Access answers to Maths RD Sharma Solutions For Class 7 Chapter 15 – Properties Of Triangle

Exercise 15.1 Page No: 15.4

1. Take three non-collinear points A. B and C on a page of your notebook. Join AB, BC and CA. What figure do you get? Name the triangle. Also, name

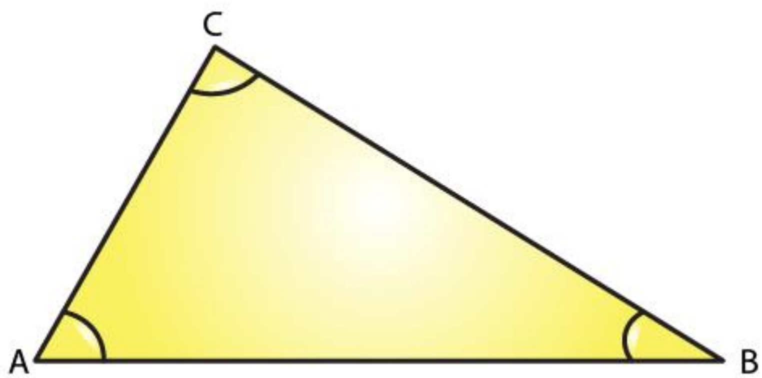
(i) The side opposite to ∠B

(ii) The angle opposite to side AB

(iii) The vertex opposite to side BC

(iv) The side opposite to vertex B.

Solution:



i) The side opposite to ∠B is AC

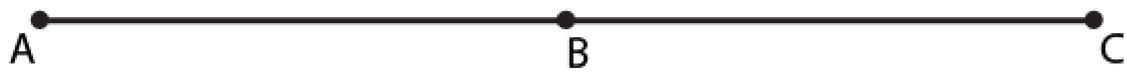
(ii) The angle opposite to side AB is ∠B

(iii) The vertex opposite to side BC is A

(iv) The side opposite to vertex B is AC

2. Take three collinear points A, B and C on a page of your note book. Join AB. BC and CA. Is the figure a triangle? If not, why?

Solution:



No, the figure is not a triangle. By definition a triangle is a plane figure formed by three non-parallel line segments

3. Distinguish between a triangle and its triangular region.

Solution:

Triangle:

A triangle is a plane figure formed by three non-parallel line segments.

Triangular region:

Whereas, it’s triangular region includes the interior of the triangle along with the triangle itself.

4. D is a point on side BC of a △CAD is joined. Name all the triangles that you can observe in the figure. How many are they?

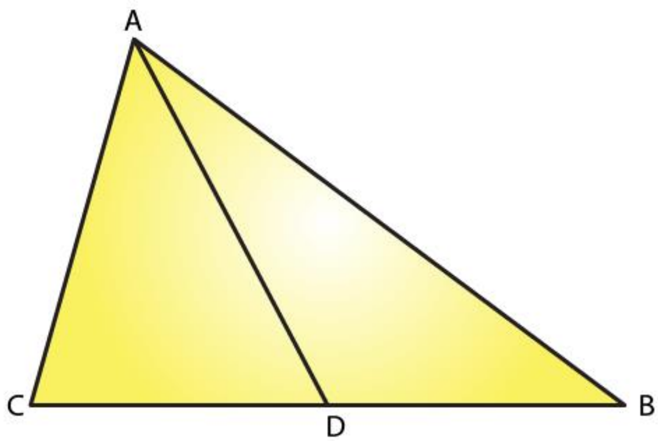


Fig. 9

Solution:

We can observe the following three triangles in the given figure

△ ABC

△ ACD

△ ADB

5. A, B. C and D are four points, and no three points are collinear. AC and BD intersect at O. There are eight triangles that you can observe. Name all the triangles

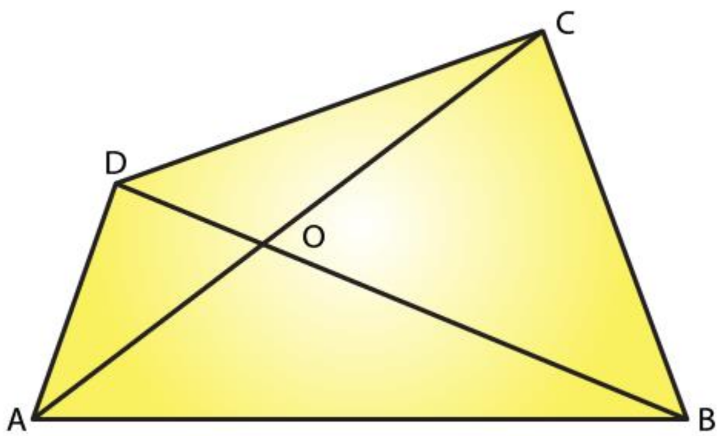


Fig. 10

Solution:

Given A, B. C and D are four points, and no three points are collinear

△ ABC

△ ABD

△ ABO

△ BCD

△ DCO

△ AOD

△ ACD

△ BCD

6. What is the difference between a triangle and triangular region?

Solution:

Triangle:

A triangle is a plane figure formed by three non-parallel line segments.

Triangular region:

Whereas, it’s triangular region includes the interior of the triangle along with the triangle itself.

7. Explain the following terms:

(i) Triangle

(a) Parts or elements of a triangle

(iii) Scalene triangle

(iv) Isosceles triangle

(v) Equilateral triangle

(vi) Acute triangle

(vii) Right triangle

(viii) Obtuse triangle

(ix) Interior of a triangle

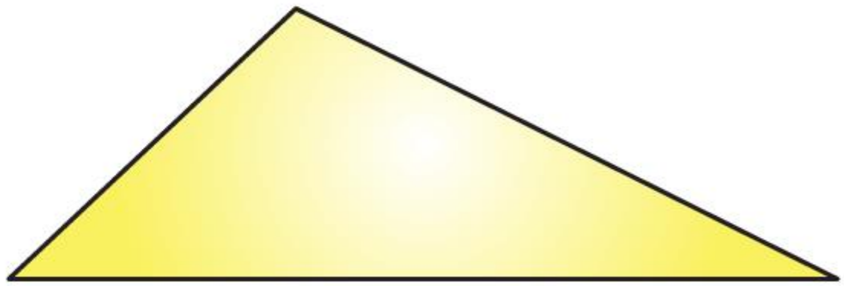
(x) Exterior of a triangle

Solution:

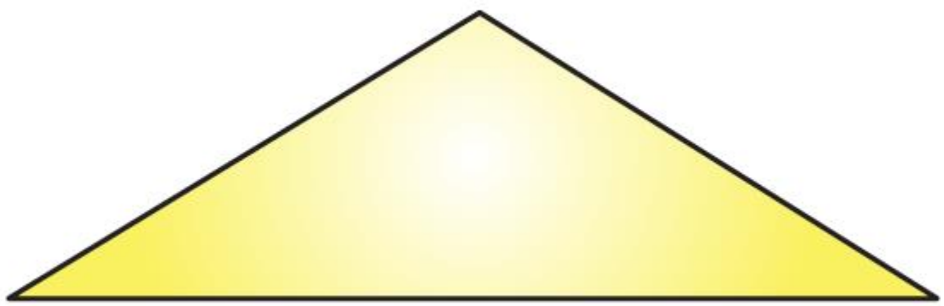
(i) A triangle is a plane figure formed by three non-parallel line segments.

(ii) The three sides and the three angles of a triangle are together known as the parts or elements of that triangle.

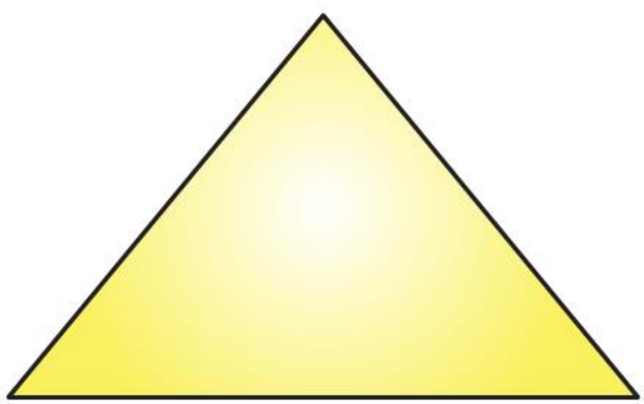
(iii) A scalene triangle is a triangle in which no two sides are equal.



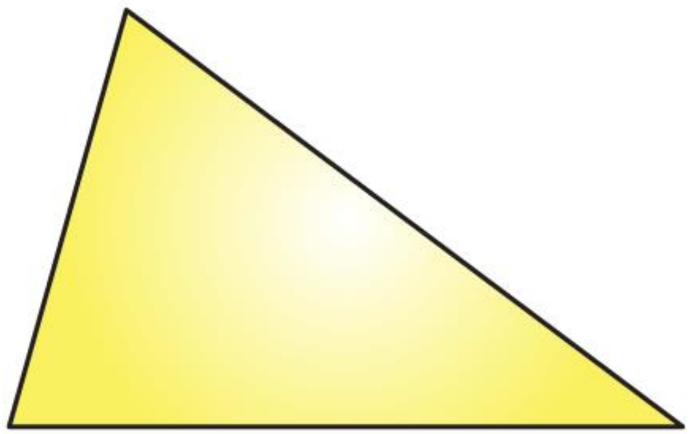
(iv) An isosceles triangle is a triangle in which two sides are equal.



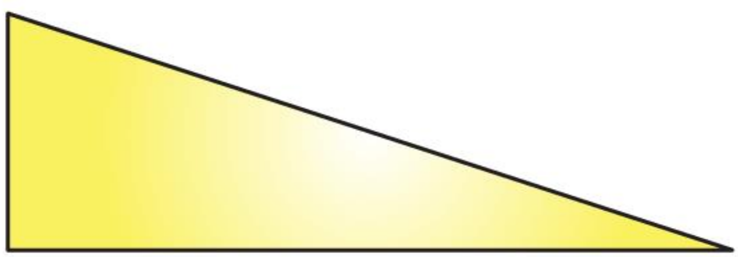
(v) An equilateral triangle is a triangle in which all three sides are equal.



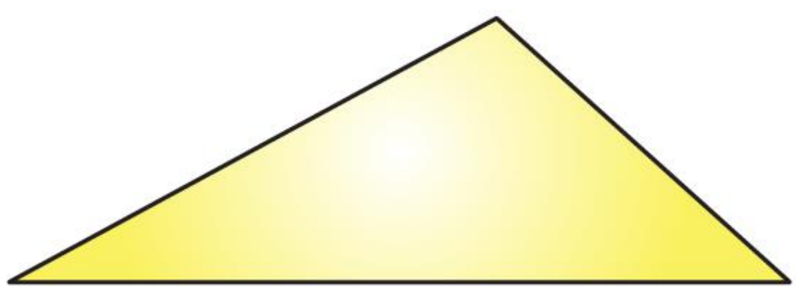
vi) An acute triangle is a triangle in which all the angles are less than 90º.



(vii) A right angled triangle is a triangle in which one angle should be equal to 90º.



viii) An obtuse triangle is a triangle in which one angle is more than 90º.



(ix) The interior of a triangle is made up of all such points that are enclosed within the triangle.

(x) The exterior of a triangle is made up of all such points that are not enclosed within the triangle.

8. In Fig. 11, the length (in cm) of each side has been indicated along the side. State for each triangle angle whether it is scalene, isosceles or equilateral:

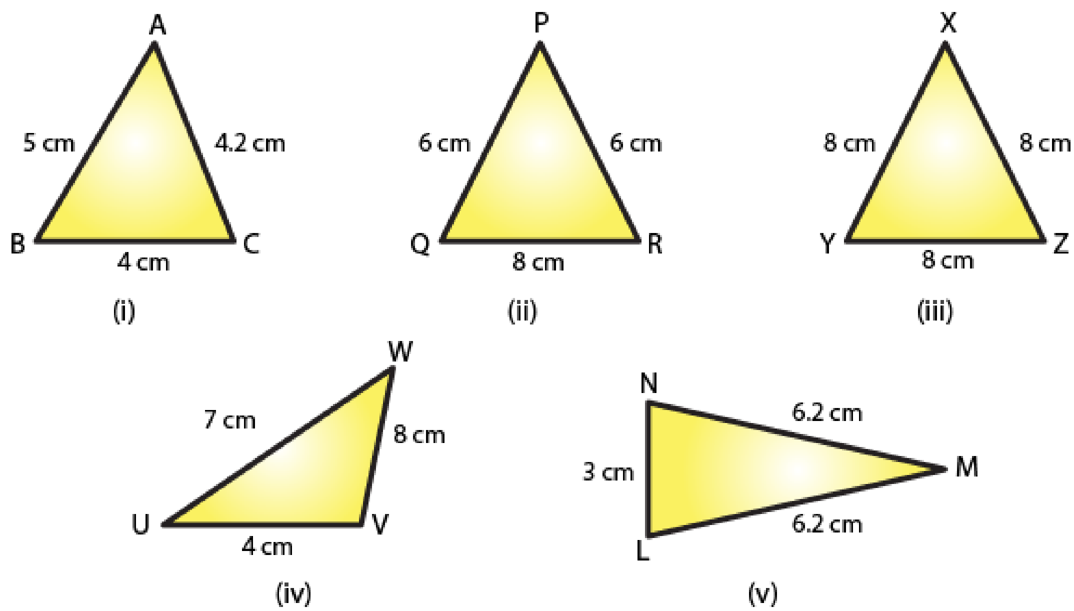


Fig. 11

Solution:

(i) The given triangle is a scalene triangle because no two sides are equal.

(ii) The given triangle is an isosceles triangle because two of its sides, viz. PQ and PR, are equal.

(iii) The given triangle is an equilateral triangle because all its three sides are equal.

(iv) The given triangle is a scalene triangle because no two sides are equal.

(v) The given triangle is an isosceles triangle because two of its sides are equal.

9. In Fig. 12, there are five triangles. The measures of some of their angles have been indicated. State for each triangle whether it is acute, right or obtuse.

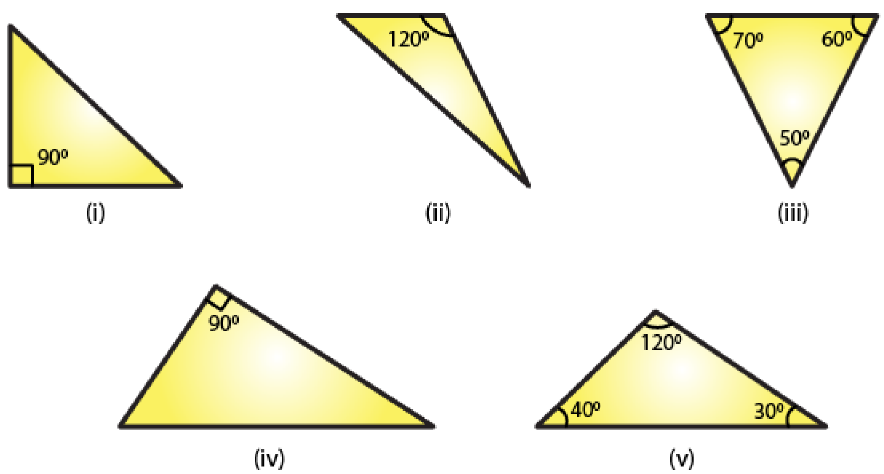


Fig. 12

Solution:

(i) The given triangle is a right triangle because one of its angles is 90^o.

(ii) The given triangle is an obtuse triangle because one of its angles is 120^o, which is greater than 90^o

(iii) The given triangle is an acute triangle because all its angles are less than 90º

(iv) The given triangle is a right triangle because one of its angles is 90º.

(v) The given triangle is an obtuse triangle because one of its angles is 110º, which is greater than 90º.

10. Fill in the blanks with the correct word/symbol to make it a true statement:

(i) A triangle has ……. sides.

(ii) A triangle has ……..vertices.

(iii) A triangle has ……..angles.

(iv) A triangle has ………parts.

(v) A triangle whose no two sides are equal is known as ………

(vi) A triangle whose two sides are equal is known as ……….

(vii) A triangle whose all the sides are equal is known as ………

(viii) A triangle whose one angle is a right angle is known as ……..

(ix) A triangle whose all the angles are of measure less than 90′ is known as ………

(x) A triangle whose one angle is more than 90′ is known as ……….

Solution:

(i) Three

(ii) Three

(iii) Three

(iv) Six

(v) A scalene triangle

(vi) An isosceles triangle

(vii) An equilateral triangle

(viii) A right triangle

(ix) An acute triangle

(x) An obtuse triangle

11. In each of the following, state if the statement is true (T) or false (F):

(i) A triangle has three sides.

(ii) A triangle may have four vertices.

(iii) Any three line-segments make up a triangle.

(iv) The interior of a triangle includes its vertices.

(v) The triangular region includes the vertices of the corresponding triangle.

(vi) The vertices of a triangle are three collinear points.

(vii) An equilateral triangle is isosceles also.

(viii) Every right triangle is scalene.

(ix) Each acute triangle is equilateral.

(x) No isosceles triangle is obtuse.

Solution:

(i) True

(ii) False

Explanation:

Any three non-parallel line segments can make up a triangle.

(iii) False.

Explanation:

Any three non-parallel line segments can make up a triangle.

(iv) False.

Explanation:

The interior of a triangle is the region enclosed by the triangle and the vertices are not enclosed by the triangle.

(v) True.

Explanation:

The triangular region includes the interior region and the triangle itself.

(vi) False.

Explanation:

The vertices of a triangle are three non-collinear points.

(vii) True.

Explanation:

In an equilateral triangle, any two sides are equal.

(viii) False.

Explanation:

A right triangle can also be an isosceles triangle.

(ix) False.

Explanation:

Each acute triangle is not an equilateral triangle, but each equilateral triangle is an acute triangle.

(x) False.

Explanation:

An isosceles triangle can be an obtuse triangle, a right triangle or an acute triangle

---

Exercise 15.2 Page No: 15.12

1. Two angles of a triangle are of measures 150º and 30º. Find the measure of the third angle.

Solution:

Given two angles of a triangle are of measures 150^o and 30^o

Let the required third angle be x

We know that sum of all the angles of a triangle $= 180^o$

$105^o + 30^o + x = 180^o$

$135^o + x = 180^o$

$x = 180^o – 135^o$

$x = 45^o$

Therefore the third angle is 45^o

2. One of the angles of a triangle is 130^o, and the other two angles are equal. What is the measure of each of these equal angles?

Solution:

Given one of the angles of a triangle is 130^o

Also given that remaining two angles are equal

So let the second and third angle be x

We know that sum of all the angles of a triangle $= 180^o$

$130^o + x + x = 180^o$

$130^o + 2x = 180^o$

$2x = 180^o – 130^o$

$2x = 50^o$

$x = 50/2$

$x = 25^o$

Therefore the two other angles are 25^o each

3. The three angles of a triangle are equal to one another. What is the measure of each of the angles?

Solution:

Given that three angles of a triangle are equal to one another

So let the each angle be x

We know that sum of all the angles of a triangle = 180^o

$x + x + x = 180^o$

$3x = 180^o$

$x = 180/3$

$x = 60^o$

Therefore angle is 60^o each

4. If the angles of a triangle are in the ratio 1: 2: 3, determine three angles.

Solution:

Given angles of the triangle are in the ratio 1: 2: 3

So take first angle as x, second angle as 2x and third angle as 3x

We know that sum of all the angles of a triangle = 180^o

$x + 2x + 3x = 180^o$

$6x = 180^o$

$x = 180/6$

$x = 30^o$

$2x = 30^o \times 2 = 60^o$

$3x = 30^o \times 3 = 90^o$

Therefore the first angle is 30^o, second angle is 60^o and third angle is 90^o.

5. The angles of a triangle are $(x - 40)^o$, $(x - 20)^o$ and $(1/2 - 10)^o$. Find the value of x.

Solution:

Given the angles of a triangle are $(x - 40)^o$, $(x - 20)^o$ and $(1/2 - 10)^o$.

We know that sum of all the angles of a triangle = 180^o

$(x - 40)^o + (x - 20)^o + (1/2 - 10)^o = 180^o$

$x + x + (x/2) – 40^o – 20^o – 10^o = 180^o$

$x + x + (x/2) – 70^o = 180^o$

$(5x/2) = 180^o + 70^o$

$(5x/2) = 250^o$

$x = (2/5) \times 250^o$

$x = 100^o$

Hence the value of x is 100°

6. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10°. Find the three angles.

Solution:

Given that angles of a triangle are arranged in ascending order of magnitude

Also given that difference between two consecutive angles is 10°

Let the first angle be x

Second angle be $x + 10^\circ$

Third angle be $x + 10^\circ + 10^\circ$

We know that sum of all the angles of a triangle $= 180^\circ$

$x + x + 10^\circ + x + 10^\circ + 10^\circ = 180^\circ$

$3x + 30 = 180$

$3x = 180 – 30$

$3x = 150$

$x = 150/3$

$x = 50^\circ$

First angle is 50°

Second angle $x + 10^\circ = 50 + 10 = 60^\circ$

Third angle $x + 10^\circ + 10^\circ = 50 + 10 + 10 = 70^\circ$

7. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30º. Determine all the angles of the triangle

Solution:

Given that two angles of a triangle are equal

Let the first and second angle be x

Also given that third angle is greater than each of those angles by 30º

Therefore the third angle is greater than the first and second by 30º = x + 30º

The first and the second angles are equal

We know that sum of all the angles of a triangle = 180º

$x + x + x + 30^\circ = 180^\circ$

$3x + 30 = 180$

$3x = 180 – 30$

$3x = 150$

$x = 150/3$

$x = 50^\circ$

Third angle = $x + 30^\circ = 50^\circ + 30^\circ = 80^\circ$

The first and the second angle is 50º and the third angle is 80º.

8. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.

Solution:

Given that one angle of a triangle is equal to the sum of the other two

Let the measure of angles be x, y, z

Therefore we can write above statement as $x = y + z$

$x + y + z = 180^o$

Substituting the above value we get

$x + x = 180^o$

$2x = 180^o$

$x = 180/2$

$x = 90^o$

If one angle is 90^o then the given triangle is a right angled triangle

9. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Solution:

Given that each angle of a triangle is less than the sum of the other two

Let the measure of angles be x, y and z

From the above statement we can write as

$x > y + z$

$y < x + z$

$z < x + y$

Therefore triangle is an acute triangle

10. In each of the following, the measures of three angles are given. State in which cases the angles can possibly be those of a triangle:

(i) 63^o, 37^o, 80^o

(ii) 45^o, 61^o, 73^o

(iii) 59^o, 72^o, 61^o

(iv) 45^o, 45^o, 90^o

(v) 30^o, 20^o, 125^o

Solution:

(i) $63^o + 37^o + 80^o = 180^o$

Angles form a triangle

(ii) 45^o, 61^o, 73^o is not equal to 180^o

Therefore not a triangle

(iii) 59^o, 72^o, 61^o is not equal to 180^0

Therefore not a triangle

(iv) $45^o + 45^o + 90^o = 180^o$

Angles form a triangle

(v) 30^o, 20^o, 125^o is not equal to 180^o

Therefore not a triangle

11. The angles of a triangle are in the ratio 3: 4: 5. Find the smallest angle

Solution:

Given that angles of a triangle are in the ratio: 3: 4: 5

Therefore let the measure of the angles be 3x, 4x, 5x

We know that sum of the angles of a triangle $=180^o$

$3x + 4x + 5x = 180^o$

$12x = 180^o$

$x = 180/12$

$x = 15^o$

Smallest angle = 3x

$= 3 \times 15^o$

$= 45^o$

Therefore smallest angle = 45^o

12. Two acute angles of a right triangle are equal. Find the two angles.

Solution:

Given that acute angles of a right angled triangle are equal

We know that Right triangle: whose one of the angle is a right angle

Let the measure of angle be x, x, 90^o

$x + x + 90^o = 180^o$

$2x = 90^o$

$x = 90/2$

$x = 45^o$

The two angles are 45^o and 45^o

13. One angle of a triangle is greater than the sum of the other two. What can you say about the measure of this angle? What type of a triangle is this?

Solution:

Given one angle of a triangle is greater than the sum of the other two

Let the measure of the angles be x, y, z

From the question we can write as

$x > y + z$ or

$y > x + z$ or

$z > x + y$

x or y or $z > 90^o$ which is obtuse

Therefore triangle is an obtuse angle

14. In the six cornered figure, (fig. 20), AC, AD and AE are joined. Find ∠FAB + ∠ABC + ∠BCD + ∠CDE + ∠DEF + ∠EFA.

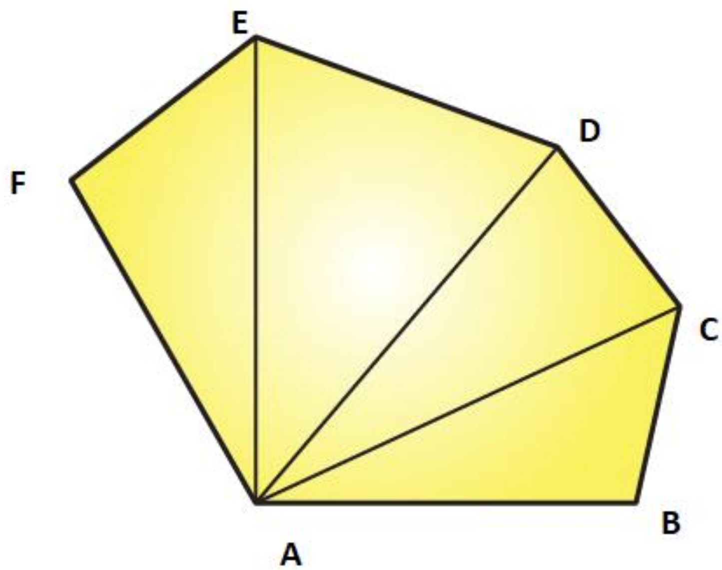


Fig. 20

Solution:

We know that sum of the angles of a triangle is 180^o

Therefore in ∆ABC, we have

∠CAB + ∠ABC + ∠BCA = 180^o …….. (i)

In∆ ACD, we have

∠DAC + ∠ACD + ∠CDA = 180^o …….. (ii)

In ∆ADE, we have

∠EAD + ∠ADE + ∠DEA =180^o ………. (iii)

In ∆AEF, we have

$\angle FAE + \angle AEF + \angle EFA = 180^o$ ………. (iv)

Adding (i), (ii), (iii), (iv) we get

$\angle CAB + \angle ABC + \angle BCA + \angle DAC + \angle ACD + \angle CDA + \angle EAD + \angle ADE + \angle DEA + \angle FAE + \angle AEF + \angle EFA = 720^o$

Therefore $\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA = 720^o$

15. Find x, y, z (whichever is required) from the figures (Fig. 21) given below:

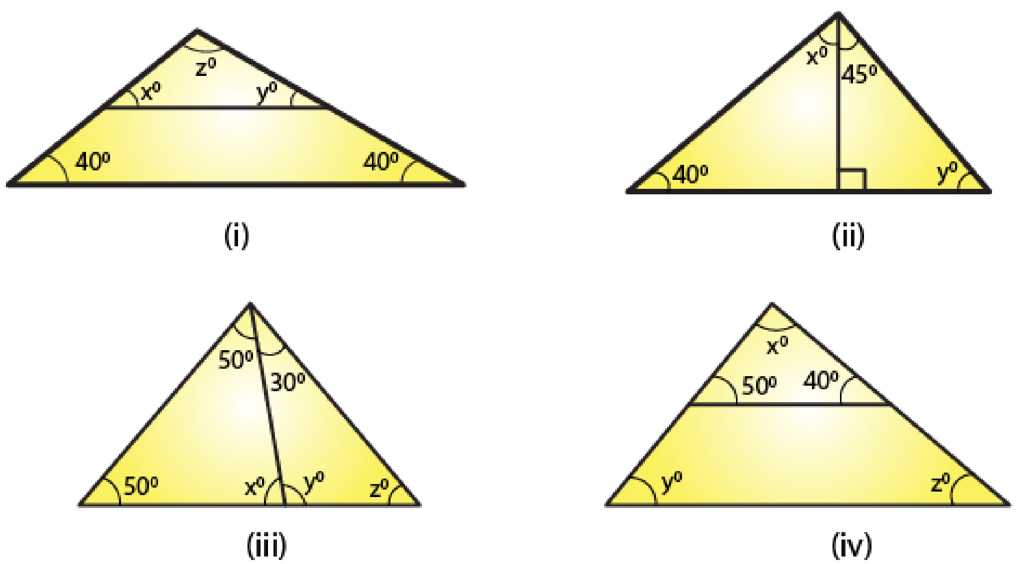


Fig.21

Solution:

(i) In △ABC and △ADE we have,

$\angle$ ADE = $\angle$ ABC [corresponding angles]

$x = 40^{o}$

$\angle AED = \angle ACB$ (corresponding angles)

$y = 30^{o}$

We know that the sum of all the three angles of a triangle is equal to 180^{o}

$x + y + z = 180^{o}$ (Angles of $\triangle ADE$)

Which means: $40^{o} + 30^{o} + z = 180^{o}$

$z = 180^{o} – 70^{o}$

$z = 110^{o}$

Therefore, we can conclude that the three angles of the given triangle are 40^{o}, 30^{o} and 110^{o}

(ii) We can see that in $\triangle ADC$, $\angle ADC$ is equal to 90^{o}.

($\triangle ADC$ is a right triangle)

We also know that the sum of all the angles of a triangle is equal to 180^{o}.

Which means: $45^{o} + 90^{o} + y = 180^{o}$ (Sum of the angles of $\triangle ADC$)

$135^{o} + y = 180^{o}$

$y = 180^{o} – 135^{o}$.

$y = 45^{o}$.

We can also say that in △ ABC, ∠ABC + ∠ACB + ∠BAC is equal to 180^o.

(Sum of the angles of △ABC)

$40^o + y + (x + 45^o) = 180^o$

$40^o + 45^o + x + 45^o = 180^o$ ($y = 45^o$)

$x = 180^o - 130^o$

$x = 50^o$

Therefore, we can say that the required angles are 45^o and 50^o.

(iii) We know that the sum of all the angles of a triangle is equal to 180^o.

Therefore, for △ABD:

∠ABD +∠ADB + ∠BAD = 180° (Sum of the angles of △ABD)

$50^o + x + 50^o = 180^o$

$100^o + x = 180^o$

$x = 180^o - 100^o$

$x = 80^o$

For △ ABC:

∠ABC + ∠ACB + ∠BAC = 180^o (Sum of the angles of △ABC)

$50^o + z + (50^o + 30^o) = 180^o$

$50^o + z + 50^o + 30^o = 180^o$

$z = 180^o – 130^o$

$z = 50^o$

Using the same argument for △ADC:

∠ADC + ∠ACD + ∠DAC = 180^o (Sum of the angles of △ADC)

$y + z + 30^o = 180^o$

$y + 50^o + 30^o = 180^o$ ($z = 50^o$)

$y = 180^o – 80^o$

$y = 100^0$

Therefore, we can conclude that the required angles are 80^o, 50^o and 100^o.

(iv) In △ABC and △ADE we have:

∠ADE = ∠ABC (Corresponding angles)

$y = 50^o$

Also, ∠AED = ∠ACB (Corresponding angles)

$z = 40^o$

We know that the sum of all the three angles of a triangle is equal to 180^o.

We can write as $x + 50^o + 40^o = 180^o$ (Angles of △ADE)

$x = 180^o – 90^o$

$x = 90^o$

Therefore, we can conclude that the required angles are 50^o, 40^o and 90^o.

16. If one angle of a triangle is 60^o and the other two angles are in the ratio 1: 2, find the angles.

Solution:

Given that one of the angles of the given triangle is 60^o.

Also given that the other two angles of the triangle are in the ratio 1: 2.

Let one of the other two angles be x.

Therefore, the second one will be 2x.

We know that the sum of all the three angles of a triangle is equal to 180^o.

$60^o + x + 2x = 180^o$

$3x = 180^o – 60^o$

$3x = 120^o$

$x = 120^o/3$

$x = 40^o$

$2x = 2 \times 40^o$

$2x = 80^o$

Hence, we can conclude that the required angles are 40^o and 80^o.

17. It one angle of a triangle is 100^{o} and the other two angles are in the ratio 2: 3. Find the angles.

Solution:

Given that one of the angles of the given triangle is 100^{o}.

Also given that the other two angles are in the ratio 2: 3.

Let one of the other two angles be 2x.

Therefore, the second angle will be 3x.

We know that the sum of all three angles of a triangle is 180^{o}.

$100^{o} + 2x + 3x = 180^{o}$

$5x = 180^{o} – 100^{o}$

$5x = 80^{o}$

$x = 80/5$

$x = 16$

$2x = 2 \times 16$

$2x = 32^{o}$

$3x = 3 \times 16$

$3x = 48^{o}$

Thus, the required angles are 32^{o} and 48^{o}.

18. In $\triangle ABC$, if $3\angle A = 4\angle B = 6\angle C$, calculate the angles.

Solution:

We know that for the given triangle, $3\angle A = 6\angle C$

$\angle A = 2\angle C$ ……. (i)

We also know that for the same triangle, $4\angle B = 6\angle C$

$\angle B = (6/4)\ \angle C$ …… (ii)

We know that the sum of all three angles of a triangle is 180^o.

Therefore, we can say that:

$\angle A + \angle B + \angle C = 180^o$ (Angles of $\triangle ABC$)…… (iii)

On putting the values of $\angle A$ and $\angle B$ in equation (iii), we get:

$2\angle C + (6/4)\ \angle C + \angle C = 180^o$

$(18/4)\ \angle C = 180^o$

$\angle C = 40^o$

From equation (i), we have:

$\angle A = 2\angle C = 2 \times 40$

$\angle A = 80^o$

From equation (ii), we have:

$\angle B = (6/4)\ \angle C = (6/4) \times 40^o$

$\angle B = 60^o$

$\angle A = 80^o$, $\angle B = 60^o$, $\angle C = 40^o$

Therefore, the three angles of the given triangle are 80^o, 60^o, and 40^o.

19. Is it possible to have a triangle, in which

(i) Two of the angles are right?

(ii) Two of the angles are obtuse?

(iii) Two of the angles are acute?

(iv) Each angle is less than 60^o?

(v) Each angle is greater than 60^o?

(vi) Each angle is equal to 60^o?

Solution:

(i) No, because if there are two right angles in a triangle, then the third angle of the triangle must be zero, which is not possible.

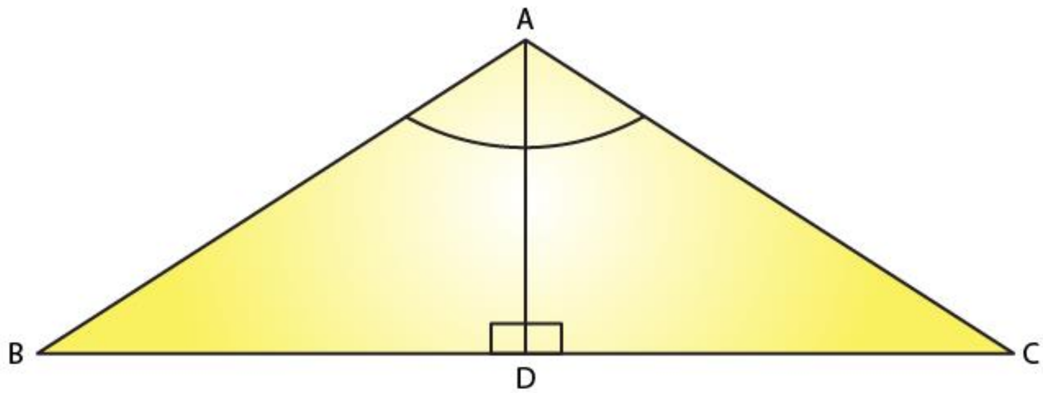
(ii) No, because as we know that the sum of all three angles of a triangle is always 180^o. If there are two obtuse angles, then their sum will be more than 180^o, which is not possible in case of a triangle.

(iii) Yes, in right triangles and acute triangles, it is possible to have two acute angles.

(iv) No, because if each angle is less than 60°, then the sum of all three angles will be less than 180°, which is not possible in case of a triangle.

20. In$\triangle$ ABC, $\angle$A = 100°, AD bisects $\angle$A and AD $\perp$ BC. Find $\angle$B

Solution:



Given that in $\triangle$ABC, $\angle$A = 100°

Also given that AD $\perp$ BC

Consider $\triangle$ABD

$\angle$BAD = 100/2   (AD bisects $\angle$A)

$\angle$BAD = 50°

$\angle$ADB = 90°   (AD perpendicular to BC)

We know that the sum of all three angles of a triangle is 180°.

Thus,

$\angle ABD + \angle BAD + \angle ADB = 180^o$ (Sum of angles of $\triangle ABD$)

Or,

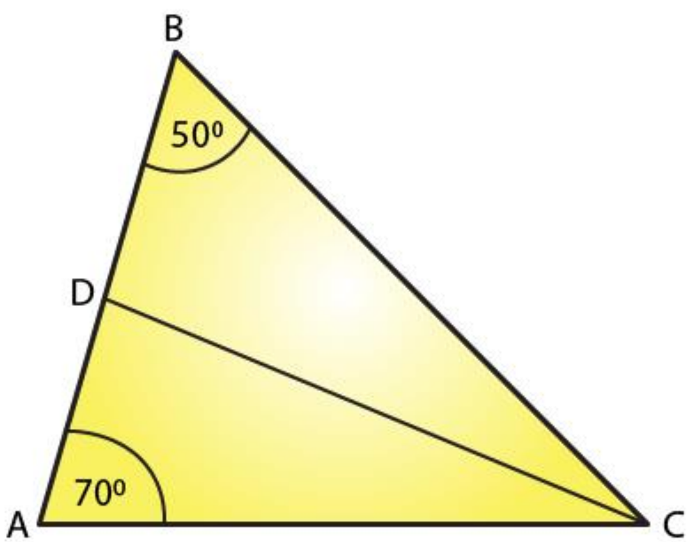
$\angle ABD + 50^o + 90^o = 180^o$

$\angle ABD = 180^o - 140^o$

$\angle ABD = 40^o$

21. In $\triangle ABC$, $\angle A = 50^o$, $\angle B = 100^o$ and bisector of $\angle C$ meets AB in D. Find the angles of the triangles ADC and BDC

Solution:



We know that the sum of all three angles of a triangle is equal to 180^o.

Therefore, for the given $\triangle ABC$, we can say that:

$\angle A + \angle B + \angle C = 180^o$ (Sum of angles of $\triangle ABC$)

$50^o + 70^o + \angle C = 180^o$

$\angle C = 180^o - 120^o$

$\angle C = 60^o$

$\angle ACD = \angle BCD = \angle C2$ (CD bisects $\angle C$ and meets AB in D.)

$\angle ACD = \angle BCD = 60/2 = 30^o$

Using the same logic for the given $\triangle ACD$, we can say that:

$\angle DAC + \angle ACD + \angle ADC = 180^o$

$50^o + 30^o + \angle ADC = 180^o$

$\angle ADC = 180^o - 80^o$

$\angle ADC = 100^o$

If we use the same logic for the given $\triangle BCD$, we can say that

$\angle DBC + \angle BCD + \angle BDC = 180^o$

$70^o + 30^o + \angle BDC = 180^o$

$\angle BDC = 180^o - 100^o$

$\angle BDC = 80^o$

Thus,

For $\triangle ADC$: $\angle A = 50^o$, $\angle D = 100^o$ $\angle C = 30^o$

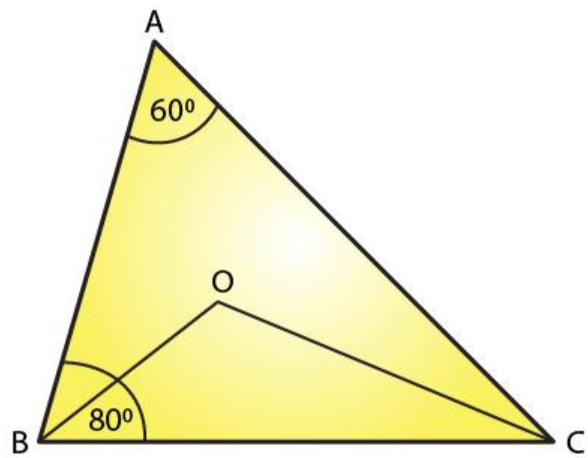
$\triangle BDC$: $\angle B = 70^o$, $\angle D = 80^o$ $\angle C = 30^o$

22. In ΔABC, $\angle$A = 60º, $\angle$B = 80º, and the bisectors of $\angle$B and $\angle$C, meet at O. Find

(i) $\angle$C

(ii) $\angle$BOC

Solution:



(i) We know that the sum of all three angles of a triangle is 180º.

Hence, for $\triangle$ABC, we can say that:

$\angle$A + $\angle$B + $\angle$C = 180º (Sum of angles of ΔABC)

60º + 80º + $\angle$C= 180º.

$\angle$C = 180º – 140º

$\angle$C = 140º.

(ii)For $\triangle$OBC,

$\angle$OBC = $\angle$B/2 = 80/2 (OB bisects $\angle$B)

$\angle$OBC = 40º

$\angle OCB = \angle C/2 = 40/2$ (OC bisects $\angle C$)

$\angle OCB = 20^o$

If we apply the above logic to this triangle, we can say that:

$\angle OCB + \angle OBC + \angle BOC = 180^o$ (Sum of angles of $\triangle OBC$)

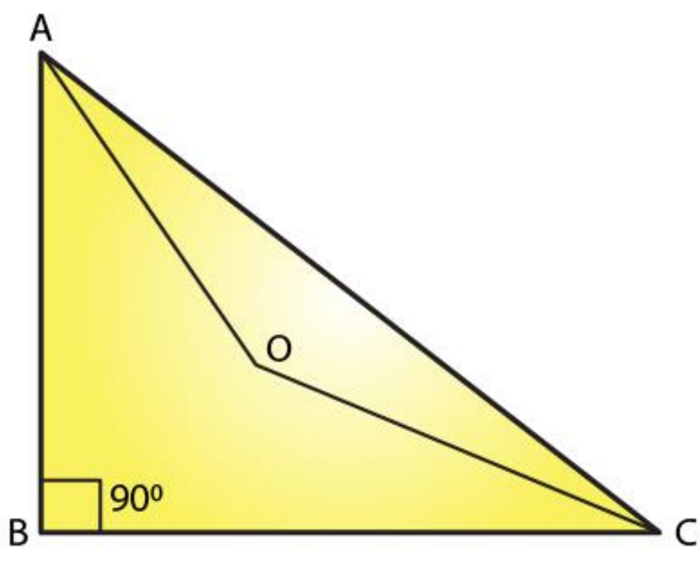
$20^o + 40^o + \angle BOC = 180^o$

$\angle BOC = 180^o – 60^o$

$\angle BOC = 120^o$

23. The bisectors of the acute angles of a right triangle meet at O. Find the angle at O between the two bisectors.

Solution:



Given bisectors of the acute angles of a right triangle meet at O

We know that the sum of all three angles of a triangle is 180º.

Hence, for △ABC, we can say that:

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 90^o + \angle C = 180^o$

$\angle A + \angle C = 180^o – 90^o$

$\angle A + \angle C = 90^o$

For △OAC:

$\angle OAC = \angle A/2$ (OA bisects LA)

$\angle OCA = \angle C/2$ (OC bisects LC)

On applying the above logic to △OAC, we get

$\angle AOC + \angle OAC + \angle OCA = 180^o$ (Sum of angles of △AOC)

$\angle AOC + \angle A2 + \angle C2 = 180^o$

$\angle AOC + \angle A + \angle C2 = 180^o$

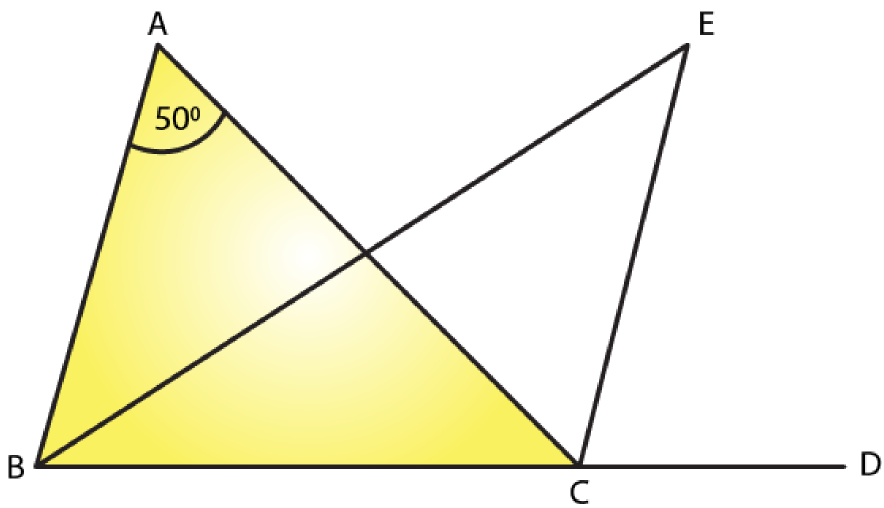
$\angle AOC + 90/2 = 180^o$

$\angle AOC = 180^o – 45^o$

$\angle AOC = 135^o$

24. In △ABC, ∠A = 50º and BC is produced to a point D. The bisectors of ∠ABC and ∠ACD meet at E. Find ∠E.

Solution:



In the given triangle,

$\angle ACD = \angle A + \angle B$. (Exterior angle is equal to the sum of two opposite interior angles.)

We know that the sum of all three angles of a triangle is 180^o.

Therefore, for the given triangle, we know that the sum of the angles = 180^o

$\angle ABC + \angle BCA + \angle CAB = 180^o$

$\angle A + \angle B + \angle BCA = 180^o$

$\angle BCA = 180^o – (\angle A + \angle B)$

But we know that EC bisects $\angle ACD$

Therefore $\angle ECA = \angle ACD/2$

$\angle ECA = (\angle A + \angle B)/2$ [$\angle ACD = (\angle A + \angle B)$]

But EB bisects $\angle ABC$

$\angle EBC = \angle ABC/2 = \angle B/2$

$\angle EBC = \angle ECA + \angle BCA$

$\angle EBC = (\angle A + \angle B)/2 + 180^o – (\angle A + \angle B)$

If we use same steps for $\triangle EBC$, then we get,

$\angle B/2 + (\angle A + \angle B)/2 + 180^o – (\angle A + \angle B) + \angle BEC = 180^o$

$\angle BEC = \angle A + \angle B – (\angle A + \angle B)/2 – \angle B/2$

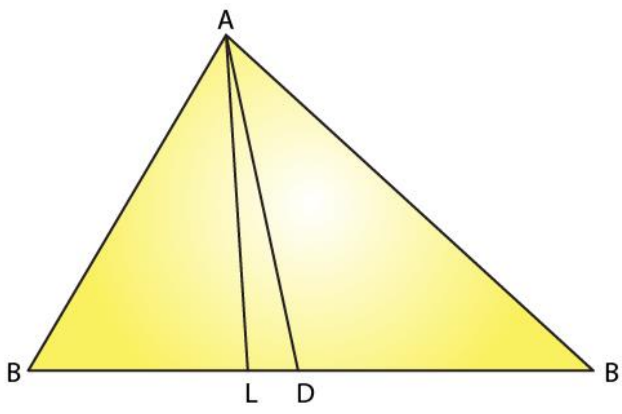
$\angle BEC = \angle A/2$

$\angle BEC = 50^o/2$

$= 25^o$

25. In $\triangle ABC$, $\angle B = 60°$, $\angle C = 40°$, $AL \perp BC$ and AD bisects $\angle A$ such that L and D lie on side BC. Find $\angle LAD$

Solution:



We know that the sum of all angles of a triangle is 180°

Consider $\triangle ABC$, we can write as

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 60^o + 40^o = 180^o$

$\angle A = 80^0$

But we know that $\angle DAC$ bisects $\angle A$

$\angle DAC = \angle A/2$

$\angle DAC = 80^o/2$

If we apply same steps for the $\triangle ADC$, we get

We know that the sum of all angles of a triangle is 180°

$\angle ADC + \angle DCA + \angle DAC = 180^o$

$\angle ADC + 40^o + 40^o = 180^o$

$\angle ADC = 180^o + 80^o$

We know that exterior angle is equal to the sum of two interior opposite angles

Therefore we have

$\angle ADC = \angle ALD + \angle LAD$

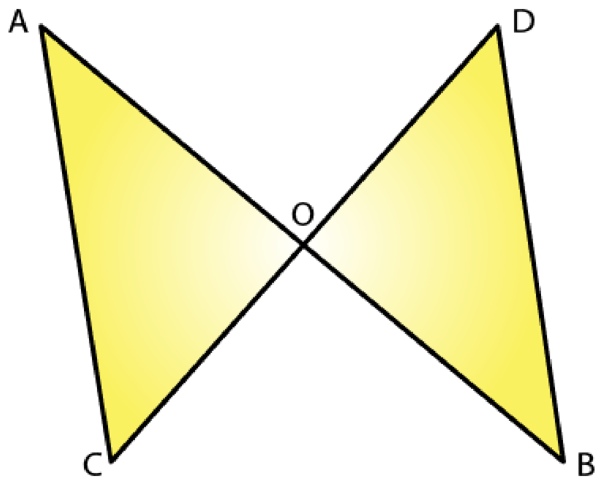
But here AL perpendicular to BC

$100^o = 90^o + \angle LAD$

$\angle LAD = 90^o$

26. Line segments AB and CD intersect at O such that AC $\parallel$ DB. It $\angle CAB = 35^o$ and $\angle CDB = 55^o$. Find $\angle BOD$.

Solution:



We know that AC parallel to BD and AB cuts AC and BD at A and B, respectively.

$\angle CAB = \angle DBA$ (Alternate interior angles)

$\angle DBA = 35^o$

We also know that the sum of all three angles of a triangle is 180^o.

Hence, for $\triangle OBD$, we can say that:

$\angle DBO + \angle ODB + \angle BOD = 180°$

$35^o + 55^o + \angle BOD = 180^o$ ($\angle DBO = \angle DBA$ and $\angle ODB = \angle CDB$)

$\angle BOD = 180^o – 90^o$

$\angle BOD = 90^o$

27. In Fig. 22, ΔABC is right angled at A, Q and R are points on line BC and P is a point such that QP ∥ AC and RP ∥ AB. Find $\angle P$

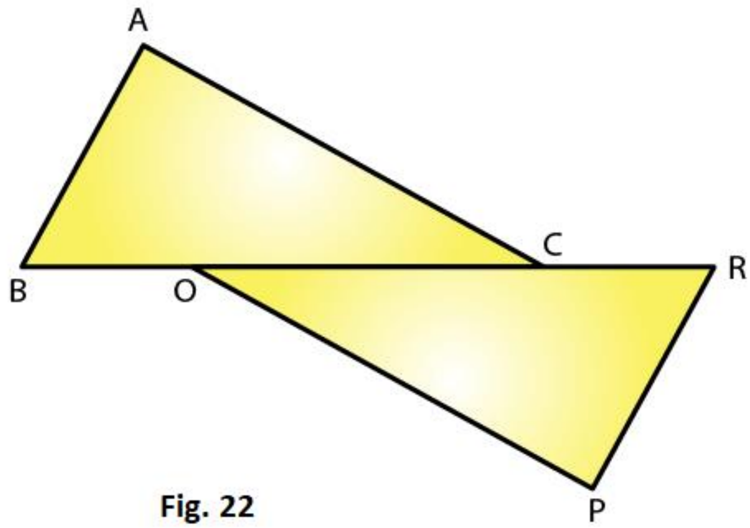


Fig. 22

Solution:

In the given triangle, AC parallel to QP and BR cuts AC and QP at C and Q, respectively.

∠QCA = ∠CQP (Alternate interior angles)

Because RP parallel to AB and BR cuts AB and RP at B and R, respectively,

∠ABC = ∠PRQ (alternate interior angles).

We know that the sum of all three angles of a triangle is 180^o.

Hence, for ∆ABC, we can say that:

∠ABC + ∠ACB + ∠BAC = 180^o

∠ABC + ∠ACB + 90^o = 180^o (Right angled at A)

∠ABC + ∠ACB = 90^o

Using the same logic for $\triangle PQR$, we can say that:

$\angle PQR + \angle PRQ + \angle QPR = 180^o$

$\angle ABC + \angle ACB + \angle QPR = 180^o$ ($\angle ABC = \angle PRQ$ and $\angle QCA = \angle CQP$)

Or,

$90^o + \angle QPR = 180^o$ ($\angle ABC + \angle ACB = 90^o$)

$\angle QPR = 90^o$

Exercise 15.3 Page No: 15.19

1. In Fig. 35, $\angle CBX$ is an exterior angle of $\triangle ABC$ at B. Name

(i) The interior adjacent angle

(ii) The interior opposite angles to exterior $\angle CBX$

Also, name the interior opposite angles to an exterior angle at A.

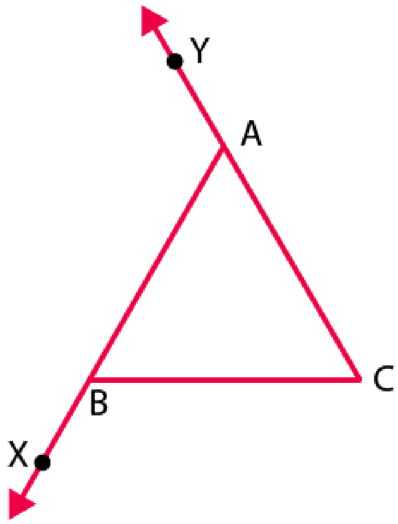


Fig. 35

Solution:

(i) The interior adjacent angle is ∠ABC

(ii) The interior opposite angles to exterior ∠CBX is ∠BAC and ∠ACB

Also the interior angles opposite to exterior are ∠ABC and ∠ACB

2. In the fig. 36, two of the angles are indicated. What are the measures of ∠ACX and ∠ACB?

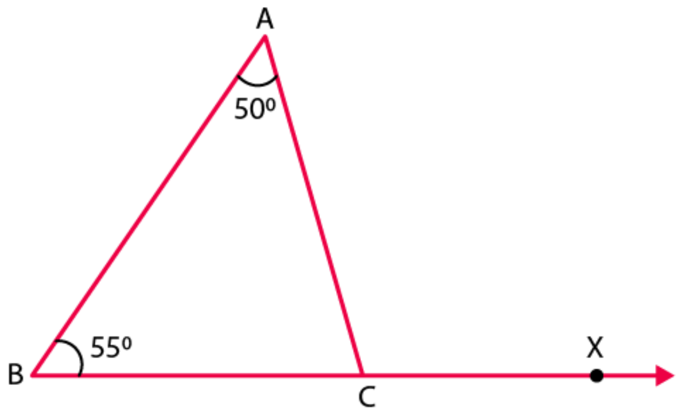


Fig. 36

Solution:

Given that in $\triangle ABC$, $\angle A = 50^o$ and $\angle B = 55^o$

We know that the sum of angles in a triangle is 180^o

Therefore we have

$\angle A + \angle B + \angle C = 180^o$

$50^o + 55^o + \angle C = 180^o$

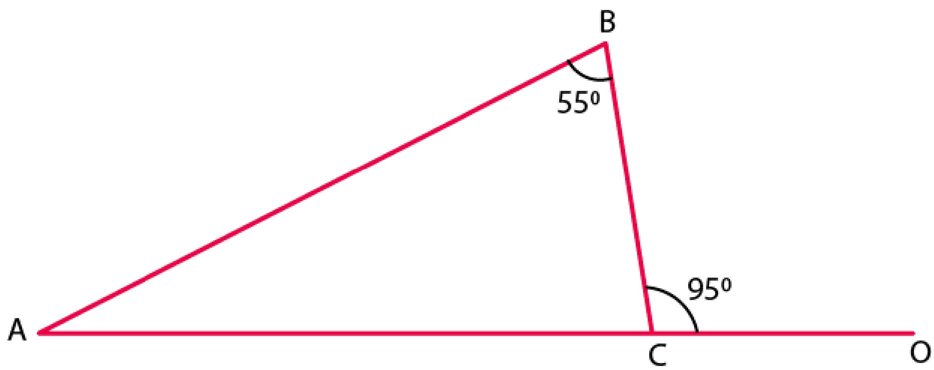
$\angle C = 75^o$

$\angle ACB = 75^o$

$\angle ACX = 180^o - \angle ACB = 180^o - 75^o = 105^o$

3. In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angles is 55°. Find all the angles of the triangle.

Solution:



We know that the sum of interior opposite angles is equal to the exterior angle.

Hence, for the given triangle, we can say that:

$\angle ABC + \angle BAC = \angle BCO$

$55^{o} + \angle BAC = 95^{o}$

$\angle BAC = 95^{o} – 95^{o}$

$\angle BAC = 40^{o}$

We also know that the sum of all angles of a triangle is 180^{o}.

Hence, for the given $\triangle ABC$, we can say that:

$\angle ABC + \angle BAC + \angle BCA = 180^{o}$

$55^{o} + 40^{o} + \angle BCA = 180^{o}$

$\angle BCA = 180^{o} – 95^{o}$

$\angle BCA = 85^{o}$

4. One of the exterior angles of a triangle is 80^{o}, and the interior opposite angles are

equal to each other. What is the measure of each of these two angles?

Solution:

Let us assume that A and B are the two interior opposite angles.

We know that $\angle A$ is equal to $\angle B$.

We also know that the sum of interior opposite angles is equal to the exterior angle.

Therefore from the figure we have,

$\angle A + \angle B = 80^\circ$

$\angle A + \angle A = 80^\circ$ (because $\angle A = \angle B$)

$2\angle A = 80^\circ$

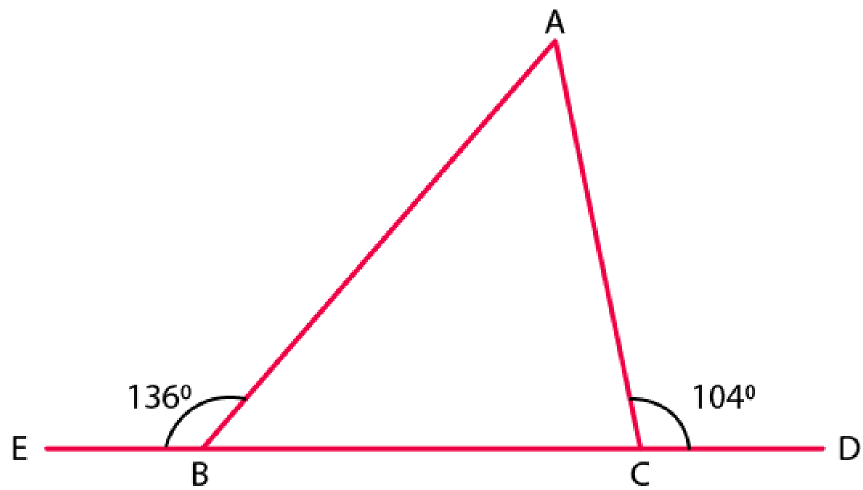
$\angle A = 40/2 = 40^\circ$

$\angle A = \angle B = 40^\circ$

Thus, each of the required angles is of 40°.

5. The exterior angles, obtained on producing the base of a triangle both ways are 104° and 136°. Find all the angles of the triangle.

Solution:



In the given figure, ∠ABE and ∠ABC form a linear pair.

$\angle ABE + \angle ABC = 180^o$

$\angle ABC = 180^o - 136^o$

$\angle ABC = 44^o$

We can also see that ∠ACD and ∠ACB form a linear pair.

$\angle ACD + \angle ACB = 180^o$

$\angle AUB = 180^o - 104^o$

$\angle ACB = 76^o$

We know that the sum of interior opposite angles is equal to the exterior angle.

Therefore, we can write as

$\angle BAC + \angle ABC = 104^o$

$\angle BAC = 104^o - 44^o = 60^o$

Thus,

$\angle ACE = 76^o$ and $\angle BAC = 60^o$

6. In Fig. 37, the sides BC, CA and BA of a $\triangle$ABC have been produced to D, E and F respectively. If $\angle ACD = 105^o$ and $\angle EAF = 45^o$; find all the angles of the $\triangle$ABC.

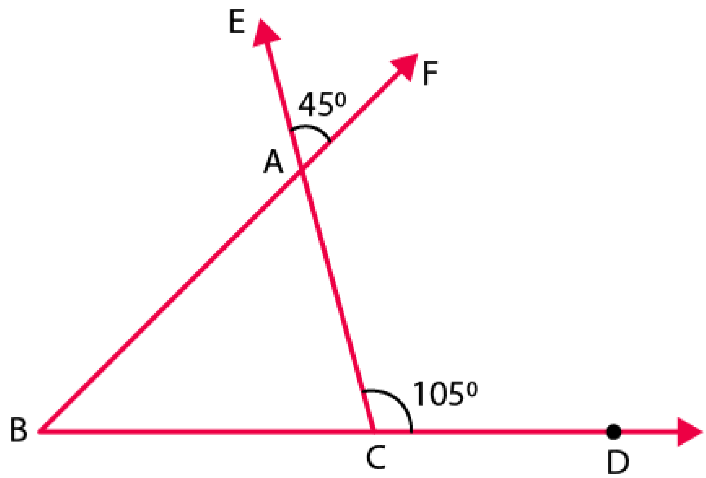


Fig. 37

Solution:

In a $\triangle$ABC, $\angle BAC$ and $\angle EAF$ are vertically opposite angles.

Hence, we can write as

$\angle BAC = \angle EAF = 45^o$

Considering the exterior angle property, we have

$\angle BAC + \angle ABC = \angle ACD = 105^o$

On rearranging we get

$\angle ABC = 105^o - 45^o = 60^o$

We know that the sum of angles in a triangle is 180º

∠ABC + ∠ACS +∠BAC = 180°

∠ACB = 75º

Therefore, the angles are 45º, 65º and 75º.

7. In Fig. 38, AC perpendicular to CE and C ∠A: ∠B: ∠C= 3: 2: 1. Find the value of ∠ECD.

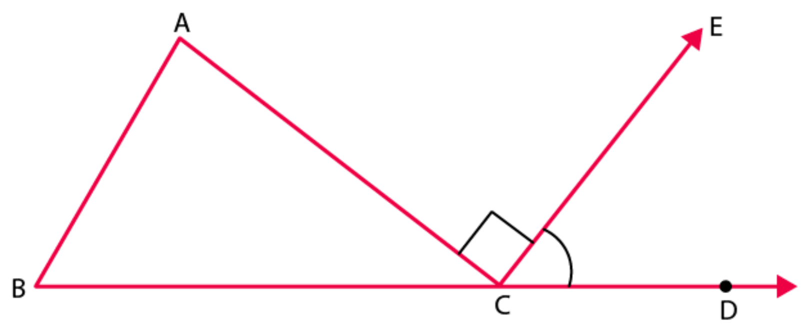


Fig. 38

Solution:

In the given triangle, the angles are in the ratio 3: 2: 1.

Let the angles of the triangle be 3x, 2x and x.

We know that sum of angles in a triangle is 180º

3x + 2x + x = 180º

6x = 180º

x = 30º

Also, ∠ACB + ∠ACE + ∠ECD = 180º

$x + 90^o + \angle ECD = 180^o$ ($\angle ACE = 90^o$)

We know that $x = 30^o$

Therefore

$\angle ECD = 60^o$

8. A student when asked to measure two exterior angles of △ABC observed that the exterior angles at A and B are of 103º and 74º respectively. Is this possible? Why or why not?

Solution:

We know that sum of internal and external angle is equal to 180^o

Internal angle at A + External angle at A = 180^o

Internal angle at A + 103^o =180^o

Internal angle at A = 77^o

Internal angle at B + External angle at B = 180^o

Internal angle at B + 74^o = 180^o

Internal angle at B = 106^o

Sum of internal angles at A and B = $77^o + 106^o = 183^o$

It means that the sum of internal angles at A and B is greater than 180^o, which cannot be possible.

9. In Fig.39, AD and CF are respectively perpendiculars to sides BC and AB of △ABC. If ∠FCD = 50º, find ∠BAD

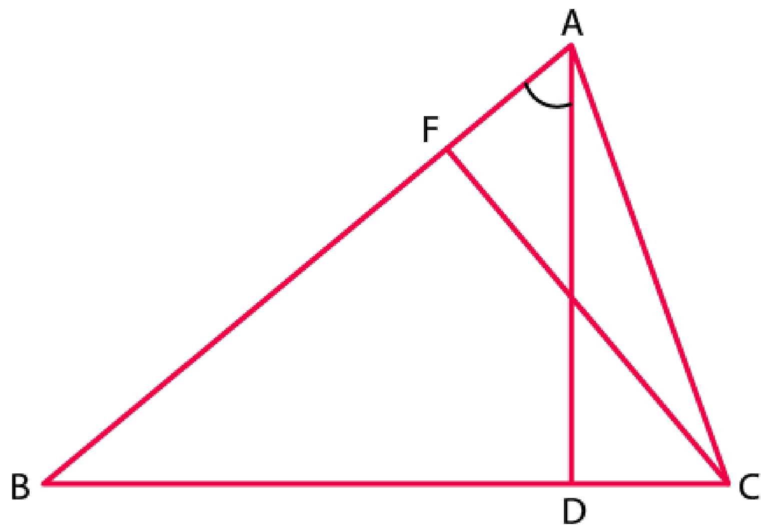


Fig. 39

Solution:

We know that the sum of all angles of a triangle is 180º

Therefore, for the given △FCB, we have

∠FCB + ∠CBF + ∠BFC = 180º

50º + ∠CBF + 90º = 180º

∠CBF = 180º – 50º – 90º = 40º

Using the above steps for △ABD, we can say that:

∠ABD + ∠BDA + ∠BAD = 180º

∠BAD = 180º – 90º – 40º = 50º

10. In Fig.40, measures of some angles are indicated. Find the value of x.

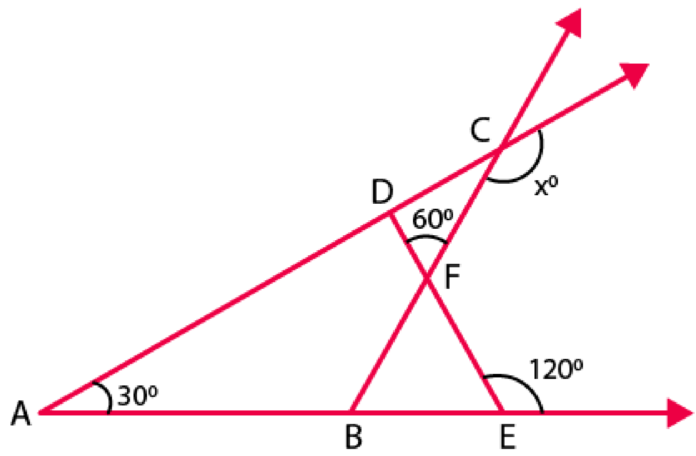


Fig. 40

Solution:

We know that the sum of the angles of a triangle is 180^{o}

From the figure we have,

$\angle AED + 120^{o} = 180^{o}$ (Linear pair)

$\angle AED = 180^{o} – 120^{o} = 60^{o}$

We know that the sum of all angles of a triangle is 180^{o}.

Therefore, for $\triangle ADE$, we have

$\angle ADE + \angle AED + \angle DAE = 180^{o}$

$60^{o} + \angle ADE + 30^{o} = 180^{o}$

$\angle ADE = 180^{o} – 60^{o} – 30^{o} = 90^{o}$

From the given figure, we have

$\angle FDC + 90^\circ = 180^\circ$ (Linear pair)

$\angle FDC = 180^\circ - 90^\circ = 90^\circ$

Using the same steps for $\triangle CDF$, we get

$\angle CDF + \angle DCF + \angle DFC = 180^\circ$

$90^\circ + \angle DCF + 60^\circ = 180^\circ$

$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Again from the figure we have

$\angle DCF + x = 180^\circ$ (Linear pair)

$30^\circ + x = 180^\circ$

$x = 180^\circ - 30^\circ = 150^\circ$

11. In Fig. 41, ABC is a right triangle right angled at A. D lies on BA produced and DE perpendicular to BC intersecting AC at F. If $\angle AFE = 130^\circ$, find

(i) $\angle BDE$

(ii) $\angle BCA$

(iii) $\angle ABC$

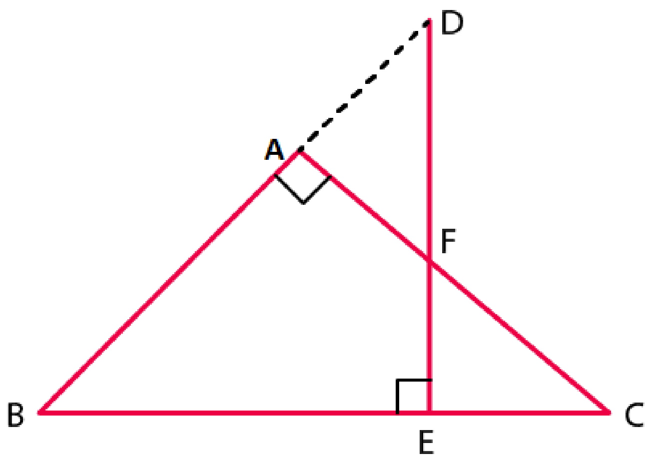


Fig. 41

Solution:

(i) Here,

$\angle BAF + \angle FAD = 180^o$ (Linear pair)

$\angle FAD = 180^o - \angle BAF = 180^o - 90^o = 90^o$

Also from the figure,

$\angle AFE = \angle ADF + \angle FAD$ (Exterior angle property)

$\angle ADF + 90^o = 130^o$

$\angle ADF = 130^o - 90^o = 40^o$

(ii) We know that the sum of all the angles of a triangle is 180^o.

Therefore, for $\triangle BDE$, we have

$\angle BDE + \angle BED + \angle DBE = 180^o$

$\angle DBE = 180^o - \angle BDE$

$\angle BED = 180^o - 90^o - 40^o = 50^o$ …. Equation (i)

Again from the figure we have,

$\angle FAD = \angle ABC + \angle ACB$ (Exterior angle property)

$90^o = 50^o + \angle ACB$

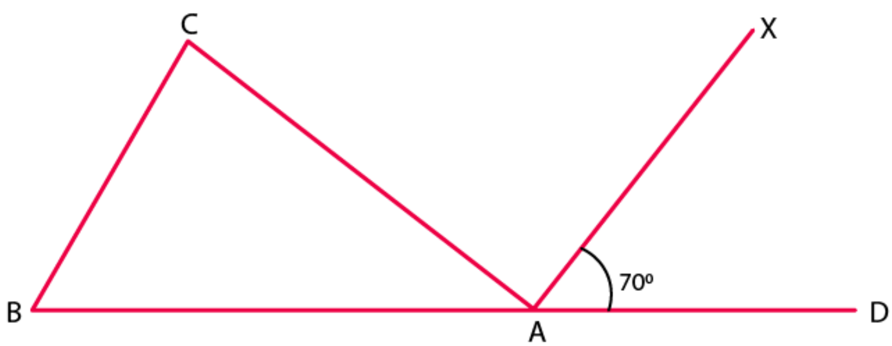
$\angle ACB = 90^o - 50^o = 40^o$

(iii) From equation we have

$\angle ABC = \angle DBE = 50^o$

12. ABC is a triangle in which $\angle B = \angle C$ and ray AX bisects the exterior angle DAC. If $\angle DAX = 70^o$. Find $\angle ACB$.

Solution:



Given that ABC is a triangle in which $\angle B = \angle C$

Also given that AX bisects the exterior angle DAC

$\angle CAX = \angle DAX$ (AX bisects $\angle CAD$)

$\angle CAX = 70^o$ [given]

$\angle CAX + \angle DAX + \angle CAB = 180^o$

$70^o + 70^o + \angle CAB = 180^o$

$\angle CAB = 180^o - 140^o$

$\angle CAB = 40^o$

$\angle ACB + \angle CBA + \angle CAB = 180^o$ (Sum of the angles of ΔABC)

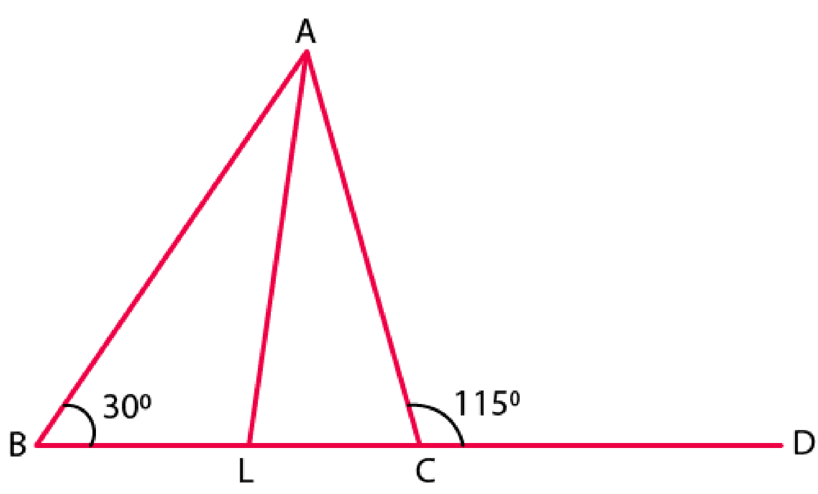
$\angle ACB + \angle ACB + 40^o = 180^o$ ($\angle C = \angle B$)

$2\angle ACB = 180^o - 40^o$

$\angle ACB = 140/2$

$\angle ACB = 70^o$

13. The side BC of $\triangle ABC$ is produced to a point D. The bisector of $\angle A$ meets side BC in L. If $\angle ABC = 30^o$ and $\angle ACD = 115^o$, find $\angle ALC$



Solution:

Given that $\angle ABC = 30^o$ and $\angle ACD = 115^o$

From the figure, we have

$\angle ACD$ and $\angle ACL$ make a linear pair.

$\angle ACD + \angle ACB = 180^o$

$115^o + \angle ACB = 180^o$

$\angle ACB = 180^o - 115^o$

$\angle ACB = 65^o$

We know that the sum of all angles of a triangle is 180^o.

Therefore, for $\triangle$ ABC, we have

$\angle ABC + \angle BAC + \angle ACB = 180^o$

$30^o + \angle BAC + 65^o = 180^o$

$\angle BAC = 85^o$

$\angle LAC = \angle BAC/2 = 85/2$

Using the same steps for $\triangle ALC$, we get

$\angle ALC + \angle LAC + \angle ACL = 180^o$

$\angle ALC + 82/2 + 65^o = 180^o$

We know that $\angle ALC = \angle ACB$

$\angle ALC = 180^o - 82/2 - 65^o$

$\angle ALC = 72\ ½^o$

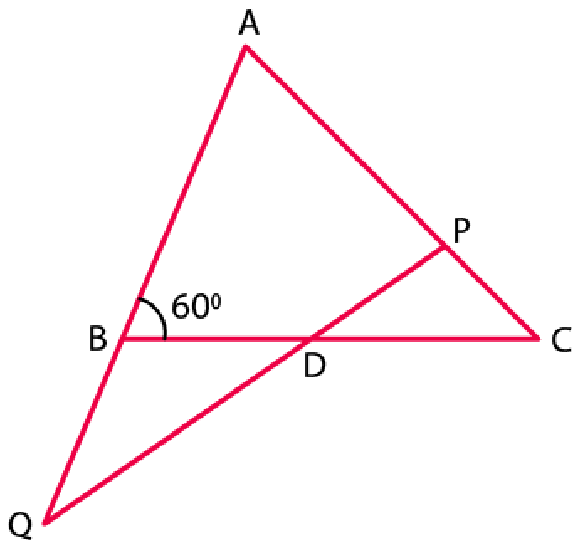
14. D is a point on the side BC of $\triangle$ABC. A line PDQ through D, meets side AC in P and

AB produced at Q. If ∠A = 80º, ∠ABC = 60º and ∠PDC = 15º, find

(i) ∠AQD

(ii) ∠APD

Solution:



From the figure we have

∠ABD and ∠QBD form a linear pair.

∠ABC + ∠QBC =180º

60º + ∠QBC = 180º

∠QBC = 120º

∠PDC = ∠BDQ (Vertically opposite angles)

∠BDQ = 75º

In △QBD:

$\angle QBD + \angle QDB + \angle BDQ = 180^o$ (Sum of angles of $\triangle QBD$)

$120^o + 15^o + \angle BQD = 180^o$

$\angle BQD = 180^o - 135^o$

$\angle BQD = 45^o$

$\angle AQD = \angle BQD = 45^o$

In $\triangle AQP$:

$\angle QAP + \angle AQP + \angle APQ = 180^o$ (Sum of angles of $\triangle AQP$)

$80^o + 45^o + \angle APQ = 180^o$

$\angle APQ = 55^o$

$\angle APD = \angle APQ$

15. Explain the concept of interior and exterior angles and in each of the figures given below. Find x and y (Fig. 42)

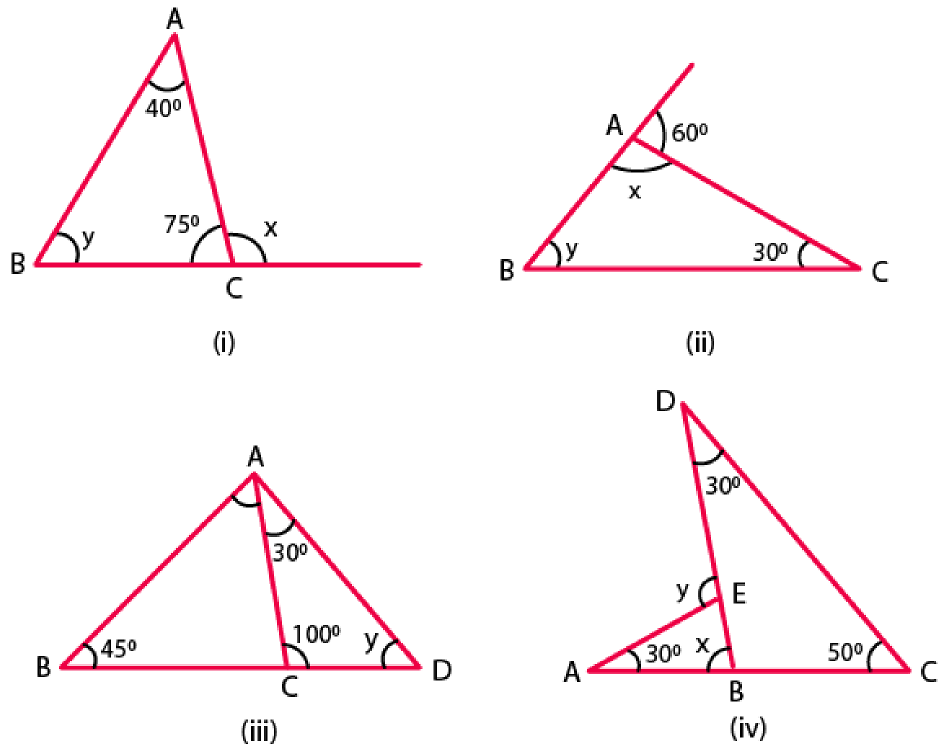


Fig. 42

Solution:

The interior angles of a triangle are the three angle elements inside the triangle.

The exterior angles are formed by extending the sides of a triangle, and if the side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Using these definitions, we will obtain the values of x and y.

(i) From the given figure, we have

$\angle ACB + x = 180^o$ (Linear pair)

$75^o + x = 180^o$

$x = 105^o$

We know that the sum of all angles of a triangle is 180^o

Therefore, for △ABC, we can say that:

$\angle BAC + \angle ABC + \angle ACB = 180^o$

$40^o + y + 75^o = 180^o$

$y = 65^o$

(ii) From the figure, we have

$x + 80^o = 180^o$ (Linear pair)

$x = 100^o$

In △ABC, we have

We also know that the sum of angles of a triangle is 180^o

$x + y + 30^o = 180^o$

$100^o + 30^o + y = 180^o$

$y = 50^o$

(iii) We know that the sum of all angles of a triangle is 180°.

Therefore, for △ACD, we have

$30^o + 100^o + y = 180^o$

$y = 50^o$

Again from the figure we can write as

$\angle ACB + 100° = 180°$

$\angle ACB = 80^{o}$

Using the above rule for $\triangle ACD$, we can say that:

$x + 45^{o} + 80^{o} = 180^{o}$

$x = 55^{o}$

(iv) We know that the sum of all angles of a triangle is 180^{o}.

Therefore, for $\triangle DBC$, we have

$30^{o} + 50^{o} + \angle DBC = 180^{o}$

$\angle DBC = 100^{o}$

From the figure we can say that

$x + \angle DBC = 180^{o}$ is a Linear pair

$x = 80^{o}$

From the exterior angle property we have

$y = 30^{o} + 80^{o} = 110^{o}$

16. Compute the value of x in each of the following figures:

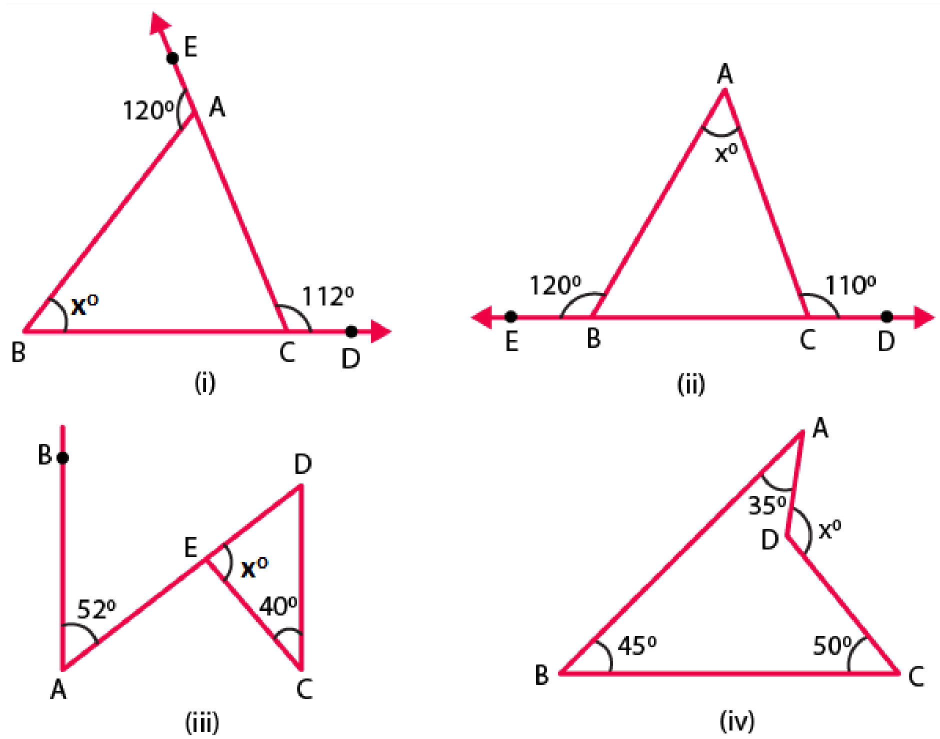


Fig. 43

Solution:

(i) From the given figure, we can write as

∠ACD + ∠ACB = 180° is a linear pair

On rearranging we get

∠ACB = 180° – 112° = 68°

Again from the figure we have,

∠BAE + ∠BAC = 180° is a linear pair

On rearranging we get,

∠BAC = 180°– 120° = 60°

We know that the sum of all angles of a triangle is 180^o.

Therefore, for $\triangle ABC$:

$x + \angle BAC + \angle ACB = 180^o$

$x = 180^o - 60^o - 68^o = 52^o$

$x = 52^o$

(ii) From the given figure, we can write as

$\angle ABC + 120^o = 180^o$ is a linear pair

$\angle ABC = 60^o$

Again from the figure we can write as

$\angle ACB + 110^o = 180^o$ is a linear pair

$\angle ACB = 70^o$

We know that the sum of all angles of a triangle is 180^o.

Therefore, consider $\triangle ABC$, we get

$x + \angle ABC + \angle ACB = 180^o$

$x = 50^o$

(iii) From the given figure, we can write as

$\angle BAD = \angle ADC = 52^o$ are alternate angles

We know that the sum of all the angles of a triangle is 180^o.

Therefore, consider $\triangle DEC$, we have

$x + 40^o + 52^o = 180^o$

$x = 88^o$

(iv) In the given figure, we have a quadrilateral and also we know that sum of all angles is quadrilateral is 360^o.

Thus,

$35^o + 45^o + 50^o +$ reflex $\angle ADC = 360^o$

On rearranging we get,

Reflex $\angle ADC = 230^o$

$230^o + x = 360^o$ (A complete angle)

$x = 130^o$

Exercise 15.4 Page No: 15.24

1. In each of the following, there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle:

(i) 5, 7, 9

(ii) 2, 10, 15

(iii) 3, 4, 5

(iv) 2, 5, 7

(v) 5, 8, 20

Solution:

(i) Given 5, 7, 9

Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side.

Here, $5 + 7 > 9$, $5 + 9 > 7$, $9 + 7 > 5$

(ii) Given 2, 10, 15

No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

(iii) Given 3, 4, 5

Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of triangle is always greater than the third side.

Here, $3 + 4 > 5$, $3 + 5 > 4$, $4 + 5 > 3$

(iv) Given 2, 5, 7

No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

Here, $2 + 5 = 7$

(v) Given 5, 8, 20

No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

Here, 5 + 8 < 20

2. In Fig. 46, P is the point on the side BC. Complete each of the following statements using symbol ‘=’,’ > ‘or ‘< ‘so as to make it true:

(i) AP… AB+ BP

(ii) AP… AC + PC

(iii) AP…. ½ (AB + AC + BC)

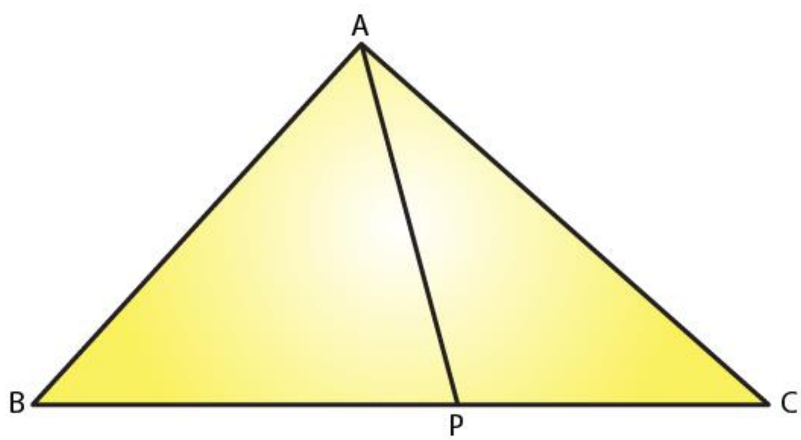


Fig. 46

Solution:

(i) In △APB, AP < AB + BP because the sum of any two sides of a triangle is greater than the third side.

(ii) In △APC, AP < AC + PC because the sum of any two sides of a triangle is greater than the third side.

(iii) AP < ½ (AB + AC + BC)

In △ABP and △ACP, we can write as

AP < AB + BP… (i) (Because the sum of any two sides of a triangle is greater than the third side)

AP < AC + PC … (ii) (Because the sum of any two sides of a triangle is greater than the third side)

On adding (i) and (ii), we have:

AP + AP < AB + BP + AC + PC

2AP < AB + AC + BC (BC = BP + PC)

AP < (AB – AC + BC)

3. P is a point in the interior of △ABC as shown in Fig. 47. State which of the following statements are true (T) or false (F):

(i) AP + PB < AB

(ii) AP + PC > AC

(iii) BP + PC = BC

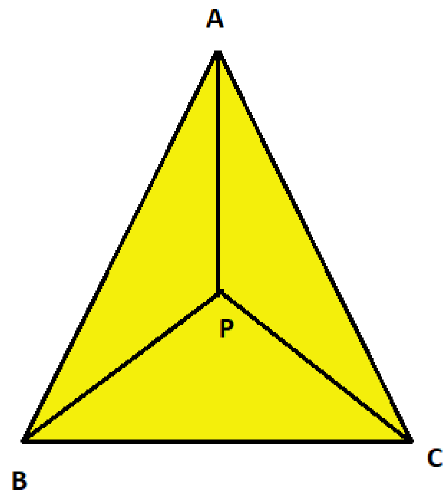


Fig. 47

Solution:

(i) False

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is not true for the given triangle.

(ii) True

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is true for the given triangle.

(iii) False

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is not true for the given triangle.

4. O is a point in the exterior of △ABC. What symbol '>','<' or '=' will you see to complete the statement OA+OB….AB? Write two other similar statements and show that OA + OB + OC > ½ (AB + BC +CA)

Solution:

We know that the sum of any two sides of a triangle is always greater than the third side, in △OAB, we have,

OA + OB > AB ….. (i)

OB + OC > BC …… (ii)

OA + OC > CA ….. (iii)

On adding equations (i), (ii) and (iii) we get:

OA + OB + OB + OC + OA + OC > AB + BC + CA

2(OA + OB + OC) > AB + BC + CA

OA + OB + OC > (AB + BC + CA)/2

Or

OA + OB + OC > ½ (AB + BC +CA)

Hence the proof.

5. In △ABC, ∠A = 100º, ∠B = 30º, ∠C = 50º. Name the smallest and the largest sides of the triangle.

Solution:

We know that the smallest side is always opposite to the smallest angle, which in this case is 30°, it is AC.

Also, because the largest side is always opposite to the largest angle, which in this case is 100°, it is BC.

Exercise 15.5 Page No: 15.30

1. State Pythagoras theorem and its converse.

Solution:

The Pythagoras Theorem:

In a right triangle, the square of the hypotenuse is always equal to the sum of the squares of the other two sides.

Converse of the Pythagoras Theorem:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle, with the angle opposite to the first side as right angle.

2. In right △ABC, the lengths of the legs are given. Find the length of the hypotenuse

(i) a = 6 cm, b = 8 cm

(ii) a = 8 cm, b = 15 cm

(iii) a = 3 cm, b = 4 cm

(iv) a = 2 cm, b =1.5 cm

Solution:

(i) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 6^2 + 8^2$

$c^2 = 36 + 64 = 100$

c = 10 cm

(ii) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 8^2 + 15^2$

$c^2 = 64 + 225 = 289$

c = 17cm

(iii) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16 = 25$

$c = 5$ cm

(iv) According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

Let c be hypotenuse and a and b be other two legs of right angled triangle

Then we have

$c^2 = a^2 + b^2$

$c^2 = 2^2 + 1.5^2$

$c^2 = 4 + 2.25 = 6.25$

$c = 2.5$ cm

3. The hypotenuse of a triangle is 2.5 cm. If one of the sides is 1.5 cm. find the length of the other side.

Solution:

Let c be hypotenuse and the other two sides be b and a

According to the Pythagoras theorem, we have

$c^2 = a^2 + b^2$

$2.52 = 1.52 + b^2$

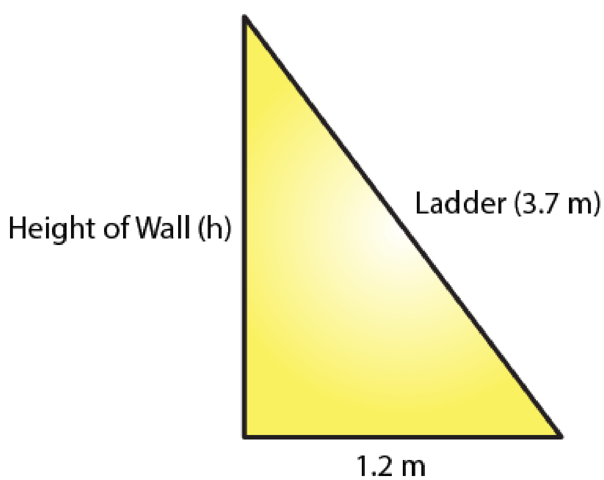
$b^2 = 6.25 - 2.25 = 4$

b = 2 cm

Hence, the length of the other side is 2 cm.

4. A ladder 3.7 m long is placed against a wall in such a way that the foot of the ladder is 1.2 m away from the wall. Find the height of the wall to which the ladder reaches.

Solution:



Let the height of the ladder reaches to the wall be h.

According to the Pythagoras theorem, we have

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

$3.72 = 1.22 + h^2$

$h^2 = 13.69 – 1.44 = 12.25$

$h = 3.5$ m

Hence, the height of the wall is 3.5 m.

5. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is right-angled triangle.

Solution:

In the given triangle, the largest side is 6 cm.

We know that in a right angled triangle, the sum of the squares of the smaller sides should be equal to the square of the largest side.

Therefore,

$3^2 + 4^2 = 9 + 16 = 25$

But, $6^2 = 36$

$3^2 + 4^2 = 25$ which is not equal to 6^2

Hence, the given triangle is not a right angled triangle.

6. The sides of certain triangles are given below. Determine which of them are right triangles.

(i) a = 7 cm, b = 24 cm and c= 25 cm

(ii) a = 9 cm, b = 16 cm and c = 18 cm

Solution:

(i) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

Here, the larger side is c, which is 25 cm.

$c^2 = 625$

Given that,

$a^2 + b^2 = 7^2 + 24^2$

$= 49 + 576$

$= 625$

$= c^2$

Thus, the given triangle is a right triangle.

(ii) We know that in a right angled triangle, the square of the largest side is equal to the sum of the squares of the smaller sides.

Here, the larger side is c, which is 18 cm.

$c^2 = 324$

Given that

$a^2 + b^2 = 9^2 + 16^2$

$= 81 + 256$

$= 337$ which is not equal to c^2

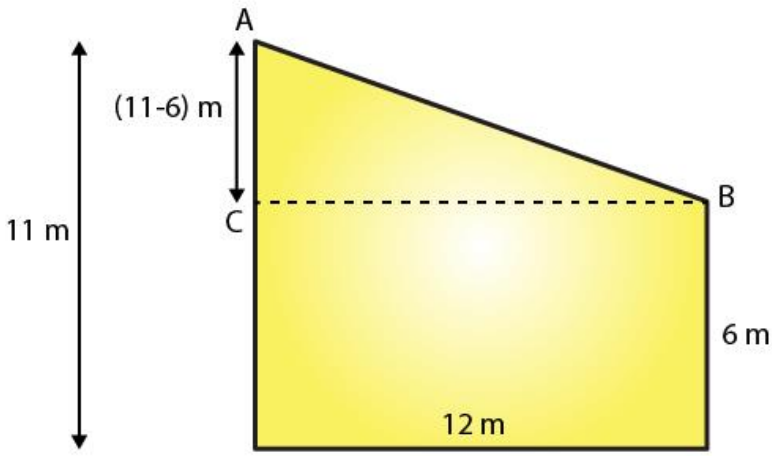
Thus, the given triangle is not a right triangle.

7. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between

their feet is 12 m. Find the distance between their tops.

(Hint: Find the hypotenuse of a right triangle having the sides (11 – 6) m = 5 m and 12 m)

Solution:



Let the distance between the tops of the poles is the distance between points A and B.

We can see from the given figure that points A, B and C form a right triangle, with AB as the hypotenuse.

By using the Pythagoras Theorem in △ABC, we get

$(11-6)^2 + 12^2 = AB^2$

$AB^2 = 25 + 144$

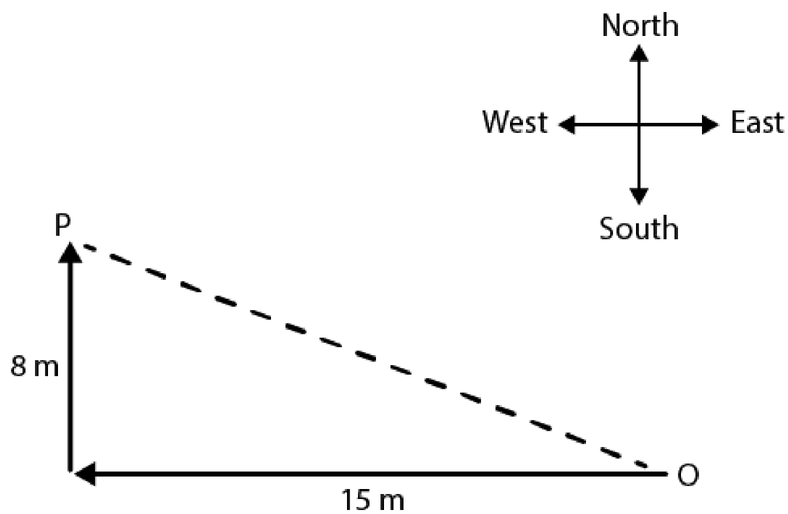
$AB^2 = 169$

$AB = 13$

Hence, the distance between the tops of the poles is 13 m.

8. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

Solution:



Given a man goes 15 m due west and then 8 m due north

Let O be the starting point and P be the final point.

Then OP becomes the hypotenuse in the triangle.

So by using the Pythagoras theorem, we can find the distance OP.

$OP^2 = 15^2 + 8^2$

$OP^2 = 225 + 64$

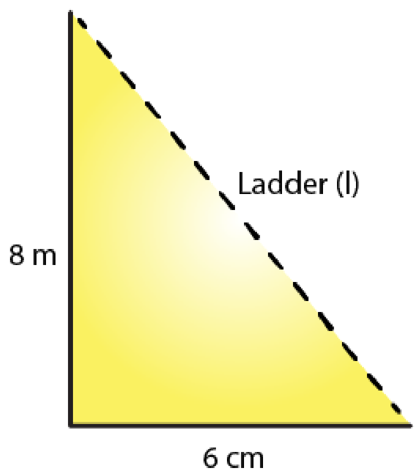
$OP^2 = 289$

OP = 17

Hence, the required distance is 17 m.

9. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach?

Solution:



Given Let the length of the ladder be L m.

By using the Pythagoras theorem, we can find the length of the ladder.

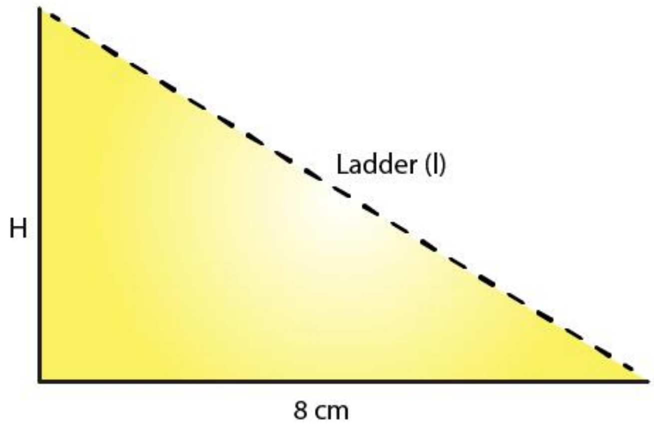
$6^2 + 8^2 = L^2$

$L^2 = 36 + 64 = 100$

L = 10

Thus, the length of the ladder is 10 m.

When ladder is shifted,



Let the height of the ladder after it is shifted be H m.

By using the Pythagoras theorem, we can find the height of the ladder after it is shifted.

$8^2 + H^2 = 10^2$

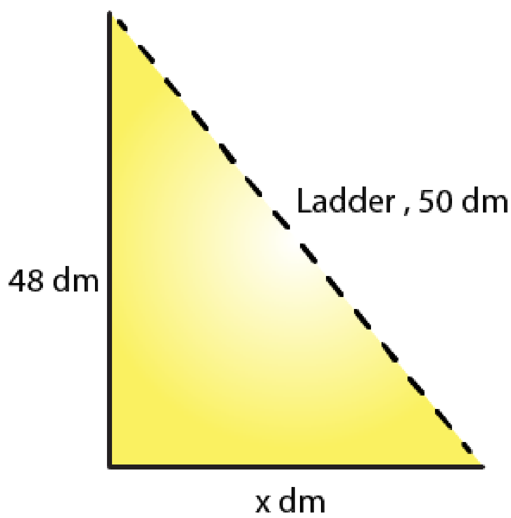
$H^2 = 100 – 64 = 36$

$H = 6$

Thus, the height of the ladder is 6 m.

10. A ladder 50 dm long when set against the wall of a house just reaches a window at a height of 48 dm. How far is the lower end of the ladder from the base of the wall?

Solution:



Given that length of a ladder is 50dm

Let the distance of the lower end of the ladder from the wall be x m.

By using the Pythagoras theorem, we get

$x^2 + 48^2 = 50^2$

$x^2 = 50^2 - 48^2$

= 2500 – 2304

= 196

H = 14 dm

Hence, the distance of the lower end of the ladder from the wall is 14 dm.

11. The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.

Solution:

According to the Pythagoras theorem, we have

$(Hypotenuse)^2 = (Base)^2 + (Height)^2$

Given that the two legs of a right triangle are equal and the square of the hypotenuse, which is 50

Let the length of each leg of the given triangle be x units.

Using the Pythagoras theorem, we get

$x^2 + x^2 = (Hypotenuse)^2$

$x^2 + x^2 = 50$

$2x^2 = 50$

$x^2 = 25$

$x = 5$

Hence, the length of each leg is 5 units.

12. Verity that the following numbers represent Pythagorean triplet:

(i) 12, 35, 37

(ii) 7, 24, 25

(iii) 27, 36, 45

(iv) 15, 36, 39

Solution:

(i) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$37^2 = 1369$

$12^2 + 35^2 = 144 + 1225 = 1369$

$12^2 + 35^2 = 37^2$

Yes, they represent a Pythagorean triplet.

(ii) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$25^2 = 625$

$7^2 + 24^2 = 49 + 576 = 625$

$7^2 + 24^2 = 25^2$

Yes, they represent a Pythagorean triplet.

(iii) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$45^2 = 2025$

$27^2 + 36^2 = 729 + 1296 = 2025$

$27^2 + 36^2 = 45^2$

Yes, they represent a Pythagorean triplet.

(iv) The condition for Pythagorean triplet is the square of the largest side is equal to the sum of the squares of the other two sides.

$39^2 = 1521$

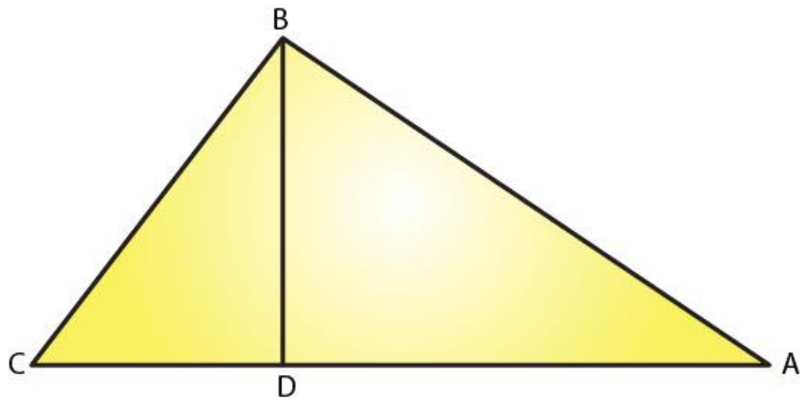
$15^2 + 36^2 = 225 + 1296 = 1521$

$15^2 + 36^2 = 39^2$

Yes, they represent a Pythagorean triplet.

13. In $\triangle ABC$, $\angle ABC = 100°$, $\angle BAC = 35°$ and $BD \perp AC$ meets side AC in D. If BD = 2 cm, find $\angle C$, and length DC.

Solution:



We know that the sum of all angles of a triangle is 180^o

Therefore, for the given $\triangle ABC$, we can say that:

$\angle ABC + \angle BAC + \angle ACB = 180^o$

$100^o + 35^o + \angle ACB = 180^o$

$\angle ACB = 180^o - 135^o$

$\angle ACB = 45^o$

$\angle C = 45^o$

On applying same steps for the $\triangle BCD$, we get

$\angle BCD + \angle BDC + \angle CBD = 180^o$

$45^o + 90^o + \angle CBD = 180^o$ ($\angle ACB = \angle BCD$ and BD parallel to AC)

$\angle CBD = 180^\circ - 135^\circ$

$\angle CBD = 45^\circ$

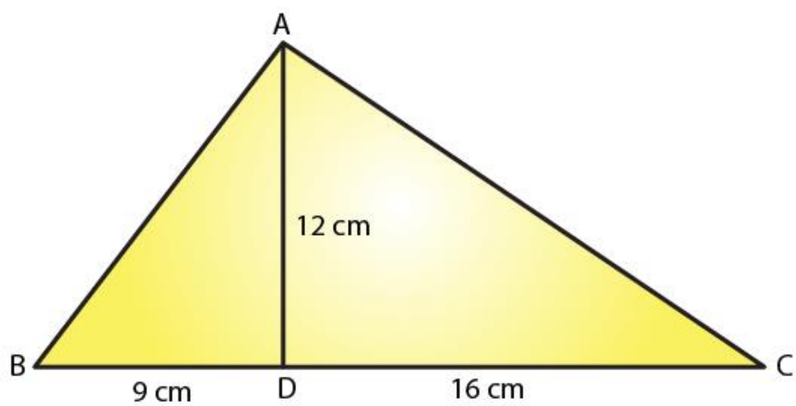
We know that the sides opposite to equal angles have equal length.

Thus, BD = DC

DC = 2 cm

14. In a △ABC, AD is the altitude from A such that AD = 12 cm. BD = 9 cm and DC = 16 cm. Examine if △ABC is right angled at A.

Solution:



Consider △ADC,

$\angle ADC = 90^\circ$ (AD is an altitude on BC)

Using the Pythagoras theorem, we get

$12^2 + 16^2 = AC^2$

$AC^2 = 144 + 256$

$= 400$

AC = 20 cm

Again consider $\triangle ADB$,

$\angle ADB = 90^o$ (AD is an altitude on BC)

Using the Pythagoras theorem, we get

$12^2 + 9^2 = AB^2$

$AB^2 = 144 + 81 = 225$

AB = 15 cm

Consider $\triangle ABC$,

$BC^2 = 25^2 = 625$

$AB^2 + AC^2 = 15^2 + 20^2 = 625$

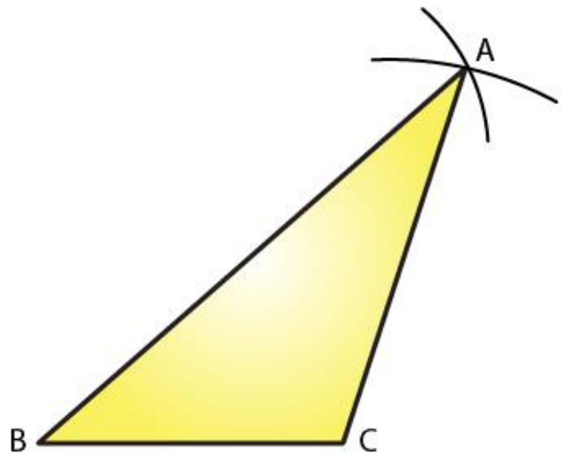
$AB^2 + AC^2 = BC^2$

Because it satisfies the Pythagoras theorem, therefore $\triangle ABC$ is right angled at A.

15. Draw a triangle ABC, with AC = 4 cm, BC = 3 cm and $\angle C = 105^o$. Measure AB. Is $(AB)^2 = (AC)^2 + (BC)^2$? If not which one of the following is true:

$(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$?

Solution:



Draw △ABC as shown in the figure with following steps.

Draw a line BC = 3 cm.

At point C, draw a line at 105° angle with BC.

Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB, which will be approximately 5.5 cm.

$AC^2 + BC^2 = 4^2 + 3^2$

$= 9 + 16$

$= 25$

$AB^2 = 5.52 = 30.25$

AB^2 is not equal to $AC^2 + BC^2$

Therefore we have

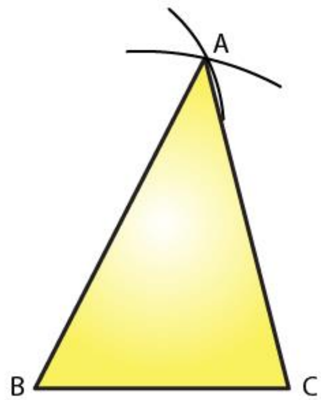
$AB^2 > AC^2 + BC^2$

16. Draw a triangle ABC, with AC = 4 cm, BC = 3 cm and ∠C = 80°. Measure AB. Is $(AB)^2$ =

$(AC)^2 + (BC)^2$? If not which one of the following is true:

$(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$?

Solution:



Draw $\triangle ABC$ as shown in the figure with following steps.

Draw a line BC = 3 cm.

At point C, draw a line at 80° angle with BC.

Take an arc of 4 cm from point C, which will cut the line at point A.

Now, join AB, it will be approximately 4.5 cm.

$AC^2 + BC^2 = 4^2 + 3^2$

$= 9 + 16$

$= 25$

$AB^2 = 4.5^2$

$= 20.25$

AB^2 not equal to $AC^2 + BC^2$

Therefore here $AB^2 < AC^2 + BC^2$