NCERT Solutions for Class 10 Maths Chapter 8 - Introduction to Trigonometry

Chapter 8 - Introduction to Trigonometry Exercise Ex. 8.1

Solution 1

In $\triangle$ABC by applying Pythagoras theorem

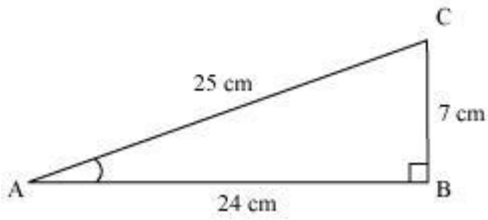
$AC^2 = AB^2 + BC^2$

$= (24)^2 + (7)^2$

$= 576 + 49$

$= 625$

$AC = \sqrt{625} = 25$ cm

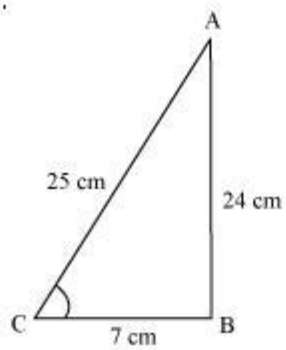


(i). $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7}{25}$

$\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$

(ii).



$\sin C = \dfrac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \dfrac{AB}{AC}$

$\cos C = \dfrac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{7}{25}$

Solution 2

In ΔPQR by applying Pythagoras theorem

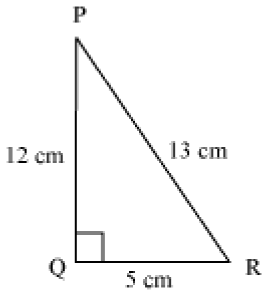
$PR^2 = PQ^2 + QR^2$

$(13)^2 = (12)^2 + QR^2$

$169 = 144 + QR^2$

$25 = QR^2$

$QR = 5$



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

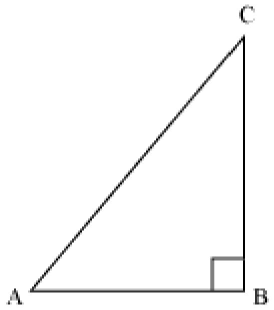
$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

Solution 3

Let ΔABC be a right angled triangle, right angled at point B.



Given that

$\sin A = \frac{3}{4}$

$\frac{BC}{AC} = \frac{3}{4}$

Let BC be 3 K so AC will be 4 K where K is a positive integer
Now applying Pythagoras theorem in ΔABC
$AC^2 = AB^2 + BC^2$
$(4\ K)^2 = AB^2 + (3\ K)^2$
$16\ K^2 - 9\ K^2 = AB^2$
$7\ K^2 = AB^2$
$AB = \sqrt{7}K$

$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}}$

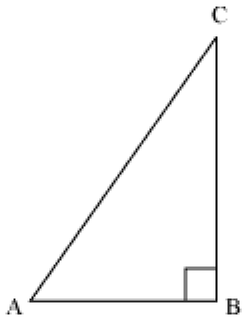
$= \frac{AB}{AC} = \frac{\sqrt{7}K}{4K} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{\text{Side opposite to } \angle A}{\text{side adjacent to } \angle A}$

$= \frac{BC}{AB} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$

Solution 4

Consider a right angled triangle, right angled at B



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

Now given that

$$\cot A = \frac{8}{15}$$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8 K so BC will be 15 K where K is a positive integer

Now applying Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$
$$= (8K)^2 + (15K)^2$$
$$= 64\ K^2 + 225\ K^2$$
$$= 289\ K^2$$

$AC = 17\ K$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

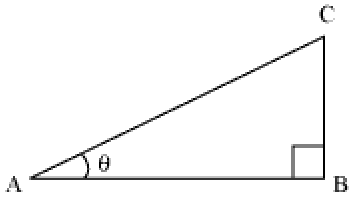
$$= \frac{15K}{17K} = \frac{15}{17}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle A}$$

$$= \frac{AC}{AB} = \frac{17}{8}$$

Solution 5

Consider a right angle triangle ΔABC right angled at point B.



$$\sec\theta = \frac{\text{hypotenuse}}{\text{Side adjacent to } \angle\theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is 13 K, AB will be 12 K, where K is a positive integer.
Now applying Pythagoras theorem in ΔABC

$(AC)^2 = (AB)^2 + (BC)^2$

$(13\ K)^2 = (12\ K)^2 + (BC)^2$

$169\ K^2 = 144\ K^2 + BC^2$

$25\ K^2 = BC^2$

$BC = 5\ K$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{5K}{13K} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12K}{13K} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{BC}{AB} = \frac{5K}{12K} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{AB}{BC} = \frac{12K}{5K} = \frac{12}{5}$$

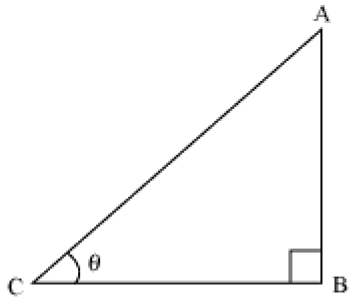
$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{AC}{BC} = \frac{13K}{5K} = \frac{13}{5}$$

Solution 6

Since $\angle A$ and $\angle B$ are acute angles, then $\angle C$ is a right angle.
cos A = cos B .... given
AC/AB = BC/AB
AC = BC
$\angle B = \angle A$ .... angles opposite to equal sides are equal in length.

Solution 7

Let us consider a right angle triangle ΔABC right angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is 7 K, AB will be 8 K, where K is a positive integer.
Now applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

$= (8\ K)^2 + (7\ K)^2$

$= 64\ K^2 + 49\ K^2$

$= 113\ K^2$

$$AC = \sqrt{113}\ K$$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8K}{\sqrt{113}\,K} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7K}{\sqrt{113}\ K} = \frac{7}{\sqrt{113}}$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

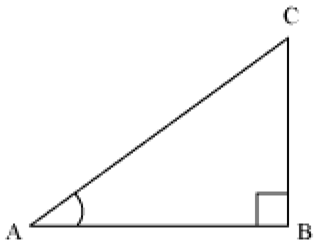
(ii) $$\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Solution 8

Given that 3cot A = 4

Or cot A $= \frac{4}{3}$

Consider a right angle triangle ΔABC right angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is 4 K, BC will be 3 K. where K is a positive integer

Now in ΔABC

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$= (4\ K)^2 + (3\ K)^2$$
$$= 16\ K^2 + 9\ K^2$$
$$= 25\ K^2$$

AC = 5 K

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$
$$= \frac{4K}{5K} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$
$$= \frac{3K}{5K} = \frac{3}{5}$$

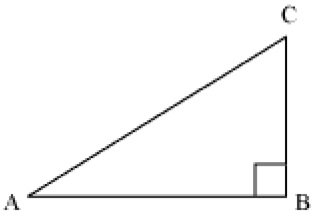
$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AB}$$
$$= \frac{3\ K}{4\ K} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$
$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Solution 9



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

If BC is K, AB will be $\sqrt{3}$ K . Where K is a positive integer

In ΔABC

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}\ K\right)^2 + (K)^2$

$= 3\ K^2 + K^2 = 4\ K^2$

$AC = 2\ K$

$\sin A = \frac{\text{Side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$

$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}\ K}{2\ K} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Side opposite to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}\ K}{2K} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{K}{2K} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

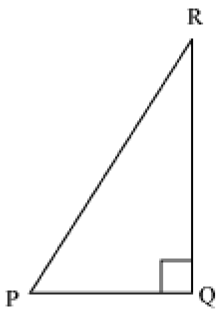
Solution 10

Given that PR + QR = 25

PQ = 5

Let PR be x

So, QR = 25 − x



Now applying Pythagoras theorem in ΔPQR

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

So, PR = 13 cm

QR = 25 − 13 = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Solution 11

(i) If $45° \leq A \leq 90°$

$1 \leq \tan A \leq$ undefined

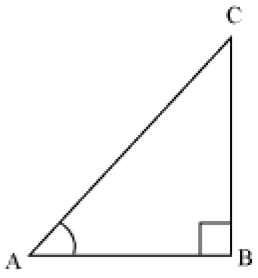
If $0° \leq A \leq 45°$

$0 \leq \tan A \leq 1$

Clearly value of tan A is not always less than 1.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be 12 K, AB will be 5 K, where K is a positive integer

Now applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

$(12\ K)^2 = (5\ K)^2 + BC^2$

$144\ K^2 = 25\ K^2 + BC^2$

$BC^2 = 119\ K^2$

$BC = 10.9\ K$

We may observe that for given two sides AC = 12 K and AB = 5 K

BC should be such that −

$AC - AB < BC < AC + AB$

$12\ K - 5\ K < BC < 12\ K + 5\ K$

$7\ K < BC < 17\ K$

But BC = 10.9 K. Clearly such a triangle is possible and hence such value of sec A is possible. Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A. Hence, the given statement is false.

(iv) cot A is not the product of cot and A but it is cotangent of $\angle A$.
Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right angle triangle

$$\sin\theta = \frac{\text{side opposite to } \angle\theta}{\text{hypotenuse}}$$

In a right angle triangle hypotenuse is always greater than the remaining two sides.
Hence such value of sin θ is not possible.
Hence, the given statement is false.

Chapter 8 - Introduction to Trigonometry Exercise Ex. 8.2

Solution 1

(i). $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii). $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii). $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2 + 2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{6} - 2\sqrt{2}\right)}{\left(2\sqrt{6} + 2\sqrt{2}\right)\left(2\sqrt{6} - 2\sqrt{2}\right)}$$

$$= \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{\left(2\sqrt{6}\right)^2 - \left(2\sqrt{2}\right)^2} = \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{24 - 8} = \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{16}$$

$$= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv). $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{3\sqrt{3} + 4}{2\sqrt{3}}} = \frac{\left(3\sqrt{3} - 4\right)}{\left(3\sqrt{3} + 4\right)}$$

$$= \frac{\left(3\sqrt{3} - 4\right)\left(3\sqrt{3} - 4\right)}{\left(3\sqrt{3} + 4\right)\left(3\sqrt{3} - 4\right)} = \frac{\left(3\sqrt{3} - 4\right)^2}{\left(3\sqrt{3}\right)^2 - (4)^2}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$$

(v). $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= 5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2$$

Solution 2

(i). $\frac{2\tan 30°}{1+\tan^2 30°}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$

$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$

Out of given alternatives only $\sin 60° = \frac{\sqrt{3}}{2}$

Hence (A) is correct.

(ii). $\frac{1-\tan^2 45°}{1+\tan^2 45°}$

$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Hence (D) is correct.

(iii). Out of given alternatives only A = 0° is correct.

As $\sin 2A = \sin 0° = 0$

$2\sin A = 2\sin 0° = 2(0) = 0$

Hence (A) is correct.

(iv). $\frac{2\tan 30°}{1-\tan^2 30°} =$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$

$= \sqrt{3}$

Out of given alternatives only $\tan 60° = \sqrt{3}$

Hence (C) is correct.

Solution 3

$\tan(A+B)=\sqrt{3}$

$\tan(A+B)=\tan 60$

A + B = 60 (i)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

tan (A - B) = tan30

A - B = 30 (ii)

Adding both equations

2A = 90

A = 45

From equation (i)

45 + B = 60

B = 15

So, ∠A = 45° and ∠B = 15°

Solution 4

(i). $\sin(A + B) = \sin A + \sin B$
Let $A = 30°$ and $B = 60°$
$\sin(A + B) = \sin(30° + 60°)$
$= \sin 90°$
$= 1$

$\sin A + \sin B = \sin 30° + \sin 60°$
$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$
Clearly $\sin(A + B) \neq \sin A + \sin B$

Hence the given statement is false.

(ii). The value of $\sin\theta$ increases as θ increases in the interval of $0° < \theta < 90°$ as
$\sin 0° = 0$
$\sin 30° = \frac{1}{2} = 0.5$
$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$
$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$
$\sin 90° = 1$
Hence the given statement is true.

(iii). $\cos 0° = 1$
$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$
$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$
$\cos 60° = \frac{1}{2} = 0.5$

$\cos 90° = 0$
We may observe that the value of $\cos\theta$ does not increase in the interval of $0° < \theta < 90°$.
Hence the given statement is false.

(iv). $\sin\theta = \cos\theta$ for all values of θ.
This is true when $\theta = 45°$
As $\sin 45° = \frac{1}{\sqrt{2}}$
$\cos 45° = \frac{1}{\sqrt{2}}$
But not true for all other values of θ.
As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$
Hence the given statement is false.

(v). $\cot A$ is not defined for $A = 0°$
Yes $\cot A$ is not defined for $A = 0°$
As $\cot A = \frac{\cos A}{\sin A}$
$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined.
Hence the given statement is true.

Chapter 8 - Introduction to Trigonometry Exercise Ex. 8.3

Solution 1

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(III) $\cos 48^\circ - \sin 42^\circ = \cos(90^\circ - 42^\circ) - \sin 42^\circ$

$= \sin 42^\circ - \sin 42^\circ$

$= 0$

(IV) $\operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$

$= \sec 59^\circ - \sec 59^\circ$

$= 0$

Solution 2

(I) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$= \tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$

$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$

$= (\cot 42^\circ \tan 42^\circ)(\cot 67^\circ \tan 67^\circ)$

$= (1)(1)$

$= 1$

(II) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

$= \cos(90^\circ - 52^\circ) \cos(90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$

$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$

$= 0$

Solution 3

Given that

tan 2A = cot (A - 18)

cot (90 - 2A) = cot (A -18)
90 - 2A = A - 18
108 = 3A
A = 36

Solution 4

Given that
tan A = cot B
tan A = tan (90 - B)
A = 90 - B
A + B = 90

Solution 5

Given that
Sec 4A = cosec (A - 20)
Cosec (90 - 4A) = cosec (A - 20)
90 - 4A = A - 20

110 = 5A
A = 22

Solution 6

We know that for a triangle ΔABC

$\angle A + \angle B + \angle C = 180°$
$\angle B + \angle C = 180° - \angle A$

$$\frac{\angle B + \angle C}{2} = 90° - \frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$= \cos\left(\frac{A}{2}\right)$$

Solution 7

sin 67 + cos 75
= sin (90 - 23) + cos (90 - 15)
= cos 23 + sin 15

Chapter 8 - Introduction to Trigonometry Exercise Ex. 8.4

Solution 1

We know that

$\text{cosec}^2 A = 1 + \cot^2 A$

$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$$

But $\sqrt{1+\cot^2 A}$ will be always positive as we are adding two positive quantities.

$$\text{So, } \sin A = \frac{1}{\sqrt{1+\cot^2 A}}$$

We know that $\tan A = \dfrac{\sin A}{\cos A}$

But $\cot A = \dfrac{\cos A}{\sin A}$

So, $\tan A = \dfrac{1}{\cot A}$

Also $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Solution 2

We know that

$$\cos A = \frac{1}{\sec A}$$
also $\sin^2 A + \cos^2 A = 1$
$\sin^2 A = 1 - \cos^2 A$
$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$
$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$
$\tan^2 A = \sec^2 A - 1$
$\tan A = \sqrt{\sec^2 A - 1}$
$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$
$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$
$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Solution 3

(i). $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$
$$= \frac{[\sin(90° - 27°)]^2 + \sin^2 27°}{[\cos(90° - 73°)]^2 + \cos^2 73°}$$
$$= \frac{[\cos 27°]^2 + \sin^2 27°}{[\sin 73°]^2 + \cos^2 73°}$$
$$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$$
$= \frac{1}{1}$ (As $\sin^2 A + \cos^2 A = 1$)
$= 1$

(ii). $\sin 25° \cos 65° + \cos 25° \sin 65°$
$= (\sin 25°)\{\cos(90° - 25°)\} + \cos 25°\{\sin(90° - 25°)\}$
$= (\sin 25°)(\sin 25°) + (\cos 25°)(\cos 25°)$
$= \sin^2 25° + \cos^2 25°$
$= 1$ (as $\sin^2 A + \cos^2 A = 1$)

Solution 4

(i) $9\sec^2 A - 9\tan^2 A$
$= 9(\sec^2 A - \tan^2 A)$
$= 9\ (1)$ (as $\sec^2 A - \tan^2 A = 1$)
$= 9$
Hence alternative (B) is correct.

(ii) $(1 + \tan\theta + \sec\theta)\ (1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\ \cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\ \cos\theta}{\sin\theta\cos\theta} = 2$$

Hence alternative (C) is correct.

(iii) (secA + tanA) (1 - sinA)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

Hence alternative (D) is correct.

(iv)

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence alternative (D) is correct.

Solution 5

(i). $\left(\operatorname{cosec}\theta - \cot\theta\right)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = \left(\operatorname{cosec}\theta - \cot\theta\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{\left(1-\cos\theta\right)^2}{\left(\sin\theta\right)^2}$$

$$= \frac{\left(1-\cos\theta\right)^2}{1-\cos^2\theta} = \frac{\left(1-\cos\theta\right)^2}{\left(1-\cos\theta\right)\left(1+\cos\theta\right)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{R.H.S.}$$

(ii). $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + \left(1+\sin A\right)^2}{\left(1+\sin A\right)\left(\cos A\right)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\left(1+\sin A\right)\left(\cos A\right)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{\left(1+\sin A\right)\left(\cos A\right)}$$

$$= \frac{1+1+2\sin A}{\left(1+\sin A\right)\left(\cos A\right)} = \frac{2+2\sin A}{\left(1+\sin A\right)\left(\cos A\right)}$$

$$= \frac{2\left(1+\sin A\right)}{\left(1+\sin A\right)\left(\cos A\right)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii). $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\theta$

$$\text{L.H.S.}=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$=\left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$=\frac{(1+\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$

$$=\frac{(1)}{(\sin\theta\cos\theta)}+\frac{(\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$$

$=\sec\theta\,\text{cosec}\,\theta+1$

= R.H.S.

(iv). $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.}=\frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$=\frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}=(\cos A+1)$$

$$=\frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$=\frac{1-\cos^2 A}{1-\cos A}=\frac{\sin^2 A}{1-\cos A}$$

= R.H.S

(v). $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\text{L.H.S} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$= \frac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

$= \operatorname{cosec} A + \cot A$

$=$ R.H.S

(vi). $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\text{L.H.S.} = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \frac{(1 + \sin A)}{\sqrt{1 - \sin^2 A}} \qquad = \frac{1 + \sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1 + \sin A}{\cos A} \qquad = \sec A + \tan A$$

$=$ R.H.S.

(vii). $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$
$$= \frac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$
$$= \frac{\sin\theta \times \left(1 - 2\sin^2\theta\right)}{\cos\theta \times \left\{2\left(1 - \sin^2\theta\right) - 1\right\}}$$
$$= \frac{\sin\theta \times \left(1 - 2\sin^2\theta\right)}{\cos\theta \times \left(1 - 2\sin^2\theta\right)}$$
$$= \tan\theta = \text{R.H.S}$$

(viii). $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$
$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$
$$= \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$
$$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$$
$$= 7 + \tan^2 A + \cot^2 A$$
$$= \text{R.H.S}$$

(ix). $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

$$\text{L.H.S} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$
$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$
$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$
$$= \frac{\left(\cos^2 A\right)\left(\sin^2 A\right)}{\sin A \cos A}$$
$$= \sin A \cos A$$

$$\text{R.H.S} = \frac{1}{\tan A + \cot A}$$
$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$
$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Hence, L.H.S = R.H.S

(x). $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\frac{1+\tan^2 A\ -\ 2\ \tan\ A}{1+\cot^2 A-2\cot A}$$

$$=\frac{\sec^2 A-\ 2\ \tan\ A}{\operatorname{cosec}^2 A-2\cot A}$$

$$=\frac{\frac{1}{\cos^2 A}-\frac{2\sin A}{\cos A}}{\frac{1}{\sin^2 A}-\frac{2\cos A}{\sin A}}=\frac{\frac{1-2\sin A\cos A}{\cos^2 A}}{\frac{1-2\sin A\cos A}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$