RD SHARMA Solutions for Class 9 Maths Chapter 15 - Circles

Chapter 15 - Circles Exercise 15.109

Question 1

If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then the radius of the circle is

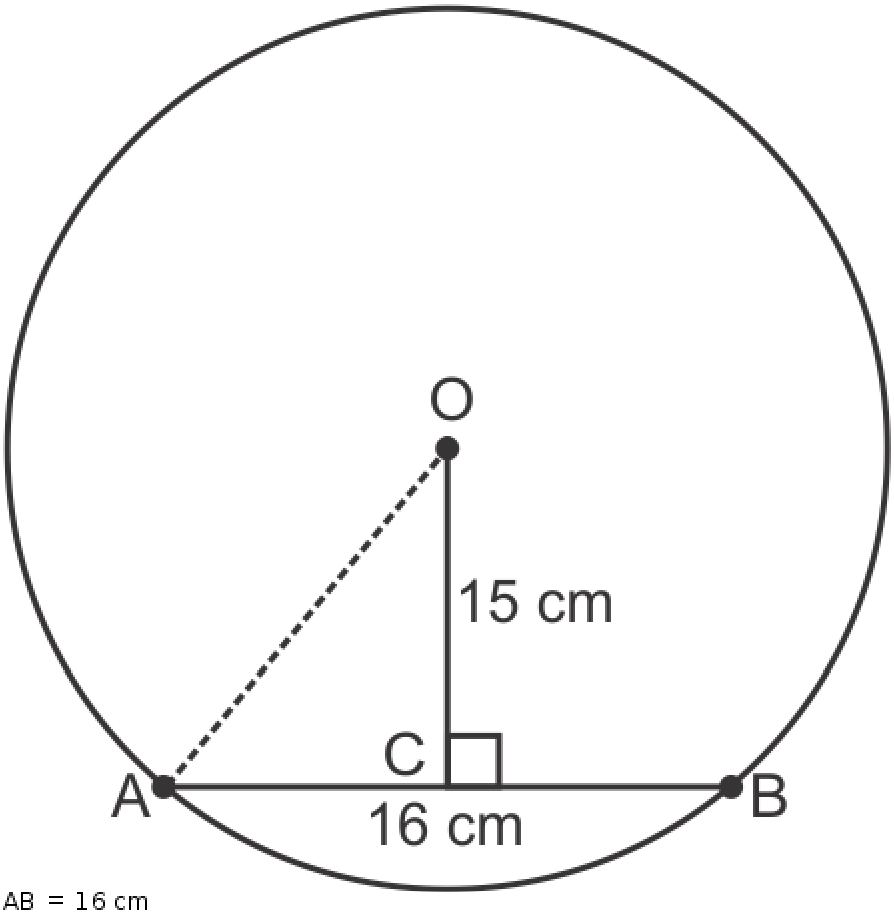
(a) 15 cm

(b) 16 cm

(c) 17 an

(d) 34 cm

Solution 1



AB = 16 cm

OC = 15 cm

C is the mid – point of AB.

$AC = BC = \frac{16}{2} = 8\text{ cm}$

Consider $\triangle OCA$,

OC = 15 cm, AC = 8 cm

$\Rightarrow OA = \sqrt{(15)^2 + (8)^2} = \sqrt{225 + 64} = \sqrt{289}$

$\Rightarrow OA = 17\text{ cm}$

Hence, correct option is (c).

Question 2

The radius of a circle is 6 cm. The perpendicular distance from the centre of the circle to the chord which is 8 cm in length, is

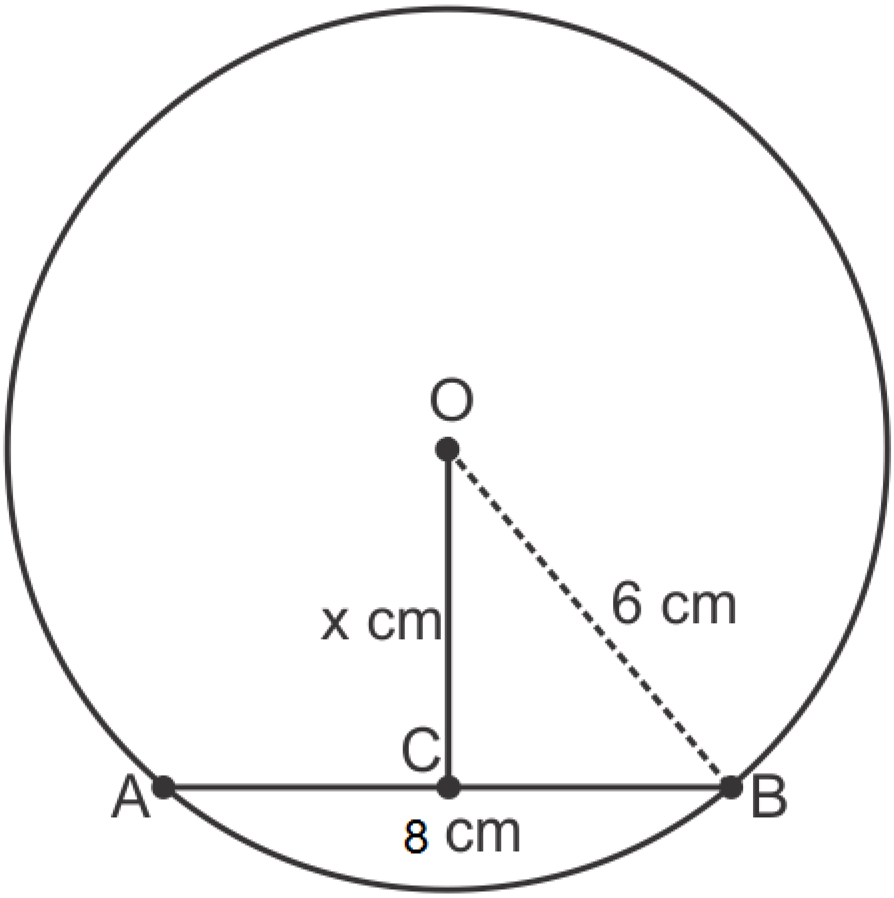
(a) $\sqrt{5}$ cm

(b) $2\sqrt{5}$ cm

(c) $2\sqrt{7}$ cm

(d) $\sqrt{7}$ cm

Solution 2



AB = 8 cm

$\Rightarrow$ AC = BC = 4 cm

Consider $\triangle$OCB, where BC = 8 cm, OB = 6 cm

Now, $(OC)^2 + (BC)^2 = (OB)^2$

$\Rightarrow (OC)^2 + 4^2 = 6^2$

$\Rightarrow (OC)^2 + 16 = 36$

$\Rightarrow (OC)^2 = 20$

$\Rightarrow OC = \sqrt{20} = 2\sqrt{5}$

Hence, correct option is (b).

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Question 3

If O is the centre of a circle of radius r and AB is a chord of the circle at a distance r/2 from O, then $\angle BAO$ =

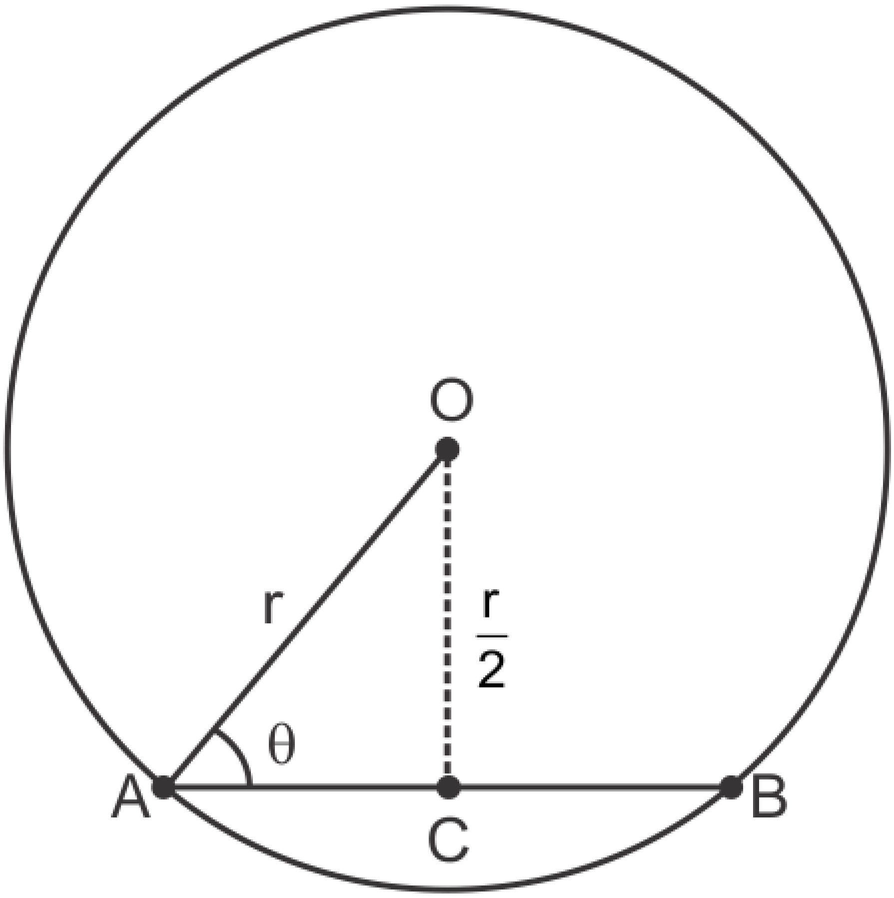
(a) 60°

(b) 45°

(c) 30°

(d) 15°

Solution 3



Let $\angle BAO = \theta$

Consider $\triangle OAC$,

$\sin\theta = \frac{OC}{OA} = \frac{r/2}{r} = \frac{1}{2} = \sin 30°$

$\Rightarrow \theta = 30°$

Hence, correct option is (c).

Question 4

ABCD is a cyclic quadrilateral such that $\angle ADB = 30°$ and $\angle DCA = 80°$, then $\angle DAB =$

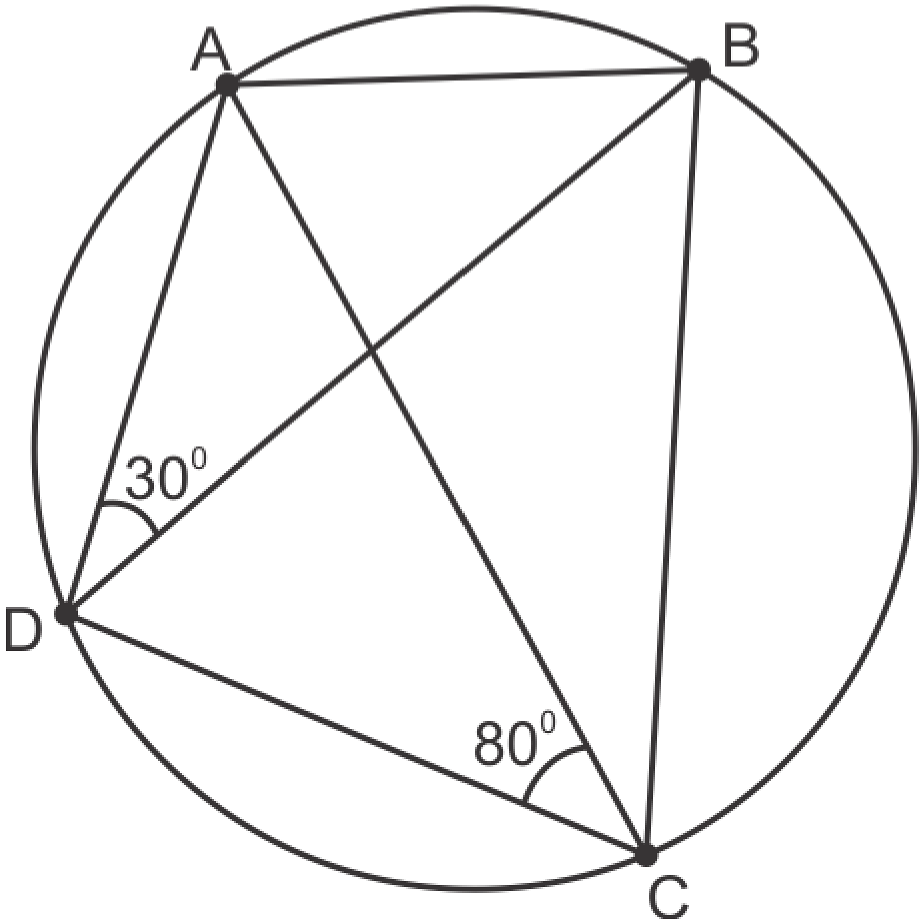
(a) 70°

(b) 100°

(c) 125°

(d) 150°

Solution 4



ABCD is a cyclic Quadrilateral.
Consider $\triangle ABD$ and $\triangle ABC$.
Both are on the same base AB and $\angle ADB$ and $\angle ACB$ are the angles in the same segment AB.

$\Rightarrow \angle ADB = \angle ACB = 30°$

$\Rightarrow \angle BCD = 80° + 30° = 110°$

In a cycle Quadrilateral, sum of opposite angles is 180°.

$\Rightarrow \angle A + \angle C = 180°$

$\Rightarrow \angle DAB + \angle BCD = 180°$

$\Rightarrow \angle DAB = 180° - 110° = 70°$

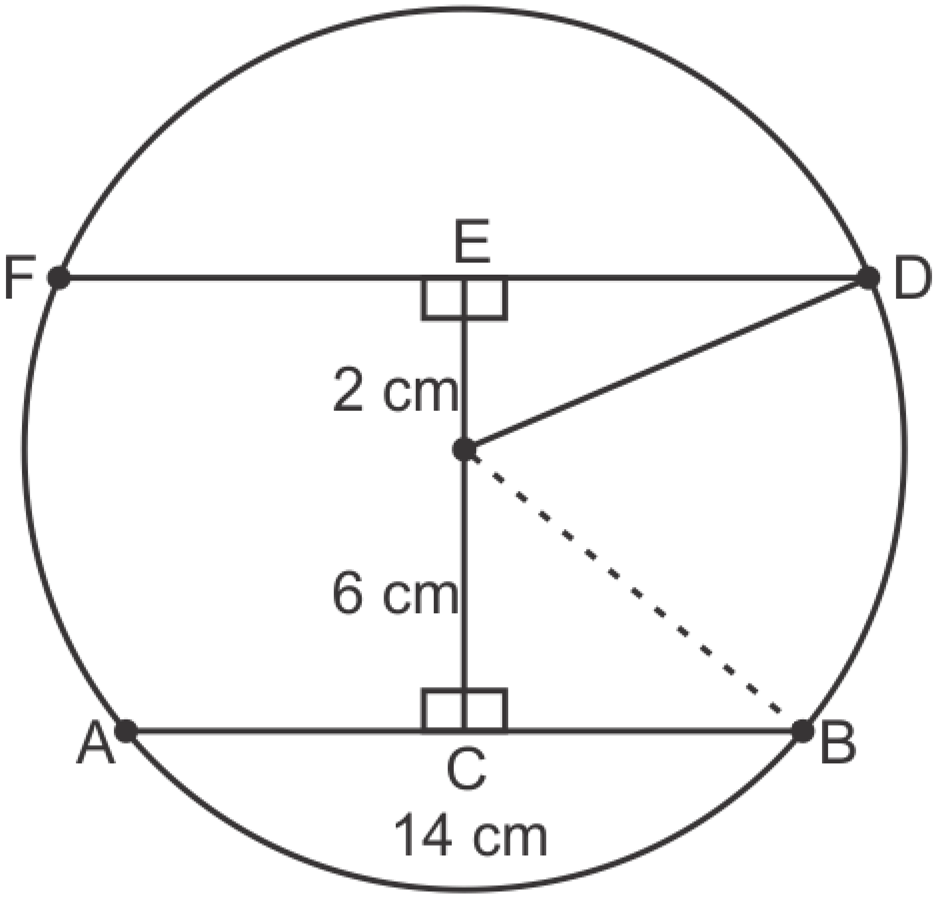
Hence, correct option is (a).

Question 5

A chord of length 14 cm is at a distance of 6 cm from the centre of a circle. The length of another chord at a distance of 2 cm from the centre of the circle is

(a) 12 cm
(b)) 14 cm
(c) 16 cm
(d) 18 cm

Solution 5



Chord AB $= 14$ cm
AC $=$ BC $= 7$ cm
OC $= 6$ cm
$\Rightarrow OB = \sqrt{7^2 + 6^2} = \sqrt{49+36} = \sqrt{85}$ cm
Consider $\triangle ODE$
$(OE)^2 + (ED)^2 = (OD)^2$
$(2)^2 + (ED)^2 = (OD)^2$
$(ED)^2 = (OD)^2 - 4$
$\Rightarrow (ED)^2 = 85 - 4 \quad (OD = OB = \sqrt{85}$ cm $-$ radii of same circle)
$\Rightarrow ED = \sqrt{81} = 9$ cm
$\Rightarrow$ Chord FD $= 9 \times 2 = 18$ cm
Hence, correct option is (d).

Question 6

One chord of a circle is known to be 10 cm. The radius of this circle must be
(a) 5 cm
(b) greater than 5 cm
(c) greater than or equal to 5 cm
(d) less than 5 cm

Solution 6

The longest chord of a circle is its diameter.
$\Rightarrow$ Diameter > 10 cm
$\Rightarrow 2 \times$ Radius > 10 cm
$\Rightarrow$ Radius > 5 cm
Hence, correct option is (b).

Question 7

ABC is a triangle with B as right angle, AC = 5 cm and AB = 4 cm. A circle is drawn with A as centre and AC as radius. The length of the chord of this circle passing through C and B is

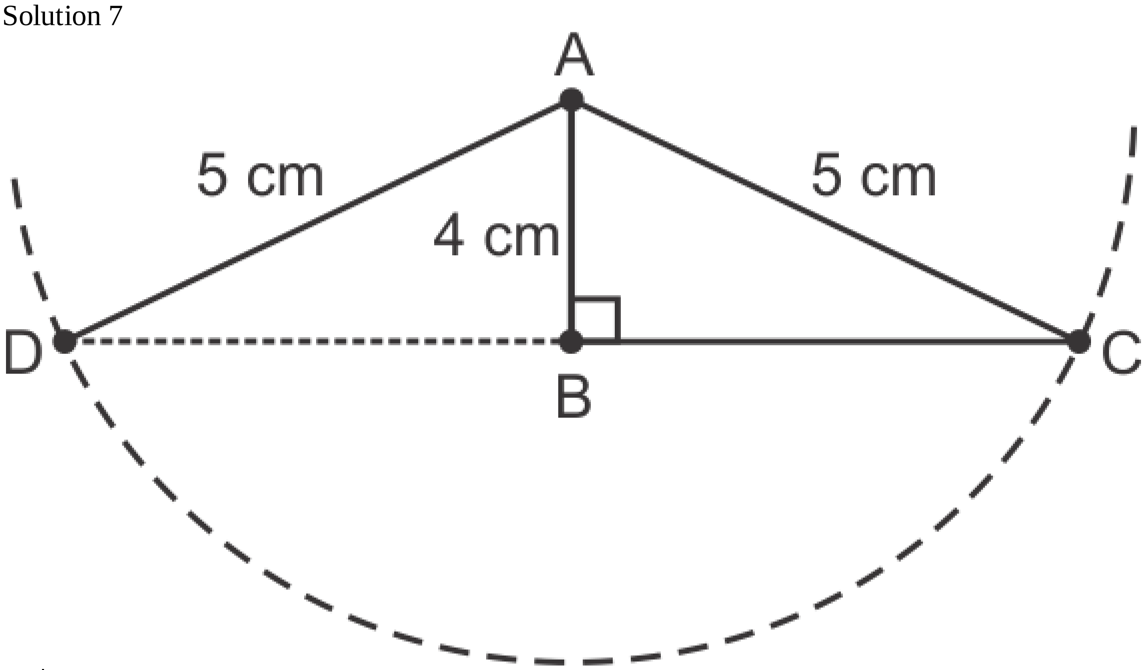
(a) 3 cm

(b) 4 cm

(c) 5 cm

(d) 6 cm

Solution 7



AD and AC are radii of same circle and CD is a chord.

Consider $\triangle ABC$,

$BC^2 = (AC)^2 - (AB)^2 = 5^2 - 4^2 = 25 - 16 = 9$

$\Rightarrow BC = 3$ cm

Chord $CD = 2 \times BC = 6$ cm

Hence, correct option is (d).

Question 8

If AB, BC and CD are equal chords of a circle with O as centre and AD diameter, then $\angle AOB =$

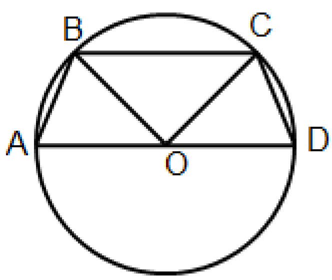
(a) 60°

(b) 90°

(c) 120°

(d) none of these

Solution 8



Chord AB = Chord BC = Chord CD

$\Rightarrow \angle AOB = \angle BOC = \angle COD$ (equal chords subtend equal angles at the center)

Now, $\angle AOB + \angle BOC + \angle COD = 180°$

$\Rightarrow \angle AOB + \angle AOB + \angle AOB = 180°$

$\Rightarrow 3\angle AOB = 180°$

$\Rightarrow \angle AOB = 60°$

Hence, correct option is (a).

Question 9

Let C be the mid – point of an arc AB of a circle such that $m\,\overset{\frown}{AB} = 183°$.

If the region bounded by the arc ACB and line segment AB is denoted by S,

then the centre O of the circle lies

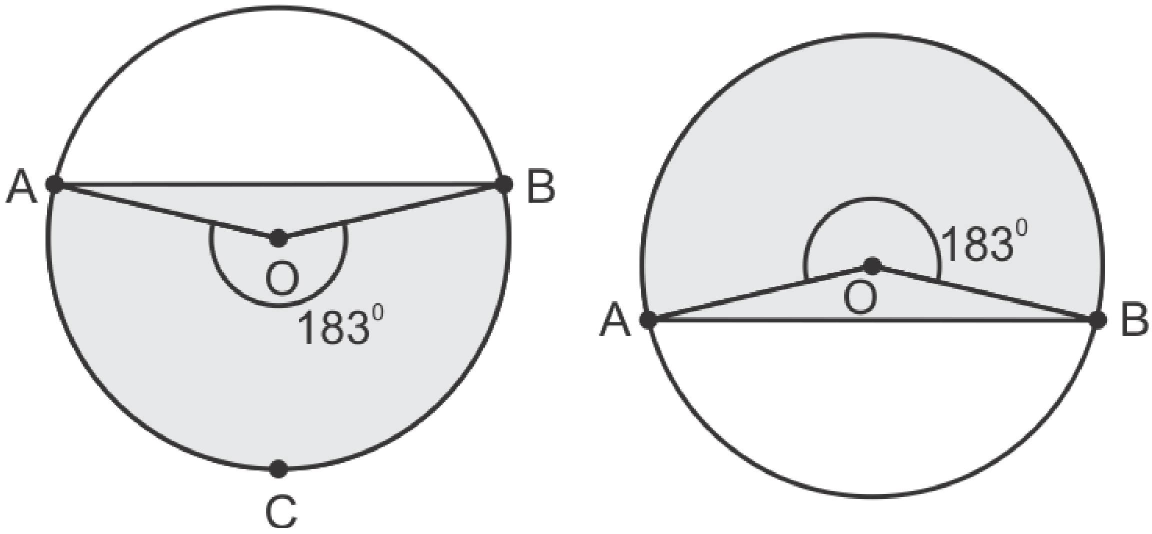
(a) in the interior of S

(b) in the exterior of S

(c) on the segment AB

(d) on AB and bisects AB

Solution 9



$m\,\overset{\frown}{AB} = 183°$

O is the centre of the circle and AB is a chord.

The Regiion bounded by arc and line segment AB is shaded.

We can see, 'O', the centre, always lie in the interior of S.

Hence, correct option is (a).

Question 10

In a circle, the major arc is 3 times the minor arc. The corresponding central angles and the degree measures of two arcs are

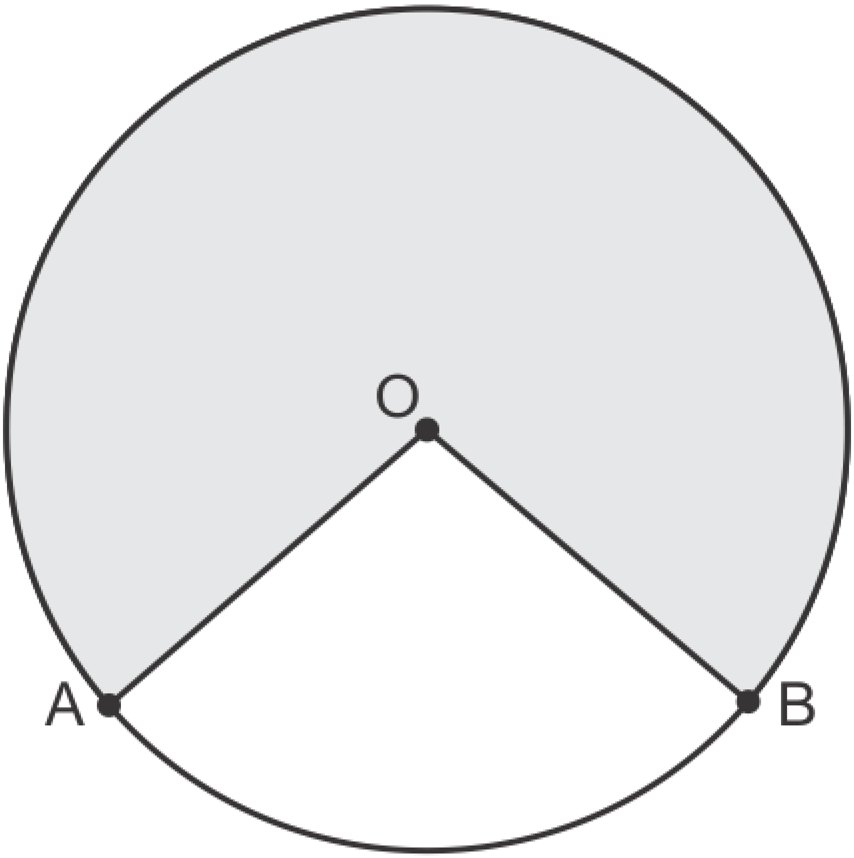
(a) 90° and 270°

(b) 90° and 90°

(c) 270° and 90°

(d) 60° and 210°

Solution 10



$$\frac{\overset{\frown}{AB}\text{minor}}{\overset{\frown}{AB}\text{major}} = \frac{1}{3} = \frac{\angle \overset{\frown}{AB}\text{minor}}{\angle \overset{\frown}{AB}\text{major}}$$

Let $\angle \overset{\frown}{AB}\text{minor} = x$

$\Rightarrow \angle \overset{\frown}{AB}\text{ major} = 3x$

Now we know $x + 3x = 360°$

$\Rightarrow 4x = 360°$

$\Rightarrow x = 90°$

$\Rightarrow 3x = 270°$

Hence, correct option is (a).

Option (c) can also be a possibility.

Question 11

If A and B are two points on a circle such that $m(\overset{\frown}{AB}) = 260°$.

A possible value for the angle subtended by arc BA at a point on the circle is

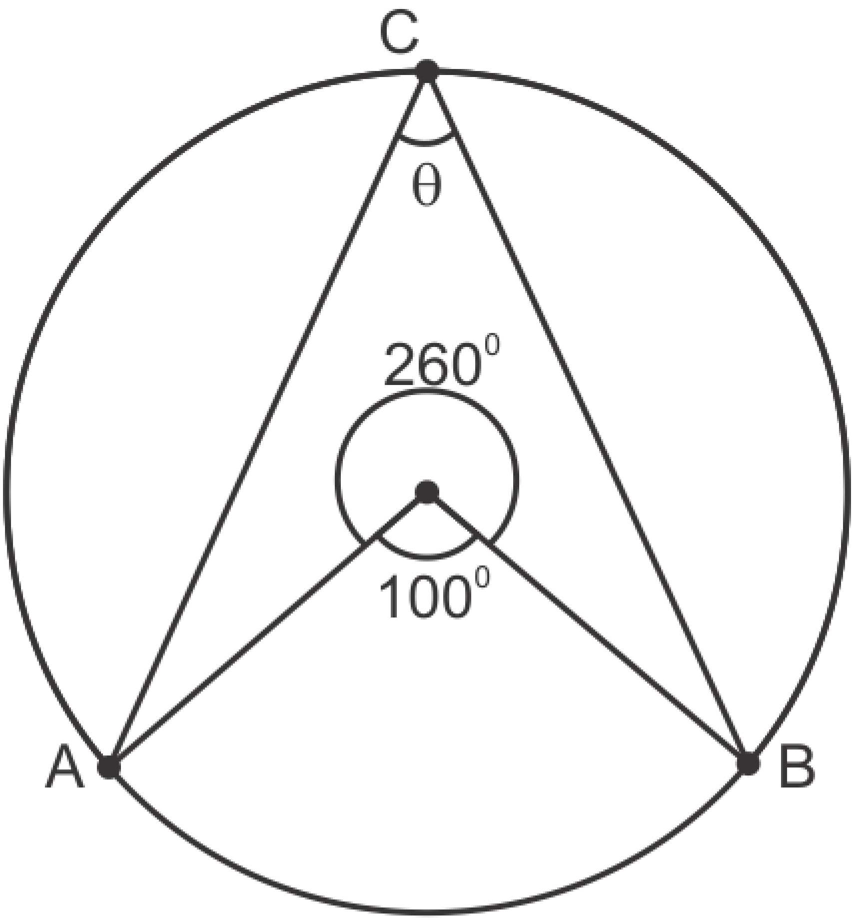
(a) 100°

(b) 75°

(c) 50°

(d) 25°

Solution 11



$m(\overset{\frown}{AB}) = 260°$

$\Rightarrow m(\overset{\frown}{BA}) = 100°$

Now Let $\overset{\frown}{BA}$ subtend an angle θ at a point C on circle.

Now, we know that angle subtend by an arc at the centre
is double the angle subtended at any point on the circle.

$\Rightarrow 100° = 2\theta$

$\Rightarrow \theta = 50°$

Hence, correct option is (c).

Question 12

An equilateral triangle ABC is inscribed in a circle with centre O. The measures of ∠BOC is

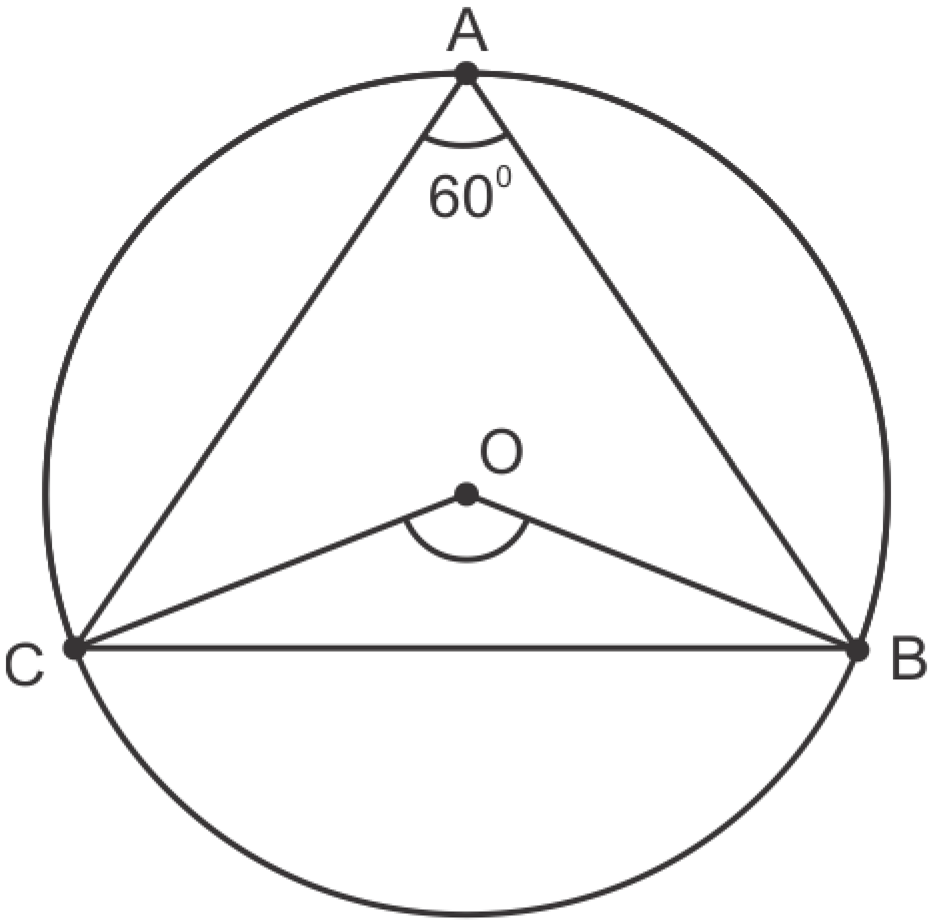
(a) 30°

(b)) 60°

(c)) 90°

(d)120°

Solution 12



$\angle BAC = 60°$ (angle of equilateral triangle)

Arc $\overset{\frown}{BC}$ makes angle $\angle BAC$ at circle and $\angle BOC$ at centre of circle.

$\Rightarrow \angle BAC = \frac{1}{2}\angle BOC$

$\Rightarrow 2 \times \angle BAC = \angle BOC$

$\Rightarrow 2 \times 60° = \angle BOC$

$\Rightarrow \angle BOC = 120°$

Hence, correct option is (d).

Question 13

If two diameters of a circle intersect each other at right angles, then quadrilateral formed joining their end points is a

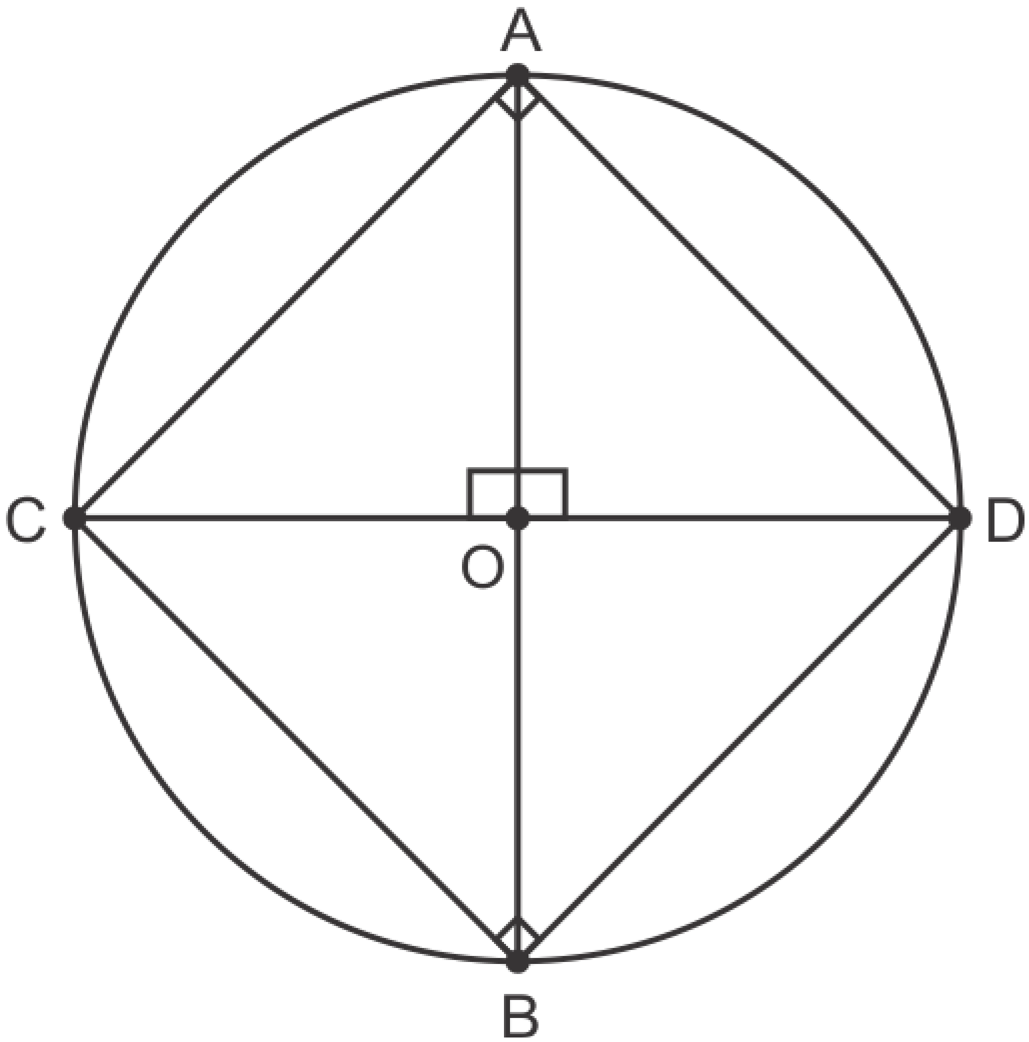
(a) rhombus

(b) rectangle

(c) parallelogram

(d) square

Solution 13



AB and CD are diameters of a circle and diameter makes 90° at any point on circle.

$\Rightarrow \angle CAD = \angle CBD = \angle BCA = \angle ADB = 90°$

Also, diagnols AB and CD are $\perp$ to each other.

Thus, ABCD is a square.

Hence, correct option is (d).

Question 14

If ABC is an arc of a circle and $\angle ABC = 135°$, then the ratio of are $\overset{\frown}{ABC}$ to the circumference is

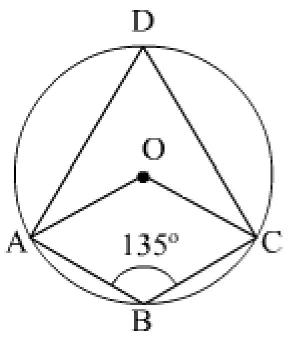
(a) 1 : 4

(b) 3 : 4

(c) 3 : 8

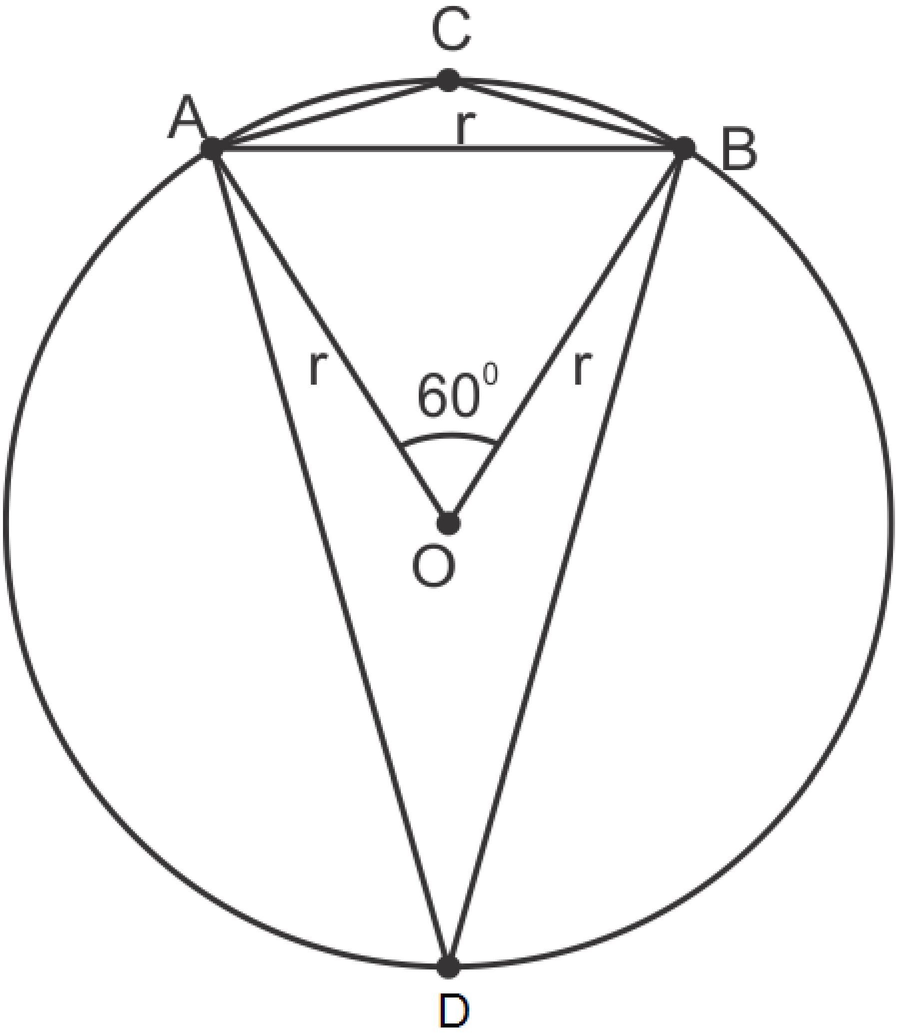
(d) 1 : 2

Solution 14



ABC is an arc of circle.
Take point D in the alternative segment and join AD and CD.
$\angle ABC = 135°$ (given)
$\angle ABC + \angle ADC = 180°$ (sum of opposite angles of cyclic quadrilateral is 180°)
$\Rightarrow \angle ADC = 180° - \angle ABC = 180° - 135° = 45°$
Now, $\angle AOC = 2 \times \angle ADC = 2 \times 45° = 90°$
$\overparen{ABC}$ = measure of the central angle $= \angle AOC = 90°$
$\Rightarrow$ Required ratio $= \dfrac{\text{arc } \overparen{ABC}}{\text{circumference}} = \dfrac{90°}{360°} = \dfrac{1}{4} = 1 : 4$
Hence, correct option is (a).

Question 15
The chord of a circle is equal to its radius. The angle subtended by this chord at the minor of the circle is
(a) 60^0
(b) 75^0
(c) 120^0
(d) 150^0

Solution 15


$\angle AOB = 60°$ (Since $\triangle AOB$ is equilateral triangle)
Now, $\angle ADB = 30°$
(Since chord AB makes 60° at centre, same chord will make half of the angle at circumference of angle made at centre)
Now $\angle ACB$ is angle made by chord at minor arc of circle.
ACBD is cyclic Quadrilateral.
$\Rightarrow \angle C + \angle D = 180°$
$\Rightarrow \angle ACB + \angle ADB = 180°$
$\Rightarrow \angle ACB = 180° - 30° = 150°$
Hence, correct option is (d).

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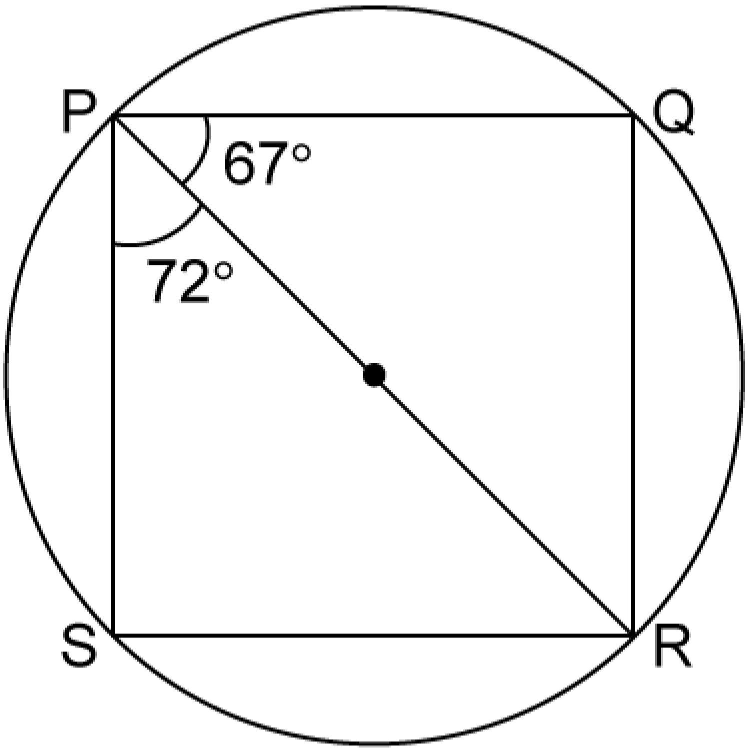
Question 16

PQRS is a cyclic quadrilateral such that PR is a diameter of the circle.
If $\angle QPR = 67°$ and $\angle SPR = 72°$, then $\angle QRS =$
(a) 41°
(b) 23°
(c) 67°
(d) 18°

Solution 16



In a cyclic quadrilateral, opposite angles are supplementary.
$\Rightarrow \angle P + \angle R = 180°$
Now, $\angle P = 67° + 72° = 139°$
Thus, $\angle R = 180° - 139° = 41°$
i.e. $\angle R = \angle QRS = 41°$
Hence, correct option is (a).

Question 17

If A, B, C are three points on a circle with centre O such that $\angle AOB = 90°$ and $OC = 120°$, then $\angle ABC =$

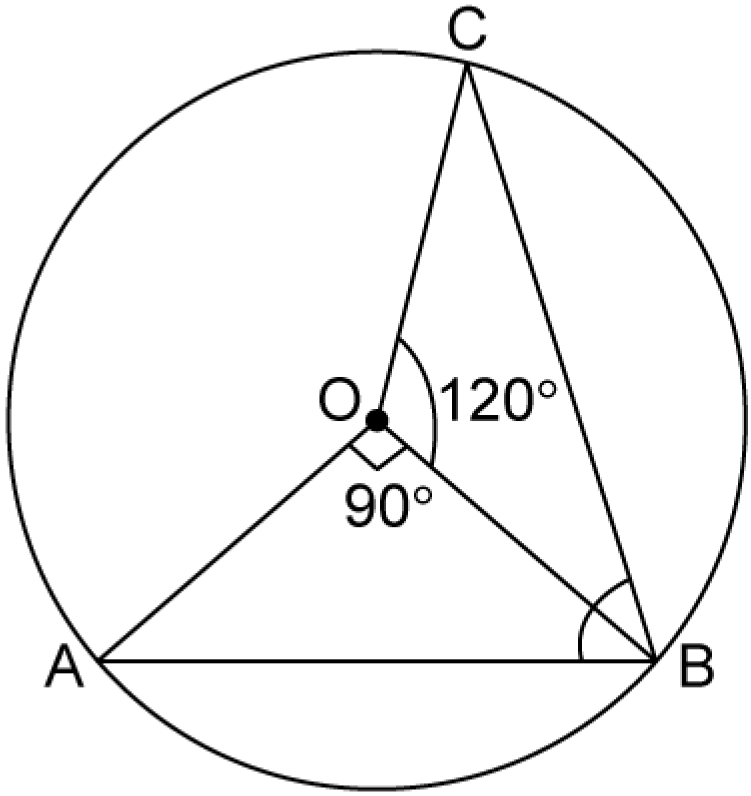
(a) 60°

(b) 75°

(c) 90°

(d) 135°

Solution 17



$\angle AOC = \angle AOB + \angle BOC = 90° + 120° = 210°$

$\angle COA = 360° - 210° = 150°$

If arc $\overparen{COA}$ makes 150° at centre, then it will make half of angle of the centre at circumference.

$\Rightarrow \angle CBA$ or $\angle ABC = \frac{150°}{2} = 75°$

Hence, correct option is (b).

Question 18

The greatest chord of a circle is called its

(a) radius

(b) secant

(c) diameter

(d) none of these

Solution 18

The greatest chord of the circle is diameter of the circle.

Hence, correct option is (c).

Question 19

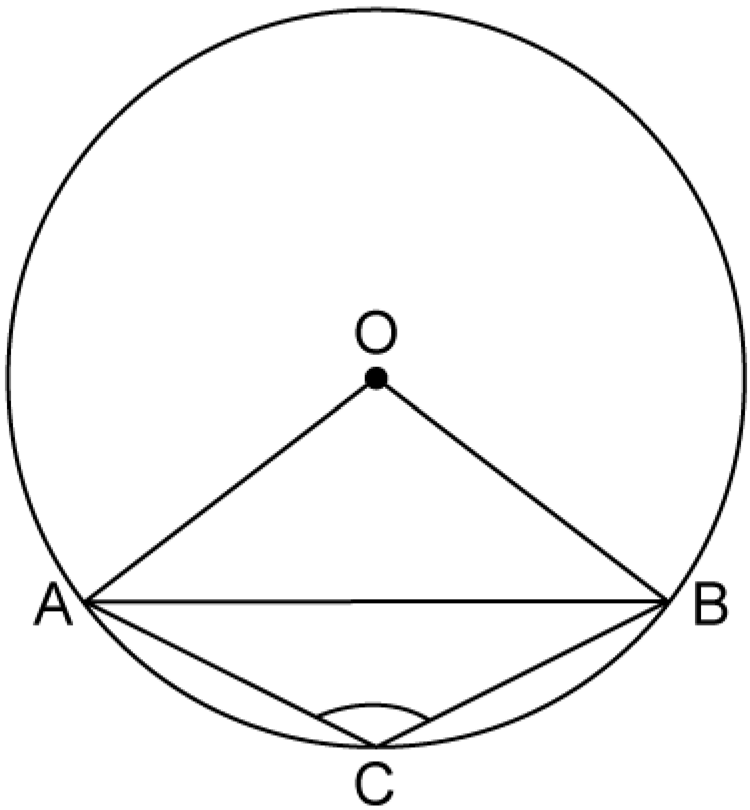
Angle formed in minor segment of a circle is

(a) acute

(b) obtuse

(c) right angle
(d) none of these

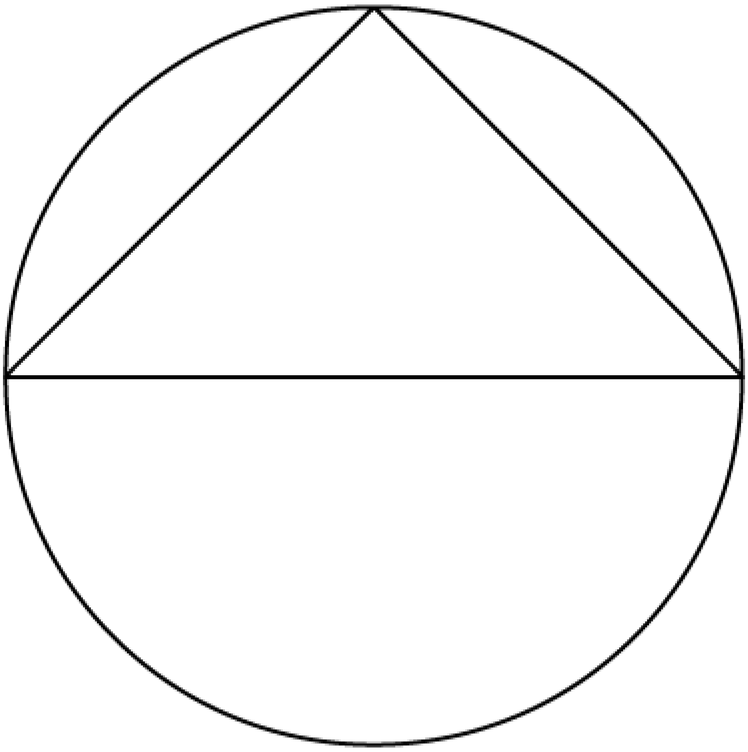
Solution 19



Angle formed in a minor segment is always a obtuse angle.
Hence, correct option is (b).

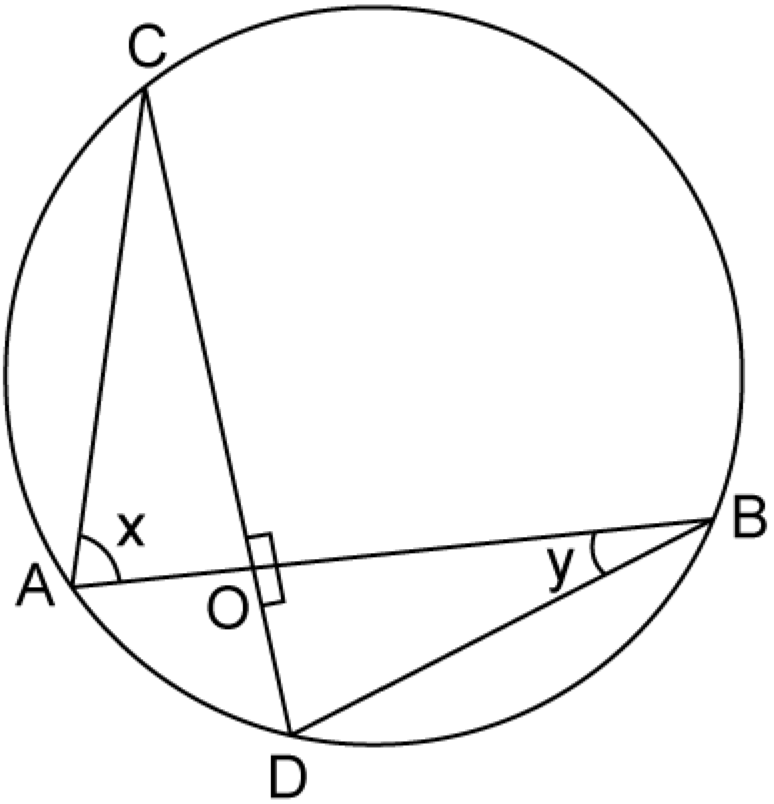
Question 20
Number of circles that can be drawn through three non-collinear points is
(a) 1
(b) 0
(c) 2
(d) 3

Solution 20

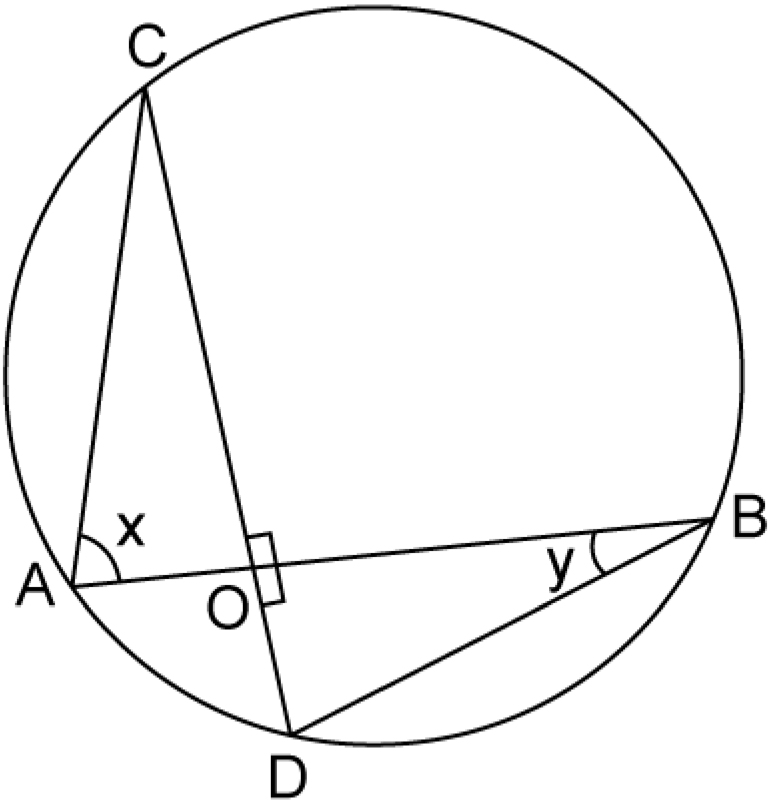


Three non-collinear points make a triangle and there is only one circle that can pass through all three points, i.e. circumcircle of that triangle.
Hence, correct option is (a).

Question 21
In figure, if chords AB and CD of the circle intersect each other at right angles, then x + y =
(a) 45^0
(b) 60^0
(c) 75^0
(d) 90^0

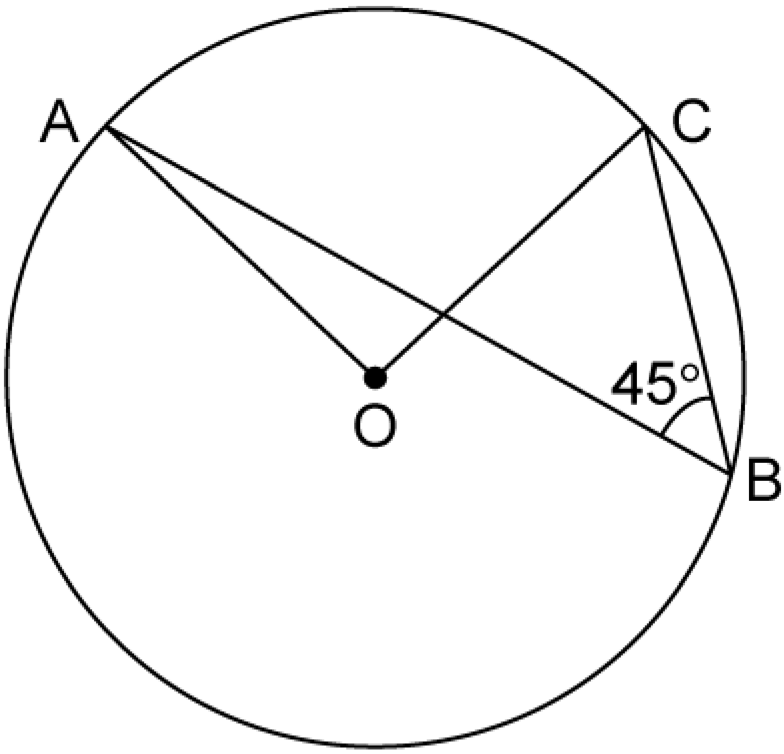


Solution 21

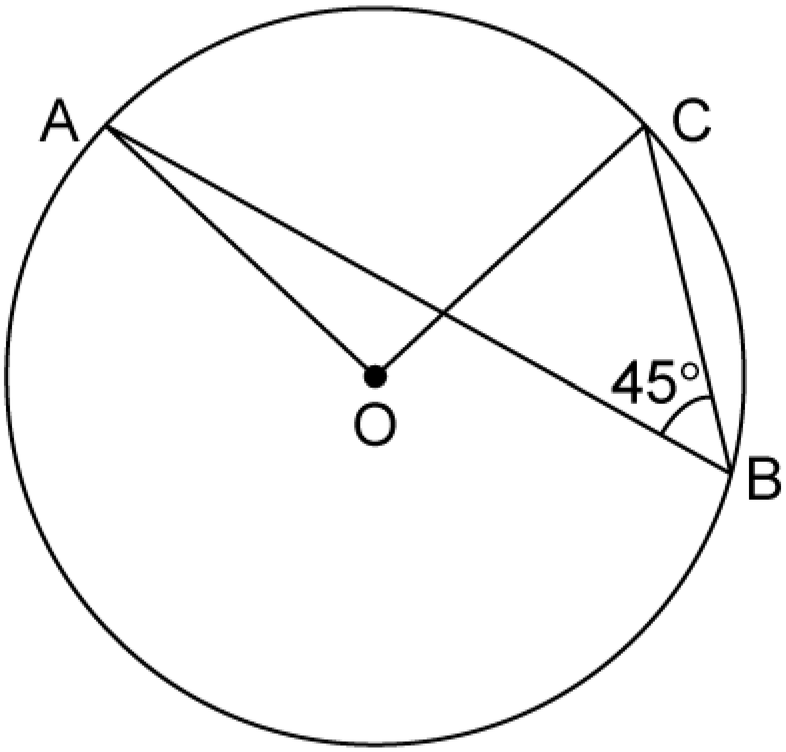


$\angle CAB = \angle CDB = x°$ ....(Both are on the same arc)
Consider $\triangle ODB$,
$\angle DOB = 90°$, $\angle OBD = y$, $\angle ODB = x$
In $\triangle ODB$,
$x + y + 90° = 180°$
$\Rightarrow x + y = 90°$
Hence, correct option is (d).

Question 22
In figure, if $\angle ABC = 45°$, then $\angle AOC =$
(a) 45°
(b) 60°
(c) 75°
(d) 90°



Solution 22



$\angle AOC$ is made by arc $\overset{\frown}{AC}$ at centre and $\angle ABC$ is made by $\overset{\frown}{AC}$ on circumference in major segment.

$$\Rightarrow \angle ABC = \frac{1}{2} \angle AOC$$

$$\Rightarrow \angle AOC = 2 \times \angle ABC = 2 \times 45° = 90°$$

Hence, correct option is (d).

Question 23

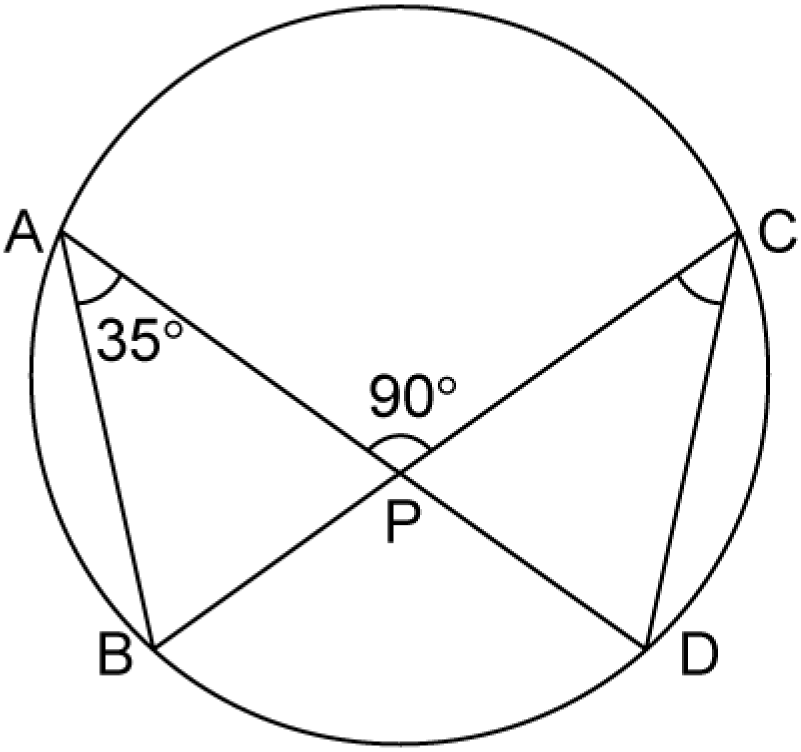
In figure, chords AD and BC intersect each other at right angles at a point P. If $\angle DAB = 35°$, then $\angle ADC =$

(a) 35^0

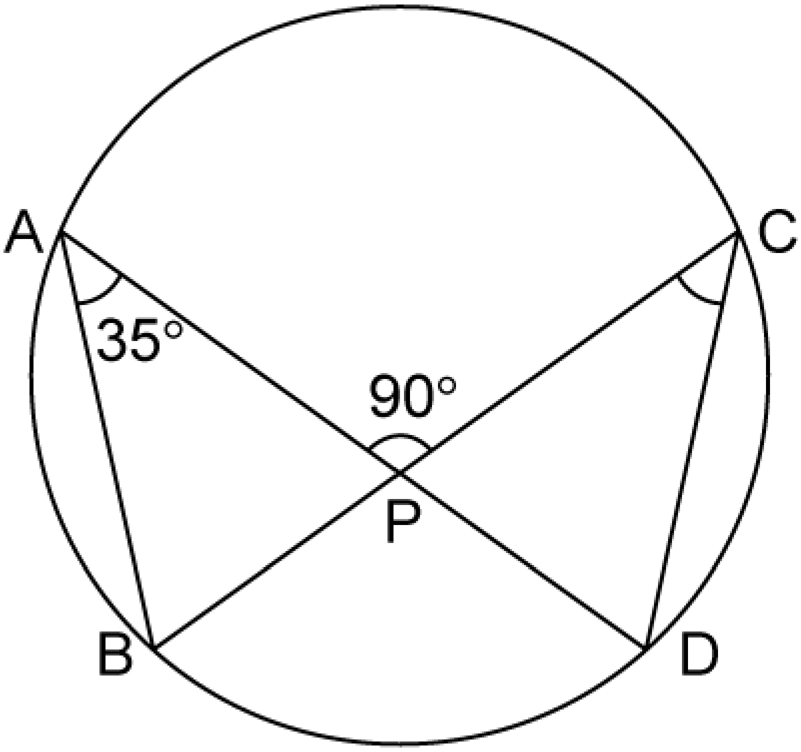
(b) 45^0

(c) 55^0

(d) 65^0



Solution 23



$\angle APC + \angle APB = 180°$

$\Rightarrow \angle APB = 180° - 90° = 90°$

In $\triangle APB$,

$\angle ABP = 180° - \angle APB - \angle BAP = 180° - 90° - 35° = 55°$

Now Arc $\overset{\frown}{AC}$ makes $\angle ABC$ and $\angle ADC$ on circle.

$\Rightarrow ABC = \angle ADC$

$\Rightarrow \angle ADC = 55°$

Hence, correct option is (c).

Question 24

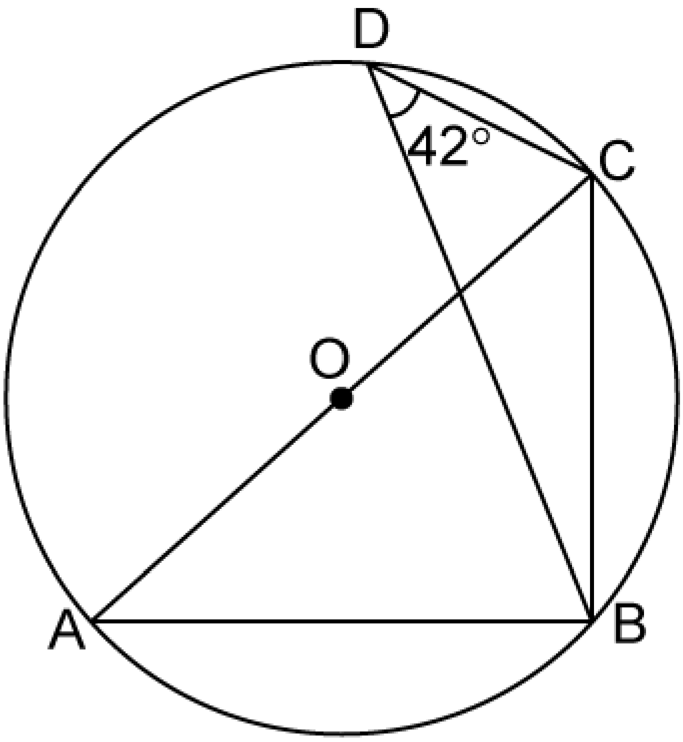
In figure, O is the center of the circle and $\angle BDC = 42°$. The measure of $\angle ACB$ is

(a) 42°

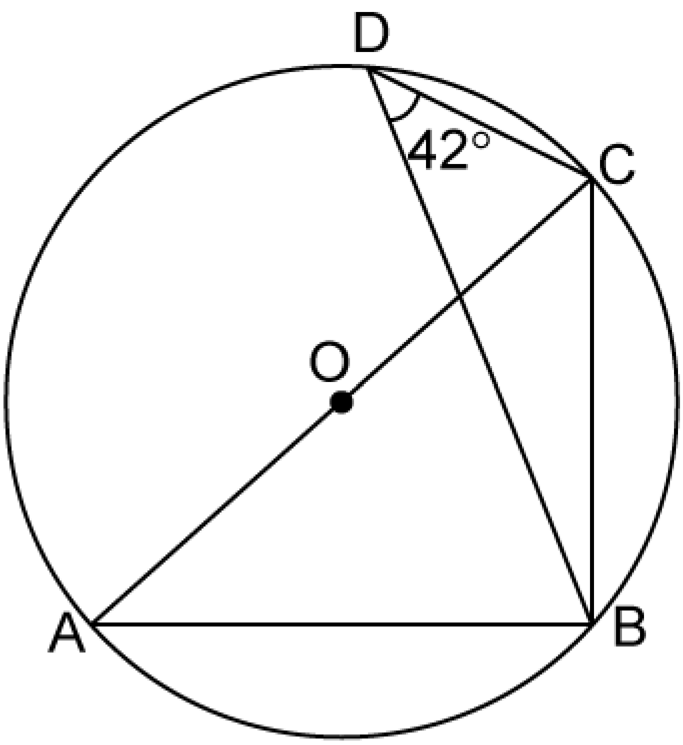
(b) 48°

(c) 58°

(c) 52°



Solution 24



$\angle ABC = 90°$ ....(Diameter AC makes 90° at circumference)
$\angle CDB = \angle CAB$ ....(angles on the same arc)
$\Rightarrow \angle CAB = 42°$
In $\triangle ABC$,
$\angle ACB = 180° - 90° - 42° = 48°$
Hence, correct option is (b).

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Question 25

In a circle with centre O, AB and CD are two diameters perpendicular to each other. The length of chord AC is

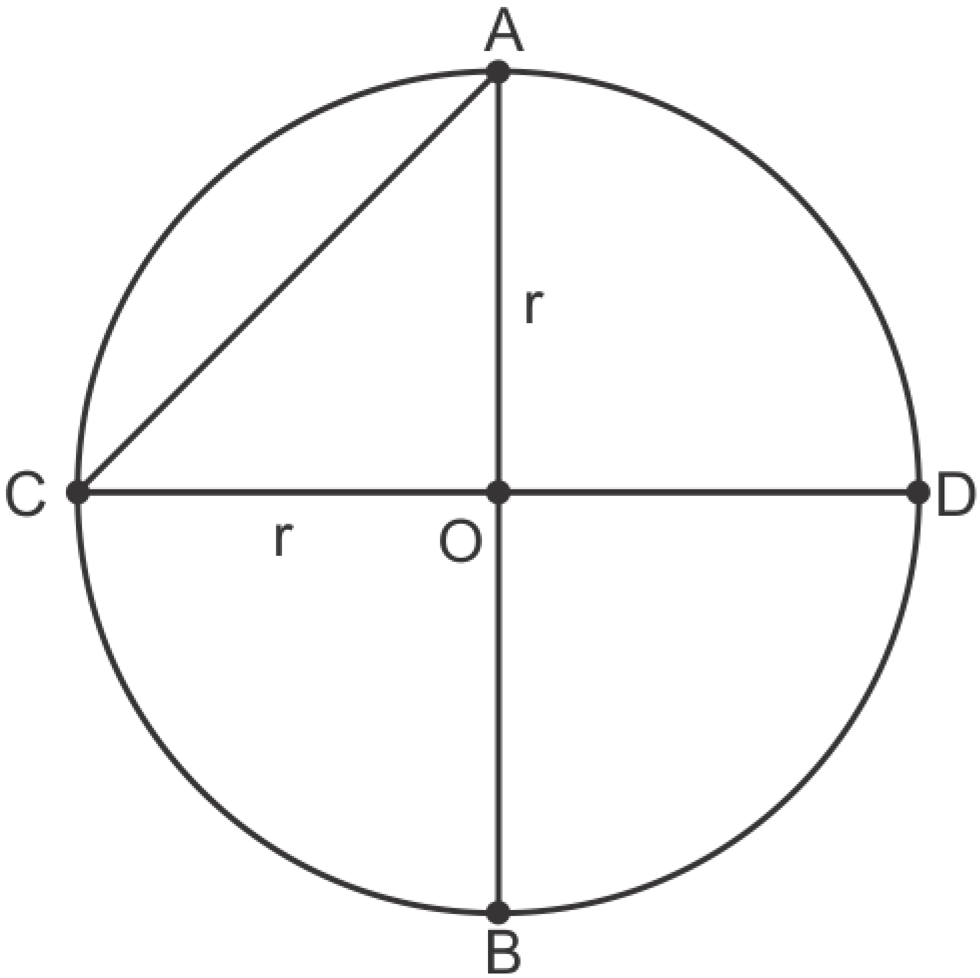
(a) 2AB

(b) $\sqrt{2}$

(c) $\frac{1}{2}$AB

(d) $\frac{1}{\sqrt{2}}$AB

Solution 25



$OC = OA = r$ (radius)

$AB = \text{Diameter} = 2r$

$$AC = \sqrt{(OA)^2 + (OC)^2}$$
$$= \sqrt{r^2 + r^2}$$
$$= \sqrt{2}r$$
$$= \sqrt{2}\left(\frac{AB}{2}\right)$$
$$\Rightarrow AC = \frac{1}{\sqrt{2}}AB$$

Hence, correct option is (d).

Question 26

Two equal circles of radius r intersect such that each passes through the centre of the other. The length of the common chord of the circles is

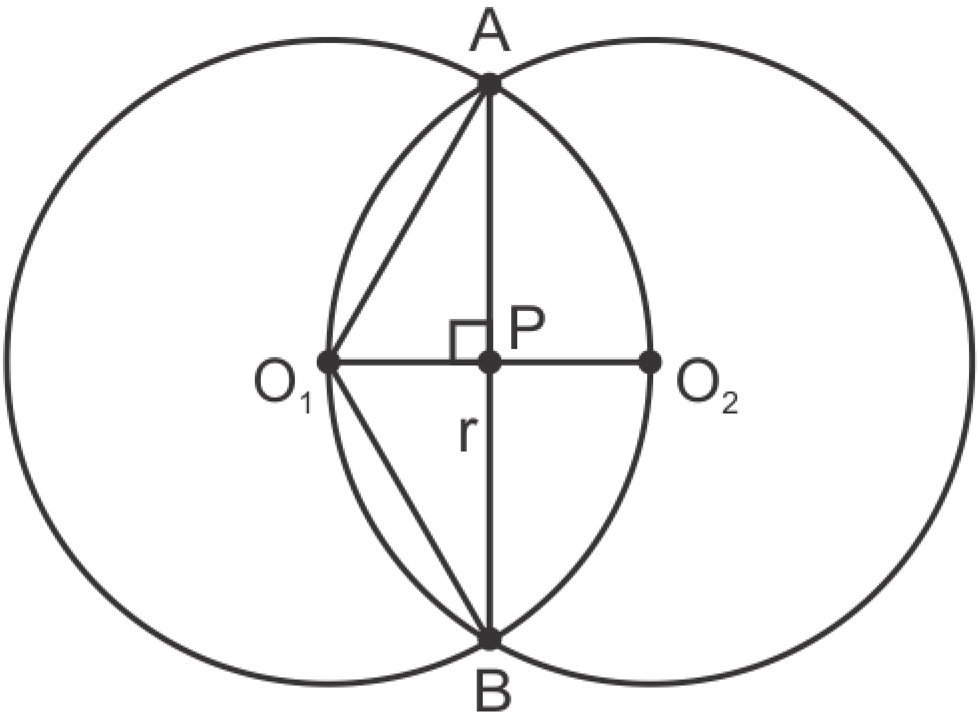
(a) $\sqrt{r}$

(b) $\sqrt{2}$ r AB

(c) $\sqrt{3}r$

(d) $\frac{\sqrt{3}}{2}r$

Solution 26



Both the circles pass through the center of each other

$\Rightarrow O_1O_2 = r$

Common chord is AB.

We know that perpendicular drawn from centre of circle to any chord bisects it.

$\Rightarrow$ P is the midpoint of AB

$\Rightarrow PA = PB$

$O_1A = r$ (radius of circle)

Consider $\triangle O_1PA$

$(O_1A)^2 = AP^2 + O_1P^2$

$\Rightarrow r^2 = AP^2 + \left(\frac{r}{2}\right)^2$ ....(P is also mid – poinat of O_1O_2)

$\Rightarrow AP^2 = r^2 - \frac{r^2}{4} = \frac{3r^2}{4}$

$\Rightarrow AP = \frac{\sqrt{3}}{2}r$

$\Rightarrow$ Length of chord $AB = 2AP = \sqrt{3}r$

Hence, correct option is (c).

Question 27

If AB is a chord of a circle, P and Q are the two points on the circle different from A and B, then

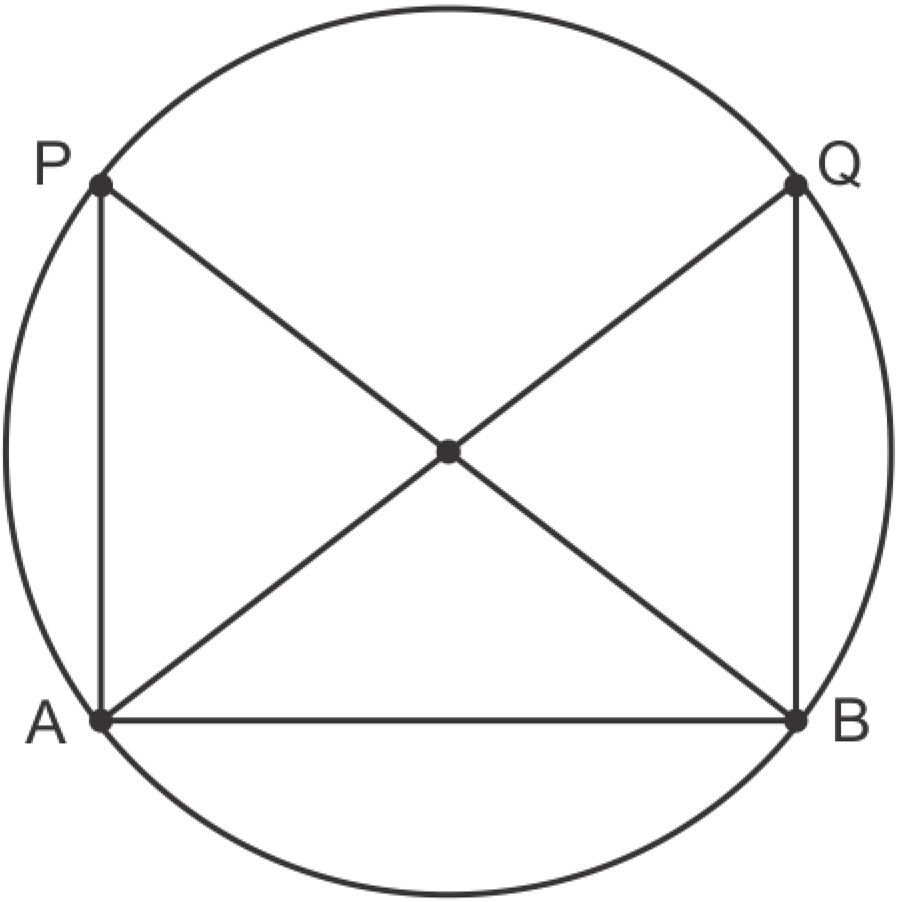
(a) $\angle APB = \angle AQB$

(b) $\angle APB + \angle AQB = 180°$ or $\angle APB = \angle AQB$

(c) $\angle APB + \angle AQB = 90°$

(d) $\angle APB + \angle AQB = 180°$

Solution 27



$\angle APB$ and $\angle AQB$ are on the same arc.

$\Rightarrow \angle APB = \angle AQB$

But, if AB = diameter, then $\angle APB = \angle AQB = 90°$

(Becasue diameter makes Right angle at any point on circumference of circle)

$\Rightarrow \angle APB + \angle AQB = 180°$

Hence, correct option is (b).

Question 28

AB and CD are two parallel chords of a circle with centre 'O' such that AB = 6 cm and CD = 12 cm. The chords are on the same side of the centre and the distance between is 3 cm. The radius of the circle is

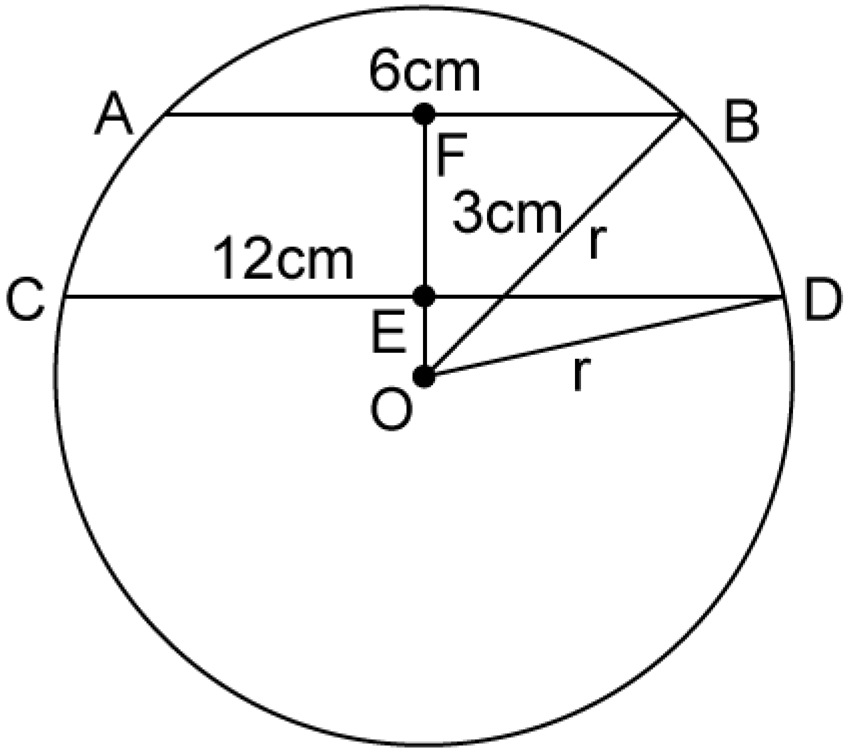
(a) 6 cm

(b) $5\sqrt{2}$ cm

(c) 7 cm

(d) $3\sqrt{5}$ cm

Solution 28



OB and OD are the radii of a circle.

In $\triangle OED$,

$r^2 = OE^2 + ED^2 = OE^2 + (6)^2$

$\Rightarrow OE = \sqrt{r^2 - 36}$ ....(1)

In $\triangle OFB$,

$r^2 = OF^2 + BF^2 = OF^2 + (3)^2$

$\Rightarrow OF = \sqrt{r^2 - 9}$ ....(2)

$OF - OE = 3$ cm (given)

$\Rightarrow \sqrt{r^2-9} - \sqrt{r^2-36} = 3$

$\Rightarrow \sqrt{r^2-9} = \sqrt{r^2-36} + 3$ ....(3)

Squarring equation (3), we have

$r^2 - 9 = r^2 - 36 + 9 + 2 \times 3\sqrt{r^2-36}$

$\Rightarrow r^2 - 9 = r^2 - 27 + 6\sqrt{r^2-36}$

$\Rightarrow 18 = 6\sqrt{r^2-36}$

$\Rightarrow 3 = \sqrt{r^2-36}$

$\Rightarrow 9 = r^2-36$

$\Rightarrow r = \sqrt{45} = 3\sqrt{5}$ cm

Hence, correct option is (d).

Question 29

In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is

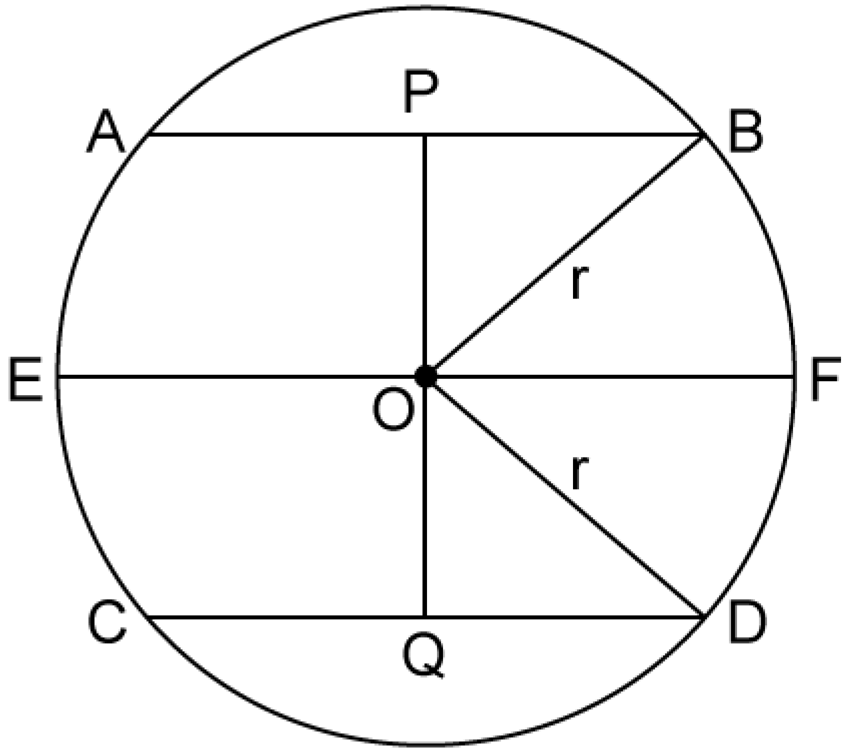
(a) 34 cm

(b) 15 cm

(c) 23 cm

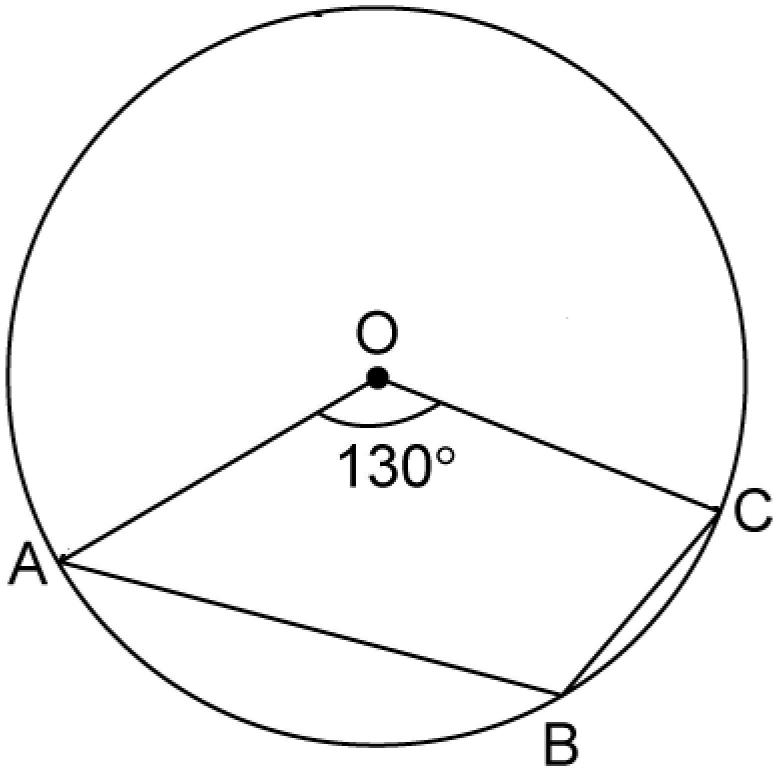
(d) 30 cm

Solution 29

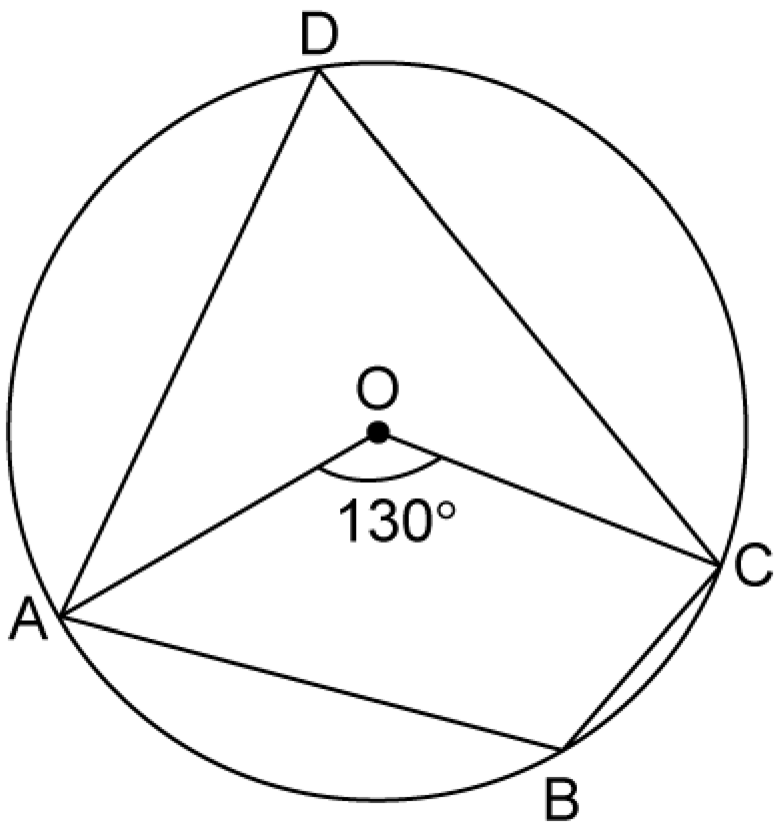


PQ = 23 cm
AB = 16 cm
$\Rightarrow$ BP = AP = 8 cm
r = 17 cm
$\Rightarrow$ EF = diameter = 2r = 34 cm
Consider $\triangle$OPB,
$r^2 = OP^2 + BP^2$
$\Rightarrow OP^2 = (17)^2 - (8)^2 = 289 - 64 = 225$
$\Rightarrow$ OP = 15 cm
$\Rightarrow$ OQ = 23 − 15 = 8 cm
Consider $\triangle$OQD,
$r^2 = OQ^2 + QD^2$
$\Rightarrow QD^2 = r^2 - OQ^2 = (17)^2 - (8)^2 = 225$
$\Rightarrow$ QD = 15 cm
$\Rightarrow$ CD = 2 $\times$ QD = 30 cm
Hence, correct option is (d).

Question 30
In figure, O is the centre of the circle such that $\angle AOC = 130°$, then $\angle ABC$ =
(a) 130°
(b) 115°
(c) 65°
(d) 165°



Solution 30



$\angle ADC = \frac{1}{2} \angle AOC$

{$\angle ADC$ and $\angle AOC$ are made by same $\overset{\frown}{AC}$ on centre and circumference}

$\Rightarrow \angle ADC = \frac{1}{2} \times 130° = 65°$

ADCB is a cyclic Quarilateral.

$\Rightarrow \angle D + \angle B = 180°$

$\Rightarrow \angle ABC = 180° - 65° = 115°$

Hence, correct option is (b).

Chapter 15 - Circles Exercise Ex. 15.1

Question 1

Fill in the blanks:

(i) All points lying inside/outside a circle are called ...... points/ ... points.

(ii) Circles having the same centre and different radii are called ... circles.

(iii) A point whose distance from the centre of a circle is greater than its radius lies in ... of the circle.

(iv) A continuous piece of a circle is ... of the circle.

(v) The longest chord of a circle is a ... of the circle.

(vi) An arc is a ... when its ends are the ends of a diameter.

(vii) Segment of a circle is the region between an arc and ... of the circle.

(viii) A circle divides the plane, on which it lies, in .... parts.

Solution 1

(i) interior/exterior

(ii) concentric

(iii) the exterior

(iv) arc

(v) diameter

(vi) semi-circle

(vii) centre

(viii) three

Question 2

Write the truth value (T/F) of the following with suitable reasons:

(i) A circle is a plane figure.

(ii) Line segment joining the centre to any point on the circle is a radius of the circle.

(iii) If a circle is divided into three equal arcs each is a major arc.

(iv) A circle has only finite number of equal chords.

(v) A chord of a circle, which is twice as long is its radius is a diameter of the circle.

(vi) Sector is the region between the chord and its corresponding arc.

(vii) The degree measure of an arc is the complement of the central angle containing the arc.

(viii) The degree measure of a semi-circle is 180^o.

Solution 2

(i) T

(ii) T

(iii) T

(iv) F

(v) T

(vi) T

(vii) F

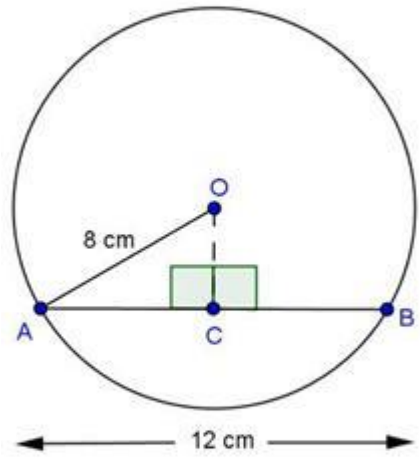
(viii) T

Chapter 15 - Circles Exercise Ex. 15.2

Question 1

The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Solution 1



Radius of circle (OA) = 8 cm

Chord (AB) = 12 cm

Draw $OC \perp AB$

We know that perpendicular from centre to chord bisects the chord

$\therefore \quad AC = BC = \frac{12}{2} = 6$ cm

Now in $\triangle OCA$, by pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow \qquad 6^2 + OC^2 = 8^2$

$\Rightarrow \qquad 36 + OC^2 = 64$

$\Rightarrow \qquad OC^2 = 64 - 36$

$\Rightarrow \qquad OC^2 = 28$

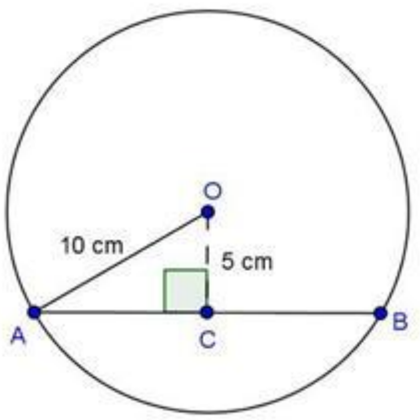
$\Rightarrow \qquad OC = \sqrt{28} = 5.291$ cm

$\therefore \quad$ The distance of chord AB from centre = 5.291 cm

Question 2

Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

Solution 2



Distance (OC) = 5 cm

Radius of circle (OA) = 10 cm

In $\triangle OCA$, by pythagoras theorem

$$AC^2 + OC^2 = OA^2$$

$\Rightarrow \quad AC^2 + 5^2 = 10^2$

$\Rightarrow \quad AC^2 + 25 = 100$

$\Rightarrow \quad AC^2 = 100 - 25 = 75$

$\Rightarrow \quad AC = \sqrt{75} = 8.66$ cm

We know that the perpendicular from centre to chord bisects the chord

$\therefore \quad AC = BC = 8.66$ cm

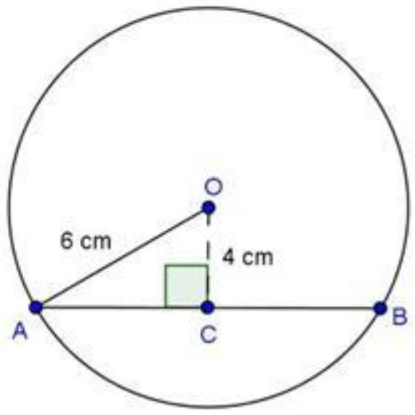
Then chord $AB = 8.66 + 8.66$

$= 17.32$ cm

Question 3

Find the length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm.

Solution 3



Radius of circle $(OA) = 6$ cm
Distance $(OC) = 4$ cm

In $\triangle OCA$, by pythagoras theorem

$$AC^2 + OC^2 = OA^2$$

$$\Rightarrow \quad AC^2 + 4^2 = 6^2$$
$$\Rightarrow \quad AC^2 + 16 = 36$$
$$\Rightarrow \quad AC^2 = 36 - 16 = 20$$

$$\Rightarrow \quad AC = \sqrt{20} = 4.47 \text{ cm}$$

We know that the perpendicular from centre to chord bisects the chord.

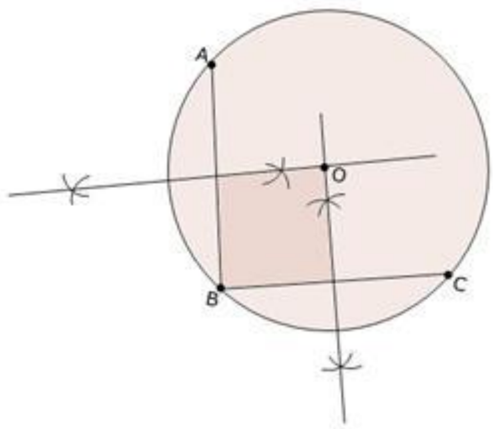
$$\therefore \quad AC = BC = 4.47 \text{ cm}$$

Then $AB = 4.47 + 4.47$
$= 8.94$ cm

Question 4

Give a method to find the centre of a given circle.

Solution 4



Steps of construction:

(1) Take three point A, B and C on the given circle.

(2) Join AB and BC.

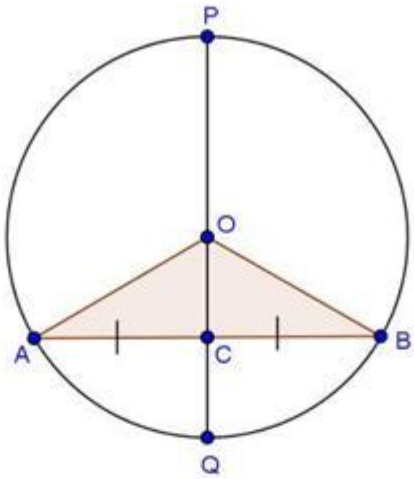
(3) Draw the perpendicular bisectors of chord AB and BC which interesect each other at O.

(4) Point O will be the required circle because we know that the perpendicular bisector of a chord always passes through the centre.

Question 5

Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Solution 5



Given:- PQ is a diameter of circle which bisects chord AB at C

To prove:- PQ bisects $\angle AOB$

Proof

In ΔAOC and ΔBOC

$OA = OB$	[Radii of circle]
$OC = OC$	[Common]
$AC = BC$	[Given]

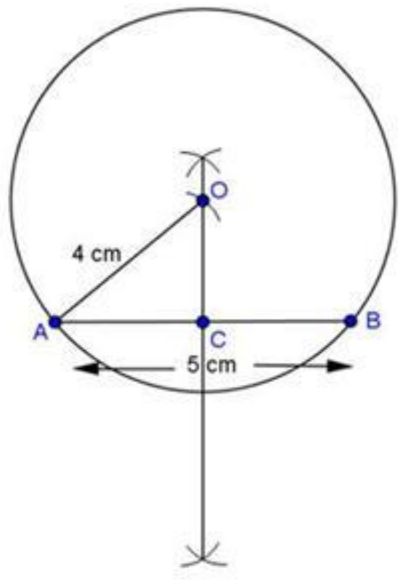
Then $\Delta AOC \cong \Delta BOC$	[by SSS condition]
$\therefore \quad \angle AOC = \angle BOC$	[c.p.c.t]

Hence, PQ bisects $\angle AOB$

Question 6

A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reason in support of your answer.

Solution 6



(1) Draw a line segment AB of 5 cm.

(2) Draw the perpendicular bisector of AB.

(3) With centre A and radius 4 cm, draw an arc which intersect the perpendicular bisector at point O then O will be the required centre.

(4) Join OA.

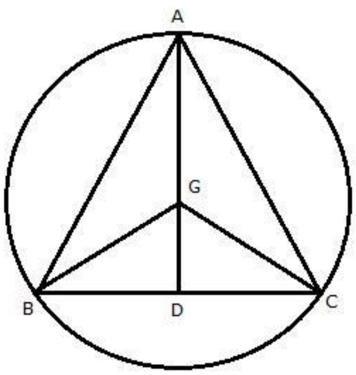
(5) With centre O and radius OA, draw a circle.

No, we cannot draw a circle of radius 2 cm passing through A and B, because when we draw an arc of radius 2 cm with centre A, the arc will not intersect the perpendicular bisector and we will not find the centre.

Question 7

An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Solution 7



Let ABC be an equilateral triangle of side 9 cm and let AD be one of its medians.
Let G be the centroid of $\triangle ABC$. Then, $AG : GD = 2 : 1$.

We know that in an equilateral triangle centroid coincides with the circumcentre.
Therefore, G is the centre of the circumcircle with circumradius GA.

Also, G is the centre and $GD \perp BC$. Therefore,
In right triangle ADB, We have

$$AB^2 = AD^2 + DB^2$$

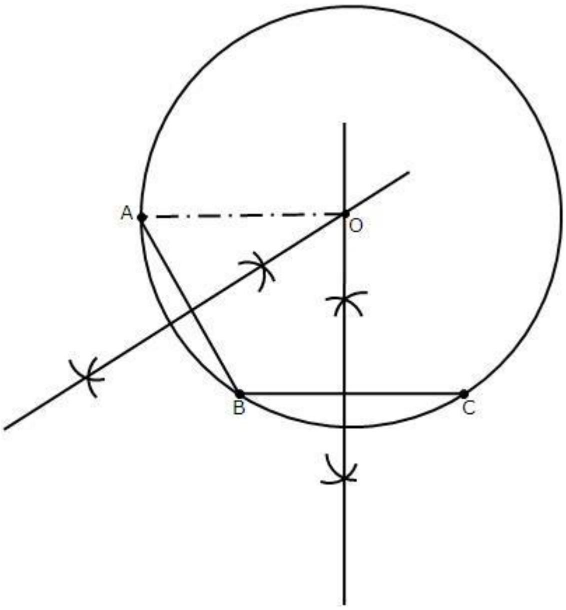
$$\Rightarrow \quad 9^2 = AD^2 + (4.5)^2$$

$$\Rightarrow \quad AD = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2} \text{ cm}$$

$$\therefore \quad \text{Radius} = AG = \frac{2}{3}AD = 3\sqrt{3} \text{ cm}$$

Question 8
Given an arc of a circle, complete the circle.

Solution 8



Steps of construction:-

(1) Take three point A, B and C on the given arc.

(2) Join AB and BC.

(3) Draw the perpendicular bisectors of chords AB and BC which intersect each other at point O
Then O will be the required centre of the required circle.

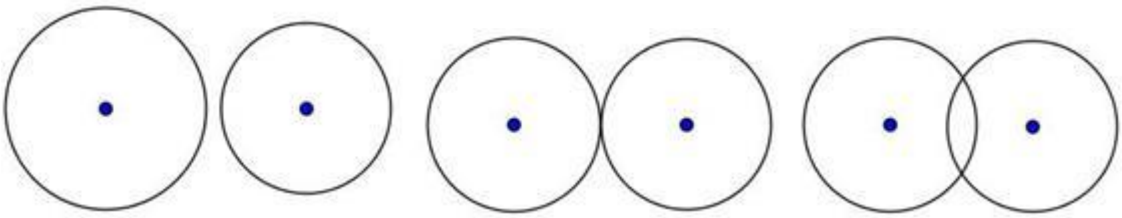
(4) Join OA.

(5) With centre O and radius OA, complete the circle.

Question 9

Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points.

Solution 9

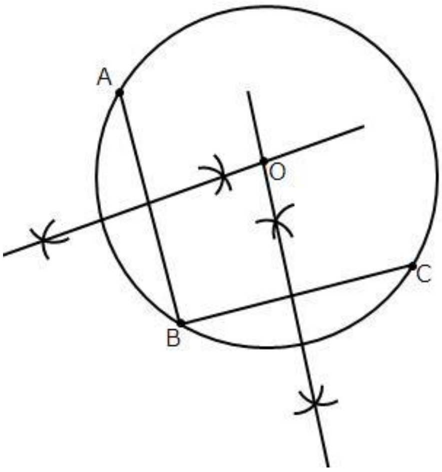


Each pair of circles have 0, 1 or 2 points in common. The maximum number of points in common is 2.

Question 10

Suppose you are given a circle. Give a construction to find its centre.

Solution 10



Steps of construction:-

(1) Take three point A, B and C on the given circle.

(2) Join AB and BC.

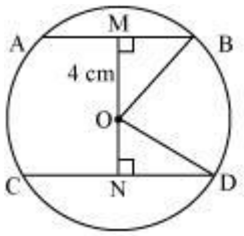
(3) Draw the perpendicular bisectors of chord AB and BC which intersect each other at O.

(4) Point O will be the required centre of the circle because we know that the perpendicular bisector of a chord always passes through the centre.

Question 11

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord form the centre?

Solution 11



Distance of smaller chord AB from centre of circle = 4 cm.
OM = 4 cm

$$MB = \frac{AB}{2} = \frac{6}{2} = 3 \text{ cm}$$

In $\triangle$OMB

$$OM^2 + MB^2 = OB^2$$
$$(4)^2 + (3)^2 = OB^2$$
$$16 + 9 = OB^2$$
$$OB = \sqrt{25}$$
$$OB = 5 \text{ cm}$$

In $\triangle$OND

OD=OB=5cm (radii of same circle)

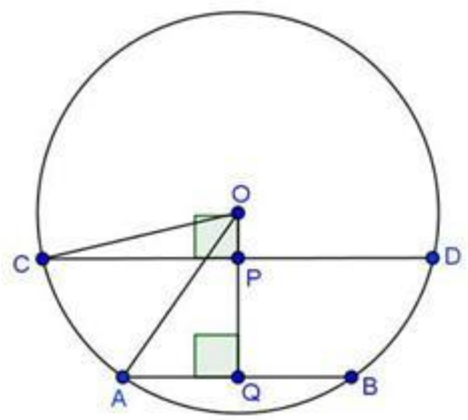
$$ND = \frac{CD}{2} = \frac{8}{2} = 4\text{cm}$$
$$ON^2 + ND^2 = OD^2$$
$$ON^2 + (4)^2 = (5)^2$$
$$ON^2 = 25 - 16 = 9$$
$$ON = 3$$

So, distance of bigger chord from centre is 3 cm.

Question 12

Two chods AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Solution 12


O
C
P
D
A
Q
B

Construction:- Draw $OP \perp CD$

Chord AB = 5 cm
Chord CD = 11 cm
Distance PQ = 3 cm

Let $OP = x$ cm
And $OC = OA = r$ cm

We know that perpendicular from centre to chord bisects it

$\therefore \quad CP = PD = \frac{11}{2}$ cm
And $AQ = BQ = \frac{5}{2}$ cm

In $\triangle OCP$, by pythagoras theorem

$$OC^2 = OP^2 + CP^2$$
$$\Rightarrow \quad r^2 = x^2 + \left(\frac{11}{2}\right)^2 \qquad ---(1)$$

In $\triangle OQA$, by pythagoras theorem

$$OA^2 = OQ^2 + AQ^2$$
$$\Rightarrow \quad r^2 = (x+3)^2 + \left(\frac{5}{2}\right)^2 \qquad ---(2)$$

Compare equation (1) and (2)

$$(x+3)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2$$
$$\Rightarrow \quad x^2 + 9 + 6x + \frac{25}{4} = x^2 + \frac{121}{4}$$
$$\Rightarrow \quad x^2 + 6x - x^2 = \frac{121}{4} - \frac{25}{4} - 9$$
$$\Rightarrow \quad 6x = 15$$
$$\Rightarrow \quad x = \frac{15}{6} = \frac{5}{2}$$

Put value of x in equation (1)

$$r^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{11}{2}\right)^2$$

$$\Rightarrow \quad r^2 = \frac{25}{4} + \frac{121}{4} = \frac{146}{4}$$

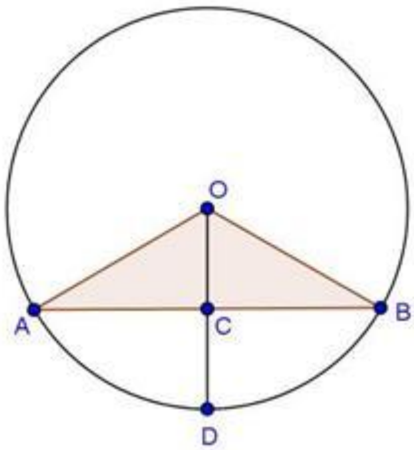
$$\Rightarrow \quad r = \frac{\sqrt{146}}{2} \text{ cm}$$

$\therefore$ Radius of circle = $\frac{\sqrt{146}}{2}$ cm

Question 13

Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Solution 13



Given:- C is the mid-point of chord AB
To prove:- D is the mid-point of arc AB

Proof
In ΔOAC and ΔOBC

$OA = OB$ [Radii of circle]

$OC = OC$ [Common]

$AC = BC$ [C is the mid-point of AB]

Then $\Delta OAC \cong \Delta OBC$ [by SSS condition]

$\therefore \angle AOC = \angle BOC$ [c.p.c.t]

$\Rightarrow \quad m\left(\overset{\frown}{AD}\right) = m\left(\overset{\frown}{BD}\right)$

$\Rightarrow \quad \overset{\frown}{AD} \cong \overset{\frown}{BD}$

Hence, D is the mid-point of arc AB

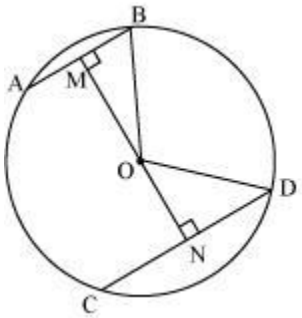
Question 14
Prove that two different circles cannot intersect each other at more than two points.

Solution 14
Suppose two different circles can intersect each other at three points then they will pass through the three common points but we know that there is one and only one circle with passes through three non-collinear points, which contradicts our supposition.

Hence, two different circles cannot intersect each other at more than two points.

Question 15
Two chords AB and CD of lengths 5 cm and 11cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution 15
Draw OM $\perp$ AB and ON $\perp$ CD. Join OB and OD



$$BM = \frac{AB}{2} = \frac{5}{2}$$

(Perpendicular from centre bisects the chord)

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let ON be x, so OM will be 6 - x

In $\triangle$ MOB

$$OM^2 + MB^2 = OB^2$$

$$(6-x)^2 + \left[\frac{5}{2}\right]^2 = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \qquad \ldots(1)$$

In $\triangle$ NOD

$$ON^2 + ND^2 = OD^2$$

$$x^2 + \left[\frac{11}{2}\right]^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \qquad \ldots(2)$$

We have OB = OD (radii of same circle)

So, from equation (1) and (2)

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$$

$$x = 1$$

From equation (2)

$$(1)^2 + \left[\frac{121}{4}\right] = OD^2$$

$$OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$$

$$OD = \frac{5}{2}\sqrt{5}$$

So, radius of circle is found to be $\frac{5}{2}\sqrt{5}$ cm.

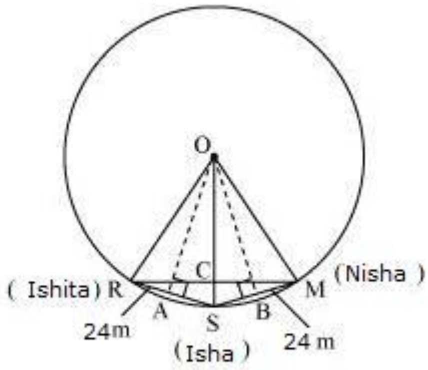
Chapter 15 - Circles Exercise Ex. 15.3

Question 1

Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha, Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is the distance between Ishita and Nisha?

Solution 1

Let R, S and M be the position of Ishita, Isha and Nisha respectively.



$AR = AS = \frac{24}{2} = 12cm$

$OR = OS = OM = 20$ m (radii of circle)

In OAR

$OA^2 + AR^2 = OR^2$

$OA^2 + (12\text{ m})^2 = (20\text{ m})^2$

$OA^2 = (400 - 144)\text{ m}^2 = 256\text{ m}^2$

$OA = 16$ m

We know that in an isosceles triangle altitude divides the base, so in ΔRSM

$\angle RCS$ will be of $90°$ and $RC = CM$

Area of $\Delta ORS = \frac{1}{2} \times OA \times RS$

$\frac{1}{2} \times RC \times OS = \frac{1}{2} \times 16 \times 24$

$RC \times 20 = 16 \times 24$

$RC = 19.2$

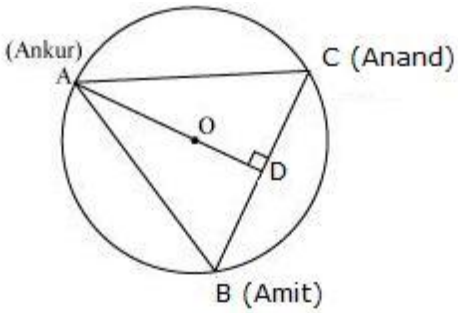
$RM = 2RC = 2(19.2) = 38.4$ m

So, distance between Ishita and Nisha is 38.4 m.

Question 2

A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Solution 2



Given that AB = BC = CA
So, ABC is an equilateral triangle
OA (radius) = 40 m.
Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ABC.
We also know that median intersect each other at the 2: 1. As AD is the median of equilateral triangle ABC, we can write:

$$\frac{OA}{OD} = \frac{2}{1}$$

$$\Rightarrow \frac{40\text{ m}}{OD} = \frac{2}{1}$$

$$\Rightarrow OD = \frac{40}{2}\text{m} = 20\text{ m}$$

$$\therefore AD = OA + OD = (40 + 20)\text{ m} = 60\text{ m}.$$

In ΔADC

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = (60)^2 + \left(\frac{AC}{2}\right)^2$$

$$AC^2 = 3600 + \frac{AC^2}{4}$$

$$\Rightarrow \frac{3}{4}AC^2 = 3600$$

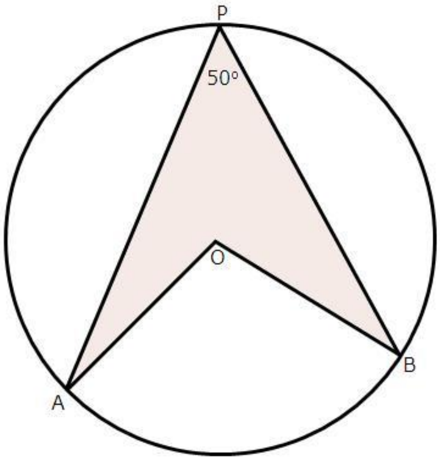
$$\Rightarrow AC^2 = 4800$$

$$\Rightarrow AC = 40\sqrt{3}\text{ m}$$

So, length of string of each phone will be $40\sqrt{3}$ m.

Chapter 15 - Circles Exercise Ex. 15.4
Question 1

In fig., O is the centre of the circle. If $\angle$APB = 50°, find $\angle$AOB and $\angle$OAB.



Solution 1

$\angle APB = 50°$

by degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \angle APB = 2 \times 50° = 100°$

Since, OA = OB [Radii of circle]

Then $\angle OAB = \angle OBA$ [Angles opposite to equal sides]

Let $\angle OAB = x$

In ΔOAB, by angle sum property

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow x + x + 100 = 180°$

$\Rightarrow 2x = 180° - 100°$

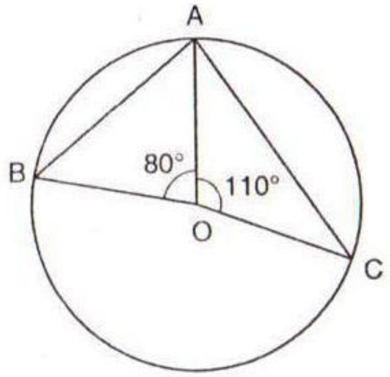
$\Rightarrow 2x = 80°$

$\Rightarrow x = \frac{80}{2} = 40°$

$\therefore \angle OAB = \angle OBA = 40°$

Question 2

In fig., O is the centre of the circle. Find $\angle$BAC.



Solution 2

We have $\angle AOB = 80°$

And $\angle AOC = 110°$

$\therefore \quad \angle AOB + \angle AOC + \angle BOC = 360°$ [Complete angle]

$\Rightarrow \quad 80° + 110° + \angle BOC = 360°$

$\Rightarrow \quad \angle BOC = 360° - 80° - 110°$

$\Rightarrow \quad \angle BOC = 170°$

By degree measure theorem

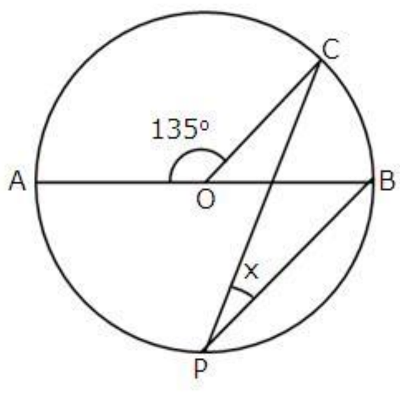
$\angle BOC = 2\angle BAC$

$\Rightarrow \quad 170° = 2\angle BAC$

$\Rightarrow \quad \angle BAC = \dfrac{170°}{2} = 85°$

Question 3(i)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(i)

$\angle AOC = 135^\circ$

$\therefore \quad \angle AOC + \angle BOC = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 135^\circ + \angle BOC = 180^\circ$

$\Rightarrow \quad \angle BOC = 180^\circ - 135^\circ = 45^\circ$

By degree measure theorem

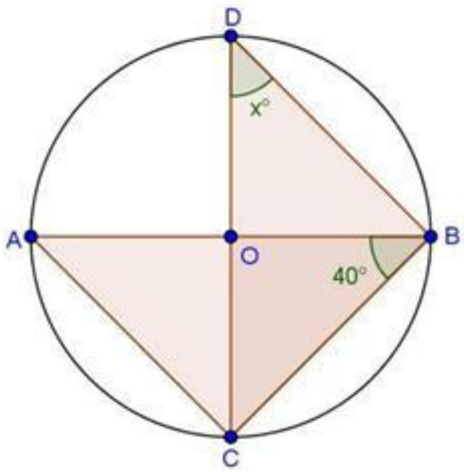
$\angle BOC = 2\angle CDB$

$\Rightarrow \quad 45^\circ = 2x$

$\Rightarrow \quad x = \frac{45^\circ}{2} = 22\frac{1}{2}^\circ$

Question 3(ii)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(ii)

We have

$\angle ABC = 40^\circ$

$\angle ACB = 90^\circ$ [Angle in semicircle]

In $\triangle ABC$, by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle CAB + 90^\circ + 40^\circ = 180^\circ$

$\Rightarrow \quad \angle CAB = 180^\circ - 90^\circ - 40^\circ$

$\Rightarrow \quad \angle CAB = 50^\circ$

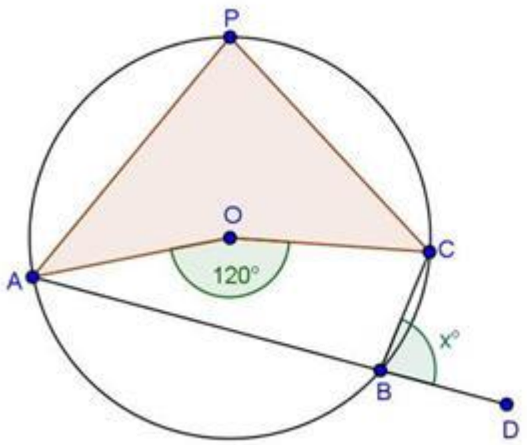
Now,

$\angle CDB = \angle CAB$ [Angle in same segment]

$\Rightarrow \quad x^\circ = 50^\circ$

Question 3(iii)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(iii)

We have

$\angle AOC = 120^\circ$

By degree measure theorem

$\angle AOC = 2\angle APC$

$\Rightarrow \quad 120^\circ = 2\angle APC$

$\Rightarrow \quad \angle APC = \dfrac{120^\circ}{2} = 60^\circ$

$\therefore \quad \angle APC + \angle ABC = 180^\circ$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow \quad 60^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 60^\circ = 120^\circ$

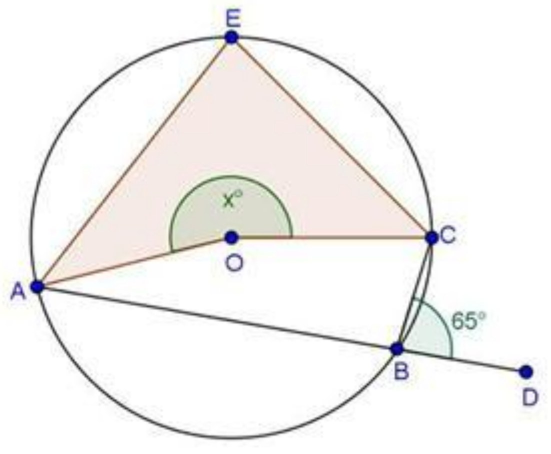
$\therefore \quad \angle ABC + \angle DBC = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 120^\circ + x = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 120^\circ = 60^\circ$

Question 3(iv)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(iv)

We have

$\angle CBD = 65^\circ$

$\therefore \quad \angle ABC + \angle CBD = 180^\circ \quad$ **[Linear pair of angles]**

$\Rightarrow \quad \angle ABC + 65^\circ = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 65^\circ = 115^\circ$

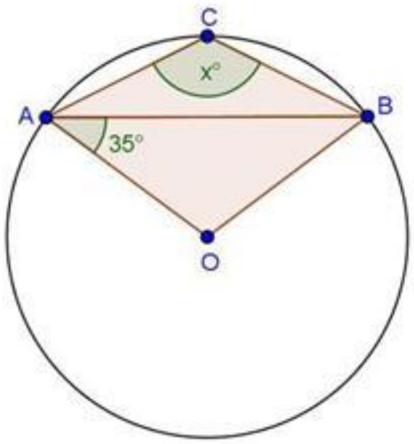
$\therefore \quad \text{Reflex } \angle AOC = 2\angle ABC \quad$ **[By degree measure theorem]**

$\Rightarrow \quad x = 2 \times 115^\circ$

$\Rightarrow \quad x = 230^\circ$

Question 3(v)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(v)

We have

$\angle OAB = 35^\circ$

Then, $\angle OBA = \angle OAB = 35^\circ$ [Angles Opposite to equal radii]

In $\triangle AOB$, by angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

$\Rightarrow \quad \angle AOB + 35^\circ + 35^\circ = 180^\circ$

$\Rightarrow \quad \angle AOB = 180^\circ - 35^\circ - 35^\circ = 110^\circ$

$\therefore \quad \angle AOB + r\,eflex\ \angle AOB = 360^\circ$ [Complete angle]

$\Rightarrow \quad 110^\circ + reflex\ \angle AOB = 360^\circ$

$\Rightarrow \quad Re\mathit{fkex}\ \angle AOB = 360^\circ - 110^\circ = 250^\circ$

By degree measure theorem

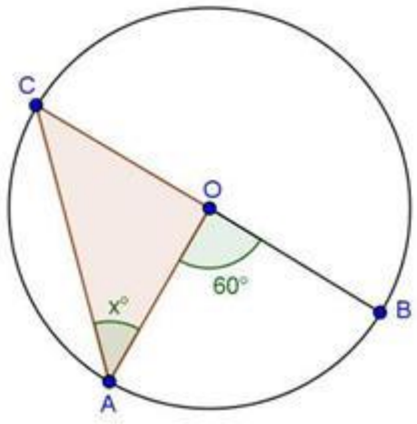
$Re\mathit{flex}\ \angle AOB = 2\angle ACB$

$\Rightarrow \quad 250^\circ = 2x$

$\Rightarrow \quad x = \frac{250^\circ}{2} = 125^\circ$

Question 3(vi)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(vi)

We have

$\angle AOB = 60^\circ$

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow \quad 60^\circ = 2\angle ACB$

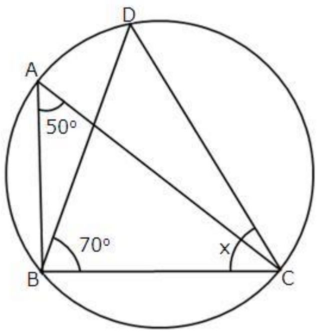
$\Rightarrow \quad \angle ACB = \frac{60^\circ}{2} = 30^\circ$

$\therefore \quad \angle OAC = \angle OCA$ **[Angles Opposite to equal radii]**

$\Rightarrow \quad x = 30^\circ$

Question 3(vii)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(vii)

We have

$\angle BAC = 50^\circ$

and $\angle DBC = 70^\circ$

$\therefore \quad \angle BDC = \angle BAC = 50^\circ$ **[Angle in same segment]**

In $\triangle BDC$**, by angle sum property**

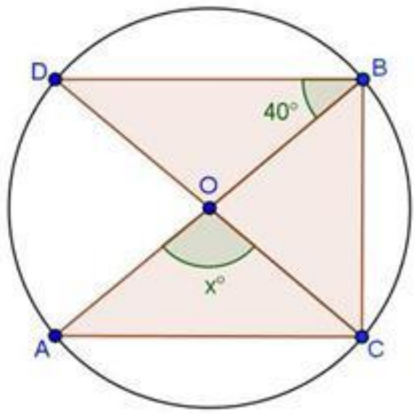
$\angle BDC + \angle BCD + \angle DBC = 180^\circ$

$\Rightarrow \quad 50^\circ + x + 70^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 50^\circ - 70^\circ = 60^\circ$

Question 3(viii)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(viii)

We have

$\angle DBO = 40°$

$\angle DBC = 90°$ **[Angle in semicircle]**

$\Rightarrow \quad \angle DBO + \angle OBC = 90°$

$\Rightarrow \quad 40° + \angle OBC = 90°$

$\Rightarrow \quad \angle OBC = 90° - 40° = 50°$

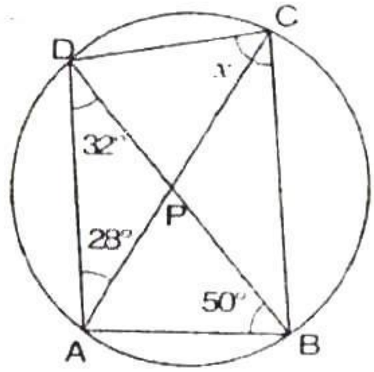
By degree measure theorem

$\angle AOC = 2\angle OBC$

$\Rightarrow \quad x = 2 \times 50° = 100°$

Question 3(ix)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(ix)

In $\triangle DAB$, by angle sum property

$\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$\Rightarrow \quad 32^\circ + \angle DAB + 50^\circ = 180^\circ$

$\Rightarrow \quad \angle DAB = 180^\circ - 32^\circ - 50^\circ$

$\Rightarrow \quad \angle DAB = 98^\circ$

Now,

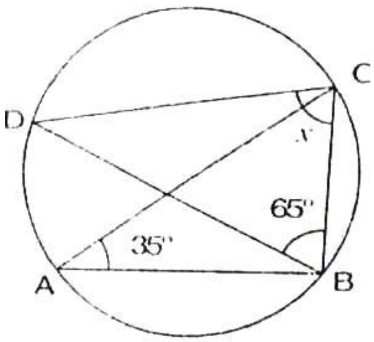
$\angle DAB + \angle DCB = 180^\circ$ **[Opposite angles of cyclic quadrilateral]**

$\Rightarrow \quad 98^\circ + x = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 98^\circ = 82^\circ$

Question 3(x)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(x)

We have

$\angle BAC = 35^\circ$

$\therefore \quad \angle BDC = \angle BAC = 35^\circ$ **[Angle in same segment]**

In $\triangle BCD$, by angle sum property

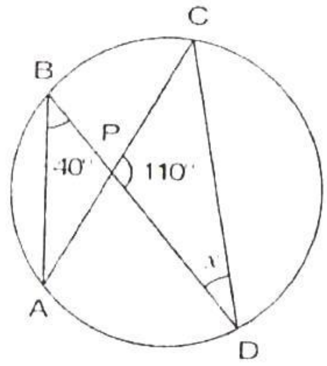
$\angle BDC + \angle BCD + \angle DBC = 180^\circ$

$\Rightarrow \quad 35^\circ + x + 65^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 35^\circ - 65^\circ = 80^\circ$

Question 3(xi)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(xi)

We have

$\angle ABD = 40^\circ$

$\therefore \quad \angle ACD = \angle ABD = 40^\circ \qquad$ [Angle in same segment]

In $\triangle PCD$, by angle sum property

$\angle PCD + \angle CPD + \angle PDC = 180^\circ$

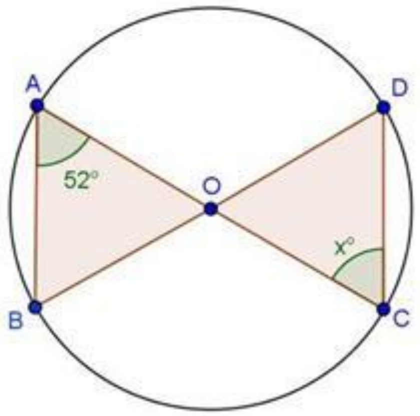
$\Rightarrow \quad 40^\circ + 110^\circ + x^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 40^\circ - 110^\circ$

$\Rightarrow \quad x = 30^\circ$

Question 3(xii)

If O is the centre of the circle. Find the value of x in the following figure:



Solution 3(xii)

We have

$\angle BAC = 52^\circ$

Then, $\angle BDC = \angle BAC = 52^\circ$ [Angle in same segment]

Since, $OD = OC$ [Radii of circle]

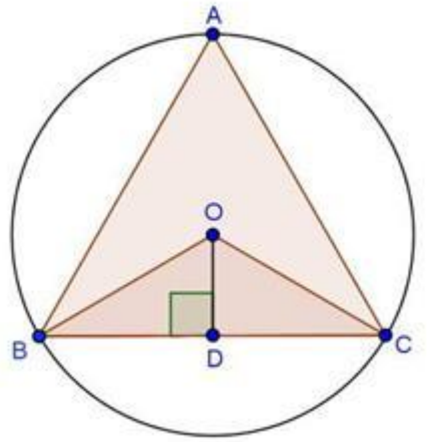
Then, $\angle ODC = \angle OCD$ [Opposite angles to equal radii]

$\Rightarrow \quad 52^\circ = x$

Question 4

O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove That $\angle BOD = \angle A$.

Solution 4



Given, O is the circumcentre of $\triangle ABC$ and $OD \perp BC$

To prove $\angle BOD = 2\angle A$

Proof

In $\triangle OBD$ and $\triangle OCD$

$\angle ODB = \angle ODC$ [Each 90°]

$OB = OC$ [Radii of circle]

$OD = OD$ [Common]

Then, $\triangle OBD \cong \triangle OCD$ [By RHS condition]

$\therefore \quad \angle BOD = \angle COD \quad ---(1)$ [c.p.c.t]

By degree measure theorem

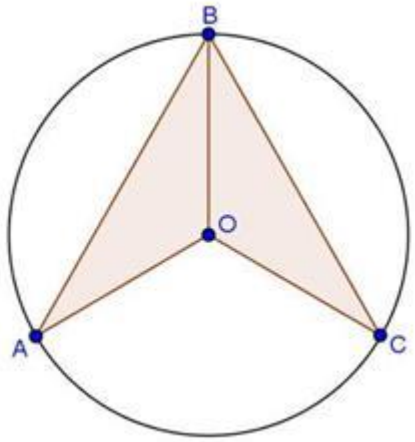
$\angle BOC = 2\angle BAC$

$\Rightarrow \quad 2\angle BOD = 2\angle BAC$ [By using (1)]

$\Rightarrow \quad \angle BOD = \angle BAC$

Question 5

In fig., O is the centre of the circle, BO is the bisector of $\angle$ABC. Show that AB = AC.



Solution 5

Given, BO is the bisector of $\angle ABC$

To prove $AB = BC$

Proof

Since, BO is the bisector of $\angle ABC$

Then, $\angle ABO = \angle CBO \qquad ---(1)$

Since, $OB = OA$ [Radii of circle]

Then, $\angle ABO = \angle OAB \qquad ---(2)$ [Opposite angles to equal sides]

Since, $OB = OC$ [Radii of circle]

Then, $\angle CBO = \angle OCB \qquad ---(3)$ [Opposite angles to equal sides]

Compare equations $(1)(2) \& (3)$

$\angle OAB = \angle OCB \qquad ---(4)$

In $\triangle OAB$ and $\triangle OCB$

$\angle OAB = \angle OCB$ [From (4)]

$\angle OBA = \angle OBC$ [Given]

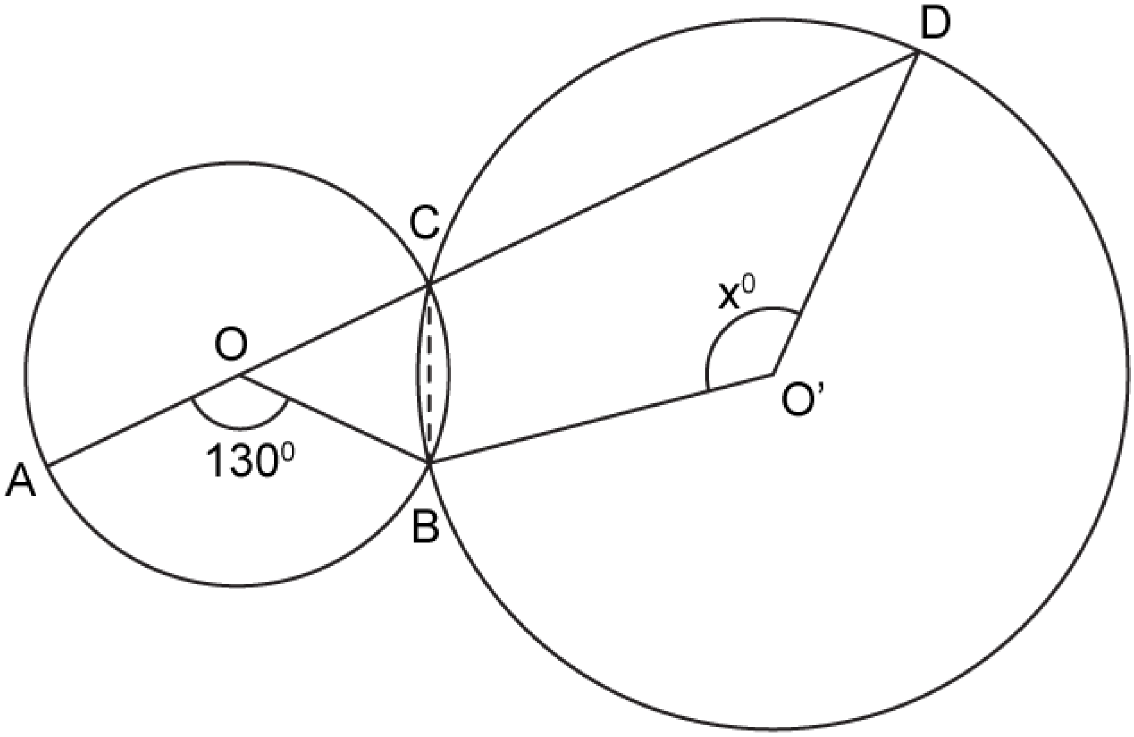
$OB = OB$ [Common]

Then, $\triangle OAB \cong \triangle OCB$ [by AAS condition]

$\therefore \quad AB = BC$ [c.p.c.t]

Question 6

In fig., O and O' are centres of two circles intersecting at B and C. ABD is straight line, find x.



Solution 6

By degree measure theorem

$\angle AOB = 2\angle ACB$

$\Rightarrow \quad 130^\circ = 2\angle ACB$

$\Rightarrow \quad \angle ACB = \frac{130^\circ}{2} = 65^\circ$

$\therefore \quad \angle ACB + \angle BCD = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 65^\circ + \angle BCD = 180^\circ$

$\Rightarrow \quad \angle BCD = 180^\circ - 65^\circ = 115^\circ$

By degree measure theorem

Reflex $\angle BO'D = 2\angle BCD$

$\Rightarrow$ Reflex $\angle BO'D = 2 \times 115^\circ = 230^\circ$

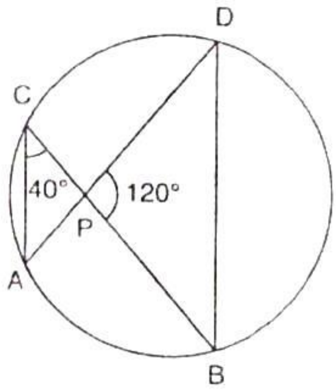
Now, reflex $\angle BO'D + \angle BO'D = 360^\circ$ [Complete angle]

$\Rightarrow \quad 230^\circ + x = 360^\circ$

$\Rightarrow \quad x = 360^\circ - 230^\circ = 130^\circ$

Question 7

In fig., if ∠ACB = 40°, ∠DPB = 120°, find ∠CBD.



Solution 7

We have,

$\angle ACB = 40°$

$\angle DPB = 120°$

$\therefore \quad \angle ADB = \angle ACB = 40°$ [Angle in same segment]

In $\triangle PDB$, by angle sum property

$\angle PDB + \angle PBD + \angle BPD = 180°$

$\Rightarrow \quad 40° + \angle PBD + 120° = 180°$

$\Rightarrow \quad \angle PBD = 180° - 40° - 120°$

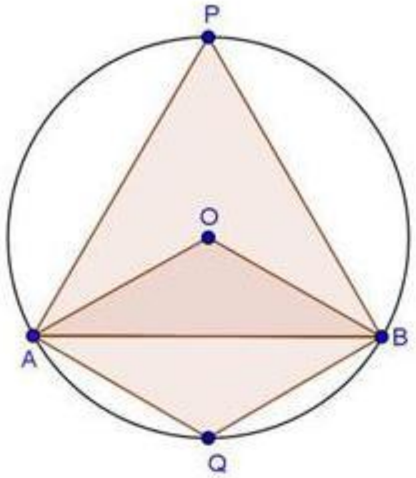
$\Rightarrow \quad \angle PBD = 20°$

$\therefore \quad \angle CBD = 20°$

Question 8

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution 8



We have,

Radius OA = Chord AB

$\Rightarrow \quad OA = OB = AB$

Then, $\triangle OAB$ is an equilateral triangle.

$\therefore \quad \angle AOB = 60^\circ$ [One angle of equilateral $\triangle$]

By degree measure theorem

$\angle AOB = 2\angle APB$

$\Rightarrow \quad 60^\circ = 2\angle APB$

$\Rightarrow \quad \angle APB = \frac{60^\circ}{2} = 30^\circ$

Now, $\angle APB + \angle AQB = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 30^\circ + \angle AQB = 180^\circ$

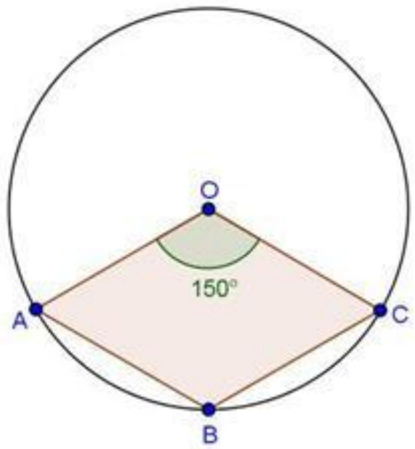
$\Rightarrow \quad \angle AQB = 180^\circ - 30^\circ = 150^\circ$

$\therefore$ Angle by chord AB at minor arc = 150°

Angle by chord AB at major arc = 30°

Question 9

In fig., it is given given that O is the centre of the circle and $\angle$AOC = 150°. Find $\angle$ABC.



Solution 9

We have $\angle AOC = 150^\circ$

$\therefore \angle AOC + \text{reflex } \angle AOC = 360^\circ$ [Complete angle]

$\Rightarrow 150^\circ + \text{reflex } \angle AOC = 360^\circ$

$\Rightarrow \text{reflex } \angle AOC = 360^\circ - 150^\circ$

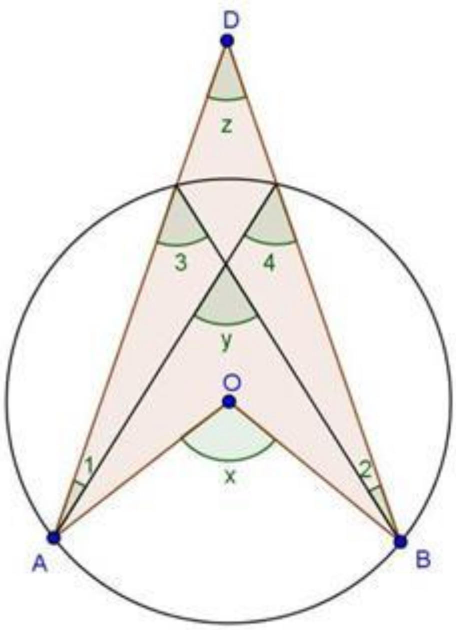
$\Rightarrow \text{reflex } \angle AOC = 210^\circ$

$\Rightarrow 2\angle ABC = 210^\circ$ [By degree measure theorem]

$\Rightarrow \angle ABC = \frac{210}{2} = 105^\circ$

Question 10

In fig., O is the centre of the circle, prove that $\angle x = \angle y + \angle z$.



Solution 10

We have, $\angle 3 = \angle 4$ [Angles in same segment]

$\therefore \quad \angle x = 2\angle 3$ [By degree measure theorem]

$\Rightarrow \quad \angle X = \angle 3 + \angle 3$

$\Rightarrow \quad \angle x = \angle 3 + \angle 4 \quad ---(1)$ $[\angle 3 = \angle 4]$

But $\angle y = \angle 3 + \angle 1$ [By exterior angle prop.]

$\Rightarrow \quad \angle 3 = \angle y - \angle 1 \quad ---(2)$

From (1) and (2)

$\angle x = \angle y - \angle 1 + \angle 4$

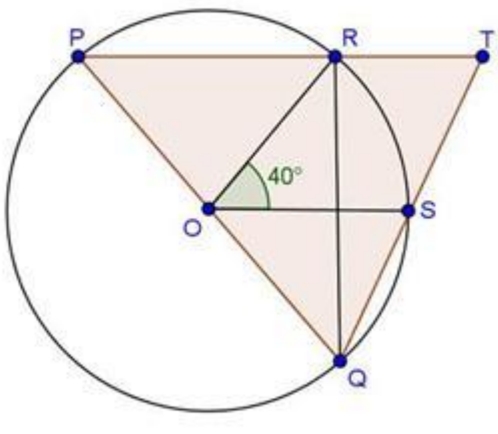
$\Rightarrow \quad \angle x = \angle y + \angle 4 - \angle 1$

$\Rightarrow \quad \angle x = \angle y + \angle z + \angle 1 - \angle 1$ [By exterior angle prop.]

$\Rightarrow \quad \angle x = \angle y + \angle z$

Question 11

in fig., O is the centre of a circle and PQ is a diameter. If $\angle$ROS = 40°, find. $\angle$RTS.



Solution 11

Since, PQ is a diameter

Then, $\angle PRQ = 90^\circ$ [Angle in semicircle]

$\therefore \quad \angle PRQ + \angle TRQ = 180^\circ$ [Linear pair of angle]

$\Rightarrow \quad \angle 90^\circ + \angle TRQ = 180^\circ$

$\Rightarrow \quad \angle TRQ = 180^\circ - 90^\circ = 90^\circ$

By degree measure theorem

$\angle ROS = 2\angle RQS$

$\Rightarrow \quad 40^\circ = 2\angle RQS$

$\Rightarrow \quad \angle RQS = \frac{40^\circ}{2} = 20^\circ$

In $\triangle RQT$, by angle sum property

$\angle RQT + \angle QRT + \angle RTS = 180^\circ$

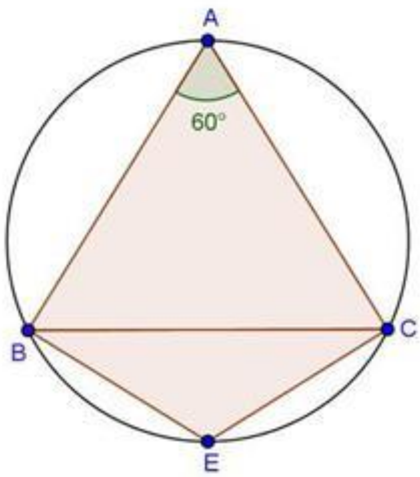
$\Rightarrow \quad 20^\circ + 90^\circ + \angle RTS = 180^\circ$

$\Rightarrow \quad \angle RTS = 180^\circ - 20^\circ - 90^\circ = 70^\circ$

Chapter 15 - Circles Exercise Ex. 15.5

Question 1

In fig., ΔABC is an equilateral triangle. Find m$\angle$BEC.



Solution 1

Since, $\triangle ABC$ is an equilateral triangle.

Then, $\angle BAC = 60^\circ$

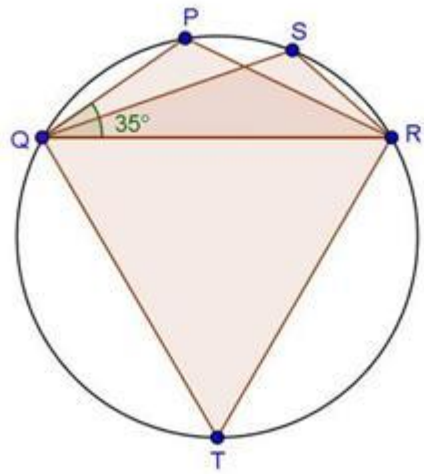
$\therefore \quad \angle BAC + \angle BEC = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 60^\circ + \angle BEC = 180^\circ$

$\Rightarrow \quad \angle BEC = 180^\circ - 60^\circ = 120^\circ$

Question 2

In fig., ΔPQR is an isosceles triangle with PQ = PR and m$\angle$PQR = 35°. find m$\angle$QSR and m$\angle$QTR.



Solution 2

We have, $\angle PQR = 35^\circ$

Since, $\triangle PQR$ is an isosceles triangle with $PQ = PR$.

Then, $\angle PQR = \angle PRQ = 35^\circ$

In $\triangle PQR$, by angle sum property

$\angle P + \angle PQR + \angle PRQ = 180^\circ$

$\Rightarrow \quad \angle P + 35^\circ + 35^\circ = 180^\circ$

$\Rightarrow \quad \angle P = 180^\circ - 35^\circ - 35^\circ = 110^\circ$

$\therefore \quad \angle QSR = \angle P = 110^\circ$ [Angles in same segment]

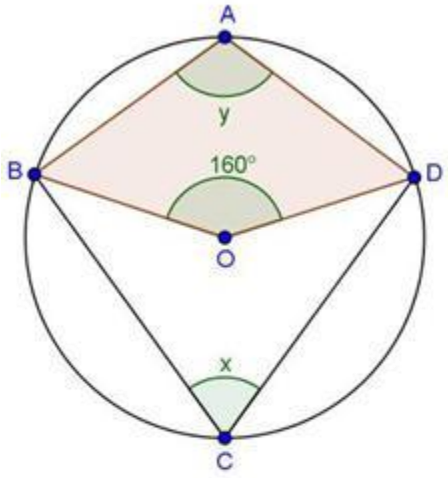
Now, $\angle QSR + \angle QTR = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 110^\circ + \angle QTR = 180^\circ$

$\Rightarrow \quad \angle QTR = 180^\circ - 110^\circ = 70^\circ$

Question 3

In fig., O is the centre of the circle. If $\angle$BOD = 160°, find the values of x and y.



Solution 3

We have, $\angle BOD = 160^\circ$

By degree measure theorem

$\angle BOD = 2\angle BCD$

$\Rightarrow \quad 160^\circ = 2 \times x$

$$\Rightarrow \quad x = \frac{160^\circ}{2} = 80^\circ$$

$\therefore \quad \angle BAD + \angle BCD = 180^\circ$ **[Opposite angles of cyclic quad.]**

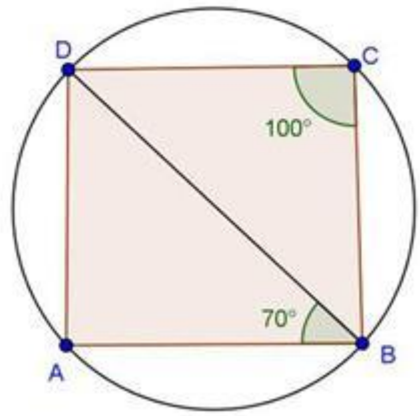
$\Rightarrow \quad y + x = 180^\circ$

$\Rightarrow \quad y + 80^\circ = 180$

$\Rightarrow \quad y = 180^\circ - 80^\circ = 100^\circ$

Question 4

In fig., ABCD is a cyclic qudrilateral. If $\angle$BCD = 100° and ABD = 70°, find $\angle$ADB.



Solution 4

We have, $\angle BCD = 100^\circ$ *and* $\angle ABD = 70^\circ$

$\therefore \quad \angle DAB + \angle BCD = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad \angle DAB + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle DAB + 180^\circ - 100^\circ = 80^\circ$

In $\triangle DAB$, by angle sum property

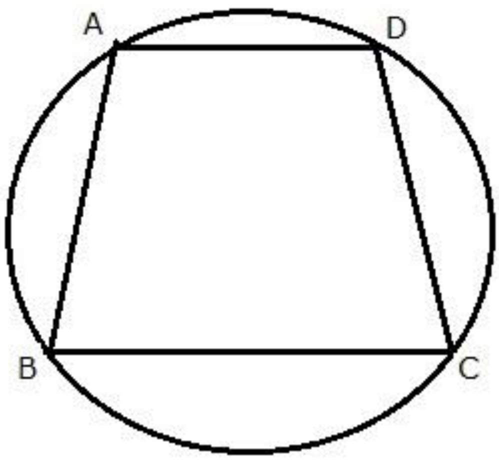
$\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$\Rightarrow \quad \angle ADB + 80^\circ + 70^\circ = 180^\circ$

$\Rightarrow \quad \angle ADB = 180^\circ - 80^\circ - 70^\circ = 30^\circ$

Question 5

If ABCD is a cyclic quadrilateral in which AD $\parallel$ BC. Prove that $\angle$B = $\angle$C.



Solution 5

Since, $ABCD$ is a cyclic quadrilateral with $AD \parallel BC$

Then, $\angle A + \angle C = 180^\circ$ $---(1)$ [Opposite angles of cyclic quad.]

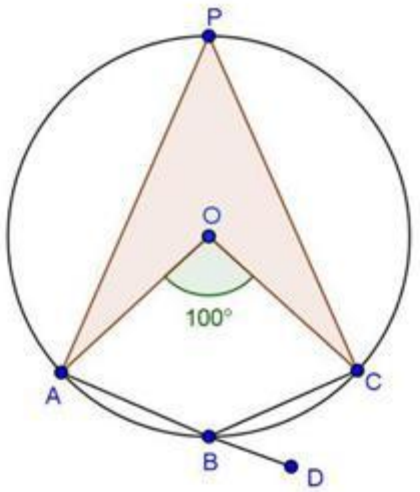
And, $\angle A + \angle B = 180^\circ$ $---(2)$ [Co-interior angles]

Compare equations (1) and (2)

$\angle B = \angle C$

Question 6

In fig., O is the centre of the circle. find $\angle$CBD.



Solution 6

We have, $\angle AOC = 100^\circ$

By degree measure theorem

$\angle AOC = 2\angle APC$

$\Rightarrow \quad 100^\circ = 2\angle APC$

$\Rightarrow \quad \angle APC = \frac{100^\circ}{2} = 50^\circ$

$\therefore \quad \angle APC + \angle ABC = 180^\circ$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 50^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 50^\circ = 130^\circ$

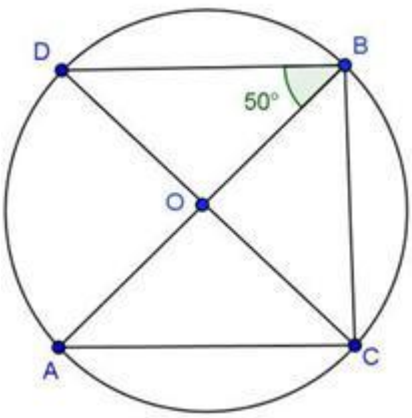
$\therefore \quad \angle ABC + \angle CBD = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 130^\circ + \angle CBD = 180^\circ$

$\Rightarrow \quad \angle CBD = 180^\circ - 130^\circ = 50^\circ$

Question 7

In fig., AB and CD are diameters of a circle with centre O. If $\angle$OBD = 50°, find $\angle$AOC.



Solution 7

We have, $\angle OBD = 50^\circ$

Since, AB and CD are diameters of circle then O is the centre of the circle.

$\therefore \quad \angle DBC = 90^\circ$ [Angle in semicircle]

$\Rightarrow \quad \angle DBO + \angle OBC = 90^\circ$
$\Rightarrow \quad 50^\circ + \angle OBC = 90^\circ$

$\Rightarrow \quad \angle OBC = 90^\circ - 50^\circ = 40^\circ$

By degree measure theorem

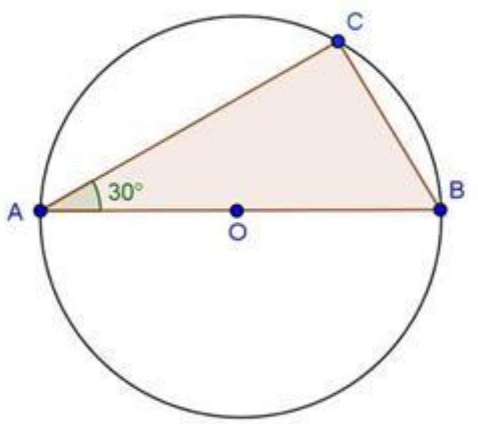
$\angle AOC = 2\angle ABC$

$\Rightarrow \quad \angle AOC = 2 \times 40^\circ = 80^\circ$

Question 8

On a semi-circle with AB as diameter, a point C is taken, so that $m(\angle CAB) = 30^\circ$.
Find $m(\angle ACB)$ and $m(\angle ABC)$.

Solution 8



We have, $\angle CAB = 30^\circ$

$\therefore \quad \angle ACB = 90^\circ$ [Angle in semicircle]

In $\triangle ABC$, by angle sum property

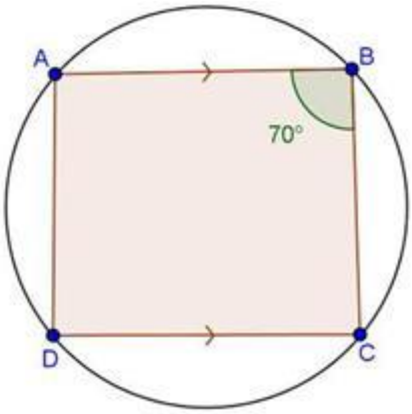
$\angle CAB + \angle ACB + \angle ABC = 180^\circ$
$\Rightarrow \quad 30^\circ + 90^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 90^\circ - 30^\circ = 60^\circ$

Question 9

In a cyclic quadrilateral $ABCD$ if $AB \parallel CD$ and $\angle B = 70^\circ$, find the remaining angles.

Solution 9



We have, $\angle B = 70^\circ$

***Since,* $ABCD$ is a cyclic quadrilateral**

Then, $\angle B + \angle D = 180^\circ$

$\Rightarrow \quad 70^\circ + \angle D = 180^\circ$

$\Rightarrow \quad \angle D = 180^\circ - 70^\circ = 110^\circ$

Since, $AB \parallel DC$

Then, $\angle B + \angle C = 180^\circ$ [Co-interior angles]

$\Rightarrow \quad 70^\circ + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 180^\circ - 70^\circ = 110^\circ$

***Now,* $\angle A + \angle C = 180^\circ$** [Opposite angles of cyclic quad.]

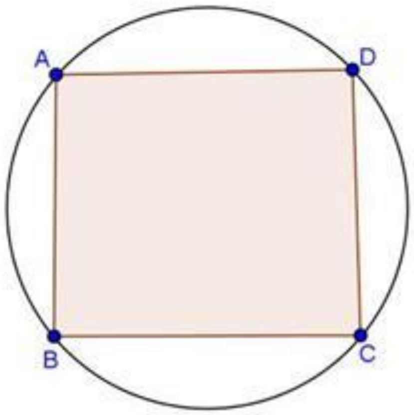
$\Rightarrow \quad \angle A + 110^\circ = 180^\circ$

$\Rightarrow \quad \angle A = 180^\circ - 110^\circ = 70^\circ$

Question 10

In a cyclic quadrilateral $ABCD$, if $m\angle A = 3(m\angle C)$. find $m\angle A$.

Solution 10



We have, $\angle A = 3\angle C$

Let $\angle C = x$
Then, $\angle A = 3x$

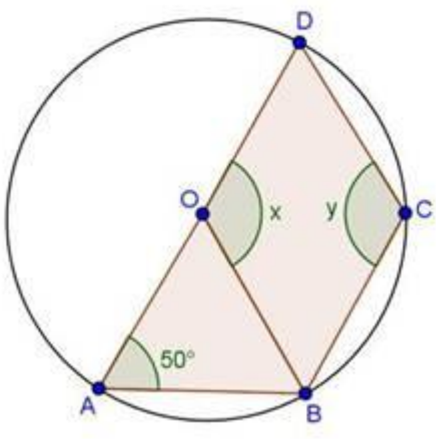
$\therefore \quad \angle A + \angle C = 180°$ [Opposite angles of cyclic quad.]

$\Rightarrow \quad 3x + x = 180°$
$\Rightarrow \quad 4x = 180°$
$\Rightarrow \quad x = \frac{180°}{4} = 45°$

$\therefore \quad \angle A = 3x$
$= 3 \times 45°$

$= 135°$

Question 11
In fig., O is the centre of the circle and DAB = 50. calculate the values of x and y.



Solution 11

We have, $\angle DAB = 50^\circ$

By degree measure theorem

$\angle BOD = 2\angle BAD$

$\Rightarrow \quad x = 2 \times 50^\circ = 100^\circ$

Since, $ABCD$ is a cyclic quadrilateral

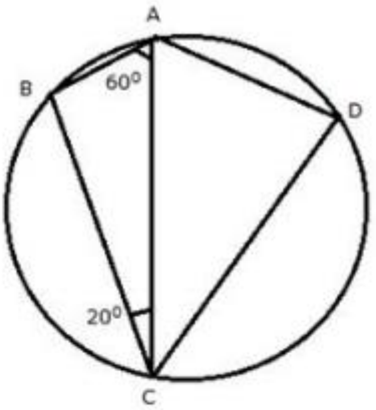
Then, $\angle A + \angle C = 180^\circ$

$\Rightarrow \quad 50^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 50^\circ = 130^\circ$

Question 12

In fig., if $\angle$BAC = 60°, and $\angle$BCA = 20°, find $\angle$ADC.



Solution 12

Using angle sum property in ΔABC,

$\angle B = 180° - (60° + 20°) = 100°$

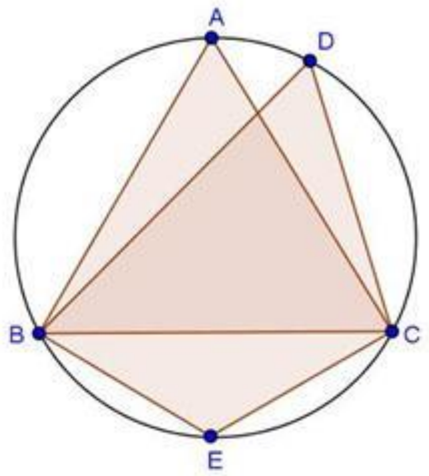
In cyclic quadrilateral ABCD, we have:

$\angle B + \angle D = 180°$

$\angle D = 180° - 100° = 80°$

Question 13

In fig., if ABC is an equilateral triangle. Find $\angle$BDC and $\angle$BEC.



Solution 13

Since, $\triangle ABC$ is an equilateral triangle

Then, $\angle BAC = 60^\circ$

$\therefore \quad \angle BDC = \angle BAC = 60^\circ$ [Angles in same segment]

Since, quad. $ABEC$ is a cyclic quadrilateral.

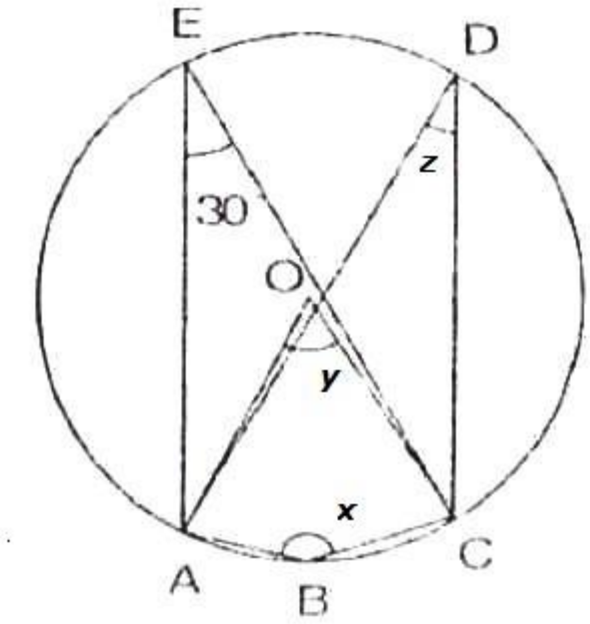
Then, $\angle BAC + \angle BEC = 180^\circ$

$\Rightarrow \quad 60^\circ + \angle BEC = 180^\circ$

$\Rightarrow \quad \angle BEC = 180^\circ - 60^\circ = 120^\circ$

Question 14

In fig., O is the centre of the circle. If $\angle$CEA = 30°, find the values of x, y and z.



Solution 14

We have, $\angle AEC = 30^\circ$

Since, quad. $ABCE$ is a cyclic quadrilateral.

Then, $\angle ABC + \angle AEC = 180^\circ$

$\Rightarrow \quad x + 30^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 30^\circ = 150^\circ$

By degree measure theorem

$\angle AOC = 2\angle AEC$

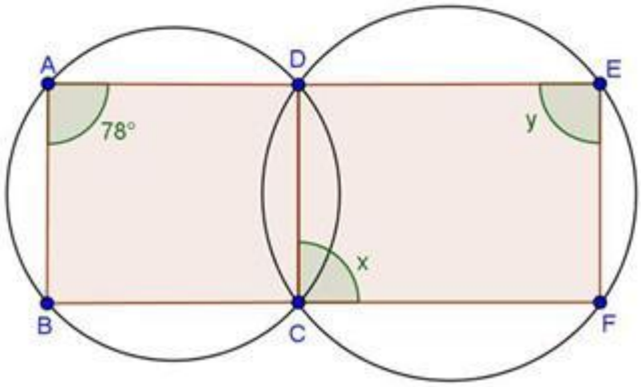
$\Rightarrow \quad y = 2 \times 30^\circ = 60^\circ$

$\therefore \quad \angle ADC = \angle AEC$ **[Angles in same segment]**

$\Rightarrow \quad z = 30^\circ$

Question 15

In fig., $\angle$BAD = 78°, $\angle$DCF = x° and DEF = y° find the values of x and y.



Solution 15

We have, $\angle BAD = 78^\circ$, $\angle DCF = x^\circ$ and $\angle DEF = y^\circ$

Since, $ABCD$ is a cyclic quadrilateral.

Then, $\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \quad 78^\circ + \angle BCD = 180^\circ$

$\Rightarrow \quad \angle BCD = 180^\circ - 78^\circ = 102^\circ$

Now, $\angle BCD + \angle DCF = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 102^\circ = x^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 102^\circ = 78^\circ$

Since, $DCFE$ is a cyclic quadrilateral

Then, $x + y = 180^\circ$

$\Rightarrow \quad 78^\circ + y = 180^\circ$

$\Rightarrow \quad y = 180^\circ - 78^\circ = 102^\circ$

Question 16

In a cyclic quadrilateral $ABCD$, if $\angle A - \angle C = 60^\circ$, prove that the smaller of two is 60°.

Solution 16

We have

$\angle A - \angle C = 60^\circ \qquad ---(1)$

Since, $ABCD$ is a cyclic quardilateral

Then $\angle A + \angle C = 180^\circ \qquad ---(2)$

Add equations (1) and (2)

$\angle A - \angle C + \angle A + \angle C = 60^\circ + 180^\circ$

$\Rightarrow \quad 2\angle A = 240^\circ$

$\Rightarrow \quad \angle A = \dfrac{240}{2} = 120^\circ$

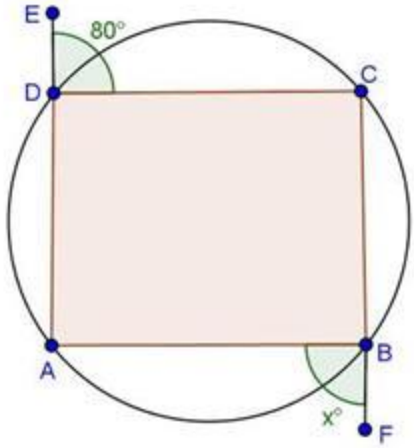
Put value of $\angle A$ in equation (2)

$120^\circ + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 180^\circ - 120^\circ = 60^\circ$

Question 17

In fig., ABCD is cyclic qudrilateral. Find the value of x.



Solution 17

$\angle EDC + \angle CDA = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 80^\circ + \angle CDA = 180^\circ$

$\Rightarrow \quad \angle CDA = 180^\circ - 80^\circ = 100^\circ$

Since, $ABCD$ is a cyclic quadrilateral.

$\angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \quad 100^\circ + \angle ABC = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 100^\circ = 80^\circ$

Now, $\angle ABC + \angle ABF = 180^\circ$ [Linear pair of angles]

$\Rightarrow \quad 80^\circ + x^\circ = 180^\circ$

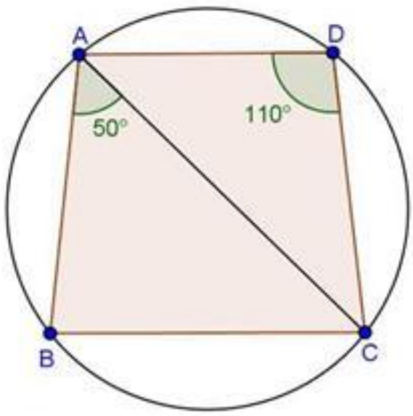
$\Rightarrow \quad x = 180^\circ - 80^\circ = 100^\circ$

Question 18(i)

$ABCD$ is a cyclic quadrilateral in which:

(i) $BC \parallel AD$, $\angle ADC = 110^\circ$ and $\angle BAC = 50^\circ$. Find $\angle DAC$.

Solution 18(i)



Since, $ABCD$ is a cyclic quadrilateral.

Then, $\angle ABC + \angle ADC = 180^\circ$

$\Rightarrow \quad \angle ABC + 110^\circ = 180^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 110^\circ = 70^\circ$

Since, $AD \parallel BC$

Then, $\angle DAB + \angle ABC = 180^\circ$ [Co-interior angles]

$\Rightarrow \quad \angle DAC + 50^\circ + 70^\circ = 180^\circ$

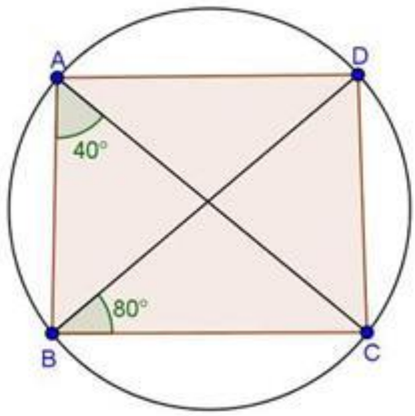
$\Rightarrow \quad \angle DAC = 180^\circ - 50^\circ - 70^\circ = 60^\circ$

Question 18(ii)

$ABCD$ is a cyclic quadrilateral in which:

(ii) $\angle DBC = 80^\circ$ and $\angle BAC = 40^\circ$. Find $\angle BCD$.

Solution 18(ii)



$\angle BAC = \angle BDC = 40^\circ$ **[Angles in same segment]**

In $\triangle BDC$, by angle sum property

$\angle DBC + \angle BCD + \angle BDC = 180^\circ$

$\Rightarrow \quad 80^\circ + \angle BCD + 40^\circ = 180^\circ$

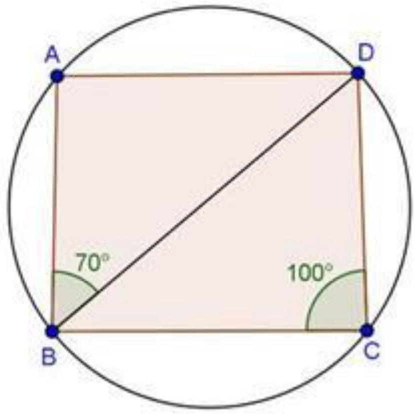
$\Rightarrow \quad \angle BCD = 180^\circ - 80^\circ - 40^\circ = 60^\circ$

Question 18(iii)

$ABCD$ is a cyclic quadrilateral in which:

(iii) $\angle BCD = 100^\circ$ and $\angle ABD = 70^\circ$. Find $\angle ADB$.

Solution 18(iii)



Since, $ABCD$ is a cyclic quadrilateral.

Then, $\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \quad \angle BAD + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle BAD = 180^\circ - 100^\circ = 80^\circ$

In $\triangle ABD$, by angle sum property

$\angle ABD + \angle ADB + \angle BAD = 180^\circ$

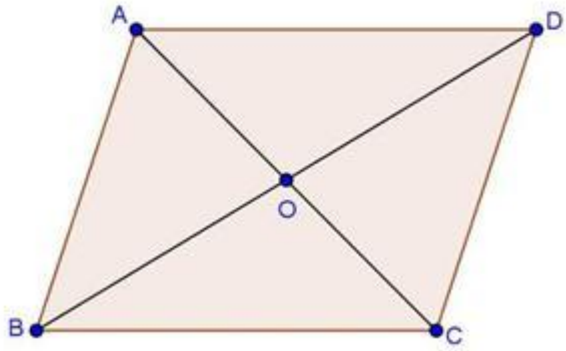
$\Rightarrow \quad 70^\circ + \angle ADB + 80^\circ = 180^\circ$

$\Rightarrow \quad \angle ADB = 180^\circ - 70^\circ - 80^\circ = 30^\circ$

Question 19

Prove that the circles described on the four sides of a rhombus as diameters, pass through the point of intersection of its diagonals.

Solution 19



Let $ABCD$ be a rhombus such that its diagonals AC and BD intersect at O.
Since, the diagonals of a rhombus intersect at right angle.

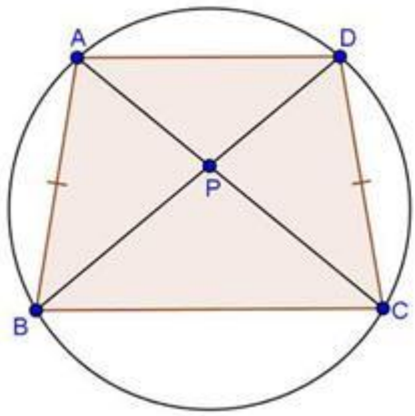
$\therefore \quad \angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$.

Now, $\angle AOB = 90^\circ \Rightarrow$ circle described on AB as diameter will pass through O.
Similarly, all the circles described on BC, AD and CD as diameters pass through O.

Question 20

If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are equal.

Solution 20



Given $ABCD$ is a cyclic quadrilateral in which $AB = DC$

To prove $AC = BD$

Proof In $\triangle PAB$ and $\triangle PDC$

$AB = DC$	[Given]
$\angle BAP = \angle CDP$	[Angles in the same segment]
$\angle PBA = \angle PCD$	[Angles in same segment]

Then, $\triangle PAB \cong \triangle PDC$ [By ASA condition]

$\therefore \quad PA = PD \quad ---(1) \quad$ [c.p.c.t]

and $\quad PC = PB \quad ---(2) \quad$ [c.p.c.t]

Add equation (1) and (2)

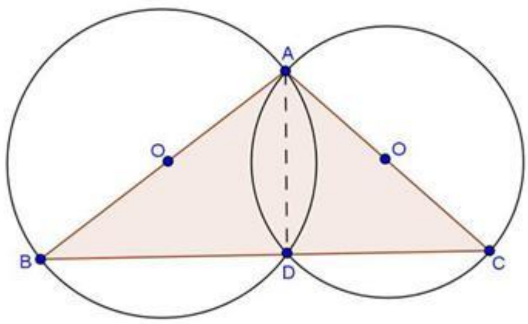
$PA + PC = PD + PB$

$\Rightarrow \quad AC = BD$

Question 21

Circles are described on the sides of a triangle as diameters. prove that the circles on any two sides intersect each other on the third side(or third side produced).

Solution 21



Since, AB is a diameter

Then, $\angle ADB = 90^\circ$ $---(1)$ [Angle in semicircle]

Since, AC is a diameter

Then, $\angle ADC = 90^\circ$ $---(2)$ [Angle in semicircle]

Add equations (1) *and* (2)

$\angle ADB + \angle ADC = 90^\circ + 90^\circ$

$\Rightarrow \quad \angle BDC = 180^\circ$

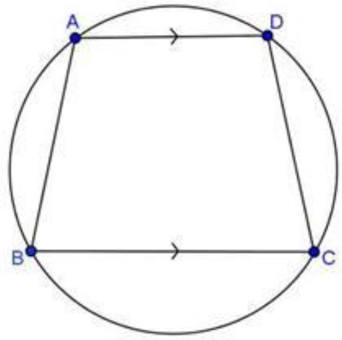
Then, BDC is a line

Hence, the circles on any two sides intersect each other on the third side.

Question 22

$ABCD$ is a cyclic trapezium with $AD \parallel BC$. If $\angle B = 70^\circ$, determine other three angles of the trapezium.

Solution 22



We have

$ABCD$ is a cyclic trapezium with $AD \parallel BC$ and $\angle B = 70°$.

Since, $ABCD$ is a cyclic quadrilateral

Then, $\angle B + \angle D = 180°$

$\Rightarrow \quad 70° + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 70° = 110°$

Since, $AD \parallel BC$

Then, $\angle A + \angle B = 180°$ [Co-interior angles]

$\Rightarrow \quad \angle A + 70° = 180°$

$\Rightarrow \quad \angle A = 180° - 70° = 110°$

Since, $ABCD$ is a cyclic quadrilateral

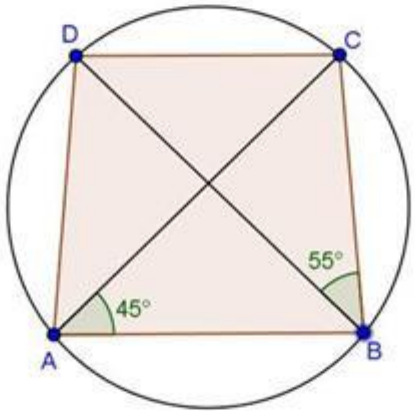
Then, $\angle A + \angle C = 180°$

$\Rightarrow \quad 110° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 110° = 70°$

Question 23

In fig., ABCD is cyclic quadrilaterial in which AC an BD are its diagonals. If $\angle$DBC = 55° and $\angle$BAC = 45°, find $\angle$BCD.



Solution 23

Since angles in the same segment of a circle are equal.

$\therefore \quad \angle CAD = \angle DBC = 55^\circ$

$\therefore \quad \angle DAB = \angle CAD + \angle BAC = 55^\circ + 45^\circ = 100^\circ$

But, $\angle DAB + \angle BCD = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$\therefore \quad \angle BCD = 180^\circ - 100^\circ = 80^\circ$

Question 24

Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

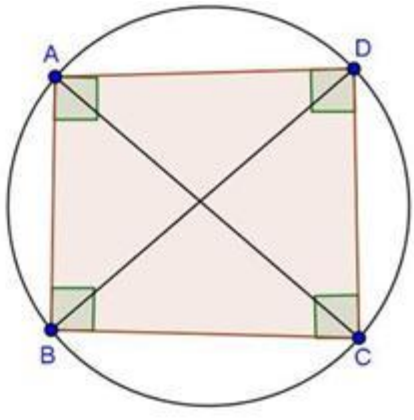
Solution 24

Let $ABCD$ be a cyclic quadrilateral, and let O be the centre of the corresponding circle.
Then, each side of quadrilateral $ABCD$ is a chord of the circle and the perpendicular bisector of a chord always passes through the centre of the circle.
So, right bisectors of the sides of quadrilateral $ABCD$ will pass through the centre O of the corresponding circle.

Question 25

Prove that the centre of the circle circumscribing the cyclic rectangle ABCD is the point of intersection of its diagonals.

Solution 25



Let O be the centre of the circle circumscribing the cyclic rectangle ABCD. Since $\angle$ABC = 90^o and AC is a chord of the circle, so, AC is a diameter of the circle. Similarly, BD is a diameter.

Hence, point of intersection of AC and BD is the centre of the circle.

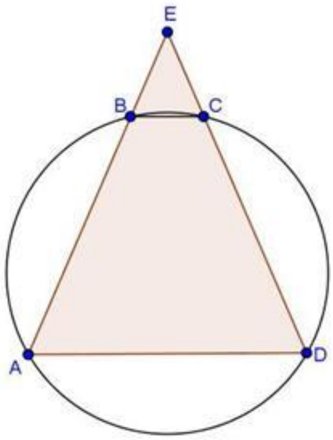
Question 26

$ABCD$ is a cyclic quadrilateral in which AB and CD when produced meet in E and $EA = ED$. Prove that:

(i) $AD \parallel BC$

(ii) $EB = EC$.

Solution 26



Given $ABCD$ is a cyclic quadrilateral in which $EA = ED$

To prove (i) $AD \parallel BC$ (ii) $EB = EC$

Proof (i) Since $EA = ED$

Then, $\angle EAD = \angle EDA$ $---(1)$ [Oppo. angles to equal sides]

Since, $ABCD$ is a cyclic quadrilateral

Then, $\angle ABC + \angle ADC = 180^\circ$

But $\angle ABC + \angle EBC = 180^\circ$ [Lilear pair of angles]

then, $\angle ADC = EBC$ $---(2)$

Compare equations (1) and (2)

$\angle EAD = \angle EBC$ $---(3)$

Since, corresponding angles are equal

Then, $BC \parallel AD$

(ii) From equation (3)

$\angle EAD = \angle EBC$ $---(3)$

Similarly $\angle EDA = \angle ECB$ $---(4)$

Compare equations $(1)(3)$ and (4)

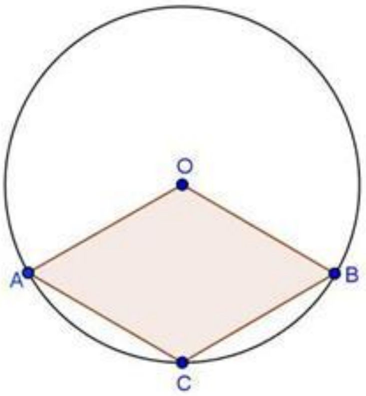
$\angle EBC = \angle ECB$

$\Rightarrow$ $EB = EC$ [Opposite angles to equal sides]

Question 27

Prove that the angle in a segment shorter than a semicircle is greater than a right angle.

Solution 27



Given:- $\angle ACB$ **is an angle in minor segment.**

To prove:- $\angle ACB > 90^\circ$

Proof:- By degree measure theorem

Reflex $\angle AOB = 2\angle ACB$

And reflex $\angle AOB > 180^\circ$

Then, $2\angle ACB > 180^\circ$

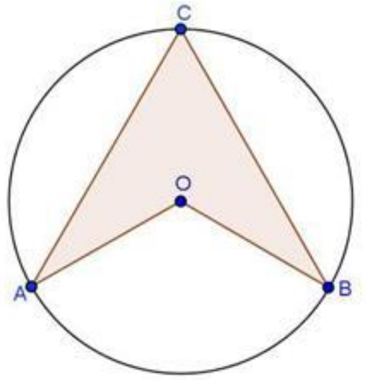
$\Rightarrow \quad \angle ACB > \dfrac{180^\circ}{2}$

$\Rightarrow \quad \angle ACB > 90^\circ$

Question 28

Prove that the angle in a segment greater than a semi-circle is less than a right angle.

Solution 28



Given:- $\angle ACB$ is an angle in major segment.

To prove:- $\angle ACB < 90^\circ$

Proof:- By degree measure theorem

$\angle AOB = 2\angle ACB$

And $\angle AOB < 180^\circ$

Then, $2\angle ACB < 180^\circ$

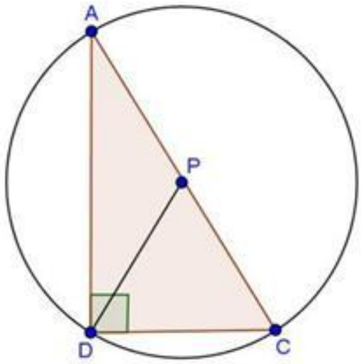
$\Rightarrow \quad \angle ACB < \frac{180^\circ}{2}$

$\Rightarrow \quad \angle ACB < 90^\circ$

Question 29

Prove that the line segment joining the mid-point of the hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

Solution 29



Let ΔABC be a right triangle right angled at B. Let P be the mid-point of hypotenuse AC. Draw a circle with centre at P and AC as a diameter.

Since, $\angle ABC = 90^\circ$. Therefore, the circle passes through B.

$\therefore \quad BP = \text{Radius}$

Also, $AP = CP = \text{Radius}$

$\therefore \quad AP = BP = CP$

Hence, $BP = \dfrac{1}{2} AC$