Access Answers of Maths NCERT class 9 Chapter 2 – Polynomials

Class 9 Maths Chapter 2 Exercise 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2 - 3x + 7$

Solution:

The equation $4x^2 - 3x + 7$ can be written as $4x^2 - 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation y^2 + can be written as $y^2 + y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression y^2 + is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation 3 + t can be written as $3t^{1/2} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,) is not a whole number. Hence, we can say that the expression 3 + t is **not** a polynomial in one variable.

(iv) y + 2/y

Solution:

The equation y + can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression y + is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1)\ x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1)\ x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\Pi/2\ x^2 + x$

Solution:

The equation $\Pi/2x^2 + x$ can be written as $(\Pi/2)\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is

, the coefficients of x^2 in $\Pi/2x^2 + x$ is $\Pi/2$.

(iv)$\sqrt{2}x$-1

Solution:

The equationx$\sqrt{2}x$-1 can be written as $0x^2 + \sqrt{2}x - 1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

, the coefficients of x^2 in$\sqrt{2}x$-1 is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

, the degree of $5t – \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3 == 3x^0$

The power of the variable here is: 0

, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x – x^3$

Solution:

The highest power of $x – x^3$ is 3

, the degree is 3

Hence, $x – x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) 1 + x

Solution:

The highest power of 1 + x is 1

, the degree is 1

Hence, 1 + x is a linear polynomial

(v) 3t

Solution:

The highest power of 3t is 1

, the degree is 1

Hence, 3t is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial (x)=$5x-4x^2+3$
(i) x= 0
(ii) x = – 1
(iii) x = 2

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When x=0

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When x= -1

$f(x)=5x-4x^2+3$

$f(-1)=5(-1) -4(-1)^2+3$

$=-5-4+3$

$=-6$

(iii) When $x=2$

$f(x)=5x-4x^2+3$

$f(2)=5(2) -4(2)^2+3$

$=10-16+3$

$=-3$

Q2. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2-(1)+1=1$

$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2-(0)^3=2$

$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$

$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1$, $x=-1/3$

Solution:

For, $x=-1/3$, $p(x)=3x+1$

$\therefore p(-1/3)=3(-1/3)+1=-1+1=0$

$\therefore -1/3$ is a zero of $p(x)$.

(ii) $p(x)=5x-\pi$, $x=4/5$

Solution:

For, $x=4/5$ $p(x)=5x–\pi$

$\therefore p(4/5)=5(4/5)–\pi=4-\pi$

$\therefore$ 4/5is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of p(x).

(iv) $p(x)=(x+1)(x–2)$, $x= -1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x–2)$

$\therefore p(-1)=(-1+1)(-1–2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2–2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) $p(x)=x^2$, $x=0$

Solution:

For, $x=0$ $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of p(x).

(vi) $p(x)=lx+m$, $x=-m/t$

Solution:

For, $x=-m/t$; $p(x)=lx+m$

$\therefore p(-m/t)=l(-m/t)+m=-m+m=0$

$\therefore -m/t$is a zero of p(x).

(vii) $p(x)=3x^2-1$,$x=-1/\sqrt{3},2/\sqrt{3}$,

Solution:

For, $x=-1/\sqrt{3},2/\sqrt{3}$,; $p(x)=3x^2-1$

$\therefore p(-1/\sqrt{3})=3(-1/\sqrt{3})^2-1=3(1/\sqrt{3})-1=1-1=0$

$\therefore p(2/\sqrt{3})=3(2/\sqrt{3})^2-1=3(4/3)-1=4-1=3\neq 0$

$\therefore --1/\sqrt{3}$ is a zero of p(x) but $2/\sqrt{3}$ is not a zero of p(x).

(viii) $p(x)=2x+1$,$x=1/2$

Solution:

For, $x=1/2$ $p(x)=2x+1$

$\therefore p(1/2)=2(1/2)+1=1+1=2\neq 0$

∴ 1/2 is not a zero of p(x).

Q4. Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5

Solution:

p(x)=x+5

⇒x+5=0

⇒x=−5

∴-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

p(x)=x−5

⇒x−5=0

⇒x=5

∴5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

p(x)=2x+5

⇒2x+5=0

⇒2x=−5

⇒x=−5/2

∴x= −5/2 is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

p(x)=3x–2

⇒3x−2=0

⇒3x=2

⇒x=2/3

∴x=2/3 is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

p(x)=3x

⇒3x=0

⇒x=0

∴0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a0

Solution:

p(x)=ax

⇒ax=0

$\Rightarrow x=0$

$\therefore x=0$ is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

$p(x)= cx + d$

$\Rightarrow cx + d =0$

$\Rightarrow x=-d/c$

$\therefore x=-d/c$ is a zero polynomial of the polynomial p(x).

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Q1. Find the remainder when x^3+3x^2+3x+1 is divided by
(i) x+1

Solution:

$x+1=0$

$\Rightarrow x=-1$

$\therefore$Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) x−1/2

Solution:

$x-1/2=0$

$\Rightarrow x= 1/2$

$\therefore$Remainder:

$p(1/2)= (1/2)^3+3(1/2)^2+3()+1$

$=1/8+3/4+3/2+1$

$=27/8$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) x+π

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-5/2$

∴Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1=-125/8+75/4-15/2+1$

$=-27/8$

Q2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a = 5a$

Q3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if 7+3x divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=-7/3$

∴Remainder:

$3(7/3)^3+7(7/3)=-343/9+(-49/3)$

$=-343+(-49)3/9$

$=-343-147/9$

$=-490/9\neq 0$

∴7+3x is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x)=x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let p(x)= $x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let p(x)= $x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

p(−1)=(−1)4+3(−1)3+3(−1)2+(−1)+1

$=1-3+3-1+1$

$=1\neq0$

∴By factor theorem, x+1 is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Solution:

Let p(x)= $x^3 - x^2 - (2 + \sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1$, g(x) = x + 1

Solution:

$p(x)= 2x^3+x^2-2x-1$, g(x) = x + 1

g(x)=0

⇒x+1=0

⇒x=−1

∴Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=-2+1+2-1$

$=0$

∴By factor theorem, $g(x)$ is a factor of $p(x)$.

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

$p(x)=x3+3\times2+3x+1$, $g(x) = x + 2$

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

∴Zero of $g(x)$ is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq0$

∴By factor theorem, $g(x)$ is not a factor of $p(x)$.

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x - 3$

Solution:

$p(x)= x^3-4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

∴Zero of $g(x)$ is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

∴By factor theorem, $g(x)$ is a factor of $p(x)$.

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of $p(x)$, then $p(1)=0$

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow1+1+k=0$

$\Rightarrow2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of $p(x)$, then $p(1)=0$

$\Rightarrow2(1)^2+k(1)+\sqrt{2} =0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

⇒k= \frac{3}{2}23

Q4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=112=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3-4=12]

$12x^2-7x+1=12x^2-4x-3x+1$

=4x (3x-1)-1(3x-1)

= (4x-1)(3x-1)

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2=6

We get 6 and 1 as the numbers [6+1=7 and 6=6]

$2x^2+7x+3 =2x^2+6x+1x+3$

=2x (x+3)+1(x+3)

= (2x+1)(x+3)

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=6= -36

We get -4 and 9 as the numbers [-4+9=5 and -4=-36]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$

$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=3= -12

We get -4 and 3 as the numbers [-4+3=-1 and -4=-12]

$3x^2 – x – 4 =3x^2–x–4$

$=3x^2–4x+3x–4$

$=x(3x–4)+1(3x–4)$

$=(3x–4)(x+1)$

Q5. Factorize:

(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

p(1) = 0

So, (x+1) is factor of p(x)

Now,

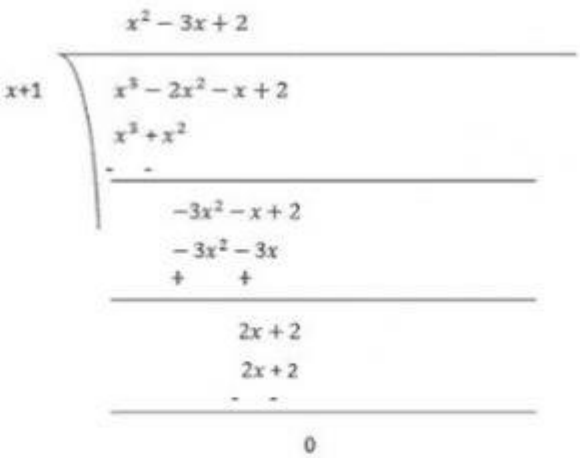
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−2+1+2$

$=0$

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x-1)-2(x-1))$

$=(x+1)(x-1)(x-2)$

(ii) x^3-3x^2-9x-5

Solution:

Let $p(x) = x^3-3x^2-9x-5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x-5)$ is factor of $p(x)$

Now,

$p(x) = x^3-3x^2-9x-5$

$p(5) = (5)^3-3(5)^2-9(5)-5$

$=125-75-45-5$

$=0$

Therefore, $(x-5)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x\text{-}5 & x^3-3x^2-9x-5 \\
 & x^3-5x^2 \\
 & - \quad + \\
\hline
 & \quad 2x^2-9x-5 \\
 & \quad 2x^2-10x \\
 & \quad - \quad + \\
\hline
 & \qquad x-5 \\
 & \qquad x-5 \\
 & \qquad - \; + \\
\hline
 & \qquad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) =(x-5)(x^2+x+x+1)$

$=(x-5)(x(x+1)+1(x+1))$

$=(x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

p(x) = $x^3+13x^2+32x+20$

p(-1) = $(-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & - \quad - \\
\hline
 & 12x^2+32x+20 \\
 & 12x^2+12x \\
 & - \quad - \\
\hline
 & 20x+20 \\
 & 20x+20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let p(y) = $2y^3+y^2-2y-1$

Factors = $2\times(-1)$= -2 are ±1 and ±2

By trial method, we find that

p(1) = 0

So, (y-1) is factor of p(y)

Now,

p(y) = $2y^3+y^2-2y-1$

p(1) = $2(1)^3+(1)^2-2(1)-1$

$=2+1-2$

$=0$

Therefore, (y-1) is the factor of p(y)

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & -\quad + \\
\hline
 & 3y^2 - 2y - 1 \\
 & 3y^2 - 3y \\
 & -\quad + \\
\hline
 & y - 1 \\
 & y - 1 \\
 & -\quad + \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1)$

$= (y-1)(2y(y+1)+1(y+1))$

$= (y-1)(2y+1)(y+1)$

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Q1. Use suitable identities to find the following products:
(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]
We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=8 and b= −10]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, x=3x, a=4 and b= −5]
We get,

$(3x+4)(3x-5) = (3x)^2+4+(-5)3x+4\times(-5)$

$=9x^2+3x(4–5)–20$

$=9x^2–3x–20$

(iv) $(y^2+3/2)(y^2–3/2)$

Solution:

Using the identity, $(x + y)(x – y) = x^2 – y^2$

[Here, $x=y^2$ and $y=3/2$]
We get,

$(y^2+3/2)(y^2–3/2) = (y^2)^2–(3/2)^2$

$=y^4–(9/4)$

Q2. Evaluate the following products without multiplying directly:
(i) 103 × 107

Solution:

$103×107=(100+3)×(100+7)$

Using identity, $[(x+a)(x+b)=x2+(a+b)x+ab$

Here, $x=100$

$a=3$

$b=7$

We get, $103×107=(100+3)×(100+7)$

$=(100)^2+(3+7)100+(3×7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95×96=(100-5)×(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, $x=100$

$a=-5$

$b=-4$

We get, $95×96=(100-5)×(100-4)$

$=(100)^2+100(-5+(-4))+(-5×-4)$

$=10000-900+20$

$=9120$

(iii) 104 × 96

Solution:

$104×96=(100+4)×(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104×96=(100+4)×(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

Q3. Factorize the following using appropriate identities:
(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2×3x×y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2×3x×y)+y^2$

$=(3x+y)^2$

$=(3x+y)(3x+y)$

(ii) $4y^2−4y+1$

Solution:

$4y^2−4y+1=(2y)^2–(2×2y×1)+12$

Using identity, $x^2 – 2xy + y^2= (x – y)^2$

Here, $x=2y$

$y=1$

$4y^2−4y+1=(2y)^2–(2×2y×1)+1^2$

$=(2y–1)^2$

$=(2y–1)(2y–1)$

(iii) $x^2–y^{2/100}$

Solution:

$x^2–y^{2/100 = x2}–(y/10)^2$

Using identity, $x^2 – y^2= (x – y) (x y)$

Here,

$x=x$

$y=y/10$

$x^2 – y^2/100= x^2–(y/10)^2$

$=(x–y/10)(x+y/10)$

Q4. Expand each of the following, using suitable identities:
(i) $(x+2y+4z)^2$
(ii) $(2x−y+z)^2$
(iii) $(−2x+3y+2z)^2$
(iv) $(3a – 7b – c)^2$
(v) $(–2x + 5y – 3z)^2$
(vi) $(a–b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$=x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$=4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$=4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= - 7b$

$z= - c$

$(3a - 7b - c)^2 = (3a)^2+(- 7b)^2+(- c)^2+(2\times 3a \times - 7b)+(2\times - 7b \times - c)+(2\times - c \times 3a)$

$=9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y= 5y$

$z= - 3z$

$(-2x+5y-3z)^2 = (-2x)^2+(5y)^2+(-3z)^2+(2\times -2x \times 5y)+(2\times 5y \times - 3z)+(2\times -3z \times -2x)$

$=4x^2 + 25y^2 + 9z^2 – 20xy – 30yz + 12zx$

(vi) $(1/4a – 1/2b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 1/4a$

$y= –1/2b$

$z= 1$

$(1/4a –1/2b +1)^2 =(1/4a)^2+(–1/2b)^2+(1)^2+(2×1/4a × –1/2b)+(2× –1/2b ×1)+(2×1×1/4a)$

$=1/16a^2+1/4b^2+1^2–2/8ab– 2/2b +2/4a$

$= 1/16a^2+1/4b^2+1–1/4ab – b +1/2a$

Q5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$
(ii) $2x^2+y^2+8z^2–2xy+4yz–8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy–24yz–16xz =(2x)^2+(3y)^2+(−4z)^2+(2×2x×3y)+(2×3y×−4z)+(2×−4z×2x)$

$=(2x+3y–4z)^2$

$=(2x+3y–4z)(2x+3y–4z)$

(ii) $2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz =(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2×-\sqrt{2}x×y)+(2×y×2\sqrt{2}z)+(2×2\sqrt{2}\sqrt{2}z×-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

Q6. Write the following cubes in expanded form:
(i) $(2x+1)^3$
(ii) $(2a−3b)^3$
(iii) $(x+1)^3$
(iv) $(x−y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3×2x×1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(2a-3b)^3=(2a)^3-(3b)^3-(3\times2a\times3b)(2a-3b)$

$=8a^3-27b^3-18ab(2a-3b)$

$=8a^3-27b^3-36a^2b+54ab^2$

(iii) $(3/2x+1)^3$

Solution:

Using identity, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(3/2x+1)^3=(3/2x)^3+1^3+(3\times3/2x\times1)(3/2x+1)$

$=27/8x^3+1+9/2x(3/2x+1)$

$=27/8x^3+1+27/4x^2+9/2x$

$=27/8x^3+27/4x^2+9/2x+1$

(iv) $(x-2/3y)^3$

Solution:

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(x-2/3y)^3=(x)^3-(2/3y)^3-(3\times x\times2/3y)(x-2/3y)$

$=(x)^3-8/27y^3-2xy(x-2/3y)$

$=(x)^3-8/27y^3-2x^2y+4/3xy^2$

Q7. Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(99)^3 = (100-1)^3$

$=(100)^3-1^3-(3\times100\times1)(100-1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(100+2)^3 = (100)^3 + 2^3 + (3×100×2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(998)^3 = (1000–2)^3$

$= (1000)^3 – 2^3 – (3×1000×2)(1000–2)$

$= 1000000000 – 8 – 6000(1000 – 2)$

$= 1000000000 – 8 - 6000000 + 12000$

$= 994011992$

Q8. Factorise each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 –1/216 −(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 = (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 = (2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$= (2a–b)^3$

$= (2a–b)(2a–b)(2a–b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iii) $27 – 125a^3 – 135a + 225a^2$

Solution:

The expression, $27 – 125a^3 – 135a + 225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$27–125a^3–135a+225a^2 = 3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$=(3–5a)^3$

$=(3–5a)(3–5a)(3–5a)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iv) 64a3–27b3–144a²b+108ab²

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2 = (4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a–3b)^3$

$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3 – 1/216 − 9/2p^2+ 1/4p$

Solution:

The expression, $27p^3 –1/216 − 9/2p^2+ 1/4p$ can be written as $(3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$27p^3 – 1/216 − 9/2p^2+ 1/4p = (3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$= (3p–(1/6))^3$

$= (3p–(1/6))(3p–(1/6))(3p–(1/6))$

Q9. Verify:
(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$
(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 =x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3–3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2–3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)–3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2–xy)$

(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

We know that,$(x–y)^3 =x^3–y^3–3xy(x–y)$

$\Rightarrow x^3−y^3=(x–y)^3+3xy(x–y)$

$\Rightarrow x^3−y^3=(x–y)[(x–y)^2+3xy]$

Taking(x+y) common$\Rightarrow x^3−y^3=(x–y)[(x^2+y^2–2xy)+3xy]$

$\Rightarrow x^3+y^3=(x–y)(x^2+y^2+xy)$

Q10. Factorize each of the following:
(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3=(3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2–xy+y^2)$

$27y^3+125z^3=(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2–(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2–15yz+25z^2)$

(ii) $64m^3–343n^3$

The expression, $64m^3–343n^3$ can be written as $(4m)^3–(7n)^3$

$64m^3–343n^3= (4m)^3–(7n)^3$

We know that, $x^3–y^3=(x–y)(x^2+xy+y^2)$

$64m^3–343n^3=(4m)^3–(7n)^3$

$=(4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$=(4m-7n)(16m^2+28mn+49n^2)$

Q11. Factorise : $27x^3+y^3+z^3–9xyz$

Solution:

The expression $27x^3+y^3+z^3–9xyz$ can be written as $(3x)^3+y^3+z^3–3(3x)(y)(z)$

$27x^3+y^3+z^3–9xyz =(3x)^3+y^3+z^3–3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$

$27x^3+y^3+z^3–9xyz =(3x)^3+y^3+z^3–3(3x)(y)(z)$

$=(3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$

$=(3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

Q12. Verify that:

$x^3+y^3+z^3–3xyz=(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Solution:

We know that,

$x^3+y^3+z^3−3xyz=(x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =\times(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$

$= (x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$

$= (x+y+z)[(x^2+y^2−2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$

$= (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Q13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,

$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$

Now, according to the question, let $(x + y + z) = 0$, then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =0$

$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

Q14. Without actually calculating the cubes, find the value of each of the following:
(i) $(−12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(−15)^3+(−13)^3$

(i) $(−12)^3+(7)^3+(5)^3$

Solution:

$(−12)^3+(7)^3+(5)^3$

Let a= −12

b= 7

c= 5

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, −12+7+5=0

$(−12)^3+(7)^3+(5)^3 = 3xyz$

=

=

(ii) $(28)^3+(−15)^3+(−13)^3$

Solution:

$(28)^3+(−15)^3+(−13)^3$

Let a= 28

b= −15

c= −13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$(28)^3+(−15)^3+(−13)^3= 3xyz$

= 0+3(28)(−15)(−13)

=16380

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2–35a+12$
(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=2512=300

We get -15 and -20 as the numbers [-15+-20=-35 and -3-4=300]

$25a^2–35a+12 =25a^2–15a−20a+12$

$=5a(5a–3)–4(5a–3)$

$=(5a–4)(5a–3)$

Possible expression for length $= 5a – 4$

Possible expression for breadth $= 5a – 3$

(ii) Area : $35y^2+13y–12$

Using the splitting the middle term method,

We have to find a number whose sum= 13 and product=3512=420

We get -15 and 28 as the numbers [-15+28=-35 and -15=420]

$35y^2+13y–12 =35y^2–15y+28y–12$

$=5y(7y–3)+4(7y–3)$

$=(5y+4)(7y–3)$

Possible expression for length $= (5y + 4)$

Possible expression for breadth $= (7y – 3)$

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:

(i) Volume : $3x^2–12x$

$3x^2–12x$ can be written as $3x(x – 4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x – 4)$

(ii) Volume : $12ky^2+8ky –20k$

$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking $4k$ out of both the terms.

$12ky^2+8ky–20k =4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]

$=4k(3y^2+5y–3y–5)$

$=4k[y(3y+5)–1(3y+5)]$

$=4k(3y+5)(y–1)$

Possible expression for length = $4k$

Possible expression for breadth = $(3y +5)$

Possible expression for height = $(y -1)$

Access Answers of Maths NCERT class 9 Chapter 2 – Polynomials

Class 9 Maths Chapter 2 Exercise 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

Solution:

The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 – 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation y^2 + can be written as $y^2 + y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression y^2 + is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation 3 + t can be written as $3t^{1/2} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,) is not a whole number. Hence, we can say that the expression 3 + t is **not** a polynomial in one variable.

(iv) y + 2/y

Solution:

The equation y + can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression y + is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1) x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1) x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\Pi/2\ x^2 + x$

Solution:

The equation $\Pi/2x^2 + x$ can be written as $(\Pi/2)\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is

, the coefficients of x^2 in $\Pi/2x^2 + x$ is $\Pi/2$.

(iv)$\sqrt{}$2x-1

Solution:

The equationx$\sqrt{}$2x-1 can be written as $0x^2 + \sqrt{}2x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

, the coefficients of x^2 in$\sqrt{2}x-1$ is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

, the degree of $5t – \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, 3== $3x^0$

The power of the variable here is: 0

, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x – x^3$

Solution:

The highest power of $x – x^3$ is 3

, the degree is 3

Hence, $x – x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) 1 + x

Solution:

The highest power of $1 + x$ is 1

, the degree is 1

Hence, $1 + x$ is a linear polynomial

(v) 3t

Solution:

The highest power of $3t$ is 1

, the degree is 1

Hence, $3t$ is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial
$(x)=5x-4x^2+3$
(i) x= 0
(ii) x = – 1
(iii) x = 2

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When x=0

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When x= -1

$f(x)=5x-4x^2+3$

$f(-1)=5(-1) -4(-1)^2+3$

$=-5–4+3$

$=-6$

(iii) When x=2

$f(x)=5x-4x^2+3$

$f(2)=5(2) -4(2)^2+3$

$=10–16+3$

$=-3$

Q2. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2–y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2–(1)+1=1$

$p(2)=(2)^2–(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2–(0)^3=2$

$p(1)=2+1+2(1)^2–(1)^3=2+1+2–1=4$

$p(2)=2+2+2(2)^2–(2)^3=2+2+8–8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x–1)(x+1)$

$\therefore p(0)=(0–1)(0+1)=(-1)(1)=–1$

$p(1)=(1–1)(1+1)=0(2)=0$

$p(2)=(2–1)(2+1)=1(3)=3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1$, $x=-1/3$

Solution:

For, $x=-1/3$, $p(x)=3x+1$

$\therefore p(-1/3)=3(-1/3)+1=-1+1=0$

$\therefore -1/3$ is a zero of $p(x)$.

(ii) $p(x)=5x-\pi$, $x=4/5$

Solution:

For, $x=4/5$ $p(x)=5x-\pi$

$\therefore p(4/5)=5(4/5)-\pi=4-\pi$

$\therefore$ 4/5is not a zero of $p(x)$.

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of $p(x)$.

(iv) $p(x)=(x+1)(x-2)$, $x= -1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) $p(x)=x^2$, $x=0$

Solution:

For, $x=0$ $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of p(x).

(vi) $p(x)=lx+m$, $x=-m/t$

Solution:

For, $x=-m/t$; $p(x)=lx+m$

$\therefore p(-m/t)=l(-m/t)+m=-m+m=0$

$\therefore -m/t$is a zero of p(x).

(vii) $p(x)=3x^2-1, x=-1/\sqrt{3}, 2/\sqrt{3}$,

Solution:

For, $x=-1/\sqrt{3}, 2/\sqrt{3}$,; $p(x)=3x^2-1$

$\therefore p(-1/\sqrt{3})=3(-1/\sqrt{3})^2-1=3(1/\sqrt{3})-1=1-1=0$

$\therefore p(2/\sqrt{3})=3(2/\sqrt{3})^2-1=3(4/3)-1=4-1=3\neq 0$

$\therefore --1/\sqrt{3}$ is a zero of p(x) but $2/\sqrt{3}$ is not a zero of p(x).

(viii) $p(x)=2x+1, x=1/2$

Solution:

For, $x=1/2$ $p(x)=2x+1$

$\therefore p(1/2)=2(1/2)+1=1+1=2\neq 0$

$\therefore$ 1/2 is not a zero of p(x).

Q4. Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5

Solution:

$p(x)=x+5$

$\Rightarrow x+5=0$

$\Rightarrow x=-5$

$\therefore$-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

$p(x)=x-5$

$\Rightarrow x-5=0$

$\Rightarrow x=5$

$\therefore$5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-5/2$

$\therefore x = -5/2$ is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

$p(x)=3x–2$

$\Rightarrow 3x-2=0$

$\Rightarrow 3x=2$

$\Rightarrow x=2/3$

$\therefore x=2/3$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

$p(x)=3x$

$\Rightarrow 3x=0$

$\Rightarrow x=0$

$\therefore 0$ is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a0

Solution:

$p(x)=ax$

$\Rightarrow ax=0$

$\Rightarrow x=0$

$\therefore x=0$ is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

$p(x)= cx + d$

$\Rightarrow cx + d =0$

$\Rightarrow x=-d/c$

$\therefore$ $x=-d/c$ is a zero polynomial of the polynomial $p(x)$.

Class 9 Maths Chapter 2 Exercise 2.3 Page: 40

Q1. Find the remainder when x^3+3x^2+3x+1 is divided by
(i) $x+1$

Solution:

$x+1=0$

$\Rightarrow x=-1$

$\therefore$Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) $x-1/2$

Solution:

$x-1/2=0$

$\Rightarrow x= 1/2$

$\therefore$Remainder:

$p(1/2\ )= (1/2)^3+3(1/2)^2+3()+1$

$=1/8+3/4+3/2+1$

$=27/8$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) x+π

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\ 5/2$

$\therefore$Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1=-125/8+75/4-15/2+1$

$=-27/8$

Q2. Find the remainder when $x^3−ax^2+6x−a$ is divided by x-a.

Solution:

Let $p(x)=x^3−ax^2+6x−a$

$x−a=0$

$\therefore x=a$

Remainder:

$p(a)= (a)^3 −a(a^2)+6(a)−a$

$=a^3−a^3+6a−a = 5a$

Q3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=−7$ only if 7+3x divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=-7/3$

$\therefore$Remainder:

$3(7/3)^3+7(7/3)= −343/9+(-49/3)$

$= −343+(-49)3/9$

$= -343-147/9$

$= -490/9\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let p(x)= x^3+x^2+x+1

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let p(x)= $x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let p(x)= $x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:
(i) $p(x)=2x^3+x^2-2x-1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2-2x-1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

∴Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=-2+1+2-1$

$=0$

∴By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, g(x) = x + 2

Solution:

p(x)=x3+3×2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

∴Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

∴By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, g(x) = x – 3

Solution:

$p(x)= x^3-4x^2+x+6$, g(x) = x -3

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

∴Zero of $g(x)$ is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

∴By factor theorem, $g(x)$ is a factor of $p(x)$.

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow 1+1+k=0$

$\Rightarrow 2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

⇒k= \frac{3}{2}23

Q4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=112=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3-4=12]

$12x^2-7x+1=12x^2-4x-3x+1$

=4x (3x-1)-1(3x-1)

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2=6

We get 6 and 1 as the numbers [6+1=7 and 6=6]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$=2x (x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=6= -36

We get -4 and 9 as the numbers [-4+9=5 and -4=-36]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$

$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=3= -12

We get -4 and 3 as the numbers [-4+3=-1 and -4=-12]

$3x^2 – x – 4 = 3x^2–x–4$

$=3x^2–4x+3x–4$

$=x(3x–4)+1(3x–4)$

$=(3x–4)(x+1)$

Q5. Factorize:
(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

$p(1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

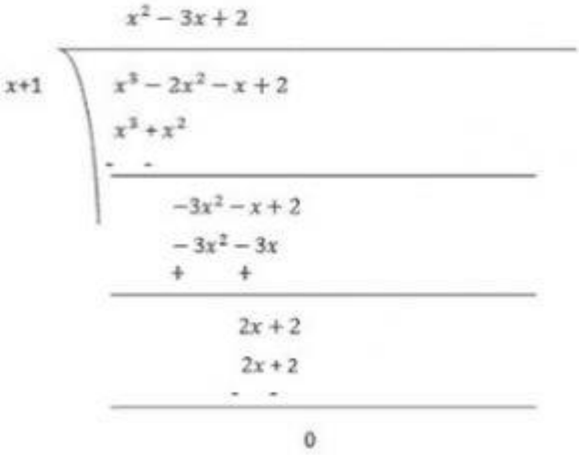
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−2+1+2$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x-5)$ is factor of $p(x)$

Now,

$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, (x-5) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x-5 & x^3-3x^2-9x-5 \\
 & x^3-5x^2 \\
 & - \quad + \\
\hline
 & \quad 2x^2-9x-5 \\
 & \quad 2x^2-10x \\
 & \quad - \quad + \\
\hline
 & \qquad x-5 \\
 & \qquad x-5 \\
 & \qquad - \quad + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) =(x-5)(x^2+x+x+1)$

$=(x-5)(x(x+1)+1(x+1))$

$=(x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, (x+1) is factor of p(x)

Now,

$p(x) = x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & 12x^2+32x+20 \\
 & 12x^2+12x \\
 & -\quad - \\
\hline
 & 20x+20 \\
 & 20x+20 \\
 & -\quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) =(x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1)= -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

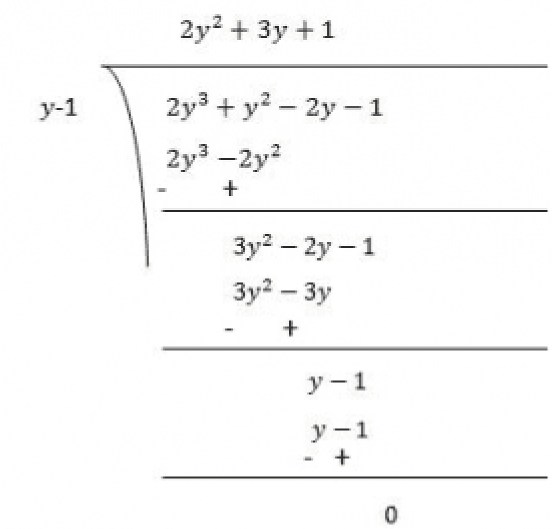
$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2–2(1)–1$

$=2+1−2$

$=0$

Therefore, (y-1) is the factor of p(y)



Now, Dividend = Divisor × Quotient + Remainder

$(y−1)(2y^2+3y+1) =(y−1)(2y^2+2y+y+1)$

$=(y−1)(2y(y+1)+1(y+1))$

$=(y−1)(2y+1)(y+1)$

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Q1. Use suitable identities to find the following products:
(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a) (x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]

We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $a=8$ and $b= -10$]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $x=3x$, $a=4$ and $b= -5$]
We get,

$(3x+4)(3x-5) = (3x)^2+4+(-5)3x+4\times(-5)$

$= 9x^2+3x(4-5)-20$

$= 9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=3/2$]
We get,

$(y^2+3/2)(y^2–3/2) = (y^2)^2–(3/2)^2$

$=y^4–(9/4)$

Q2. Evaluate the following products without multiplying directly:
(i) 103 × 107

Solution:

103×107=(100+3)×(100+7)

Using identity, [(x+a)(x+b)=x2+(a+b)x+ab

Here, x=100

a=3

b=7

We get, 103×107=(100+3)×(100+7)

$=(100)^2+(3+7)100+(3\times7))$

=10000+1000+21

=11021

(ii) 95 × 96

Solution:

95×96=(100-5)×(100-4)

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, x=100

a=-5

$b=-4$

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii) 104 × 96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

Q3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2×3x×y)+y^2$

$=(3x+y)^2$

$=(3x+y)(3x+y)$

(ii) $4y^2−4y+1$

Solution:

$4y^2−4y+1=(2y)^2–(2×2y×1)+12$

Using identity, $x^2 – 2xy + y^2= (x – y)^2$

Here, $x=2y$

$y=1$

$4y^2−4y+1=(2y)^2–(2×2y×1)+1^2$

$=(2y–1)^2$

$=(2y–1)(2y–1)$

(iii) $x^2–y^{2/100}$

Solution:

$x^2–y^{2/100 = x_2}–(y/10)^2$

Using identity, $x^2 – y^2= (x – y) (x y)$

Here,

$x=x$

$y=y/10$

$x^2 – y^2/100= x^2–(y/10)^2$

$=(x–y/10)(x+y/10)$

Q4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii) $(-2x+3y+2z)^2$
(iv) $(3a - 7b - c)^2$
(v) $(-2x + 5y - 3z)^2$
(vi) $(a-b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$=x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$=4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$=4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= -7b$

$z= -c$

$(3a - 7b - c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times 3a \times -7b)+(2\times -7b \times -c)+(2\times -c \times 3a)$

$=9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= –2x$

$y= 5y$

$z= – 3z$

$(–2x+5y–3z)^2 =(–2x)^2+(5y)^2+(–3z)^2+(2\times–2x \times 5y)+(2\times 5y \times– 3z)+(2\times–3z \times–2x)$

$=4x^2 + 25y^2 + 9z^2– 20xy–30yz+12zx$

(vi) $(1/4a – 1/2b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 1/4a$

$y= –1/2b$

$z= 1$

$(1/4a –1/2b +1)^2 =(1/4a)^2+(–1/2b)^2+(1)^2+(2\times1/4a \times –1/2b)+(2\times –1/2b \times1)+(2\times1\times1/4a)$

$=1/16a^2+1/4b^2+1^2–2/8ab– 2/2b +2/4a$

$= 1/16a^2+1/4b^2+1–1/4ab – b +1/2a$

Q5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$
(ii) $2x^2+y^2+8z^2–2xy+4yz–8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy–24yz–16xz =(2x)^2+(3y)^2+(−4z)^2+(2×2x×3y)+(2×3y×−4z)+(2×−4z×2x)$

$=(2x+3y–4z)^2$

$=(2x+3y–4z)(2x+3y–4z)$

(ii) $2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz =(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2×−\sqrt{2}x×y)+(2×y×2\sqrt{2}z)+(2×2\sqrt{2}\,2\sqrt{2}z×−\sqrt{2}x)$

$=(−\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(−\sqrt{2}x+y+2\sqrt{2}z)(−\sqrt{2}x+y+2\sqrt{2}z)$

Q6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$
(iii) $(x+1)^3$
(iv) $(x-y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times2x\times1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(2a-3b)^3=(2a)^3-(3b)^3-(3\times2a\times3b)(2a-3b)$

$=8a^3-27b^3-18ab(2a-3b)$

$=8a^3-27b^3-36a^2b+54ab^2$

(iii) $(3/2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(3/2x+1)^3 =(3/2x)^3+1^3+(3\times3/2x\times1)(3/2x+1)$

$=27/8x^3+1+9/2x(3/2x+1)$

$=27/8x^3+1+27/4x^2+9/2x$

$=27/8x^3+27/4x^2+9/2x+1$

(iv) $(x-2/3y)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(x-2/3y)^3 = (x)^3-(2/3y)^3-(3\times x\times 2/3y)(x-2/3y)$

$=(x)^3-8/27y^3-2xy(x- 2/3y)$

$=(x)^3-8/27y^3-2x^2y+4/3xy^2$

Q7. Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(99)^3 = (100-1)^3$

$=(100)^3-1^3-(3\times 100\times 1)(100-1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(100+2)^3 =(100)^3+2^3+(3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(998)^3 =(1000–2)^3$

$=(1000)^3–2^3–(3\times1000\times2)(1000–2)$

$= 1000000000 – 8 – 6000(1000 – 2)$

$= 1000000000 – 8- 6000000 + 12000$

$= 994011992$

Q8. Factorise each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 –1/216 −(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 = (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$=(2a+b)^3$

$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 = (2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$=(2a–b)^3$

$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iii) $27 – 125a^3 – 135a + 225a^2$

Solution:

The expression, $27 – 125a^3 – 135a + 225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$27–125a^3–135a+225a^2 = 3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$=(3–5a)^3$

$=(3–5a)(3–5a)(3–5a)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iv) 64a3–27b3–144a²b+108ab²

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2 = (4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a–3b)^3$

$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3$ – 1/216 − 9/2p²+ 1/4p

Solution:

The expression, $27p^3 –1/216 − 9/2p^2+ 1/4p$ can be written as $(3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$27p^3 – 1/216 − 9/2p^2+ 1/4p = (3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$= (3p–(1/6))^3$

$= (3p–(1/6))(3p–(1/6))(3p–(1/6))$

Q9. Verify:
(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$
(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 =x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3–3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2–3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)–3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2–xy)$

(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

We know that,$(x–y)^3 =x^3–y^3–3xy(x–y)$

$\Rightarrow x^3-y^3=(x–y)^3+3xy(x–y)$

$\Rightarrow x^3-y^3=(x–y)[(x–y)^2+3xy]$

Taking(x+y) common$\Rightarrow x^3-y^3=(x–y)[(x^2+y^2–2xy)+3xy]$

$\Rightarrow x^3+y^3=(x–y)(x^2+y^2+xy)$

Q10. Factorize each of the following:
(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3=(3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2–xy+y^2)$

$27y^3+125z^3=(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2–(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2–15yz+25z^2)$

(ii) $64m^3–343n^3$

The expression, $64m^3–343n^3$ can be written as $(4m)^3–(7n)^3$

$64m^3–343n^3 = (4m)^3–(7n)^3$

We know that, $x^3–y^3=(x–y)(x^2+xy+y^2)$

$64m^3–343n^3=(4m)^3–(7n)^3$

$=(4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$=(4m-7n)(16m^2+28mn+49n^2)$

Q11. Factorise : $27x^3+y^3+z^3–9xyz$

Solution:

The expression $27x^3+y^3+z^3–9xyz$ can be written as $(3x)^3+y^3+z^3–3(3x)(y)(z)$

$27x^3+y^3+z^3–9xyz \ =(3x)^3+y^3+z^3–3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$

$27x^3+y^3+z^3–9xyz \ =(3x)^3+y^3+z^3–3(3x)(y)(z)$

$=(3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$

$=(3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

Q12. Verify that:

$x^3+y^3+z^3–3xyz=(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Solution:

We know that,

$x^3+y^3+z^3−3xyz=(x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =\times(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$

$= (x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$

$= (x+y+z)[(x^2+y^2−2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$

$= (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Q13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,

$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$

Now, according to the question, let (x + y + z) = 0,

then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =0$

$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(-15)^3+(-13)^3$

(i) $(-12)^3+(7)^3+(5)^3$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let a= -12

b= 7

c= 5

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$(-12)^3+(7)^3+(5)^3 = 3xyz$

=

=

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a= 28

b= -15

c= -13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$(28)^3+(-15)^3+(-13)^3= 3xyz$

$= 0+3(28)(-15)(-13)$

=16380

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2–35a+12$
(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=2512=300

We get -15 and -20 as the numbers [-15+-20=-35 and -3-4=300]

$25a^2–35a+12 =25a^2–15a−20a+12$

$=5a(5a–3)–4(5a–3)$

$=(5a–4)(5a–3)$

Possible expression for length = 5a – 4

Possible expression for breadth = 5a – 3

(ii) Area : $35y^2+13y–12$

Using the splitting the middle term method,

We have to find a number whose sum= 13 and product=3512=420

We get -15 and 28 as the numbers [-15+28=-35 and -15=420]

$35y^2+13y–12 =35y^2–15y+28y–12$

$=5y(7y-3)+4(7y-3)$

$=(5y+4)(7y-3)$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y - 3)$

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume : $3x^2-12x$
(ii) Volume : $12ky^2+8ky-20k$

Solution:

(i) Volume : $3x^2-12x$

$3x^2-12x$ can be written as $3x(x - 4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x - 4)$

(ii) Volume : $12ky^2+8ky-20k$

$12ky^2+8ky-20k$ can be written as $4k(3y^2+2y-5)$ by taking $4k$ out of both the terms.

$12ky^2+8ky-20k = 4k(3y^2+2y-5)$

[Here, $3y^2+2y-5$ can be written as $3y^2+5y-3y-5$ using splitting the middle term method.]
$=4k(3y^2+5y-3y-5)$

$=4k[y(3y+5)-1(3y+5)]$

$=4k(3y+5)(y-1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)

Access Answers of Maths NCERT class 9 Chapter 2 – Polynomials

Class 9 Maths Chapter 2 Exercise 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2 – 3x + 7$

Solution:

The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 – 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation y^2 + can be written as $y^2 + y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression y^2 + is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation 3 + t can be written as $3t^{1/2} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,) is not a whole number. Hence, we can say that the expression 3 + t is **not** a polynomial in one variable.

(iv) y + 2/y

Solution:

The equation y + can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression y + is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1) x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1) x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\Pi/2\ x^2 + x$

Solution:

The equation $\Pi/2x^2 + x$ can be written as $(\Pi/2)\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is

, the coefficients of x^2 in $\Pi/2x^2+x$ is $\Pi/2$.

(iv)$\sqrt{2}$x-1

Solution:

The equationx$\sqrt{2}$x-1 can be written as $0x^2+\sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

, the coefficients of x^2 in$\sqrt{2}x-1$ is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

, the degree of $5t - \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, 3== $3x^0$

The power of the variable here is: 0

, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x – x^3$

Solution:

The highest power of $x – x^3$ is 3

, the degree is 3

Hence, $x – x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) 1 + x

Solution:

The highest power of 1 + x is 1

, the degree is 1

Hence, 1 + x is a linear polynomial

(v) 3t

Solution:

The highest power of 3t is 1

, the degree is 1

Hence, 3t is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial
$(x)=5x-4x^2+3$
(i) $x= 0$
(ii) $x = -1$
(iii) $x = 2$

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When $x=0$

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When $x= -1$

$f(x)=5x-4x^2+3$

$f(-1)=5(-1) -4(-1)^2+3$

$=-5-4+3$

$=-6$

(iii) When $x=2$

$f(x)=5x-4x^2+3$

$f(2)=5(2)-4(2)^2+3$

$=10–16+3$

$=-3$

Q2. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2–y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2–(1)+1=1$

$p(2)=(2)^2–(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2–(0)^3=2$

$p(1)=2+1+2(1)^2–(1)^3=2+1+2–1=4$

$p(2)=2+2+2(2)^2–(2)^3=2+2+8–8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x–1)(x+1)$

$\therefore p(0)=(0–1)(0+1)=(-1)(1)=–1$

$p(1)=(1–1)(1+1)=0(2)=0$

$p(2)=(2–1)(2+1)=1(3)=3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1$, $x=-1/3$

Solution:

For, $x=-1/3$, $p(x)=3x+1$

$\therefore p(-1/3)=3(-1/3)+1=-1+1=0$

$\therefore -1/3$ is a zero of p(x).

(ii) $p(x)=5x–\pi$, $x=4/5$

Solution:

For, $x=4/5$ $p(x)=5x–\pi$

$\therefore p(4/5)=5(4/5)–\pi=4-\pi$

$\therefore$ 4/5is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of $p(x)$.

(iv) $p(x)=(x+1)(x-2)$, $x= -1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of $p(x)$.

(v) $p(x)=x^2$, $x=0$

Solution:

For, $x=0$ $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of $p(x)$.

(vi) $p(x)=lx+m$, $x=-m/t$

Solution:

For, $x=-m/t$; $p(x)=lx+m$

$\therefore p(-m/t)=l(-m/t)+m=-m+m=0$

$\therefore -m/t$is a zero of $p(x)$.

(vii) $p(x)=3x^2-1, x=-1/\sqrt{3}, 2/\sqrt{3},$

Solution:

For, $x=-1/\sqrt{3}, 2/\sqrt{3},$; $p(x)=3x^2-1$

$\therefore p(-1/\sqrt{3})=3(-1/\sqrt{3})^2-1=3(1/\sqrt{3})-1=1-1=0$

$\therefore p(2/\sqrt{3})=3(2/\sqrt{3})^2-1=3(4/3)-1=4-1=3\neq0$

$\therefore --1/\sqrt{3}$ is a zero of p(x) but $2/\sqrt{3}$ is not a zero of p(x).

(viii) $p(x)=2x+1, x=1/2$

Solution:

For, $x=1/2$ $p(x)=2x+1$

$\therefore p(1/2)=2(1/2)+1=1+1=2\neq0$

$\therefore$ 1/2 is not a zero of p(x).

Q4. Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5

Solution:

$p(x)=x+5$

$\Rightarrow x+5=0$

$\Rightarrow x=-5$

$\therefore$-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

$p(x)=x-5$

$\Rightarrow x-5=0$

$\Rightarrow x=5$

$\therefore 5$ is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-5/2$

$\therefore x= -5/2$ is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

$p(x)=3x-2$

$\Rightarrow 3x-2=0$

$\Rightarrow 3x=2$

$\Rightarrow x=2/3$

$\therefore x=2/3$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

$p(x)=3x$

$\Rightarrow 3x=0$

$\Rightarrow x=0$

∴0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a0

Solution:

p(x)=ax

⇒ax=0

⇒x=0

∴x=0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

p(x)= cx + d

⇒ cx + d =0

⇒x=-d/c

∴ x=-d/c is a zero polynomial of the polynomial p(x).

Class 9 Maths Chapter 2 Exercise 2.3 Page: 40

Q1. Find the remainder when x^3+3x^2+3x+1 is divided by
(i) x+1

Solution:

x+1=0

⇒x=−1

∴Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) $x-1/2$

Solution:

$x-1/2=0$

$\Rightarrow x= 1/2$

$\therefore$Remainder:

$p(1/2\ )= (1/2)^3+3(1/2)^2+3()+1$

$=1/8+3/4+3/2+1$

$=27/8$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) $x+\pi$

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\ 5/2$

∴Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1=-125/8+75/4-15/2+1$

$=-27/8$

Q2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a=5a$

Q3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if 7+3x divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=-7/3$

$\therefore$Remainder:

$3(7/3)^3+7(7/3)= -343/9+(-49/3)$

$= -343+(-49)3/9$

$= -343-147/9$

$= -490/9\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

$\therefore$By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1\neq 0$

∴By factor theorem, x+1 is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:
(i) $p(x)=2x^3+x^2–2x–1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2–2x–1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2–2(-1)–1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3×2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

∴By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3–4x^2+x+6$, g(x) = x – 3

Solution:

$p(x)= x^3–4x^2+x+6$, g(x) = x -3

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

∴Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

∴By factor theorem, g(x) is a factor of p(x).

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow (1)^2+(1)+k=0$

$\Rightarrow 1+1+k=0$

$\Rightarrow 2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2} =0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

⇒k= \frac{3}{2}23

Q4. Factorize:

(i) $12x^2–7x+1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=112=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3-4=12]

$12x^2–7x+1=12x^2-4x-3x+1$

$=4x\ (3x-1)-1(3x-1)$

$=(4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2=6

We get 6 and 1 as the numbers [6+1=7 and 6=6]

$2x^2+7x+3 =2x^2+6x+1x+3$

$=2x\ (x+3)+1(x+3)$

$=(2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=6= -36

We get -4 and 9 as the numbers [-4+9=5 and -4=-36]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$

$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=3= -12

We get -4 and 3 as the numbers [-4+3=-1 and -4=-12]

$3x^2 – x – 4 =3x^2–x–4$

$=3x^2–4x+3x–4$

$=x(3x–4)+1(3x–4)$

$=(3x–4)(x+1)$

Q5. Factorize:
(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

p(1) = 0

So, (x+1) is factor of p(x)

Now,

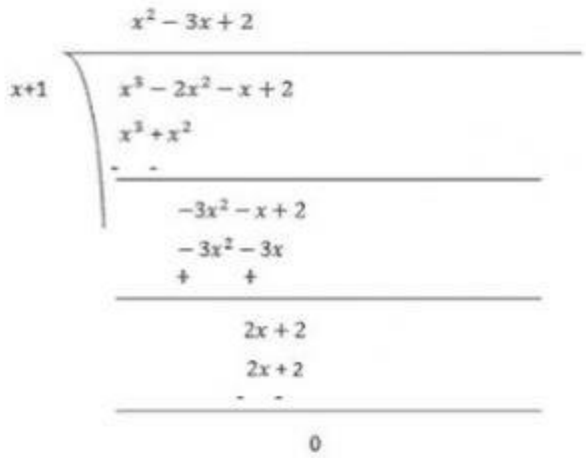
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−2+1+2$

$=0$

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

p(5) = 0

So, (x-5) is factor of p(x)

Now,

$p(x) = x^3 – 3x^2 – 9x – 5$

$p(5) = (5)^3 – 3(5)^2 – 9(5) – 5$

$= 125 − 75 − 45 − 5$

$= 0$

Therefore, (x-5) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x-5 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 - 5x^2 \\
 & - \quad + \\
\hline
 & \quad 2x^2 - 9x - 5 \\
 & \quad 2x^2 - 10x \\
 & \quad - \quad + \\
\hline
 & \qquad x - 5 \\
 & \qquad x - 5 \\
 & \qquad - \quad + \\
\hline
 & \qquad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x−5)(x^2+2x+1) = (x−5)(x^2+x+x+1)$

$= (x−5)(x(x+1)+1(x+1))$

$= (x−5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let p(x) = $x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

p(-1) = 0

So, (x+1) is factor of p(x)

Now,

p(x) = $x^3+13x^2+32x+20$

p(-1) = $(-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2+32x+20 \\
 & \quad 12x^2+12x \\
 & \quad - \quad - \\
\hline
 & \qquad 20x+20 \\
 & \qquad 20x+20 \\
 & \qquad - \quad - \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) =(x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors = $2\times(-1)= -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

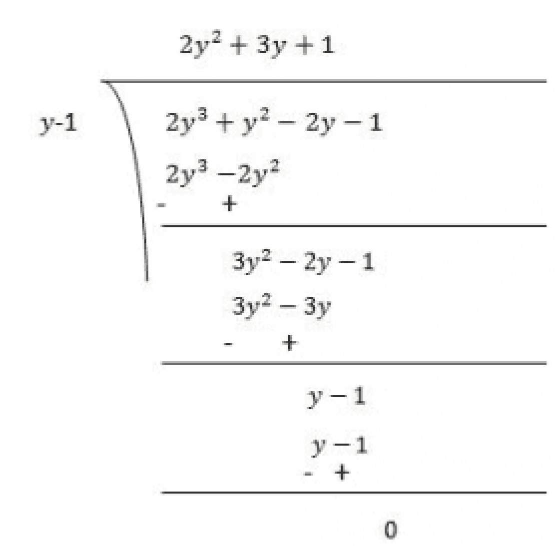
$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$=2+1-2$

$=0$

Therefore, $(y-1)$ is the factor of $p(y)$



Now, Dividend = Divisor × Quotient + Remainder

$(y-1)(2y^2+3y+1) =(y-1)(2y^2+2y+y+1)$

$=(y-1)(2y(y+1)+1(y+1))$

$=(y-1)(2y+1)(y+1)$

Exercise 2.5 Page: 48

Q1. Use suitable identities to find the following products:
(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]
We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$=x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=8 and b= −10]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$=x^2+(8-10)x-80$

$=x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $x=3x$, $a=4$ and $b= -5$]
We get,

$(3x+4)(3x-5) =(3x)^2+4+(-5)3x+4\times(-5)$

$=9x^2+3x(4-5)-20$

$=9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=3/2$]
We get,

$(y^2+3/2)(y^2-3/2) = (y^2)^2-(3/2)^2$

$=y^4-(9/4)$

Q2. Evaluate the following products without multiplying directly:
(i) 103 × 107

Solution:

$103\times107=(100+3)\times(100+7)$

Using identity, $[(x+a)(x+b)=x2+(a+b)x+ab$

Here, $x=100$

$a=3$

$b=7$

We get, $103\times107=(100+3)\times(100+7)$

$=(100)^2+(3+7)100+(3\times7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95\times96=(100-5)\times(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, $x=100$

$a=-5$

$b=-4$

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii) 104 × 96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

Q3. Factorize the following using appropriate identities:
(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

$=(3x+y)^2$

$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+12$

Using identity, $x^2 - 2xy + y^2= (x - y)^2$

Here, $x=2y$

$y=1$

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$

$=(2y-1)^2$

$=(2y-1)(2y-1)$

(iii) $x^2-y^{2/100}$

Solution:

$x^2 – y^{2/100 = x_2} – (y/10)^2$

Using identity, $x^2 – y^2 = (x – y)(x\ y)$

Here,

$x=x$

$y=y/10$

$x^2 – y^2/100 = x^2 – (y/10)^2$

$=(x – y/10)(x+y/10)$

Q4. Expand each of the following, using suitable identities:
(i) $(x+2y+4z)^2$
(ii) $(2x−y+z)^2$
(iii) $(−2x+3y+2z)^2$
(iv) $(3a – 7b – c)^2$
(v) $(–2x + 5y – 3z)^2$
(vi) $(a–b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2×x×2y)+(2×2y×4z)+(2×4z×x)$

$=x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2 =(2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$=4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2 =(-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$=4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= -7b$

$z= -c$

$(3a - 7b - c)^2 =(3a)^2+(-7b)^2+(-c)^2+(2\times3a \times-7b)+(2\times-7b \times-c)+(2\times-c \times3a)$

$=9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y= 5y$

$z= -3z$

$(-2x+5y-3z)^2 =(-2x)^2+(5y)^2+(-3z)^2+(2\times-2x \times 5y)+(2\times 5y \times-3z)+(2\times-3z \times-2x)$

$=4x^2 + 25y^2 + 9z^2 - 20xy-30yz+12zx$

(vi) $(1/4a - 1/2b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 1/4a$

$y= -1/2b$

$z = 1$

$(1/4a - 1/2b + 1)^2 = (1/4a)^2 + (-1/2b)^2 + (1)^2 + (2 \times 1/4a \times -1/2b) + (2 \times -1/2b \times 1) + (2 \times 1 \times 1/4a)$

$= 1/16a^2 + 1/4b^2 + 1^2 - 2/8ab - 2/2b + 2/4a$

$= 1/16a^2 + 1/4b^2 + 1 - 1/4ab - b + 1/2a$

Q5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$
(ii) $2x^2+y^2+8z^2-2xy+4yz-8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz = (2x)^2+(3y)^2+(-4z)^2+(2\times 2x\times 3y)+(2\times 3y\times -4z)+(2\times -4z\times 2x)$

$=(2x+3y-4z)^2$

$=(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz = (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times 2\sqrt{2}z)+(2\times 2\sqrt{2}\,2\sqrt{2}z\times-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

Q6. Write the following cubes in expanded form:
(i) $(2x+1)^3$
(ii) $(2a-3b)^3$
(iii) $(x+1)^3$
(iv) $(x-y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times 2x\times 1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(2a-3b)^3=(2a)^3-(3b)^3–(3\times 2a\times 3b)(2a–3b)$

$=8a^3–27b^3–18ab(2a–3b)$

$=8a^3–27b^3–36a^2b+54ab^2$

(iii) $(3/2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(3/2x+1)^3 =(3/2x)^3+1^3+(3×3/2x×1)(3/2x+1)$

$=27/8x^3+1+9/2x(3/2x+1)$

$=27/8x^3+1+27/4x^2+9/2x$

$=27/8x^3+27/4x^2+9/2x+1$

(iv) $(x−2/3y)^3$

Solution:

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(x−2/3y)^3 =(x)^3–(2/3y)^3–(3×x×2/3y)(x–2/3y)$

$=(x)^3–8/27y^3–2xy(x– 2/3y)$

$=(x)^3–8/27y^3–2x^2y+4/3xy^2$

Q7. Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(99)^3 = (100–1)^3$

$=(100)^3–1^3–(3×100×1)(100–1)$

$= 1000000 – 1 – 300(100 – 1)$

$= 1000000 – 1 – 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(100+2)^3 =(100)^3+2^3+(3×100×2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(998)^3 =(1000–2)^3$

$=(1000)^3–2^3–(3×1000×2)(1000–2)$

$= 1000000000 – 8 – 6000(1000 – 2)$

$= 1000000000 – 8- 6000000 + 12000$

$= 994011992$

Q8. Factorise each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 –1/216 −(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 =(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$=(2a+b)^3$

$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 =(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$=(2a–b)^3$

$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iii) $27 - 125a^3 - 135a + 225a^2$

Solution:

The expression, $27 - 125a^3 - 135a + 225a^2$ can be written as $3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$27-125a^3-135a+225a^2 = 3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$=(3-5a)^3$

$=(3-5a)(3-5a)(3-5a)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iv) $64a3-27b3-144a^2b+108ab^2$

Solution:

The expression, $64a^3-27b^3-144a^2b+108ab^2$ can be written as $(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3-27b^3-144a^2b+108ab^2 = (4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a-3b)^3$

$=(4a-3b)(4a-3b)(4a-3b)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(v) $27p^3 - 1/216 - 9/2p^2 + 1/4p$

Solution:

The expression, $27p^3 - 1/216 - 9/2p^2 + 1/4p$ can be written as $(3p)^3 - (1/6)^3 - 3(3p)^2(1/6) + 3(3p)(1/6)^2$

$27p^3 - 1/216 - 9/2p^2 + 1/4p = (3p)^3 - (1/6)^3 - 3(3p)^2(1/6) + 3(3p)(1/6)^2$

$= (3p - (1/6))^3$

$= (3p - (1/6))(3p - (1/6))(3p - (1/6))$

Q9. Verify:
(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Solutions:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We know that, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$

$\Rightarrow x^3 + y^3 = (x+y)[(x+y)^2 - 3xy]$

Taking(x+y) common $\Rightarrow x^3 + y^3 = (x+y)[(x^2 + y^2 + 2xy) - 3xy]$

$\Rightarrow x^3 + y^3 = (x+y)(x^2 + y^2 - xy)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)[(x-y)^2 + 3xy]$

Taking(x+y) common⇒$x^3-y^3=(x–y)[(x^2+y^2–2xy)+3xy]$

⇒$x^3+y^3=(x–y)(x^2+y^2+xy)$

Q10. Factorize each of the following:
(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3=(3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2–xy+y^2)$

$27y^3+125z^3=(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2–(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2–15yz+25z^2)$

(ii) $64m^3–343n^3$

The expression, $64m^3–343n^3$ can be written as $(4m)^3–(7n)^3$

$64m^3–343n^3= (4m)^3–(7n)^3$

We know that, $x^3–y^3=(x–y)(x^2+xy+y^2)$

$64m^3–343n^3=(4m)^3–(7n)^3$

$=(4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$=(4m-7n)(16m^2+28mn+49n^2)$

Q11. Factorise : $27x^3+y^3+z^3–9xyz$

Solution:

The expression $27x^3+y^3+z^3–9xyz$ can be written as $(3x)^3+y^3+z^3–3(3x)(y)(z)$

$27x^3+y^3+z^3–9xyz = (3x)^3+y^3+z^3–3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$

$27x^3+y^3+z^3–9xyz = (3x)^3+y^3+z^3–3(3x)(y)(z)$

$=(3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$

$=(3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

Q12. Verify that:

$x^3+y^3+z^3–3xyz=(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Solution:

We know that,

$x^3+y^3+z^3−3xyz=(x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz = \times(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$

$= (x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$

$= (x+y+z)[(x^2+y^2−2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$

$= (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Q13. If $x + y + z = 0$, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,

$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$

Now, according to the question, let $(x + y + z) = 0$,

then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =0$

$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

Q14. Without actually calculating the cubes, find the value of each of the following:
(i) $(−12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(−15)^3+(−13)^3$

(i) $(−12)^3+(7)^3+(5)^3$

Solution:

$(−12)^3+(7)^3+(5)^3$

Let a= −12

b= 7

c= 5

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, −12+7+5=0

$(−12)^3+(7)^3+(5)^3 = 3xyz$

=

=

(ii) $(28)^3+(−15)^3+(−13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a= 28

b= −15

c= −13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$(28)^3+(-15)^3+(-13)^3= 3xyz$

= 0+3(28)(−15)(−13)

=16380

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2–35a+12$
(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=2512=300

We get -15 and -20 as the numbers [-15+-20=-35 and -3-4=300]

$25a^2–35a+12 =25a^2–15a−20a+12$

=5a(5a–3)–4(5a–3)

=(5a–4)(5a–3)

Possible expression for length = $5a - 4$

Possible expression for breadth = $5a - 3$

(ii) Area : $35y^2+13y-12$

Using the splitting the middle term method,

We have to find a number whose sum= 13 and product=3512=420

We get -15 and 28 as the numbers [-15+28=-35 and -15=420]

$35y^2+13y-12 = 35y^2-15y+28y-12$

$=5y(7y-3)+4(7y-3)$

$=(5y+4)(7y-3)$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y - 3)$

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume : $3x^2-12x$
(ii) Volume : $12ky^2+8ky-20k$

Solution:

(i) Volume : $3x^2-12x$

$3x^2-12x$ can be written as $3x(x - 4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x – 4)$

(ii) Volume : $12ky^2+8ky –20k$

$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking 4k out of both the terms.

$12ky^2+8ky–20k =4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$

$=4k[y(3y+5)–1(3y+5)]$

$=4k(3y+5)(y–1)$

Possible expression for length = 4k

Possible expression for breadth = $(3y +5)$

Possible expression for height = $(y -1)$

Access Answers of Maths NCERT class 9 Chapter 2 – Polynomials

Class 9 Maths Chapter 2 Exercise 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2 – 3x + 7$

Solution:

The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation y^2 + can be written as $y^2 + y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression y^2 + is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation 3 + t can be written as $3t^{1/2} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,) is not a whole number. Hence, we can say that the expression 3 + t is **not** a polynomial in one variable.

(iv) y + 2/y

Solution:

The equation y + can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression y + is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1)x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1)x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\Pi/2\ x^2 + x$

Solution:

The equation $\Pi/2x^2 + x$ can be written as $(\Pi/2)\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is

, the coefficients of x^2 in $\Pi/2x^2 + x$ is $\Pi/2$.

(iv)$\sqrt{2}$x-1

Solution:

The equationx$\sqrt{2}$x-1 can be written as $0x^2 + \sqrt{2}x - 1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

, the coefficients of x^2 in$\sqrt{2}$x-1 is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

, the degree of $5t – \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3== 3x^0$

The power of the variable here is: 0

, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x – x^3$

Solution:

The highest power of $x – x^3$ is 3

, the degree is 3

Hence, $x – x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) $1 + x$

Solution:

The highest power of $1 + x$ is 1

, the degree is 1

Hence, 1 + x is a linear polynomial

(v) 3t

Solution:

The highest power of 3t is 1

, the degree is 1

Hence, 3t is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial $(x)=5x-4x^2+3$
(i) x= 0
(ii) x = – 1
(iii) x = 2

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When $x=0$

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When $x= -1$

$f(x)=5x-4x^2+3$

$f(-1)=5(-1) -4(-1)^2+3$

$=-5-4+3$

$=-6$

(iii) When $x=2$

$f(x)=5x-4x^2+3$

$f(2)=5(2) -4(2)^2+3$

$=10-16+3$

$=-3$

Q2. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2-(1)+1=1$

$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2-(0)^3=2$

$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$

$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1$, $x=-1/3$

Solution:

For, $x=-1/3$, $p(x)=3x+1$

$\therefore p(-1/3)=3(-1/3)+1=-1+1=0$

$\therefore -1/3$ is a zero of p(x).

(ii) $p(x)=5x-\pi$, $x=4/5$

Solution:

For, $x=4/5$ $p(x)=5x-\pi$

$\therefore p(4/5)=5(4/5)-\pi=4-\pi$

$\therefore$ 4/5is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of p(x).

(iv) $p(x)=(x+1)(x-2)$, $x= -1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) $p(x)=x^2$, $x=0$

Solution:

For, $x=0$ $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of $p(x)$.

(vi) $p(x)=lx+m, x=-m/t$

Solution:

For, $x=-m/t$; $p(x)=lx+m$

$\therefore p(-m/t)=l(-m/t)+m=-m+m=0$

$\therefore -m/t$is a zero of $p(x)$.

(vii) $p(x)=3x^2-1, x=-1/\sqrt{3}, 2/\sqrt{3}$,

Solution:

For, $x=-1/\sqrt{3},2/\sqrt{3}$,; $p(x)=3x^2-1$

$\therefore p(-1/\sqrt{3})=3(-1/\sqrt{3})^2-1=3(1/\sqrt{3})-1=1-1=0$

$\therefore p(2/\sqrt{3})=3(2/\sqrt{3})^2-1=3(4/3)-1=4-1=3\neq 0$

$\therefore --1/\sqrt{3}$ is a zero of $p(x)$ but $2/\sqrt{3}$ is not a zero of $p(x)$.

(viii) $p(x)=2x+1, x=1/2$

Solution:

For, $x=1/2$ $p(x)=2x+1$

$\therefore p(1/2)=2(1/2)+1=1+1=2\neq 0$

$\therefore$ 1/2 is not a zero of $p(x)$.

Q4. Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5

Solution:

$p(x)=x+5$

$\Rightarrow x+5=0$

$\Rightarrow x=-5$

$\therefore$-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

$p(x)=x-5$

$\Rightarrow x-5=0$

$\Rightarrow x=5$

$\therefore$5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-5/2$

$\therefore x=-5/2$ is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

p(x)=3x–2

⇒3x−2=0

⇒3x=2

⇒x=2/3

∴x=2/3 is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

p(x)=3x

⇒3x=0

⇒x=0

∴0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a0

Solution:

p(x)=ax

⇒ax=0

⇒x=0

∴x=0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

p(x)= cx + d

⇒ cx + d =0

⇒x=-d/c

∴ x=-d/c is a zero polynomial of the polynomial p(x).

Class 9 Maths Chapter 2 Exercise 2.3 Page: 40

Q1. Find the remainder when x^3+3x^2+3x+1 is divided by
(i) x+1

Solution:

$x+1=0$

$\Rightarrow x=-1$

∴Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) x−1/2

Solution:

$x-1/2=0$

$\Rightarrow x= 1/2$

∴Remainder:

$p(1/2)= (1/2)^3+3(1/2)^2+3()+1$

$=1/8+3/4+3/2+1$

$=27/8$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) x+π

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\ 5/2$

$\therefore$Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1=-125/8+75/4-15/2+1$

$=-27/8$

Q2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)= (a)^3 -a(a^2)+6(a)-a$

$=a^3-a^3+6a-a = 5a$

Q3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if $7+3x$ divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=-7/3$

$\therefore$Remainder:

$3(7/3)^3+7(7/3)= -343/9+(-49/3)$

$= -343+(-49)3/9$

$= -343-147/9$

$= -490/9\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let p(x)= x^3+x^2+x+1

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(−1)=(−1)^3+(−1)^2+(−1)+1$

$=−1+1−1+1$

$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let p(x)= $x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(−1)=(−1)^4+(−1)^3+(−1)^2+(−1)+1$

$=1−1+1−1+1$

$=1≠0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let p(x)= $x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

p(−1)=(−1)4+3(−1)3+3(−1)2+(−1)+1

$=1−3+3−1+1$

$=1≠0$

∴By factor theorem, x+1 is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 – x^2 – (2 +\sqrt{2})x +\sqrt{2}$

Solution:

Let $p(x)= x^3 – x^2 – (2 + \sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(−1)=(−1)^3–(−1)^2–(2+\sqrt{2})(−1)+\sqrt{2}$

$=−1−1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 – x^2 – (2 +\sqrt{2})x +\sqrt{2}$

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:
(i) $p(x)=2x^3+x^2–2x–1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2–2x–1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=−1$

∴Zero of g(x) is -1.

Now,

$p(−1)=2(−1)^3+(−1)^2–2(−1)–1$

$=−2+1+2−1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

$p(x)=x3+3\times2+3x+1$, $g(x) = x + 2$

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3–4x^2+x+6$, $g(x) = x – 3$

Solution:

$p(x)= x^3–4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow1+1+k=0$

$\Rightarrow2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow2(1)^2+k(1)+\sqrt{2} =0$

$\Rightarrow2+k+\sqrt{2}=0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 – 3x + k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2 – 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

⇒k= \frac{3}{2}23

Q4. Factorize:

(i) $12x^2 – 7x + 1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=112=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3-4=12]

$12x^2 – 7x + 1 = 12x^2 - 4x - 3x + 1$

=4x (3x-1)-1(3x-1)

= (4x-1)(3x-1)

(ii) $2x^2 + 7x + 3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2=6

We get 6 and 1 as the numbers [6+1=7 and 6=6]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$=2x(x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=6= -36

We get -4 and 9 as the numbers [-4+9=5 and -4=-36]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$

$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=3= -12

We get -4 and 3 as the numbers [-4+3=-1 and -4=-12]

$3x^2 – x – 4 = 3x^2–x–4$

$= 3x^2–4x+3x–4$

$= x(3x–4)+1(3x–4)$

$= (3x–4)(x+1)$

Q5. Factorize:
(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

$p(1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

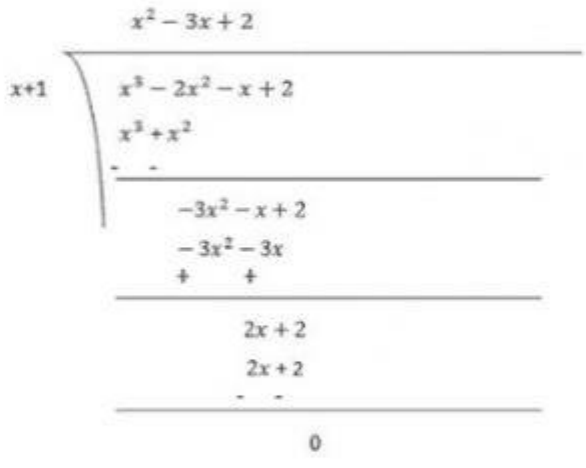
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−2+1+2$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x-5)$ is factor of $p(x)$

Now,

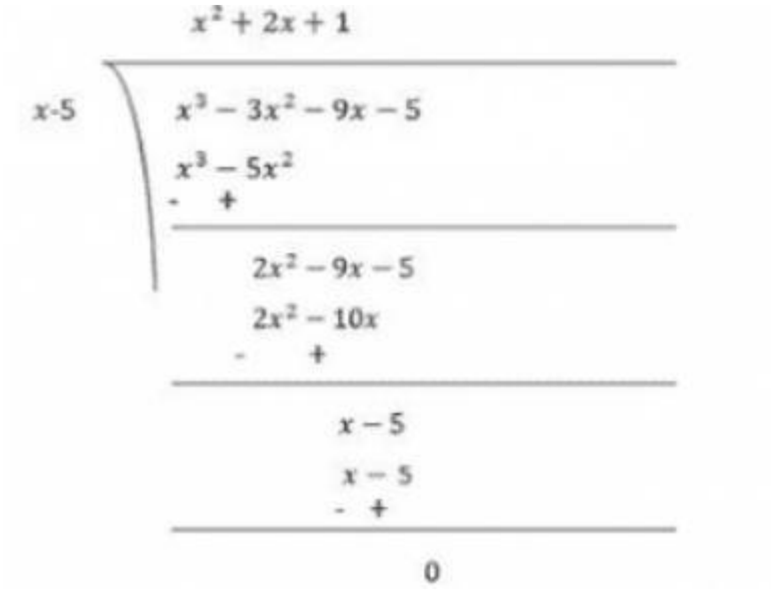
$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$

$= (x-5)(x(x+1)+1(x+1))$

$= (x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$p(x) = x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & -\quad - \\
\hline
 & 12x^2+32x+20 \\
 & 12x^2+12x \\
 & -\quad - \\
\hline
 & 20x+20 \\
 & 20x+20 \\
 & -\quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) =(x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1)= -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

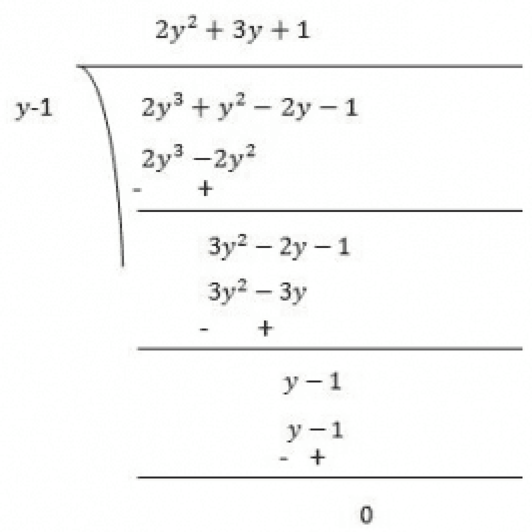
$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2–2(1)–1$

$=2+1−2$

$=0$

Therefore, (y-1) is the factor of p(y)



Now, Dividend = Divisor × Quotient + Remainder

$(y−1)(2y^2+3y+1) =(y−1)(2y^2+2y+y+1)$

$=(y−1)(2y(y+1)+1(y+1))$

$=(y−1)(2y+1)(y+1)$

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Q1. Use suitable identities to find the following products:
(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a) (x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]

We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a) (x + b) = x^2 + (a + b)x + ab$

[Here, $a=8$ and $b= -10$]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a) (x + b) = x^2 + (a + b)x + ab$

[Here, $x=3x$, $a=4$ and $b= -5$]
We get,

$(3x+4)(3x-5) = (3x)^2+4+(-5)3x+4\times(-5)$

$= 9x^2+3x(4-5)-20$

$= 9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x + y) (x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=3/2$]
We get,

$(y^2+3/2)(y^2–3/2) = (y^2)^2–(3/2)^2$

$=y^4–(9/4)$

Q2. Evaluate the following products without multiplying directly:
(i) 103 × 107

Solution:

$103×107=(100+3)×(100+7)$

Using identity, $[(x+a)(x+b)=x2+(a+b)x+ab$

Here, $x=100$

$a=3$

$b=7$

We get, $103×107=(100+3)×(100+7)$

$=(100)^2+(3+7)100+(3×7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95×96=(100-5)×(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, $x=100$

$a=-5$

$b=-4$

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii) 104×96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

Q3. Factorize the following using appropriate identities:
(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

$=(3x+y)^2$

$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+12$

Using identity, $x^2 - 2xy + y^2= (x - y)^2$

Here, $x=2y$

$y=1$

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$

$=(2y-1)^2$

$=(2y-1)(2y-1)$

(iii) $x^2-y^{2/100}$

Solution:

$x^2-y^{2/100\ =\ x_2}-(y/10)^2$

Using identity, $x^2 - y^2= (x - y)\ (x\ y)$

Here,

$x=x$

$y=y/10$

$x^2 - y^2/100= x^2-(y/10)^2$

$=(x-y/10)(x+y/10)$

Q4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii) $(-2x+3y+2z)^2$
(iv) $(3a - 7b - c)^2$
(v) $(-2x + 5y - 3z)^2$
(vi) $(a-b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$=x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2=(2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$=4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2=(-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$=4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= -7b$

$z= -c$

$(3a - 7b - c)^2=(3a)^2+(-7b)^2+(-c)^2+(2\times 3a \times -7b)+(2\times -7b \times -c)+(2\times -c \times 3a)$

$=9a^2 + 49b^2 + c^2- 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= –2x$

$y= 5y$

$z= – 3z$

$(–2x+5y–3z)^2 =(–2x)^2+(5y)^2+(–3z)^2+(2×–2x × 5y)+(2× 5y ×– 3z)+(2×–3z ×–2x)$

$=4x^2 + 25y^2 + 9z^2– 20xy–30yz+12zx$

(vi) $(1/4a – 1/2b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 1/4a$

$y= –1/2b$

$z= 1$

$(1/4a –1/2b +1)^2 =(1/4a)^2+(–1/2b)^2+(1)^2+(2×1/4a × –1/2b)+(2× –1/2b ×1)+(2×1×1/4a)$

$=1/16a^2+1/4b^2+1^2–2/8ab– 2/2b +2/4a$

$= 1/16a^2+1/4b^2+1–1/4ab – b +1/2a$

Q5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$
(ii) $2x^2+y^2+8z^2–2xy+4yz–8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy–24yz–16xz =(2x)^2+(3y)^2+(−4z)^2+(2×2x×3y)+(2×3y×−4z)+(2×−4z×2x)$

$=(2x+3y–4z)^2$

$=(2x+3y–4z)(2x+3y–4z)$

(ii) $2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz =(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2×-\sqrt{2}x×y)+(2×y×2\sqrt{2}z)+(2×2\sqrt{2}z×-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

Q6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$
(iii) $(x+1)^3$
(iv) $(x-y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times2x\times1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(2a-3b)^3=(2a)^3-(3b)^3–(3\times2a\times3b)(2a–3b)$

$=8a^3–27b^3–18ab(2a–3b)$

$=8a^3–27b^3–36a^2b+54ab^2$

(iii) $(3/2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(3/2x+1)^3 =(3/2x)^3+1^3+(3\times3/2x\times1)(3/2x+1)$

$=27/8x^3+1+9/2x(3/2x+1)$

$=27/8x^3+1+27/4x^2+9/2x$

$=27/8x^3+27/4x^2+9/2x+1$

(iv) $(x-2/3y)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(x-2/3y)^3 = (x)^3-(2/3y)^3-(3\times x\times 2/3y)(x-2/3y)$

$=(x)^3-8/27y^3-2xy(x-2/3y)$

$=(x)^3-8/27y^3-2x^2y+4/3xy^2$

Q7. Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(99)^3 = (100-1)^3$

$=(100)^3-1^3-(3\times 100\times 1)(100-1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(100+2)^3 = (100)^3 + 2^3 + (3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(998)^3 = (1000–2)^3$

$= (1000)^3 – 2^3 – (3\times1000\times2)(1000–2)$

$= 1000000000 – 8 – 6000(1000 – 2)$

$= 1000000000 – 8- 6000000 + 12000$

$= 994011992$

Q8. Factorise each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 –1/216 −(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 =(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$=(2a+b)^3$

$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 =(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$=(2a–b)^3$

$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iii) $27 – 125a^3 – 135a + 225a^2$

Solution:

The expression, $27 – 125a^3 – 135a + 225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$27–125a^3–135a+225a^2 = 3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$=(3–5a)^3$

$=(3–5a)(3–5a)(3–5a)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iv) 64a3–27b3–144a^{2}b+108ab^2

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2 = (4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a–3b)^3$

$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3 – 1/216 − 9/2p^2+ 1/4p$

Solution:

The expression, $27p^3 –1/216 − 9/2p^2+ 1/4p$ can be written as $(3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$27p^3 – 1/216 − 9/2p^2+ 1/4p = (3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$= (3p–(1/6))^3$

$= (3p–(1/6))(3p–(1/6))(3p–(1/6))$

Q9. Verify:
(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$
(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 =x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3–3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2–3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)–3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2–xy)$

(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

We know that,$(x–y)^3 =x^3–y^3–3xy(x–y)$

$\Rightarrow x^3-y^3=(x–y)^3+3xy(x–y)$

$\Rightarrow x^3-y^3=(x–y)[(x–y)^2+3xy]$

Taking(x+y) common$\Rightarrow x^3-y^3=(x–y)[(x^2+y^2–2xy)+3xy]$

$\Rightarrow x^3+y^3=(x–y)(x^2+y^2+xy)$

Q10. Factorize each of the following:
(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 = (3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2-xy+y^2)$

$27y^3+125z^3 = (3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3$

The expression, $64m^3-343n^3$ can be written as $(4m)^3-(7n)^3$

$64m^3-343n^3 = (4m)^3-(7n)^3$

We know that, $x^3-y^3=(x-y)(x^2+xy+y^2)$

$64m^3-343n^3 = (4m)^3-(7n)^3$

$=(4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$=(4m-7n)(16m^2+28mn+49n^2)$

Q11. Factorise : $27x^3+y^3+z^3-9xyz$

Solution:

The expression $27x^3+y^3+z^3-9xyz$ can be written as $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$

$=(3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$

$=(3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

Q12. Verify that:

$x^3+y^3+z^3–3xyz=(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Solution:

We know that,

$x^3+y^3+z^3−3xyz=(x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =\times(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$

$= (x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$

$= (x+y+z)[(x^2+y^2−2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$

$= (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Q13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,

$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$

Now, according to the question, let (x + y + z) = 0,

then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =0$

$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(-15)^3+(-13)^3$

(i) $(-12)^3+(7)^3+(5)^3$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let $a= -12$

$b= 7$

$c= 5$

We know that if $x + y + z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$(-12)^3+(7)^3+(5)^3 = 3xyz$

=

=

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let $a= 28$

$b= -15$

$c= -13$

We know that if $x + y + z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $x + y + z = 28 – 15 – 13 = 0$

$(28)^3+(-15)^3+(-13)^3= 3xyz$

$= 0+3(28)(-15)(-13)$

=16380

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2–35a+12$
(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=2512=300

We get -15 and -20 as the numbers [-15+-20=-35 and -3-4=300]

$25a^2–35a+12 =25a^2–15a-20a+12$

$=5a(5a–3)–4(5a–3)$

$=(5a–4)(5a–3)$

Possible expression for length = 5a – 4

Possible expression for breadth = 5a – 3

(ii) Area : $35y^2+13y–12$

Using the splitting the middle term method,

We have to find a number whose sum= 13 and product=3512=420

We get -15 and 28 as the numbers [-15+28=-35 and -15=420]

$35y^2+13y–12 =35y^2–15y+28y–12$

$=5y(7y–3)+4(7y–3)$

$=(5y+4)(7y–3)$

Possible expression for length $= (5y + 4)$

Possible expression for breadth $= (7y – 3)$

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:

(i) Volume : $3x^2–12x$

$3x^2–12x$ can be written as $3x(x – 4)$ by taking $3x$ out of both the terms.

Possible expression for length $= 3$

Possible expression for breadth $= x$

Possible expression for height $= (x – 4)$

(ii) Volume : $12ky^2+8ky –20k$

$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking $4k$ out of both the terms.

$12ky^2+8ky–20k =4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$

$=4k[y(3y+5)–1(3y+5)]$

$=4k(3y+5)(y–1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)

Access Answers of Maths NCERT class 9 Chapter 2 – Polynomials

Class 9 Maths Chapter 2 Exercise 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2 - 3x + 7$

Solution:

The equation $4x^2 - 3x + 7$ can be written as $4x^2 - 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation $y^2 +$ can be written as $y^2 + y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2 +$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation 3 + t can be written as $3t^{1/2} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,) is not a whole number. Hence, we can say that the expression 3 + t is **not** a polynomial in one variable.

(iv) y + 2/y

Solution:

The equation y + can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression y + is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1) x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1) x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\Pi/2\ x^2 +x$

Solution:

The equation $\Pi/2x^2 +x$ can be written as $(\Pi/2)\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is

, the coefficients of x^2 in $\Pi/2x^2+x$ is $\Pi/2$.

(iv)$\sqrt{2}x-1$

Solution:

The equationx$\sqrt{2}x-1$ can be written as $0x^2+\sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

, the coefficients of x^2 in$\sqrt{2}x-1$ is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

, the degree of $5t - \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, 3== $3x^0$

The power of the variable here is: 0

, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x - x^3$

Solution:

The highest power of $x - x^3$ is 3

, the degree is 3

Hence, $x - x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) $1 + x$

Solution:

The highest power of $1 + x$ is 1

, the degree is 1

Hence, $1 + x$ is a linear polynomial

(v) 3t

Solution:

The highest power of 3t is 1

, the degree is 1

Hence, 3t is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial
$(x)=5x-4x^2+3$
(i) $x= 0$
(ii) $x = -1$
(iii) $x = 2$

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When $x=0$

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When $x= -1$

$f(x)=5x-4x^2+3$

$f(-1)=5(-1) -4(-1)^2+3$

$=-5-4+3$

$=-6$

(iii) When $x=2$

$f(x)=5x-4x^2+3$

$f(2)=5(2)-4(2)^2+3$

$=10-16+3$

$=-3$

Q2. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2-(1)+1=1$

$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2-(0)^3=2$

$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$

$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1$, $x=-1/3$

Solution:

For, $x=-1/3$, $p(x)=3x+1$

$\therefore p(-1/3)=3(-1/3)+1=-1+1=0$

$\therefore -1/3$ is a zero of p(x).

(ii) $p(x)=5x-\pi$, $x=4/5$

Solution:

For, $x=4/5$ $p(x)=5x-\pi$

$\therefore p(4/5)=5(4/5)-\pi=4-\pi$

$\therefore$ 4/5is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of $p(x)$.

(iv) $p(x)=(x+1)(x–2)$, $x= -1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x–2)$

$\therefore p(-1)=(-1+1)(-1–2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2–2)=(3)(0)=0$

$\therefore -1,2$ are zeros of $p(x)$.

(v) $p(x)=x^2$, $x=0$

Solution:

For, $x=0$ $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of $p(x)$.

(vi) $p(x)=lx+m$, $x=-m/t$

Solution:

For, $x=-m/t$; $p(x)=lx+m$

$\therefore p(-m/t)=l(-m/t)+m=-m+m=0$

$\therefore -m/t$is a zero of $p(x)$.

(vii) $p(x)=3x^2-1, x=-1/\sqrt{3}, 2/\sqrt{3},$

Solution:

For, $x=-1/\sqrt{3}, 2/\sqrt{3},$; $p(x)=3x^2-1$

$\therefore p(-1/\sqrt{3})=3(-1/\sqrt{3})^2-1=3(1/\sqrt{3})-1=1-1=0$

$\therefore p(2/\sqrt{3})=3(2/\sqrt{3})^2-1=3(4/3)-1=4-1=3\neq0$

$\therefore --1/\sqrt{3}$ is a zero of p(x) but $2/\sqrt{3}$ is not a zero of p(x).

(viii) $p(x)=2x+1, x=1/2$

Solution:

For, $x=1/2$ $p(x)=2x+1$

$\therefore p(1/2)=2(1/2)+1=1+1=2\neq0$

$\therefore$ 1/2 is not a zero of p(x).

Q4. Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5

Solution:

$p(x)=x+5$

$\Rightarrow x+5=0$

$\Rightarrow x=-5$

$\therefore$-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

$p(x)=x-5$

$\Rightarrow x-5=0$

$\Rightarrow x=5$

$\therefore 5$ is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-5/2$

$\therefore x=-5/2$ is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

$p(x)=3x-2$

$\Rightarrow 3x-2=0$

$\Rightarrow 3x=2$

$\Rightarrow x=2/3$

$\therefore x=2/3$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

$p(x)=3x$

$\Rightarrow 3x=0$

$\Rightarrow x=0$

∴0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a0

Solution:

p(x)=ax

⇒ax=0

⇒x=0

∴x=0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

p(x)= cx + d

⇒ cx + d =0

⇒x=-d/c

∴ x=-d/c is a zero polynomial of the polynomial p(x).

Class 9 Maths Chapter 2 Exercise 2.3 Page: 40

Q1. Find the remainder when x^3+3x^2+3x+1 is divided by
(i) x+1

Solution:

x+1=0

⇒x=−1

∴Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) $x-1/2$

Solution:

$x-1/2=0$

$\Rightarrow x= 1/2$

$\therefore$Remainder:

$p(1/2)= (1/2)^3+3(1/2)^2+3()+1$

$=1/8+3/4+3/2+1$

$=27/8$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) $x+\pi$

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\ 5/2$

$\therefore$Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1=-125/8+75/4-15/2+1$

$=-27/8$

Q2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a = 5a$

Q3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if $7+3x$ divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=-7/3$

$\therefore$Remainder:

$3(7/3)^3+7(7/3)= -343/9+(-49/3)$

$= -343+(-49)3/9$

$= -343-147/9$

$= -490/9\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

$\therefore$By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq 0$

$\therefore$By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1\neq 0$

$\therefore$By factor theorem, x+1 is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

$\therefore$By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 +\sqrt{2})x +\sqrt{2}$

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:
(i) $p(x)=2x^3+x^2–2x–1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2–2x–1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2–2(-1)–1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3×2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

∴By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, g(x) = x – 3

Solution:

$p(x)= x^3-4x^2+x+6$, g(x) = x -3

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

∴Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

∴By factor theorem, g(x) is a factor of p(x).

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow (1)^2+(1)+k=0$

$\Rightarrow 1+1+k=0$

$\Rightarrow 2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2} =0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k = \sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

⇒k= \frac{3}{2}23

Q4. Factorize:

(i) $12x^2$–7x+1

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=112=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3-4=12]

$12x^2$–7x+1=$12x^2$-4x-3x+1

=4x (3x-1)-1(3x-1)

= (4x-1)(3x-1)

(ii) $2x^2$+7x+3

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2=6

We get 6 and 1 as the numbers [6+1=7 and 6=6]

$2x^2$+7x+3 =$2x^2$+6x+1x+3

=2x (x+3)+1(x+3)

= (2x+1)(x+3)

(iii) $6x^2$+5x-6

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=6= -36

We get -4 and 9 as the numbers [-4+9=5 and -4=-36]

$6x^2+5x-6=6x^2+9x-4x-6$

$=3x(2x+3)-2(2x+3)$

$=(2x+3)(3x-2)$

(iv) $3x^2-x-4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=3= -12

We get -4 and 3 as the numbers [-4+3=-1 and -4=-12]

$3x^2-x-4=3x^2-x-4$

$=3x^2-4x+3x-4$

$=x(3x-4)+1(3x-4)$

$=(3x-4)(x+1)$

Q5. Factorize:
(i) x^3-2x^2-x+2

Solution:

Let $p(x)=x^3-2x^2-x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

p(1) = 0

So, (x+1) is factor of p(x)

Now,

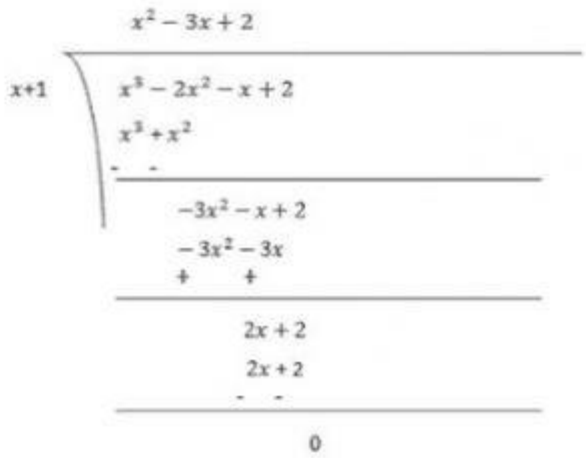
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−2+1+2$

$=0$

Therefore, (x+1) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

p(5) = 0

So, (x-5) is factor of p(x)

Now,

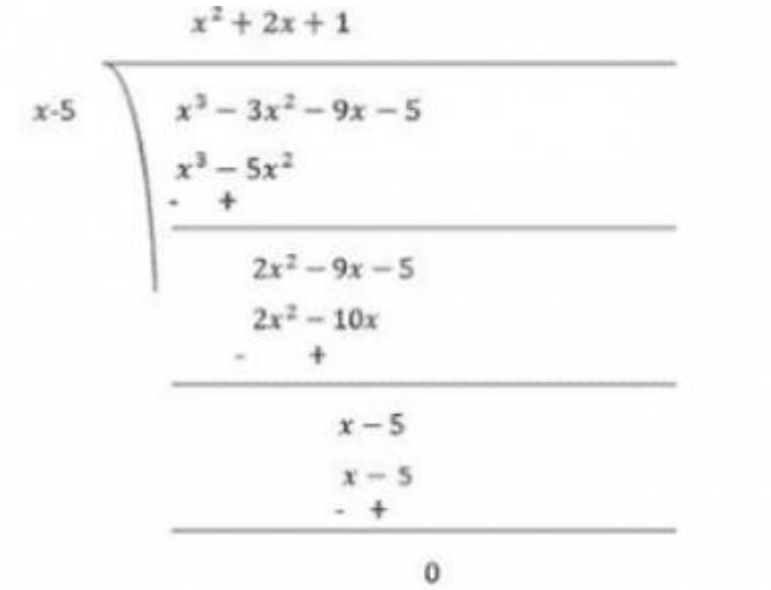
p(x) = $x^3–3x^2–9x–5$

p(5) = $(5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x−5)(x^2+2x+1) =(x−5)(x^2+x+x+1)$

$=(x−5)(x(x+1)+1(x+1))$

$=(x−5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let p(x) = $x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

p(-1) = 0

So, (x+1) is factor of p(x)

Now,

p(x) = $x^3+13x^2+32x+20$

p(-1) = $(-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, (x+1) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
\hline
x+1 & x^3+13x^2+32x+20 \\
 & x^3+x^2 \\
 & - \quad - \\
\hline
 & 12x^2+32x+20 \\
 & 12x^2+12x \\
 & - \quad - \\
\hline
 & 20x+20 \\
 & 20x+20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) =(x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2–2y–1$

Solution:

Let $p(y) = 2y^3+y^2–2y–1$

Factors = $2×(−1)= -2$ are $±1$ and $±2$

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

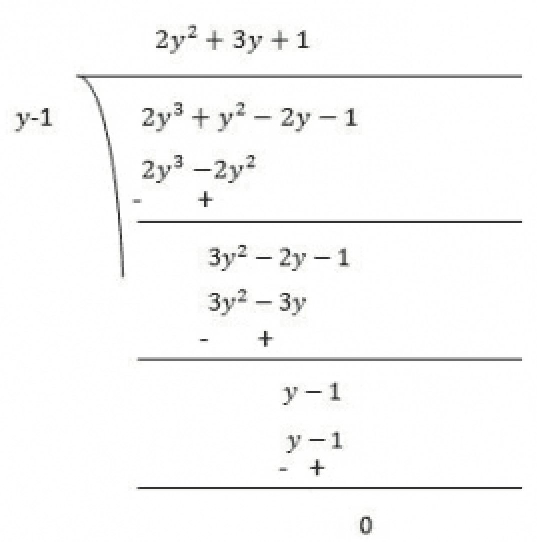
$p(y) = 2y^3+y^2–2y–1$

$p(1) = 2(1)^3+(1)^2–2(1)–1$

$=2+1−2$

$=0$

Therefore, $(y-1)$ is the factor of $p(y)$



Now, Dividend = Divisor × Quotient + Remainder

$(y−1)(2y^2+3y+1) =(y−1)(2y^2+2y+y+1)$

$=(y-1)(2y(y+1)+1(y+1))$

$=(y-1)(2y+1)(y+1)$

Exercise 2.5 Page: 48

Q1. Use suitable identities to find the following products:
(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]
We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a=8 and b= −10]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $x=3x$, $a=4$ and $b= -5$]
We get,

$(3x+4)(3x-5) =(3x)^2+4+(-5)3x+4\times(-5)$

$=9x^2+3x(4-5)-20$

$=9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=3/2$]
We get,

$(y^2+3/2)(y^2-3/2) = (y^2)^2-(3/2)^2$

$=y^4-(9/4)$

Q2. Evaluate the following products without multiplying directly:
(i) 103 × 107

Solution:

$103\times107=(100+3)\times(100+7)$

Using identity, $[(x+a)(x+b)=x2+(a+b)x+ab$

Here, $x=100$

$a=3$

$b=7$

We get, $103\times107=(100+3)\times(100+7)$

$=(100)^2+(3+7)100+(3\times7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95\times96=(100-5)\times(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, $x=100$

$a=-5$

$b=-4$

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii) 104 × 96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

Q3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

$=(3x+y)^2$

$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+12$

Using identity, $x^2 - 2xy + y^2= (x - y)^2$

Here, $x=2y$

$y=1$

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$

$=(2y-1)^2$

$=(2y-1)(2y-1)$

(iii) $x^2-y^{2/100}$

Solution:

$x^2 – y^{2/100 = x_2} –(y/10)^2$

Using identity, $x^2 – y^2 = (x – y) (x y)$

Here,

$x=x$

$y=y/10$

$x^2 – y^2/100= x^2–(y/10)^2$

$=(x–y/10)(x+y/10)$

Q4. Expand each of the following, using suitable identities:
(i) $(x+2y+4z)^2$
(ii) $(2x−y+z)^2$
(iii) $(−2x+3y+2z)^2$
(iv) $(3a – 7b – c)^2$
(v) $(–2x + 5y – 3z)^2$
(vi) $(a–b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 =x^2+(2y)^2+(4z)^2+(2×x×2y)+(2×2y×4z)+(2×4z×x)$

$=x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2 =(2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$=4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2 =(-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$=4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 3a$

$y= -7b$

$z= -c$

$(3a - 7b - c)^2 =(3a)^2+(-7b)^2+(-c)^2+(2\times3a \times-7b)+(2\times-7b \times-c)+(2\times-c \times3a)$

$=9a^2 + 49b^2 + c^2- 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= -2x$

$y= 5y$

$z= -3z$

$(-2x+5y-3z)^2 =(-2x)^2+(5y)^2+(-3z)^2+(2\times-2x \times 5y)+(2\times 5y \times-3z)+(2\times-3z \times-2x)$

$=4x^2 + 25y^2 + 9z^2- 20xy-30yz+12zx$

(vi) $(1/4a - 1/2b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 1/4a$

$y= -1/2b$

$z = 1$

$(1/4a - 1/2b + 1)^2 = (1/4a)^2 + (-1/2b)^2 + (1)^2 + (2\times 1/4a \times -1/2b) + (2\times -1/2b \times 1) + (2\times 1\times 1/4a)$

$= 1/16a^2 + 1/4b^2 + 1^2 - 2/8ab - 2/2b + 2/4a$

$= 1/16a^2 + 1/4b^2 + 1 - 1/4ab - b + 1/2a$

Q5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$
(ii) $2x^2+y^2+8z^2-2xy+4yz-8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz = (2x)^2+(3y)^2+(-4z)^2+(2\times 2x\times 3y)+(2\times 3y\times -4z)+(2\times -4z\times 2x)$

$=(2x+3y-4z)^2$

$=(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz = (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2\times-\sqrt{2}x\times y)+(2\times y\times 2\sqrt{2}z)+(2\times 2\sqrt{}2\sqrt{2}z\times-\sqrt{2}x)$

$=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(-\sqrt{2}x+y+2\sqrt{2}z)(-\sqrt{2}x+y+2\sqrt{2}z)$

Q6. Write the following cubes in expanded form:
(i) $(2x+1)^3$
(ii) $(2a-3b)^3$
(iii) $(x+1)^3$
(iv) $(x-y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times 2x\times 1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(2a-3b)^3=(2a)^3-(3b)^3–(3\times 2a\times 3b)(2a–3b)$

$=8a^3–27b^3–18ab(2a–3b)$

$=8a^3–27b^3–36a^2b+54ab^2$

(iii) $(3/2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(3/2x+1)^3 =(3/2x)^3+1^3+(3×3/2x×1)(3/2x+1)$

$=27/8x^3+1+9/2x(3/2x+1)$

$=27/8x^3+1+27/4x^2+9/2x$

$=27/8x^3+27/4x^2+9/2x+1$

(iv) $(x−2/3y)^3$

Solution:

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(x−2/3y)^3 =(x)^3–(2/3y)^3–(3×x×2/3y)(x–2/3y)$

$=(x)^3–8/27y^3–2xy(x– 2/3y)$

$=(x)^3–8/27y^3–2x^2y+4/3xy^2$

Q7. Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(99)^3 = (100–1)^3$

$=(100)^3–1^3–(3×100×1)(100–1)$

$= 1000000 – 1 – 300(100 – 1)$

$= 1000000 – 1 – 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(100+2)^3 =(100)^3+2^3+(3×100×2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(998)^3 =(1000–2)^3$

$=(1000)^3–2^3–(3×1000×2)(1000–2)$

$= 1000000000 – 8 – 6000(1000 – 2)$

$= 1000000000 – 8- 6000000 + 12000$

$= 994011992$

Q8. Factorise each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 –1/216 −(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 =(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$=(2a+b)^3$

$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 =(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$=(2a–b)^3$

$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iii) $27 - 125a^3 - 135a + 225a^2$

Solution:

The expression, $27 - 125a^3 - 135a + 225a^2$ can be written as $3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$27-125a^3-135a+225a^2 = 3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$=(3-5a)^3$

$=(3-5a)(3-5a)(3-5a)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(iv) $64a3-27b3-144a^2b+108ab^2$

Solution:

The expression, $64a^3-27b^3-144a^2b+108ab^2$ can be written as $(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3-27b^3-144a^2b+108ab^2 = (4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a-3b)^3$

$=(4a-3b)(4a-3b)(4a-3b)$

Here, the identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ is used.

(v) $27p^3 - 1/216 - 9/2p^2 + 1/4p$

Solution:

The expression, $27p^3 - 1/216 - 9/2p^2 + 1/4p$ can be written as $(3p)^3 - (1/6)^3 - 3(3p)^2(1/6) + 3(3p)(1/6)^2$

$27p^3 - 1/216 - 9/2p^2 + 1/4p = (3p)^3 - (1/6)^3 - 3(3p)^2(1/6) + 3(3p)(1/6)^2$

$= (3p - (1/6))^3$

$= (3p - (1/6))(3p - (1/6))(3p - (1/6))$

Q9. Verify:
(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Solutions:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We know that, $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$

$\Rightarrow x^3 + y^3 = (x+y)[(x+y)^2 - 3xy]$

Taking$(x+y)$ common $\Rightarrow x^3 + y^3 = (x+y)[(x^2 + y^2 + 2xy) - 3xy]$

$\Rightarrow x^3 + y^3 = (x+y)(x^2 + y^2 - xy)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that,$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$\Rightarrow x^3 - y^3 = (x-y)[(x-y)^2 + 3xy]$

Taking(x+y) common⇒$x^3-y^3=(x–y)[(x^2+y^2–2xy)+3xy]$

⇒$x^3+y^3=(x–y)(x^2+y^2+xy)$

Q10. Factorize each of the following:
(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3 =(3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2–xy+y^2)$

$27y^3+125z^3 =(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2–(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2–15yz+25z^2)$

(ii) $64m^3–343n^3$

The expression, $64m^3–343n^3$ can be written as $(4m)^3–(7n)^3$

$64m^3–343n^3 = (4m)^3–(7n)^3$

We know that, $x^3–y^3=(x–y)(x^2+xy+y^2)$

$64m^3–343n^3 =(4m)^3–(7n)^3$

$=(4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$=(4m-7n)(16m^2+28mn+49n^2)$

Q11. Factorise : $27x^3+y^3+z^3–9xyz$

Solution:

The expression $27x^3+y^3+z^3–9xyz$ can be written as $(3x)^3+y^3+z^3–3(3x)(y)(z)$

$27x^3+y^3+z^3–9xyz \; =(3x)^3+y^3+z^3–3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$

$27x^3+y^3+z^3–9xyz \; =(3x)^3+y^3+z^3–3(3x)(y)(z)$

$=(3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$

$=(3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

Q12. Verify that:

$x^3+y^3+z^3–3xyz=(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Solution:

We know that,

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =\times(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$

$= (x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$

$= (x+y+z)[(x^2+y^2-2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$

$= (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Q13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,

$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$

Now, according to the question, let $(x + y + z) = 0$,

then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =0$

$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

Q14. Without actually calculating the cubes, find the value of each of the following:
(i) $(−12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(−15)^3+(−13)^3$

(i) $(−12)^3+(7)^3+(5)^3$

Solution:

$(−12)^3+(7)^3+(5)^3$

Let $a= −12$

$b= 7$

$c= 5$

We know that if $x + y + z = 0$, then $x^3+y^3+z^3=3xyz$.

Here, $−12+7+5=0$

$(−12)^3+(7)^3+(5)^3 = 3xyz$

=

=

(ii) $(28)^3+(−15)^3+(−13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a= 28

b= −15

c= −13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$(28)^3+(-15)^3+(-13)^3= 3xyz$

= 0+3(28)(−15)(−13)

=16380

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2–35a+12$
(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=2512=300

We get -15 and -20 as the numbers [-15+-20=-35 and -3-4=300]

$25a^2–35a+12 =25a^2–15a−20a+12$

=5a(5a–3)–4(5a–3)

=(5a–4)(5a–3)

Possible expression for length = $5a - 4$

Possible expression for breadth = $5a - 3$

(ii) Area : $35y^2+13y-12$

Using the splitting the middle term method,

We have to find a number whose sum= 13 and product=3512=420

We get -15 and 28 as the numbers [-15+28=-35 and -15=420]

$35y^2+13y-12 = 35y^2-15y+28y-12$

$=5y(7y-3)+4(7y-3)$

$=(5y+4)(7y-3)$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y - 3)$

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume : $3x^2-12x$
(ii) Volume : $12ky^2+8ky-20k$

Solution:

(i) Volume : $3x^2-12x$

$3x^2-12x$ can be written as $3x(x - 4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x – 4)$

(ii) Volume : $12ky^2+8ky –20k$

$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking 4k out of both the terms.

$12ky^2+8ky–20k =4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$

$=4k[y(3y+5)–1(3y+5)]$

$=4k(3y+5)(y–1)$

Possible expression for length = $4k$

Possible expression for breadth = $(3y +5)$

Possible expression for height = $(y -1)$

Access Answers of Maths NCERT class 9 Chapter 2 – Polynomials

Class 9 Maths Chapter 2 Exercise 2.1

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2 – 3x + 7$

Solution:

The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation y^2 + can be written as $y^2 + y^0$

Since *y* is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression y^2 + is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation 3 + t can be written as $3t^{1/2} + \sqrt{2}t$

Though, *t* is the only variable in the given equation, the powers of *t* (i.e.,) is not a whole number. Hence, we can say that the expression 3 + t is **not** a polynomial in one variable.

(iv) y + 2/y

Solution:

The equation y + can be written as $y+2y^{-1}$

Though, *y* is the only variable in the given equation, the powers of *y* (i.e.,-1) is not a whole number. Hence, we can say that the expression y + is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:
(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1) x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 – x^2 + x^3$

Solution:

The equation $2 – x^2 + x^3$ can be written as $2 + (–1) x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

, the coefficients of x^2 in $2 – x^2 + x^3$ is -1.

(iii) $\Pi/2\ x^2 + x$

Solution:

The equation $\Pi/2x^2 + x$ can be written as $(\Pi/2)\ x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is

, the coefficients of x^2 in $\Pi/2x^2 + x$ is $\Pi/2$.

(iv)$\sqrt{2}$x-1

Solution:

The equationx$\sqrt{2}$x-1 can be written as $0x^2 + \sqrt{2}x - 1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

, the coefficients of x^2 in$\sqrt{2}$x-1 is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg., $4x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $5x^3 + 4x^2 + 7x= 5x^3 + 4x^2 + 7x^1$

The powers of the variable x are: 3, 2, 1

, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 – y^2$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4 – y^2$,

The power of the variable y is: 2

, the degree of $4 – y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t – \sqrt{7}$

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t – \sqrt{7}$,

The power of the variable y is: 1

, the degree of $5t – \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, $3 == 3x^0$

The power of the variable here is: 0

, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Solution:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x – x^3$

Solution:

The highest power of $x – x^3$ is 3

, the degree is 3

Hence, $x – x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) $1 + x$

Solution:

The highest power of $1 + x$ is 1

, the degree is 1

Hence, 1 + x is a linear polynomial

(v) 3t

Solution:

The highest power of 3t is 1

, the degree is 1

Hence, 3t is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial
$(x)=5x-4x^2+3$
(i) x= 0
(ii) x = – 1
(iii) x = 2

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When $x=0$

$f(0)=5(0)+4(0)^2+3$

$=3$

(ii) When $x= -1$

$f(x)=5x-4x^2+3$

$f(-1)=5(-1) -4(-1)^2+3$

$=-5–4+3$

$=-6$

(iii) When $x=2$

$f(x)=5x-4x^2+3$

$f(2)=5(2) -4(2)^2+3$

$=10–16+3$

$=-3$

Q2. Find p(0), p(1) and p(2) for each of the following polynomials:
(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2–y+1$

$\therefore p(0)=(0)^2-(0)+1=1$

$p(1)=(1)^2–(1)+1=1$

$p(2)=(2)^2–(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$

$\therefore p(0)=2+0+2(0)^2-(0)^3=2$

$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$

$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) $p(x)=x^3$

Solution:

$p(x)=x^3$

$\therefore p(0)=(0)^3=0$

$p(1)=(1)^3=1$

$p(2)=(2)^3=8$

(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$

$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$

$p(1)=(1-1)(1+1)=0(2)=0$

$p(2)=(2-1)(2+1)=1(3)=3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1$, $x=-1/3$

Solution:

For, $x=-1/3$, $p(x)=3x+1$

$\therefore p(-1/3)=3(-1/3)+1=-1+1=0$

$\therefore -1/3$ is a zero of p(x).

(ii) $p(x)=5x-\pi$, $x=4/5$

Solution:

For, $x=4/5$ $p(x)=5x-\pi$

$\therefore p(4/5)=5(4/5)-\pi=4-\pi$

$\therefore$ 4/5is not a zero of p(x).

(iii) $p(x)=x^2-1$, $x=1, -1$

Solution:

For, $x=1, -1$;

$p(x)=x^2-1$

$\therefore p(1)=1^2-1=1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of p(x).

(iv) $p(x)=(x+1)(x-2)$, $x=-1, 2$

Solution:

For, $x=-1,2$;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) $p(x)=x^2$, $x=0$

Solution:

For, $x=0$ $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of $p(x)$.

(vi) $p(x)=lx+m, x=-m/t$

Solution:

For, $x=-m/t$; $p(x)=lx+m$

$\therefore p(-m/t)=l(-m/t)+m=-m+m=0$

$\therefore -m/t$is a zero of $p(x)$.

(vii) $p(x)=3x^2-1, x=-1/\sqrt{3}, 2/\sqrt{3},$

Solution:

For, $x=-1/\sqrt{3}, 2/\sqrt{3},$; $p(x)=3x^2-1$

$\therefore p(-1/\sqrt{3})=3(-1/\sqrt{3})^2-1=3(1/\sqrt{3})-1=1-1=0$

$\therefore p(2/\sqrt{3})=3(2/\sqrt{3})^2-1=3(4/3)-1=4-1=3\neq 0$

$\therefore --1/\sqrt{3}$ is a zero of $p(x)$ but $2/\sqrt{3}$ is not a zero of $p(x)$.

(viii) $p(x)=2x+1, x=1/2$

Solution:

For, $x=1/2$ $p(x)=2x+1$

$\therefore p(1/2)=2(1/2)+1=1+1=2\neq 0$

$\therefore$ 1/2 is not a zero of $p(x)$.

Q4. Find the zero of the polynomial in each of the following cases:
(i) p(x) = x + 5

Solution:

$p(x)=x+5$

$\Rightarrow x+5=0$

$\Rightarrow x=-5$

$\therefore$-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

$p(x)=x-5$

$\Rightarrow x-5=0$

$\Rightarrow x=5$

$\therefore$5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-5/2$

$\therefore x=-5/2$ is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

$p(x)=3x–2$

$\Rightarrow 3x-2=0$

$\Rightarrow 3x=2$

$\Rightarrow x=2/3$

$\therefore x=2/3$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Solution:

$p(x)=3x$

$\Rightarrow 3x=0$

$\Rightarrow x=0$

$\therefore 0$ is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a0

Solution:

$p(x)=ax$

$\Rightarrow ax=0$

$\Rightarrow x=0$

$\therefore x=0$ is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c ≠ 0, c, d are real numbers.

Solution:

$p(x)= cx + d$

$\Rightarrow cx + d =0$

$\Rightarrow x = -d/c$

$\therefore$ $x = -d/c$ is a zero polynomial of the polynomial p(x).

Class 9 Maths Chapter 2 Exercise 2.3 Page: 40

Q1. Find the remainder when x^3+3x^2+3x+1 is divided by
(i) x+1

Solution:

$x+1=0$

$\Rightarrow x=-1$

$\therefore$Remainder:

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

$=0$

(ii) x−1/2

Solution:

$x-1/2=0$

$\Rightarrow x= 1/2$

$\therefore$Remainder:

$p(1/2\ )= (1/2)^3+3(1/2)^2+3()+1$

$=1/8+3/4+3/2+1$

$=27/8$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) x+π

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\ 5/2$

$\therefore$Remainder:

$(-5/2)^3+3(-5/2)^2+3(-5/2)+1=-125/8+75/4-15/2+1$

$=-27/8$

Q2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)= (a)^3 -a(a^2)+6(a)-a$

$=a^3-a^3+6a-a = 5a$

Q3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if $7+3x$ divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=-7/3$

$\therefore$Remainder:

$3(7/3)^3+7(7/3)= -343/9+(-49/3)$

$= -343+(-49)3/9$

$= -343-147/9$

$= -490/9\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1\neq0$

∴By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$

$=1-3+3-1+1$

$=1\neq0$

∴By factor theorem, x+1 is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 – x^2 – (2 +\sqrt{2})x +\sqrt{2}$

Solution:

Let $p(x)= x^3 – x^2 – (2 + \sqrt{2})x +\sqrt{2}$

The zero of x+1 is -1.

$p(−1)=(−1)^3–(−1)^2–(2+\sqrt{2})(−1)+\sqrt{2}$

$=−1−1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 – x^2 – (2 +\sqrt{2})x +\sqrt{2}$

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:
(i) $p(x)=2x^3+x^2–2x–1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2–2x–1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=−1$

∴Zero of g(x) is -1.

Now,

$p(−1)=2(−1)^3+(−1)^2–2(−1)–1$

$=−2+1+2−1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

$p(x)=x3+3\times2+3x+1$, $g(x) = x + 2$

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x - 3$

Solution:

$p(x)= x^3-4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

Q3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:
(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow1+1+k=0$

$\Rightarrow2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow2(1)^2+k(1)+\sqrt{2} =0$

$\Rightarrow2+k+\sqrt{2}=0$

$\Rightarrow k = -(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 – 3x + k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2 – 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

⇒k= \frac{3}{2}23

Q4. Factorize:

(i) $12x^2 – 7x + 1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-7 and product=112=12

We get -3 and -4 as the numbers [-3+-4=-7 and -3-4=12]

$12x^2 – 7x + 1 = 12x^2 - 4x - 3x + 1$

$= 4x\ (3x-1) - 1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2 + 7x + 3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2=6

We get 6 and 1 as the numbers [6+1=7 and 6=6]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$=2x(x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=6= -36

We get -4 and 9 as the numbers [-4+9=5 and -4=-36]

$6x^2+5x-6=6x^2+ 9x – 4x – 6$

$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $3x^2 – x – 4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=3= -12

We get -4 and 3 as the numbers [-4+3=-1 and -4=-12]

$3x^2 – x – 4 = 3x^2–x–4$

$=3x^2–4x+3x–4$

$=x(3x–4)+1(3x–4)$

$=(3x–4)(x+1)$

Q5. Factorize:
(i) $x^3–2x^2–x+2$

Solution:

Let $p(x)=x^3–2x^2–x+2$

Factors of 2 are ± 1 and $\pm$ 2

By trial method, we find that

$p(1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

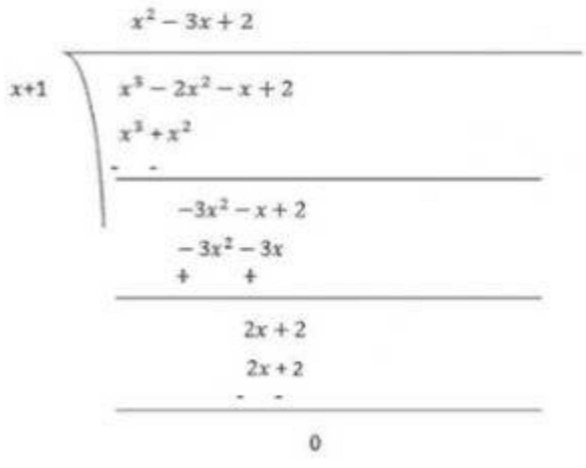
$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−2+1+2$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x-5)$ is factor of $p(x)$

Now,

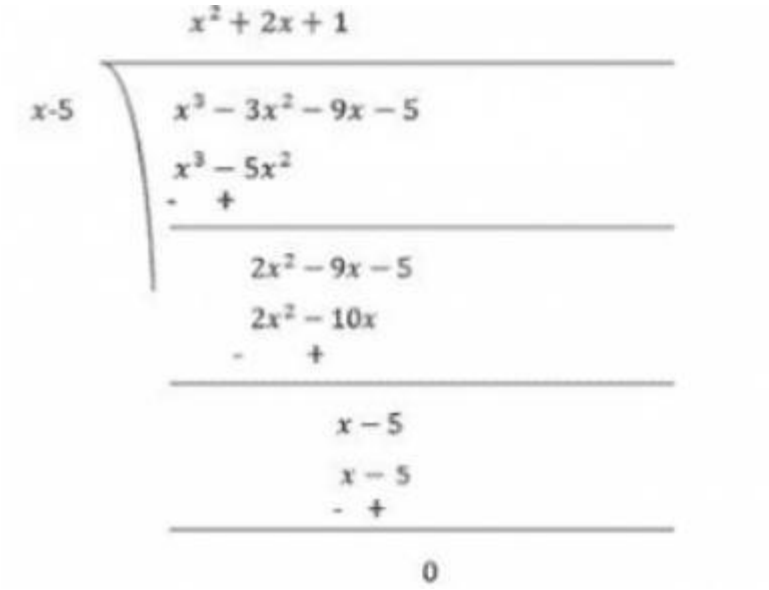
$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$

$= (x-5)(x(x+1)+1(x+1))$

$= (x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, (x+1) is factor of p(x)

Now,

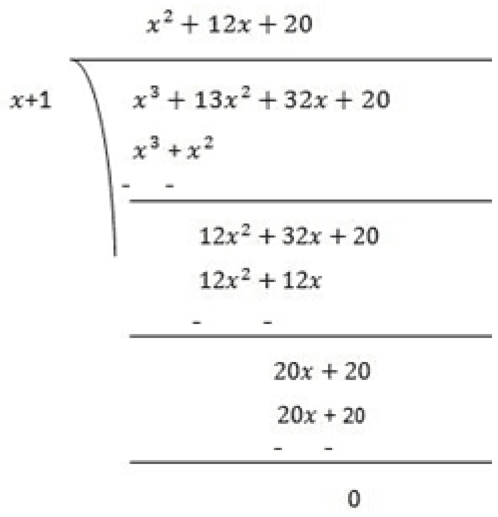
$p(x) = x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1) = -2$ are ± 1 and ± 2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

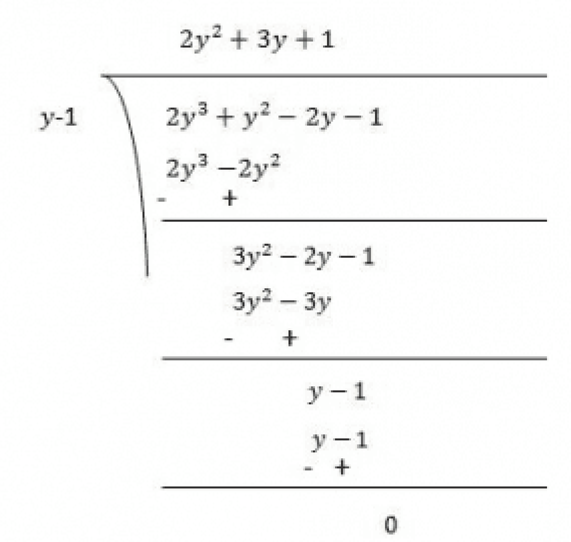
$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2–2(1)–1$

$=2+1−2$

$=0$

Therefore, (y-1) is the factor of p(y)



Now, Dividend = Divisor × Quotient + Remainder

$(y−1)(2y^2+3y+1) =(y−1)(2y^2+2y+y+1)$

$=(y−1)(2y(y+1)+1(y+1))$

$=(y−1)(2y+1)(y+1)$

Exercise 2.5 Page: 48

Q1. Use suitable identities to find the following products:
(i) (x + 4) (x + 10)

Solution:

Using the identity, $(x + a) (x + b) = x^2 + (a + b)x + ab$

[Here, a=4 and b=10]

We get,

$(x+4)(x+10) = x^2+(4+10)x+(4\times10)$

$= x^2+14x+40$

(ii) (x + 8) (x – 10)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $a=8$ and $b= -10$]
We get,

$(x+8)(x-10) = x^2+(8+(-10))x+(8\times(-10))$

$= x^2+(8-10)x-80$

$= x^2-2x-80$

(iii) (3x + 4) (3x – 5)

Solution:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, $x=3x$, $a=4$ and $b= -5$]
We get,

$(3x+4)(3x-5) = (3x)^2+4+(-5)3x+4\times(-5)$

$= 9x^2+3x(4-5)-20$

$= 9x^2-3x-20$

(iv) $(y^2+3/2)(y^2-3/2)$

Solution:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x=y^2$ and $y=3/2$]
We get,

$(y^2+3/2)(y^2–3/2) = (y^2)^2–(3/2)^2$

$=y^4–(9/4)$

Q2. Evaluate the following products without multiplying directly:
(i) 103 × 107

Solution:

$103×107=(100+3)×(100+7)$

Using identity, $[(x+a)(x+b)=x2+(a+b)x+ab$

Here, $x=100$

$a=3$

$b=7$

We get, $103×107=(100+3)×(100+7)$

$=(100)^2+(3+7)100+(3×7))$

$=10000+1000+21$

$=11021$

(ii) 95 × 96

Solution:

$95×96=(100-5)×(100-4)$

Using identity, $[(x-a)(x-b)=x^2+(a+b)x+ab$

Here, $x=100$

$a=-5$

$b=-4$

We get, $95\times96=(100-5)\times(100-4)$

$=(100)^2+100(-5+(-4))+(-5\times-4)$

$=10000-900+20$

$=9120$

(iii) 104×96

Solution:

$104\times96=(100+4)\times(100–4)$

Using identity, $[(a+b)(a-b)= a^2-b^2]$

Here, $a=100$

$b=4$

We get, $104\times96=(100+4)\times(100–4)$

$=(100)^2–(4)^2$

$=10000–16$

$=9984$

Q3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

Solution:

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

Using identity, $x^2 + 2xy + y^2= (x + y)^2$

Here, $x=3x$

$y=y$

$9x^2+6xy+y^2=(3x)^2+(2\times3x\times y)+y^2$

$=(3x+y)^2$

$=(3x+y)(3x+y)$

(ii) $4y^2-4y+1$

Solution:

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+12$

Using identity, $x^2 - 2xy + y^2= (x - y)^2$

Here, $x=2y$

$y=1$

$4y^2-4y+1=(2y)^2-(2\times2y\times1)+1^2$

$=(2y-1)^2$

$=(2y-1)(2y-1)$

(iii) $x^2-y^{2/100}$

Solution:

$x^2-y^{2/100} = x_2-(y/10)^2$

Using identity, $x^2 - y^2= (x - y) (x y)$

Here,

$x=x$

$y=y/10$

$x^2 - y^2/100= x^2-(y/10)^2$

$=(x-y/10)(x+y/10)$

Q4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$
(ii) $(2x-y+z)^2$
(iii) $(-2x+3y+2z)^2$
(iv) $(3a - 7b - c)^2$
(v) $(-2x + 5y - 3z)^2$
(vi) $(a-b+1)^2$

Solutions:

(i) $(x+2y+4z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=x$

$y=2y$

$z=4z$

$(x+2y+4z)^2 = x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$

$=x^2+4y^2+16z^2+4xy+16yz+8xz$

(ii) $(2x-y+z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x=2x$

$y=-y$

$z=z$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

(iii) $(-2x+3y+2z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x$

$y=3y$

$z=2z$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a - 7b - c)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = 3a$

$y = -7b$

$z = -c$

$(3a - 7b - c)^2 = (3a)^2+(-7b)^2+(-c)^2+(2\times 3a \times -7b)+(2\times -7b \times -c)+(2\times -c \times 3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab+14bc-6ca$

(v) $(-2x + 5y - 3z)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= –2x$

$y= 5y$

$z= – 3z$

$(–2x+5y–3z)^2 =(–2x)^2+(5y)^2+(–3z)^2+(2×–2x × 5y)+(2× 5y ×– 3z)+(2×–3z ×–2x)$

$=4x^2 + 25y^2 + 9z^2– 20xy–30yz+12zx$

(vi) $(1/4a – 1/2b+1)^2$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x= 1/4a$

$y= –1/2b$

$z= 1$

$(1/4a –1/2b +1)^2 =(1/4a)^2+(–1/2b)^2+(1)^2+(2×1/4a × –1/2b)+(2× –1/2b ×1)+(2×1×1/4a)$

$=1/16a^2+1/4b^2+1^2–2/8ab– 2/2b +2/4a$

$= 1/16a^2+1/4b^2+1–1/4ab – b +1/2a$

Q5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$
(ii) $2x^2+y^2+8z^2–2xy+4yz–8xz$

Solutions:

(i) $4x^2+9y^2+16z^2+12xy–24yz–16xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2+9y^2+16z^2+12xy–24yz–16xz =(2x)^2+(3y)^2+(−4z)^2+(2×2x×3y)+(2×3y×−4z)+(2×−4z×2x)$

$=(2x+3y–4z)^2$

$=(2x+3y–4z)(2x+3y–4z)$

(ii) $2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz$

Solution:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2+y^2+8z^2–2\sqrt{2}xy+4\sqrt{2}yz–8xz =(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2×−\sqrt{2}x×y)+(2×y×2\sqrt{2}z)+(2×2\sqrt{2}z×−\sqrt{2}x)$

$=(−\sqrt{2}x+y+2\sqrt{2}z)^2$

$=(−\sqrt{2}x+y+2\sqrt{2}z)(−\sqrt{2}x+y+2\sqrt{2}z)$

Q6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$
(iii) $(x+1)^3$
(iv) $(x-y)^3$

Solutions:

(i) $(2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(2x+1)^3=(2x)^3+1^3+(3\times2x\times1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Solution:

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(2a-3b)^3=(2a)^3-(3b)^3–(3\times2a\times3b)(2a–3b)$

$=8a^3–27b^3–18ab(2a–3b)$

$=8a^3–27b^3–36a^2b+54ab^2$

(iii) $(3/2x+1)^3$

Solution:

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(3/2x+1)^3 =(3/2x)^3+1^3+(3\times3/2x\times1)(3/2x+1)$

$=27/8x^3+1+9/2x(3/2x+1)$

$=27/8x^3+1+27/4x^2+9/2x$

$=27/8x^3+27/4x^2+9/2x+1$

(iv) $(x-2/3y)^3$

Solution:

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(x-2/3y)^3 = (x)^3-(2/3y)^3-(3\times x\times 2/3y)(x-2/3y)$

$=(x)^3-8/27y^3-2xy(x- 2/3y)$

$=(x)^3-8/27y^3-2x^2y+4/3xy^2$

Q7. Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Solutions:

(i) $(99)^3$

Solution:

We can write 99 as 100–1

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(99)^3 = (100-1)^3$

$=(100)^3-1^3-(3\times 100\times 1)(100-1)$

$= 1000000 - 1 - 300(100 - 1)$

$= 1000000 - 1 - 30000 + 300$

$= 970299$

(ii) $(102)^3$

Solution:

We can write 102 as 100+2

Using identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

$(100+2)^3 =(100)^3+2^3+(3\times100\times2)(100+2)$

$= 1000000 + 8 + 600(100 + 2)$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solution:

We can write 99 as 1000–2

Using identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$

$(998)^3 =(1000–2)^3$

$=(1000)^3–2^3–(3\times1000\times2)(1000–2)$

$= 1000000000 – 8 – 6000(1000 – 2)$

$= 1000000000 – 8- 6000000 + 12000$

$= 994011992$

Q8. Factorise each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$
(ii) $8a^3–b^3–12a^2b+6ab^2$
(iii) $27 – 125a^3 – 135a + 225a^2$
(iv) $64a^3–27b^3–144a^2b+108ab^2$
(v) $27p^3 –1/216 −(9/2)p^2+(1/4)p$

Solutions:

(i) $8a^3+b^3+12a^2b+6ab^2$

Solution:

The expression, $8a^3+b^3+12a^2b+6ab^2$ can be written as $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$8a^3+b^3+12a^2b+6ab^2 = (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$=(2a+b)^3$

$=(2a+b)(2a+b)(2a+b)$

Here, the identity, $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ is used.

(ii) $8a^3–b^3–12a^2b+6ab^2$

Solution:

The expression, $8a^3–b^3−12a^2b+6ab^2$ can be written as $(2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$8a^3–b^3−12a^2b+6ab^2 = (2a)^3–b^3–3(2a)^2b+3(2a)(b)^2$

$=(2a–b)^3$

$=(2a–b)(2a–b)(2a–b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iii) $27 – 125a^3 – 135a + 225a^2$

Solution:

The expression, $27 – 125a^3 – 135a + 225a^2$ can be written as $3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$27–125a^3–135a+225a^2 = 3^3–(5a)^3–3(3)^2(5a)+3(3)(5a)^2$

$=(3–5a)^3$

$=(3–5a)(3–5a)(3–5a)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(iv) 64a3–27b3–144a²b+108ab²

Solution:

The expression, $64a^3–27b^3–144a^2b+108ab^2$ can be written as $(4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$64a^3–27b^3–144a^2b+108ab^2 = (4a)^3–(3b)^3–3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a–3b)^3$

$=(4a–3b)(4a–3b)(4a–3b)$

Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.

(v) $27p^3$ – 1/216 − 9/2p²+ 1/4p

Solution:

The expression, $27p^3 –1/216 − 9/2p^2+ 1/4p$ can be written as $(3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$27p^3 – 1/216 − 9/2p^2+ 1/4p = (3p)^3–(1/6)^3–3(3p)^2(1/6)+3(3p)(1/6)^2$

$= (3p–(1/6))^3$

$= (3p–(1/6))(3p–(1/6))(3p–(1/6))$

Q9. Verify:
(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$
(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

Solutions:

(i) $x^3+y^3=(x+y)(x^2–xy+y^2)$

We know that, $(x+y)^3 = x^3+y^3+3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)^3–3xy(x+y)$

$\Rightarrow x^3+y^3=(x+y)[(x+y)^2–3xy]$

Taking(x+y) common $\Rightarrow x^3+y^3=(x+y)[(x^2+y^2+2xy)–3xy]$

$\Rightarrow x^3+y^3=(x+y)(x^2+y^2–xy)$

(ii) $x^3–y^3=(x–y)(x^2+xy+y^2)$

We know that,$(x–y)^3 = x^3–y^3–3xy(x–y)$

$\Rightarrow x^3-y^3=(x–y)^3+3xy(x–y)$

$\Rightarrow x^3-y^3=(x–y)[(x–y)^2+3xy]$

Taking(x+y) common$\Rightarrow x^3-y^3=(x–y)[(x^2+y^2–2xy)+3xy]$

$\Rightarrow x^3+y^3=(x–y)(x^2+y^2+xy)$

Q10. Factorize each of the following:
(i) $27y^3+125z^3$
(ii) $64m^3–343n^3$

Solutions:

(i) $27y^3+125z^3$

The expression, $27y^3+125z^3$ can be written as $(3y)^3+(5z)^3$

$27y^3+125z^3=(3y)^3+(5z)^3$

We know that, $x^3+y^3=(x+y)(x^2–xy+y^2)$

$27y^3+125z^3=(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2–(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2–15yz+25z^2)$

(ii) $64m^3–343n^3$

The expression, $64m^3–343n^3$ can be written as $(4m)^3–(7n)^3$

$64m^3–343n^3 = (4m)^3–(7n)^3$

We know that, $x^3–y^3=(x–y)(x^2+xy+y^2)$

$64m^3–343n^3=(4m)^3–(7n)^3$

$=(4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$=(4m-7n)(16m^2+28mn+49n^2)$

Q11. Factorise : $27x^3+y^3+z^3–9xyz$

Solution:

The expression $27x^3+y^3+z^3–9xyz$ can be written as $(3x)^3+y^3+z^3–3(3x)(y)(z)$

$27x^3+y^3+z^3–9xyz \ =(3x)^3+y^3+z^3–3(3x)(y)(z)$

We know that, $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$

$27x^3+y^3+z^3–9xyz \ =(3x)^3+y^3+z^3–3(3x)(y)(z)$

$=(3x+y+z)(3x)^2+y^2+z^2–3xy–yz–3xz$

$=(3x+y+z)(9x^2+y^2+z^2–3xy–yz–3xz)$

Q12. Verify that:

$x^3+y^3+z^3–3xyz=(x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Solution:

We know that,

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =\times(x+y+z)[2(x^2+y^2+z^2–xy–yz–xz)]$

$= (x+y+z)(2x^2+2y^2+2z^2–2xy–2yz–2xz)$

$= (x+y+z)[(x^2+y^2-2xy)+(y^2+z^2–2yz)+(x^2+z^2–2xz)]$

$= (x+y+z)[(x–y)^2+(y–z)^2+(z–x)^2]$

Q13. If x + y + z = 0, show that $x^3+y^3+z^3=3xyz$.

Solution:

We know that,

$x^3+y^3+z^3=3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$

Now, according to the question, let $(x + y + z) = 0$,

then, $x^3+y^3+z^3=3xyz =(0)(x^2+y^2+z^2–xy–yz–xz)$

$\Rightarrow x^3+y^3+z^3–3xyz =0$

$\Rightarrow x^3+y^3+z^3 =3xyz$

Hence Proved

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(-15)^3+(-13)^3$

(i) $(-12)^3+(7)^3+(5)^3$

Solution:

$(-12)^3+(7)^3+(5)^3$

Let a= −12

b= 7

c= 5

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, $-12+7+5=0$

$(-12)^3+(7)^3+(5)^3 = 3xyz$

=

=

(ii) $(28)^3+(-15)^3+(-13)^3$

Solution:

$(28)^3+(-15)^3+(-13)^3$

Let a= 28

b= −15

c= −13

We know that if x + y + z = 0, then $x^3+y^3+z^3=3xyz$.

Here, x + y + z = 28 – 15 – 13 = 0

$(28)^3+(-15)^3+(-13)^3= 3xyz$

$= 0+3(28)(-15)(-13)$

=16380

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2–35a+12$
(ii) Area : $35y^2+13y–12$

Solution:

(i) Area : $25a^2–35a+12$

Using the splitting the middle term method,

We have to find a number whose sum= -35 and product=2512=300

We get -15 and -20 as the numbers [-15+-20=-35 and -3-4=300]

$25a^2–35a+12 =25a^2–15a−20a+12$

$=5a(5a–3)–4(5a–3)$

$=(5a–4)(5a–3)$

Possible expression for length = 5a – 4

Possible expression for breadth = 5a – 3

(ii) Area : $35y^2+13y–12$

Using the splitting the middle term method,

We have to find a number whose sum= 13 and product=3512=420

We get -15 and 28 as the numbers [-15+28=-35 and -15=420]

$35y^2+13y–12 =35y^2–15y+28y–12$

$=5y(7y–3)+4(7y–3)$

$=(5y+4)(7y–3)$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y – 3)$

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume : $3x^2–12x$
(ii) Volume : $12ky^2+8ky–20k$

Solution:

(i) Volume : $3x^2–12x$

$3x^2–12x$ can be written as $3x(x – 4)$ by taking $3x$ out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = $(x – 4)$

(ii) Volume : $12ky^2+8ky –20k$

$12ky^2+8ky –20k$ can be written as $4k(3y^2+2y–5)$ by taking $4k$ out of both the terms.

$12ky^2+8ky–20k =4k(3y^2+2y–5)$

[Here, $3y^2+2y–5$ can be written as $3y^2+5y–3y–5$ using splitting the middle term method.]
$=4k(3y^2+5y–3y–5)$

$=4k[y(3y+5)–1(3y+5)]$

$=4k(3y+5)(y–1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)