# RD SHARMA Solutions for Class 12-science Maths Chapter 26 - Scalar Triple Product

## Chapter 26 - Scalar Triple Product Exercise Ex. 26.1

Question 1(i)

Evaluate $\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]$

Solution 1(i)

We have

$$\begin{aligned}\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right] &= \left(\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{j}\times\hat{k}\right)\cdot\hat{i}+\left(\hat{k}\times\hat{i}\right)\cdot\hat{j}\\ &= \hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}\\ &= 1+1+1\\ &= 3\end{aligned}$$

Therefore, $\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]=3$

Question 1(ii)

Evaluate $\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]$

Solution 1(ii)

We have

$$\begin{aligned}\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right] &= \left(2\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{i}\times\hat{k}\right)\cdot\hat{j}+\left(\hat{k}\times\hat{j}\right)\cdot 2\hat{i}\\ &= 2\hat{k}\cdot\hat{k}+\left(-\hat{j}\right)\cdot\hat{j}+\left(-\hat{i}\right)\cdot 2\hat{i}\\ &= 2-1-2\\ &= -1\end{aligned}$$

Therefore, $\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]=-1$

Question 2(i)

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when

$\vec{a}=2\hat{i}-3\hat{j}, \vec{b}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=3\hat{i}-\hat{k}$

Solution 2(i)

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}$$

$$=2(-1-0)+3(-1+3)$$

$$=-2+6$$

$$=4$$

Therefore, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=4$

Question 2(ii)

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when

$\vec{a}=\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=\hat{j}+\hat{k}$

Solution 2(ii)

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & -1\\ 0 & 1 & 1\end{vmatrix}$$

$$=1(1+1)+2(2+0)+3(2-0)$$

$$=2+4+6$$

$$=12$$

Therefore, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=12$

Question 3(i)

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors

$\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}, \vec{c}=3\hat{i}-\hat{j}+2\hat{k}$

Solution 3(i)

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & 3 & 4\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}$$

$$=2(4-1)-3(2+3)+4(-1-6)$$

$$=6-15-28$$

$$=-9-28$$

$$=-37$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|-37|=37$ cubic unit.

Question 3(ii)

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}, \vec{c}=3\hat{i}-\hat{j}-2\hat{k}$

Solution 3(ii)

Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}, \vec{c}=3\hat{i}-\hat{j}-2\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$
\begin{aligned}
\left[\vec{a}\ \vec{b}\ \vec{c}\right] &= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix} \\
&= 2(-4-1)+3(-2+3)+4(-1-6) \\
&= -10+3-28 \\
&= -10-25 \\
&= -35
\end{aligned}
$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = |-35| = 35$ cubic unit.

Question 3(iii)

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a}=11\hat{i}, \vec{b}=2\hat{j}, \vec{c}=13\hat{k}$

Solution 3(iii)

Let $\vec{a}=11\hat{i}, \vec{b}=2\hat{j}, \vec{c}=13\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$
\begin{aligned}
\left[\vec{a}\ \vec{b}\ \vec{c}\right] &= \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix} \\
&= 11(26-0)+0+0 \\
&= 286
\end{aligned}
$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = |286| = 286$ cubic unit.

Question 3(iv)

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

Solution 3(iv)

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}$$
$$=1(1-2)-1(-1-1)+1(2+1)$$
$$=-1+2+3$$
$$=4$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|4|=4$ cubic unit.

Question 4(i)

Show that each of the following triads of vectors are coplanar:
$\vec{a}=\hat{i}+2\hat{j}-\hat{k}, \vec{b}=3\hat{i}+2\hat{j}+7\hat{k}, \vec{c}=5\hat{i}+6\hat{j}+5\hat{k}$

Solution 4(i)

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

Here,

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & 2 & -1\\ 3 & 2 & 7\\ 5 & 6 & 5\end{vmatrix}$$
$$=1(10-42)-2(15-35)-1(18-10)$$
$$=-32+40-8$$
$$=0$$

Hence, the given vectors are coplanar.

Question 4(ii)

Show that each of the following triads of vectors are coplanar:
$\vec{a}=-4\hat{i}-6\hat{j}-2\hat{k}, \vec{b}=-\hat{i}+4\hat{j}+3\hat{k}, \vec{c}=-8\hat{i}-\hat{j}+3\hat{k}$

Solution 4(ii)

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.

Here,

$$\begin{aligned}\left[\vec{a}\ \vec{b}\ \vec{c}\right] &= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} \\ &= -4(12+3)+6(-3+24)-2(1+32) \\ &= -60+126-66 \\ &= 0\end{aligned}$$

Hence, the given vectors are coplanar.

Question 4(iii)

Show that each of the following triads of vectors are coplanar:

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Solution 4(iii)

We know that three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff their scalar triple product is zero i.e. $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.

Here,

$$\begin{aligned}\left[\vec{a}\ \vec{b}\ \vec{c}\right] &= \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} \\ &= 1(15-12)+2(-10+4)+3(6-3) \\ &= 3-12+9 \\ &= 0\end{aligned}$$

Hence, the given vectors are coplanar.

Question 5(i)

Find the value of $\lambda$ such that the following vectors are coplanar:

$\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Solution 5(i)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0$$

$$= 1(\lambda - 1) + 1(2\lambda + \lambda) + 1(-2 - \lambda)$$
$$= \lambda - 1 + 3\lambda - 2 - \lambda$$
$$3 = 3\lambda$$
$$1 = \lambda$$

Question 5(ii)

Find the value of $\lambda$ such that the following vectors are coplanar:
$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{c} = \lambda\hat{i} + \lambda\hat{j} + 5\hat{k}$

Solution 5(ii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix} = 0$$

$$= 2(10 + 3\lambda) + 1(5 + 3\lambda) + 1(\lambda - 2\lambda)$$
$$= 20 + 6\lambda + 5 + 3\lambda - \lambda$$
$$-25 = 8\lambda$$
$$\lambda = -\frac{25}{8}$$

Question 5(iii)

Find the value of $\lambda$ such that the following vectors are coplanar:
$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 3\hat{i} + \lambda\hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$

Solution 5(iii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix}=0$$

$$=1(2\lambda-2)-2(6-1)-3(6-\lambda)$$
$$=2\lambda-2-12+2-18+3\lambda$$
$$=5\lambda-30$$
$$30=5\lambda$$
$$\lambda=6$$

Question 5(iv)

Find the value of $\lambda$ such that the following vectors are coplanar:
$\vec{a}=\hat{i}+3\hat{j}, \vec{b}=5\hat{k}, \vec{c}=\lambda\hat{i}-\hat{j}$

Solution 5(iv)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix}=0$$

$$=1(0+5)-3(0-5\lambda)+0$$
$$=5+15\lambda$$
$$-5=15\lambda$$
$$\lambda=-\frac{1}{3}$$

Question 6

Show that the four points having position vectors $6\hat{i}-7\hat{j}, 16\hat{i}-19\hat{j}-4\hat{k}, 3\hat{j}-6\hat{k}, 2\hat{i}+5\hat{j}+10\hat{k}$ are not co-planer

Solution 6

Let

$OA = 6\hat{i} - 7\hat{j}, OB = 16\hat{i} - 19\hat{j} - 4\hat{k}, OC = 3\hat{j} - 6\hat{k}, OD = 2\hat{i} + 5\hat{j} + 10\hat{k}$

$AB = OB - OA = 16\hat{i} - 25\hat{j} - 4\hat{k}$

$AC = OC - OA = -16\hat{i} - 16\hat{j} + 2\hat{k}$

$CD = OD - OC = 2\hat{i} + 2\hat{j} + 16\hat{k}$

$AD = OD - OA = -4\hat{i} + 12\hat{j} + 10\hat{k}$

The four points are co-planer if vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are co-planer.

$$\begin{vmatrix} 16 & -25 & -4 \\ -16 & -16 & 2 \\ -4 & 12 & 10 \end{vmatrix} = 16(-160-24) + 25(-160+8) - 4(-144+64)$$

$$\neq 0$$

Hence the points are not coplanar.

Question 7

*Show that the points* $A(-1,4,-3)$, $B(3,2,-5)$, $C(-3,8,-5)$ *and* $D(-3,2,1)$ *are co-planar.*

Solution 7

$AB = \text{position vector of } B - \text{position vector of } A$

$\quad = 4\hat{i} - 2\hat{j} - 2\hat{k}$

$AC = \text{position vector of } C - \text{position vector of } A$

$\quad = -2\hat{i} + 4\hat{j} - 2\hat{k}$

$AD = \text{position vector of } D - \text{position vector of } A$

$\quad = -2\hat{i} - 2\hat{j} + 4\hat{k}$

*The four points are co-planar if the vectors are co-planar.*

$$\text{Thus,} \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix} = 4[16-4] + 2[-8-4] - 2[4+8] = 48 - 24 - 24 = 0$$

*Hence proved.*

Question 8

*Show that the folur points whose position vectors are* $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{i} - 6\hat{k}$, $2\hat{i} - 5\hat{j} + 10\hat{k}$

Solution 8

$Let\ OA = 6\hat{i} - 7\hat{j},\ OB = 16\hat{i} - 19\hat{j} - 4\hat{k},\ OC = 3\hat{i} - 6\hat{k},\ OD = 2\hat{i} - 5\hat{j} + 10\hat{k}$

$Thus,$

$AB = OB - OA = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$AC = OC - OA = -3\hat{i} + 7\hat{j} - 6\hat{k}$

$AD = OD - OA = -4\hat{i} + 2\hat{j} + 10\hat{k}$

$The\ four\ points\ are\ co-planar\ if\ vectors\ AB,\ AC\ and\ AD\ are\ co-planar.$

$Thus,\ we\ have$

$$\begin{vmatrix} 10 & -12 & -4 \\ -3 & 7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 10(70+12) + 12(-30-24) - 4(-6+28) = 820 - 648 - 88$$

Question 9

Find the value of $\lambda$ for which the four points with position vectors $-\hat{j}-\hat{k}, 4\hat{i}+5\hat{j}+\lambda\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

Solution 9

Let

Position vector of $A = -\hat{j} - \hat{k}$

Position vector of $B = 4\hat{i} + 5\hat{j} + \lambda\hat{k}$

Position vector of $C = 3\hat{i} + 9\hat{j} + 4\hat{k}$

Position vector of $D = -4\hat{i} + 4\hat{j} + 4\hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$\overline{AB} = 4\hat{i} + 6\hat{j} + (\lambda+1)\hat{k}$

$\overline{AC} = 3\hat{i} + 10\hat{j} + 5\hat{k}$

$\overline{AD} = -4\hat{i} + 5\hat{j} + 5\hat{k}$

$$\begin{vmatrix} 4 & 6 & (\lambda+1) \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$$

$4(50-25) - 6(15+20) + (\lambda+1)(15+40) = 0$

$100 - 210 + 55 + 55\lambda = 0$

$55\lambda = 55$

$\lambda = 1$

Question 10

Prove that $(\vec{a}-\vec{b}).\{(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\} = 0$

Solution 10

$$
\begin{aligned}
&(\vec{a}-\vec{b}).\{(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\}\\
&=\left[(\vec{a}-\vec{b})\quad(\vec{b}-\vec{c})\quad(\vec{c}-\vec{a})\right]\\
&=\left[a\quad(\vec{b}-\vec{c})\quad(\vec{c}-\vec{a})\right]+\left[-b\quad(\vec{b}-\vec{c})\quad(\vec{c}-\vec{a})\right]\\
&=6[a\quad b\quad c]-6[a\quad b\quad c]\\
&=0
\end{aligned}
$$

Question 11

$\vec{a},\vec{b},\vec{c}$ are the position vector of points A,B,C; prove that $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane of the triangle ABC.

Solution 11

If $\vec{a}$ represents the sides AB , If $\vec{b}$ represents the sides BC, If $\vec{c}$ represents the sides AC of the triang leABC.

$\vec{a}\times\vec{b}$ is perpendicular to the plane of the triang leABC.

$\vec{b}\times\vec{c}$ is perpendicular to the plane of the triangleABC.

$\vec{c}\times\vec{a}$ is perpendicular to the plane of the triang leABC.

Hence $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane of the triangleABC.

Question 12(i)

let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$ ,find $c_3$ which makes $\vec{a},\vec{b},\vec{c}$ coplanar

Solution 12(i)

$\vec{a},\vec{b},\vec{c}$ are coplanar if

$$
\begin{aligned}
&[a\quad b\quad c]=0\\
&\begin{vmatrix}1&1&1\\1&0&0\\1&2&c_3\end{vmatrix}=0\\
&0-1(c_3)+1(2)=0\\
&c_3=2
\end{aligned}
$$

Question 12(ii)

let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$.
Then, If $c_2=-1$ and $c_3=1$, show that no value of $c_1$ can make $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

Solution 12(ii)

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$.

$c_2 = -1$ and $c_3 = 1$,

If $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar, then their scalar triple product is zero.

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$\Rightarrow -c_3 + c_2 = 0$

$\Rightarrow c_2 = c_3$

But this is a contradiction as it is given that $c_2 = -1$ and $c_3 = 1$.

Hence, no value of $c_1$ can make the vectors coplanar.

Question 13

Find $\lambda$ for which the points $A(3,2,1), B(4,\lambda,5), C(4,2,-2)$ and $D(6,5,-1)$ are coplanar

Solution 13

Let

Position vector of $OA = 3\hat{i}+2\hat{j}+\hat{k}$

Position vector of $OB = 4\hat{i}+\lambda\hat{j}+5\hat{k}$

Position vector of $OC = 4\hat{i}+2\hat{j}-2\hat{k}$

Position vector of $OD = 6\hat{i}+5\hat{j}-\hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$$\overline{AB} = \hat{i}+(\lambda-2)\hat{j}+4\hat{k}$$
$$\overline{AC} = \hat{i}+0\hat{j}-3\hat{k}$$
$$\overline{AD} = 3\hat{i}+3\hat{j}-2\hat{k}$$

$$\begin{vmatrix} 1 & (\lambda-2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$1(9)-(\lambda-2)(-2+9)+4(3-0) = 0$$
$$9-7\lambda+14+12 = 0$$
$$7\lambda = 35$$
$$\lambda = 5$$

# Chapter 26 - Scalar Triple Product Exercise MCQ

Question 1

If $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, then which of the following is correct?

(a) $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

(b) $[\vec{a}\ \vec{b}\ \vec{c}] = 1$

(c) $[\vec{a}\ \vec{b}\ \vec{c}] = 3$

(d) $[\vec{b}\ \vec{c}\ \vec{a}] = 1$

Solution 1

Correct option: (a)

Given that $\vec{a}, \vec{b}, \vec{c}$ are on the same plane.

$$\Rightarrow \left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$$

Question 2

The value of $[\vec{a} - \vec{b}\ \vec{b} - \vec{c}\ \vec{c} - \vec{a}]$, where $|\vec{a}|=1,\ |\vec{b}|=5,\ |\vec{c}|=3$, is

(a) 0

(b) 1

(c) 6

(d) none of these

Solution 2

Correct option: (a)

$$\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right] - \left[\vec{b}\ \vec{c}\ \vec{a}\right]$$

$$\Rightarrow \left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{a}\ \vec{c}\ \vec{b}\right]$$

$$\Rightarrow \left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right] - \left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

$$\Rightarrow \left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right] = 0$$

Question 3

If $\vec{a},\ \vec{b},\ \vec{c}$ are three non-coplanar mutually perpendicular unit vectors, then $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$, is

(a) $\pm 1$

(b) 0

(c) -2

(d) 2

Solution 3

Correct option: (a)

$$\left[\vec{a}\,\vec{b}\ \vec{c}\right] = \left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$$

$$\left[\vec{a}\,\vec{b}\ \vec{c}\right] = \left|\vec{a}\times\vec{b}\right| \text{ or } \left[\vec{a}\,\vec{b}\ \vec{c}\right] = -\left|\vec{a}\times\vec{b}\right| \quad \left(\left|\vec{c}\right| = 1, \text{ same directions or opposite}\right)$$

Hence,

$$\left[\vec{a}\,\vec{b}\ \vec{c}\right] = \left|\vec{a}\right|\left|\vec{b}\right|\sin\frac{\pi}{2} = 1$$

$$\text{Or } -\left|\vec{a}\right|\left|\vec{b}\right|\sin\frac{\pi}{2} = -1$$

Question 4

If $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$, is

(a) 2

(b) 3

(c) 0

(d) none of these

Solution 4

Correct option: (c)

$\vec{r} \cdot \vec{a} = 0$

$\Rightarrow$ either $\vec{a} = 0$ or both are perpendicular to each other.

If $\vec{a}, \vec{b}, \vec{c}$ are zero $\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

and

if three vectors are non $-$ zero

$\Rightarrow$ they are coplanar and perpendicular to $\vec{r}$.

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

Question 5

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the expression $\left(\vec{a} - \vec{b}\right) \cdot \left\{\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)\right\}$ equals

(a) $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(b) $2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(c) $[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$

(d) none of these

Solution 5

Correct option: (d)

$$\left(\vec{a} - \vec{b}\right) \cdot \left\{\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)\right\}$$

$$= \left(\vec{a} - \vec{b}\right) \cdot \left(\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}\right)$$

$$= \left(\vec{a} - \vec{b}\right) \cdot \left(\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\right)$$

$$= \vec{a} \cdot \left(\vec{b} \times \vec{c}\right) - \vec{a} \cdot \left(\vec{b} \times \vec{a}\right) + \vec{a} \cdot \left(\vec{c} \times \vec{a}\right) - \vec{b} \cdot \left(\vec{b} \times \vec{c}\right) + \vec{b} \cdot \left(\vec{b} \times \vec{a}\right) - \vec{b} \cdot \left(\vec{c} \times \vec{a}\right)$$

$$= \left[\vec{a}\ \vec{b}\ \vec{c}\right] - 0 + 0 - 0 + 0 - \left[\vec{b}\ \vec{c}\ \vec{a}\right]$$

$$= \left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{b}\ \vec{a}\ \vec{c}\right]$$

$$= \left[\vec{a}\ \vec{b}\ \vec{c}\right] - \left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

$$= 0$$

Question 6

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors, then $\dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\dfrac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$

is equal to

(a) 0

(b) 2

(c) 1

(d) none of these

Solution 6

Correct option:(a)

$$\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$$

$$=\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{\vec{b}\cdot(\vec{c}\times\vec{a})}+\frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$$

$$=\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{b}\ \vec{c}\ \vec{a}]}+\frac{[\vec{b}\ \vec{a}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]}$$

$$=\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{-[\vec{b}\ \vec{a}\ \vec{c}]}-\frac{[\vec{b}\ \vec{a}\ \vec{c}]}{[\vec{b}\ \vec{a}\ \vec{c}]}$$

$$=\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]}-\frac{[\vec{b}\ \vec{a}\ \vec{c}]}{[\vec{b}\ \vec{a}\ \vec{c}]}$$

$= 1 - 1$

$= 0$

Question 7

Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$,

then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

(a) 0

(b) 1

(c) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

(d) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

Solution 7

Correct option: (c)

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$

$$= \left(\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}\right)^2$$

$$= \left(\left|\left(\vec{a} \times \vec{b}\right)\right|\right)^2 \qquad \left(\begin{matrix} \because \text{All are unit vectors} \\ \text{and } \cos 0 = 1 \end{matrix}\right)$$

$$= \left(\left|\vec{a}\right|\left|\vec{b}\right| \sin\frac{\pi}{6}\right)^2$$

$$= \frac{\left|\vec{a}\right|^2 \left|\vec{b}\right|^2}{4}$$

Question 8

If $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} - 2\hat{k}$, then the volume of the parallelopiped with conterminous edges $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ is

(a) 2

(b) 1

(c) -1

(d) 2

Solution 8

$\vec{a} + \vec{b} = 5\hat{i} - 7\hat{j} + 10\hat{k}$

$\vec{b} + \vec{c} = 8\hat{i} - 7\hat{j} + 3\hat{k}$

$\vec{c} + \vec{a} = 7\hat{i} - 6\hat{j} + 3\hat{k}$

Volume of parallelopiped $= \vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$

$$= \begin{vmatrix} 5 & -7 & 10 \\ 8 & -7 & 3 \\ 7 & -6 & 3 \end{vmatrix}$$

$$= 5(-21 + 18) + 7(24 - 21) + 10(-48 + 49)$$

$$= -15 + 21 + 10$$

$$= 16$$

NOTE: Answer not matching with back answer.

Question 9

If $[2\vec{a} + 4\vec{b}\ \ \vec{c}\ \ \vec{d}] = \lambda[\vec{a}\ \ \vec{c}\ \ \vec{d}] + \mu[\vec{b}\ \ \vec{c}\ \ \vec{d}]$, then $\lambda+\mu=$

(a) 6

(b) -6

(c) 10

(d) 8

Solution 9

Correct option: (a)

$$\left[2\vec{a}+4\vec{b}\ \ \vec{c}\ \ \vec{d}\right] = \left[2\vec{a}\ \ \vec{c}\ \ \vec{d}\right] + \left[4\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$$

$$\left[2\vec{a}+4\vec{b}\ \ \vec{c}\ \ \vec{d}\right] = \left[2\vec{a}\ \ \vec{c}\ \ \vec{d}\right] + \left[4\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$$

$$\left[2\vec{a}+4\vec{b}\ \ \vec{c}\ \ \vec{d}\right] = 2\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right] + 4\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$$

$\lambda = 2,\ \mu = 4$

$\Rightarrow \lambda + \mu = 6$

Question 10

$[\vec{a}\ \vec{b}\ \ \vec{a} \times \vec{b}] + \left(\vec{a} \cdot \vec{b}\right)^2 =$

(a) $|\vec{a}|^2\,|\vec{b}|^2$

(b) $|\vec{a} + \vec{b}|^2$

(c) $|\vec{a}|^2 + |\vec{b}|^2$

(d) $2|\vec{a}|^2 + |\vec{b}|^2$

Solution 10

Correct option: (a)

$$\left[\vec{a}\ \vec{b}\ \ \overrightarrow{a \times b}\right] + \left(\vec{a} \cdot \vec{b}\right)^2$$

$$= \left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \cdot \vec{b}\right)^2$$

$$= \left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a} \cdot \vec{b}\right)^2$$

$$= \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 \sin^2\theta + \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 \cos^2\theta$$

$$= \left|\vec{a}\right|^2 \left|\vec{b}\right|^2$$

Question 11

If the vectors $4\hat{i}+11\hat{j}+m\hat{k}, 7\hat{i}+2\hat{j}+6\hat{k}$ and $\hat{i}+5\hat{j}+4\hat{k}$ are coplanar, then m =

(a) 0

(b) 38

(c) -10

(d) 10

Solution 11

Correct option: (d)

If given vectors are coplanar then

$$\begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0$$

$$4(8-30) - 11(28-6) + m(35-2) = 0$$

$$-88 - 242 + 33m = 0$$

$$-330 + 33m = 0$$

$$m = 10$$

NOTE: Answer not matching with back answer.

Question 12

For non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ the relation $|(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}|\ |\vec{b}|\ |\vec{c}|$ holds good, if

(a) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = 0$

(b) $\vec{a}\cdot\vec{b} = 0 = \vec{c}\cdot\vec{a}$

(c) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

(d) $\vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

Solution 12

Correct option: (c)

$$|(\vec{a}\times\vec{b})\cdot\vec{c}| = |(\vec{a}\times\vec{b})||\vec{c}|\cos\alpha$$

For $\alpha = 0$ or $\pi$

$$|(\vec{a}\times\vec{b})\cdot\vec{c}| = |(\vec{a}\times\vec{b})||\vec{c}|$$

$$|(\vec{a}\times\vec{b})\cdot\vec{c}| = |(\vec{a}\times\vec{b}\sin\beta)||\vec{c}|$$

For $\beta = \frac{\pi}{2}$

$$|(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$$

Hence, given relation holds good if

$\vec{a}\cdot\vec{b} = 0, \vec{b}\cdot\vec{c} = 0, \vec{c}\cdot\vec{a} = 0$

Question 13

$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}+\vec{c}\right)=$

(a) 0

(b) $-[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(c) $2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(d) $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

Solution 13

Correct option: (d)

$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}+\vec{c}\right)$

$=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}\times\vec{a}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}+\vec{c}\times\vec{c}\right)$

$=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}\times\vec{a}+0+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}+0\right)$

$=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}\times\vec{a}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}-\vec{b}\times\vec{c}\right)$

$=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}\times\vec{a}+\vec{c}\times\vec{a}\right)$

$=\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{a}\cdot\left(\vec{c}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

$=0+0+0+\left[\vec{b}\ \ \vec{c}\ \vec{a}\right]$

$=\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

Question 14

If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, then

$(\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$ equals

(a) 0

(b) $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(c) $2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(d) $-[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

Solution 14

Correct option: (d)

$\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)\right]$

$=\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left(\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}\right)$

$=\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left(\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}\right)$

$=0+0+\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]+\left[\vec{b}\ \ \vec{a}\ \ \vec{c}\right]+0+0+0+\left[\vec{c}\ \ \vec{b}\ \ \vec{a}\right]+0$

$=\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]-\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]-\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$

$=-\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$

NOTE: Answer not matching with back answer.

Question 15

$(\vec{a}+2\vec{b}-\vec{c})\cdot\{(\vec{a}-\vec{b})\times(\vec{a}-\vec{b}-\vec{c})\}$ is equal to

(a) $[\vec{a}\ \vec{b}\ \vec{c}]$

(b) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(c) $3[\vec{a}\ \vec{b}\ \vec{c}]$

(d) 0

Solution 15

Correct option: (c)

$$(\vec{a}+2\vec{b}-\vec{c})\cdot\{(\vec{a}-\vec{b})\times(\vec{a}-\vec{b}-\vec{c})\}$$
$$=(\vec{a}+2\vec{b}-\vec{c})\cdot(\vec{a}\times\vec{a}-\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{a}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c})$$
$$=(\vec{a}+2\vec{b}-\vec{c})\cdot(-\vec{a}\times\vec{b}-\vec{a}\times\vec{c}+\vec{a}\times\vec{b}+\vec{b}\times\vec{c})$$
$$=(\vec{a}+2\vec{b}-\vec{c})\cdot(-\vec{a}\times\vec{c}+\vec{b}\times\vec{c})$$
$$=[a\ b\ c]+2[a\ b\ c]$$
$$=3[a\ b\ c]$$

# Chapter 26 - Scalar Triple Product Exercise Ex. 26VSAQ

Question 1

Write the value of $[2\hat{i}\quad 3\hat{j}\quad 4\hat{k}]$

Solution 1

$$[2\hat{i}\quad 3\hat{j}\quad 4\hat{k}]$$
$$=2\times3\times4[\hat{i}\quad\hat{j}\quad\hat{k}]$$
$$=24\times1$$
$$=24$$

Question 2

Write the value of $[\hat{i}+\hat{j}\quad \hat{j}+\hat{k}\quad \hat{k}+\hat{i}]$

Solution 2

$$\left[\hat{i}+\hat{j}\quad \hat{j}+\hat{k}\quad \hat{k}+\hat{i}\right]$$
$$=\left[\hat{i}\quad \hat{j}+\hat{k}\quad \hat{k}+\hat{i}\right]+\left[\hat{j}\quad \hat{j}+\hat{k}\quad \hat{k}+\hat{i}\right]$$
$$=\left(\left[\hat{i}\quad \hat{j}\quad \hat{k}+\hat{i}\right]+\left[\hat{i}\quad \hat{k}\quad \hat{k}+\hat{i}\right]\right)+\left(\left[\hat{j}\quad \hat{j}\quad \hat{k}+\hat{i}\right]+\left[\hat{j}\quad \hat{k}\quad \hat{k}+\hat{i}\right]\right)$$
$$=\left[\hat{i}\quad \hat{j}\quad \hat{i}\right]+\left[\hat{i}\quad \hat{j}\quad \hat{k}\right]+\left[\hat{i}\quad \hat{k}\quad \hat{k}\right]+\left[\hat{i}\quad \hat{k}\quad \hat{i}\right]+\left[\hat{j}\quad \hat{j}\quad \hat{k}\right]+\left[\hat{j}\quad \hat{j}\quad \hat{i}\right]+\left[\hat{j}\quad \hat{k}\quad \hat{i}\right]+\left[\hat{j}\quad \hat{k}\quad \hat{k}\right]$$
$$=0+\left[\hat{i}\quad \hat{j}\quad \hat{k}\right]+0+0+0+0+\left[\hat{i}\quad \hat{j}\quad \hat{k}\right]+o$$
$$=2$$

Question 3

3. write the value of $\left[\hat{i}-\hat{j}\quad \hat{j}-\hat{k}\quad \hat{k}-\hat{i}\right]$

Solution 3

$$\left[\hat{i}-\hat{j}\quad \hat{j}-\hat{k}\quad \hat{k}-\hat{i}\right]$$
$$=\left[\hat{i}\quad \hat{j}-\hat{k}\quad \hat{k}-\hat{i}\right]-\left[\hat{j}\quad \hat{j}-\hat{k}\quad \hat{k}-\hat{i}\right]$$
$$=\left[\hat{i}\quad \hat{j}\quad \hat{k}-\hat{i}\right]-\left[\hat{i}\quad \hat{k}\quad \hat{k}-\hat{i}\right]-\left[\hat{j}\quad \hat{j}\quad \hat{k}-\hat{i}\right]+\left[\hat{j}\quad \hat{k}\quad \hat{k}-\hat{i}\right]$$
$$=\left[\hat{i}\quad \hat{j}\quad \hat{k}\right]-\left[\hat{i}\quad \hat{j}\quad \hat{i}\right]-\left[\hat{i}\quad \hat{k}\quad \hat{k}\right]+\left[\hat{i}\quad \hat{k}\quad \hat{i}\right]-0+0-\left[\hat{j}\quad \hat{k}\quad \hat{i}\right]$$
$$=\left[\hat{i}\quad \hat{j}\quad \hat{k}\right]-0-0+0+0-0+0-\left[\hat{i}\quad \hat{j}\quad \hat{k}\right]$$
$$=0$$

Question 4

4.find the value of a for which the vectors $\overline{\alpha}=\hat{i}+2\hat{j}+\hat{k},\overline{\beta}=a\hat{i}+\hat{j}+\widehat{2k},\overline{\gamma}=\hat{i}+2\hat{j}+a\hat{k}$ are coplanar

Solution 4

$$\begin{vmatrix}1 & 2 & 1\\ a & 1 & 2\\ 1 & 2 & a\end{vmatrix}=0$$
$$1(a-4)-2\left(a^2-2\right)+1(2a-1)=0$$
$$a-4-2a^2+4+2a-1=0$$
$$2a^2-3a+1=0$$
$$(2a-1)(a-1)=0$$
$$a=\frac{1}{2},1$$

Question 5

.

5.find the volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j},\hat{i}+2\widehat{j},\hat{i}+\hat{j}+\pi\hat{k}$.

Solution 5

Volume of the parallelepiped is given by

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = 1(2\pi - 0) - 1(\pi - 0) + 0$$
$$= 2\pi - \pi$$
$$= \pi$$

Thus, volume of the parallelopiped is $\pi$ cubic units.

Question 6

*If a, b are non − collinear vectors, then find the value of*

$$[\vec{a}\ \vec{b}\ \hat{i}]\hat{i} + [\vec{a}\ \vec{b}\ \hat{j}]\hat{j} + [\vec{a}\ \vec{b}\ \hat{k}]\hat{k}$$

Solution 6

*For any vector $\vec{r}$, we have*

$$\vec{r} = (\vec{r}.\hat{i})\hat{i} + (\vec{r}.\hat{j})\hat{j} + (\vec{r}.\hat{k})\hat{k}$$

*Replacing $\vec{r}$ by $\vec{a} \times \vec{b}$, we have*

$$\vec{a} \times \vec{b} = |(\vec{a} \times \vec{b}) \cdot \hat{i}|\hat{i} + |(\vec{a} \times \vec{b}) \cdot \hat{j}|\hat{j} + |(\vec{a} \times \vec{b}) \cdot \hat{k}|\hat{k}$$
$$\Rightarrow \vec{a} \times \vec{b} = [\vec{a}\ \vec{b}\ \hat{i}]\hat{i} + [\vec{a}\ \vec{b}\ \hat{j}]\hat{j} + [\vec{a}\ \vec{b}\ \hat{k}]\hat{k}$$

Question 7

the vectors $(\sec^2 A)\hat{i} + \hat{j} + \hat{k}, \hat{i} + (\sec^2 B)\hat{j} + \hat{k}, \hat{i} + \hat{j} + (\sec^2 C)\hat{k}$ are coplanar,then the value of

$\operatorname{cosec}^2 A + \operatorname{cosec}^2 B + \operatorname{cosec}^2 C$

Solution 7

$$\begin{vmatrix} \sec^2 A & 1 & 1 \\ 1 & \sec^2 B & 1 \\ 1 & 1 & \sec^2 C \end{vmatrix} = 0$$

$$\sec^2 A(\sec^2 B \sec^2 C - 1) - 1(\sec^2 C - 1) + 1(1 - \sec^2 B) = 0$$
$$\sec^2 A \sec^2 B \sec^2 C - \sec^2 A + (1 - \sec^2 B) + (1 - \sec^2 C) = 0$$
$$\sec^2 A \sec^2 B \sec^2 C - (\sec^2 A + \sec^2 B + \sec^2 C) = 2$$
$$\operatorname{cosec}^2 A + \operatorname{cosec}^2 B + \operatorname{cosec}^2 C = 2$$

Question 8

For any two vectors $\vec{a}$ and $\vec{b}$ of magnitude 3and 4 respectively, write the value of

$[\vec{a}\ \ \vec{b}\ \ \vec{a}\times\vec{b}] + (\vec{a}.\vec{b})^2$

Solution 8

$$\left[\vec{a}\ \ \vec{b}\ \ \vec{a}\times\vec{b}\right]+\left(\vec{a}.\vec{b}\right)^2$$
$$=0+(12)^2$$
$$=144$$

Question 9

If $\left[3\vec{a}+7\vec{b}\ \ \vec{c}\ \ \vec{d}\right]=\lambda\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+\mu\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$,then find the value of $\lambda+\mu$

Solution 9

$$\left[3\vec{a}+7\vec{b}\ \ \vec{c}\ \ \vec{d}\right]=\lambda\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+\mu\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$$
$$\left[3\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+\left[7\vec{b}\ \ \vec{c}\ \ \vec{d}\right]=\lambda\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+\mu\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$$
$$3\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+7\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]=\lambda\left[\vec{a}\ \ \vec{c}\ \ \vec{d}\right]+\mu\left[\vec{b}\ \ \vec{c}\ \ \vec{d}\right]$$

Comparing ,we get

$$\lambda=3$$
$$\mu=7$$
$$\lambda+\mu=10$$

Question 10

If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors,then find the value of $\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left(\vec{c}\times\vec{a}\right).\vec{b}}+\frac{\vec{b}.\left(\vec{a}\times\vec{c}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}$

Solution 10

$$\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left(\vec{c}\times\vec{a}\right).\vec{b}}+\frac{\vec{b}.\left(\vec{a}\times\vec{c}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}$$
$$=\frac{[a\ \ b\ \ c]}{[c\ \ a\ \ b]}+\frac{[b\ \ a\ \ c]}{[c\ \ a\ \ b]}$$
$$=\frac{[a\ \ b\ \ c]}{[c\ \ a\ \ b]}-\frac{[a\ \ b\ \ c]}{[c\ \ a\ \ b]}$$
$$=0$$

Question 11

Find $\vec{a}.(\vec{b}\times\vec{c})$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$.

Solution 11

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = \hat{i}(4-1) - \hat{j}(-5) + \hat{k}(-1-6)$$

$= 3\hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{a}.(\vec{b} \times \vec{c}) = (2\hat{i} + \hat{j} + 3\hat{k}).(3\hat{i} + 5\hat{j} - 7\hat{k})$

$= 6 + 5 - 21 = -10$

.