RD SHARMA Solutions for Class 9 Maths Chapter 17 - Heron's Formula

Chapter 17 - Heron's Formula Exercise 17.24

Question 1

The side of triangle are 16 cm, 30 cm, 34 cm, its area is

(a) $225\ cm^2$

(b) $225\sqrt{3}cm^2$

(c) $225\sqrt{2}cm^2$

(d) $450\ cm^2$

Solution 1

Let a = 16 cm, b = 30 cm, c = 34 cm

$$\text{Semi-perimeter of a triangle} = \frac{a+b+c}{2} = \frac{16+30+34}{2} = 40$$

Now, $s-a = 24$ cm, $s-b = 10$ cm and $s-c = 6$ cm

By Heron's formula,

$$\begin{aligned}\text{Area of a triangle} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{40\times24\times10\times6}\\ &= \sqrt{4\times10\times4\times6\times10\times6}\\ &= \sqrt{4^2\times10^2\times6^2}\\ &= 4\times10\times6\\ &= 240\ \text{cm}^2\end{aligned}$$

Note: Correct option not given

Question 2

The base of an isosceles right triangle is 30 cm. Its area is

(a) $225\ cm^2$

(b) $225\sqrt{3}\ cm^2$

(c) $225\sqrt{2}\ cm^2$

(d) $450\ cm^2$

Solution 2

Let ABC be an isosceles Δ, right angled at A.

$\Rightarrow AB = AC$ and $\angle CAB = 90°$

$\Rightarrow (AB)^2 + (AC)^2 = (BC)^2 = (30)^2$

$\Rightarrow 2(AB)^2 = (30)^2$

$\Rightarrow (AB)^2 = \frac{900}{2} = AC$

$$\begin{aligned}\text{Area of } \Delta ABC &= \frac{1}{2}\times AB\times AC\\ &= \frac{1}{2}\times(AB)^2 \qquad (\because AB = AC)\\ &= \frac{1}{2}\times\frac{900}{2}\\ &= 225\ \text{cm}^2\end{aligned}$$

Hence, correct option is (a).

Question 3

The sides of a triangle are 7 cm, 9 cm, and 14 cm. Its area is

$(a) 12\sqrt{5} cm^2$
$(b) 12\sqrt{3} cm^2$
$(c) 24\sqrt{5} cm^2$
$(d) 63 cm^2$

Solution 3

Let a = 7 cm, b = 9 cm, c = 14 cm

$$\text{Semi-perimeter} = s = \frac{a+b+c}{2} = \frac{7+9+14}{2} = 15 \text{ cm}$$

$s - a = 15 - 7 = 8$ cm, $s - b = 15 - 9 = 6$ cm and $s - c = 15 - 14 = 1$ cm

$$\begin{aligned}\text{Area of a triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15 \times 8 \times 6 \times 1} \\ &= \sqrt{5 \times 3 \times 4 \times 2 \times 3 \times 2} \\ &= 12\sqrt{5} \text{ cm}^2\end{aligned}$$

Hence, correct option is (a).

Question 4

The sides of a triangle are 325 m, 300 m and 125 m. Its area is
(a) 18750 m^2
(b) 37500 m^2
(c) 97500 m^2
(d) 48750 m^2

Solution 4

a = 325 m, b = 300 m, c = 125 m

$$s = \frac{a+b+c}{2} = \frac{325+300+125}{2} = 375 \text{ m}$$

$s - a = 50$ m, $s - b = 75$ m, $s - c = 250$ m

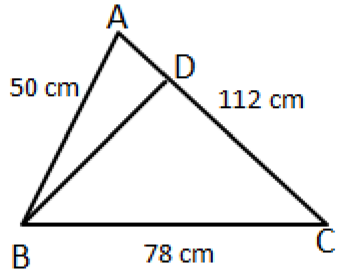
$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{375 \times 50 \times 75 \times 250} \\ &= \sqrt{15 \times 25 \times 25 \times 2 \times 3 \times 25 \times 25 \times 10} \\ &= \sqrt{\underline{25 \times 25} \times \underline{25 \times 25} \times \underline{30 \times 30}} \\ &= 25 \times 25 \times 30 \\ &= 18750 \text{ m}^2\end{aligned}$$

Hence, correct option is (a).

Question 5

The sides of a triangle are 50 cm, 78 cm and 112 cm. The smallest altitude is
(a) 20 cm
(b) 30 cm
(c) 40 cm
(d) 50 cm

Solution 5



The smallest altitude is $\perp$ drawn to the largest side of a $\triangle$ from opposite point.
i.e. BD

$$\text{Area of } \triangle = \frac{1}{2} \times Ac \times BD = \frac{1}{2} \times 112 \times BD = 56 \times BD$$

$$s = \frac{50+78+112}{2} = 120 \text{ cm}$$

$$s - AB = 70 \text{ cm}, \ s - BC = 42 \text{ cm}, \ s - AC = 8 \text{ cm}$$

$$\text{Area} = \sqrt{s(s-AB)-(s-BC)(s-AC)} = \sqrt{120 \times 70 \times 42 \times 8} = 1680 \text{ cm}^2$$

$$\text{Now, } 56 \times BD = 1680 \text{ cm}^2$$

$$\Rightarrow BD = \frac{1680}{56} = 30 \text{ cm}$$

Hence, correct option is (b).

Chapter 17 - Heron's Formula Exercise 17.25

Question 6

The sides of a triangle are 11 m, 60 m and 61 m. The altitude to the smallest side is

(a) 11 m
(b) 66 m
(c) 50 m
(d) 60 m

Solution 6

$$\text{Area of } \triangle = \frac{1}{2} \text{ Base } \times \text{Height}$$

The smallest side is 11 m

$$\Rightarrow \text{Area } = \frac{1}{2} \times 11 \times \text{Height} \quad ....(1)$$

$$\text{Area by Heron's Formula } = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s \ \frac{11+60+61}{2} = 66 \text{ m}$$

$$\Rightarrow \text{Area } = \sqrt{66 \times 55 \times 6 \times 5} = 330 \text{ m}^2$$

From eq (1)

$$330 = \frac{1}{2} \times 11 \times \text{height}$$

$$\Rightarrow \text{Height} = \frac{2 \times 330}{11} = 60 \text{ m}$$

Hence, correct option is (d).

Question 7

The sides of triangle are 11 cm, 15 cm and 16 cm. The altitude to the largest side is

(a) $30\sqrt{7}$ cm

(b) $\frac{15\sqrt{7}}{2}$ cm

(c) $\frac{15\sqrt{7}}{4}$ cm

(d) 30 cm

Solution 7

$$s = \frac{11+15+16}{2} = 21 \text{ cm}$$

$$\text{Area of } \triangle = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \times 10 \times 6 \times 5} = 30\sqrt{7} \text{ cm}^2$$

Also if we choose largest side and its Altitude, the area would be

$$A = \frac{1}{2} \times \text{largest side} \times h$$

$$\Rightarrow \frac{1}{2} \times 16 \times h = 30\sqrt{7}$$

$$\Rightarrow h = \frac{30\sqrt{7}}{8} = \frac{15}{4}\sqrt{7} \text{ cm}$$

Hence, correct option is (c).

Question 8

The base and hypotenuse of a right triangle are respectively 5 cm and 13 cm long. it is area is:

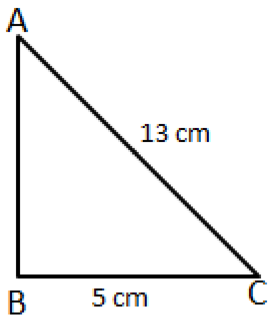
(a) 25 cm^2

(b) 28 cm^2

(c) 30 cm^2

(d) 40 cm^2

Solution 8



$$AB = \sqrt{(13)^2 - (5)^2} = 12 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

Hence, correct option is (c).

Question 9

The length of each side of an equilateral triangle of area $4\sqrt{3}$ cm^2, is

(a) 4 cm

(b) $\frac{4}{\sqrt{3}}$ cm

(c) $\frac{\sqrt{3}}{4}$ cm

(d) 3 cm

Solution 9

If side of an equilateral triangle is 'a', then its

Area $= \frac{\sqrt{3}}{4}a^2$

Now $\frac{\cancel{\sqrt{3}}}{4}a^2 = 4\sqrt{3}$

$\Rightarrow a^2 = 4^2$

$\Rightarrow a = 4$ cm

Hence, correct option is (a).

Question 10

If the area of an isosceles right triangle is 8 cm^2, what is the perimeter of the triangle?

(a) $8+\sqrt{2}$cm

(b) $8+4\sqrt{2}$cm

(c) $4+8\sqrt{2}$ cm

(d) $12\sqrt{2}$cm

Solution 10

Let each of the two equal sides of an isosceles right triangle be a cm.

Then, third side $= a\sqrt{2}$ cm

Area of $\triangle = \frac{1}{2} \times a \times a$

$\Rightarrow 8 = \frac{a^2}{2}$

$\Rightarrow a^2 = 16$

$\Rightarrow a = 4$ cm

$\Rightarrow$ Perimeter $\Rightarrow a + a + a\sqrt{2} = 4 + 4 + 4\sqrt{2} = 8 + 4\sqrt{2}$ cm

Hence, correct option is (b).

Question 11

The lengths of the sides of $\triangle ABC$ are consecutive integers.

If $\triangle ABC$ has the same perimeter as an equilateral triangle with a side of length 9 cm, what is the length of the shortest side of $\triangle ABC$?

(a) 4

(b) 6

(c) 8

(d) 10

Solution 11

Let the sides of $\triangle ABC$ be n, n + 1, n + 2.

$\Rightarrow$ Perimeter $= n + n + 1 + n + 2$

$\Rightarrow (9+9+9) = 3n + 3$

$\Rightarrow 27 = 3n + 3$

$\Rightarrow 3n = 24$

$\Rightarrow n = 8$ cm

Thus, the shortest side is 8 cm.

Hence, correct option is (c).

Question 12

In figure, the ratio of AD to DC is 3 to 2. If the area of $\triangle ABC$ is 40 cm^2, what is the area of $\triangle ABC$?

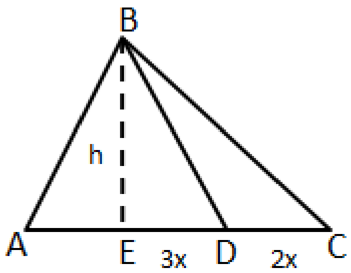
(a)16 cm^2

(b) 24 cm^2

(c) 30 cm^2

(d) 36 cm^2

Solution 12



$\frac{AD}{DC} = \frac{3}{2}$

Let AD = 3x and DC = 2x

Area of $\triangle ABC = \frac{1}{2} \times AC \times BE$ (BE = h)

$\Rightarrow 40 = \frac{1}{2} \times 5x \times h$

$\Rightarrow 80 = 5xh$

$\Rightarrow xh = 16 \text{ cm}^2$ ....(1)

Now Area of $\triangle ABD = \frac{1}{2} \times 3x \times h = \frac{3xh}{2} = \frac{3}{2} \times 16 = 24 \text{ cm}^2$

Area of $\triangle BDC$ = Area of $\triangle ABC -$ Area of $\triangle ABD = 40 - 24 = 16 \text{ cm}^2$

Hence, correct option is (a).

Question 13

If the length of the median of an equilateral triangle is x cm, then itis area is

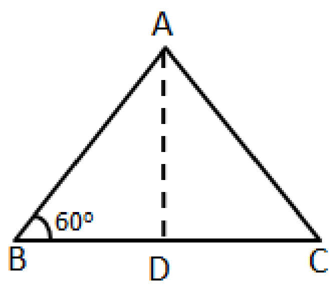
(a) x^2

(b) $\frac{\sqrt{3}}{2}x^2$

(c) $\frac{x^2}{\sqrt{3}}$

(d) $\frac{x^2}{2}$

Solution 13



Let the side of equilateral $\triangle ABC$ be a cm

The median of equilateral triangle is its altitude drawn from A to BC.

(i.e. the height of $\triangle$ over Base BC)

$\Rightarrow AD = a\sin 60°$

$\Rightarrow x = \frac{a\sqrt{3}}{2}$ [AD = x (given)

$\Rightarrow a = \frac{2x}{\sqrt{3}}$

Area of equilateral $\triangle$ of side a

$= \frac{\sqrt{3}a^2}{4}$

$= \frac{\sqrt{3}}{4}\left(\frac{2x}{\sqrt{3}}\right)^3$

$= \frac{x^2}{\sqrt{3}}$

Hence, correct option is (c).

Question 14

If every side of a triangle is doubled, then increase in the area of the triangle is

(a) $100\sqrt{2}\%$

(b) 200%

(c) 300%

(d) 400%

Solution 14

$s = \frac{a+b+c}{2}, \quad A = \sqrt{s(s-a)(s-b)(s-c)}$

Now, if $a' = 2a,\ b' = 2b$ and $c' = 2c$

then, $s' = \frac{a'+b'+c'}{2} = \frac{2a+2b+2c}{2} = 2s$

$A' = \sqrt{s'(s'-a')(s'-b')(s'-c')}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow A' = 4A$

$\Rightarrow$ Increase in Area $= \frac{4A - A}{A} \times 100\ \% = 300\%$

Hence, correct option is (c).

Question 15

A square and an equilateral triangle have equal perimeters. If diagonal of the square is $12\sqrt{2}$cm, then the area of the triangle is

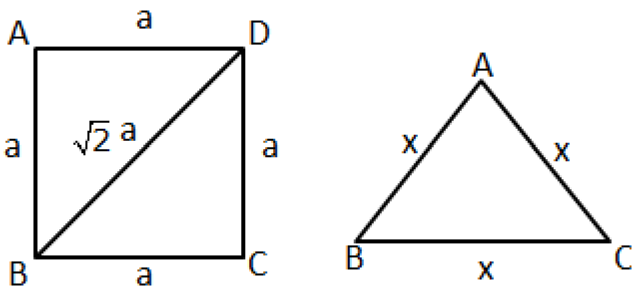
(a) $24\sqrt{2}\text{cm}^2$

(b) $24\sqrt{3}\text{cm}^2$

(c) $48\sqrt{3}\text{cm}^2$

(d) $64\sqrt{3}\text{cm}^2$

Solution 15



If side of a square is a cm

then, its diagonal $= \sqrt{2}a$ cm

But diagonal $= 12\sqrt{2}$ cm

$\Rightarrow \sqrt{2}a = 12\sqrt{2}$

$\Rightarrow a = 12$ cm

$\Rightarrow$ Perimeter of a square $= 4a = 4 \times 12 = 48$ cm

Now, perimeter of an equilateral triangle with side $x = 3x$ cm

But, perimeter of equilateral triangle = Perimeter of square

$\Rightarrow 3x = 48$

$\Rightarrow x = 16$ cm

Now, Area of equilateral $\triangle = \frac{\sqrt{3}x^2}{4} = \frac{\sqrt{3}}{4} \times 16 \times 16 = 64\sqrt{3}\text{cm}^2$

Hence, correct option is (d).

Chapter 17 - Heron's Formula Exercise Ex. 17.1

Question 1

Find the area of the triangle whose sides are respectively 150 cm, 120 cm and 200 cm.

Solution 1

Let $a = 150cm$

$b = 120cm$

$c = 200cm$

If s denotes semi-permieter

then $2s = a + b + c$

$\Rightarrow s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(200 + 120 + 150)$

$= 235cm$

Now, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [by heron's formula]

$= \sqrt{235(235-150)(235-120)(235-200)}cm^2$

$= \sqrt{235 \times 85 \times 115 \times 35}cm^2$

$= 8966.56cm^2$

Question 2

Find the area of the triangle whose sides are $9cm$, $12cm$ and $15cm$.

Solution 2

Let $a = 9cm$
$b = 12cm$
$c = 15cm$

Since, $2s = a + b + c$

$$\Rightarrow s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(9 + 12 + 15)$$
$$= \frac{1}{2}(36) = 18cm$$

Now, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [by heron's formula]

$$= \sqrt{18(18-9)(18-12)(18-15)}$$
$$= \sqrt{18 \times 9 \times 6 \times 3}$$
$$= 54cm^2$$

Question 3

Find the area of the triangle two sides of which are $18cm$ and $10cm$ and the perimeter is $42cm$.

Solution 3

Let the third side be c.

Here $a = 18cm$
$b = 10cm$

Perimeter $= a + b + c = 42$

$$\Rightarrow c = 42 - (a + b)$$
$$\Rightarrow c = 42 - 18 - 10$$
$$\Rightarrow c = 14cm$$

Now,

$$2s = a + b + c$$
$$\Rightarrow s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(42) = 21cm$$

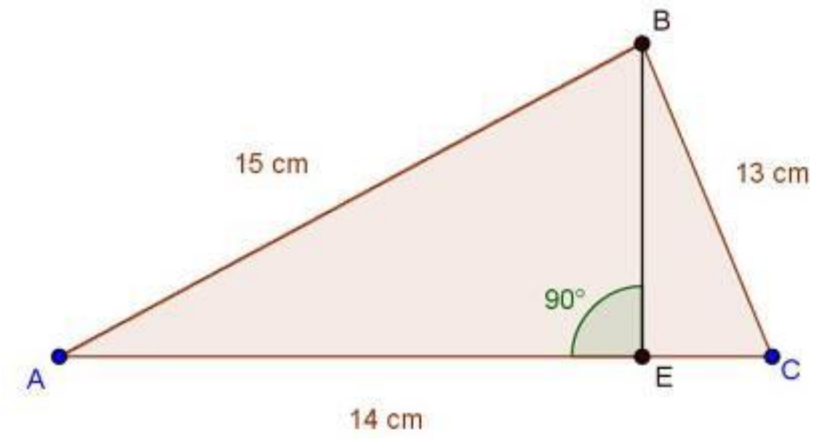
Now, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [by heron's formula]

$$= \sqrt{21(21-18)(21-10)(21-14)}$$
$$= \sqrt{21 \times 3 \times 11 \times 7}$$
$$= 21\sqrt{11}cm^2$$

Question 4

In a $\triangle ABC$, $AB = 15cm$, $BC = 13cm$ and $AC = 14cm$. Find the area of $\triangle ABC$ and hence its altitude on AC.

Solution 4



We have,

$AB = 15cm$
$BC = 13cm$
$AC = 14cm$

$Now,$
$\text{Perimeter} = 2s = AB + BC + AC$

$$\Rightarrow s = \frac{1}{2}(AB + BC + AC)$$
$$= \frac{1}{2}(42) = 21cm$$

$$\therefore \text{Area of Triangle} = \sqrt{s(s-a)(s-b)(s-c)} \qquad [\text{by heron's formula}]$$
$$= \sqrt{21(21-15)(21-13)(21-14)}$$
$$= \sqrt{21 \times 6 \times 8 \times 7}$$
$$= \sqrt{21 \times 8 \times 42} = 84cm^2$$

Let BE be perpendicular on AC.

Now, area of triangle $= 84cm^2$

$$\Rightarrow \frac{1}{2} \times BE \times AC = 84 \qquad \left[ar\ of\ \Delta = \frac{1}{2} \times b \times h\right]$$
$$\Rightarrow BE = \frac{84 \times 2}{14} = 12cm$$

$\therefore$ Length of altitude on AC is $12cm$.

Question 5

The perimeter of triangular field is 540 m and its sides are in the ratio 25:17:12. Find the area of the triangle.

Solution 5

The sides of the triangular field are in the ratio 25:17:12.
Let the sides of triangle be 25x, 17x, and 12x.
Perimeter of this triangle = 540 m

$$25x + 17x + 12x = 540 \text{ m}$$
$$54x = 540 \text{ m}$$
$$x = 10 \text{ m}$$

Sides of triangle will be 250 m, 170 m, and 120 m.

Semi-perimeter (s) = $\frac{\text{Perimeter}}{2} = \frac{540 \text{ m}}{2} = 270 \text{ m}$

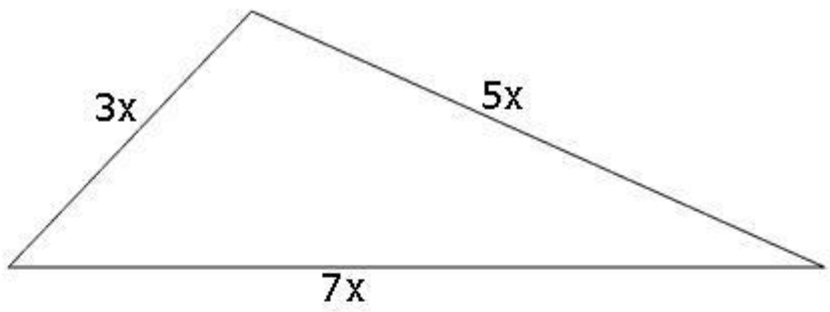
By Heron's formula:

So, area of the triangle is 9000 m^2.

Question 6

The perimeter of right triangle is 300m. If its sides are in the ratio 3 : 5 : 7. Find the area of the triangle.

Solution 6



Suppose that the sides, in metres, are 3x, 5x and 7x (See fig.,) .

Then, we know that 3x + 5x + 7x = 300 (perimeter of the triangle)

Therefore, 15x = 300, which gives x = 20.

So the sides of the triangles are 3×20m, 5×20m and 7×20m

i.e., 60m, 100m and 140m.

Can you now find the area [Using Heron's formula]?

We have $s = \frac{60 + 100 + 140}{2}$ m $= 150$m

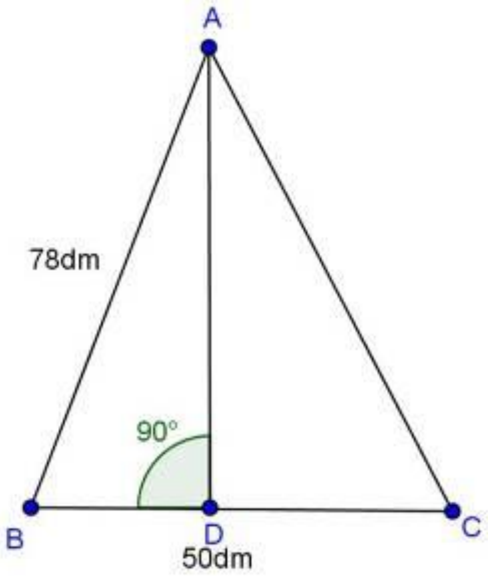
and area will be $= \sqrt{150(150-60)(150-100)(150-140)} \text{ m}^2$

$$= \sqrt{150 \times 90 \times 50 \times 10} \text{ m}^2$$
$$= 1500\sqrt{3} \text{ m}^2$$

Question 7

The perimeter of a triangular field is 240 dm. If two of its sides are 78 dm and 50 dm, find the length of the perpendicular on the side of length 50 dm from the opposite vertex.

Solution 7



Let ABC be a triangle.
Here, $AB = 78dm, BC = 50dm$

Now, Perimeter $= 240dm$
$\Rightarrow AB + BC + CA = 240dm$
$\Rightarrow AC = 240 - 78 - 50$
$\Rightarrow AC = 112dm$

Now, $2s = AB + BC + CA$
$\Rightarrow 2s = 240$
$\Rightarrow s = 120dm$

$$\therefore \text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} \quad [\text{by heron's formula}]$$
$$= \sqrt{120(120-78)(120-50)(120-112)}$$
$$= \sqrt{120 \times 42 \times 70 \times 8}$$
$$= 1680dm^2$$

Let AD be perpendicular on BC.

$$ar\ of \triangle ABC = \frac{1}{2} \times AD \times BC \quad \left(ar\ of \triangle = \frac{1}{2} \times b \times h\right)$$
$$\Rightarrow \frac{1}{2} \times AD \times BC = 1680$$
$$\Rightarrow AD = \frac{2 \times 1680}{50} = 67.2dm$$

Question 8

A triangle has sides 35 cm, 54 cm and 61 cm long. Find its area. Also, find the smallest of its altitudes.

Solution 8

Let $a = 35cm$

$b = 54cm$

$c = 61cm$

Now, $2s = a + b + c$

$\Rightarrow s = \frac{1}{2}(35 + 54 + 61)$

$\Rightarrow s = 75cm$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [by heron's formula]

$= \sqrt{75(75-35)(75-54)(75-61)}$

$= \sqrt{75 \times 40 \times 21 \times 14}$

$= 939.14cm^2$

The altitude will be smallest when the side corresponding to it is longest.

Here Longest side is 61cm

$\therefore \frac{1}{2} \times h \times 61 = 939.14$ $\left[ar\ of\ \Delta = \frac{1}{2} \times b \times h\right]$

$\Rightarrow h = \frac{939.14 \times 2}{61} = 30.79$ cm

Hence the lenght of the smallest altitude is 30.79 cm.

Question 9

The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find the area of the triangle and the height corresponding to the longest side.

Solution 9

Let the sides be $3x$, $4x$, and $5x$

Now,

Perimeter= 144

$\Rightarrow 3x + 4x + 5x = 144$

$\Rightarrow 12x = 144$

$\Rightarrow x = 12$

$\therefore$ Sides are $36cm$, $48cm$ and $60cm$

Now $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(36+60+48) = 72cm$

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [by heron's formula]

Area of triangle $= \sqrt{72(72-36)(72-48)(72-60)}$

$= 864cm^2$

Let l be the Altitude corresponding to longest side.

$\therefore \frac{1}{2} \times 60 \times l = 864$ $(ar\ of\ \Delta = \frac{1}{2} \times b \times h)$

$\Rightarrow l = \frac{864 \times 2}{60} = 28.8cm$

Hence the altitude corresponding to longest side $= 28.8cm$

Question 10

The perimeter of an isosceles triangle is 42 cm and its base is (3/2) times each of the equal sides. Find the length of each side of the triangle, area of the triangle and the height of the triangle.

Solution 10

Let'x'be the measure of each equal sides

$\therefore$ Base= $\frac{3}{2}x$.

$\therefore x + x + \frac{3}{2}x = 42$ $\qquad$ $[\because$ permieter= 42cm$]$

$\Rightarrow \frac{7}{2}x = 42$

$\Rightarrow x = 12cm$

$\therefore$ Sides are $12cm$, $12cm$ and $18cm$.

Now, $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(42) = 21cm$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ $\qquad$ [by heron's formula]

$= \sqrt{21(21-12)(21-12)(21-18)}$

$= \sqrt{21 \times 9 \times 9 \times 3}$

$= 71.42cm^2$

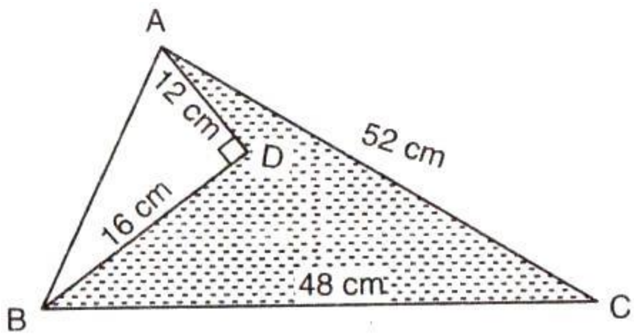
Now, area of triangle= $71.42cm^2$

$\Rightarrow \frac{1}{2} \times 18 \times \text{h} = 71.42$ $\qquad$ [where h is the height]

$\Rightarrow \text{h} = \frac{71.42 \times 2}{18} = 7.937cm$

Question 11

Find the area of the shaded region in fig.12.12



Solution 11

$$\text{Area of shaded region } ADBC = \text{Area of } \Delta ABC - \text{Area of } \Delta ADB.$$

Now in ΔADB

$$AB^2 = AD^2 + BD^2 \qquad [\text{by pythagoras theorem}]$$

$$\Rightarrow AB = \sqrt{12^2 + 16^2}$$

$$\Rightarrow AB = \sqrt{144 + 256}$$

$$\Rightarrow AB = \sqrt{400} = 20 \text{ cm}$$

$$\text{area } \Delta ADB = \frac{1}{2} \times 16 \times 12 = 96 cm^2 \qquad \left[ar \text{ of } \Delta = \frac{1}{2} \times b \times h\right]$$

Now,

$$\text{In } \Delta ABC, s = \frac{1}{2}(AB + BC + CA)$$

$$= \frac{1}{2}(52 + 48 + 20)$$

$$= 60 cm$$

$$\therefore \text{Area of } \Delta ABC = \sqrt{s(s - AB)(s - BC)(s - CA)} \qquad [\text{by heron's formula}]$$

$$= \sqrt{60(60 - 20)(60 - 48)(60 - 52)}$$

$$= \sqrt{60 \times 40 \times 12 \times 8}$$

$$= 480 cm^2$$

$$\therefore \text{Area of shaded region} = ar(\Delta ABC) - ar(\Delta ADB)$$

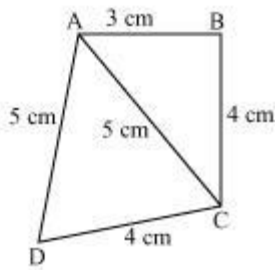
$$= (480 - 96) \text{cm}^2 = 384 \text{cm}^2$$

Chapter 17 - Heron's Formula Exercise Ex. 17.2

Question 1

Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Solution 1



For $\triangle$ABC

$AC^2 = AB^2 + BC^2$

$(5)^2 = (3)^2 + (4)^2$

So, $\triangle$ABC is a right angle triangle, right angled at point B.

Area of $\triangle$ABC $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$

For $\triangle$ ADC

Perimeter = 2s = AC + CD + DA = (5 + 4 + 5) cm = 14 cm

s = 7 cm

By Heron's formula

Area of triangle $\sqrt{s(s-a)(s-b)(s-c)}\ \text{cm}^2$

$$\text{Area of } \Delta ADC = \left[\sqrt{7(7-5)(7-5)(7-4)}\right]\text{cm}^2$$
$$= \left(\sqrt{7\times2\times2\times3}\right)\text{cm}^2$$
$$= 2\sqrt{21}\ \text{cm}^2$$
$$= (2\times4.583)\ \text{cm}^2$$
$$= 9.166\ \text{cm}^2$$

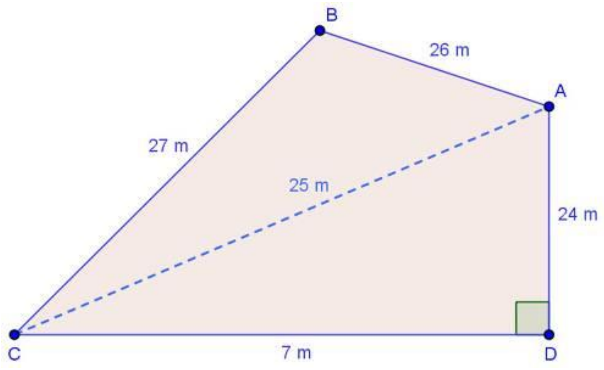
Area of ABCD = Area of $\triangle$ ABC + Area of $\triangle$ ACD

= (6 + 9.166) cm^2 = 15.166 cm^2 = 15.2 cm^2 (approximately)

Question 2

The sides of a quadrangular field, taken in order are $26m$, $27m$, $7m$ and $24m$ respectively. The angle contained by the last two sides is a right angle. Find its area.

Solution 2



$AB = 26m$
$BC = 27m$
$CD = 7m$
$DA = 24m$

Diagonal AC is joined

Now in $\triangle ADC$

$AC^2 = AD^2 + CD^2$ [by pythagoras theorem]

$\Rightarrow AC = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25m$

Now for area of $\triangle ABC$

$S = \frac{1}{2}(AB + BC + CA) = \frac{1}{2}(26 + 27 + 25) = 39m$

$$\begin{aligned}\text{Area}(\triangle \text{ABC}) &= \sqrt{s(s - AB)(s - BC)(s - CA)} \quad \text{[by heron's formula]}\\ &= \sqrt{39(39 - 26)(39 - 27)(39 - 25)}\\ &= \sqrt{39 \times 14 \times 13 \times 12}\\ &= 291.849m^2\end{aligned}$$

Now for area of $\triangle ADC$, $s = \frac{1}{2}(AD + CD + AC) = \frac{1}{2}(25 + 24 + 7) = 28m$

$$\begin{aligned}\therefore \text{Area}(\triangle \text{ADC}) &= \sqrt{s(s - AD)(s - DC)(s - CA)} \quad \text{[by heron's formula]}\\ &= \sqrt{28(28 - 24)(28 - 7)(28 - 25)}\\ &= 84m^2\end{aligned}$$

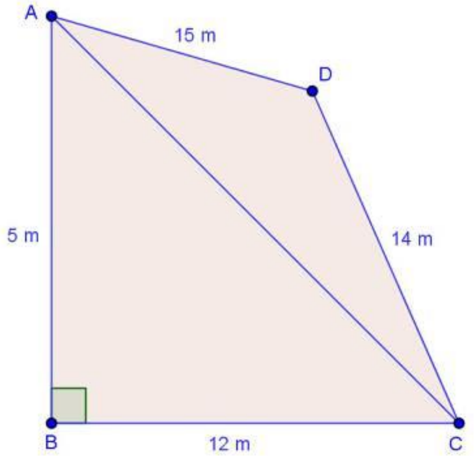
Therefore area of quadrangular field

$$\begin{aligned}\text{ABCD} &= \text{area}(\triangle ABC) + \text{area}(\triangle ADC)\\ &= 291.849 + 84\\ &= 375.8m^2\end{aligned}$$

Question 3

The sides of a quadrilateral, taken in order are 5, 12, 14 and 15 metres respectively, and the angle contained by the first two sides is a right angle. Find its area.

Solution 3



Given that,

$AB = 5m$

$BC = 12m$

$CD = 14m$ and $DA = 15m$

Join AC

$$\text{ar}(\triangle ABC) = \frac{1}{2} \times AB \times BC \quad \left(ar\ of\ \triangle = \frac{1}{2} \times b \times h \right)$$

$$= \frac{1}{2} \times 5 \times 12 = 30m^2$$

In $\triangle ABC$

$AC^2 = AB^2 + BC^2$ [by pythagoras theorem]

$\Rightarrow AC = \sqrt{5^2 + 12^2} = \sqrt{169} = 13m$

Now in $\triangle ADC$

Let $2s$ be the perimeter

$\therefore 2s = (AD + DC + AC)$

$\Rightarrow s = \frac{1}{2}(15 + 14 + 13) = \frac{1}{2} \times 42 = 21m$

$$\therefore \text{ar}(\triangle \text{ADC}) = \sqrt{s(s - AD)(s - DC)(s - AC)} \quad \text{[by heron's formula]}$$

$$= \sqrt{21(21 - 15)(21 - 14)(21 - 13)}$$

$$= \sqrt{21 \times 6 \times 7 \times 8}$$

$$= 84m^2$$

$\therefore$ Area of quadrilateral $ABCD = \text{ar}(\triangle ABC) + \text{ar}(\triangle ADC)$

$$= 30 + 84 = 114m^2$$

Question 4

A park, in the shape of a quadrilateral ABCD, has $\angle C$ = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Solution 4

Let us join BD.

In $\triangle$ BCD applying Pythagoras theorem

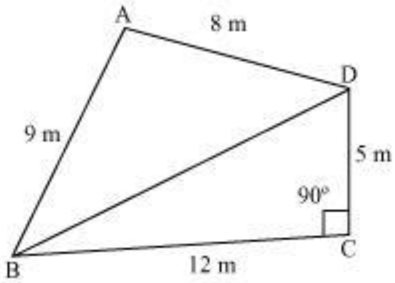
$BD^2 = BC^2 + CD^2$

$= (12)^2 + (5)^2$

$= 144 + 25$

$BD^2 = 169$

BD = 13 m



Area of $\triangle$ BCD

$$= \frac{1}{2} \times BC \times CD = \left[\frac{1}{2} \times 12 \times 5\right] m^2 = 30\ m^2$$

For $\triangle$ ABD

$$s = \frac{\text{perimeter}}{2} = \frac{(9+8+13)\,cm}{2} = 15\ cm$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

By Heron's formula

$$= \left[\sqrt{15(15-9)(15-8)(15-13)}\right] m^2$$

Area of triangle

$$= \left(\sqrt{15 \times 6 \times 7 \times 2}\right) m^2$$

$$= 6\sqrt{35}\ m^2$$

$$= (6 \times 5.916) m^2$$

$$= 35.496\ m^2$$

Area of park = Area of $\triangle$ ABD + Area of $\triangle$ BCD

$= 35.496 + 30\ m^2$

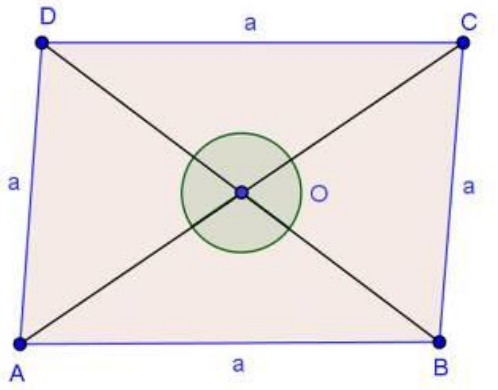
$= 65.496\ m^2$

$= 65.5\ m^2$ (approximately)

Question 5

Find the area of a rhombus whose perimeter is 80 m and one of whose diagonal is 24 m.

Solution 5



$$\text{Given that perimeter of rhombus} = 80m$$
$$\Rightarrow 4a = 80m \qquad [\text{perimeter of rhombus} = 4 \times \text{side}]$$
$$\Rightarrow a = 20m$$

$$\text{Let } AC = 24m$$
$$\therefore OA = \frac{1}{2}AC = \frac{1}{2} \times 24 = 12m$$

$$\text{In } \triangle AOB$$
$$OB^2 = AB^2 - OA^2 \qquad [\text{pythagoras theorem}]$$
$$\Rightarrow OB = \sqrt{20^2 - 12^2} = \sqrt{400 - 144} = \sqrt{256} = 16m$$

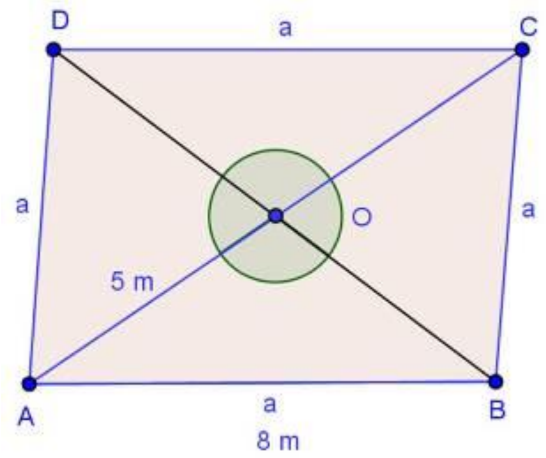
$$\text{Also } BO = OD \qquad [\text{Diagonal of a rhombus bisect each other at } 90^\circ]$$
$$\therefore BD = 2OB = 2 \times 16 = 32m$$

$$\therefore \text{Area of rhombus} = \frac{1}{2} \times 32 \times 24 \qquad \left[\text{Area of rhombus} = \frac{1}{2} \times BD \times AC\right]$$
$$= 384m^2$$

Question 6

A rhombus sheet, whose perimeter is 32 m and whose one diagonal is 10 m long, is painted on both sides at the rate of Rs 5 per m^2. Find the cost of painting.

Solution 6



Since perimeter = $32m$

$\Rightarrow 4a = 32m$ [perimeter of rhombus = 4 × side]

$\Rightarrow a = 8m$

Let, $AC = 10 \Rightarrow OA = \frac{1}{2}AC = \frac{1}{2} \times 10 = 5m$

$\therefore OB^2 = AB^2 - OA^2$ [by pythagoras theorem]

$\Rightarrow OB = \sqrt{8^2 - 5^2} = \sqrt{64 - 25} = \sqrt{39}m$

Now, $BD = 2OB = 2\sqrt{39}m$

$\therefore$ Area of sheet $= \frac{1}{2} \times BD \times AC = \frac{1}{2} \times 2\sqrt{39} \times 10 = 10\sqrt{39}m^2$

$\therefore$ Cost of printing on both sides at the rate of Rs 5 per m^2

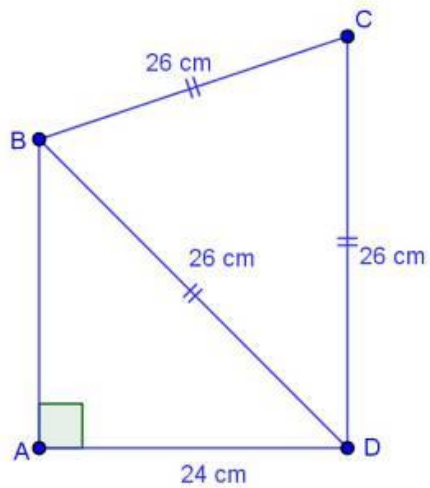
$= Rs\ 2 \times 10\sqrt{39} \times 5$

$= Rs\ 625.00$

Question 7

Find the area of a quadrilateral $ABCD$ in which $AD = 24cm$, $\angle BAD = 90°$ and BCD forms an equilateral triangle whose each side is equal to $26cm$.

Solution 7



In $\triangle BAD$,

$BA^2 = BD^2 - AD^2$ [pythagoras theorem]

$\Rightarrow BA = \sqrt{26^2 - 24^2} = \sqrt{676 - 576} = \sqrt{100} = 10cm$

Area $\triangle BAD = \frac{1}{2} \times BA \times AD = \frac{1}{2} \times 10 \times 24$

$= 120cm^2$

Area of $\triangle BCD = \frac{\sqrt{3}}{4} \times (26)^2 = 292.37cm^2$

$\therefore$ Area of quadrilateral $ABCD$ = area of $\triangle BAD$ + area of $\triangle BCD$

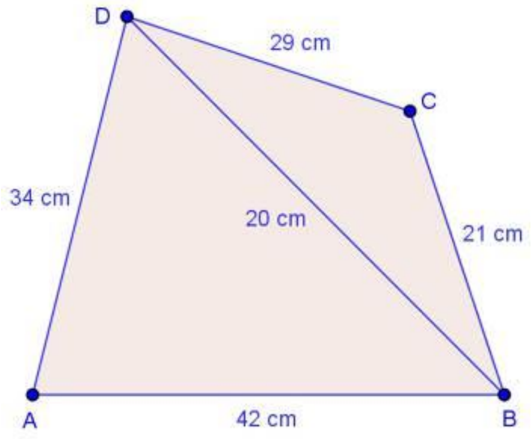
$= 120 + 292.37$

$= 412.37cm^2$

Question 8

Find the area of a quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and diagonal BD = 20 cm.

Solution 8



Area of quadrilateral ABCD = area of $\triangle ABD$ + area of BCD

Now, area of $\triangle ABD$

Let $2s = AB + BD + DA$ [perimeter of triangle ABD]

$$\Rightarrow s = \frac{1}{2}(AB + BD + DA) = \frac{1}{2}(34 + 20 + 42) = \frac{1}{2} \times 96 = 48cm$$

$$\therefore \text{ Ar of}(\triangle ABD) = \sqrt{s(s - AB)(s - BD)(s - DA)} \quad \text{[by heron's formula]}$$

$$= \sqrt{48(48 - 42)(48 - 20)(48 - 34)}$$

$$= 336cm^2$$

Also for area of $\triangle BCD$,

Let $2s = BC + CD + DB$ [perimeter of triangle BCD]

$$\Rightarrow s = \frac{1}{2}(BC + CD + DB) = \frac{1}{2}(29 + 21 + 20) = 35cm$$

$$\therefore \text{ Ar of}(\triangle BCD) = \sqrt{s(s - BC)(s - CD)(s - DB)} \quad \text{[by heron's formula]}$$

$$= \sqrt{35(35 - 21)(35 - 29)(35 - 20)}$$

$$= 210cm^2$$

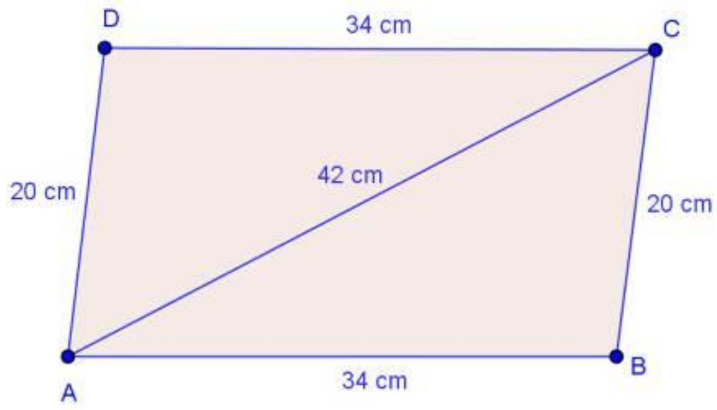
$$\therefore \text{ Area of quadrilateral } ABCD = 336 + 210$$

$$= 546cm^2$$

Question 09

The adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm. Find the area of the parallelogram.

Solution 09



Area of parallelogram

$= \text{area of } \triangle ADC + \text{area of } ABC$ [Diagonal of a parallelogram divides it into two congruent $\triangle$]

$= 2 \times [\text{area of } \triangle ABC]$

Now for area of $\triangle ABC$

Let $2s = AB + BC + CA$ [perimeter of triangle ABC]

$\Rightarrow s = \frac{1}{2}(34 + 20 + 42) = \frac{1}{2} \times 96 = 48cm$

$\therefore$ Area of $(\triangle ABC) = \sqrt{s(s - AB)(s - BC)(s - AC)}$ [heron's formula]

$= \sqrt{48(48 - 34)(48 - 20)(48 - 42)}$

$= \sqrt{48 \times 14 \times 28 \times 6}$

$= 336cm^2$

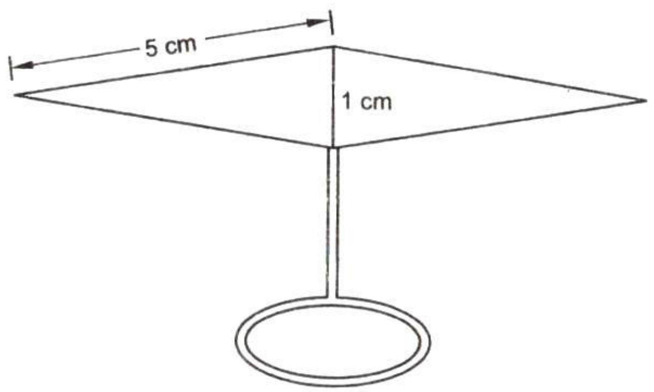
$\therefore$ Area of parallelogram $ABCD = 2 \times [\text{area of } \triangle ABC]$

$= 2 \times 336 = 672cm^2$

Question 10

Find the area of the blades of the magnetic compass shown in fig.

(Take √11 = 3.32)



Solution 10

Area of the blades of magnetic compass = area of $\triangle ADB$ + area of $\triangle CDB$

Now for area of $\triangle ADB$

Let, $2s = AD + DB + BA$ (perimeter of $\triangle ADB$)

$\Rightarrow s = \frac{1}{2}(5 + 1 + 5) = \frac{11}{2} cm$

$$\text{Now, area of} (\triangle \text{ADB}) = \sqrt{s(s - AD)(s - DB)(s - BA)}$$
$$= \sqrt{\frac{11}{2}\left(\frac{11}{2} - 5\right)\left(\frac{11}{2} - 1\right)\left(\frac{11}{2} - 5\right)}$$
$$= 2.49 cm^2$$

Also, ar of $\triangle$ ADB = ar of $\triangle CDB$

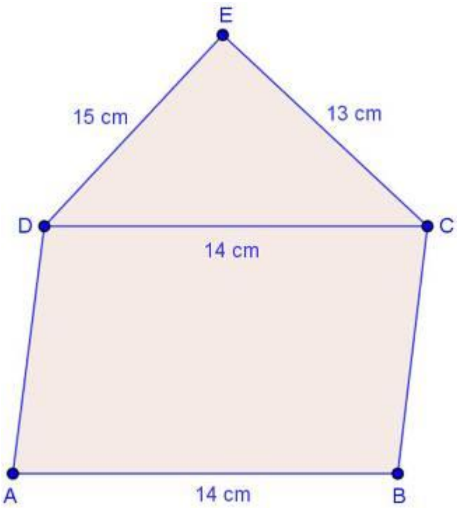
$\therefore$ Area of blades of the magnetic field = $2 \times$ (area of $\triangle ADB$)

$$= 2 \times 2.49 = 4.98 cm^2$$

Question 11

A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 13 cm, 14 cm and 15 cm and the parallelogram stands on the base 14 cm, find the height of the parallelogram.

Solution 11



Let h be the height of parallelogram $ABCD$.

Given $2s = (DC + CE + ED)$ (perimeter of $\triangle DCE$)

$\Rightarrow s = \frac{1}{2}(DC + CE + ED)$

$= \frac{1}{2}(15 + 13 + 14) = \frac{1}{2} \times 42 = 21 cm$

Area of $\triangle DCE = \sqrt{s(s - DC)(s - CE)(s - ED)}$ [Heron's formula]

$= \sqrt{21(21 - 14)(21 - 13)(21 - 15)}$

$= \sqrt{21 \times 7 \times 8 \times 6}$

$= 84 cm^2$

Its give that area of $\triangle DCE$ = area of $ABCD$

$\Rightarrow$ area of Parallelogram $ABCD = 84 cm^2$

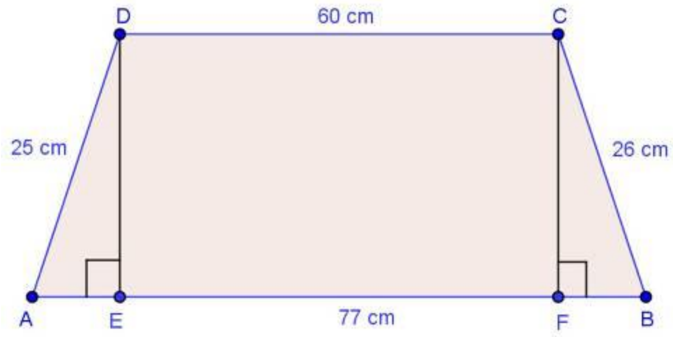
$\Rightarrow 24 \times h = 84$ [area of Parallelogram = base×height]

$\Rightarrow h = 6$ cm

Question 12

Two parallel side of a trapezium are $60 cm$ and $77 cm$ and other sides are $25 cm$ and $26 cm$. Find the area of the trapezium.

Solution 12



Given that,

$AB = 77cm$

$CD = 60cm$

$BC = 26cm$

$AD = 25cm$

Now, $DE \perp AB$ and $CF \perp AB$ is drawn.

$\therefore EF = DC = 60cm$

Let $AE = x$

$\Rightarrow BF = 77 - 60 - x = (17 - x)$

In $\triangle ADE$, $DE^2 = AD^2 - AE^2$ [pythagoras theorem]

$= 25^2 - x^2$

And in $\triangle BCF$, $CF^2 = BC^2 - BF^2$ [pythagoras theorem]

$= 26^2 - (17 - x)^2$

But $DE = CF \Rightarrow DE^2 = CF^2$

$\Rightarrow 25^2 - x^2 = 26^2 - (17 - x)^2$

$\Rightarrow 25^2 - x^2 = 26^2 - (289 + x^2 - 34x)$ $\left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$\Rightarrow 625 - x^2 = 676 - 289 - x^2 + 34x$

$\Rightarrow 34x = 238$

$\Rightarrow x = 7$

$\therefore DE = \sqrt{25^2 - x^2} = \sqrt{25^2 - 7^2} = \sqrt{576} = 24cm$

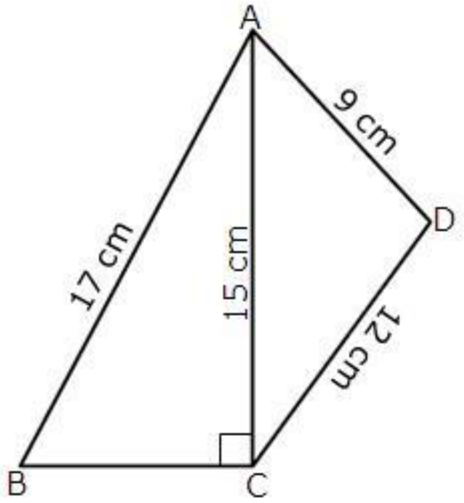
$\therefore$ Area of trapezium $= \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$

$= \frac{1}{2} \times (60 + 77) \times 24 = 1644cm^2$

Question 13

Find the perimeter and area of the quadrilateral ABCD in which AB= 17 cm, AD = 9 cm, CD = 12 cm, $\angle ACB = 90^0$ and AC = 15 cm.

Solution 13



Here,

$$BC = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8 \text{ cm}$$

Now, area of triangle ABC = $\frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2$

For area of triangle ACD,

Let a = 15 cm, b = 12 cm and c = 9 cm

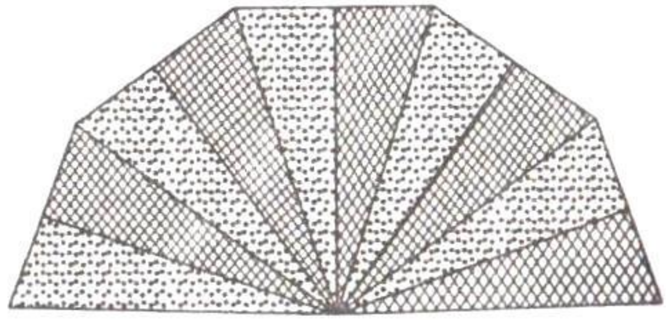
Therefore, s = $\frac{15 + 12 + 9}{2} = \frac{36}{2} = 18$ cm

$$\begin{aligned} \text{Area of ACD} &= \sqrt{S(S-a)(S-b)(S-c)} \\ &= \sqrt{18(18-15)(18-12)(18-9)} \\ &= \sqrt{18 \times 3 \times 6 \times 9} \\ &= \sqrt{18 \times 18 \times 3 \times 3} \\ &= 18 \times 3 = 54 \text{ cm}^2 \end{aligned}$$

Thus, the area of quadrilateral ABCD = 60 + 54 = 114 cm^2.

Question 14

A hand fan is made by stitching 10 equal size triangular strips of two different types of paper as shown in fig., The dimensions of equal strips are 25 cm, 25 cm and 14 cm. Find the area of each type of paper needed to make the hand fan.



Solution 14

Area of each equal strips = area of $\triangle AOB$

Now for area of $\triangle AOB$

Let $2S = AO + OB + BA$ $\quad$ [perimeter of $\triangle AOB$]

$$\Rightarrow S = \frac{1}{2}(AO + OB + BA) = \frac{1}{2}(25 + 25 + 14) = 32cm$$

$$\therefore \text{ Area of } (\triangle \text{AOB}) = \sqrt{s(s - AO)(s - OB)(s - BA)} \quad [\text{Heron's formula}]$$

$$= \sqrt{32(32 - 25)(32 - 25)(32 - 14)}$$

$$= \sqrt{32 \times 7 \times 7 \times 18}$$

$$= 168\, cm^2$$

$$\therefore \text{ Area of each type of paper needed to make the hand fan } = 5 \times (\text{area of } \triangle AOB)$$

$$= 5 \times 168 = 840cm^2$$