

## NCERT Solutions for Class 12- Maths Chapter 3 - Matrices

### Chapter 3 - Matrices Exercise Ex. 3.1

#### Solution 1

(i) In the given matrix, the number of rows is 3 and the number of columns is 4.

Therefore, the order of the matrix is  $3 \times 4$ .

(ii) Since the order of the matrix is  $3 \times 4$ , there are  $3 \times 4 = 12$  elements in it.

(iii)  $a_{13} = 19$ ,  $a_{21} = 35$ ,  $a_{33} = -5$ ,  $a_{24} = 12$ ,  $a_{23} = 5/2$

#### Solution 2

We know that if a matrix is of the order  $m \times n$ , it has  $mn$  elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)

Hence, the possible orders of a matrix having 24 elements are:

$1 \times 24$ ,  $24 \times 1$ ,  $2 \times 12$ ,  $12 \times 2$ ,  $3 \times 8$ ,  $8 \times 3$ ,  $4 \times 6$ , and  $6 \times 4$

(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are  $1 \times 13$  and  $13 \times 1$ .

#### Solution 3

We know that if a matrix is of the order  $m \times n$ , it has  $mn$  elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6), and (6, 3)

Hence, the possible orders of a matrix having 18 elements are:

$1 \times 18$ ,  $18 \times 1$ ,  $2 \times 9$ ,  $9 \times 2$ ,  $3 \times 6$ , and  $6 \times 3$

(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are  $1 \times 5$  and  $5 \times 1$ .

#### Solution 4

$$\text{Let } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

(i)

$$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = 2,$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2},$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = 8$$

$$\text{Therefore, } A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii)

$$a_{11} = \frac{1}{1} = 1,$$

$$a_{12} = \frac{1}{2},$$

$$a_{21} = \frac{2}{1} = 2,$$

$$a_{22} = \frac{2}{2} = 1$$

$$\text{Therefore, } A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

(iii)

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2},$$

$$a_{21} = \frac{(2+2)^2}{2} = 8,$$

$$a_{22} = \frac{(2+4)^2}{2} = 18$$

$$\text{Therefore, } A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}.$$

## Solution 5

In general, a  $3 \times 4$  matrix is given by  $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$$(i) a_{ij} = \frac{1}{2}|-3i + j|, i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4$$

$$\therefore a_{11} = \frac{1}{2}|-3 \times 1 + 1| = \frac{1}{2}|-3 + 1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{1}{2}|-6 + 1| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}|-3 \times 3 + 1| = \frac{1}{2}|-9 + 1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}|-3 + 2| = \frac{1}{2}|-1| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = \frac{1}{2}|-6 + 2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$

$$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{1}{2}|-9 + 2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3 \times 1 + 3| = \frac{1}{2}|-3 + 3| = 0$$

$$a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{1}{2}|-6 + 3| = \frac{1}{2}|-3| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}|-3 \times 3 + 3| = \frac{1}{2}|-9 + 3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}|-3 + 4| = \frac{1}{2}|1| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}|-3 \times 2 + 4| = \frac{1}{2}|-6 + 4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{34} = \frac{1}{2}|-3 \times 3 + 4| = \frac{1}{2}|-9 + 4| = \frac{1}{2}|-5| = \frac{5}{2}$$

Therefore, the required matrix is  $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii)  $a_{ij} = 2i - j$ ,  $i = 1, 2, 3$  and  $j = 1, 2, 3, 4$

$$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

$$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$$

$$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$$

$$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$$

$$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$$

$$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$$

$$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$$

Therefore, the required matrix is  $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

**Solution 6**

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$x = 1, y = 4, \text{ and } z = 3$$

$$(ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$x + y = 6, xy = 8, 5 + z = 5$$

$$\text{Now, } 5 + z = 5 \Rightarrow z = 0$$

We know that:

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$\Rightarrow (x - y)^2 = 36 - 32 = 4$$

$$\Rightarrow x - y = \pm 2$$

Now, when  $x - y = 2$  and  $x + y = 6$ , we get  $x = 4$  and  $y = 2$

When  $x - y = -2$  and  $x + y = 6$ , we get  $x = 2$  and  $y = 4$

$\therefore x = 4, y = 2, \text{ and } z = 0$  or  $x = 2, y = 4, \text{ and } z = 0$

$$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$x + y + z = 9 \dots (1)$$

$$x + z = 5 \dots (2)$$

$$y + z = 7 \dots (3)$$

From (1) and (2), we have:

$$y + 5 = 9$$

$$\Rightarrow y = 4$$

Then, from (3), we have:

$$4 + z = 7$$

$$\Rightarrow z = 3$$

$$\therefore x + z = 5$$

$$\Rightarrow x = 2$$

$$\therefore x = 2, y = 4, \text{ and } z = 3$$

**Solution 7**

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:

$$a - b = -1 \dots (1)$$

$$2a - b = 0 \dots (2)$$

$$2a + c = 5 \dots (3)$$

$$3c + d = 13 \dots (4)$$

From (2), we have:

$$b = 2a$$

Then, from (1), we have:

$$a - 2a = -1$$

$$\Rightarrow a = 1$$

$$\Rightarrow b = 2$$

Now, from (3), we have:

$$2 \times 1 + c = 5$$

$$\Rightarrow c = 3$$

From (4) we have:

$$3 \times 3 + d = 13$$

$$\Rightarrow 9 + d = 13 \Rightarrow d = 4$$

$$\therefore a = 1, b = 2, c = 3, \text{ and } d = 4$$

## Solution 8

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore,  $A = [a_{ij}]_{m \times n}$  is a square matrix, if  $m = n$ .

## Solution 9

The correct answer is B.

$$\text{Let } \begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

Equating the corresponding elements, we get:

$$3x + 7 = 0 \Rightarrow x = -\frac{7}{3}$$

$$5 = y - 2 \Rightarrow y = 7$$

$$y + 1 = 8 \Rightarrow y = 7$$

$$2 - 3x = 4 \Rightarrow x = -\frac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of  $x$ , which is not possible.

Hence, it is not possible to find the values of  $x$  and  $y$  for which the given matrices are equal.

### Solution 10

The correct answer is D.

The given matrix of the order  $3 \times 3$  has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is  $2^9 = 512$

## Chapter 3 - Matrices Exercise Ex. 3.2

### Solution 1

$$(i) A+B=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}+\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix}=\begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

$$(ii) A-B=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}-\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii)

$$\begin{aligned} 3A-C &= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}-\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3\times 2 & 3\times 4 \\ 3\times 3 & 3\times 2 \end{bmatrix}-\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix}-\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix} \end{aligned}$$

(iv) Matrix  $A$  has 2 columns. This number is equal to the number of rows in matrix  $B$ . Therefore,  $AB$  is defined as:

$$\begin{aligned} AB &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix} \\ &= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix}=\begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix} \end{aligned}$$



(iv) Matrix  $A$  has 2 columns. This number is equal to the number of rows in matrix  $B$ . Therefore,  $AB$  is defined as:

$$\begin{aligned} AB &= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix} \\ &= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix} \end{aligned}$$

(v) Matrix  $B$  has 2 columns. This number is equal to the number of rows in matrix  $A$ . Therefore,  $BA$  is defined as:

$$\begin{aligned} BA &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{bmatrix} \\ &= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix} \end{aligned}$$

Solution 2

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

$$(ii) \begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

$$(iii) \begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

$$(iv) \begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (\because \sin^2 x + \cos^2 x = 1)$$

### Solution 3

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

$$ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

$$\text{iii) } \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

$$\text{iv) } \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

$$\text{v) } \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & -0+2 & -1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\text{vi) } \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1+9 & -9+0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

**Solution 4**

$$\begin{aligned}
A+B &= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} \\
B-C &= \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \\
A+(B-C) &= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 1+(-1) & 2+(-2) & -3+0 \\ 5+4 & 0+(-1) & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \\
(A+B)-C &= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1-(-2) & 4-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}
\end{aligned}$$

Hence, we have verified that  $A+(B-C)=(A+B)-C$ .

### Solution 5

$$\begin{aligned}
3A-5B &= 3 \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5 \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix} \\
&= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

### Solution 6

$$\begin{aligned} & \cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} + \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (\because \cos^2 \theta + \sin^2 \theta = 1) \end{aligned}$$

### Solution 7

(i)

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \dots(1)$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \dots(2)$$

Adding equations (1) and (2), we get:

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\text{Now, } X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

(ii)

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad \dots(1)$$

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \quad \dots (2)$$

Multiplying equation (1) with 2, we get:

$$2(2X + 3Y) = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \quad \dots (3)$$

Multiplying equation (2) with 3, we get:

$$3(3X + 2Y) = 3 \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\Rightarrow 9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \quad \dots(4)$$

From (3) and (4), we have:

$$(4X + 6Y) - (9X + 6Y) = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\therefore X = -\frac{1}{5} \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

$$\begin{aligned}
\text{Now, } 2X + 3Y &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \\
\Rightarrow 2 \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \\
\Rightarrow 3Y &= \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} \\
\Rightarrow 3Y &= \begin{bmatrix} 2 - \frac{4}{5} & 3 + \frac{24}{5} \\ 4 + \frac{22}{5} & 0 - 6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}
\end{aligned}$$

$$\therefore Y = \frac{1}{3} \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

**Solution 8**

$$\begin{aligned}
2X + Y &= \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \\
\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \\
\Rightarrow 2X &= \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix} \\
\Rightarrow 2X &= \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}
\end{aligned}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

**Solution 9**

$$\begin{aligned}
2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}
\end{aligned}$$

Comparing the corresponding elements of these two matrices, we have:

$$2 + y = 5$$

$$\Rightarrow y = 3$$

$$2x + 2 = 8$$

$$\Rightarrow x = 3$$

$$\therefore x = 3 \text{ and } y = 3$$

### Solution 10

$$\begin{aligned}
2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} &= 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} &= \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} &= \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}
\end{aligned}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x + 3 = 9$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$2y = 12$$

$$\Rightarrow y = 6$$

$$2z - 3 = 15$$

$$\Rightarrow 2z = 18$$

$$\Rightarrow z = 9$$

$$2t + 6 = 18$$

$$\Rightarrow 2t = 12$$

$$\Rightarrow t = 6$$

$$\therefore x = 3, y = 6, z = 9, \text{ and } t = 6$$



### Solution 11

$$x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x - y = 10 \text{ and } 3x + y = 5$$

Adding these two equations, we have:

$$5x = 15$$

$$\Rightarrow x = 3$$

$$\text{Now, } 3x + y = 5$$

$$\Rightarrow y = 5 - 3x$$

$$\Rightarrow y = 5 - 9 = -4$$

$$\therefore x = 3 \text{ and } y = -4$$

### Solution 12

$$\begin{aligned}
3\begin{bmatrix} x & y \\ z & w \end{bmatrix} &= \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} &= \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}
\end{aligned}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$3y = 6 + x + y$$

$$\Rightarrow 2y = 6 + x = 6 + 2 = 8$$

$$\Rightarrow y = 4$$

$$3w = 2w + 3$$

$$\Rightarrow w = 3$$

$$3z = -1 + z + w$$

$$\Rightarrow 2z = -1 + w = -1 + 3 = 2$$

$$\Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, \text{ and } w = 3$$

**Solution 13**

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} & F(x)F(y) \\ &= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= F(x+y) \end{aligned}$$

$$\therefore F(x)F(y) = F(x+y)$$

Solution 14

(i)

$$\begin{aligned} & \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix} \\ &= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix} \\ &= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \end{aligned}$$

$$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(ii)

$$\begin{aligned} & \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix} \\ &= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \\ & \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \\ & \therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{aligned}$$

Solution 15

We have  $A^2 = A \times A$

$$\begin{aligned}
 A^2 = AA &= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{bmatrix} \\
 &= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix} \\
 &= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}
 \end{aligned}$$

$$\therefore A^2 - 5A + 6I$$

$$\begin{aligned}
 &= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}
 \end{aligned}$$

Solution 16

$$\begin{aligned}
 A^2 = AA &= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}
 \end{aligned}$$

$$\text{Now } A^3 = A^2 \cdot A$$

$$\begin{aligned}
 &= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix} \\
 &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}
 \end{aligned}$$

$$\therefore A^3 - 6A^2 + 7A + 2I$$

$$\begin{aligned}
 &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} \\
 &= \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O
 \end{aligned}$$

$$\therefore A^3 - 6A^2 + 7A + 2I = O$$

### Solution 17

$$\begin{aligned} A^2 = A \cdot A &= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 3(3) + (-2)(4) & 3(-2) + (-2)(-2) \\ 4(3) + (-2)(4) & 4(-2) + (-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \end{aligned}$$

Now  $A^2 = kA - 2I$

$$\begin{aligned} \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements, we have:

$$3k - 2 = 1$$

$$\Rightarrow 3k = 3$$

$$\Rightarrow k = 1$$

Thus, the value of  $k$  is 1.

### Solution 18



On the L.H.S.

$$I + A$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix} \quad \dots(1) \end{aligned}$$

On the R.H.S.

$$\begin{aligned} &(I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ &= \left( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} \right) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha + \sin \alpha \tan \frac{\alpha}{2} & -\sin \alpha + \cos \alpha \tan \frac{\alpha}{2} \\ -\cos \alpha \tan \frac{\alpha}{2} + \sin \alpha & \sin \alpha \tan \frac{\alpha}{2} + \cos \alpha \end{bmatrix} \quad \dots(2) \end{aligned}$$

$$\begin{aligned}
&= \begin{bmatrix} 1 - 2 \sin^2 \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} & -2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \left( 2 \cos^2 \frac{\alpha}{2} - 1 \right) \tan \frac{\alpha}{2} \\ - \left( 2 \cos^2 \frac{\alpha}{2} - 1 \right) \tan \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} + 1 - 2 \sin^2 \frac{\alpha}{2} \end{bmatrix} \\
&= \begin{bmatrix} 1 - 2 \sin^2 \frac{\alpha}{2} + 2 \sin^2 \frac{\alpha}{2} & -2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} - \tan \frac{\alpha}{2} \\ -2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \tan \frac{\alpha}{2} + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2 \sin^2 \frac{\alpha}{2} + 1 - 2 \sin^2 \frac{\alpha}{2} \end{bmatrix} \\
&= \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}
\end{aligned}$$

Thus, from (1) and (2), we get L.H.S. = R.H.S.

**Solution 19**

**(a)** Let Rs  $x$  be invested in the first bond. Then, the sum of money invested in the second bond will be Rs  $(30000 - x)$ .

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\left[ \begin{array}{cc} x & (30000 - x) \end{array} \right] \left[ \begin{array}{c} \frac{5}{100} \\ \frac{7}{100} \end{array} \right] = 1800 \quad \left[ \text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

**(b)** Let Rs  $x$  be invested in the first bond. Then, the sum of money invested in the second bond will be Rs  $(30000 - x)$ .

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\left[ \begin{array}{cc} x & (30000 - x) \end{array} \right] \left[ \begin{array}{c} \frac{5}{100} \\ \frac{7}{100} \end{array} \right] = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond

**Solution 20**

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60, and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12 \begin{bmatrix} 10 & 8 & 10 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$$= 12 [10 \times 80 + 8 \times 60 + 10 \times 40]$$

$$= 12 (800 + 480 + 400)$$

$$= 12 (1680)$$

$$= 20160$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

### Solution 21

Matrices  $P$  and  $Y$  are of the orders  $p \times k$  and  $3 \times k$  respectively.

Therefore, matrix  $PY$  will be defined if  $k = 3$ . Consequently,  $PY$  will be of the order  $p \times k$ .

Matrices  $W$  and  $Y$  are of the orders  $n \times 3$  and  $3 \times k$  respectively.

Since the number of columns in  $W$  is equal to the number of rows in  $Y$ , matrix  $WY$  is well-defined and is of the order  $n \times k$ .

Matrices  $PY$  and  $WY$  can be added only when their orders are the same.

However,  $PY$  is of the order  $p \times k$  and  $WY$  is of the order  $n \times k$ . Therefore, we must have  $p = n$ .

Thus,  $k = 3$  and  $p = n$  are the restrictions on  $n$ ,  $k$ , and  $p$  so that  $PY + WY$  will be defined

### Solution 22

The correct answer is B.

Matrix  $X$  is of the order  $2 \times n$ .

Therefore, matrix  $7X$  is also of the same order.

Matrix  $Z$  is of the order  $2 \times p$ , i.e.,  $2 \times n$  [Since  $n = p$ ]

Therefore, matrix  $5Z$  is also of the same order.

Now, both the matrices  $7X$  and  $5Z$  are of the order  $2 \times n$ .

Thus, matrix  $7X - 5Z$  is well-defined and is of the order  $2 \times n$

## Chapter 3 - Matrices Exercise Ex. 3.3

### Solution 1

(i) Let  $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ , then  $A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let  $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ , then  $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let  $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$ , then  $A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

## Solution 2

We have:

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i)

$$A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that  $(A + B)' = A' + B'$

(ii)

$$A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have verified that  $(A - B)' = A' - B'$ .

### Solution 3

(i) It is known that  $A = (A')'$

Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Thus, we have verified that  $(A + B)' = A' + B'$ .

(ii)

$$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Thus, we have verified that  $(A - B)' = A' - B'$ .

#### Solution 4

We know that  $A = (A')'$

$$\therefore A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

#### Solution 5

(i)

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$\text{Now, } A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}, B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, we have verified that  $(AB)' = B'A'$ .

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$$\text{Now, } A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that  $(AB)' = B'A'$ .

## Solution 6



(i)

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos \alpha)(\cos \alpha) + (-\sin \alpha)(-\sin \alpha) & (\cos \alpha)(\sin \alpha) + (-\sin \alpha)(\cos \alpha) \\ (\sin \alpha)(\cos \alpha) + (\cos \alpha)(-\sin \alpha) & (\sin \alpha)(\sin \alpha) + (\cos \alpha)(\cos \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that  $A'A = I$ .

**Solution 7**

(ii)

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned} & \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\sin \alpha)(\sin \alpha) + (-\cos \alpha)(-\cos \alpha) & (\sin \alpha)(\cos \alpha) + (-\cos \alpha)(\sin \alpha) \\ (\cos \alpha)(\sin \alpha) + (\sin \alpha)(-\cos \alpha) & (\cos \alpha)(\cos \alpha) + (\sin \alpha)(\sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that  $A'A = I$ .

**Solution 8**

(i) We have:

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$$\therefore A' = A$$

Hence,  $A$  is a symmetric matrix.

(ii)

We have:

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$$\therefore A' = -A$$

Hence,  $A$  is a skew-symmetric matrix.

**Solution 9**

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$(i) \quad A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

Hence,  $(A + A')$  is a symmetric matrix.

(ii)

$$A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$$

Hence,  $(A - A')$  is a skew-symmetric matrix.

**Solution 10**

The given matrix is  $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

**Solution 11**

(i)

$$\text{Let } A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$$

Thus,  $P = \frac{1}{2}(A + A')$  is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

Thus,  $Q = \frac{1}{2}(A - A')$  is a skew-symmetric matrix.

Representing  $A$  as the sum of  $P$  and  $Q$ :

$$P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

(ii)

$$\text{Let } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$$

Thus,  $P = \frac{1}{2}(A + A')$  is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$$

Thus,  $Q = \frac{1}{2}(A - A')$  is a skew-symmetric matrix.

Representing  $A$  as the sum of  $P$  and  $Q$ :

$$P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$$

(iii)

$$\text{Let } A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} = P$$

Thus,  $P = \frac{1}{2}(A + A')$  is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -Q$$

Thus,  $Q = \frac{1}{2}(A - A')$  is a skew-symmetric matrix.

Representing  $A$  as the sum of  $P$  and  $Q$ :

$$P + Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$$



(iv)

$$\text{Let } A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}, \text{ then } A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\text{Now } A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$$

Thus,  $P = \frac{1}{2}(A + A')$  is a symmetric matrix.

$$\text{Now, } A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$$

$$\text{Let } Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$\text{Now, } Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -Q$$

Thus,  $Q = \frac{1}{2}(A - A')$  is a skew-symmetric matrix.

Representing  $A$  as the sum of  $P$  and  $Q$ :

$$P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$$

Solution 12

The correct answer is A.

$A$  and  $B$  are symmetric matrices, therefore, we have:

$$A' = A \text{ and } B' = B \quad \dots(1)$$

$$\begin{aligned} \text{Consider } (AB - BA)' &= (AB)' - (BA)' && \left[ (A - B)' = A' - B' \right] \\ &= B'A' - A'B' && \left[ (AB)' = B'A' \right] \\ &= BA - AB && [\text{by (1)}] \\ &= -(AB - BA) \end{aligned}$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus,  $(AB - BA)$  is a skew-symmetric matrix.

### Solution 13

The correct answer is B.

$$\begin{aligned} A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ \Rightarrow A' &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \end{aligned}$$

$$\text{Now, } A + A' = I$$

$$\begin{aligned} \therefore \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2 \cos \alpha & 0 \\ 0 & 2 \cos \alpha \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements of the two matrices, we have:

$$2 \cos \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\therefore \alpha = \frac{\pi}{3}$$

## Chapter 3 - Matrices Exercise Ex. 3.4

### Solution 1

$$\text{Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad \left( R_2 \rightarrow \frac{1}{5} R_2 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

## Solution 2

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

## Solution 3

$$\text{Let } A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - 3R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

#### Solution 4

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left( R_1 \rightarrow \frac{1}{2} R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 5R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + 3R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad (R_2 \rightarrow -2R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

#### Solution 5

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left( R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 7R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad (R_2 \rightarrow 2R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

Solution 6

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

We know that  $AI = I$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left( R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_2 - 5R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow 2R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

### Solution 7

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

We know that  $AI = I$

$$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad (C_2 \rightarrow C_2 - C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

### Solution 8

$$\text{Let } A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 3R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

### Solution 9

$$\text{Let } A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - 3R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

### Solution 10

$$\text{Let } A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$$

Now,  $\det(A) = |A| = 6 - 4 = 2 \neq 0$

Therefore, inverse of A exist.

We know that,  $A = AI$

$$\begin{aligned}
&\Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&\Rightarrow \begin{bmatrix} 3 & 0 \\ -4 & \frac{2}{3} \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & 1 \end{bmatrix} \dots\dots (C_2 \rightarrow C_2 + \frac{1}{3} C_1) \\
&\Rightarrow \begin{bmatrix} 3 & 0 \\ 0 & \frac{2}{3} \end{bmatrix} = A \begin{bmatrix} 3 & \frac{1}{3} \\ 6 & 1 \end{bmatrix} \dots\dots (C_1 \rightarrow C_1 + 6C_2) \\
&\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{2}{3} \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{3} \\ 2 & 1 \end{bmatrix} \dots\dots (C_1 \rightarrow \frac{1}{3} C_1) \\
&\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} \dots\dots (C_2 \rightarrow \frac{3}{2} C_2) \\
&\Rightarrow A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}
\end{aligned}$$

### Solution 11

$$\text{Let } A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

We know that  $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \quad (C_2 \rightarrow C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \quad \left( C_1 \rightarrow \frac{1}{2} C_1 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$



### Solution 12

$$\text{Let } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left( R_1 \rightarrow \frac{1}{6} R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 + 2R_1)$$

Now, in the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Therefore,  $A^{-1}$  does not exist

### Solution 13

$$\text{Let } A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A \quad (R_2 \rightarrow R_2 + R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

### Solution 14

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

We know that  $IA = A$

$$\therefore \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying  $R_1 \rightarrow R_1 - \frac{1}{2}R_2$ , we have:

$$\begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

Now, in the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Therefore,  $A^{-1}$  does not exist

**Solution 15**

$$\text{Let } A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad \left(R_2 \rightarrow \frac{1}{5}\right)$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ -\frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} A \quad (R_1 \rightarrow R_1 + R_2 \text{ and } R_3 \rightarrow R_3 + 2R_2)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 1 & -2 \end{bmatrix} A \quad (R_3 \rightarrow R_3 + 3R_1)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad \left( R_3 \rightarrow \frac{1}{5}R_3 \right)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & 0 & -\frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A \quad (R_1 \rightarrow (-1)R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Solution 16

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying  $R_2 \rightarrow R_2 + 3R_1$  and  $R_3 \rightarrow R_3 - 2R_1$ , we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying  $R_1 \rightarrow R_1 + 3R_3$  and  $R_2 \rightarrow R_2 + 8R_3$ , we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying  $R_3 \rightarrow R_3 + R_2$ , we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying  $R_3 \rightarrow \frac{1}{25}R_3$ , we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying  $R_1 \rightarrow R_1 - 10R_3$ , and  $R_2 \rightarrow R_2 - 21R_3$ , we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

**Solution 17**

$$\text{Let } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

We know that  $A = IA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying  $R_1 \rightarrow \frac{1}{2}R_1$ , we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying  $R_2 \rightarrow R_2 - 5R_1$ , we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying  $R_3 \rightarrow R_3 - R_2$ , we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying  $R_3 \rightarrow 2R_3$ , we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying  $R_1 \rightarrow R_1 + \frac{1}{2}R_3$ , and  $R_2 \rightarrow R_2 - \frac{5}{2}R_3$ , we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

## Solution 18

**Answer: D**

We know that if  $A$  is a square matrix of order  $m$ , and if there exists another square matrix  $B$  of the same order  $m$ , such that  $AB = BA = I$ , then  $B$  is said to be the inverse of  $A$ . In this case, it is clear that  $A$  is the inverse of  $B$ .

Thus, matrices  $A$  and  $B$  will be inverses of each other only if  $AB = BA = I$

## Chapter 3 - Matrices Exercise Misc. Ex.

### Solution 1



It is given that  $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show:  $P(n): (aI + bA)^n = a^n I + na^{n-1}bA, n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For  $n = 1$ , we have:

$$P(1): (aI + bA) = aI + ba^0 A = aI + bA$$

Therefore, the result is true for  $n = 1$ .

Let the result be true for  $n = k$ .

That is,

$$P(k): (aI + bA)^k = a^k I + ka^{k-1}bA$$

Now, we prove that the result is true for  $n = k + 1$ .

Consider

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= (a^k I + ka^{k-1}bA)(aI + bA) \\ &= a^{k+1} I + ka^k bAI + a^k bIA + ka^{k-1}b^2 A^2 \\ &= a^{k+1} I + (k+1)a^k bA + ka^{k-1}b^2 A^2 \quad \dots(1) \end{aligned}$$

$$\text{Now, } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

From (1), we have:

$$\begin{aligned} (aI + bA)^{k+1} &= a^{k+1} I + (k+1)a^k bA + O \\ &= a^{k+1} I + (k+1)a^k bA \end{aligned}$$

Therefore, the result is true for  $n = k + 1$ .

Thus, by the principle of mathematical induction, we have:

$$(aI + bA)^n = a^n I + na^{n-1}bA \text{ where } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, n \in \mathbf{N}$$

**Solution 2**

It is given that  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

To show:  $P(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For  $n = 1$ , we have:

$$P(1): \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

Therefore, the result is true for  $n = 1$ .

Let the result be true for  $n = k$ .

That is  $P(k): A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$

Now, we prove that the result is true for  $n = k + 1$ .

Now,  $A^{k+1} = A \cdot A^k$

$$\begin{aligned} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \\ &= \begin{bmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{bmatrix} \\ &= \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix} \end{aligned}$$

Therefore, the result is true for  $n = k + 1$ .

Thus by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$$

Solution 3

It is given that  $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove:  $P(n): A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For  $n = 1$ , we have:

$$P(1): A^1 = \begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Therefore, the result is true for  $n = 1$ .

Let the result be true for  $n = k$ .

That is,

$$P(k): A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}, n \in \mathbf{N}$$

Now, we prove that the result is true for  $n = k + 1$ .

Consider

$$A^{k+1} = A^k \cdot A$$

$$\begin{aligned} &= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3(1+2k) - 4k & -4(1+2k) + 4k \\ 3k + 1 - 2k & -4k - 1(1-2k) \end{bmatrix} \\ &= \begin{bmatrix} 3 + 6k - 4k & -4 - 8k + 4k \\ 3k + 1 - 2k & -4k - 1 + 2k \end{bmatrix} \\ &= \begin{bmatrix} 3 + 2k & -4 - 4k \\ 1 + k & -1 - 2k \end{bmatrix} \\ &= \begin{bmatrix} 1 + 2(k+1) & -4(k+1) \\ 1 + k & 1 - 2(k+1) \end{bmatrix} \end{aligned}$$

Therefore, the result is true for  $n = k + 1$ .

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbf{N}$$

**Solution 4**

It is given that  $A$  and  $B$  are symmetric matrices. Therefore, we have:

$$A' = A \text{ and } B' = B \quad \dots(1)$$

$$\begin{aligned} \text{Now, } (AB - BA)' &= (AB)' - (BA)' & \left[ (A - B)' = A' - B' \right] \\ &= B'A' - A'B' & \left[ (AB)' = B'A' \right] \\ &= BA - AB & [\text{Using (1)}] \\ &= -(AB - BA) \end{aligned}$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus,  $(AB - BA)$  is a skew-symmetric matrix.

### Solution 5

We suppose that  $A$  is a symmetric matrix, then  $A' = A \dots (1)$

Consider

$$\begin{aligned} (B'AB)' &= \{B'(AB)\}' \\ &= (AB)'(B')' & \left[ (AB)' = B'A' \right] \\ &= B'A'(B) & \left[ (B')' = B \right] \\ &= B'(A'B) \\ &= B'(AB) & [\text{Using (1)}] \end{aligned}$$

$$\therefore (B'AB)' = B'AB$$

Thus, if  $A$  is a symmetric matrix, then  $B'AB$  is a symmetric matrix.

Now, we suppose that  $A$  is a skew-symmetric matrix.

Then,  $A' = -A$

Consider

$$\begin{aligned} (B'AB)' &= [B'(AB)]' = (AB)'(B')' \\ &= (B'A')B = B'(-A)B \\ &= -B'AB \end{aligned}$$

$$\therefore (B'AB)' = -B'AB$$

Thus, if  $A$  is a skew-symmetric matrix, then  $B'AB$  is a skew-symmetric matrix.

Hence, if  $A$  is a symmetric or skew-symmetric matrix, then  $B'AB$  is a symmetric or skew-symmetric matrix accordingly

## Solution 6

It is given that  $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Now,  $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-xz+xz & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$6y^2 = 1 \Rightarrow y = \pm \frac{1}{\sqrt{6}}$$

$$3z^2 = 1 \Rightarrow z = \pm \frac{1}{\sqrt{3}}$$

## Solution 7

We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6(0) + 2(2) + 4(x)] = O$$

$$\Rightarrow [4 + 4x] = [0]$$

$$\therefore 4 + 4x = 0$$

$$\Rightarrow x = -1$$

Thus, the required value of  $x$  is  $-1$ .

**Solution 8**

It is given that  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\begin{aligned}\therefore A^2 &= A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\therefore \text{L.H.S.} &= A^2 - 5A + 7I \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= O = \text{R.H.S.}\end{aligned}$$

$$\therefore A^2 - 5A + 7I = O$$

**Solution 9**

We have:

$$\begin{aligned}
 & \begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O \\
 & \Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O \\
 & \Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O \\
 & \Rightarrow [x(x-2) - 40 + 2x - 8] = O \\
 & \Rightarrow [x^2 - 2x - 40 + 2x - 8] = [0] \\
 & \Rightarrow [x^2 - 48] = [0] \\
 & \therefore x^2 - 48 = 0 \\
 & \Rightarrow x^2 = 48 \\
 & \Rightarrow x = \pm 4\sqrt{3}
 \end{aligned}$$

## Solution 10

(a) The unit sale prices of  $x$ ,  $y$ , and  $z$  are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00.

Consequently, the total revenue in market **I** can be represented in the form of a matrix as:

$$\begin{aligned}
 & \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \\
 & = 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00 \\
 & = 25000 + 3000 + 18000 \\
 & = 46000
 \end{aligned}$$

The total revenue in market **II** can be represented in the form of a matrix as:

$$\begin{aligned}
 & \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \\
 & = 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00 \\
 & = 15000 + 30000 + 8000 \\
 & = 53000
 \end{aligned}$$

Therefore, the total revenue in market **I** is Rs 46000 and the same in market **II** is Rs 53000.

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(b) The unit cost prices of  $x$ ,  $y$ , and  $z$  are respectively given as Rs 2.00, Rs 1.00, and 50 paise.

Consequently, the total cost prices of all the products in market **I** can be represented in the form of a matrix as:

$$\begin{aligned} & \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} \\ &= 10000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50 \\ &= 20000 + 2000 + 9000 \\ &= 31000 \end{aligned}$$

Since the total revenue in market **I** is Rs 46000, the gross profit in this market is (Rs 46000 – Rs 31000) Rs 15000.

The total cost prices of all the products in market **II** can be represented in the form of a matrix as:

$$\begin{aligned} & \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} \\ &= 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50 \\ &= 12000 + 20000 + 4000 \\ &= \text{Rs } 36000 \end{aligned}$$

Since the total revenue in market **II** is Rs 53000, the gross profit in this market is (Rs 53000 – Rs 36000) Rs 17000

**Solution 11**

It is given that:

$$X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a  $2 \times 3$  matrix and the one given on the L.H.S. of the equation is a  $2 \times 3$  matrix. Therefore,  $X$  has to be a  $2 \times 2$  matrix.

$$\text{Now, let } X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c = -7, \quad 2a+5c = -8, \quad 3a+6c = -9$$

$$b+4d = 2, \quad 2b+5d = 4, \quad 3b+6d = 6$$

$$\text{Now, } a+4c = -7 \Rightarrow a = -7-4c$$

$$\therefore 2a+5c = -8 \Rightarrow -14-8c+5c = -8$$

$$\Rightarrow -3c = 6$$

$$\Rightarrow c = -2$$

$$\therefore a = -7-4(-2) = -7+8 = 1$$

$$\text{Now, } b+4d = 2 \Rightarrow b = 2-4d$$

$$\therefore 2b+5d = 4 \Rightarrow 4-8d+5d = 4$$

$$\Rightarrow -3d = 0$$

$$\Rightarrow d = 0$$

$$\therefore b = 2-4(0) = 2$$

$$\text{Thus, } a = 1, b = 2, c = -2, d = 0$$

$$\text{Hence, the required matrix } X \text{ is } \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.$$

**Solution 12**

$A$  and  $B$  are square matrices of the same order such that  $AB = BA$ .

To prove:  $P(n): AB^n = B^n A, n \in \mathbb{N}$

For  $n = 1$ , we have:

$$P(1): AB = BA \quad [\text{Given}]$$

$$\Rightarrow AB^1 = B^1 A$$

Therefore, the result is true for  $n = 1$ .

Let the result be true for  $n = k$ .

$$P(k): AB^k = B^k A \quad \dots(1)$$

Now, we prove that the result is true for  $n = k + 1$ .

$$\begin{aligned} AB^{k+1} &= AB^k \cdot B \\ &= (B^k A) B && [\text{By (1)}] \\ &= B^k (AB) && [\text{Associative law}] \\ &= B^k (BA) && [AB = BA \text{ (Given)}] \\ &= (B^k B) A && [\text{Associative law}] \\ &= B^{k+1} A \end{aligned}$$

Therefore, the result is true for  $n = k + 1$ .

Thus, by the principle of mathematical induction, we have  $AB^n = B^n A, n \in \mathbb{N}$ .

Now, we prove that  $(AB)^n = A^n B^n$  for all  $n \in \mathbb{N}$

For  $n = 1$ , we have:

$$(AB)^1 = A^1 B^1 = AB$$

Therefore, the result is true for  $n = 1$ .

Let the result be true for  $n = k$ .

$$(AB)^k = A^k B^k \quad \dots(2)$$

Now, we prove that the result is true for  $n = k + 1$ .

$$\begin{aligned} (AB)^{k+1} &= (AB)^k \cdot (AB) \\ &= (A^k B^k) \cdot (AB) && [\text{By (2)}] \\ &= A^k (B^k A) B && [\text{Associative law}] \\ &= A^k (AB^k) B && [AB^n = B^n A \text{ for all } n \in \mathbb{N}] \\ &= (A^k A) \cdot (B^k B) && [\text{Associative law}] \\ &= A^{k+1} B^{k+1} \end{aligned}$$

Therefore, the result is true for  $n = k + 1$ .

Thus, by the principle of mathematical induction, we have  $(AB)^n = A^n B^n$ , for all natural numbers

**Solution 13**

**Answer: C**

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\begin{aligned} \therefore A^2 = A \cdot A &= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} \\ &= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^2 = I \Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$\alpha^2 + \beta\gamma = 1$$

$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

**Solution 14**

**Answer: B**

If  $A$  is both symmetric and skew-symmetric matrix, then we should have

$$A' = A \text{ and } A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = O$$

$$\Rightarrow 2A = O$$

$$\Rightarrow A = O$$

Therefore,  $A$  is a zero matrix.

**Solution 15**

**Answer: C**

$$(I + A)^3 - 7A = I^3 + A^3 + 3I^2A + 3A^2I - 7A$$

$$= I + A^3 + 3A + 3A^2 - 7A$$

$$= I + A^2 \cdot A + 3A + 3A - 7A \quad [A^2 = A]$$

$$= I + A \cdot A - A$$

$$= I + A^2 - A$$

$$= I + A - A$$

$$= I$$

$$\therefore (I + A)^3 - 7A = I$$

