RD SHARMA Solutions for Class 9 Maths Chapter 21 - Surface Areas and Volume of a Sphere

Chapter 21 - Surface Areas and Volume of a Sphere Exercise 21.26

Question 1

In a sphere the number of faces is

(a) 1

(b) 2

(c) 3

(d) 4

Solution 1

Sphere has only one surface i.e. curved surface, so number of faces = 1

Hence, correct option is (a).

Question 2

The total surface area of a hemisphere of radius r is

(a) $\prod r^2$

(b) $2\prod r^2$

(c) $3\prod r^2$

(d) $4\prod r^2$

Solution 2

A hemisphere has two surfaces: one top surface and other curved surface.

T.S.A. = $2\prod r^2 + (\prod r^2)$ {Area of Top-face = $\prod r^2$}

= $3\prod r^2$

Question 3

The ratio of the total surface area of a sphere and a hemisphere of same radius is

(a) 2 : 1

(b) 3 : 2

(c) 4 : 1

(d) 4 : 3

Solution 3

Total Surface Area of Sphere $= 4\pi r^2$

Total surface Area of Hemisphere $= 3\pi r^2$

$\therefore$ Required Ratio $= \frac{4\pi r^2}{3\pi r^2} = \frac{4}{3} = 4:3$

Hence, correct option is (d).

Question 4

A sphere and a cube are of the same height. The ratio of their volumes is

(a) 3 : 4

(b) 21 : 11

(c) 4 : 3

(d) 11 : 21

Solution 4

Height of sphere = Diameter = 2r
Height of cube = Side of cube = Height of sphere = 2r

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cube $= (2r)^3 = 8r^3$

Ratio of their volumes $= \frac{\frac{4}{3}\pi r^3}{8r^3} = \frac{\pi}{6} = \frac{22}{7 \times 6} = \frac{11}{21} = 11 : 21$

Hence, correct option is (d).

Question 5

The largest sphere is cut off from a cube of side 6 cm. The volume of the sphere will be
(a) $27\prod\ cm^3$
(b) $36\prod\ cm^3$
(c) $108\prod\ cm^3$
(d) $12\prod\ cm^3$

Solution 5

The largest sphere that can be cut from a cube of side 6 cm will have
its diameter = side of cube
i.e. $2r = 6\ cm \Rightarrow r = 3\ cm$

Volume of that sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3 \times 3 \times 3 = 36\pi\ cm^3$

Hence, correct option is (b).

Question 6

A cylindrical rod whose height is 8 times of its radius is melted and recast into spherical balls of same radius. The number of balls will be
(a) 4
(b) 3
(c) 6
(d) 8

Solution 6

Volume of cylindrical rod $= \pi r^2 h$
$= \pi r^2 (8r)$ [h = 8r (given)]
$= 8\pi r^3$

Now, if spherical balls have same radius, then the volume of one ball $= \frac{4}{3}\pi r^3$

$\therefore$ No. of balls $= \frac{\text{Volume of Cylindrical Rod}}{\text{Volume of one Rod}} = \frac{8\pi r^3}{\frac{4}{3}\pi r^3} = 6$

Hence, correct option is (c).

Question 7

If the ratio of volumes of two sphere is 1 : 8, then the ratio of their surface areas is
(a) 1 : 2
(b) 1 : 4
(c) 1 : 8
(d) 1 : 16

Solution 7

Volume of sphere $= \frac{4}{3}\pi r^3 = V$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{1}{8}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{1}{2}$$

Now, Surface Area of Sphere $= 4\pi r^2 = S$

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 1:4$$

Hence, correct option is (b).

Question 8

If the surface area of a sphere is $144\prod$ m^2, then its volume (in m^3) is

(a) $288\prod$

(b) $316\prod$

(c) $300\prod$

(d) $188\prod$

Solution 8

Surface Area of Sphere $\Rightarrow 4\pi r^2 = 144\pi$

$\Rightarrow r^2 = 36 \Rightarrow r = 6$

Volume of Sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi\ m^3$

Hence, correct option is (a).

Question 9

If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the surface area of each ball (in sq. cm) is

(a) $100\prod$

(b) $75\prod$

(c) $60\prod$

(d) $50\prod$

Solution 9

Volume of solid sphere $= \frac{4}{3}\pi(10)^3 = \frac{4000\pi}{3}\ cm^3$

Volume 8 solid spheres of radius (say) $r = 8 \times \frac{4}{3}\pi r^3 = \frac{32\pi r^3}{3}\ cm^3$

Now, $\frac{32\pi r^3}{3} = \frac{4000\pi}{3}$

$\Rightarrow r = \left(\frac{1000}{8}\right)^{1/3} = \frac{10}{2} = 5\ cm$

Surface Area of each small ball $= 4\pi r^2 = 4\pi(5)^2 = 100\pi\ cm^2$

Hence, correct option is (a).

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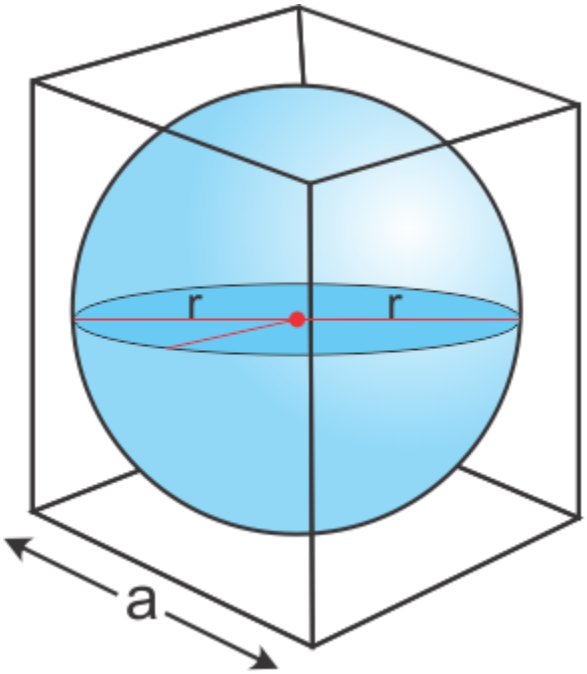
Question 10

If a sphere is inscribed in a cube, then the ratio of the ratio of the volume of the sphere to the volume of the cube is

(a) $\prod$: 2
(b) $\prod$: 3
(c) $\prod$: 4
(d) $\prod$: 6

Solution 10



Edge of cube $= a$

$\Rightarrow$ Volume of cube $= a^3$

If Sphere is inscribed inside cube then $a = 2r \Rightarrow r = \frac{a}{2}$

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{\pi}{6}a^3$

Ratio of volume of sphere to volume of cube $= \frac{\frac{\pi}{6}a^3}{a^3} = \frac{\pi}{6}$

Hence, correct option is (d).

Question 11

If a solid sphere of radius r is melted and cast into the shape of a solid cone of height r, then the radius of the base of the cone is

(a) 2r
(b) 3r
(c) r
(d) 4r

Solution 11

Volume of sphere $= \frac{4}{3}\pi r^3$

Sphere casted into a cone of height r.

Let the radius of cone = R

$\therefore$ Volume of cone $= \frac{1}{3}\pi R^2(r)$

Volume of cone = Volume of sphere

$\Rightarrow \frac{1}{3}\pi R^2 r = \frac{4}{3}\pi r^3$

$\Rightarrow R^2 = 4r^2$

$\Rightarrow R = 2r$

Hence, correct option is (a).

Question 12

A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is r, then the volume of the cylinder is

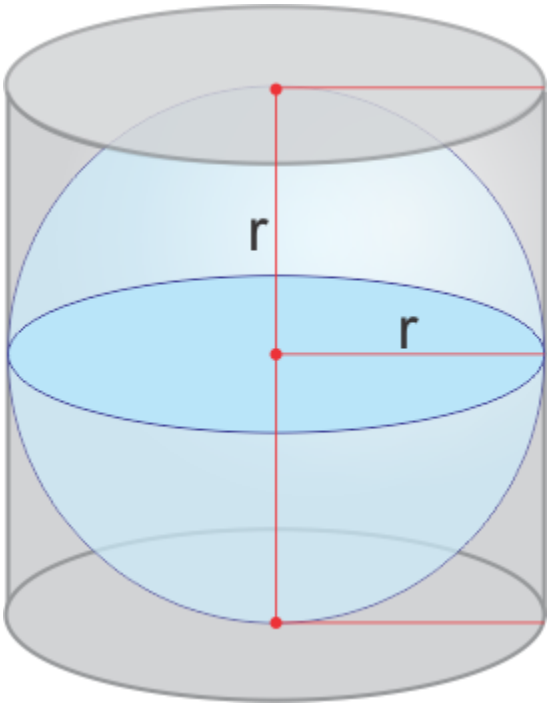
(a) $4\pi r^3$

(b) $\frac{8}{3}\pi r^3$

(c) $2\pi r^3$

(d) $8\pi r^3$

Solution 12



Radius of sphere = r

Sphere touches cylinder at Top, Base and Lateral Surface.

Then,

2r = height of cylinder = h

r = Radius of cylinder

Volume of cylinder = $\prod r^2 h$

$= \prod r^2(2r)$

$= 2\prod r^3$

Hence, correct option is (c).

Question 13

The ratio between the volume of a sphere and volume of a circumscribing right circular cylinder is

(a) 2 : 1
(b) 1 : 1
(c) 2 : 3
(d) 1 : 2

Solution 13

Volume of sphere of radius r $= \frac{4}{3}\pi r^3 = V_1$ ....(1)

If a cylinder is circumscribing the sphere, then
diameter of cylinder = diameter of sphere
height of cylinder = diameter of sphere
$\Rightarrow$ Radius of cylinder = Radius of sphere
Height of cylinder = 2r
Volume of cylinder $= V_2 = \pi r^2 h$

$= \pi r^2 (2r)$

$\Rightarrow V_2 = 2\pi r^3$ ....(2)

Dividing equation (1) and (2)

$$\frac{V_1}{V_2} = \frac{4/3\ \pi r^3}{2\pi r^3}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{2}{3}$$

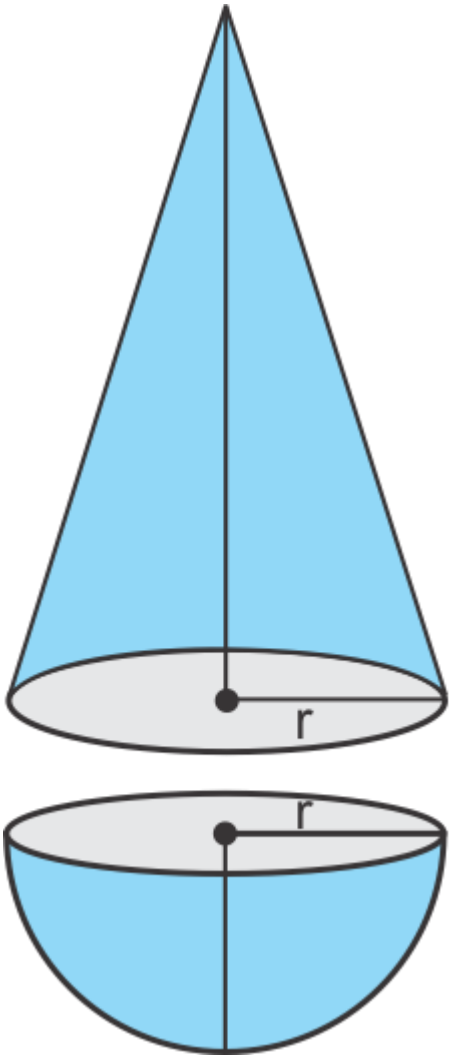
Hence, correct option is (c).

Question 14

A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is

(a) 1 : 2
(b) 2 : 1
(c) 4 : 1
(d) $\sqrt{2}$: 1

Solution 14



Let the radius of cone = r

Then, radius of hemisphere = r (Both have equal bases)

$V_{cone} = \frac{1}{3}\pi r^2 h$

$V_{Hemisphere} = \frac{2}{3}\pi r^3$

$V_{cone} = V_{Hemisphere}$

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow h = 2r$

$\Rightarrow \frac{h}{r} = \frac{2}{1}$ {r = height of hemisphere}

Hence, correct option is (b).

Question 15

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is

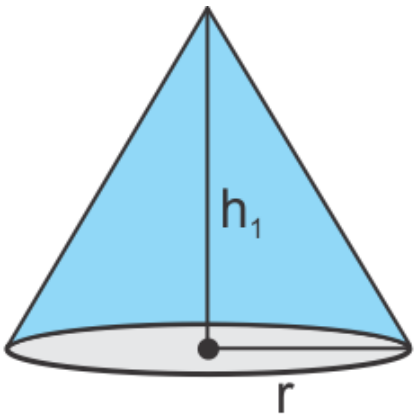
(a) 1 : 2 : 3

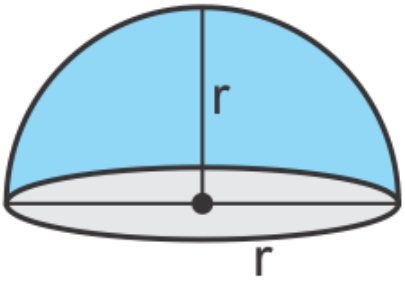
(b) 2 : 1 : 3

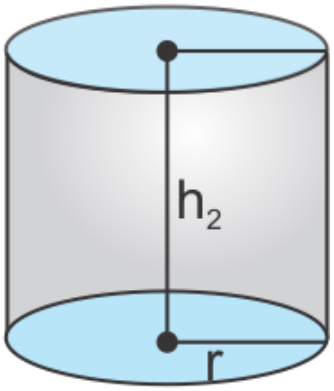
(c) 2 : 3 : 1

(d) 3 : 2 : 1

Solution 15







Cone **Hemisphere** **Cylinder**

If all of these have equal bases, then their radii are equal.
Their heights are same. (given)

$r = h_1 = h_2$

$V_{cone} = \frac{1}{3}\pi r^2 h_1 = \frac{1}{3}\pi r^2(r) = \frac{1}{3}\pi r^3$

$V_{hemisphere} = \frac{2}{3}\pi r^3$

$V_{cylinder} = \pi r^2 h_2 = \pi r^2 (r) = \pi r^3$

$V_{cone} : V_{hemisphere} : V_{cylinder} = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$

Hence, correct option is (a).

Chapter 21 - Surface Areas and Volume of a Sphere Exercise Ex. 21.1

Question 1

Find the surface area of a sphere of radius.

(i) $10.5cm$ (ii) $5.6cm$ (iii) $14cm$

Solution 1

(i) $r = 10.5cm$

$$S.A = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times (10.5)^2$$
$$= 1386cm^2$$

(ii) $r = 5.6cm$

$$S.A = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times (5.6)^2$$
$$= 394.24cm^2$$

(iii) $r = 14cm$

$$S.A = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times (14)^2$$
$$= 2464cm^2$$

Question 2

Find the surface area of a sphere of diameter.

(i) $14cm$ (ii) $21cm$ (iii) $3.5cm$

Solution 2

(i) $D = 14cm$

$$r = \frac{D}{2} = \frac{14}{2} = 7cm$$

$$\therefore S.A = 4\pi r^2 = 4 \times \frac{22}{7} \times (7)^2$$
$$= 616cm^2$$

(ii) $D = 21cm$

$$r = \frac{D}{2} = \frac{21}{2} = 10.5cm$$

$$\therefore S.A = 4\pi r^2 = 4 \times \frac{22}{7} \times (10.5)^2$$
$$= 1386cm^2$$

(iii) $D = 3.5cm$

$$r = \frac{3.5}{2} cm$$

$$\therefore S.A = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{3.5^2}{2^2}$$
$$= 38.5cm^2$$

Question 3

Find the total surface area of a hemisphere and a solid hemisphere each of radius $10cm$ (Use $\pi = 3.14$)

Solution 3

Total surface area of a hemisphere $= 2\pi r^2$

$$= 2 \times 3.14 \times (10)^2$$
$$= 628cm^2$$

Total surface area of a solid hemisphere $= 3\pi r^2$

$$= 3 \times 3.14 \times (10)^2$$
$$= 942cm^2$$

Question 4

The surface area of a sphere is $5544cm^2$, find its diameter.

Solution 4

$\therefore$ Surface area $= 5544 cm^2$

$\Rightarrow \quad 4\pi r^2 = 5544$

$\Rightarrow \quad r^2 = \frac{5544}{4\pi}$

$\Rightarrow \quad r = 21 cm$

$\therefore$ Diameter $= 2r = 42 cm$

Question 5

A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 4 per 100 cm^2.

Solution 5

Inner radius (r) of hemispherical bowl $= \left(\frac{10.5}{2}\right) cm = 5.25$ cm

Surface area of hemispherical bowl $= 2\pi r$

$= \left[2 \times \frac{22}{7} \times (5.25)^2\right] cm^2$

$= 173.25 cm^2$

Cost of tin-plating 100 cm^2 area = Rs 4

Cost of tin-plating 173.25 cm^2 area $= Rs\left(\frac{4 \times 173.25}{100}\right) =$ Rs 6.93

Thus, the cost of tin-plating the inner side of hemispherical bowl is Rs 6.93.

Question 6

The dome of a building is in the form of a hemisphere. Its radius is 63 *dm*. Find the cost of painting it at the rate of Rs 2 per *sq.m*.

Solution 6

Radius of dome $= 63$ dm

$= 6.3$ m

Inner surface area of dome $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 6.3^2$

$= 249.48 m^2$

Now, cost of $1 m^2$ = Rs 2

$\therefore$ Cost of $249.48 m^2$ $= Rs[2 \times 249.48]$

$=$ Rs 498.96

Question 7

Assuming the earth to be a sphere of radius 6370 *km*, how many square kilometres is area of the land, if three-fourth of the earth's surface is covered by water?

Solution 7

$\frac{3}{4}^{th}$ of earth surface is covered by water.

$\therefore \frac{1}{4}^{th}$ earth surface is covered by land.

$\therefore$ Surface area covered by land $= \frac{1}{4} \times 4\pi r^2$

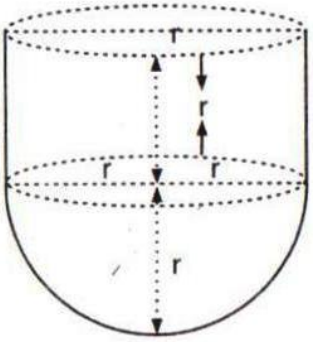
$= \frac{1}{4} \times 4 \times \frac{22}{7} \times (6370)^2$

$= 127527400 km^2$

Question 8

A cylinder of same height and radius is placed on the top of a hemisphere. Find the curved surface area of the shape if the length of the shape be $7cm$

Solution 8



Given: Length of the shape $= 7$ cm

But Length $= r + r$

$\Rightarrow r + r = 7$ cm

$\Rightarrow r = 3.5$ cm

Also, $h = r$

$\therefore$ Total surface area of shape $= 2\pi rh + 2\pi r^2$

$= 2\pi r \times r + 2\pi r^2$

$= 2\pi r^2 + 2\pi r^2$

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154 cm^2$

Question 9

The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Solution 9

Let diameter of the Earth is d. Then, diameter of moon will be $\frac{d}{4}$.

Radius of earth $= \frac{d}{2}$

Radius of moon $= \frac{d}{8}$

Surface area of moon $= 4\pi\left(\frac{d}{8}\right)^2$

Surface area of earth $= 4\pi\left(\frac{d}{2}\right)^2$

Required ratio $= \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = \frac{4}{64} = \frac{1}{16}$

Thus, the required ratio of the surface areas is 1:16.

Question 10

A hemi-spherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 cm, find the cost of painting, if given the cost of painting is Rs 5 per 100 cm^2.

Solution 10

Since only the rounded surface of the dome is to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of the dome = 17.6 m. Therefore, 17.6 = 2 π r.

$$2 \times \frac{22}{7} r = 17.6 \text{ m}$$

So, the radius of the dome = $17.6 \times \frac{7}{2 \times 22}$ m = 2.8 m

The curved surface area of the dome = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 2.8 \times 2.8 \text{ m}^2$$

$$= 49.28 \text{ m}^2$$

Now, cost of painting 100 cm^2 is Rs 5.

So, cost of painting 1 m^2 = Rs 500

Therefore, cost of painting the whole dome

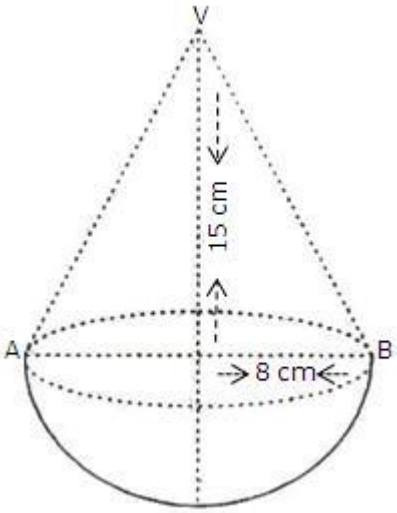
= Rs 500 × 49.28

= Rs 24640

Question 11

A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 16*cm* and its height is 15*cm*. Find the cost of painting the toy at Rs 7 per 100cm^2.

Solution 11



Diameter of cone = $16cm$

$\therefore$ Radius of cone = $8cm$

Height of cone = $15cm$

Slant height of cone $= \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{289}$

$= 17cm$

$\therefore$ Total curved surface area of toy

$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 8 \times 17 + 2 \times \frac{22}{7} \times 8^2$

$= \frac{5808}{7} cm^2$

Now, Cost of $100cm^2$ = Rs 7

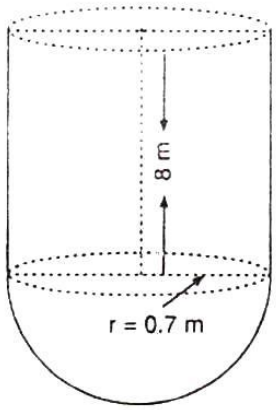
$\therefore$ $1cm^2$ = Rs $\frac{7}{100}$

Hence, Cost of $\frac{5808}{7} cm^2$ = Rs $\left(\frac{5808}{7} \times \frac{7}{100}\right)$ = Rs 58.08

Question 12

A storage tank consists of a circular cylinder with a hemisphere adjoined on either end. If the external diameter of the cylinder be $1.4m$ and its length be $8m$, find the cost of painting it on the outside at the rate of Rs 10 per m^2.

Solution 12



$$\text{Diameter of cylinder} = 1.4m$$

$$\therefore \text{Radius of cylinder} = \frac{1.4}{2} = 0.7m$$

$$\text{Height of cylinder} = 8m$$

$$\therefore \text{Surface area of tank} = 2\pi rh + 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 0.7 \times 8 + 2 \times \frac{22}{7} \times 0.7^2$$

$$= \frac{176}{5} + \frac{77}{25}$$

$$= \frac{957}{25} = 38.28m^2$$

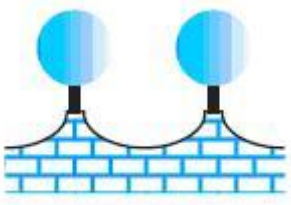
$$\text{Now, cost of } 1m^2 = \text{Rs } 10$$

$$\therefore \text{Cost of } 38.28m^2 = \text{Rs}[10 \times 38.28]$$

$$= \text{Rs } 382.80$$

Question 13

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the given figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Solution 13

Radius (r) of a wooden sphere = $\left(\frac{21}{2}\right)$ cm = 10.5cm

Surface area of a wooden sphere = $4\pi r^2 = \left[4 \times \frac{22}{7} \times (10.5)^2\right] cm^2 = 1386 cm^2$

Radius (r') of cylindrical support = 1.5 cm
Height (h) of cylindrical support = 7 cm

CSA of cylindrical support = $2\ \pi r'h\ \left[2 \times \frac{22}{7} \times 1.5 \times 7\right] = 66\ cm^2$

Area of circular end of cylindrical support = $\pi r^2 \left[\frac{22}{7} \times (1.5)^2\right] = 7.07\ cm^2 = 7.07\ cm^2$

Area to be painted silver = [8 × (1386 - 7.07)] cm^2
= (8 × 1378.93) cm^2 = 11031.44 cm^2
Cost occurred in painting silver colour = Rs (11031.44 × 0.25) = Rs 2757.86

Area to painted black = (8 × 66) cm^2 = 528 cm^2
Cost occurred in painting black colour = Rs (528 × 0.05) = Rs 26.40

∴ Total cost occurred in painting = Rs (2757.86 + 26.40) = Rs 2784.26

Chapter 21 - Surface Areas and Volume of A Sphere Exercise Ex. 21.2

Question 1

Find the volume of sphere whose radius is:

(i) 2*cm* (ii) 3.5*cm* (iii) 10.5*cm*

Solution 1

(i) $r = 2cm$

$$\therefore \text{Volume} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 2^3$$
$$= 33.52cm^3$$

(ii) $r = 3.5cm$

$$\therefore \text{Volume} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (3.5)^3$$
$$= 179.666cm^3$$

(iii) $r = 10.5cm$

$$\therefore \text{Volume} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$$
$$= 4851cm^3$$

Question 2

Find the volume of sphere whose diameter is:

(i) $14cm$ (ii) $3.5dm$ (iii) $2.1m$

Solution 2

(i) $D = 14cm$

$\Rightarrow r = 7cm$

$\therefore \text{Volume} = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7^3$

$= 1437.33cm^3$

(ii) $D = 3.5dm$

$\Rightarrow r = \frac{3.5}{2}dm$

$\therefore \text{Volume} = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{3.5}{2}\right)^3$

$= 22.46dm^3$

(iii) $D = 2.1m$

$\Rightarrow r = \frac{2.1}{2}m$

$\therefore \text{Volume} = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{2.1}{2}\right)^3$

$= 4.851m^3$

Question 3

A hemispherical tank has inner radius of $2.8m$. Find its capacity in litres.

Solution 3

Radius of tank $= 2.8m$

$\therefore \text{Capacity} = \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (2.8)^3$

$= 45.994m^3$

$\therefore$ Capacity in litres = 45994 litres $[1m^3 = 1000l]$

Question 4

A hemispherical bowl is made of steel $0.25cm$ thick. The inside radius of the bowl is $5cm$. Find the volume of steel used in making the bowl.

Solution 4

Inner radius $= 5cm$

Outer radius $= 5 + 0.25$

$= 5.25$

$\therefore$ Volume of steel used = Outer volume−inner volume

$$= \frac{2}{3}\pi\left(R^3 - r^3\right)$$

$$= \frac{2}{3} \times \frac{22}{7}\left[5.25^3 - 5^3\right]$$

$$= 41.282cm^3$$

Question 5

How many bullets can be made out of a cube of lead, whose edge measure 22cm, each bullet being $2cm$ in diameter?

Solution 5

Cube edge $= 22cm$

$\therefore$ Volume of cube $= (22)^3$

$= 10648cm^3$

And,

Volume of each bullet $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{2}{2}\right)^3$$

$$= \frac{4}{3} \times \frac{22}{7}$$

$$= \frac{88}{21}cm^3$$

$\therefore$ No of bullets $= \frac{\text{Volume of cube}}{\text{Volume of bullet}}$

$$= \frac{10648}{\frac{88}{21}} = 2541$$

Question 6

A shopkeeper has one laddoo of radius $5cm$. With the same material, how many laddoos of radius $2.5cm$ can be made.

Solution 6

Volume of laddoo having radius $= 5cm$

$$i.e\ v_1 = \frac{4}{3}\pi r^3$$

$$v_1 = \frac{4}{3} \times \frac{22}{7} \times (5)^3$$

$$v_1 = \frac{11000}{21} cm^3$$

Also volume of laddoo having radius 2.5cm

$$i.e.,\ v_2 = \frac{4}{3}\pi r^3$$

$$v_2 = \frac{4}{3} \times \frac{22}{7} \times (2.5)^3$$

$$v_2 = \frac{1375}{21} cm^3$$

$$\therefore \text{No of laddoos} = \frac{v_1}{v_2} = \frac{11000}{1375} = 8$$

Question 7

A spherical ball of lead $3cm$ in diameter is melted and recast in to three spherical balls. If diameters of two balls be $\frac{3}{2}cm$ and $2cm$. Find the diameter of the third ball.

Solution 7

Volume of lead ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3$

$\therefore$ According to question,

Volume of lead ball $= \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi\left(\frac{2}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$\Rightarrow \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi\left(\frac{2}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$

$\Rightarrow \frac{27}{8} = \frac{27}{64} + \frac{8}{8} + \frac{d^3}{8}$

$\Rightarrow \frac{d^3}{8} = -\left(\frac{27}{64} + 1\right) + \frac{27}{8}$

$\Rightarrow \frac{d^3}{8} = \frac{-(27+64)+216}{64}$

$\Rightarrow \frac{d^3}{8} = \frac{125}{64}$

$\Rightarrow \frac{d}{2} = \frac{5}{4}$

$\Rightarrow d = \frac{10}{4} = 2.5cm$

Hence $d = 2.5cm$

Question 8

A sphere of radius $5cm$ is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}cm$. Find the radius of the cylinder.

Solution 8

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(5)^3$

$\therefore$ Volume of water rises in cylinder = volume of sphere

Let r' be the radius of cylinder

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow r'^2 \times \frac{5}{3} = \frac{4}{3}(5)^3$

$\Rightarrow r'^2 = 4 \times 5^2 \text{ cm}^2 = 100 \text{ cm}^2$

$\Rightarrow r' = 10 \text{ cm}$

Question 9

If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Solution 9

Let V_1 and V_2 be the volumes of first sphere and second sphere respectively.

Radius of first sphere = r

Radius of second sphere = 2r

$$\therefore \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (2r)^3} = \frac{1}{8}$$

Question 10

A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Solution 10

Volume of water in the hemispherical bowl = volume of water in the cylinder

Let 'h' be the height to which water rises in the cylinder.

$$\Rightarrow \frac{2}{3}\pi {r_1}^3 = \pi {r_2}^2 h$$

$$\Rightarrow h = \frac{2r_1^3}{3r_2^2} = \frac{2(3.5)^3}{3(7)^2}$$

$$\Rightarrow h = \frac{7}{12} cm$$

Question 11

A cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4cm. Calculate the radius of the base of the cylinder.

Solution 11

$$\text{Height of cylinder} = \frac{2}{3}\text{diameter}$$

$$= \frac{2}{3} \times 2r$$

$$= \frac{4}{3} r$$

Volume of cylinder = Volume of sphere

$$\Rightarrow \pi r^2 \times \frac{4}{3} r = \frac{4}{3}\pi (4)^3$$

$$\Rightarrow r^3 = 4^3$$

$$\Rightarrow r = 4cm$$

Question 12

A vessel in the form of a hemispherical bowl is full of water.The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively $6cm$ and $4cm$. Find the height of water in the cylinder.

Solution 12

Volume of water in hemispherical bowl = Volume of cylinder

$$\Rightarrow \frac{2}{3}\pi(6)^3 = \pi(4)^2 h$$

Where, h = height of cylinder

$$\Rightarrow h = \frac{2(6)^3}{3(4)^2} = 9$$

$\therefore$ Height of cylinder = $9cm$

Question 13

The diameter of a coper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, find its diameter.

Solution 13

Diameter of the sphere = $18cm$

$\therefore$ Radius of the sphere = $9cm$

Length of the wire = $108m$

$= 10800cm$

Volume of cylinder = Volume of sphere

$$\Rightarrow \pi r_1^2 h = \frac{4}{3}\pi r_2^3$$

$$\Rightarrow r_1^2 \times 10800 = \frac{4}{3} \times 9 \times 9 \times 9$$

$$\Rightarrow r_1^2 = \frac{4 \times 3 \times 9 \times 9}{10800}$$

$$\Rightarrow r_1^2 = 0.09$$

$$\Rightarrow r_1 = 0.3 \text{ cm}$$

$\therefore$ Diameter = 2×0.3 cm = 0.6 cm

Question 14

The diameter of a sphere is $6cm$. It is melted and drawn into a wire of diameter $0.2cm$.Find the length of the wire.

Solution 14

Diameter of sphere = $6cm$
Radius of sphere = $3cm$
Diameter of the wire=$0.2cm$
Radius of the wire = $0.1cm$

Volume of sphere = Volume of *wire*

$$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$
$$\Rightarrow \frac{4}{3}\times 3\times 3\times 3 = 0.1\times 0.1\times h$$
$$\Rightarrow \frac{4\times 3\times 3}{0.1\times 0.1} = h$$
$$\Rightarrow 3600 = h$$
$$\Rightarrow 36m = h$$

$\therefore$ Length of wire = $36m$

Question 15

The radius of the internal and external surfaces of a hollow spherical shell are $3cm$ and $5cm$ respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}cm$. Find the diameter of the cylinder.

Solution 15

Internal radius of the sphere = $3cm$
External Radius of the sphere = $5cm$

Height of cylinder = $2\frac{2}{3}cm$
$= \frac{8}{3}cm$

Volume of spherical shell = Volume of the cylinder

$$\Rightarrow \frac{4}{3}\pi\left(r_2^3 - r_1^3\right) = \pi r_3^2 h$$
$$\Rightarrow \frac{4}{3}\left(5^3 - 3^3\right) = r_3^2\times\frac{8}{3}$$
$$\Rightarrow \frac{4}{3}\times\left(125 - 27\right) = r_3^2\times\frac{8}{3}$$
$$\Rightarrow \frac{4\times 98\times 3}{3\times 8} = r_3^2$$
$$\Rightarrow 49 = r_3^2$$
$$\Rightarrow \sqrt{49} = r_3$$
$$\Rightarrow 7cm = r_3$$

$\therefore$ Diameter of the cylinder = $14cm$

Question 16

A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Solution 16

Radius of the hemisphere = $7cm$

Height of cone = $49cm$

Volume of hemisphere = Volume of cone

$$\Rightarrow \frac{2}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 \times h$$

$$\Rightarrow \frac{2}{3} \times 7^3 = \frac{1}{3} r_2^2 \times 49$$

$$\Rightarrow \frac{2 \times 7 \times 7 \times 7 \times 3}{3 \times 49} = r_2^2$$

$$\Rightarrow r_2^2 = 14$$

$$\Rightarrow r_2 = 3.74cm$$

$\therefore$ Radius of the base = $3.74cm$

Question 17

A hollow sphere of internal and external radii $2cm$ and $4cm$ respectively is melted into a cone of base radius $4cm$. Find the height and slant height of the cone.

Solution 17

$$V_{cone} = V_{sphere}$$

$$\Rightarrow \frac{1}{3}\pi R^2 H = \frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3$$

$$\Rightarrow \frac{1}{3}\pi R^2 H = \frac{4}{3}\pi \left(R_2^3 - R_1^3\right)$$

$$\Rightarrow 4^2 H = 4\left(4^3 - 2^3\right)$$

$$\Rightarrow 16H = 4 \times (64 - 8)$$

$$\Rightarrow H = \frac{4 \times 56}{16} = 14cm$$

Slant height = $\sqrt{R^2 + H^2}$

$$\Rightarrow l = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212} = 14.56cm$$

Question 18

A metallic sphere of radius $10.5cm$ is melted and thus recast into small cones, each of radius $3.5cm$ and height $3cm$. Find how many cones are obtained.

Solution 18

Let the number of cones obtained be x

$$V_s = x \times V_{cone}$$

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = x \times \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5 = x \times \frac{1}{3}\pi 3.5 \times 3.5 \times 3$$

$$\Rightarrow \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3} = x$$

$$\Rightarrow x = 126$$

$\therefore$ Number of cones = 126

Question 19

A cone and hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Solution 19

$$V_{cone} = V_{hemisphere}$$

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad r^2 h = 2r^3$$

$$\Rightarrow \quad h = 2r$$

$$\Rightarrow \quad h : r = 2r : r = 2 : 1$$

Question 20

The largest sphere is carved out of a cube of side 10.5cm. Find the volume of the sphere.

Solution 20

We have,

Diameter of the largest sphere = 10.5cm

$$2r = 10.5$$

$$r = 5.25cm$$

$$\therefore \text{ Volume of sphere} = \frac{4}{3} \times \frac{22}{7} \times 5.25^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25$$

$$= \frac{11 \times 441}{8} cm^3$$

$$= 606.375 cm^3$$

Question 21

A cube of side 4 cm contained a sphere touching its sides. Find the volume of the gap in between.

Solution 21

Length of the side of the cube = 4 cm

Volume of the cube = $(4\text{ cm})^3 = 64\text{ cm}^3$

Diameter of the sphere = Length of the side of the cube = 4 cm

$\therefore$ Radius of sphere = 2 cm

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times(2)^3 = 33.52\text{ cm}^3$

$\therefore$ Volume of the gap = Volume of cube − Volume of sphere

$= 64\text{ cm}^3 - 33.52\text{ cm}^3 = 30.48\text{ cm}^3$

Question 22

A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Solution 22

Inner radius (r_1) of hemispherical tank = 1 m

Thickness of hemispherical tank = 1 cm = 0.01 m

Outer radius (r_2) of hemispherical tank = (1 + 0.01) m = 1.01 m

Volume of iron used to make the tank = $\frac{2}{3}\pi\left(r_2^3 - r_1^3\right)$

$$= \left[\frac{2}{3}\times\frac{22}{7}\times\{(1.01)^3 - (1)^3\}\right]\text{ m}^3$$

$$= \left[\frac{44}{21}\times\{1.030301 - 1\}\right]\text{ m}^3$$

$$= 0.06348\text{ m}^3 \quad \text{(approximately)}$$

Question 23

A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Solution 23

Radius (r) of capsule $= \left(\frac{3.5}{2}\right)\text{mm} = 1.75\text{ mm}$

Volume of spherical capsule $= \frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3}\times\frac{22}{7}\times(1.75)^3\right]\text{ mm}^3$$

$$= 22.458\text{ mm}^3$$

Thus, approximately 22.46 mm^3 of medicine is required to fill the capsule.

Question 24

The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Solution 24

Let diameter of earth be d. So, radius earth will be $\frac{d}{2}$.

Then, diameter of moon will be $\frac{d}{4}$. So, radius of moon will be $\frac{d}{8}$.

Volume of moon = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{8}\right)^3 = \frac{1}{512}\times\frac{4}{3}\pi d^3$

Volume of earth = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{8}\times\frac{4}{3}\pi d^3$

$$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{1}{512}\times\frac{4}{3}\pi d^3}{\frac{1}{8}\times\frac{4}{3}\pi d^3} = \frac{1}{64}$$

Thus, the volume of moon is $\frac{1}{64}$ of volume of earth.

Question 25

A cone and hemisphere have equal bases and equal volumes.
Find the ratio of their heights.

Solution 25

Volume of cone = volume of hemisphere [given]

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow r^2 h = 2r^3$

$\Rightarrow h = 2r$

$\Rightarrow \frac{h}{r} = \frac{1}{1}\times 2 = \frac{2}{1}$

$\therefore$ Ratio of the their height is 2:1

Question 26

A cylinderical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball?

Solution 26

Let r be the radius of the iron ball.

Then, Volume of iron ball = volume of water raised in the tub

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 h$$
$$\Rightarrow \frac{4}{3} r^3 = (16)^2 \times 9$$
$$\Rightarrow r^3 = 64 \times 9 \times 3$$
$$\Rightarrow r^3 = 1728$$
$$\Rightarrow r = 12 \text{ cm}$$

Therefore, radius of the ball = 12 cm

Question 27

A cylinder of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm. Find the radius of the ball. (Use π = 22/7)

Solution 27

Radius of cylinder = $12cm$

Raised in height = 6.75

Volume of water raised = volume of sphere

$$\Rightarrow \pi r_1^2 h = \frac{4}{3}\pi r_2^3$$
$$\Rightarrow 12 \times 12 \times 6.75 = \frac{4}{3} r_2^3$$
$$\Rightarrow \frac{12 \times 12 \times 6.75 \times 3}{4} = r_2^3$$
$$\Rightarrow 729 = r_2^3$$
$$\Rightarrow 9^3 = r_2^3$$
$$\Rightarrow 9cm = r_2$$

$\therefore$ Radius of sphere is $9cm$.

Question 28

A cylinderical jar of radius $6cm$ contains oil. Iron spheres each of radius $1.5cm$ are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimetres?

Solution 28

Radius of the cylinder = $6cm$
Level of oil raised = $2cm$
Radius of each sphere = $1.5cm$

$$\text{Number of sphere} = \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$= \frac{\pi r_1^2 h}{\frac{4}{3}\pi r_2^3}$$

$$= \frac{r_1^2 h}{\frac{4}{3} r_2^3}$$

$$= \frac{6 \times 6 \times 2}{\frac{4}{3} \times 1.5 \times 1.5 \times 1.5}$$

$$= \frac{6 \times 6 \times 2 \times 3}{4 \times 1.5 \times 1.5 \times 1.5}$$

Number of sphere = 16

Question 29

A measuring jar of internal diameter $10cm$ is partially filled with water. Four equal spherical balls of diameter $2cm$ each are dropped in it and they sink down in water completely.What will be the change in the level of water in the jar?

Solution 29

Diameter of jar = $10cm$
Radius of jar = $5cm$

Let the level of water raised by $"h"$
Diameter of spherical ball = $2cm$
Radius of the ball = $1cm$

Volume of jar = 4(Volume of spherical)

$$\Rightarrow \pi r_1^2 h = 4\left(\frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow r_1^2 h = 4 \times \frac{4}{3} r_2^3$$

$$\Rightarrow 5 \times 5 \times h = 4 \times \frac{4}{3} \times 1 \times 1 \times 1$$

$$\Rightarrow h = \frac{4 \times 4 \times 1}{3 \times 5 \times 5}$$

$$\Rightarrow h = \frac{16}{75} cm$$

$\therefore$ Height of water in jar = $\frac{16}{75} cm$

Question 30

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1:2:3.

Solution 30

We know that,

$V_{cone} : V_{hemisphere} : V_{cylinder}$

$\frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

Multiplying by 3

or, $\pi r^2 h : 2\pi r^3 : 3\pi r^2 h$

or, $\pi r^3 : 2\pi r^3 : 3\pi r^3$ $\quad \left[\because r = h \;\therefore r^2 h = r^3\right]$

or 1 : 2 : 3

Question 31

A cylinderical tub of radius 12*cm* contains water to a depth of 20*cm*. A spherical form ball is dropped into the tub and thus the level of water is raised by 6.75*cm*.What is the radius of the ball?

Solution 31

Let r cm be the radius of the ball.

Then, volume of ball = volume of water raised

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi (12)^2 \times 6.75$

$\Rightarrow \quad r^3 = \frac{144 \times 6.75 \times 3}{4}$

$\Rightarrow \quad r^3 = 729$

$\Rightarrow \quad r = 9cm$

Thus, radius of the ball = 9 cm

Question 32

A sphere, a cylinder and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere. Find the ratio of their volumes.

Solution 32

Let r be the common radius thus,
h = height of the cone = height of the cylinder = $2r$

Let,

V_1 = volume of sphere = $\frac{4}{3}\pi r^3$

V_2 = volume of cylinder = $\pi r^2 \times 2r = 2\pi r^3$

V_3 = volume of the cone = $\frac{1}{3}\pi r^2 \times 2r = \frac{2}{3}\pi r^3$

Now,

$$V_1 : V_2 : V_3 = \frac{4}{3}\pi r^3 : 2\pi r^3 : \frac{2}{3}\pi r^3$$
$$= 4 : 6 : 2$$
$$= 2 : 3 : 1$$