RD SHARMA Solutions for Class 12-science Maths Chapter 33 - Binomial Distribution

Chapter 33 - Binomial Distribution Exercise Ex. 33.1

Question 1

There, are 6% defective items in a large bulk of items. Find the probability that a sample of 8 items will include not more than one defective items.

Solution 1

Let p denote the probability of having defective item, so

$$p = 6\% = \frac{6}{100} = \frac{3}{50}$$

So, $q = 1 - p$

$$= 1 - \frac{3}{50} \qquad [\text{Since } p + q = 1]$$

$$= \frac{47}{50}$$

Let X denote the number of defective items in a sample of 8 items. Then, the probability of getting r defective bulks is

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = {}^{8}C_r \left(\frac{3}{50}\right)^r \left(\frac{47}{50}\right)^{8-r} \qquad ---(1)$$

Therefore, probability of getting not more then one defective item

$$= P(X = 0) + P(X = 1)$$

$$= {}^{8}C_0 \left(\frac{3}{50}\right)^0 \left(\frac{47}{50}\right)^{8-0} + {}^{8}C_1 \left(\frac{3}{50}\right)^1 \left(\frac{47}{50}\right)^{8-1} \qquad [\text{Using equation } (1)]$$

$$= 1.1.\left(\frac{47}{50}\right)^8 + 8.\frac{3}{50}.\left(\frac{47}{50}\right)^7$$

$$= \left(\frac{47}{50}\right)^7 \left(\frac{47}{50} + \frac{24}{50}\right)$$

$$= \left(\frac{71}{50}\right)\left(\frac{47}{50}\right)^7$$

$$= (1.42) \times (0.94)^7$$

The required probability is,

$$(1.42) \times (0.94)^7$$

Question 2

A coin is tossed 5 times. What is the probability of getting at least 3 heads.

Solution 2

Probability of getting head on one throw of coin $= \frac{1}{2}$

So, $p = \frac{1}{2}$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

The coin is tossed 5 times. Let X denote the number of getting head as 5 tosses of coins.
So probability of getting r heads in n tosses of coin is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting at least 3 heads

$$= P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 . \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 \qquad [\text{Using } (1)]$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 .1$$

$$= \frac{5.4}{2} . \left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5 + 1.\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [10 + 5 + 1]$$

$$= 16.\frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is $= \frac{1}{2}$

Question 3

A coin is tossed 5 times. What is the probability that tail appears an odd number of times.

Solution 3

Let p be the probability getting tail on a toss of a fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the number tail obtained on the toss of coin 5 times. So probability of getting r tails in n tosses of coin is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{5}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting tail an odd number of times

$$= P\left(X = 1\right) + P\left(X = 3\right) + P\left(X = 5\right)$$

$$= {}^{5}C_{3}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{5-1} + {}^{5}C_{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{5-3} + {}^{5}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{0} \qquad \left[\text{Using } (1)\right]$$

$$= 5.\left(\frac{1}{2}\right)^{5} + \frac{5.4}{2}.\left(\frac{1}{2}\right)^{5} + 1.\left(\frac{1}{2}\right)^{5}$$

$$= \left(\frac{1}{2}\right)^{5}\left[5 + 10 + 1\right]$$

$$= 16\left(\frac{1}{2}\right)^{5}$$

$$= 16.\frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is $= \frac{1}{2}$

Question 4

A pair of dice is thrown 6 times. If getting a total of 9 is considered a success, what is the probability of at least 5 successes.

Solution 4

Let p be the probability of getting a sum of 9 and it considered as success.
Sum of a 9 on a pair of dice

$$= \{(3,6), (4,5), (5,4), (6,3)\}$$

So, $p = \frac{4}{36}$

$$p = \frac{1}{9}$$

$$q = 1 - \frac{1}{9}$$

$$q = \frac{8}{9} \qquad [\text{Since } p + q = 1]$$

Let X denote the number of success in throw of a pair of dice 6 times. So probability of getting r success out of n is given by

$$P(X = r) = {}^nC_r p^r q^{n-r} \qquad ---(1)$$

Probability of getting at least 5 success

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{1}{9}\right)^5 \left(\frac{8}{9}\right)^{6-5} + {}^6C_6 \left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^{6-6} \qquad [\text{Using } (1)]$$

$$= 6\left(\frac{1}{9}\right)^5 \left(\frac{8}{9}\right)^1 + 1.\left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^0$$

$$= \left(\frac{1}{9}\right)^5 \left[\frac{48}{9} + \frac{1}{9}\right]$$

$$= \frac{49}{9} \times \left(\frac{1}{9}\right)^5$$

$$= \frac{49}{9^6}$$

So,

Required probability $= \frac{49}{9^6}$

Question 5

A fair coin is tossed 6 times. What is the probability of getting at least 3 heads.

Solution 5

Let p be the probability of getting head in a throw of coin. So,

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the number of heads on tossing the coin 6 times. Probability of getting r in tossing the coin n times is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r} \qquad ---(1)$$

Probability of getting at least three heads

$$= P\left(X = 3\right) + P\left(X = 4\right) + P\left(X = 5\right) + P\left(X = 6\right)$$

$$= 1 - \left[P\left(X = 0\right) + P\left(X = 1\right) + P\left(X = 2\right)\right]$$

$$= 1 - \left[{}^{6}C_{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{6-0} + {}^{6}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{6-1} + {}^{6}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{6-2}\right] \qquad \left[\text{Using } (1)\right]$$

$$= 1 - \left[1.\left(\frac{1}{2}\right)^{6} + 6\left(\frac{1}{2}\right)^{6} + \frac{6.5}{2}.\left(\frac{1}{2}\right)^{6}\right]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^{6}\left(1 + 6 + 15\right)\right]$$

$$= 1 - \left[\frac{22}{64}\right]$$

$$= \frac{64 - 22}{64}$$

$$= \frac{42}{64}$$

$$= \frac{21}{32}$$

Required probability $= \frac{21}{32}$

Question 6

Find the probability of 4 turning up at least once in two tosses of a fair die.

Solution 6

Let p denote the 4 turning up in a toss of a fair die, so

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6}$$

$$q = \frac{5}{6} \qquad [\text{Since } p + q = 1]$$

Let X denote the variable showing the number of turning 4 up in 2 tosses of die.
Probability of getting 4, r times in n tosses of a die is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^2C_r \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{2-r} \qquad ---(1)$$

Probability of getting 4 at least once in tow tosses of a fair die

$$= P(X = 1) + P(X = 2)$$

$$= 1 - P(X = 0)$$

$$= 1 - \left[{}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{2-0}\right] \qquad [\text{Using } (1)]$$

$$= 1 - \left[1.1.\left(\frac{5}{6}\right)^2\right]$$

$$= 1 - \left[\frac{25}{36}\right]$$

$$= \frac{36 - 25}{36}$$

$$= \frac{11}{36}$$

So,

Required probability $= \dfrac{11}{36}$

Question 7

A coin is tossed 5 times. What is the probability that head appears an even number of times.

Solution 7

Let p denote the probability of getting head in a toss of fair coin. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

Let X denote the variable representing number of heads on 5 tosses of a fair coin.
Probability of getting r an n tosses of a fair coin, so

$$P\left(X = r\right) = {}^{n}C_r p^r q^{n-r}$$

$$P\left(X = r\right) = {}^{5}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability of getting head on an even number of tosses of coin

$$= P\left(X = 0\right) + P\left(X = 2\right) + P\left(X = 4\right)$$

$$= {}^{5}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} + {}^{5}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} + {}^{5}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} \qquad \left[\text{Using } (1)\right]$$

$$= 1.1.\left(\frac{1}{2}\right)^5 + \frac{5.4}{2}.\left(\frac{1}{2}\right)^5 + 5.\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 \left[1 + 10 + 5\right]$$

$$= 16 \times \frac{1}{32}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Question 8

The probability of a man hitting of target is $\frac{1}{4}$. If he fires 7 times, what is the probability of his hitting the target at least twice.

Solution 8

Let p be the probability of hitting the target, so

$$p = \frac{1}{4}$$

$$q = 1 - p \qquad [\text{Since } p + q = 1]$$

$$= 1 - \frac{1}{4}$$

$$q = \frac{3}{4}$$

Let X denote the variable representing the number of times hittintg the target out of 7 fires. Probability of hitting the target r times out of n fires is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^7C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{7-r} \qquad ---(1)$$

Probability of hitting the target at least twice

$$= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[{}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{7-0} + {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{7-1}\right] \qquad [\text{Using } (1)]$$

$$= 1 - \left[1.1.\left(\frac{3}{4}\right)^7 + 7.\frac{1}{4}.\left(\frac{3}{4}\right)^6\right]$$

$$= 1 - \left(\frac{3}{4}\right)^6 \left(\frac{3}{4} + \frac{7}{4}\right)$$

$$= 1 - \left(\frac{3}{4}\right)^6 \left(\frac{10}{4}\right)$$

$$= 1 - \frac{7290}{16384}$$

$$= \frac{9194}{16384}$$

$$= \frac{4547}{8192}$$

Required Probability = $\frac{4547}{8192}$

Question 9

Assume that on an average one telephone number out of 15 called between 2 P.M. and 3 P.M. on week days is busy. What is the probability that if six randomly selected telephone numbers are called, at least 3 of them will be busy.

Solution 9

Let the probability of one telephone number out of 15 is busy between 2 PM and 3 PM be 'p'. then
P = 1/15 ; probability that number is not busy, q = 1-p

Q = 14/16. Binomial distribution is $\left(\frac{14}{15}+\frac{1}{15}\right)^6$
Since 6 numbers are called we find the probability for none of the numbers are busy is P(0)
One number is busy P(1); Two numbers are busy is P(2)
Three numbers are busy is P(3); Four numbers are busy is P(4) ; Five numbers are busy is P(5); Six numbers are busy is P(6).

$P(0) = {}^6C_0 \left(\frac{14}{15}\right)^6$

$P(1) = {}^6C_1 \left(\frac{14}{15}\right)^5 \left(\frac{1}{15}\right)^1$

$P(2) = {}^6C_2 \left(\frac{14}{15}\right)^4 \left(\frac{1}{15}\right)^2$

$P(3) = {}^6C_3 \left(\frac{14}{15}\right)^3 \left(\frac{1}{15}\right)^3$

$P(4) = {}^6C_4 \left(\frac{14}{15}\right)^2 \left(\frac{1}{15}\right)^4$

$P(5) = {}^6C_5 \left(\frac{14}{15}\right)^1 \left(\frac{1}{15}\right)^5$

$P(6) = {}^6C_6 \left(\frac{14}{15}\right)^0 \left(\frac{1}{15}\right)^6$

Probability that at least 3 of the numbers will be busy

P(3) + P(4) + P(5) + P(6) = 0.05

Question 10

If getting 5 or 6 in a throw of an unbiased die is a success and the random variable 'X' denotes the number of successes in six throws of the die, find $P(X \geq 4)$.

Solution 10

p denote the probability of success
p = Probability of getting 5 or 6 in a throw of die.

$$= \frac{2}{6}$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in six throws of a dic. Probability of getting r success in six throws of an unbiased dic is given by

$$P(X = r) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$= {}^{6}C_{r}\left(\frac{1}{3}\right)^{r}\left(\frac{2}{3}\right)^{6-r} \qquad ---(1)$$

$$P(X \geq 4)$$

$$= P(X = 4) + P(X = 5) + P(X = 6)$$

$$= {}^{6}C_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{6-4} + {}^{6}C_{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{6-5} + {}^{6}C_{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{6-6}$$

$$= \frac{6.5}{2}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{2} + 6\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right) + 1.\left(\frac{1}{3}\right)^{6}.1$$

$$= 15.\frac{1}{81}.\frac{4}{9} + 6.\frac{1}{243}.\frac{2}{3} + \frac{1}{729}$$

$$= \frac{60}{729} + \frac{12}{729} + \frac{1}{729}$$

$$= \frac{73}{729}$$

Required probability $= \dfrac{73}{729}$

Question 11

Eight coins are thrown simultaneously. Find the chance of obtaining at least six heads.

Solution 11

Let p denote the probability of getting head on a throw of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{1}{2}$$

Let X denote the variable representing the number of getting heads on throw of 8 coins.
Probability of getting r heads in a throw of n coins is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \qquad ---(1)$$

Probability of getting at least six heads

$$= P(X = 6) + P(X = 7) + P(X = 8)$$

$$= {}^8C_6 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{8-6} + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{8-7} + {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{8-8} \qquad [\text{Using } (1)]$$

$$= \frac{8.7}{2}\left(\frac{1}{2}\right)^8 + 8\left(\frac{1}{2}\right)^8 + 1.\left(\frac{1}{2}\right)^8.1$$

$$= \left(\frac{1}{2}\right)^8 [28 + 8 + 1]$$

$$= \frac{1}{256}(37)$$

$$= \frac{37}{256}$$

Required probability $= \frac{37}{256}$

Question 12

Five cards are drawn successively with replacement from a well shuffled pack of 52 cards. What is the probability that

(i) all the five cards are spades? (ii) only 3 cards are spades?

(iii) none is spade?

Solution 12

Let p denote the probability of getting one spade out of a deck of 52 cards, so

$$p = \frac{13}{52}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{3}{4}$$

Let X denote the radom variable of number of spades out of 5 cards. Probability of getting r spades out of n cards is given by

$$P\left(X = r\right) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$= {}^{5}C_{r}\left(\frac{1}{4}\right)^{r}\left(\frac{3}{4}\right)^{5-r} \qquad ---(1)$$

(i)

Probability of getting all five spades

$$= P\left(X = 5\right)$$

$$= {}^{5}C_{5}\left(\frac{1}{4}\right)^{5}\left(\frac{3}{4}\right)^{5-5}$$

$$= \frac{1}{1024}$$

Probability of getting 5 spades $= \frac{1}{1024}$

(ii)

Probability of getting only 3 spades

$$= P\left(X = 3\right)$$

$$= {}^{5}C_{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{5-3}$$

$$= \frac{5.4}{2}\left(\frac{1}{64}\right)\left(\frac{9}{16}\right)$$

$$= \frac{45}{512}$$

Probability of getting 3 spades $= \frac{45}{512}$

(iii)

Probability that none is spade

$$= P(X = 0)$$

$$= {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0}$$

$$= \frac{243}{1024}$$

Probability of getting non spade $= \frac{243}{1024}$

Question 13

A bag contains 7 red, 5 white and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that

(i) none is white? (ii) all are white? (iii) any two are white?

Solution 13

Let p be the probability of getting 1 white ball out of 7 red, 5 white and 8 black balls. So

$$p = \frac{5}{20}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{3}{4}$$

Let X denote the random variable of number of selecting white ball with replacement out of 4 balls. Probability of getting r white balls out of n balls is given by

$$P\left(X = r\right) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$= {}^{4}C_{r}\left(\frac{1}{4}\right)^{r}\left(\frac{3}{4}\right)^{4-r} \qquad ---(1)$$

(i)

Probability of getting none white ball

$$= P\left(X = 0\right)$$

$$= {}^{4}C_{0}\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{4-0} \qquad \left[\text{Using } (1)\right]$$

$$= \left(\frac{3}{4}\right)^{4}$$

$$= \frac{81}{256}$$

Probability of getting none white ball $= \frac{81}{256}$

(ii)

Probability of getting all white balls

$$= P\left(X = 4\right)$$

$$= {}^{4}C_{4}\left(\frac{1}{4}\right)^{4}\left(\frac{3}{4}\right)^{4-0}$$

$$= \left(\frac{1}{4}\right)^{4}$$

$$= \frac{1}{256}$$

Probability of getting all white balls $= \frac{1}{256}$

(iii)

Probability of getting any two are white

$$= P(X = 2)$$

$$= {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{4-2}$$

$$= \frac{4.3}{2} \cdot \frac{1}{16} \cdot \frac{9}{16}$$

$$= \frac{27}{128}$$

Probability of getting any two are white balls $= \frac{27}{128}$

Question 14

A box contains 100 tickets each bearing one of the numbers from 1 to 100. If 5 tickets are drawn successively with replacement from the box, find the probability that all the tickets bear numbers divisible by 10.

Solution 14

Let p denote the probability of getting a ticket bearing number divisible by 10, So

$$p = \frac{10}{100} \qquad \left[\begin{array}{l}\text{Since there are } 10, 20, 30, 40, 50, 60, 70, 80,\\ 90, 100 \text{ which are divisible by } 10\end{array}\right]$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Let X denote the variable representing the number of tickets bearing a number divisible by 10 out of 5 tickets. Probability of getting r tickets bearing a number divisible by 10 out of n tickets is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r} \qquad ---(1)$$

Probability of getting all the tickets bearing a number divisible by 10

$$= {}^5C_5 \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^{5-5} \qquad [\text{Using } (1)]$$

$$= 1. \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^0$$

$$= \left(\frac{1}{10}\right)^5$$

Required probability $= \left(\frac{1}{10}\right)^5$

Question 15

A bag contains 10 balls each marked wiht one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Solution 15

Let p denote the probability of getting a ball marked with 0. So

$$p = \frac{1}{10} \qquad \left[\text{Since balls are marked with } 0,1,2,3,4,5,6,7,8,9\right]$$

$$q = 1 - \frac{1}{10} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{9}{10}$$

Let X denote the variable presenting the number of balls marked with 0 out of four balls drawn. Probability of drawing r balls out of n balls that are marked 0 is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^4C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{4-r} \qquad ---(1)$$

Probability of getting none balls marked with 0

$$= P(X = 0)$$

$$= {}^4C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{4-0}$$

$$= 1.1. \left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^4$$

Probability of getting none balls marked with 0 $= \left(\frac{9}{10}\right)^4$

Question 16

There are 5 percent defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more that one defective item.

Solution 16

Let p denote the probability of getting one defective item out of hundred. So

$$p = 5\% \qquad \left[\text{Since } 5\% \text{ are defective items}\right]$$

$$= \frac{5}{100}$$

$$p = \frac{1}{20}$$

$$q = 1 - \frac{1}{20} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{19}{20}$$

Let X denote the random variable representing the number of defective items out of 10 items. Probability of getting r defective items out of n items selected is given by,

$$P\left(X = r\right) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$= {}^{10}C_{r}\left(\frac{1}{20}\right)^{r}\left(\frac{19}{20}\right)^{10-r} \qquad ---\left(1\right)$$

Probability of getting not more than one defective items

$$= P\left(X = 0\right) + P\left(X = 1\right)$$

$$= {}^{10}C_{0}\left(\frac{1}{20}\right)^{0}\left(\frac{19}{20}\right)^{10-0} + {}^{10}C_{1}\left(\frac{1}{20}\right)^{1}\left(\frac{19}{20}\right)^{10-1}$$

$$= 1.1.\left(\frac{19}{20}\right)^{10} + 10.\frac{1}{20}\left(\frac{19}{20}\right)^{9}$$

$$= \left(\frac{19}{20}\right)^{9}\left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \frac{29}{20}\left(\frac{19}{20}\right)^{9}$$

The required probability $= \frac{29}{20}\left(\frac{19}{20}\right)^{9}$

Question 17

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

(i) none (ii) not more than one (iii) more than one

(iv) at least fuse after 150 days of use.

Solution 17

Let p denote the probability that one bulb produced will fuse after 150 days, so

$$p = 0.05$$

$$= \frac{5}{100} \qquad \left[\text{It is given}\right]$$

$$p = \frac{1}{20}$$

$$q = 1 - \frac{1}{20} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{19}{20}$$

Let X denote the number of fuse bulb out of 5 bulbs. Probability that r bulbs out of n will fuse in 150 days is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{5}C_{r} \left(\frac{1}{20}\right)^{r} \left(\frac{19}{20}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that none is fuse $= P\left(X = 0\right)$

$$= {}^{5}C_{0} \left(\frac{1}{20}\right)^{0} \left(\frac{19}{20}\right)^{5-0}$$

$$= \left(\frac{19}{20}\right)^{5}$$

Probability that none will fuse $= \left(\frac{19}{20}\right)^{5}$

(ii)

Probability that not more than 1 will fuse

$$= P\left(X = 0\right) + P\left(X = 1\right)$$

$$= \left(\frac{19}{20}\right)^{5} + {}^{5}C_{1} \left(\frac{1}{20}\right)^{1} \left(\frac{19}{20}\right)^{5-1}$$

$$= \left(\frac{19}{20}\right)^{4} \left[\frac{19}{20} + \frac{5}{20}\right]$$

$$= \left(\frac{24}{20}\right) \left(\frac{19}{20}\right)^{4}$$

Probability not more than one will fuse $= \left(\frac{6}{5}\right) \left(\frac{19}{20}\right)^{4}$

(iii)

Probability that more than one will fuse

$$= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$
$$= 1 - \left[P(X = 0) + P(X = 1)\right]$$
$$= 1 - \left[\frac{6}{5}\left(\frac{19}{20}\right)^4\right]$$

Probability that more than one will fuse $= 1 - \left[\frac{6}{5}\left(\frac{19}{20}\right)^4\right]$

(iv)

Probability that that at least one will fuse

$$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$
$$= 1 - P(X = 0)$$
$$= 1 - \left[{}^5C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^{5-0}\right]$$
$$= 1 - \left[\left(\frac{19}{20}\right)^5\right]$$

Probability that that at least one will fuse $= 1 - \left(\frac{19}{20}\right)^5$

Question 18

Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Solution 18

A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed.

$$\therefore p = \text{P}\left(\text{right-handed}\right) = \frac{9}{10}$$
$$q = \text{P}\left(\text{left-handed}\right) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

= 1 − P (more than 6 are right-handed)

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r \left(0.9\right)^r \left(0.1\right)^{10-r}$$

Question 19

A bag contains 7 green, 4 white and 5 red balls. If four balls are drawn one by one with replacement, what is the probability that one is red?

Solution 19

Let p denote the probability of getting 1 red ball out of 7 green, 4 white and 5 red balls, so

$$p = \frac{5}{16}$$

$$q = 1 - \frac{5}{16} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{11}{16}$$

Let X denote the number of red balls drawn out of four balls. Probability of getting r red balls out of n drawn balls is given by

$$P\left(X = r\right) = {}^{n}C_r p^r q^{n-r}$$

$$= {}^{4}C_r \left(\frac{5}{16}\right)^r \left(\frac{11}{16}\right)^{4-r} \qquad ---(1)$$

Probability of getting one red ball

$$= P\left(X = 1\right)$$

$$= {}^{4}C_1 \left(\frac{5}{16}\right)^1 \left(\frac{11}{16}\right)^{4-1}$$

$$= 4.\left(\frac{5}{16}\right)\left(\frac{11}{16}\right)^3$$

$$= \left(\frac{5}{4}\right)\left(\frac{11}{16}\right)^3$$

Required probability $= \left(\frac{5}{4}\right)\left(\frac{11}{16}\right)^3$

Question 20

A bag contains 2 white, 3 red and 4 blue balls. Two balls are drawn at random from the bag. If X denotes the number of white balls among the two balls drawn, describe the probability distribution of X.

Solution 20

Here, A bag has 2 white, 3 red and 4 blue balls and 2 balls are drawn one-by one without replacement.

X denote the number of white balls out of 2 balls drawn. So

X	$P(X)$
0	$\frac{7}{9}\times\frac{6}{8}=\frac{21}{36}$
1	$\frac{7}{9}\times\frac{2}{8}\times 2=\frac{14}{36}$
2	$\frac{2}{9}\times\frac{1}{8}=\frac{1}{36}$

$$\Sigma P(X)=\frac{21}{36}+\frac{7}{18}+\frac{1}{36}$$
$$=\frac{36}{36}$$
$$\Sigma P(X)=1$$

Question 21

An urn contains 4 white and three red balls. Find the probability distribution of the number of red balls in three draws, with replacement form the urn.

Solution 21

Here, the urn contains 4 white and 3 red balls. Three balls are drawn with replacement.

Let X denote the number of red balls drawn out of 3 drawn balls, so probability distribution is given by

X	$P(X)$
0	${}^3C_0\left(\frac{3}{7}\right)^0\left(\frac{4}{7}\right)^{3-0}=\left(\frac{4}{7}\right)^3=\frac{64}{343}$
1	${}^3C_1\left(\frac{3}{7}\right)^1\left(\frac{4}{7}\right)^{3-1}=3.\left(\frac{3}{7}\right)\left(\frac{4}{7}\right)^2=\frac{144}{343}$
2	${}^3C_2\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)^{3-2}=3.\left(\frac{3}{7}\right)^2\left(\frac{4}{7}\right)=\frac{108}{343}$
3	${}^3C_3\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^{3-0}=\left(\frac{3}{7}\right)^3=\frac{27}{343}$

$$\Sigma P(X) = \frac{64}{343} + \frac{144}{343} + \frac{108}{343} + \frac{27}{343}$$
$$= \frac{343}{343}$$
$$= 1$$

Question 22

Find the probability distribution of the number of doublets in 4 throws of a pair of dice. Also, find the mean and variance of this distribution.

Solution 22

Let p be the probability of getting doublet is a throw of a pair of dice, so

$$p = \frac{6}{36} \qquad \left[\text{Since } (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \text{ are doublets}\right]$$
$$p = \frac{1}{6}$$
$$q = 1 - \frac{1}{6} \qquad \left[\text{Since } p + q = 1\right]$$
$$= \frac{5}{6}$$

Let X denote the number of getting doublets out of 4 times. So probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{4-0} = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$
1	${}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{4-1} = 4\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 = \frac{500}{1296}$
2	${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{4-2} = \frac{4.3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{150}{1296}$
3	${}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{4-3} = \frac{4.3}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) = \frac{20}{1296}$
4	${}^4C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{4-4} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$

$$\Sigma P(X) = \frac{625}{1296} + \frac{500}{1296} + \frac{150}{1296} + \frac{20}{1296} + \frac{1}{1296}$$
$$= \frac{1296}{1296}$$
$$= 1$$

Question 23

Find the probability distribution of the number of sixes in three tosses of a die.

Solution 23

We know that, probability of getting 6 in a throw of die $= \frac{1}{6}$

And not getting 6 $= \frac{5}{6}$

Here, die is thrown 3 time. Let X denote the number of times getting 6 out of 3 tosses of a die, then probability distribution is given by,

X	$P(X)$
0	${}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{3-0}=\left(\frac{5}{6}\right)^3=\frac{125}{216}$
1	${}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{3-1}=3\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)^2=\frac{25}{72}$
2	${}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{3-2}=3\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)=\frac{5}{72}$
3	${}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{3-3}=\left(\frac{1}{6}\right)^3=\frac{1}{216}$

$$\Sigma P(X)=\frac{125}{216}+\frac{75}{216}+\frac{15}{216}+\frac{1}{216}$$
$$=\frac{216}{216}$$

$$\Sigma P(X)=1$$

Question 24

A coin is tossed 5 times. If X is the number of heads observed, find the probability distribution of X.

Solution 24

We know that, probability of getting head in a toss of coin $p = \frac{1}{2}$

Probability of not getting head $q = 1 - \frac{1}{2}$

$$q = \frac{1}{2}$$

The coin is tossed 5 times. Let X denote the number of times head occur is 5 tosses.

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$$

Probability distribution is given by

X	$P(X)$
0	${}^5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{5-0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$
1	${}^5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{5-1} = 5.\left(\frac{1}{2}\right)^5 = \frac{5}{32}$
2	${}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{5-2} = \frac{5.4}{2}\left(\frac{1}{2}\right)^5 = \frac{10}{32}$
3	${}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3} = \frac{5.4}{2}\left(\frac{1}{2}\right)^5 = \frac{10}{32}$
4	${}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{5-4} = 5\left(\frac{1}{2}\right)^5 = \frac{5}{32}$
5	${}^5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{5-5} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$$\Sigma P(X) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32}$$
$$= \frac{32}{32}$$

$$\Sigma P(X) = 1$$

Question 25

An unbiased die is thrown twice. A success is getting a number greater than 4. Find the probability distribution of the number of successes.

Solution 25

Let p be the probability of a getting a number greater than 4 in a toss of die, so

$$p = \frac{2}{6} \qquad \left[\text{Since, numbers greater than 4 coin a die } = 5,6\right]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in 2 throws of a die. Probability of getting r success in n thrown of a die is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^2C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{2-r} \qquad ---(1)$$

Probability distribution of number of success is given by

X	$P(X)$
0	${}^2C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{2-0} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$
1	${}^2C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{2-1} = 2.\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$
2	${}^2C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{2-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$$\Sigma P(X) = \frac{4}{9} + \frac{4}{9} + \frac{1}{9}$$

$$= \frac{1}{9}$$

$$\Sigma P(X) = 1$$

Question 26

A man wins a rupee for head and loses a rupee for tail when coin is tossed. Suppose that he tosses once and quits if he wins but tries once more if he loses on the first toss. Find the probability distribution of the number of rupees the man wins.

Solution 26

Let n denote the number of throws required to get a head and X denote the amount won/lost.

He may get head on first toss or lose first and 2nd toss or lose first and won second toss probability distribution for X

Number of throws (n) :	1	2	2
Amount won/lost (X) :	1	0	-2
Probability $P(X)$:	$\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

So probability distribution is given by

X	$P(X)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
-2	$\frac{1}{4}$

Question 27

Five dice are thrown simultaneously. If the occurrence of 3,4 or 5 in a single die is considered a success, find the probability of at least 3 successes.

Solution 27

Let p denote the probability of getting 3,4 or 5 in a throw of die. So

p = probability of success

$= \frac{3}{6}$

$p = \frac{1}{2}$

$q = 1 - \frac{1}{2}$ $\qquad$ [Since $p + q = 1$]

$q = \frac{1}{2}$

Let X denote the number of success in throw of 5 dice simultaneously. Probability of getting r success out of n throws of die is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \qquad ---(1)$$

Probability getting at least 3 success

$$= P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5}$$

$$= \frac{5.4}{2}\left(\frac{1}{2}\right)^5 + 5.\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [10 + 5 + 1]$$

$$= \frac{16}{32}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Question 28

The items produced by a company contain 10% defective items. Show that the probability of getting 2 defective items in a sample of 8 items is $\frac{28 \times 9^6}{10^8}$.

Solution 28

Let p denote the probability of getting defective items out of 100 items, so

$$p = 10\%$$
$$= \frac{10}{100}$$

$$p = \frac{1}{10}$$
$$q = 1 - \frac{1}{10} \qquad \left[\text{Since } p + q = 1\right]$$
$$q = \frac{9}{10}$$

Let X denote the number of defective items drawn out of 8 items. Probability of getting r defective items out of a sample of 8 items is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$
$$= {}^{8}C_{r} \left(\frac{1}{10}\right)^{r} \left(\frac{9}{10}\right)^{8-r} \qquad ---(1)$$

Probability of getting 2 defective items

$$= P\left(X = 2\right)$$
$$= {}^{8}C_{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{8-2}$$
$$= \frac{8 \times 7}{2} \left(\frac{1}{10}\right)^{2} \left(\frac{9}{10}\right)^{6}$$
$$= \frac{28 \times 9^{6}}{10^{8}}$$

Required probability $= \dfrac{28 \times 9^{6}}{10^{8}}$

Question 29

A card is drawn and replaced in an ordinary pack of 52 cards. How many times must a card be drawn so that (i) there is at least an even chance of drawing a heart,

(ii) the probability of drawing a heart is greater than $\frac{3}{4}$?

Solution 29

Let p denote the probability of drawing a heart from a deck of 52 cards, so

$$p = \frac{13}{52} \qquad \left[\because \text{ There are 13 hearts in deck}\right]$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{3}{4}$$

Let the card is drawn n times. So Binomial distribution is given by

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

where X denote the number of spades drawn and $r = 0, 1, 2, 3, \ldots n$

(i)

We have to find the smallest value of n for which $P\left(X = 0\right)$ is less than $\frac{1}{4}$

$$P\left(X = 0\right) < \frac{1}{4}$$

$${}^{n}C_{0}\left(\frac{1}{1}\right)^{0}\left(\frac{3}{4}\right)^{n-0} < \frac{1}{4}$$

$$\left(\frac{3}{4}\right)^{n} < \frac{1}{4}$$

Put $n = 1, \left(\frac{3}{4}\right) \not< \frac{1}{4}$

$$n = 2, \left(\frac{3}{4}\right)^{2} \not< \frac{1}{4}$$

$$n = 3, \left(\frac{3}{4}\right)^{3} \not< \frac{1}{4}$$

So, smallest value of $n = 3$

$\therefore$ We must draw cards at least 3 times

(ii)

Given, the probability of drawing a heart $> \frac{3}{4}$

$$1 - P(X = 0) > \frac{3}{4}$$

$$1 - {}^{n}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{n-0} > \frac{3}{4}$$

$$1 - \left(\frac{3}{4}\right)^n > \frac{3}{4}$$

$$1 - \frac{3}{4} > \left(\frac{3}{4}\right)^n$$

$$\frac{1}{4} > \left(\frac{3}{4}\right)^n$$

For $n = 1$, $\left(\frac{3}{4}\right)^1 \not< \frac{1}{4}$

$n = 2$, $\left(\frac{3}{4}\right)^2 \not< \frac{1}{4}$

$n = 3$, $\left(\frac{3}{4}\right)^3 \not< \frac{1}{4}$

$n = 4$, $\left(\frac{3}{4}\right)^4 \not< \frac{1}{4}$

$n = 5$, $\left(\frac{3}{4}\right)^5 \not< \frac{1}{4}$

So, card must be drawn 5 times.

Question 30

The mathematics department has 8 graduate assistants who are assigned to the same office. Each assistant is just likely to study at home as in the office. How many desks must be there in the office so that each assistant ha a desk at least 90% of the time?

Solution 30

Here $\quad x = 8, p = \frac{1}{2}, q = \frac{1}{2}$

Let there be k desks and X be the number of students studying in office.

Then we want that

$$P(X \le k) > .90$$

$$\Rightarrow \quad P(X > k) < .10$$

$$\Rightarrow \quad P(X = k+1, k+2, \ldots 8) < .10$$

Clearly $P(X > 6) = P(X = 7 \text{ or } X = 8)$

$$= {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8$$

$$= .04$$

and $\quad P(X > 5) = P(X = 6,\ X = 7 \text{ or } X = 8)$

$$= .15$$

$$\therefore \quad P(X > 6) < 0.10$$

$\Rightarrow$ If there are 6 desks then there is at least 90% chance for every graduate assistant to get a desk.

Question 31

An unbiased coin is tossed 8 times. Find, by using binomial distribution, the probability of getting at least 6 heads.

Solution 31

Binomial Distribution formula is given by

$P(x) = {}^nC_x\, p^x\, q^{n-x}$, where $x = 0, 1, 2, \ldots n$

Let x = No. of heads in a toss

We need probability of 6 or more heads

$X = 6, 7, 8$

Here $p = ½$ and $q = ½$

$P(6)$ = Prob of getting 6 heads, 2 tails $= {}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2$

$P(7)$ = Prob of getting 7 heads, 1tails $= {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1$

$P(8)$ = Prob of getting 8 heads, 0 tails $= {}^8C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$

The probability of getting at least 6 heads (not more than 2 tails) is then

$${}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2 + {}^8C_1 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_2 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{256} + 8\frac{1}{256} + 28\frac{1}{256} = \frac{37}{256}$$

Question 32

Six coins are tossed simultaneously. Find the probability of getting

(i) 3 heads (ii) no heads (iii) at least one head

Solution 32

Let p represents the probability of getting head in a toss of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{2}$$

Let X denote the random variable representing the number heads in 6 tosses of coin. Probability of getting r sixes in n tosses of a fair coin is given by,

$$P\left(X = r\right) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{6}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{6-r} \qquad ---(1)$$

(i)

Probability of getting 3 heads

$$= P\left(X = 3\right)$$

$$= {}^{6}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{6-3}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{3}$$

$$= \frac{20}{64}$$

Probability of getting 3 heads $= \frac{20}{64} = \frac{5}{16}$

(ii)

Probability of getting no heads

$$= P\left(X = 0\right)$$

$$= {}^{6}C_{0} \left(\frac{1}{2}\right)^{0} \left(\frac{1}{2}\right)^{6-0}$$

$$= \left(\frac{1}{2}\right)^{6}$$

$$= \frac{1}{64}$$

Probability of getting no heads $= \frac{1}{64}$

(iii)

Probability of getting at least one head

$$= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$
$$= 1 - P(X = 0)$$
$$= 1 - \frac{1}{64}$$
$$= \frac{63}{64}$$

Probability of getting at least one head $= \frac{63}{64}$

Question 33

Suppose that a radio tube inserted into a certain type of set has probability 0.2 of functioning more than 500 hours. If we test 4 tubes at random what is the probability that exactly three of these tubes function for more than 500 hours?

Solution 33

Let p be the probability that a tube function for more than 500 hours. So

$$p = 0.2$$
$$p = \frac{1}{5}$$
$$q = 1 - \frac{1}{5} \qquad \left[\text{Since } p + q = 1\right]$$
$$= \frac{4}{5}$$

Let X denote the random variable representing the number of tube that functions for more than 500 hours out of 4 tubes. Probability of functioning r tubes out n tubes selected for more than 500 hours is given by,

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$
$$= {}^{4}C_{r} \left(\frac{1}{5}\right)^{r} \left(\frac{4}{5}\right)^{4-r} \qquad ---(1)$$

Probability that exactly 3 tube will function for more than 500 hours

$$= {}^{4}C_{3} \left(\frac{1}{5}\right)^{3} \left(\frac{4}{5}\right)^{4-3}$$
$$= 4.\left(\frac{1}{5}\right)^{3} \left(\frac{4}{5}\right)$$
$$= \frac{16}{625}$$

Required probability $= \frac{16}{625}$

$= 0.0256$

Question 34

The probability that a certain kind of component will survive a given shock test is $\frac{3}{4}$.

Find the probability that among 5 components tested

(i) exactly 2 will survive (ii) at most 3 will survive

Solution 34

Let p be the probability that component survive the shock test. So

$$p = \frac{3}{4}$$

$$q = 1 - \frac{3}{4} \qquad \left[\text{Since } p + q = 1\right]$$

$$q = \frac{1}{4}$$

Let X denote the random variable representing the number of components that survive shock test out of 5 components. Probability of that r components that survive shock test out of n components is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$= {}^{5}C_{r} \left(\frac{3}{4}\right)^{r} \left(\frac{1}{4}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that exactly 2 will survive the shock test

$$= P(X = 2)$$

$$= {}^{5}C_{2} \left(\frac{3}{4}\right)^{2} \left(\frac{1}{4}\right)^{5-2}$$

$$= \frac{5.4}{2} \left(\frac{9}{16}\right) \left(\frac{1}{64}\right)$$

$$= \frac{45}{512} \quad = 0.0879$$

Probability that exactly 2 survive $= 0.0879$

(ii)

Probability that at most 3 will survive

$$= P(X = 0) + P(X = 1) + P(X = 3) + P(X = 4)$$

$$= 1 - [P(X = 4) + P(X = 5)]$$

$$= 1 - \left[{}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^{5-4} + {}^5C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^{5-5}\right]$$

$$= 1 - \left[5.\frac{81}{1024} + \frac{243}{1024}\right]$$

$$= 1 - \left[\frac{405 + 243}{1024}\right]$$

$$= \frac{1024 - 648}{1024}$$

$$= \frac{376}{1024} = 0.3672$$

$$= \frac{376}{1024} = 0.3672$$

Probability that at most 3 will survive $= 0.3672$

Question 35

Assume that the pobability that a bomb dropped from an aeroplane will strike a certain target is 0.2. If 6 bombs are dropped find the probability that (i) exactly 2 will strike the target. (ii) at least 2 will strike the target.

Solution 35

Probability that bomb strikes a target p = 0.2
Probability that a bomb misses the target = 0.8
n = 6
let x = number of bombs that strike the target
P(x=2) = exactly 2 bombs strike the target

$$= {}^6C_2\left(\frac{2}{10}\right)^2 \times \left(\frac{8}{10}\right)^4 = 15\times\frac{16384}{10^6} = 0.24576$$

P(x≥2) = at least 2 bombs strike the target

$$= 1 - P(x<2)$$

$$= 1 - [P(x=0)+P(x=1)]$$

$$= 1 - [\ {}^6C_0\left(\frac{2}{10}\right)^0 \times \left(\frac{8}{10}\right)^6 + {}^6C_1\left(\frac{2}{10}\right)^1 \times \left(\frac{8}{10}\right)^5]$$

$$= 1 - [0.0.262144 + 0.393216] = 1 - 0.65536$$
$$= 0.34464$$

Question 36

It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that
(i) none contract the disease (ii) more than 3 contract the disease.

Solution 36

Let p be the probability that a mouse get contract the desease. So

$$p = 40\%$$
$$= \frac{40}{100}$$
$$= \frac{2}{5}$$
$$q = 1 - \frac{2}{5} \qquad [\text{Since } p + q = 1]$$
$$q = \frac{3}{5}$$

Let X denote the variable representing number of mice contract the disease out of 5 mice.
Probability the r mice get contract the disease out of n mice inoculated is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$= {}^5C_r \left(\frac{2}{5}\right)^r \left(\frac{3}{5}\right)^{5-r} \qquad ---(1)$$

(i)

Probability that none contract the disease $= P(X = 0)$

$$= {}^5C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{5-0}$$
$$= \left(\frac{3}{5}\right)^5$$

Probability that none contract the disease $= \left(\frac{3}{5}\right)^5$

(ii)

Probability that more than 3 contract disease

$$= P(X = 4) + P(X = 5)$$
$$= {}^5C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^{5-4} + {}^5C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^{5-5}$$
$$= 5.\left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)^5$$
$$= \left(\frac{2}{5}\right)^4 \left[3 + \frac{2}{5}\right]$$
$$= \frac{17}{5}\left(\frac{2}{5}\right)^4$$

Probability that more than 3 contract disease

$$= \frac{17}{5}\left(\frac{2}{5}\right)^4$$

Question 37

An experiment succeeds twice as often as it fails. Find the probability that in the next 6 trials there will be at least 4 successes.

Solution 37

Let p be the probability of success is experiments, q be the probability of failure,

Given, $P = 2q$

but $p + q = 1$

$2q + q = 1$

$3q = 1$

$q = \frac{1}{3}$

$p = \frac{2}{3}$

Let X denote the random variable representing the number of success out of 6 experiments.
Probability of getting r success out of n experiments is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r} \qquad ---(1)$$

Probability of getting at least 4 success

$$= P(X = 4) + P(X = 5) + P(X = 6)$$

$$= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^{6-4} + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^{6-5} + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^{6-6}$$

$$= \frac{6 \times 5}{2}\left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + 6\left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + \left(\frac{2}{3}\right)^6$$

$$= \left(\frac{2}{3}\right)^4 \left[\frac{15}{9} + \frac{4}{3} + \frac{4}{9}\right]$$

$$= \left(\frac{2}{3}\right)^4 \left[\frac{15 + 12 + 4}{9}\right]$$

$$= \left(\frac{31}{9}\right)\left(\frac{2}{3}\right)^4$$

$$= \frac{496}{729}$$

Required probability $= \frac{496}{729}$

Question 38

In a hospital, there are 20 kidney dialysis machines and that the chance of any one of them to be out of service during a day is 0.02. Determine the probability that exactly 3 machines will be out of service on the same day.

Solution 38

Let x = number of out of service machines
p = probability that machine will be out of service on the same day
= 2/100
q = probability that machine will be in service on the same day
= 8/100
P(x=3) = probability exactly 3 machines will be out of service on the same day

$P(x=3) = {}^{20}C_3 \times \left(\frac{2}{100}\right)^3 \left(\frac{8}{100}\right)^0 = 1140 \times 0.000008$
$= 0.00912$
For low probability events Poisson' distribution is used instead of Binomial distribution. Then,
$\lambda = np = 20 \times 0.02 = 0.4$

$P(x=r) = \frac{e^{-\lambda} \times \lambda^3}{r!}$

$P(x=3) = \frac{e^{-0.4} \times 0.4^3}{3!} = 0.6703 \times 0.064/6 = 0.0071$

Question 39

The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

(i) none will graduate,

(ii) only one will graduate,

(iii) all will graduate.

Solution 39

Let p be the probability that a student entering a university will graduate, so

$$p = 0.4$$
$$q = 1 - 0.4 \qquad \left[\text{Since } p + q = 1\right]$$
$$= 0.6$$

Let X denote the random variable representing the number of students entering a university will graduate out of 3 students of university. Probability that r students will graduate out of n entering the university is given by

$$P\left(X = r\right) = {}^{n}C_{r}p^{r}\left(q\right)^{n-r}$$
$$= {}^{3}C_{r}\left(0.4\right)^{r}\left(0.6\right)^{3-r} \qquad ---(1)$$

(i)

Probability that none will graduate

$$= P\left(X = 0\right)$$
$$= {}^{3}C_{0}\left(0.4\right)^{0}\left(0.6\right)^{3-0}$$
$$= \left(0.6\right)^{3}$$
$$= 0.216$$

Probability that none will graduate $= 0.216$

(ii)

Probability that one will graduate

$$= P\left(X = 1\right)$$
$$= {}^{3}C_{1}\left(0.4\right)^{1}\left(0.6\right)^{3-1}$$
$$= 3 \times \left(0.4\right)\left(0.36\right)$$
$$= 0.432$$

Probability that only one will graduate $= 0.432$

(iii)

Probability that all will graduate

$$= P\left(X = 3\right)$$
$$= {}^{3}C_{3}\left(0.4\right)^{3}\left(0.6\right)^{3-3}$$
$$= \left(0.4\right)^{3}$$
$$= 0.064$$

Probability that all will graduate $= 0.064$

Question 40

Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs.Find the probability that there is at least one defective egg.

Solution 40

Let X denote the number of defective eggs in the 10 eggs drawn.
Since the drawing is done with replacement, the trials are Bernoulli trials.

Clearly, X has the binomial distribution with n=10 and $p = \frac{10}{100} = \frac{1}{10}$

Therefore, $q = 1 - \frac{1}{10} = \frac{9}{10}$

Now, P(at least one defective egg) $= P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 \left(\frac{9}{10}\right)^{10} = 1 - \frac{9^{10}}{10^{10}}$$

Question 41

In a 20-question true-false examination suppose a student tosses a fair coin to determine his answer to each question. If the coin fails a heads, he answer 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution 41

Let p be the probability of answering a true. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

$$= \frac{1}{2}$$

Thus the probability that he answers at least 12 questions correctly among 20 questions is

$$P(X \geq 12) = P(X = 12) + P(X = 13) + P(X = 14) + P(X = 15) + P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)$$

$$= \left(\frac{1}{2}\right)^{20} \left\{ {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20} \right\}$$

$$= \frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Therefore, the required answer is

$$\frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Question 42

Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

Solution 42

X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Then, } P(X = x) = {}^{n}C_{x} q^{n-x} p^{x}$$

$$= {}^{6}C_{x} \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^{x}$$

$$= {}^{6}C_{x} \left(\frac{1}{2}\right)^{6}$$

It can be seen that $P(X = x)$ will be maximum, if ${}^{6}C_{x}$ will be maximum.

$$\text{Then, } {}^{6}C_{0} = {}^{6}C_{6} = \frac{6!}{0! \cdot 6!} = 1$$

$$ {}^{6}C_{1} = {}^{6}C_{5} = \frac{6!}{1! \cdot 5!} = 6$$

$$ {}^{6}C_{2} = {}^{6}C_{4} = \frac{6!}{2! \cdot 4!} = 15$$

$$ {}^{6}C_{3} = \frac{6!}{3! \cdot 3!} = 20$$

The value of ${}^{6}C_{3}$ is maximum. Therefore, for $x = 3$, $P(X = x)$ is maximum.

Thus, $X = 3$ is the most likely outcome.

Question 43

On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution 43

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with n = 5 and $p = \frac{1}{3}$

$$\therefore P(X = x) = {}^{n}C_{x} q^{n-x} p^{x}$$
$$= {}^{5}C_{x} \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^{x}$$

P (guessing more than 4 correct answers) = P(X ≥ 4)

$$= P(X = 4) + P(X = 5)$$
$$= {}^{5}C_{4} \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^{4} + {}^{5}C_{5} \left(\frac{1}{3}\right)^{5}$$
$$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243}$$
$$= \frac{10}{243} + \frac{1}{243}$$
$$= \frac{11}{243}$$

Question 44

A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will in a prize (a) at least once (b) exactly once (c) at least twice?

Solution 44

Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with n = 50 and $p = \frac{1}{100}$

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

$$\therefore \mathrm{P}(\mathrm{X} = x) = {}^{n}\mathrm{C}_x q^{n-x} p^x = {}^{50}\mathrm{C}_x \left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^x$$

(a) P (winning at least once) = P (X ≥ 1)

$$= 1 - \mathrm{P}(\mathrm{X} < 1)$$

$$= 1 - \mathrm{P}(\mathrm{X} = 0)$$

$$= 1 - {}^{50}\mathrm{C}_0 \left(\frac{99}{100}\right)^{50}$$

$$= 1 - 1 \cdot \left(\frac{99}{100}\right)^{50}$$

$$= 1 - \left(\frac{99}{100}\right)^{50}$$

(b) P (winning exactly once) = P(X = 1)

$$= {}^{50}C_1\left(\frac{99}{100}\right)^{49}\cdot\left(\frac{1}{100}\right)^{1}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P (at least twice) = P(X ≥ 2)

$$= 1 - P(X < 2)$$
$$= 1 - P(X \le 1)$$
$$= 1 - \left[P(X = 0) + P(X = 1)\right]$$
$$= \left[1 - P(X = 0)\right] - P(X = 1)$$
$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\cdot\left(\frac{99}{100}\right)^{49}$$
$$= 1 - \left(\frac{99}{100}\right)^{49}\cdot\left[\frac{99}{100} + \frac{1}{2}\right]$$
$$= 1 - \left(\frac{99}{100}\right)^{49}\cdot\left(\frac{149}{100}\right)$$
$$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

Question 45

The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target atleast once is more than 0.99 ?

Solution 45

Let the shooter fire n times.
n fires are Bernoulli trials.

In each trial, p= probability of hitting the target= $\frac{3}{4}$

And q = probability of not hitting the target= $1-\frac{3}{4}=\frac{1}{4}$

Then, $P(X=x)={}^nC_x q^{n-x} p^x={}^nC_x\left(\frac{1}{4}\right)^{n-x}\left(\frac{3}{4}\right)^x={}^nC_x\frac{3^x}{4^n}$

Now, given that

P (hitting the target atleast once) > 0.99

i.e. $P(x\geq 1)>0.99$

$\Rightarrow \quad 1-P(x=0)>0.99$

$\Rightarrow \quad 1-{}^nC_0\frac{1}{4^n}>0.99$

$\Rightarrow \quad {}^nC_0\frac{1}{4^n}<0.01$

$\Rightarrow \quad \frac{1}{4^n}<0.01$

$\Rightarrow \quad 4^n>\frac{1}{0.01}=100$

The minimum value of n to satisfy this inequality is 4

Thus, the shooter must fire 4 times.

Question 46

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Solution 46

Let the man toss the coin n times. The n tosses are n Bernoulli trials.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

$$\Rightarrow p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$$

$$\therefore P(X = x) = {}^nC_x p^{n-x} q^x = {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x = {}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that,

P (getting at least one head) > $\frac{90}{100}$

P ($x \geq 1$) > 0.9

1 − P (x = 0) > 0.9

$$1 - {}^nC_0 \cdot \frac{1}{2^n} > 0.9$$

$$ {}^nC_0 \cdot \frac{1}{2^n} < 0.1$$

$$\frac{1}{2^n} < 0.1$$

$$2^n > \frac{1}{0.1}$$

$$2^n > 10 \qquad \ldots(1)$$

The minimum value of n that satisfies the given inequality is 4.

Thus, the man should toss the coin 4 or more than 4 times.

Question 47

How many times must a man toss a fair coin so that the probability of having at least one head is more than 80%?

Solution 47

Let the man toss the coin n times.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

So,

$$p=\frac{1}{2}$$
$$q=1-\frac{1}{2} \qquad [\text{Since } p+q=1]$$
$$=\frac{1}{2}$$

$$\therefore P(X=x)={}^nC_x p^{n-x} q^x$$
$$={}^nC_x \left(\frac{1}{2}\right)^{n-x}\left(\frac{1}{2}\right)^x$$
$$={}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that

$$p(\text{getting at least one head})>\frac{80}{100}$$
$$P(x\geq 1)>0.8$$
$$1-P(x=0)>0.8$$
$$1-{}^nC_0\cdot\frac{1}{2^n}>0.8$$
$${}^nC_0\cdot\frac{1}{2^n}<0.2$$
$$\frac{1}{2^n}<0.2$$
$$2^n>\frac{1}{0.2}$$
$$2^n>5$$

The minimum value of n that satisfies the given inequality is 3.
Thus, the man should toss the coin 3 or more than 3 times.

Question 48

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes.

Solution 48

Let p be the probability of getting a doublet in a throw of a pair of dice, so

$$p = \frac{6}{36} \qquad \left[\text{Since } (1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\right]$$
$$= \frac{1}{6}$$
$$q = 1 - \frac{1}{6} \qquad [\text{Since } p + q = 1]$$
$$= \frac{5}{6}$$

Let X denote the number of getting doublets i.e. success out of 4 times. So, probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{4-0} = \left(\frac{5}{6}\right)^4$
1	${}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{4-1} = 4\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 = \frac{2}{3}\left(\frac{5}{6}\right)^3$
2	${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{4-2} = \frac{4\cdot3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{25}{216}$
3	${}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{4-3} = \frac{4\cdot3}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) = \frac{5}{324}$
4	${}^4C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{4-4} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$

Question 49

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution 49

Let p be the probability of defective bulbs, so

$$p = \frac{6}{30}$$
$$= \frac{1}{5}$$
$$q = 1 - \frac{1}{5} \qquad [\text{Since } p + q = 1]$$
$$= \frac{4}{5}$$

Here, 4 bulbs is drawn at random with replacement. So, probability distribution is given by

X	$P(X)$
0	${}^4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{4-0} = \frac{256}{625}$
1	${}^4C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{4-1} = \frac{4}{5} \times \frac{4^3}{5^3} = \frac{256}{625}$
2	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^{4-2} = \frac{6}{5^2} \times \frac{4^2}{5^2} = \frac{96}{625}$
3	${}^4C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^{4-3} = \frac{4}{5^3} \times \frac{4}{5} = \frac{16}{625}$
4	${}^4C_4\left(\frac{1}{5}\right)^4\left(\frac{6}{5}\right)^{4-4} = 1 \cdot \frac{1}{625} = \frac{1}{625}$

Question 50

Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws.

Solution 50

Here success is a score which is multiple of 3 i.e. 3 or 6.

$$\therefore p(3 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$$

The probability of r successes in 10 throws is given by

$$P(r) = {}^{10}C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{10-r}$$

Now P(at least 8 successes) = P(8) + P(9) + P(10)

$$= {}^{10}C_8\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^2 + {}^{10}C_9\left(\frac{1}{3}\right)^9\left(\frac{2}{3}\right)^1 + {}^{10}C_{10}\left(\frac{1}{3}\right)^{10}\left(\frac{2}{3}\right)^0$$
$$= \frac{1}{3^{10}}[45 \times 4 + 10 \times 2 + 1]$$
$$= \frac{201}{3^{10}}$$

Question 51

A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

Solution 51

Here success is an odd number i.e. 1,3 or 5.

$\therefore p(1,3 \text{ or } 5) = \frac{3}{6} = \frac{1}{2}$

The probability of r successes in 5 throws is given by

$P(r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$

Now P(exactly 3 times) = P(3)

$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= \frac{10}{2^5}$

$= \frac{5}{16}$

Question 52

The probability of a man hitting a target is 0.25. He shoots 7 times. What is the probability of his hitting at least twice?

Solution 52

Pr obablity of a man hitting a target is 0.25.

$\therefore p = 0.25 = \frac{1}{4}, \quad q = 1 - p = \frac{3}{4}$

The probability of r successes in 7 shoots is given by

$P(r) = {}^7C_r (0.25)^r (0.75)^{7-r}$

Now P(at least twice) = 1 − P(less than 2)

$= 1 - {}^7C_0 (0.25)^0 (0.75)^7 + {}^7C_1 (0.25)^1 (0.75)^6$

$= 1 - \frac{3^7}{4^7} + 7 \times \frac{3^6}{4^7}$

$= \frac{4547}{8192}$

Question 53

A factory produces bulbs. The probability that one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

i. none of the bulbs is defective.

ii. exactly two bulls are defective.

iii. more than 8 bulbs work properly.

Solution 53

Probablity of a bulb to be defective is $\frac{1}{50}$.

$\therefore p = \frac{1}{50}, \; q = 1 - p = \frac{49}{50}$

The probability of r defective bulbs in 10 bulbs is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{50}\right)^r \left(\frac{49}{50}\right)^{10-r}$$

(i) P(none of the bulb is defective) = P(0)

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10}$$

$$= \left(\frac{49}{50}\right)^{10}$$

(ii) P(exactly two bulbs are defective) = P(2)

$$= {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= 45 \times \frac{(49)^8}{(50)^{10}}$$

(iii) P(more than 8 bulbs work properly)

= P(at most two bulbs are defective)

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^9 + {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= \left(\frac{49}{50}\right)^{10} + 10 \times \frac{(49)^9}{(50)^{10}} + 45 \times \frac{(49)^8}{(50)^{10}}$$

$$= \frac{(49)^8}{(50)^{10}}\left[(49)^2 + 490 + 45\right]$$

$$= \frac{(49)^8 \times 2936}{(50)^{10}}$$

Note: Answer given in the book is incorrect.

Chapter 33 - Binomial Distribution Exercise Ex. 33.2

Question 1

Can the mean of a binomial distribution be less than its variance ?

Solution 1

Let X be a binomial variate with parameters n and p.

$$\text{Mean} = np$$
$$\text{Varience} = npq$$

$$\begin{aligned}\text{Mean} - \text{Variance} &= np - npq \\ &= np(1-q) \\ &= np.p \\ &= np^2\end{aligned}$$

$$\text{Mean} - \text{Variance} > 0$$
$$\text{Mean} > \text{Variance}$$

So, mean can never be less than varience.

Question 2

Determine the binomial distribution whose mean is 9 and variance $\frac{9}{4}$.

Solution 2

Let X denote the variance with parameters n and p

$$p+q=1$$
$$q=1-p$$

Goven, Mean $= np = 9$ $\quad ---(\text{i})$

Variance $= npq = \frac{9}{4}$ $\quad ---(\text{ii})$

$$\frac{npq}{np}=\frac{\frac{9}{4}}{9} \quad \left[\text{By dividing (i) by (ii)}\right]$$
$$q=\frac{1}{4}$$

So, $p = 1-q$
$$= 1-\frac{1}{4}$$
$$p=\frac{3}{4}$$

Put p in equation (i),

$$n\left(\frac{3}{4}\right)=9$$
$$\Rightarrow \quad n=\frac{36}{3}$$

So, $n = 12$

The distribution is given by

$$= {}^{n}C_{r}p^{r}(q)^{n-r}$$

$$P(X=r) = {}^{12}C_{r}\left(\frac{3}{4}\right)^{r}\left(\frac{1}{4}\right)^{12-r}$$
for $r = 0,1,2,..12$

Question 3

If the mean and variance of a binomial distribution are respectively 9 and 6, find the distribution.

Solution 3

Let n and p be parameters of binomial distribution. Here

$$\text{Mean } = np = 9 \qquad ---(\text{i})$$

$$\text{Variance } = npq = 6 \qquad ---(\text{ii})$$

$$\frac{npq}{np} = \frac{6}{9}$$

$$q = \frac{2}{3}$$

So, $p = 1 - \frac{2}{3}$ $\quad$ $[\text{Since } p + q = 1]$

$$p = \frac{1}{3}$$

Using equation (i), $np = 9$

$$n\left(\frac{1}{3}\right) = 9$$

$$n = 27$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{27}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{27-r}$$

$$r = 0, 1, 2, \ldots 27$$

Question 4

Find the binomial distribution when the sum of its mean and variance for 5 trials is 4.8.

Solution 4

Given that,

$$n = 5$$

Also, $\quad \text{Mean} + \text{Varience} = 4.8$

$$np + npq = 4.8$$
$$np(1+q) = 4.8$$
$$5p(1+q) = 4.8$$
$$5(1-q)(1+q) = 4.8 \qquad [\text{Since } p+q=1]$$
$$5(1-q^2) = 4.8$$
$$1-q^2 = \frac{4.8}{5}$$
$$q^2 = 1 - \frac{4.8}{5}$$
$$= \frac{0.2}{5}$$
$$q^2 = \frac{1}{25}$$
$$q = \frac{1}{5}$$

$$\Rightarrow \quad p = 1-q$$
$$= 1 - \frac{1}{5}$$
$$p = \frac{4}{5}$$

So, $\quad n = 5,\ p = \frac{4}{5},\ q = \frac{1}{5}$

Here binomial distribution is

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=r) = 5C_r \left(\frac{4}{5}\right)^r \left(\frac{1}{5}\right)^{5-r}$$
$$r = 0, 1, 2, 3, \ldots 5$$

Question 5

Determine the binomial distribution whose mean is 20 and variance 16.

Solution 5

Given that,

$$\text{Mean } = np = 20 \qquad ---(\text{i})$$

$$\text{Variance } = npq = 16 \qquad ---(\text{ii})$$

Let n and p be the parameters of distribution dividing equation (ii) by (i)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $\quad p = 1 - q \qquad \left[\text{Since } p + q = 1\right]$

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put p in equation (i),

$$np = 20$$

$$n\left(\frac{1}{5}\right) = 20$$

$$n = 20 \times 5$$

$$n = 100$$

So, binomial distribution is given by

$$P\left(X = r\right) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$P\left(X = r\right) = {}^{100}C_{r}\left(\frac{1}{5}\right)^{r}\left(\frac{4}{5}\right)^{100-r}$$

$$r = 0, 1, 2, 3, \ldots 100$$

Question 6

In a binomial distribution the sum and product of the mean and the variance are $\frac{25}{3}$ and $\frac{50}{3}$ respectively. Find the distribution.

Solution 6

Let n and p be the parameters of distribution binomial distribution. So

$$q = 1 - p \qquad \text{as } p + q = 1$$

$$\text{Mean} + \text{Variance} = \frac{25}{3}$$

$$np + npq = \frac{25}{3}$$

$$np(1+q) = \frac{25}{3}$$

$$np = \frac{25}{3(1+q)} \qquad ---(1)$$

$$\text{Mean} \times \text{Variance} = \frac{50}{3}$$

$$np \times npq = \frac{50}{3}$$

$$n^2p^2q = \frac{50}{3}$$

$$\left[\frac{25}{3(1+q)}\right]^2 .q = \frac{50}{3} \qquad [\text{Using } (1)]$$

$$625q = \frac{50}{3}\left[9(1+q)^2\right]$$

$$625q = 150(1+q)^2$$

$$25q = 6(1+q)^2$$

$$6 + 6q^2 + 12q - 25q = 0$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$3q(2q-3) - 2(2q-3) = 0$$

$$(2q-3)(3q-2) = 0$$

$$\Rightarrow \qquad 2q - 3 = 0 \qquad \text{or} \qquad 3q - 2 = 0$$

$$\Rightarrow \qquad q = \frac{3}{2} \qquad \text{or} \qquad q = \frac{2}{3}$$

Since $q \le 1$, so

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Put the value of p and q in equation

$$np = \frac{25}{3(1+q)}$$

$$3np(1+q) = 25$$

$$3n\left(\frac{1}{3}\right)\left(1+\frac{2}{3}\right) = 25$$

$$3(n)\left(\frac{1}{3}\right)\left(\frac{5}{3}\right) = 25$$

$$n = \frac{75}{5}$$

$$n = 15$$

So, binomial distribution is given by

$$P(X = r) = {}^{n}C_{r}p^{r}q^{n-r}$$

$$P(X = r) = {}^{15}C_{r}\left(\frac{1}{3}\right)^{r}\left(\frac{2}{3}\right)^{15-r}$$

as $r = 0, 1, 2, \ldots 15$

Question 7

The mean of a binomial distribution is 20, and the standard deviation 4. Calculate parameters of the binomial distribution.

Solution 7

Let n and p be the parameters of binomial distribution.

Given that,

$$\text{Mean } = np - 20 \qquad ---(\text{i})$$

$$\text{Standard deviation } = \sqrt{npq} = 4$$

Squaring both the sides,

$$npq = 16 \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

So, $p = 1 - q$ $\qquad$ [Since $p + q = 1$]

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put value of p in equation (i),

$$np = 20$$

$$\frac{n}{5} = 20$$

$$n = 100$$

$$p = \frac{1}{5}$$

Question 8

If the probability of a defective bolt is 0.4, find the (i) mean, (ii) standard deviation, for the distribution of bolts in a total of 400 bolts.

Solution 8

Let p denotes the probability of selecting a defective bolt, so

$$p = 0.1$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Given, $n = 400$

(i)

$$\text{Mean} = np$$

$$= 400 \times \frac{1}{10}$$

$$\text{Mean} = 40$$

(ii)

$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{400 \times \frac{1}{10} \times \frac{9}{10}}$$

$$= \sqrt{36}$$

$$\text{Standard deviation} = 6$$

Question 9

Find the binomial distribution whose mean is 5 and variance $\frac{10}{3}$.

Solution 9

Let n and p be the parameters of binomial distribution.

Given, Mean $= np = 5$ ---(i)

$$\text{Variance } = npq = \frac{10}{3} \qquad \text{---(ii)}$$

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{\frac{10}{3}}{5}$$

$$q = \frac{2}{3}$$

So, $p = 1 - q$ [Since $p + q = 1$]

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Put the value of p in equation (i),

$$np = 5$$
$$n = 5 \times 3$$
$$n = 15$$

Hence, the binomial distribution is given by

$$P(X = r) = {}^{n}C_r p^r q^{n-r}$$

$$P(X = r) = {}^{15}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{15-r}$$
$$r = 0, 1, 2, \ldots 15$$

Question 10

If on an average 9 ships out of 10 arrive safely to ports, find the mean and S.D. of ships returning safely out of a total of 500 ships.

Solution 10

Let p be the probability of a ship returning safely to parts, so

$$p = \frac{9}{10}$$

$$q = 1 - \frac{9}{10} \qquad [\text{Since } p + q = 1]$$

$$q = \frac{1}{10}$$

Given, $n = 500$

$$\text{Mean} = np$$

$$= 500 \times \frac{9}{10}$$

$$\text{Mean} = 450$$

$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{500 \times \frac{9}{10} \times \frac{1}{10}}$$

$$= \sqrt{45}$$

$$= 6.71$$

Mean $= 450$

Standard deviation $= 6.71$

Question 11

The mean and variance of a binomial variate with parameters n and p are 16 and 8 respectively. Find $P(X = 0)$, $P(X = 1)$ and $P(X \geq 2)$.

Solution 11

Given that, parameters for binomial distribution are n and p.

Also, Mean $= np = 16$ $\quad --- (i)$

Variance $= npq = 8$ $\quad --- (ii)$

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{8}{16}$$

$$q = \frac{1}{2}$$

So, $p = 1 - \frac{1}{2}$ $\quad [\text{as } p + q = 1]$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$n = 32$$

Hence, binomial distribution is given by,

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P(X = r) = {}^{32}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{32-r} \quad --- (iii)$$

$$P(X = 0)$$

$$= {}^{32}C_{0} \left(\frac{1}{2}\right)^{0} \left(\frac{1}{2}\right)^{32-0} \quad [\text{Using (iii)}]$$

$$= \left(\frac{1}{2}\right)^{32}$$

$$P(X = 1)$$

$$= {}^{32}C_{1} \left(\frac{1}{2}\right)^{1} \left(\frac{1}{2}\right)^{32-1}$$

$$= 32 . \frac{1}{2} \left(\frac{1}{2}\right)^{31}$$

$$= \left(\frac{1}{2}\right)^{27}$$

$$P(X \geq 2)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - \left[\left(\frac{1}{2}\right)^{32} + \left(\frac{1}{2}\right)^{27}\right]$$
$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{1}{32} + 1\right)$$
$$= 1 - \left(\frac{1}{2}\right)^{27}\left(\frac{33}{32}\right)$$
$$= 1 - \frac{33}{2^{32}}$$

Hence

$$P(X = 0) = \left(\frac{1}{2}\right)^{32},\ P(X = 1) = \left(\frac{1}{2}\right)^{27},\ P(X \geq 2) = 1 - \frac{33}{2^{32}}$$

Question 12

In eight throws of a die 5 or 6 is considered a success, find the mean number of successes and the standard deviation.

Solution 12

Let p be the probability of success in a single throw of die

$$p = \frac{2}{6} \qquad \text{[Since success is occurance of 5 or 6]}$$
$$p = \frac{1}{3}$$
$$q = 1 - \frac{1}{3} \qquad [\text{Since } p + q = 1]$$
$$q = \frac{2}{3}$$

Given, $n = 8$

$$\text{Mean} = np = \frac{8}{3} = 2.66$$

$$\text{Standard deviation} = \sqrt{npq} = \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}} = \frac{4}{3} = 1.33$$

Mean $= 2.66$, Standard deviation $= 1.33$

Question 13

Find the expected number of boys in a family with 8 children, assuming the sex distribution to be equally probable.

Solution 13

Let n and p be the parameters of binomial distribution.
Let p = probability of having a boy in the family

Given, $p = q$

Since, $p + q = 1$

$$p + p = 1$$
$$2p = 1$$
$$p = \frac{1}{2}$$
$$q = \frac{1}{2}$$
$$n = 8$$

$$\begin{aligned}\text{The expected number of boys } &= np \\ &= 8 \times \frac{1}{2} \\ &= 4\end{aligned}$$

The expected number of boys $= 4$

Question 14

The probability is 0.02 that an item produced by a factory is defective. A shipment of 10.000 items is sent to its warehouse. Find the expected number of defective items and the standard deviation.

Solution 14

Let p denot the probability of a defective item produced in the factory, so

$$p = 0.02$$
$$= \frac{2}{100}$$
$$p = \frac{1}{50}$$
$$q = 1 - \frac{1}{50} \qquad [\text{Since } p + q = 1]$$
$$= \frac{49}{50}$$

Given $n = 10{,}000$

Expected number of defective item $= np$

$$= 10000 \times \frac{1}{50}$$
$$= 200$$

Standard deviation $= \sqrt{npq}$

$$= \sqrt{10000 \times \frac{1}{50} \times \frac{49}{50}}$$
$$= 14$$

Expected No. of defective items $= 200$

Standard deviation $= 14$

Question 15

A dice is thrown thrice. A success is 1 or 6 in a throw. Find the mean and variance of the number of successes.

Solution 15

Let p be the probability of success, so

$$p = \frac{2}{6} \quad \left[\text{Since success in occurance of 1 or 6 on the die}\right]$$

$$p = \frac{1}{3}$$

Given, $n = 3$

$$q = 1 - p \quad \left[\text{Since } p + q = 1\right]$$

$$= 1 - \frac{1}{3}$$

$$q = \frac{2}{3}$$

$$\text{Mean} = np$$

$$= 3\left(\frac{1}{3}\right)$$

$$= 1$$

$$\text{Variance} = npq$$

$$= 3 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$$

$$= \frac{2}{3}$$

Mean = 1

Variance $= \frac{2}{3}$

Question 16

If a random variable X follows binomial distribution with mean 3 and variance $\frac{3}{2}$, find $P(X \le 5)$.

Solution 16

Let n and p be the parameters of binomial distribution
Given,

$$\text{Mean } = np = 3 \qquad ---(i)$$

$$\text{Variance } = npq = \frac{3}{2} \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{\frac{3}{2}}{3}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \qquad [\text{as } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i)

$$np = 3$$

$$n\left(\frac{1}{2}\right) = 3$$

$$n = 6$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P(X = r) = {}^{6}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{6-r} \qquad ---(iii)$$

$$P(X \le 5)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^{6}C_{6} \left(\frac{1}{2}\right)^{6} \left(\frac{1}{2}\right)^{6-6}, \qquad [\text{Using (iii)}]$$

$$= 1 - \left(\frac{1}{2}\right)^{6}$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

$$P(X \le 5) = \frac{63}{64}$$

Question 17

If X follows binomial distribution with mean 4 and variance 2 find $P(X \ge 5)$.

Solution 17

Let n and p be the parameters of binomial distribution.

Given,

$$\text{Mean } = np = 4 \qquad ---(i)$$

$$\text{Variance } = npq = 2 \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{2}{4}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 4$$

$$n\left(\frac{1}{2}\right) = 4$$

$$n = 8$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

$$P(X = r) = {}^{8}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{8-r} \qquad ---(iii)$$

$$P(X \geq 5)$$

$$= P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$= {}^{8}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{3} + {}^{8}C_{6}\left(\frac{1}{2}\right)^{6}\left(\frac{1}{2}\right)^{2} + {}^{8}C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right) + {}^{8}C_{8}\left(\frac{1}{2}\right)^{8}$$

$$[\text{Using equation (iii)}]$$

$$= \frac{8 \times 7 \times 6}{3 \times 2}\left(\frac{1}{2}\right)^{8} + \frac{8 \times 7}{2}\left(\frac{1}{2}\right)^{8} + 8\left(\frac{1}{2}\right)^{8} + \left(\frac{1}{2}\right)^{8}$$

$$= \left(\frac{1}{2}\right)^{8}[56 + 28 + 8 + 1]$$

$$= \frac{93}{256}$$

$$P(X \geq 5) = \frac{93}{256}$$

Question 18

The mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively. Find $P(X \geq 1)$.

Solution 18

Let n and p be the parameters of binomial distribution.

Given,

$$\text{Mean } = np = \frac{4}{3} \qquad ---(\text{i})$$

$$\text{Variance } = npq = \frac{8}{9} \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{\frac{8}{9}}{\frac{4}{3}}$$

$$q = \frac{2}{3}$$

$$p = 1 - \frac{2}{3} \qquad [\text{Since } p + q = 1]$$

$$p = \frac{1}{3}$$

Put the value of p in equation (i),

$$np = \frac{4}{3}$$

$$n\left(\frac{1}{3}\right) = \frac{4}{3}$$

$$n = 4$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^4C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{4-r} \qquad ---(\text{iii})$$

$$P(X \geq 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{4-0} \qquad [\text{Using } (\text{iii})]$$

$$= 1 - \left(\frac{2}{3}\right)^4$$

$$= 1 - \frac{16}{81}$$

$$= \frac{65}{81}$$

$$P(X \geq 1) = \frac{65}{81}$$

Question 19

If the sum of the mean and variance of a binomial distribution for 6 trials is $\frac{10}{3}$, Find the distribution.

Solution 19

Let n and p be the parameters of binomial distribution,

Given, $n = 6$

$$\text{Mean} + \text{Variance} = \frac{10}{3}$$

$$np + npq = \frac{10}{3}$$

$$6p + 6pq = \frac{10}{3}$$

$$6p\left(1+q\right) = \frac{10}{3}$$

$$6\left(1-q\right)\left(1+q\right) = \frac{10}{3} \qquad \left[\text{Since } p + q = 1\right]$$

$$6\left(1-q^2\right) = \frac{10}{3}$$

$$1 - q^2 = \frac{10}{18}$$

$$-q^2 = \frac{5}{9} - 1$$

$$-q^2 = -\frac{4}{9}$$

$$q^2 = \frac{4}{9}$$

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Hence, the binomial distribution is given by,

$$P\left(X = r\right) = {}^nC_r p^r q^{n-r}$$

$$P\left(X = r\right) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$$

as $r = 0, 1, 2, \ldots 6$

Question 20

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes and hence find its mean.

Solution 20

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$
Total number of outcomes $= 36$
Let p be the probability of success therefore

$$p = 6/36 = 1/6$$

Let q be the probability of failure therefore $q = 1 - p = 1 - 1/6 = 5/6$
Since the dice is thrown 4 times, $n = 4$
Let X be the random variable for getting doublet, therefore X can take at max 4 values.

$$P(X=0) = {}^4C_0 p^0 q^4 = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X=1) = {}^4C_1 p^1 q^3 = 4 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 = \frac{500}{1296}$$

$$P(X=2) = {}^4C_2 p^2 q^2 = \frac{4 \cdot 3}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 = \frac{150}{1296}$$

$$P(X=3) = {}^4C_3 p^3 q^1 = 4 \cdot \left(\frac{1}{6}\right)^3 \cdot \frac{5}{6} = \frac{20}{1296}$$

$$P(X=4) = {}^4C_4 p^4 q^0 = 1 \cdot \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = \frac{1}{1296}$$

Mean

$$\mu = \sum_{i=1}^{4} X_i P(X_i) = 0 \cdot \frac{625}{1296} + 1 \cdot \frac{500}{1296} + 2 \cdot \frac{150}{1296} + 3 \cdot \frac{20}{1296} + 4 \cdot \frac{1}{1296}$$

$$= \frac{500 + 300 + 60 + 4}{1296} = \frac{54}{81} = \frac{2}{3}$$

Hence the mean is $= \dfrac{2}{3}$

Question 21

Find the probability distribution of the number of doublets in three rows of a pair of dice and hence find its mean.

Solution 21

Throwing a doublet i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$
Total number of outcomes $=36$
Let p be the probability of success therefore

$$p=6/36=1/6$$

Let q be the probability of failure therefore $q=1-p=1-1/6=5/6$
Since there is three rows of dice so $n=3$
Let X be the random variable for getting doublet, therefore X can take at max 3 values.

$$P(X=0)={}^3C_0p^0q^3=\left(\frac{5}{6}\right)^3=\frac{125}{216}$$

$$P(X=1)={}^3C_1p^1q^2=3\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^2=\frac{75}{216}$$

$$P(X=2)={}^3C_2p^2q^1=3\cdot\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)=\frac{15}{216}$$

$$P(X=3)={}^3C_3p^3q^0=\left(\frac{1}{6}\right)^3=\frac{1}{216}$$

Mean

$$\mu=\sum_{i=1}^{3}X_iP(X_i)=0\cdot\frac{125}{216}+1\cdot\frac{75}{216}+2\cdot\frac{15}{216}+3\cdot\frac{1}{216}$$

$$=\frac{75+30+3}{216}=\frac{108}{216}=\frac{1}{2}$$

Hence the mean is $=\dfrac{1}{2}$

Question 22

From a lot of 15 bulbs which include 5 defective, sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence, find the mean of the distribution.

Solution 22

Out of 15 bulbs 5 are defective.

Hence, the probability that the drawn bulb is defective is

$$P(\text{Defective}) = \frac{5}{15} = \frac{1}{3}$$

$$P(\text{Not defective}) = \frac{10}{15} = \frac{2}{3}$$

Let X denote the number of defective bulbs out of 4.

Then, X follows binomial distribution with

$n = 4$, $p = \frac{1}{3}$ and $q = \frac{2}{3}$ such that

$$P(X = r) = {}^4C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{4-r}; r = 0, 1, 2, 3, 4$$

$$\text{Mean} = \sum_{r=0}^{4} rP(r) = 1 \times {}^4C_1\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + 2 \times {}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$$

$$+3 \times {}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right) + 4 \times {}^4C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0$$

$$= \frac{32}{81} + \frac{48}{81} + \frac{24}{81} + \frac{4}{81} = \frac{108}{81} = \frac{4}{3}$$

Question 23

A die is thrown three times. Let X be' the number of twos seen'. Find the expectation of X.

Solution 23

Let p be the probablity of getting 2 when a dice is thrown.

Then $p = \frac{1}{6}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{6}$.

$\therefore$ Expectation $= E(X) = np = 3 \times \frac{1}{6} = \frac{1}{2}$

Question 24

A die is thrown twice. A 'success' is getting an even number on a toss. Find the variance of number of successes.

Solution 24

Let p be the probablity of getting an even number on the toss when a dice is thrown.

Let q be the probablity of not getting an even number on the toss when a dice is thrown.

Then $p = \frac{3}{6} = \frac{1}{2}$ and $q = \frac{1}{2}$

Clearly, X follows binomial distribution with $n = 2$, $p = \frac{1}{2}$.

$\therefore$ Variance $= npq = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

Question 25

Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability of the number spades. Hence, find the mean of the distribution.

Solution 25

Let p be the probablity of getting a spade card.

Let q be the probablity of getting a spade card.

Then $p = \frac{13}{52} = \frac{1}{4}$ and $q = \frac{3}{4}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{4}$ and $q = \frac{3}{4}$.

Probablity distribution is given by,

$$P(X = r) = {}^3C_r\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{3-r};\ r = 0, 1, 2$$

$$\therefore \text{Mean} = np = 3 \times \frac{1}{4} = \frac{3}{4}$$

Chapter 33 - Binomial Distribution Exercise MCQ

Question 1

In a box containing 100 bulls, 10 are defective. What is the probability that out of a sample of 5 bulls , none is defective

a. $\left(\frac{9}{10}\right)^5$

b. $\frac{9}{10}$

c. 10^{-5}

d. $\left(\frac{1}{2}\right)^2$

Solution 1

Correct option: (a)

Let X denotes the number of bulls defective.

Total bulls = 100

Out of 100 bulls 10 are defective.

Probability of defective bulls $= \frac{10}{100} = \frac{1}{10} = p$

Probability of non – defective bulls

$= 1 - \frac{10}{100} = \frac{9}{10} = q$

$n = 5, p = \frac{1}{10}, q = \frac{9}{10}, X = 0$

$$P(X = 0) = {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5$$

$$P(X = 0) = \left(\frac{9}{10}\right)^5$$

Question 2

If in a binormial distribution n=4, $P(X = 0) = \frac{16}{81}$,

then $P(X = 4)$ equals

a. $\frac{1}{16}$

b. $\frac{1}{81}$

c. $\frac{1}{27}$

d. $\frac{1}{8}$

Solution 2

Correct option: (b)

Given n = 4, $P(X = 0) = \frac{16}{81}$

$P(X = 0) = \frac{16}{81}$

$^5C_0 p^0 q^4 = \frac{16}{81}$

$q^4 = \frac{16}{81}$

$q = \frac{2}{3} \Rightarrow p = \frac{1}{3}$

$\Rightarrow P(X = 4) = {}^5C_4\left(\frac{1}{3}\right)^4 = \frac{1}{81}$

Question 3

A rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds, he must fire in order to have more than 50% chance of hitting it at least once is

a. 11
b. 9
c. 7
d. 5

Solution 3

Correct option: (c)

$\text{Given } p = \frac{1}{10} \Rightarrow q = \frac{9}{10}$

Let n be the number of rounds.

$P(x \geq 1) = 1 - P(X = 0)$

$\Rightarrow P(x \geq 1) \geq 0.5$

$\Rightarrow 1 - P(X = 0) \geq 0.5$

$\Rightarrow P(X = 0) \leq 0.5$

$\Rightarrow 0.9^n \leq 0.5$

Using log table,

$n \leq 6.572 \approx 7$

He must fire in order to have more than
50% chance of hitting the target at least once.

Question 4

A fair coin is tossed fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, the probability of getting two heads is

a. $15/2^8$
b. $2/15$
c. $15/2^{13}$
d. none of these

Solution 4

Correct option: (c)

Let X be the number of heads.

$p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$.........(i)

$P(X = 7) = P(X = 9)$

${}^nC_7 p^7 q^{n-7} = {}^nC_9 p^9 q^{n-9}$

$\frac{{}^nC_7}{{}^nC_9} = \frac{q^{n-9}}{q^{n-7}} \times \frac{p^9}{p^7}$

$\frac{\frac{n!}{7!(n-7)!}}{\frac{n!}{9!(n-9)!}} = q^{-2}p^2$

$\frac{9!(n-9)!}{7!(n-7)!} = \frac{p^2}{q^2}$

$\frac{9 \times 8 \times 7!(n-9)!}{7!(n-7)(n-8)(n-9)!} = 1.....[\because \text{from (i)}]$

$9 \times 8 = (n-7)(n-8)$

Comparing both sides,

$n - 7 = 9 \Rightarrow n = 16$

$\Rightarrow P(X = 2) = {}^{16}C_2 \times 0.5^2 \times 0.5^{14}$

$\Rightarrow P(X = 2) = \frac{15}{2^{13}}$

Question 5

A fair coin is tossed 100 times. The probability of getting tails an odd number of times is

a. 1/2

b. 1/8

c. 3/8

d. None of these

Solution 5

Correct option: (a)

A fair coin tossed 100 times then probability of odd or even numbers are same and equals $= \frac{1}{2}$

$\Rightarrow$ The probability of getting tails an odd number of times is also $\frac{1}{2}$.

Question 6

A fair die is thrown twenty times. The probability that on the tenth throw the fourth six appears is

a. $\dfrac{{}^{20}C_{10} \times 5^6}{6^{20}}$

b. $\dfrac{120 \times 5^7}{6^{10}}$

c. $\dfrac{84 \times 5^6}{6^{10}}$

d. none of these

Solution 6

Correct option: (c)

A fair die is thrown then probability of getting 6 is $p = \dfrac{1}{6}$.

$\Rightarrow q = \dfrac{5}{6}$

To find probability that on tenth throw 4^{th} six appears,
in the first nine throw 3 six should appear.

$$\text{Required probability} = P(3 \text{ six in first } 9 \text{ throw}) \times P(\text{a six in tenth throw})$$

$$\text{Required probability} = {}^9C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^6 \times \frac{1}{6}$$

$$\text{Required probability} = \frac{84 \times 5^6}{6^{10}}$$

Question 7

If X is a binomial variate with parameters n and p, where $0<p<1$ such that $\dfrac{P(X=r)}{P(X=n-r)}$ is indendent of n and r,

then p equals

a. 1/2
b. 1/3
c. 1/4
d. None of these

Solution 7

Correct option: (a)

Consider,

$P(X = r) = kP(X = n - r)$

Using ${}^nC_r = {}^nC_{n-r}$, $q = 1 - p$

$p^r q^{n-r} = kp^{n-r} q^r$

$p^r (1-p)^{n-r} = kp^{n-r} (1-p)^r$

$p^{2r-n} = k(1-p)^{2r-n}$

$\left(\frac{p}{q}\right)^{2r-n} = k$

When $p = q$ then $k = 1$

$\Rightarrow p = q = \frac{1}{2}$

Question 8

Let X denote the number of times heads occur in n tosses of a fair coin. If P (X = 4), P(X = 5) and P(X=6) are in AP; the value of n is

a. 7, 14
b. 10, 14
c. 12, 7
d. 14, 12

Solution 8

Correct option: (a)

X denotes the number of times heads occurs.

$P(x=4), P(x=5), P(x=6)$ are in AP

$\Rightarrow 2P(x=5) = P(x=4)P(x=6)$

$\Rightarrow 2\,{}^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5} = {}^nC_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{n-4} x\, {}^nC_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{n-6}$

$\Rightarrow 2\,{}^nC_5\left(\frac{1}{2}\right)^n = {}^nC_4\left(\frac{1}{2}\right)^n + {}^nC_6\left(\frac{1}{2}\right)^n$

$\Rightarrow 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow \frac{2n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$

$\Rightarrow \frac{2}{5 \times 4!(n-5)(n-6)!} = \frac{1}{4!(n-4)(n-5)(n-6)!} + \frac{1}{6 \times 5 \times 4!(n-6)!}$

$\Rightarrow \frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}$

$\Rightarrow \frac{2}{5(n-5)} = \frac{30+(n-4)(n-5)}{30(n-4)(n-5)}$

$\Rightarrow 12(n-4) = 30 + (n-4)(n-5)$

$\Rightarrow 12(n-4) - (n-4)(n-5) = 30$

$\Rightarrow (n-4)(12-n+5) = 30$

$\Rightarrow (n-4)(17-n) = 30$

Check with options by putting value of n.

$\Rightarrow n = 7,\ 14$

Question 9

One hundred identical coins, each with probability p of showing heads are tossed once. If 0 < p

a. 1/2

b. 51/101

c. 49/101

d. None of these

Solution 9

Correct option: (b)

$$n = 100, \; p = q = \frac{1}{2}$$
$$P(X = 50) = P(X = 51)$$
$$\Rightarrow {}^{100}C_{50}\, p^{50} q^{50} = {}^{100}C_{51}\, p^{51} q^{49}$$
$$\Rightarrow \frac{p}{q} = \frac{{}^{100}C_{50}}{{}^{100}C_{51}}$$
$$\text{Using } \frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{r}{n-r+1}$$
$$\Rightarrow \frac{p}{q} = \frac{51}{50}$$
$$\Rightarrow \frac{p}{1-p} = \frac{51}{50}$$
$$\Rightarrow p = \frac{51}{101}$$

Question 10
A fair coin is tossed 99 times. If X is the number of times heads occur, then P (X = r) is maximum when r is
a. 49, 50
b. 50, 51
c. 51,52
d. None of these

Solution 10
Correct option: (a)

$$\text{When a coin is tossed } p = q = \frac{1}{2}$$
$$\Rightarrow P(X = r) = {}^{n}C_{r} \times 0.5^{n}$$
Coin is tossed 99 times.
For odd number of n maximum terms at
$$r = \frac{n-1}{2} \text{ and } r = \frac{n+1}{2}$$
$$n = 99 \Rightarrow r = 49 \text{ or } 50$$

Question 11
The least number of times a fair coin must be tossed so that the probability of getting at least one head is at least 0.8, is
a. 7
b. 6
c. 5
d. 3

Solution 11
Correct option: (d)

A fair coin is tossed. $\Rightarrow p = q = \frac{1}{2}$

$P(X \geq 1) \geq 0.8$

$\Rightarrow 1 - P(0) \geq 0.8$

$\Rightarrow P(0) = 0.2$

$\Rightarrow \left(\frac{1}{2}\right)^n = 0.2$

$\Rightarrow 2^{-n} = 0.2$

$\Rightarrow 2^n \geq 5$

$\Rightarrow n \geq 3$

Question 12

If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than 1 is
a. 2/3
b. 4/5
c. 7/8
d. 15/16

Solution 12

Correct option: (d)

$E(X) = 2,\ V(X) = 1$

$np = 2,\ npq = 1$

$\Rightarrow q = \frac{1}{2} = p$

$\Rightarrow n = 4$

$P(X \geq 1) = 1 - P(X = 0)$

$P(X \geq 1) = 1 - \left(\frac{1}{2}\right)^4$

$P(X \geq 1) = 1 - \frac{1}{16} = \frac{15}{16}$

Question 13

A biased coin with probability p, 0< p
a. 1/3
b. 2/3
c. 2/5
d. 3/5

Solution 13

Correct option: (a)

p is the probability of getting head.

q = 1 − p is the probability of getting tail.

The number of tosses required is even.

$$\Rightarrow qp + q^3p + q^5p + q^7p + q^9p \ldots\ldots\ldots\ldots$$

$$\Rightarrow qp\left(\frac{1}{1-q^2}\right)$$

$$\Rightarrow \frac{(1-p)p}{1-(1-p)^2}$$

$$\Rightarrow \frac{(1-p)p}{1-\left(1-2p+p^2\right)}$$

$$\Rightarrow \frac{1-p}{2-p}$$

Given $\frac{1-p}{2-p} = \frac{2}{5}$

$$\Rightarrow p = \frac{1}{3}$$

Question 14

If X follows a binomial distribution with parameters n=8 and p=1/2, then p (|X-4|≤ 2) equals

a. $\frac{118}{128}$

b. $\frac{119}{128}$

c. $\frac{117}{128}$

d. none of these

Solution 14

Correct option: (b)

$$n = 8,\ p = \frac{1}{2} = q$$

$$P(|x-4|) \le 2$$

$$\Rightarrow -2 \le x - 4 \le 2$$

$$\Rightarrow 4 - 2 \le x \le 2 + 4$$

$$\Rightarrow 2 \le x \le 6$$

$$P(2 \le x \le 6) = P(2) + P(3) + P(4) + P(5) + P(6)$$

$$P(2 \le x \le 6) = {}^8C_2\left(\frac{1}{2^8}\right) + {}^8C_3\left(\frac{1}{2^8}\right) + {}^8C_4\left(\frac{1}{2^8}\right) + {}^8C_5\left(\frac{1}{2^8}\right) + {}^8C_6\left(\frac{1}{2^8}\right)$$

$$= \frac{119}{128}$$

Question 15
If X follows a binomial distribution with parameters n=100 and p=1/3, then P(X=r) is maximum when r=
a. 32
b. 34
c. 33
d. 31

Solution 15
Correct option: (c)

$n = 100, \ p = \frac{1}{3} \Rightarrow q = \frac{2}{3}$

$np = \frac{100}{3} = 33 + \frac{1}{3}$

$\Rightarrow$ Probability is maximum at 33.

Question 16
A fair die is tossed eight times. The probability that a third six is observed in the eight throw is

a. $\frac{{}^7C_2 \times 5^5}{6^7}$

b. $\frac{{}^7C_2 \times 5^5}{6^8}$

c. $\frac{{}^7C_2 \times 5^5}{6^6}$

d. none of these

Solution 16
Correct option: (b)

Probability of getting $6 = p = \frac{1}{6}, q = \frac{5}{6}$

Probability of getting third six in eight throw.

= Probability of getting 2 sixes in first seven throw +
Probability of getting six in eight throw

$= \left({}^7C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 \right) \left(\frac{1}{6}\right)$

$= \frac{{}^7C_2 \times 5^5}{6^8}$

Question 17
Fifteen coupons are numbered 1 to 15. Seven coupons are selected at random, one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is

a. $\left(\frac{3}{5}\right)^7$

b. $\left(\frac{1}{15}\right)^7$

c. $\left(\frac{8}{15}\right)^7$

d. none of these

Solution 17

Correct option: (d)

The sample space = 15^7 for selecting seven coupons from 15 coupons.

Maximum number on selected coupon is 9 can be made by 9^7 ways.

A number selected on second card is less than 9 can be made by 8^7 ways.

Required probability = $\frac{9^7 - 8^7}{15^7}$

Question 18

A five-digit number is written down at random. The probability that the number is divisible by 5 and no two consecutive digits are identical, is

a. $\frac{1}{5}$

b. $\frac{1}{5}\left(\frac{9}{10}\right)^3$

c. $\left(\frac{3}{5}\right)^4$

d. none of these

Solution 18

Correct option: (d)

If last digit is either 0 or 5 then the number is divisible by 5.
Case : 1
Last digit is 0.
First three places can be selected by $9 \times 9 \times 9 = 729$ ways.
If $c = 0$ then three places can be selected by $9 \times 8 \times 1 = 72$
If $c \neq 0$ then $729 - 72 = 657$
Fourth place has 8 choices $= 657 \times 8 = 5256$
Total $= 72 + 5256 = 5904$
Case : 2
If $c = 5$
First place other than 5
then first three places can be filled in $8 \times 8 \times 1 = 64$
If first place is 5 then first three places can be filled
in $1 \times 9 \times 1 = 9$ ways
If third place is other than 5 then $729 - 64 - 9 = 656$ ways.
For fourth place has 8 choices.
As per required condition $= (64 + 9) \times 9 + 656 \times 8 = 5905$

$$\text{required probability} = \frac{5904 + 5905}{9 \times 10 \times 10 \times 10 \times 10} = \frac{11809}{90000}$$

NOTE: Answer not matching with back answer.

Question 19

A coin is tossed 10 times. The probability of getting exactly six heads is

a. $\frac{512}{513}$

b. $\frac{105}{512}$

c. $\frac{100}{153}$

d. ${}^{10}C_6$

Solution 19

Correct option: (b)

$$n = 10,\ X = 6,\ p = q = \frac{1}{2}$$

$$P(X = 6) = {}^{10}C_6 \left(\frac{1}{2}\right)^{10} = \frac{105}{512}$$

Question 20

The mean and variance of a binominal distribution are 4 and 3 respectively, then the probability of getting exactly six success in this distribution, is

a. $^{16}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^{6}$

b. $^{16}C_6\left(\frac{1}{4}\right)^{6}\left(\frac{3}{4}\right)^{10}$

c. $^{12}C_6\left(\frac{1}{20}\right)\left(\frac{3}{4}\right)^{6}$

d. $^{12}C_6\left(\frac{1}{4}\right)^{6}\left(\frac{3}{4}\right)^{6}$

Solution 20

Correct option: (b)

$np = 4, npq = 3$

$\Rightarrow q = \frac{3}{4}, p = \frac{1}{4}, n = 16$

$P(X = 6) = {}^{16}C_6\left(\frac{1}{4}\right)^{6}\left(\frac{3}{4}\right)^{10}$

Question 21

In a binomial distribution, the probability of getting success is 1/4 and standard deviation is 3. Then, its mean is

a. 6
b. 8
c. 12
d. 10

Solution 21

Correct option: (c)

$p = \frac{1}{4}, \sqrt{npq} = 3$

$\Rightarrow q = \frac{3}{4}, npq = 9$

$\Rightarrow \text{Mean} = np = \frac{9}{q}$

$\Rightarrow \text{Mean} = 9 \times \frac{4}{3} = 12$

Question 22

A coin is tossed 4 times. The probability that at least one head turns up, is

a. $\frac{1}{16}$

b. $\frac{2}{16}$

c. $\frac{14}{16}$

d. $\frac{15}{16}$

Solution 22

Correct option: (d)

$$n = 4,\ p = q = \frac{1}{2}$$

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X \geq 1) = 1 - \left(\frac{1}{2}\right)^4$$

$$P(X \geq 1) = \frac{15}{16}$$

Question 23

For a binominal variate X, if n = 3 and P (X =1)= 8 P (X=3), then p =

a. 4/5
b. 1/5
c. 1/3
d. 2/3

Solution 23

$$n = 3,$$

$$P(X = 1) = 8P(X = 3)$$

$$^3C_1 pq^2 = 8 \times {}^3C_3 p^3$$

$$3q^2 = 8p^2$$

$$3(1 - p)^2 = 8p^2$$

$$3\left(1 - 2p + p^2\right) = 8p^2$$

$$3 - 6p + 3p^2 = 8p^2$$

$$5p^2 + 6p - 3 = 0$$

$$p = \frac{-6 \pm \sqrt{96}}{10}$$

NOTE: Answer not matching with back answer.

Question 24

A coin is tossed n times. The probability of getting at least once is greater than 0.8. Then, the least value of n, is

a. 2
b. 3
c. 4
d. 5

Solution 24

Correct option: (b)

A fair coin is tossed. $\Rightarrow p = q = \frac{1}{2}$

$P(X \geq 1) \geq 0.8$

$\Rightarrow 1 - P(0) \geq 0.8$

$\Rightarrow P(0) = 0.2$

$\Rightarrow \left(\frac{1}{2}\right)^n = 0.2$

$\Rightarrow 2^{-n} = 0.2$

$\Rightarrow 2^n \geq 5$

$\Rightarrow n \geq 3$

Question 25

The probability of selecting a male or a female is same.

If the probability that in an office of n person $(n-1)$ Males being selected is $\frac{3}{2^{10}}$, the value of n is

a. 5
b. 3
c. 10
d. 12

Solution 25

Correct option: (d)

X represents number of males.

$p = q = \frac{1}{2}$

$P(n-1) = \frac{3}{2^{10}}$

${}^nC_{n-1} p^{n-1} q = \frac{3}{2^{10}}$

$n\left(\frac{1}{2}\right)^{n-1} \left(\frac{1}{2}\right)^n = \frac{3}{2^{10}}$

$n\left(\frac{1}{2}\right)^n = \frac{1}{4} \times \frac{3 \times 4}{2^{10}}$

$n\left(\frac{1}{2}\right)^n = 12\left(\frac{1}{2}\right)^{12}$

$\Rightarrow n = 12$

Question 26

A box contains 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

a. $\left(\frac{9}{10}\right)^5$

b. $\frac{1}{2}\left(\frac{9}{10}\right)^4$

c. $\frac{1}{2}\left(\frac{9}{10}\right)^5$

d. $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Solution 26

Correct option: (d)

$$p = \frac{10}{100} = \frac{1}{10},\ q = \frac{90}{100} = \frac{9}{10},\ n = 5$$

$$P(X \le 1) = P(0) + P(1)$$

$$P(X \le 1) = \left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^4$$

$$P(X \le 1) = \left(\frac{9}{10}\right)^5 + \left(\frac{1}{2}\right)\left(\frac{9}{10}\right)^4$$

Question 27

Suppose a random variable X follows the binomial distribution with parameters n and p, where $0 < p < 1$. If $\frac{P(X = r)}{P(X = n - r)}$ is independent of n and r, then p equal

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{5}$

d. $\frac{1}{7}$

Solution 27

Correct option: (a)

Consider,

$P(X = r) = kP(X = n - r)$

Using ${}^nC_r = {}^nC_{n-r}$, $q = 1 - p$

$p^r q^{n-r} = kp^{n-r} q^r$

$p^r (1 - p)^{n-r} = kp^{n-r} (1 - p)^r$

$p^{2r-n} = k(1 - p)^{2r-n}$

$\left(\frac{p}{q}\right)^{2r-n} = k$

When $p = q$ then $k = 1$

$\Rightarrow p = q = \frac{1}{2}$

Question 28

The probability that a person is not a swimmer is 0.3. the probability that out of 5 persons 4 are swimmers is

a. ${}^5C_4 (0.7)^4 (0.3)$

b. ${}^5C_1 (0.7)(0.3)^4$

c. ${}^5C_4 (0.7)(0.3)^4$

d. $(0.7)^4 (0.3)$

Solution 28

Correct option: (a)

Given that a person is not a swimmer $\Rightarrow q = 0.3$

$\Rightarrow p = 0.7$

$n = 5, X = 4$

$P(x = 4) = {}^5C_4 \times 0.7^4 \times 0.3$

Question 29

Which one is not a requirement of a binomial distribution?

a. There are 2 outcomes for each trial

b. There is a fixed number of trials

c. The outcomes must be dependent on each other

d. The probability of success must be the same for all the trials.

Solution 29

Correct option: (c)

In Binomial distribution trails are independent.

Question 30

The probability of guessing correctly at least 8 out of 10 answer of a true false types examination is

a. $\frac{7}{64}$

b. $\frac{7}{128}$

c. $\frac{45}{1024}$

d. $\frac{7}{41}$

Solution 30

Correct option: (b)

$n = 10, p = q = \frac{1}{2}$

$P(X \geq 8) = P(8) + P(9) + P(10)$

$P(X \geq 8) = {}^{10}C_8\left(\frac{1}{2}\right)^{10} + {}^{10}C_9\left(\frac{1}{2}\right)^{10} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$

$P(X \geq 8) = \frac{45 + 10 + 1}{2^8}$

$P(X \geq 8) = \frac{56}{256} = \frac{7}{128}$

Chapter 33 - Binomial Distribution Exercise Ex. 33VSAQ

Question 1

In a binomial distribution, if $n = 20, q = 0.75$, then write its mean.

Solution 1

Given,

$n = 20,\ q = 0.75$

We know that

$$\begin{aligned} \text{Mean} &= np \\ &= n(1-q) \qquad [\text{Since } p + q = 1] \\ &= 20(1 - 0.75) \\ &= 20(0.25) \\ &= 5 \end{aligned}$$

Mean $= 5$

Question 2

If in a binomial distribution mean is 5 and variance is 4, write the number of trials.

Solution 2

Let n and p be the parameters of binomial distribution. So

Given

$$\text{Mean } = np = 5 \qquad ---(\text{i})$$
$$\text{Variance } = npq = 4 \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{4}{5}$$
$$q = \frac{4}{5}$$
$$p = 1 - \frac{4}{5} \qquad [\text{Since } p + q = 1]$$
$$p = \frac{1}{5}$$

Put the value of p in equation (i),

$$np = 5$$
$$n\left(\frac{1}{5}\right) = 5$$
$$n = 25$$

Number of trials $= 25$

Question 3

In a group of 200 items, if the probability of getting a defective item is 0.2, write the mean of the distribution.

Solution 3

Let p be the probability of defective item, so

$$p = 0.2$$
$$p = \frac{1}{5}$$
$$q = 1 - \frac{1}{5} \qquad [\text{Since } p + q = 1]$$
$$q = \frac{4}{5}$$

Given, $n = 200$

$$\begin{aligned}\text{Mean } &= np \\ &= 200\left(\frac{1}{5}\right) \\ &= 40\end{aligned}$$

$$\text{Mean } = 40$$

Question 4

If the mean of a binomial distribution is 20 and its standard deviation is 4, find p.

Solution 4

Let n and p be the parameters of binomial distribution. So

Given

$$\text{Mean } = np = 20 \qquad ---(\text{i})$$

$$\text{Standard deviation } = \sqrt{npq} = 4$$

Squaring both the sides,

$$npq = 16 \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

$$p = 1 - q \qquad [\text{Since } p + q = 1]$$

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Question 5

If mean of a binomial distribution is 10 and its standard deviation is 2, write the value of q.

Solution 5

Let n and p be the parameters of binomial distribution.

Given

$$\text{Mean } = np = 10 \qquad ---(\text{i})$$

$$\text{Standard Deviation } = \sqrt{npq} = 2$$

Squaring both the sides,

$$npq = 4 \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{4}{10}$$

$$q = \frac{2}{5}$$

$$\Rightarrow q = 0.4$$

Question 6

If the mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, find $P(X = 1)$.

Solution 6

Let n and p be the parameters of binomial distribution,

Given,

$$\text{Mean } = np = 4 \qquad ---(i)$$

$$\text{Variance } = npq = 2 \qquad ---(ii)$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{2}{4}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \qquad [\text{Since } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 4$$

$$n\left(\frac{1}{2}\right) = 4$$

$$n = 8$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \qquad ---(iii)$$

$$P(X = 1)$$

$$= {}^8C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{8-1} \qquad [\text{Using equation (iii)}]$$

$$= 8\left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^5$$

$$= \frac{1}{32}$$

$$P(X = 1) = \frac{1}{32}$$

Question 7

If the mean and variance of a binomial variate X are 2 and 1 respectively, find $P(X > 1)$.

Solution 7

Mean = np = 2
Variance = npq = 1
q = ½
p = ½
n = 4

probability distribution $= \left(\frac{1}{2}+\frac{1}{2}\right)^4$

Question 8

If in a binomial distribution $n = 4$ and $P(X = 0) = \frac{16}{81}$, find q.

Solution 8

Given that,

$$n = 4,\ P(X = 0) = \frac{16}{81}$$

We know that,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$
$$P(X = r) = {}^4C_r (p)^r (q)^{4-r}$$
$$P(X = 0) = {}^4C_0 (1-q)^0 (q)^{4-0}$$

$$\frac{16}{81} = 1.1.q^4$$
$$q^4 = \left(\frac{2}{3}\right)^4$$

$$q = \frac{2}{3}$$

Question 9

If the mean and variance of a binomial distribution are 4 and 3 respectively, find the probability of no-success.

Solution 9

Let p be the probability of success and n be the number of trials

Given,

$$\text{Mean } = np = 4 \qquad ---(\text{i})$$

$$\text{Variance } = npq = 3 \qquad ---(\text{ii})$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{3}{4}$$

$$q = \frac{3}{4}$$

$$p = 1 - q \qquad [\text{Since } p + q = 1]$$

$$= 1 - \frac{3}{4}$$

$$p = \frac{1}{4}$$

Put the value of p in equation (i),

$$np = 4$$

$$n\left(\frac{1}{4}\right) = 4$$

$$n = 16$$

Hence, binomial distribution is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{16}C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{16-r} \qquad ---(\text{iii})$$

$$P(\text{No success})$$

$$= P(X = 0)$$

$$= {}^{16}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{16-0} \qquad [\text{Using (iii)}]$$

$$= \left(\frac{3}{4}\right)^{16}$$

$$P(\text{No success}) = \left(\frac{3}{4}\right)^{16}$$

Question 10

If for a binomial distribution p (x = 1) = p (x = 2) = α, write p (x = 4) in terms of α.

Solution 10

Binomial distribution ${}^nC_x\, p^x\, q^{(n-x)}$
Let x be the discrete variable, n the sample size
$P(x=1) = {}^nC_1\, p^1\, q^{(n-1)}$
$P(x=2) = {}^nC_2\, p^2\, q^{(n-2)}$
Given $P(x=1) = P(x=2) = \alpha$
${}^nC_1\, p^1\, q^{(n-1)} = {}^nC_2\, p^2\, q^{(n-2)} = \alpha$

$npq^{(n-1)} = \alpha \Rightarrow q^n = \frac{\alpha}{n} \times \frac{q}{p}$

$P(x=4) = {}^nC_4\, p^4\, q^{(n-4)} = {}^nC_4\, p^4\, \frac{q^n}{q^4}$

$= {}^nC_4\, p^4\, \frac{1}{q^4} \times \frac{\alpha}{n} \times \frac{q}{p}$

$= {}^nC_4\, \alpha \times \left(\frac{p}{q}\right)^3$