Access NCERT Solutions for Class 11 Maths Chapter 5

Exercise 5.1 Page No: 103

Express each of the complex number given in the Exercises 1 to 10 in the form a + ib.

1. (5i) (-3/5i)

Solution:

(5i) (-3/5i) = 5 x (-3/5) x i^2

= -3 x -1 [i^2 = -1]

= 3

Hence,

(5i) (-3/5i) = 3 + i0

2. $i^9 + i^{19}$

Solution:

$i^9 + i^{19} = (i^2)^4 . i + (i^2)^9 . i$

= $(-1)^4$. i + $(-1)^9$ .i

= 1 x i + -1 x i

= i – i

= 0

Hence,

$i^9 + i^{19}$ = 0 + i0

3. i^{-39}

Solution:

$i^{-39} = 1/ i^{39} = 1/ i^{4 \times 9 + 3} = 1/ (1^9 \times i^3) = 1/ i^3 = 1/ (-i)$ [$i^4 = 1$, $i^3 = -I$ and $i^2 = -1$]

Now, multiplying the numerator and denominator by i we get

i^{-39} = 1 x i / (-i x i)

= i/ 1 = i

Hence,

i^{-39} = 0 + i

4. 3(7 + *i*7) + *i*(7 + *i*7)

Solution:

3(7 + *i*7) + *i*(7 + *i*7) = 21 + *i*21 + *i*7 + i^2 7

= 21 + *i*28 – 7 [i^2 = -1]

= 14 + *i*28

Hence,

$3(7 + i7) + i(7 + i7) = 14 + i28$

5. (1 – *i*) – (–1 + *i*6)

Solution:

$(1 - i) - (-1 + i6) = 1 - i + 1 - i6$

$= 2 - i7$

Hence,

$(1 - i) - (-1 + i6) = 2 - i7$

$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$ **6.**

Solution:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$

$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$

$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$

$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$

$$=\frac{-19}{5}-\frac{21}{10}i$$

Hence,

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)=\frac{-19}{5}-\frac{21}{10}i$$

7. $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Solution:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Hence,

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)=\frac{17}{3}+i\frac{5}{3}$$

8. $(1 – i)^4$

Solution:

$(1 – i)^4 = [(1 – i)^2]^2$

$= [1 + i^2 – 2i]^2$

$= [1 – 1 – 2i]^2$ $[i^2 = -1]$

$= (-2i)^2$

$= 4(-1)$

$= -4$

Hence, $(1 – i)^4 = -4 + 0i$

9. $(1/3 + 3i)^3$

Solution:

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad [i^3=-i]$$
$$=\frac{1}{27}-27i+i-9 \qquad [i^2=-1]$$
$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$
$$=\frac{-242}{27}-26i$$

Hence, $(1/3 + 3i)^3 = -242/27 – 26i$

10. $(-2 – 1/3i)^3$

Solution:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Hence,

$(-2 – 1/3i)^3 = -22/3 – 107/27i$

Find the multiplicative inverse of each of the complex numbers given in the Exercises 11 to 13.

11. 4 – 3i

Solution:

Let's consider $z = 4 – 3i$

Then,

= 4 + 3*i* and

$\overline{z}$

$|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Thus, the multiplicative inverse of 4 – 3*i* is given by z^{-1}

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

12. √5 + 3*i*

Solution:

Let's consider *z* = √5 + 3*i*

$$\text{Then, } \overline{z} = \sqrt{5} - 3i \text{ and } |z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5 + 9 = 14$$

$|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Thus, the multiplicative inverse of √5 + 3*i* is given by z^{-1}

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

13. – i

Solution:

Let's consider *z* = –*i*

$$\text{Then, } \overline{z} = i \text{ and } |z|^2 = 1^2 = 1$$

$|z|^2 = 1^2 = 1$

Thus, the multiplicative inverse of –*i* is given by z^{-1}

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{i}{1} = i$$

14. Express the following expression in the form of a + ib:

$$\frac{\left(3 + i\sqrt{5}\right)\left(3 - i\sqrt{5}\right)}{\left(\sqrt{3} + \sqrt{2}i\right) - \left(\sqrt{3} - i\sqrt{2}\right)}$$

Solution:

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$=\frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad [(a+b)(a-b)=a^2-b^2]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad [i^2=-1]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Hence,

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})} = 0+\frac{-7\sqrt{2}i}{2}$$

Exercise 5.2 Page No: 108

Find the modulus and the arguments of each of the complex numbers in Exercises 1 to 2.

1. z = – 1 – i √3

Solution:

Given,
$z = -1 - i\sqrt{3}$
Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$
On squaring and adding, we get
$$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$$
$$r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$$
$$r^2 = 4 \qquad [\cos^2\theta + \sin^2\theta = 1]$$
$$r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$
Thus, modulus = 2
So, we have
$2\cos\theta = -1$ and $2\sin\theta = -\sqrt{3}$
$$\cos\theta = \frac{-1}{2} \text{ and } \sin\theta = \frac{-\sqrt{3}}{2}$$
As the values of both sin θ and cos θ are negative, θ lies in III Quadrant,
$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$
Therefore, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

2. z = -√3 + i

Solution:

Given,
$z = -\sqrt{3} + i$
Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$
On squaring and adding , we get
$$r^2\cos^2\theta + r^2\sin^2\theta = (-\sqrt{3})^2 + 1^2$$
$$r^2 = 3 + 1 = 4 \qquad [\cos^2\theta + \sin^2\theta = 1]$$
$$r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$
Thus, modulus = 2
So,
$2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$
$$\cos\theta = \frac{-\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$
$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$
Therefore, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Convert each of the complex numbers given in Exercises 3 to 8 in the polar form:

3. 1 – i

Solution:

Given complex number,

$1 - i$

Let $r \cos \theta = 1$ and $r \sin \theta = -1$

On squaring and adding, we get

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1^2 + (-1)^2$$

$$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} = \text{Modulus} \quad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

So,

$$1 - i = r \cos \theta + i r \sin \theta = \sqrt{2} \cos\left(-\frac{\pi}{4}\right) + i\sqrt{2} \sin\left(-\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right]$$

Hence, this is the required polar form.

Given complex number,
$1 - i$
Let $r \cos \theta = 1$ and $r \sin \theta = -1$
On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} = \text{Modulus} \quad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = 1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

So,

$$1 - i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

Hence, this is the required polar form.

4. – 1 + i

Solution:

Given complex number,
$-1 + i$
Let $r \cos \theta = -1$ and $r \sin \theta = 1$
On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$

$$\cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Hence, it can be written as

$$-1 + i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

5. – 1 – i

Solution:

Given complex number,

$-1-i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$

$$r^2\left(\cos^2\theta+\sin^2\theta\right) = 1+1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r>0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

Hence, it can be written as

$$-1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

Given complex number,

$-1-i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$

$$r^2\left(\cos^2\theta+\sin^2\theta\right) = 1+1$$

$$r^2 = 2$$

$$r = \sqrt{2} \qquad [\text{Conventionally, } r>0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

Hence, it can be written as

$$-1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

6. – 3

Solution:

Given complex number,

–3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 9$$

$$r^2 = 9$$

$$r = \sqrt{9} = 3 \qquad [\text{Conventionally, } r > 0]$$

So,

$$3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$

$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$

$$\therefore \theta = \pi$$

Hence, it can be written as

$$-3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$$

This is the required polar form.

7. 3 + *i*

Solution:

Given complex number,

$\sqrt{3} + i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = 3 + 1$$

$$r^2 = 4$$

$$r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$

So,

$$2\cos\theta = \sqrt{3} \text{ and } 2\sin\theta = 1$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$$

Hence, it can be written as

$$\sqrt{3} + i = r\cos\theta + ir\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6}$$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

Given complex number,

$\sqrt{3}+i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 3+1$

$r^2 = 4$

$r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$

So,

$2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$ and $\sin\theta = \dfrac{1}{2}$

$\therefore \theta = \dfrac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$

Hence, it can be written as

$$\sqrt{3}+i = r\cos\theta + i\,r\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6}$$
$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

8. ***i***

Solution:

Given complex number, i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$

$r^2 = 1$

$r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$

So,

$\cos\theta = 0$ and $\sin\theta = 1$

$\therefore \theta = \dfrac{\pi}{2}$

Hence, it can be written as

$$i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

Exercise 5.3 Page No: 109

Solve each of the following equations:

1. $x^2 + 3 = 0$

Solution:

Given quadratic equation,

$x^2 + 3 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 0$, and $c = 3$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = 0^2 - 4 \times 1 \times 3 = -12$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2\times 1} = \frac{\pm\sqrt{12}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

$$\therefore\ x = \frac{\pm 2\sqrt{3}\,i}{2} = \pm\sqrt{3}\,i$$

2. $2x^2 + x + 1 = 0$

Solution:

Given quadratic equation,

$2x^2 + x + 1 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 2$, $b = 1$, and $c = 1$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = 1 - 8 = -7$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\times 2} = \frac{-1 \pm \sqrt{7}\,i}{4} \qquad \left[\sqrt{-1} = i\right]$$

3. $x^2 + 3x + 9 = 0$

Solution:

Given quadratic equation,

$x^2 + 3x + 9 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 3$, and $c = 9$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 9 = 9 - 36 = -27$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-27}}{2(1)} = \frac{-3 \pm 3\sqrt{-3}}{2} = \frac{-3 \pm 3\sqrt{3}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

4. $-x^2 + x - 2 = 0$

Solution:

Given quadratic equation,

$-x^2 + x - 2 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = -1$, $b = 1$, and $c = -2$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm \sqrt{7}\,i}{-2} \qquad \left[\sqrt{-1} = i\right]$$

5. $x^2 + 3x + 5 = 0$

Solution:

Given quadratic equation,

$x^2 + 3x + 5 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 3$, and $c = 5$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

6. $x^2 - x + 2 = 0$

Solution:

Given quadratic equation,

$x^2 - x + 2 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -1$, and $c = 2$

So, the discriminant of the given equation is

$D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1} = \frac{1 \pm \sqrt{7}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

7. $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Solution:

Given quadratic equation,

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$

So, the discriminant of the given equation is

$D = b^2 - 4ac = (1)^2 - 4 \times \sqrt{2} \times \sqrt{2} = 1 - 8 = -7$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times \sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

8. $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Solution:

Given quadratic equation,

$\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = \sqrt{3}$, $b = -\sqrt{2}$, and $c = 3\sqrt{3}$

So, the discriminant of the given equation is

$D = b^2 - 4ac = (-\sqrt{2})^2 - 4 \times \sqrt{3} \times 3\sqrt{3} = 2 - 36 = -34$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\left(-\sqrt{2}\right) \pm \sqrt{-34}}{2 \times \sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}\,i}{2\sqrt{3}} \qquad \left[\sqrt{-1} = i\right]$$

9. $x^2 + x + 1/\sqrt{2} = 0$

Solution:

Given quadratic equation,

$x^2 + x + 1/\sqrt{2} = 0$

It can be rewritten as,

$\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = \sqrt{2}$, $b = \sqrt{2}$, and $c = 1$

So, the discriminant of the given equation is

$D = b^2 – 4ac = (\sqrt{2})^2 – 4 \times \sqrt{2} \times 1 = 2 – 4\sqrt{2} = 2(1 – 2\sqrt{2})$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\sqrt{2} \pm \sqrt{2-4\sqrt{2}}}{2 \times \sqrt{2}} = \frac{-\sqrt{2} \pm \sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$= \left(\frac{-\sqrt{2} \pm \sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right) \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{-1 \pm \left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\sqrt{2} \pm \sqrt{2-4\sqrt{2}}}{2 \times \sqrt{2}} = \frac{-\sqrt{2} \pm \sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$= \left(\frac{-\sqrt{2} \pm \sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right) \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{-1 \pm \left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

10. $x^2 + x/\sqrt{2} + 1 = 0$

Solution:

Given quadratic equation,

$x^2 + x/\sqrt{2} + 1 = 0$

It can be rewritten as,

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$

So, the discriminant of the given equation is

$D = b^2 – 4ac = (1)^2 – 4 \times \sqrt{2} \times \sqrt{2} = 1 – 8 = -7$

Hence, the required solutions are:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

Miscellaneous Exercise Page No: 112

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

1.

Solution:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$

$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$

$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$

$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$

$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$

$$=\left[-1+\frac{i}{i^2}\right]^3$$

$$=\left[-1-i\right]^3$$

$$=\left(-1\right)^3\left[1+i\right]^3$$

$$=-\left[1^3+i^3+3\cdot1\cdot i\left(1+i\right)\right]$$

$$=-\left[1+i^3+3i+3i^2\right]$$

$$=-\left[1-i+3i-3\right]$$

$$=-\left[-2+2i\right]$$

$$=2-2i$$

2. For any two complex numbers z_1 and z_2, prove that
Re (z_1z_2) = Re z_1 Re z_2 – Im z_1 Im z_2

Solution:

Lets's assume $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ as two complex numbers

Product of these complex numbers, z_1z_2

$$z_1z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$$

$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

Now,

$$\operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$

$$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

3. Reduce to the standard form

Solution:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$

$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad [\text{On multiplying numerator and denominator by } (14 + 5i)]$$

$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2 - (5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$

$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

Hence, this is the required standard form.

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

4.

Solution:

Given,

$$x - iy = \sqrt{\frac{a - ib}{c - id}}$$

$$= \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}} \left[\text{On multiplying numerator and deno min ator by } (c + id)\right]$$

$$= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}}$$

So,

$$(x - iy)^2 = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

$$x^2 - y^2 - 2ixy = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

On comparing real and imaginary parts, we get

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2}, \quad -2xy = \frac{ad - bc}{c^2 + d^2} \qquad (1)$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \qquad \left[\text{Using } (1)\right]$$

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}$$

$$= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}$$

$$= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}$$

$$= \frac{a^2 + b^2}{c^2 + d^2}$$

- Hence Proved

5. Convert the following in the polar form:

(i) $\frac{1 + 7i}{(2 - i)^2}$ **, (ii)** $\frac{1 + 3i}{1 - 2i}$

Solution:

(i) Here, $z=\dfrac{1+7i}{(2-i)^2}$

$=\dfrac{1+7i}{(2-i)^2}=\dfrac{1+7i}{4+i^2-4i}=\dfrac{1+7i}{4-1-4i}$

$=\dfrac{1+7i}{3-4i}\times\dfrac{3+4i}{3+4i}=\dfrac{3+4i+21i+28i^2}{3^2+4^2}$ [Multiplying by its conjugate in the numerator and denominator]

$=\dfrac{3+4i+21i-28}{3^2+4^2}=\dfrac{-25+25i}{25}$

$=-1+i$

Let $r\cos\theta=-1$ and $r\sin\theta=1$

On squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=1+1$

$r^2(\cos^2\theta+\sin^2\theta)=2$

$r^2=2$ $\quad[\cos^2\theta+\sin^2\theta=1]$

$r=\sqrt{2}$ $\quad[\text{Conventionally, } r>0]$

So,

$\sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow\cos\theta=\dfrac{-1}{\sqrt{2}}$ and $\sin\theta=\dfrac{1}{\sqrt{2}}$

$\therefore\theta=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ $\quad[\text{As } \theta \text{ lies in II quadrant}]$

Expressing as, $z=r\cos\theta+i\,r\sin\theta$

$=\sqrt{2}\cos\dfrac{3\pi}{4}+i\sqrt{2}\sin\dfrac{3\pi}{4}=\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$

Therefore, this is the required polar form.

(ii) Let, $z = \frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i+3i-6}{1+4}$$

$$= \frac{-5+5i}{5} = -1+i$$

Now,

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$r^2(\cos^2\theta + \sin^2\theta) = 2$

$r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow r = \sqrt{2}$ $\quad$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$$\cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$ [As θ lies in II quadrant]

Expressing as, $z = r\cos\theta + i\, r\sin\theta$

$$z = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

Therefore, this is the required polar form.

Solve each of the equation in Exercises 6 to 9.

6. $3x^2 - 4x + 20/3 = 0$

Solution:

Given quadratic equation, **$3x^2 - 4x + 20/3 = 0$**

It can be re-written as: $9x^2 - 12x + 20 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 9$, $b = -12$, and $c = 20$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18}$$

$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3}i$$

7. $x^2 - 2x + 3/2 = 0$

Solution:

Given quadratic equation, $x^2 - 2x + 3/2 = 0$

It can be re-written as $2x^2 - 4x + 3 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -4$, and $c = 3$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}\,i}{4} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{2 \pm \sqrt{2}\,i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

8. $27x^2 - 10x + 1 = 0$

Solution:

Given quadratic equation, $27x^2 - 10x + 1 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 27$, $b = -10$, and $c = 1$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}\,i}{54}$$

$$= \frac{5 \pm \sqrt{2}\,i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27} i$$

9. $21x^2 - 28x + 10 = 0$

Solution:

Given quadratic equation, $21x^2 - 28x + 10 = 0$

On comparing it with $ax^2 + bx + c = 0$, we have

$a = 21$, $b = -28$, and $c = 10$

So, the discriminant of the given equation will be

$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$

Hence, the required solutions are

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}\, i}{42}$$

$$= \frac{28 \pm 2\sqrt{14}\, i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42} i = \frac{2}{3} \pm \frac{\sqrt{14}}{21} i$$

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$$

10. If $z_1 = 2 - i$, $z_2 = 1 + i$, find

Solution:

Given, $z_1 = 2 - i$, $z_2 = 1 + i$

So,

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad [i^2 = -1]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Hence, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

11.

Solution:

Given,

$$a+ib=\frac{(x+i)^2}{2x^2+1}$$

$$=\frac{x^2+i^2+2xi}{2x^2+1}$$

$$=\frac{x^2-1+i2x}{2x^2+1}$$

$$=\frac{x^2-1}{2x^2+1}+i\left(\frac{2x}{2x^2+1}\right)$$

Comparing the real and imaginary parts, we have

$$a=\frac{x^2-1}{2x^2+1} \text{ and } b=\frac{2x}{2x^2+1}$$

$$\therefore a^2+b^2=\left(\frac{x^2-1}{2x^2+1}\right)^2+\left(\frac{2x}{2x^2+1}\right)^2$$

$$=\frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$$

$$=\frac{x^4+1+2x^2}{(2x^2+1)^2}$$

$$=\frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence proved,

$$a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$$

Given,

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$

$$= \frac{x^2 + i^2 + 2xi}{2x^2+1}$$

$$= \frac{x^2 - 1 + i2x}{2x^2+1}$$

$$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$

Comparing the real and imaginary parts, we have

$$a = \frac{x^2-1}{2x^2+1} \text{ and } b = \frac{2x}{2x^2+1}$$

$$\therefore a^2 + b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2$$

$$= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x+1)^2}$$

$$= \frac{x^4 + 1 + 2x^2}{(2x^2+1)^2}$$

$$= \frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence proved,

$$a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$$

12. Let $z_1 = 2 - i$, $z_2 = -2 + i$. Find

(i) $\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right)$**, (ii)** $\mathrm{Im}\left(\frac{1}{z_1 \overline{z_1}}\right)$

Solution:

Given,

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\overline{z}_1 = 2 + i$

$$\therefore \frac{z_1 z_2}{\overline{z}_1} = \frac{-3+4i}{2+i}$$

On multiplying numerator and denominator by (2 - *i*), we get

$$\frac{z_1 z_2}{\overline{z}_1} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$

$$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

Comparing the real parts, we have

$$\text{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$$

(ii) $\frac{1}{z_1 \overline{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2+(1)^2} = \frac{1}{5}$

On comparing the imaginary part, we get

$$\text{Im}\left(\frac{1}{z_1 \overline{z}_1}\right) = 0$$

$$\frac{1+2i}{1-3i}$$

13. Find the modulus and argument of the complex number

Solution:

Let $z = \frac{1+2i}{1-3i}$, then

$$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$$

$$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$$

Let $z = r\cos\theta + ir\sin\theta$

So,

$$r\cos\theta = \frac{-1}{2} \text{ and } r\sin\theta = \frac{1}{2}$$

On squaring and adding, we get

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \qquad [\text{Conventionally, } r > 0]$$

$$r = \frac{1}{\sqrt{2}}$$

Now,

$$\frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

14. Find the real numbers *x* and *y* if (*x* – *iy*) (3 + 5*i*) is the conjugate of – 6 – 24*i*.

Solution:

Let's assume z = (*x* – *iy*) (3 + 5*i*)

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x+5y) + i(5x-3y)$$

$$\therefore \overline{z} = (3x+5y) - i(5x-3y)$$

Also given, $\overline{z} = -6 - 24i$

And,

(3x + 5y) – *i*(5x – 3y) = -6 -24*i*

On equating real and imaginary parts, we have

3x + 5y = -6 …… (i)

5x – 3y = 24 …… (ii)

Performing (i) x 3 + (ii) x 5, we get

(9x + 15y) + (25x – 15y) = -18 + 120

34x = 102

x = 102/34 = 3

Putting the value of *x* in equation (i), we get

3(3) + 5y = -6

5y = -6 – 9 = -15

y = -3

Therefore, the values of *x* and *y* are 3 and –3 respectively.

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}$$

15. Find the modulus of

Solution:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$

$$=\frac{4i}{2}=2i$$

$$\therefore\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

$$\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$$

16. If (x + iy)^3 = u + iv, then show that

Solution:

Given,

$$(x+iy)^3 = u+iv$$

$$x^3 + (iy)^3 + 3\cdot x\cdot iy(x+iy) = u+iv$$

$$x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u+iv$$

$$x^3 - iy^3 + 3x^2yi - 3xy^2 = u+iv$$

$$(x^3 - 3xy^2) + i(3x^2y - y^3) = u+iv$$

On equating real and imaginary parts, we get

$$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$$

$$\frac{u}{x}+\frac{v}{y} = \frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$$

$$= \frac{x(x^2-3y^2)}{x}+\frac{y(3x^2-y^2)}{y}$$

$$= x^2-3y^2+3x^2-y^2$$

$$= 4x^2-4y^2$$

$$= 4(x^2-y^2)$$

$$\therefore \frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$$

Hence proved.

Given,

$$(x+iy)^3 = u+iv$$

$$x^3 + (iy)^3 + 3\cdot x\cdot iy(x+iy) = u+iv$$

$$x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u+iv$$

$$x^3 - iy^3 + 3x^2yi - 3xy^2 = u+iv$$

$$(x^3 - 3xy^2) + i(3x^2y - y^3) = u+iv$$

On equating real and imaginary parts, we get

$$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$$

$$\frac{u}{x}+\frac{v}{y} = \frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$$

$$= \frac{x(x^2-3y^2)}{x}+\frac{y(3x^2-y^2)}{y}$$

$$= x^2-3y^2+3x^2-y^2$$

$$= 4x^2-4y^2$$

$$= 4(x^2-y^2)$$

$$\therefore \frac{u}{x}+\frac{v}{y} = 4(x^2-y^2)$$

Hence proved.

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$$

17. If α and β are different complex numbers with |β| = 1, then find

Solution:

Let α = *a* + *ib* and β = *x* + *iy*

Given, $|\beta| = 1$

So, $\sqrt{x^2+y^2} = 1$

$\Rightarrow x^2+y^2 = 1 \quad \ldots \text{(i)}$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$= \frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad [\text{Using (1)}]$$

$$= 1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$

18. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$

Solution:

$$|1-i|^x = 2^x$$
$$\left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$
$$\left(\sqrt{2}\right)^x = 2^x$$
$$2^{\frac{x}{2}} = 2^x$$
$$\frac{x}{2} = x$$
$$x = 2x$$
$$2x - x = 0$$
$$x = 0$$

Therefore, 0 is the only integral solution of the given equation.

Hence, the number of non-zero integral solutions of the given equation is 0.

19. If (a + ib) (c + id) (e + if) (g + ih) = A + iB, then show that

(a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = $A^2 + B^2$.

Solution:

Given,

$$(a+ib)(c+id)(e+if)(g+ih) = A + iB$$
$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$
$$\Rightarrow |(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \qquad [|z_1 z_2| = |z_1||z_2|]$$
$$\sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

On squaring both sides, we get

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Hence proved.

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

20. If, then find the least positive integral value of m.

Solution:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m = 1$$

$$\left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\left(\frac{2i}{2}\right)^m = 1$$

$$i^m = 1$$

Hence, $m = 4k$, where k is some integer.

Thus, the least positive integer is 1.

Therefore, the least positive integral value of *m* is 4 (= 4 × 1).