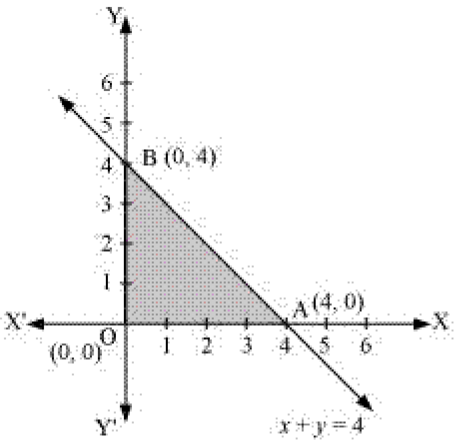
NCERT Solutions for Class 12-science Maths Chapter 12 - Linear Programming

Chapter 12 - Linear Programming Exercise Ex. 12.1

Solution 1

The feasible region determined by the constraints, $x + y \le 4$, $x \ge 0$, $y \ge 0$, is as follows.



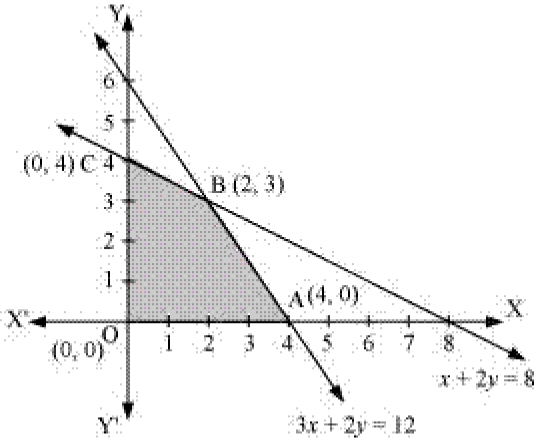
The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

Corner point	**$Z = 3x + 4y$**	
O(0, 0)	0	
A(4, 0)	12	
B(0, 4)	16	→ Maximum

Therefore, the maximum value of Z is 16 at the point B (0, 4).

Solution 2

The feasible region determined by the system of constraints, $x+2y\le 8$, $3x+2y\le 12$, $x \ge 0$, and $y \ge 0$, is as follows.



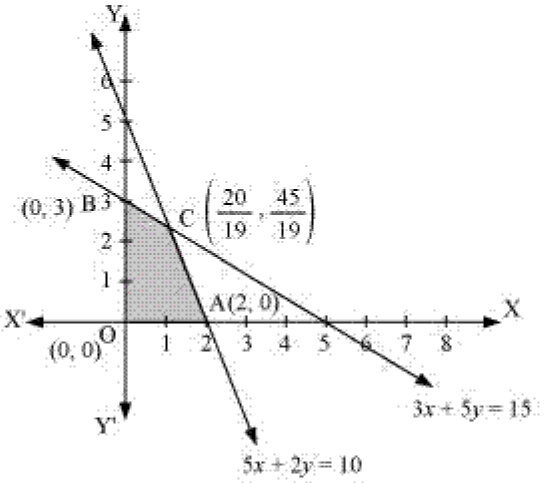
The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4). The values of Z at these corner points are as follows.

Corner point	**Z = −3*x* + 4*y***	
O(0, 0)	0	
A(4, 0)	−12	→ Minimum
B(2, 3)	6	
C(0, 4)	16	

Therefore, the minimum value of Z is −12 at the point (4, 0).

Solution 3

The feasible region determined by the system of constraints, $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, and $y \geq 0$, are as follows.



The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $C\left(\frac{20}{19}, \frac{45}{19}\right)$.

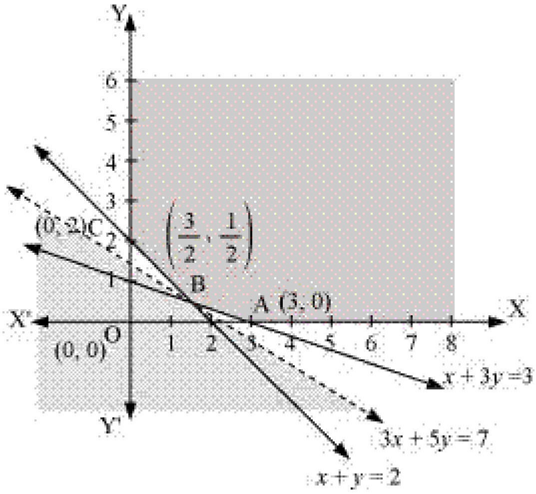
The values of Z at these corner points are as follows.

Corner point	Z = 5x + 3y	
O(0, 0)	0	
A(2, 0)	10	
B(0, 3)	9	
$C\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$	→ Maximum

Therefore, the maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

Solution 4

The feasible region determined by the system of constraints, $x+3y \geq 3, x+y \geq 2$, and x, $y \geq 0$, is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2}, \frac{1}{2}\right)$, and C (0, 2).

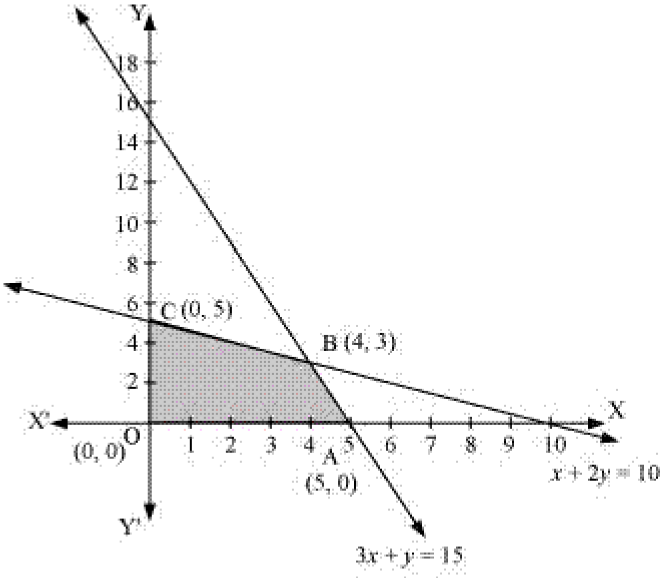
The values of Z at these corner points are as follows.

Corner point	$Z = 3x + 5y$	
A(3, 0)	9	
$B\left(\frac{3}{2}, \frac{1}{2}\right)$	7	→ Smallest
C(0, 2)	10	

Therefore, the minimum value of Z is 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.

Solution 5

The feasible region determined by the constraints, $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5).

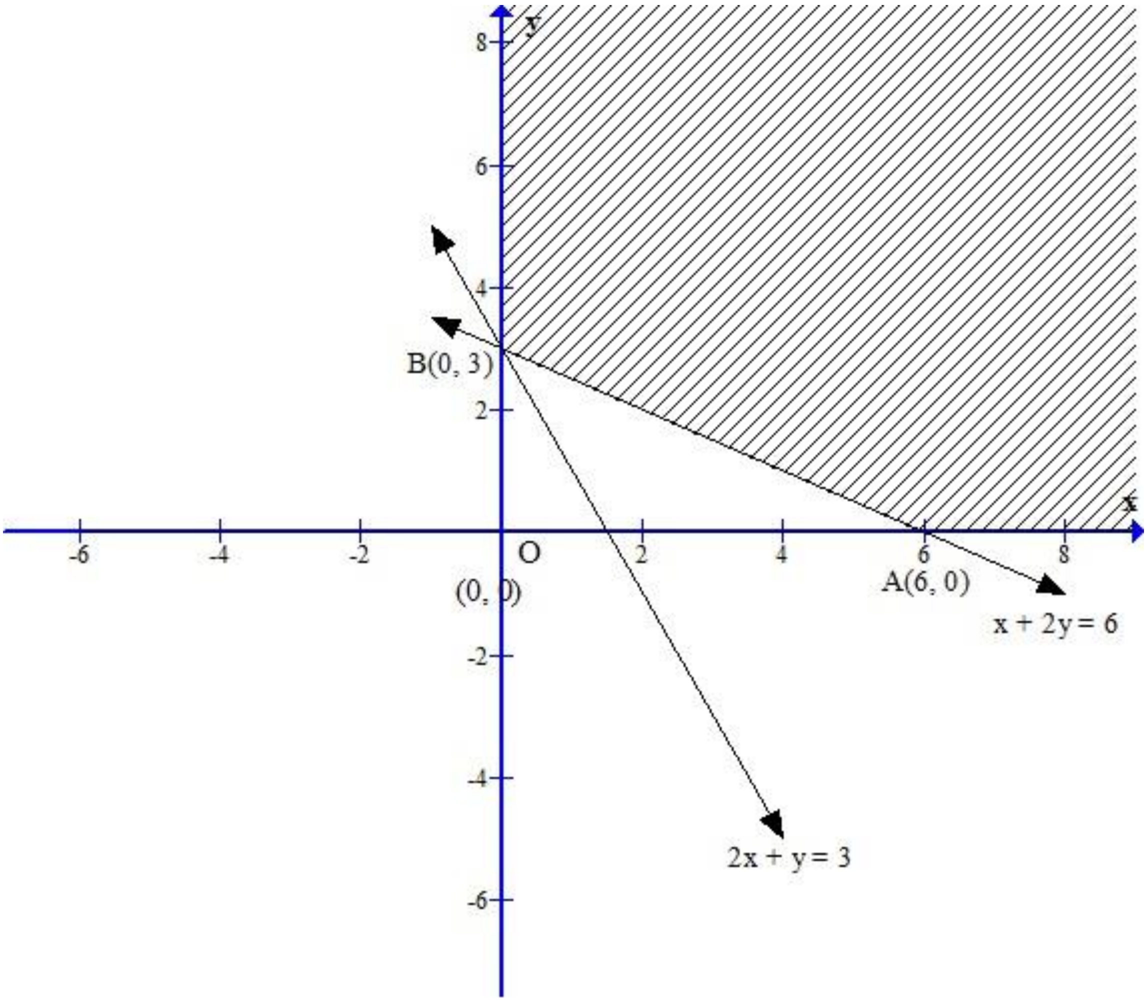
The values of Z at these corner points are as follows.

Corner point	**Z = 3*x* + 2*y***	
A(5, 0)	15	
B(4, 3)	18	→ Maximum
C(0, 5)	10	

Therefore, the maximum value of Z is 18 at the point (4, 3).

Solution 6

The feasible region determined by the constraints, $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$ and $y \geq 0$, is as follos.



The corner points of the feasible region are A (6, 0) and B(0, 3).

The values of Z at these corner points are as follows.

Corner point	Z = x + 2y
A(6, 0)	6
B(0, 3)	6

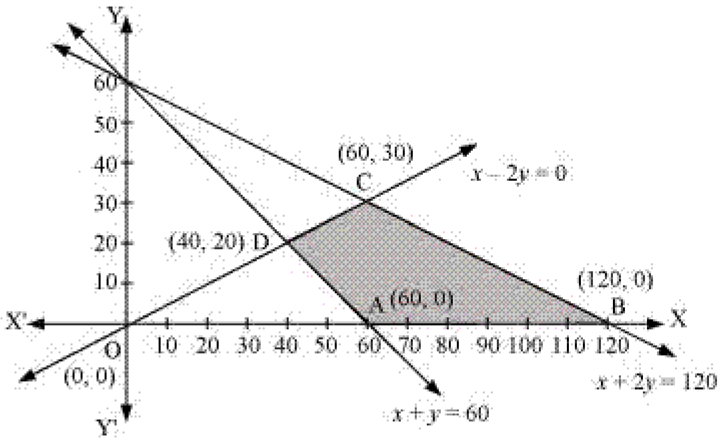
It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line $x + 2y = 6$, then Z = 6

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line, $x + 2y = 6$

Solution 7

The feasible region determined by the constraints, $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20).

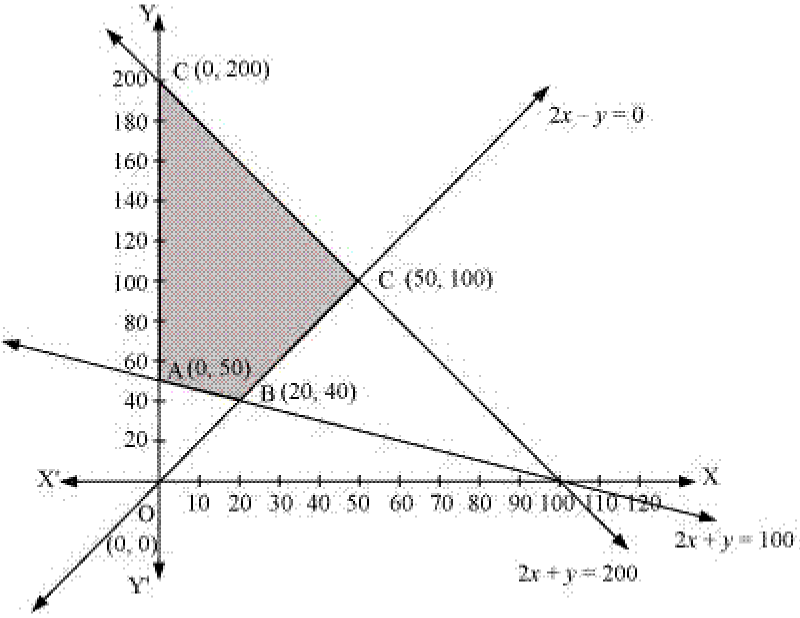
The values of Z at these corner points are as follows.

Corner point	$Z = 5x + 10y$	
A(60, 0)	300	→ Minimum
B(120, 0)	600	→ Maximum
C(60, 30)	600	→ Maximum
D(40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

Solution 8

The feasible region determined by the constraints, $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100), and D(0, 200).

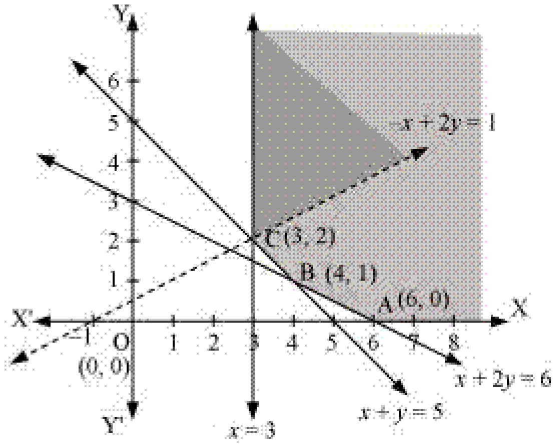
The values of Z at these corner points are as follows.

Corner point	$Z = x + 2y$	
A(0, 50)	100	→ Minimum
B(20, 40)	100	→ Minimum
C(50, 100)	250	
D(0, 200)	400	→ Maximum

The maximum value of Z is 400 at (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

Solution 9

The feasible region determined by the constraints,
$x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,$ and $y \geq 0,$ is as follows.



It can be seen that the feasible region is unbounded.

The values of Z at corner points A (6, 0), B (4, 1), and C (3, 2) are as follows.

Corner point	**Z = −*x* + 2*y***
A(6, 0)	Z = − 6
B(4, 1)	Z = − 2
C(3, 2)	Z = 1

As the feasible region is unbounded, therefore, Z = 1 may or may not be the maximum value.

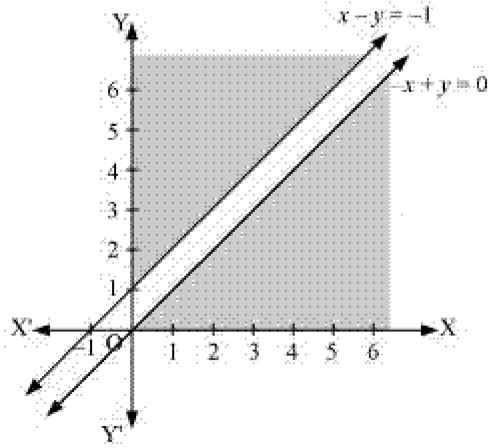
For this, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half plane has points in common with the feasible region or not.

The resulting feasible region has points in common with the feasible region.

Therefore, Z = 1 is not the maximum value. Z has no maximum value.

Solution 10

The region determined by the constraints, $x - y \le -1, -x + y \le 0, x, y \ge 0,$ is as follows.



There is no feasible region and thus, Z has no maximum value.

Chapter 12 - Linear Programming Exercise Ex. 12.2

Solution 1

Let the mixture contain x kg of food P and y kg of food Q. Therefore,

$x \ge 0$ and $y \ge 0$

The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B. Therefore, the constraints are

$3x + 4y \ge 8$

$5x + 2y \ge 11$

Total cost, Z, of purchasing food is, $Z = 60x + 80y$

The mathematical formulation of the given problem is

Minimise $Z = 60x + 80y$ ... (1)

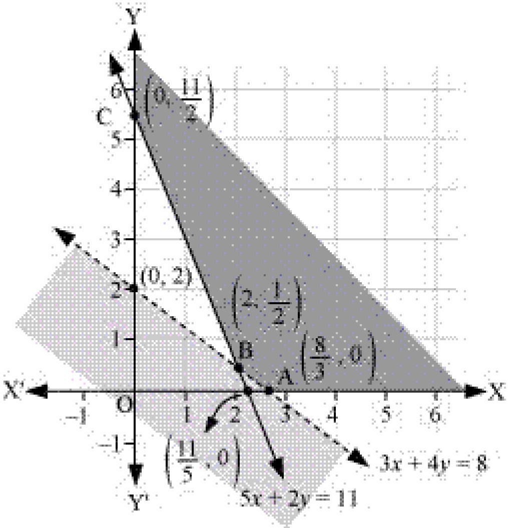
subject to the constraints,

$3x + 4y \ge 8$ ... (2)

$5x + 2y \ge 11$ ... (3)

$x, y \ge 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3},0\right)$, $B\left(2,\frac{1}{2}\right)$, and $C\left(0,\frac{11}{2}\right)$.

The values of Z at these corner points are as follows.

Corner point	$Z = 60x + 80y$	
$A\left(\frac{8}{3},0\right)$	160	$\Big\}\rightarrow$ Minimum
$B\left(2,\frac{1}{2}\right)$	160	
$C\left(0,\frac{11}{2}\right)$	440	

As the feasible region is unbounded, therefore, 160 may or may not be the minimum value of Z.

For this, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 4y < 8$

Therefore, the minimum cost of the mixture will be Rs 160 at the line segment joining the points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$.

Solution 2

Let there be x cakes of first kind and y cakes of second kind. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Flour (g)	**Fat (g)**
Cakes of first kind, *x*	200	25
Cakes of second kind, *y*	100	50
Availability	5000	1000

$\therefore 200x + 100y \leq 5000$

$\Rightarrow 2x + y \leq 50$

$25x + 50y \leq 1000$

$\Rightarrow x + 2y \leq 40$

Total numbers of cakes, Z, that can be made are, Z = x + y

The mathematical formulation of the given problem is

Maximize Z = x + y ... (1)

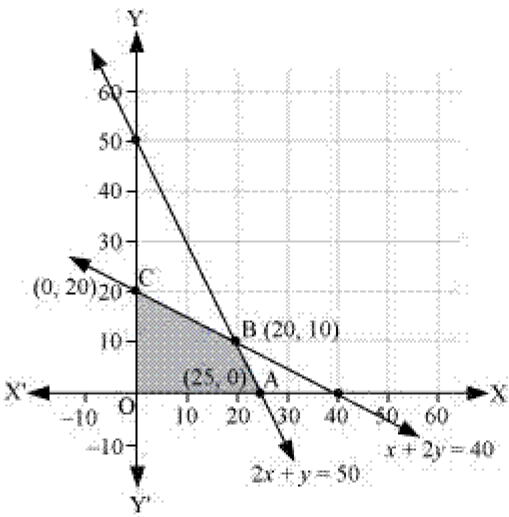
subject to the constraints,

$2x + y \leq 50$ ...(2)

$x + 2y \leq 40$ ...(3)

$x, y \geq 0$ ...(4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

Corner point	**$Z = x + y$**	
A(25, 0)	25	
B(20, 10)	30	→ Maximum
C(0, 20)	20	
O(0, 0)	0	

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

Solution 3

(i) Let the number of rackets and the number of bats to be made be x and y respectively.

The machine time is not available for more than 42 hours.

$\therefore 1.5x + 3y \leq 42$ ...(1)

The craftsman's time is not available for more than 24 hours.

$\therefore 3x + y \leq 24$ ...(2)

The factory is to work at full capacity. Therefore,

$1.5x + 3y = 42$

$3x + y = 24$

On solving these equations, we obtain

$x = 4$ and $y = 12$

Thus, 4 rackets and 12 bats must be made.

(ii) The given information can be complied in a table as follows.

	Tennis Racket	**Cricket Bat**	**Availability**
Machine Time (h)	1.5	3	42
Craftsman's Time (h)	3	1	24

$1.5x + 3y \leq 42$

$3x + y \leq 24$

$x, y \geq 0$

The profit on a racket is Rs 20 and on a bat is Rs 10.

$\therefore Z = 20x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 20x + 10y$ ... (1)

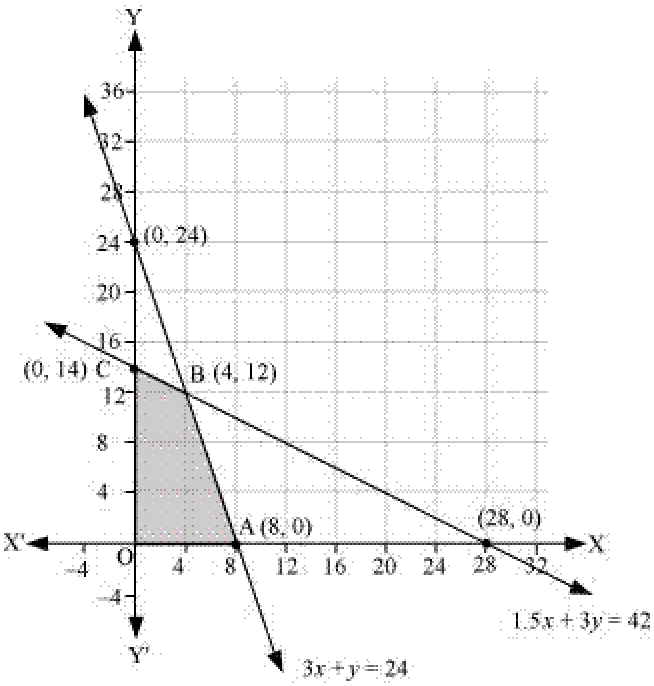
subject to the constraints,

$1.5x + 3y \leq 42$ ... (2)

$3x + y \leq 24$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (8, 0), B (4, 12), C (0, 14), and O (0, 0).

The values of Z at these corner points are as follows.

Corner point	$Z = 20x + 10y$	
A(8, 0)	160	
B(4, 12)	200	→ Maximum
C(0, 14)	140	
O(0, 0)	0	

Thus, the maximum profit of the factory when it works to its full capacity is Rs 200.

Solution 4

Let the manufacturer produce x packages of nuts and y packages of bolts. Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Nuts	**Bolts**	**Availability**
Machine A (h)	1	3	12
Machine B (h)	3	1	12

The profit on a package of nuts is Rs 17.50 and on a package of bolts is Rs 7. Therefore, the constraints are

$x + 3y \leq 12$

$3x + y \leq 12$

Total profit, Z = $17.5x + 7y$

The mathematical formulation of the given problem is

Maximise Z = $17.5x + 7y$ ... (1)

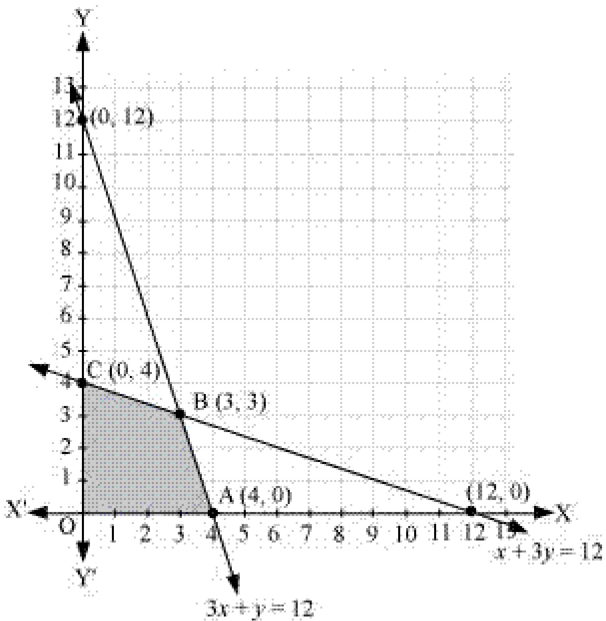
subject to the constraints,

$x + 3y \leq 12$ ... (2)

$3x + y \leq 12$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (4, 0), B (3, 3), and C (0, 4).

The values of Z at these corner points are as follows.

Corner point	Z = 17.5*x* + 7*y*	
O(0, 0)	0	
A(4, 0)	70	
B(3, 3)	73.5	→ Maximum
C(0, 4)	28	

The maximum value of Z is Rs 73.50 at (3, 3).

Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50.

Solution 5

Let the factory manufacture *x* screws of type A and *y* screws of type B on each day. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Screw A	Screw B	Availability
Automatic Machine (min)	4	6	4 × 60 = 240
Hand Operated Machine (min)	6	3	4 × 60 = 240

The profit on a package of screws A is Rs 7 and on the package of screws B is Rs 10. Therefore, the constraints are

$4x+6y \leq 240$

$6x+3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 7x + 10y$ ... (1)

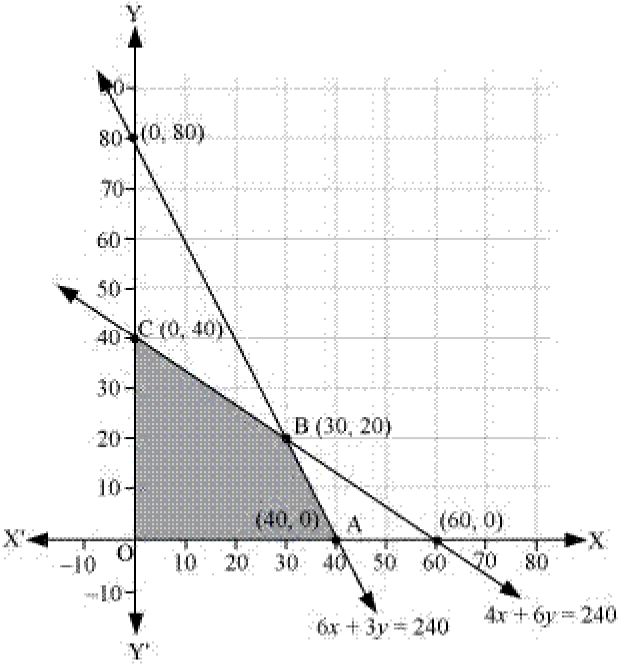
subject to the constraints,

$4x+6y \leq 240$ ... (2)

$6x+3y \leq 240$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is



The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

Corner point	**Z = 7*x* + 10*y***	
A(40, 0)	280	
B(30, 20)	410	→ Maximum
C(0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

Solution 6

Let the cottage industry manufacture x pedestal lamps and y wooden shades. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, $Z = 5x + 3y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 3y$ ... (1)

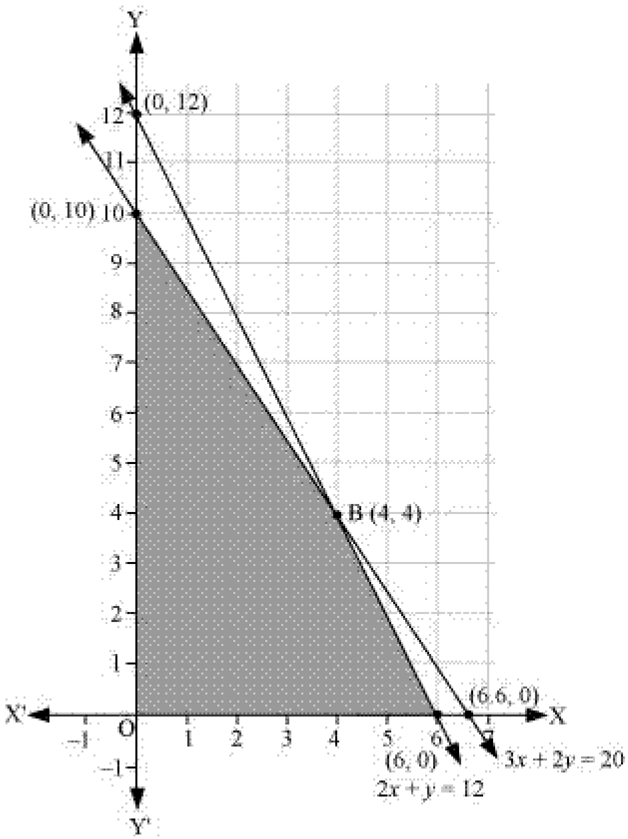
subject to the constraints,

$2x + y \leq 12$ ... (2)

$3x + 2y \leq 20$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	Z = 5*x* + 3*y*	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

Solution 7

Let the company manufacture x souvenirs of type A and y souvenirs of type B. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Type A	Type B	Availability
Cutting (min)	5	8	3 × 60 + 20 =200
Assembling (min)	10	8	4 × 60 = 240

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are

$5x + 8y \leq 200$

$10x + 8y \leq 240$ i.e., $5x + 4y \leq 120$

Total profit, Z = $5x + 6y$

The mathematical formulation of the given problem is

Maximize Z = $5x + 6y$ ... (1)

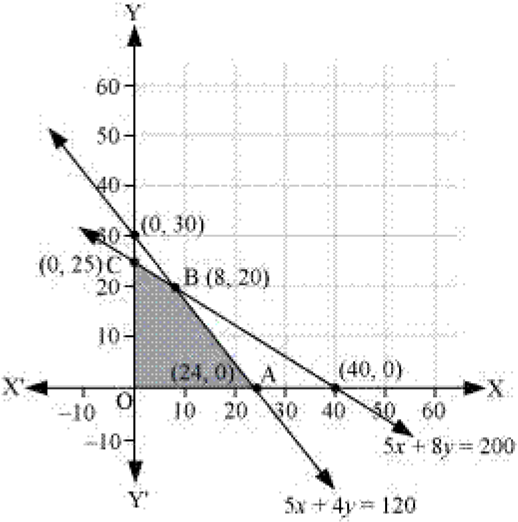
subject to the constraints,

$5x + 8y \leq 200$ ... (2)

$5x + 4y \leq 120$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

Corner point	***Z* = 5*x* + 6*y***	
A(24, 0)	120	
B(8, 20)	160	→ Maximum
C(0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

Solution 8

Let the merchant stock x desktop models and y portable models. Therefore,

$x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs 25000 and of a portable model is Rs 4000. However, the merchant can invest a maximum of Rs 70 lakhs.

$$\therefore 25000x + 40000y \leq 7000000$$

$$5x + 8y \leq 1400$$

The monthly demand of computers will not exceed 250 units.

$$\therefore x + y \leq 250$$

The profit on a desktop model is Rs 4500 and the profit on a portable model is Rs 5000.

Total profit, Z = $4500x + 5000y$

Thus, the mathematical formulation of the given problem is

$$\text{Maximum } Z = 4500x + 5000y \qquad \text{...(1)}$$

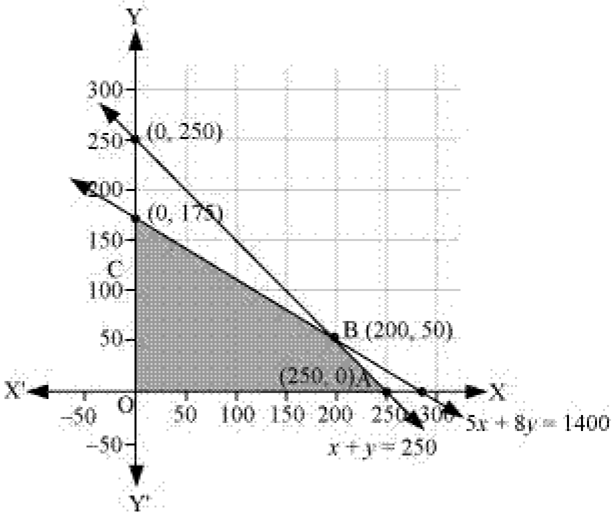
subject to the constraints,

$$5x + 8y \leq 1400 \qquad \text{...(2)}$$

$$x + y \leq 250 \qquad \text{...(3)}$$

$$x,\ y \geq 0 \qquad \text{...(4)}$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

Corner point	$Z = 4500x + 5000y$	
A(250, 0)	1125000	
B(200, 50)	1150000	→ Maximum
C(0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

Solution 9

Let the diet contain x units of food F_1 and y units of food F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Vitamin A (units)	**Mineral (units)**	**Cost per unit (Rs)**
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4 per unit and of Food F_2 is Rs 6 per unit. Therefore, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

Minimise $Z = 4x + 6y$ ... (1)

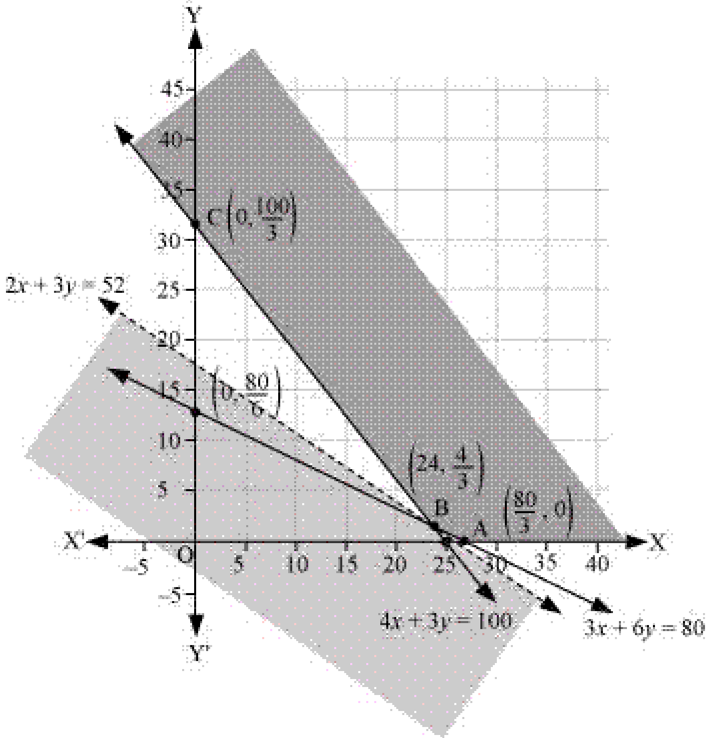
subject to the constraints,

$3x + 6y \geq 80$ ... (2)

$4x + 3y \geq 100$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{80}{3}, 0\right)$, $B\left(24, \frac{4}{3}\right)$, and $C\left(0, \frac{100}{3}\right)$.

The values of Z at these corner points are as follows.

Corner point	**Z = 4*x* + 6*y***	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	→ Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.

Solution 10

Let the farmer buy x kg of fertilizer F_1 and y kg of fertilizer F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs/kg)
F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

$\Rightarrow$ 10% of x + 5% of $y \geq 14$

$$\frac{x}{10}+\frac{y}{20}\geq 14$$
$$2x+y\geq 280$$

F_1 consists of 6% phosphoric acid and F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

$\Rightarrow$ 6% of x + 10% of $y \geq 14$

$$\frac{6x}{100}+\frac{10y}{100}\geq 14$$
$$3x+5y\geq 700$$

Total cost of fertilizers, $Z = 6x + 5y$

The mathematical formulation of the given problem is

Minimize $Z = 6x + 5y$ ... (1)

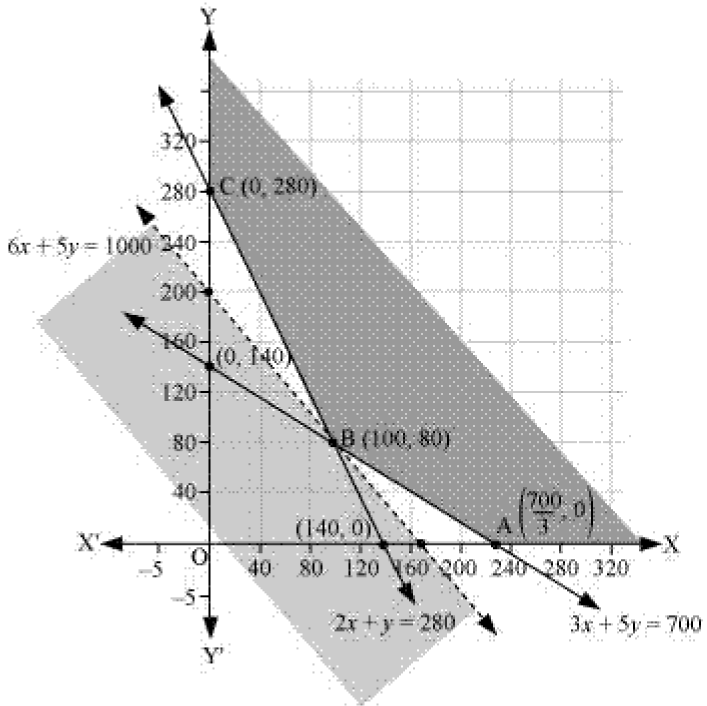
subject to the constraints,

$2x + y \geq 280$ ... (2)

$3x + 5y \geq 700$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3},0\right), B(100,80)$, and $C(0,280)$.

The values of Z at these points are as follows.

Corner point	Z = 6*x* + 5*y*	
$A\left(\frac{700}{3},0\right)$	1400	
B(100, 80)	1000	→ Minimum
C(0, 280)	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with

$6x + 5y < 1000$

Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost. The minimum cost is Rs 1000.

Solution 11

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points, (3, 4) and (0, 5).

Value of Z at (3, 4) = Value of Z at (0, 5)

$p(3) + q(4) = p(0) + q(5)$

$3p + 4q = 5q$

$q = 3p$

Hence, the correct answer is D.

Chapter 12 - Linear Programming Exercise Misc. Ex.

Solution 1

Let the diet contain x and y packets of foods P and Q respectively. Therefore,

$x \geq 0$ and $y \geq 0$

The mathematical formulation of the given problem is as follows.

Maximize $z = 6x + 3y$ ... (1)

subject to the constraints,

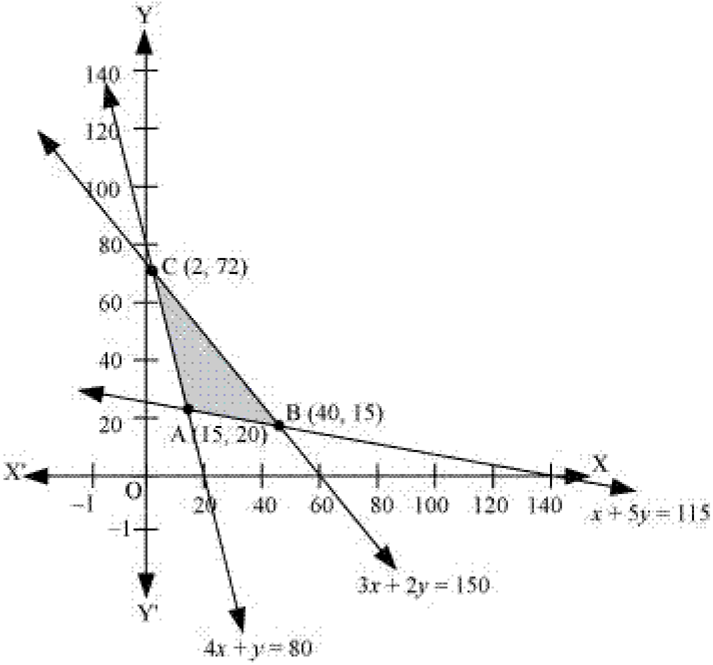
$4x + y \geq 80$ ...(2)

$x + 5y \geq 115$ ...(3)

$3x + 2y \leq 150$ ...(4)

$x, y \geq 0$ ...(5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (15, 20), B (40, 15), and C (2, 72).

The values of z at these corner points are as follows.

Corner point	$z = 6x + 3y$	
A(15, 20)	150	
B(40, 15)	285	→ Maximum
C(2, 72)	228	

Thus, the maximum value of z is 285 at (40, 15).

Therefore, to maximize the amount of vitamin A in the diet, 40 packets of food P and 15 packets of food Q should be used. The maximum amount of vitamin A in the diet is 285 units.

Solution 2

Let the farmer mix x bags of brand P and y bags of brand Q.
The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Vitamin C (units/kg)	Cost (Rs./kg)
Food P	3	2.5	2	250
Food Q	1.5	11.25	3	200
Requirement (units/kg)	18	45	24	

The given problem can be formulated as follows.

Minimize $z = 250x + 200y$ ... (1)

subject to the constraints,

$$3x + 1.5y \geq 18 \quad ...(2)$$

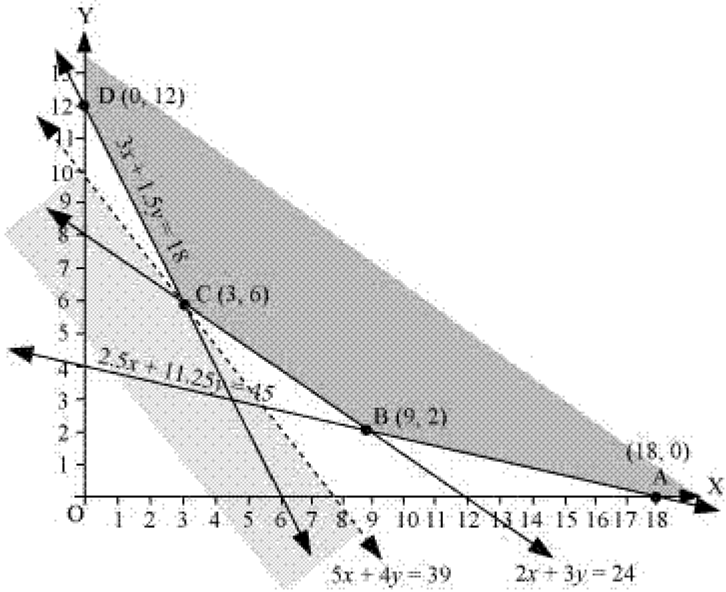
$$2.5x + 11.25y \geq 45 \quad ...(3)$$

$$2x + 3y \geq 24 \quad ...(4)$$

$$x,\ y \geq 0 \quad ...(5)$$

The feasible region determined by the system of constraints is as follows.

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6), and D (0, 12).

The values of z at these corner points are as follows.

Corner point	**$z = 250x + 200y$**	
A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	→ Minimum
D (0, 12)	2400	

As the feasible region is unbounded, therefore, 1950 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 4y < 39$

Therefore, the minimum value of z is 2000 at (3, 6).

Thus, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimize the cost to Rs 1950.

Solution 3

Let the mixture contain x kg of food X and y kg of food Y.

The mathematical formulation of the given problem is as follows.

Minimize $z = 16x + 20y$ ... (1)

subject to the constraints,

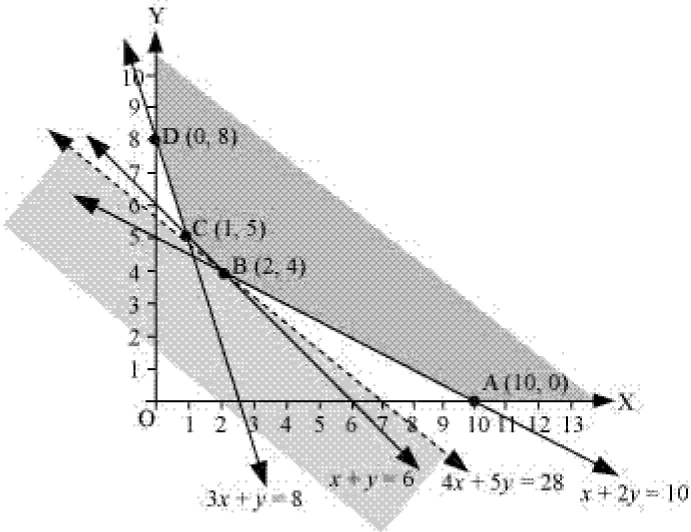
$$x + 2y \geq 10 \qquad \ldots(2)$$

$$x + y \geq 6 \qquad \ldots(3)$$

$$3x + y \geq 8 \qquad \ldots(4)$$

$$x,\ y \geq 0 \qquad \ldots(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5), and D (0, 8).

The values of z at these corner points are as follows.

Corner point	$z = 16x + 20y$	
A (10, 0)	160	
B (2, 4)	112	→ Minimum
C (1, 5)	116	
D (0, 8)	160	

As the feasible region is unbounded, therefore, 112 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, $16x + 20y < 112$ or $4x + 5y < 28$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $4x + 5y < 28$

Therefore, the minimum value of z is 112 at (2, 4).

Thus, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is Rs 112.

Solution 4

Let x and y toys of type A and type B respectively be manufactured in a day.

The given problem can be formulated as follows.

Maximize $z = 7.5x + 5y$ ... (1)

subject to the constraints,

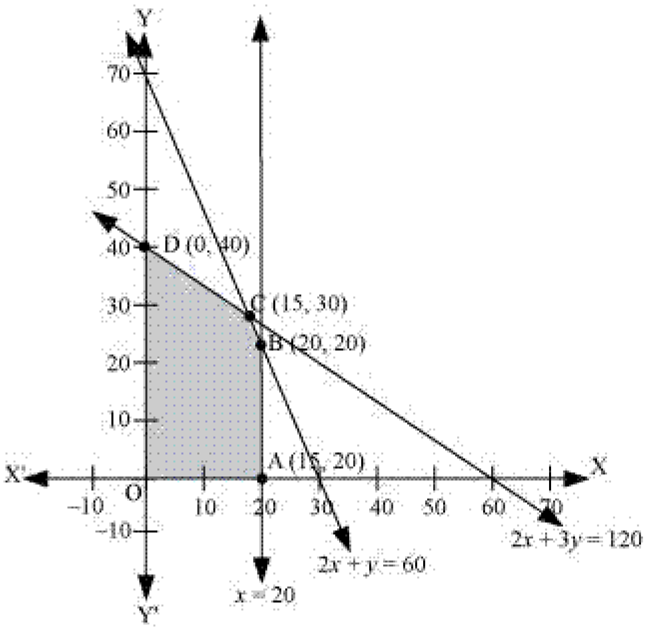
$$2x + y \le 60 \quad ...(2)$$

$$x \le 20 \quad ...(3)$$

$$2x + 3y \le 120 \quad ...(4)$$

$$x, y \ge 0 \quad ...(5)$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 0), B (20, 20), C (15, 30), and D (0, 40).

The values of z at these corner points are as follows.

Corner point	**Z = 7.5x + 5y**	
A(20, 0)	150	
B(20, 20)	250	
C(15, 30)	262.5	→ Maximum
O(0, 40)	200	

The maximum value of z is 262.5 at (15, 30).

Thus, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximize the profit.

Solution 5

Let the airline sell x tickets of executive class and y tickets of economy class.

The mathematical formulation of the given problem is as follows.

Maximize $z = 1000x + 600y$ ... (1)

subject to the constraints,

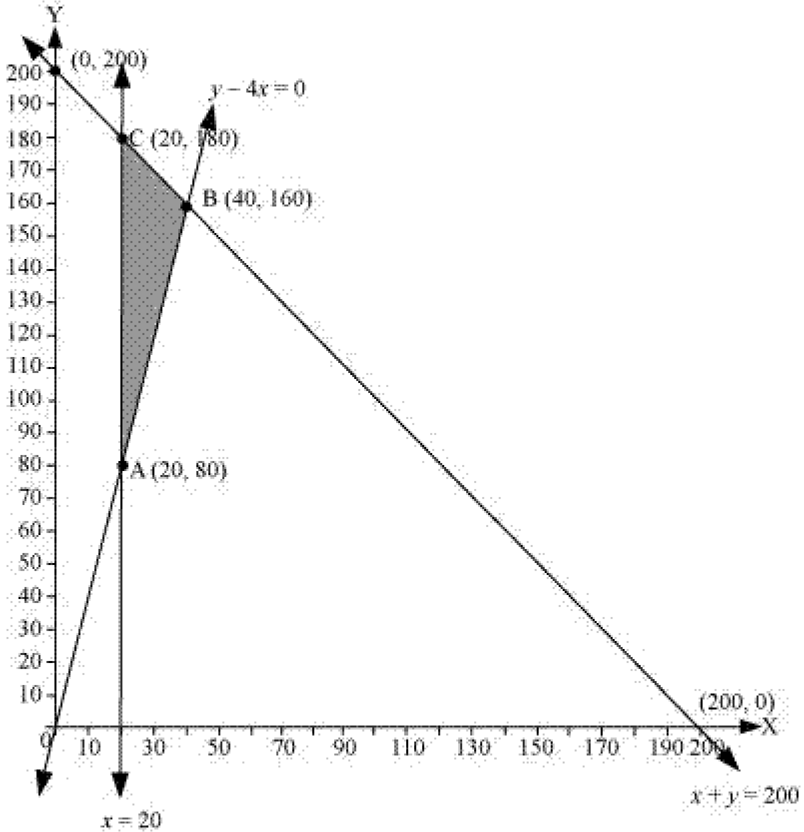
$$x + y \leq 200 \quad ...(2)$$

$$x \geq 20 \quad ...(3)$$

$$y - 4x \geq 0 \quad ...(4)$$

$$x, y \geq 0 \quad ...(5)$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of z at these corner points are as follows.

Corner point	$z = 1000x + 600y$	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of z is 136000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

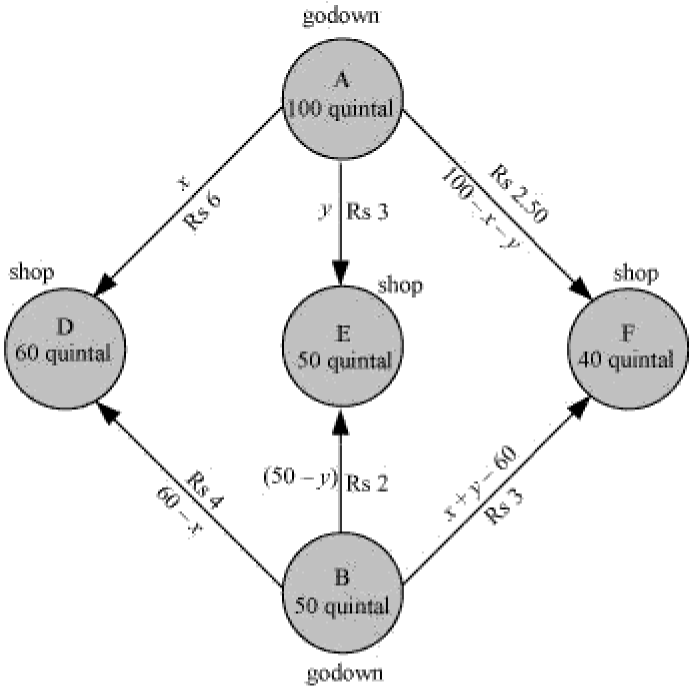
Solution 6

Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, $(100 - x - y)$ will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining $(60 - x)$ quintals will be transported from godown B.

Similarly, $(50 - y)$ quintals and $40 - (100 - x - y) = (x + y - 60)$ quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows.



$$x \geq 0,\ y \geq 0,\ \text{and } 100 - x - y \geq 0$$
$$\Rightarrow x \geq 0,\ y \geq 0,\ \text{and } x + y \leq 100$$

$$60 - x \geq 0,\ 50 - y \geq 0,\ \text{and } x + y - 60 \geq 0$$
$$\Rightarrow x \leq 60,\ y \leq 50,\ \text{and } x + y \geq 60$$

Total transportation cost z is given by,

$$\begin{aligned} z &= 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60) \\ &= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180 \\ &= 2.5x + 1.5y + 410 \end{aligned}$$

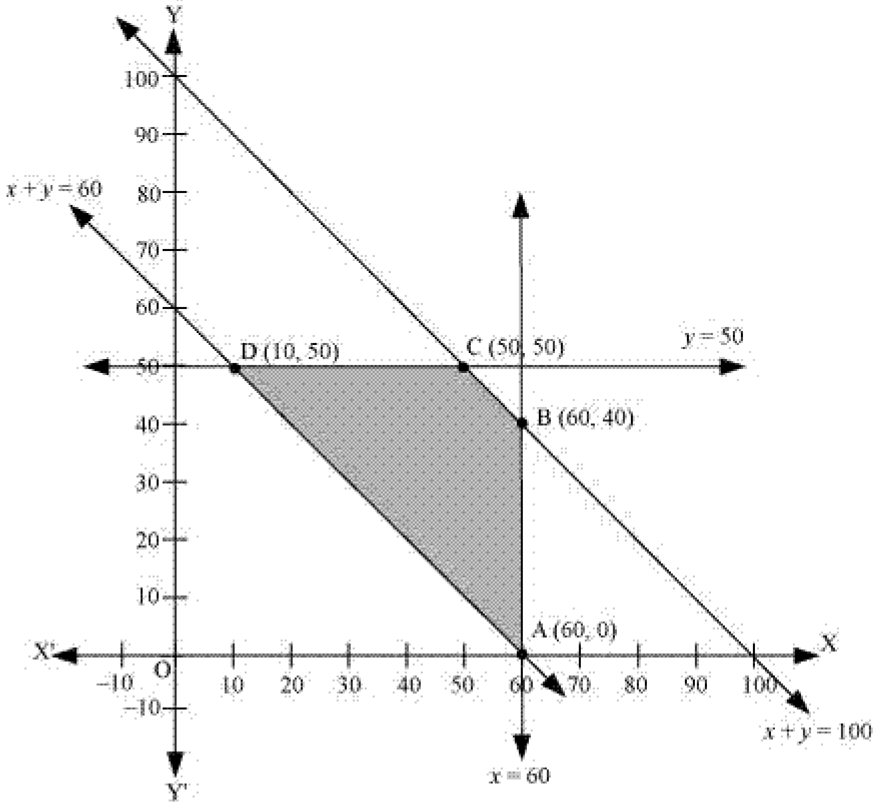
The given problem can be formulated as

Minimize $z = 2.5x + 1.5y + 410$ ... (1)

subject to the constraints,

$$x + y \leq 100 \quad \ldots(2)$$
$$x \leq 60 \quad \ldots(3)$$
$$y \leq 50 \quad \ldots(4)$$
$$x + y \geq 60 \quad \ldots(5)$$
$$x,\ y \geq 0 \quad \ldots(6)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (60, 0), B (60, 40), C (50, 50), and D (10, 50).

The values of z at these corner points are as follows.

Corner point	**$z = 2.5x + 1.5y + 410$**	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	→ Minimum

The minimum value of z is 510 at (10, 50).

Thus, the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals, and 40 quintals respectively and from B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively.

The minimum cost is Rs 510.

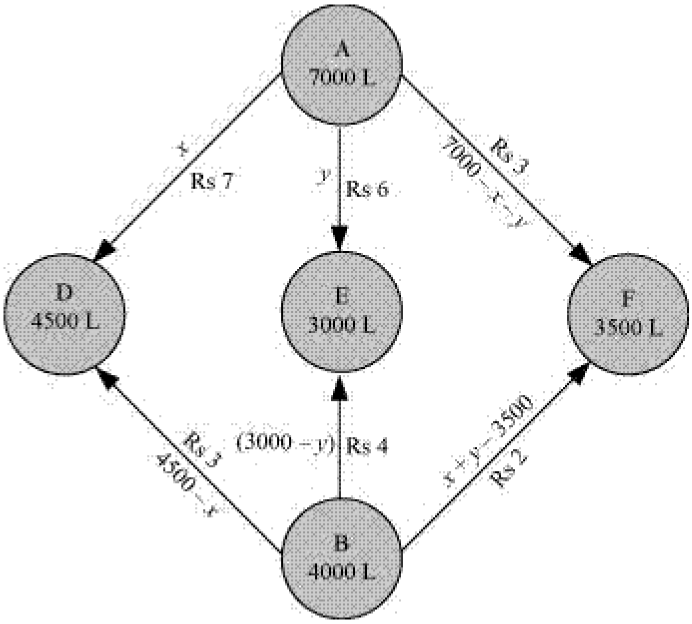
Solution 7

Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, $(7000 - x - y)$ will be supplied from A to petrol pump F.

The requirement at petrol pump D is 4500 L. Since x L are transported from depot A, the remaining $(4500 - x)$ L will be transported from petrol pump B.

Similarly, $(3000 - y)$ L and $3500 - (7000 - x - y) = (x + y - 3500)$ L will be transported from depot B to petrol pump E and F respectively.

The given problem can be represented diagrammatically as follows.



$x \geq 0,\ y \geq 0,$ and $(7000 - x - y) \geq 0$

$\Rightarrow x \geq 0,\ y \geq 0,$ and $x + y \leq 7000$

$$4500 - x \geq 0,\ 3000 - y \geq 0,\ \text{and } x + y - 3500 \geq 0$$
$$\Rightarrow x \leq 4500,\ y \leq 3000,\ \text{and } x + y \geq 3500$$

Cost of transporting 10 L of petrol = Re 1

Cost of transporting 1 L of petrol = $\text{Rs } \frac{1}{10}$

Therefore, total transportation cost is given by,

$$z = \frac{7}{10} \times x + \frac{6}{10} y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$
$$= 0.3x + 0.1y + 3950$$

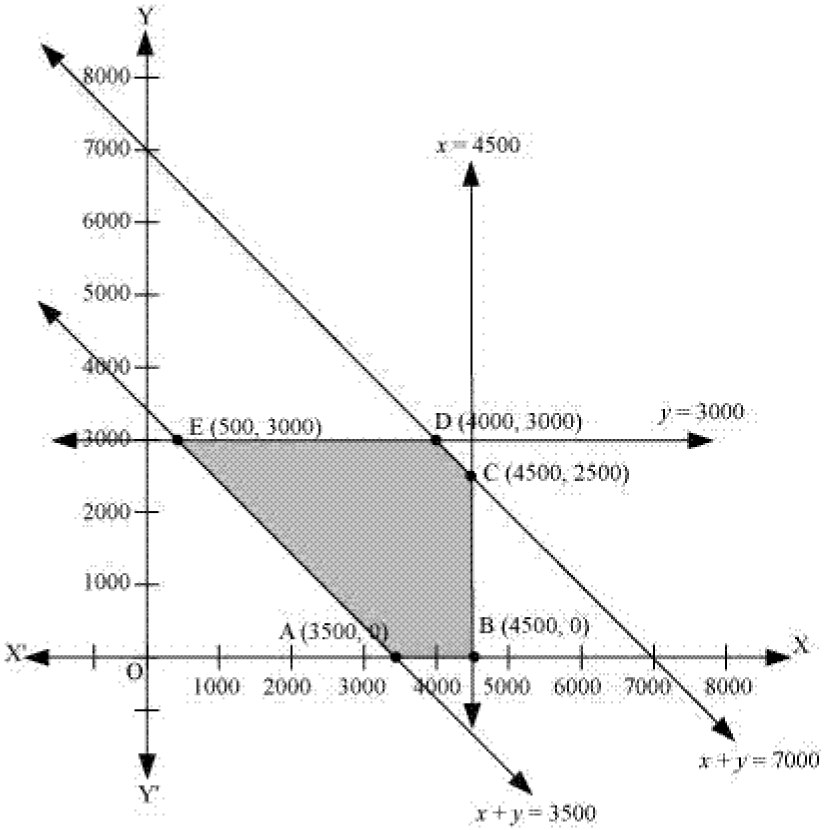
The problem can be formulated as follows.

Minimize $z = 0.3x + 0.1y + 3950$ ... (1)

subject to the constraints,

$$x + y \leq 7000 \quad \ldots(2)$$
$$x \leq 4500 \quad \ldots(3)$$
$$y \leq 3000 \quad \ldots(4)$$
$$x + y \geq 3500 \quad \ldots(5)$$
$$x,\ y \geq 0 \quad \ldots(6)$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000), and E (500, 3000).

The values of z at these corner points are as follows.

Corner point	$z = 0.3x + 0.1y + 3950$	
A (3500, 0)	5000	
B (4500, 0)	5300	
C (4500, 2500)	5550	
D (4000, 3000)	5450	
E (500, 3000)	4400	→ Minimum

The minimum value of z is 4400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E, and F respectively.

The minimum transportation cost is Rs 4400.

Solution 8

Let the fruit grower use x bags of brand P and y bags of brand Q.

The problem can be formulated as follows.

Minimize $z = 3x + 3.5y$ ... (1)

subject to the constraints,

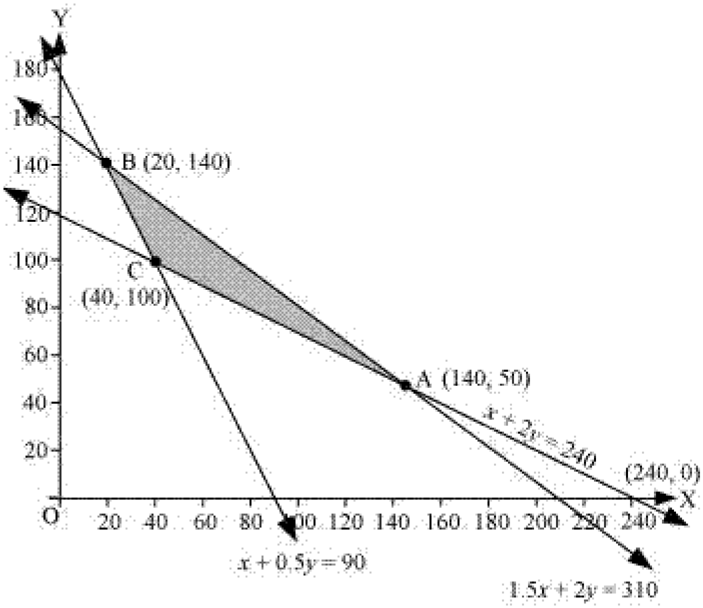
$$x + 2y \geq 240 \qquad ...(2)$$

$$x + 0.5y \geq 90 \qquad ...(3)$$

$$1.5x + 2y \leq 310 \qquad ...(4)$$

$$x, y \geq 0 \qquad ...(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (240, 0), B (140, 50), and C (20, 140).

The values of z at these corner points are as follows.

Corner point	**$z = 3x + 3.5y$**	
A (140, 50)	595	
B (20, 140)	550	
C (40, 100)	470	→ Minimum

The maximum value of z is 470 at (40, 100).

Thus, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimize the amount of nitrogen.

The minimum amount of nitrogen added to the garden is 470 kg.

Solution 9

Let the fruit grower use x bags of brand P and y bags of brand Q.

The problem can be formulated as follows.

Maximize $z = 3x + 3.5y$ ... (1)

subject to the constraints,

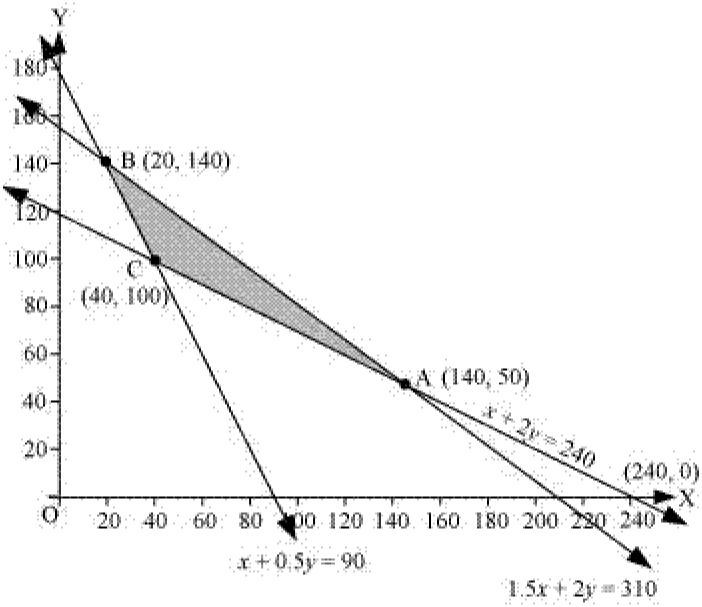
$$x + 2y \geq 240 \qquad ...(2)$$

$$x + 0.5y \geq 90 \qquad ...(3)$$

$$1.5x + 2y \leq 310 \qquad ...(4)$$

$$x, y \geq 0 \qquad ...(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (140, 50), B (20, 140), and C (40, 100).

The values of z at these corner points are as follows.

Corner point	**$z = 3x + 3.5y$**	
A (140, 50)	595	→ Maximum
B (20, 140)	550	
C (40, 100)	470	

The maximum value of z is 595 at (140, 50).

Thus, 140 bags of brand P and 50 bags of brand Q should be used to maximize the amount of nitrogen.

The maximum amount of nitrogen added to the garden is 595 kg.

Solution 10

Let x and y be the number of dolls of type A and B respectively that are produced per week.

The given problem can be formulated as follows.

Maximize $z = 12x + 16y$ ... (1)

subject to the constraints,

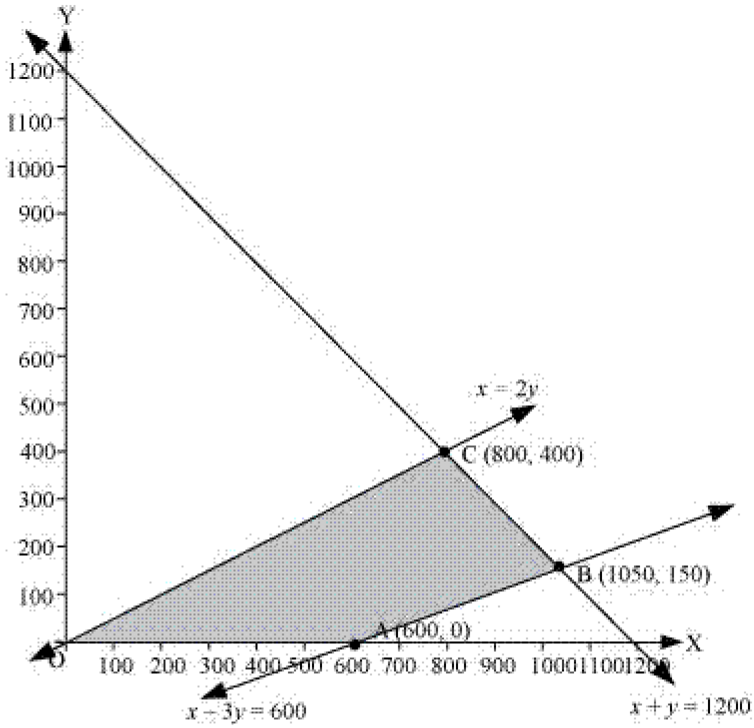
$$x + y \leq 1200 \qquad ...(2)$$

$$y \leq \frac{x}{2} \Rightarrow x \geq 2y \qquad ...(3)$$

$$x - 3y \leq 600 \qquad ...(4)$$

$$x, y \geq 0 \qquad ...(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (600, 0), B (1050, 150), and C (800, 400).

The values of z at these corner points are as follows.

Corner point	**$z = 12x + 16y$**	
A (600, 0)	7200	
B (1050, 150)	15000	
C (800, 400)	16000	→ Maximum

The maximum value of z is 16000 at (800, 400).

Thus, 800 and 400 dolls of type A and type B should be produced respectively to get the maximum profit of Rs 16000.