NCERT Solutions for Class 10 Maths Chapter 7 - Coordinate Geometry

Chapter 7 - Coordinate Geometry Exercise Ex. 7.1

Solution 1

(i) Distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore distance between $(2, 3)$ and $(4, 1)$ is given by

$$D = \sqrt{(2-4)^2 + (3-1)^2} = \sqrt{(-2)^2 + (2)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

(ii) Distance between $(-5, 7)$ and $(-1, 3)$ is given by

$$D = \sqrt{(-5-(-1))^2 + (7-3)^2} = \sqrt{(-4)^2 + (4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

(iii) Distance between (a, b) and $(-a, -b)$ is given by

$$D = \sqrt{(a-(-a))^2 + (b-(-b))^2}$$
$$= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$$

Concept Insights: In the ordered pair (a, b) order is important coordinate a represent x coordinate and b represent y coordinate

Solution 2

Distance between points $(0, 0)$ and $(36, 15)$

Now Distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$$
$$= \sqrt{1296 + 225} = \sqrt{1521} = 39$$

In section 7.2, A is (4,0) and B is (6,0).

$AB^2 = (6 - 4)^2 - (0 - 0)^2 = 4$

AB = 2

Solution 3

Three points are collinear if they lie on a line i.e one point lies in between any other two points.

Let $A = (1,5), B = (2,3), c = (-2,-11)$

Therefore $AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$

$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16+196} = \sqrt{212}$

$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$

Here sum of the distances of any two points is not equal to third point
Therefore points (1, 5), (2, 3) and (− 2, − 11) are not collinear.

Solution 4

Three non collinear points will represent the vertices of an isosceles triangle, if its two sides are of equal length.

Let $A = (5,-2), B = (6,4), C = (7,-2)$

$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$

$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$

Here AB = BC

As two sides are equal in length therefore *ABC* is an isosceles triangle.

Solution 5

From the figure coordinates of points A, B, C and D are
A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1)

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal } AC = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal } BD = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$

Here, all sides of this square are of same length and also diagonals are of same length.
So, ABCD is a square and hence Champa was correct.

Concept Insight: For the Vertices of square all sides & both the diagonals are equal.

Solution 6

(i). Let, $A = (-1,-2), B = (1,0), C = (-1,2), D(-3,0)$

$$AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal } AC = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal } BD = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

Here, all sides of this quadrilateral are of same length and also diagonals are of same length. So, given points are vertices of a square.

(ii) Let $A = (-3, 5)$, $B = (3, 1)$, $C = (0, 3)$, $D = (-1, -4)$

$$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

Here all sides of this quadrilateral are of different length. So, we can say that it is only a general quadrilateral not specific like square, rectangle etc.

(iii) Let $A = (4, 5)$, $B = (7, 6)$, $C = (4, 3)$, $D = (1, 2)$

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$\text{Diagonal BD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

Here opposite sides of this quadrilateral are of same length but diagonals are of different length. So, given points are vertices of a parallelogram.

Concept Insight: Recall the properties of various quadrilaterals.

Solution 7

We have to find point on x axis. So, its y coordinate will be 0.
Let point on x-axis be $(x, 0)$

Distance between $(x, 0)$ and $(2, -5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$

Distance between $(x, 0)$ and $(-2, 9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$

By given condition these distances are equal in measure.

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$8x = 25 - 81$$

$$8x = -56$$

$$x = -7$$

Therefore the point is $(-7, 0)$.

Solution 8

Given that distance between $(2, -3)$ and $(10, y)$ is 10

Therefore using distance formula $\sqrt{(2-10)^2 + (-3-y)^2} = 10$

$$\sqrt{(-8)^2 + (3+y)^2} = 10$$

$$64 + (y+3)^2 = 100$$

$$(y+3)^2 = 36$$

$$y + 3 = \pm 6$$

$y + 3 = 6$ or $y + 3 = -6$

Therefore $y = 3$ or -9

Concept Insights: Any point on y axis will have x coordinate zero.

Solution 9

Given Q (0, 1) is equidistant from P (5, − 3) and R (x, 6) so

PQ = QR

$\sqrt{(5-0)^2+(-3-1)^2}=\sqrt{(0-x)^2+(1-6)^2}$

$\sqrt{(5)^2+(-4)^2}=\sqrt{(-x)^2+(-5)^2}$

$\sqrt{25+16}=\sqrt{x^2+25}$

$41=x^2+25$

$16=x^2$

$x=\pm4$

So, point R is (4, 6) or (− 4, 6)

When point R is (4, 6)

$PR=\sqrt{(5-4)^2+(-3-6)^2}=\sqrt{1^2+(-9)^2}=\sqrt{1+81}=\sqrt{82}$

$QR=\sqrt{(0-4)^2+(1-6)^2}=\sqrt{(-4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$

When point R is (− 4, 6)

$PR=\sqrt{(5-(-4))^2+(-3-6)^2}=\sqrt{(9)^2+(-9)^2}=\sqrt{81+81}=9\sqrt{2}$

$QR=\sqrt{(0-(-4))^2+(1-6)^2}=\sqrt{(4)^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$

Solution 10

Point (x, y) is equidistant from (3, 6) and (−3, 4)

Therefore $\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x-(-3))^2+(y-4)^2}$

$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$

$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$

$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$

$36-16=6x+6x+12y-8y$

$20=12x+4y$

$3x+y=5$

Chapter 7 - Coordinate Geometry Exercise Ex. 7.2

Solution 1

Let P(x, y) be the required point. Using the section formula which says Coordinates of points P, dividing the line segment joining A(x_1,y_1) & B(x_2,y_2) internally in the ratio m:n

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$, we get

$$x=\frac{2\times4+3\times(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

$$y=\frac{2\times(-3)+3\times7}{2+3}=\frac{-6+21}{5}=\frac{15}{5}=3$$

Therefore point is (1, 3).

Concept Insight: The key idea here is to identify m and n with point A & n with point B

Solution 2

Trisection means division into three equal parts. So we need to find two points such that they divide the line segment in three equal parts.

Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e. AP = PQ = QB

Therefore point P divides AB internally in ratio 1:2

$$x_1=\frac{1\times(-2)+2\times4}{1+2},\quad y_1=\frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1=\frac{-2+8}{3}=\frac{6}{3}=2,\quad y_1=\frac{-3-2}{3}=\frac{-5}{3}$$

$$\text{So, } P(x_1,y_1)=\left(2,-\frac{5}{3}\right)$$

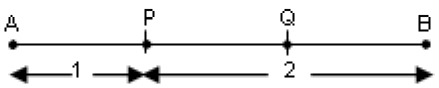
Point Q divides AB internally in ratio 2:1

$$x_2=\frac{2\times(-2)+1\times4}{2+1},\quad y_2=\frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2=\frac{-4+4}{3}=0,\qquad y_2=\frac{-6-1}{3}=\frac{-7}{3}$$

$$Q(x_2,y_2)=\left(0,-\frac{7}{3}\right)$$

Concept Insights: **Trisection** means line segment divided into 3 equal parts (1:2 ratio)



Find coordinates of point P (divided into 1:2) ratio.

Now, find coordinates of point Q by applying section formula with ratio 2:1(PB divided into 1:1 ratio by point Q)

Solution 3

Given that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e. $\frac{1}{4} \times 100 = 25\,\text{m}$ from the starting point of 2nd line.

So, coordinates of this point G is (2, 25).

Similarly Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e. $\frac{1}{5} \times 100 = 20\,\text{m}$ from the starting point of 8th line.

So, coordinates of this point R is (8, 20).

Now distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{36+25} = \sqrt{61}\,\text{m}$$

Now the point at which Rashmi should post her blue flag is the midpoint of line joining these points. Let this point be A (x, y).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \; y = \frac{45}{2} = 22.5$$

So, $A(x, y) = (5, 22.5)$

So, Rashmi should post her blue flag at 22.5m on 5th line.

Solution 4

Let the ratio in which line segment joining (−3, 10) and (6, −8) is divided by point (−1, 6) is k:1.

$$\text{So,} \quad -1 = \frac{6k-3}{k+1}$$

$$-k - 1 = 6k - 3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

Therefore the required ratio is $2:7$.

Concept Insight: Assume the ratio as k:1 and not as m:n otherwise we will get one equation in two unknowns.

Solution 5

If the ratio in which P divides AB is k:1 , then the co-ordinates of the point P will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right).$$

Let the ratio in which line segment joining A (1, −5) and (− 4, 5) is divided by x axis be $k:1$.

Therefore, coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$

We know that y coordinate of any point on x axis is 0.

Therefore $\frac{5k-5}{k+1} = 0$

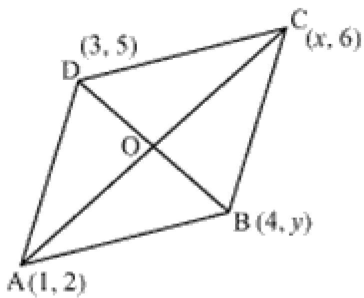
$k = 1$

Therefore x-axis divide it in ratio 1:1.

Division point $= \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right) = \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$

Concept Insight : Assume the ratio as k:1 and not as m:n otherwise we will get one equation in two unknowns.
Use the fact that y coordinate is zero

Solution 6



Let (1, 2), (4, y), (x, 6) and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD.
Diagonals of a parallelogram bisects each other so, O is midpoint of AC and BD
If O is midpoint of AC, then coordinate of O are

$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$

If O is midpoint of BD, then coordinates of O are

$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$

Since both coordinates are of same point O.

Therefore $\frac{x+1}{2} = \frac{7}{2}$ and $4 = \frac{5+y}{2}$

$x + 1 = 7$ and $5 + y = 8$

$x = 6$ and $y = 3$

Concept Insight:

Use the property of a parallelogram that the diagonals of a Parallelogram bisects each other for finding the values of a and y.

Solution 7

Let coordinates of point A be (x, y)

Mid point of diameter AB is the centre of circle $(2, -3)$

$$(2, -3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

$$\frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$x + 1 = 4$ and $y + 4 = -6$

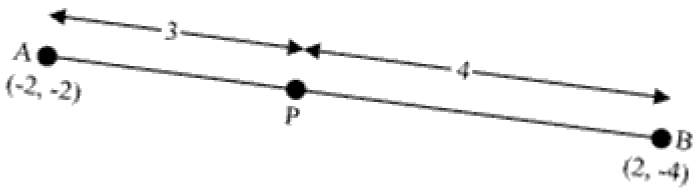
$x = 3$ and $y = -10$

Therefore coordinates of A are $(3, -10)$

Solution 8

coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$



The coordinates of point A and B are $(-2, -2)$ and $(2, -4)$ respectively.

Since $AP = \frac{3}{7}AB$

Therefore AP: PB = 3:4

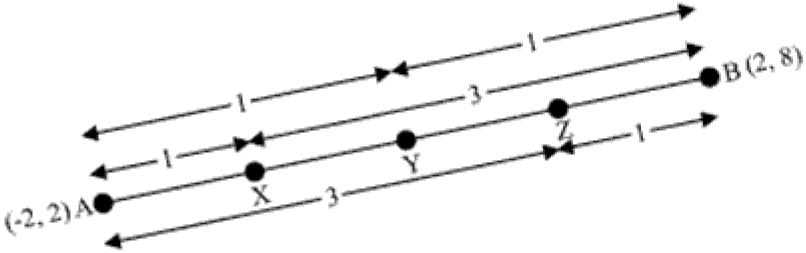
So, point P divides the line segment AB in a ratio 3:4.

$$\text{Coordinates of P} = \left(\frac{3 \times 2 + 4 \times (-2)}{3+4}, \frac{3 \times (-4) + 4 \times (-2)}{3+4}\right)$$

$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Solution 9



From the figure we have points X, Y, Z are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of X} = \left(\frac{1\times 2+3\times(-2)}{1+3}, \frac{1\times 8+3\times 2}{1+3}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

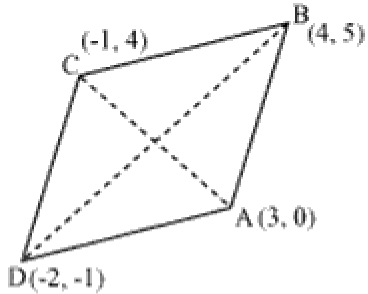
$$\text{Coordinates of Y} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$

$$= (0, 5)$$

$$\text{Coordinates of Z} = \left(\frac{3\times 2+1\times(-2)}{3+1}, \frac{3\times 8+1\times 2}{3+1}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

Solution 10



Let (3, 0), (4, 5), (− 1, 4) and (− 2, −1) are the vertices A, B, C, D of a rhombus ABCD.

Length of diagonal AC = $\sqrt{\left[3-(-1)\right]^2+(0-4)^2}$

$= \sqrt{16+16} = 4\sqrt{2}$

Length of diagonal BD = $\sqrt{\left[4-(-2)\right]^2+\left[5-(-1)\right]^2}$

$= \sqrt{36+36} = 6\sqrt{2}$

Therefore area of rhombus ABCD = $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

= 24 square units

Concept Insights: Use the result Area of a rhombus = $\frac{1}{2}$(product of its diagonals) and diagonals are formed by joining opposite vertices.

Chapter 7 - Coordinate Geometry Exercise Ex. 7.3

Solution 1

(i) Area of a triangle is given by -

$$\text{area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{area of given triangle} = \frac{1}{2}\left[2\{0-(-4)\} + (-1)\{(-4)-(3)\} + 2(3-0)\right]$$

$$= \frac{1}{2}\{8+7+6\}$$

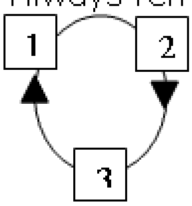
$$= \frac{21}{2} \text{ square units}$$

$$\text{(ii) Area of given triangle} = \frac{1}{2}\left[(-5)\{(-5)-(2)\} + 3(2-(-1)) + 5\{-1-(-5)\}\right]$$

$$= \frac{1}{2}\{35+9+20\}$$

$$= 32 \text{ square units}$$

Concept Insight:
Always remember that the coordinate points are rotating clockwise



$(1 \rightarrow 2 \rightarrow 3)$ 1st term, $(2 \rightarrow 3 \rightarrow 1)$ 2nd term & $(3 \rightarrow 1 \rightarrow 2)$ 3rd term.

Also area of a region is always a positive quantity and hence absolute value must be taken.

Solution 2

(i) For collinear points, area of triangle formed by them is zero.

So, for points (7, −2) (5, 1) and (3, *k*), area = 0

$$\frac{1}{2}\left[7\{1-k\} + 5\{k-(-2)\} + 3\{(-2)-1\}\right] = 0$$

$$7 - 7k + 5k + 10 - 9 = 0$$

$$-2k + 8 = 0$$

$$k = 4$$

(ii) For collinear points, area of triangle formed by them is zero.

So, for points (8, 1) (*k*, −4), (2, −5), area = 0

$$\frac{1}{2}\left[8\{-4-(-5)\} + k\{(-5)-(1)\} + 2\{1-(-4)\}\right] = 0$$

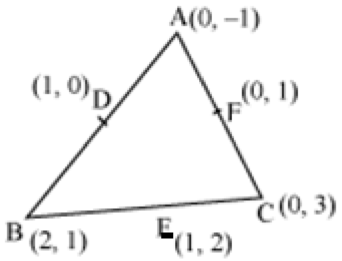
$$8 - 6k + 10 = 0$$

$$6k = 18$$

$$k = 3$$

Concept Insight: Only three non collinear points can give a triangle.

Solution 3



Let vertices of the triangle be A (0, –1), B (2, 1), C (0, 3)
Let D, E, F are midpoints of the sides of this triangle. Coordinates of D, E, and F are given by –

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$E = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$F = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta DEF = \frac{1}{2}\{1(2-1) + 1(1-0) + 0(0-2)\}$$

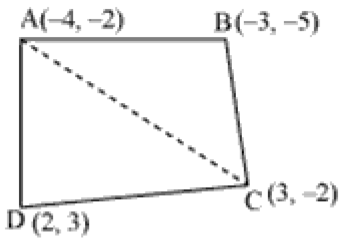
$$= \frac{1}{2}(1+1) = 1 \text{ square units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[0(1-3) + 2\{3-(-1)\} + 0(-1-1)\right]$$

$$= \frac{1}{2}\{8\} = 4 \text{ square units}$$

Therefore the required ratio = 1 : 4

Solution 4



Let vertices of the quadrilateral be A (− 4, −2), B (−3, −5), C (3, −2) and D (2, 3). Join AC to form two triangles ΔABC and ΔACD

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\{(-5)-(-2)\} + (-3)\{(-2)-(-2)\} + 3\{(-2)-(-5)\}\right]$$

$$= \frac{1}{2}(12 + 0 + 9) = \frac{21}{2} \text{ square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\{(-2)-(3)\} + 3\{(3)-(-2)\} + 2\{(-2)-(-2)\}\right]$$

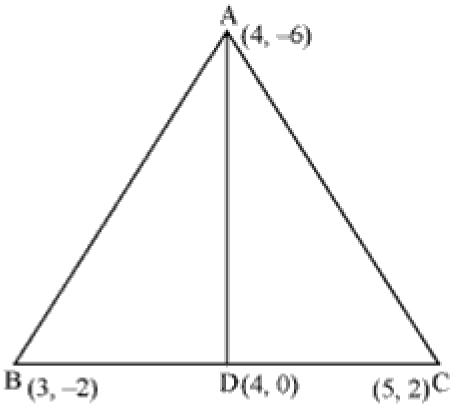
$$= \frac{1}{2}\{20 + 15 + 0\} = \frac{35}{2} \text{ square units}$$

$$\text{Area of } \square ABCD = \text{area of } \Delta ABC + \text{area of } \Delta ACD$$

$$= \frac{21}{2} + \frac{35}{2} = 28 \text{ square units}$$

Concept Insight: Join either point A & C or B & D (not both)
Compute Area of triangle separately & then add.

Solution 5



Let vertices of the triangle be A (4, – 6), B (3, – 2), C (5, 2)
Let D be the midpoint of side BC of ΔABC. So AD is the median in ΔABC.

Coordinates of point D = $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ = (4,0)

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta ABD = \frac{1}{2}[(4)\{(-2)-(0)\} + (3)\{(0)-(-6)\} + (4)\{(-6)-(-2)\}]$$

$$= \frac{1}{2}(-8 + 18 - 16) = -3 \text{ square units}$$

But area can not be negative. So area of ΔABD is 3 square units.

$$\text{Area of } \Delta ADC = \frac{1}{2}[(4)\{0-(2)\} + (4)\{(2)-(-6)\} + (5)\{(-6)-(0)\}]$$

$$= \frac{1}{2}(-8 + 32 - 30) = -3 \text{ square units}$$

But area can not be negative. So area of ΔADC is 3 square units.
Clearly median AD has divided ΔABC in two triangles of equal areas.

Chapter 7 - Coordinate Geometry Exercise Ex. 7.4

Solution 1

If the ratio in which P divides AB is $k : 1$, then the coordinates of the point P will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right).$$

Let the given line divides the line segment joining the points A (2, − 2) and B (3, 7) in a ratio k: 1.

The coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\frac{6k+4+7k-2-4k-4}{k+1}=0$$

$$9k-2=0$$

$$k=\frac{2}{9}$$

Solution 2

If the given points are collinear the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$\text{Area} = \frac{1}{2}[x(2-0)+1(0-y)+7(y-2)]$$

$$0=\frac{1}{2}[2x-y+7y-14]$$

$$0=\frac{1}{2}[2x+6y-14]$$

$$2x+6y-14=0$$

$$x+3y-7=0$$

This is the required relation between x and y.

Solution 3

Let O (x, y) be the centre of circle. And let (6, − 6), (3, − 7) and (3, 3) are A, B, and C points on the circumference of circle.

$$OA = \sqrt{(x-6)^2 + (y+6)^2}$$

$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$

$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

OA = OB (Radius of circle)

$$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$

$$-6x - 2y + 14 = 0$$

$$3x + y = 7 \qquad (1)$$

OA = OC (Radius of circle)

$$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

$$x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$

$$-6x + 18y + 54 = 0$$

$$-3x + 9y = -27 \qquad (2)$$

Adding equation (1) and (2)

$10y = -20$

$y = -2$

From equation (1)

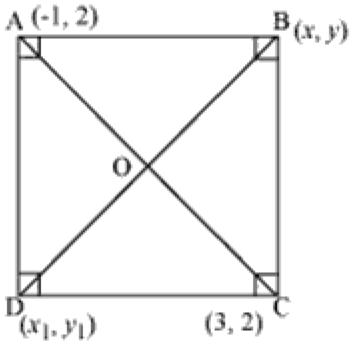
$3x - 2 = 7$

$3x = 9$

$x = 3$

So, the centre of circle is (3, − 2)

Solution 4



Let □ABCD be a square having (−1, 2), (3, 2) as vertices A and C respectively and (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.
We know that the sides of a square are equal to each other
AB = BC

$$\sqrt{(x+1)^2+(y-2)^2}=\sqrt{(x-3)^2+(y-2)^2}$$

$$x^2+2x+1+y^2-4y+4=x^2+9-6x+y^2+4-4y$$

$$8x=8$$

$$x=1$$

We know that in a square all interior angles are of 90º.
So in ΔABC
$AB^2 + BC^2 = AC^2$

$$\left(\sqrt{(1+1)^2+(y-2)^2}\right)^2+\left(\sqrt{(1-3)^2+(y-2)^2}\right)^2=\left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

$4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$
$2y^2 + 16 - 8y = 16$
$2y^2 - 8y = 0$
$y(y - 4) = 0$
$y = 0$ or 4
We know that in a square diagonals are of equal length and bisect each other at 90º. Let O be the mid point of AC so it will also be the mid point of BD.

Coordinate of point O = $\left(\frac{-1+3}{2}, \frac{2+2}{2}\right)$

$\left(\frac{1+x_1}{2}, \frac{y+y_1}{2}\right) = (1, 2)$

$\frac{1+x_1}{2} = 1$

$1 + x_1 = 2$

$x_1 = 1$

$\frac{y+y_1}{2} = 2$

$y + y_1 = 4$
If $y = 0$
$y_1 = 4$
If $y = 4$
$y_1 = 0$
So coordinates of other vertices are (1, 0) (1, 4)

Solution 5

(I) Taking A as origin, we will take AD as x axis and AB as y axis. Now we may observe that coordinates of point P, Q and R are (4, 6), (3, 2), (6, 5)

$$\text{Area of triangle} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[4(2-5) + 3(5-6) + 6(6-2)\right]$$

$$= \frac{1}{2}\left[-12 - 3 + 24\right]$$

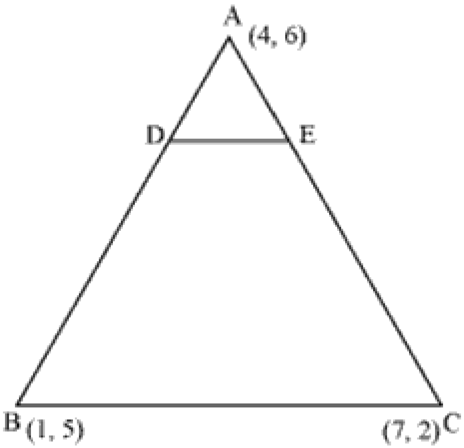
$$= \frac{9}{2} \text{ square units}$$

(II) Taking C as origin and CB as x axis and CD as y axis the coordinates of vertices P, Q, R are (12, 2), (13, 6), (10, 3).

$$\text{Area of triangle} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 + y_2)\right]$$
$$= \frac{1}{2}\left[12(6-3) + 13(3-2) + 10(2-6)\right]$$
$$= \frac{1}{2}\left[36 + 13 - 40\right]$$
$$= \frac{9}{2} \text{ square units}$$

Area of the triangle is same in both the cases.

Solution 6



$$\text{Given}: \frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$
$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} = 4$$
$$\Rightarrow \frac{AB - AD}{AD} = \frac{AC - AE}{AE} = 4 - 1$$
$$\Rightarrow \frac{BD}{AD} = \frac{EC}{AE} = \frac{3}{1}$$
$$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC} = \frac{1}{3}$$

So, D and E are two points on side AB and AC respectively such that they divide side AB and AC in the ratio 1:3

Coordinates of the point P(x,y) which divides the line segment joining the points points A(x_1,y_1) and B(x_2,y_2) internally in the ratio m_1:m_2 are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\text{Coordinates of Point D} = \left(\frac{1\times1+3\times4}{1+3}, \frac{1\times5+3\times6}{1+3}\right)$$

$$= \left(\frac{13}{4}, \frac{23}{4}\right)$$

$$\text{Coordinates of point E} = \left(\frac{1\times7+3\times4}{1+3}, \frac{1\times2+3\times6}{1+3}\right)$$

$$= \left(\frac{19}{4}, \frac{20}{4}\right)$$

$$\text{Area of a triangle} = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\text{Area of } \Delta ADE = \frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

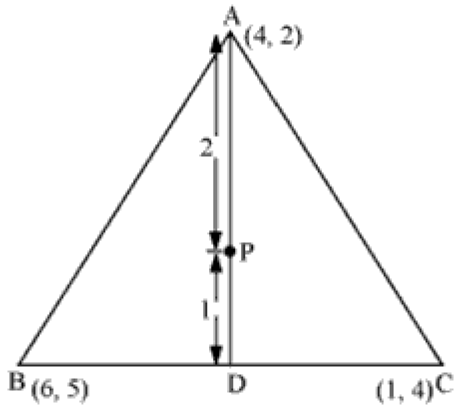
$$= \frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right] = \frac{1}{2}\left[\frac{48-52+19}{16}\right] = \frac{15}{32} \text{ square units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[4(5-2)+1(2-6)+7(6-5)\right]$$

$$= \frac{1}{2}\left[12-4+7\right] = \frac{15}{2} \text{ square units}$$

Clearly the ratio between the areas of ΔADE and of ΔABC is 1:16.

Solution 7



(i) Median AD of the triangle will divide the side BC in two equal parts. So D is the midpoint of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii)Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right)$$

(iii)Median BE of the triangle will divide the side AC in two equal parts. So E is the midpoint of side AC.

Coordinates of E $= \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Point Q divides the side BE in a ratio 2:1.

Coordinates of Q $= \left(\frac{2 \times \frac{5}{2} + 1 \times 6}{2+1}, \frac{2 \times 3 + 1 \times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Median CF of the triangle will divide the side AB in two equal parts. So F is the midpoint of side AB.

Coordinates of F $= \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Point R divides the side CF in a ratio 2:1.

Coordinates of R $= \left(\frac{2 \times 5 + 1 \times 1}{2+1}, \frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv)

Now we may observe that coordinates of point P, Q, R are same.
So, all these are representing same point on the plane i.e. centroid of the triangle.

(v) Now consider a triangle ΔABC having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3).
Median AD of the triangle will divide the side BC in two equal parts. So D is the midpoint of side BC.

Coordinates of D $= \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

Let centroid of this triangle is O.
Point O divides the side AD in a ratio 2:1.

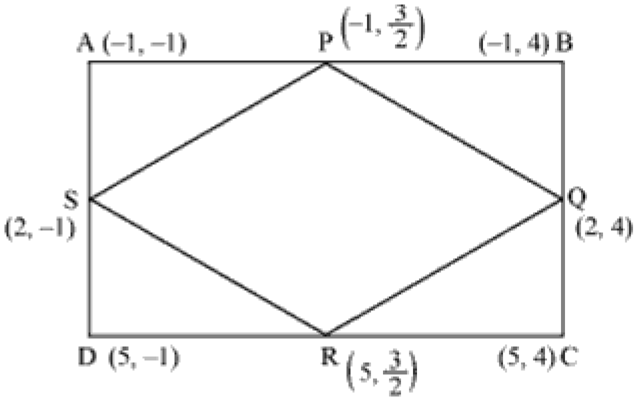
Coordinates of O $= \left(\frac{2 \times \frac{x_2 + x_3}{2} + 1 \times x_1}{2+1}, \frac{2 \times \frac{y_2 + y_3}{2} + 1 \times y_1}{2+1}\right)$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Note:
Coordinates of the point P(x,y) which divides the line segment joining the points points A(x_1,y_1) and B(x_2,y_2) internally in the ratio m_1:m_2 are

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Solution 8



$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

Here all sides of given quadrilateral is of same measure but the diagonals are of different lengths. So, □PQRS is a rhombus.