RD SHARMA Solutions for Class 9 Maths Chapter 14 - Areas of Parallelograms and Triangles

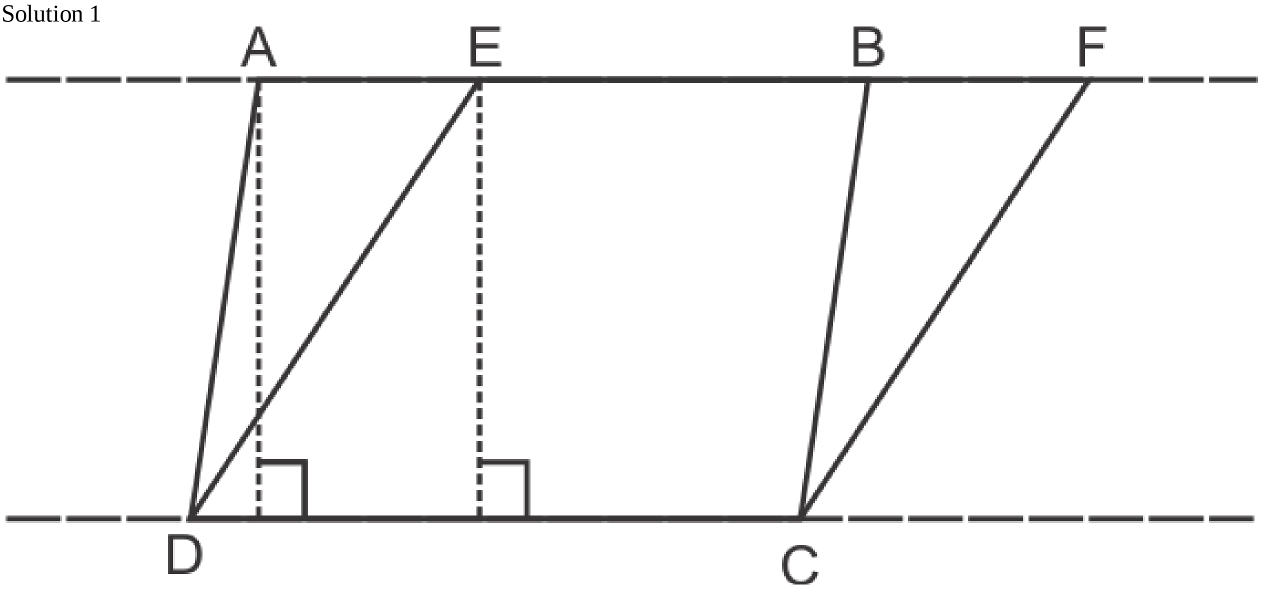
Chapter 14 - Areas of Parallelograms and Triangles Exercise 14.60

Question 1

Two parallelograms are on the same base and between the same parallels. The ratio of their areas is

(a) 1 : 2
(b) 2 : 1
(c) 1 : 1
(d) 3 : 1

Solution 1



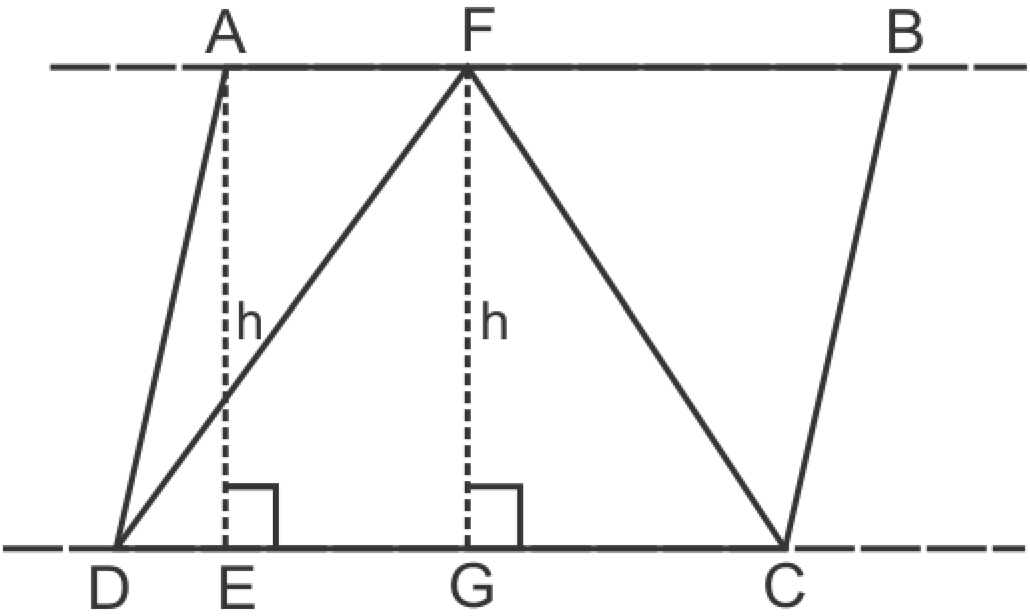
Area of parallelogram = Base × height
Base = Length of base
Height = distance between Base and Side parallel to it
In figure, there are two Parallelograms.
Base of both is same, and because both lie under same parallels that's why height is also same.
Thus, the Ratio of Areas of both parallelogram = 1 : 1
Hence, correct option is (c).

Question 2

A triangle and a parallelogram are on the same base and between the same parallels.
The ratio of the areas of triangle and parallelogram is

(a) 1 : 1
(b) 1 : 2
(c) 2 : 1
(d) 1 : 3

Solution 2



Area of a triangle DFC $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times DC \times FG = \frac{1}{2} \times DC \times h$

Area of Parallelogram ABCD $= \text{base} \times \text{height} = DC \times AE = DC \times h$

Required Ratio $= \dfrac{\frac{1}{2} \times \cancel{DC \times h}}{\cancel{DC \times h}} = 1 : 2$

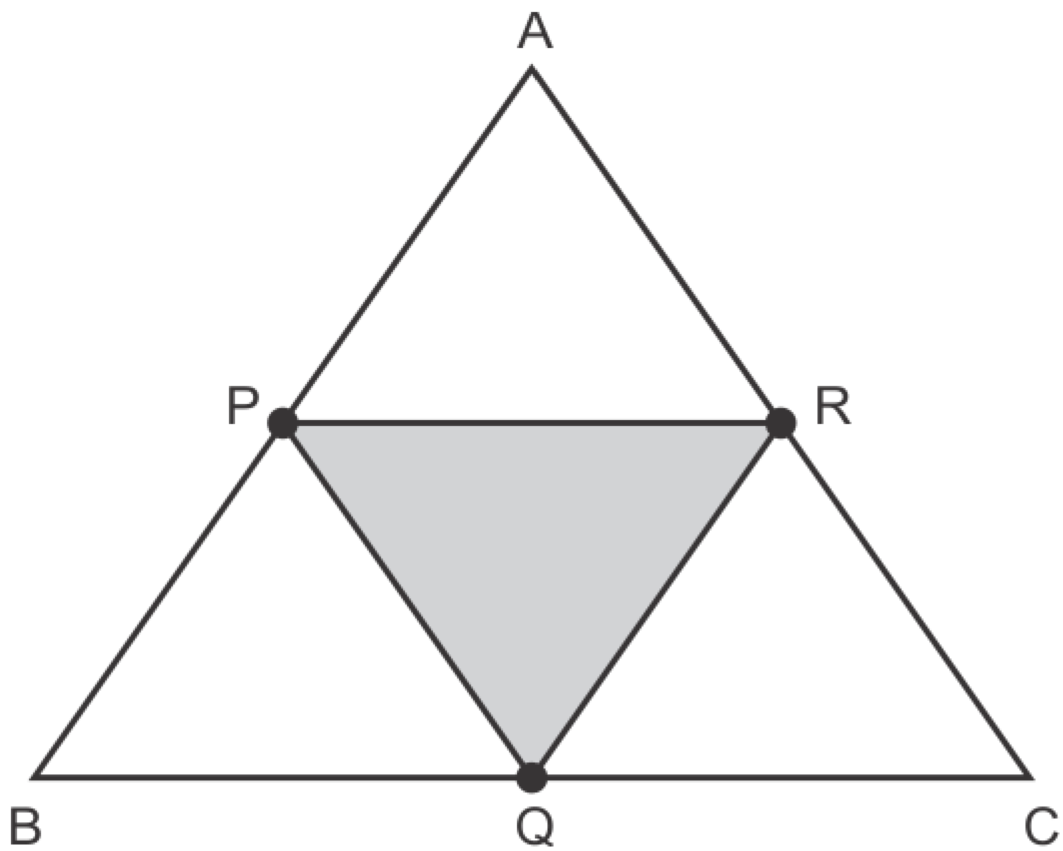
Hence, correct option is (b).

Question 3

Let ABC be a triangle of area 24 sq. units and PQR be the triangle formed by the mid-points of sides of ΔABC. Then the area of ΔPQR is

(a) 12 sq. units
(b) 6 sq. units
(c) 4 sq. units
(d) 3 sq. units

Solution 3



When a triangle is formed by joining the mid-points of sides of a triangle, the triangle formed is Congruent to triangles formed around that.

i.e. ΔPQR is congruent to ΔRPA, ΔQBP & ΔCQR.

Hence, Area of all four triangles formed inside ΔABC is same.

So (4 × Area of any one Δ) = Area of ΔABC

4 × (Area of ΔPQR) = 24 sq. units

Area of ΔPQR = 6 sq. units

Hence, correct option is (b).

Question 4

The median of a triangle divides it into two

(a) congruent triangles
(b) isosceles triangles
(c) right triangles
(d) triangles of equal areas

Solution 4

A median divides the base in two equal parts but height of a triangle remains the same.

Now, since bases and heights are equal, areas of both Δs are equal.

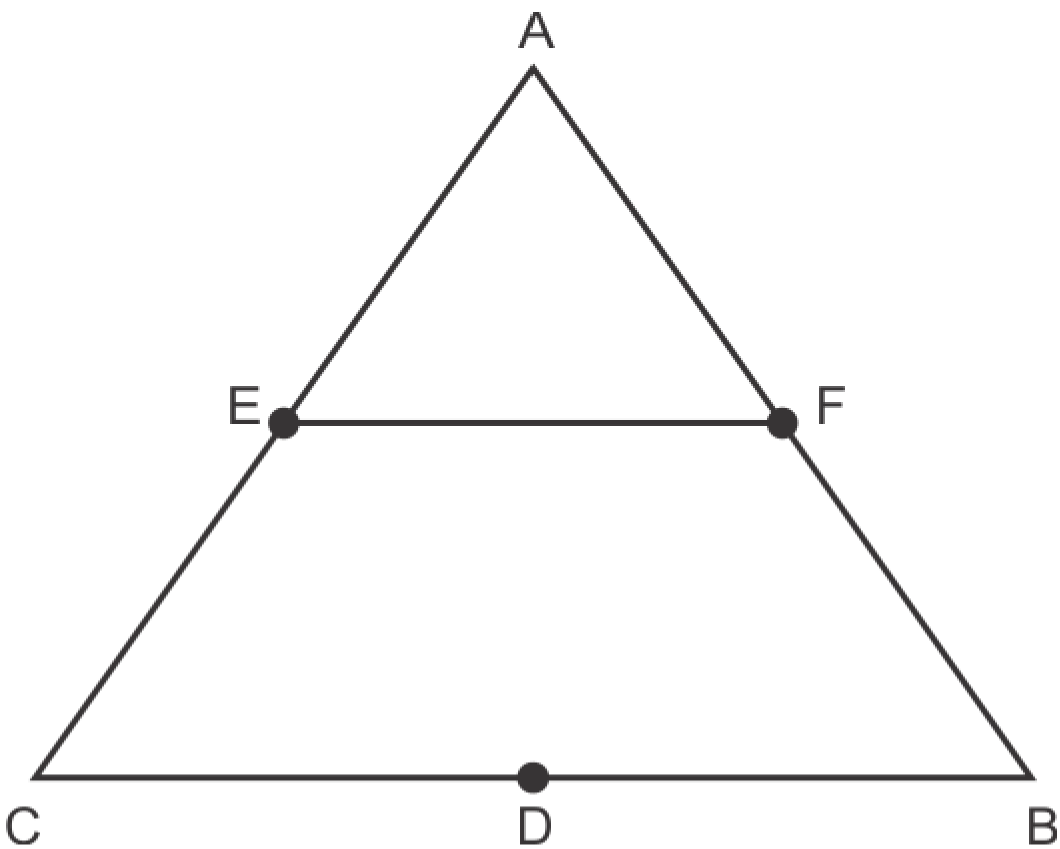
Hence, correct option is (d).

Question 5

In a ΔABC, D, E, F are the mid-points of sides BC, CA and AB respectively. If ar(ΔABC) = 16 cm^2 , then ar(trapezium FBCE) =

(a) 4 cm^2
(b) 8 cm^2
(c) 12 cm^2
(d) 10 cm^2

Solution 5



Area of $\triangle ABC$ = Area of $\triangle AEF$ + Area of trapazium FBCE

We know that any triangle formed by joining the mid − points of sides of triangle,

has area $= \frac{1}{4} \times (\text{Parent } \triangle)$

$\Rightarrow$ Area of $\triangle AEF = \frac{1}{4} \times Ar(\triangle ABC) = \frac{1}{4} \times 16\ cm^2 = 4\ cm^2$

$\Rightarrow$ Area of Trapezium $= (16 - 4)\ cm^2 = 12\ cm^2$

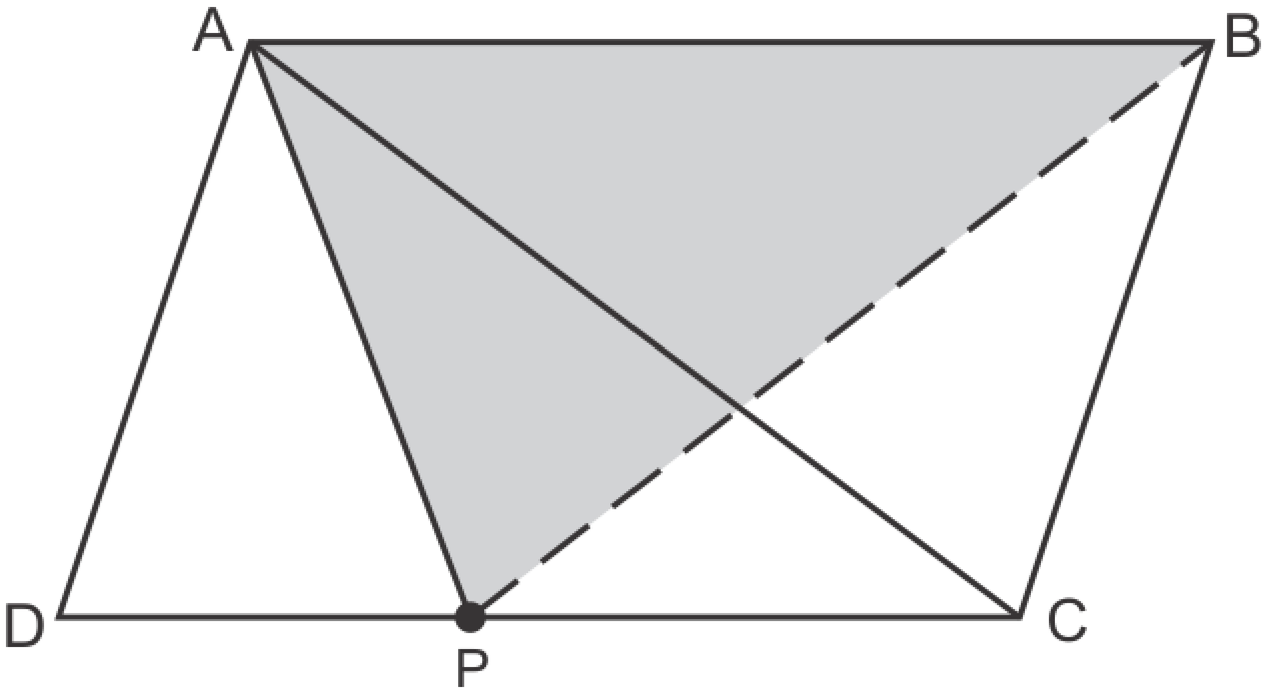
Hence, correct option is (c).

Question 6

ABCD is a parallelogram. P is any point on CD. If ar (ΔDPA) = 15 cm^2 and ar(ΔAPC) = 20 cm^2, then ar(ΔAPB) =

(a) 15 cm^2
(b) 20 cm^2
(c) 35 cm^2
(d) 30 cm^2

Solution 6



Area of trapazium ABCP $=$ Area of $\triangle$APB $+$ ar of $\triangle$BPC ...(1)

$\triangle$APC and $\triangle$BPC have same base PC and are between same parallels.

$\Rightarrow$ Area of $\triangle$APC $=$ Area of $\triangle$BPC $= 20\text{ cm}^2$ ...(2)

From figure, $\text{Ar}(\triangle ADP) + \text{Ar}(\triangle APC) = \frac{1}{2}\text{Ar}(\|^{gm}\ ABCD)$

$\Rightarrow \text{Ar}(\|^{gm}\ ABCD) = 2(20 + 15) = 70\text{ cm}^2$

Area of trapazium ABCP $= \text{Ar}(\|^{gm}\ ABCD) - \text{Ar}(\triangle ADP) = 70 - 15 = 55\text{ cm}^2$

$\Rightarrow$ Area of $\triangle$APB $=$ Area of trapezium ABCP $-$ Area of $\triangle$BPC

$= (55 - 20)\text{ cm}^2$ [From (1)]

$= 35\text{ cm}^2$

Hence, correct option is (c).

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Question 7

The area of the figure formed by joining the mid-points of the adjacent sides of a rhombus with diagonals 16 cm and 12 cm is

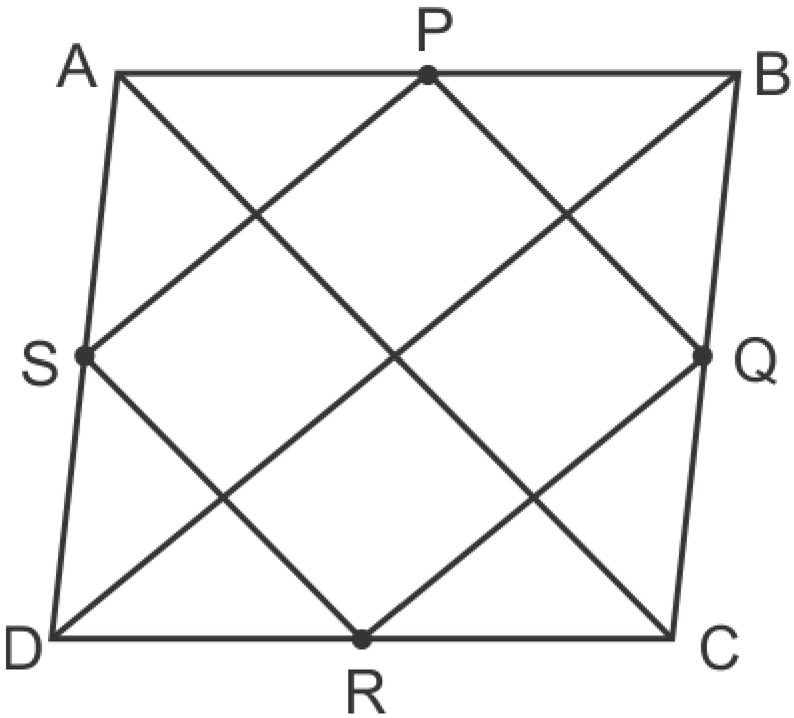
(a) 28 cm^2

(b) 48 cm^2

(c) 96 cm^2

(d) 24 cm^2

Solution 7



$AC = 12$ cm and $BD = 16$ cm (given)

Now, consider $\triangle ABC$.

P and Q are mid – points of sides AB and BC

So line joining them will be parallel to the third side AC and equal to $\frac{1}{2}AC$.

$\Rightarrow PQ = \frac{1}{2}AC = 6$ cm

Similiary, in $\triangle ABD$, PS $\parallel$ BD

$\Rightarrow PS = \frac{1}{2}BD = 8$ cm

Now we know that by joining mid – points of adjacent sides of a Rhombus, we get a Rectangle whose sides are PQ and PS.

$\Rightarrow$ Area $= PQ \times PS = 6 \times 8 = 48\ cm^2$

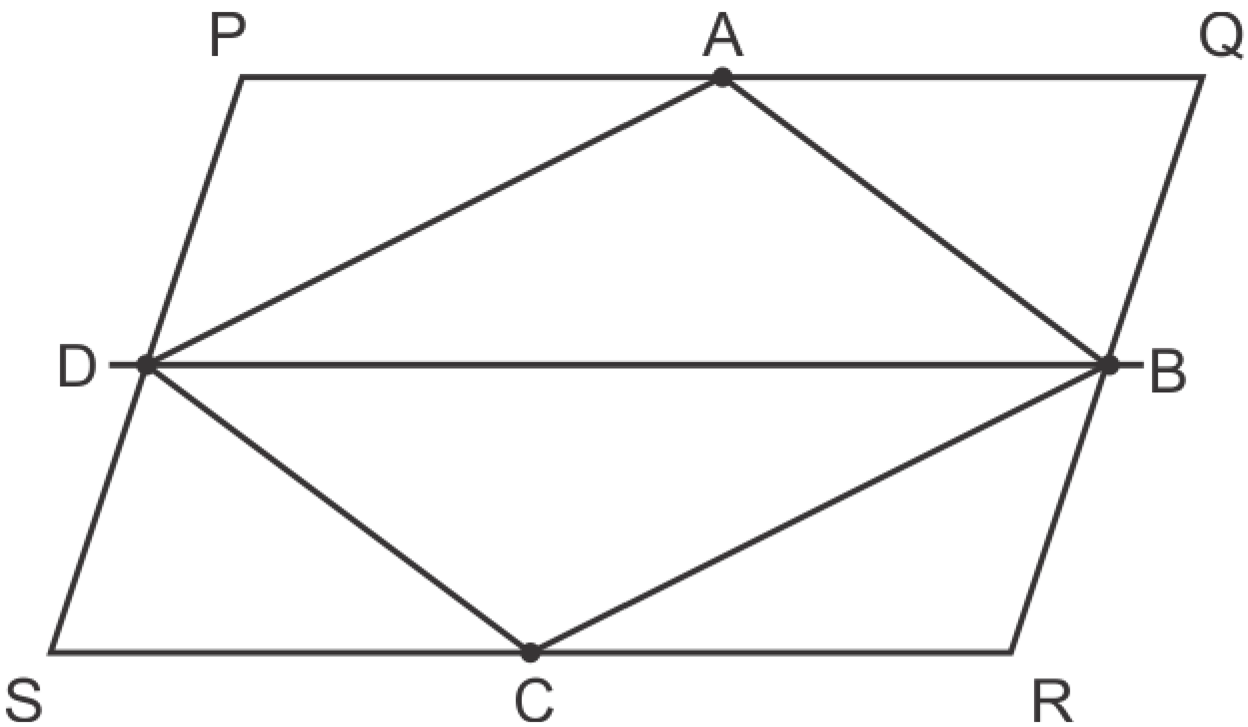
Hence, correct option is (b).

Question 8

A, B, C, D are mid-points of sides of parallelogram PQRS. If ar(PQRS) = 36 cm^2, then ar(ABCD) =

(a) 24 cm^2
(b) 18 cm^2
(c) 30 cm^2
(d) 36 cm^2

Solution 8



D and B are joined.

DB || PQ || RS

Now, consider parallelogram PQBD and parallelogram DBRS.

In PQBD, $\triangle ABD$ has same base and same height as parallelogram PQBD.

So, Area of $\triangle ABD = \frac{1}{2} \times Ar(PQBD)$

Similarly, Area of $\triangle CDB = \frac{1}{2} \times Ar(RSDB)$

$$\text{Area of }(ABCD) = \text{Area of } \triangle ABD + \text{Area of } \triangle CDB$$

$$= \frac{1}{2}[Ar(PQBD) + Ar(RSDB)]$$

$$= \frac{1}{2}\text{Area of PQRS}$$

$$= \frac{1}{2} \times 36$$

$$= 18 \text{ cm}^2$$

Hence, correct option is (b).

Question 9

The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is

(a) a rhombus of area 24 cm^2

(b) a rectangle of area 24 cm^2

(c) a square of area 26 cm^2

(d) a trapezium of area 14 cm^2

Solution 9

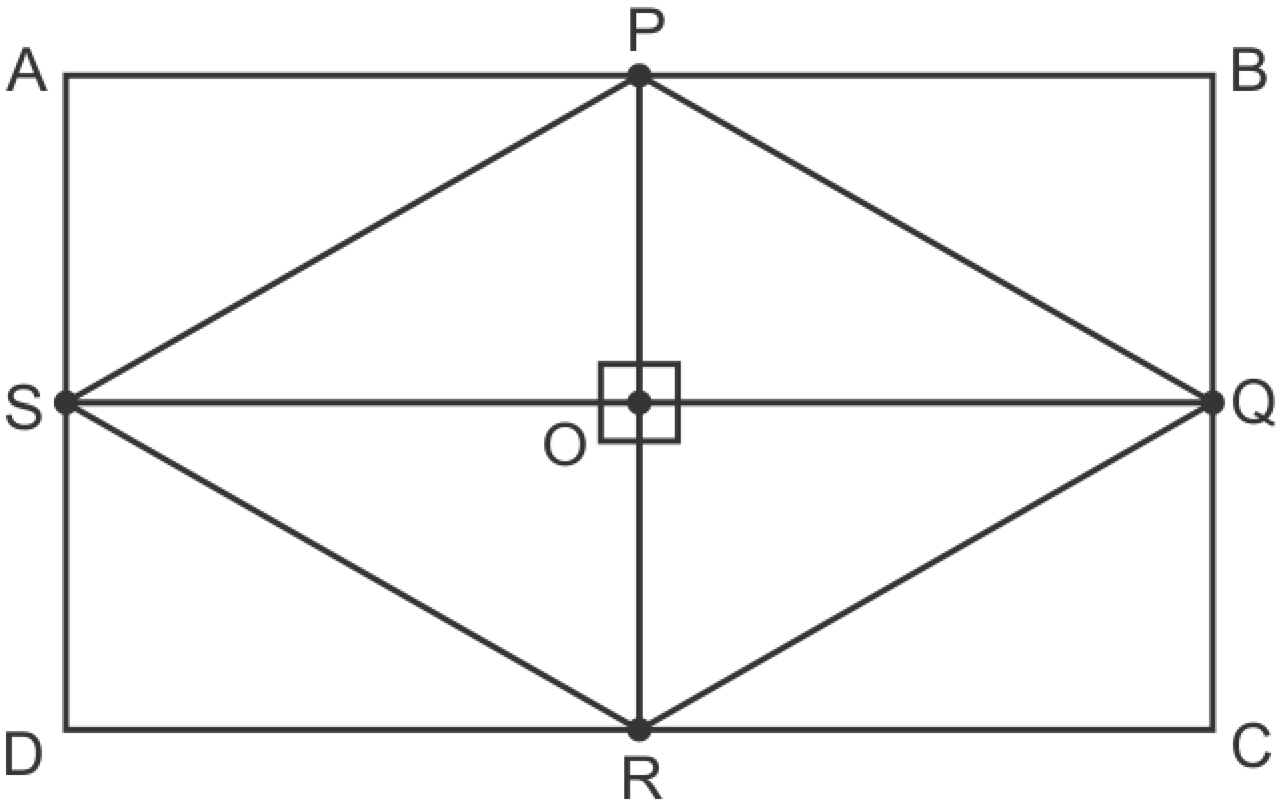


Figure obtained by joining the mid – points of adjacent sides of rectangle ABCD is a rhombus PQRS.

AB = 8 cm, AD = 6 cm

QS and PR are diagonals of Rhombus PQRS.

QS = AB = 8 cm

PR = AD = 6 cm

Ar(Rhombus PQRS) = 4 × Area of $\triangle$POR

$$= 4 \times \frac{1}{2} \times OQ \times OP \quad \left(\begin{array}{l}\triangle POQ \text{ is a Right } \triangle \\ OQ = \frac{QS}{2},\ OP = \frac{PR}{2}\end{array}\right)$$

$$= \frac{\cancel{4}^{2}}{2} \times 4 \times 3$$

$\Rightarrow$ Ar(Rhombus PQRS) $= 24\ cm^2$

Hence, correct option is (a).

Question 10

If AD is median of ΔABC and P is a point on AC such that ar(ΔADP) : ar (ΔABC) = 2 : 3, then ar (ΔPDC) : ar(ΔABC) is

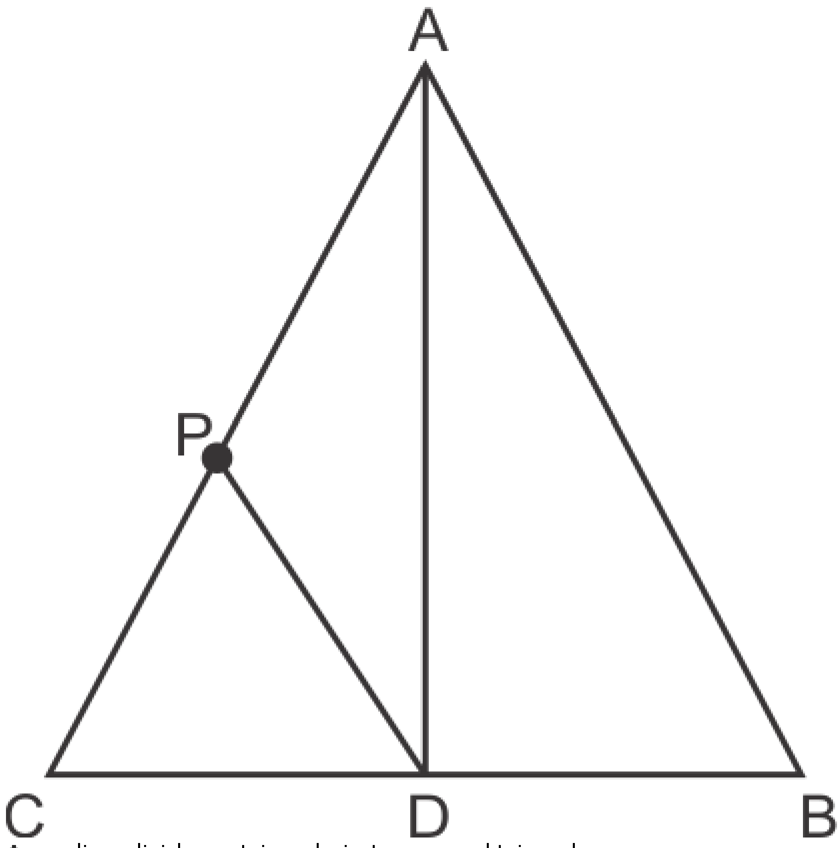
(a) 1 : 5

(b) 1 : 5

(c) 1 : 6

(d) 3 : 5

Solution 10



A median divides a triangle in two equal triangles.

$\Rightarrow Ar(\Delta ABD) = Ar(\Delta ADC)$

$Ar(\triangle PDC) = Ar(\Delta ADC) - Ar(\Delta ADP)$

$\Rightarrow Ar(\Delta PDC) = Ar(\Delta ABD) - Ar(\Delta ADP) \quad ....(1)$

Also,

$Ar(\Delta ABC) = 2 \times Ar(\Delta ABD)$

Dividing equation (1) by $Ar(\Delta ABC)$, we get

$$\frac{Ar(\Delta PDC)}{Ar(\Delta ABC)} = \frac{Ar(\Delta ABD)}{Ar(\Delta ABC)} - \frac{Ar(\Delta ADP)}{Ar(\Delta ABC)}$$

$$\Rightarrow \frac{Ar(\Delta PDC)}{Ar(\Delta ABC)} = \frac{\cancel{Ar(\Delta ABD)}}{2\cancel{Ar(\Delta ABD)}} - \frac{Ar(\Delta ADP)}{2Ar(\Delta ABD)}$$

$$= \frac{1}{2} - \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}$$

$$= \frac{1}{6}$$

$$= 1 : 6$$

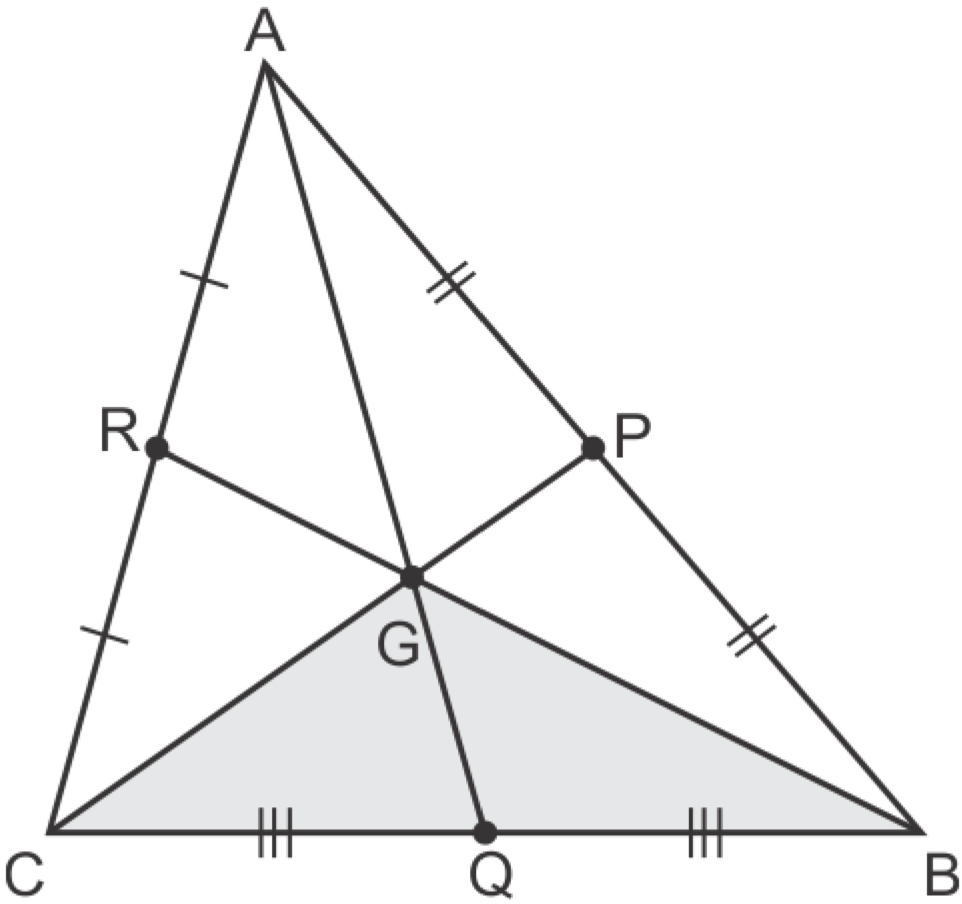
Hence, correct option is (c).

Question 11

Medians of ΔABC intersect at G. If ar(ΔABC) = 27 cm^2, then ar(ΔBGC) =

(a) 6 cm^2
(b) 9 cm^2
(c) 12 cm^2
(d) 18 cm^2

Solution 11



AQ, CP and RB are medians of $\triangle ABC$.

Consider $\triangle ACP$ & $\triangle ACQ$

$Ar(\triangle ACP) = \frac{27}{2} cm^2$ {Median divides a $\triangle$ into two equal Area}

$Ar(\triangle ACQ) = \frac{27}{2} cm^2$ {Median divides a $\triangle$ into two equal Area}

$\Rightarrow Ar(\triangle ACP) = Ar(ACQ)$

$Ar(\triangle AGC)$ is common in both the triangles.

$\Rightarrow Ar(\triangle CGQ) = Ar(\triangle AGP) \quad ....(1)$

Similiarly $Ar(\triangle ABR) = \frac{27}{2} cm^2 = Ar(\triangle AQB)$

$Ar(\triangle AGB)$ is common in both the triangels.

$\Rightarrow Ar(\triangle ARG) = Ar(\triangle GQB) \quad ....(2)$

From figure GR, GP, GQ are also medians for $\triangle AGC$, $\triangle AGB$ & $\triangle CGB$ respectively.

$\Rightarrow Ar(\triangle AGC) + Ar(\triangle AGB) + Ar(\triangle CGB) = 27 cm^2$

$\Rightarrow 2Ar(\triangle ARG) + 2Ar(\triangle AGP) + Ar(\triangle BGC) = 27 cm^2$

$\Rightarrow 2(Ar(\triangle ARG) + Ar(\triangle AGP)) + Ar(\triangle BGC) = 27 cm^2$

From equauations (1) and (2),

$2[Ar(\triangle GQB) + Ar(\triangle CGQ)] + Ar(\triangle BGC) = 27 cm^2$

$\Rightarrow 2[Ar(\triangle BGC)] + Ar(\triangle BGC) = 27 cm^2$

$\Rightarrow Ar(\triangle BGC) = 9 cm^2$

Hence, correct option is (b).

Question 12

In a ΔABC if D and E are mid-points of BC and AD respectively such that $ar(\Delta AEC) = 4cm^2$, then $ar(\Delta BEC) =$

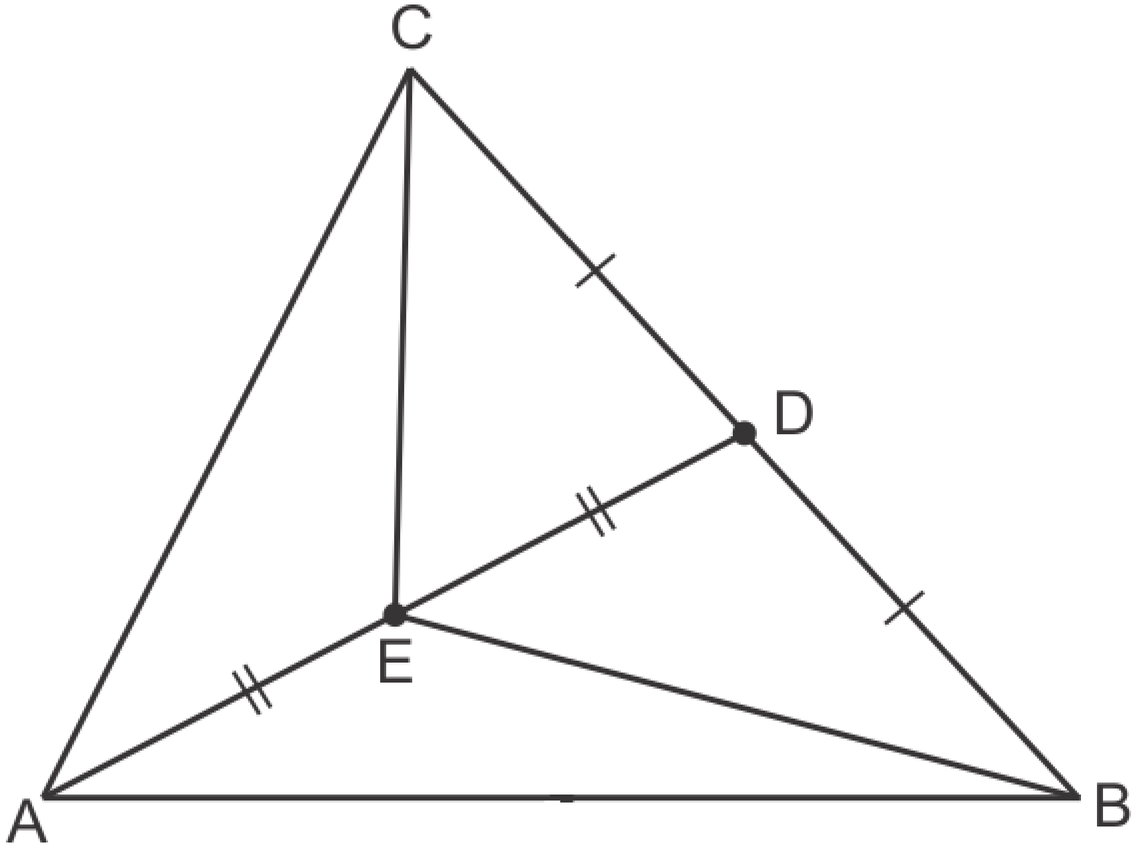
(a) 4 cm^2

(b) 6 cm^2

(c) 8 cm^2

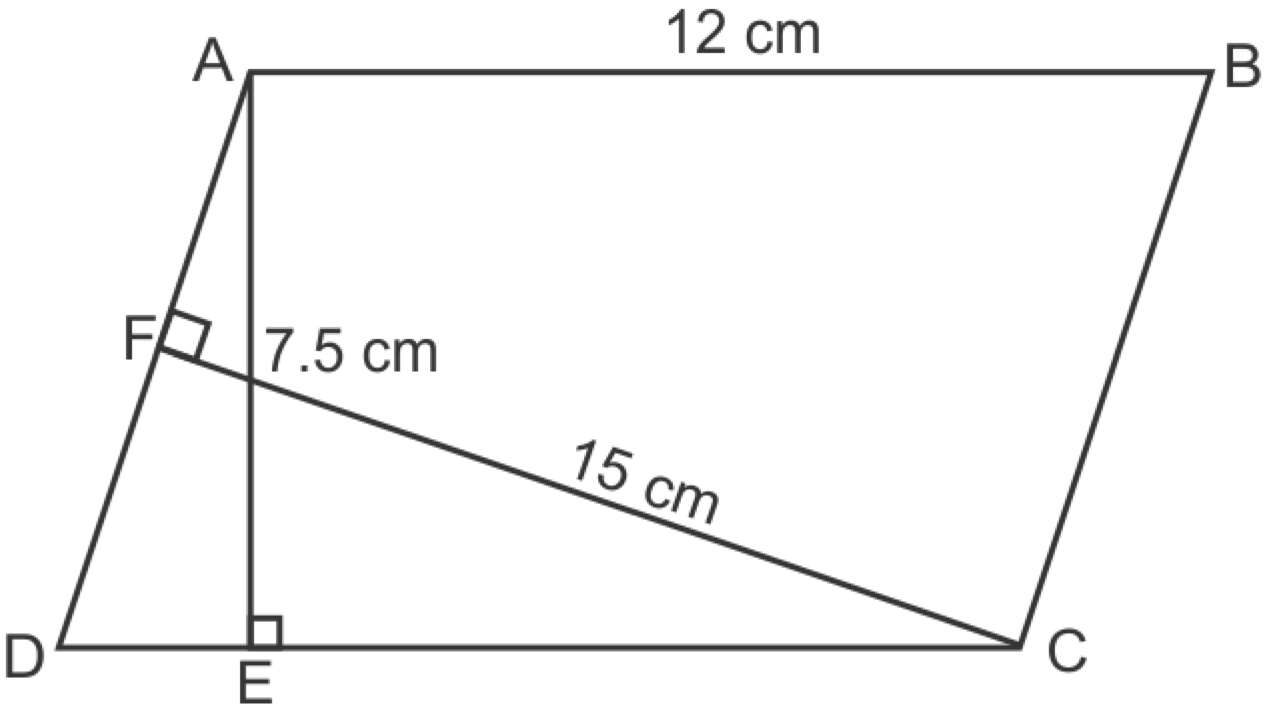
(d) 12 cm^2

Solution 12

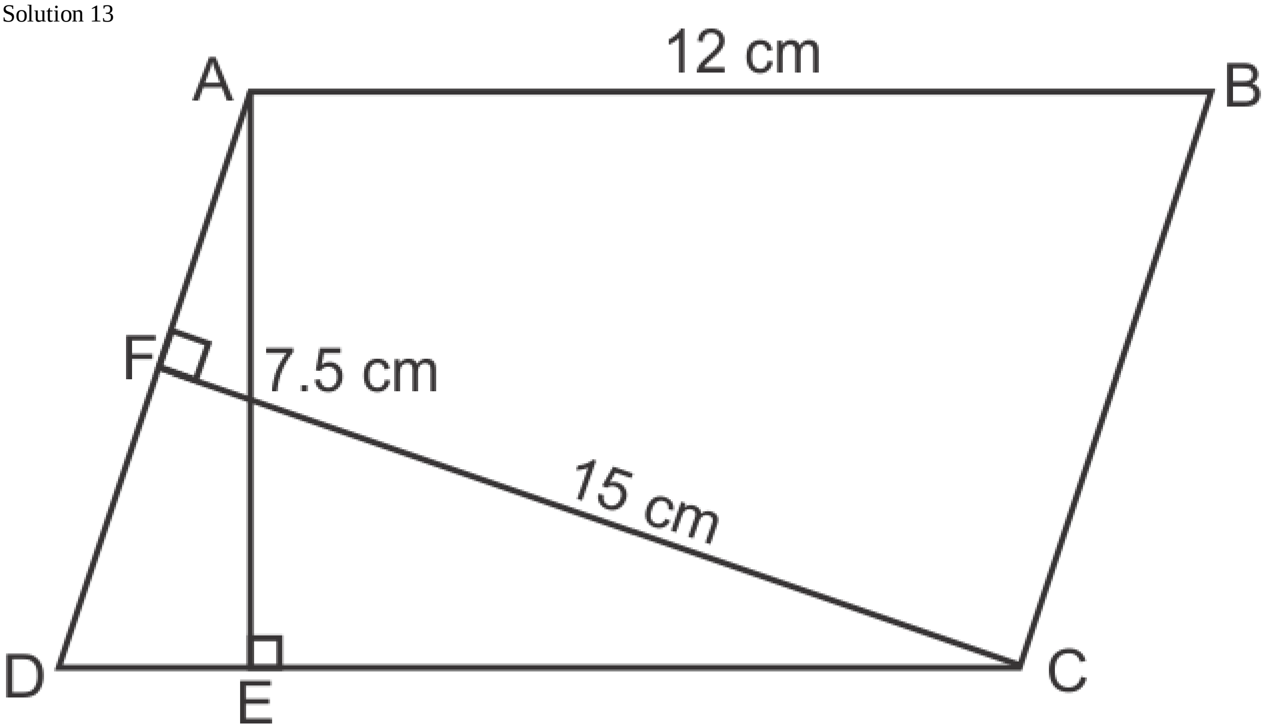


E is the mid-point of AD and CE is median of ΔACD.
Hence Ar(ΔAEC) = Ar(ΔCED) = 4 cm^2 ....(1)
(Median divides a Δ in two two equal Areas)
Also AD is median of ΔABC and and ED is median of ΔBEC.
So Ar(ΔBED) = Ar(CED) = 4 cm^2 [From eq (1)]
So Ar(ΔBEC) = Ar(ΔBED) + Ar(ΔCED) = 4 + 4 = 8 cm^2
Hence, correct option is (c).

Question 13
In figure, ABCD is a parallelogram. If AB = 12 cm, AE = 7.5 cm, CF = 15 cm, then AD =
(a) 3 cm
(b) 6 cm
(c) 8 cm
(d) 10.5 cm



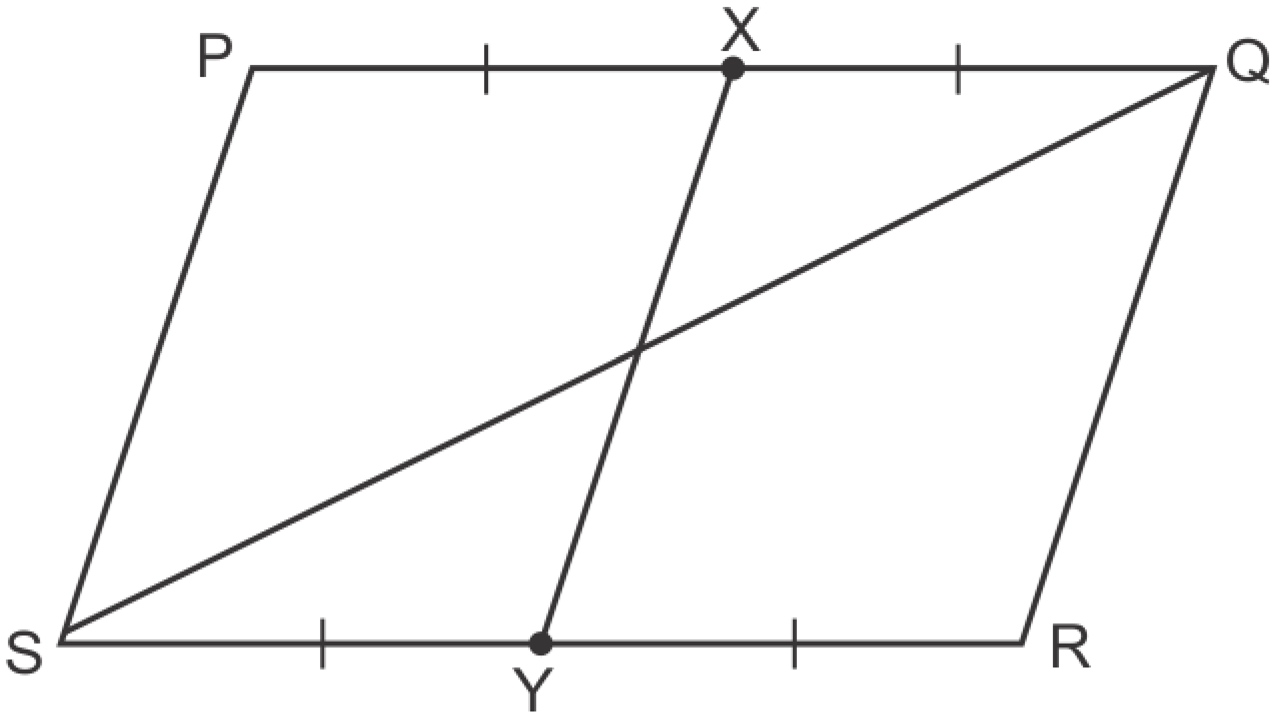
Solution 13



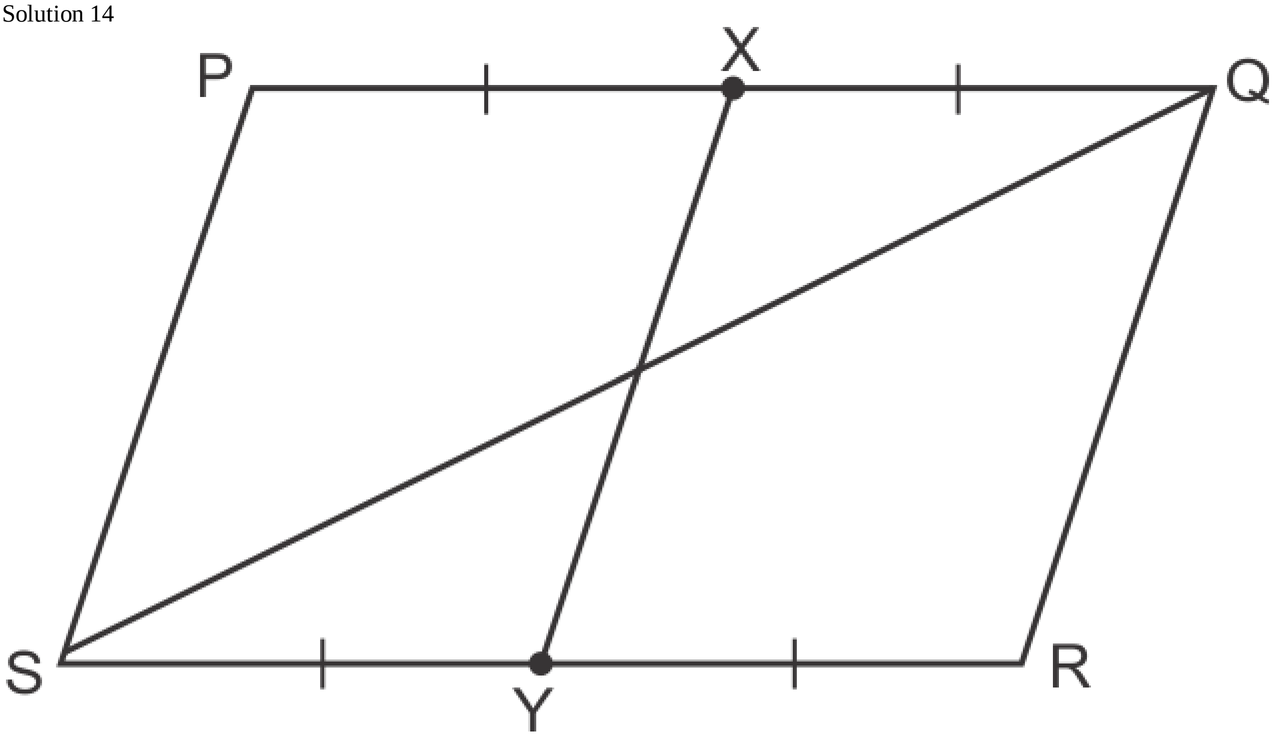
Area of parallelogram = AD × FC = AB × AE
Thus,
AD × 15 = 12 × 7.5
AD = 6 cm
Hence, correct option is (b).

Question 14

In figure, PQRS is a parallelogram. If X and Y are mid-points of PQ and SR respectively and diagonal SQ is joined. The ratio ar(||^{gm} XQRY) : ar (ΔQSR) =
(a) 1 : 4
(b) 2 : 1
(c) 1 : 2
(d) 1 : 1



Solution 14



Diagonal SQ divides $||^{gm}$ in two equal areas.

Hence Ar($\triangle$QSR) $= \frac{1}{2}$ Ar(PQRS)

Also XY divides the $||^{gm}$ into two equal parts.

Hence, Area($||^{gm}$XQRY) $= \frac{1}{2}$ Ar (PQRS)

Thus, Ratio of Ar($||^{gm}$XQRY) : Ar($\triangle$QSR) $= 1 : 1$

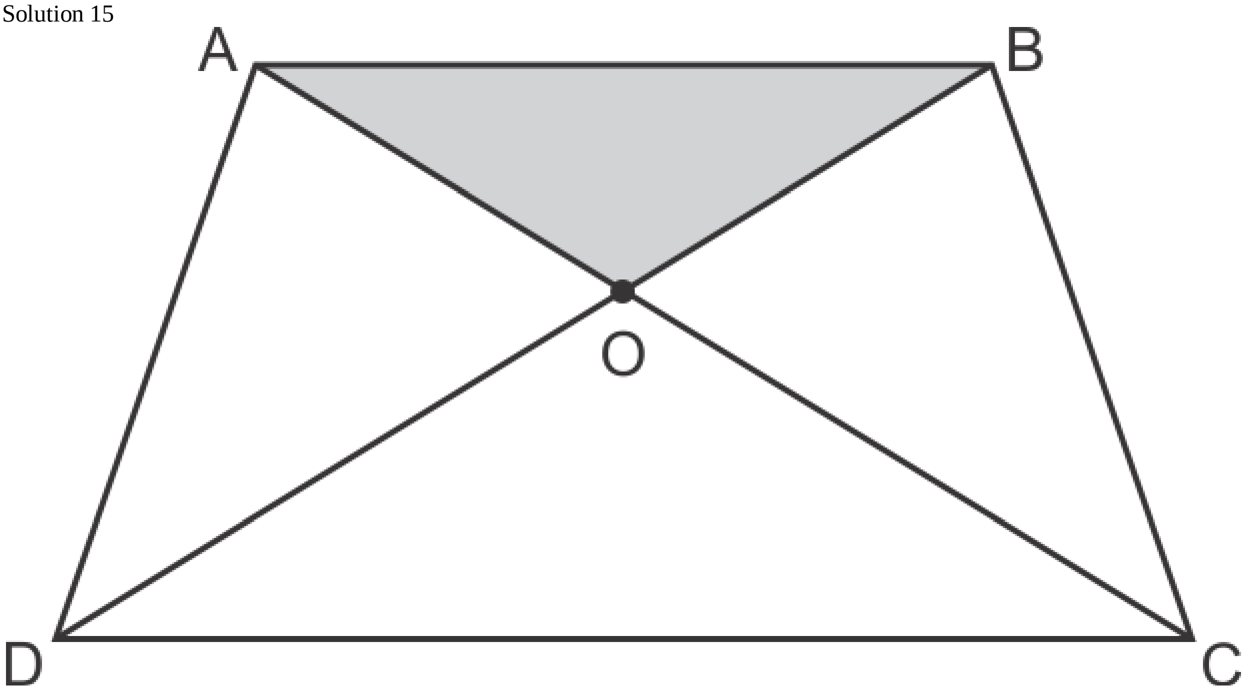
Hence, correct option is (d).

Question 15

Diagonal AC and BD of trapezium ABCD, in which AB || DC, intersect each other at O. The triangle which is equal in area of ΔAOD is

(a) ΔAOB
(b) ΔBOC
(c) ΔDOC
(d) ΔADC

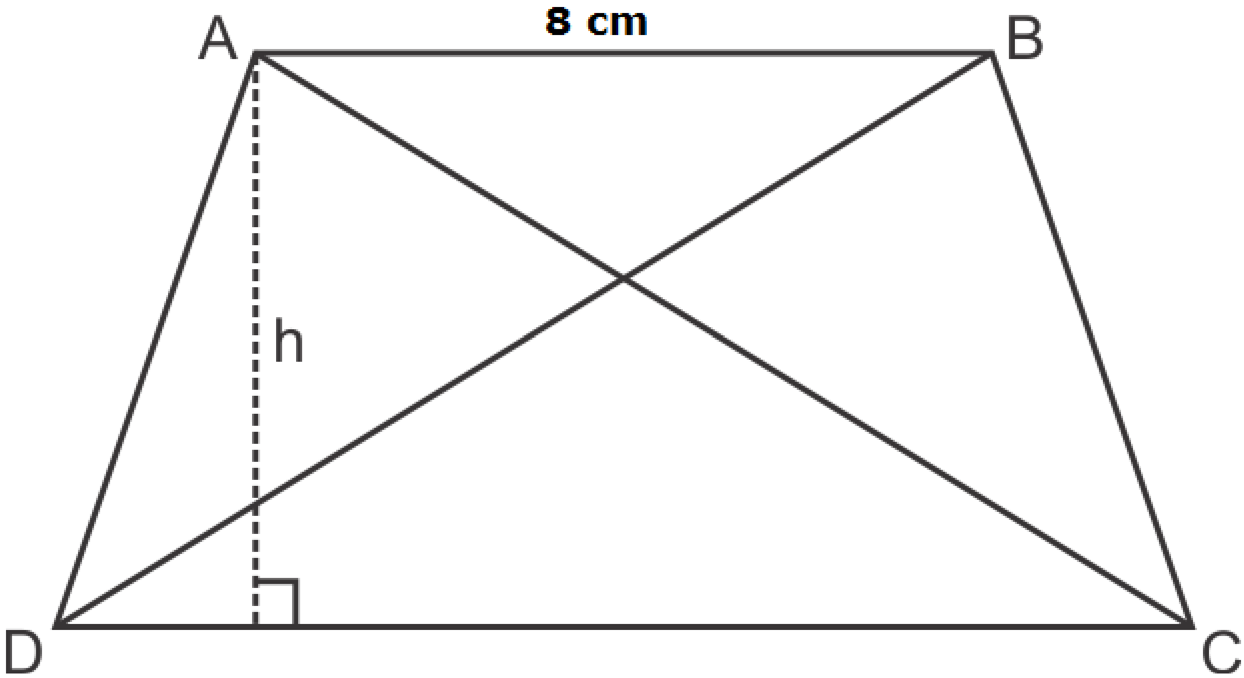
Solution 15



ΔABD & ΔABC have same base AB and are between same parallels.
Then,
Ar(ΔABD) = Ar(ΔABC)
But Ar(ΔAOB) is common in both.
Thus, Ar(ΔAOD) = Ar(ΔBOC)
Hence, correct option is (b).

Question 16
ABCD is a trapezium in which AB || DC. If ar(ΔABD)= 24 cm^2 and AB = 8 cm, then height of ΔABC is
(a) 3 cm
(b) 4 cm
(c) 6 cm
(d) 8 cm

Solution 16



$\triangle ABD$ & $\triangle ABC$ are on same base AB and are between same parallels.

$\Rightarrow Ar(\triangle ABD) = Ar(\triangle ABC)$

$Ar(\triangle ABD) = \frac{1}{2} \times 8 \times h \quad = 24 \text{ cm}^2$

$\Rightarrow h = 6 \text{ cm}$

Now $Ar(\triangle ABC) = \frac{1}{2} \times 8 \times h \quad = 24 \text{ cm}^2$

$\Rightarrow h = 6 \text{ cm}$

Hence, correct option is (c).

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Question 19

The mid – points of the sides of a triangle ABC along with any of the vertices as the fourth point make a parallelogram of a area equal to

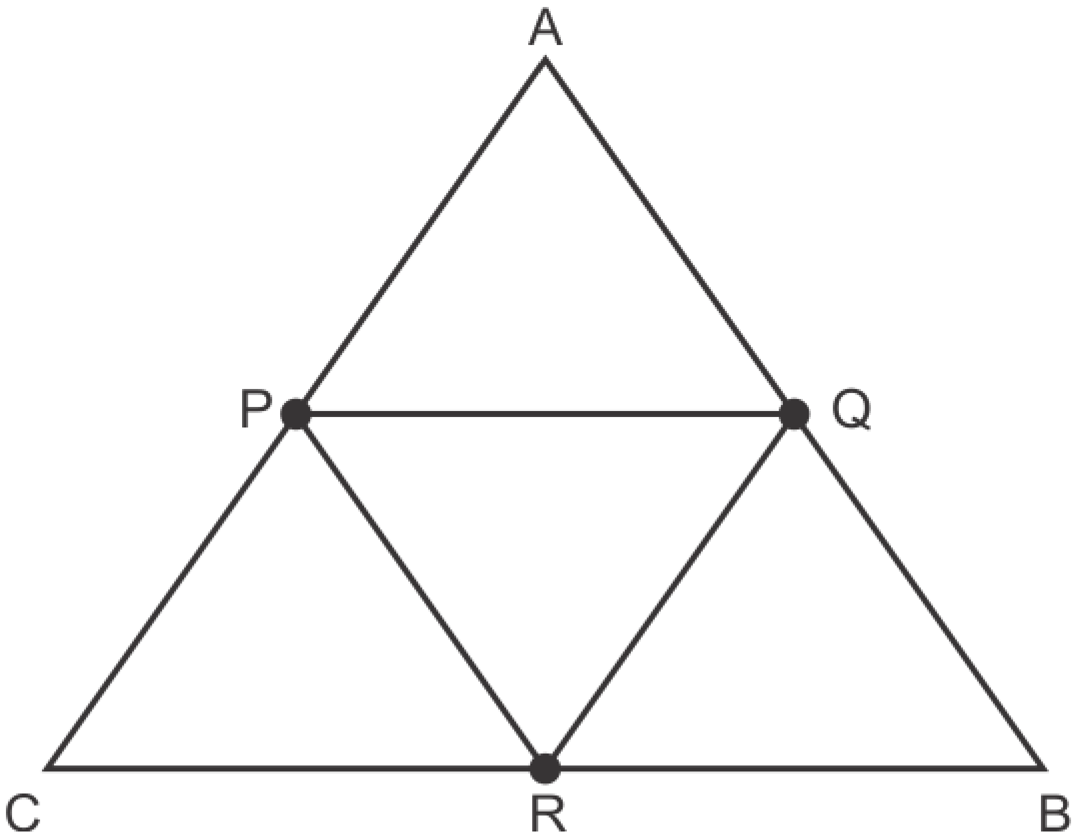
(a) $ar(\triangle ABC)$

(b) $\frac{1}{2} ar(\triangle ABC)$

(c) $\frac{1}{3} ar(\triangle ABC)$

(d) $\frac{1}{4} ar(\triangle ABC)$

Solution 19



AQRP is a required parallelogram by by joining the mid – points.
All 4 triangles formed are congruent and are equal in area.

So area of any one $\triangle = \frac{1}{4}Ar(\triangle ABC)$

$Ar(\triangle APQ) + Ar(\triangle PQR) = \frac{1}{2}Ar(\triangle ABC)$

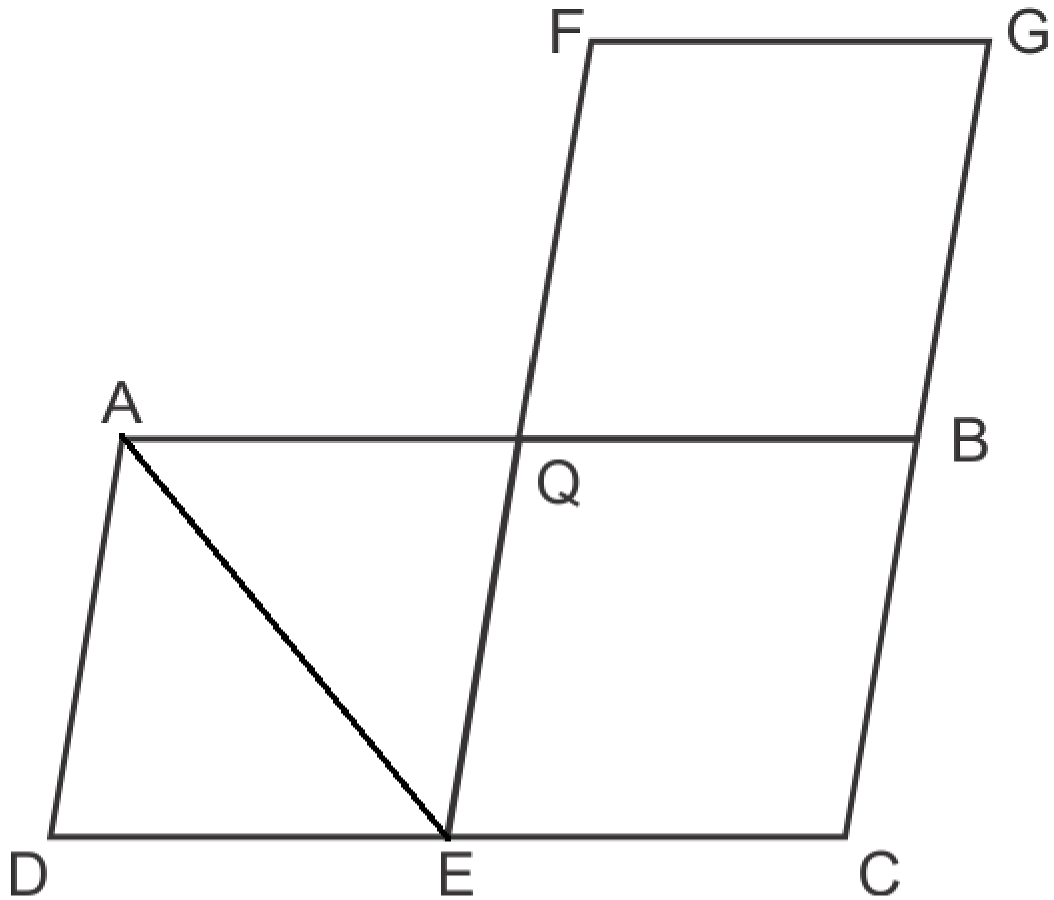
$\Rightarrow Ar(AQRP) = \frac{1}{2}Ar(\triangle ABC)$

Hence, correct option is (b).

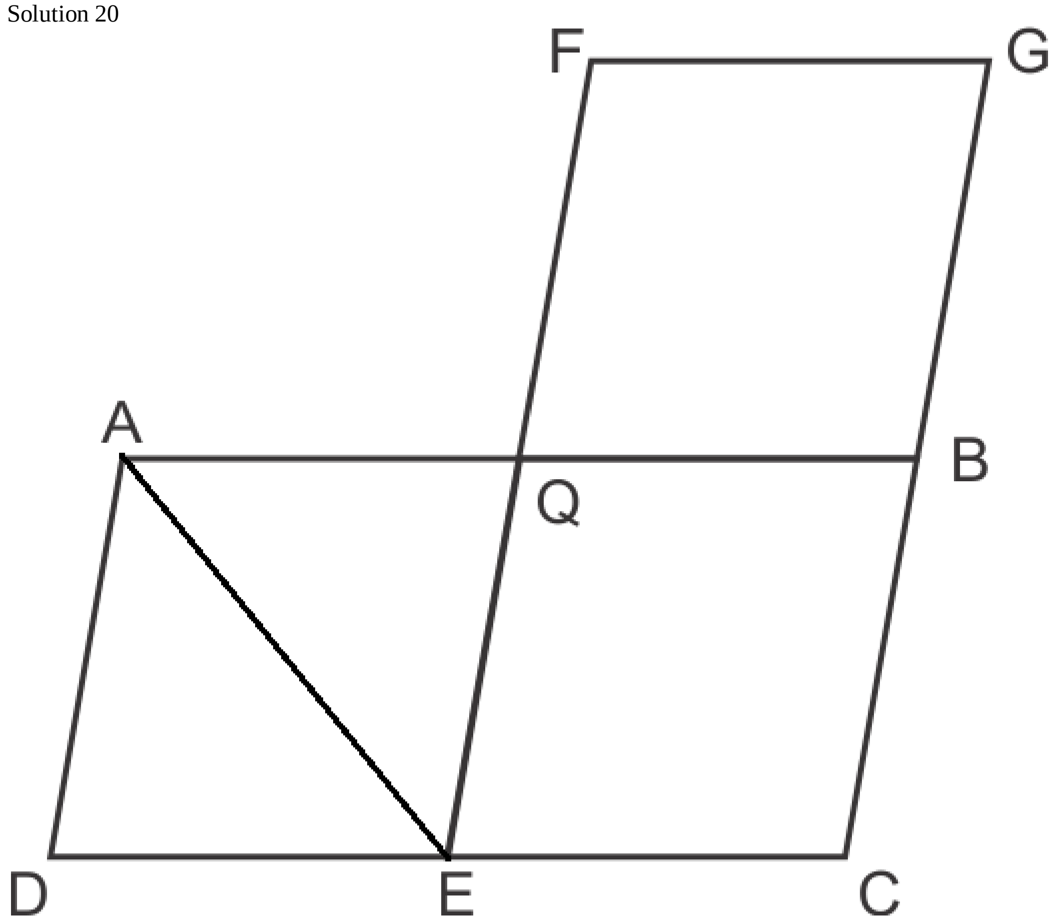
Question 20

In figure, ABCD and FECG are parallelograms equal in area. If ar(ΔAQE) = 12 cm^2, then ar($\|^{gm}$ FGBQ) =

(a) 12 cm^2
(b) 20 cm^2
(c) 24 cm^2
(d) 36 cm^2



Solution 20



$\text{Ar}(\parallel^{gm} ABCD) = \text{Ar}(\parallel^{gm} FECG)$

Ar of $(\parallel^{gm} QBCE)$ is common in both,

$\Rightarrow \text{Ar}(\parallel^{gm} AQED) = \text{Ar}(\parallel^{gm} FGBQ)$ ....(1)

Now AE is diagonal of AQED.

$\Rightarrow \text{Ar}(\triangle AQE) = \frac{1}{2}\text{Ar}(\parallel^{gm} AQED)$

$\Rightarrow 12 \text{ cm}^2 = \frac{1}{2}\text{Ar}(\parallel^{gm} AQED)$

$\Rightarrow \text{Ar}(\parallel^{gm} AQED) = 2 \times 12 \text{ cm} = 24 \text{ cm}^2$

$\Rightarrow \text{Ar}(\parallel^{gm} FGBQ) = 24 \text{ cm}^2$ [From (1)]

Hence, correct option is (c).

Question 17

ABCD is a trapezium with parallel sides AB = a and DC = b. If E and F are mid-points of non-parallel sides AD and BC respectively, then the ratio of area of quadrilaterals ABFE and EFCD is

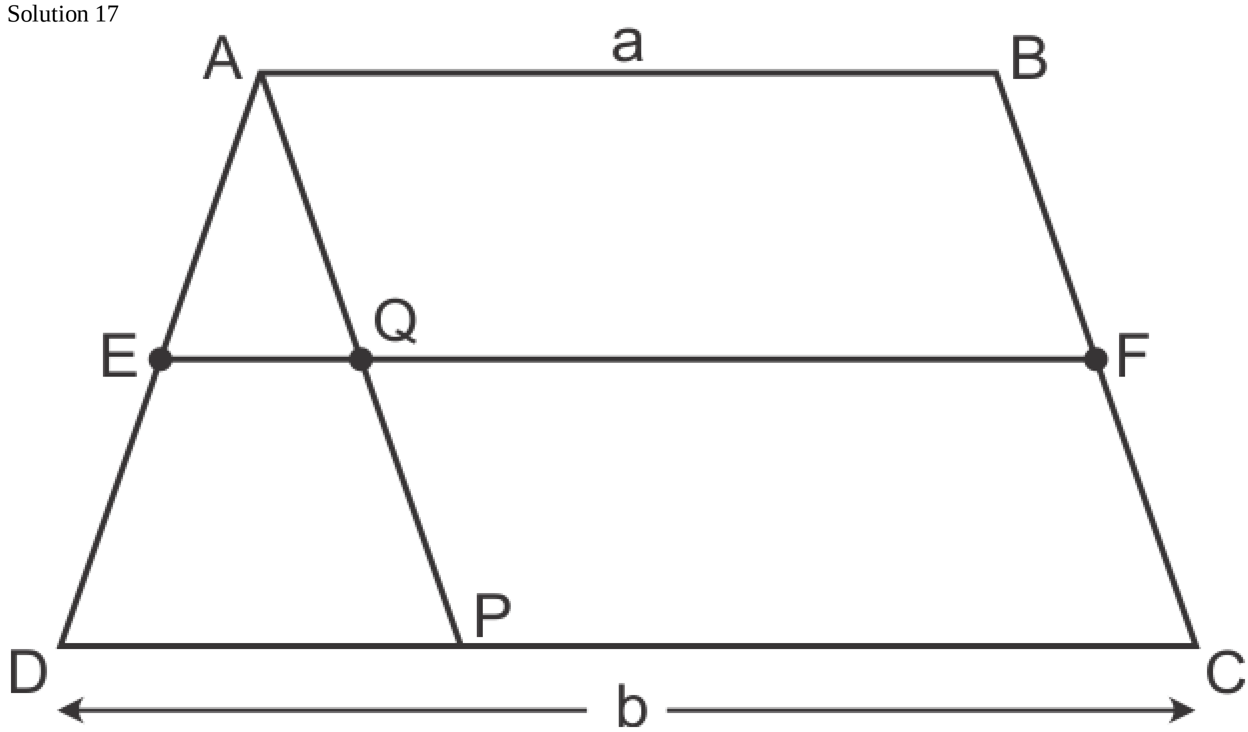
(a) a : b

(b) (a + 3b) : (3a + b)

(c) (3a + b) : (a + 3b)

(d) (2a + b) : (3a + b)

Solution 17



AP is drawn parallel to BC.
ABCP is a parallelogram.
AB = PC = a
DP = DC − PC = b − a

$$\text{Area (ABFE)} = \text{Ar}(\triangle AQE) + \text{Ar}(\|^{gm}\ ABFQ)$$
$$= \frac{1}{4}(\text{Ar}(\triangle ADP)) + \frac{1}{2}\text{Ar}(\|^{gm}\ ABCP)$$
$$= \frac{1}{4} \times \frac{1}{2} \times (b - a)h + \frac{1}{2} \times a \times h$$
$$= \frac{(3a + b)h}{8}$$

Siimiliarly, Area of trapazium EFCD

$$= \text{Ar(EQPD)} + \text{Ar}(\|^{gm}\ QFCP)$$
$$= \frac{3}{4}\text{Ar}(\triangle ADP) + \frac{1}{2}(\|^{gm}\ ABCP)$$
$$= \frac{3}{4} \times \frac{1}{2} \times (b - a) \times h + \frac{1}{2} \times a \times h$$
$$= \frac{(3b + a)h}{8}$$

$$\Rightarrow \text{Ratio of Ar (Quad ABFE)} : \text{Ar(Quad EFCD)} = \frac{(3a+b)h}{8} : \frac{(3b+a)h}{8}$$
$$= (3a + b) : (a + 3b)$$

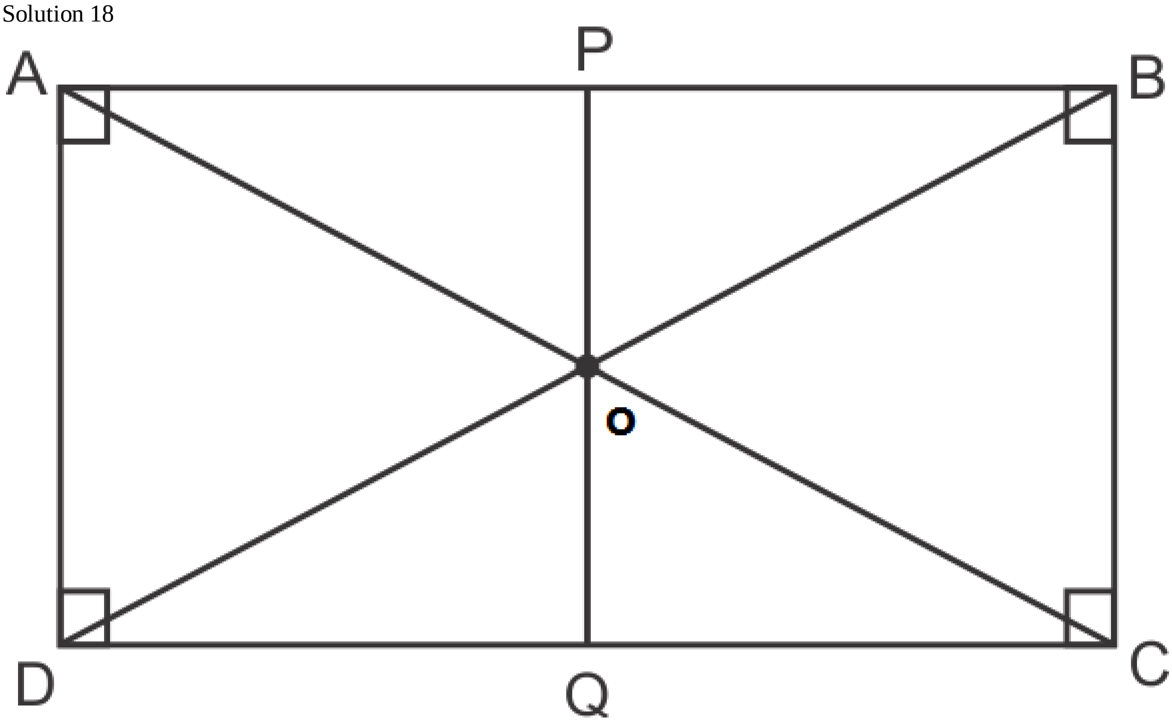
Hence, correct option is (c).

Question 18

ABCD is a rectangle with O as any point in its interior. If ar (ΔAOD) = 3 cm^2 and ar (ΔBOC) = 6 cm^2, then area of rectangle ABCD is

(a) 9 cm^2
(b) 12 cm^2
(c) 15 cm^2
(d) 18 cm^2

Solution 18



A line PQ is drawn from AB parallel to AD & BC.

Now, $\triangle AOD$ has height $= AP$

And, $\triangle BOC$ has height $= BP$

Area of $\triangle AOD = \frac{1}{2} \times AD \times AP = 3\ cm^2$

$\Rightarrow AD \times AP = 6\ cm^2$ ....(1)

$Ar(\triangle BOC) = \frac{1}{2} \times BC \times BP = 6\ cm^2$

$\Rightarrow BC \times BP = 12\ cm^2$ ....(2)

Adding equations (1) and (2), we get

$AD \times AP + BC \times BP = 18\ cm^2$

$\Rightarrow AD \times AP + AD \times BP = 18\ cm^2 \quad (AD = BC)$

$\Rightarrow AD(AP + BP) = 18\ cm^2$

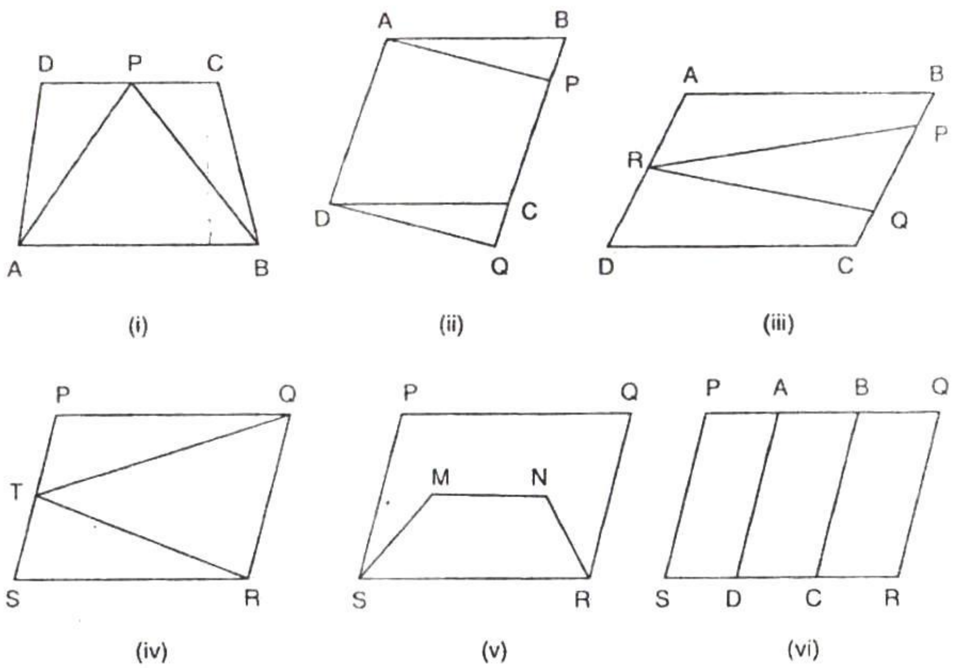
$\Rightarrow AD \times AB = 18\ cm^2 =$ Area of rectangle ABCD

Hence, correct option is (d).

Chapter 14 - Areas of Parallelograms and Triangles Exercise Ex. 14.1

Question 1

Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and two parallels:



Solution 1

(i) ΔAPB and trapezium ABCD are on the same base AB and between the same parallels AB and CD.

(ii) Parallelograms ABCD and APQD are on the same base AD and between the same parallels AD and BQ.

(iii) Parallelogram ABCD and ΔPQR are between the same parallels AD and BC but they are not on the same base.

(iv) ΔQRT and parallelogram PQRS are on the same base QR and between the same parallels QR and PS.

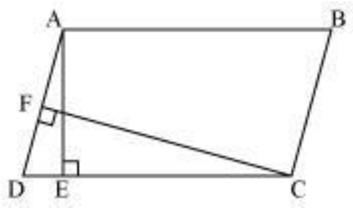
(v) Parallelogram PQRS and trapezium SMNR are on the same base SR but they are not between the same parallels.

(vi) Parallelograms PQRS, AQRD, BQRC are between the same parallels. Also, parallelograms PQRS, BPSC and APSD are between the same parallels.

Chapter 14 - Areas of Parallelograms and Triangles Exercise Ex. 14.2

Question 1

In the given figure, ABCD is parallelogram, AE $\perp$ DC and CF $\perp$ AD. If AB = 16 cm. AE = 8 cm and CF = 10 cm, find AD.



Solution 1

In parallelogram ABCD, CD = AB = 16 cm [Opposite sides of a parallelogram are equal]

We know that,

Area of parallelogram = Base x corresponding attitude

Area of parallelogram ABCD = CD x AE = AD x CF

16 cm x 8 cm = AD x 10 cm

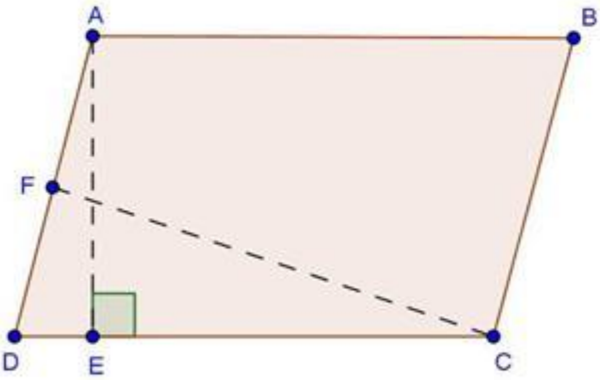
AD = $\frac{16 \times 8}{10}$ cm = 12.8 cm.

Thus, the length of AD is 12.8 cm.

Question 2

In Q. No. 1, if AD = 6 cm, CF = 10 cm, and AE = 8, find AB.

Solution 2



Area of parallelogram $ABCD = AD \times CF$ $\quad --- (1)$

Again area of parallelogram $ABCD = DC \times AE$ $\quad --- (2)$

Compare equation (1) and equation (2)

$AD \times CF = DC \times AE$

$\Rightarrow \quad 6 \times 10 = DC \times 8$

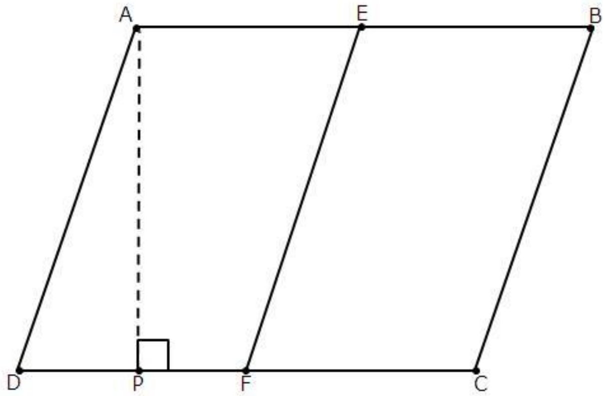
$\Rightarrow \quad DC = \dfrac{6 \times 10}{8} = 7.5 \text{ cm}$

$\therefore \quad AB = DC = 7.5 \text{ cm}$ $\quad$ [Opposite sides of $\|$gm]

Question 3

Let $ABCD$ be a parallelogram of area 124 cm^2. If E and F are the mid-points of sides AB and CD respectively, then, find the area of parallelogram $AEFD$.

Solution 3



Given,

Area of parallelogram ABCD = 124 cm^2

Construction : Draw AP $\perp$ DC

Proof:-

Area of parallelogram AEFD = DF $\times$ AP ...(1)

And area of parallelogram EBCF = FC $\times$ AP ...(2)

And DF = FC ...(3) [F is the mid-point of DC]

Compare equation (1), (2) and (3)

Area of parallelogram AEFD = area of parallelogram EBCF

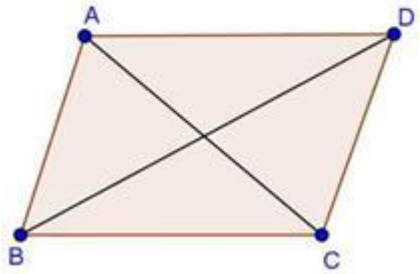
Therefore, Area of parallelogram AEFD $= \dfrac{\text{Area of parallelogram ABCD}}{2} = \dfrac{124}{2} = 62 \text{ cm}^2$

Question 4

If $ABCD$ is a parallelogram, the prove that

$$ar(\triangle ABD) = ar(\triangle BCD) = ar(\triangle ABC) = ar(\triangle ACD) = \frac{1}{2} ar(\|^{gm}\ ABCD)$$

Solution 4



Given: - $ABCD$ is a parallelogram.

To prove: - $ar(\triangle ABD) = ar(\triangle BCD) = ar(\triangle ABC) = ar(\triangle ACD) = \frac{1}{2} ar(||^{gm}\ ABCD)$

Proof: - We know that diagonal of a parallelogram divides it into two equal triangles.

Since, AC is the diagonal

Then, $ar(\triangle ABC) = ar(\triangle ACD) = \frac{1}{2} ar(||^{gm}\ ABCD) \quad ---(1)$

Since, BD is the diagonal

Then, $ar(\triangle ABD) = ar(\triangle BCD) = \frac{1}{2} ar(||^{gm}\ ABCD) \quad ---(2)$

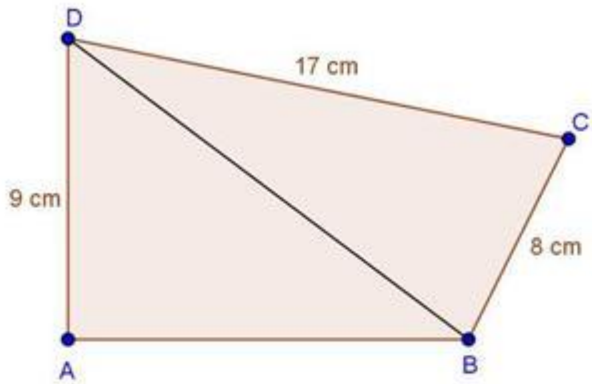
Compare equation (1) and (2)

$\therefore \quad ar(\triangle ABC) = ar(\triangle ACD) = ar(\triangle ABD) = ar(\triangle BCD) = \frac{1}{2} ar(||^{gm}\ ABCD)$

Chapter 14 - Areas of Parallelograms and Triangles Exercise Ex. 14.3

Question 1

In fig., compute the area of quadrilateral ABCD.



Solution 1

In $\triangle BCD$, we have

$CD^2 = BD^2 + BC^2$

$\Rightarrow \quad (17)^2 = BD^2 + (8)^2$

$\Rightarrow \quad BD^2 = 289 - 64$

$\Rightarrow \quad BD = 15$

In $\triangle ABD$, we have

$BD^2 = AB^2 + AD^2$

$\Rightarrow \quad (15)^2 = AB^2 + (9)^2$

$\Rightarrow \quad AB^2 = 225 - 81 = 144$

$\Rightarrow \quad AB = 12$

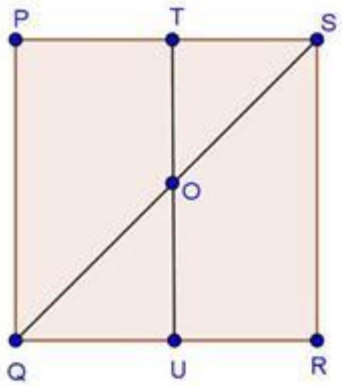
$\therefore \quad ar(\text{quad. } ABCD) = ar(\triangle ABD) + ar(\triangle BCD)$

$\Rightarrow \quad ar(\text{quad. } ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68 = 112 \text{ cm}^2$

$\Rightarrow \quad \text{ar(quad. ABCD)} = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15) = (54 + 60)\text{cm}^2 = 114\text{cm}^2$

Question 2

In the fig., PQRS is a square and T and U are, respectively, the mid-points of PS and QR. Find the area of ΔOTS if PQ = 8 cm.



Solution 2

Since, T and U are the mid-points of PS and QR respectively.

$\therefore \quad TU \parallel PQ$

$\Rightarrow \quad TO \parallel PQ$

Thus, in $\triangle PQS$, T is the mid-point of PS and $TO \parallel PQ$.

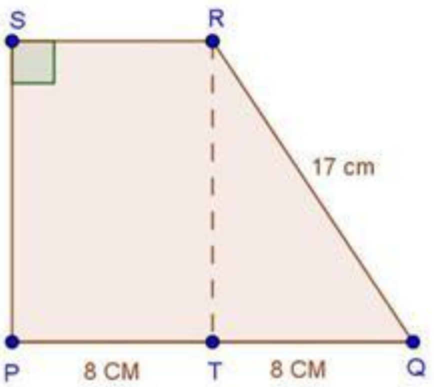
$\therefore \quad TO = \frac{1}{2} PQ = 4$ cm

Also, $TS = \frac{1}{2} PS = 4$ cm

$\therefore \quad ar(\triangle OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4)\ \text{cm}^2 = 8\ \text{cm}^2$

Question 3

Compute the area of trapezium PQRS in fig.



Solution 3

We have,

$ar(\text{trap. } PQRS) = ar(\text{rect. } PSRT) + ar(\triangle QRT)$

$\Rightarrow \quad ar(\text{trap. } PQRS) = PT \times RT + \frac{1}{2}(QT \times RT) = 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$

In $\triangle QRT$, we have,

$QR^2 = QT^2 + RT^2$

$\Rightarrow \quad RT^2 = QR^2 - QT^2$

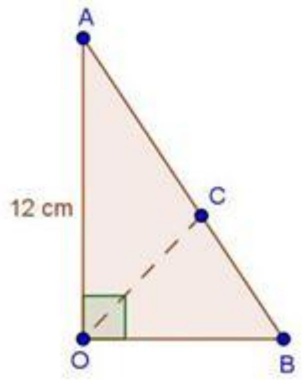
$\Rightarrow \quad RT^2 = (17)^2 - (8)^2 = 225$

$\Rightarrow \quad RT = 15$

Hence, $ar(\text{trap. } PQRS) = 12 \times 15\ \text{cm}^2 = 180\ \text{cm}^2$

Question 4

In fig., $\angle AOB = 90$, AC = BC, OA = 12 cm and OC = 6.5 cm. find the area of ΔAOB



Solution 4

Since, the mid-point of the hypotenuse of a right triangle is equidistant from the vertices.

$\therefore \quad CA = CB = OC$

$\Rightarrow \quad CA = CB = 6.5$ cm

$\Rightarrow \quad AB = 13$ cm

In right triangle OAB, we have

$AB^2 = OB^2 + OA^2$

$\Rightarrow \quad 13^2 = OB^2 + 12^2$

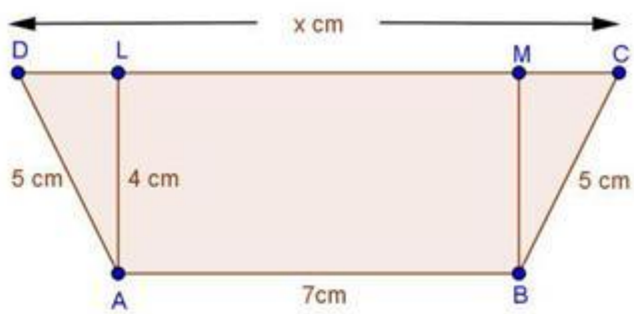
$\Rightarrow \quad OB^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow \quad OB = 5$

$\therefore \quad ar(\Delta AOB) = \frac{1}{2}(OA \times OB) = \frac{1}{2}(12 \times 5) = 30 \text{ cm}^2$

Question 5

In fig., ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.



Solution 5

Draw $AL \perp DC$, $BM \perp DC$. Then,

$AL = BM = 4\ cm$ and $LM = 7$ cm

In $\triangle ADL$, we have

$AD^2 = AL^2 + DL^2$

$\Rightarrow \quad 25 = 16 + DL^2$

$DL^2 = 25 - 16$

$= 9$

$\Rightarrow \quad DL = 3$

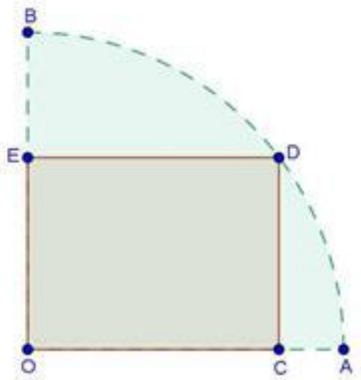
Similarly, $MC = \sqrt{BC^2 - BM^2} = \sqrt{25 - 16} = 3$ cm

$\therefore \quad x = CD = CM + ML + LD = (3 + 7 + 3)$ cm $= 13$ cm

$ar\,(\text{trap.}ABCD) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7 + 13) \times 4\ \text{cm}^2 = 40\ \text{cm}^2$

Question 6

In fig., OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = $2\sqrt{5}$, find the area of the rectangle.



Solution 6

We have, OD = 10 cm and OE = $2\sqrt{5}$cm

$\therefore\ OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{(10)^2 - (2\sqrt{5})^2} = 4\sqrt{5}$ cm

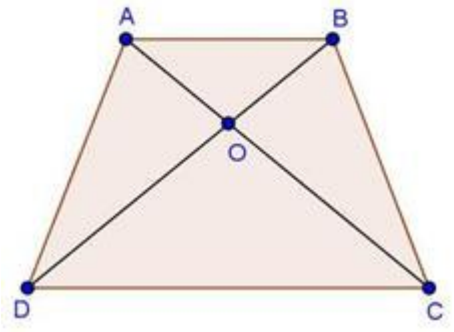
$\therefore$ or $(\text{rect OCDE}) = OE \times DE = 2\sqrt{5} \times 4\sqrt{5}\ \text{cm}^2$

$= 8 \times 5\,\text{cm}^2$

$= 40\,\text{cm}^2$

Question 7

In fig., ABCD is a trapezium in which AB $\parallel$ DC. PRove that ar (ΔAOD) = ar (ΔBOC)



Solution 7

Given:- $ABCD$ is a trapezium with $AB \parallel DC$.

To prove:- $ar(\Delta AOD) = ar(\Delta BOC)$.

Proof:-

Since, ΔADC and ΔBDC are on the same base DC and between same parallels AB and DC

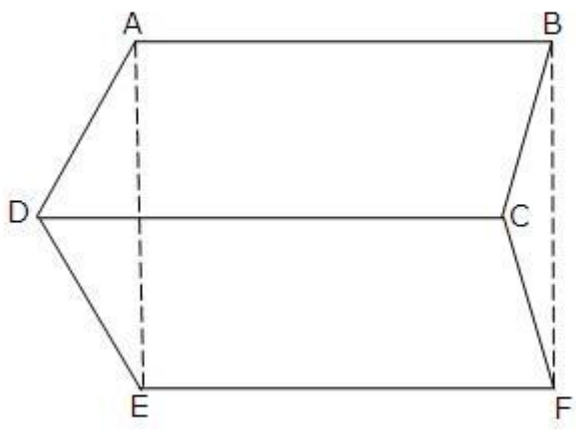
Then, $ar(\Delta ADC) = ar(\Delta BDC)$

$$\Rightarrow \quad ar(\Delta AOD) + ar(\Delta DOC) = ar(\Delta BOC) + ar(\Delta DOC)$$

$$\Rightarrow \quad ar(\Delta AOD) = ar(\Delta BOC)$$

Question 8

In figure, ABCD, ABFE and CDEF are parallelograms. Prove that ar(ΔADE) = ar(ΔBCF)



Solution 8

ABCD is a $||^{gm}$ $\Rightarrow$ AD = BC

CDEF is a $||^{gm}$ $\Rightarrow$ DE = CF

ABFE is a $||^{gm}$ $\Rightarrow$ AE = BF

Thus, in Δs ADE and BCF, we have

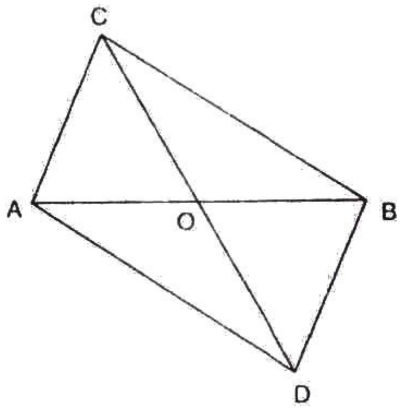
AD = BC, DE = CF and AE = BF

So, by SSS criterion of congruence, we have

$\Delta ADE \cong \Delta\Delta BCF$
$\therefore ar(\Delta ADE) = ar(\Delta BCF)$

Question 9

In fig., ABC and ABD are two triangles on the base Ab. If the line segment CD is bisected by AB at O, show that ar (Δ ABC) = ar (Δ ABD).



Solution 9

Given:- CD is bisected at O by AB

To prove:- $ar(\triangle ABC) = ar(\triangle ABD)$

Construction:- Draw $CP \perp AB$ and $DQ \perp AB$

Proof:-

$ar(\triangle ABC) = \frac{1}{2} \times AB \times CP$ ---(1)

$ar(\triangle ABD) = \frac{1}{2} \times AB \times DQ$ ---(2)

In $\triangle CPO$ and $\triangle DQO$

$\angle CPO = \angle DQO$ [Each $90°$]

$CO = DO$ [Given]

$\angle COP = \angle DOQ$ [Vertically opposite angles]

Then, $\triangle CPO \cong DQO$ [By AAS condition]

$\therefore \quad CP = DQ$ ---(3) [c.p.c.t]

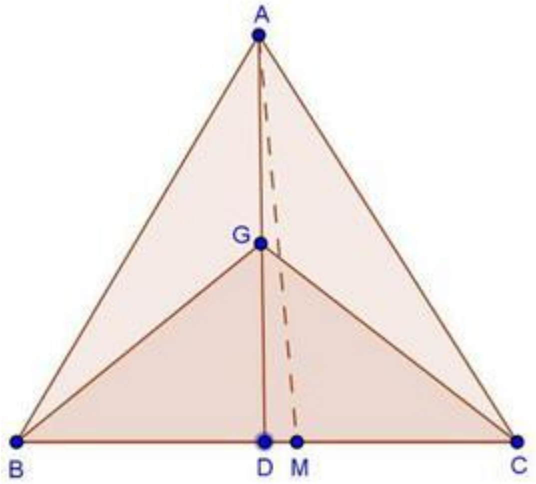
Compare equations (1) (2) and (3)

$ar(\triangle ABC) = ar(\triangle ABD)$

Question 10

If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of madian AD, prove that $ar(\triangle BGC) = 2ar(\triangle AGC)$.

Solution 10



Draw $AM \perp BC$.

Since, AD is the median of $\triangle ABC$.

$\therefore \quad BD = DC$

$\Rightarrow \quad BD \times AM = DC \times AM$

$\Rightarrow \quad \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$

$\Rightarrow \quad ar(\triangle ABD) = ar(\triangle ACD) \qquad \ldots(i)$

In $\triangle BGC$, GD is the median.

$\therefore \quad ar(BGD) = ar(CGD) \qquad \ldots(ii)$

In $\triangle ACD$, CG is a median.

$\therefore \quad ar(AGC) = ar(\triangle CGD) \qquad \ldots(iii)$

From (ii) and (iii), we have,

$ar(\triangle BGD) = ar(\triangle AGC)$

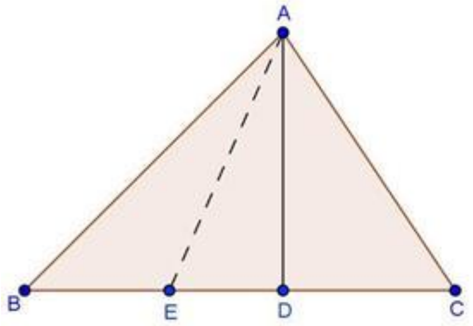
But, $ar(\triangle BGC) = 2\,ar(BGD)$

$\therefore \quad ar(BGC) = 2\,ar(\triangle AGC)$

Question 11

A point D is taken on the side BC of a $\triangle ABC$ such that $BD = 2DC$. Prove that $ar(\triangle ABD) = 2ar(\triangle ADC)$.

Solution 11



Given:- In $\triangle ABC$, $BD = 2DC$

To prove:- $ar(\triangle ABD) = 2ar(\triangle ADC)$

Construction:- Take a point E on BD such that $BE = ED$

Proof:- since, $BE = ED$ and $BD = 2DC$
Then, $BE = ED = DC$

We know that median of a $\triangle$ divides it into two equal triangles.

$\therefore$ In $\triangle ABD$, AE is a median
Then, $ar(\triangle ABD) = 2ar(\triangle AED)$ $\quad ---(1)$

In $\triangle AEC$, AD is a median

Then, $ar(\triangle AED) = ar(\triangle ADC)$ $\quad ---(2)$

Compare equation (1) and (2)

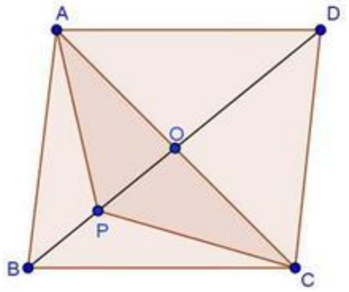
$ar(\triangle ABD) = 2ar(\triangle ADC)$.

Question 12

$ABCD$ is a parallelogram whose diagonals intersect at O. If P is any point on BO, Prove that:

(i) $ar(\triangle ADO) = ar(\triangle CDO)$ (ii) $ar(\triangle ABP) = ar(\triangle CBP)$.

Solution 12



Given:- $ABCD$ is a parallelogram.

To prove:- (i) $ar(\triangle ADO) = ar(\triangle CDO)$
(ii) $ar(\triangle ABP) = ar(\triangle CBP)$

Proof:- We know that diagonals of a parallelogram bisect each other.

$\therefore \quad AO = OC$ and $BO = OD$

(i) In $\triangle DAC$, since DO is a medain
Then, $ar(\triangle ADO) = ar(\triangle CDO)$

(ii) In $\triangle BAC$, since BO is a median
Then, $ar(\triangle BAO) = ar(\triangle BCO) \qquad ---(1)$

In $\triangle PAC$, since PO is a median
Then, $ar(\triangle PAO) = ar(\triangle PCO) \qquad ---(2)$

Subtract equation (2) from (1)

$ar(\triangle BAO) - ar(\triangle PAO) = ar(\triangle BCO) - ar(\triangle PCO)$

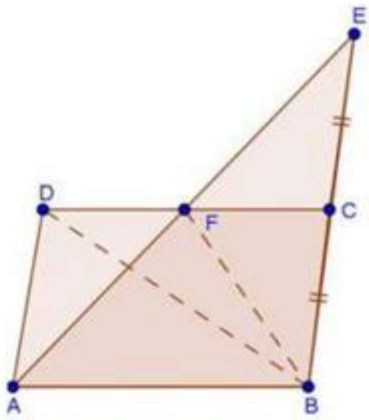
$\Rightarrow \quad ar(\triangle ABP) = ar(\triangle CBP)$

Question 13

$ABCD$ is a parallelogram in which BC is produced to E such that $CE = BC$. AE intersects CD at F.

(i) Prove that $ar(\triangle ADF) = ar(\triangle ECF)$
(ii) If the area of $\triangle DFB = 3\ \text{cm}^2$, find the area of $||^{gm}$ $ABCD$.

Solution 13



In triangles ADF and ECF, we have

$\angle ADF = \angle ECF$ [Alternate interior angles, Since AD || BE]
$AD = EC$ [Since AD = BC = CE]
And $\angle DFA = \angle CFE$ [Vertically opposite angles.]

So, by AAS congruence criterion, we have
$\triangle ADF \cong ECF$

$\Rightarrow \quad ar(\triangle ADF) = ar(\triangle ECF)$ and $DF = CF$

Now, $DF = CF$

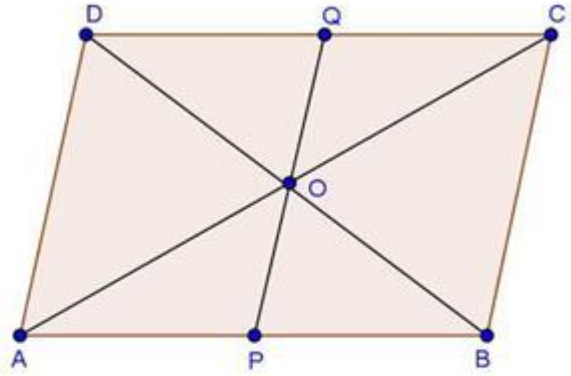
$\Rightarrow \quad BF$ is a median in $\triangle BCD$
$\Rightarrow \quad ar(\triangle BCD) = 2ar(\triangle BDF)$

$\Rightarrow \quad ar(\triangle BCD) = 2 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$

Hence, $ar(\|^{gm}\ ABCD) = 2ar(\triangle BCD) = 2 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$

Question 14
$ABCD$ is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersect AB at P and DC at Q. prove that $ar(\triangle POA) = ar(\triangle QOC)$.

Solution 14



In triangles POA and QOC, we have

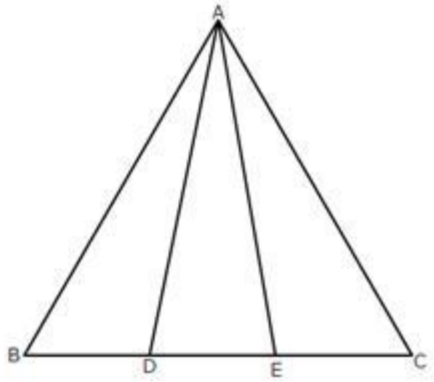
$\angle AOP = \angle COQ$	[Vertically opposite angles]
$OA = OC$	[Diagonals of a parallelogram bisect each other]
$\angle PAC = \angle QCA$	[AB ∥ DC; alternate angles]

So, by ASA congruence criterion, we have
$\triangle POA \cong QOC$

$ar(\triangle POA) = ar(\triangle QOC)$

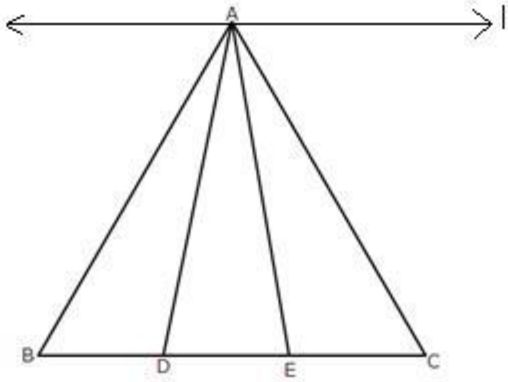
Question 15

In fig., D and E are two points on BC such that BD = DE = EC. Show that ar (ΔABD) = ar (ΔADE) = ar(ΔAEC).



Solution 15

Draw a line *l* through A parallel to BC.



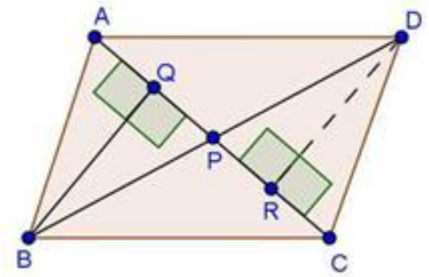
Given that, BD = DE = EC.
We observe that the triangles ABD, ADE and AEC are on the equal bases and between the same parallels *l* and BC. Therefore, their areas are equal.

Hence, ar (ΔABD) = ar (ΔADE) = ar(ΔAEC).

Question 16

Diagonals AC and BD of a quadrilateral $ABCD$ intersect each other at P. Show that:
$ar(\Delta APB) \times ar(\Delta CPD) = ar(\Delta APD) \times ar(\Delta BPC)$.

Solution 16



Construction:- Draw $BQ \perp AC$ and $DR \perp AC$

Proof:-

$L.H.S$

$$= ar(\Delta APB) \times ar(\Delta CPD)$$

$$= \left[\frac{1}{2} \times AP \times BQ\right] \times \left[\frac{1}{2} \times PC \times DR\right]$$

$$= \left[\frac{1}{2} \times PC \times BQ\right] \times \left[\frac{1}{2} \times AP \times DR\right]$$

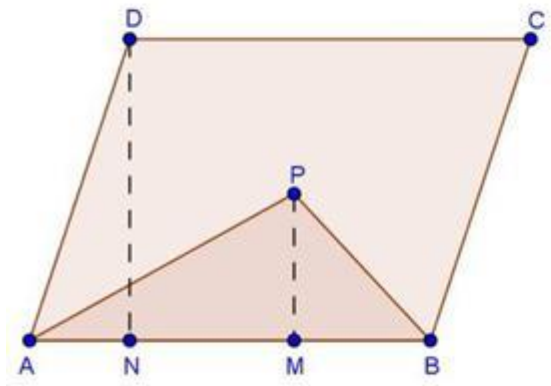
$$= ar(\Delta BPC) \times ar(\Delta APD)$$

$$= R.H.S$$

Question 17

If P is any point in the interior of a parallelogram $ABCD$, then prove that area of the triangle APB is less than half the area of parallelogram.

Solution 17



Draw $DN \perp AB$ and $PM \perp AB$.

Now,

$ar\left(\|^{gm}\ ABCD\right) = AB \times DN,\ ar\left(\triangle APB\right) = \frac{1}{2}\left(AB \times PM\right)$

Now, $PM < DN$

$\Rightarrow \quad AB \times PM < AB \times DN$

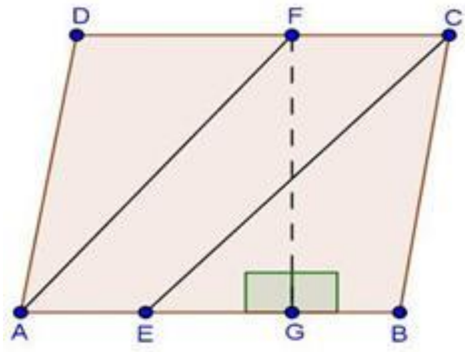
$\Rightarrow \quad \frac{1}{2}\left(AB \times PM\right) < \frac{1}{2}\left(AB \times DN\right)$

$\Rightarrow \quad ar\left(\triangle APB\right) < \frac{1}{2} ar\left(\|^{gm}\ ABCD\right)$

Question 18

$ABCD$ is a parallelogram. E is a point on BA such that $BE = 2EA$ and F is a point on DC Such that $DF = 2FC$. Prove that $AECF$ is a parallelogram whose area is one third of the area of parallelogram $ABCD$.

Solution 18



Construction:- Draw $FG \perp AB$

Proof:- We have,

$BE = 2EA$ and $DF = 2FC$

$$\Rightarrow \quad AB - AE = 2EA \text{ and } DC - FC = 2FC$$
$$\Rightarrow \quad AB = 3EA \text{ and } DC = 3FC$$

$$\Rightarrow \quad AE = \frac{1}{3} AB \text{ and } FC = \frac{1}{3} DC \qquad ---(1)$$

But $AB = DC$ $\qquad$ [Opposite sides of $\|^{gm}$]

Then, $AE = FC$

Thus $AE = FC$ and $AE \parallel FC$

Then, $AECF$ is a parallelogram

Now, $ar\left(\|^{gm} AECF\right) = AE \times FG$

$$\Rightarrow \quad ar\left(\|^{gm} AECF\right) = \frac{1}{3} AB \times FG \qquad [\text{From } (1)]$$
$$\Rightarrow \quad 3ar\left(\|^{gm} AECF\right) = AB \times FG \qquad ---(2)$$

And $ar\left(\|^{gm} ABCD\right) = AB \times FG$ $\qquad ---(3)$

Compare equations (2) and (3)

$$3ar\left(\|^{gm} AECF\right) = ar\left(\|^{gm} ABCD\right)$$

$$\Rightarrow \quad ar\left(\|^{gm} AECF\right) = \frac{1}{3} ar\left(\|^{gm} ABCD\right)$$

Question 19

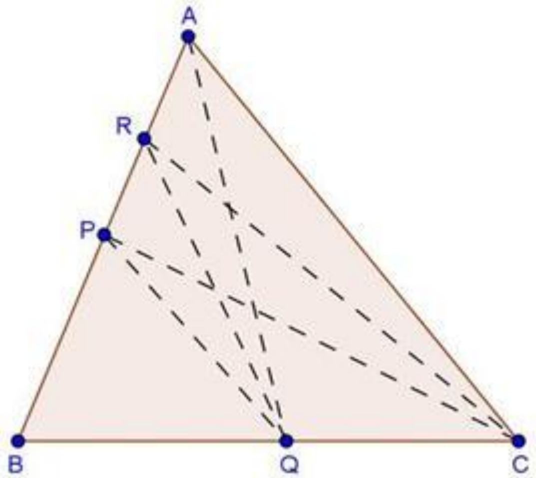
In a $\triangle ABC$, P and Q are respectively the mid-points of AB and BC and R is the mid-point of AP. Prove that:

(i) $ar(\triangle PBQ) = ar(\triangle ARC)$

(ii) $ar(\triangle PRQ) = \frac{1}{2} ar(\triangle ARC)$

(iii) $ar(\triangle RQC) = \frac{3}{8} ar(\triangle ABC)$

Solution 19



(i)

We know that each median of a triangle divides it into two triangles of equal area.

Since, CR is a median of $\triangle CAP$

$\therefore \quad ar(\triangle CRA) = \frac{1}{2} ar(\triangle CAP) \quad \ldots (i)$

Also, CP is a median of $\triangle CAB$.

$\therefore \quad ar(\triangle CAP) = ar(\triangle CPB) \quad \ldots (ii)$

From (i) and (ii), we get

$\therefore \quad ar(\triangle ARC) = \frac{1}{2} ar(\triangle CPB) \quad \ldots (iii)$

PQ is a median of $\triangle PBC$.

$\therefore \quad ar(\triangle CPB) = 2ar(\triangle PBQ) \quad \ldots (iv)$

From (iii) and (iv), we get

$\therefore \quad ar(\triangle ARC) = ar(\triangle PBQ) \quad \ldots (v)$

(ii)

Since, QP and QR medians of $\triangle$'s QAB and QAP respectively.

$$\therefore \quad ar(\triangle QAP) = ar(\triangle QBP) \quad \ldots (vi)$$

And $ar(\triangle QAP) = 2ar(\triangle QRP) \quad \ldots (vii)$

From (vi) and (vii), we have

$$ar(\triangle PRQ) = \frac{1}{2} ar(\triangle PBQ) \quad \ldots (viii)$$

From (v) and (viii), we get

$$ar(\triangle PRQ) = \frac{1}{2} ar(\triangle ARC)$$

(iii)

Since, CR is a median of $\triangle CAP$.

$$\begin{aligned} \therefore \quad ar(\triangle ARC) &= \frac{1}{2} ar(\triangle CAP) \\ &= \frac{1}{2}\left\{\frac{1}{2} ar(ABC)\right\} \\ &= \frac{1}{4} ar(\triangle ABC) \end{aligned}$$

Since, RQ is a median of $\triangle RBC$.

$$\begin{aligned} \therefore \quad ar(\triangle RQC) &= \frac{1}{2} ar(\triangle RBC) \\ &= \frac{1}{2}\{ar(\triangle ABC) - ar(\triangle ARC)\} \\ &= \frac{1}{2}\left\{ar(\triangle ABC) - \frac{1}{4} ar(\triangle ABC)\right\} \\ &= \frac{3}{8} ar(\triangle ABC) \end{aligned}$$

Question 20

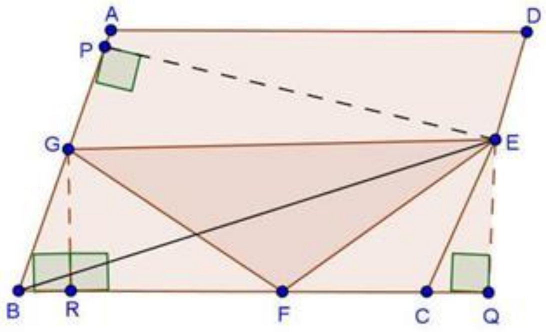
$ABCD$ is a parallelogram, G is the point on AB such that $AG = 2\ GB$, E is a point on DC such that $CE = 2DE$ and F is the point of BC such that $BF = 2FC$. prove that:

(i) $ar(ADEG) = ar(GBCE)$ (ii) $ar(\triangle EGB) = \frac{1}{6} ar(ABCD)$

(iii) $ar(\triangle EFC) = \frac{1}{2} ar(\triangle EBF)$ (iv) $ar(\triangle EBG) = \frac{3}{2} ar(\triangle EFC)$

(v) Find what portion of the area of parallelogram is the area of ΔEFG.

Solution 20


A
D
P
G
E
B
R
F
C
Q

Given:- $ABCD$ is a parallelogram and $AG = 2GB$, $CE = 2DE$ and $BF = 2FC$

To prove:-

(i) $ar(ADEG) = ar(GBCE)$

(ii) $ar(\triangle EGB) = \frac{1}{6} ar(ABCD)$

(iii) $ar(\triangle EFC) = \frac{1}{2} ar(\triangle EBF)$

(iv) $ar(\triangle EBG) = \frac{3}{2} ar(EFC)$

(v) Find what portion of the area of parallelogram is the area of $\triangle FEG$

Construction:- Draw $EP \perp AB$ and $EQ \perp BC$

Proof:- We have,

$AG = 2GB$ and $CE = 2DE$ and $BF = 2FC$

$\Rightarrow \quad AB - GB = 2GB$ and $CD - DE = 2DE$ and $BC - FC = 2FC$

$\Rightarrow \quad AB = 3GB$ and $CD = 3DE$ and $BC = 3FC$

$$\Rightarrow \quad GB = \frac{1}{3}AB \text{ and } DE = \frac{1}{3}CD \text{ and } FC = \frac{1}{3}BC \qquad ---(1)$$

(i) $ar(ADEG) = \frac{1}{2}(AG + DE) \times EP$

$$\Rightarrow \quad ar(ADEG) = \frac{1}{2}\left(\frac{2}{3}AB + \frac{1}{3}CD\right) \times EP \qquad [\text{By using (1)}]$$

$$\Rightarrow \quad ar(ADEG) = \frac{1}{2}\left(\frac{2}{3}AB + \frac{1}{3}AB\right) \times EP \qquad [\because AB = CD]$$

$$\Rightarrow \quad ar(ADEG) = \frac{1}{2} \times AB \times EP \qquad ---(2)$$

And $ar(GBCE) = \frac{1}{2}(GB + CE) \times EP$

$$\Rightarrow \quad ar(GBCE) = \frac{1}{2}\left[\frac{1}{3}AB + \frac{2}{3}CD\right] \times EP \qquad [\text{By using (1)}]$$

$$\Rightarrow \quad ar(GBCE) = \frac{1}{2}\left[\frac{1}{3}AB + \frac{2}{3}AB\right] \times EP \qquad [\because AB = CD]$$

$$\Rightarrow \quad ar(GBCE) = \frac{1}{2} \times AB \times EP \qquad ---(3)$$

Compare equation (2) and (3)

$ar(ADEG) = ar(GBCE)$

(ii) $ar(\Delta EGB) = \frac{1}{2} \times GB \times EP$

$$\Rightarrow \quad ar(\Delta EGB) = \frac{1}{2} \times \frac{1}{3} AB \times EP \qquad [\text{By using } (1)]$$

$$= \frac{1}{6} AB \times EP$$

$$= \frac{1}{6} ar\left(||^{gm}\ ABCD\right)$$

(iii) $ar(\Delta EFC) = \frac{1}{2} \times FC \times EQ \qquad ---(4)$

And $ar(\Delta EBF) = \frac{1}{2} \times BF \times EQ$

$$\Rightarrow \quad ar(\Delta EBF) = \frac{1}{2} \times 2FC \times EQ \qquad [BF = 2FC \text{ given}]$$

$$\Rightarrow \quad ar(\Delta EBF) = FC \times EQ \qquad ---(5)$$

Compare equation (4) and (5)

$ar(\Delta EFC) = \frac{1}{2} \times ar(\Delta EBF)$

(iv) from (ii) part

$ar(\Delta EGB) = \frac{1}{6} ar\left(||^{gm}\ ABCD\right) \qquad ---(6)$

from (iii) part

$ar(\Delta EFC) = \frac{1}{2} ar(\Delta EBF)$

$$\Rightarrow \quad ar(\Delta EFC) = \frac{1}{2} ar(\Delta EBF)$$

$$\Rightarrow \quad ar(\Delta EFC) = \frac{1}{3} ar(\Delta EBC)$$

$$\Rightarrow \quad ar(\Delta EFC) = \frac{1}{3} \times \frac{1}{2} CE \times EP$$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{2}{3} CD \times EP$$

$$= \frac{1}{6} \times \frac{2}{3} \times ar\left(||^{gm}\ ABCD\right)$$

$$\Rightarrow \quad ar(\triangle EFC) = \frac{2}{3} \times ar(\triangle EGB) \qquad [\text{By using } (6)]$$

$$\Rightarrow \quad ar(\triangle EGB) = \frac{3}{2} ar(\triangle EFC)$$

$$(v)\ ar(\triangle EFG) = ar(\text{trap. } BGEC) - ar(\triangle BGF) \qquad ---(7)$$

$$\text{Now, } ar(\text{trap. } BGEC) = \frac{1}{2}(GB + EC) \times EP$$

$$= \frac{1}{2}\left(\frac{1}{3}AB + \frac{2}{3}CD\right) \times EP$$

$$= \frac{1}{2} AB \times EP$$

$$= \frac{1}{2} ar(||^{gm}\ ABCD)$$

$$ar(\triangle EFC) = \frac{1}{9} ar(||^{gm}\ ABCD) \qquad [\text{From iv part}]$$

$$\text{And } ar(\triangle BGF) = \frac{1}{2} BF \times GR$$

$$= \frac{1}{2} \times \frac{2}{3} BC \times GR$$

$$= \frac{2}{3} \times \frac{1}{2} BC \times GR$$

$$= \frac{2}{3} \times ar(\triangle GBC)$$

$$= \frac{2}{3} \times \frac{1}{2} GB \times EP$$

$$= \frac{1}{3} \times \frac{1}{3} AB \times EP$$

$$= \frac{1}{9} AB \times EP$$

$$= \frac{1}{9} ar(||^{gm}\ ABCD)$$

$$\therefore \quad \text{From } (7)$$

$$ar(\triangle EFG) = \frac{1}{2} ar(||^{gm}\ ABCD) - \frac{1}{9} ar(||^{gm}\ ABCD) - \frac{1}{9} ar(||^{gm}\ ABCD) = \frac{5}{18} ar(||^{gm}\ ABCD)$$

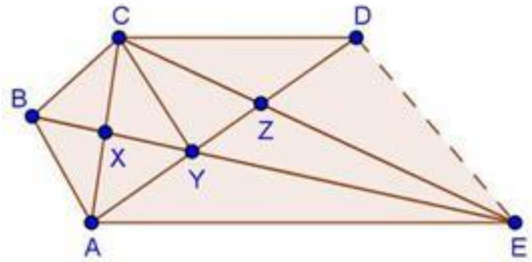
Question 21

In fig., CD $\parallel$ AE and CY $\parallel$ BA.

(i) Name a triangle equal in area of ΔCBX

(ii) Prove that ar (ΔZDE) = ar (ΔCZA)

(iii) Prove that ar (BCZY) = ar (ΔEDZ)



Solution 21

Since, ΔBCA and ΔBYA are on the same base BA and between same parallels BA and CY

Then, $ar(\Delta BCA) = ar(\Delta BYA)$

$\Rightarrow \quad ar(\Delta CBX) + ar(\Delta BXA) = ar(\Delta BXA) + ar(\Delta AXY)$

$\Rightarrow \quad ar(\Delta CBX) = ar(\Delta AXY) \qquad ---(1)$

Since, ΔACE and ΔADE are on the same base AE and between same parallels CD and AE

Then, $ar(\Delta ACE) = ar(\Delta ADE)$

$\Rightarrow \quad ar(\Delta CZA) + ar(\Delta AZE) = ar(\Delta AZE) + ar(\Delta DZE)$

$\Rightarrow \quad ar(\Delta CZA) = (\Delta DZE) \qquad ---(2)$

Since, ΔCBY and ΔCAY are on the same base CY and between same parallels BA and CY

Then, $ar(\Delta CBY) = ar(\Delta CAY)$

Adding $ar(\Delta CYZ)$ on both sides

$\Rightarrow \quad ar(\Delta CBY) + ar(\Delta CYZ) = ar(\Delta CAY) + ar(\Delta CYZ)$

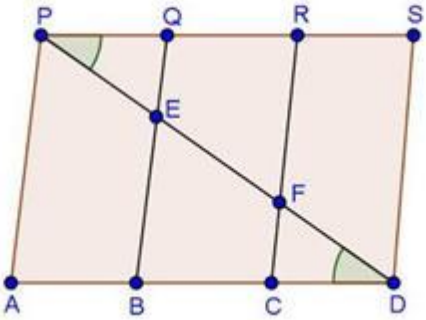
$\Rightarrow \quad ar(\ BCZY) = ar(\Delta CZA) \qquad ---(3)$

Compare equation (2) and (3)

$ar(\ BCZY) = ar(\Delta DZE)$

Question 22

In fig., PSDA is a parallelogram in which PQ = QR = RS and AP $\parallel$ BQ $\parallel$ CR. Prove that ar(Δ PQE) = ar (Δ CFD).



Solution 22

Since, $AP \parallel BQ \parallel CR \parallel DS$ and $AD \parallel PS$

$\therefore \quad PQ = CD \qquad$ ---(i)

In $\triangle BED$, C is the mid-point of BD and $CF \parallel BE$

$\therefore \quad F$ is the mid-point of ED

$\Rightarrow \quad EF = FD$

Similarly, $EF = PE$

$\therefore \quad PE = FD \qquad$ ---(2)

In $\triangle$'s PQE and CFD, we have

$PE = FD$

$\angle EPQ = \angle FDC \qquad$ [Alternate angles]

And, $PQ = CD$.

So,by SAS congruence criterion, we have

$\triangle PQE \cong \triangle DCF$

$\Rightarrow \quad ar(\triangle PQE) = ar(\triangle DCF)$

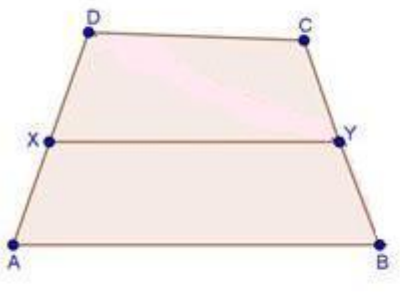
Question 23

In fig., ABCD is a trapezium in which AB DC and DC = 40 cm and AB = 60 cm. If X and Y are, respectively, the mid - points of AD and BC, prove that:

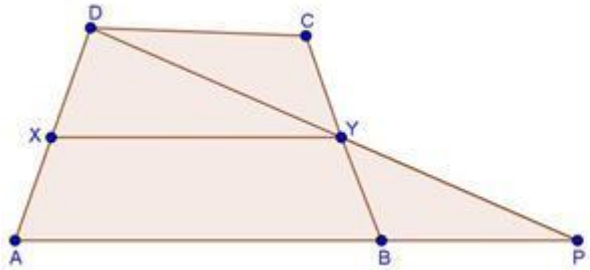
(i) XY = 50 cm

(ii) DCYX is a trapezium

(iii) ar (trap. DCYX) = (9/11)ar (trap.(XYBA)



Solution 23



(i) Join DY and produce it to meet AB produced at P.

In $\triangle$'s BYP and CYD, we have

$\angle BYP = \angle CYD$ [Vertically opp. angles]

$\angle DCY = \angle PBY$ [$\because DC \parallel AP$]

And, $BY = CY$

So, by ASA congruence criterion, we have

$\triangle BYP \cong \triangle CYD$

$\Rightarrow \quad DY = YP$ and $DC = BP$

$\Rightarrow \quad Y$ is the mid-point of DP

Also, X is the mid-point of AD

$\therefore \quad XY \parallel AP$ and $XY = \frac{1}{2}AP$

$\Rightarrow \quad XY = \frac{1}{2}(AB + BP)$

$\Rightarrow \quad XY = \frac{1}{2}(AB + DC)$

$\Rightarrow \quad XY = \frac{1}{2}(60 + 40)$ cm $= 50$ cm

(ii) We have,

$XY \parallel AP$ [As proved above]

$\Rightarrow \quad XY \parallel AB$ and $AB \parallel DC$

$\Rightarrow \quad XY \parallel DC$

$\Rightarrow \quad DCYX$ is a trapezium

(iii) Since, X and Y are the mid-points of AD and BC respectively. Therefore, trapezium $DCYX$ and $ABYX$ are of the same height, say h cm.

Now,

$$ar(\text{trap. } DCYX) = \frac{1}{2}(DC + XY) \times h = \frac{1}{2}(40 + 50)h \text{ cm}^2 = 45h \text{ cm}^2$$

$$ar(\text{trap. } ABYX) = \frac{1}{2}(AB + XY) \times h = \frac{1}{2}(60 + 50)h \text{ cm}^2 = 55h \text{ cm}^2$$

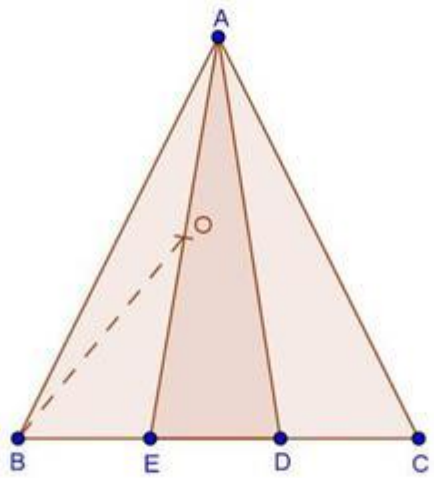
$$\therefore \quad \frac{ar(\text{trap. } DCYX)}{ar(\text{trap. } ABYX)} = \frac{45h}{55h} = \frac{9}{11}$$

$$\Rightarrow \quad ar(\text{trap. } DCYX) = \frac{9}{11} ar(\text{trap. } ABYX)$$

Question 24

D is the mid-point of side BC of $\triangle ABC$ and E is the mid-point of BD. If O is the mid-point of AE, prove that $ar(\triangle BOE) = \frac{1}{8} ar(\triangle ABC)$.

Solution 24



Since, AD and AE are medians of $\triangle ABC$ and $\triangle ABD$ respectively.

$$\therefore \quad ar(\triangle ABD) = \frac{1}{2} ar(\triangle ABC) \qquad ---(i)$$

$$ar(\triangle ABE) = \frac{1}{2} ar(\triangle ABD) \qquad ---(ii)$$

OB is a median of $\triangle ABE$.

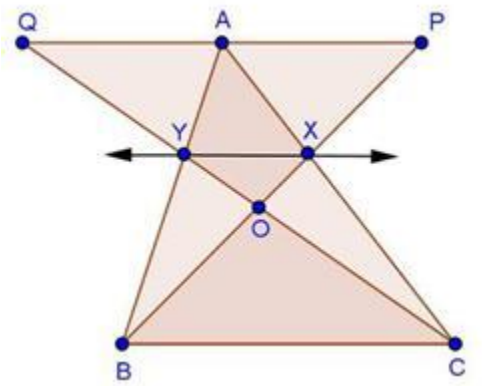
$$\therefore \quad ar(\triangle BOE) = \frac{1}{2} ar(\triangle ABE) \qquad ---(iii)$$

From (i), (ii) and (iii), we have

$$ar(\triangle BOE) = \frac{1}{8} ar(\triangle ABC)$$

Question 25

In fig., X and Y are the mid-points of AC and AB respectively, QP $\parallel$ BC and CYQ and BXP are straight lines. Prove that ar(Δ ABP) = ar (Δ ACQ)



Solution 25

Since, X and Y are the mid-points AC and AB respectively.

$\therefore \quad XY \parallel BC$

Clearly, triangles BYC and BXC are on the same base BC and between the same parallels XY and BC.

$$\therefore \quad ar(\Delta BYC) = ar(\Delta BXC)$$
$$\Rightarrow \quad ar(\Delta BYC) - ar(\Delta BOC) = ar(\Delta BXC) - ar(\Delta BOC)$$
$$\Rightarrow \quad ar(\Delta BOY) = ar(\Delta COX)$$
$$\Rightarrow \quad ar(\Delta BOY) + ar(\Delta XOY) = ar(\Delta COX) + ar(\Delta XOY)$$

$$\Rightarrow \quad ar(\Delta BXY) = ar(\Delta CXY) \qquad ---(i)$$

We observe that the quadrilaterals $XYAP$ and $XYAQ$ are on the same base XY and between the same parallels XY and PQ.

$$\therefore \quad ar(\text{quad } XYAP) = ar(\text{quad } XYQA) \qquad ---(ii)$$

Adding (i) and (ii), we get

$$ar(\Delta BXY) + ar(\text{quad } XYAP) = ar(\Delta CXY) + ar(\text{quad } XYQA)$$

$$\Rightarrow \quad ar(\Delta ABP) = ar(\Delta ACQ)$$

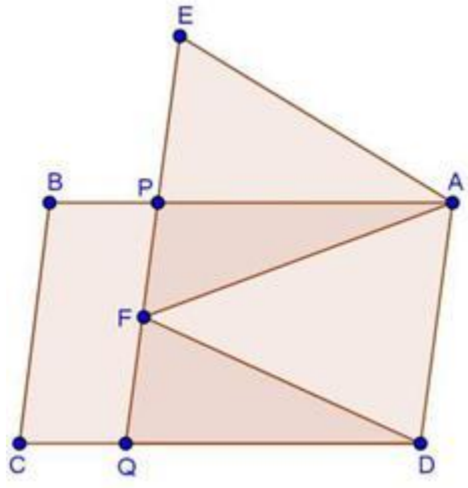
Question 26

In fig., ABCD and AEFD are two parallelograms. Prove that

(i) PE = FQ

(ii) ar(ΔAPE) : ar(ΔPFA) = ar Δ(QFD) : ar (ΔPFD)

(iii) ar(ΔPEA) = ar (ΔQFD)



Solution 26

Given:- $ABCD$ and $AEFD$ are two parallelograms

To prove:- (i) $PE = FQ$

(ii) $\frac{ar(\triangle APE)}{ar(\triangle PFA)} = \frac{ar\triangle(QFD)}{ar(\triangle PFD)}$

(iii) $ar(\triangle PEA) = ar(\triangle QFD)$

Proof:-(i) In $\triangle EPA$ and $\triangle FQD$

$\angle PEA = \angle QFD$	[Corresponding angles]
$\angle EPA = \angle FQD$	[Corresponding angles]
$PA = QD$	[Opp. sides of $\|^{gm}$]
Then, $\triangle EPA \cong \triangle FQD$	[By AAS condition]

$\therefore \quad EP = FQ$ [c.p.c.t]

(ii) Since, $\triangle PEA$ and $\triangle QFD$ stand on equal bases PE and FQ and lie between the same parallels EQ and AD

$\therefore \quad ar(\triangle PEA) = ar(\triangle QFD) \quad ---(1)$

Since, $\triangle PFA$ and $\triangle PFD$ stand on the same base PF and lie between the same parallels PF and AD

$\therefore \quad ar(\triangle PFA) = ar(\triangle PFD) \quad ---(2)$

Divide equation (1) by equation (2)

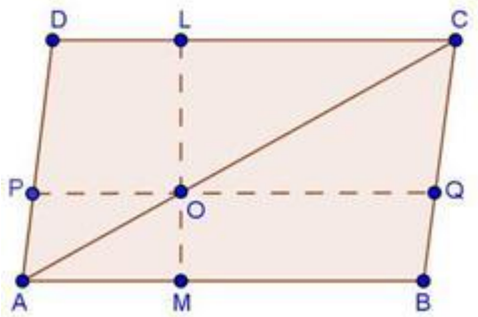
$$\frac{ar(\triangle PEA)}{ar(\triangle PFA)} = \frac{ar\triangle(QFD)}{ar(\triangle PFD)}$$

(iii) From (i) part $\triangle EPA \cong \triangle FQD$

Then, $ar(\triangle EPA) = ar(\triangle FQD)$

Question 27

In fig. ABCD is a $\|^{gm}$. O is any point on AC. PQ $\|$ AB and LM $\|$ AD. Prove that ar($\|^{gm}$ DLOP) = ar ($\|^{gm}$ BMOQ).



Solution 27

Since, a diagonal of a parallelogram divides it into two triangles of equal area.

$\therefore \quad ar(\triangle ADC) = ar(\triangle ABC)$

$\Rightarrow \quad ar(\triangle APO) + ar(\|^{gm}\ DLOP) + ar(\triangle OLC)$

$= ar(\Delta AOM) + ar(\|gm\ BMOQ) + ar(\Delta OQC) \quad ---(i)$

Since, AO and OC are diagonals of parallelograms $AMOP$ and $OQCL$ respectively.

$\therefore \quad ar(\triangle APO) = ar(\triangle AMO) \quad ---(ii)$

And, $ar(\triangle OLC) = ar(\triangle OQC) \quad ---(iii)$

Subtracting (ii) and (iii) from (i), we get

$ar(\|^{gm}\ DLOP) = ar(\|^{gm}\ BMOQ)$

Question 28

In a $\triangle ABC$, if L and M are points on AB and AC respectively such that $LM \parallel BC$. Prove that:

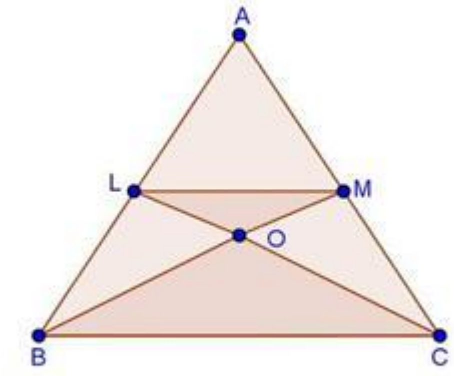
(i) $ar(\triangle LCM) = ar(\triangle LBM)$

(ii) $ar(\triangle LBC) = ar(MBC)$

(iii) $ar(\triangle ABM) = ar(\triangle ACL)$

(iv) $ar(\triangle LOB) = ar(\triangle MOC)$.

Solution 28



(i) Clearly, triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

$\therefore \quad ar(\Delta LMB) = ar(\Delta LMC) \qquad ---(i)$

(ii) We observe that triangles LBC and MBC are on the same base BC and between the same parallels LM and BC.

$\therefore \quad ar(\Delta LBC) = ar(\Delta MBC) \qquad ---(ii)$

(iii) We have,

$ar(\Delta LMB) = ar(\Delta LMC) \qquad$ [From (i)]

$\Rightarrow \quad ar(\Delta ALM) + ar(\Delta LMB) = ar(\Delta ALM) + ar(\Delta LMC)$

$\Rightarrow \quad ar(\Delta ABM) = ar(\Delta ACL)$

(iv) We have,

$ar(\Delta LBC) = ar(\Delta MBC) \qquad$ [From (i)]

$\Rightarrow \quad ar(\Delta LBC) - ar(\Delta BOC) = ar(\Delta MBC) - ar(\Delta BOC)$

$\Rightarrow \quad ar(\Delta LOB) = ar(\Delta MOC)$

Question 29

In fig., ABC and BDE are two equilateral triangles such that D is the mid-point of BC. AE intersects BC in F. Prove that

(i) ar (ΔBDE) = $\frac{1}{4}$ ar Δ(ABC)

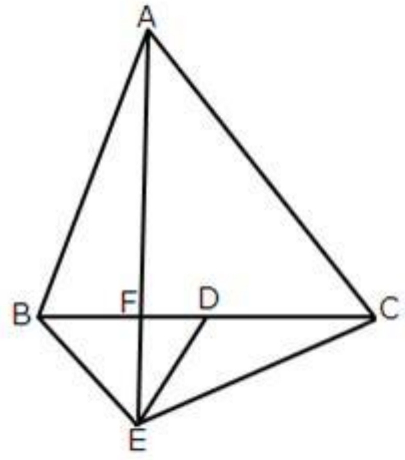
(ii) ar(ΔBDE) = $\frac{1}{2}$ ar(ΔBAE)

(iii) ar (ΔBFE) = ar(ΔAFD)

(iv) ar(ΔABC) = 2 ar(ΔBEC)

(v) ar (ΔFED) = $\frac{1}{8}$ ar(AFC)

(vi) ar(Δ BFE) = 2 ar (Δ EFD)



Solution 29

Given, ABC and BDE are two equilateral triangles.

Let AB = BC = CA = x. Then, BD = $\frac{x}{2}$ = DE = BE

(i) We have,

ar(Δ ABC) = $\frac{\sqrt{3}}{4}$ x^2

ar (Δ BDE) = $\frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{1}{4}\times\frac{\sqrt{3}}{4}x^2$

$\Rightarrow$ar(Δ BDE) = $\frac{1}{4}$ ar (Δ ABC)

(ii) It is given that triangles ABC and BED are equilateral triangles.

$\angle$ACB = $\angle$DBE = 60^o

$\Rightarrow$BE||AC(Since, alternate angles are equal)

Triangles BAE and BEC are on the same base BE and between the same parallels BE and AC.

$\therefore$ ar (Δ BAE) = ar(Δ BEC)

$\Rightarrow$ar (Δ BAE) =2 ar (Δ BDE)

[$\because$ ED is a median of Δ EBC $\therefore$ ar(Δ BEC) = 2ar(Δ BDE)]

$\Rightarrow$ar (Δ BDE) = $\frac{1}{2}$ ar(Δ BAE)

(iii) Since Δ ABC and Δ BDE are equilateral triangles.

$\therefore$ $\angle$ABC = 60^o and $\angle$BDE = 60^o

$\Rightarrow$ $\angle$ABC = $\angle$BDE

$\Rightarrow$AB||DE(Since, alternate angles are equal)

Triangles BED and AED are on the same base ED and between the same parallels AB and DE.

$\therefore$ ar (Δ BED) = ar(Δ AED)

$\Rightarrow$ar (Δ BED) ar(Δ EFD) = ar(Δ AED) ar(Δ EFD)

$\Rightarrow$ar(Δ BEF) = ar(Δ AFD)

(iv) Since ED is a median of Δ BEC

$\therefore$ ar (Δ BEC) = 2 ar (Δ BDE)

$\Rightarrow$ ar (Δ BEC) = $2 \times \frac{1}{4}$ ar (Δ ABC)[From (i), ar (Δ BDE) = $\frac{1}{4}$ ar (Δ ABC)]

$\Rightarrow$ ar(Δ BEC) = $\frac{1}{2}$ ar (Δ ABC)

$\Rightarrow$ ar (Δ ABC) = 2 ar (Δ BEC)

(v) Let h be the height of vertex E, corresponding to the side BD in triangle BDE.

Let H be the height of vertex A, corresponding to the side BC in triangle ABC.

From part (i),

ar(Δ BDE) = $\frac{1}{4}$ ar (Δ ABC)

$$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4}\left(\frac{1}{2} \times BC \times H\right)$$

$$\Rightarrow BD \times h = \frac{1}{4}(2BD \times H)$$

$$\Rightarrow h = \frac{1}{2}H \quad \ldots (1)$$

From part (iii),

ar (Δ BFE) = ar (Δ AFD)

$$= \frac{1}{2} \times FD \times H$$

$$= \frac{1}{2} \times FD \times 2h$$

$$= 2\left(\frac{1}{2} \times FD \times h\right)$$

$$= 2ar(\Delta EFD)$$

(vi) ar (Δ AFC) = ar (Δ AFD) + ar (Δ ADC)

= ar (Δ BFE) + $\frac{1}{2}$ ar (Δ ABC)

(Using part (iii); and AD is the median of Δ ABC)

= ar (Δ BFE) + $\frac{1}{2} \times$ 4 ar (Δ BDE)(Using part (i))

= ar (Δ BFE) + 2 ar (Δ BDE) (2)

Now, from part (v),

ar (Δ BFE) = 2ar (Δ FED) (3)

ar (Δ BDE) = ar (Δ BFE) + ar (Δ FED)

= 2 ar (Δ FED) + ar (Δ FED)

= 3 ar (Δ FED) (4)

From (2), (3) and (4), we get,

ar (Δ AFC) = 2ar (Δ FED) + 2 $\times$3 ar (Δ FED) = 8 ar (Δ FED)

Hence, ar (Δ FED) = $\frac{1}{8}$ ar(AFC)

Now, from

Question 30

If fig., ABC is a right triangle right angled at A, BCED, ACFG and ABMN are square on the sides BC, CA and AB respectively. Line segment AX $\perp$ DE meets BC at Y. Show that

(i) ΔMBC $\cong$ ΔABD

(ii) ar (BYXD) = 2ar(ΔMBC)

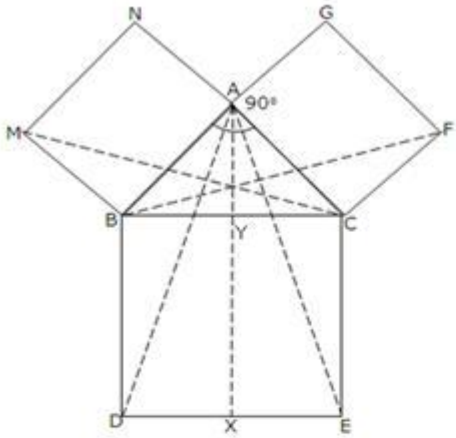
(iii) ar(BYXD) = ar(ABMN)

(iv) ΔFCB $\cong$ ΔACE

(v) ar(CYXE) = 2ar (ΔFCB)

(vi) ar(CYXE) = ar (ACFG)

(vii) ar(BCED) = ar (ABMN) + ar (ACFG)



Solution 30

(i) In ΔMBC and ΔABD, we have

MB = AB

BC = BD

And $\angle$MBC = $\angle$ABD

[$\because$ $\angle$MBC and $\angle$ABD are obtained by adding $\angle$ABC to a right angle]

So, by SAS congruence criterion, we have

ΔMBC $\cong$ ΔABD

$\Rightarrow$ar (ΔMBC) = ar(ΔABD) (1)

(ii) Clearly, triangle ABD and rectangle BYXD are on the same base BD and between the same parallels AX and BD.

$\therefore$ ar(ΔABD) = $\frac{1}{2}$ar (rect. BYXD)

$\Rightarrow$ar (rect. BYXD) = 2 ar(ΔABD)

$\Rightarrow$ar (rect. BYXD) = 2 ar (ΔMBC)...(2)

[$\because$ ar (ΔABD) = ar (ΔMBC), from (1)]

(iii) Since triangle MBC and square MBAN are on the same base MB and between the same parallels MB and NC.

$\therefore$ 2 ar (ΔMBC) = ar (MBAN) (3)

From (2) and (3), we have

ar (sq. MBAN) = ar(rec. BYXD)

(iv) In triangles FCB and ACE, we have

FC = AC
CB = CE
And, $\angle$FCB = $\angle$ACE
[$\because$ $\angle$FCB and $\angle$ACE are obtained by adding $\angle$ACB to a right angle]
So, by SAS congruence criterion, we have
ΔFCB $\cong$ ΔACE
(v) We have,
ΔFCB $\cong$ ΔACE
$\Rightarrow$ar (ΔFCB) = ar (ΔACE)
Clearly, ΔACE and rectangle CYXE are on the same base CE ad between the same parallels CE and AX.
$\therefore$ 2 ar (ΔACE) = ar (CYXE)
$\Rightarrow$2 ar (ΔFCB) = ar (CYXE) (4)
(vi) Clearly, ΔFCB and rectangle FCAG are on the same base FC and between the same parallels FC and BG.
$\therefore$ 2ar (ΔFCB) = ar(FCAG) (5)
From (4) and (5), we get
ar(CYXE) = ar (ACFG)
(vii) Applying Pythagoras theorem in ΔACB, we have
$BC^2 = AB^2 + AC^2$
$\Rightarrow BC \times BD = AB \times MB + AC \times FC$
$\Rightarrow ar(BCED) = ar(ABMN) + ar(ACFG)$