3 - 9$$
$$= 27 - 3 - 9$$
$$= 27 - 12$$

$$= 15$$

$$\Rightarrow \qquad 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$$

Question 17

Prove that:

$$\left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$$

Solution 17

We have,

$$\left(\frac{1}{4}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}$$

$$=\left(\frac{1}{2^2}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 1+\left(\frac{3^2}{4^2}\right)^{-\frac{1}{2}}$$

$$=\left(2^{-2}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 1+\left(\frac{3^{2\times\frac{-1}{2}}}{4^{2\times\frac{-1}{2}}}\right)$$

$$=2^{(-2)\times(-2)}-3\times 8^{\frac{2}{3}}+\left(\frac{3^{-1}}{4^{-1}}\right)$$

$$=2^4-3\times 2^{3\times\frac{2}{3}}+\frac{4}{3}$$

$$=2^4-3\times 2^2+\frac{4}{3}$$

$$=2^4-3\times 4+\frac{4}{3}$$

$$=16-12+\frac{4}{3}$$

$$=4+\frac{4}{3}=\frac{12+4}{3}$$

$$=\frac{16}{3}$$

$$\Rightarrow\qquad \left(\frac{1}{4}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}=\frac{16}{3}$$

Question 18

Prove that:

$$\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}}\times 5^{\frac{3}{5}}}\div\frac{3^{\frac{4}{3}}\times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}}\times 6}=10$$

Solution 18

We have,

$$\frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6}$$

$$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \times \frac{4^{-\frac{3}{5}} \times 6}{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times \left(2^2\right)^{\frac{1}{4}} \times \left(2^2\right)^{-\frac{3}{5}} \times (2 \times 3)}{(2 \times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{\left(2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{-\frac{6}{5}} \times 2^1\right) \times \left(3^{\frac{1}{3}} \times 3^1\right)}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}$$

$$= \frac{\left(2 \times 2^{-\frac{6}{5}} \times 2\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{2^{-\frac{1}{5}} \times \left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)}$$

$$= \frac{\left(2 \times 2^{-\frac{6}{5}} \times 2 \times 2^{\frac{1}{5}}\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{\left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)}$$

$$= \frac{(2)^{1-\frac{6}{5}+1+\frac{1}{5}} \times (3)^{\frac{1}{3}+1-\frac{4}{3}}}{(5)^{-\frac{1}{5}+\frac{3}{5}-\frac{7}{5}}}$$

$$= \frac{(2)^{2-\frac{6}{5}+\frac{1}{5}} \times (3)^{\frac{1+3-4}{3}}}{(5)^{\frac{-1+3-7}{5}}}$$

$$= \frac{(2)^{2-\frac{5}{5}} \times (3)^{\frac{0}{3}}}{(5)^{-\frac{5}{5}}}$$

$$= \frac{(2)^{2-1} \times (3)^0}{(5)^{-1}}$$

$$= 2^1 \times 1 \times 5^1$$

$$= 10$$

Question 19

Prove:

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}=\frac{3}{2}$$

Solution 19

We have,

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}$$

$$=\frac{1}{2}+\frac{1}{(0.01)^{\frac{1}{2}}}-(3^3)^{\frac{2}{3}}$$

$$=\frac{1}{2}+\frac{1}{(0.1)^{2\times\frac{1}{2}}}-3^{3\times\frac{2}{3}}$$

$$=\frac{1}{2}+\frac{1}{0.1}-3^2$$

$$=\frac{1}{2}+10-9$$

$$=\frac{1}{2}+1=\frac{3}{2}$$

$$\Rightarrow \quad \sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}=\frac{3}{2}$$

Question 20

Prove that:

$$\frac{2^n+2^{n-1}}{2^{n+1}-2^n}=\frac{3}{2}$$

Solution 20

We have,

$$\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n \times 2^{-1}}{2^n \times 2^1 - 2^n}$$

$$= \frac{2^n\left[1 + 2^{-1}\right]}{2^n\left[2 - 1\right]}$$

$$= \frac{1 + \frac{1}{2}}{1}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

$$\Rightarrow \qquad \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$$

Question 21

Prove that:

$$\left(\frac{64}{125}\right)^{-2/3} + \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0 = \frac{61}{16}$$

Solution 21

$$\text{L.H.S.} = \left(\frac{64}{125}\right)^{-2/3} + \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^{0}$$

$$= \left(\frac{4^3}{5^3}\right)^{-2/3} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{1/4}} + 1$$

$$= \frac{4^{3\times\left(\frac{-2}{3}\right)}}{5^{3\times\left(\frac{-2}{3}\right)}} + \frac{1}{\frac{4^{4\times\frac{1}{4}}}{5^{4\times\frac{1}{4}}}} + 1$$

$$= \frac{4^{-2}}{5^{-2}} + \frac{1}{\frac{4}{5}} + 1$$

$$= \frac{5^2}{4^2} + \frac{5}{4} + 1$$

$$= \frac{25 + 20 + 16}{16}$$

$$= \frac{61}{16}$$

$$= \text{R.H.S.}$$

Question 22

Prove that:

$$\frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} = 28\sqrt{2}$$

Solution 22

We have,

$$\frac{3^{-3}\times 6^2\times\sqrt{98}}{5^2\times\sqrt[3]{\frac{1}{25}}\times(15)^{-\frac{4}{3}}\times 3^{\frac{1}{3}}}$$

$$=\frac{3^{-3}\times 36\times\sqrt{7\times 7\times 2}}{5^2\times\left(\frac{1}{25}\right)^{\frac{1}{3}}\times(15)^{-\frac{4}{3}}\times 3^{\frac{1}{3}}}$$

$$=\frac{3^{-3}\times 36\times 7\sqrt{2}}{5^2\times\left(\frac{1}{5^{2\times\frac{1}{3}}}\right)\times\frac{1}{(15)^{\frac{4}{3}}}\times 3^{\frac{1}{3}}}$$

$$=\frac{3^{-3}\times 36\times 7\sqrt{2}}{5^2\times 5^{-\frac{2}{3}}\times\frac{1}{(5\times 3)^{\frac{4}{3}}}\times 3^{\frac{1}{3}}}$$

$$=\frac{3^{-3}\times 36\times 7\sqrt{2}}{\left(5^2\times 5^{-\frac{2}{3}}\times 5^{-\frac{4}{3}}\right)\times 3^{-\frac{4}{3}}\times 3^{\frac{1}{3}}}$$

$$=\frac{3^{-3}\times 36\times 7\sqrt{2}\times 3^{\frac{4}{3}}\times 3^{-\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}}$$

$$=\frac{3^{-3}\times 36\times 7\sqrt{2}\times 3^{\frac{4}{3}}\times 3^{-\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}}$$

$$=\frac{3^{-3+\frac{4}{3}-\frac{1}{3}}\times 36\times 7\sqrt{2}}{5^0}$$

$$=3^{-3+\left(\frac{4-1}{3}\right)}\times 36\times 7\sqrt{2}$$

$$=3^{-3+\frac{3}{3}}\times 36\times 7\sqrt{2}$$

$$=3^{-3+1}\times 36\times 7\sqrt{2}$$

$$=3^{-2}\times 36\times 7\sqrt{2}$$

$$=\frac{1}{9}\times 36\times 7\sqrt{2}$$

$$=4\times 7\sqrt{2}$$

$$=28\sqrt{2}$$

$$\Rightarrow\quad\frac{3^{-3}\times 6^2\times\sqrt{98}}{5^2\times\sqrt[3]{\frac{1}{25}}\times(15)^{-\frac{4}{3}}\times 3^{\frac{1}{3}}}=28\sqrt{2}$$

Question 23

Prove that:

$$\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}=-\frac{3}{2}$$

Solution 23

We have,

$$\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}$$

$$=\frac{1-\frac{1}{0.1}}{\left(\frac{8}{3}\right)\left(\frac{3}{2}\right)^3+(-3)^1}$$

$$=\frac{1-10}{\frac{8}{3}\times\frac{3^3}{2^3}-3}$$

$$=\frac{-9}{3^2-3}$$

$$=\frac{-9}{9-3}=-\frac{9}{6}=-\frac{3}{2}$$

Question 24

Show that:

$$\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=1$$

Solution 24

$$\text{L.H.S.}=\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}$$

Multiplying the numerators and denominators of two terms on L.H.S. by $x^b$ and $x^a$ respectively, we obtain

$$\text{L.H.S.}=\frac{x^b}{x^b+x^{a-b+b}}+\frac{x^a}{x^a+x^{b-a+a}}$$

$$=\frac{x^b}{x^b+x^a}+\frac{x^a}{x^a+x^b}$$

$$=\frac{x^b+x^a}{x^b+x^a}$$

$$=1$$

$$=\text{R.H.S.}$$

Question 25

Show that:

$$\left[\left\{\frac{x^{a(a-b)}}{x^{a(a+b)}}\right\} \div \left\{\frac{x^{b(b-a)}}{x^{b(b+a)}}\right\}\right]^{a+b} = 1$$

Solution 25

$$\left[\left\{\frac{x^{a(a-b)}}{x^{a(a+b)}}\right\} \div \left\{\frac{x^{b(b-a)}}{x^{b(b+a)}}\right\}\right]^{a+b} = 1$$

$$\text{L.H.S.} = \left[\left\{\frac{x^{a(a-b)}}{x^{a(a+b)}}\right\} \div \left\{\frac{x^{b(b-a)}}{x^{b(b+a)}}\right\}\right]^{a+b}$$

$$= \left[\frac{x^{a(a-b)}}{x^{a(a+b)}} \times \frac{x^{b(b+a)}}{x^{b(b-a)}}\right]^{a+b}$$

$$= \left[\frac{x^{a^2-ab}}{x^{a^2+ab}} \times \frac{x^{b^2+ab}}{x^{b^2-ab}}\right]^{a+b}$$

$$= \left[\frac{x^{a^2-ab+b^2+ab}}{x^{a^2+ab+b^2-ab}}\right]^{a+b}$$

$$= \left[\frac{x^{a^2+b^2}}{x^{a^2+b^2}}\right]^{a+b}$$

$$= [1]^{a+b}$$

$$= 1$$

$$= \text{R.H.S.}$$

Question 26

Show that:

$$\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}} = 1$$

Solution 26

$$\text{L.H.S.} = \left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$$

$$= \left(x^{\frac{1}{a-b}\times\frac{1}{a-c}}\right)\left(x^{\frac{1}{b-c}\times\frac{1}{b-a}}\right)\left(x^{\frac{1}{c-a}\times\frac{1}{c-b}}\right)$$

$$= \left(x^{\frac{1}{a-b}\times\frac{1}{a-c}+\frac{1}{b-c}\times\frac{1}{b-a}+\frac{1}{c-a}\times\frac{1}{c-b}}\right)$$

$$= \left(x^{-\frac{1}{(a-b)(c-a)}-\frac{1}{(b-c)(a-b)}-\frac{1}{(c-a)(b-c)}}\right)$$

$$= \left(x^{\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}}\right)$$

$$= \left(x^{\frac{-b+c-c+a-a+b}{(a-b)(b-c)(c-a)}}\right)$$

$$= \left(x^0\right)$$

$$= 1$$

$$= \text{R.H.S.}$$

Question 27

Show that:

$$\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b} \left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c} \left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c} = 1$$

Note: Question modified

Solution 27

$$\text{L.H.S.} = \left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b} \left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c} \left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c}$$

$$= \left(x^{a^2+b^2-ab}\right)^{a+b} \left(x^{b^2+c^2-bc}\right)^{b+c} \left(x^{c^2+a^2-ac}\right)^{a+c}$$

$$= \left(x^{(a+b)(a^2+b^2-ab)}\right)\left(x^{(b+c)(b^2+c^2-bc)}\right)\left(x^{(a+c)(a^2+c^2-ac)}\right)$$

$$= \left(x^{a^3-b^3}\right)\left(x^{b^3-c^3}\right)\left(x^{c^3-a^3}\right)$$

$$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$$

$$= x^0$$

$$= 1$$

$$= \text{R.H.S.}$$

Note: Question modified

Question 28

Show that:

$(x^{a-b})^{a+b}(x^{b-c})^{b+c}(x^{c-a})^{c+a} = 1$

Solution 28

$$\text{L.H.S.} = \left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a}$$

$$= \left(x^{(a-b(a+b)}\right)\left(x^{(b-c)(b+c)}\right)\left(x^{(c-a)(c+a)}\right)$$

$$= \left(x^{a^2-b^2}\right)\left(x^{b^2-c^2}\right)\left(x^{c^2-a^2}\right)$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

$$= \text{R.H.S.}$$

Question 29

Show that:

$$\left\{\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right\}^{\frac{a}{a+1}} = x$$

Solution 29

$$\text{L.H.S.} = \left\{\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right\}^{\frac{a}{a+1}}$$

$$= \left(x^{a-\frac{1}{a}}\right)^{\frac{1}{a-1}\times\frac{a}{a+1}}$$

$$= \left(x^{\frac{a^2-1}{a}}\right)^{\frac{a}{(a-1)(a+1)}}$$

$$= \left(x^{\frac{a^2-1}{a}}\right)^{\frac{a}{a^2-1}}$$

$$= x^{\frac{a^2-1}{a}\times\frac{a}{a^2-1}}$$

$$= x$$

$$= \text{R.H.S.}$$

Question 30

Show that:

$$\left(\frac{a^{x+1}}{a^{y+1}}\right)^{x+y}\left(\frac{a^{y+2}}{a^{z+2}}\right)^{y+z}\left(\frac{a^{z+3}}{a^{x+3}}\right)^{z+x} = 1$$

Solution 30

$$\text{L.H.S.} = \left(\frac{a^{x+1}}{a^{y+1}}\right)^{x+y}\left(\frac{a^{y+2}}{a^{z+2}}\right)^{y+z}\left(\frac{a^{z+3}}{a^{x+3}}\right)^{z+x}$$

$$= \left(a^{x+1-(y+1)}\right)^{x+y}\left(a^{y+2-(z+2)}\right)^{y+z}\left(a^{z+3-(x+3)}\right)^{z+x}$$

$$= \left(a^{x+1-y-1}\right)^{x+y}\left(a^{y+2-z-2}\right)^{y+z}\left(a^{z+3-x-3}\right)^{z+x}$$

$$= \left(a^{x-y}\right)^{x+y}\left(a^{y-z}\right)^{y+z}\left(a^{z-x}\right)^{z+x}$$

$$= \left(a^{(x-y)(x+y)}\right)\left(a^{(y-z)(y+z)}\right)\left(a^{(z-x)(z+x)}\right)$$

$$= \left(a^{x^2-y^2}\right)\left(a^{y^2-z^2}\right)\left(a^{z^2-x^2}\right)$$

$$= a^{x^2-y^2+y^2-z^2+z^2-x^2}$$

$$= a^0$$

$$= 1$$

$$= \text{R.H.S.}$$

Question 31

Show that:

$$\left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a} = 1$$

Solution 31

$$\text{L.H.S.} = \left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a}$$

$$= \left(3^{a-b}\right)^{a+b}\left(3^{b-c}\right)^{b+c}\left(3^{c-a}\right)^{c+a}$$

$$= \left(3^{(a-b)(a+b)}\right)\left(3^{(b-c)(b+c)}\right)\left(3^{(c-a)(c+a)}\right)$$

$$= \left(3^{a^2-b^2}\right)\left(3^{b^2-c^2}\right)\left(3^{c^2-a^2}\right)$$

$$= 3^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= 3^0$$

$$= 1$$

$$= \text{R.H.S.}$$

Question 32

If $2^x = 3^y = 12^z$, show that $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$.

Solution 32

Let $2^x = 3^y = 12^z = k$

Then, $2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$ and $12 = k^{\frac{1}{z}}$

Now,

$12 = k^{\frac{1}{z}}$

$\Rightarrow 2^2 \times 3 = k^{\frac{1}{z}}$

$\Rightarrow \left(k^{\frac{1}{x}}\right)^2 \times k^{\frac{1}{y}} = k^{\frac{1}{z}}$

$\Rightarrow k^{\frac{2}{x}} \times k^{\frac{1}{y}} = k^{\frac{1}{z}}$

$\Rightarrow k^{\frac{2}{x}+\frac{1}{y}} = k^{\frac{1}{z}}$

$\Rightarrow \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$

Question 33

If $2^x = 3^y = 6^{-z}$, show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$.

Solution 33

Let $2^x = 3^y = 6^{-z} = k$

Then, $2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$ and $6 = k^{-\frac{1}{z}}$

Now,

$6 = k^{-\frac{1}{z}}$

$\Rightarrow 2 \times 3 = k^{-\frac{1}{z}}$

$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$

$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{-\frac{1}{z}}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Question 34

If $a^x = b^y = c^z$ and $b^2 = ac$, then show that $y = \frac{2zx}{z+x}$.

Solution 34

Let $a^x = b^y = c^z = k$

Then, $a = k^{\frac{1}{x}}$, $b = k^{\frac{1}{y}}$ and $c = k^{\frac{1}{z}}$

Now,

$b^2 = ac$

$$\Rightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\Rightarrow \frac{2}{y} = \frac{z+x}{zx}$$

$$\Rightarrow y = \frac{2zx}{z+x}$$

Question 35

If $3^x = 5^y = (75)^z$, show that $z = \frac{xy}{2x+y}$.

Solution 35

Let $3^x = 5^y = (75)^z = k$

Then, $3 = k^{\frac{1}{x}}$, $5 = k^{\frac{1}{y}}$ and $75 = k^{\frac{1}{z}}$

Now,

$$75 = k^{\frac{1}{z}}$$

$$\Rightarrow 3 \times 5^2 = k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x}} \times \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{2}{y}} = k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x}+\frac{2}{y}} = k^{\frac{1}{z}}$$

$$\Rightarrow \frac{1}{x} + \frac{2}{y} = \frac{1}{z}$$

$$\Rightarrow \frac{y+2x}{xy} = \frac{1}{z}$$

$$\Rightarrow z = \frac{xy}{2x+y}$$

Question 36

If $27^x = \frac{9}{3^x}$, find $x$.

Solution 36

We have,

$$27^x = \frac{9}{3^x}$$

$\Rightarrow \quad \left(3^3\right)^x = \frac{3^2}{3^x}$

$\Rightarrow \quad 3^{3x} = \frac{3^2}{3^x}$

$\Rightarrow \quad 3^{3x} \times 3^x = 3^2$

$\Rightarrow \quad 3^{3x+x} = 3^2$

$\Rightarrow \quad 3^{4x} = 3^2$

On equating the exponents, we get

$4x = 2$

$\Rightarrow \quad x = \frac{2}{4} = \frac{1}{2}$

Hence, $x = \frac{1}{2}$

Question 37

Find the values of $x$ if:

$$2^{5x} \div 2^x = \sqrt[5]{2^{20}}$$

Solution 37

We have,

$$2^{5x} \div 2^x = \sqrt[5]{2^{20}}$$

$\Rightarrow \quad \frac{2^{5x}}{2^x} = \left(2^{20}\right)^{\frac{1}{5}}$

$\Rightarrow \quad 2^{5x} \times 2^{-x} = 2^{20 \times \frac{1}{5}}$

$\Rightarrow \quad 2^{4x} = 2^4$

On equating the exponents, we get,

$4x = 4$

$\Rightarrow \quad x = \frac{4}{4} = 1$

Hence, $x = 1$

Question 38
Find the value of $x$ if:

$$\left(2^3\right)^4 = \left(2^2\right)^x$$

Solution 38
We have,

$$\left(2^3\right)^4 = \left(2^2\right)^x$$

$\Rightarrow \quad 2^{3\times 4} = 2^{2\times x}$

$\Rightarrow \quad 2^{12} = 2^{2x}$

On comparing the exponents, we get,

$2x = 12$

$\Rightarrow \quad x = \frac{12}{2} = 6$

Hence, $x = 6$

Question 39
Find the value of $x$ if:

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

Solution 39

We have,

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

$$\Rightarrow \quad \left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{5^3}{3^3}$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^{-x} \left(\frac{5}{3}\right)^{2x} = \left(\frac{5}{3}\right)^3$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^{-x+2x} = \left(\frac{5}{3}\right)^3$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^3$$

On equating the exponents, we get,

$x = 3$

Question 40

Find the value of $x$ if:

$$5^{x-2} \times 3^{2x-3} = 135$$

Solution 40

We have,

$$5^{x-2} \times 3^{2x-3} = 135$$

$$\Rightarrow \quad 5^{x-2} \times 3^{2x-3} = 5 \times 3^3$$

On equating the exponents,
we get,

$x - 2 = 1$ and $2x - 3 = 3$

$$\Rightarrow \quad x = 3$$

Hence, $x = 3$

Question 41

Find the value of $x$ if:

$$2^{x-7} \times 5^{x-4} = 1250$$

Solution 41

We have,

$2^{x-7} \times 5^{x-4} = 1250$

$\Rightarrow \quad 2^{x-7} \times 5^{x-4} = 2^1 \times 5^4$

On equating the exponents,
we get,

$x - 7 = 1$ and $x - 4 = 4$

$\Rightarrow \quad x = 8$

Hence, $x = 8$

Question 42

Find the value of x if:

$$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$$

Solution 42

$$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$\Rightarrow 4^{\frac{1}{3}\times\left(2x+\frac{1}{2}\right)} = \frac{1}{2^5}$$

$$\Rightarrow \left(2^2\right)^{\frac{1}{3}\times\frac{4x+1}{2}} = \frac{1}{2^5}$$

$$\Rightarrow 2^{2\times\frac{1}{3}\times\frac{4x+1}{2}} = 2^{-5}$$

$$\Rightarrow 2^{\frac{4x+1}{3}} = 2^{-5}$$

$$\Rightarrow \frac{4x+1}{3} = -5$$

$$\Rightarrow 4x + 1 = -15$$

$$\Rightarrow 4x = -16$$

$$\Rightarrow x = -4$$

Question 43

Find the value of x if:

$5^{2x+3} = 1$

Solution 43

$5^{2x+3} = 1$

$\Rightarrow 5^{2x} \times 5^3 = 1$

$\Rightarrow 5^{2x} \times 125 = 1$

$\Rightarrow 5^{2x} = \frac{1}{125}$

$\Rightarrow 5^{2x} = \frac{1}{5^3}$

$\Rightarrow 5^{2x} = 5^{-3}$

$\Rightarrow 2x = -3$

$\Rightarrow x = -\frac{3}{2}$

Question 44

Find the value of x if:

$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

Solution 44

$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

$\Rightarrow 13^{\sqrt{x}} = 256 - 81 - 6$

$\Rightarrow 13^{\sqrt{x}} = 169$

$\Rightarrow 13^{\sqrt{x}} = 13^2$

$\Rightarrow \sqrt{x} = 2$

$\Rightarrow x = 4$

Question 45

Find the value of x if:

$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$

Solution 45

$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{1}{2}\times(x+1)} = \frac{5^3}{3^3}$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{5}{3}\right)^3$

$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{3}{5}\right)^{-3}$

$\Rightarrow \frac{x+1}{2} = -3$

$\Rightarrow x + 1 = -6$

$\Rightarrow x = -7$

Question 46

If $x = 2^{1/3} + 2^{2/3}$, show that $x^3 - 6x = 6$.

Solution 46

$$x = 2^{1/3} + 2^{2/3}$$

$$\Rightarrow x^3 = \left(2^{1/3} + 2^{2/3}\right)^3$$

$$\Rightarrow x^3 = \left(2^{1/3}\right)^3 + \left(2^{2/3}\right)^3 + 3 \times 2^{1/3} \times 2^{2/3}\left(2^{1/3} + 2^{2/3}\right)$$

$$\Rightarrow x^3 = 2 + 2^2 + 3 \times 2\left(2^{1/3} + 2^{2/3}\right)$$

$$\Rightarrow x^3 = 6 + 6\left(2^{1/3} + 2^{2/3}\right)$$

$$\Rightarrow x^3 = 6 + 6x$$

$$\Rightarrow x^3 - 6x = 6$$

Question 47

Determine $(8x)^x$, if $9^{x+2} = 240 + 9^x$.

Solution 47

$$9^{x+2} = 240 + 9^x$$

$$\Rightarrow 9^x \times 9^2 = 240 + 9^x$$

$$\Rightarrow 9^x \times 81 = 240 + 9^x$$

$$\Rightarrow 9^x \times 81 - 9^x = 240$$

$$\Rightarrow 9^x (81 - 1) = 240$$

$$\Rightarrow 9^x \times 80 = 240$$

$$\Rightarrow 9^x = 3$$

$$\Rightarrow \left(3^2\right)^x = 3$$

$$\Rightarrow 3^{2x} = 3$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$\therefore\ (8x)^x = \left(8 \times \frac{1}{2}\right)^{\frac{1}{2}} = (4)^{\frac{1}{2}} = 2^{2 \times \frac{1}{2}} = 2$$

Question 48

If $3^{x+1} = 9^{x-2}$, find the value of $2^{1+x}$.

Solution 48

$3^{x+1} = 9^{x-2}$

$\Rightarrow 3^x \times 3 = 9^x \times 9^{-2}$

$\Rightarrow 3^x \times 3 = (3^2)^x \times \frac{1}{9^2}$

$\Rightarrow 3^x \times 3 = 3^{2x} \times \frac{1}{(3^2)^2}$

$\Rightarrow 3^x \times 3 = 3^{2x} \times \frac{1}{3^4}$

$\Rightarrow \frac{3^{2x}}{3^x} = 3 \times 3^4$

$\Rightarrow 3^{2x-x} = 3^5$

$\Rightarrow 3^x = 3^5$

$\Rightarrow x = 5$

$\therefore\ 2^{1+x} = 2^{1+5} = 2^6 = 64$

Question 49

If $3^{4x} = (81)^{-1}$ and $10^{1/y} = 0.0001$, find the value of $2^{-x+4y}$.

Solution 49

$3^{4x} = (81)^{-1}$

$\Rightarrow 3^{4x} = \frac{1}{81}$

$\Rightarrow 3^{4x} = \frac{1}{3^4}$

$\Rightarrow 3^{4x} = 3^{-4}$

$\Rightarrow 4x = -4$

$\Rightarrow x = -1$

And, $10^{1/y} = 0.0001$

$\Rightarrow 10^{1/y} = \frac{1}{10000}$

$\Rightarrow 10^{1/y} = \frac{1}{10^4}$

$\Rightarrow 10^{1/y} = 10^{-4}$

$\Rightarrow \frac{1}{y} = -4$

$\Rightarrow y = -\frac{1}{4}$

$\therefore\ 2^{-x+4y} = 2^{-(-1)+4\times\left(-\frac{1}{4}\right)} = 2^{1-1} = 2^0 = 1$

Question 50

If $5^{3x} = 125$ and $10^y = 0.001$ find x and y.

Solution 50

$5^{3x} = 125$

$\Rightarrow 5^{3x} = 5^3$

$\Rightarrow 3x = 3$

$\Rightarrow x = 1$

And, $10^y = 0.001$

$\Rightarrow 10^y = \frac{1}{1000}$

$\Rightarrow 10^y = \frac{1}{10^3}$

$\Rightarrow 10^y = 10^{-3}$

$\Rightarrow y = -3$

Hence, $x = 1$ and $y = -3$

Question 51

Solve the equation:

$3^{x+1} = 27 \times 3^4$

Solution 51

$3^{x+1} = 27 \times 3^4$

$\Rightarrow 3^{x+1} = 3^3 \times 3^4$

$\Rightarrow 3^{x+1} = 3^7$

$\Rightarrow x + 1 = 7$

$\Rightarrow x = 6$

Question 52

Solve the equation:

$4^{2x} = \left(\sqrt[3]{16}\right)^{-6/y} = \left(\sqrt{8}\right)^2$

Solution 52

$$4^{2x} = \left(\sqrt[3]{16}\right)^{-6/y} = \left(\sqrt{8}\right)^2$$

Consider, $4^{2x} = \left(\sqrt{8}\right)^2$

$\Rightarrow \left(2^2\right)^{2x} = \left(\sqrt{2^3}\right)^2$

$\Rightarrow 2^{4x} = \left(2^{3\times\frac{1}{2}}\right)^2$

$\Rightarrow 2^{4x} = 2^{3\times\frac{1}{2}\times 2}$

$\Rightarrow 2^{4x} = 2^3$

$\Rightarrow 4x = 3$

$\Rightarrow x = \dfrac{3}{4}$

Now, consider $\left(\sqrt[3]{16}\right)^{-6/y} = \left(\sqrt{8}\right)^2$

$\Rightarrow \left(\sqrt[3]{2^4}\right)^{-6/y} = \left(\sqrt{2^3}\right)^2$

$\Rightarrow \left(2^{4\times\frac{1}{3}}\right)^{-6/y} = \left(2^{3\times\frac{1}{2}}\right)^2$

$\Rightarrow 2^{4\times\frac{1}{3}\times\left(-\frac{6}{y}\right)} = 2^{3\times\frac{1}{2}\times 2}$

$\Rightarrow 2^{-\frac{8}{y}} = 2^3$

$\Rightarrow -\dfrac{8}{y} = 3$

$\Rightarrow y = -\dfrac{8}{3}$

Hence, $x = \dfrac{3}{4}$ and $y = -\dfrac{8}{3}$

Question 53

Solve the equation

$3^{x-1} \times 5^{2y-3} = 225$

Solution 53

$3^{x-1} \times 5^{2y-3} = 225$

$\Rightarrow 3^{x-1} \times 5^{2y-3} = 9 \times 25$

$\Rightarrow 3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$

$\Rightarrow x - 1 = 2$ and $2y - 3 = 2$

$\Rightarrow x = 3$ and $y = \dfrac{5}{2}$

Question 54

Solve the equation:

$8^{x+1} = 16^{y+2}$ and, $\left(\dfrac{1}{2}\right)^{3+x} = \left(\dfrac{1}{4}\right)^{3y}$

Solution 54

$$8^{x+1} = 16^{y+2}$$
$$\Rightarrow (2^3)^{x+1} = (2^4)^{y+2}$$
$$\Rightarrow 2^{3x+3} = 2^{4y+8}$$
$$\Rightarrow 3x + 3 = 4y + 8$$
$$\Rightarrow 3x - 4y = 5 \quad ....(i)$$

And, $\left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{4}\right)^{3y}$
$$\Rightarrow (2^{-1})^{3+x} = \left(\frac{1}{2^2}\right)^{3y}$$
$$\Rightarrow 2^{-3-x} = (2^{-2})^{3y}$$
$$\Rightarrow 2^{-3-x} = 2^{-6y}$$
$$\Rightarrow -3 - x = -6y$$
$$\Rightarrow x - 6y = -3 \quad ....(ii)$$
Multiplying eqn (ii) by 3, we get
$$3x - 18y = -9 \quad ....(iii)$$
Subtracting eqn (iii) from (i), we get
$$14y = 14 \Rightarrow y = 1$$
$$\Rightarrow 3x - 4(1) = 5 \quad ....[\text{From (i)}]$$
$$\Rightarrow 3x = 9 \Rightarrow x = 3$$
Hence, $x = 3$ and $y = 1$

Question 55

Solve the equation:

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

Solution 55

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$
$$\Rightarrow (2^2)^{x-1} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^x$$
$$\Rightarrow 2^{2x-2} \times 2^{-3+2x} = 2^{-3x}$$
$$\Rightarrow 2^{2x-2-3+2x} = 2^{-3x}$$
$$\Rightarrow 2^{4x-5} = 2^{-3x}$$
$$\Rightarrow 4x - 5 = -3x$$
$$\Rightarrow 7x = 5$$
$$\Rightarrow x = \frac{5}{7}$$

Question 56

Solve the equation:

$$\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}$$, where a, b are distinct positive primes.

Solution 56

$$\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{b}{a}\right)^{1-2x}$$

$$\Rightarrow \left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-1+2x}$$

$$\Rightarrow \frac{1}{2} = -1 + 2x$$

$$\Rightarrow 2x = \frac{1}{2} + 1$$

$$\Rightarrow 2x = \frac{3}{2}$$

$$\Rightarrow x = \frac{3}{4}$$

Question 57

If a and b are distinct positive primes such that $\sqrt[3]{a^6 b^{-4}} = a^x b^{2y}$, find x and y.

Solution 57

$$\sqrt[3]{a^6 b^{-4}} = a^x b^{2y}$$

$$\Rightarrow \left(a^6 b^{-4}\right)^{\frac{1}{3}} = a^x b^{2y}$$

$$\Rightarrow a^{6\times\frac{1}{3}} \times b^{-4\times\frac{1}{3}} = a^x \times b^{2y}$$

$$\Rightarrow a^2 \times b^{-\frac{4}{3}} = a^x \times b^{2y}$$

$$\Rightarrow 2 = x \text{ and } -\frac{4}{3} = 2y$$

$$\Rightarrow x = 2 \text{ and } y = -\frac{2}{3}$$

Question 58

If a and b are different positive primes such that

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right) = a^x b^y$$, find x and y.

Solution 58

$$\left(\frac{a^{-1}b^{2}}{a^{2}b^{-4}}\right)^{7} \div \left(\frac{a^{3}b^{-5}}{a^{-2}b^{3}}\right) = a^{x}b^{y}$$

$$\Rightarrow \left(a^{-1-2} \times b^{2+4}\right)^{7} \div \left(a^{3+2} \times b^{-5-3}\right) = a^{x}b^{y}$$

$$\Rightarrow \left(a^{-3} \times b^{6}\right)^{7} \div \left(a^{5} \times b^{-8}\right) = a^{x}b^{y}$$

$$\Rightarrow \left(a^{-21} \times b^{42}\right) \div \left(a^{5} \times b^{-8}\right) = a^{x}b^{y}$$

$$\Rightarrow \frac{a^{-21} \times b^{42}}{a^{5} \times b^{-8}} = a^{x}b^{y}$$

$$\Rightarrow a^{-21-5} \times b^{42+8} = a^{x} \times b^{y}$$

$$\Rightarrow a^{-26} \times b^{50} = a^{x} \times b^{y}$$

$$\Rightarrow x = -26 \text{ and } y = 50$$

Question 59

If a and b are different positive primes such that $(a + b)^{-1}(a^{-1} + b^{-1}) = a^{x}b^{y}$, find x + y + 2.

Solution 59

$$(a+b)^{-1}(a^{-1} + b^{-1}) = a^{x}b^{y}$$

$$\Rightarrow \frac{1}{a+b} \times \left(\frac{1}{a} + \frac{1}{b}\right) = a^{x}b^{y}$$

$$\Rightarrow \frac{1}{a+b} \times \frac{a+b}{ab} = a^{x}b^{y}$$

$$\Rightarrow \frac{1}{ab} = a^{x}b^{y}$$

$$\Rightarrow (ab)^{-1} = a^{x}b^{y}$$

$$\Rightarrow a^{-1} \times b^{-1} = a^{x} \times b^{y}$$

$$\Rightarrow x = -1 \text{ and } y = -1$$

$$\therefore\ x + y + 2 = -1 - 1 + 2 = 0$$

Question 60

If $2^{x} \times 3^{y} \times 5^{z} = 2160$, find x, y and z. Hence, compute the value of $3^{x} \times 2^{-y} \times 5^{-z}$.

Solution 60

$$2^{x} \times 3^{y} \times 5^{z} = 2160$$

By prime factorisation, we have

$$2160 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^{4} \times 3^{3} \times 5$$

$$\Rightarrow 2^{x} \times 3^{y} \times 5^{z} = 2^{4} \times 3^{3} \times 5$$

$$\Rightarrow x = 4,\ y = 3 \text{ and } z = 1$$

$$\begin{aligned}\therefore\ 3^{x} \times 2^{-y} \times 5^{-z} &= 3^{x} \times \frac{1}{2^{y}} \times \frac{1}{5^{z}} \\ &= 3^{4} \times \frac{1}{2^{3}} \times \frac{1}{5} \\ &= 81 \times \frac{1}{8} \times \frac{1}{5} \\ &= \frac{81}{40}\end{aligned}$$

Question 61

If 1176 = $2^a \times 3^b \times 7^c$, find the values of a, b and c. hence, compute the value of $2^a \times 3^b \times 7^{-c}$ as a fraction.

Solution 61

$1176 = 2^a \times 3^b \times 7^c$

By prime factorisation, we have

$1176 = 2 \times 2 \times 2 \times 3 \times 7 \times 7 = 2^3 \times 3 \times 7^2$

$\Rightarrow 2^3 \times 3 \times 7^2 = 2^a \times 3^b \times 7^c$

$\Rightarrow a = 3,\ b = 1$ and $c = 2$

$$\therefore\ 2^a \times 3^b \times 7^{-c} = 2^3 \times 3^1 \times 7^{-2}$$

$$= 8 \times 3 \times \frac{1}{7^2}$$

$$= 24 \times \frac{1}{49}$$

$$= \frac{24}{49}$$

Question 62

Simplify :

$$\left(\frac{x^{a+b}}{x^c}\right)^{a-b}\left(\frac{x^{b+c}}{x^a}\right)^{b-c}\left(\frac{x^{c+a}}{x^b}\right)^{c-a}$$

Solution 62

$$\left(\frac{x^{a+b}}{x^c}\right)^{a-b}\left(\frac{x^{b+c}}{x^a}\right)^{b-c}\left(\frac{x^{c+a}}{x^b}\right)^{c-a}$$

$$= \left(x^{a+b-c}\right)^{a-b}\left(x^{b+c-a}\right)^{b-c}\left(x^{c+a-b}\right)^{c-a}$$

$$= x^{(a-b)(a+b-c)} \times x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)}$$

$$= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-bc-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab}$$

$$= x^{a^2-ac-b^2+bc} \times x^{b^2-ab-c^2+ac} \times x^{c^2-bc-a^2+ab}$$

$$= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab}$$

$$= x^0$$

$$= 1$$

Question 63

Simplify:

$$\sqrt[lm]{\frac{x^l}{x^m}} \times \sqrt[mn]{\frac{x^m}{x^n}} \times \sqrt[nl]{\frac{x^n}{x^l}}$$

Solution 63

$$\sqrt[lm]{\frac{x^l}{x^m}} \times \sqrt[mn]{\frac{x^m}{x^n}} \times \sqrt[nl]{\frac{x^n}{x^l}}$$

$$= \left(\frac{x^l}{x^m}\right)^{\frac{1}{lm}} \times \left(\frac{x^m}{x^n}\right)^{\frac{1}{mn}} \times \left(\frac{x^n}{x^l}\right)^{\frac{1}{nl}}$$

$$= \left(x^{l-m}\right)^{\frac{1}{lm}} \times \left(x^{m-n}\right)^{\frac{1}{mn}} \times \left(x^{n-l}\right)^{\frac{1}{nl}}$$

$$= x^{\frac{l-m}{mn}} \times x^{\frac{m-n}{mn}} \times x^{\frac{n-l}{nl}}$$

$$= x^{\frac{l-m}{mn}+\frac{m-n}{mn}+\frac{n-l}{nl}}$$

$$= x^{\frac{n(l-m)+l(m-n)+m(n-l)}{lmn}}$$

$$= x^{\frac{nl-mn+lm-nl+mn-lm}{lmn}}$$

$$= x^0$$

$$= 1$$

Question 64

Show that:

$$\frac{\left(a+\frac{1}{b}\right)^m \times \left(a-\frac{1}{b}\right)^n}{\left(b+\frac{1}{a}\right)^m \times \left(b-\frac{1}{a}\right)^n} = \left(\frac{a}{b}\right)^{m+n}$$

Solution 64

$$\text{L.H.S.} = \frac{\left(a+\frac{1}{b}\right)^m \times \left(a-\frac{1}{b}\right)^n}{\left(b+\frac{1}{a}\right)^m \times \left(b-\frac{1}{a}\right)^n}$$

$$= \frac{\left(\frac{ab+1}{b}\right)^m \times \left(\frac{ab-1}{b}\right)^n}{\left(\frac{ab+1}{a}\right)^m \times \left(\frac{ab-1}{a}\right)^n}$$

$$= \frac{\frac{(ab+1)^m}{b^m} \times \frac{(ab-1)^n}{b^n}}{\frac{(ab+1)^m}{a^m} \times \frac{(ab-1)^n}{a^n}}$$

$$= \frac{(ab+1)^m}{b^m} \times \frac{(ab-1)^n}{b^n} \times \frac{a^m}{(ab+1)^m} \times \frac{a^n}{(ab-1)^n}$$

$$= \frac{a^{m+n}}{b^{m+n}}$$

$$= \left(\frac{a}{b}\right)^{m+n}$$

$= \text{R.H.S.}$

Question 65

If $a = x^{m+n}y^{l}$, $b = x^{n+l}y^{m}$ and $c = x^{l+m}y^{n}$, prove that $a^{m-n}b^{n-l}c^{l-m} = 1$.

Solution 65

$a = x^{m+n}y^{l}$, $b = x^{n+l}y^{m}$ and $c = x^{l+m}y^{n}$

Now,

$$\begin{aligned}
\text{L.H.S.} &= a^{m-n}\,b^{n-l}\,c^{l-m} \\
&= \left(x^{m+n}y^{l}\right)^{m-n} \times \left(x^{n+l}y^{m}\right)^{n-l} \times \left(x^{l+m}y^{n}\right)^{l-m} \\
&= x^{(m+n)(m-n)} \times y^{l(m-n)} \times x^{(n+l)(n-l)} \times y^{m(n-l)} \times x^{(l+m)(l-m)} \times y^{n(l-m)} \\
&= x^{m^2-n^2} \times y^{lm-nl} \times x^{n^2-l^2} \times y^{mn-lm} \times x^{l^2-m^2} \times y^{nl-mn} \\
&= x^{m^2-n^2+n^2-l^2+l^2-m^2} \times y^{lm-nl+mn-lm+nl-mn} \\
&= x^{0} \times y^{0} \\
&= 1 \times 1 \\
&= 1 \\
&= \text{R.H.S.}
\end{aligned}$$

Question 66

If $x = a^{m+n}$, $y = a^{n+l}$ and $z = a^{l+m}$, prove that $x^{m}y^{n}z^{l} = z^{n}y^{l}z^{m}$.

Solution 66

$x = a^{m+n}$, $y = a^{n+l}$ and $z = a^{l+m}$

$$\begin{aligned}
\text{L.H.S.} &= x^{m}y^{n}z^{l} \\
&= \left(a^{m+n}\right)^{m} \times \left(a^{n+l}\right)^{n} \times \left(a^{l+m}\right)^{l} \\
&= a^{m^2+mn} \times a^{n^2+nl} \times a^{l^2+lm} \\
&= a^{m^2+mn+n^2+nl+l^2+lm} \\
&= a^{mn+n^2} \times a^{nl+l^2} \times a^{lm+m^2} \\
&= a^{n(m+n)} \times a^{l(n+l)} \times a^{m(l+m)} \\
&= \left(a^{(m+n)}\right)^{n} \times \left(a^{(n+l)}\right)^{l} \times \left(a^{(l+m)}\right)^{m} \\
&= x^{n}y^{l}z^{m}
\end{aligned}$$