RD SHARMA Solutions for Class 12-science Maths Chapter 31 - Probability

Chapter 31 - Probability Exercise Ex. 31.1

Question 1

Ten cards numbered 1 through 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Solution 1

The sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Let,

A = Number on the card drawn is even number

$A = \{2, 4, 6, 8, 10\}$

B = Number on the card greater than 4

$B = \{4, 5, 6, 7, 8, 9, 10\}$

$A \cap B = \{4, 6, 8, 10\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{4}{7}$$

Required probability $= \frac{4}{7}$

Question 2

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

Solution 2

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be

$S = \{(b, b), (b, g), (g, b), (g, g)\}$

Let A be the event that both children are girls.

$$\therefore A = \{(g, g)\}$$

(i) Let B be the event that the youngest child is a girl.

$$\therefore B = [(b, g), (g, g)]$$
$$\Rightarrow A \cap B = \{(g, g)\}$$
$$\therefore P(B) = \frac{2}{4} = \frac{1}{2}$$
$$P(A \cap B) = \frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by P (A|B).

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

(ii) Let C be the event that at least one child is a girl.

$$\therefore C = \{(b, g), (g, b), (g, g)\}$$
$$\Rightarrow A \cap C = \{g, g\}$$

$$\Rightarrow P(C) = \frac{3}{4}$$
$$P(A \cap C) = \frac{1}{4}$$

The conditional probability that both are girls, given that at least one child is a girl, is given by P(A|C).

$$\text{Therefore, } P(A | C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Question 3

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Solution 3

A = Two numbers on two dice are different

$$= \{(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$(2,1),(2,3),(2,4),(2,5),(2,6)$$
$$(3,1),(3,2),(3,4),(3,5),(3,6)$$
$$(4,1),(4,2),(4,3),(4,5),(4,6)$$
$$(5,1),(5,2),(5,3),(5,4),(5,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5)\}$$

B = Sum of numbers on the dice is 4

$B = \{(1,3),(2,2),(3,1)\}$

$A \cap B = \{(1,3),(3,1)\}$

$$\text{Required probability } = P\left(\frac{B}{A}\right)$$
$$= \frac{n(A \cap B)}{n(A)}$$
$$= \frac{2}{30}$$

$$\text{Required probability } = \frac{1}{15}$$

Question 4

A coin is tossed three times, if head occurs on first two tosses, find the probability of getting head on third toss.

Solution 4

A = Head on the first two toss on three tosses of coin
$A = \{HHT, HHH\}$

B = Getting ahead on third toss
$B = \{HHH, HTH, THH, TTH\}$

$A \cap B = \{HHH\}$

$$\text{Required probability } = P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)}$$

$$\text{Required probability } = \frac{1}{2}$$

Question 5

A die is thrown three times, find the probability that 4 appears on the third toss if it is given that 6 and 5 appear respectively on first two tosses.

Solution 5

A = 4 appears on third toss, if a die is thrown three times

$$= \{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)$$
$$(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)$$
$$(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)$$
$$(4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)$$
$$(5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)$$
$$(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}$$

B = 6 and 5 appears respectively on first two tosses, if die is tosses three times
$B = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$

$A \cap B = \{(6,5,4)\}$

$$\text{Required probability } = P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{6}$$

$$\text{Required probability } = \frac{1}{6}$$

Question 6

Compute $P\left(\frac{A}{B}\right)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

Solution 6

Given, $P(B) = 0.5$, $P(A \cap B) = 0.32$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{0.32}{0.5}$$

$$= \frac{32}{50}$$

$$= \frac{16}{25}$$

$$P\left(\frac{A}{B}\right) = \frac{16}{25}$$

Question 7

If $P(A) = 0.4$, $P(B) = 0.3$ and $P\left(\frac{B}{A}\right) = 0.5$, find $P(A \cap B)$ and $P\left(\frac{A}{B}\right)$.

Solution 7

Given, $P(A) = 0.4$, $P(B) = 0.3$ and $P\left(\frac{B}{A}\right) = 0.5$

We know that,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$0.5 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.5 \times 0.4$$

$$P(A \cap B) = 0.2$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

Question 8

If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{11}{30}$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$.

Solution 8

Given, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$, $P(A \cup B) = \frac{11}{30}$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{11}{30} = \frac{1}{3} + \frac{1}{5} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{11}{30}$$

$$= \frac{10 + 6 - 11}{30}$$

$$= \frac{5}{30}$$

$$P(A \cap B) = \frac{1}{6}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5}}$$

$$P\left(\frac{A}{B}\right) = \frac{5}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$P\left(\frac{B}{A}\right) = \frac{\frac{1}{6}}{\frac{1}{3}}$$

$$= \frac{1}{6} \times \frac{3}{1}$$

$$= \frac{1}{2}$$

$$P\left(\frac{A}{B}\right) = \frac{5}{6},\ P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Question 9

A couple has two children. Find the probability that both the children are (i) males, if it is known that at least one of the children is male. (ii) females, if it is known that the elder child is a female.

Solution 9

Given, Couple has two children.

(i)

A = Both are male

$A = \{M_1M_2\}$

B = Atleast one is male

$B = \{M_1M_2, M_1F_2, F_1M_2\}$

$A \cap B = \{M_1M_2\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{3}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{3}$$

(ii)

A = Both are Females

$A = \{F_1F_2\}$

B = Elder child is female

$B = \{F_1M_2, F_1F_2\}$

$A \cap B = \{F_1F_2\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{2}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

Chapter 31 - Probability Exercise Ex. 31.2

Question 1

From a pack of 52 cards, two are drawn one by one without replacement. Find the probability that both of them are kings.

Solution 1

A = first card is king

B = second card is also king

Probability of getting two kings (without replacement)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{4}{52} \times \frac{3}{51}$ [Since, 4 kings out of 52 cards.]

$= \frac{1}{13} \times \frac{1}{17}$

$= \frac{1}{221}$

Required probability $= \frac{1}{221}$

Question 2

From a pack of 52 cards, 4 are drawn one by one without replacement. Find the probability that all are aces.

Solution 2

A = first card Ace

B = second card Ace

C = third card Ace

D = fourth card Ace

P (All four drawn are Ace, without replacement)

$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)P\left(\frac{D}{A \cap B \cap C}\right)$

$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49}$ [Since, there are four Ace in 52 cards]

$= \frac{1}{270725}$

Required probability $= \frac{1}{270725}$

Question 3

Find the chance of drawing 2 white balls in succession from a bag containing 5 red and 7 white balls, the ball first drawn not being replaced.

Solution 3

Bag contains 5 red and 7 white balls

A = first ball white

B = second ball white

P (2 white balls drawn without replacement)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{7}{12} \times \frac{6}{11}$

$= \frac{7}{22}$

Required probability $= \frac{7}{22}$

Question 4

A bag contains 25 tickets, numbered from 1 to 25. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

Solution 4

Tickets are numbered from 1 to 25

$\Rightarrow$ Total number of tickets $= 25$

Number of tickets with even numbers on it

$= 12 \qquad \{2,4,6,8,10,12,14,16,18,20,22,24\}$

A = first ticket with even number

B = second ticket with even number

P (Both tickets will show even number, without replacement)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{12}{25} \times \frac{11}{24}$

$= \frac{11}{50}$

Required probability $= \frac{11}{50}$

Question 5

From a deck of cards, three cards are drawn one by one without replacement. Find the probability that each time it is a card of spade.

Solution 5

We know that, Deck of 52 cards contains 13 spades.

A = first card is spade

B = second card spade

C = third card spade

P (3 cards drawn without replacement are spade)

$= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$

$= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}$

$= \frac{11}{850}$

Required probability $= \frac{11}{850}$

Question 6(i)

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that both are kings

Solution 6(i)

In a deck of 52 cards, there are 4 kings. Two cards are drawn without replacement

A = first card is king

B = second card is king

P (Both drawn cards are king)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{4}{52} \times \frac{3}{51}$

$= \frac{1}{221}$

Required probability $= \frac{1}{221}$

Question 6(ii)

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that the first is a king and the second is an ace

Solution 6(ii)

We know that, there are 4 kings and 4 ace in a pack of 52 cards.
Two cards are drawn without replacement

A = first card is king
B = second card an ace

P (The first card is a king and second is an ace)

$= P(A) P\left(\frac{B}{A}\right)$

$= \frac{4}{52} \times \frac{4}{51}$

$= \frac{4}{663}$

Required probability $= \frac{4}{663}$

Question 6(iii)

Two cards are drawn without replacement from a pack of 52 cards. Find the probability that the first is a heart and second is red.

Solution 6(iii)

There are 13 heart and 26 red cards
Hearts are also red .

A = first card is heart
B = second card is red

P (first card is heart and second is red)

$= P(A).P\left(\frac{B}{A}\right)$

$= \frac{13}{52} \times \frac{25}{51}$

$= \frac{25}{204}$

Required probability $= \frac{25}{204}$

Question 7

A bag contains 20 tickets, numbered from 1 to 20. Two tickets are drawn without replacement. What is the probability that the first ticket has an even number and the second an odd number.

Solution 7

Total number of tickets are 20 numbered from 1, 2, 3, ...20.

Number of tickets with even numbers

$= 10$ [Since, even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

Number of tickets with odd numbers

$= 10$ [Since, odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19]

Two cards are drawn without replacement.

A = tickets with even numbers

B = tickets with odd numbers

P (first ticket has even number and second has odd number)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{10}{20} \cdot \frac{10}{19}$

$= \frac{5}{19}$

Required probability $= \frac{5}{19}$

Question 8

An urn contains 3 white, 4 red and 5 black balls. Two balls are drawn one by one without replacement. What is the probability that at least one ball is black?

Solution 8

Urn contains 3 white, 4 red and 5 black balls. Total balls $= 12$

Two balls are drawn without replacement

A = first ball is black

B = second ball is black

$$P\left(\text{Atleast one ball is black}\right)$$
$$= P\left(A \cup B\right)$$
$$= 1 - P\left(\overline{A \cup B}\right)$$
$$= 1 - P\left(\overline{A} \cap \overline{B}\right)$$
$$= 1 - P\left(\overline{A}\right) P\left(\overline{B / A}\right)$$
$$= 1 - \left(\frac{7}{12} \times \frac{6}{12}\right)$$
$$= 1 - \frac{7}{22}$$
$$= \frac{15}{22}$$

Required probability $= \frac{15}{22}$

Question 9

A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find the probability that none is red.

Solution 9

Bag contains 5 white, 7 red and 3 black balls.

Total number of balls $= 15$

Three balls are drawn without replacement

A = first ball is red

B = second ball is red

C = Third balls is red

$$P\left(\text{Three balls are drawn, non is red}\right)$$
$$= P\left(\overline{A}\right) P\left(\frac{\overline{B}}{\overline{A}}\right) P\left(\frac{\overline{C}}{\overline{A} \cap \overline{B}}\right)$$
$$= \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \qquad \left[\text{Since, number of non red balls } = 5 + 3 = 8\right]$$
$$= \frac{8}{65}$$

Required probability $= \frac{8}{65}$

Question 10

A card is drawn from a well-shuffled deck of 52 cards and then a second card is drawn. Find the probability that the first card is a heart and the second card is a diamond if the first card is not replaced.

Solution 10

Two cards are drawn from a pack of 52 cards without replacement.

There are 13 heart and 13 diamond in pack

A = first card is heart

B = second card is diamond

P (first card heart and second diamond)

$= P(A)P\left(\frac{B}{A}\right)$

$= \frac{13}{52} \times \frac{13}{51}$

$= \frac{13}{204}$

Required probability $= \frac{13}{204}$

Question 11

An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is probability that both drawn balls are black?

Solution 11

Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$ or $P(EF)$.

Now $P(E) = P$ (black ball in first draw) $= \frac{10}{15}$

Also given that the first ball drawn is black, i.e, event E has occurred, now there are 9 black balls and five white balls left in the urn. Therefore, the probability that the second ball drawn is black, given that the ball in the first draw is black, is nothing but the conditional probability of F given that E has occurred.

i.e., $P(F \mid E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E)\ P(F \mid E)$

$= \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$

Multiplication rule of probability for more than two events if E.F and G are three events of sample space, we have

$P(E \cap F \cap G) = P(E)\ P(F \mid E)\ P(E)\ P(G \mid)\ (G \cap F) = P(E)\ P(F \mid E)\ P\ (G \mid EF)$

Similarly, the multiplication rule of probability can be extended for four or more events.

The following example illustrates the extension of multiplication rule of probability for three events.

Question 12

Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?

Solution 12

Let K denote the event that the card drawn is king
and A be the event that the card drawn is an ace.
We are to find $P(KKA)$.

Now, $P(K) = \frac{4}{52}$

Also, $P(K/K)$ is the probability of second king with the condition that one king has already been drawn.
Now, there are 3 kings in (52-1) = 51 cards.

$\therefore \quad P(K/K) = \frac{3}{51}$

Lastly, $P(A/KK)$ is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn.
Now, there are four aces in left 50 cards.

$\therefore \quad P(A/KK) = \frac{4}{50}$

By multiplication law of probability, we have

$$P(KKA) = P(K)P(K/K)P(A/KK)$$
$$= \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} = \frac{2}{5525}$$

Question 13

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Solution 13

There are 15 oranges out of which 12 are good and 3 are bad.

Three oranges selected without replacement are drawn and if they found good the box is approved for sale.

A = first orange good

B = second orange good

B = third orange good

P (All three oranges are good)

$= P(A) P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)$

$= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$

$= \frac{44}{91}$

Required probability $= \frac{44}{91}$

Question 14

A bag contains 4 white, 7 black and 5 red balls. Three balls are drawn one after the other without replacement. Find the probability that the balls drawn are white, black and red respectively.

Solution 14

Given bag contains 4 white, 7 black and 5 red balls.

Total number of balls $= 16$

Three balls are drawn without replacement

A = first ball is white

B = second ball is black

C = Third balls is red

P (Three balls drawn are white, Black, red respectively)

$= P(A) P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)$

$= \frac{4}{16} \times \frac{7}{15} \times \frac{5}{14}$

$= \frac{1}{24}$

Required probability $= \frac{1}{24}$

Chapter 31 - Probability Exercise Ex. 31.3

Question 1

If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, find $P\left(\frac{A}{B}\right)$.

Solution 1

Given,

$$P(A) = \frac{7}{13},\ P(B) = \frac{9}{13} \text{ and } P(A \cap B) = \frac{4}{13}$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{13}}{\frac{9}{13}}$$

$$= \frac{4}{9}$$

$$P\left(\frac{A}{B}\right) = \frac{4}{9}$$

Question 2

If A and B are events such that $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$.

Solution 2

Given,

$$P(A) = 0.6,\ P(B) = 0.3 \text{ and } P(A \cap B) = 0.2$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.2}{0.3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

and, $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.2}{0.6}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{3}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3},\ P\left(\frac{B}{A}\right) = \frac{1}{3}$$

Question 3

If A and B are two events such that $P(A \cap B) = 0.32$ and $P(B) = 0.5$, find $P\left(\frac{A}{B}\right)$.

Solution 3

Given,

$$P(A \cap B) = 0.32 \text{ and } P(B) = 0.5$$

We know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.32}{0.5}$$

$$= \frac{16}{25}$$

$$= 0.64$$

$$P\left(\frac{A}{B}\right) = 0.64$$

Question 4

If $P(A) = 0.4$, $P(B) = 0.8$, $P\left(\frac{B}{A}\right) = 0.6$. Find $P\left(\frac{A}{B}\right)$ and $P(A \cup B)$.

Solution 4

Given,

$$P(A) = 0.4,\ P(B) = 0.8,\ P\left(\frac{B}{A}\right) = 0.6$$

We know that,

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$0.6 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.6 \times 0.4$$

$$P(A \cap B) = 0.24$$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{0.24}{0.8}$$

$$P\left(\frac{A}{B}\right) = 0.3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24$$

$$P(A \cap B) = 0.96$$

$$P\left(\frac{A}{B}\right) = 0.3,\quad P(A \cap B) = 0.96$$

Question 5(i)

If A and B are two events such that

$$P(A) = \frac{1}{3},\ P(B) = \frac{1}{4} \text{ and } P(A \cup B) = \frac{5}{12},\ \text{find } P\left(\frac{A}{B}\right) \text{ and } P\left(\frac{B}{A}\right).$$

Solution 5(i)

Given,

$$P(A) = \frac{1}{3},\ P(B) = \frac{1}{4},\ P(A \cup B) = \frac{5}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{12} = \frac{1}{3} + \frac{1}{4} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{5}{12}$$

$$= \frac{4 + 3 - 5}{12}$$

$$P(A \cap B) = \frac{2}{12}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{2}{12}}{\frac{1}{4}}$$

$$= \frac{2}{12} \times \frac{4}{1}$$

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{2}{12}}{\frac{1}{3}}$$

$$= \frac{2}{12} \times \frac{3}{1}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Hence,

$$P\left(\frac{A}{B}\right) = \frac{2}{3}$$

$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Question 5(ii)

If A and B are two events such that

$$P(A) = \frac{6}{11},\ P(B) = \frac{5}{11} \text{ and } P(A \cup B) = \frac{7}{11},\ \text{find } P(A \cap B),\ P\left(\frac{A}{B}\right),\ P\left(\frac{B}{A}\right)$$

Solution 5(ii)

Given,

$$P\left(A\right)=\frac{6}{11},\ P\left(B\right)=\frac{5}{11}\ \text{ and }\ P\left(A\cup B\right)=\frac{7}{11}$$

$$P\left(A\cup B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)$$
$$P\left(A\cap B\right)=P\left(A\right)+P\left(B\right)-P\left(A\cup B\right)$$
$$=\frac{6}{11}+\frac{5}{11}+\frac{7}{11}$$
$$P\left(A\cap B\right)=\frac{4}{11}$$

We know that,

$$P\left(\frac{A}{B}\right)=\frac{P\left(A\cap B\right)}{P\left(B\right)}$$
$$=\frac{\frac{4}{11}}{\frac{5}{11}}$$
$$=\frac{4}{5}$$

$$P\left(\frac{B}{A}\right)=\frac{P\left(A\cap B\right)}{P\left(A\right)}$$
$$=\frac{\frac{4}{11}}{\frac{6}{11}}$$
$$=\frac{2}{3}$$

Hence,

$$P\left(\frac{A}{B}\right)=\frac{4}{5},\ P\left(\frac{B}{A}\right)=\frac{2}{3}$$

Question 5(iii)

If A and B are two events such that

$$P\left(A\right)=\frac{7}{13},\ P\left(B\right)=\frac{9}{13}\ \text{ and }\ P\left(A\cap B\right)=\frac{4}{13},\ \text{ find } P\left(\frac{A'}{B}\right).$$

Solution 5(iii)

Given,

$$P(A) = \frac{7}{13},\ P(B) = \frac{9}{13},\ P(A \cap B) = \frac{4}{13}$$

Since, $P(A' \cap B) = P(B) - P(A \cap B)$

$$= \frac{9}{13} - \frac{4}{13}$$

$$P(A' \cap B) = \frac{5}{13}$$

$$P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)}$$

$$= \frac{\frac{5}{13}}{\frac{9}{13}}$$

$$= \frac{5}{9}$$

$$P\left(\frac{A'}{B}\right) = \frac{5}{9}$$

Question 5(iv)

If A and B are two events such that

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, find $P(A/B)$, $P(B/A)$, $P(\overline{A}/B)$ and $P(\overline{A}/\overline{B})$.

Solution 5(iv)

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$,

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$$P(\overline{A}/B) = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}} = \frac{1}{4}$$

$$P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{1 - P(\overline{A \cup B})}{P(A) - P(A \cap B)} = \frac{1 - P(A) - P(B) + P(A \cap B)}{P(A) - P(A \cap B)} = \frac{1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}}{\frac{1}{2} - \frac{1}{4}} = \frac{5}{4}$$

Question 6

If A and B are two events such that $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, find $P(A \cup B)$.

Solution 6

Given,

$$2P(A) = P(B) = \frac{5}{13}$$

$$2P(A) = \frac{5}{13}$$

$$\Rightarrow \quad P(A) = \frac{5}{26}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{2}{5} = \frac{P(A \cap B)}{\frac{5}{13}}$$

$$P(A \cap B) = \frac{2}{5} \times \frac{5}{13}$$

$$P(A \cap B) = \frac{2}{13}$$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$= \frac{5 + 10 - 4}{26}$$

$$= \frac{11}{26}$$

$$P(A \cup B) = \frac{11}{26}$$

Question 7

If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

(i) $P(A \cap B)$ (ii) $P\left(\frac{A}{B}\right)$ (iii) $P\left(\frac{B}{A}\right)$

Solution 7

Given,

$$P(A) = \frac{6}{11},\ P(B) = \frac{5}{11},\ P(A \cup B) = \frac{7}{11}$$

(i)

Since, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$$

$$P(A \cap B) = \frac{4}{11}$$

(ii)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{11}}{\frac{5}{11}}$$

$$P\left(\frac{A}{B}\right) = \frac{4}{5}$$

(iii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{4}{11}}{\frac{6}{11}}$$

$$P\left(\frac{B}{A}\right) = \frac{2}{3}$$

Question 8

A coin is tossed three times. Find $P\left(\frac{A}{B}\right)$ in each of the following:

(i) A = Heads on third toss, B = Heads on first two tosses

(ii) A = At least two heads, B = At most two heads

(iii) A = At most two tails, B = At least one tail.

Solution 8

Sample space for three coins is given by

$$\{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$$

(i)

A = Head on third toss

$A = \{HHH, HTH, THH, TTH\}$

B = Head on first two toss

$B = \{HHH, HHT\}$

$(A \cap B) = \{HHH\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{2}$

(ii)

A = At least two heads

$A = \{HHH, HHT, HTH, THH\}$

B = At most two heads

$B = \{HHT, HTT, THT, TTT, HTH, THH, TTH\}$

$(A \cap B) = \{HHT, HTH, THH\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P\left(\frac{A}{B}\right) = \frac{3}{7}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{3}{7}$

(iii)

A = At most two tails

$A = \{HHH, HTH, THT, TTH, HHT, THT, HTT\}$

B = At least one tail

$B = \{HTH, THH, TTH, HHT, HTT, THT, TTT\}$

$(A \cap B) = \{HTH, THT, TTH, HHT, THT, HTT\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{6}{7}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{6}{7}$

Question 9

Two coins are tossed once. Find $P\left(\frac{A}{B}\right)$ in each of the following:

(i) A = Tail appears on one coin, B = One coin shows head.

(ii) A = No tail appears, B = No head appears.

Solution 9

Sample space of two coins

$\{HH, HT, TH, TT\}$

(i)

A = Tail appears on one coin

$A = \{HT, TH\}$

B = One coin shows head

$B = \{HT, TH\}$

$(A \cap B) = \{HT, TH\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{2}{2}$$
$$P\left(\frac{A}{B}\right) = 1$$

Hence, $P\left(\frac{A}{B}\right) = 1$

(ii)

A = No tail appears

$A = \{HH\}$

B = No head appears

$B = \{TT\}$

$(A \cap B) = \{\ \}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0}{1}$$
$$= 0$$

$$P\left(\frac{A}{B}\right) = 0$$

Question 10

A die is thrown three times. Find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$, if

A = 4 appears on the third toss

B = 6 and 5 appear respectively on first two tosses.

Solution 10

Die is thrown three times.

A = 4 appears on the third toss

$$A = \{(1,1,4),(1,2,4),(1,3,4),(1,4,4),(1,5,4),(1,6,4)$$
$$(2,1,4),(2,2,4),(2,3,4),(2,4,4),(2,5,4),(2,6,4)$$
$$(3,1,4),(3,2,4),(3,3,4),(3,4,4),(3,5,4),(3,6,4)$$
$$(4,1,4),(4,2,4),(4,3,4),(4,4,4),(4,5,4),(4,6,4)$$
$$(5,1,4),(5,2,4),(5,3,4),(5,4,4),(5,5,4),(5,6,4)$$
$$(6,1,4),(6,2,4),(6,3,4),(6,4,4),(6,5,4),(6,6,4)\}$$

B = 6 and 5 appear respectively on first two tosses

$$B = \{(6,5,1),(6,5,5),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

$$(A \cap B) = \{(6,5,4)\}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{1}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{1}{36}$$

Hence, $P\left(\frac{A}{B}\right) = \frac{1}{6}$, $P\left(\frac{B}{A}\right) = \frac{1}{36}$

Question 11

Mother, father and son line up at random for a family picture. If A and B are two events given by

A = Son on one end, B = Father in the middle, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$.

Solution 11

There are three person for photograph father (F), mother (M), son (S).

Sample space $= \{FMS, FSM, MFS, MSF, SFM, SMF\}$

A = Son on one end
$A = \{SFM, SMF, MFS, FMS\}$

B = Father in the middle
$B = \{MFS, SFM\}$

$(A \cap B) = \{MFS, SFM\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{2}{2}$$
$$P\left(\frac{A}{B}\right) = 1$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{2}{4}$$
$$P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Hence, $P\left(\frac{A}{B}\right) = 1$, $P\left(\frac{B}{A}\right) = \frac{1}{2}$

Question 12

A dice is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Solution 12

The sample space of the experiment is $\{(1,1)(1,2)(1,3)\ldots(6,6)\}$ conisting of 36 outcomes.

$$P(A) = P(\text{Sum} = 6) = \frac{5}{36}$$

$$P(B) = P(4 \text{ appears at least once}) = \frac{11}{36}$$

$$\text{Now,} \quad P\left(\frac{B}{A}\right) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{P(\text{sum is 6 and 4 has appeared at least once})}{P(A)}$$

$$= \frac{\frac{2}{36}}{\frac{5}{36}}$$

$$= \frac{2}{5}$$

Question 13

Two dice are thrown. Find the probability that the numbers appeared has the sum 8, if it is known that the second die a always exhibits 4.

Solution 13

Two dice are thrown.

A = Sum on the dice is 8

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

B = Second die always exhibits 4

$B = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$

$(A \cap B) = \{(4,4)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{6}$$

$$\text{Required probability} = \frac{1}{6}$$

Question 14

A pair of dice is thrown. Find the probability of getting 7 as the sum, if it is known that the second die always exhibits an odd number.

Solution 14

Here two dice are thrown

A = Getting 7 as sum on two dice

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = Second die exhibits an odd number

$$B = \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)$$
$$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)$$
$$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

$$(A \cap B) = \{(2,5),(4,3),(6,1)\}$$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{3}{18}$$
$$= \frac{1}{6}$$

Hence, Required probability $= \frac{1}{6}$

Question 15

A pair of dice is thrown. Find the probability of getting 7 as the sum if it is known that the second die alsways exhibits a prime number.

Solution 15

A pair of dice is thrown.

A = Getting 7 as sum number on 2 dice.

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

B = Second die always exhibits prime number

$$B = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)$$
$$(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)$$
$$(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

[Since, there are 2, 3, 5 prime number on a die]

$(A \cap B) = \{(2,5),(4,3),(5,2)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{3}{18}$$
$$= \frac{1}{6}$$

Hence, Required probability $= \frac{1}{6}$

Question 16

A die is rolled. If the outcome is an odd number, what is the probability that it is prime?

Solution 16

A die is rolled.

A = A prime number on die

$A = \{2, 3, 5\}$

B = An odd number on die

$B = (1, 3, 5)$

$(A \cap B) = \{3, 5\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{2}{3}$$

Required probability $= \frac{2}{3}$

Question 17

A pair of dice is thrown. Find the probability of getting the sum 8 or more, if 4 appears on the first die.

Solution 17

A pair of dice is thrown

A = Getting sum 8 or more

= Getting sum 8,9,10,11 or 12 on the pair of dice

$$A = \{(2,6),(3,5),(4,4),(5,3),(6,2),(3,6)$$
$$(4,5),(5,4),(6,3),(4,6),(5,5),(6,4)$$
$$(5,6),(6,5),(6,6)$$

B = 4 on first die

$$B = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$$

$$(A \cap B) = \{(4,4),(4,5),(4,6)\}$$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

Required probability $= \dfrac{1}{2}$

Question 18

Find the probability that the sum of the numbers showing on two dice is 8, given that at least one die does not show five.

Solution 18

Two dice are thrown

A = Sum of the numbers on dice is 8

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

B = At least one die does not show five

$$B = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,1),(2,2)\\ (2,3),(2,4),(2,6),(3,1),(3,2),(3,3),(3,4)\\ (3,6),(4,1),(4,2),(4,3),(4,4),(4,6),(6,1)\\ (6,2),(6,3),(6,4),(6,6)\}$$

$(A \cap B) = \{(2,6),(4,6),(6,2)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{3}{25}$$

Required probability $= \frac{3}{25}$

Question 19

Two numbers are selected at random from integers 1 through 9. If the sum is even, find the probability that both the numbers are odd.

Solution 19

Two numbers are selected at random from integers 1 through 9.

A = Both numbers are odd

$$A = \{(3,1),(5,1),(7,1),(9,1),(3,5),(3,7),(9,3),(5,3),(5,7),(5,9)\\ (7,3),(7,5),(7,9),(9,3),(9,5),(9,7)\}$$

B = Sum of both numbers is even

= Sum of both numbers is 2, 4, 6, 8, 10, 12, 14, 16 or 18

$$= \{(1,3),(1,5),(2,4),(1,7),(2,6),(3,5),(1,9),(2,8),\\ (3,7),(4,6),(7,5),(8,4),(9,3),(8,6),(9,5),(9,7)\}$$

$(A \cap B) = \{(1,3),(1,5),(1,7),(3,5),(1,9),(3,7),(7,5),(9,3),(9,5),(9,7)\}$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{10}{16}$$

Required probability $= \frac{5}{8}$

Question 20

A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?

Solution 20

A die is thrown twice

A = The number 5 has appeared at least once

$$A = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,6)\}$$

B = Sum of the numbers is 8

$$= \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$$

$$(A \cap B) = \{(3,5),(5,3)\}$$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{2}{5}$$

Required probability $= \frac{2}{5}$

Question 21

Two dice are thrown and it is known that the first die shows a 6. Find the probability that the sum of the numbers showing on the dice is 7.

Solution 21

Two dice are thrown

A = Sum of the numbers showing on the dice is 7

$$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$$

B = First die shows a 6

$$= \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$(A \cap B) = \{(6,1)\}$$

$$P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{1}{6}$$

Required probability $= \frac{1}{6}$

Question 22

A pair of dice is thrown. Let E be the event that the sum is greater than or equal to 10 and F be the event "5 appears on the first-die". Find $P\left(\frac{E}{F}\right)$. If F is the event "5 appears on at least one die", find $P\left(\frac{E}{F}\right)$.

Solution 22

A pair of die is thrown

E = Sum is greater than or equal to 10

$= \{(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}$

Case I:

F = 5 appears on first die

$= \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$E \cap F = \{(5,5),(5,6)\}$

$$P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{2}{6}$$

$$P\left(\frac{E}{F}\right) = \frac{1}{3}$$

Case II:

F = 5 appears on at least one die

$= \{(1,5),(2,5),(3,5),(4,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$E \cap F = \{(5,5),(5,6),(6,5)\}$

$$P\left(\frac{E}{F}\right) = \frac{n(E \cap F)}{n(F)}$$

$$= \frac{3}{11}$$

$$P\left(\frac{E}{F}\right) = \frac{3}{11}$$

Question 23

The probablility that a student selected at random from a class will pass in Mathematics is $\frac{4}{5}$, and the probability that he/she passes in Mathematics and Computer Science is $\frac{1}{2}$. What is the probability that he/she will pass in Computer Science if it is known that he/she has passed in Mathematics?

Solution 23

Given,

Probability to pass mathematics (M)

$$P(M) = \frac{4}{5}$$

Probability to pass in mathematics (M) and computer Science (C)

$$P(M \cap C) = \frac{1}{2}$$

To find, $P\left(\frac{C}{M}\right)$

We know that,

$$P\left(\frac{C}{M}\right) = \frac{P(M \cap C)}{P(M)}$$

$$= \frac{\frac{1}{2}}{\frac{4}{5}}$$

$$= \frac{1}{2} \times \frac{5}{4}$$

$$= \frac{5}{8}$$

Required probability $= \frac{5}{8}$

Question 24

The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probability that he will buy a shirt given that he buys a trouser is 0.4. Find the probability that he will buy both a shirt and a trouser. Find also the probability that he will buy a trouser given that he buys a shirt.

Solution 24

Given,

Probability that a person buys a shirt $(S) = P(S) = 0.2$

Probability that he buys a trouser $(T) = P(T) = 0.3$

$$P\left(\frac{S}{T}\right) = 0.4$$

We know that,

$$P\left(\frac{S}{T}\right) = \frac{P(S \cap T)}{P(T)}$$

$$0.4 = \frac{P(S \cap T)}{0.3}$$

$$P(S \cap T) = 0.4 \times 0.3$$

$$P(S \cap T) = 0.12$$

Probability that he buys a shirt and a trouser both $= 0.12$

$$P\left(\frac{T}{S}\right) = \frac{P(S \cap T)}{P(S)}$$

$$= \frac{0.12}{0.2}$$

$$P\left(\frac{T}{S}\right) = \frac{12}{20}$$

$$= \frac{3}{5}$$

$$= 0.6$$

Probability that he buys a trouser given that he buys a shirt $= 0.6$

Question 25

In a school there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl?

Solution 25

Total students = 1000
Number of girls = 430
% of girls in class XII = 10%

Let A = Student chosen studies in class XII
B = Student chosen is a girl

Then $P(B) = \frac{430}{1000}$

$P(A \cap B) = \frac{43}{1000}$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{43}{430} = \frac{1}{10}$

Question 26

Ten cards numbered 1 through 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Solution 26

Total no. of cards = 10

Let A = drawn number is more than 3
B = drawn number is even

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

Now $P(A) = \frac{7}{10}$

$P(A \cap B) = \frac{4}{10}$

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{4}{7}$

Question 27

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that
(i) the youngest is a girl
(ii) at least one is girl.

Solution 27

(i) Let 'A' be the event that both the children born are girls.
Let 'B' be the event that the youngest is a girl.
We have to find conditional probability P(A/B).

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$A \subset B \Rightarrow A \cap B = A$$

$$\Rightarrow P(A \cap B) = P(A) = P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(B) = P(BG) + P(GG) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\text{Hence, } P(A/B) = \frac{1/4}{1/2} = \frac{1}{2}$$

(ii) Let 'A' be the event that both the children born are girls.
Let 'B' be the event that at least one is a girl.
We have to find the conditional probability P(A/B).

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$A \subset B \Rightarrow A \cap B = A$$

$$\Rightarrow P(A \cap B) = P(A) = P(GG) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(B) = 1 - P(BB) = 1 - \frac{1}{2} \times \frac{1}{2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\text{Hence, } P(A/B) = \frac{1/4}{3/4} = \frac{1}{3}$$

Chapter 31 - Probability Exercise Ex. 31.4

Question 1(i)

A coin is tossed thrice and all the eight outcomes are assumed equally likely.
State whether events A and B are independent if,

A = the first throw results in head, B = the last throw results in tail

Solution 1(i)

A coin is tossed thrice

Sample space $= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = The first throw results in head

$A = \{HHT, HTH, HHH, HTT\}$

B = The last throw in tail

$B = \{HHT, HTT, THT, TTT\}$

$A \cap B = \{HHT, HTT\}$

$$P(A) = \frac{4}{8} = \frac{1}{2}$$
$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$
$$P(A).P(B) = \frac{1}{2}.\frac{1}{2}$$
$$P(A).P(B) = \frac{1}{4}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Question 1(ii)

A coin is tossed thrice and all the eight outcomes are assumed equally likely.

State whether events A and B are independent if,

A = the number of heads is odd, B = the number of tails is odd

Solution 1(ii)

Sample space for a coin thrown thrice is

$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = the number of head is odd

$A = \{HTT, THT, TTH, HHH\}$

B = the number of tails is odd

$B = \{THH, HTH, HHT, TTT\}$

$A \cap B = \{\ \} = \varnothing$

$P(A) = \frac{4}{8} = \frac{1}{2}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{0}{8} = 0$

$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

$P(A).P(B) \neq P(A \cap B)$

So, A and B are not independent events.

Question 1(iii)

A coin is tossed thrice and all the eight outcomes are assumed equally likely.

State whether events A and B are independent if,

A = the number of heads is two, B = the last throw results in head

Solution 1(iii)

Sample space for throwing a coin thrice

$= \{HHT, HTT, THT, TTT, HHH, HTH, THH, TTH\}$

A = the number of heads is two

$A = \{HHT, THH, HTH\}$

B = the last throw results in head

$B = \{HHH, HTH, THH, TTH\}$

$A \cap B = \{THH, HTH\}$

$P(A) = \frac{3}{8}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4}$

$P(A).P(B) = \frac{3}{8} \times \frac{1}{2}$

$= \frac{3}{16}$

$P(A).P(B) \neq P(A \cap B)$

So, A and B are not independent events.

Question 2

Prove that in throwing a pair of dice, the occurrence of the number 4 on the first die is independent of the occurrence of 5 on the second die.

Solution 2

A pair of dice are thrown. It has 36 elements in its sample space.

A = Occurence of number 4 on first die

$A = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

B = Occurence of 5 on second die

$B = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$

$A \cap B = \{(4,5)\}$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$
$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = \frac{1}{36}$$

$$P(A).P(B) = \frac{1}{6} \times \frac{1}{6}$$
$$= \frac{1}{36}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Question 3(i)

A card is drawn from a pack of 52 cards so that each card is equally likely to be selected. State whether events A and B are independent if,

A = the card drawn is a king or queen

B = the card drawn is a queen or jack

Solution 3(i)

A card is drawn from 52 cards
It has 4 kings, 4 Queen, 4 Jack

A = the card drawn is a king or a queen

$$P(A) = \frac{4+4}{52}$$
$$= \frac{8}{52}$$
$$P(A) = \frac{2}{13}$$

B = the card drawn is a queen or a jack

$$P(B) = \frac{4+4}{52}$$
$$= \frac{8}{52}$$
$$= \frac{2}{13}$$

$A \cap B$ = The card drawn is a queen

$$P(A \cap B) = \frac{4}{52}$$
$$= \frac{1}{13}$$

$$P(A)P(B) = \frac{2}{13} \times \frac{2}{13}$$
$$= \frac{4}{169}$$

$$P(A)P(B) \neq P(A \cap B)$$

Hence, A and B are not independent.

Question 3(ii)
A card is drawn from a pack of 52 cards so that each card is equally likely to be selected. State whether events A and B are independent if,

A = the card drawn is black, B = the card drawn is a king

Solution 3(ii)

A card is drawn from pack of 52 cards
There are 26 black and four kings in which 2 kings are black.

A = the card drawn is black

$$P(A) = \frac{26}{52}$$
$$P(A) = \frac{1}{2}$$

B = the card drawn is a king

$$P(B) = \frac{4}{52}$$
$$= \frac{1}{13}$$

$A \cap B$ = The card drawn is a black king

$$P(A \cap B) = \frac{2}{52} = \frac{1}{26}$$

$$P(A)P(B) = \frac{1}{2} \times \frac{1}{13}$$
$$= \frac{1}{26}$$

$$P(A).P(B) = P(A \cap B)$$

So, A and B are independent events.

Question 3(iii)

A card is drawn from a pack of 52 cards so that each card is equally likely to be selected. State whether events A and B are independent if,

A = the card drawn is spade, B = the card drawn in an ace

Solution 3(iii)

A card is drawn from a pack of 52 cards
There are 13 spades and 4 Ace in which one card is ace of spade

A = the card drawn is spade

$$P(A) = \frac{13}{52}$$
$$P(A) = \frac{1}{4}$$

B = the card drawn is an ace

$$P(B) = \frac{4}{52}$$
$$P(B) = \frac{1}{13}$$

$A \cap B$ = The card drawn is an ace of spade

$$P(A \cap B) = \frac{1}{52}$$

$$P(A).P(B) = \frac{1}{4} \times \frac{1}{13}$$
$$= \frac{1}{52}$$

$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

Question 4

A coin is tossed three times. Let the events A, B and C be defined as follows:
A = first toss is head, B = second toss is head, and
C = exactly two heads are tossed in a row.
Check the independence of (i) A and B (ii) B and C (iii) C and A

Solution 4

A coin is tossed three times,

Sample space $= \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

A = first toss is Head

$A = \{HHH, HHT, HTH, HTT\}$

$$P(A) = \frac{4}{8}$$

$$P(A) = \frac{1}{2}$$

B = second toss is Head

$= \{HHH, HHT, THH, THT\}$

$$P(B) = \frac{4}{8}$$

$$P(B) = \frac{1}{2}$$

C = exactly two Head in a row

$C = \{HHT, THH\}$

$$P(C) = \frac{2}{8}$$

$$P(C) = \frac{1}{4}$$

$A \cap B = \{HHH, HHT\}$

$$P(A \cap B) = \frac{2}{8}$$

$$= \frac{1}{4}$$

$B \cap C = \{HHT, THH\}$

$$P(B \cap C) = \frac{2}{8}$$

$$P(B \cap C) = \frac{1}{4}$$

$A \cap C = \{HHT\}$

$$P(A \cap C) = \frac{1}{8}$$

(i)

$$P(A).P(B) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$
$$P(A).P(B) = P(A \cap B)$$

Hence, A and B are independent events.

(ii)

$$P(B)\,P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(B)\,P(C) \neq P(B \cap C)$$

So, B and C are not independent events.

(iii)

$$P(A).P(C) = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{8}$$
$$P(A).P(C) = P(A \cap C)$$

Hence, A and C are independent events.

Question 5

If A and B be two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$, show that A and B are independent events.

Solution 5

Given,

$$P\left(A\right) = \frac{1}{4}, P\left(B\right) = \frac{1}{3} \text{ and } P\left(A \cup B\right) = \frac{1}{2}$$

We know that,

$$P\left(A \cap B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cup B\right)$$
$$= \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$
$$= \frac{3 + 4 - 6}{12}$$
$$P\left(A \cap B\right) = \frac{1}{12}$$

$$P\left(A\right) P\left(B\right) = \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}$$
$$P\left(A\right) P\left(B\right) = P\left(A \cap B\right)$$

Hence, A and B are independent events.

Question 6

Given two independent events A and B such that $P\left(A\right) = 0.3$ and $P\left(B\right) = 0.6$, Find

(i) $P\left(A \cap B\right)$ (ii) $P\left(A \cap \overline{B}\right)$ (iii) $P\left(\overline{A} \cap B\right)$ (iv) $P\left(\overline{A} \cap \overline{B}\right)$

(v) $P\left(A \cup B\right)$ (vi) $P\left(\frac{A}{B}\right)$ (vii) $P\left(\frac{B}{A}\right)$

Solution 6

Given that A and B are independent events and $P\left(A\right) = 0.3,\ P\left(B\right) = 0.6$

(i)

$$P\left(A \cap B\right) = P\left(A\right)P\left(B\right) \qquad \left[\text{Since, } A \text{ and } B \text{ are indenpendent events}\right]$$
$$= 0.3 \times 0.6$$

$$P\left(A \cap B\right) = 0.18$$

(ii)

$$P\left(A \cap \overline{B}\right) = P\left(A\right) - P\left(A \cap B\right)$$
$$= 0.3 - 0.18$$

$$P\left(A \cap \overline{B}\right) = 0.12$$

(iii)

$$P\left(\overline{A} \cap B\right) = P\left(B\right) - P\left(A \cap B\right)$$
$$= 0.6 - 0.18$$

$$P\left(\overline{A} \cap B\right) = 0.42$$

(iv)

$$P\left(\overline{A} \cap \overline{B}\right) = P\left(A\right) P\left(B\right)$$
$$= \left[1 - P\left(A\right)\right]\left[1 - P\left(B\right)\right]$$
$$= \left(1 - 0.3\right)\left(1 - 0.6\right)$$
$$= 0.7 \times 0.4$$

$$P\left(\overline{A} \cap \overline{B}\right) = 0.28$$

(v)

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$
$$= 0.3 + 0.6 - 0.18$$

$$P\left(A \cup B\right) = 0.72$$

(vi)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.18}{0.6}$$

$$P\left(\frac{A}{B}\right) = 0.3$$

(vii)

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.18}{0.3}$$

$$P\left(\frac{B}{A}\right) = 0.6$$

Question 7

If $P(\text{not } B) = 0.65$, $P(A \cup B) = 0.85$, and A and B are independent events, then find $P(A)$.

Solution 7

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since A, B are independent

$\therefore$ $P(A \cap B) = P(A).P(B)$

Also $P(\text{not } B) = 0.65 \Rightarrow$ $P(B) = 0.35$

Hence, we have

$$0.85 = P(A) + 0.35 - P(A)(0.35)$$

$\Rightarrow$ $0.5 = P(A)[1 - 0.35]$

$\Rightarrow$ $\frac{0.5}{.65} = P(A)$

$\Rightarrow$ $P(A) = 0.77$

Question 8

If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$,

then find $P(B)$.

Solution 8

We are given

$PP\left(\overline{A} \cap B\right) = \frac{2}{15}$

$P\left(A \cap \overline{B}\right) = \frac{1}{6}$

Since A, B are independent,

$\therefore \quad P\left(\overline{A}\right)P\left(B\right) = \frac{2}{15} \Rightarrow \quad \left[1 - P\left(A\right)\right]P\left(B\right) = \frac{2}{15} \qquad ---(i)$

and $\quad P\left(A\right)P\left(\overline{B}\right) = \frac{1}{6} \Rightarrow \quad P\left(A\right)\left[1 - P\left(B\right)\right] = \frac{1}{6} \qquad ---(ii)$

From (i) we get

$$P\left(B\right) = \frac{2}{15} \times \frac{1}{1 - P\left(A\right)}$$

Substituting this value in equation (ii) we get,

$$P\left(A\right)\left[1 - \frac{2}{15\left(1 - P\left(A\right)\right)}\right] = \frac{1}{6}$$

$$\Rightarrow \quad P\left(A\right)\left[\frac{15\left(1 - P\left(A\right)\right) - 2}{15\left(1 - P\left(A\right)\right)}\right] = \frac{1}{6}$$

$$\Rightarrow \quad 6P\left(A\right)\left(13 - 15P\left(A\right)\right) = 15\left(1 - P\left(A\right)\right)$$

$$\Rightarrow \quad 2P\left(A\right)\left(13 - 15P\left(A\right)\right) = 5 - 5P\left(A\right)$$

$$\Rightarrow \quad 26P\left(A\right) - 30\left[P\left(A\right)\right]^2 + 5P\left(A\right) - 5 = 0$$

$$\Rightarrow \quad -30\left[P\left(A\right)\right]^2 + 31P\left(A\right) - 5 = 0$$

This is a quadratic equation in $x = P\left(A\right)$ given as

$$-30x^2 + 31x - 5 = 0$$

$$\Rightarrow \quad 30x^2 - 31x + 5 = 0$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = +30,$ $b = -31,$ $c = +5$

$$\Rightarrow \quad x = \frac{31 \pm \sqrt{(-31)^2 - 4(30)(5)}}{60}$$
$$= \frac{31 \pm \sqrt{961 - 600}}{60}$$
$$= \frac{31 \pm 19}{60}$$
$$= \frac{50}{60}, \frac{12}{60}$$
$$= \frac{5}{6}, \frac{1}{5}$$

$\therefore \quad P(A) = \frac{5}{6}$ or $\frac{1}{5}$

Now

$$P(A)[1 - P(B)] = \frac{1}{6}$$

Putting $P(A) = \frac{5}{6}$

$$\frac{5}{6}[1 - P(B)] = \frac{1}{6}$$
$$1 - P(B) = \frac{1}{5}$$
$$P(B) = 1 - \frac{1}{5}$$
$$P(B) = \frac{4}{5}$$

Putting $P(A) = \frac{1}{5}$

$$\frac{1}{5}[1 - P(B)] = \frac{1}{6}$$
$$1 - P(B) = \frac{5}{6}$$
$$P(B) = 1 - \frac{5}{6}$$
$$P(B) = \frac{1}{6}$$

Hence $P(B) = \frac{4}{5}$ or $\frac{1}{6}$

Question 9

A and B are two independent events. The probability that A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. Find the probability of occurrence of two events.

Solution 9

Given,

$$P\left(A \cap B\right) = \frac{1}{6}$$

$$P\left(\overline{A} \cap \overline{B}\right) = \frac{1}{3}$$

We know that,

$$P\left(\overline{A} \cap \overline{B}\right) = P\left(A\right)P\left(B\right)$$

$$\frac{1}{3} = \left(1 - P\left(A\right)\right)\left(1 - P\left(B\right)\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A\right)P\left(B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + P\left(A \cap B\right)$$

$$\frac{1}{3} = 1 - P\left(B\right) - P\left(A\right) + \frac{1}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{1}{1} + \frac{1}{6} - \frac{1}{3}$$

$$= \frac{6 + 1 - 2}{6}$$

$$P\left(A\right) + P\left(B\right) = \frac{5}{6}$$

$$P\left(A\right) = \frac{5}{6} - P\left(B\right) \qquad ---\left(i\right)$$

Given, $P\left(A \cap B\right) = \frac{1}{6}$

$$P\left(A\right)P\left(B\right) = \frac{1}{6}$$

$$\left[\frac{5}{6} - P\left(B\right)\right]P\left(B\right) = \frac{1}{6} \qquad \left[\text{Using equation } \left(i\right)\right]$$

$\Rightarrow \quad \frac{5}{6}P\left(B\right) - \left\{P\left(B\right)\right\}^2 = \frac{1}{6}$

$\Rightarrow \quad \left\{P\left(B\right)\right\}^2 - \frac{5}{6}P\left(B\right) + \frac{1}{6} = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 5P\left(B\right) + 1 = 0$

$\Rightarrow \quad 6\left\{P\left(B\right)\right\}^2 - 3P\left(B\right) - 2P\left(B\right) + 1 = 0$

$\Rightarrow \quad 3P\left(B\right)\left[2P\left(B\right) - 1\right] - 1\left[2P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad \left[2P\left(B\right) - 1\right]\left[3P\left(B\right) - 1\right] = 0$

$\Rightarrow \quad 2P\left(B\right) - 1 = 0 \text{ or } 3P\left(B\right) - 1 = 0$

$\Rightarrow \quad P(B) = \frac{1}{2}$ or $P(B) = \frac{1}{3}$

$\Rightarrow$ Using equation (i),

$$P(B) = \frac{1}{2} \qquad \Rightarrow P(A) = \frac{5}{6} - \frac{1}{2} = \frac{1}{3}$$
$$P(B) = \frac{1}{3} \qquad \Rightarrow P(A) = \frac{5}{6} - \frac{1}{3} = \frac{1}{2}$$

Hence, $P(B) = \frac{1}{2}, P(A) = \frac{1}{3}$ or $P(B) = \frac{1}{3}, P(A) = \frac{1}{2}$

Question 10

If A and B are two independent events such that $P(A \cup B) = 0.60$ and $P(A) = 0.2$ find $P(B)$.

Solution 10

Given, A and B are independent events and $P(A \cup B) = 0.60$, $P(A) = 0.2$

A and B are independent events,

So, $P(A \cap B) = P(A)P(B)$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.6 = 0.2 + P(B) - P(A)P(B)$$
$$0.6 - 0.2 = P(B) - 0.2P(B)$$
$$0.4 = 0.8P(B)$$
$$P(B) = \frac{0.4}{0.8}$$

$$P(B) = 0.5$$

Question 11

A die is tossed twice. Find the probability of getting a number greater than 3 on each toss.

Solution 11

A die is tossed twice.

Let A = Getting a number greater than 3 on first toss
B = Getting a number greater than 3 on second toss

$P(A) = \frac{3}{6}$ [Since, number greater than 3 on die are 4, 5, 6.]

$P(A) = \frac{1}{2}$

$P(B) = \frac{3}{6}$

$P(B) = \frac{1}{2}$

P (Getting a number greater than 3 on each toss)
$= P(A \cap B)$ [Since, A and B are independent events]
$= P(A)P(B)$
$= \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{4}$

Required Probability $= \frac{1}{4}$

Question 12

Given the probability that A can solve a problem is $\frac{2}{3}$ and the probability that B can solve the same problem is $\frac{3}{5}$. Find the probability that none of the two will be able to solve the problem.

Solution 12

Given,

Probability that A can solve a problem $= \frac{2}{3}$

$\Rightarrow \quad P(A) = \frac{2}{3}$

$\Rightarrow \quad P(\overline{A}) = 1 - \frac{2}{3}$

$P(\overline{A}) = \frac{1}{3}$

Probability that B can solve the same problem $= \frac{3}{5}$

$\Rightarrow \quad P(B) = \frac{3}{5}$

$\Rightarrow \quad P(\overline{B}) = 1 - \frac{3}{5}$

$P(\overline{B}) = \frac{2}{5}$

P(None of them solve the problem)

$= P(\overline{A} \cap \overline{B})$

$= P(\overline{A}).P(\overline{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Required probability $= \frac{2}{15}$

Question 13

An unbiased die is tossed twice. Find the probability of getting 4, 5, or 6 on the first toss and 1, 2, 3 or 4 on the second toss.

Solution 13

Given an unbiased die is tossed twice

A = Getting 4,5 or 6 on the first toss

B = 1,2,3 or 4 on second toss

$$\Rightarrow \quad P(A) = \frac{3}{6}$$

$$P(A) = \frac{1}{2}$$

and, $P(B) = \frac{4}{6}$

$$P(B) = \frac{2}{3}$$

P (Getting 4,5 or 6 on the first toss and 1,2,3 or 4 on second toss)

$= P(A \cap B)$

$= P(A)P(B)$

$= \frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3}$

Required probability $= \frac{1}{3}$

Question 14

A bag contains 3 red and 2 black balls. One ball is drawn from it at random. Its colour is noted and then it is put back in the bag. A second draw is made and the same procedure is repeated. Find the probability of drawing (i) two red balls, (ii) two black balls, (iii) first red and second black ball.

Solution 14

Given bag contains 3 red and 2 black balls.

A = Getting one red ball

$\Rightarrow \quad P(A) = \frac{3}{5}$

B = Getting one black ball

$\Rightarrow \quad P(B) = \frac{2}{5}$

(i)

P (Getting two red balls)

$= P(A)\,P(A)$

$= \frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

P (Getting two red balls) $= \frac{9}{25}$

(ii)

P (Getting two black balls)

$= P(B).P(B)$

$= \frac{2}{5} \times \frac{2}{5}$

$= \frac{4}{25}$

P (Getting two black balls) $= \frac{4}{25}$

(iii)

P (Getting first red and second black ball)

$= P(A)\,P(B)$

$= \frac{3}{5} \times \frac{2}{5}$

$= \frac{6}{25}$

P (Getting first red and second black ball) $= \frac{6}{25}$

Question 15

Three cards are drawn with replacement from a well shuffled pack of cards. Find the probability that the cards drawn are king, queen and jack.

Solution 15

Three cards are drawn with replacement consider,

A = drawing a king

B = drawing a queen

C = drawing a jack

$\Rightarrow \quad P(A) = \frac{4}{52}$ [Since there are 4 kings]

$P(A) = \frac{1}{13}$

$\Rightarrow \quad P(B) = \frac{4}{52}$ [Since there are 4 queens]

$P(B) = \frac{1}{13}$

$\Rightarrow \quad P(C) = \frac{4}{52}$ [Since there are 4 jacks]

$P(C) = \frac{1}{13}$

P(Cards drawn are king, queen and jack)

$= P(A \cap B \cap C) + P(A \cap C \cap B) + P(B \cap A \cap C)$

$P(B \cap C \cap A) + P(C \cap A \cap B) + P(C \cap B \cap A)$

[Since order of drawing them may be different]

$= P(A)P(B)P(C) + P(A)P(C)P(B) + P(B)P(A)P(C)$

$+P(B)P(C)P(A) + P(C)P(A)P(B) + P(C)P(B)P(A)$

$= \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13} + \frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13}$

$= \left(\frac{1}{13}\cdot\frac{1}{13}\cdot\frac{1}{13}\right) \times 6$

$= \frac{6}{2197}$

Required probability $= \frac{6}{2197}$

Question 16

An article manufactured by a company consists of two parts X and Y. In the process of manufacture of the part X, 9 out of 100 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part Y. Calculate the probability that the assembled product will not be defective.

Solution 16

Given,

Prat X has 9 out of 100 defective

$\Rightarrow$ Part X has 91 out of 100 non defective

Part Y has 5 out of 100 defective

$\Rightarrow$ Part Y has 95 out of 100 non defective

Consider,

X = A non defective part X

Y = A non defective part Y

$\Rightarrow$ $P(X) = \frac{91}{100}$ abd $P(Y) = \frac{95}{100}$

$= P$ (Assembled product will not be defective)

$= P$ (Niether X defective nor Y defective)

$= P(X \cap Y)$

$= P(X)P(Y)$

$= \frac{91}{100} \times \frac{95}{100}$

$= 0.8645$

Required probability $= 0.8645$

Question 17

The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. What is the probability that the target will be hit, if each one of A and B shoots at the target?

Solution 17

Given,

Probability that A hits a target $= \frac{1}{3}$

$\Rightarrow \quad P(A) = \frac{1}{3}$

Probability that B hits the target $= \frac{2}{5}$

$\Rightarrow \quad P(B) = \frac{2}{5}$

P (Target will be hit)
$= 1 - P$ (target will not be hit)
$= 1 - P$ (Niether A non B hits the target)
$= 1 - P\left(\overline{A} \cap \overline{B}\right)$
$= 1 - P\left(\overline{A}\right) P\left(\overline{B}\right)$
$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\overline{B}\right)\right]$
$= 1 - \left[1 - \frac{1}{3}\right]\left[1 - \frac{2}{5}\right]$
$= 1 - \frac{2}{3} \cdot \frac{3}{5}$
$= 1 - \frac{2}{5}$
$= \frac{3}{5}$

Required probability $= \frac{3}{5}$

Question 18

An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?

Solution 18

Given,

An anti aircraft gun can take a maximum 4 shots at an enemy plane

Consider,

A = Hitting the plane at first shot
B = Hetting the plane at second shot
C = Hetting the plane at third shot
D = Hetting the plane at fourth shot

$\Rightarrow$ $P(A) = 0.4$, $P(B) = 0.3$, $P(C) = 0.2$, $P(D) = 0.1$

P (Gun hits the plane)
= 1 – P (Gun does not hit the plane)
= 1 – P (Non of the four shots hit the plane)

$$= 1 - P\left(\overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D}\right)$$
$$= 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)P\left(\overline{C}\right)P\left(\overline{D}\right)$$
$$= 1 - \left[1 - P(A)\right]\left[1 - P\left(\overline{B}\right)\right]\left[1 - P(C)\right]\left[1 - P(D)\right]$$
$$= 1 - [1 - 0.4][1 - 0.3][1 - 0.2][1 - 0.1]$$
$$= 1 - (0.6)(0.7)(0.8)(0.9)$$
$$= 1 - 0.3024$$
$$= 0.6976$$

Required probability = 0.6976

Question 19

The odds against a certain event are 5 to 2 and the odds in favour of another event, independent to the former are 6 to 5. Find the probability that (a) at least one of the events will occur, and (b) none of the events will occur.

Solution 19

Given,

The odds against a certain event (say, A) are 5 to 2

$$\Rightarrow \quad P\left(\overline{A}\right) = \frac{5}{5+2}$$

$$P\left(\overline{A}\right) = \frac{5}{7}$$

The odds in favour of another event (say, B) are 6 to 5

$$\Rightarrow \quad P\left(B\right) = \frac{6}{5+6}$$

$$P\left(B\right) = \frac{6}{11}$$

$$P\left(\overline{B}\right) = 1 - \frac{6}{11}$$

$$P\left(\overline{B}\right) = \frac{5}{11}$$

(a)

P (At least one of the events will occur)

$= 1 - P$ (None of events occur)

$= 1 - P\left(\overline{A} \cap \overline{B}\right)$

$= 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)$ [Since events are independent]

$= 1 - \frac{5}{7} \times \frac{5}{11}$

$= 1 - \frac{25}{77}$

$= \frac{52}{77}$

Required probability $= \frac{52}{77}$

(b)

P (None of the events will occur)

$= P\left(\overline{A} \cap \overline{B}\right)$

$= P\left(\overline{A}\right)P\left(\overline{B}\right)$

$= \frac{5}{7} \times \frac{5}{11}$

$= \frac{25}{11}$

Required probability $= \frac{25}{77}$

Question 20

A die is thrown thrice. Find the probability of getting an odd number at least once.

Solution 20

Given, A die is thrown thrice.

Consider,

A = Getting an odd number in a throw of die

$P(A) = \frac{3}{6}$ [Since there are 1,3,5 odd number on die]

$P(A) = \frac{1}{2} \quad \Rightarrow P(\overline{A}) = \frac{1}{2}$

P (Getting an odd number at least once)

$= 1 - P$ (Getting no odd number)

$= 1 - P(\overline{A} \cap \overline{A} \cap \overline{A})$

$= 1 - P(\overline{A})P(\overline{A})P(\overline{A})$

$= 1 - \frac{1}{2}.\frac{1}{2}.\frac{1}{2}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Required probability $= \frac{7}{8}$

Question 21

Two balls are drawn at random with replacement from a box containing 10 black and 8red balls. Find the probability that

(i) Both balls are red

(ii) First ball is black and second is red.

(iii) one of them is black and other is red

Solution 21

The box contains 10 black balls and 8red balls.

Then $P(\text{black ball}) = \frac{10}{18}$

$P(\text{red ball}) = \frac{8}{18}$

(i) $P(\text{Both balls are red}) = \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$

(ii) $P(\text{ First ball is black and second is red})$

$$= \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}$$

(iii) P (one of them is black and other is red)

$$= \frac{10}{18} \cdot \frac{8}{18} + \frac{8}{18} \cdot \frac{10}{18}$$
$$= 2\left(\frac{20}{81}\right)$$
$$= \frac{40}{81}$$

Question 22

An urn contains 4 red and 7 black balls. Two balls are drawn at random with replacement. Find the probability of getting (i) 2 red balls (ii) 2 blue balls (iii) one red and one blue ball.

Solution 22

Given, Urn contains 4 red and 7 black balls.
Two balls drawn at random with replacement.

Consider,

R = Getting one red ball from urn.

$P(R) = \frac{4}{11}$

B = Getting one blue ball from urn.

$P(B) = \frac{7}{11}$

(i)

P (Getting 2 red balls)

$= P(R).P(R)$

$= \frac{4}{11} \times \frac{4}{11}$

$= \frac{16}{121}$

Required probability $= \frac{16}{121}$

(ii)

P (Getting two blue balls)

$= P(B)P(B)$

$= \frac{7}{11} \times \frac{7}{11}$

$= \frac{49}{121}$

Required probability $= \frac{49}{121}$

(iii)

P (Getting one red and one blue ball)

$= P(R)P(B) + P(B)P(R)$

$= \frac{4}{11} \times \frac{7}{11} + \frac{7}{11} \times \frac{4}{11}$

$= \frac{28}{121} + \frac{28}{121}$

$= \frac{56}{121}$

Required probability $= \frac{56}{121}$

Question 23

The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively.

Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time.

Write atleast one advantage of coming to school in time.

Solution 23

Given that the events 'A coming in time' and 'B coming in time' are independent.

Let 'A' denote the event of 'A coming in time'.

Then, '$\bar{A}$' denotes the complementary event of A.

Similarly we define B and $\bar{B}$.

$P(\text{only one coming in time}) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$...(since A and B are independent events)

$= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

The advantage of coming to school in time is that you will not miss any part of the lecture and will be able to learn more.

Question 24

Two dice are thrown together and the total score is noted. The event E, F and G are "a total 4", "a total of 9 or more", and "a total divisible by 5", respectively. Calculate P (E), P(F) and P(G) and decide which pairs of events, if any, are independent.

Solution 24

$$S = \left\{\begin{array}{l}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{array}\right\}$$

$n(S) = 36$

E be the event of geting a total of 4.

$E = \{(1,3),(3,1),(2,2)\}$

$n(E) = 3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

F be the event of geting a total of 9 or more.

$F = \{(3,6),(6,3),(4,5),(5,4),(4,6),(6,4),(5,5),(5,6),(6,5),(6,6)\}$

$n(F) = 10$

$$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

G be the event of geting a total divisible by 5.

$G = \{(1,4),(4,1),(2,3),(3,2),(4,6),(6,4),(5,5)\}$

$n(G) = 7$

$$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$$

No pair is independent.

Question 25

Let A and B be two independent events such that P (A) = p_1 and P (B) = p_2. Describe in words the events whose probabilities are:

(i) p_1p_2 (ii) $(1 - p_1)p_2$ (iii) $1-(1- p_1)(1 - p_2)$ (iv) $p_1 + p_2 = 2p_1p_2$

Solution 25

Events are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence or non-occurrence of the other.

$(i) p_1 p_2 = P(A)P(B)$
$\Rightarrow$ Both A and B occour.

$(ii) (1-p_1)p_2 = (1-P(A))P(B) = P(\overline{A})P(B)$
$\Rightarrow$ Event A does not occour, but event B occours.

$(iii) 1-(1-p_1)(1-p_2) = [1-(1-P(A))(1-P(B))] = (1-P(\overline{A})P(\overline{B}))$
$\Rightarrow$ At least one of the events A or B occours.

$(iv) p_1 + p_2 = 2p_1 p_2$
$\Rightarrow P(A)+P(B) = 2P(A)P(B)$
$\Rightarrow P(A)+P(B)-2P(A)P(B) = 0$
$\Rightarrow P(A)-P(A)P(B)+P(B)-P(A)P(B) = 0$
$\Rightarrow P(A)(1-P(B))+P(B)(1-P(A)) = 0$
$\Rightarrow P(A)P(\overline{B})+P(B)P(\overline{A}) = 0$
$\Rightarrow P(A)P(\overline{B}) = P(B)P(\overline{A})$
$\Rightarrow$ Exactly one of A and B occours.

Chapter 31 - Probability Exercise Ex. 31.5

Question 1

A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from each bag, find the probability that these two balls are of the same colour.

Solution 1

There are two bags.

One bag (1) Contain 6 black and 3 white balls

other bag (2) Contain 5 black and 4 white balls

One ball is drawn from each bag

$$P(\text{One black from bag 1}) = \frac{6}{9}$$
$$P(B_1) = \frac{2}{3}$$

$$P(\text{One black from bag 2}) = \frac{5}{9}$$
$$P(B_2) = \frac{5}{9}$$

$$P(\text{One white from bag 1}) = \frac{3}{9}$$
$$P(W_1) = \frac{1}{3}$$

$$P(\text{One white from bag 2}) = \frac{4}{9}$$
$$P(W_2) = \frac{4}{9}$$

$$
\begin{aligned}
&P(\text{Two balls of same colour})\\
&= P\left[(W_1 \cap W_2) \cup (B_1 \cap B_2)\right]\\
&= P(W_1 \cap W_2) + P(B_1 \cap B_2)\\
&= P(W_1)P(W_2) + P(B_1)P(B_2)\\
&= \frac{1}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{5}{9}\\
&= \frac{4}{27} + \frac{10}{27}\\
&= \frac{14}{27}
\end{aligned}
$$

$$\text{Required probability } = \frac{14}{27}$$

Question 2

A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A ball is drawn from each bag. Find the probability that one is red and the other is black.

Solution 2

There are two bags.

Bag (1) contain 3 red and 5 black balls

Bag (2) contain 6 red and 4 black balls

$$P(\text{One red ball from bag 1}) = \frac{3}{8}$$
$$P(R_1) = \frac{3}{8}$$

$$P(\text{One black ball from bag 1}) = \frac{5}{8}$$
$$P(B_1) = \frac{5}{8}$$

$$P(\text{One red ball from bag 1}) = \frac{6}{10}$$
$$P(R_2) = \frac{3}{5}$$

$$P(\text{One black ball from bag 2}) = \frac{4}{10}$$
$$P(B_2) = \frac{2}{5}$$

One ball is drawn from each bag.

$$\begin{aligned}
&P(\text{One ball is red and the other is black})\\
&= P\left[(R_1 \cap B_2) \cup (B_1 \cap R_2)\right]\\
&= P(R_1 \cap B_2) + P(B_1 \cap R_2)\\
&= P(R_1)P(B_2) + P(B_1)P(R_2)\\
&= \frac{3}{8} \times \frac{2}{5} + \frac{5}{8} \times \frac{3}{5}\\
&= \frac{6}{40} + \frac{15}{40}\\
&= \frac{21}{40}
\end{aligned}$$

$$\text{Required probability} = \frac{21}{40}$$

Question 3

Two balls are drawn at random with replacment from a box containing 10 black and 8 red balls. Find the probability that (i) both the ball are red

(ii) first ball is black and second is red. (iii) one of them is black and other is red.

Solution 3

Given, box contains 10 black and 8 red balls.
Two balls are drawn with replacement.

(i)

$$\begin{aligned}
&P\,(\text{Both the balls are red})\\
&= P\,(R_1 \cap R_2)\\
&= P\,(R_1)\,P\,(R_2)\\
&= \frac{8}{18} \times \frac{8}{18}\\
&= \frac{16}{81}
\end{aligned}$$

Required probability = $\frac{16}{81}$

(ii)

$$\begin{aligned}
&P\,(\text{first ball is black and second is red})\\
&= P\,(B \cap R)\\
&= P\,(B)\,P\,(R)\\
&= \frac{10}{18} \times \frac{8}{18}\\
&= \frac{20}{81}
\end{aligned}$$

Required probability = $\frac{20}{81}$

(iii)

$$\begin{aligned}
&P\,(\text{one of them red and other black})\\
&= P\,((B \cap R) \cup (R \cap B))\\
&= P\,(B \cap R) + P\,(R \cap B)\\
&= P\,(B)P\,(R) + P\,(R)P\,(B)\\
&= \frac{10}{18} \times \frac{8}{18} + \frac{8}{18} \times \frac{10}{18}\\
&= \frac{20+20}{81}\\
&= \frac{40}{81}
\end{aligned}$$

Required probability = $\frac{40}{81}$

Question 4

Two cards are drawn successively without replacement from a well-shuffled deck of cards. Find the probability of exactly one ace.

Solution 4

Two cards are drawn without replacement.

There are total 4 ace.

A = Getting Ace

$$\begin{aligned}
&\text{P(Exactly one ace out of 2 cards)} \\
&= P\left(\left(A \cap \overline{A}\right) \cup \left(\overline{A} \cap A\right)\right) \\
&= P(A)P\left(\frac{\overline{A}}{A}\right) + P\left(\overline{A}\right)P\left(\frac{A}{\overline{A}}\right) \\
&= \frac{4}{52}\cdot\frac{48}{51} + \frac{48}{52}\cdot\frac{4}{51} \\
&= \frac{96}{663} \\
&= \frac{32}{221}
\end{aligned}$$

Required probability = $\frac{32}{221}$

Question 5

A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in narrating the same incident?

Solution 5

Given,

A speaks truth in 75% cases.

B speaks truth in 80% cases.

$$P(A) = \frac{75}{100} \quad \Rightarrow P(\overline{A}) = \frac{25}{100}$$

$$P(B) = \frac{80}{100} \quad \Rightarrow P(\overline{B}) = \frac{20}{100}$$

$P(A \text{ and } B \text{ contradict each other})$

$= P\left[\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right]$

$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$

$= P(A) P\left(\overline{B}\right) + P\left(\overline{A}\right) P(B)$

$= \frac{75}{100} \cdot \frac{20}{100} + \frac{25}{100} \cdot \frac{80}{100}$

$= \frac{1500}{1000} + \frac{2000}{10000}$

$= \frac{3500}{10000}$

$= 35\%$

Required probability $= 35\%$

Question 6

Kamal and Monica appeared for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Monika's selection is $\frac{1}{5}$. Find the probability that

(i) both of them will be selected

(ii) none of them will be selected

(iii) at least one of them will be selected

(iv) only one of them will be selected.

Solution 6

Given,

Probability of selection of Kamal $(K) = \frac{1}{3}$

$$P(K) = \frac{1}{3}$$

Probability of selection of Monika $(M) = \frac{1}{5}$

$$P(M) = \frac{1}{5}$$

(i)

$P(\text{Both of them selected})$

$= P(K \cap M)$

$= P(K)\, P(M)$

$= \frac{1}{3} \cdot \frac{1}{5}$

$= \frac{1}{15}$

Required probability $= \frac{1}{15}$

(ii)

$P(\text{None of them will be selected})$

$= P(\overline{K} \cap \overline{M})$

$= P(\overline{K})\, P(\overline{M})$

$= [1 - P(K)][1 - P(M)]$

$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$

$= \frac{2}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

Required probability $= \frac{8}{15}$

(iii)

$$P\left(\text{At least one of them selected}\right)$$
$$= 1 - P\left(\text{None of them selected}\right)$$
$$= 1 - P\left(\overline{M} \cap \overline{K}\right)$$
$$= 1 - P\left(\overline{M}\right).P\left(\overline{K}\right)$$
$$= 1 - \left[1 - P\left(M\right)\right]\left[1 - P\left(K\right)\right]$$
$$= 1 - \left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{3}\right)$$
$$= 1 - \frac{4}{5}.\frac{2}{3}$$
$$= 1 - \frac{8}{15}$$
$$= \frac{7}{15}$$

$$\text{Required probability } = \frac{7}{15}$$

(iv)

$$P\left(\text{Only one of them will be selected}\right)$$
$$= P\left[\left(K \cap \overline{M}\right) \cup \left(\overline{K} \cap M\right)\right]$$
$$= P\left(K \cap \overline{M}\right) + P\left(\overline{K} \cap M\right)$$
$$= P\left(K\right)P\left(\overline{M}\right) + P\left(\overline{K}\right)P\left(M\right)$$
$$= \frac{1}{3}\left[1 - P\left(M\right)\right] + \left[1 - P\left(K\right)\right]\frac{1}{5}$$
$$= \frac{1}{3}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{3}\right].\frac{1}{5}$$
$$= \frac{1}{3}.\frac{4}{5} + \frac{2}{3}.\frac{1}{5}$$
$$= \frac{4}{15} + \frac{2}{15}$$
$$= \frac{6}{15}$$
$$= \frac{2}{5}$$

$$\text{Required probability } = \frac{2}{5}$$

Question 7

A bag contains 3 white, 4 red and 5 black balls. Two balls are drawn one after the other, without replacement. What is the probability that one is white and the other is black?

Solution 7

Bag contain 3 white, 4 red, 5 black balls.
Two balls are drawn without replacement.

$$P\left(\text{One ball is white and other black}\right)$$
$$= P\left[\left(W \cap B\right) \cup \left(B \cap W\right)\right]$$
$$= P\left(W \cap B\right) + P\left(B \cap W\right)$$
$$= P\left(W\right) P\left(\frac{B}{W}\right) + P\left(B\right) P\left(\frac{W}{B}\right)$$
$$= \frac{3}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{3}{11}$$
$$= \frac{15}{132} + \frac{15}{132}$$
$$= \frac{30}{132}$$
$$= \frac{5}{22}$$

Required probability $= \frac{5}{22}$

Question 8

A bag contains 8 red and 6 green balls. Three balls are drawn one after another without replacement. Find the probability that at least two balls drawn are green.

Solution 8

A bag contains 8 red and 6 green balls.
Three balls are drawn without replacement

$$P\left(\text{at least 2 balls are green}\right)$$
$$= P\left[\left(G_1 \cap G_2 \cap R_1\right) \cup \left(G_1 \cap R_1 \cap G_2\right) \cup \left(R_1 \cap G_1 \cap G_2\right) \cup \left(G_1 \cap G_2 \cap G_3\right)\right]$$
$$= P\left(G_1 \cap G_2 \cap R_1\right) + P\left(G_1 \cap R_1 \cap G_2\right) + P\left(R_1 \cap G_1 \cap G_2\right) + P\left(G_1 \cap G_2 \cap G_3\right)$$
$$= P\left(G_1\right) P\left(\frac{G_2}{G_1}\right) P\left(\frac{R_1}{G_1 \cap G_2}\right) + P\left(G_1\right) P\left(\frac{R_1}{G_1}\right) P\left(\frac{G_2}{R_1 \cap G_1}\right) +$$
$$P\left(R_1\right) P\left(\frac{G_1}{R_1}\right) P\left(\frac{G_2}{G_1 \cap R_1}\right) + P\left(G_1\right) P\left(\frac{G_2}{G_1}\right) P\left(\frac{G_3}{G_1 \cap G_2}\right)$$
$$= \frac{6}{14} \times \frac{5}{13} \times \frac{8}{12} + \frac{6}{14} \times \frac{8}{13} \times \frac{5}{12} + \frac{8}{14} \times \frac{6}{13} \times \frac{5}{12} + \frac{6}{14} \times \frac{5}{13} \times \frac{4}{12}$$
$$= \frac{1}{14} \times \frac{1}{13} \times \frac{1}{12} \times \left(240 + 240 + 240 + 120\right)$$
$$= \frac{840}{14 \times 13 \times 12}$$
$$= \frac{5}{13}$$

Required probability $= \frac{5}{13}$

Question 9

Arun and Tarun appeared for an interview for two vacancies. The probability of Arun's selection is $\frac{1}{4}$ and that of Tarun's rejection is $\frac{2}{3}$. Find the probability that at least and of them will be selected.

Solution 9

Given, Probability of Arun's (A) selection $= \frac{1}{4}$

$$P(A) = \frac{1}{4}$$

Probability of Tarun's (T) rejection $= \frac{2}{3}$

$$P(\overline{T}) = \frac{2}{3}$$

$$P(\overline{A}) = 1 - P(A)$$

$$\Rightarrow \quad P(\overline{A}) = 1 - \frac{1}{4}$$

$$\Rightarrow \quad P(\overline{A}) = \frac{3}{4}$$

$$P(T) = 1 - P(\overline{T})$$

$$\Rightarrow \quad P(T) = 1 - \frac{2}{3}$$

$$\Rightarrow \quad P(T) = \frac{1}{3}$$

P(At least one of them will be selected)

$= 1 - P$(None of them selected)

$= 1 - P(\overline{A} \cap \overline{T})$

$= 1 - P(\overline{A})P(\overline{T})$

$= 1 - \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{2}$

Required probability $= \frac{1}{2}$

Question 10

A and B toss a coin alternately till one of them gets a head and wins the game, If A starts the game, find the probability that B will win the game.

Solution 10

Let E be event of occuring head in a toss of fair coin.

$$P(E) = \frac{1}{2}$$

$$P(\overline{E}) = \frac{1}{2}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw

$$= P(E) = \frac{1}{2}$$

Probability that A wins in 3rd throw

$$= P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^3$$

Probability that A wins in 5th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^5$$

Hence,

Probability of winning A

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \ldots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{1}{2}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

Probability that B wins $= 1 - P(A \text{ wins})$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

Required probability $= \frac{1}{3}$

Question 11

Two cards are drawn from a well shuffled pack of 52 cards, one after another without replacement. Find the probability that one of these is red card and the other a black card?

Solution 11

Two cards are drawn without replacement from a pack of 52 cards.
There are 26 black and 26 red cards

P (one red and other black card)

$$= P\left[(R \cap B) \cup (B \cap R)\right]$$

$$= P(R \cap B) + P(B \cap R)$$

$$= P(R)P\left(\frac{B}{R}\right) + P(B)P\left(\frac{R}{B}\right)$$

$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$$

$$= \frac{13}{51} + \frac{13}{51}$$

$$= \frac{26}{51}$$

Required probability $= \frac{26}{51}$

Question 12

Tickets are numbered from 1 to 10. Two tickets are drawn one after the other at random. Find the probability that the number on one of the tickets is a multiple of 5 and on the other a multiple of 4.

Solution 12

Tickets are numbered from 1 to 10.
Two tickets are drawn.

Consider, A = Multiple of 5
B = Multiple of 4

$P(A) = \frac{2}{10}$ $\quad$ [Since 5, 10 are multiple of 5]

$P(A) = \frac{1}{5}$

$P(B) = \frac{2}{10}$

$P(B) = \frac{1}{5}$ $\quad$ [Since 4, 8 are multiple of 4]

P (One number multiple of 5 and other multiple of 4)

$= P\left[(A \cap B) \cup (B \cap A)\right]$

$= P(A \cap B) + P(B \cap A)$

$= P(A) P\left(\frac{B}{A}\right) + P(B) P\left(\frac{A}{B}\right)$

$= \frac{1}{5} \times \frac{2}{9} + \frac{1}{5} \times \frac{2}{9}$

$= \frac{4}{45}$

Required probability $= \frac{4}{45}$

Question 13

In a family, the husband tells a lie in 30% cases and the wife in 35% cases. Find the probability that both contradict each other on the same fact.

Solution 13

Given, In a family Husband (H) tells a lie in 30% cases and Wife (W) tells a lie in 35%

$P(H) = 30\%$, $P(\overline{H}) = 70\%$

$P(W) = 35\%$, $P(\overline{W}) = 65\%$

$$
\begin{aligned}
&P(\text{Both contradict each other}) \\
&= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right] \\
&= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right) \\
&= P(H)\,P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W) \\
&= \frac{30}{100} \times \frac{65}{100} + \frac{70}{100} \times \frac{35}{100} \\
&= \frac{1950 + 2450}{10000} \\
&= \frac{4400}{10000} \\
&= 0.44
\end{aligned}
$$

Required probability $= 0.44$

Question 14

A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that

(a) both of them will be selected?

(b) only one of them will be selected?

(c) none of them will be selected?

Solution 14

Given, Probability of Husband's (H) selection $= \frac{1}{7}$

$$P(H) = \frac{1}{7}$$

Probability of Wife's (W) selection $= \frac{1}{5}$

$$P(W) = \frac{1}{5}$$

(a)

P(Both of them will be selected)

$= (H \cap W)$

$= P(H)P(W)$

$= \frac{1}{7} \times \frac{1}{5}$

$= \frac{1}{35}$

Required probability $= \frac{1}{35}$

(b)

P(Only one of them will be selected)

$= P\left[\left(H \cap \overline{W}\right) \cup \left(\overline{H} \cap W\right)\right]$

$= P\left(H \cap \overline{W}\right) + P\left(\overline{H} \cap W\right)$

$= P(H)\,P\left(\overline{W}\right) + P\left(\overline{H}\right)P(W)$

$= P(H)\left[1 - P(W)\right] + \left[1 - P(H)\right]P(W)$

$= \frac{1}{7}\left[1 - \frac{1}{5}\right] + \left[1 - \frac{1}{7}\right]\frac{1}{5}$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$

$= \frac{10}{35}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

(c)

$$P\left(\text{None of them selected}\right)$$
$$= \left(\overline{H} \cap \overline{W}\right)$$
$$= P\left(\overline{H}\right) P\left(\overline{W}\right)$$
$$= \left(1 - P\left(H\right)\right)\left(1 - P\left(W\right)\right)$$
$$= \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{5}\right)$$
$$= \frac{6}{7} \times \frac{4}{5}$$
$$= \frac{24}{35}$$

Required probability $= \frac{24}{35}$

Question 15

A bag contains 7 white, 5 black and 4 red balls. Four balls are drawn without replacement. Find the probability that at least three balls are black.

Solution 15

A bag contains 7 white, 5 black and 4 red balls.

Four balls are drawn without replacement

$$P\left(\text{At least three balls are black}\right)$$
$$= P\left(\text{3 black balls and one not black or 4 black balls}\right)$$
$$= P\left(\text{3 black and one not black}\right) + P\left(\text{4 black balls}\right)$$
$$= \frac{{}^{5}C_{3} \times {}^{11}C_{1}}{{}^{16}C_{4}} + \frac{{}^{5}C_{4}}{{}^{16}C_{4}}$$
$$= \frac{\frac{5!}{3!2!} \times 11 + \frac{5!}{4!1!}}{\frac{16!}{4!12!}} \qquad \left[\text{Since } {}^{n}C_{r} = \frac{n!}{r!\left(n-r\right)!}\right]$$
$$= \frac{\frac{5.4}{2} \times 11 + 5}{\frac{16.15.14.13}{4.3.2}}$$
$$= \frac{\left(110 + 5\right)}{1820}$$
$$= \frac{115}{1820}$$
$$= \frac{23}{364}$$

Required probability $= \frac{23}{364}$

Question 16

$A, B,$ and C are independent witness of an event which is known to have occurred. A speaks the truth three times out of four, B four times out of five and C five times out of six. What is the probability that the occurrence will be reported truthfully by majority of three witnesses?

Solution 16

Given,

A speaks truth 3 out of four times
B speaks truth 4 out of five times
C speads truth 5 out of six times.

$$\Rightarrow \quad P(A) = \frac{3}{4},\ P(B) = \frac{4}{5},\ P(C) = \frac{5}{6}$$

$$\begin{aligned}
&P(\text{Reported truth fully by majority of witnesses})\\
&= P\left(\left(A \cap B \cap \overline{C}\right) \cup \left(A \cap \overline{B} \cap C\right) \cup \left(\overline{A} \cap B \cap C\right) \cup \left(A \cap B \cap C\right)\right)\\
&= P\left(A \cap B \cap \overline{C}\right) + P\left(A \cap \overline{B} \cap C\right) + P\left(\overline{A} \cap B \cap C\right) + P\left(A \cap B \cap C\right)\\
&= P(A)P(B)P(\overline{C}) + P(A)P(\overline{B})P(C) + P(\overline{A})P(B)P(C) + P(A)P(B)P(C)\\
&= P(A)P(B)(1 - P(C)) + P(A)(1 - P(B))P(C) + (1 - P(A))P(B)P(C) + P(A)P(B)P(C)\\
&= \frac{3}{4} \times \frac{4}{5}\left(1 - \frac{5}{6}\right) + \frac{3}{4}\left(1 - \frac{4}{5}\right)\frac{5}{6} + \left(1 - \frac{3}{4}\right)\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}\\
&= \frac{3}{4}.\frac{4}{5}.\frac{1}{6} + \frac{3}{4}.\frac{1}{5}.\frac{5}{6} + \frac{1}{4}.\frac{4}{5}.\frac{5}{6} + \frac{3}{4}.\frac{4}{5}.\frac{5}{6}\\
&= \frac{1}{10} + \frac{1}{8} + \frac{1}{6} + \frac{1}{2}\\
&= \frac{12 + 15 + 20 + 60}{120}\\
&= \frac{107}{120}
\end{aligned}$$

Required probability $= \frac{107}{120}$

Question 17

A bag contains 4 white balls and 2 black balls. Another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that

(i) Both are white

(ii) Both are black

(iii) One is white and one is black

Solution 17

Bag A has 4 white balls and 2 black balls;
Bag B has 3 white balls and 5 black balls.

(i) $P\left(A_W \text{ and } B_W\right) = P\left(A_W\right) P\left(B_W\right) = \frac{4}{6} \cdot \frac{3}{8} = \frac{1}{4}$

(ii) $P\left(A_B \text{ and } B_B\right) = P\left(A_B\right) P\left(B_B\right) = \frac{2}{6} \cdot \frac{5}{8} = \frac{5}{24}$

(iii) $P\left(A_W \text{ and } B_B \text{ or } A_B \text{ and } B_W\right) = P\left(A_W\right) P\left(B_B\right) + P\left(A_B\right) P\left(B_W\right)$

$$= \frac{4}{6} \cdot \frac{5}{8} + \frac{2}{6} \cdot \frac{3}{8}$$

$$= \frac{20}{48} + \frac{6}{48}$$

$$= \frac{26}{48} = \frac{13}{24}$$

Question 18

A bag contains 4 white, 7 black, and 5 red balls. 4 balls are drawn with replacement. What is the probability that at least two are white?

Solution 18

Number of white balls = 4
Number of black balls = 7
Number of red balls = 5
Total balls = 16

Number of ways in which 4 balls can be drawn from 16 balls = ${}^{16}C_4$
Let A = getting at least two white ball = getting 2, 3, 4 white balls

Number of ways of choosing 2 white balls = ${}^{4}C_2 \times {}^{12}C_2$

Number of ways of choosing 3 white balls = ${}^{4}C_3 \times {}^{12}C_1$

Number of ways of choosing 4 white balls = ${}^{4}C_4 \times {}^{12}C_0$

$$\therefore \quad P\left(A\right) = \frac{{}^{4}C_2 \times {}^{12}C_2 + {}^{4}C_3 \times {}^{12}C_1 + {}^{4}C_4 \times {}^{12}C_0}{{}^{16}C_4} = \frac{67}{256}$$

Question 19

Three cards are drawn with replacement from a well shufled pack of cards. Find the probability that the cards are a king, a queen and a jack.

Solution 19

Three cards are drawn with replacement from a pack of cards.
There are 4 Kings, 4 Queens, 5 Jacks.

P (1 King, 1 Queen, 1 Jack)

$= P\left((K \cap Q \cap J) \cup (K \cap J \cap Q) \cup (J \cap K \cap Q) \cup (J \cap Q \cap K) \cup (Q \cap K \cap L) \cup (Q \cap J \cap K)\right)$
$= P(K \cap Q \cap J) + P(K \cap J \cap Q) + P(J \cap K \cap Q) + P(J \cap Q \cap K) + P(Q \cap K \cap J) + P(Q \cap J \cap K)$
$= P(K)P(Q)P(J) + P(K)P(J)P(Q) + P(J)P(K)P(Q) + P(J)P(Q)P(K) + P(Q)P(K)P(J)$
$+ P(Q)P(J)P(K)$
$= \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52} + \frac{4}{52}.\frac{4}{52}.\frac{4}{52}$
$= \frac{6}{13.13.13}$
$= \frac{6}{2197}$

Required probability = $\frac{6}{2197}$

Question 20

A bag contains 4 red and 5 black balls, a second bag contains 3 red and 7 black balls. One ball is drawn at random from each bag; find the probability that the (i) balls are of different colours (ii) balls are of the same colour.

Solution 20

Given, Bag(1) contains 4 red and 5 black balls.

Bag (2) contains 3 red and 7 black balls

One ball is drawn at random from each bag.

(i)

P (Balls are of different colours)

$= P\left((R_1 \cap B_2) \cup (B_1 \cap R_2)\right)$

$= P(R_1 \cap B_2) + P(B_1 \cap R_2)$

$= P(R_1) P(B_2) + P(B_1) P(R_2)$

$= \frac{4}{9} \cdot \frac{7}{10} + \frac{5}{9} \cdot \frac{9}{10}$

$= \frac{28}{90} + \frac{15}{90}$

$= \frac{43}{93}$

(ii)

P (Balls are of the same colour)

$= P\left((B_1 \cap B_2) \cup (R_1 \cap R_2)\right)$

$= P(B_1 \cap B_2) + P(R_1 \cap R_2)$

$= P(B_1) P(B_2) + P(R_1) P(R_2)$

$= \frac{5}{9} \cdot \frac{7}{10} + \frac{4}{9} \cdot \frac{3}{10}$

$= \frac{35}{90} + \frac{12}{90}$

$= \frac{47}{90}$

Required probability $= \frac{47}{90}$

Question 21

A can hit a target 3 times in 6 shots, B : 2 times in 6 shots and C : 4 times in 4 shots. They fix a volley. What is the probability that at least 2 shots hit?

Solution 21

Let A be the event that "A hits the target",
B be the event that "B hits the target" and
C be the event that "C hits the target".
Then A, B and C are independent events such that
$P(A) = \frac{3}{6} = \frac{1}{2}$; $P(B) = \frac{2}{6} = \frac{1}{3}$; $P(C) = \frac{4}{4} = 1$

The target is hit by at least 2 shots in the following mutually exclusive ways :
(i) A hits, B hits and C does not hit, i.e., $A \cap B \cap C^-$
(ii) A hits, B does not hit and C hits, i.e., $A \cap B^- \cap C$
(iii) A does not hit, B hits and C hits, i.e., $A^- \cap B \cap C$
(iv) A hits, B hits and C hits, i.e., $A \cap B \cap C$
Hence, by the addition theorem for mutually exclusive events, the probability that at least 2 shots hit.

$= P(i) + P(ii) + P(iii) + P(iv)$
$= P(A \cap B \cap C^-) + P(A \cap B^- \cap C) + P(A^- \cap B \cap C) + P(A \cap B \cap C)$
$= P(A)\,P(B)\,P(C^-) + P(A)\,P(B^-)\,P(C) + P(A^-)\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$
$= P(A)\,P(B)\,[1 - P(C)] + P(A)\,[1 - P(B)]\,P(C) + [1 - P(A)]\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$
$= \frac{1}{2} \times \frac{1}{3} \times (1-1) + \frac{1}{2} \times \left(1 - \frac{1}{3}\right) \times 1 + \left(1 - \frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= \frac{1}{2} \times \frac{1}{3} \times 0 + \frac{1}{2} \times \left(\frac{2}{3}\right) \times 1 + \left(\frac{1}{2}\right) \times \frac{1}{3} \times 1 + \frac{1}{2} \times \frac{1}{3} \times 1$
$= 0 + \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6}$
$= \frac{2}{3}$

Question 22

The probability of student A passing an examination is $\frac{2}{9}$ and of student B passing is $\frac{5}{9}$. Assuming the two events: 'A passes','B passes as independent, find the probability of : (i) only A passing the examination (ii) only one of them passing the examination.

Solution 22

Given,

The probability of A passing exam $= \frac{2}{9}$

The probability of B passing exam $= \frac{5}{9}$

And they are independent.

$\Rightarrow \quad P(A) = \frac{2}{9},\ P(B) = \frac{5}{9}$

(i)

P (Only A passing the exam)

$= P\left(A \cap \overline{B}\right)$

$= P(A).P(B)$

$= P(A).\left(1 - P(B)\right)$

$= \frac{2}{9}\left(1 - \frac{5}{9}\right)$

$= \frac{2}{9}\left(\frac{4}{9}\right)$

$= \frac{8}{81}$

(ii)

P (Only one of them passing exam)

$= P\left(\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right)$

$= P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$

$= P(A)P\left(\overline{B}\right) + P\left(\overline{A}\right)P(B)$

$= P(A)\left(1 - P(B)\right) + \left(1 - P(A)\right)P(B)$

$= \frac{2}{9}\left(1 - \frac{5}{9}\right) + \left(1 - \frac{2}{9}\right)\frac{5}{9}$

$= \frac{2}{9}.\frac{4}{9} + \frac{7}{9}.\frac{5}{9}$

$= \frac{8}{81} + \frac{35}{81}$

$= \frac{43}{81}$

Required probability $= \frac{43}{81}$

Question 23

There are three urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B conatins 5 red balls and 4 black balls. Urn C contains 4 red and 4 black balls. One ball is drawn from each of these urns. What is the probability that 3 balls drawn consist of 2 red balls and a black ball?

Solution 23

Given,

Urn A contains 4 red (R_1) and 3 black (B_1) balls

Urn B contains 5 red (R_2) and 4 black (B_2) balls

Urn C contains 4 red (R_3) and 4 black (B_3) balls.

$$P\,(3 \text{ balls drawn consists of 2 red and a black ball})$$
$$= P\left[(R_1 \cap R_2 \cap R_3) \cup (R_1 \cap B_2 \cap R_3) \cup (B_1 \cap R_2 \cap R_3)\right]$$
$$= P\,(R_1 \cap R_2 \cap R_3) + P\,(R_1 \cap B_2 \cap R_3) + P\,(B_1 \cap R_2 \cap R_3)$$
$$= P\,(R_1) + P\,(R_2) + P\,(R_3) + P\,(R_1) + P\,(B_2) + P\,(R_3) + P\,(B_1) + P\,(R_2) + P\,(R_3)$$
$$= \frac{4}{7}.\frac{5}{9}.\frac{4}{8} + \frac{4}{7}.\frac{4}{9}.\frac{4}{8} + \frac{3}{7}.\frac{5}{9}.\frac{4}{8}$$
$$= \frac{80+64+60}{504}$$
$$= \frac{204}{504}$$
$$= \frac{17}{42}$$

Required probability $= \frac{17}{42}$

Question 24

X is taking up subjects – Mathematics, Physics and Chemistry in the examination. His probabilities of getting grade A in these subjects are 0.2, 0.3 and 0.5 respectively. Find the probability that he gets.

(i) Grade A in all subjects (ii) Grade A in no subject

(iii) Grade A in two subjects.

Solution 24

Given,

Probability of getting A grade in mathematics $(m) = 0.2$

$\Rightarrow \quad P(m) = 0.2$

Probability of getting A grade in physics $(p) = 0.3$

$\Rightarrow \quad P(p) = 0.3$

Probability of getting A grade in chemistry $(p) = 0.5$

$\Rightarrow \quad P(c) = 0.5$

(i)

$P(\text{Getting } A \text{ grade in all subjects})$
$= P(m \cap p \cap c)$
$= P(m) + P(p) + P(c)$
$= 0.2 \times 0.3 \times 0.5$
$= 0.03$

Required probability $= 0.03$

(ii)

$P(\text{Getting } A \text{ in no subject})$
$= P(\overline{m} \cap \overline{p} \cap \overline{c})$
$= P(\overline{m}) + P(\overline{p}) + P(\overline{c})$
$= (1 - P(m))(1 - P(p))(1 - P(c))$
$= (1 - 0.2)(1 - 0.3)(1 - 0.5)$
$= (0.8)(0.7)(0.5)$
$= 0.28$

Required probability $= 0.28$

(iii)

$P(\text{Getting } A \text{ grade in two subjects})$
$= P((m \cap p \cap \overline{c}) \cup (m \cap \overline{p} \cap c) \cup (\overline{m} \cap p \cap c))$
$= P(m)P(p)P(\overline{c}) + P(m)P(\overline{p})P(c) + P(\overline{m})P(p)P(c)$
$= P(m)P(p)(1 - P(c)) + P(m)(1 - P(p))P(c) + (1 - P(m))P(p)P(c)$
$= (0.2)(0.3)(1 - 0.5) + (0.2)(1 - 0.3)(0.5) + (1 - 0.2)(0.3)(0.5)$

$= (0.2)(0.3)(0.5) + (0.2)(0.7)(0.5) + (0.8)(0.3)(0.5)$
$= 0.03 + 0.07 + 0.12$
$= 0.22$

Required probability $= 0.22$

Question 25

A and B take turns in throwing two dice, the first to throw 9 being awarded the prize. Show that their chance of winning are in the ratio $9:8$.

Solution 25

Sum of 9 can be obtained by

$$E = \{(3,6),(4,5),(5,4),(6,3)\}$$

Probability of throwing 9 = $\frac{4}{36}$

$$P(E) = \frac{1}{9},\ P(\overline{E}) = \frac{8}{9}$$

$$\Rightarrow \quad P(A) = P(B) = \frac{1}{9}$$

$$\Rightarrow \quad P(\overline{A}) = P(\overline{B}) = \frac{8}{9}$$

A and B take turns in throwing two dice.
Let A starts the game.

$$P(A \text{ wins the game})$$
$$= P\left(A \cup \overline{A} \cap \overline{B} \cap A \cup \overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A \cup \ldots\right)$$
$$= P(A) + P\left(\overline{A} \cap \overline{B} \cap A\right) + P\left(\overline{A} \cap \overline{B} \cap \overline{A} \cap \overline{B} \cap A\right) + \ldots$$
$$= P(A) + P(\overline{A})P(\overline{B})P(A) + P(\overline{A})P(\overline{B})P(\overline{A})P(\overline{B})P(A) + \ldots$$
$$= \frac{1}{9} + \frac{8}{9}.\frac{8}{9}.\frac{1}{9} + \frac{8}{9}.\frac{8}{9}.\frac{8}{9}.\frac{8}{9}.\frac{1}{9} + \ldots$$
$$= \frac{1}{9}\left[1 + \left(\frac{8}{9}\right)^2 + \left(\frac{8}{9}\right)^4 + \ldots\right]$$
$$= \frac{1}{9}\left[\frac{1}{1 - \left(\frac{8}{9}\right)^2}\right] \qquad \left[\text{Since for a G.P. with first term 9 and common ratio} \atop S_\infty = \frac{a}{1-r}\right.$$
$$= \frac{1}{9}\left[\frac{1}{1 - \frac{64}{81}}\right]$$
$$= \frac{1}{9}\left[\frac{81}{81 - 64}\right]$$
$$= \frac{9}{17}$$

$$P(B \text{ wins the game}) = 1 - P(A \text{ wins the game})$$
$$= 1 - \frac{9}{17}$$

$= \frac{8}{17}$

Chances of winning of $A : B$

$= \frac{9}{17} : \frac{8}{17}$

$= 9 : 8$

Chances of winning $A : B = 9 : 8$

Question 26

A, B and C in order toss a coin. The one to throw a head wins. What are their respective chances of winning assuming that the game may continue indefinitely?

Solution 26

Let E be event of getting a head.

$$P(E) = \frac{1}{2} \qquad \Rightarrow P(\overline{E}) = \frac{1}{2}$$

If A starts the game,

$\Rightarrow$ A wins the game in 1st, 4th, 7th,... toss of coin.

$$P(A \text{ wins})$$
$$= P\left(E \cup \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup \ldots\right)$$
$$= P(\overline{E}) + P\left(\overline{E} \cap \overline{E} \cap \overline{E} \cap E\right) + P\left(\overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E\right) + \ldots$$
$$= P(E) + P(\overline{E})P(\overline{E})P(\overline{E})P(E) + P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E) + \ldots$$
$$= \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots$$
$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^7 + \ldots$$
$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \ldots\right]$$
$$= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{1}{2}\right)^3}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$
$$= \frac{1}{2}\left[\frac{1}{1 - \frac{1}{8}}\right]$$
$$= \frac{1}{2}\left[\frac{8}{7}\right]$$
$$= \frac{4}{7}$$

B wins in 2nd, 5th, 8th,... toss of coin

$$P(B \text{ wins})$$
$$= P\left(\overline{E} \cap E \cup \overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup \ldots\right)$$
$$= P\left(\overline{E} \cap E\right) + P\left(\overline{E} \cap \overline{E} \cap \overline{E} \cap \overline{E} \cap E \cup\right) + \ldots$$
$$= P(\overline{E})P(E) + P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E) + \ldots$$
$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \left(\frac{1}{2}\right)^2 \left[1 + \left(\frac{1}{2}\right)^3 + \ldots\right]$$

$$= \frac{1}{4}\left[\frac{1}{1-\left(\frac{1}{2}\right)^3}\right] \qquad \left[\begin{array}{l}\text{Since for G.P.} \\ S_\infty = \dfrac{a}{1-r}\end{array}\right]$$

$$= \frac{1}{4}\left[\frac{1}{1-\frac{1}{8}}\right]$$

$$= \frac{1}{4}\left[\frac{8}{7}\right]$$

$$= \frac{2}{7}$$

$$P\left(C \text{ wins}\right) = 1 - P\left(A \text{ wins}\right) - P\left(B \text{ wins}\right)$$

$$= 1 - \frac{4}{7} - \frac{2}{7}$$

$$= \frac{1}{7}$$

Probabilities of winning A, B and C are $\frac{4}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively.

Question 27

Three persons A, B, C throw a die in succession till one gets a 'six' and wins the game. Find their respective probabilities of winning.

Solution 27

Let E be the even of getting a six

$$P(E) = \frac{1}{6}$$
$$P(\overline{E}) = \frac{5}{6}$$

A wins if he gets a six in 1st or 4th or 7th... throw

A wins in first throw $= P(E) = \frac{1}{6}$

A wins in 4th throw if he fails in 1^{st}, B fails in 2^{nd}, C fails in 3^{rd} throw.

Probability of winning A in 4th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$
$$= \left(\frac{5}{6}\right)^3 . \frac{1}{6}$$

Similarly, Probability of winning A in 7th thrwo $= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$

$$= \left(\frac{5}{6}\right)^6 . \frac{1}{6}$$

Hence, probability of winning of A

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^3 . \frac{1}{6} + \left(\frac{5}{6}\right)^6 . \frac{1}{6} + \ldots$$
$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\right]$$
$$= \frac{1}{6}\left[\frac{1}{1 - \left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Using } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$
$$= \frac{1}{6}\left[\frac{1}{1 - \frac{125}{216}}\right]$$
$$= \frac{1}{6} \times \frac{216}{91}$$
$$= \frac{36}{91}$$

B wins if he gets a six in 2nd or 5th or 8th ...throw.

B wins in 2nd throw $= P\left(\bar{E}\right)P\left(E\right)$

$$= \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$$

B wins in 5th throw if A fails in first, B fails in 2nd, C fails in 3rd, A fails in 4th.

Probability of winning B in 5th throw

$$= P\left(\bar{E}\right)P\left(\bar{E}\right)P\left(\bar{E}\right)P\left(\bar{E}\right)P\left(E\right)$$

$$= \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$$

Probability of winning B in 8th thrwo

$$= \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

Hence, probability of winning B

$$= \left(\frac{5}{6}\right)\frac{1}{6} + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)$$

$$= \frac{5}{6}.\frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \ldots\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\frac{125}{216}}\right]$$

$$= \frac{5}{36}\times\left[\frac{216}{91}\right]$$

$$= \frac{30}{91}$$

Probability of winning $C = 1 - P\left(A \text{ wins}\right) - P\left(B \text{ wins}\right)$

$$= 1 - \frac{36}{91} - \frac{30}{91}$$

$$= \frac{25}{91}$$

The respective probabilities of winning of A, B and C are $\frac{36}{91}$, $\frac{30}{91}$ and $\frac{25}{91}$.

Question 28

A and B take turns in throwing two dice, the first to throw 10 being awarded the prize, show that if A has the first throw, their chance of winning are in the ratio $12:11$.

Solution 28

Let E be events of throwing 10 on a pair of dice,

$$E = \{(4,6),(5,5),(6,4)\}$$

$$P(E) = \frac{3}{37}$$

$$P(E) = \frac{1}{12}$$

$$P(\overline{E}) = \frac{11}{12}$$

A wins the game in first or 3rd or 5th throw, ...

Probability that A wins in first throw $= P(E) = \frac{1}{12}$

Probability that A wins in 3rd throw

$$= P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right)$$

Probability that A wins in 5th throw

$$= P(\overline{E})P(\overline{E})P(\overline{E})P(\overline{E})P(E)$$

$$= \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$$

Hence,

Probability of winning A

$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) + \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$$

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \dots\right]$$

$$= \frac{1}{12}\left[\frac{1}{1-\left(\frac{11}{12}\right)^2}\right] \qquad \left[\text{Since } S_\infty = \frac{a}{1-r} \text{ for G.P.}\right]$$

$$= \frac{1}{12}\left[\frac{1}{1-\frac{121}{144}}\right]$$
$$= \frac{1}{12} \times \frac{144}{23}$$
$$= \frac{12}{23}$$

Probability of winning B
$$= 1 - P\left(\text{winning } A\right)$$
$$= 1 - \frac{12}{23}$$
$$= \frac{11}{23}$$

Chances of winning A and B are $\frac{12}{23}$ and $\frac{11}{23}$ respectively
or in $12:11$.

Question 29

There are 3 red and 5 black balls in bag 'A' and 2 red and 3 black balls in bag 'B'. One ball is drawn from bag 'A' and two from bag 'B'. Find the probability that out of the 3 balls drawn one is red and 2 are black.

Solution 29

Bag A has 3 red and 5 black balls
Bag B has 2 red and 3 black balls

One ball is drawn from bag A and two from bag B.

P(One red from bag A and 2 black from bag B Or one black from bag A and 1 red and one black from bag B)
$$= P\left(R_1 \cap \left(2B_2\right)\right) + P\left(B_1 \cap R_2 \cap B_2\right)$$
$$= P\left(R_1\right) P\left(2B_2\right) + P\left(B_1\right) P\left(R_2\right) P\left(B_2\right)$$
$$= \frac{3}{8} \cdot \frac{{}^3C_2}{{}^5C_2} + \frac{5}{8} \cdot \frac{2}{5} \cdot \frac{3}{4}$$
$$= \frac{3}{8} \cdot \frac{3}{\left(\frac{5.4}{2}\right)} + \frac{5}{8} \cdot \frac{2}{5} \cdot \frac{3}{4}$$
$$= \frac{18}{160} + \frac{30}{160}$$
$$= \frac{48}{160} = \frac{3}{10}$$

Required probability $= \frac{3}{10}$

Question 30

Fatima and John appear in an interview for two vacancies in the same post. The probability of Fatima's selection is $\frac{1}{7}$ and that of John's selection is $\frac{1}{5}$. What is the probability that

(i) both of them will be selected?

(ii) only one of them will be selected?

(iii) none of them will be selected?

Solution 30

Given,

Probability of Fatima's (F) selection $= \frac{1}{7}$

$P(F) = \frac{1}{7} \quad \Rightarrow P(\bar{F}) = \frac{6}{7}$

Probability of John's (J) selection $= \frac{1}{5}$

$P(J) = \frac{1}{5} \quad \Rightarrow P(\bar{J}) = \frac{4}{5}$

(i)

P (Both of them selected)

$= P(F \cap J)$

$= P(F).P(J)$

$= \frac{1}{7} \times \frac{1}{5}$

$= \frac{1}{35}$

Required probability $= \frac{1}{35}$

(ii)

P (only one of them selected)

$= P\left(\left(F \cap \bar{J}\right) \cup \left(\bar{F} \cap J\right)\right)$

$= P(F)P(\bar{J}) + P(\bar{F})P(J)$

$= \frac{1}{7} \times \frac{4}{5} + \frac{6}{7} \times \frac{1}{5}$

$= \frac{4+6}{35}$

$= \frac{10}{35}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

(iii)

$$P(\text{None of them selected})$$
$$= P(\bar{F} \cap \bar{J})$$
$$= P(\bar{F})P(\bar{J})$$
$$= \frac{6}{7} \times \frac{4}{5}$$
$$= \frac{24}{35}$$

Required probability = $\frac{24}{35}$

Question 31

A bag contains 8 marbles of which 3 are blue and 5 are red. One marble is drawn at random, its colour is noted and the marble is replaced in the bag. A marble is again drawn from the bag and its colour is noted. Find the probability that the marble will be

(i) blue followed by red.

(ii) blue and red in any order.

(iii) of the same colour.

Solution 31

Bag contains 3 blue, 5 red marble. One marble is drawn, its colour noted and replaced, then again a marble drawn and its colour is noted.

(i)

$$
\begin{aligned}
&P(\text{Blue followed by red})\\
&= P(B \cap R)\\
&= P(B)\,P(R)\\
&= \frac{3}{8} \times \frac{5}{8}\\
&= \frac{15}{64}
\end{aligned}
$$

Required probability = $\frac{15}{64}$

(ii)

$$
\begin{aligned}
&P(\text{Blue and red in any order})\\
&= P((B \cap R) \cup (R \cap B))\\
&= P(B \cap R) + P(R \cap B)\\
&= P(B)\,P(R) + P(R)\,P(B)\\
&= \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}\\
&= \frac{30}{64}\\
&= \frac{15}{32}
\end{aligned}
$$

Required probability = $\frac{15}{32}$

(iii)

$$
\begin{aligned}
&P(\text{of the same colour})\\
&= P((R_1 \cap R_2) \cup (B_1 \cap B_2))\\
&= P(R_1)\,P(R_2) + P(B_1)\,P(B_2)\\
&= \frac{5}{8} \times \frac{5}{8} + \frac{3}{8} \times \frac{3}{8}\\
&= \frac{25+9}{64}\\
&= \frac{34}{64}\\
&= \frac{17}{32}
\end{aligned}
$$

Required probability = $\frac{17}{32}$

Question 32

An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting

(i) 2 red balls

(ii) 2blue balls

(iii) One red and one blue ball.

Solution 32

An urn contains 7 red and 4 blue balls.
Two balls are drawn with replacement.

(i)

P (Getting 2 red balls)
$= P(R_1 \cap R_2)$
$= P(R_1).P(R_2)$
$= \frac{7}{11} \times \frac{7}{11}$
$= \frac{49}{121}$

Required probability = $\frac{49}{121}$

(ii)

P (Getting 2 blue balls)
$= P(B_1 \cap B_2)$
$= P(B_1).P(B_2)$
$= \frac{4}{11} \times \frac{4}{11}$
$= \frac{16}{121}$

Required probability = $\frac{16}{121}$

(iii)

P (Getting one red and one blue ball)
$= P((R \cap B) \cup (B \cap R))$
$= P(R)P(B) + P(B)P(R)$
$= \frac{7}{11} \times \frac{4}{11} + \frac{4}{11} \times \frac{7}{11}$
$= \frac{28+28}{121}$
$= \frac{56}{121}$

Required probability = $\frac{56}{121}$

Question 33

A card is drawn from a well-shuffled deck of 52 cards. The outcome is noted, the card is replaced and the deck reshuffled. Another card is then drawn from the deck.

(i) What is the probability that both the cards are of the same suit?

(ii) What is the probability that the first card is an ace and the second card is a red queen?

Solution 33

A card is drawn, out come noted, the card is replaced, pack reshuffled, another card is drawn.

(i)

We know that, there are four suits club (C), spade (S), heart (H) diamand (D), each contains 13 cards.

$$
\begin{aligned}
& P\,(\text{Both the cards are of same suit}) \\
&= P\left(\left(C_1 \cap C_2\right) \cup \left(S_1 \cap S_2\right) \cup \left(H_1 \cap H_2\right) \cup \left(D_1 \cap D_2\right)\right) \\
&= P\left(C_1 \cap C_2\right) + P\left(S_1 \cap S_2\right) + P\left(H_1 \cap H_2\right) + P\left(D_1 \cap D_2\right) \\
&= P\left(C_1\right) P\left(C_2\right) + P\left(S_1\right) P\left(S_2\right) + P\left(H_1\right) P\left(H_2\right) + P\left(D_1\right) P\left(D_2\right) \\
&= \frac{13}{52}.\frac{13}{52} + \frac{13}{52}.\frac{13}{52} + \frac{13}{52}.\frac{13}{52} + \frac{13}{52}.\frac{13}{52} \\
&= \left(\frac{1}{4}.\frac{1}{4}\right)^4 \\
&= \frac{1}{4}
\end{aligned}
$$

Required probability = $\frac{1}{4}$

(ii)

We know that, there are four ace and 2 red queens.

$$
\begin{aligned}
& P\,(\text{first card an ace and second card a red queen}) \\
&= P\,(\text{Getting an ace})\, P\,(\text{Getting a red queen}) \\
&= \frac{4}{52} \times \frac{2}{52} \\
&= \frac{1}{338}
\end{aligned}
$$

Required probability = $\frac{1}{338}$

Question 34

Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among 100 students, what is the probability that: (i) you both enter the same section? (ii) you both enter the different section?

Solution 34

(i)

Out of 100 students two friends can enter the sections in $^{100}C_2$ ways.

Let A = event both enter in section A(40 students)

B = event both enter in section B(60 students)

$$P(A) = \frac{^{40}C_2}{^{100}C_2},\ P(B) = \frac{^{60}C_2}{^{100}C_2}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{^{40}C_2 + ^{60}C_2}{^{100}C_2}$$

$$= \frac{\frac{40 \times 39}{2} + \frac{60 \times 59}{2}}{\frac{100 \times 99}{2}}$$

$$= \frac{780 + 1770}{4950}$$

$$= \frac{2550}{4950}$$

$$= \frac{17}{33}$$

$$P(\text{Both enter same section}) = \frac{17}{33}$$

(ii)

$$P(\text{Both enter different section})$$

$$= 1 - P(\text{Both enter same section})$$

$$= 1 - \frac{17}{33}$$

$$= \frac{16}{33}$$

$$P(\text{Both enter different section}) = \frac{16}{32}$$

Question 35

In a hockey match, both teams *A* and *B* scored same number of goals upto the end of the game, so to decide the winner, the refree asked both the captains to throw a die alternately and decide that the team, whose captain gets a first six, will be declared the winner. If the captain of team *A* was asked to start, find their respective probabilities of winning the match and state whether the decision of the refree was fair or not.

Solution 35

Probability of getting six in any toss of a dice $= \frac{1}{6}$

Probability of not getting six in any toss of a dice $= \frac{5}{6}$

A and B toss the die alternatively.
Hence probability of A's win

$$= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}\bar{A}\bar{B}A) + \ldots\ldots.$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots..$$

$$= \frac{1/6}{1 - (5/6)^2} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

Similarly, probability of B's win

$$= P(\bar{A}B) + P(\bar{A}\bar{B}\bar{A}B) + P(\bar{A}\bar{B}\bar{A}\bar{B}\bar{A}B) + \ldots\ldots$$

$$= \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{5}{6} \times \frac{1}{6} + \ldots..$$

$$= \frac{\frac{5}{6} \times \frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{5}{36} \times \frac{36}{11} = \frac{5}{11}$$

Since the probabilities are not equal,
the decision of the refree was not a fair one.

Chapter 31 - Probability Exercise Ex. 31.6

Question 1

A bag A contains 5 white and 6 black balls. Another bag B contains 4 white and 3 black balls. A ball is transferred from bag A to the bag B and then a ball is taken out of the second bag. Find the probability of this ball being black.

Solution 1

Given,

Bag A contains 5 white and 6 black balls

Bag B contains 4 white and 3 black balls.

There are two ways of transferring a ball from bag A to bag B

I- By transferring one white ball from bag A to bag B then drawing one black ball from bag B.

II- By transferring one black ball from bag A to bag B, then drawing one black from bag B.

Let, E_1, E_2 and A be events as below:-

E_1 = One white ball drawn from bag A

E_2 = One black ball drawn from bag B

A = One black ball drawn from bag B

$P(E_1) = \frac{5}{11}$

$P(E_2) = \frac{6}{11}$

$P(A \mid E_1) = \frac{3}{8}$ [Since, E_1 has increased one white ball in bag B]

$P\left(\frac{A}{E_2}\right) = \frac{4}{8}$ [Since, E_2 has increased one black ball in bag B]

By the law of total probability.

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{5}{11} \times \frac{3}{8} + \frac{6}{11} \times \frac{4}{8}$$

$$= \frac{15}{88} + \frac{24}{88}$$

$$= \frac{39}{88}$$

Required probability = $\frac{39}{88}$.

Question 2

A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probabitlity that it is a silver coin?

Solution 2

Purse (I) Contains 2 silver and 4 copper coins

Purse (II) Contains 4 silver and 3 copper coins

One coin is drawn from one of the two purse and it is silver

Let, E_1, E_2 and A are defined as

E_1 = Selecting purse I

E_2 = Selecting purse II

A = Drawing a silver coin

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since, there are only 2 purses]

$P(A \mid E_1) = P(\text{A} \mid \text{silver coin from purse I})$

$= \frac{2}{6}$

$= \frac{1}{3}$

$P\left(\frac{A}{E_2}\right) = P(\text{A} \mid \text{silver coin from purse II})$

$= \frac{4}{7}$

By the law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{4}{7}$$

$$= \frac{1}{6} + \frac{4}{14}$$

$$= \frac{7+12}{42}$$

$$= \frac{19}{42}$$

Required probability = $\frac{19}{42}$.

Question 3

One bag contains 4 yellow and 5 red balls. Another bag contains 6 yellow and 3 red balls. A ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is yellow.

Solution 3

Bag I contains 4 yellow and 5 red balls
Bag II contains 6 yellow and 3 red balls
Transfer can be done in two ways:-
I- A yellow ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.
II-A red ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.
Let E_1, E_2 and A be events as:

E_1 = One yellow ball drawn from bag I
E_2 = One red ball drawn from bag I
A = One yellow ball drawn from bag II.

$$P\left(E_1\right) = \frac{4}{9}$$

$$P\left(E_2\right) = \frac{5}{9}$$

$$P\left(A \mid E_1\right) = \frac{7}{10} \qquad \left[\text{Since } E_1 \text{ has increased one yellow ball in bag II}\right]$$

$$P\left(\frac{A}{E_2}\right) = \frac{6}{10} \qquad \left[\text{Since } E_2 \text{ has increased one red ball in bag II}\right]$$

By law of total probability,

$$P\left(A\right) = P\left(E_1\right)P\left(A \mid E_1\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right)$$

$$= \frac{4}{9}\times\frac{7}{10} + \frac{5}{9}\times\frac{6}{10}$$

$$= \frac{28+30}{90}$$

$$= \frac{58}{90}$$

$$= \frac{29}{45}$$

Required probability = $\frac{29}{45}$.

Question 4

A bag contains 3 white and 2 black balls and another bag contains 2 white and 4 black balls. One bag is chosen at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is white.

Solution 4

Bag I contains 3 white and 2 black balls
Bag II contains 2 white and 4 black balls
One bag is chosen at random, then one ball is drawn and it is white.
Let E_1, E_2 and A be events as:

E_1 = Selecting bag I
E_2 = Selecting bag II
A = Drawing one white ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are only 2 bags]}$$

$$P(A \mid E_1) = P[\text{Drawing a white ball from bag I}]$$
$$= \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing a white ball from bag II}]$$
$$= \frac{2}{6}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$
$$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{6}$$
$$= \frac{3}{10} + \frac{2}{12}$$
$$= \frac{18+10}{60}$$
$$= \frac{28}{60}$$
$$= \frac{7}{15}$$

Required probability $= \frac{7}{15}$.

Question 5

The contents of three bags I, II and III are as follows:
Bag I: 1 white, 2 black and 3 red balls,
Bag II: 2 white, 1 black and 1 red, and
Bag III: 4 white, 5 black and 3 red balls.
A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

Solution 5

Given,
Bag I contains 1 white, 2 black and 3 red balls
Bag II contains 2 white, 1 black and 1 red balls
Bag III contains 4 white, 5 black and 3 red balls.
A bag is chosen at random, then one red and one white ball is drawn.
Let E_1, E_2, E_3 and A be events as:

E_1 = Selecting bag I
E_2 = Selecting bag II
E_3 = Selecting bag III
A = Drawing one red and one white ball

$$P(E_1) = \frac{1}{3}$$

$$P(E_2) = \frac{1}{3}$$

$$P(E_3) = \frac{1}{3} \qquad \text{[Since there are only three bags]}$$

$$P(A \mid E_1) = P[\text{Drawing one red and one white ball from bag I}]$$

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$= \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing one red and one white ball from bag II}]$$

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$

$$= \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$= \frac{1}{3}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing one red and one white ball from bag III}]$$

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

$$= \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$= \frac{2}{11}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$= \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}$$

$$= \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$$

$$= \frac{33 + 55 + 30}{495}$$

$$= \frac{118}{495}$$

Required probability $= \frac{118}{495}$.

Question 6

An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the sum of the numbers obtained is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8?

Solution 6

An unbiased coin is tossed, then

I:- If head occurs, pair of dice is rolled and sum on them is either 7 or 8.

II:- If tail occurs, a card is drawn from cards numbered 2,3,...,12 and is 7 or 8.

Let E_1, E_2, A be events as

E_1 = Head occurs on the coin

E_2 = Tail occurs on the coin

A = Noted number is 7 or 8

$P\left(E_1\right) = \frac{1}{2}$

$P\left(E_2\right) = \frac{1}{2}$

$P\left(A \mid E_1\right) = P\left[\text{Pair of dice shows 7 or 8 as sum}\right]$

$\left[\text{Sum on dice is 7 or 8 when } (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (2,6), (3,5), (4,4), (5,3), (6,2)\right]$

$P\left(A \mid E_1\right) = \frac{11}{36}$

$P\left(\frac{A}{E_2}\right) = P\left[\text{7 or 8 on card drawn from 11 cards numbered } 2, 3, 4, \ldots, 12\right]$

$= \frac{2}{11}$

By law of total probability,

$$P\left(A\right) = P\left(E_1\right)P\left(A \mid E_1\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{11}$$

$$= \frac{11}{72} + \frac{2}{22}$$

$$= \frac{121 + 72}{792}$$

$$= \frac{193}{792}$$

Required probability $= \frac{193}{792}$.

Question 7

A factory has two machines A and B. Past records show that the machine A produced 60% of the items of output and machine B produced 40% of the items. Further 2% of the items produced by machine A were defective and 1% produced by machine B were defective. If an item is drawn at random, what is the probability that it is defective?

Solution 7

Let E_1, E_2, A be defined as,

E_1 = Item produced by machine A

E_2 = Item produced by machine B

A = The item drawn is defective

$$P\left(E_1\right) = 60\%$$
$$= \frac{60}{100}$$
$$P\left(E_2\right) = 40\%$$
$$= \frac{40}{100}$$
$$P\left(A \mid E_1\right) = P\left[\text{Defective item from machine } A\right]$$
$$= 2\%$$
$$= \frac{2}{100}$$
$$P\left(\frac{A}{E_2}\right) = P\left[\text{Defective item from machine } B\right]$$
$$= 1\%$$
$$= \frac{1}{100}$$

By law of total probability,

$$P\left(A\right) = P\left(E_1\right)P\left(A \mid E_1\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right)$$
$$= \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1}{100}$$
$$= \frac{120 + 40}{10000}$$
$$= \frac{160}{10000}$$
$$= 0.016$$

Required probability = 0.016.

Question 8

The bag A contains 8 white and 7 black balls while the bag B contains 5 white and 4 black balls. One ball is randomly picked up from the bag A and mixed up with the balls in bag B. Then a ball is randomly drawn out from it. Find the probability that ball drawn is white.

Solution 8

Bag A contains 8 white and 7 black balls

Bag B contains 5 white and 4 black balls

Transfer can be done in two ways:-

I-A white ball is transferred from bag A to bag B and then one white ball is drawn from bag B.

II-A black ball is transferred from bag A to bag B, then one white ball is drawn from bag B.

Let E_1, E_2 and A be events as:-

E_1 = One white ball from bag A

E_2 = One black ball from bag A

A = One white ball from bag B

$$P(E_1) = \frac{8}{15}$$

$$P(E_2) = \frac{7}{15}$$

$$P(A \mid E_1) = \frac{6}{10} \qquad \left[\text{Since } E_1 \text{ has increased white balls in bag } B\right]$$

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10} \qquad \left[\text{Since } E_2 \text{ has increased black ball in bag } B\right]$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{8}{15} \times \frac{6}{10} + \frac{7}{15} \times \frac{5}{10}$$

$$= \frac{48}{150} + \frac{35}{150}$$

$$= \frac{83}{150}$$

Required probability = $\frac{83}{150}$.

Question 9

A bag contains 4 white and 5 black balls and another bag contains 3 white and 4 black balls. A ball is taken out from the first bag and without seeing its colour is put in the second bag. A ball is taken out from the later. Find the probability that the ball drawn is white.

Solution 9

There are two bags.

Bag (1) contain 4 white and 5 black balls

Bag (2) contain 3 white and 4 black balls.

A ball is taken from bag (i) and without seeing its colour is put in second bag. Then a ball is drawn from bag 2 and is white in colour.

$$P\left(\text{White ball from bag 1}\right) = \frac{4}{9}$$

$$P\left(W_1\right) = \frac{4}{9}$$

$$P\left(\text{Black ball from bag 1}\right) = \frac{5}{9}$$

$$P\left(B_1\right) = \frac{5}{9}$$

$$P\left(\text{White ball from bag 2 given } B_1 \text{ transfer}\right)$$

$$P\left(\frac{W_2}{B_1}\right) = \frac{3}{8}$$

$$P\left(\text{White from bag 2 given } W_1 \text{ transfer}\right)$$

$$P\left(\frac{W_2}{W_1}\right) = \frac{4}{8}$$

$$= \frac{1}{2}$$

$$P\left(\text{White from bag 2}\right)$$

$$= P\left(B_1\right)P\left(\frac{W_2}{B_1}\right) + P\left(W_1\right)P\left(\frac{W_2}{W_1}\right)$$

$$= \frac{5}{9} \times \frac{3}{8} + \frac{4}{9} \times \frac{1}{2}$$

$$= \frac{15}{72} + \frac{4}{18}$$

$$= \frac{31}{72}$$

$$\text{Required probability } = \frac{31}{72}$$

Question 10

One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

Solution 10

There are two bags.
Bag (1) contain 4 white and 5 black balls
Bag (2) contain 6 white and 7 black balls.

A ball is taken from bag (1) and without seeing its colour is put in bag (2). Then a ball is drawn from bag (2) and is found white in colour.

$$P(\text{1 white ball from bag 1}) = \frac{4}{9}$$
$$P(W_1) = \frac{4}{9}$$

$$P(\text{1 black ball from bag 1}) = \frac{5}{9}$$
$$P(B_1) = \frac{5}{9}$$

$$P(\text{1 white ball from bag 2 given } W_1 \text{ is put in bag 2})$$
$$P\left(\frac{W_2}{W_1}\right) = \frac{7}{14}$$
$$P\left(\frac{W_2}{W_1}\right) = \frac{1}{2}$$

$$P(\text{1 white ball from bag 2 given } B_1 \text{ is put in bag 2})$$
$$P\left(\frac{W_2}{B_1}\right) = \frac{6}{14}$$

$$P(\text{1 white from bag 2})$$
$$= P(W_1)P\left(\frac{W_2}{W_1}\right) + P(B_1)P\left(\frac{W_2}{B_1}\right)$$
$$= \frac{4}{9}\times\frac{1}{2} + \frac{5}{9}\times\frac{6}{14}$$
$$= \frac{4}{18} + \frac{30}{126}$$
$$= \frac{58}{126}$$
$$= \frac{29}{63}$$

Required probability $= \frac{29}{63}$

Question 11

An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two are drawn from first urn and put into the second urn and then a ball is drawn from the latter. Find the probability that it is a white ball.

Solution 11

Urn '1' — Urn '2'

Urn '1'	Urn '2'
10W 3B	3W 5B

Let $U1_{2W}$, $U1_{1W1B}$, $U1_{2B}$ be the events of transferring 2 white balls, 1 white & 1black ball, 2 black balls from first Urn1 to second Urn2.

$P(U1_{2W}) = {}^{10}C_2/{}^{13}C_2 = 45/78$

$P(U1_{1W1B}) = {}^{10}C_1{}^{3}C_1/{}^{13}C_2 = 10\text{x}3/78$

$P(U1_{2B}) = {}^{3}C_2/{}^{13}C_2 = 3/78$

Let $U2_W$ be the event that a white ball is drawn from the Urn 2. There are three scenarios for Urn 2 based on the events $U1_{2W}$ $U1_{1W1B}$ $U1_{2B}$

	5W	4W	3W
	5B	6B	7B
Total	10	10	10

$P(U1_{2W}U2_W) = \dfrac{{}^{5}C_1}{{}^{10}C_1} = 1/2$

$P(U1_{1W1B}U2_W) = \dfrac{{}^{4}C_1}{{}^{10}C_1} = 2/5$

$P(U1_{2B}U2_W) = \dfrac{{}^{3}C_1}{{}^{10}C_1} = 3/10$

$P(U2_W) = P(U1_{2W}\,U2_W) + P(U1_{1W1B}\,U2_W) + P(U1_{2B}U2_W)$

$= P(U1_{2W})\text{x}\,P(U1_{2W}\,U2_W) + P(U1_{1W1B})\text{x}\,P(U1_{1W1B}\,U2_W) +$

$P(U1_{2B})\text{x}\,P(U1_{2B}U2_W)$

$= \frac{45}{78}\times\frac{1}{2}+\frac{30}{78}\times\frac{2}{5}+\frac{3}{78}\times\frac{3}{10} = \frac{114}{780} = \frac{59}{130}$

Question 12

A bag contains 6 red and 8 black balls and another bag contains 8 red and 6 black balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag. Find the probability that the ball drawn is red in colur.

Solution 12

Given,

Bag (1) contains 6 red (R_1) and 8 black (B_1) balls

Bag (2) contains 8 red (R_2) and 6 black (B_2) balls

A ball is drawn from the first bag and without noticing its colour is put in the bag (2). Then a ball is drawn from second bag and it is red.

$$P\left(\text{One red ball from bag 2}\right)$$
$$= P\left(\left(B_1 \cap R_2\right) \cup \left(R_1 \cap R_2\right)\right)$$
$$= P\left(B_1 \cap R_2\right) + P\left(R_1 \cap R_2\right)$$
$$= P\left(B_1\right) P\left(\frac{R_2}{B_1}\right) + P\left(R_1\right) P\left(\frac{R_2}{R_1}\right)$$
$$= \frac{8}{14} \cdot \frac{8}{15} + \frac{6}{14} \cdot \frac{9}{15}$$
$$= \frac{64 + 54}{210}$$
$$= \frac{118}{210}$$
$$= \frac{59}{105}$$

Required probability $= \frac{59}{105}$

Question 13

There machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective.

Solution 13

Let D be the event that the picked up tube is defective.
Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3 respectively.

$$P(D) = P(A_1)P(D \mid A_1) + P(A_2)P(D \mid A_2) + P(A_3)P(D \mid A_3) \ldots.(i)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, P(A_2) = \frac{25}{100} = \frac{1}{4}, P(A_3) = \frac{25}{100} = \frac{1}{4}$$
$$P(D \mid A_1) = P(D \mid A_2) = \frac{4}{100} = \frac{1}{25}$$
$$P(D \mid A_3) = \frac{5}{100} = \frac{1}{20}$$

Putting these values in (i), we get
$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$
$$P(D) = \frac{17}{400}$$

Chapter 31 - Probability Exercise Ex. 31.7

Question 1

The contents of urns, I, II, III ar as follows:
Urn I: 1 white, 2 black and 3 red balls
Urn II: 2 white, 1 black and 1 red balls
Urn III: 4 white, 5 black and 3 red balls.
One urn is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from Urns, I, II, III?

Solution 1

Urn I contains 1 white, 2 balck and 3 red balls
Urn II contains 2 white, 1 black and 1 red balls
Urn III contains 4 white, 5 black and 3 red balls.
Consider E_1, E_2, E_2 and A be events as:-

E_1 = Selecting unr I

E_2 = Selecting urn II

E_3 = Selecting urn III

A = Drawing 1 white and 1 red balls

$$P(E_1) = \frac{1}{3}$$

$$P(E_2) = \frac{1}{3}$$

$$P(E_3) = \frac{1}{3} \qquad \text{[Since there are 3 urns]}$$

$$P(A \mid E_1) = P[\text{Drawing 1 red and 1 white from urn I}]$$

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$= \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 1 red and 1 white from urn II}]$$

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$

$$= \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$= \frac{1}{3}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing 1 red and 1 white from urn III}]$$

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

$$= \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$= \frac{2}{11}$$

We have to find,

$$P\left(\text{They come from urn I}\right) = P\left(\frac{E_1}{A}\right)$$

$$P\left(\text{They come from urn II}\right) = P\left(\frac{E_2}{A}\right)$$

$$P\left(\text{They come from urn III}\right) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right)P\left(\frac{A}{E_1}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right)+P\left(E_2\right)P\left(\frac{A}{E_2}\right)+P\left(E_3\right)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{3}+\frac{2}{11}}$$

$$= \frac{\frac{1}{5}}{\frac{36+55+30}{165}}$$

$$= \frac{1}{5}\times\frac{165}{118}$$

$$= \frac{33}{118}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{5} + \frac{1}{3} + \frac{2}{11}}$$

$$= \frac{\frac{1}{3}}{\frac{33 + 55 + 30}{165}}$$

$$= \frac{1}{3} \times \frac{165}{118}$$

$$= \frac{55}{118}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{11}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}}$$

$$= \frac{\frac{2}{11}}{\frac{1}{5} + \frac{1}{3} + \frac{2}{11}}$$

$$= \frac{2}{11} \times \frac{165}{118}$$

$$= \frac{30}{118}$$

Therefore, required probability = $\frac{33}{118}, \frac{55}{118}, \frac{30}{118}$.

Question 2

A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

Solution 2

Bag A contains 2 white and 3 red balls
Bag B contains 4 white and 5 red balls.
Consider E_1, E_2 and A events as:-

E_1 = Selecting bag A
E_2 = Selecting bag B
A = Drawing one red ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are 2 bags]}$$

$$P(A \mid E_1) = P[\text{Drawing one red ball from bag } A]$$
$$= \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing one red ball from bag } B]$$
$$= \frac{5}{9}$$

To find,

$$P(\text{Drawn, one red ball is from bag } B) = P\left(\frac{E_2}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{3}{5} + \frac{5}{9}}$$

$$= \frac{\frac{5}{9}}{\frac{27+25}{45}}$$

$$= \frac{5}{9} \times \frac{45}{52} = \frac{25}{52}$$

Required probability $= \frac{25}{52}$.

Question 3

Three urns contain 2 white and 3 black balls; 3 white and 2 black balls and 4 white and 1 black ball respectively. One ball is drawn from an urn chosen at random and it was found to be white. Find the probability that it was drawn from the first urn.

Solution 3

Urn I contains 2 white and 3 black balls
Urn II contains 3 white and 2 black balls
Urn III contains 4 white and 1 black balls
Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting urn I
E_2 = Selecting urn II
E_3 = Selecting urn III
A = A white balls is drawn

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$

$P(E_3) = \frac{1}{2}$ [Since there are 3 urns]

$P(A \mid E_1) = P[\text{Drawing 1 white ball from urn I}]$

$= \frac{2}{5}$

$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 1 white ball from urn II}]$

$= \frac{3}{5}$

$P\left(\frac{A}{E_3}\right) = P[\text{Drawing one whtie ball from urn III}]$

$= \frac{4}{5}$

To find,

$P(\text{Drawn one white ball from urn I}) = P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{5}}{\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{4}{5}}$$

$$= \frac{\frac{2}{10}}{\frac{2+3+4}{10}}$$

$$= \frac{2}{9}$$

Required probability $= \frac{2}{9}$.

Question 4

The contents of three urns are as follows:
Urn 1: 7 white, 3 black balls,
Urn 2: 4 white, 6 black balls, and
Urn 3: 2 white, 8 black balls.
One of these urns is chosen at random with probabilities 0.20, 0.60 and 0.20 respectively. From the chosen urn two balls are drawn at random without replacment. If both these balls are white, what is the probability that these came from urn 3?

Solution 4

Urn I contains 7 white and 3 black balls

Urn II contains 4 white and 6 black balls

Urn III contains 2 white and 8 black balls

Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting urn I

E_2 = Selecting urn II

E_3 = Selecting urn III

A = Drawing 2 white balls without replacement.

Given,

$$P(E_1) = 0.20$$

$$P(E_2) = 0.60$$

$$P(E_3) = 0.20$$

$$P(A \mid E_1) = P[\text{Drawing 2 white ball from urn I}]$$

$$= \frac{{}^7C_2}{{}^{10}C_2}$$

$$= \frac{\frac{7 \times 6}{2}}{\frac{10 \times 9}{2}}$$

$$= \frac{7}{15}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing 2 white ball from urn II}]$$

$$= \frac{{}^4C_2}{{}^{10}C_2}$$

$$= \frac{\frac{4 \times 3}{2}}{\frac{10 \times 9}{2}}$$

$$= \frac{12}{90}$$

$$= \frac{2}{15}$$

$$P\left(\frac{A}{E_3}\right) = P[\text{Drawing 2 whtie ball from urn III}]$$

$$= \frac{^2C_2}{^{10}C_2}$$

$$= \frac{1}{\frac{10\times 9}{2}}$$

$$= \frac{1}{45}$$

To find,

$$P\left(\text{2 white balls drawn are from urn III}\right) = P\left(\frac{E_3}{A}\right)$$

By baye's theorem,

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.2\times\frac{1}{45}}{0.2\times\frac{7}{15}+0.6\times\frac{2}{15}+0.2\times\frac{1}{45}}$$

$$= \frac{\frac{2}{450}}{\frac{14}{150}+\frac{12}{150}+\frac{2}{450}}$$

$$= \frac{\frac{2}{450}}{\frac{42+36+2}{450}}$$

$$= \frac{2}{80}$$

$$= \frac{1}{40}$$

Required probability $= \frac{1}{40}$.

Question 5

Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3,4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die?

Solution 5

Consider the following events:

E_1 = Getting 1 or 2 in a throw of die,

E_2 = Getting 3, 4, 5 or 6 in a throw of die,

A = Getting exactly one tail

Clearly,

$$P(E_1) = \frac{2}{6} = \frac{1}{3}, P(E_2) = \frac{4}{6} = \frac{2}{3}, P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2}$$

Required probablity $= P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{8}{11}$$

Question 6

Two groups are competing for the positions of the Board of Directors of a Corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Solution 6

Consider the following events:

E_1 = First group wins, E_2 = Second group wins, A = New product is introduced.

It is given that

$$P(E_1) = 0.6, P(E_2) = 0.4, P(A/E_1) = 0.7, P(A/E_2) = 0.3$$

Required probability $= P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$

$$= \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3} = \frac{12}{54} = \frac{2}{9}$$

Hence required probability is $\frac{2}{9}$

Question 7

Suppose 5 men out of 100 and 25 women out of 1000 are good orators. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women.

Solution 7

Given, 5 man out of 100 and 25 women out of 1000 are good orators.

Consider E_1, E_2 and A events as:-

E_1 = Selected person is male

E_2 = Selected perosn is female

E_3 = Selected person is an orator

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$ [Since number of males and females are equal]

$P(A \mid E_1) = P$ (Selecting a male orator)

$= \frac{5}{100}$

$= \frac{1}{20}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a frmale orator)

$= \frac{25}{1000}$

$= \frac{1}{40}$

To find, P (Orator selected is a male) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{20}}{\frac{1}{2}\times\frac{1}{20}+\frac{1}{2}\times\frac{1}{40}}$$

$$= \frac{\frac{1}{40}}{\frac{1}{40}+\frac{1}{80}}$$

$$= \frac{1}{40}\times\frac{80}{3}$$

$$= \frac{2}{3}$$

Required probability $= \frac{2}{3}$.

Question 8

A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON?

Solution 8

Consider events E_1, E_2 and A events as:-

E_1 = Letters come from LONDON

E_2 = Letters come from CLIFTON

E_3 = Two consecutive letters visible on the envelope are ON

$P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$

[Since letters came either from LONDON or CLIFTON]

$P(A \mid E_1) = P$ (Two consecutive letters ON from LONDON)

$= \frac{2}{5}$

[Since LONDON has 2-ON and 5 letters we consider one 'ON' as one letter]

$P\left(\frac{A}{E_2}\right) = P$ (Two consecutive letters ON from CLIFTON)

$= \frac{1}{6}$

[Since CLIFTON has one 'ON' nad 6 letters considering ON as one letter]

(i) To find, P (ON visible are from LONDON) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$

$$= \frac{\frac{2}{10}}{\frac{2}{10}+\frac{1}{12}}$$

$$= \frac{2}{10}\times\frac{60}{17}$$

$$= \frac{12}{17}$$

$$P\left(\frac{E_1}{A}\right) = \frac{12}{17}$$

Required probability $= \frac{12}{17}$

(ii) $P\left(\frac{E_2}{A}\right) = \dfrac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$

$$= \frac{\frac{1}{2}\times\frac{1}{6}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$

$$= \frac{\frac{1}{12}}{\frac{2}{10}+\frac{1}{12}}$$

$$= \frac{1}{12}\times\frac{60}{17}$$

$$= \frac{5}{17}$$

Required probability = $\frac{5}{17}$.

Question 9

In a class, 5% of the boys and 10% of the girls have an IQ of more than 50. In this class, 60% of the students are boys. If a student is selected at random and found to have an IQ of more than 150, find the probability that the student is a boy.

Solution 9

Consider E_1, E_2 and A events as:-

E_1 = Selected student is boy

E_2 = Selected student is girl

E_3 = A student with IQ more that 150 is selected

$P\left(E_1\right) = \frac{60}{100}$

$P\left(E_2\right) = \frac{40}{100}$

$P\left(A \mid E_1\right) = P$ (Selected boy has IQ more than 150)

$= \frac{5}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selected girl has IQ more than 150)

$= \frac{10}{100}$

To find, P (Selected student with IQ more than 150 is a boy) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right) P\left(\frac{A}{E_1}\right)}{P\left(E_1\right) P\left(\frac{A}{E_1}\right) + P\left(E_2\right) P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{60}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{5}{100} + \frac{40}{100} \times \frac{10}{100}}$$

$$= \frac{300}{300 + 400}$$

$$= \frac{300}{700}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

Question 10

A factory has three machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% and Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found to be defective. What is the probability that this defective bolt has been produced by machine?

Solution 10

Consider E_1, E_2, E_3 and A as:-

E_1 = Bolt produced by machine X

E_2 = Bolt produced by machine Y

E_3 = Bolt produced by machine Z

A = A bolt drawn is defective.

$P(E_1) = \frac{1000}{6000} = \frac{1}{6}$

$P(E_2) = \frac{2000}{6000} = \frac{1}{3}$

$P(E_3) = \frac{3000}{6000} = \frac{1}{2}$

$P(A \mid E_1) = P$ (Drawing defective bolt from machine X)

$= \frac{1}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Drawing defective bolt from machine Y)

$= \frac{1.5}{100}$

$= \frac{3}{200}$

$P\left(\frac{A}{E_3}\right) = P$ (Drawing defective bolt from machine Z)

$= \frac{2}{100}$

To find, P (Defective bolt drawn is produced by machine X) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{200} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{\frac{1}{600}}{\frac{1}{600} + \frac{3}{600} + \frac{1}{100}}$$

$$= \frac{1}{10}$$

Required probability $= \frac{1}{10}$.

Question 11

An insurance company issued 3000 scooters, 4000 cars and 5000 trucks. The probabilities of the accident involving a scooter, a car and a truck are 0.02, 0.03 and 0.04 respectively. One of the insured vehicles meet with an accident. Find the probability that it is a (i) scooter (ii) car (iii) truck.

Solution 11

Let E_1, E_2, E_3 and A be the events defined as follows

$E_1 =$ scooters

$E_2 =$ cars

$E_3 =$ trucks

$A =$ vehicle meet with an accident

Since there are 12000 vehicles, therefore

$$P(E_1)=\frac{3000}{12000}=\frac{1}{4}, P(E_2)=\frac{4000}{12000}=\frac{1}{3}, P(E_3)=\frac{5000}{12000}=\frac{5}{12}$$

It is given that $P(A/E_1)$=Probability that the accident involves a scooter $=0.02$

Similarly $P(A/E_2)=0.03$ and $P(A/E_3)=0.04$

(i)

We are required to find $P(E_1/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} \\ &= \frac{\frac{1}{4}\times 0.02}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04} \\ &= \frac{3}{19} \end{aligned}$$

(ii)

We are required to find $P(E_2/A)$ i.e. given that the vehicle meet with an accident is a car

By Baye's rule

$$\begin{aligned} P(E_2/A) &= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} \\ &= \frac{\frac{1}{3}\times 0.03}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04} \\ &= \frac{6}{19} \end{aligned}$$

(iii)

We are required to find $P(E_3/A)$ i.e. given that the vehicle meet with an accident is a scooter

By Baye's rule

$$\begin{aligned} P(E_3/A) &= \frac{P(E_3)P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} \\ &= \frac{\frac{5}{12}\times 0.04}{\frac{1}{4}\times 0.02+\frac{1}{3}\times 0.03+\frac{5}{12}\times 0.04} \\ &= \frac{10}{19} \end{aligned}$$

Question 12

Suppose we have four boxes A, B, C and D containing coloured marbles as given below:

Box	Colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it.
If the marble is red, what is the probability that it was drawn from that box A? Box B? Box C?

Solution 12

We need to find

$$P\left(\frac{A}{\text{Red}}\right),\ P\left(\frac{B}{\text{Red}}\right),\ P\left(\frac{C}{\text{Red}}\right)$$

Now,

$$P\left(\frac{A}{\text{Red}}\right)=\frac{P\left(\frac{\text{Red}}{\text{A}}\right)P(A)}{P\left(\frac{\text{Red}}{A}\right)P(A)+P\left(\frac{\text{Red}}{B}\right)P(B)+P\left(\frac{\text{Red}}{C}\right)P(C)+P\left(\frac{\text{Red}}{D}\right)P(D)}$$

$$=\frac{\frac{1}{10}\times\frac{1}{4}}{\frac{1}{10}\times\frac{1}{4}+\frac{6}{10}\times\frac{1}{4}+\frac{8}{10}\times\frac{1}{4}+0}$$

$$=\frac{1}{1+6+8}=\frac{1}{15}$$

Similarly

$$P\left(\frac{B}{\text{Red}}\right)=\frac{6}{15}$$

$$P\left(\frac{C}{\text{Red}}\right)=\frac{8}{15}$$

Question 13

A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that was produced by A?

Solution 13

Let E_1, E_2, and E_3 be the respective events of the time consumed by machines A, B, and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$
$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$
$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X|E_1) = 1\% = \frac{1}{100}$$
$$P(X|E_2) = 5\% = \frac{5}{100}$$
$$P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by P $(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|X) = \frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1) + P(E_2)\cdot P(X|E_2) + P(E_3)\cdot P(X|E_3)}$$
$$= \frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{2}\cdot\frac{1}{100} + \frac{3}{10}\cdot\frac{5}{100} + \frac{1}{5}\cdot\frac{7}{100}}$$
$$= \frac{\frac{1}{100}\cdot\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2} + \frac{3}{2} + \frac{7}{5}\right)}$$
$$= \frac{\frac{1}{2}}{\frac{17}{5}}$$
$$= \frac{5}{34}$$

Question 14

An item is manufactured by three machine A, B and C. out of the total number of items manufactured during a specified period, 50% are manufacture on machine A 30% on B and 20% on C. 2% of the items produced on A and 2% of items produced on B are defective and 3% of these produced on C are defective.

All the items stored at one godown. One items is drawn at random and is found to be defective. What is the probability that it was manufactured on machine A?

Solution 14

Consider the following events:

E_1 = Item is produced by machine A,

E_2 = Item is produced by machine B,

E_3 = Item is produced by machine C,

A = Item is defective

Clearly,

$$P(E_1) = \frac{50}{100} = \frac{1}{2}, P(E_2) = \frac{30}{100} = \frac{3}{10}, P(E_2) = \frac{20}{100} = \frac{1}{5}$$

$$P(A/E_1) = \frac{2}{100}, P(A/E_2) = \frac{2}{100}, P(A/E_2) = \frac{3}{100}$$

Re quired probablity $= P(E_1/A)$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{100}}{\frac{1}{2} \times \frac{2}{100} + \frac{3}{10} \times \frac{2}{100} + \frac{1}{5} \times \frac{3}{100}}$$

$$= \frac{5}{11}$$

Question 15

There are three coins. One is two-headed coin (having head on both faces), another is biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tail 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

Solution 15

Let E_1, E_2, E_3 be the events that we choose the first coin, second coin, and third coin respectively in a random toss.

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$$

Let A denote the event when the toss shows heads.

It is given that

$$P(A/E_1) = 1, P(A/E_2) = 0.75, P(A/E_3) = .60$$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} =$$

$$= \frac{\frac{1}{3}(1)}{\frac{1}{3}(1) + \frac{1}{3}(0.75) + \frac{1}{3}(0.60)} = \frac{1/3}{(1/3) + (1/4) + (1/5)}$$

$$= \frac{1/3}{47/60} = \frac{20}{47}$$

Question 16

In a factory, machine A produces 30% of the total output, machine B produces 25% and the machine C produces the remaining output. If defective items produced by machines A, B and C are 1%, 1.2% respectively. Three machines working output and found to be defective. Find the probability that it was produced by machine B ?

Solution 16

Consider events E_1, E_2, E_3 and A as:-

E_1 = Selecting product from machine A

E_2 = Selecting product from machine B

E_3 = Selecting product from machine C

A = Selecting a standard quality product

$$P(E_1) = \frac{30}{100}$$

$$P(E_2) = \frac{25}{100}$$

$$P(E_3) = \frac{45}{100}$$

$$P(A \mid E_1) = P(\text{Selecting defective product from machine } A)$$
$$= \frac{1}{100}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Selecting defective product from machine } B)$$
$$= \frac{1.2}{100}$$

$$P\left(\frac{A}{E_3}\right) = P(\text{Selecting defective product from machine } C)$$
$$= \frac{2}{100}$$

To find, $P(\text{Selecting defective product is produced by machine } B)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{12}{1000}}{\frac{30}{100} \times \frac{1}{100} + \frac{25}{100} \times \frac{12}{1000} + \frac{45}{100} \times \frac{2}{100}}$$

$$= \frac{300}{300 + 300 + 900}$$

$$= \frac{300}{1500}$$

$$= \frac{1}{5}$$

Required probability $= \frac{1}{5}$.

Question 17

A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant 40%. Out of that 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

Solution 17

Let E_1, E_2 and A be events as:-

E_1 = Selecting bicycle from first plant

E_2 = Selecting bicycle from second plant

A = Selecting a standard quality bicycle

$P\left(E_1\right) = \frac{60}{100}$

$P\left(E_2\right) = \frac{40}{100}$

$P\left(A \mid E_1\right) = P$ (Selecting standard quality bicycle from first plant)

$= \frac{80}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting standard quality bicycle from second plant)

$= \frac{90}{100}$

To find, P (Selected standard quality bicycle is from second plant) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P\left(E_2\right)P\left(\frac{A}{E_2}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{40}{100}\times\frac{90}{100}}{\frac{60}{100}\times\frac{80}{100}+\frac{40}{100}\times\frac{90}{100}}$$

$$= \frac{3600}{4800+3600}$$

$$= \frac{3600}{8400}$$

$$= \frac{3}{7}$$

Required probability $= \frac{3}{7}$.

Question 18

Three urns A, B and C contain 6 red and 4 white; 2red and 6 white; and 1 red and 5 white balls redpectively. An urn is chosen at random and a ball is drawn. If the ball drawn is found to be red, find the probability that the ball was drawn from urn A.

Solution 18

Urn A contains 6 red and 4 white balls
Urn B contains 2 red and 6 white balls
Urn C contains 1 red and 5 white balls
Consider E_1, E_2, E_3 and A events as:-

E_1 = Selecting urn A
E_2 = Selecting urn B
E_3 = Selecting urn C
A = Selecting a red ball

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$ [Since there are three urns]

$P(A \mid E_1) = P$ (Selecting a red ball from urn A)

$= \frac{6}{10}$

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a red ball from urn B)

$= \frac{2}{8}$

$= \frac{1}{4}$

$P\left(\frac{A}{E_3}\right) = P$ (Selecting a red ball from urn C)

$= \frac{1}{6}$

To find, P (Selected red ball is from urn A) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{6}}$$

$$= \frac{\frac{3}{5}}{\frac{3}{5}+\frac{1}{4}+\frac{1}{6}}$$

$$= \frac{\frac{3}{5}}{\frac{36+15+10}{60}}$$

$$= \frac{3}{5}\times\frac{60}{61}$$

$$= \frac{36}{61}$$

Required probability = $\frac{36}{61}$.

Question 19

In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian?

Solution 19

Let E_1, E_2, E_3 be the events that the people are smokers and non−vegetarian, smokers and vegetarian, and non−smokers and vegetarian respectively.

$P(E_1) = \frac{2}{5}, P(E_2) = \frac{1}{4}, P(E_3) = \frac{7}{20}$

Let A denote the event that the person has the special chest disease.

It is given that

$P(A/E_1) = 0.35, P(A/E_2) = 0.20, P(A/E_3) = 0.10$

We have to find $P(E_1/A)$.

By Baye's theorem

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} =$$

$$= \frac{\frac{2}{5}(0.35)}{\frac{2}{5}(0.35)+\frac{1}{4}(0.20)+\frac{7}{20}(0.10)} = \frac{7/50}{(7/50)+(1/20)+(7/200)}$$

$$= \frac{7/50}{9/40} = \frac{28}{45}$$

Question 20

A factory has three machines A, B and C, which produce 100,200 and 300 items of a particular type daily. The machines produce 2%, 3% and 5% defective items respectively. One day when the production was over, an item was picked up randomly and it was found to be defective. Find the probability that it was produced by machine A.

Solution 20

Let E_1, E_2, E_3 and A be events as:-

E_1 = Selecting product from machine A

E_2 = Selecting product from machine B

E_3 = Selecting product from machine C

A = Selecting a defective product

$P(E_1) = \frac{100}{600} = \frac{1}{6}$

$P(E_2) = \frac{200}{600} = \frac{1}{3}$

$P(E_3) = \frac{300}{600} = \frac{1}{2}$

$P(A \mid E_1) = P$ (Selecting a defective item from machine A)

$= \frac{2}{100}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting a defective item from machine B)

$= \frac{3}{100}$

$P\left(\frac{A}{E_3}\right) = P$ (Selecting a defective item machine C)

$= \frac{5}{100}$

To find, P (Selected defective item is produced by machine A) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{100}}{\frac{1}{6} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{5}{100}}$$

$$= \frac{\frac{2}{600}}{\frac{2}{600} + \frac{3}{300} + \frac{5}{200}}$$

$$= \frac{2}{600} \times \frac{600}{23}$$

$$= \frac{2}{23}$$

Required probability $= \frac{2}{23}$.

Question 21

A bag contains 1 white and 6 red balls, and a second bag contains 4 white and 3 red balls. One of the bags is picked up at random and a ball is randomly drawn from it, and is found to be white in colour. Find the probability that the drawn ball was from the first bag.

Solution 21

Bag I contains 1 white and 6 red balls

Bag II contains 4 white and 3 red balls

Let E_1, E_2 and A events be:-

E_1 = Selecting bag I

E_2 = Selecting bag II

A = Selecting a white ball

$P\left(E_1\right) = \frac{1}{2}$

$P\left(E_2\right) = \frac{1}{2}$ [Since there are two bags]

$P\left(A \mid E_1\right) = P$ (Selecting 1 white ball from bag I)

$= \frac{1}{7}$

$P\left(\frac{A}{E_2}\right) = P$ (Selecting 1 white ball from bag II)

$= \frac{4}{7}$

To find, P (Drawn white ball is from bag I) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right)P\left(\frac{A}{E_1}\right)}{P\left(E_1\right)P\left(\frac{A}{E_1}\right) + P\left(E_2\right)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{7}}{\frac{1}{2}\times\frac{1}{7} + \frac{1}{2}\times\frac{4}{7}}$$

$$= \frac{\frac{1}{14}}{\frac{1}{14} + \frac{4}{14}}$$

$$= \frac{1}{5}$$

Required probability $= \frac{1}{5}$.

Question 22

In a certain college, 4% of boys and 1% of girls are taller than 1.75 meters. Further more, 60% of the students in the college are girls. A student selected at random from the college is found to be taller than 1.75 meters. Find the probability that the selected student is girl.

Solution 22

Consider the following events

E_1 = The selected student is a girl

E_2 = The selected student is not a girl

A = The student is taller than 1.75 meters

We have,

$$P(E_1) = 60\% = \frac{60}{100} = 0.6$$

$$P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$$

$P(A/E_1)$ = Probability that the student is taller than 1.75 meters given that the student is a girl

$$P(A/E_1) = \frac{1}{100} = 0.01$$

And

$P(A/E_2)$ = Probability that the student is taller than 1.75 meters given that the student is not a girl

$$P(A/E_2) = \frac{4}{100} = 0.04$$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.6 \times 0.01}{0.6 \times 0.01 + 0.4 \times 0.04}$$

$$= \frac{\frac{6}{1000}}{\frac{22}{1000}}$$

$$= \frac{3}{11}$$

Question 23

For A, B and C the chances of being selected as the manager of a firm are in the ratio $4:1:2$ respectively. The respective probabilities for them to introduce a radical change in marketing startegy are 0.3, 0.8 and 0.5. If the change does take place, find the probability that it is due to the appointment of B or C.

Solution 23

Let E_1, E_2, E_3 and A be events as:-

E_1 = A is appointed

E_2 = B is appointed

E_3 = C is appointed

A = A change does take place

$P(E_1) = \frac{4}{7}$

$P(E_2) = \frac{1}{7}$

$P(E_3) = \frac{2}{7}$

$P(A \mid E_1) = P$ (Changes take place by A)

$= 0.3$

$P\left(\frac{A}{E_2}\right) = P$ (Changes take place by B)

$= 0.8$

$P\left(\frac{A}{E_3}\right) = P$ (Changes take place by C)

$= 0.5$

To find, P (Changes were taken place by B or C) $= P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{7}\times\frac{8}{10} + \frac{2}{7}\times\frac{5}{10}}{\frac{4}{7}\times\frac{3}{10} + \frac{1}{7}\times\frac{8}{10} + \frac{2}{7}\times\frac{5}{10}}$$

$$= \frac{\frac{18}{70}}{\frac{30}{70}}$$

$$= \frac{18}{30}$$

$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$.

Question 24

An insurance company insured 2000 scooters and 300 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motorcycle is 0.02. An insured vehicle met whith an accident. Find the probability that the accidented vehicle was a motorcycle.

Solution 24

Let E_1, E_2 and A be events as:-

E_1 = Vehicle is scooter

E_2 = Vehicle is motorcycle

A = An insured met with accident

$P(E_1) = \frac{2000}{5000} = \frac{2}{5}$

$P(E_2) = \frac{3000}{5000} = \frac{3}{5}$

$P(A \mid E_1) = P$ (Accident of scooter)

$= 0.01$

$P\left(\frac{A}{E_2}\right) = P$ (Accident of motorcycle)

$= 0.02$

To find, P (Accident vehicle was motorcycle) $= P\left(\frac{E_2}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5} \times \frac{2}{100}}{\frac{2}{5} \times \frac{1}{100} + \frac{3}{5} \times \frac{2}{100}}$$

$$= \frac{\frac{6}{500}}{\frac{2}{500} + \frac{6}{500}}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

Required probability $= \frac{3}{4}$.

Question 25

Of the students in a college, it is known that 60% reside in a hostel and 40% do not reside in hostel. Previous year results report that 30% of students residing in hostel attain A grade and 20% of ones not residing in hostel attain A grade in their annual examination. At the end of the year, one students is chosen at random from the college and he has an A grade. What is the probability that the selected student is a hosteller?

Solution 25

Consider the following events

E_1 = The selected student is a hosteller

E_2 = The selected student is not a hosteller.

A = The student has an A grade.

We have,

$$P(E_1) = 30\% = \frac{30}{100} = 0.3$$

$$P(E_2) = 20\% = \frac{20}{100} = 0.2$$

$P(A/E_1)$ = Probability that the student has an A grade given that the student is a hosteller

$$P(A/E_1) = \frac{60}{100} = 0.6$$

And

$P(A/E_2)$ = Probability that the student has an A grade given that the student is not a hosteller

$$P(A/E_2) = \frac{40}{100} = 0.4$$

Now,

Required probability

$$= P(E_1/A)$$

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{0.3 \times 0.6}{0.3 \times 0.6 + 0.2 \times 0.4}$$

$$= \frac{\frac{18}{100}}{\frac{26}{100}}$$

$$= \frac{9}{13}$$

Question 26

There are three coins. One is two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Solution 26

Let E_1, E_2, and E_3 be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$$\therefore P(A|E_1) = P(\text{coin showing heads, given that it is a two-headed coin}) = 1$$

Probability of heads coming up, given that it is a biased coin= 75%

$$\therefore P(A|E_2) = P(\text{coin showing heads, given that it is a biased coin}) = \frac{75}{100} = \frac{3}{4}$$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$$\therefore P(A|E_3) = P(\text{coin showing heads, given that it is an unbiased coin}) = \frac{1}{2}$$

The probability that the coin is two-headed, given that it shows heads, is given by

$P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)}$$

$$= \frac{1}{\frac{9}{4}}$$

$$= \frac{4}{9}$$

Question 27

Assume that the chances of the patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Solution 27

Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$$\therefore P(A) = 0.40$$

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = 0.40 \times 0.70 = 0.28$$

$$P(A|E_2) = 0.40 \times 0.75 = 0.30$$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P $(E_1|A)$.

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30}$$

$$= \frac{14}{29}$$

Question 28

Coloured balls are distributed in four boxes as shown in the following table:

Box	Colour			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III.

Solution 28

We need to find

$$P\left(\frac{\text{Box III}}{\text{Black}}\right)$$

$$= \frac{P\left(\frac{\text{Black}}{\text{Box III}}\right)P(\text{Box III})}{P\left(\frac{\text{Black}}{\text{Box III}}\right)P(\text{Box III})+P\left(\frac{\text{Black}}{\text{Box II}}\right)P(\text{Box II})+P\left(\frac{\text{Black}}{\text{Box I}}\right).P(\text{Box I})+P\left(\frac{\text{Black}}{\text{Box Iv}}\right).P(\text{Box Iv})}$$

$$= \frac{\frac{1}{7}\times\frac{1}{4}}{\frac{1}{7}\times\frac{1}{4}+\frac{2}{8}\times\frac{1}{4}+\frac{3}{18}\times\frac{1}{4}+\frac{4}{13}\times\frac{1}{4}}$$

$$= \frac{\frac{1}{7}}{\frac{1}{7}+\frac{1}{4}+\frac{1}{6}+\frac{4}{13}}$$

$$= \frac{1}{7}\times\frac{7\times4\times6\times13}{4\times6\times13+7\times6\times13+7\times4\times13+7\times4\times6}$$

$$= \frac{4\times6\times13}{4\times6\times13+7\times6\times13+7\times4\times13+7\times4\times6}$$

$= 0.165$

Question 29

If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.

Solution 29

Let A be the event that the machine produces 2 acceptable items.

Also let B_1 be the event of correct set up and B_2 represent the event of incorrect set up.

Now, $P(B_1) = 0.8$, $P(B_2) = 0.2$

$$P(A/B_1) = 0.9 \times 0.9 \quad \text{and} \quad P(A/B_2) = 0.4 \times 0.4$$

$$\text{Therefore, } P(B_1/A) = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{0.8 \times 0.9 \times 0.9}{0.8 \times 0.9 \times 0.9 + 0.2 \times 0.4 \times 0.4} = \frac{648}{680} = 0.95$$

Question 30

By examining the chest X-ray, probability that T.B is detected when a person is actually suffering is 0.99. The probability that the doctor diagnoses incorrectly that a person has T.B. on the basis of X-ray is 0.001. In a certain city 1 in 100 persons suffers from T.B. A person is selected at random is diagnosed to have T.B. What is the chance that he actually has T.B.?

Solution 30

Consider events E_1, E_2 and A as

E_1 = The person selected is actually having T.B.

E_2 = The person selected is not having T.B.

E_3 = The person diagnosed to have T.B.

Given,

$$P(E_1) = \frac{1}{1000}$$

$$P(E_2) = \frac{999}{1000}$$

$P(A \mid E_1) = P$ (Person diagnosed to have T.B. and he is actually having T.B.)

$= 0.99$

$P\left(\frac{A}{E_2}\right) = P$ (Person diagnosed to have T.B. and he is not a actually having T.B.)

$= 0.001$

To find, P (Person diagnosed to have T.B. is actually having T.B.) $= P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{1000} \times 0.99}{\frac{1}{1000} \times 0.99 + \frac{999}{1000} \times 0.001}$$

$$= \frac{990}{990 + 999}$$

$$= \frac{990}{1989}$$

$$= \frac{110}{221}$$

Required probability $= \frac{110}{221}$.

Question 31

A test for detection of a particular disease is not fool proof. The test will correctly detect the disease 90% of the time, but will incorrectly detect the disease 1% of the time. For a large population of which an estimated 0.2% have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease?

Solution 31

Consider events E_1, E_2 and A as:-

E_1 = The selected person actually has disease

E_2 = The selected person has no disease

A = Selected person has disease

$$P(E_1) = \frac{0.2}{100}$$
$$= \frac{2}{1000}$$
$$P(E_2) = \frac{998}{1000}$$
$$P(A \mid E_1) = \frac{90}{100}$$
$$P\left(\frac{A}{E_2}\right) = \frac{1}{100}$$

To find, P (Person has disease is actually diseased) = $P\left(\frac{E_1}{A}\right)$.

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$
$$= \frac{\frac{2}{1000} \times \frac{90}{100}}{\frac{2}{1000} \times \frac{90}{100} + \frac{998}{1000} \times \frac{1}{100}}$$
$$= \frac{180}{180 + 998}$$
$$= \frac{180}{1178}$$
$$= \frac{90}{589}$$

Required probability = $\frac{90}{589}$.

Question 32

Let d_1, d_2, d_3 be three mutually exclusive diseases. Let S be the set of observable symptoms of these diseases. A doctor has the following information from a random sample of 5000 patients:

1800 had disease d_1, 2100 has disease d_2 and the others had disease d_3.

1500 patients with disease d_1, 1200 patients with disease d_2 and 900 patients with disease d_3 showed the symptom.

Which of the diseases is the patient most likely to have?

Solution 32

Let E_1, E_2, E_3 and A be events as:-

E_1 = Patient has disease d_1

E_2 = Patient has disease d_2

E_3 = Patient has disease D_3

A = Selected patient has symptom S.

$$P\left(E_1\right) = \frac{1800}{5000} = \frac{18}{50}$$

$$P\left(E_2\right) = \frac{2100}{5000} = \frac{21}{50}$$

$$P\left(E_3\right) = \frac{1100}{5000} = \frac{11}{50}$$

$P\left(A \mid E_1\right) = P\left(\text{Patient with disease } d_1 \text{ and shows symptom } S\right)$

$$= \frac{1500}{1800}$$

$$= \frac{5}{6}$$

$P\left(\frac{A}{E_2}\right) = P\left(\text{Patient with disease } d_2 \text{ and symptom } S\right)$

$$= \frac{1200}{2100}$$

$$= \frac{4}{7}$$

$P\left(\frac{A}{E_3}\right) = P\left(\text{Patient with disease } d_3 \text{ and symptom } S\right)$

$$= \frac{900}{1100}$$

$$= \frac{9}{11}$$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right) P\left(\frac{A}{E_1}\right)}{P\left(E_1\right) P\left(\frac{A}{E_1}\right) + P\left(E_2\right) P\left(\frac{A}{E_2}\right) + P\left(E_3\right) P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{5}{6} \times \frac{18}{50}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$

$$= \frac{3}{10} \times \frac{50}{36}$$
$$= \frac{5}{12}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$
$$= \frac{\frac{21}{50} \times \frac{4}{7}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$
$$= \frac{\frac{6}{25}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$
$$= \frac{6}{25} \times \frac{50}{36}$$
$$= \frac{1}{3}$$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$
$$= \frac{\frac{11}{50} \times \frac{9}{11}}{\frac{5}{6} \times \frac{18}{50} + \frac{21}{50} \times \frac{4}{7} + \frac{11}{50} \times \frac{9}{11}}$$
$$= \frac{\frac{9}{50}}{\frac{3}{10} + \frac{6}{25} + \frac{9}{50}}$$
$$= \frac{9}{50} \times \frac{50}{36}$$
$$= \frac{1}{4}$$

So, probabilities of d_1, d_2, d_3 diseases are $\frac{5}{12}, \frac{1}{3}, \frac{1}{4}$ respectively.

Hence, the patient is most likely to have d_1 diseased.

Question 33

A is known to speak truth 3 times out of 5 times. He throws a die and reports that it is 1. Find the probability that it is actually 1.

Solution 33

Let E_1, E_2 and A be events as:-

E_1 = 1 occurs on die

E_2 = 1 does not occur on die

A = The man reports that it is one

$P(E_1) = \frac{1}{6}$

$P(E_2) = \frac{5}{6}$

$P\left(\frac{A}{E_1}\right) = P$ (He reports one when occurs on die)

$= P$ (He speaks truth)

$= \frac{3}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (He reports one when 1 has not occured)

$= P$ (He does not speak truth)

$= 1 - \frac{3}{5}$

$= \frac{2}{5}$

To find, P (It is actually 1 when he reported that it is one on die) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{5}}{\frac{1}{6} \times \frac{3}{5} + \frac{5}{6} \times \frac{2}{5}}$$

$$= \frac{\frac{3}{30}}{\frac{3}{30} + \frac{10}{30}}$$

$$= \frac{3}{13}$$

Required probability $= \frac{3}{13}$.

Question 34

A speaks the truth 8 times out of 10 times. A die is tossed. He reports that it was 5. What is the probability that it was actually 5?

Solution 34

Let E_1, E_2 and A events be as:-

E_1 = 5 occurs on die

E_2 = 5 does not occur on die

A = He reports that it was 5

$P(E_1) = \frac{1}{6}$

$P(E_2) = \frac{5}{6}$

$P(A \mid E_1) = P$ (He reports 5 when 5 occurs on die)

$= P$ (He speaks truth)

$= \frac{8}{10}$

$= \frac{4}{5}$

$P\left(\frac{A}{E_2}\right) = P$ (He reports 5 when 5 does not occur on die)

$= P$ (He does not speak truth)

$= \frac{1}{5}$

To find, P (It was actually 5 when he reports that it is five) $= P\left(\frac{E_1}{A}\right)$

By baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}}$$

$$= \frac{\frac{4}{30}}{\frac{4}{30} + \frac{5}{30}}$$

$$= \frac{4}{9}$$

Required probability $= \frac{4}{9}$.

Question 35

In answering a question on a multiple choice test a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that a student knows the answer given that he answered it correctly?

Solution 35

$P(\text{Knows}) = \frac{3}{4}$

$P(\text{Guesses}) = \frac{1}{4}$

$P\left(\frac{\text{Correct}}{\text{Guesses}}\right) = \frac{1}{4}$

We need to find

$$P\left(\frac{\text{Knows}}{\text{Correctly}}\right) = \frac{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows})}{P\left(\frac{\text{Correctly}}{\text{knows}}\right)P(\text{Knows}) + P\left(\frac{\text{Correctly}}{\text{Guesses}}\right)P(\text{Guesses})}$$

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$= \frac{\frac{3}{4}}{\frac{12+1}{16}}$$

$$= \frac{12}{13}$$

Question 36

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (that is, if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Solution 36

Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Since E_1 and E_2 are events complimentary to each other,

$P(E_1) + P(E_2) = 1$

$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

Let A be the event that the blood test result is positive.

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

$$P(A|E_1) = P(\text{result is positive given the person has disease}) = 99\% = 0.99$$
$$P(A|E_2) = P(\text{result is positive given that the person has no disease}) = 0.5\% = 0.005$$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$
$$= \frac{0.001\times 0.99}{0.001\times 0.99 + 0.999\times 0.005}$$
$$= \frac{0.00099}{0.00099 + 0.004995}$$
$$= \frac{0.00099}{0.005985}$$
$$= \frac{990}{5985}$$
$$= \frac{110}{665}$$
$$= \frac{22}{133}$$

Chapter 31 - Probability Exercise MCQ

Question 1

If one ball is drawn at random from each of three boxes containing 3 white and 1 black, 2 white and black, 1 white and 3 black balls, then the probability that 2 white and 1 black balls will be draws is

a. $\frac{13}{32}$

b. $\frac{1}{4}$

c. $\frac{1}{32}$

d. $\frac{3}{16}$

Solution 1

Correct option: (a)

Total balls in first box = 3 white + 1 black = 4

Total balls in second box = 2 white + 2 black = 4

Total balls in third box = 1 white + 3 black = 4

Probability of 2 white and 1 black

$= P(WWB) + P(WBW) + P(BWW)$

$= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}$

$= \frac{18+6+2}{64} = \frac{13}{32}$

Question 2

A and B draw two cards each, one after another, from a pack of well-shuffled pack of 52 cards. The probability that all the four cards drawn are of the same suit is

a. $\frac{44}{85 \times 49}$

b. $\frac{11}{85 \times 49}$

c. $\frac{13 \times 24}{17 \times 25 \times 49}$

d. none of these

Solution 2

Correct option: (a)

Total cards = 52

probability of getting 4 cards are of same suit

$= 4 \times \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49}$

$= \frac{44}{85 \times 49}$

Question 3

A and B are two events such that P (A) = 0.25 and P (B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is

a. 0.39

b. 0.25

c. 0.11

d. none of these

Solution 3

Correct option: (a)

$P(A) = 0.25,\ P(B) = 0.5,\ P(A \cap B) = 0.14$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.25 + 0.5 - 0.14$

$P(A \cup B) = 0.61$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$P(\bar{A} \cap \bar{B}) = 1 - 0.61$

$P(\bar{A} \cap \bar{B}) = 0.39$

Question 4

The probabilities of a student getting I, II and III division in an examination are $\frac{1}{10}, \frac{3}{5}$ and $\frac{1}{4}$ respectively.

The probability that the student fails in the examination is

a. $\frac{197}{200}$

b. $\frac{27}{100}$

c. $\frac{83}{100}$

d. none of these

Solution 4

Correct option: (b)

$P(A) = \frac{1}{10},\ P(B) = \frac{3}{5},\ P(C) = \frac{1}{4}$

Required probability $= P(\bar{A} \cap \bar{B} \cap \bar{C})$

Required probability $= P(\bar{A})P(\bar{B})P(\bar{C})$

Required probability $= (1 - P(A))(1 - P(B))(1 - P(C))$

Required probability $= \left(1 - \frac{1}{10}\right)\left(1 - \frac{3}{5}\right)\left(1 - \frac{1}{4}\right)$

Required probability $= \frac{27}{100}$

Question 5

Indian play two matches each with West Indies and Australia. In any match the probabilities of India getting 0,1and 2 points are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

a. 0.0875

b. 1/16
c. 0.1125
d. None of these

Solution 5

Correct option: (a)

India can get at least 7 points by 5 ways.

case 1 – 2 + 2 + 2 + 2 = 8

case 2 – 2 + 2 + 2 + 1 = 7

case 3 – 2 + 2 + 1 + 2 = 7

case 4 – 2 + 1 + 2 + 2 = 7

case 5 – 1 + 2 + 2 + 2 = 7

$$P(x \geq 7) = 0.5 \times 0.5 \times 0.5 \times 0.5 + 4(0.5 \times 0.5 \times 0.5 \times 0.5)$$

$$P(x \geq 7) = 0.0875$$

Question 6

Three faces of an ordinary dice are yellow, two faces are red and one face is blue. The dice is rolled 3 times. The probability that yellow red and blue face appear in the first second and third throws respectively, is

a. $\frac{1}{36}$

b. $\frac{1}{6}$

c. $\frac{1}{30}$

d. none of these

Solution 6

Correct option: (a)

Given that 3 yellow faces, 2 red faces, one blue face.

Total faces = 6

$$P(Y) = \frac{3}{6}, P(R) = \frac{2}{6}, P(B) = \frac{1}{6}$$

$$\text{Required probability} = \frac{3}{6} \times \frac{2}{6} \times \frac{1}{6} = \frac{1}{36}$$

Question 7

The probability that a leap year will have 53 Friday or 53 Saturday is

a. $\frac{2}{7}$

b. $\frac{3}{7}$

c. $\frac{4}{7}$

d. $\frac{1}{7}$

Solution 7
Correct option: (b)

Non – leap year has 365 days = 52 weeks + 1

366 days in leap year.

We want to find probability of 53 Fridays or 53 Saturday.

Favourable cases = $\{(\text{Thursday}, \text{Friday}), (\text{Friday}, \text{Saturday}), (\text{Saturday}, \text{Sunday})\}$

Required probability = $\frac{3}{7}$

Question 8
A person writes 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is

a. $\frac{1}{4}$

b. $\frac{11}{24}$

c. $\frac{15}{24}$

d. $\frac{23}{24}$

Solution 8
Correct option: (d)

Total number of ways of 4 letters can be placed

in 4 envelopes = 4! = 24

Probability of a letter placed in the right envelope

is $\frac{1}{24}$.

Probability that all letters are not placed in the right

envelope = $1 - \frac{1}{24} = \frac{23}{24}$

Question 9
A speaks truth in 75% cases and B speaks truth in 80% cases. Probability that they contradict each other in a statement, is

a. $\frac{7}{20}$

b. $\frac{13}{20}$

c. $\frac{3}{5}$

d. $\frac{2}{5}$

Solution 9
Correct option: (a)

Let A be the event that A speaks truth.

$\Rightarrow P(A) = \frac{75}{100} = \frac{3}{4} \Rightarrow P(\overline{A}) = \frac{1}{4}$

Let B be the event that B speaks truth.

$\Rightarrow P(B) = \frac{80}{100} = \frac{4}{5} \Rightarrow P(\overline{B}) = \frac{1}{5}$

Required probability $= P(A)P(\overline{B}) + P(B)P(\overline{A})$

Required probability $= \frac{3}{4} \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{4} = \frac{7}{20}$

Question 10

Three integers are chosen at random from the first 20 integers. The probability that their product is even is

a. $\frac{2}{19}$

b. $\frac{3}{29}$

c. $\frac{17}{19}$

d. $\frac{4}{19}$

Solution 10

Correct option: (c)

Required probability that product of two integers should be even.

10 integers are odd out of first 20 integers.

Required probability = 1 − Probability of product is odd

Product of three integers is odd if two numbers are odd.

Required probability $= 1 - \frac{10}{20} \times \frac{9}{19} \times \frac{8}{18} = \frac{17}{19}$

Question 11

Out of 30 consecutive integers, 2 are chosen at random. The probability that their sum is odd, is

a. $\frac{14}{29}$

b. $\frac{16}{29}$

c. $\frac{15}{29}$

d. $\frac{10}{29}$

Solution 11

Correct option: (c)

To find sum of two integers should be odd,
we should have one integer should be even
and other should be odd.
Given that 30 consecutive integers then 15
are even and 15 are odd.

$$\text{Required probability} = P\left(1^{st} \text{ even } 2^{nd} \text{ odd}\right) + P\left(1^{st} \text{ odd } 2^{nd} \text{ even}\right)$$

$$\text{Required probability} = \frac{15}{30} \times \frac{15}{29} + \frac{15}{30} \times \frac{15}{29}$$

$$\text{Required probability} = \frac{15}{29}$$

Question 12

A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected random wise, the probability that it is black or red ball is

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. $\frac{5}{12}$

d. $\frac{2}{3}$

Solution 12

Correct option: (d)

Total number of balls = 5 black + 4 white + 3 red = 12

Probability of getting black or red = Probability of black + probability of red

$$\text{Probability of getting black or red} = \frac{5}{12} + \frac{3}{12}$$

$$\text{Probability of getting black or red} = \frac{2}{3}$$

Question 13

Two dice are thrown simultaneously. The probability of getting a pair of aces is

a. $\frac{1}{36}$

b. $\frac{1}{3}$

c. $\frac{1}{6}$

d. none of these

Solution 13

Correct option: (a)

Required probability = probability of ace in first throw

+ probability of ace in second throw

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Question 14

An urn contains 9 balls two of which are red, three blue and four black. Three balls are drawn at random. The probability that they are of the same colour is

a. $\frac{5}{84}$

b. $\frac{3}{9}$

c. $\frac{3}{7}$

d. $\frac{7}{17}$

Solution 14

Correct option: (a)

Total number of balls = 2 red + 3 blue + 4 black = 9

Red balls are only three hence we can not draw 3 balls out of 2.

Probability of same colour ball

$$= \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} + \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7}$$

$$= \frac{5}{84}$$

Question 15

A coin is tossed three times. If events A and B defined as A = Two heads come, B = Last should be head. Then , A and B are

a. independent

b. dependent

c. both

d. mutually exclusive

Solution 15

Correct option: (b)

A coin is tossed three times.

$S = \{HHH, HHT, HTH, THH, HTT, TTH, THT, TTT\}$

$A = \{HHT, HTH, THH\} \Rightarrow P(A) = \frac{3}{8}$

$B = \{HHH, HTH, THH, TTH\} \Rightarrow P(B) = \frac{4}{8}$

$A \cap B = \{HTH, THH\} \Rightarrow P(A \cap B) = \frac{2}{8}$

Consider,

$P(A) \times P(B) = \frac{3}{8} \times \frac{4}{8} = \frac{3}{4} \neq P(A \cap B)$

$\Rightarrow$ A and B are dependent.

Question 16

Five persons entered the life cabin on the ground floor of an 8 floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floors beginning with the first, then probability of all 5 persons leaving different floor is

a. $\frac{^7P_5}{7^5}$

b. $\frac{7^5}{^7P_5}$

c. $\frac{6}{^7P_5}$

d. $\frac{^5P_5}{5^5}$

Solution 16

Correct option: (a)

Five persons can leave different floors by $^7P_5$ ways.

Possible ways of leaving the lift = 7^5

Required probability = $\frac{^7P_5}{7^5}$

Question 17

A box contains 10 goods articles and 6 with defects. One item is drawn at random. The probability that it is either or has a defect is

a. $\frac{64}{64}$

b. $\frac{49}{64}$

c. $\frac{40}{64}$

d. $\frac{24}{64}$

Solution 17

Correct option: (a)

As both the events are complementary

$\Rightarrow P(A \cap B) = 0$ also $P(A \cup B) = 1$

Hence, probablity that either good artide or defect items is 1.

Question 18

A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, the probability that it is rusted or is a nail is

a. $\frac{3}{16}$

b. $\frac{5}{16}$

c. $\frac{11}{16}$

d. $\frac{14}{16}$

Solution 18

Correct option: (c)

Given that 6 nails and 10 nuts.

$\Rightarrow P(\text{a nail}) = \frac{6}{16}$

Rusted items $= 5 + 3 = 8 \Rightarrow P(\text{Rusted items}) = \frac{8}{16}$

Out of these half rusted nails $= \frac{6}{2} = 3$

$\Rightarrow P(\text{rusted item and nail}) = \frac{3}{16}$

Required probability $= P(\text{rusted item}) + P(\text{a nail}) - P(\text{rusted item and nail})$

Required probability $= \frac{8}{16} + \frac{6}{16} - \frac{3}{16} = \frac{11}{16}$

Question 19

A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that these are of the same colour is

a. $\frac{5}{108}$

b. $\frac{18}{108}$

c. $\frac{30}{108}$

d. $\frac{48}{108}$

Solution 19

Correct option: (d)

Total number of balls = 5 brown + 4 white = 9

$$\text{Required probability} = \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8} = \frac{4}{9}$$

$$\Rightarrow \frac{4 \times 12}{9 \times 12} = \frac{48}{108}$$

Question 20

If S is the sample space and $P(A) = \frac{1}{3}P(B)$ and $S = A \cup B$,

where A and B are two mutually exclusive events, then

$P(A) =$

a. 1/4

b. 1/2

c. 3/4

d. 3/8

Solution 20

Correct option: (a)

$$P(A) = \frac{1}{3}P(B)$$

$$\Rightarrow P(B) = 3P(A) \quad .....(i)$$

A and B are two mutually exclusive events.

$$\Rightarrow P(A \cap B) = 0, \ P(A \cup B) = 1$$

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

$$\Rightarrow P(A) + 3P(A) = 1 \ldots\ldots\ldots\ldots (\text{from (i)})$$

$$\Rightarrow 4P(A) = 1$$

$$\Rightarrow P(A) = \frac{1}{4}$$

Question 21

If A and B are two event, then $P(\overline{A} \cap B) =$

a. $P(\overline{A})P(\overline{B})$

b. $1 - P(A) - P(B)$

c. $P(A) + P(B) - P(A \cap B)$

d. $P(B) - P(A \cap B)$

Solution 21
Correct option: (d)

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

Question 22
If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(\overline{A}) + P(\overline{B}) =$

a. 0.3
b. 0.5
c. 0.7
d. 0.9

Solution 22
Correct option: (d)

If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

$$\Rightarrow P(A) + P(B) = 1.1$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - [P(A) + P(B)]$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - 1.1$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 0.9$$

Question 23
A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then, the probability chosen to be white is
a. 2/15
b. 7/15
c. 8/15
d. 14/15

Solution 23
Correct option: (c)

Total balls in bag X = 2 white + 3 black = 5

Total balls in bag Y = 4 white + 2 black = 6

If a bag selected at random then probability of

selecting bag = $\frac{1}{2}$

$$\text{Required probability} = \frac{1}{2} \times \left(\begin{matrix}\text{Probability of getting white ball}\\ \text{from X}\end{matrix}\right) + \frac{1}{2} \times \left(\begin{matrix}\text{Probability of getting white ball}\\ \text{from Y}\end{matrix}\right)$$

$$\text{Required probability} = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{5}$$

$$\text{Required probability} = \frac{8}{15}$$

Question 24

Two persons A and B turn in throwing a pair of dice. The first person to throw 9 from both dice will be awarded the prize. If A throws first, then the probability that B wins the game is

a. 9/17

b. 8/17

c. 8/9

d. 1/9

Solution 24

Correct option: (b)

From throw of two dice 9 can be obtained

$\{(3,6),(4,5),(5,4),(6,3)\}$

$$\Rightarrow \text{Probability of getting } 9 = \frac{4}{36}$$

$$\Rightarrow \text{Probability of not getting } 9 = \frac{32}{36}$$

$$\text{Probability of B winning} = P\left(9 \text{ in } 2^{nd} \text{ throw}\right) + P\left(9 \text{ in } 4^{ndth} \text{ throw}\right)$$

$$\text{Probability of B winning} = \frac{8}{9} \times \frac{1}{9} + \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} + \ldots.$$

$$\text{Probability of B winning} = \frac{8}{81}\left(\frac{1}{1-\frac{64}{81}}\right) = \frac{8}{17}$$

Question 25

The probability that in a year of 22nd century chosen at random, there will be 53 Sundays, is

a. 3/28

b. 2/28

c. 7/28

d. 5/28

Solution 25

$$\text{Probability of 53 Sundays in non-leap year} = \frac{1}{7}$$

$$\text{Probability of 53 Sundays in leap year} = \frac{2}{7}$$

In 22^{nd} century there will be 24 leap years.

$$\text{Probability of leap year} = \frac{24}{100}$$

$$\text{Probability of non-leap year} = \frac{76}{100}$$

Probability of 53 Sundays

$$= P(\text{non-leap year}) \times P(\text{53 Sundays in a non-leap year})$$

$$+ P(\text{leap year}) \times P(\text{53 Sundays in a leap year})$$

$$= \frac{76}{100} \times \frac{1}{7} + \frac{24}{100} \times \frac{2}{7} = \frac{31}{175}$$

NOTE: Answer not matching with back answer.

Question 26

From a set of 100 cards numbered 1 to 100, one card is draw at random. The probability that the number obtained on the card is divisible by 6 or 8 but not by 24 is

a. 6/25
b. 1/4
c. 1/6
d. 2/5

Solution 26

Correct option: (a)

Number of cards divisible by 6 = 16

$$\Rightarrow P(A) = \frac{16}{100}$$

Number of cards divisible by 8 = 12

$$\Rightarrow P(B) = \frac{12}{100}$$

Number of cards divisible by 24 = 4

$$\Rightarrow P(A \cap B) = \frac{4}{100}$$

$$P(A \cup B) = \frac{16}{100} + \frac{12}{100} - \frac{4}{100}$$

$$P(A \cup B) = \frac{6}{25}$$

Question 27

If A and B are two events such that $P(A) = \frac{4}{5}$,

and $P(A \cap B) = \frac{7}{10}$, then $P(B / A) =$

a. 1/10

b. 1/8
c. 7/8
d. 17/20

Solution 27
Correct option: (c)

$$P(A) = \frac{4}{5}, P(A \cap B) = \frac{7}{10}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{7}{8}$$

Question 28
If A and B are two events associated to a random experiment such that $P(A \cap B) = \frac{7}{10}$ and $P(B) = 17/20$, then $P(A/B) =$

a. 14/17
b. 17/20
c. 7/8
d. 1/8

Solution 28
Correct option: (a)

$$P(A \cap B) = \frac{7}{10}, P(B) = \frac{17}{20}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) = \frac{\frac{7}{10}}{\frac{17}{20}} = \frac{14}{17}$$

Question 29
Associated to a random experiment two events A and B are such that P (B) = 3/5, P (A/B) = 1/2 and (A ∪ B) = 4/5. The Value of P(A) is

a. $\frac{3}{10}$

b. $\frac{1}{2}$

c. $\frac{1}{10}$

d. $\frac{3}{5}$

Solution 29

Correct option: (b)

$$P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}, P(A \cup B) = \frac{4}{5}$$

$$P(A/B) = \frac{1}{2}$$

$$\frac{P(A \cap B)}{P(B)} = \frac{1}{2}$$

$$\frac{P(A \cap B)}{\frac{3}{5}} = \frac{1}{2}$$

$$P(A \cap B) = \frac{3}{10}$$

$$P(A) + P(B) - P(A \cup B) = \frac{3}{10}$$

$$P(A) + \frac{3}{5} - \frac{4}{5} = \frac{3}{10}$$

$$P(A) = \frac{1}{2}$$

Question 30

If P(A) = 3/10, P(B) = 2/5 and P(A ∪ B) = 3/5, then P(A/B) + P(B/A) equals

a. 1/4
b. 7/2
c. 5/12
d. 1/3

Solution 30

Correct option: (b)

$$P(A) = \frac{3}{10}, P(B) = \frac{2}{5}, P(A \cup B) = \frac{3}{5}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$P(A \cap B) = \frac{3}{10} + \frac{2}{5} - \frac{3}{5}$$

$$P(A \cap B) = \frac{1}{10}$$

$$P(A/B) + P(B/A) = \frac{P(A \cap B)}{P(B)} + \frac{P(A \cap B)}{P(A)}$$

$$P(A/B) + P(B/A) = \frac{\frac{1}{10}}{\frac{2}{5}} + \frac{\frac{1}{10}}{\frac{3}{10}}$$

$$P(A/B) + P(B/A) = \frac{7}{12}$$

Note: option is modified.

Question 31

Let $P(A) = 7/13, P(B) = 9/13$ and $P(A \cap B) = 4/13$.

Then, $P(\overline{A}/B) =$

a. 5/9
b. 4/9
c. 4/13
d. 6/13

Solution 31

Correct option: (a)

$$P(A) = \frac{7}{13}, P(B) = \frac{9}{13}, P(A \cap B) = \frac{4}{13}$$

$$P(\overline{A}/B) = \frac{P(\overline{A} \cap B)}{P(B)}$$

$$\Rightarrow P(\overline{A}/B) = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$\Rightarrow P(\overline{A}/B) = \frac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}}$$

$$\Rightarrow P(\overline{A}/B) = \frac{5}{9}$$

Question 32

If $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$,

then $P(\overline{A}/\overline{B})\ P(\overline{B}/\overline{A})$ is equal to

a. $\frac{5}{6}$

b. $\frac{5}{7}$

c. $\frac{25}{42}$

d. 1

Solution 32

Correct option: (c)

$$P(A)=\frac{2}{5}, P(B)=\frac{3}{10} \text{ and } P(A\cap B)=\frac{1}{5}$$

$$P(A\cap B)=\frac{1}{5}$$

$$P(A)+P(B)-P(A\cup B)=\frac{1}{5}$$

$$\frac{2}{5}+\frac{3}{10}-P(A\cup B)=\frac{1}{5}$$

$$P(A\cup B)=\frac{1}{2}$$

$$P(\overline{A}/\overline{B})\,P(\overline{B}/\overline{A})=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}\frac{P(\overline{A}\cap\overline{B})}{P(\overline{A})}$$

$$P(\overline{A}/\overline{B})\,P(\overline{B}/\overline{A})=\frac{\left[P(\overline{A\cup B})\right]^2}{P(\overline{B})P(\overline{A})}$$

$$P(\overline{A}/\overline{B})\,P(\overline{B}/\overline{A})=\frac{\left[1-P(A\cup B)\right]^2}{P(\overline{B})P(\overline{A})}$$

$$P(\overline{A}/\overline{B})\,P(\overline{B}/\overline{A})=\frac{\left[1-\frac{1}{2}\right]^2}{\frac{7}{10}\times\frac{3}{5}}$$

$$P(\overline{A}/\overline{B})\,P(\overline{B}/\overline{A})=\frac{25}{42}$$

Question 33

If A and B are two events such that $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$,

$P(A/B)=\frac{1}{4}$, then $P(\overline{A}\cap\overline{B})$ equals

a. $\frac{1}{12}$

b. $\frac{3}{4}$

c. $\frac{1}{4}$

d. $\frac{3}{16}$

Solution 33

Correct option: (c)

$$P(A)=\frac{1}{2}, P(B)=\frac{1}{3}, P(A/B)=\frac{1}{4}$$

$$P(A/B)=\frac{1}{4}$$

$$\frac{P(A\cap B)}{P(B)}=\frac{1}{4}$$

$$\frac{P(A\cap B)}{\frac{1}{3}}=\frac{1}{4}$$

$$P(A\cap B)=\frac{1}{12}$$

$$P(A/B)=\frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow P(A/B)=\frac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$$

$$P(\overline{A}\cap\overline{B})=P(\overline{A\cup B})$$

$$P(\overline{A}\cap\overline{B})=1-P(A\cup B)$$

$$P(\overline{A}\cap\overline{B})=1-\left[P(A)+P(B)-P(A\cap B)\right]$$

$$P(\overline{A}\cap\overline{B})=1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right]$$

$$P(\overline{A}\cap\overline{B})=\frac{1}{4}$$

Question 34

Let A and B are two events such that $P(A)=\frac{3}{8}, P(B)=\frac{5}{8}$ and $P(A\cup B)=\frac{3}{4}$. Then $P(A/B)P(\overline{A}/B)$ is equals to

a. $\frac{2}{5}$

b. $\frac{3}{8}$

c. $\frac{3}{20}$

d. $\frac{6}{25}$

Solution 34

Correct option: (d)

$P(A) = \frac{3}{8}, P(B) = \frac{5}{8}, P(A \cup B) = \frac{3}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{1}{4}$

$P(A/B)P(\overline{A}/B) = \frac{P(A \cap B)}{P(B)} \times \frac{P(\overline{A} \cap B)}{P(B)}$

$P(A/B)P(\overline{A}/B) = \frac{P(A \cap B)}{P(B)} \times \frac{P(B) - P(A \cap B)}{P(B)}$

$P(A/B)P(\overline{A}/B) = \frac{\frac{1}{4}}{\frac{5}{8}} \times \frac{\left(\frac{5}{8} - \frac{1}{4}\right)}{\frac{5}{8}}$

$P(A/B)P(\overline{A}/B) = \frac{6}{25}$

Question 35

If $P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$,

then $P(\overline{A \cup B}) + P(\overline{A} \cup B) =$

a. $\frac{1}{5}$

b. $\frac{4}{5}$

c. $\frac{1}{2}$

d. 1

Solution 35

Correct option: (d)

$P(B) = \frac{3}{5}, P(A/B) = \frac{1}{2}, P(A \cup B) = \frac{4}{5}$

Consider,

$P(A/B) = \frac{1}{2}$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$

$\Rightarrow \frac{P(A \cap B)}{\frac{3}{5}} = \frac{1}{2}$

$\Rightarrow P(A \cap B) = \frac{3}{10}$

$P(\overline{A \cup B}) + P(\overline{A} \cup B)$

$= 1 - P(A \cap B) + P(B) - P(A \cap B)$

$= 1 - \frac{3}{10} + \frac{3}{5} - \frac{3}{10}$

$= 1$

Question 36

If P(A) = 0.4, P(B) = 0.8 and P(B/A)=0.6, then
P(A ∪ B)=

a. 0.24
b. 0.3
c. 0.48
d. 0.96

Solution 36

Correct option: (d)

$P(A) = 0.4, P(B) = 0.8, P(B/A) = 0.6$

Consider,

$P(B/A) = 0.6$

$\frac{P(A \cap B)}{P(A)} = 0.6$

$\frac{P(A \cap B)}{0.4} = 0.6$

$\Rightarrow P(A \cap B) = 0.24$

$\Rightarrow P(A) + P(B) - P(A \cup B) = 0.24$

$\Rightarrow 0.4 + 0.8 - P(A \cup B) = 0.24$

$\Rightarrow P(A \cup B) = 0.96$

Question 37

If $P(B)=\frac{3}{5}, P(A/B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$,

then $P(B/\overline{A})=$

a. $\frac{1}{5}$

b. $\frac{3}{10}$

c. $\frac{1}{2}$

d. $\frac{3}{5}$

Solution 37

Correct option: (d)

$P(B)=\frac{3}{5}, P(A/B)=\frac{1}{2}, P(A\cup B)=\frac{4}{5}$

Consider,

$$P(A/B)=\frac{1}{2}$$

$$\frac{P(A\cap B)}{P(B)}=\frac{1}{2}$$

$$\frac{P(A\cap B)}{\frac{3}{5}}=\frac{1}{2}$$

$$P(A\cap B)=\frac{3}{10}$$

$$P(A)+P(B)-P(A\cup B)=\frac{3}{10}$$

$$P(A)+\frac{3}{5}-\frac{4}{5}=\frac{3}{10}$$

$$P(A)=\frac{1}{2}$$

$$P(B/\overline{A})=\frac{P(B\cap\overline{A})}{P(\overline{A})}$$

$$P(B/\overline{A})=\frac{\frac{3}{5}-\frac{3}{10}}{1-\frac{1}{2}}$$

$$P(B/\overline{A})=\frac{3}{5}$$

Question 38

If A and B are two events such that $P(A) = 0.4, P(B) = 0.3$ and $P(A \cup B) = 0.5$, then $P(\bar{B} \cap A)$ equals

a. $\frac{2}{3}$

b. $\frac{1}{2}$

c. $\frac{3}{10}$

d. $\frac{1}{5}$

Solution 38

Correct option: (d)

$P(A) = 0.4, P(B) = 0.3$ and $P(A \cup B) = 0.5,$

$P(\bar{B} \cap A) = P(A) - P(A \cap B)$

$P(\bar{B} \cap A) = P(A) - [P(A) + P(B) - P(A \cup B)]$

$P(\bar{B} \cap A) = P(A \cup B) - P(B)$

$P(\bar{B} \cap A) = 0.5 - 0.3$

$P(\bar{B} \cap A) = 0.2$

$P(\bar{B} \cap A) = \frac{1}{5}$

Question 39

If A and B are two events such that $A \neq \phi$, $B = \phi$, then

a. $P(A/B) = \frac{P(A \cap B)}{P(B)}$

b. $P(A/B) = P(A)\ P(B)$

c. $P(A/B) = P(B/A) = 1$

d. $P(A/B) = \frac{P(A)}{P(B)}$

Solution 39

Correct option: (a)

If A and B are two events such that $A \neq \phi, B = \phi$, then

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

Question 40

If A and B are two events such that $P(A) \neq 0$ and $P(B) \neq 1$,

then $P(\bar{A}/\bar{B})$

a. $1 - P(A/B)$

b. $1 - P(\bar{A}/B)$

c. $\dfrac{1 - P(A \cup B)}{P(\bar{B})}$

d. $\dfrac{P(\bar{A})}{P(B)}$

Solution 40

Correct option: (c)

A and B are two events such that $P(A) \neq 0$ and $P(B) \neq 1$,

$$\Rightarrow P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$$

$$\Rightarrow P(\bar{A}/\bar{B}) = \frac{P(\overline{A \cap B})}{P(\bar{B})}$$

$$\Rightarrow P(\bar{A}/\bar{B}) = \frac{1 - P(A \cup B)}{P(\bar{B})}$$

Question 41

If the events A and B are independent, then P(A ∩ B) is equal to

a. P(A) + P(B)

b. P(A) - P(B)

c. P(A) P(B)

d. $\dfrac{P(A)}{P(B)}$

Solution 41

Correct option: (c)

P(A ∩ B)= P(A) P(B) for independent events.

Question 42

If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(\bar{A} \cap \bar{B})$ equals

a. $\frac{4}{15}$

b. $\frac{8}{45}$

c. $\frac{1}{3}$

d. $\frac{2}{9}$

Solution 42

Correct option: (d)

$$P(A) = \frac{3}{5},\ P(B) = \frac{4}{9}$$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$P(\bar{A} \cap \bar{B}) = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$P(\bar{A} \cap \bar{B}) = 1 - \left[\frac{3}{5} + \frac{4}{9} - \frac{3}{5} \times \frac{4}{9}\right] \qquad (\because \text{A and B are independent})$$

$$P(\bar{A} \cap \bar{B}) = 1 - \frac{7}{9}$$

$$P(\bar{A} \cap \bar{B}) = \frac{2}{9}$$

Question 43

If A and B are two independent events such that P(A) = 0.3, P(A ∪ B) = 0.5, then P(A/B) - P(B/A) =

a. $\frac{2}{7}$

b. $\frac{3}{35}$

c. $\frac{1}{70}$

d. $\frac{1}{7}$

Solution 43

Correct option: (c)

Given that $P(A) = 0.3$, $P(A \cup B) = 0.5$, A and B are independent.

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

Let $P(B) = x$

$\Rightarrow 0.5 = 0.3 + x - 0.3x$

$\Rightarrow 0.2 = 0.7x$

$\Rightarrow x = \frac{2}{7} = P(B)$

$\Rightarrow P(A \cap B) = P(A) \times P(B)$

$\Rightarrow P(A \cap B) = \frac{3}{10} \times \frac{2}{7} = \frac{3}{35}$

$P(A / B) - P(B / A)$

$= \frac{P(A \cap B)}{P(B)} - \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{3}{35}}{\frac{2}{7}} - \frac{\frac{3}{35}}{\frac{3}{10}}$

$= \frac{3}{10} - \frac{2}{7}$

$= \frac{1}{70}$

Question 44

A flash light has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is

a. $\frac{3}{28}$

b. $\frac{1}{14}$

c. $\frac{9}{64}$

d. $\frac{33}{56}$

Solution 44

Correct option: (a)

A flash light has 8 batteries out of which 3 are dead.

Two batteries are selected out of $8 \Rightarrow n(S) = {}^8C_2 = 28$

Let A be the event of getting both dead batteries.

$n(A) = {}^3C_2 = 3$

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{28}$

Question 45
A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is

a. $\frac{15}{29}$

b. $\frac{15}{56}$

c. $\frac{45}{196}$

d. $\frac{135}{392}$

Solution 45
Correct option: (b)

Total balls = 5 red + 3 blue = 8

Let R be the event of getting red ball.

B be the event of getting a blue ball.

Required probability

$= P(BBR) + P(BRB) + P(RBB)$

$= \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6}$

$= \frac{15}{56}$

Question 46
A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability that exactly two of the three balls were red, the first ball being red, is

a. $\frac{1}{3}$

b. $\frac{4}{7}$

c. $\frac{15}{28}$

d. $\frac{5}{28}$

Solution 46
Correct option: (b)

Total number of balls = 5 red + 3 blue = 8

Probability of getting exactly two red balls given that first ball should be red.

required probability $= P(R_2B_2 / R_1) + P(B_1R_2 / R_1)$

required probability $= \frac{4}{7} \times \frac{3}{6} + \frac{3}{7} \times \frac{4}{6} = \frac{4}{7}$

Question 47

In a college 30% students fail in physics, 25% fails in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics if she has failed in Mathematics in

a. $\frac{1}{10}$

b. $\frac{1}{3}$

c. $\frac{2}{5}$

d. $\frac{9}{20}$

Solution 47

Correct option: (c)

Let A be the event that students failed in Physics.

B be the event that students failed in Mathematics.

Given that $P(A) = 30\% = \frac{30}{100}$

$P(B) = 25\% = \frac{25}{100}$

$P(A \cap B) = 10\% = \frac{10}{100}$

Required probability is given by $P(A/B)$.

$$\Rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{10}{100}}{\frac{25}{100}} = \frac{2}{5}$$

Question 48

Three persons A, B and C fire a target in turn starting with A. their probabilities of hitting the target are 0.4, 0.3 and 0.2 respectively. The probability of two hits is

a. 0.024

b. 0.452

c. 0.336

d. 0.138

Solution 48

$P(A) = 0.4, P(B) = 0.3, P(C) = 0.2$

$\Rightarrow P(\overline{A}) = 0.6, P(\overline{B}) = 0.7, P(\overline{C}) = 0.8$

$P(\text{Two hits}) = P(A)P(B)P(\overline{C}) + P(A)P(\overline{B})P(C) + P(\overline{A})P(B)P(C)$

$= 0.4 \times 0.3 \times 0.8 + 0.4 \times 0.7 \times 0.2 + 0.6 \times 0.3 \times 0.2$

$= 0.188$

NOTE: Answer not matching with back answer.

Question 49

A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability of their making common error is $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

a. $\frac{10}{13}$

b. $\frac{13}{120}$

c. $\frac{1}{40}$

d. $\frac{1}{12}$

Solution 49

Correct option: (a)

Let E_1 be the event that Both A and B solve the problem.

A and B are independent.

$$\Rightarrow P(E_1) = P(A) \times P(B)$$

$$\Rightarrow P(E_1) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Let E_2 both A and B got wrong solution.

$$P(E_2) = \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{4}\right) = \frac{1}{2}$$

Let E be the event getting same answer.

$$P(E / E_1) = 1, \; P(E / E_2) = \frac{1}{20}$$

$$\Rightarrow P(E_1 / E) = \frac{\frac{1}{12} \times 1}{\frac{1}{12} \times 1 + \frac{1}{2} \times \frac{1}{20}} = \frac{10}{13}$$

Question 50

Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability that both cards are queen is

a. $\frac{1}{13} \times \frac{1}{13}$

b. $\frac{1}{13} + \frac{1}{13}$

c. $\frac{1}{13} \times \frac{1}{17}$

d. $\frac{1}{13} \times \frac{4}{5}$

Solution 50

Correct option: (a)

Two cards are drawn from 52 cards.
Let, E_1 be the event that getting queen in first draw
and E_2 be the event that getting queen in second draw.

$$P(E_1 \cap E_2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$$

Question 51
A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

a. $\frac{167}{168}$

b. $\frac{1}{28}$

c. $\frac{2}{21}$

d. $\frac{3}{28}$

Solution 51
Correct option: (d)

Total balls in a box = 3 orange + 3 green + 2 blue = 8
Three balls are drawn at random from the box then

$$\text{sample space } n(S) = {}^8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Let A be the event that drawing 2 green and one blue ball.

$$n(A) = {}^3C_2 \times {}^2C_1 = 6$$

$$P(A) = \frac{6}{56} = \frac{3}{28}$$

Question 52
If two events are independent, then
a. they must be mutually exclusive
b. the sum of their probabilities must be equal to 1
c. (a) and (b) both are correct
d. none of the above is correct

Solution 52
Correct option: (d)

For two independent events $P(A \cap B) = P(A) \times P(B)$

$$\Rightarrow P(A \cap B) \neq 0 \Rightarrow P(A) + P(B) \neq 1$$

Question 53
Two dice are thrown. If it is known that the sum of the numbers on the dice was less than 6, the probability of getting a sum 3, is

a. $\frac{1}{18}$

b. $\frac{5}{18}$

c. $\frac{1}{5}$

d. $\frac{2}{5}$

Solution 53

Correct option: (c)

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$n(S) = 36$

Let A be the event that sum of the numbers on dice was less than 6.

$A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$

$n(A) = 10$

Let B be the event that getting sum 3.

$B = \{(1,2),(2,1)\} \Rightarrow n(B) = 2$

$A \cap B = \{(1,2),(2,1)\} \Rightarrow n(A \cap B) = 2$

$P(A/B) = \frac{n(A \cap B)}{n(B)}$

$\Rightarrow P(A/B) = \frac{2}{10} = \frac{1}{5}$

Question 54

If A and B are such that $P(A \cup B) = \frac{5}{9}$ and $P(\overline{A} \cup \overline{B}) = \frac{2}{3}$, then $P(\overline{A}) + P(\overline{B}) =$

a. $\frac{9}{10}$

b. $\frac{10}{9}$

c. $\frac{8}{9}$

d. $\frac{9}{8}$

Solution 54

Correct option: (b)

$P(A \cup B) = \frac{5}{9}$, $P(\overline{A} \cup \overline{B}) = \frac{2}{3}$

Consider,

$$P(\overline{A} \cup \overline{B}) = P\overline{(A \cap B)}$$

$$\Rightarrow P(\overline{A} \cup \overline{B}) = \frac{2}{3}$$

$$\Rightarrow 1 - P(A \cap B) = \frac{2}{3}$$

$$\Rightarrow P(A \cap B) = \frac{1}{3}$$

$$\Rightarrow P(A) + P(B) - P(A \cup B) = \frac{1}{3}$$

$$\Rightarrow P(A) + P(B) - \frac{5}{9} = \frac{1}{3}$$

$$\Rightarrow P(A) + P(B) = \frac{8}{9}$$

$$P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - [P(A) + P(B)]$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - \frac{8}{9}$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = \frac{10}{9}$$

Question 55

If A and B are two events such that $P(A/B) = p, P(A) = p,$

$P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{9}$, then P =

a. $\frac{2}{3}$

b. $\frac{3}{5}$

c. $\frac{1}{3}$

d. $\frac{3}{4}$

Solution 55

Correct option: (c)

$P(A/B) = p, P(A) = p,\ P(B) = \frac{1}{3}, P(A \cup B) = \frac{5}{9}$

Consider,

$P(A/B) = p$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = p$

$\Rightarrow \frac{P(A) + P(B) - P(A \cup B)}{P(B)} = p$

$\Rightarrow \frac{p + \frac{1}{3} - \frac{5}{9}}{\frac{1}{3}} = p$

$\Rightarrow p + \frac{1}{3} - \frac{5}{9} = \frac{p}{3}$

$\Rightarrow \frac{-2}{9} = \frac{p}{3} - p$

$\Rightarrow \frac{-2}{3}p = \frac{-2}{9}$

$\Rightarrow p = \frac{1}{3}$

Question 56

A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number of the die and a spade card is

a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{1}{8}$

d. $\frac{3}{4}$

Solution 56

Correct option: (c)

A sample space when a die is thrown.

$S_1 = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S_1) = 6$

Let A be the event that getting even number.

$A = \{2, 4, 6\} \Rightarrow n(A) = 3$

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

A card is selected from a deck of 52 cards.

$n(S_2) = {}^{52}C_1 = 52$

Let B be the event that getting spade card.

$n(B) = {}^{13}C_1 = 13 \Rightarrow P(B) = \frac{13}{52} = \frac{1}{4}$

Required probability $= P(A) \times P(B)$

$= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Question 57

Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{2}{3}$

d. $\frac{4}{7}$

Solution 57

Correct option: (d)

$S = \{GBB, GGB, GBG, GGG, BGG, BGB, BBG, BBB\}$

Let E_1 be the event that choosing a family with a girl as eldest child.

E_2 be the event that choosing a family with at least one girl.

$E_1 = \{GBB, GGB, GBG, GGG\}$

$E_2 = \{GBB, GGB, GBG, GGG, BGG, BGB, BBG\}$

$n(E_1) = 4,\ n(E_2) = 7,\ n(A \cap B) = 4$

$\Rightarrow P(A / B) = \frac{n(A \cap B)}{n(B)} = \frac{4}{7}$

Question 58

Let A and B be two events. If P(A) = 0.2, P(B) = 0.4,
P(A ∪ B) = 0.6, then P(A/B) is equal to

a. 0.8
b. 0.5
c. 0.3
d. 0

Solution 58

Correct option: (d)

$$P(A) = 0.2,\ P(B) = 0.4,\ P(A \cup B) = 0.6$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A / B) = \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$$

$$\Rightarrow P(A / B) = \frac{0.2 + 0.4 - 0.6}{P(B)}$$

$$\Rightarrow P(A / B) = 0$$

Question 59

Let A and B be two events such that P(A)=0.6,
P(B)=0.2 and P(A/B)=0.5. Then $P(\overline{A}/\overline{B})$ equal

a. $\frac{1}{10}$
b. $\frac{3}{10}$
c. $\frac{3}{8}$
d. $\frac{6}{7}$

Solution 59

Correct option: (c)

Given that $P(A) = 0.6$, $P(B) = 0.2$, $P(A/B) = 0.5$
Consider,

$$P(A/B) = 0.5$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = 0.5$$

$$\Rightarrow \frac{P(A \cap B)}{0.2} = 0.5$$

$$\Rightarrow P(A \cap B) = 0.1$$

$$\Rightarrow P(A) + P(B) - P(A \cup B) = 0.1$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - 0.1$$

$$\Rightarrow P(A \cup B) = 0.7$$

Now, $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

$$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$$

$$\Rightarrow P(\overline{A} \cap \overline{B}) = 0.3$$

To find

$$P(\overline{A} / \overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$$

$$P(\overline{A} / \overline{B}) = \frac{0.3}{0.8}$$

$$P(\overline{A} / \overline{B}) = \frac{3}{8}$$

NOTE: Answer not matching with back answer.

Chapter 31 - Probability Exercise Ex. 31VSAQ

Question 1

A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Write the probability that the number is divisible by 5.

Solution 1

Given digits are 1, 2, 3, 5

Total number of four digit numbers with no repetitions

$= {}^4P_4$

$= \dfrac{4!}{(4-4)!}$ $\quad\left[\because \text{Since } {}^nP_r = \dfrac{n!}{(n-r)!} \text{ and } 0! = 1\right]$

$= 4 \times 3 \times 2 \times 1$

$= 24$

Number of four digit numbers divisible by 5 with no repetitions

$= {}^3P_3 \times 1$ $\quad$ [Since unit place is fixed with 5 at unit place]

$= \dfrac{3!}{(3-3)!}$

$= 3 \times 2 \times 1$

$= 6$

P (The number divisible by 5)

$= \dfrac{6}{24}$

$= \dfrac{1}{4}$

Required probability $= \dfrac{1}{4}$.

Question 2

When three dice are thrown, write the probability of getting 4 or 5 on each of the dice simultaneously.

Solution 2

three dice are thrown

Given, 4 or 5 can occur on each dice simultaneously

E = 4 or 5 on each of the dice simultaneously

$= \{(4, 4, 4,), (4, 4, 5), (4, 5, 4), (4, 5, 5), (5, 4, 4), (5, 4, 5), (5, 5, 4), (5, 5, 5)\}$

$n(E) = 8$

$n(S) = 216$

$P(E) = \dfrac{8}{216}$

$= \dfrac{1}{27}$

Required probability $= \dfrac{1}{27}$.

Question 3

Three digit numbers are formed with the digits 0, 2, 4, 6 and 8. Write the probability of forming a three digit number with the same digits.

Solution 3

Three digit number are formed using 0, 2, 4, 6, 8.
Total number of 3 digit numbers

$= 4 \times 5 \times 5$ $\quad$ [0 can not occur at thousand's place]

$= 100$

Total number of 3 digit numbers with same digits

$= 4$ $\quad$ [Since 22, 444, 666, 888]

Requird probability $= \frac{4}{100}$

$= \frac{1}{25}$

Required probability $= \frac{1}{25}$.

Question 4

A ordinary cube has four plane faces, one face marked 2 and another face marked 3, find the probability of getting a total of 7 in 5 throws.

Solution 4

Given, Ordinary cube with four plane fales, 2 marked on one face and 3 on other face.
Total elements in sample space of 5 throws of a die

$= 6 \times 6 \times 6 \times 6 \times 6$

$n(S) = 6^5$

E = Sum of numbers on die thrown five times is 7
Event E is possible when out of 5 throws die shows 2, 2, 3 on any three throws.

$n(E) = {}^3P_3 \times 5$

$= 3! \times 5$

$= 3 \times 2 \times 5$

$= 30$

$P(E) = \frac{30}{6^5}$

$P(E) = \frac{5}{6^4}$

Question 5

Three numbers are chosen from 1 to 20. Find the probability that they are consecutive.

Solution 5

S = Three numbers are chosen from 1 to 20

$n(S) = {}^{20}C_3$

E = Group of three consecutive numbers between 1 and 20

$n(E) = 18$

$$\left\{\begin{matrix}(1,2,3),(2,3,4),(3,4,5),(4,5,6),(5,6,7),(7,8,9),(8,9,10),(9,10,11),(10,11,12),(11,12,13), \\ (12,13,14),(13,14,15),(14,15,16),(15,16,17),(16,17,18),(17,18,19),(18,19,20)\end{matrix}\right\}$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{18}{{}^{20}C_3}$$

Required probability = $\frac{18}{{}^{20}C_3}$.

Question 6

6 boys and 6 girls sit in a row at random. Find the probability that all the girls sit together.

Solution 6

There are 6 boys and 6 girls to sit in a row

S = Sitting all 12 boys and girls in a row

$n(S)$ = Number of ways of occuping 12 places by 12 persons

$= {}^{12}P_{12}$

$= 12!$

E = 6 girls sitting together in the row with 6 boys

$n(E)$ = Number of ways to sit 6 girls together with 6 boys

= Number of ways 6 girls sit at 6 places together $\times$ Group of six girls and one boy occupies 7 places

$= {}^{6}P_{6} \times {}^{7}P_{7}$

$= 6! \times 7!$

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{6!\,7!}{12!}$$

$$= \frac{6\times5\times4\times3\times2\times1}{12\times11\times10\times9\times8}$$

$$= \frac{720}{132\times720}$$

$$= \frac{1}{132}$$

Required probability = $\frac{1}{132}$.

Question 7

If A and B are two independent events such that $P(a) = 0.3$ and $P\left(A \cup \overline{B}\right) = 0.8$. Find $P(B)$.

Solution 7

Given, A and B are independant events $P(A) = 0.3, p\left(A \cup \overline{B}\right) = 0.8$

We know that,

$$\overline{\left(A \cup \overline{B}\right)} = \left(\overline{A} \cap B\right)$$

$$P\left(\overline{A} \cap B\right) = 1 - P\left(A \cup \overline{B}\right)$$

$$P\left(\overline{A}\right)P(B) = 1 - 0.8$$

$$\left(1 - P(A)\right)P(B) = 0.2$$

$$(1 - 0.3)P(B) = 0.2$$

$$(0.7)P(B) = 0.2$$

$$P(B) = \frac{0.2}{0.7}$$

$$= \frac{2}{7}$$

$$P(B) = \frac{2}{7}.$$

Question 8

An unbiased die with face marked 1, 2, 3, 4, 5, 6 is rolled four times. Out of 4 face values obtained, find the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5.

Solution 8

Die is rolled 4 times

$n(S)$ = Number of elements in sample space

$$n(S) = 6 \times 6 \times 6 \times 6$$

Given, Numbers obtained on face are not less than 2 and greater than 5.

E = Obtaining 2, 3, 4, 5 on die in four throw

$$n(E) = 4 \times 4 \times 4 \times 4$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{4 \times 4 \times 4 \times 4}{6 \times 6 \times 6 \times 6}$$

$$= \frac{16}{81}$$

Required probability $= \frac{16}{81}$.

Question 9

If A and B are two events write the expression for the probability of occurrence of exactly one of two events.

Solution 9

Given,

A and B are two events

P (Occurance of exactly one of them)

$= P\left[\left(\overline{A} \cap B\right) \cup \left(A \cap \overline{B}\right)\right]$

$= P\left(\overline{A} \cap B\right) + \left(A \cap \overline{B}\right)$

$= \left[P\left(B\right) - P\left(A \cap B\right)\right] + \left[P\left(A\right) - P\left(A \cap B\right)\right]$

$= P\left(A\right) + P\left(B\right) - 2P\left(A \cap B\right)$

Required expression $= P\left(A\right) + P\left(B\right) - 2P\left(A \cap B\right)$

Question 10

Write the probability that a number selected at random from the set of first 100 natural numbers is a cube.

Solution 10

Total number of cubes in first 100 natural numbers = 4 [Since 1, 8, 27, 64 are cubes]

$P\left(\text{Number is cube}\right) = \frac{4}{100}$

$= \frac{1}{25}$

Required probability $= \frac{1}{25}$.

Question 11

In a competition A, B and C are participating. The probability that A wins is twice that of B, the probability that B wins is twice that of C. Find the probability that A losses.

Solution 11

Let $P(A \text{ wins}) = x$

So, $P(B \text{ wins}) = 2x$

$P(A \text{ wins}) = 2P(B \text{ wins})$

$= 2(2x)$

$P(A \text{ wins}) = 4x$

$P(A \text{ wins}) + P(B \text{ wins}) + P(C \text{ wins}) = 1$

$\Rightarrow \quad 4x + 2x + x = 1$

$\Rightarrow \quad 7x = 1$

$\Rightarrow \quad x = \frac{1}{7}$

$P(A \text{ wins}) = 4x$

$= \frac{4}{7}$

$P(A \text{ losses}) = 1 - P(A \text{ wins})$

$= 1 - \frac{4}{7}$

$= \frac{3}{7}$

Required probability $= \frac{3}{7}$.

Question 12

If A, B, C are mutually exclusive and exhaustive events associated to a random experiment, then write the value of $P(A) + P(B) + P(C)$.

Solution 12

Since A, B, C are mutually exclusive

$P(A \cap B) = 0, P(B \cap C) = 0, P(C \cap A) = 0, P(A \cap B \cap C) = 0$

Since A, B, C are mutually exhaustive

$P(A \cup B \cup C) = 1$

We know that,

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$1 = P(A) + P(B) + P(C) - 0 - 0 - 0 + 0$

$1 = P(A) + P(B) + P(C)$

Question 13

If two events A and B are such that $P(\bar{A}) = 0.3, P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$,
find $P(B / \bar{A} \cap \bar{B})$

Solution 13

According to Baye's Theorem

$$P(B/\overline{A}\cap\overline{B})=\frac{P(B\cap(\overline{A}\cap\overline{B}))}{P(\overline{A}\cap\overline{B})}$$
$$=\frac{P(B\cap\overline{(A\cup B)})}{P(\overline{A}\cap\overline{B})}$$
$$=\frac{P(\overline{\overline{B}}\cap\overline{(A\cup B)})}{P(\overline{A}\cap\overline{B})}$$
$$=\frac{P(\overline{B}\cup(A\cup B))}{P(\overline{A}\cap\overline{B})}$$

Now $\overline{B}\cup B=\overline{U}=\phi$

So $P(\overline{B}\cup(A\cup B))=\phi$

Therefore

$$P(B/\overline{A}\cap\overline{B})=0$$

Question 14

If A and B are two independent events, then write $P\left(A\cap\overline{B}\right)$ in terms of $P(A)$ and $P(B)$.

Solution 14

$$P\left(A\cap\overline{B}\right)=P(A)\times P\left(\overline{B}\right)$$
$$=P(A)\times(1-P(B))$$
$$P\left(A\cap\overline{B}\right)=P(A)-P(A)P(B)$$

Question 15

If $P(A)=0.3, P(B)=0.6, P\left(\frac{B}{A}\right)=0.5$. Find $P(A\cup B)$.

Solution 15

Given,

$$P(A)=0.3,\ P(B)=0.6,\ P\left(\frac{B}{A}\right)=0.5$$
$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$
$$0.5=\frac{P(A\cap B)}{0.3}$$
$$P(A\cap B)=0.5\times 0.3$$
$$P(A\cap B)=0.15$$
$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=0.3+0.6-0.15$$
$$=0.75$$
$$P(A\cup B)=0.75.$$