Access answers to RD Sharma Solutions for Class 11 Maths Chapter 5 – Trigonometric Functions

EXERCISE 5.1 PAGE NO: 5.18

Prove the following identities:

1. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Solution:

Let us consider LHS: $\sec^4 x - \sec^2 x$

$(\sec^2 x)^2 - \sec^2 x$

By using the formula, $\sec^2 \theta = 1 + \tan^2 \theta$.

$(1 + \tan^2 x)^2 - (1 + \tan^2 x)$

$1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$

$\tan^4 x + \tan^2 x$

= RHS

∴ LHS = RHS

Hence proved.

2. $\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$

Solution:

Let us consider LHS: $\sin^6 x + \cos^6 x$

$(\sin^2 x)^3 + (\cos^2 x)^3$

By using the formula, $a^3 + b^3 = (a + b) (a^2 + b^2 - ab)$

$(\sin^2 x + \cos^2 x) [(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$

By using the formula, $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x$

$1^2 - 3\sin^2 x \cos^2 x$

$1 - 3\sin^2 x \cos^2 x$

= RHS

∴ LHS = RHS

Hence proved.

3. (cosec x – sin x) (sec x – cos x) (tan x + cot x) = 1

Solution:

Let us consider LHS: (cosec x – sin x) (sec x – cos x) (tan x + cot x)

By using the formulas

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\left(\frac{1}{\sin x} - \sin x\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$\frac{1-\sin^2 x}{\sin x} \times \frac{1-\cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

By using the formula,

$\mathrm{Sin}^2 x + \cos^2 x = 1;$

$$\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

4. cosec x (sec x – 1) – cot x (1 – cos x) = tan x – sin x

Solution:

Let us consider LHS: cosec x (sec x – 1) – cot x (1 – cos x)

By using the formulas

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\frac{1}{\sin x}\left(\frac{1}{\cos x}-1\right)-\frac{\cos x}{\sin x}(1-\cos x)$$

$$\frac{1}{\sin x}\left(\frac{1-\cos x}{\cos x}\right)-\frac{\cos x}{\sin x}(1-\cos x)$$

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1}{\cos x}-\cos x\right)$$

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1-\cos^2 x}{\cos x}\right)$$

By using the formula, $1 - \cos^2 x = \sin^2 x$;

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{\sin^2 x}{\cos x}\right)$$

$$(1-\cos x)\left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\sin x}{\cos x}-\sin x$$

$$\tan x-\sin x$$

= RHS

∴ LHS = RHS

Hence Proved.

5.

$$\frac{1-\sin x\cos x}{\cos x(\sec x-\operatorname{cosec} x)}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}=\sin x$$

Solution:

Let us consider the LHS:

$$\frac{1-\sin x\cos x}{\cos x(\sec x-\operatorname{cosec} x)}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}$$

By using the formula,

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

Now,

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{1}{\cos x}-\frac{1}{\sin x}\right)} \times \frac{(\sin x)^2-(\cos x)^2}{(\sin x)^3+(\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{\sin x-\cos x}{\cos x\sin x}\right)} \times \frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x\,(1-\sin x\cos x)}{\sin x-\cos x} \times \frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x\,(1-\sin x\cos x)}{1} \times \frac{1}{[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x\,(1-\sin x\cos x) \times \frac{1}{(1-\sin x\cos x)}$$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{1}{\cos x}-\frac{1}{\sin x}\right)} \times \frac{(\sin x)^2-(\cos x)^2}{(\sin x)^3+(\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{\sin x-\cos x}{\cos x\sin x}\right)} \times \frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x\,(1-\sin x\cos x)}{\sin x-\cos x} \times \frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x\,(1-\sin x\cos x)}{1} \times \frac{1}{[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x\,(1-\sin x\cos x) \times \frac{1}{(1-\sin x\cos x)}$$

$\sin x$

= RHS

∴ LHS = RHS

Hence Proved.

6.

$$\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-tan x}=(\sec x \; cosec \; x+1)$$

Solution:

Let us consider the LHS:

$$\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-tan x}$$

By using the formula,

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\frac{\frac{\sin x}{\cos x}}{1-\frac{\cos x}{\sin x}}+\frac{\frac{\cos x}{\sin x}}{1-\frac{\sin x}{\cos x}}$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x-\cos x}{\sin x}}+\frac{\frac{\cos x}{\sin x}}{\frac{\cos x-\sin x}{\cos x}}$$

$$\frac{\sin^2 x}{\cos x(\sin x-\cos x)}-\frac{\cos^2 x}{\sin x(\sin x-\cos x)}$$

$$\frac{\sin^3 x-\cos^3 x}{\sin x\cos x(\sin x-\cos x)}$$

By using the formula, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\frac{(\sin x - \cos x)\,[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x\,(\sin x - \cos x)}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$\frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

By using the formula,

cosec θ = 1/sin θ,

sec θ = 1/cos θ;

cosec x × sec x + 1

sec x cosec x + 1

=RHS

∴ LHS = RHS

Hence Proved.

7.

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Solution:

Let us consider LHS:

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

By using the formula $a^3 \pm b^3 = (a \pm b)(a^2 + b^2 \mp ab)$

$$\frac{(\sin x + \cos x)\,[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x} + \frac{(\sin x - \cos x)\,[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

1 – sinx cosx + 1 + sinx cosx

2

= RHS

∴ LHS = RHS

Hence Proved.

8. $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2 = 1$

Solution:

Let us consider LHS:

$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$

Expanding the above equation we get,

$[(\sec x \sec y)^2 + (\tan x \tan y)^2 + 2 (\sec x \sec y) (\tan x \tan y)] - [(\sec x \tan y)^2 + (\tan x \sec y)^2 + 2 (\sec x \tan y) (\tan x \sec y)]$ $[\sec^2 x \sec^2 y + \tan^2 x \tan^2 y + 2 (\sec x \sec y) (\tan x \tan y)] - [\sec^2 x \tan^2 y + \tan^2 x \sec^2 y + 2 (\sec^2 x \tan^2 y) (\tan x \sec y)]$

$\sec^2 x \sec^2 y - \sec^2 x \tan^2 y + \tan^2 x \tan^2 y - \tan^2 x \sec^2 y$

$\sec^2 x (\sec^2 y - \tan^2 y) + \tan^2 x (\tan^2 y - \sec^2 y)$

$\sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$

We know, $\sec^2 x - \tan^2 x = 1$.

$\sec^2 x \times 1 - \tan^2 x \times 1$

$\sec^2 x - \tan^2 x$

1 = RHS

∴ LHS = RHS

Hence proved.

9.

$$\frac{\cos x}{1-\sin x} = \frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

Solution:

Let us Consider RHS:

$$\frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

$$\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}$$

$$\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}\times\frac{(1+\cos x)+(\sin x)}{(1+\cos x)+(\sin x)}$$

$$\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{1+\cos^2 x+2\cos x+\sin^2 x+2\sin x+2\sin x\cos x}{1+\cos^2 x+2\cos x-\sin^2 x}$$

We know, $\sin^2 x+\cos^2 x=1$.

$$\frac{1+1+2\cos x+2\sin x+2\sin x\cos x}{(1-\sin^2 x)+\cos^2 x+2\cos x}$$

We know, $1-\cos^2 x=\sin^2 x$.

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$\frac{2 + 2\cos x + 2\sin x + 2\sin x\cos x}{2\cos^2 x + 2\cos x}$$

$$\frac{2 + 2\cos x + 2\sin x + 2\sin x\cos x}{\cos^2 x + \cos^2 x + 2\cos x}$$

$$\frac{1 + \cos x + \sin x + \sin x\cos x}{\cos x\,(\cos x + 1)}$$

$$\frac{1(1 + \cos x) + \sin x\,(\cos x + 1)}{\cos x\,(\cos x + 1)}$$

$$\frac{(1 + \sin x)(\cos x + 1)}{\cos x\,(\cos x + 1)}$$

$$\frac{1 + \sin x}{\cos x} \times \frac{\cos x}{\cos x}$$

$$\frac{(1 + \sin x)\cos x}{\cos^2 x}$$

We know, $1 - \sin^2 x = \cos^2 x$.

$$\frac{(1 + \sin x)\cos x}{1 - \sin^2 x}$$

$$\frac{(1 + \sin x)\cos x}{(1 - \sin x)(1 + \sin x)}$$

$$\frac{\cos x}{1 - \sin x}$$

= LHS

∴ LHS = RHS

Hence Proved.

10.

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x} = \frac{1 - 2\sin^2 x\cos^2 x}{\sin x\cos x}$$

Solution:

Let us consider LHS:

$$\frac{\tan^3 x}{1+\tan^2 x}+\frac{\cot^3 x}{1+\cot^2 x}$$

By using the formulas,

1 + $\tan^2$x = $\sec^2$x and 1 + $\cot^2$x = cosec^2x

$$\frac{\tan^3 x}{\sec^2 x}+\frac{\cot^3 x}{\mathrm{cosec}^2 x}$$

$$\frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}}+\frac{\frac{\cos^3 x}{\sin^3 x}}{\frac{1}{\sin^2 x}}$$

$$\frac{\sin^3 x}{\cos x}+\frac{\cos^3 x}{\sin x}$$

$$\frac{\sin^4 x+\cos^4 x}{\cos x \sin x}$$

$$\frac{(\sin^2 x)^2+(\cos^2 x)^2}{\cos x \sin x}$$

We know, $a^2 + b^2 = (a + b)^2 - 2ab$

$$\frac{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x}{\sin x\cos x}$$

$$\frac{1^2-2\sin^2 x\cos^2 x}{\sin x\cos x}$$

$$\frac{1-2\sin^2 x\cos^2 x}{\sin x\cos x}$$

= RHS

∴ LHS = RHS

Hence Proved.

11.

$$1-\frac{\sin^2 x}{1+\cot x}-\frac{\cos^2 x}{1+\tan x}=\sin x\cos x$$

Solution:

Let us consider LHS:

$$1-\frac{\sin^2 x}{1+\cot x}-\frac{\cos^2 x}{1+\tan x}$$

By using the formula,

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$1-\frac{\sin^2 x}{1+\frac{\cos x}{\sin x}}-\frac{\cos^2 x}{1+\frac{\sin x}{\cos x}}$$

$$1-\frac{\sin^3 x}{\sin x+\cos x}-\frac{\cos^3 x}{\sin x+\cos x}$$

$$\frac{\sin x+\cos x-(\sin^3 x+\cos^3 x)}{\sin x+\cos x}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{\sin x+\cos x-((\sin x+\cos x)(\sin x)^2+(\cos x)^2-\sin x\cos x))}{\sin x+\cos x}$$

$$\frac{(\sin x+\cos x)(1-\sin^2 x-\cos^2 x+\sin x\cos x)}{\sin x+\cos x}$$

$$1-(\sin^2 x+\cos^2 x)+\sin x\cos x$$

We know, $\sin^2 x + \cos^2 x = 1$.

1 – 1 + sin x cos x

Sin x cos x

= RHS

∴ LHS = RHS

Hence proved.

12.

$$\left(\frac{1}{\sec^2 x-\cos^2 x}+\frac{1}{cosec^2 x-\sin^2 x}\right)\sin^2 x\cos^2 x=\frac{1-\sin^2 x\cos^2 x}{2+\sin^2 x\cos^2 x}$$

Solution:

Let us consider LHS:

$$\left(\frac{1}{\sec^2 x-\cos^2 x}+\frac{1}{cosec^2 x-\sin^2 x}\right)\sin^2 x\cos^2 x$$

By using the formula,

cosec θ = 1/sin θ,

sec θ = 1/cos θ;

$$\left(\frac{1}{\frac{1}{\cos^2 x}-\cos^2 x}+\frac{1}{\frac{1}{\sin^2 x}-\sin^2 x}\right)\sin^2 x\cos^2 x$$

$$\left(\frac{\cos^2 x}{1-\cos^4 x}+\frac{\sin^2 x}{1-\sin^4 x}\right)\sin^2 x\cos^2 x$$

$$\left(\frac{\cos^2 x(1-\sin^4 x)+\sin^2 x(1-\cos^4 x)}{(1-\cos^4 x)(1-\sin^4 x)}\right)\sin^2 x\cos^2 x$$

$$\left(\frac{\cos^2 x-\cos^2 x\sin^4 x+\sin^2 x-\sin^2 x\cos^4 x}{(1+\sin^2 x)(1-\sin^2 x)(1+\cos^2 x)(1-\cos^2 x)}\right)\sin^2 x\cos^2 x$$

We know, $\sin^2 x+\cos^2 x=1$.

$$\left(\frac{1-\cos^2 x\sin^4 x-\sin^2 x\cos^4 x}{(1+\sin^2 x)\cos^2 x(1+\cos^2 x)\sin^2 x}\right)\sin^2 x\cos^2 x$$

$$\left(\frac{1-\cos^2 x\sin^2 x(\sin^2 x+\cos^2 x)}{(1+\sin^2 x)(1+\cos^2 x)}\right)$$

$$\frac{1-\sin^2 x\cos^2 x}{2+\sin^2 x\cos^2 x}$$

= RHS

∴ LHS = RHS

Hence proved.

13. (1 + tan α tan β)^2 + (tan α – tan β)^2 = sec^2 α sec^2 β

Solution:

Let us consider LHS: (1 + tan α tan β)^2 + (tan α – tan β)^2

1+ tan^2 α tan^2 β + 2 tan α tan β + tan^2 α + tan^2 β – 2 tan α tan β

$1 + \tan^2 \alpha \tan^2 \beta + \tan^2 \alpha + \tan^2 \beta$

$\tan^2 \alpha (\tan^2 \beta + 1) + 1 (1 + \tan^2 \beta)$

$(1 + \tan^2 \beta) (1 + \tan^2 \alpha)$

We know, $1 + \tan^2 \theta = \sec^2 \theta$

So,

$\sec^2 \alpha \sec^2 \beta$

= RHS

∴ LHS = RHS

Hence proved.

EXERCISE 5.2 PAGE NO: 5.25

1. Find the values of the other five trigonometric functions in each of the following:

(i) cot x = 12/5, x in quadrant III

(ii) cos x = -1/2, x in quadrant II

(iii) tan x = 3/4, x in quadrant III

(iv) sin x = 3/5, x in quadrant I

Solution:

(i) cot x = 12/5, x in quadrant III

In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.

By using the formulas,

tan x = 1/cot x

= 1/(12/5)

= 5/12

$\text{cosec } x = -\sqrt{(1 + \cot^2 x)}$

$= -\sqrt{(1 + (12/5)^2)}$

$= -\sqrt{(25+144)}/25$

$= -\sqrt{(169/25)}$

= -13/5

sin x = 1/cosec x

= 1/(-13/5)

= -5/13

$\cos x = -\sqrt{(1 - \sin^2 x)}$

$= -\sqrt{(1 - (-5/13)^2)}$

$= -\sqrt{(169-25)/169}$

$= -\sqrt{(144/169)}$

$= -12/13$

$\sec x = 1/\cos x$

$= 1/(-12/13)$

$= -13/12$

∴ $\sin x = -5/13$, $\cos x = -12/13$, $\tan x = 5/12$, $\operatorname{cosec} x = -13/5$, $\sec x = -13/12$

(ii) $\cos x = -1/2$, x in quadrant II

In second quadrant, sin x and cosec x are positive. tan x, cot x, cos x, sec x are negative.

By using the formulas,

$\sin x = \sqrt{(1 - \cos^2 x)}$

$= \sqrt{(1 - (-1/2)^2)}$

$= \sqrt{(4-1)/4}$

$= \sqrt{(3/4)}$

$= \sqrt{3}/2$

$\tan x = \sin x/\cos x$

$= (\sqrt{3}/2)/(-1/2)$

$= -\sqrt{3}$

$\cot x = 1/\tan x$

$= 1/-\sqrt{3}$

$= -1/\sqrt{3}$

$\operatorname{cosec} x = 1/\sin x$

$= 1/(\sqrt{3}/2)$

$= 2/\sqrt{3}$

$\sec x = 1/\cos x$

$= 1/(-1/2)$

$= -2$

∴ $\sin x = \sqrt{3}/2$, $\tan x = -\sqrt{3}$, $\operatorname{cosec} x = 2/\sqrt{3}$, $\cot x = -1/\sqrt{3}$ $\sec x = -2$

(iii) tan x = 3/4, x in quadrant III

In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.

By using the formulas,

sin x = $\sqrt{(1 - \cos^2 x)}$

$= -\sqrt{(1-(-4/5)^2)}$

$= -\sqrt{(25-16)/25}$

$= -\sqrt{(9/25)}$

= – 3/5

cos x = 1/sec x

= 1/(-5/4)

= -4/5

cot x = 1/tan x

= 1/(3/4)

= 4/3

cosec x = 1/sin x

= 1/(-3/5)

= -5/3

sec x = $-\sqrt{(1 + \tan^2 x)}$

$= -\sqrt{(1+(3/4)^2)}$

$= -\sqrt{(16+9)/16}$

$= -\sqrt{(25/16)}$

= -5/4

∴ sin x = -3/5, cos x = -4/5, cosec x = -5/3, sec x = -5/4, cot x = 4/3

(iv) sin x = 3/5, x in quadrant I

In first quadrant, all trigonometric ratios are positive.

So, by using the formulas,

tan x = sin x/cos x

= (3/5)/(4/5)

= 3/4

cosec x = 1/sin x

$= 1/(3/5)$

$= 5/3$

$\cos x = \sqrt{(1-\sin^2 x)}$

$= \sqrt{(1 – (-3/5)^2)}$

$= \sqrt{(25-9)/25}$

$= \sqrt{(16/25)}$

$= 4/5$

$\sec x = 1/\cos x$

$= 1/(4/5)$

$= 5/4$

$\cot x = 1/\tan x$

$= 1/(3/4)$

$= 4/3$

$\therefore \cos x = 4/5, \tan x = 3/4, \operatorname{cosec} x = 5/3, \sec x = 5/4, \cot x = 4/3$

2. If sin x = 12/13 and lies in the second quadrant, find the value of sec x + tan x.

Solution:

Given:

Sin x = 12/13 and x lies in the second quadrant.

We know, in second quadrant, sin x and cosec x are positive and all other ratios are negative.

By using the formulas,

$\text{Cos } x = \sqrt{(1-\sin^2 x)}$

$= – \sqrt{(1-(12/13)^2)}$

$= – \sqrt{(1- (144/169))}$

$= – \sqrt{(169-144)/169}$

$= -\sqrt{(25/169)}$

$= – 5/13$

We know,

$\tan x = \sin x/\cos x$

$\sec x = 1/\cos x$

Now,

tan x = (12/13)/(-5/13)

= -12/5

sec x = 1/(-5/13)

= -13/5

Sec x + tan x = -13/5 + (-12/5)

= (-13-12)/5

= -25/5

= -5

∴ Sec x + tan x = -5

3. If sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2 find the value of 8 tan x -√5 sec y.

Solution:

Given:

sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2

We know that, x is in second quadrant and y is in third quadrant.

In second quadrant, cos x and tan x are negative.

In third quadrant, sec y is negative.

By using the formula,

cos x = – $\sqrt{}$(1-$\sin^2$ x)

tan x = sin x/cos x

Now,

cos x = – $\sqrt{}$(1-$\sin^2$ x)

= – $\sqrt{}$(1 – $(3/5)^2$)

= – √(1 – 9/25)

= – √((25-9)/25)

= – √(16/25)

= – 4/5

tan x = sin x/cos x

= (3/5)/(-4/5)

= 3/5 × -5/4

$= -3/4$

We know that $\sec y = -\sqrt{(1+\tan^2 y)}$

$= -\sqrt{(1 + (1/2)^2)}$

$= -\sqrt{(1 + 1/4)}$

$= -\sqrt{((4+1)/4)}$

$= -\sqrt{(5/4)}$

$= -\sqrt{5}/2$

Now, $8 \tan x - \sqrt{5} \sec y = 8(-3/4) - \sqrt{5}(-\sqrt{5}/2)$

$= -6 + 5/2$

$= (-12+5)/2$

$= -7/2$

$\therefore 8 \tan x - \sqrt{5} \sec y = -7/2$

4. If sin x + cos x = 0 and x lies in the fourth quadrant, find sin x and cos x.

Solution:

Given:

Sin x + cos x = 0 and x lies in fourth quadrant.

Sin x = -cos x

Sin x/cos x = -1

So, tan x = -1 (since, tan x = sin x/cos x)

We know that, in fourth quadrant, cos x and sec x are positive and all other ratios are negative.

By using the formulas,

$\operatorname{Sec} x = \sqrt{(1 + \tan^2 x)}$

Cos x = 1/sec x

$\operatorname{Sin} x = -\sqrt{(1- \cos^2 x)}$

Now,

$\operatorname{Sec} x = \sqrt{(1 + \tan^2 x)}$

$= \sqrt{(1 + (-1)^2)}$

$= \sqrt{2}$

Cos x = 1/sec x

$= 1/\sqrt{2}$

$\sin x = -\sqrt{(1 - \cos^2 x)}$

$= -\sqrt{(1 - (1/\sqrt{2})^2)}$

$= -\sqrt{(1 - (1/2))}$

$= -\sqrt{((2-1)/2)}$

$= -\sqrt{(1/2)}$

$= -1/\sqrt{2}$

∴ $\sin x = -1/\sqrt{2}$ and $\cos x = 1/\sqrt{2}$

5. If cos x = -3/5 and π<x<3π/2 find the values of other five trigonometric functions and hence evaluate $\frac{cosec\ x + cot\ x}{sec\ x - tan\ x}$

Solution:

Given:

$\cos x = -3/5$ and $\pi < x < 3\pi/2$

We know that in the third quadrant, tan x and cot x are positive and all other rations are negative.

By using the formulas,

$\sin x = -\sqrt{(1-\cos^2 x)}$

Tan x = sin x/cos x

Cot x = 1/tan x

Sec x = 1/cos x

Cosec x = 1/sin x

Now,

$\sin x = -\sqrt{(1-\cos^2 x)}$

$= -\sqrt{(1-(-3/5)^2)}$

$= -\sqrt{(1-9/25)}$

$= -\sqrt{((25-9)/25)}$

$= -\sqrt{(16/25)}$

$= -4/5$

Tan x = sin x/cos x

= (-4/5)/(-3/5)

= -4/5 × -5/3

= 4/3

Cot x = 1/tan x

= 1/(4/3)

= 3/4

Sec x = 1/cos x

= 1/(-3/5)

= -5/3

Cosec x = 1/sin x

= 1/(-4/5)

= -5/4

$\therefore \dfrac{cosec\ x + cot\ x}{sec\ x - tan\ x}$

= [(-5/4) + (3/4)] / [(-5/3) – (4/3)]

= [(-5+3)/4] / [(-5-4)/3]

= [-2/4] / [-9/3]

= [-1/2] / [-3]

= 1/6

EXERCISE 5.3 PAGE NO: 5.39

1. Find the values of the following trigonometric ratios:

(i) sin 5π/3

(ii) sin 17π

(iii) tan 11π/6

(iv) cos (-25π/4)

(v) tan 7π/4

(vi) sin 17π/6

(vii) cos 19π/6

(viii) sin (-11π/6)

(ix) cosec (-20π/3)

(x) tan (-13π/4)

(xi) cos 19π/4

(xii) sin 41π/4

(xiii) cos 39π/4

(xiv) sin 151π/6

Solution:

(i) sin $5\pi/3$

$5\pi/3 = (5\pi/3 \times 180)^o$

$= 300^o$

$= (90\times3 + 30)^o$

Since, 300^o lies in IV quadrant in which sine function is negative.

$\sin 5\pi/3 = \sin (300)^o$

$= \sin (90\times3 + 30)^o$

$= -\cos 30^o$

$= -\sqrt{3}/2$

(ii) sin 17π

$\text{Sin } 17\pi = \sin 3060^o$

$= \sin (90\times34 + 0)^o$

Since, 3060^o lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.

$\text{Sin } 17\pi = \sin (90\times34 + 0)^o$

$= -\sin 0^o$

$= 0$

(iii) tan $11\pi/6$

$\tan 11\pi/6 = (11/6 \times 180)^o$

$= 330^o$

Since, 330^o lies in the IV quadrant in which tangent function is negative.

$\tan 11\pi/6 = \tan (300)^o$

$= \tan (90\times3 + 60)^o$

$= -\cot 60^o$

$= -1/\sqrt{3}$

(iv) cos $(-25\pi/4)$

$\cos (-25\pi/4) = \cos (-1125)^o$

$= \cos (1125)^o$

Since, 1125^o lies in the I quadrant in which cosine function is positive.

$\cos (1125)^o = \cos (90\times12 + 45)^o$

$= \cos 45^o$

$= 1/\sqrt{2}$

(v) tan 7π/4

$\tan 7\pi/4 = \tan 315^o$

$= \tan (90\times3 + 45)^o$

Since, 315^o lies in the IV quadrant in which tangent function is negative.

$\tan 315^o = \tan (90\times3 + 45)^o$

$= -\cot 45^o$

$= -1$

(vi) sin 17π/6

$\sin 17\pi/6 = \sin 510^o$

$= \sin (90\times5 + 60)^o$

Since, 510^o lies in the II quadrant in which sine function is positive.

$\sin 510^o = \sin (90\times5 + 60)^o$

$= \cos 60^o$

$= 1/2$

(vii) cos 19π/6

$\cos 19\pi/6 = \cos 570^o$

$= \cos (90\times6 + 30)^o$

Since, 570^o lies in III quadrant in which cosine function is negative.

$\cos 570^o = \cos (90\times6 + 30)^o$

$= -\cos 30^o$

$= -\sqrt{3}/2$

(viii) sin (-11π/6)

$\sin (-11\pi/6) = \sin (-330^o)$

$= -\sin (90\times3 + 60)^o$

Since, 330^o lies in the IV quadrant in which the sine function is negative.

$\sin (-330^o) = -\sin (90\times3 + 60)^o$

$= -(-\cos 60^o)$

$= -(-1/2)$

$= 1/2$

(ix) cosec (-20π/3)

cosec (-20π/3) = cosec $(-1200)^o$

= – cosec $(1200)^o$

= – cosec $(90×13 + 30)^o$

Since, 1200^o lies in the II quadrant in which cosec function is positive.

cosec $(-1200)^o$ = – cosec $(90×13 + 30)^o$

= – sec 30^o

= -2/√3

(x) tan (-13π/4)

tan (-13π/4) = tan $(-585)^o$

= – tan $(90×6 + 45)^o$

Since, 585^o lies in the III quadrant in which the tangent function is positive.

tan $(-585)^o$ = – tan $(90×6 + 45)^o$

= – tan 45^o

= -1

(xi) cos 19π/4

cos 19π/4 = cos 855^o

= cos $(90×9 + 45)^o$

Since, 855^o lies in the II quadrant in which the cosine function is negative.

cos 855^o = cos $(90×9 + 45)^o$

= – sin 45^o

= – 1/√2

(xii) sin 41π/4

sin 41π/4 = sin 1845^o

= sin $(90×20 + 45)^o$

Since, 1845^o lies in the I quadrant in which the sine function is positive.

sin 1845^o = sin $(90×20 + 45)^o$

= sin 45^o

= 1/√2

(xiii) cos 39π/4

$\cos 39\pi/4 = \cos 1755^o$

$= \cos (90\times19 + 45)^o$

Since, 1755^o lies in the IV quadrant in which the cosine function is positive.

$\cos 1755^o = \cos (90\times19 + 45)^o$

$= \sin 45^o$

$= 1/\sqrt{2}$

(xiv) $\sin 151\pi/6$

$\sin 151\pi/6 = \sin 4530^o$

$= \sin (90\times50 + 30)^o$

Since, 4530^o lies in the III quadrant in which the sine function is negative.

$\sin 4530^o = \sin (90\times50 + 30)^o$

$= -\sin 30^o$

$= -1/2$

2. prove that:
(i) $\tan 225^o \cot 405^o + \tan 765^o \cot 675^o = 0$

(ii) $\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6 = 1/2$

(iii) $\cos 24^o + \cos 55^o + \cos 125^o + \cos 204^o + \cos 300^o = 1/2$

(iv) $\tan (-125^o) \cot (-405^o) - \tan (-765^o) \cot (675^o) = 0$

(v) $\cos 570^o \sin 510^o + \sin (-330^o) \cos (-390^o) = 0$

(vi) $\tan 11\pi/3 - 2 \sin 4\pi/6 - 3/4 \operatorname{cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 - 4\sqrt{3})/2$

(vii) $3 \sin \pi/6 \sec \pi/3 - 4 \sin 5\pi/6 \cot \pi/4 = 1$

Solution:

(i) $\tan 225^o \cot 405^o + \tan 765^o \cot 675^o = 0$

Let us consider LHS:

$\tan 225° \cot 405° + \tan 765° \cot 675°$

$\tan (90° \times 2 + 45°) \cot (90° \times 4 + 45°) + \tan (90° \times 8 + 45°) \cot (90° \times 7 + 45°)$

We know that when n is odd, cot → tan.

$\tan 45° \cot 45° + \tan 45° [-\tan 45°]$

$\tan 45° \cot 45° – \tan 45° \tan 45°$

$1 \times 1 – 1 \times 1$

$1 – 1$

0 = RHS

∴ LHS = RHS

Hence proved.

(ii) $\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6 = 1/2$

Let us consider LHS:

$\sin 8\pi/3 \cos 23\pi/6 + \cos 13\pi/3 \sin 35\pi/6$

$\sin 480° \cos 690° + \cos 780° \sin 1050°$

$\sin (90° \times 5 + 30°) \cos (90° \times 7 + 60°) + \cos (90° \times 8 + 60°) \sin (90° \times 11 + 60°)$

We know that when n is odd, sin → cos and cos → sin.

$\cos 30° \sin 60° + \cos 60° [-\cos 60°]$

$\sqrt{3}/2 \times \sqrt{3}/2 – 1/2 \times 1/2$

$3/4 – 1/4$

$2/4$

$1/2$

= RHS

∴ LHS = RHS

Hence proved.

(iii) $\cos 24^o + \cos 55^o + \cos 125^o + \cos 204^o + \cos 300^o = 1/2$

Let us consider LHS:

$\cos 24^o + \cos 55^o + \cos 125^o + \cos 204^o + \cos 300^o$

$\cos 24° + \cos (90° \times 1 – 35°) + \cos (90° \times 1 + 35°) + \cos (90° \times 2 + 24°) + \cos (90° \times 3 + 30°)$

We know that when n is odd, cos → sin.

$\cos 24° + \sin 35° – \sin 35° – \cos 24° + \sin 30°$

$0 + 0 + 1/2$

$1/2$

= RHS

∴ LHS = RHS

Hence proved.

(iv) tan (-125^o) cot (-405^o) – tan (-765^o) cot (675^o) = 0

Let us consider LHS:

tan (-125^o) cot (-405^o) – tan (-765^o) cot (675^o)

We know that tan (-x) = -tan (x) and cot (-x) = -cot (x).

[-tan (225°)] [-cot (405°)] – [-tan (765°)] cot (675°)
tan (225°) cot (405°) + tan (765°) cot (675°)

tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)

tan 45° cot 45° + tan 45° [-tan 45°]

1 × 1 + 1 × (-1)

1 – 1

0

= RHS

∴ LHS = RHS

Hence proved.

(v) cos 570^o sin 510^o + sin (-330^o) cos (-390^o) = 0

Let us consider LHS:

cos 570^o sin 510^o + sin (-330^o) cos (-390^o)

We know that sin (-x) = -sin (x) and cos (-x) = +cos (x).

cos 570^o sin 510^o + [-sin (330^o)] cos (390^o)

cos 570^o sin 510^o – sin (330^o) cos (390^o)

cos (90° × 6 + 30°) sin (90° × 5 + 60°) – sin (90° × 3 + 60°) cos (90° × 4 + 30°)

We know that cos is negative at 90° + θ i.e. in Q_2 and when n is odd, sin → cos and cos → sin.

-cos 30° cos 60° – [-cos 60°] cos 30°

-cos 30° cos 60° + cos 60° cos 30°

0

= RHS

∴ LHS = RHS

Hence proved.

(vi) $\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \text{ cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 – 4\sqrt{3})/2$

Let us consider LHS:

$\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \text{ cosec}^2 \pi/4 + 4 \cos^2 17\pi/6$

$\tan (11 \times 180^o)/3 – 2 \sin (4 \times 180^o)/6 – 3/4 \text{ cosec}^2 180^o/4 + 4 \cos^2 (17 \times 180^o)/6$

$\tan 660^o – 2 \sin 120^o – 3/4 (\text{cosec } 45^o)^2 + 4 (\cos 510^o)^2$

$\tan (90° \times 7 + 30°) – 2 \sin (90° \times 1 + 30°) – 3/4 [\text{cosec } 45°]^2 + 4 [\cos (90° \times 5 + 60°)]^2$

We know that tan and cos is negative at 90° + θ i.e. in Q_2 and when n is odd, tan → cot, sin → cos and cos → sin.

$[-\cot 30°] – 2 \cos 30° – 3/4 [\text{cosec } 45°]^2 + [-\sin 60°]^2$
$– \cot 30° – 2 \cos 30° – 3/4 [\text{cosec } 45°]^2 + [\sin 60°]^2$

$-\sqrt{3} – 2\sqrt{3}/2 – 3/4 (\sqrt{2})^2 + 4 (\sqrt{3}/2)^2$

$-\sqrt{3} – \sqrt{3} – 6/4 + 12/4$

$(3 – 4\sqrt{3})/2$

= RHS

∴ LHS = RHS

Hence proved.

(vii) $3 \sin \pi/6 \sec \pi/3 – 4 \sin 5\pi/6 \cot \pi/4 = 1$

Let us consider LHS:

$3 \sin \pi/6 \sec \pi/3 – 4 \sin 5\pi/6 \cot \pi/4$

$3 \sin 180^o/6 \sec 180^o/3 – 4 \sin 5(180^o)/6 \cot 180^o/4$

$3 \sin 30° \sec 60° – 4 \sin 150° \cot 45°$

$3 \sin 30° \sec 60° – 4 \sin (90° \times 1 + 60°) \cot 45°$

We know that when n is odd, sin → cos.

$3 \sin 30° \sec 60° – 4 \cos 60° \cot 45°$

$3 (1/2) (2) – 4 (1/2) (1)$

$3 – 2$

1

= RHS

∴ LHS = RHS

Hence proved.

3. Prove that:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

(iii)

$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)} = 1$$

(iv)

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\} = 2\, cot\, x$$

(v)

$$\frac{tan(\frac{\pi}{2} - x)\, sec(\pi - x)\, sin(-x)}{sin(\pi + x)\, cot(2\pi - x)\, cosec(\frac{\pi}{2} - x)} = 1$$

Solution:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

Let us consider LHS:

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)}$$

$$\frac{cos\, x\, cosec\, x[-cot\, x]}{[-cosec\, x]\, cos\, x\, cot\, x}$$

$$\frac{-cos\, x cosec\, x\, cot\, x}{-cosec\, x\, cos\, x\, cot\, x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(ii)

$$\frac{cosec\,(90^o+x)+cot(450^o+x)}{cosec(90^o-x)+tan(180^o-x)}+\frac{tan(180^o+x)+sec(180^o-x)}{tan(360^o+x)-sec(-x)}=2$$

Let us consider LHS:

$$\frac{cosec\,(90^o+x)+cot(450^o+x)}{cosec(90^o-x)+tan(180^o-x)}+\frac{tan(180^o+x)+sec(180^o-x)}{tan(360^o+x)-sec(-x)}$$

$$\frac{cosec\,(90^o+x)+cot(90^o\times 5+x)}{cosec(90^o-x)+tan(90^o\times 2-x)}+\frac{tan(90^o\times 2+x)+sec(90^o\times 2-x)}{tan(90^o\times 4+x)-sec(-x)}$$

We know that when n is odd, cosec → sec and also sec (-x) = sec x.

$$\frac{sec\,x+cot(90^o\times 5+x)}{cosec(90^o-x)+tan(90^o\times 2-x)}+\frac{tan(90^o\times 2+x)+sec(90^o\times 2-x)}{tan(90^o\times 4+x)-sec(-x)}$$

$$\frac{sec\,x-tan\,x}{sec\,x-tan\,x}+\frac{tan\,x-sec\,x}{tan\,x-sec\,x}$$

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\frac{sin(\pi+x)\,cos(\frac{\pi}{2}+x)\,tan(\frac{3\pi}{2}-x)\,cot(2\pi-x)}{sin(2\pi-x)\,cos(2\pi+x)\,cosec(-x)\,sin(\frac{3\pi}{2}-x)}=1$$

Let us consider LHS:

$$\frac{sin(\pi+x)\,cos(\frac{\pi}{2}+x)\,tan(\frac{3\pi}{2}-x)\,cot(2\pi-x)}{sin(2\pi-x)\,cos(2\pi+x)\,cosec(-x)\,sin(\frac{3\pi}{2}-x)}$$

$$\frac{sin(180^o-x)\,cos(90^o+x)\,tan(270^o-x)\,cot(360^o-x)}{sin(360^o-x)\,cos(360^o+x)\;\;cosec(-x)\,sin(270^o-x)}$$

We know that cosec (-x) = - cosec x.

$$\frac{sin(90^o \times 2 - x)\, cos(90^o \times 1 + x)\, tan(90^o \times 3 - x)\, cot(90^o \times 4 - x)}{sin(90^o \times 4 - x)\, cos(90^o \times 4 + x)\, [-cosec(x)]\, sin(90^o \times 3 - x)}$$

We know that when n is odd, cos → sin, tan → cot and sin → cos.

$$\frac{(-sin\, x)(-sin\, x)\, cot\, x\, (-cot\, x)}{(-sin\, x)\, cos\, x\, (-cosec\, x)(-cos\, x)}$$

$$\frac{sin^2 x\, cot^2 x}{sin\, x\, cosec\, x\, cos\, x\, cos\, x}$$

$$\frac{sin^2 x \ \times \frac{cos^2 x}{sin^2 x}}{sin\, x \ \times \frac{1}{sin\, x} \times cos^2 x}$$

$$\frac{cos^2 x}{cos^2 x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(iv)

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\} = 2\, cot\, x$$

Let us consider LHS:

$$\left\{1 + cot\, x - sec(\frac{\pi}{2} + x)\right\} \left\{1 + cot\, x + sec(\frac{\pi}{2} + x)\right\}$$

{1 + cot x – (-cosec x)} {1 + cot x + (-cosec x)}

{1 + cot x + cosec x} {1 + cot x – cosec x}

{(1 + cot x) + (cosec x)} {(1 + cot x) – (cosec x)}

By using the formula, (a + b) (a – b) = $a^2 - b^2$

$(1 + \cot x)^2 - (\text{cosec } x)^2$

1 + $\cot^2 x$ + 2 cot x – $\text{cosec}^2 x$

We know that 1 + $\cot^2 x = \text{cosec}^2 x$

$\text{cosec}^2 x$ + 2 cot x – $\text{cosec}^2 x$

2 cot x = RHS

∴ LHS = RHS

Hence proved.

(v)

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)} = 1$$

Let us consider LHS:

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)}$$

$$\frac{tan(90^o-x)\ sec(180^o-x)\ sin(-x)}{sin(180^o+x)\ cot(360^o-x)cosec(90^o-x)}$$

We know that sin (-x) = -sin x.

$$\frac{tan(90^o\times 1-x)\ sec(90^o\times 2-x)\ [-sin(x)]}{sin(90^o\times 2+x)\ cot(90^o\times 4-x)cosec(90^o\times 1-x)}$$

We know that when n is odd, tan → cot and cosec → sec.

$$\frac{(cot\,x)(-sec\,x)(-sin\,x)}{(-sin\,x)(-cot\,x)(sec\,x)}$$

$$\frac{cot\,x\ sec\,x\ sin\,x}{sin\,x\ cot\,x\ sec\,x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

4. Prove that: $\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9 = 2$

Solution:

Let us consider LHS:

$\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9$

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 7\pi/18 + \sin^2 8\pi/18$

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 (\pi/2 – 2\pi/18) + \sin^2 (\pi/2 – \pi/18)$

We know that when n is odd, sin → cos.

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \cos^2 2\pi/18 + \cos^2 2\pi/18$

when rearranged,

$\sin^2 \pi/18 + \cos^2 2\pi/18 + \sin^2 \pi/18 + \cos^2 2\pi/18$

We know that $\sin^2 + \cos^2 x = 1$.

So,

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.