RD SHARMA Solutions for Class 12-science Maths Chapter 28 - Straight line in space

Chapter 28 - Straight line in space Exercise Ex. 28.1

Question 1

Find the vector and Cartesian equations of the line through the point (5, 2, -4) and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.

Solution 1

Vector equation of a line

is $\vec{r}=\vec{a}+\lambda\vec{b}$

The Cartesian equation of a line is

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{a_2}=\frac{x-x_3}{a_3}$$

Using the above formula,

Vector equation of the line,

$\vec{r}=(5\hat{i}+2\hat{j}-4\hat{k})+\lambda(3\hat{i}+2\hat{j}-8\hat{k})$

The Cartesian equation of the line

$$\frac{x-5}{3}=\frac{y-2}{2}=\frac{z+4}{-8}$$

Question 2

Find the vector equation of the line passing through the points (-1, 0, 2) and (3, 4, 6).

Solution 2

The direction ratios of the line are

$(3+1, 4-0, 6-2)=(4,4,4)$

Since the line passes through $(-1,0,2)$

The vector equation of the line,

$\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow \vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow \vec{r}=(-\vec{i}+0\vec{j}+2\vec{k})+\lambda(4\vec{i}+4\vec{j}+4\vec{k})$

$\therefore$ The vector equation of the line,

$\vec{r}=(-\vec{i}+0\vec{j}+2\vec{k})+\lambda(4\vec{i}+4\vec{j}+4\vec{k})$

Question 3

Find the vector equation of a line which is parallel to the vector $2\hat{i}-\hat{j}+3\hat{k}$ and which passes through the point $(5,-2,4)$, Also, reduce it to cartesian form.

Solution 3

We know that, vector equation of line passing through a fixed point $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar}$$

Here, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

So, equation of required line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, so

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (5 + 2\lambda)\hat{i} + (-2 - \lambda)\hat{j} + (4 + 3\lambda)\hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$x = 5 + 2\lambda,\ y = -2 - \lambda,\ z = 4 + 3\lambda$$

$$\Rightarrow \quad \frac{x-5}{2} = \lambda,\ \frac{y+2}{-0} = \lambda,\ \frac{z-4}{3} = \lambda$$

Cortesian form of equation of the line is,

$$\frac{x-5}{2} = \frac{y+2}{-0} = \frac{z-4}{3}$$

Question 4

A line passes through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and is in the direction of $3\hat{i} + 4\hat{j} - 5\hat{k}$. Find equations of the line in vector and cartesian form.

Solution 4

We know that, equation of line passing through a vector $\vec{a}$ and parallel to a vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}, \text{ where } \lambda \text{ is scalar,}$$

Here, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

Required equation of line is,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$x\hat{i} + y\hat{j} + z\hat{k} = (2 + 3\lambda)\hat{i} + (-3 + 4\lambda)\hat{j} + (4 - 5\lambda)\hat{k}$$

On equating coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$\Rightarrow \quad 2 + 3\lambda = x,\ -3 + 4\lambda = y,\ 4 - 5\lambda = z$

$\Rightarrow \quad \dfrac{x-2}{3} = \lambda,\ \dfrac{y+3}{4} = \lambda,\ \dfrac{z-4}{-5} = z$

So, cartesian form of equation of the line is

$$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$$

Question 5

$ABCD$ is a parallelogram. The position vectors of the points A, B and C are respectively, $4\hat{i} + 5\hat{j} - 10\hat{k}$, $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-\hat{i} + 2\hat{j} + \hat{k}$. Find the vector equation of the line BD. Also, reduce it to cartesian form.

Solution 5

$ABCD$ is a parallelogram.

$\Rightarrow$ AC and BD bisect each other at point O (say).

Position vector of point $O = \dfrac{\vec{a}+\vec{c}}{2}$

$$= \frac{\left(4\hat{i}+5\hat{j}-10\hat{k}\right)+\left(-\hat{i}+2\hat{j}+\hat{k}\right)}{2}$$

$$= \frac{3\hat{i}+7\hat{j}-9\hat{k}}{2}$$

Let position vector of point O and B are represented by $\vec{o}$ and $\vec{b}$.

Equation of the line BD is the line passing through O and B is given by

$$\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right) \qquad \left[\begin{array}{l}\text{Since equation of the line passing through}\\ \text{two points } \vec{a} \text{ and } \vec{b}\end{array}\right]$$

$$\vec{r}=\vec{b}+\lambda\left(\vec{o}-\vec{b}\right)$$

$$=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(\frac{3\hat{i}+7\hat{j}-9\hat{k}}{2}-2\hat{i}-3\hat{j}+4\hat{k}\right)$$

$$\vec{r}=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}-9\hat{k}-4\hat{i}+6\hat{j}-8\hat{k}\right)$$

$$\vec{r}=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)+\lambda\left(-\hat{i}+13\hat{j}-17\hat{k}\right)$$

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)=\left(2-\lambda\right)\hat{i}+\left(-3+13\lambda\right)\hat{j}+\left(4-17\lambda\right)\hat{k}$$

Equation the coefficients of $\hat{i},\hat{j},\hat{k}$, so

$\Rightarrow$ $x=2-\lambda,\ y=-3-13\lambda,\ z=4-17\lambda$

$\Rightarrow$ $\dfrac{x-2}{-1}=\lambda,\ \dfrac{y+3}{13}=\lambda,\ \dfrac{z-4}{-17}=\lambda$

So equation of the line BD in cortesian form,

$$\frac{x-2}{-1}=\frac{y+3}{13}=\frac{z-4}{-17}$$

Question 6

Find the vector form as well as in cartesian form, the equation of the line passing through the points $A\left(1,2,-1\right)$ and $B\left(2,1,1\right)$.

Solution 6

We know that, equation of line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = A(1, 2, -1)$
$(x_2, y_2, z_2) = B(2, 1, 1)$

Using equation (i), equation of line AB,

$$\frac{x - 1}{2 - 1} = \frac{y - 2}{1 - 2} = \frac{z + 1}{1 + 1}$$

$$\frac{x - 1}{1} = \frac{y - 2}{-1} = \frac{z + 1}{2} = \lambda \text{ (say)}$$

$$x = \lambda + 1,\ y = -\lambda + 2,\ z = 2\lambda - 1$$

Vector form of equation of line AB is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 1)\hat{i} + (-\lambda + 2)\hat{j} + (2\lambda - 1)\hat{k}$$

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

Question 7

Find the vector equation for the line which passes through the point $(1, 2, 3)$ and parallel to the vector $\hat{i} - 2\hat{j} + 3\hat{k}$. Reduce the corresponding equation in cartesian form.

Solution 7

We know that vector equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is given by,

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, required vector equation of line is,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$$

Now,

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (1 + \lambda)\hat{i} + (2 - 2\lambda)\hat{j} + (3 + 3\lambda)\hat{k}$$

Equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$\Rightarrow \quad x = 1 + \lambda,\ y = 2 - 2\lambda,\ z = 3 + 3\lambda$

$\Rightarrow \quad x - 1 = \lambda,\ \dfrac{y - 2}{2} = \lambda,\ \dfrac{z - 3}{3} = \lambda$

So, required equation of line is cortesian form,

$$\frac{x - 1}{1} = \frac{y - 2}{-2} = \frac{z - 3}{3}$$

Question 8

Find the vector equation of a line passing through $(2, -1, 1)$ and parallel to the line whose equations are $\dfrac{x - 3}{2} = \dfrac{y + 1}{7} = \dfrac{z - 2}{-3}$.

Solution 8

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (2, -1, 1)$ and

Given line $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$ is prallel to required line.

$\Rightarrow \quad a = 2\mu,\ b = 7\mu,\ c = -3\mu$

So, equation of required line using equation (i),

$$\frac{x-2}{2\mu} = \frac{y+1}{7\mu} = \frac{z-1}{-3\mu}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = 2\lambda + 2,\ y = 7\lambda - 1,\ z = -3\lambda + 1$$

So, $x\hat{i} + y\hat{j} + z\hat{k} = (2\lambda + 2)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 1)\hat{k}$

$$\vec{r} = \left(2\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 7\hat{j} - 3\hat{k}\right)$$

Question 9

The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

Solution 9

The Cartesian equation of the line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} \qquad \ldots (1)$$

The given line passes through the point (5, −4, 6). The position vector of this point is
$\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$

Also, the direction ratios of the given line are 3, 7, and 2.

This means that the line is in the direction of vector, $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

It is known that the line through position vector $\vec{a}$ and in the direction of the vector $\vec{b}$ is given by the equation, $\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$$\Rightarrow \vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} + 2\hat{k}\right)$$

This is the required equation of the given line in vector form.

Question 10

Find the cartesian equation of a line passing through $(1,-1,2)$ and parallel to the line whose equations are

$$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$$

Also, reduce the equation obtained in vector form.

Solution 10

We know that, equation of a line passing through a point (x_1, y_1, z_1) and having direction ratios proportional to a, b, c is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1)=(1,-1,2)$ and

Given line $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$ is prallel to required line, so

$$\Rightarrow \quad a=\mu,\ b=2\mu,\ c=-2\mu$$

So, equation of required line using equation (i) is,

$$\frac{x-1}{\mu}=\frac{y+1}{2\mu}=\frac{z-2}{-2\mu}$$

$$\Rightarrow \quad \frac{x-1}{1}=\frac{y+1}{2}=\frac{z-2}{-2}=\lambda \text{ (say)}$$

$$x=\lambda+1,\ y=2\lambda-1,\ z=-2\lambda+2$$

So, $x\hat{i}+y\hat{j}+z\hat{k}=(\lambda+1)\hat{i}+(2\lambda-1)\hat{j}+(-2\lambda+2)\hat{k}$

$$\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}-2\hat{k}\right)$$

Question 11

Find the direction cosines of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Also, reduce it to vector form.

Solution 11

Given, line is,

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$

$$\Rightarrow \quad \frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=\lambda \ (\text{say})$$

$$x=-2\lambda+4,\ y=6\lambda,\ z=-3\lambda+1$$

So, $x\hat{i}+y\hat{j}+z\hat{k}=(-2\lambda+4)\hat{i}+(6\lambda)\hat{j}+(-3\lambda+1)\hat{k}$

$$\vec{r}=\left(4\hat{i}+\hat{k}\right)+\lambda\left(-2\hat{i}+6\hat{j}-3\hat{k}\right)$$

Direction ratios of the line are $=-2,\ 6,\ -3$

Direction cosines of the line are,

$$\frac{a}{\sqrt{a^2+b^2+c^2}},\ \frac{b}{\sqrt{a^2+b^2+c^2}},\ \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \quad \frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}},\ \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}},\ \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$$

$$\Rightarrow \quad \frac{-2}{7},\ \frac{6}{7},\ \frac{-3}{7}$$

Question 12

The cartesian equations of a line are $x=ay+b$, $z=cy+d$. Find its direction ratios and reduce it to vector form.

Solution 12

$$x = ay + b,$$
$$z = cy + d$$
$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} = \lambda(say)$$

So DR's of line are (a, 1, c)

From above equation, we can write

$$x = a\lambda + b$$
$$y = \lambda$$
$$z = c\lambda + d$$

So vector equation of line is

$$x\hat{i} + y\hat{j} + z\hat{k} = (b\hat{i} + d\hat{k}) + \lambda\left(a\hat{i} + \hat{j} + c\hat{k}\right)$$

Question 13

Find the vector equation of a line passing through the point with position vector $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$. Also, find the cartesian equivalent of this equation.

Solution 13

We know that, equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is,

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$

and, $\vec{b} =$ line joining $\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

$$= \left(2\hat{i} + \hat{j} + 2\hat{k}\right) - \left(\hat{i} - \hat{j} + 4\hat{k}\right)$$

$$= 2\hat{i} - \hat{i} + \hat{j} + \hat{j} + 2\hat{k} - 4\hat{k}$$

$$= \hat{i} + 2\hat{j} - 2\hat{k}$$

Equation of the line is

$$\vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$$

For cortesion form of equation put $x\hat{i} + y\hat{j} + z\hat{k}$,

$$x\hat{i} + y\hat{j} + z\hat{k} = (1 + \lambda)\hat{i} + (-2 + 2\lambda)\hat{j} + (-3 - 2\lambda)\hat{k}$$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$, so

$$x = 1 + \lambda,\ y = -2 + 2\lambda,\ z = -3 - 2\lambda$$

$$\Rightarrow \quad \frac{x-1}{1} = \lambda,\ \frac{y+2}{2} = \lambda,\ \frac{z+3}{-2} = \lambda$$

So, $\dfrac{x-1}{1} = \dfrac{y+2}{2} = \dfrac{z+3}{-2}$

Question 14

Find the points on the line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2}$ at a distance of 5 units from the point $P(1,3,3)$.

Solution 14

Given, line is $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2} = \lambda$ (say)

General point Q on line is $(3\lambda - 2,\ 2\lambda - 1,\ 2\lambda + 3)$

Distance of point P from $Q = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$PQ = \sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2}$$

$$\Rightarrow \quad (5)^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2$$

$$\Rightarrow \quad 25 = 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2$$

$$\Rightarrow \quad 17\lambda^2 - 34\lambda = 0$$

$$\Rightarrow \quad 17\lambda(\lambda - 2) = 0$$

$$\Rightarrow \quad \lambda = 0 \text{ or } 2$$

So, points on the line are $(3(0) - 2,\ 2(0) - 1,\ 2(0) + 3)$

$$(3(2) - 2,\ 2(2) - 1,\ 2(2) + 3)$$

$$= (-2,\ -1,\ 3),\quad (4, 3, 7)$$

Question 15

Show that the points whose position vectors are $-2\hat{i} + 3\hat{j}, \hat{i} + 2\hat{j} + 3\hat{k},$ and $7\hat{i} - \hat{k}$ are collinear.

Solution 15

Let the given points are A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively, so

$$\vec{a} = -2\hat{i} + 3\hat{j},\ \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{c} = 7\hat{i} - \hat{k}$$

We know that, equation of a line passing through $\vec{a}$ and $\vec{b}$ are,

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$= (-2\hat{i} + 3\hat{j}) + \lambda\left((\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j})\right)$$

$$= (-2\hat{i} + 3\hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k} + 2\hat{i} - 3\hat{j})$$

$$\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} + 3\hat{k}) \qquad ---\text{(i)}$$

If A, B, C are collinear then $\vec{c}$ must satisfy equation (i),

$$7\hat{i} - \hat{k} = (-2 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + (3\lambda)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$,

$$-2 + 3\lambda = 7 \qquad \Rightarrow \lambda = 3$$

$$3 - \lambda = 0 \qquad \Rightarrow \lambda = 3$$

$$3\lambda = -1 \qquad \Rightarrow \lambda = -\frac{1}{3}$$

Since, value of λ are not equal, so,

Given points are not collinear.

Question 16

Find the cartesian and vector equations of a line which passes through the point $(1, 2, 3)$ and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

Solution 16

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Question 17

The cartesian equations of a line are $3x + 1 = 6y - 2 = 1 - z$. Find the fixed point through which it passes, its direction ratios and also its vector equation.

Solution 17

Given equation of line is,

$$3x + 1 = 6y - 2 = 1 - z$$

Dividing all by 6,

$$\frac{3x+1}{6} = \frac{6y-2}{6} = \frac{1-z}{6}$$

$$\Rightarrow \quad \frac{3x}{6} + \frac{1}{6} = \frac{6y}{6} - \frac{2}{6} = \frac{1}{6} - \frac{z}{6}$$

$$\Rightarrow \quad \frac{1}{2}x + \frac{1}{6} = y - \frac{1}{3} = -\frac{z}{6} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{2}\left(x + \frac{1}{3}\right) = 1\left(y - \frac{1}{3}\right) = +\frac{1}{6}(z-1)$$

$$\Rightarrow \quad \frac{x + \frac{1}{3}}{2} = \frac{y - \frac{1}{3}}{1} = \frac{z-1}{-6} = \lambda \text{ (say)} \qquad ---(i)$$

Comparing it with equation of line passing through (x_1, y_1, z_1) and direction ratios $a, b, c,$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow \quad (x_1, y_1, z_1) = \left(-\frac{1}{3}, \frac{1}{3}, 1\right)$$

$$a = 2,\ b = 1,\ -6$$

So, direction ratios of the line are $= 2, 1, -6$

From equation (i),

$$x = \left(2\lambda - \frac{1}{3}\right),\ y = \left(\lambda + \frac{1}{3}\right),\ z = (-6\lambda + 1)$$

So, vector equation of the given line is,

$$x\hat{i} + y\hat{j} + z\hat{k} = \left(2\lambda - \frac{1}{3}\right)\hat{i} + \left(\lambda + \frac{1}{3}\right)\hat{j} + (-6\lambda + 1)\hat{k}$$

$$\vec{r} = \left(-\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 6\hat{k}\right)$$

Chapter 28 - Straight line in space Exercise Ex. 28.2

Question 1

Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$; $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$; $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

Solution 1

$$\text{Let } l_1 = \frac{12}{13}, m_1 = -\frac{3}{13}, n_1 = -\frac{4}{13}$$
$$l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_1 = \frac{3}{13}$$
$$l_3 = \frac{3}{13}, m_3 = -\frac{4}{13}, n_3 = \frac{12}{13}$$

$$l_1 l_2 + m_1 m_2 + n_1 n_2$$
$$= \frac{12}{13} \times \frac{4}{13} + (-\frac{3}{13}) \times \frac{12}{13} + (-\frac{4}{13}) \times \frac{3}{13}$$
$$= \frac{48 - 36 - 12}{169} = 0$$
$$l_2 l_3 + m_2 m_3 + n_2 n_3$$
$$= \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times (-\frac{4}{13}) + \frac{3}{13} \times \frac{12}{13}$$
$$= \frac{12 - 48 + 36}{169} = 0$$
$$l_1 l_3 + m_1 m_3 + n_1 n_3$$
$$= \frac{12}{13} \times \frac{3}{13} + (-\frac{3}{13}) \times (-\frac{4}{13}) + (-\frac{4}{13}) \times \frac{12}{13}$$
$$= \frac{36 + 12 - 48}{169} = 0$$
$\therefore$ The lines are mutually perpendicular.

Question 2

Show that the line through the points (1, -1, 2) and (3, 4, -2) is perpendicular to the line through points (0, 3, 2) and (3, 5, 6).

Solution 2

The direction ratios of a line passing through the points
$(1, -1, 2)$ and $(3, 4, -2)$ are
$(3 - 1, 4 + 1, -2 - 2)$
$= (2, 5, -4)$

The direction ratios of a line passing through the points
$(0, 3, 2)$ and $(3, 5, 6)$ are
$(3 - 0, 5 - 3, 6 - 2)$
$= (3, 2, 4)$

Angle between the lines
$$\cos\theta = \frac{(a_1 a_2 + b_1 b_2 + c_1 c_2)}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$\cos\theta = \frac{[2 \times 3 + 5 \times 2 + (-4) \times 4]}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$
$$\cos\theta = \frac{0}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$
$$\cos\theta = 0$$
$$\theta = \frac{\pi}{2}$$
The lines are mutually perpendicular.

Question 3

Show that the line through the points (4, 7, 8) and (2, 3, 4) is parallel to the line through the points (-1, -2, 1) and (1, 2, 5).

Solution 3

The direction ratios of a line passing through the points

(4,7,8) and (2,3,4) are

$(4-2, 7-3, 8-4)$

$= (2, 4, 4)$

The direction ratios of a line passing through the points

$(-1, -2, 1)$ and $(1, 2, 5)$ are

$(-1-1, -2-2, 1-5)$

$= (-2, -4, -4)$

The direction ratios are proportional.

$$\frac{2}{-2} = \frac{4}{-4} = \frac{4}{-4}$$

Hence, the lines are mutually parallel.

Question 4

Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

Solution 4

The Cartesian equation of a line passing through (x_1, y_1, z_1) and with direction ratios (a_1, b_1, c_1)

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

The Cartesian equation of a line passing through $(-2, 4, -5)$ and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Question 5

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Solution 5

Given equations of lines are $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

Clearly,

$7 \times 1 + (-5) \times 2 + 1 \times 3$

$= 7 - 10 + 3$

$= 0$

$\therefore$ Lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Question 6

Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, -1) and (4, 3, -1).

Solution 6

The direction ratios of a line joining the origin to the point $(2, 1, 1)$

are $(2-0, 1-0, 1-0) = (2, 1, 1)$

The direction ratios of a line joining $(3, 5, -1)$ and $(4, 3, -1)$

are $(4-3, 3-5, -1+1) = (1, -2, 0)$

Angle between the lines

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

$$\cos\theta = \frac{2\times 1 + 1\times(-2) + 1\times 0}{\sqrt{2^2 + 1^2 + 1^2}\sqrt{1^2 + (-2)^2 + 0^2}}$$

$$\cos\theta = \frac{0}{\sqrt{6}\sqrt{5}}$$

$$\cos\theta = 0$$

$$\therefore \theta = \frac{\pi}{2}$$

The lines are mutually perpendicular.

Question 7

Find the equation of a line parallel to x-axis and passing through the origin.

Solution 7

Vector equation of a line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

The direction cosines of the $x-$axis are $(1, 0, 0)$. Equation of a line parallel to the $x-$axis and passing through the origin is

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(1\hat{i} + 0\hat{j} + 0\hat{k})$$

$$\vec{r} = \lambda\hat{i}$$

Question 8(i)

Find the angle between the pair of lines:

$$\vec{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) \text{ and } \vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) - \mu\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)$$

Solution 8(i)

We know that, If Q be the angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, then

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|} \qquad ---(i)$$

Here, $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$

and, $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) - \mu(2\hat{i} + 4\hat{j} - 4\hat{k})$

$\Rightarrow \quad \vec{a_1} = 4\hat{i} - \hat{j}, \quad \vec{b_1} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b_2} = 2\hat{i} + 4\hat{j} - 4\hat{k}$

$$|\vec{b_1}| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$$

$$|\vec{b_2}| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$$

Let θ be the angle between given lines. So using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|}$$

$$= \frac{(\hat{i} + 2\hat{j} - 2\hat{k})(2\hat{i} + 4\hat{j} - 4\hat{k})}{3.6}$$

$$= \frac{2 + 8 + 8}{18}$$

$$\cos\theta = 1$$

$$\theta = 0°$$

Question 8(ii)

Find the angle between the pair of lines:

$$\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \text{ and } \vec{r} = (5\hat{j} - 2\hat{k}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

Solution 8(ii)

We know that, angle between two lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$, is given by

$$\cos\theta = \frac{\vec{b_1}\vec{b_2}}{\left|\vec{b_1}\right| \cdot \left|\vec{b_2}\right|} \qquad ---(i)$$

Given lines are,

$$\vec{r} = \left(3\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$
$$\vec{r} = \left(5\hat{j} - 2\hat{k}\right) + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \qquad \vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}, \; \vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\left|\vec{b_1}\right| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$
$$\left|\vec{b_2}\right| = \sqrt{(3)^2 + (2)^2 + (6)^2} = 7$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\vec{b_1}\vec{b_2}}{\left|\vec{b_1}\right| \cdot \left|\vec{b_2}\right|}$$
$$= \frac{\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)}{3.7}$$
$$= \frac{3 + 4 + 12}{21}$$
$$= \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Question 8(iii)

Find the angle between the pair of lines:

$$\vec{r} = \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right) \text{ and } \vec{r} = 2\hat{j} + \mu\left[\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right]$$

Solution 8(iii)

We know that, angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, is given by

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|} \qquad ---(i)$$

Equation of given lines are,

$$\vec{r} = \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right) \text{ and}$$

$$\vec{r} = 2\hat{j} + \mu\left[\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right]$$

$$\Rightarrow \quad \vec{b_1} = \left(\hat{i} + \hat{j} + 2\hat{k}\right), \ \vec{b_2} = \left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$$

$$= \frac{\left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right)}{\sqrt{(1)^2 + (1)^2 + (2)^2}\sqrt{\left(\sqrt{3} - 1\right)^2 + \left(-\sqrt{3} - 1\right)^2 + (4)^2}}$$

$$= \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6}.\sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 16}}$$

$$= \frac{6}{\sqrt{6}.2\sqrt{6}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Question 9(i)

Find the angle between the pair of lines:

$$\frac{x + 4}{3} = \frac{y - 1}{5} = \frac{z + 3}{4} \text{ and } \frac{x + 1}{1} = \frac{y - 4}{1} = \frac{z - 5}{2}$$

Solution 9(i)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Here, given lines are,

$$\frac{x+4}{3}=\frac{y-1}{5}=\frac{z+3}{4} \text{ and } \frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$$

$\Rightarrow \quad a_1=3,\ b_1=5,\ c_1=4,\ a_2=1,\ b_2=1,\ c_2=2$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta=\frac{(3)(1)+(5)(1)+(4)(2)}{\sqrt{(3)^2+(5)^2+(4)^2}\sqrt{(1)^2+(1)^2+(2)^2}}$$

$$=\frac{3+5+8}{\sqrt{50}\sqrt{6}}$$

$$=\frac{16}{10\sqrt{3}}$$

$$\cos\theta=\frac{8}{5\sqrt{3}}$$

$$\theta=\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

Question 9(ii)

Find the angle between the pair of lines:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \text{ and } \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

Solution 9(ii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given, equation of lines are,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \text{ and } \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

$$\Rightarrow \quad a_1=2,\ b_1=3,\ c_1=-3,\ a_2=-1,\ b_2=8,\ c_2=4$$

Let θ be the angle between two given lines, so using equation (i),

$$\cos\theta=\frac{(2)(-1)+(3)(8)+(-3)(4)}{\sqrt{(2)^2+(3)^2+(-3)^2}\sqrt{(-1)^2+(8)^2+(4)^2}}$$

$$=\frac{-2+24-12}{\sqrt{22}\sqrt{81}}$$

$$\cos\theta=\frac{10}{9\sqrt{22}}$$

$$\theta=\cos^{-1}\left(\frac{10}{9\sqrt{22}}\right)$$

Question 9(iii)

Find the angle between the pair of lines:

$$\frac{5-x}{-2}=\frac{y+3}{1}=\frac{1-z}{3} \text{ and } \frac{x}{3}=\frac{1-y}{-2}=\frac{z+5}{-1}$$

Solution 9(iii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(i)$$

Given lines are,

$$\frac{5-x}{-2}=\frac{y+3}{1}=\frac{1-z}{3} \text{ and } \frac{x}{3}=\frac{1-y}{-2}=\frac{z+5}{-1}$$

$$\Rightarrow \quad \frac{x-5}{2}=\frac{y+3}{1}=\frac{z-1}{-3} \text{ and } \frac{x}{3}=\frac{y-1}{2}=\frac{z+5}{-1}$$

$$\Rightarrow \quad a_1=2,\ b_1=1,\ c_1=-3,\ a_2=3,\ b_2=2,\ c_2=-1$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta=\frac{(2)(3)+(1)(2)+(-3)(-1)}{\sqrt{(2)^2+(1)^2+(-3)^2}\sqrt{(3)^2+(2)^2+(-1)^2}}$$

$$=\frac{6+2+3}{\sqrt{14}\sqrt{14}}$$

$$\cos\theta=\frac{11}{14}$$

$$\theta=\cos^{-1}\left(\frac{11}{14}\right)$$

Question 9(iv)

Find the angle between the pair of lines:

$$\frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-5}{2}$$

Solution 9(iv)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Equation of given lines are,

$$\frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-5}{2}$$

$$\Rightarrow \quad \frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{\frac{y-3}{3}}{\frac{3}{2}}=\frac{z-5}{2}$$

$$\Rightarrow \quad a_1=3,\ b_1=-2,\ c_1=0,\ a_2=1,\ b_2=\frac{3}{2},\ c_2=2$$

Let θ be the angle between given lines, so from equation (i),

$$\cos\theta=\frac{(3)(1)+(-2)\left(\frac{3}{2}\right)+(0)(2)}{\sqrt{(3)^2+(-2)^2+(0)^2}\sqrt{(1)^2+\left(\frac{3}{2}\right)^2+(2)^2}}$$

$$=\frac{3-3+0}{\sqrt{38}\sqrt{\frac{29}{4}}}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Question 9(v)

Find the angle between the pair of lines:

$$\frac{x-5}{1}=\frac{2y+6}{-2}=\frac{z-3}{1} \text{ and } \frac{x-2}{3}=\frac{y+1}{4}=\frac{z-6}{5}$$

Solution 9(v)

$$\frac{x-5}{1}=\frac{2y+6}{-2}=\frac{z-3}{1} \text{ and } \frac{x-2}{3}=\frac{y+1}{4}=\frac{z-6}{5}$$

$\hat{a}=\hat{i}-2\hat{j}+\hat{k}, \hat{b}=3\hat{i}+4\hat{j}+5\hat{k}$ are the vectors parallel to above lines

$$\therefore \text{angle between } \hat{a} \text{ and } \hat{b} \rightarrow \cos\theta=\frac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$$

$$\cos\theta=\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j}+5\hat{k})}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=\frac{3-8+5}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=0$$

$$\cos\theta=0 \rightarrow \theta=90^\circ$$

Question 9(vi)

find the angle between the following pairs of line :

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

Solution 9(vi)

$$\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$$

$\hat{a}=2\hat{i}+7\hat{j}-3\hat{k}, \hat{b}=-1\hat{i}+4\hat{j}+4\hat{k}$ are the vectors parallel to above lines

$$\therefore \text{angle between } \hat{a} \text{ and } \hat{b} \rightarrow \cos\theta=\frac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$$

$$\cos\theta=\frac{(2\hat{i}+7\hat{j}-3\hat{k})\cdot(-1\hat{i}+2\hat{j}+4\hat{k})}{|(2\hat{i}+7\hat{j}-3\hat{k})||(-1\hat{i}+2\hat{j}+4\hat{k})|}=0$$

$$\cos\theta=0 \rightarrow \theta=90^\circ$$

Question 10(i)

Find the angle between the pair of lines with direction ratios proportional to

5, −12, 13 and −3, 4, 5

Solution 10(i)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Here, $a_1=5,\ b_1=-12,\ c_1=13$

$a_2=-3,\ b_2=4,\ c_2=5$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(5)(-3)+(-12)(4)+(13)(5)}{\sqrt{(5)^2+(-12)^2+(13)^2}\sqrt{(-3)^2+(4)^2+(5)^2}}$$

$$=\frac{-15-48+65}{\sqrt{169\times2}\sqrt{25\times2}}$$

$$=\frac{2}{65\times2}$$

$$\cos\theta=\frac{1}{65}$$

$$\theta=\cos^{-1}\left(\frac{1}{65}\right)$$

Question 10(ii)

Find the angle between the pair of lines with direction ratios proportional to

2, 2, 1 and 4, 1, 8

Solution 10(ii)

We know that, angle (θ) between lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Here, $a_1=2,\ b_1=2,\ c_1=1$
$a_2=4,\ b_2=1,\ c_2=8$

Let θ be required angle, so using equation (i),

$$\cos\theta=\frac{(2)(4)+(2)(1)+(1)(8)}{\sqrt{(2)^2+(2)^2+(1)^2}\sqrt{(4)^2+(1)^2+(8)^2}}$$

$$=\frac{8+2+8}{3.9}$$

$$=\frac{18}{27}$$

$$\cos\theta=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Question 10(iii)

Find the angle between the pair of lines with direction ratios proportional to

$1,2,-2$ and $-2,2,1$

Solution 10(iii)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Here, $a_1=1,\ b_1=2,\ c_1=-2$
$a_2=-2,\ b_2=2,\ c_2=1$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(1)(-2)+(2)(2)+(-2)(1)}{\sqrt{(1)^2+(2)^2+(-2)^2}\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$=\frac{-2+4-2}{3.3}$$

$$=\frac{0}{9}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Question 10(iv)

find the angle between the pairs of lines with directions ratios proposal to a, b, c and b - c, c - a, a - b.

Solution 10(iv)

a, b, c and $b - c$, $c - a$, $a - b$ are direction ratios
these are the vectors with above direction ratios
$\hat{x} = a\hat{i} + b\hat{j} + c\hat{k}, \hat{y} = (b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}$
are the vectors parallel to two given lines
$\therefore$ angle between the lines with above
direction ratios are $\hat{x}$ and $\hat{y} \rightarrow \cos\theta = \dfrac{\hat{x} \cdot \hat{y}}{|\hat{x}||\hat{y}|}$

$$\cos\theta = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot ((b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k})}{|(a\hat{i} + b\hat{j} + c\hat{k})||(b - c)\hat{i} + (c - a)\hat{j} + (a - b)\hat{k}|}$$

$$= \frac{a(b - c) + b(c - a) + c(a - b)}{\sqrt{a^2 + b^2 + c^2}\sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}}$$

$$= \frac{ab - ac + bc - ba + ca - cb}{\sqrt{a^2 + b^2 + c^2}\sqrt{(b - c)^2 + (c - a)^2 + (a - b)^2}} = 0$$

$\cos\theta = 0 \rightarrow \theta = 90^\circ$

Question 11

Find the angle between two lines, one of which has direction ratios 2, 2, 1 while the other one is obtained by joining the points $(3, 1, 4)$ and $(7, 2, 12)$.

Solution 11

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Here, Direction ratios of first line is 2, 2, 1

$\Rightarrow \quad a_1=2,\ b_1=2,\ c_1=1$

Direction ratios of the line joining $(3,1,4)$ and $(7,2,12)$ is given by

$= (7-3),\ (2-1),\ (12-4)$

$= 4, 1, 8$

$\Rightarrow \quad a_2=4,\ b_2=1,\ c_2=8$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(2)(4)+(2)(1)+(1)(8)}{\sqrt{(2)^2+(2)^2+(1)^2}\sqrt{(4)^2+(1)^2+(8)^2}}$$

$$=\frac{8+2+8}{3.9}$$

$$\cos\theta=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Question 12

Find the equation of the line passing through the point $(1,2,-4)$ and parallel to the line $\frac{x-3}{4}=\frac{y-5}{2}=\frac{z+1}{3}$.

Solution 12

We know that equation of a line passing through (x_1, y_1, z_1) and direction
a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, -4)$
and required line is parallel to the given line

$$\frac{x - 3}{4} = \frac{y - 5}{2} = \frac{z + 1}{3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $4, 2, 3$
$\Rightarrow$ $a = 4\lambda,\ b = 2\lambda,\ c = 3\lambda$

So, required equation of the line is

$$\Rightarrow \frac{x - 1}{4\lambda} = \frac{y - 2}{2\lambda} = \frac{z + 4}{3\lambda}$$

$$\Rightarrow \frac{x - 1}{4} = \frac{y - 2}{2} = \frac{z + 4}{3}$$

Question 13

Find the equations of the line passing through the point $(-1, 2, 1)$ and parallel to the line $\frac{2x - 1}{4} = \frac{3y + 5}{2} = \frac{2 - z}{3}$

Solution 13

We know that, equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (-1, 2, 1)$

and required line is parallel to the given line

$$\frac{2x - 1}{4} = \frac{3y + 5}{2} = \frac{2 - z}{3}$$

$$\Rightarrow \quad \frac{x - \frac{1}{2}}{2} = \frac{y + \frac{5}{3}}{\frac{2}{3}} = \frac{z - 2}{-3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $2, \frac{2}{3}, -3$

$$\Rightarrow \quad a = 2\lambda,\ b = \frac{2}{3}\lambda,\ c = -3\lambda$$

So, required equation of the line using equation (i),

$$\frac{x + 1}{2\lambda} = \frac{y - 2}{\frac{2}{3}\lambda} = \frac{z - 1}{-3\lambda}$$

$$\Rightarrow \quad \frac{x + 1}{2} = \frac{y - 2}{\frac{2}{3}} = \frac{z - 1}{-3}$$

Question 14

Find the equation of the line passing through the point $(2, -1, 3)$ and parallel to the line $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 5\hat{k})$.

Solution 14

We know that equation of a line passing through the point $\bar{a}$ and is the direction of vector $\bar{b}$ is

$$\vec{r} = \bar{a} + \lambda\bar{b} \qquad ---(\text{i})$$

Here, $\bar{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

and given that the required line is parallel to

$$\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

$$\Rightarrow \qquad \bar{b} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

So, required equation of the line using equation (i) is

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda'\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

where λ' is a scalar such that $\lambda' = \lambda.\mu$

Question 15

Find the equations of the line passing through the point $(2, 1, 3)$ and perpendicular to the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \text{ and } \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

Solution 15

We know that, equation of a line passing through (x_1, y_1, z_1) with direction ratios a, b, c is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

So, equation of required line passing through $(2,1,3)$ is

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \qquad ---(1)$$

Given that line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ is perpendicular to line (i), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(2) + (c)(3) = 0$$
$$a + 2b + 3c = 0 \qquad ---(2)$$

And line $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ is perpendicular to line (i), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(-3) + (b)(2) + (c)(5) = 0$$
$$-3a + 2b + 5c = 0 \qquad ---(3)$$

Solving equation (2) and (3) by cross multiplication,

$$\frac{a}{(2)(5)-(2)(3)} = \frac{b}{(-3)(3)-(1)(5)} = \frac{c}{(1)(2)-(-3)(2)}$$

$$\Rightarrow \quad \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$
$$\Rightarrow \quad \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$
$$\Rightarrow \quad \frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (Say)}$$
$$\Rightarrow \quad a = 2\lambda,\ b = -7\lambda,\ c = 4\lambda$$

Using a, b, c in equation (i),

$$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$$

Question 16

Find the equation of the line passing through the point $\hat{i} + \hat{j} - 3\hat{k}$ and perpendicular to the lines $\vec{r} = \hat{i} + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$.

Solution 16

We know that equation of a line passing through a point with position vector $\vec{\alpha}$ and perpendiculat to $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right) \qquad ---(i)$$

Here, $\vec{\alpha} = \left(\hat{i} + \hat{j} - 3\hat{k}\right)$

and required line is perpendicular to

$$\vec{r} = \hat{i} + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\Rightarrow \qquad \vec{b_1} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right),\ \vec{b_2} = \hat{i} + \hat{j} + \hat{k}$$

Now,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(1 + 3\right) - \hat{j}\left(2 + 3\right) + \hat{k}\left(2 - 1\right)$$

$$\vec{b_1} \times \vec{b_2} = 4\hat{i} - 5\hat{j} + \hat{k}$$

Using equation, required equation of line is

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right)$$

$$\vec{r} = \left(\hat{i} + \hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} + \hat{k}\right)$$

Question 17

Find the equation of the line passing through the point $\left(1, -1, 1\right)$ and perpendicular to the lines joining the points $\left(4, 3, 2\right)$, $\left(1, -1, 0\right)$ and $\left(1, 2, -1\right)$, $\left(2, 1, 1\right)$.

Solution 17

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios as a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ----(1)$$

So, equation of a line passing through $(1, -1, 1)$ is

$$\frac{x - 1}{a} = \frac{y + 1}{b} = \frac{z - 1}{c} \qquad ---(2)$$

Now, Directions ratios of the line joining $A(4, 3, 2)$ and $B(1, -1, 0)$

$= (1 - 4), (-1 - 3), (0 - 2)$

$\Rightarrow$ Direction ratios of line $AB = -3, -4, -2$

and, Directions ratios of the line joining $C(1, 2, -1)$ and $D(2, 1, 1)$

$= (2 - 1), (1 - 2), (1 + 1)$

$\Rightarrow$ Direction ratios of line $CD = 1, -1, 2$

Given that, line AB is perpendicular to line (2), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(-3) + (b)(-4) + (c)(-2) = 0$$
$$-3a + 4b - 2c = 0$$
$$3a + 4b + 2c = 0 \qquad ---(3)$$

and, line CD is also perpendicular to line (2), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-1) + (c)(2) = 0$$
$$a - b + 2c = 0 \qquad ---(4)$$

Solving equation (3) and (4) using cross multiplication,

$$\frac{a}{(4)(2) - (-1)(2)} = \frac{b}{(1)(2) - (3)(2)} = \frac{c}{(3)(-1) - (4)(1)}$$

$\Rightarrow$ $\dfrac{a}{8 + 2} = \dfrac{b}{2 - 6} = \dfrac{c}{-3 - 4}$

$\Rightarrow$ $\dfrac{a}{10} = \dfrac{b}{-4} = \dfrac{c}{-7} = \lambda$ (Say)

$\Rightarrow$ $a = 10\lambda,\ b = -4\lambda,\ c = -7\lambda$

Using a, b, c in equation (2) to get required equation of line,

$$\frac{x-1}{10\lambda} = \frac{y+1}{-4\lambda} = \frac{z-1}{-7\lambda}$$

$$\Rightarrow \quad \frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7}$$

Question 18

Determine the equations of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines

$$\frac{x-8}{8} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Solution 18

We know that equation of a line passing through a point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

So, equation of required line passing through $(1, 2, -4)$ is

$$\frac{x - 1}{a} = \frac{y - 2}{b} = \frac{z + 4}{c} \qquad ---(1)$$

Given that, line $\frac{x - 8}{8} = \frac{y + 9}{-16} = \frac{z - 10}{7}$ is perpendicular to line (1), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow \quad (a)(8) + (b)(-16) + (c)(7) = 0$$

$$\Rightarrow \quad 8a - 16b + 7c = 0 \qquad ----(2)$$

also, line $\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$ is perpendicular to line (1), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow \quad (3)(a) + (8)(b) + (-5)(c) = 0$$

$$\Rightarrow \quad 3a + 8b - 5c = 0 \qquad ----(3)$$

Solving equation (2) and (3) by cross-multiplication,

$$\frac{a}{(-16)(-5) - (8)(7)} = \frac{b}{(3)(7) - (8)(-5)} = \frac{c}{(8)(8) - (3)(-16)}$$

$$\Rightarrow \quad \frac{a}{80 - 56} = \frac{b}{21 + 40} = \frac{c}{64 + 48}$$

$$\Rightarrow \quad \frac{a}{24} = \frac{b}{61} = \frac{c}{112} = \lambda \ (\text{Say})$$

$$\Rightarrow \quad a = 24\lambda,\ b = 61\lambda,\ c = 112\lambda$$

Put a, b, c in equation (1) to get required equation of the line, so

$$\frac{x - 1}{24\lambda} = \frac{y - 2}{61\lambda} = \frac{z + 4}{112\lambda}$$

$$\Rightarrow \quad \frac{x - 1}{24} = \frac{y - 2}{61} = \frac{z + 4}{112}$$

Question 19

Show that the lines

$$\frac{x - 5}{7} = \frac{y + 2}{-5} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

are perpendicular to each other.

Solution 19

Equation of lines are,

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$$

and, $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

Now, $a_1a_2 + b_1b_2 + c_1c_2$

$= (7)(1) + (-5)(2) + (1)(3)$

$= 7 - 10 + 3$

$= 0$

So, given lines are perpendicular.

Question 20

Find the vector equation of the line passing through the point $(2, -1, -1)$ which is parallel to the line $6x - 2 = 3y + 1 = 2z - 2$.

Solution 20

We know that, equation of a line passing through the point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(1)$$

So, equation of line passing through $(2, -1, -1)$ is

$$\frac{x - 2}{a} = \frac{y + 1}{b} = \frac{z + 1}{c} \qquad ---(2)$$

Line (2) is parallel to given line,

$$6x - 2 = 3y + 1 = 2z - 2$$

$$\Rightarrow \quad \frac{6x - 2}{6} = \frac{3y + 1}{6} = \frac{2z - 2}{6}$$

$$\Rightarrow \quad \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{2}}{2} = \frac{z - \frac{1}{3}}{3}$$

So, $a = \lambda$, $b = 2\lambda$, $c = 3\lambda$

Using a, b, c in equation (2) to get required equation of line,

$$\frac{x - 2}{\lambda} = \frac{y + 1}{2\lambda} = \frac{z + 1}{3\lambda}$$

$$\Rightarrow \quad \frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z + 1}{3} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad x = \lambda + 2,\ y = 2\lambda - 1,\ z = 3\lambda - 1$$

So,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

Question 21

If the lines $\frac{x - 1}{-3} = \frac{y - 2}{2\lambda} = \frac{z - 3}{2}$ and $\frac{x - 1}{3\lambda} = \frac{y - 1}{1} = \frac{z - 6}{-5}$ are perpendicular, find the value of λ.

Solution 21

Given that lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular

$$\therefore (-3)\times 3\lambda + 2\lambda \times 1 + 2\times(-5) = 0$$
$$\Rightarrow -9\lambda + 2\lambda - 10$$
$$\Rightarrow \lambda = -\frac{10}{7}$$

Question 22

If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (−4, 3, −6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Solution 22

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (−4, 3, −6), and (2, 9, 2) respectively.

The direction ratios of AB are (4 − 1) = 3, (5 − 2) = 3, and (7 − 3) = 4

The direction ratios of CD are (2 −(− 4)) = 6, (9 − 3) = 6, and (2 −(−6)) = 8

It can be seen that, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{2}$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either 0° or 180°.

Question 23

Find the value of λ so that the following lines are perpendicular to each other.

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1},\ \frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

Solution 23

Given equation of line are,

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1} \text{ and}$$

$$\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$$

$$\Rightarrow \quad \frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1} \qquad ---(1)$$

$$\text{and,} \quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \qquad ---(2)$$

Given that line (1) and (2) are perpendicular,

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(5\lambda+2)(1) + (-5)(2\lambda) + (1)(3) = 0$$

$$5\lambda + 2 - 10\lambda + 3 = 0$$

$$-5\lambda + 5 = 0$$

$$\lambda = \frac{5}{5}$$

$$\lambda = 1$$

Question 24

Find the direction cosines of the line $\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$.

Also, find the vector equation of the line through the point A$(-1, 2, 3)$ and parallel to the given line.

Solution 24

The direction ratios of the line are

$$\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$$

2,6,6

The direction cosines of the line are

$$l = \frac{2}{\sqrt{2^2+6^2+6^2}} = \frac{2}{\sqrt{76}}$$

$$m = \frac{6}{\sqrt{2^2+6^2+6^2}} = \frac{6}{\sqrt{76}}$$

$$n = \frac{6}{\sqrt{2^2+6^2+6^2}} = \frac{6}{\sqrt{76}}$$

$$\left(\frac{2}{\sqrt{76}}, \frac{6}{\sqrt{76}}, \frac{6}{\sqrt{76}}\right)$$

$\therefore$ Vector equation of the line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = (-\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(2\vec{i} + 6\vec{j} + 6\vec{k})$$

Chapter 28 - Straight line in space Exercise Ex. 28.3

Question 1

Show that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect and find their point of intersection.

Solution 1

We have equation of first line,

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda \quad (\text{Say}) \qquad ---(1)$$

General point on line (1) is

$$(\lambda,\ 2\lambda+2,\ 3\lambda-3)$$

Another line is,

$$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} = \mu \quad (\text{Say}) \qquad ---(2)$$

General point on line (2) is

$$(2\mu+2,\ 3\mu+6,\ 4\mu+3)$$

If lines (1) and (2) intersect then they have a common point, so for same value of λ and μ, we must have,

$$\lambda = 2\mu + 2 \quad \Rightarrow \lambda - 2\mu = 2 \qquad ---(3)$$

$$2\lambda + 2 = 3\mu + 6 \Rightarrow 2\lambda - 4\mu = 4 \qquad ---(4)$$

$$3\lambda - 3 = 4\mu + 3 \Rightarrow 3\lambda - 4\mu = 6 \qquad ---(5)$$

Now, solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 2\lambda - 4\mu = 4 \\ \underset{(-)}{2\lambda} \underset{(+)}{-\ 3\mu} = \underset{(-)}{4} \\ \hline -\ \mu = 0 \end{array}$$

$$\Rightarrow \qquad \mu = 0$$

Put $\mu = 0$ in equation (3),

$$\lambda - 2\mu = 2$$

$$\lambda - 2(0) = 2$$

$$\lambda = 2$$

Put λ and μ in equation (5),

$$3\lambda - 4\mu = 6$$

$$3(2) - 4(0) = 6$$

$$6 = 6$$

LHS = RHS

So, given lines intersect each other

Coordinates of point of intersection

$$= \left(2\mu + 2,\ 3\mu + 6,\ 4\mu + 3\right)$$

$$= \left(2\left(0\right) + 2,\ 3\left(0\right) + 6,\ 4\left(0\right) + 3\right)$$

$$= \left(2, 6, 3\right)$$

So, coordinates of point of intersection $= \left(2, 6, 3\right)$

Question 2

Show that the lines $\dfrac{x-1}{3} = \dfrac{y+1}{2} = \dfrac{z-1}{5}$ and $\dfrac{x+2}{4} = \dfrac{y-1}{3} = \dfrac{z+1}{-2}$ do not intersect.

Solution 2

We have equation of first line,

$$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}=\lambda \text{ (Say)} \qquad ---(1)$$

General point on line (1) is

$$(3\lambda+1,\ 2\lambda-1,\ 5\lambda+1)$$

Another line is,

$$\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}=\mu \text{ (Say)} \qquad ---(2)$$

General point on line (2) is,

$$(4\mu-2,\ 3\mu+1,\ -2\mu-1)$$

If lines (1) and (2) intersect, then they have a common point, so for same value of λ and μ, we must have,

$$3\lambda+1=4\mu-2 \Rightarrow 3\lambda-4\mu=-3 \qquad ---(3)$$

$$2\lambda-1=3\mu+1 \Rightarrow 2\lambda-3\mu=2 \qquad ---(4)$$

$$5\lambda+1=-2\mu-1 \Rightarrow 5\lambda+2\mu=-2 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 6\lambda-8\mu=-6 \\ 6\lambda-9\mu=6 \\ (-)\quad(+)\quad(-) \\ \hline \mu=-12 \end{array}$$

Put the value of μ in equation (3),

$$3\lambda-4(-12)=-3$$

$$3\lambda+48=-3$$

$$3\lambda=-3-48$$

$$3\lambda=-51$$

$$\lambda=\frac{-51}{3}$$

$$\lambda=-17$$

Put the value of λ and μ in equation (5),

$$5\lambda+2\mu=-2$$

$$5(-17)+2(-12)=-2$$

$$-85-24=-2$$

$$-109\neq-2$$

$$\text{LHS}\neq\text{RHS}$$

So, the value of λ and μ obtained by solving equation (3) and (4) does not satisfy equation (5), so

Given lines are not intersecting.

Question 3

Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Find their point of intersection.

Solution 3

Given equation of first line is

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \text{ (Say)} \qquad ---(1)$$

General point on line (1) is

$$(3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$$

Another equation of line is

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \text{ (Say)} \qquad ---(2)$$

General point on line (2) is,

$$(\mu + 2,\ 3\mu + 4,\ 5\mu + 6)$$

If lines (1) and (2) are intersecting then, they have a common point. So for same value of λ and μ, we must have,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3 \qquad ---(3)$$

$$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7 \qquad ---(4)$$

$$7\lambda - 5 = 5\mu + 6 \Rightarrow 7\lambda - 5\mu = 11 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$\begin{array}{r} 15\lambda - 5\mu = 15 \\ 15\lambda - 9\mu = 21 \\ (-)\quad (+)\quad (-) \\ \hline 4\mu = -6 \end{array}$$

$$\mu = \frac{-3}{2}$$

Put the value of μ in equation (3),

$$3\lambda - \mu = 3$$

$$3\lambda - \left(-\frac{3}{2}\right) = 3$$

$$3\lambda = 3 - \frac{3}{2}$$

$$\lambda = \frac{1}{2}$$

Put the value of λ and μ in equation (5),

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2} + \frac{15}{2} = 11$$

$$\frac{22}{2} = 11$$

$$11 = 11$$

LHS $\neq$ RHS

Since, the values of λ and μ obtained by solving (3) and (4) satisfy equation (5), Hence

Given lines intersect each other.

Point of intersection $= (3\lambda - 1,\ 5\lambda - 3,\ 7\lambda - 5)$

$$= \left\{\frac{3}{2} - 1,\ \left(\frac{5}{2} - 3\right),\ \left(\frac{7}{2} - 5\right)\right\}$$

$$= \left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$$

Point of intersection is $\left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$.

Question 4

Prove that the lines through $A(0,-1,-1)$ and $B(4,5,1)$ intersects the line through $C(3,9,4)$ and $D(-4,4,4)$. Also find their point of intersection.

Solution 4

Equation of the line passing through $A(0,-1,-1)$ and $B(4,5,1)$ is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-0}{4-0}=\frac{y+1}{5+1}=\frac{z+1}{1+1}$$

$$\frac{x}{4}=\frac{y+1}{6}=\frac{z+1}{2}=\lambda \;(\text{say})$$

So, general point on line AB is

$$(4\lambda,\ 4\lambda,\ 2\lambda-1)$$

Now, equation of the line passing through $C(3,9,4)$ and $D(-4,4,4)$ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}$$

$$\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}=\mu \;(\text{say})$$

So, general point on line CD is

$$(-7\mu+3,\ -5\mu+9,\ 0.\mu+4)$$

$$(-7\mu+3,\ -5\mu+9,\ 4)$$

If lines AB and CD intersect, there must be a common point to them. So we have to find λ and μ such that

$$4\lambda=-7\mu+3 \quad\Rightarrow 4\lambda+7\mu=3 \qquad ---(1)$$

$$6\lambda-1=-5\mu+9 \quad\Rightarrow 6\lambda+5\mu=10 \qquad ---(2)$$

$$2\lambda-1=4 \quad\Rightarrow 2\lambda-1=4 \qquad ---(3)$$

From equation (3),

$$2\lambda=4+1$$

$$\lambda=\frac{5}{2}$$

Put $\lambda=\frac{5}{2}$ in equation (2),

$$6\left(\frac{5}{2}\right)+5\mu=10$$

$$5\mu=10-15$$

$$5\mu=-5$$

$$\mu=-1$$

Now, put values of λ and μ in equation (1),

$$4\lambda + 7(\mu) = 3$$
$$4\left(\frac{5}{2}\right) + 7(-1) = 3$$
$$10 - 7 = 3$$
$$3 = 3$$

LHS $\neq$ RHS

Since, the values of λ and μ by solving (2) and (3) satisfy equation (1), so

Line AB and CD are intersecting lines

Point of intersection of AB and CD

$$= (-7\mu + 3,\ -5\mu + 9,\ 4)$$
$$= (-7(-1) + 3,\ -5(-1) + 9,\ 4)$$
$$= (7 + 3,\ 5 + 9,\ 4)$$
$$= (10, 14, 4)$$

So, point of intersection of AB and CD $= (10, 14, 4)$.

Question 5

Prove that the line $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect and find their point of intersection.

Solution 5

Given equations of lines are

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$
$$\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

If these lines intersect, they must have a common point, so, for some value of λ and μ we must have,

$$\left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right) = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$
$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$$1 + 3\lambda = 4 + 2\mu \quad \Rightarrow 3\lambda - 2\mu = 3 \quad ---(1)$$
$$1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \quad ---(2)$$
$$-1 = -1 + 3\mu \quad \Rightarrow \mu = 0 \quad ---(3)$$

Put the value of λ and μ in equation (1),

$$3\lambda - 2\mu = 3$$
$$3(1) - 2(0) = 3$$
$$3 = 3$$
$$\text{LHS} = \text{RHS}$$

The value of λ and μ satisfy equation (1), so

Lines are intersecting.

Put value of λ in equation (1) to get point of intersection

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + (1)\left(3\hat{i} - \hat{j}\right)$$
$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$
$$= 4\hat{i} - \hat{k}$$

So, point of intersection is $(4, 0, -1)$.

Question 6(i)

Determine whether the pair of lines intersect or not:

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) \text{ and } \vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

Solution 6(i)

Given equations of lines are

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

If these lines intersect each other, there must be some common point, so, we must have λ and μ such that

$$\left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

$$\left(1 + 2\lambda\right)\hat{i} - \hat{j} + \lambda\hat{k} = \left(2 + \mu\right)\hat{i} + \left(-1 + \mu\right)\hat{j} - \mu\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$$1 + 2\lambda = 2 + \mu \qquad \Rightarrow 2\lambda - \mu = 1 \qquad ---(1)$$

$$-1 = -1 + \mu \qquad \Rightarrow \mu = 0 \qquad ---(2)$$

$$\lambda = -\mu \qquad \Rightarrow \lambda = 0 \qquad ---(3)$$

Put value of λ and μ in equation (1),

$$2\lambda - \mu = 1$$

$$2(0) - (0) = 1$$

$$0 = 1$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ form equation (2) and (3) does not satisfy equation (1),

Hence, given lines do not intersect each other.

Question 6(ii)

Determine whether the pair of lines intersect or not:

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \ \frac{x+1}{5} = \frac{y-2}{1}, \ z = 3$$

Solution 6(ii)

Given, equations of first line is

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \ (\text{say}) \qquad ---(1)$$

General point on line (1) is

$$(2\lambda+1,\ 3\lambda-1,\ \lambda)$$

Another equation of line is

$$\frac{x-1}{5} = \frac{y-2}{1},\ z=3 \qquad ---(2)$$

$$\frac{x-1}{5} = \frac{y-2}{1} = \mu,\ (\text{say}),\ z=3$$

General point on line (2) is

$$(5\mu+1,\ \mu+2,\ 3)$$

If line (1) and (2) intersect each other then, there is a common point to them, so, we must have value of λ and μ such that

$$2\lambda+1 = 5\mu+1 \quad \Rightarrow 2\lambda-5\mu=0 \qquad ---(3)$$

$$3\lambda-1 = \mu+2 \quad \Rightarrow 3\lambda-\mu=3 \qquad ---(4)$$

$$\lambda = 3 \quad \Rightarrow \lambda = 3 \qquad ---(5)$$

Put value of λ in equation (4),

$$3\lambda-\mu=3$$
$$3(3)-\mu=3$$
$$-\mu=3-9$$
$$\mu=6$$

Put the value of λ and μ in equation (3), so

$$2\lambda-5\mu=0$$
$$2(3)-5(6)=0$$
$$6-30=0$$
$$-24\neq 0$$
$$\text{LHS}\neq\text{RHS}$$

Since the values of λ and μ obtained from equation (4) and (5) does not satisfy equation (3), so,

Given lines are not intersecting.

Question 6(iii)

Determine whether the pair of lines intersect or not:

$$\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0},\ \frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}$$

Solution 6(iii)

Given, equations of first line is,

$$\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}=\lambda\ \text{(say)} \qquad ---(1)$$

General point on line (1) is,

$$(3\lambda+1,\ -\lambda+1,\ -1)$$

Another equation of line is

$$\frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}=\mu\ \text{(say)} \qquad ---(2)$$

General point on line (2) is,

$$(2\mu+4,\ 0,\ 3\mu-1)$$

If line (1) and (2) intersecting then there must be a common point, so, we must have the value of λ and μ as

$$3\lambda+1=2\mu+4 \quad \Rightarrow 3\lambda-2\mu=3 \qquad ---(1)$$

$$-\lambda+1=0 \quad \Rightarrow \lambda=1 \qquad ---(2)$$

$$3\mu-1=-1 \quad \Rightarrow \mu=0 \qquad ---(3)$$

Put the value of λ and μ in equation (1), so

$$3\lambda-2\mu=3$$

$$3(1)-2(0)=3$$

$$3=3$$

$$\text{LHS}\neq\text{RHS}$$

Since the values of λ and μ obtained by equation (2) and (3) satisfy equation (1), so,

Given lines are intersecting.

Question 6(iv)

Determine whether the pair of lines intersect or not:

$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5},\ \frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$$

Solution 6(iv)

Given, equation of line is

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} = \lambda \text{ (say)} \qquad ---(1)$$

General point on line (1) is,

$$(4\lambda + 5,\ 4\lambda + 7,\ -5\lambda - 3)$$

Another equation of line is,

$$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} = \mu \text{ (say)} \qquad ---(2)$$

General point on line (2) is

$$(7\mu + 8,\ \mu + 4,\ 3\mu + 5)$$

If line (1) and (2) intersecting, then there must have some common point to them, so, we must have value of λ and μ such that

$$4\lambda + 5 = 7\mu + 8 \quad \Rightarrow 4\lambda - 7\mu = 3 \qquad ---(3)$$

$$4\lambda + 5 = \mu + 4 \quad \Rightarrow 4\lambda - \mu = -3 \qquad ---(4)$$

$$-5\lambda - 3 = 3\mu + 5 \quad \Rightarrow -5\lambda - 3\mu = 8 \qquad ---(5)$$

Solving equation (3) and (4) to find λ and μ,

$$\begin{array}{r} 4\lambda - 7\mu = 3 \\ 4\lambda - \mu = -3 \\ (-) \quad (+) \quad (-) \\ \hline -6\mu = 6 \\ \mu = -1 \end{array}$$

Put value of λ in equation (3),

$$4\lambda - 7\mu = 3$$

$$4\lambda - 7(-1) = 3$$

$$4\lambda = 3 - 7$$

$$\lambda = -1$$

Put the value of λ and μ in equation (5),

$$-5\lambda - 3\mu = 8$$

$$-5(-1) - 3(-1) = 8$$

$$5 + 3 = 8$$

$$\text{LHS} = \text{RHS}$$

Since, values of λ and μ obtained by solving equation (3) and (4) satisfy equation (5), so,

Given lines are intersecting.

Question 7

Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ are intersecting. Hence, find their point of intersection.

Solution 7

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect eachother, then the shortest distance between the lines should be zero.

Here,

$\vec{a_1} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a_2} = 5\hat{i} - 2\hat{j}$

$\vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$(\vec{b_1} \times \vec{b_2}) = \left|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{matrix}\right|$$

$= \vec{i}(12 - 4) - \vec{j}(6 - 6) + \vec{k}(2 - 6)$

$= 8\vec{i} - 0\vec{j} - 4\vec{k}$

$(\vec{a_2} - \vec{a_1}) = (5\hat{i} - 2\hat{j} - 3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 4\hat{j} + 4\hat{k})$

$$\text{Shortest Distance, } d = \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$= \left|\frac{(8\vec{i} - 0\vec{j} - 4\vec{k}).(2\hat{i} - 4\hat{j} + 4\hat{k})}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{8 \times 2 - 0 \times 4 + (-4) \times 4}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{0}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right| = 0$$

Since the shortest distance is zero, the lines are intersect each other.

Point of intersection of the lines,

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Lines in the Cartesian form,

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$$

$x = \lambda + 3, y = 2\lambda + 2, z = 2\lambda - 4$

$$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = \mu$$

$x = 3\mu + 5, y = 2\mu - 2, z = 6\mu$

From coordinates of x,

$\lambda + 3 = 3\mu + 5$

$\lambda = 3\mu + 2$.....(i)

From coordinates of y,

$2\lambda + 2 = 2\mu - 2$

$\lambda = \mu - 2$.......(ii)

Solving (i) and (ii),

$\lambda = -4, \mu = -2$

Coordinates of the point of intersection,

$x = 3(-2) + 5, y = 2(-2) - 2, z = 6(-2)$

$x = -1, y = -6, z = -12$

$(-1, -6, -12)$

Chapter 28 - Straight line in space Exercise Ex. 28.4

Question 1

Find the perpendicular distance of the point $(3, -1, 11)$ from the line

$$\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4}.$$

Solution 1

Let the foot of the perpendicular drawn from $P(3,-1,11)$ to the line

$\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}$ is Q ,so we have to find length of PQ.Q is general point on the line

$$\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}=\lambda \quad (say)$$

Co-ordinate of $Q=(2\lambda,-3\lambda+2,4\lambda+3)$

Direction ratios of the given line $=2.-3,4$

Since PQ is perpendicular to the given line therefore

$$\begin{aligned}
&a1a2+b1b2+c1c2=0\\
&\Rightarrow 2(2\lambda-3)+(-3)(-3\lambda+3)+4(4\lambda-8)=0\\
&\Rightarrow 4\lambda-6+9\lambda-9+16\lambda-32=0\\
&\Rightarrow 29\lambda-47=0\\
&\Rightarrow \lambda=\frac{47}{29}
\end{aligned}$$

Therefore co-ordinates of Q

$$\begin{aligned}
&=2\left(\frac{47}{29}\right),-3\left(\frac{47}{29}\right)+2,4\left(\frac{47}{29}\right)+3\\
&=\frac{94}{29},\frac{-83}{29},\frac{275}{29}
\end{aligned}$$

Distance between P and Q is

$$\begin{aligned}
&=\sqrt{\left(\frac{94}{29}-3\right)^2+\left(\frac{-83}{29}+1\right)^2+\left(\frac{275}{29}-11\right)^2}\\
&=\sqrt{\left(\frac{94-87}{2}\right)^2+\left(\frac{-83+29}{2}\right)^2+\left(\frac{275-319}{2}\right)^2}\\
&=\sqrt{\left(\frac{7}{29}\right)^2+\left(\frac{-54}{29}\right)^2+\left(\frac{-44}{29}\right)^2}\\
&=\sqrt{\frac{49}{841}+\frac{2916}{841}+\frac{1936}{841}}\\
&=\sqrt{\frac{4901}{841}}
\end{aligned}$$

Required distance $=\sqrt{\frac{4901}{841}}$ units

Question 2

Find the perpendicular distance of the point $(1,0,0)$ from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$.
Also, find the coordinates of the foot of the perpendicular.

Solution 2

Let foot of the perpendicular drawn from the point $P(1,0,0)$ to the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is Q. We have to find length of PQ.

Q is a general point on the line,

$$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=\lambda \quad (\text{say})$$

Coordinate of $Q=(2\lambda+1,-3\lambda-1,8\lambda-10)$

Direction ratios line PQ are

$$=(2\lambda+1-1),\ (-3\lambda-1-0),\ (8\lambda-10-0)$$

$$\Rightarrow \quad =(2\lambda),\ (-3\lambda-1),\ (8\lambda-10)$$

Since, line PQ is perpendicular to the given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(2)(2\lambda)+(-3)(-3\lambda-1)+8(8\lambda-10)=0$$
$$4\lambda+9\lambda+3+64\lambda-80=0$$
$$77\lambda-77=0$$
$$\lambda=1$$

Therefore, coordinate of Q is $(2\lambda+1,\ -3\lambda-1,\ 8\lambda-10)$

$$=(2(1)+1,\ -3(1)-1,\ 8(1)-10)$$
$$=(3,-4,-2)$$

$$PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$
$$=\sqrt{(1-3)^2+(0+4)^2+(0+2)^2}$$
$$=\sqrt{4+16+4}$$
$$=\sqrt{24}$$
$$=2\sqrt{6}$$

So, foot of perpendicular $=(3,-4,-2)$

length of perpendicular $=2\sqrt{6}$ units

Question 3

Find the foot of the perpendicular drawn from the point $A(1,0,3)$ to the line joining the points $B(4,7,1)$ and $C(3,5,3)$.

Solution 3

Let the foot of the perpendicular drawn from $A(1,0,3)$ to the line joining the points $B(4,7,1)$

And $C(3,5,3)$ be D

Equation of line passing through $B(4,7,1)$ and $C(3,5,3)$ is

$$\frac{x-x1}{x2-x1}=\frac{y-y1}{y2-y1}=\frac{z-z1}{z2-z1}$$
$$\Rightarrow \frac{x-4}{3-4}=\frac{y-7}{5-7}=\frac{z-1}{3-1}$$
$$\Rightarrow \frac{x-4}{-1}=\frac{y-7}{-2}=\frac{z-1}{2}=\lambda \text{ (say)}$$

Direction ratio of AD are

$$(-\lambda+4-1),(-2\lambda+7-0),(2\lambda+1-3)$$
$$=(-\lambda+3),(-2\lambda+7),(2\lambda-2)$$

Line AD is perpendicular to BC so

$$a1a2+b1b2+c1c2=0$$
$$\Rightarrow(-1)(-\lambda+3)+(-2)(-2\lambda+7)+2(2\lambda-2)=0$$
$$\Rightarrow \lambda-3+4\lambda-14+4\lambda-4=0$$
$$\Rightarrow 9\lambda-21=0$$
$$\Rightarrow \lambda=\frac{21}{9}$$

Co-ordinates of D are

$$=\left(-\frac{21}{9}+4,(-2)\left(\frac{21}{9}+7\right),2\left(\frac{21}{9}+1\right)\right)$$
$$=\left(\frac{15}{9},\frac{21}{9},\frac{51}{9}\right)$$
$$=\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$$

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Question 4

$A(1,0,4)$, $B(0,-11,3)$, $C(2,-3,1)$ are three points and D is the foot of perpendicular from A on BC. FInd the coordinates of D.

Solution 4

Given that D is the foot of perpendicular from $A(1,0,4)$ on BC, so

Equation of line passing through B, C is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-0}{2-0}=\frac{y+11}{-3+11}=\frac{z-3}{1-3}$$

$$\Rightarrow \quad \frac{x}{2}=\frac{y+11}{8}=\frac{z-3}{-2}=\lambda \ \text{(say)}$$

Coordinate of $D = (2\lambda,\ 8\lambda - 11,\ -2\lambda + 3)$

Direction ratios of $AD = 2\lambda - 1,\ 8\lambda - 11 - 0,\ -2\lambda + 3 - 4$

$$= (2\lambda - 1),\ (8\lambda - 11),\ (-2\lambda - 1)$$

Since, line AD is perpendicular on BC, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow \quad (2)(2\lambda - 1) + (8)(8\lambda - 11) + (-2)(-2\lambda - 1) = 0$$

$$\Rightarrow \quad 4\lambda - 2 + 64\lambda - 88 + 4\lambda + 2 = 0$$

$$\Rightarrow \quad 72\lambda - 88 = 0$$

$$\Rightarrow \quad \lambda = \frac{88}{72}$$

$$\lambda = \frac{11}{9}$$

Coordinate of $D = (2\lambda,\ 8\lambda - 11,\ -2\lambda + 3)$

$$= \left(2\left(\frac{11}{9}\right),\ 8\left(\frac{11}{9}\right) - 11,\ -2\left(\frac{11}{9}\right) + 3\right)$$

Coordinate of $D = \left(\frac{22}{9}, \frac{-11}{9}, \frac{5}{9}\right)$

Question 5

Find the foot of perpendicular from the point $(2, 3, 4)$ to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.
Also, fint the perpendicular distance from the given point to the line.

Solution 5

Let foot of the perpendicular from $P\,(2, 3, 4)$ is θ on the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$, so

Equation of given line is,

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$
$$\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=\lambda \quad \text{(say)}$$

Coordinate of $Q=\left(-2\lambda+4,\ 6\lambda,\ -3\lambda+1\right)$

Direction ratios of $PQ=\left(-2\lambda+4-2\right),\ \left(6\lambda-3\right),\ \left(-3\lambda+1-4\right)$
$$=\left(-2\lambda+2\right),\ \left(6\lambda-3\right),\ \left(-3\lambda-3\right)$$

Line PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$\left(-2\right)\left(-2\lambda+2\right)+\left(6\right)\left(6\lambda-3\right)+\left(-3\right)\left(-3\lambda-3\right)=0$$
$$4\lambda-4+36\lambda-18+9\lambda+9=0$$
$$49\lambda-13=0$$
$$\lambda=\frac{13}{49}$$

Coordinate of $Q=\left(-2\lambda+4,\ 6\lambda,\ -3\lambda+1\right)$

$$=\left(-2\left(\frac{13}{49}\right)+4,\ 6\left(\frac{13}{49}\right),\ -3\left(\frac{13}{49}\right)+1\right)$$
$$=\left(\frac{-26+196}{49},\ \frac{78}{49},\ \frac{-39+49}{49}\right)$$

Coordinate of $Q=\left(\frac{170}{49},\ \frac{78}{49},\ \frac{10}{49}\right)$

$$PQ=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2+\left(z_1-z_2\right)^2}$$
$$=\sqrt{\left(\frac{170}{49}-2\right)^2+\left(\frac{78}{49}-3\right)^2+\left(\frac{10}{49}-4\right)^2}$$
$$=\sqrt{\left(\frac{72}{49}\right)^2+\left(\frac{69}{49}\right)^2+\left(-\frac{168}{49}\right)^2}$$
$$=\sqrt{\frac{5184+4761+34596}{2401}}$$
$$=\sqrt{\frac{44541}{2401}}$$
$$=\sqrt{\frac{909}{49}}$$

$$= \frac{3\sqrt{101}}{49}$$

Perpendicular distance from $(2, 3, 4)$ to given line is $\frac{3\sqrt{101}}{49}$ units.

Question 6

Find the equation of the perpendicular drawn from the point $P(2, 4, -1)$ to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}.$$

Also, write down the coordinates of the foot of the perpendicular from P.

Solution 6

Let θ be the foot of the perpendicular drawn from $P(2, 4, -1)$ to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

Given line is $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$ (say)

Coordinate of Q (General point on the line)

$= (\lambda - 5,\ 4\lambda - 3,\ -9\lambda + 6)$

Direction ratios of PQ are

$= (\lambda - 5 - 2),\ (4\lambda - 3 - 4),\ (-9\lambda + 6 + 1)$

$= \lambda - 7,\ 4\lambda - 7,\ -9\lambda + 7$

Line PQ is perpendicular to the given line, so

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(1)(\lambda - 7) + (4)(4\lambda - 7) + (-9)(-9\lambda + 7) = 0$

$\lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$

$98\lambda - 98 = 0$

$\lambda = 1$

$\therefore$ Coordinate of $Q = (\lambda - 5,\ 4\lambda - 3,\ -9\lambda + 6)$

$= (1 - 5,\ 4(1) - 3,\ -9(1) + 6)$

Coordinate of foot of perpendicular $= (-4, 1, -3)$

So, equation of the perpendicular PQ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \quad \frac{x-2}{-4-2} = \frac{y-4}{1-4} = \frac{z+1}{-3+1}$$

$$\Rightarrow \quad \frac{x-2}{-6} = \frac{y-4}{-3} = \frac{z+1}{-2}$$

Question 7

Find the length of the perpendicular drawn from the point $(5, 4, -1)$ to the line $\vec{r} = \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k})$.

Solution 7

Let foot of the perpendicular drawn from $P(5,4,-1)$ to the given line is Q, so
Given equation of line is,

$$\vec{r} = \hat{i} + \lambda\left(2\hat{i} + 9\hat{j} + 5\hat{k}\right)$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = (1+2\lambda)\hat{i} + (9\lambda)\hat{j} + (5\lambda)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$

$\Rightarrow \quad x = 1 + 2\lambda,\ y = 9\lambda,\ z = 5\lambda$

$\Rightarrow \quad \dfrac{x-1}{2} = \lambda,\ \dfrac{y}{9} = \lambda,\ \dfrac{z}{5} = \lambda$

$\Rightarrow \quad \dfrac{x-1}{2} = \dfrac{y}{9} = \dfrac{z}{5} = \lambda \ \text{(say)}$

Coordinate of $Q = (2\lambda + 1,\ 9\lambda,\ 5\lambda)$

Direction ratios of line PQ are

$$(2\lambda + 1 - 5),\ 9\lambda - 4,\ 5\lambda + 1$$

$\Rightarrow \quad 2\lambda - 4,\ 9\lambda - 4,\ 5\lambda + 1$

Since, line PQ is perpendicular to the given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad (2)(2\lambda - 4,) + (9)(9\lambda - 4) + 5(5\lambda + 1) = 0$

$\Rightarrow \quad 4\lambda - 8 + 81\lambda - 36 + 25\lambda + 5 = 0$

$\Rightarrow \quad 110\lambda - 39 = 0$

$\Rightarrow \quad \lambda = \dfrac{39}{110}$

Coordinate of $Q = (2\lambda + 1,\ 9\lambda,\ 5\lambda)$

$$= \left(2\left(\frac{39}{110}\right) + 1,\ 9\left(\frac{39}{110}\right),\ 5\left(\frac{39}{110}\right)\right)$$

$$= \left(\frac{188}{110},\ \frac{351}{110},\ \frac{195}{110}\right)$$

Length of perpendicular = PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{\left(5 - \frac{188}{110}\right)^2 + \left(4 - \frac{351}{110}\right)^2 + \left(-1 - \frac{195}{110}\right)^2}$$

$$= \sqrt{\frac{131044 + 7921 + 93025}{12100}}$$

$$= \sqrt{\frac{231990}{12100}}$$

$$PQ = \sqrt{\frac{2109}{110}} \text{ units}$$

Question 8

Find the foot of the perpendicular drawn for the point $\hat{i} + 6\hat{j} + 3\hat{k}$ to the line $\vec{r} = \hat{j} + 2\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$. Also find the length of the perpendicular

Solution 8

Let position vector of foot of perpendicular drown from $P\left(\hat{i} + 6\hat{j} + 3\hat{k}\right)$ on $\vec{r} = \left(\hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ be $Q\left(\vec{q}\right)$. So

Q is on the line $\vec{r} = \left(\hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

So, Position vector of Q $= (\lambda)\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= \left\{\lambda\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}\right\} - \left(\hat{i} + 6\hat{j} + 3\hat{k}\right)$$

$$= (\lambda - 1)\hat{i} + (1 + 2\lambda - 6)\hat{j} + (2 + 3\lambda - 3)\hat{k}$$

$$\overrightarrow{PQ} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$$

Here, $\overrightarrow{PQ}$ is perpendicular to given line

So,

$$\left\{(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}\right\}\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 0$$

$\Rightarrow \quad (\lambda - 1)(1) + (2\lambda - 5)(2) + (3\lambda - 1)(3) = 0$

$\Rightarrow \quad \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow \quad 14\lambda - 14 = 0$

$\Rightarrow \quad \lambda = 1$

Position vector of Q $= \left(\hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$= \left(\hat{j} + 2\hat{k}\right) + (1)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

Foot of perpendicular $= \hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= \left(\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 6\hat{j} + 3\hat{k}\right)$$

$$= \hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 6\hat{j} - 3\hat{k}$$

$$= -3\hat{i} + 2\hat{k}$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{(-3)^2 + (2)^2}$$

$$= \sqrt{13} \text{ units}$$

Length of perpendicular $= \sqrt{13}$ units

Question 9

Find the equation of the perpendicular drawn from the point $P(-1, 3, 2)$ to the line $\vec{r} = \left(2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$. Also find the coordinates of the foot of the perpendicular from P.

Solution 9

Let Q be the perpendicular drown from $P\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ on the line

$$\vec{r}=\left(2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

Let the position vector of Q be

$$\left(2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

$$(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}$$

$$\overrightarrow{PQ}=\text{Position vector of } Q-\text{Position vector of } P$$

$$=\left\{(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}\right\}-\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$$

$$=(2\lambda+1)\hat{i}+(2\lambda-3)\hat{j}+(3+3\lambda-2)\hat{k}$$

$$\overrightarrow{PQ}=(2\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(3\lambda+1)\hat{k}$$

Since, $\overrightarrow{PQ}$ is perpendicular to the given line, so

$$\left\{(2\lambda+1)\hat{i}+(\lambda-1)\hat{j}+(3\lambda+1)\hat{k}\right\}\left(2\hat{i}+\hat{j}+3\hat{k}\right)=0$$

$$(2\lambda+1)(2)+(\lambda-1)(1)+(3\lambda+1)3=0$$

$$4\lambda+2+\lambda-1+9\lambda+3=0$$

$$14\lambda+4=0$$

$$\lambda=-\frac{4}{14}$$

$$\lambda=-\frac{2}{7}$$

Position vector of $Q=(2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3+3\lambda)\hat{k}$

$$=2\left(-\frac{2}{7}\right)\hat{i}+\left(2-\frac{2}{7}\right)\hat{j}+\left(3+3\left(-\frac{2}{7}\right)\right)\hat{k}$$

$$=-\frac{4}{7}\hat{i}+\frac{12}{7}\hat{j}+\frac{15}{7}\hat{k}$$

Coordinates of foot of the perpendicular $=\left(-\frac{4}{7},\frac{12}{7},\frac{15}{7}\right)$

Equation of PQ is

$$\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$$

$$\Rightarrow\quad \vec{r}=\left(-\hat{i}+3\hat{j}+2\hat{k}\right)+\lambda\left(\left(-\frac{4}{7}\hat{i}+\frac{12}{7}\hat{j}+\frac{15}{7}\hat{k}\right)-\left(-\hat{i}+3\hat{j}+2\hat{k}\right)\right)$$

$$=\left(-\hat{i}+3\hat{j}+2\hat{k}\right)+\lambda\left(\frac{3}{7}\hat{i}-\frac{9}{7}\hat{j}+\frac{1}{7}\hat{k}\right)$$

$$\vec{r}=\left(-\hat{i}+3\hat{j}+2\hat{k}\right)+\mu\left(3\hat{i}-9\hat{j}+\hat{k}\right)$$

Question 10

Find the foot of the perpendicular from $(0,2,7)$ to the line

$$\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$$

Solution 10

Let foot of the perpendicular drawn from $(0,2,7)$ to the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$ be Q.

Given equation of the line is

$$\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}=\lambda \ \text{(say)}$$

Coordinate of Q is $(-\lambda-2,\ 3\lambda+1,\ -2\lambda+3)$

Direction ratios of PQ are $(-\lambda-2-0),\ (3\lambda+1-2),\ (-2\lambda+3-7)$

$$=(-\lambda-2),\ (3\lambda-1),\ (-2\lambda-4)$$

Since, PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(-1)(-\lambda-2)+(3)(3\lambda-1)+(-2)(-2\lambda-4)=0$$

$\Rightarrow \quad \lambda+2+9\lambda-3+4\lambda+8=0$

$\Rightarrow \quad 14\lambda+7=0$

$$\lambda=-\frac{1}{2}$$

Foot of the perpendicular $=(-\lambda-2,\ 3\lambda+1,\ -2\lambda+3)$

$$=\left(-\left(-\frac{1}{2}\right)-2,\ 3\left(-\frac{1}{2}\right)+1,\ -2\left(-\frac{1}{2}\right)+3\right)$$

Foot of the perpendicular $=\left(-\frac{3}{2},-\frac{1}{2},4\right)$

Question 11

Find the foot of the perpendicular from $(1,2,-3)$ to the line

$$\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$$

Solution 11

Let foot of the perpendicular from $P(1,2,-3)$ to the line $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$ be Q

Given equation of the line is

$$\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}=\lambda$$

$\Rightarrow \quad x=2\lambda-1,\ y=-2\lambda+3,\ z=-\lambda$

Coordinate of Q $(2\lambda-1,\ -2\lambda+3,\ -\lambda)$

Direction ratios of PQ are

$$(2\lambda-1-1),\ (-2\lambda+3-2),\ (-\lambda+3)$$

$\Rightarrow \quad (2\lambda-2),\ (-2\lambda+1),\ (-\lambda+3)$

Let PQ is perpendicular to given line, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$\Rightarrow \quad (2)(2\lambda-2)+(-2)(-2\lambda+1)+(-1)(-\lambda+3)=0$

$\Rightarrow \quad 4\lambda-4+4\lambda-2+\lambda-3=0$

$\Rightarrow \quad 9\lambda-9=0$

$\Rightarrow \quad \lambda=1$

Coordinate of foot of perpendicular

$$=(2\lambda-1,\ -2\lambda+3,\ -\lambda)$$
$$=(2(1)-1,\ -2(1)+3,\ -1)$$

$$=(1,1,-1)$$

Question 12

Find the equation of line passing through the points $A(0,6,-9)$ and $B(-3,-6,3)$. If D is the foot of perpendicular drawn from a point $C(7,4,-1)$ on the line AB, then find the coordinates of the point D and the equation of line CD.

Solution 12

Equation of line AB is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-0}{-3-0}=\frac{y-6}{-6-6}=\frac{z+9}{3+9}$$

$$\Rightarrow \quad \frac{x}{-3}=\frac{y-6}{-12}=\frac{z+9}{12}=\lambda \ (\text{say})$$

Coordinate of point $D = \left(-3\lambda, -12\lambda+6, 12\lambda-9\right)$

Direction ratios of $CD = \left(-3\lambda-7\right), \left(-12\lambda+6-4\right), \left(12\lambda-9+1\right)$

$$= \left(-3\lambda-7\right), \left(-12\lambda+2\right), \left(12\lambda-8\right)$$

Line CD is perpendicular to line AB, so

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow \quad \left(-3\right)\left(-3\lambda-7\right)+\left(-12\right)\left(-12\lambda+2\right)+\left(12\right)\left(12\lambda-8\right)=0$$

$$\Rightarrow \quad 9\lambda+21+144\lambda-24+144\lambda-96=0$$

$$\Rightarrow \quad 297\lambda-99=0$$

$$\Rightarrow \quad \lambda=\frac{1}{3}$$

Coordinate of $D = \left(-3\lambda, -12\lambda+6, 12\lambda-9\right)$

$$= \left(-3\left(\frac{1}{3}\right), -12\left(\frac{1}{3}\right)+6, 12\left(\frac{1}{3}\right)-9\right)$$

Coordinate of $D = \left(-1, 2, -5\right)$

Equation of CD is,

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \quad \frac{x-7}{-1-7}=\frac{y-4}{2-4}=\frac{z+1}{-5+1}$$

$$\Rightarrow \quad \frac{x-7}{-8}=\frac{y-4}{-2}=\frac{z+1}{-4}$$

or $$\frac{x-7}{4}=\frac{y-4}{1}=\frac{z+1}{2}$$

Question 13

Find the distance of the point (2, 4, -1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

Solution 13

Let P ≡ (2, 4, -1).

In order to find the distance we need to find a point Q on the line.

We see that line is passing through the point Q(-5, -3, 6).

So, let take this point as required point.

Also line is parallel to the vector $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$.

Now, $\overrightarrow{PQ} = \left(-5\hat{i} - 3\hat{j} + 6\hat{k}\right) - \left(2\hat{i} + 4\hat{j} - \hat{k}\right) = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$$\vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ -7 & -7 & 7 \end{vmatrix} = -35\hat{i} + 56\hat{j} + 21\hat{k}$$

$$\left|\vec{b} \times \overrightarrow{PQ}\right| = \sqrt{1225+3136+441} = \sqrt{4802}$$

$$\left|\vec{b}\right| = \sqrt{1+16+81} = \sqrt{98}$$

$$d = \frac{\left|\vec{b} \times \overrightarrow{PQ}\right|}{\left|\vec{b}\right|} = \frac{\sqrt{4802}}{\sqrt{98}} = 7$$

Question 14

Find the coordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, -1, 3) and C (2, -3, -1).

Solution 14

Let L be the foot of the perpendicular drawn from A(1, 8, 4) on the line joining the points B (0, -1, 3) and C (2, -3, -1).

Equation of the line passing through the points B (0, -1, 3) and C (2, -3, -1) is given by,

$\vec{r} = \vec{b} + \lambda\left(\vec{c} - \vec{b}\right)$

$\vec{r} = (0 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k}$

Let positon vector of L be,

$\vec{r} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k}$ ....(i)

Then, $\overrightarrow{AL}$ = Position vector of L - position vector of A

$\Rightarrow \overrightarrow{AL} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} - \left(1\hat{i} + 8\hat{j} + 4\hat{k}\right)$

$\Rightarrow \overrightarrow{AL} = (-1 + 2\lambda)\hat{i} + (-9 - 2\lambda)\hat{j} + (-1 - 4\lambda)\hat{k}$

Since $\overrightarrow{AL}$ is perpendicular to the given line which is parallel to $\vec{b} = 2\hat{i} - 2\hat{j} - 4\hat{k}$

$\therefore \quad \overrightarrow{AL} \cdot \vec{b} = 0$

$\Rightarrow 2(-1 + 2\lambda) - 2(-9 - 2\lambda) - 4(-1 - 4\lambda) = 0$

$\Rightarrow -2 + 4\lambda + 18 + 4\lambda + 4 + 16\lambda = 0$

$\Rightarrow 24\lambda = -20$

$\Rightarrow \lambda = \frac{-5}{6}$

Putting valure of $\lambda = \frac{-5}{6}$ in (i) we get

$\vec{r} = -\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{19}{3}\hat{k}$

Coordinates of the foot of the perpendicular are $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$.

Chapter 28 - Straight line in space Exercise Ex. 28.5

Question 1(i)

Find the shortest distance between the pair of lines whose vector equation is:

$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$

and, $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$

Solution 1(i)

We know that, shortest distance between lines

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are.

$$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) + \mu\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_1} = \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right), \ \overrightarrow{b_1} = 3\hat{i} - \hat{j} + \hat{k}$$

$$\overrightarrow{a_2} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right), \ \overrightarrow{b_2} = \left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

Now,

$$\overrightarrow{a_2} - \overrightarrow{a_1} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) - \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right)$$

$$= -3\hat{i} - 7\hat{j} + 6\hat{k} - 3\hat{i} - 8\hat{j} - 3\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}\left(-4 - 2\right) - \hat{j}\left(12 + 3\right) + \hat{k}\left(6 - 3\right)$$

$$= \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right).\left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)$$

$$= \left(-6\right)\left(6\right) + \left(-15\right)\left(-15\right) + \left(3\right)\left(3\right)$$

$$= 36 + 225 + 9$$

$$= 270$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{\left(-6\right)^2 + \left(-15\right)^2 + \left(3\right)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

Substituting values of $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|$ and $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \frac{270}{\sqrt{270}}$$
$$= \sqrt{270}$$

$$\text{S.D.} = 3\sqrt{30} \text{ units}$$

Question 1(ii)

Find the shortest distance between the pair of lines whose vector equation is:

$$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 7\hat{k}\right)$$

and, $\vec{r} = -\hat{i} - \hat{j} - \hat{k} + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$

Solution 1(ii)

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 7\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$$

$\Rightarrow$ $\overrightarrow{a_1} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right)$, $\overrightarrow{b_1} = \left(\hat{i} - 2\hat{j} + 7\hat{k}\right)$

$\overrightarrow{a_2} = \left(-\hat{i} - \hat{j} - \hat{k}\right)$, $\overrightarrow{b_2} = \left(7\hat{i} - 6\hat{j} + \hat{k}\right)$

So, $\overrightarrow{a_2} - \overrightarrow{a_1} = \left(-\hat{i} - \hat{j} - \hat{k}\right) - \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right)$

$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$$= -4\hat{i} - 6\hat{j} - 8\hat{k} = -2\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ 7 & -6 & 1 \end{vmatrix}$$

$$= \hat{i}\left(-2 + 42\right) - \hat{j}\left(1 - 49\right) + \hat{k}\left(-6 + 14\right)$$

$$= 40\hat{i} + 48\hat{j} + 8\hat{k}$$

$$= 8\left(5\hat{i} + 6\hat{j} + \hat{k}\right)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left\{-2\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)\right\}\left\{8\left(5\hat{i} + 6\hat{j} + \hat{k}\right)\right\}$$

$$= -16\left[(2)(5) + (3)(6) + (4)(1)\right]$$

$$= -16\left[10 + 18 + 4\right]$$

$$= -16 \times 32$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -512$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = 8\sqrt{(5)^2 + (6)^2 + (1)^2}$$

$$= 8\sqrt{25 + 36 + 1}$$

$$= 8\sqrt{62}$$

Substituting values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get required shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-512}{8\sqrt{62}}\right|$$

$$\text{S.D.} = \frac{512}{\sqrt{3968}}$$

Question 1(iii)

Find the shortest distance between the pair of lines whose vector equation is:

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

and, $\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$

Solution 1(iii)

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \text{ and}$$
$$\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

Now, $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$= 2\hat{i} + 4\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$
$$= \hat{i}\left(15 - 16\right) - \hat{j}\left(10 - 12\right) + \hat{k}\left(8 - 9\right)$$
$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (1)(-1) + (2)(2) + (2)(-1)$$
$$= -1 + 4 - 2$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 1$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{1 + 4 + 1}$$
$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{6}$$

Substituting values of $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{1}{\sqrt{6}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Question 1(iv)

Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

Solution 1(iv)

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-t)\hat{k}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Above equations can be rewritten as

$$\vec{r} = (i - 2j + 3k) + t(-i + j - k)$$

$$\vec{r} = (i - j - k) + s(i + 2j - 2k)$$

Shortest distance is given by $\left| \frac{(a_2 - a_1) \cdot (b_1 \times b_2)}{|(b_1 \times b_2)|} \right|$

$$(b_1 \times b_2) = -3j - 3k$$

$$(a_2 - a_1) = j - 4k$$

$$(a_2 - a_1) \cdot (b_1 \times b_2) = 9$$

$$|(b_1 \times b_2)| = 3\sqrt{2}$$

Shortest distance is $\frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$

Question 1(v)

Find the shortest distance between the pair of lines whose vector equation is:

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$$

and, $\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

Solution 1(v)

We know that, the shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

$$\vec{r} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$$

$\Rightarrow$ $\vec{r} = \left(-\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$ and

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

$\Rightarrow$ $\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

So, $\overrightarrow{a_1} = \left(-\hat{i} + \hat{j} - \hat{k}\right)$, $\overrightarrow{b_1} = \left(\hat{i} + \hat{j} - \hat{k}\right)$ and

$\overrightarrow{a_2} = \left(\hat{i} - \hat{j} + 2\hat{k}\right)$, $\overrightarrow{b_2} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(\hat{i} - \hat{j} + 2\hat{k}\right) - \left(-\hat{i} + \hat{j} - \hat{k}\right)$$

$$= \hat{i} - \hat{j} + 2\hat{k} + \hat{i} - \hat{j} + \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = 2\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}\,(1 + 2) - \hat{j}\,(1 - 1) + \hat{k}\,(2 + 1)$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 3\hat{i} + 3\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(2\hat{i} - 2\hat{j} + 3\hat{k}\right)\left(3\hat{i} + 3\hat{k}\right)$$

$$= (2)(3) + (-2)(0) + (3)(3)$$

$$= 6 + 0 + 9$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 15$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{18}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = 3\sqrt{2}$$

Substituting values of $\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|$ in equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\text{S.D.} = \frac{5}{\sqrt{2}} \text{ units}$$

Question 1(vi)

Find the shortest distance between the pair of lines whose vector equation is:

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 5\hat{j} + 2\hat{k}\right);\ \vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$

Solution 1(vi)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 5\hat{j} + 2\hat{k}\right)$ and

$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$

$\Rightarrow$ $\vec{a_1} = \left(2\hat{i} - \hat{j} - \hat{k}\right)$, $\vec{b_1} = \left(2\hat{i} - 5\hat{j} + 2\hat{k}\right)$ and

$\vec{a_2} = \left(\hat{i} + 2\hat{j} + \hat{k}\right)$, $\vec{b_2} = \left(\hat{i} - \hat{j} + \hat{k}\right)$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} + 2\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} - \hat{k}\right)$$
$$= \hat{i} + 2\hat{j} + \hat{k} - 2\hat{i} + \hat{j} + \hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= \hat{i}\left(-5 + 2\right) - \hat{j}\left(2 - 2\right) + \hat{k}\left(-2 + 5\right)$$
$$= -3\hat{i} + 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = \left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)\left(-3\hat{i} + 3\hat{k}\right)$$
$$= \left(-1\right)\left(-3\right) + \left(3\right)\left(0\right) + \left(2\right)\left(3\right)$$
$$= 3 + 0 + 6$$
$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = 9$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{\left(-3\right)^2 + \left(3\right)^2}$$
$$= \sqrt{9 + 9}$$
$$\left|\vec{b_1} \times \vec{b_2}\right| = 3\sqrt{2}$$

Substituting the values of $\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{9}{3\sqrt{2}}\right|$$

$$\text{S.D.} = \frac{3}{\sqrt{2}}$$

Question 1(vii)

Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

Solution 1(vii)

Given,

$$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ------(i)$$

and

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right) \qquad ------(ii)$$

Comparing (i) and (ii) with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ respectively, we get

$$\vec{a}_1 = \hat{i} + \hat{j}, \qquad \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \qquad \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$\therefore \qquad \vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

and $\qquad \vec{b}_1 \times \vec{b}_2 = \left(2\hat{i} - \hat{j} + \hat{k}\right) \times \left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

So, $\qquad \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9 + 1 + 49} = \sqrt{59}$

Hence, the shortest distance between the lines l_1 and l_2 is given by

$$d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right) . \left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

Question 1(viii)

Find the shortest distance between the following pair of lines whose vector equations are:

$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Solution 1(viii)

The equation of lines are

$\vec{r} = (8+3\lambda)\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

The lines pass through $\vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$ and $\vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$

and parallel to vectors, $\vec{b_1} = 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k}$ and $\vec{b_2} = 3\mu\hat{i} + 8\mu\hat{j} - 5\mu\hat{k}$

$\vec{a_1} - \vec{a_2} = -7\hat{i} - 38\hat{j} + 5\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

So, $(\vec{a_1} - \vec{a_2}) \cdot (\vec{b_1} \times \vec{b_2}) = -168 - 1368 + 360 = -1176$

$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{576 + 1296 + 5184} = 84$

$$\text{S.D.} = \left| \frac{(\vec{a_1} - \vec{a_2}) \cdot (\vec{b_1} \times \vec{b_2})}{\left|\vec{b_1} \times \vec{b_2}\right|} \right| = \left| \frac{-1176}{84} \right| = 14$$

Question 2(i)

Find the shortest distance between the pair of lines whose Cartesian equation is:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

Solution 2(i)

Given lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \ \text{(say)}$$

$$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 4\lambda + 3$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (4\lambda + 3)\hat{k}$$
$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right),\ \vec{b_1} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

and, $$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5} = \mu \ \text{(say)}$$

$$x = 3\mu + 2,\ y = 4\mu + 3,\ z = 5\mu + 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (3\mu + 2)\hat{i} + (4\mu + 3)\hat{j} + (5\mu + 5)\hat{k}$$
$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right),\ \vec{b_2} = \left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

We know that, the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$
$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\begin{aligned}\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) &= \left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right) \\ &= (1)(-1) + (1)(2) + (2)(-1) \\ &= -1 + 2 - 2\end{aligned}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = -1$$

$$\begin{aligned}\left|\vec{b_1} \times \vec{b_2}\right| &= \sqrt{(-1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{1 + 4 + 1} \\ &= \sqrt{6}\end{aligned}$$

Using the values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-1}{\sqrt{3}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Question 2(ii)

Find the shortest distance between the pair of lines whose Cartesian equation is:

$$\frac{x-1}{2} = \frac{y+1}{3} = z \text{ and } \frac{x+1}{3} = \frac{y-2}{1}; z = 2$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 3\hat{j} - 7\hat{k}$$

$$\begin{aligned}\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) &= \left(2\hat{i} - 3\hat{j} - 2\hat{k}\right)\left(-\hat{i} + 3\hat{j} - 7\hat{k}\right) \\ &= (2)(-1) + (-3)(3) + (-2)(-7) \\ &= -2 - 9 + 14\end{aligned}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 3$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (-7)^2} = \sqrt{59}$$

Substitute the value of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{3}{\sqrt{59}}\right|$$

$$\text{S.D.} = \frac{3}{\sqrt{59}} \text{ units}$$

Solution 2(ii)

Given equations of line are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \quad (\text{say})$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + \lambda\hat{k}$$
$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

$\Rightarrow \quad \overrightarrow{a_1} = \hat{i} - \hat{j},\ \overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

and, $\quad \dfrac{x+1}{3} = \dfrac{y-2}{1} = \mu,\ z = 2$

$\Rightarrow \quad x = 3\mu - 1,\ y = \mu + 2,\ z = 2$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (3\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$
$$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(3\hat{i} + \hat{j})$$

$\Rightarrow \quad \overrightarrow{a_2} = (-\hat{i} + 2\hat{j} + 2\hat{k}),\ \overrightarrow{b_2} = (3\hat{i} + \hat{j})$

We know that, the shortest distance between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by,

$$\text{S.D.} = \left| \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right| \qquad ---(i)$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (\hat{i} - \hat{j}) - (-\hat{i} + 2\hat{j} + 2\hat{k})$$
$$= \hat{i} - \hat{j} + \hat{i} - 2\hat{j} - 2\hat{k}$$
$$(\overrightarrow{a_2} - \overrightarrow{a_1}) = 2\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}(0 - 1) - \hat{j}(0 - 3) + \hat{k}(2 - 9)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -\hat{i} + 3\hat{j} - 7\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(2\hat{i} - 3\hat{j} - 2\hat{k}\right)\left(-\hat{i} + 3\hat{j} - 7\hat{k}\right)$$
$$= (2)(-1) + (-3)(3) + (-2)(-7)$$
$$= -2 - 9 + 14$$
$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 3$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (-7)^2} = \sqrt{59}$$

Substitute the value of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{3}{\sqrt{59}}\right|$$

$$\text{S.D.} = \frac{3}{\sqrt{59}} \text{ units}$$

Question 2(iii)

Find the shortest distance between the pair of lines whose Cartesian equation is:

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} \text{ and } \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}$$

Solution 2(iii)

Given equation of lines are,

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = -\lambda + 1,\ y = \lambda - 2,\ z = -2\lambda + 3$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= (-\lambda + 1)\hat{i} + (\lambda - 2)\hat{j} + (-2\lambda + 3)\hat{k}$$

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$\Rightarrow \quad \vec{a_1} = (\hat{i} - 2\hat{j} + 3\hat{k}),\ \vec{b_1} = (-\hat{i} + \hat{j} - 2\hat{k})$

and, $\quad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2} = \mu \ \text{(say)}$

$\Rightarrow \quad x = \mu + 1,\ y = 2\mu - 1,\ z = -2\mu - 1$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (-2\mu - 1)\hat{k}$$

$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$$

$\Rightarrow \quad \vec{a_2} = (\hat{i} - \hat{j} - \hat{k}),\ \vec{b_2} = (\hat{i} + 2\hat{j} - 2\hat{k})$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k}$$

$$= \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(-2 + 4) - \hat{j}(2 + 2) + \hat{k}(-2 - 1)$$

$$(\vec{b_1} \times \vec{b_2}) = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{29}$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right) = \left(\hat{j}-4\hat{k}\right)\left(2\hat{i}-4\hat{j}-3\hat{k}\right)$$
$$= (0)(2)+(1)(-4)+(-4)(-3)$$
$$= 0-4+12$$
$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right) = 8$$

We know that, shortest distance between $\vec{r} = \overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2}+\lambda\overrightarrow{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

So, shortest distance between given lines is

$$\text{S.D.} = \left|\frac{8}{\sqrt{29}}\right|$$

$$\text{S.D.} = \frac{8}{\sqrt{29}} \text{ units}$$

Question 2(iv)

Find the shortest distance between the pair of lines whose Cartesian equation is:

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}; \quad \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

Solution 2(iv)

Given equation of lines are,

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}=\lambda \quad (\text{say})$$

$\Rightarrow \quad x=\lambda+3,\ y=-2\lambda+5,\ z=\lambda+7$

$$\Rightarrow \quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$=(\lambda+3)\hat{i}+(-2\lambda+5)\hat{j}+(\lambda+7)\hat{k}$$
$$\vec{r}=\left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}+\hat{k}\right)$$

$\Rightarrow \quad \overrightarrow{a_1}=\left(3\hat{i}+5\hat{j}+7\hat{k}\right),\ \overrightarrow{b_1}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$

and, $\quad \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}=\mu \quad (\text{say})$

$\Rightarrow \quad x=7\mu-1,\ y=-6\mu-1,\ z=\mu-1$

$$\Rightarrow \quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$=(7\mu-1)\hat{i}+(-6\mu-1)\hat{j}+(\mu-1)\hat{k}$$
$$\vec{r}=\left(-\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(7\hat{i}-6\hat{j}+\hat{k}\right)$$

$\Rightarrow \quad \overrightarrow{a_2}=\left(-\hat{i}-\hat{j}-\hat{k}\right),\ \overrightarrow{b_2}=7\hat{i}-6\hat{j}+\hat{k}$

We know that, shortest distance between $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ and $\vec{r}=\overrightarrow{a_2}+\lambda\overrightarrow{b_2}$ is given by,

$$\text{S.D.}=\left|\frac{\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right| \qquad ---(\text{i})$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=\left(-\hat{i}-\hat{j}-\hat{k}\right)-\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$$
$$=-\hat{i}-\hat{j}-\hat{k}-3\hat{i}-5\hat{j}-7\hat{k}$$
$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=-4\hat{i}-6\hat{j}-8\hat{k}$$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 7 & -6 & 1\end{vmatrix}$$
$$=\hat{i}(-2+6)-\hat{j}(1-7)+\hat{k}(-6+14)$$
$$\overrightarrow{b_1}\times\overrightarrow{b_2}=4\hat{i}+6\hat{j}+8\hat{k}$$

$$\left|\vec{b_1}\times\vec{b_2}\right| = \sqrt{(4)^2+(6)^2+(8)^2}$$
$$= \sqrt{16+36+64}$$
$$= \sqrt{116}$$
$$= 2\sqrt{29}$$

$$\left(\vec{a_2}-\vec{a_1}\right)\left(\vec{b_1}\times\vec{b_2}\right) = \left(-4\hat{i}-6\hat{j}-8\hat{k}\right)\left(4\hat{i}+6\hat{j}+8\hat{k}\right)$$
$$= (-4)(4)+(-6)(6)+(-8)(8)$$
$$= -16-36-64$$
$$= -116$$

Substituting the values of $\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)$ and $\left|\vec{b_1}\times\vec{b_2}\right|$ in equation (i) to get the shortest distance between the two given lines, so

$$\text{S.D.} = \left|\frac{-116}{2\sqrt{29}}\right|$$
$$= \frac{58}{\sqrt{29}}$$

$$\text{S.D.} = 2\sqrt{29} \text{ units}$$

Question 3(i)

By computing the shortest distance determine whether the pairs of lines intersect or not:

$$\vec{r} = \left(\hat{i}-\hat{j}\right)+\lambda\left(2\hat{i}+\hat{k}\right);\ \vec{r} = \left(2\hat{i}-\hat{j}\right)+\mu\left(\hat{i}+\hat{j}-\hat{k}\right)$$

Solution 3(i)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$$

$$\Rightarrow \qquad \overrightarrow{a_1} = \left(\hat{i} - \hat{j}\right),\ \overrightarrow{b_1} = \left(2\hat{i} + \hat{k}\right)$$

and, $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

$$\Rightarrow \qquad \overrightarrow{a_2} = \left(2\hat{i} - \hat{j}\right),\ \overrightarrow{b_2} = \left(\hat{i} + \hat{j} - \hat{k}\right)$$

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(\text{i})$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} - \hat{j}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 2\hat{i} - \hat{j} - \hat{i} + \hat{j}$$
$$= \hat{i}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(-2 - 1\right) + \hat{k}\left(2 - 0\right)$$
$$= -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(\hat{i}\right)\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$$
$$= (1)(-1) + (0)(3) + (0)(2)$$
$$= -1 + 0 + 0$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -1$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (2)^2}$$
$$= \sqrt{1 + 9 + 4}$$
$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{14}$$

So, shortest distance between the given lines using equation (1) is,

$$\text{S.D.} = \left|\frac{-1}{\sqrt{14}}\right|$$

$$= \frac{1}{\sqrt{14}} \text{ units}$$

$S.D \neq 0$

Since, shortest distance between lines is not zero, so lines are not intersecting.

Question 3(ii)

By computing the shortest distance determine whether the pairs of lines intersect or not:

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right);\ \vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

Solution 3(ii)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\Rightarrow \quad \overrightarrow{a_1} = \left(\hat{i} + \hat{j} - \hat{k}\right),\ \overrightarrow{b_1} = \left(3\hat{i} - \hat{j}\right)$$

and, $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$

$$\Rightarrow \quad \overrightarrow{a_2} = \left(4\hat{i} - \hat{k}\right),\ \overrightarrow{b_2} = \left(2\hat{i} + 3\hat{k}\right)$$

We know that, shortest distance between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(\text{i})$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(4\hat{i} - \hat{k}\right) - \left(\hat{i} + \hat{j} - \hat{k}\right)$$
$$= 4\hat{i} - \hat{k} - \hat{i} - \hat{j} + \hat{k}$$
$$= 3\hat{i} - \hat{j}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$
$$= \hat{i}\left(-3 - 0\right) - \hat{j}\left(9 - 0\right) + \hat{k}\left(0 + 2\right)$$
$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{\left(-3\right)^2 + \left(-9\right)^2 + \left(2\right)^2}$$
$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{9 + 81 + 4}$$
$$= \sqrt{94}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(3\hat{i} - \hat{j}\right)\left(-3\hat{i} - 9\hat{j} + 2\hat{k}\right)$$
$$= \left(3\right)\left(-3\right) + \left(-1\right)\left(-9\right) + \left(0\right)\left(2\right)$$
$$= -9 + 9 + 0$$
$$= 0$$

Using $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|$ in equation (i) to get shortest distance between given lines, so

$$S.D. = \left|\frac{0}{\sqrt{94}}\right|$$

$$S.D = 0$$

Since, shortest distance between the given lines is zero, so lines are intersecting.

Question 3(iii)

By computing the shortest distance determine whether the pairs of lines intersect or not:

$$\frac{x-1}{2} = \frac{y+1}{3} = z; \quad \frac{x+1}{5} = \frac{y-2}{1}; \; z = 2$$

Solution 3(iii)

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \ (\text{say})$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + (\lambda)\hat{k}$$
$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

$\Rightarrow \quad \overrightarrow{a_1} = (\hat{i} - \hat{j}),\ \overrightarrow{b_1} = (2\hat{i} + 3\hat{j} + \hat{k})$

and, $\quad \frac{x+1}{5} = \frac{y-2}{1} = \mu \ (\text{say}),\ z = 2$

$\Rightarrow \quad x = 5\mu - 1,\ y = \mu + 2,\ z = 2$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (5\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$
$$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j})$$

$\Rightarrow \quad \overrightarrow{a_2} = (-\hat{i} + 2\hat{j} + 2\hat{k}),\ \overrightarrow{b_2} = (5\hat{i} + \hat{j})$

We know that, the shortest distance between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left| \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right| \qquad ---(\text{i})$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (-\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} - \hat{j})$$
$$= -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j}$$
$$= -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}(0 - 1) - \hat{j}(0 - 5) + \hat{k}(2 - 15)$$
$$\overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)=\left(-2\hat{i}+3\hat{j}+2\hat{k}\right)\left(-\hat{i}+5\hat{j}-13\hat{k}\right)$$
$$=(-2)(-1)+(3)(5)+(2)(-13)$$
$$=-9$$

$$\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{(-1)^2+(5)^2+(-13)^2}$$
$$=\sqrt{1+25+169}$$
$$=\sqrt{195}$$

Substituting the value of $\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.}=\left|\frac{-9}{\sqrt{195}}\right|$$

$$=\frac{9}{\sqrt{195}}\text{ units}$$

Since, shortest distance between given lines is not zero, so lines are not intersecting.

Question 3(iv)

By computing the shortest distance determine whether the pairs of lines intersect or not:

$$\frac{x-5}{4}=\frac{y-7}{-5}=\frac{z+3}{-5};\quad\frac{x-8}{7}=\frac{y-7}{1}=\frac{z-5}{3}$$

Solution 3(iv)

Given lines are,

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = 4\lambda + 5,\ y = -5\lambda + 7,\ z = -5\lambda - 3$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (4\lambda + 5)\hat{i} + (-5\lambda + 7)\hat{j} + (-5\lambda - 3)\hat{k}$$
$$\vec{r} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_1} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right),\ \overrightarrow{b_1} = \left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$$

and, $$\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3} = \mu \ \text{(say)}$$

$\Rightarrow \quad x = 7\mu + 8,\ y = \mu + 7,\ 3\mu + 5$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (7\mu + 8)\hat{i} + (\mu + 7)\hat{j} + (3\mu + 5)\hat{k}$$
$$\vec{r} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) + \mu\left(7\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_2} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right),\ \overrightarrow{b_2} = \left(7\hat{i} + \hat{j} + 3\hat{k}\right)$$

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) - \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right)$$
$$= 8\hat{i} + 7\hat{j} + 5\hat{k} - 5\hat{i} - 7\hat{j} + 3\hat{k}$$
$$= 3\hat{i} + 8\hat{k}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$
$$= \hat{i}(-15 + 5) - \hat{j}(12 + 35) + \hat{k}(4 + 35)$$
$$= -10\hat{i} - 47\hat{j} + 39\hat{k}$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)=\left(3\hat{i}+8\hat{k}\right)\left(-10\hat{i}-47\hat{j}+39\hat{k}\right)$$
$$=(3)(-10)+(0)(-4)+(8)(39)$$
$$=-30+312$$
$$=282$$

Using equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.}=\left|\frac{282}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right|$$

$$\text{S.D.}\neq 0$$

Since, the shortest distance between given lines is not equal to zero, so

Given lines are not intersecting.

Question 4(i)

Find the shortest distance between the pairs of parallel lines whose equations is:

$$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right) \text{ and } \vec{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(-\hat{i}+\hat{j}-\hat{k}\right)$$

Solution 4(i)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right) \quad ---(1)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - \hat{k}\right)$$
$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$
$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu'\left(\hat{i} - \hat{j} + \hat{k}\right) \quad ---(2)$$

These two lines passes through the points having position vectors $\overrightarrow{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ respectively and both are parallel to the vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

We know that, shortest distance between parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\vec{b}}{\left|\vec{b}\right|}\right| \quad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= \hat{i}\left(-3 - 4\right) - \hat{j}\left(1 + 4\right) + \hat{k}\left(-1 + 3\right)$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{\left(-7\right)^2 + \left(-5\right)^2 + \left(2\right)^2}$$
$$= \sqrt{49 + 25 + 4}$$
$$= \sqrt{78}$$

$$\left|\vec{b}\right| = \sqrt{\hat{i} - \hat{j} + \hat{k}}$$
$$= \sqrt{\left(1\right)^2 + \left(-1\right)^2 + \left(1\right)^2}$$
$$\left|\vec{b}\right| = \sqrt{3}$$

Using $\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between parallel lines, so

$$\text{S.D.} = \frac{\sqrt{78}}{\sqrt{3}}$$

$$\text{S.D.} = \sqrt{\frac{78}{3}}$$

$$\text{S.D.} = \sqrt{26}\ \text{units}$$

Question 4(ii)

Find the shortest distance between the pairs of parallel lines whose equations is:

$$\vec{r} = \left(\hat{i}+\hat{j}\right)+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)\ \text{and}\ \vec{r} = \left(2\hat{i}+\hat{j}-\hat{k}\right)+\mu\left(4\hat{i}-2\hat{j}+2\hat{k}\right)$$

Solution 4(ii)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(1)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(4\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + 2\mu\left(2\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu'\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(2)$$

So, $\overrightarrow{a_1} = \left(\hat{i} + \hat{j}\right)$, $\overrightarrow{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

We know that, the shortest distance between the parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\vec{b}$ is given by

$$\text{S.D.} = \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} + \hat{j} - \hat{k}\right) - \left(\hat{i} + \hat{j}\right)$$

$$= 2\hat{i} + \hat{j} - \hat{k} - \hat{i} - \hat{j}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \hat{i} - \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(1 + 2\right) + \hat{k}\left(-1 - 0\right)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = -\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{\left(-1\right)^2 + \left(-3\right)^2 + \left(-1\right)^2}$$

$$= \sqrt{1 + 9 + 1}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{11}$$

$$\left|\vec{b}\right| = \sqrt{\left(2\right)^2 + \left(-1\right)^2 + \left(1\right)^2}$$

$$= \sqrt{4 + 1 + 1}$$

$$\left|\vec{b}\right| = \sqrt{6}$$

Using $\left|\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\times\vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{\sqrt{11}}{\sqrt{6}}$$

$$\text{S.D.} = \sqrt{\frac{11}{6}} \text{ units}$$

Question 5

Find the equations of the lines joining the following pairs of vertices and then find the shortest distance between the lines

(i) $(0,0,0)$ and $(1,0,2)$ (ii) $(1,3,0)$ and $(0,3,0)$

Solution 5

Equation of line passing through $(0,0,0)$ and $(1,0,2)$ is given by $\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$

$$\vec{r} = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \lambda\left((1-0)\hat{i} + (0-0)\hat{j} + (2-0)\hat{k}\right)$$

$$\vec{r} = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{k}\right) \qquad ---(1)$$

Equation of another line passing through $(1,3,0)$ and $(0,3,0)$ is

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + \mu\left((0-1)\hat{i} + (3-3)\hat{j} + (0-0)\hat{k}\right)$$

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + \mu\left(-\hat{i}\right) \qquad ---(2)$$

From equation (1) and (2)

$$\overrightarrow{a_1} = \left(0.\hat{i} + 0.\hat{j} + 0.\hat{k}\right),\ \overrightarrow{b_1} = \left(\hat{i} + 2\hat{k}\right)$$

$$\overrightarrow{a_2} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right), \qquad \overrightarrow{b_2} = -\hat{i}$$

We know that, shortest distance between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \qquad ---(3)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(\hat{i} + 3\hat{j} + 0.\hat{k}\right) - \left(0.\hat{i} + 0.\hat{j} + 0\hat{k}\right)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(\hat{i} + 3\hat{j}\right)$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0+2) + \hat{k}(-2)$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -2\hat{j}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(\hat{i} + 3\hat{j}\right)\left(-2\hat{j}\right)$$

$$= (1)(0) + (3)(-2)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -6$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-2)^2}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = 2$$

Using $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|$ in equation (1) to get shortest distance between the lines, so

$$\text{S.D.} = \left|\frac{-6}{2}\right|$$

$$\text{S.D.} = 3 \text{ units}$$

Question 6

Write the vector equations of the following lines and hence determine the distance between them

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6};\ \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Solution 6

Given equations of lines are,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}=\lambda \quad (\text{say})$$

$\Rightarrow \quad x=2\lambda+1,\ y=3\lambda+2,\ z=6\lambda-4$

$$\Rightarrow \quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$=(2\lambda+1)\hat{i}+(3\lambda+2)\hat{j}+(6\lambda-4)\hat{k}$$
$$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

$\Rightarrow \quad \overrightarrow{a_1}=\hat{i}+2\hat{j}-4\hat{k},\ \vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$

Another equation of line is,

$$\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}=\mu \quad (\text{say})$$

$\Rightarrow \quad x=4\mu+3,\ y=6\mu+3,\ 12\mu-5$

$$\Rightarrow \quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$=(4\mu+3)\hat{i}+(6\mu+3)\hat{j}+(12\mu-5)\hat{k}$$
$$=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+\mu\left(4\hat{i}+6\hat{j}+12\hat{k}\right)$$
$$=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+2\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$
$$\vec{r}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+\mu'\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

$\Rightarrow \quad \overrightarrow{a_2}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right),\ \vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$

We know that, shortest distance between parallel lines $\vec{r}=\overrightarrow{a_1}+\lambda\vec{b}$ and $\vec{r}=\overrightarrow{a_2}+\mu\vec{b}$ is given by

$$\text{S.D.}=\frac{\left|\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\times\vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(i)$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)-\left(\hat{i}+2\hat{j}-4\hat{k}\right)$$
$$=3\hat{i}+3\hat{j}-5\hat{k}-\hat{i}-2\hat{j}+4\hat{k}$$
$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=2\hat{i}+\hat{j}-\hat{k}$$

$$\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix}$$

$$=\hat{i}(6+3)-\hat{j}(12+2)+\hat{k}(6-2)$$

$$=9\hat{i}-14\hat{j}+4\hat{k}$$

$$\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b}\right|=\sqrt{(9)^2+(-14)^2+(4)^2}$$

$$=\sqrt{81+196+16}$$

$$=\sqrt{293}$$

$$\left|\vec{b}\right|=\sqrt{(2)^2+(3)^2+(6)^2}$$

$$=\sqrt{4+9+36}$$

$$=\sqrt{49}$$

$$\left|\vec{b}\right|=7$$

Using $\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.}=\frac{\sqrt{293}}{7}\text{ units}$$

Question 7(i)

Find the shortest distance between the lines

$\vec{r}=(\vec{i}+2\vec{j}+\vec{k})+\lambda(\vec{i}-\vec{j}+\vec{k})$ and $\vec{r}=2\vec{i}-\vec{j}-\vec{k}+\mu(2\vec{i}+\vec{j}+2\vec{k})$

Solution 7(i)

Here,

$a_1=\hat{i}+2\hat{j}+\hat{k}$

$b_1=\hat{i}-\hat{j}+\hat{k}$

$a_2=2\hat{i}-\hat{j}-\hat{k}$

$b_2=2\hat{i}+\hat{j}+2\hat{k}$

$\bar{a}_2-\bar{a}_1=2\hat{i}-\hat{j}-\hat{k}-\hat{i}-2\hat{j}-\hat{k}=\hat{i}-3\hat{j}-2\hat{k}$

$b_1\times b_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}=\hat{i}(-2-1)-\hat{j}(2-2)+\hat{k}(1+2)=-3\hat{i}+3\hat{k}$

The shortest distance between the two lines,

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\bar{a}_2-\bar{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$d=\left|\frac{(-3\hat{i}+3\hat{k}).(\hat{i}-3\hat{j}-2\hat{k})}{|-3\hat{i}-3\hat{k}|}\right|=\left|\frac{-3-6}{\sqrt{(-3)^2+(-3)^2}}\right|=\frac{9}{3\sqrt{2}}$$

The shortest distance between the two lines $=\dfrac{3}{\sqrt{2}}$ units

Question 7(ii)

Find the shortest distance between the lines

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

Solution 7(ii)

Here,

$\vec{a}_1=-\hat{i}-\hat{j}-\hat{k}$

$\vec{a}_2=3\hat{i}+5\hat{j}+7\hat{k}$

$\vec{b}_1=7\hat{i}-6\hat{j}+\hat{k}$

$\vec{b}_2=\hat{i}-2\hat{j}+\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$$

$=\hat{i}(-6+2)-\hat{j}(7-1)+\hat{k}(-14+6)$

$=-4\hat{i}-6\hat{j}-8\hat{k}$

$\vec{a}_2-\vec{a}_1=\hat{i}(3+1)+\hat{j}(5+1)+\hat{k}(7+1)$

$=4\hat{i}+6\hat{j}+8\hat{k}$

The shortest distance between two lines,

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|b_1\times b_2|}\right|$$

$$=\left|\frac{(-4\hat{i}-6\hat{j}-8\hat{k}).(4\hat{i}+6\hat{j}+8\hat{k})}{\sqrt{(-4)^2+(-6)^2+(-8)^2}}\right|$$

$$=\left|\frac{-16-36-64}{\sqrt{116}}\right|$$

$$=\left|\frac{-116}{\sqrt{116}}\right|$$

$=2\sqrt{29}$ units

Question 7(iii)

Find the shortest distance between the lines

$$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(\hat{i}-3\hat{j}+2\hat{k}) \text{ and } \vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$$

Solution 7(iii)

Here,

$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k},$

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k},$

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2$

$= (\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

Shortest distance between the two lines $= \left|\frac{(a_2 - a_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$= \left|\frac{(3\hat{i} + 3\hat{j} + 3\hat{k}).(-9\hat{i} + 3\hat{j} + 9\hat{k})}{|-9\hat{i} + 3\hat{j} + 9\hat{k}|}\right|$

$= \left|\frac{3 \times (-9) + 3 \times 3 + 3 \times 9}{\sqrt{(-9)^2 + 3^2 + 9^2}}\right|$

$= \left|\frac{-27 + 9 + 27}{\sqrt{(-9)^2 + 3^2 + 9^2}}\right|$

$= \left|\frac{9}{\sqrt{171}}\right| = \frac{3}{\sqrt{19}}$ units

Question 7(iv)

Find the shortest distance between the lines

$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

Solution 7(iv)

Here,

$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - \hat{k}$

$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k}$

$= -10\hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$

$= 8\hat{i} + 8\hat{j} + 4\hat{k}$

$$\text{Shortest Distance} = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{|8\hat{i} - 8\hat{j} + 4\hat{k}|}\right|$$

$$= \left|\frac{(-10) \times 8 + (-2) \times 8 + (-3) \times 4}{\sqrt{8^2 + (-8)^2 + (-4)^2}}\right|$$

$$= \left|\frac{-80 - 16 - 12}{\sqrt{64 + 64 + 16}}\right| = \left|\frac{-108}{\sqrt{144}}\right| = 9 \text{ units}$$

Question 8

Find the distance between the lines l_1 and l_2 given by

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and}$$
$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Solution 8

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$

$$= 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$$

$$= 9\hat{i} - 14\hat{j} + 4\hat{k}$$

Shortest distance betweeen 2lines

$$= \left|\frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|}\right|$$

$$= \left|\frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\left|\sqrt{2^2 + 3^2 + 6^2}\right|}\right|$$

$$= \left|\frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\sqrt{49}}\right|$$

$$= \left|\frac{\sqrt{9^2 + (-14)^2 + 4^2}}{\sqrt{49}}\right|$$

$$= \left|\frac{\sqrt{293}}{\sqrt{49}}\right| = \frac{\sqrt{293}}{7} \text{units}$$

Chapter 28 - Straight line in space Exercise MCQ

Question 1

The angle between the straight lines

$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4} \text{ and } \frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3} \text{ is}$$

a. 45°
b. 30°
c. 60°
d. 90°

Solution 1

Correct option: (d)

$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4} \text{ and } \frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$$

Let the direction ratios $\vec{a}1 = 2\hat{i} + 5\hat{j} + 4\hat{k}$, $\vec{a}2 = \hat{i} + 2\hat{j} - 3\hat{k}$

Let θ be the angle between the lines.

$$\cos\theta = \frac{\vec{a}_1 \cdot \vec{a}_2}{|\vec{a}1||\vec{a}2|}$$

$$\cos\theta = \frac{(2\hat{i} + 5\hat{j} + 4\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{2^3 + 5^2 + 4^2}\sqrt{1^2 + 2^2 + 3^2}}$$

$$\cos\theta = 0$$

$$\Rightarrow \theta = 90°$$

Question 2

The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are

a. coincident
b. skew
c. intersecting
d. parallel

Solution 2

Correct option: (a)

$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$

Direction ratios of $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are $1,2,3$.

Direction ratios of $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are $-2,-4,-6$.

$\Rightarrow \frac{-2}{2},\frac{-4}{2},\frac{-6}{2}$

$\Rightarrow -1,-2,-3$

Direction ratios of both lines are same but on opposite directions.

Hence, they are coincident.

Question 3

The direction ratios of the line perpendicular to the lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ and $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are proportional to

a. 4, 5, 7
b. 4, -5, 7
c. 4, -5, -7
d. -4, 5, 7

Solution 3

Correct option: (a)

Direction ratios of line $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ are

$a_1=2,\ b_1=-3,\ c_1=1.$

Direction ratios of line $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are

$a_2=1,\ b_2=2,\ c_2=-2.$

The direction ratios perpendicular to both the lines

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 1\\ 1 & 2 & -2\end{vmatrix}$$

$$\vec{a}\times\vec{b}=(6-2,\ 4+1,\ 4+3)$$

$$\vec{a}\times\vec{b}=(4,\ 5,\ 7)$$

Question 4

The angle between the lines

$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{2}$ and $\frac{x-1}{-\sqrt{3}-1}=\frac{y-1}{\sqrt{3}-1}=\frac{z-1}{4}$ is

a. $\cos^{-1}\left(\frac{1}{65}\right)$

b. $\frac{\pi}{6}$

c. $\frac{\pi}{3}$

d. $\frac{\pi}{4}$

Solution 4

Correct option: (c)

Direction ratios of the line $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{2}$ are 1, 1, 2

Direction ratios of the line $\frac{x-1}{-\sqrt{3}-1}=\frac{y-1}{\sqrt{3}-1}=\frac{z-1}{4}$ are $-\sqrt{3}-1, \sqrt{3}-1, 4$

Let angle between the lines be θ.

$$\cos\theta=\frac{\left(-\sqrt{3}-1+\sqrt{3}-1+8\right)}{\sqrt{\left(-\sqrt{3}-1\right)^2+\left(\sqrt{3}-1\right)^2+4}\sqrt{1+1+4}}$$

$$\cos\theta=\frac{6}{12}$$

$$\cos\theta=\frac{1}{2}$$

$$\Rightarrow\theta=\frac{\pi}{3}$$

Question 5

The direction of the lines x - y + z - 5= 0 = x - 3y - 6 are proportional to

a. 3, 1, -2

b. 2, -4, 1

c. $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$

d. $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

Solution 5

Correct option: (a)

$x - y + x - 5 = 0$

$x - 3y - 6 = 0$

Solving both equations,

$\Rightarrow y = \frac{x-6}{3} = \frac{-z-1}{2}$

$\Rightarrow \frac{x-6}{3} = y = \frac{-z-1}{2}$

Direction ratios of the given line are proportional to $3, 1, -2$.

Question 6

The perpendicular distance of the point $P(1,2,3)$ from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

a. 7
b. 5
c. 0
d. none of these

Solution 6

Correct option: (a)

The perpendicular distance of the point $P(1,2,3)$ from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

The line passes through points $P(1,2,3)$ and $R(6,7,7)$ having direction ratios $3, 2, -2$.

$$\text{perpendicular to both vector} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -2 \\ 5 & 5 & 4 \end{vmatrix}$$

$$\text{perpendicular to both vector} = (8+10)\hat{i} - (12+10)\hat{j} + (15-10)\hat{k}$$

$$\text{perpendicular to both vector} = 18\hat{i} - 22\hat{j} + 5\hat{k}$$

$$\Rightarrow \text{Perpendicular distance} = \sqrt{18^2 + 22^2 + 5^2} = \sqrt{833}$$

$$d = \frac{\sqrt{833}}{\sqrt{17}} = 7$$

Question 7

The equation of the line passing through the points

$a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is

a. $\vec{r}=\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right)+\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$

b. $\vec{r}=\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right)-t\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$

c. $\vec{r}=a_1(1-t)\hat{i}+a_2(1-t)\hat{j}+a_3(1-t)\hat{k}+t\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$

d. none of these

Solution 7

Correct option: (c)

The equation of the line passing through the points

$a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is

$\vec{r}=\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right)+t\left[\left(b_1-a_1\right)\hat{i}+\left(b_2-a_2\right)\hat{j}+\left(b_3-a_3\right)\hat{k}\right]$

$\vec{r}=\vec{r}=a_1(1-t)\hat{i}+a_2(1-t)\hat{j}+a_3(1-t)\hat{k}+t\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$

Question 8

If a line makes angles α, β and γ with the axes respectively, then cos 2 α + cos 2 β + cos 2 γ =

a. -2
b. -1
c. 1
d. 2

Solution 8

Correct option: (b)

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma$

$=2\cos^2\alpha-1+2\cos^2\beta-1+2\cos^2\gamma-1$

$=2\cos^2\alpha+2\cos^2\beta+2\cos^2\gamma-3$

$=2\left(\cos^2\alpha+\cos^2\beta+\cos^2\gamma\right)-3$

$=2\times 1-3$

$=-1$

NOTE: Answer not matching with back answer.

Question 9

If the direction ratios of a line are proportional to 1, -3, 2, then its direction cosines are

a. $\frac{1}{\sqrt{14}},-\frac{3}{\sqrt{14}},\frac{2}{\sqrt{14}}$

b. $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$

c. $-\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}},\frac{2}{\sqrt{14}}$

d. $-\frac{1}{\sqrt{14}},-\frac{2}{\sqrt{14}},-\frac{3}{\sqrt{14}}$

Solution 9

Correct option: (a)

Direction cosines of the ratios 1, −3, 2 are given by

$$\frac{1}{\sqrt{1+9+4}}, \frac{-3}{\sqrt{1+9+4}}, \frac{2}{\sqrt{1+9+4}}$$

$$\Rightarrow \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$$

Question 10

If a line makes angle $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with x-axis and y-axis respectively, then the angle made by the line with z-axis is

a. $\pi/2$

b. $\pi/3$

c. $\pi/4$

d. $5\pi/12$

Solution 10

Correct option: (b)

Angle made by the line with x − axis is $\alpha = \frac{\pi}{3}$.

Angle made by the line with y − axis is $\beta = \frac{\pi}{4}$.

Angle made by the line with z − axis is λ.

$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma = \frac{1}{4}$$

$$\Rightarrow \cos\gamma = \frac{1}{2}$$

$$\Rightarrow \gamma = \frac{\pi}{3}$$

Question 11

The projections of a line segment on X, Y and Z axes are 12, 4 and 3 respectively. The length and direction cosines of the line segment are

a. $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

b. $19; \frac{12}{19}, \frac{4}{19}, \frac{3}{19}$

c. $11; \frac{12}{11}, \frac{14}{11}, \frac{3}{11}$

d. none of these

Solution 11

Correct option: (a)

Length of direction cosines = $\sqrt{12^2 + 4^2 + 3^2} = 13$

direction cosines = $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

Question 12

The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are

a. parallel
b. intersecting
c. skew
d. coincident

Solution 12

Correct option: (d)

The lines Direction ratios of the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are 1,2,3

The lines Direction ratios of the line $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are

$-2, -4, -6$

Let angle between the lines be θ.

$$\cos\theta = \frac{-2-8-18}{\sqrt{1+4+9}\sqrt{4+16+36}}$$

$\cos\theta = -1$

$\theta = 180°$

The line are coincident.

NOTE: Answer not matching with back answer.

Question 13

The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is

a. parallel to x-axis
b. parallel to y-axis
c. parallel to z-axis
d. perpendicular to z-axis

Solution 13

Correct option: (d)

Given straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0} = \lambda$

Consider, $\frac{z-1}{0} = \lambda \Rightarrow z = 1$

Intersection of a line and z – axis is at $(0,0,1)$.

Direction cosines of the line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

from origin are 3, 1, 0.

Direction cosines of a point $(3,1,0)$ are $(0,0,1)$.

Let θ be the angle between z axis and straight line given.

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^o$

$\Rightarrow$ Given straight line is perpendicular to z axis.

Question 14

The shortest distance between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and, $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is

a. $\sqrt{30}$

b. $2\sqrt{30}$

c. $5\sqrt{30}$

d. $3\sqrt{30}$

Solution 14

Correct option: (d)

$$\left(a_2 - a_1\right)\cdot\left(b_1 \times b_2\right) = \left(-6\hat{i} - 15\hat{j} + 3\hat{k}\right)\cdot\left(\left(3\hat{i} - 1\hat{j} + \hat{k}\right) \times \left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)\right)$$

$$\left(a_2 - a_1\right)\cdot\left(b_1 \times b_2\right) = \begin{vmatrix} -6 & -15 & 3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$\left(a_2 - a_1\right)\cdot\left(b_1 \times b_2\right) = -6(-4-2) + 15(12+3) + 3(6-3)$$

$$\left(a_2 - a_1\right)\cdot\left(b_1 \times b_2\right) = 36 + 225 + 9$$

$$\left(a_2 - a_1\right)\cdot\left(b_1 \times b_2\right) = 270$$

$$\Rightarrow d = \frac{\left(a_2 - a_1\right)\cdot\left(b_1 \times b_2\right)}{\left|b_1 \times b_2\right|}$$

$$d = \frac{270}{\sqrt{270}}$$

$$d = \sqrt{270}$$

$$d = 3\sqrt{30}$$

Chapter 28 - Straight line in space Exercise Ex. 28VSAQ

Question 1

Write the cartesian and vector equations of X-axis.

Solution 1

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(\text{i})$$

We know that, x-axis passes through origin $(0,0,0)$ and direction ratios of the x-axis are 1,0,0, so

Equation of x-axis using eqation (i)

$$\frac{x - 0}{1} = \frac{y - 0}{0} = \frac{z - 0}{0} = \lambda \text{ (say)}$$

$$\Rightarrow \qquad x = \lambda,\ y = 0,\ z = 0$$

So, vector equation of x-axis is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda)\hat{i} + (0)\hat{j} + (0)\hat{k}$$

$$x = \lambda\hat{i}$$

Question 2

Write the cartesian and vector equations of Y-axis.

Solution 2

We know that, equation of a line passing through (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(\text{i})$$

We know that, y-axis passes through origin $(0,0,0)$ and its direction ratios are 0,1,0 so,

Equation of y-axis using eqation (i)

$$\frac{x - 0}{0} = \frac{y - 0}{1} = \frac{z - 0}{0} = \lambda \text{ (say)}$$

$$\Rightarrow \qquad x = 0,\ y = \lambda,\ z = 0$$

So, vector equation of y-axis is

$$x\hat{i} + y\hat{j} + z\hat{k} = (0)\hat{i} + (\lambda)\hat{j} + (0)\hat{k}$$

$$y = \lambda\hat{j}$$

Question 3

Write the cartesian and vector equations of Z-axis.

Solution 3

We know that, equation of a line passing through (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

We know that, z-axis passes through origin $(0, 0, 0)$ and direction ratios of the z-axis are $0, 0, 1$.

Equation of z-axis is

$$\frac{x - 0}{0} = \frac{y - 0}{0} = \frac{z - 0}{1} = \lambda \text{ (say)}$$

$$\Rightarrow \qquad x = 0,\ y = 0,\ z = \lambda$$

So, vector equation of z-axis is,

$$x\hat{i} + y\hat{j} + z\hat{k} = (0)\hat{i} + (0)\hat{j} + (\lambda)\hat{k}$$

$$z = \lambda\hat{k}$$

Question 4

Write the vector equation of a line passing through a point having position vector $\vec{\alpha}$ and parallel to vector $\vec{\beta}$.

Solution 4

We know that equation of a line passing through a point having position vector $\vec{a}$ and parallel to vector $\vec{b}$ is given by

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here, $\vec{a} = \vec{\alpha}$ and $\vec{b} = \vec{\beta}$

So, required vector equation is

$$\vec{r} = \vec{\alpha} + \lambda\vec{\beta}$$

Question 5

Cartesian equations of a line AB are $\frac{2x - 1}{2} = \frac{4 - y}{7} = \frac{z + 1}{2}$

Write the direction ratios of the line parallel to AB.

Solution 5

$$\frac{2x-1}{2}=\frac{4-y}{7}=\frac{z+1}{2}\rightarrow\frac{x-\frac{1}{2}}{1}=\frac{y-4}{-7}=\frac{z+1}{2}$$

The direction ratios of the given line is propotional

to the direction ratios of the line parallel to the given line $\rightarrow(1,-7,2)$

$$\rightarrow\left(\frac{1}{\sqrt{54}},\frac{-7}{\sqrt{54}},\frac{2}{\sqrt{54}}\right)$$

Question 6

Write the direction cosines of the line whose cartesian equations are

$6x-2=3y+1=2z-4$.

Solution 6

Given equation of line are

$6x-2=3y+1=2z-4$

Dividing by 6,

$$\frac{6x-2}{6}=\frac{3y+1}{6}=\frac{2z-4}{6}$$

$$\Rightarrow\quad \frac{6x}{6}=\frac{2}{6}=\frac{3y}{6}+\frac{1}{6}=\frac{2z}{6}-\frac{4}{6}$$

$$\Rightarrow\quad x-\frac{1}{3}=\frac{y}{2}+\frac{1}{6}=\frac{z}{3}-\frac{2}{3}$$

$$\Rightarrow\quad \frac{x-\frac{1}{3}}{1}=\frac{1}{2}\left(y+\frac{1}{3}\right)=\frac{1}{3}(z-2)$$

$$\Rightarrow\quad \frac{x-\frac{1}{3}}{1}=\frac{y+\frac{1}{3}}{2}=\frac{z-2}{3}$$

$\Rightarrow$ Direction ratios of given line are 1,2,3

Direction cosines of given line are

$$\frac{1}{\sqrt{(1)^2+(2)^2+(3)^2}},\frac{2}{\sqrt{(1)^2+(2)^2+(3)^2}},\frac{3}{\sqrt{(1)^2+(2)^2+(3)^2}}$$

$$\Rightarrow\quad \frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$$

So, direction cosines of given line are $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$

Question 7

Write the direction cosines of the line $\frac{x-2}{2}=\frac{2y-5}{-3}, z=2$.

Solution 7

Given equation of line is

$$\frac{x-2}{2} = \frac{2y-5}{-3},\ z = 2$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-\frac{5}{2}}{\frac{-3}{2}},\ z = 2$$

$\Rightarrow$ Direction ratios of given line are $2, \frac{-3}{2}, 0$

$\Rightarrow$ Direction cosines of given line are

$$\frac{2}{\sqrt{(2)^2+\left(\frac{-3}{2}\right)^2+(0)^2}},\ \frac{\frac{-3}{2}}{\sqrt{(2)^2+\left(\frac{-3}{2}\right)^2+(0)^2}},\ \frac{0}{\sqrt{(2)^2+\left(\frac{-3}{2}\right)^2+(0)^2}}$$

$$\Rightarrow \quad \frac{2}{\sqrt{\frac{25}{4}}},\ \frac{\frac{-3}{2}}{\sqrt{\frac{25}{4}}},\ \frac{0}{\sqrt{\frac{25}{4}}}$$

$$\Rightarrow \quad \frac{2\times 2}{5},\ \frac{-3}{2}\times\frac{2}{5},\ 0\times\frac{2}{5}$$

$$\Rightarrow \quad \frac{4}{5},\ \frac{-3}{5},\ 0$$

So, direction cosines of the given line are $\frac{4}{5}, \frac{-3}{5}, 0$

Question 8

Write the coordinate axis to which the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-1}{0}$ is perpendicular.

Solution 8

Given equation of line is

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-1}{0}$$

We know that, equation of z-axis is

$$\frac{x-0}{0}=\frac{y-0}{0}=\frac{z-0}{1}$$

Now, $a_1a_2+b_1b_2+c_1c_2$

$= (3)(0)+(4)(0)+(0)(1)$

$= 0+0+0$

$= 0$

$\because \quad a_1a_2+b_1b_2+c_1c_2 = 0$

So, given line is perpendicular to z-axis

Question 9

Write the angle between the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z-2}{1}$ and $\frac{x-1}{1}=\frac{y}{2}=\frac{z-1}{3}$.

Solution 9

Given equations of lines are

$$\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z-2}{1} \qquad ---(1)$$

and, $$\frac{x-1}{1}=\frac{y}{2}=\frac{z-1}{3} \qquad ---(2)$$

$\Rightarrow \quad a_1 = 7,\ b_1 = -5,\ c_1 = 1$

$a_2 = 1,\ b_2 = 2,\ c_2 = 3$

We know that angle (θ) between two lines is given by

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_3^2}}$$

$$\cos\theta = \frac{(7)(1)+(-5)(2)+(1)(3)}{\sqrt{(7)^2+(-5)^2+(1)^2}\sqrt{(1)^2+(2)^2+(3)^2}}$$

$$= \frac{7-10+3}{\sqrt{75}\sqrt{14}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Question 10

Write the direction cosines of the line whose cartesian equations are $2x = 3y = -z$.

Solution 10

Given equations of lines is

$$2x = 3y = -z$$

Dividing by 6,

$$\frac{2x}{6} = \frac{3y}{6} = \frac{-z}{6}$$

$$\Rightarrow \quad \frac{x}{3} = \frac{y}{2} = \frac{-z}{6}$$

$\Rightarrow$ Direction ratios of given line are 3, 2, −6

Direction cosines of given line are

$$\frac{3}{\sqrt{(3)^2+(2)^2+(-6)^2}}, \frac{2}{\sqrt{(3)^2+(2)^2+(-6)^2}}, \frac{-6}{\sqrt{(3)^2+(2)^2+(-6)^2}}$$

$$\Rightarrow \quad \frac{3}{\sqrt{9+4+36}}, \frac{2}{\sqrt{9+4+36}}, \frac{-6}{\sqrt{9+4+36}}$$

$$\Rightarrow \quad \frac{3}{\sqrt{49}}, \frac{2}{\sqrt{49}}, \frac{-6}{\sqrt{49}}$$

$$\Rightarrow \quad \frac{3}{7}, \frac{2}{7}, \frac{-6}{7}$$

So, direction cosines of given line are $\frac{3}{7}, \frac{2}{7}, \frac{-6}{7}$

Question 11

Write the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$.

Solution 11

Given equations of lines are,

$$2x = 3y = -z$$

$$\frac{2x}{6} = \frac{3y}{6} = \frac{-z}{6}$$

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6} \qquad ---(1)$$

and, $6x = -y = -4z$

$$\frac{6x}{12} = \frac{-y}{12} = \frac{-4z}{12}$$

$$\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3} \qquad ---(2)$$

We know that, angle (θ) between two lines is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_3^2}}$$

$$\cos\theta = \frac{(3)(2) + (2)(-12) + (-6)(-3)}{\sqrt{(3)^2 + (2)^2 + (-6)^2}\sqrt{(2)^2 + (-12)^2 + (-3)^2}}$$

$$= \frac{6 - 24 + 18}{\sqrt{49}\sqrt{157}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Question 12

Write the value of λ for which the lines $\frac{x-3}{-3} = \frac{y+2}{2\lambda} = \frac{z+4}{2}$ and $\frac{x+1}{3\lambda} = \frac{y-2}{1} = \frac{z+6}{-5}$ are perpendicular to each other.

Solution 12

Given that, lines

$$\frac{x-3}{-3} = \frac{y+2}{2\lambda} = \frac{z+4}{2} \qquad ---(1)$$

and, $$\frac{x+1}{3\lambda} = \frac{y-2}{1} = \frac{z+6}{-5} \qquad ---(2)$$

are perpendicular, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(-3)(3\lambda) + (2\lambda)(1) + (2)(-5) = 0$$
$$-9\lambda + 2\lambda - 10 = 0$$
$$-7\lambda - 10 = 0$$
$$-7\lambda = 10$$

$$\lambda = \frac{-10}{7}$$

Question 13

Write the formula for the shortest distance between the lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$.

Solution 13

Given equation of line are

$$\vec{r} = \overrightarrow{a_1} + \lambda\vec{b} \text{ and } \vec{r} = \overrightarrow{a_2} + \mu\vec{b}$$

Clearly, lines are parallel

and, we know that shortest distance (S.D.) between two parallel lines is given by,

$$\text{S.D.} = \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right|}{\left|\vec{b}\right|}$$

Question 14

Write the condition for the lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ to be intersectiong.

Solution 14

We know that, shortest distance (S.D.) between two lines $\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda \overrightarrow{b_2}$ is given by,

$$\text{S.D.} = \left| \frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right| \qquad ---(1)$$

Here, linere $\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda \overrightarrow{b_2}$ are intersectiong lines, so

S.D. = 0

$$\Rightarrow \quad \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} = 0 \qquad \left[\text{Using equation } (1)\right]$$

$$\Rightarrow \quad \left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \cdot \left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)\right| = 0$$

Question 15

The cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

Solution 15

Given equations of lines is,

$$\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$$

$$\Rightarrow \quad \frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{y+2}{2} = \frac{z-3}{3}$$

$\Rightarrow$ Direction ratios of the line are $\frac{\sqrt{3}}{2}, 2, 3$

Direction cosines of given line are

$$\frac{\frac{\sqrt{3}}{2}}{\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+(2)^2+(3)^2}}, \frac{2}{\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+(2)^2+(3)^2}}, \frac{3}{\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+(2)^2+(3)^2}}$$

$$\Rightarrow \quad \frac{\frac{\sqrt{3}}{2}}{\sqrt{\frac{55}{4}}}, \frac{2}{\sqrt{\frac{55}{4}}}, \frac{3}{\sqrt{\frac{55}{4}}}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{55}}, \frac{2\times 2}{\sqrt{55}}, \frac{3\times 2}{\sqrt{55}}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$$

So, direction cosines of the given line $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$.

Question 16

16.

If the equations of a line AB are $\frac{3-x}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, write the direction ratios of a line parallel to AB.

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Solution 16

From the given equation

$\frac{3-x}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$,we can find the direction ratios

Therefore the direction ratios will be

$$\frac{x-3}{-1}=\frac{y-2}{2}=\frac{5-z}{-4}$$

So the direction ratios will be

$$(-1,2,-4)$$

Question 17

17. Write the vector equation of a line given by $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

Solution 17

The given line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

The direction ratios of the line is $(3,7,2)$

We know that the vector equation of a line is $\vec{r}=\vec{a}+\overline{\lambda b}$

$$\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$$
$$\vec{b}=3\hat{i}+4\hat{j}+\hat{k}$$

Thus the vector equation will be

$$\vec{a}+\lambda\vec{b}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+\hat{k}\right)$$

Question 18

18. The equations of a line are given $\frac{4-x}{3}=\frac{y+3}{3}=\frac{z+2}{6}$. Write the direction cosines of a line parallel to the above line.

.

Solution 18

$$\frac{4-x}{3}=\frac{y+3}{3}=\frac{z+2}{6}\rightarrow\frac{x-4}{-1}=\frac{y+3}{1}=\frac{z+2}{2}$$

$(-1,1,2)$ are direction ratios

$$\sqrt{(-1)^2+(1)^2+(2)^2}=\sqrt{1+1+4}=\sqrt{6}$$

l,m,n are direction cosines

$$l=\frac{-1}{\sqrt{6}}$$

$$m=\frac{1}{\sqrt{6}}$$

$$n=\frac{2}{\sqrt{6}}$$

Question 19

Find the cartesian equations of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

Solution 19

The Cartesian equation of a line is,

$$\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$$

$$\frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$$

The direction ratios of the given line are $3, -5, 6$.

Since the lines are parallel, the direction ratios of the required line are $3, -5, 6$.

Cartesian equation of a line is,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

The Cartesian equation of a line passing through $(-2, 4, -5)$ is

$$\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$$

Question 20

Find the angle between the lines

$\vec{r}=(2\hat{i}-5\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$

and $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$.

Solution 20

$\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$
$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

The direction ratios of the first line are $a_1 = 3, b_1 = 2, c_1 = 6$.
The direction ratios of the second line are $a_2 = 1, b_2 = 2, c_2 = 2$.

The angle between the two lines,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a^2 + b^2 + c^2}\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{3 \times 1 + 2 \times 2 + 6 \times 2}{\sqrt{3^2 + 2^2 + 6^2}\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{3 + 4 + 12}{\sqrt{49}\sqrt{9}}$$

$$= \frac{19}{7 \times 3}$$

$$= \frac{19}{21}$$

$$\theta = \cos^{-1}(\frac{19}{21})$$