RD SHARMA Solutions for Class 9 Maths Chapter 1 - Number Systems

Chapter 1 - Number Systems Exercise 1.40

Question 1

Which one of the following is a correct statement?
(a) Decimal Expansion of a Rational number is terminating.
(b) Decimal Expansion of a Rational number is Non-terminating.
(c) Decimal Expansion of a irrational number is terminating.
(d) Decimal Expansion of a irrational number is Non-terminating and Non-Repeating.

Solution 1

Decimal Expansion of a Rational number is not only terminating,

it can be either terminating like $\frac{1}{2} = 0.5$ or non – terminating Repeating like

$\frac{1}{3} = 0.3333333....$ So option (a) is not true alone.

Now we know that Non – Terminating numbers are of two types :
one is Non – Terminating Repeating and other is Non – terminating Non – Repeating.
The Decimal expansion of a Rational number matches one of it's Kind
i.e Non – terminating Repeating of Non – terminating numbers.
So Rational number does not consist both the kinds of Non – terminating numbers
Hence, they are not Non – terminating numbers.
An irrational number is always Non – terminating in nature,but again not of both of it's kin
The Decimal Expansion of an irrational number is Non – terminating Non – Repeating in N
So From all above points and theory we can conclude an Irrational number is
Non – terminating but Non – Repeating in nature
i.e. $\sqrt{2} = 1.4142135623730$ ......
So, option (d) is correct.

Question 2

Which one of the following statement is true?
(a) The sum of two irrational numbers is always an irrational number.
(b) The sum of two irrational numbers is always a rational number.
(c) The sum of two irrational numbers may be a rational number or an irrational number
(d) The sum of two irrational numbers is always an Integer.

Solution 2

If two irrational numbers i.e. $\sqrt{2}, \sqrt{5}, 2+\sqrt{3}, 2-\sqrt{3}$ etc. are added
it is not necessary that sum comes out to be an irrational number always,
or a rational number always...

Since $\sqrt{2} + \sqrt{5}$ = an irrational number

$2+\cancel{\sqrt{3}} + 2-\cancel{\sqrt{3}} = 4 =$ a rational number

So we see that $\sqrt{2}$ and $\sqrt{5}$ are irrational numbers and their sum is also irrational.

But $2+\sqrt{3}$ and $2-\sqrt{3}$ are also irrational numbers, and their sum is a rational number '4'.

So sum of two irrational numbers can be either an irrational number or a rational number depending which numbers are being added.

So options (a) and (b) are totally wrong, beacause they are not 'always ' true.

Option (c) is correct because sum can be either irrational or rational and option (c) is verifying this statement.

Option (d) − again it is not always true, if we add two irrational numbers like $2+\sqrt{3}$ and $2-\sqrt{3}$.

Sum is an integer = 4, but if we add $\sqrt{3}$ and $\sqrt{3}$, sum is $2\sqrt{3}$ which is not an integer but again an irrational number.

So option (d) is also incorrect.

Hence,correct option is (c).

Question 3

Which of the following is a correct statement?

(a) Sum of two irrational numbers is always irrational.

(b) Sum of a rational and irrational number is always irrational.

(c) Square of an irrational number is always a Rational number.

(d) Sum of two Rational numbers can never be an integer.

Solution 3

Option (a) is incorrect, because sum of two irrational numbers is not an irrational number a
It can also be a rational number

i.e. if we add $2+\sqrt{3}$ and $2-\sqrt{3}$, sum comes out to be $2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}} = 4$, which is a ratio

Option (b) is correct

If a rational number is added to an irrational number means to a Non − terminating non − r
the sum will also be non − terminating and Non − repeating number, i.e an irrational numb

Example: a rational number '2' and an irrational no '$\sqrt{3}$' is added, sum $= 2+\sqrt{3}$ which is a
and non − repeating number, hence an irrational number always.

Option (c) is incorrect

Square of an irrational number is not necessarily a rational number. Again it can be either

i.e $\left(\sqrt{2}\right)^2 = 2$ (Rational)

$\left(2+\sqrt{3}\right)^2 = 4+3+2\times 2\sqrt{3} = 7+4\sqrt{3}$ (irrational)

Option (d) is incorrect

Sum of two rational numbers can be an integer and a rational number both.

i.e $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ (Rational number)

$\frac{1}{2}+\frac{1}{2}=1$ (Interger)

Hence, correct option is (b).

Question 4

Which of the following statements is true?

(a) Product of two irrational numbers is always irrational
(b) Product of a rational and an irrational number is always irrational
(c) Sum of two irrational numbers can never be irrational
(d) Sum of an integer and a rational number can never be an integer

Solution 4

Option (a):
Product of two irrational numbers is not always irrational,
it can be also rational sometimes.
When an irrational number is multiplied to itself, or multiplied by another irrational, that product becomes a perfect square.
Example:

$\sqrt{2} \times \sqrt{2} = 2$ (Rational)

$\sqrt{2} \times \sqrt{8} = \sqrt{16} = \pm 4$ (Rational)

So option (a) is incorrect.

Option (b) is correct because when a rational number is multiplied to an irrational number, it can not make an irrational number terminating or Non – terminating Repeating. Product again becomes a Non – terminating Non – Repeating number.

as : $2 \times \sqrt{3} = 2\sqrt{3}$

$$\frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}}$$

So, product of a rational number and an irrational number is always an irrational, because irrational number is just changed in magnitude not in properties.

option (c) is incorrect
Sum of two irrational numbers can be an irrational number.
i.e. if we add $\sqrt{2}$ and $\sqrt{3}$, we will get $\sqrt{2} + \sqrt{3}$ which is alos an irrational .
So option (c) is incorrect.

Option (d) is incorrect
Sum of an integer and a rational number can be a interger.
Because all integers are rational numbers and also we can say some rational numbers are integers. So their sum with integer would be a integer
i.e. $2 + 3 = 5$
Hence, correct option is (b).

Question 5
Which of the following is irrational?

(a) $\sqrt{\frac{4}{9}}$

(b) $\frac{4}{5}$

(c) $\sqrt{7}$

(d) $\sqrt{81}$

Solution 5

Option (a) is incorrect because $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \pm\frac{2}{3}$ (Rational)

Option (b) is also incorrect as $\frac{4}{5}$ is in the form of $\frac{P}{Q}$ $(Q \neq 0)$, (Rational)

Option (c) is correct beacuse $\sqrt{7}$ is a non – terminating and Non – Repeating number.

Option (d) is also incorrect because $\sqrt{81} = \pm 9$ (Rational)

Hence, correct option is (c).

Question 6

Which of the following is irrational?

(a) 0.14

(b) $0.14\overline{16}$

(c) $0.\overline{1416}$

(d) 0.1014001400014..

Solution 6

Correct option (d)

Option (a): $0.14 = \frac{14}{100}$, which is a Rational number

Option (b): $0.14\overline{16}$ is non – terminating but repeating, hence a rational number

Option (c): $0.\overline{1416}$ is Non – terminating but repeating, hence a rational number

Now, option (d): 0.1014001400014.... is non – terminating as well as non – repating numb

which is irrational in nature.

Hence, correct option is (d).

Question 7

Which of the following is rational?

(a) $\sqrt{3}$

(b) π

(c) $\frac{4}{0}$

(d) $\frac{0}{4}$

Solution 7

Option (a): $\sqrt{3} = 1.732.....$ = Non − terminating and non − repeating number, hence irratio

Option (b): $\pi = 3.14.....$ also can not be terminated to $\frac{p}{q}$ form,

and is non − terminating and non − repeating in nature.
Hence, irrational.

Option (c): $\frac{4}{0}$ is not a rational number because this is in the form $\frac{p}{q}$,

where p and q are integers but $q = 0$

Option (d): $\frac{0}{4}$ follows the difinition of rational number.

Hence, correct option is (d).

Question 8
The number 0.318564318564318564......... is:
(a) a natural number
(b) an integer
(c) a rational number
(d) an irrational number

Solution 8
$0.318564318564318564..... = 0.\overline{318564}$ is a Non − terminating Repeating Number.
Hence, it is a rational number.
So, correct option is (c).

Question 9
If n is a natural number, then $\sqrt{n}$ is

(a) always a natural number
(b) always an rational number
(c) always an irrational number
(d) sometimes a natural number and sometimes an irrational number.

Solution 9

Option (a) is incorrect because $\sqrt{n}$ can not be always a natural number

i.e. if $n = 2$, $\sqrt{n} = \sqrt{2}$ (not a natural no.)

Option (b) is incorrect similiarly, if $n = 2, 5, ...$ or any odd no, or not perfect square,

$\sqrt{n} = \sqrt{2}, \sqrt{5}, \sqrt{7}$ are Non − terminating and non − repeating, so irrational in na

So, not always a rational number.

Option (c) is also incorrect. $\sqrt{n}$ can also be rational or say a natural number.

If $n = 4, 9, 16......$ or any perfect square number then $\sqrt{n} = 2, 3, 4 .....$ natural nu

Option (d) is fully correct because if n is any odd number or non − perfect square number

then $\sqrt{n}$ would be irrational, but if n is a perfect square number then $\sqrt{n}$ will be a natural number.

if $n = 2, 3, 5, 8, ... \sqrt{n} = \sqrt{2}, \sqrt{3}, \sqrt{8}$ ....(irrational)

if $n = 4, 9, 16............ = 2, 3, 4$ ....(Natural number)

Hence, correct option is (d).

Chapter 1 - Number Systems Exercise 1.41

Question 1

Which of the following numbers can be represented as non-terminating repeating decimals?

(a) $\frac{39}{24}$

(b) $\frac{3}{16}$

(c) $\frac{3}{11}$

(d) $\frac{137}{25}$

Solution 1

$$Option\ (a):\ \frac{39}{24} = 1.625 = Terminating\ Decimal$$

$$Option\ (b):\ \frac{3}{16} = 0.1875 = Terminating\ Decimal$$

$$Option\ (c):\ \frac{3}{11} = 0.27272727....... = Non-terminating\ decimal$$

$$Option\ (d):\ \frac{137}{25} = 5.48 = Terminating\ Decimal$$

Hence, option (c) is correct.

Question 2

Every point on a number line represents:

(a) a unique real number

(b) a natural number

(c) a rational number

(d) an irrational number

Solution 2

On number line, we have $-\infty$ to ∞ numbers,
consisting $-\infty.....-4, -3, -2, -1, 0, 1, 2, 3, 4......\infty$,
1.12, 1.14 and 1.41406532, 3.146201286295......... etc.
That means on number line, there are natural numbers (1, 2, 3, 4....), integers,
rational numbers $\frac{1}{2}, \frac{1}{3}$, 1.33333, irrational numbers 1.4148625385...
But if we see every number as a complete family, it becomes
Real numbers (any number which can be represent on Real axes)
So, every point on the number line represents a unique real number
which contains every type.
Hence, correct option is (a).

Question 3

Which of the following is irrational?
(a) 0.15
(b) 0.01516
$0.\overline{1516}$
(c)
(d) 0.5015001500015

Solution 3

Option (a): $0.15 = \frac{15}{100}$ = Rational number
Option (b): $0.01516 = \frac{1516}{100000}$ = Rational number
Option (c): $0.\overline{1516}$ is a Non – termating Repeating number = Rational Number
Option (d): 0.5015001500015.. is a Non – terminating, Non – Repeating decimal number,
so is a irrational number.
Hence, Correct option is (d).

Question 4

The number $1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(a) $\frac{14}{9}$

(b) $\frac{14}{11}$

(c) $\frac{14}{13}$

(d) $\frac{14}{15}$

Solution 4

Let $x = 1.\overline{27} = 1.272727....$ (1)

Now, $100x = 127.272727... = 127.\overline{27}$ (2)

Subtracting equation (1) from (2), we get

$99x = 126$

$\Rightarrow x = \frac{126}{99} = \frac{14}{11}$

Hence, option (b) is correct.

Question 5

The number $0.\overline{3}$ in the form $\frac{p}{q}$, where p *and* q are integers and $q \neq 0$, is

(a) $\frac{33}{100}$

(b) $\frac{3}{10}$

(c) $\frac{1}{3}$

(d) $\frac{3}{100}$

Solution 5

Let $x = 0.\overline{3} = 0.33333.......$ (1)

Now, $10x = 3.33333...... = 3.\overline{3}.....$ (2)

Subtracting equation (1) from (2), we get

$9x = 3$

$\Rightarrow x = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow 0.\overline{3} = \frac{1}{3}$

Hence, option (c) is correct.

Question 6

$0.3\overline{2}$ when expressed in form of $\frac{p}{q}$ (p, q are integers, $q \neq 0$), is

(a) $\frac{8}{25}$

(b) $\frac{29}{90}$

(c) $\frac{32}{99}$

(d) $\frac{32}{199}$

Solution 6

Let $x = 0.3\overline{2} = 0.322222.......$ (1)

Now, $10x = 3.22222...... = 3.\overline{2}$ (2),

$100x = 32.2222....... = 32.\overline{2}$ (3)

subtracting equation (2) from (3), we get

$90x = 29$

$\Rightarrow x = \dfrac{29}{90}$

Hence, option (b) is correct.

Question 7

$23.\overline{43}$ when expressed in the form $\dfrac{p}{q}$, (p, q are integers, $q \neq 0$) is

(a) $\dfrac{2320}{99}$

(b) $\dfrac{2343}{100}$

(c) $\dfrac{2343}{999}$

(d) $\dfrac{2320}{199}$

Solution 7

Let $x = 23.\overline{43} = 23.434343........$ (1)

Now, $100x = 2343.43333........$ (2)

Subtracting equation (1) from (2), we get

$99x = 2320$

$\Rightarrow x = \dfrac{2320}{99}$

Hence, option (a) is correct.

Question 8

$0.\overline{001}$ when expressed in the form $\dfrac{p}{q}$ (p, q are integers, $q \neq 0$) is

(a) $\dfrac{1}{1000}$

(b) $\dfrac{1}{100}$

(c) $\dfrac{1}{1999}$

(d) $\dfrac{1}{999}$

Solution 8

Let $x = 0.\overline{001} = 0.001001001.....$ (1)

Now, $1000x = 001.001001001.....$ (2)

Subtracting equation (1) from (2), we get

$999x = 1$

$\Rightarrow x = \frac{1}{999}$

Hence, correct option is (d).

Question 9

The value of $0.\overline{23} + 0.\overline{22}$ is

(a) $0.\overline{45}$

(b) $0.\overline{43}$

(c) $0.4\overline{5}$

(d) 0.45

Solution 9

Let $x = 0.\overline{23} = 0.232323......$ (1)

$y = 0.\overline{22} = 0.222222.....$ (2)

Adding equations (1) and (2), we get

$x + y = 0.454545 = 0.\overline{45}$

$\Rightarrow 0.\overline{23} + 0.\overline{22} = 0.\overline{45}$

Hence, option (a) is correct.

Question 10

An irrational number between 2 and 2.5 is

(a) $\sqrt{11}$

(b) $\sqrt{5}$

(c) $\sqrt{22.5}$

(d) $\sqrt{12.5}$

Solution 10

$\sqrt{4} = 2$ and $\sqrt{6.25} = 2.5$

Option (a), (c) and (d): $\sqrt{11}$, $\sqrt{22.5}$ and $\sqrt{12.5}$, all are greater than $\sqrt{6.25}$

$\Rightarrow$ Out of interval (2, 2.5)

Option (b): $\sqrt{4} < \sqrt{5} < \sqrt{6.25} \Rightarrow$ lies in the interval (2, 2.5)

Hence, option (b) is correct.

Question 11

The number of consecutive zeros in $2^3 \times 3^4 \times 5^4 \times 7$, is

(a) 3
(b) 2
(c) 4
(d) 5

Solution 11

$5 \times 2 = 10 \Rightarrow$ one 5 and one 2 make one zero, so $5 \times 2 \times 5 \times 2 = 100$

Number of pairs of 5 and 2 will be equal to the number of consecutive zeros in the given number.

In the given number, there are three 2's and four 5's.

So number of pairs of 5 and 2 are only three.

So there will be three consecutive zeros in the given number.

So, option (a) is correct.

Chapter 1 - Number Systems Exercise 1.42

Question 1

The smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal is

(a) $\frac{1}{10}$

(b) $\frac{3}{10}$

(c) 3

(d) 30

Solution 1

$\frac{1}{3} = 0.333333.........$(a Non – terminating number)

Now, if we remove 3 from denominator it will terminate.

So, if we multiply by $\frac{3}{10}$

i.e. $\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{10} = \frac{1}{10} = 0.1$ (terminates after one place of decimal)

By multiplying by $\frac{1}{10}$, 3 does not replaces.

By multiplying by 3, we get 1, which is not terminating after one place of decimal

And, by multiplying by 30, we get 10, again not terminating after one place of decimal.

So, correct answer is (b).

Chapter 1 - Number Systems Exercise Ex. 1.1

Question 1

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0?

Solution 1

Yes zero is a rational number as it can be represented in the $\frac{p}{q}$ form, where p and q are integers and q $\neq$ 0 as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Concept Insight: Key idea to answer this question is "every integer is a rational number and zero is a non negative integer".

Also 0 can be expressed in $\frac{p}{q}$ form in various ways as 0 divided by any number is 0. simplest is $\frac{0}{1}$.

Question 2
Find five rational numbers between 1 and 2.

Solution 2
Recall that to find a rational number between r and s, you can add r and s and divide the sum by that is $\frac{r+s}{2}$ lies between r and s. So, $\frac{3}{2}$ is a number between 1 and 2. you can proceed in this manner to find four more rational numbers between 1 and 2. These four numbers are $\frac{5}{4}$, $\frac{11}{8}$, $\frac{13}{8}$ and $\frac{7}{4}$

Question 3
Find six rational numbers between 3 and 4.

Solution 3
There are infinite rational numbers in between 3 and 4.

3 and 4 can be represented as $\frac{21}{7}$ and $\frac{28}{7}$ respectively.

Now rational numbers between 3 and 4 are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$$

Concept Insight: Since there are infinite number of rational numbers between any two numbers so the answer is not unique here. The trick is to convert the number to equivalent $\frac{p}{q}$ form by multiplying and dividing by the number atleast 1 more than the rational numbers to be inserted.

Question 4
Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Solution 4
There are infinite rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

$$\frac{3}{5} = \frac{3\times10}{5\times10} = \frac{30}{50}$$

$$\frac{4}{5} = \frac{4\times10}{5\times10} = \frac{40}{50}$$

Now rational numbers between are $\frac{3}{5}$ and $\frac{4}{5}$

$\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$

Concept Insight: Since there are infinite number of rational numbers between any two numbers so the answer is not unique here. The trick is to convert the number to equivalent $\frac{p}{q}$ form by multiplying and dividing by the number at least 1 more than the rational numbers required.

Alternatively for any two rational numbers a and b, $\frac{a+b}{2}$ is also a rational number which lies between a and b.

Question 5

Are the following statements true or false? Give reasons for you answer.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

(iv) Every natural number is a whole number.

(v) Every integer is whole number.

(vi) Every rational number is whole number.

Solution 5

(i) False

(ii) True

(iii) False

(iv)True

(v) False

(vi) False

Chapter 1 - Number Systems Exercise Ex. 1.2

Question 1

Express the given rational numbers as decimal:

(i) $\frac{42}{100}$

(ii) $\frac{327}{500}$

(iii) $\frac{15}{4}$

Solution 1

(i)

We have,

$$\frac{42}{100} = 0.42$$

$\therefore \frac{42}{100} = 0.42$

(ii)

By long division , we have

$$\begin{array}{r|l} & 0.654 \\ 500 & 3270 \\ & \underline{-3000} \\ & 2700 \\ & \underline{-2500} \\ & 2000 \\ & \underline{-2000} \\ & 0 \end{array}$$

$\therefore \frac{327}{500} = 0.654$

(iii)

By long division , we have

$$\begin{array}{r|l} & 3.75 \\ 4 & 15 \\ & \underline{-12} \\ & 30 \\ & \underline{-28} \\ & 20 \\ & \underline{-20} \\ & 0 \end{array}$$

$\therefore \frac{15}{4} = 3.75$

Question 2

Express the follwoing rational numbers as decimals:

(i) $\frac{2}{3}$

(ii) $-\frac{4}{9}$

(iii) $\frac{-2}{15}$

(iv) $-\frac{22}{13}$

(v) $\frac{437}{999}$

(vi) $\frac{33}{26}$

Solution 2

(i)

By long division , we have

$$\begin{array}{r} 0.66\ldots \\ 3\overline{)20} \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

$\therefore \quad \frac{2}{3} = 0.\overline{6}$

(ii)

By long division , we have

$$\begin{array}{r} 0.44\ldots \\ 9\overline{)40} \\ \underline{-36} \\ 40 \\ \underline{-36} \\ 4 \end{array}$$

$\therefore \quad \frac{4}{9} = 0.\overline{4}$

$\therefore \quad -\frac{4}{9} = -0.\overline{4}$

(iii)

By long division , we have

$$
\begin{array}{r|l}
 & 0.133\ldots \\ \hline
15 & 20 \\
 & -15 \\ \hline
 & \;\;50 \\
 & -45 \\ \hline
 & \;\;\;50 \\
 & \;-45 \\ \hline
 & \;\;\;\;5
\end{array}
$$

$\therefore \quad \dfrac{2}{15} = 0.1\bar{3}$

Hence, $-\dfrac{2}{15} = -0.1\bar{3}$

(iv)

By long division , we have

$$
\begin{array}{r|l}
 & 1.69230769\ldots \\ \hline
13 & 22 \\
 & -13 \\ \hline
 & \;\;90 \\
 & -78 \\ \hline
 & \;120 \\
 & -117 \\ \hline
 & \;\;\;\;30 \\
 & \;\;-26 \\ \hline
 & \;\;\;\;\;40 \\
 & \;\;\;-39 \\ \hline
 & \;\;\;\;\;100 \\
 & \;\;\;\;\;-91 \\ \hline
 & \;\;\;\;\;\;\;90 \\
 & \;\;\;\;\;\;-78 \\ \hline
 & \;\;\;\;\;\;120 \\
 & \;\;\;\;\;\;-117 \\ \hline
 & \;\;\;\;\;\;\;\;\;3
\end{array}
$$

We observe that the remainders start repeating after 6 divisions.

$\therefore \quad \dfrac{22}{13} = 1.\overline{692307}$

Hence, $-\dfrac{22}{13} = -1.\overline{692307}$

(v)

By long division , we have

$$\begin{array}{r} 0.437\ldots \\ 999\overline{)4370} \\ -3996 \\ \hline 3740 \\ -2997 \\ \hline 7430 \\ -6993 \\ \hline 437 \end{array}$$

$\therefore \quad \frac{437}{999} = 0.437437\ldots = 0.\overline{437}$

Hence, $\frac{437}{999} = 0.\overline{437}$

(vi)

By long division , we have

$$
\begin{array}{r}
1.26923076\ldots \\
26\overline{)33} \\
-26 \\
\hline
70 \\
-52 \\
\hline
180 \\
-156 \\
\hline
240 \\
-234 \\
\hline
60 \\
-52 \\
\hline
80 \\
-78 \\
\hline
200 \\
-182 \\
\hline
180 \\
-156 \\
\hline
24
\end{array}
$$

$\therefore \quad \frac{33}{26} = 1.2692307692307\ldots$

$\Rightarrow \quad \frac{33}{26} = 1.2\overline{692307}$

Hence, $\frac{33}{26} = 1.2\overline{692307}$

Question 3

Look at several examples of rational numbers in the form $\frac{p}{q}\left(q \neq 0\right)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

Solution 3

A rational number $\frac{p}{q}$ is a terminating decimal only, when prime factors of q are 2 and 5 only. Therefore, $\frac{p}{q}$ is a terminating decimal only, when prime factorization of q must have only powers of 2 or 5 or both.

Chapter 1 - Number Systems Exercise Ex. 1.3

Question 1

Express the given decimals in the form $\frac{p}{q}$:

(i) 0.39

(ii) 0.750

(iii) 2.15

(iv) 7.010

(v) 9.90

(vi) 1.0001

Solution 1

(i)
We have,

$$0.39 = \frac{39}{100}$$

Hence, $0.39 = \frac{39}{100}$

(ii)
We have,

$$0.750 = \frac{750}{1000} = \frac{3}{4}$$

$$\Rightarrow \qquad 0.750 = \frac{3}{4}$$

Hence, $0.750 = \frac{3}{4}$

(iii)
We have,

$$2.15 = \frac{215}{100}$$

$$= \frac{43}{20}$$

$$\Rightarrow \qquad 2.15 = \frac{43}{20}$$

(iv)
We have,

$$7.010 = \frac{7010}{1000}$$

$$= \frac{701}{100}$$

Hence, $7.010 = \frac{701}{100}$

(v)
We have,

$$9.90 = \frac{990}{100}$$

$$= \frac{99}{10}$$

Hence, $9.90 = \frac{99}{10}$

(vi)
We have,

$$1.0001 = \frac{10001}{10000}$$

Hence, $1.0001 = \frac{10001}{10000}$

Question 2

Express each of the followimg decimals in the form $\frac{p}{q}$

(i) $0.\overline{4}$
(ii) $0.\overline{37}$
(iii)$0.\overline{54}$
(iv) $0.\overline{621}$
(v) $125.\overline{3}$
(vi)$4.\overline{7}$
(vii)0.47

Solution 2

(i)

Let $x = 0.\overline{4}$

Now, $x = 0.\overline{4} = 0.444...$ $\quad ---(1)$

Multiplying both sides of equation (1) by 10, we get,

$10x = 4.444...$ $\quad ---(2)$

Subtracting equation (1) by (2)

$\therefore \quad 10x - x = 4.444... - 0.444...$

$\Rightarrow \quad 9x = 4$

$\Rightarrow \quad x = \frac{4}{9}$

Hence, $0.\overline{4} = \frac{4}{9}$

(ii)

Let $x = 0.\overline{37}$

Now, $x = 0.3737...$ $\quad ---(1)$

Multiplying equation (1) by 10.

$\therefore \quad 10x = 3.737...$ $\quad ---(2)$

Multiplying equation (2) by 10.

$100x = 37.3737...$ $\quad ---(3)$

Subtracting equation (1) by equation (3)

$\therefore \quad 100x - x = 37$

$\Rightarrow \quad 99x = 37$

$\Rightarrow \quad x = \frac{37}{99}$

Hence, $0.\overline{37} = \frac{37}{99}$

(iii)

Let $x = 0.\overline{54}$

$\Rightarrow \qquad x = 0.5454... \qquad ---(1)$

Multiplying equation (1) by 100,
we get,

$100x = 54.5454... \qquad ---(2)$

Subtracting equation (1) by equation (2)

$\therefore \qquad 100x - x = 54$

$\Rightarrow \qquad 99x = 54$

$\Rightarrow \qquad x = \frac{54}{99} = \frac{6}{11}$

Hence, $0.\overline{54} = \frac{6}{11}$

(iv)

Let $x = 0.\overline{621}$

Now, $x = 0.621621... \qquad ---(1)$

Multiplying equation (1) by 1000,

$\therefore \qquad 1000x = 621.621621... \qquad ---(2)$

Subtracting equation (1) by equation (2)

$\therefore \qquad 1000x - x = 621$

$\Rightarrow \qquad 999x = 621$

$\Rightarrow \qquad x = \frac{621}{999} = \frac{69}{111} = \frac{23}{37}$

Hence, $0.\overline{621} = \frac{23}{37}$

(v)

Let $x = 125.\overline{3}$

$\Rightarrow \quad x = 125.33... \qquad ---(1)$

Multiplying equation (1) by 10,

$\therefore \quad 10x = 1253.33... \qquad ---(2)$

Subtracting equation (1) by equation (2)

$\therefore \quad 10x - x = 1253.33... - 125.33...$

$\Rightarrow \quad 9x = 1128$

$\Rightarrow \quad x = \frac{1128}{9} = \frac{376}{3}$

Hence, $x = \frac{376}{3}$

(vi)

Let $x = 4.\overline{7}$

$\Rightarrow \quad x = 4.77... \qquad ---(1)$

Multiplying equation (1) by 10,

$\therefore \quad 10x = 47.77... \qquad ---(2)$

Subtracting equation (1) by equation (2)

$\therefore \quad 10x - x = 47.77... - 4.77...$

$\Rightarrow \quad 9x = 43$

$\Rightarrow \quad x = \frac{43}{9}$

Hence, $4.\overline{7} = \frac{43}{9}$

(vii)

$0.4\overline{7} = 0.4777...$
Let $x = 0.4777 ...$(i)
$10x = 4.777...$
$100x = 47.777 ...$(ii)
(ii) – (i) gives
$99x = 43$
$x = \frac{43}{99}$

Chapter 1 - Number Systems Exercise Ex. 1.4

Question 1

Define an irrational number.

Solution 1

A number which can neither be expressed as a terminating decimal nor as a repeating decimal, is called an irrational number. For example,

1.01001000100001...

Question 2

Explain, how irrational numbers differ from rational numbers?

Solution 2

A number which can neither be expressed as a terminating decimal nor as a repeating decimal, is called an irrational number. For example,

0.33033003300033...

On the other hand, every rational number is expresible either as a terminating decimal or as a repeating decimal. For examples, $3.2\overline{4}$ and 6.2876 are rational numbers

Question 3

Examine whether $\sqrt{7}$ is rational or irrational.

Solution 3

$\sqrt{7}$ is not a perfect square root, so it is an irrational number.

Question 4

Examine whether $\sqrt{4}$ is rational or irrational.

Solution 4

We have,

$\sqrt{4} = 2 = \frac{2}{1}$

$\therefore \sqrt{4}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

The decimal representation of $\sqrt{4}$ is 2.0.

Question 5

Examine whether $2+\sqrt{3}$ is rational or irrational.

Solution 5

2 is a rational number, whereas $\sqrt{3}$ is an irrational number.

Because, sum of a rational number and an irrational number is an irrational number, so $2+\sqrt{3}$ is an irrational number.

Question 6

Examine whether $\sqrt{3}+\sqrt{2}$ is rational or irrational.

Solution 6

$\sqrt{2}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{2}$ is an irrational number.

Question 7

Examine whether $\sqrt{3}+\sqrt{5}$ is rational or irrational.

Solution 7

$\sqrt{5}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{5}$ is an irrational number.

Question 8

Examine whether $\left(\sqrt{2}+2\right)^2$ is rational or irrational.

Solution 8

We have,

$$\left(\sqrt{2}-2\right)^2 = \left(\sqrt{2}\right)^2 - 2\times\sqrt{2}\times 2 + (2)^2$$
$$= 2 - 4\sqrt{2} + 4$$
$$= 6 - 4\sqrt{2}$$

Now, 6 is a rational number, whereas $4\sqrt{2}$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number.

So, $6 - 4\sqrt{2}$ is an irrational number.

$\therefore$ $\left(\sqrt{2}-2\right)^2$ is an irrational number.

Question 9

Examine whether $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ is rational or irrational.

Solution 9

We have,

$$\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right) = (2)^2 - \left(\sqrt{2}\right)^2 \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= 4 - 2$$

$$= 2 = \frac{2}{1}$$

Since, 2 is a rational number.

$\therefore \quad \left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ is a rational number.

Question 10

Examine whether $\left(\sqrt{2}+\sqrt{3}\right)^2$ is rational or irrational.

Solution 10

We have,

$$\left(\sqrt{2}+\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + 2\times\sqrt{2}\times\sqrt{3} + \left(\sqrt{3}\right)^2$$

$$= 2 + 2\sqrt{6} + 3$$

$$= 5 + 2\sqrt{6}$$

The sum of a rational number and an irrational number is an irrational number, so $5+2\sqrt{6}$ is an irrational number.

$\therefore \quad \left(\sqrt{2}+\sqrt{3}\right)^2$ is an irrational number.

Question 11

Examine whether $\sqrt{5}-2$ is rational or irrational.

Solution 11

The difference of a rational number and an irrational number is an irrational number.

$\therefore \quad \sqrt{5}-2$ is an irrational number.

Question 12

Examine whether $\sqrt{23}$ is rational or irrational.

Solution 12

$\sqrt{23} = 4.79583152331\ ......$

As decimal expansion of this number is non-terminating non recurring. So it is an irrational number.

Question 13

Examine whether $\sqrt{225}$ is rational or irrational.

Solution 13

$\sqrt{225} = 15 = \frac{15}{1}$

Rational number as it can be represented in $\frac{p}{q}$ form.

Question 14

Examine whether 0.3796 is rational or irrational.

Solution 14

0.3796

As decimal expansion of this number is terminating, so it is a rational number.

Question 15

Examine whether 7.478478... is rational or irrational.

Solution 15

$7.478478 \ldots\ldots\ldots.. = 7.\overline{478}$

As decimal expansion of this number is non terminating recurring so it is a rational number.

Question 16

Examine whether 1.101001000100001... is rational or irrational.

Solution 16

1.10100100010000 …….

Question 17

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

$\sqrt{4}$

Solution 17

We have,

$\sqrt{4} = 2 = \frac{2}{1}$

$\sqrt{4}$ can be written in the form of $\frac{p}{q}$, so it is a rational number.

Its decimal representation is 2.0.

Question 18

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

$3\sqrt{18}$

Solution 18

We have,

$$3\sqrt{18} = 3\sqrt{2 \times 3 \times 3}$$
$$= 3 \times 3\sqrt{2}$$
$$= 9\sqrt{2}$$

Since, the product of a rational and an irrational is an irrational number.

$\therefore$ $9\sqrt{2}$ is an irrational

$\Rightarrow$ $3\sqrt{18}$ is an irrational number.

Question 19

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

$\sqrt{1.44}$

Solution 19

We have,

$$\sqrt{1.44} = \sqrt{\frac{144}{100}}$$
$$= \frac{12}{10}$$
$$= 1.2$$

Every terminating decimal is a rational number, so 1.2 is a rational number.

Its decimal representation is 1.2.

Question 20

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

$\sqrt{\frac{9}{27}}$

Solution 20

We have,

$$\sqrt{\frac{9}{27}} = \frac{3}{\sqrt{27}} = \frac{3}{\sqrt{3\times3\times3}}$$

$$= \frac{3}{3\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

Quotient of a rational and an irrational number is irrational number, so $\frac{1}{\sqrt{3}}$ is an irrational number.

$\Rightarrow \quad \sqrt{\frac{9}{27}}$ is an irrational number.

Question 21

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

$-\sqrt{64}$

Solution 21

We have,

$$-\sqrt{64} = -\sqrt{8\times8}$$

$$= -8$$

$$= -\frac{8}{1}$$

$-\sqrt{64}$ can be expressed in the form of $\frac{p}{q}$, so $-\sqrt{64}$ is a rational number.

Its decimal representation is – 8.0.

Question 22

Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

$\sqrt{100}$

Solution 22

We have,

$\sqrt{100} = 10$

$= \frac{10}{1}$

$\sqrt{100}$ can be expressed in the form of $\frac{p}{q}$. so $\sqrt{100}$ is a rational number.

The decimal representation of $\sqrt{100}$ is 10.0.

Question 23

In the following equations, find which variables x, y, z etc. represent rational or irrational numbers:

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = 0.04$ (iv) $u^2 = \frac{17}{4}$

(v) $v^2 = 3$ (vi) $w^2 = 27$ (vii) $t^2 = 0.4$

Solution 23

(i)
We have,
$x^2 = 5$

Taking square root on both sides.

$\Rightarrow \quad \sqrt{x^2} = \sqrt{5}$

$\Rightarrow \quad x = \sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii)
We have,
$y^2 = 9$

$\Rightarrow \quad y = \sqrt{9}$

$= 3$

$= \frac{3}{1}$

$\sqrt{9}$ can be expressed in the form of $\frac{p}{q}$, so it a rational number.

(iii)
We have,
$z^2 = 0.04$

Taking square root on both sides, we get,

$\sqrt{z^2} = \sqrt{0.04}$

$\Rightarrow \quad z = \sqrt{0.04}$

$= 0.2$

$= \frac{2}{10}$

$= \frac{1}{5}$

z can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iv)
We have,
$u^2 = \frac{17}{4}$

Taking square root on both sides, we get,
$\sqrt{u^2} = \sqrt{\frac{17}{4}}$

$\Rightarrow \quad u = \sqrt{\frac{17}{2}}$

Quotient of an irrational and a rational number is irrational, so u is an irrational number.

(v)
We have,
$v^2 = 3$

Taking square root on both sides, we get,
$\sqrt{v^2} = \sqrt{3}$

$\Rightarrow \quad v = \sqrt{3}$

$\sqrt{3}$ is not a perfect square root, so v is an irrational number.

(vi)
We have,

$w^2 = 27$

Taking square root on both sides, we get,

$\sqrt{w^2} = \sqrt{27}$

$\Rightarrow \quad w = \sqrt{3 \times 3 \times 3}$

$= 3\sqrt{3}$

Product of a rational and an irrational is irrational number, so w is an irrational number.

(vii)
We have,
$t^2 = 0.4$

Taking square root on both sides, we get,
$\sqrt{t^2} = \sqrt{0.4}$

$\Rightarrow \quad t = \sqrt{\frac{4}{10}}$

$= \frac{2}{\sqrt{10}}$

Since, quotient of a rational and an irrational number is irrational number, so t is an irrational number.

Question 24
Give two rational numbers lying between 0.232332333233332... and 0.212112111211112.

Solution 24

Let, a = 0.212112111211112

And, b = 0.232332333233332...

Clearly, $a < b$ because in the second decimal place a has digit 1 and b has digit 3

If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b.

Let,

x = 0.22

y = 0.22112211...

Then,

$a < x < y < b$

Hence, x, and y are required rational numbers.

Question 25

Give two rational numbers lying between 0.515115111511115... and 0.5353353335...

Solution 25

Let, a = 0.515115111511115...

And, b = 0.5353353335...

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore, $a < b$. So if we consider rational numbers

x = 0.52

y = 0.52052052...

We find that,

$a < x < y < b$

Hence, x, and y are required rational numbers.

Question 26

Find one irrational number between 0.2101 and 0.2222... = $0.\bar{2}$.

Solution 26

Let, $a = 0.2101$
And, $b = 0.2222...$

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore $a < b$. in the third decimal place a has digit 0. so, if we consider irrational number

$x = 0.211011001100011...$

We find that,
$a < x < b$

Hence, x is required irrational number.

Question 27
Find a rational numbar and also an irrational number lying between the numbers 0.3030030003... and 0.3010010001...

Solution 27
Let, $a = 0.3010010001$
And, $b = 0.3030030003...$

We observe that in the third decimal place a has digit 1 and b has digit 3, therefore $a < b$. in the third decimal place a has digit 1. so, if we consider rational and irrational numbers

$x = 0.302$
$y = 0.302002000200002...$

We find that,
$a < x < b$
And, $a < y < b$

Hence, x and y are required rational and irrational numbers respectively.

Question 28
Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solution 28
$$\frac{5}{7} = 0.\overline{714285}$$
$$\frac{9}{11} = 0.\overline{81}$$

3 irrational numbers are -
0.73073007300073000073 ... ...
0.75075007500075000075 ... ...
0.79079007900079000079 ... ...

Concept Insight: There is infinite number of rational and irrational numbers between any two rational numbers. Convert the number into its decimal form to find irrationals between them.

Alternatively following result can be used to answer
Irrational number between two numbers x and y

$$= \begin{cases} \sqrt{xy}, \text{ if } x \text{ and } y \text{ both are irrational numbers} \\ \sqrt{xy}, \text{ if } x \text{ is rational number and } y \text{ is irrational number} \\ \sqrt{xy}, \text{ if } x \times y \text{ is not a perfect square and } x, y \text{ both are rational numbers} \end{cases}$$

Question 29

Give an example each, of two irrational numbers whose:

(i) Difference is a rational number.
(ii) Difference is an irrational number.
(iii) Sum is a rational number.
(iv) Sum is an irrational number.
(v) Product is a rational number.
(vi) Product is an irrational number.
(vii) Quotient is a rational number.
(viii) Quotient is an irrational number.

Solution 29

(i)

$\sqrt{3}$ is an irrational number.

Now, $\left(\sqrt{3}\right) - \left(\sqrt{3}\right) = 0$

0 is the rational number.

(ii)

Let two irrational numbers are $5\sqrt{2}$ and $\sqrt{2}$

Now, $\left(5\sqrt{2}\right) - \left(\sqrt{2}\right) = 4\sqrt{2}$

$4\sqrt{2}$ is an irrational number.

(iii)

Let two irrational numbers are $\sqrt{11}$ and $-\sqrt{11}$

Now, $\left(\sqrt{11}\right) + \left(-\sqrt{11}\right) = 0$

0 is a rational number.

(iv)

Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

Now, $\left(4\sqrt{6}\right) + \left(\sqrt{6}\right) = 5\sqrt{6}$

$5\sqrt{6}$ is an irrational number.

(v)

Let two irrational numbers are $2\sqrt{3}$ and $\sqrt{3}$

Now, $2\sqrt{3} \times \sqrt{3} = 2 \times 3$

$= 6$

6 is a rational number.

(vi)

Let two irrational numbers are $\sqrt{2}$ and $\sqrt{5}$

Now, $\sqrt{2} \times \sqrt{5} = \sqrt{10}$

$\sqrt{10}$ is an irrational number.

(vii)

Let two irrational numbers are $3\sqrt{6}$ and $\sqrt{6}$

Now, $\frac{3\sqrt{6}}{\sqrt{6}} = 3$

3 is a rational number.

(viii)

Let two irrational numbers are $\sqrt{6}$ and $\sqrt{2}$

Now, $\frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{3 \times 2}}{\sqrt{2}}$

$= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2}}$

$= \sqrt{3}$

$\sqrt{3}$ is an irrational number.

Question 30

Find two irrational numbers between 0.5 and 0.55.

Solution 30

Let, $a = 0.5 = 0.50$
And, $b = 0.55$

We observe that in the second decimal place a has digit 0 and b has digit 5, therefore $a < b$. so, if we consider irrational numbers

$x = 0.51051005100051...$
$y = 0.530535305353530...$

We find that,
$a < x < y < b$

Hence, x and y are required irrational numbers.

Question 31
Find two irrational numbers lying between 0.1 and 0.12.

Solution 31
Let, $a = 0.1 = 0.10$
And, $b = 0.12$

We observe that in the second decimal place a has digit 0 and b has digit 2, therefore $a < b$. so, if we consider irrational numbers

$x = 0.11011001100011...$
$y = 0.111011110111110...$

We find that,
$a < x < y < b$

Hence, x and y are required irrational numbers.

Question 32
Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Solution 32

If possible, let $\sqrt{3}+\sqrt{5}$ be a rational number equal to x. Then,

$x = \sqrt{3}+\sqrt{5}$

$$\Rightarrow \quad x^2 = \left(\sqrt{3}+\sqrt{5}\right)^2$$

$$\Rightarrow \quad x^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{3}\times\sqrt{5}$$

$$= 3+5+2\sqrt{15}$$

$$= 8+2\sqrt{15}$$

$$\Rightarrow \quad x^2 - 8 = 2\sqrt{15}$$

$$\Rightarrow \quad \frac{x^2-8}{2} = \sqrt{15}$$

Now, x is rational

$\Rightarrow \quad x^2$ is rational

$\Rightarrow \quad \dfrac{x^2-8}{2}$ is rational

$\Rightarrow \quad \sqrt{15}$ is rational

But, $\sqrt{15}$ is irrational.

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3}+\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{3}+\sqrt{5}$ is an irrational number.

Chapter 1 - Number Systems Exercise Ex. 1.5

Question 1

Complete the following sentences:

(i) Every point on the number line corresponds to a ___ number which may be either ____ or_____.

(ii) The decimal form of an irrational number is neither ______ nor ______.

(iii) The decimal representation of a rational number is either ____ or _____.

(iv) Every real number is either ______ number or ______ number.

Solution 1

(i) Real, rational, irrartional.

(ii) terminating, repeating.

(iii) terminating, non-terminating and reccuring.

(iv) rational, an irrational.

Question 2

Find whether the following sentences are true or false:

(i) Every real number is either rational or irrational.

(ii) π is an irrational number.

(iii) Irrational numbers cannot be represented by points on the number line.

Solution 2

(i) True

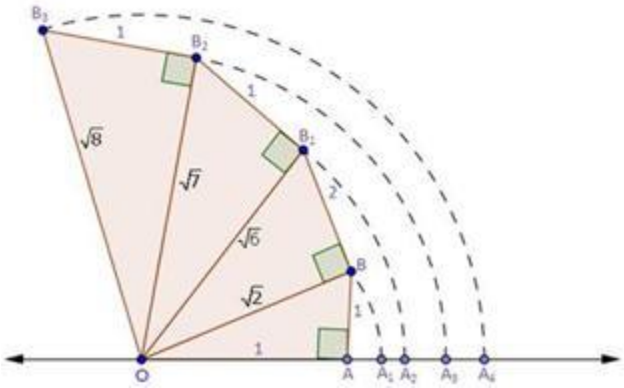
(ii) True

(iii) False

Question 3

Represent $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$ on the number line.

Solution 3



Draw a number line and mark a point O, representing zero, on it. Suppose point A represents 1 as shown. Then, $OA = 1$.

Draw a right triangle OAB such that $AB = OA = 1$.

By pythagoras theorem, we have

$$(OB)^2 = (OA)^2 + (AB)^2$$
$$\Rightarrow \quad OB^2 = 1^2 + 1^2$$
$$\Rightarrow \quad OB^2 = 1 + 1 = 2$$

$$\Rightarrow \quad OB = \sqrt{2}$$

Now, draw a circle with centre O and radius OB. We find that the circle cuts the number line at A.
Clearly, $OA_1 = OB =$ Radius of the circle $= \sqrt{2}$

Thus, A_1 represents $\sqrt{2}$ on the number line.

Now, draw a right triangle OBB_1 such that $BB_1 = 2$.

Again by pythagoras theorem, we have,

$$OB_1^2 = OB^2 + BB_1^2$$

$$\Rightarrow \quad OB_1^2 = \left(\sqrt{2}\right)^2 + (2)^2$$
$$\Rightarrow \quad OB_1^2 = 2 + 4 = 6$$
$$\Rightarrow \quad OB_1^2 = 6$$

$$\Rightarrow \quad OB_1 = \sqrt{6}$$

Now, draw a circle with centre O and radius OB_1. We find that the circle cuts the number line at A_2.

Clearly, $OA_2 = OB_1$ = Radius of circle = $\sqrt{6}$

Thus, A_2 represents $\sqrt{6}$ on the number line.

Now, draw a right angle triangle OB_1B_2 such that $B_1B_2 = 1$.

By pythagoras theorem, we have,

$$OB_2^2 = OB_1^2 + B_1B_2^2$$

$$\Rightarrow \quad OB_2^2 = \left(\sqrt{6}\right)^2 + (1)^2$$

$$\Rightarrow \quad OB_2^2 = 6 + 1 = 7$$

$$\Rightarrow \quad OB_2 = \sqrt{7}$$

Now, draw a circle with centre O and radius OB_2. We find that the circle cuts the number line at A_3.

Clearly, $OA_3 = OB_2$ = Radius of circle = $\sqrt{7}$

Thus, A_3 represents $\sqrt{7}$ on the number line.

Now, again draw a right triangle OB_2B_3 such that $B_2B_3 = 1$.

By pythagoras theorem, we have,

$$OB_3^2 = OB_2^2 + B_2B_3^2$$

$$\Rightarrow \quad OB_3^2 = \left(\sqrt{7}\right)^2 + (1)^2$$

$$\Rightarrow \quad OB_3^2 = 7 + 1 = 8$$

$$\Rightarrow \quad OB_3 = \sqrt{8}$$

Now, draw a circle with centre O and radius OB_3. We find that the circle cuts the number line at A_4.

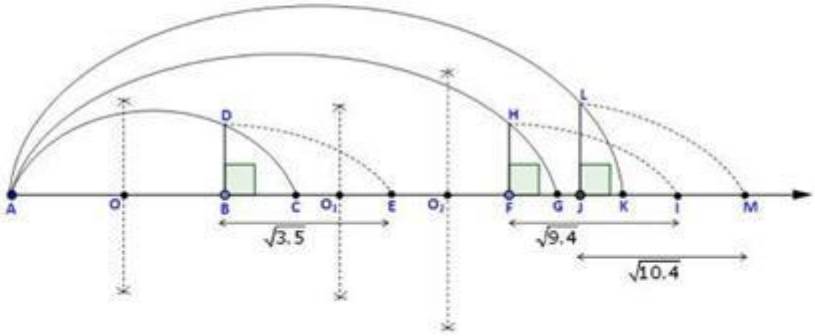
Clearly, $OA_4 = OB_3$ = Radius of circle = $\sqrt{8}$

Thus, A_4 represents $\sqrt{8}$ on the number line.

Question 4

Represent $\sqrt{3.5}$, $\sqrt{9.4}$, $\sqrt{10.5}$ on the real number line.

Solution 4



In order to represent $\sqrt{3.5}$ on number line, we follow the following steps:

(1) Draw a line and mark a point A on it.

(2) Mark a point B on the line drawn in step 1 such that $AB = 3.5$ cm.

(3) Mark a point C on AB produced such that $BC = 1$ unit.

(4)Find mid-point of AC. Let the mid-point be O.

(5) Taking O as the centre and $OC = OA$ as radius draw a semi-circle; also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D.

(6)Taking B as the centre and BD as radius draw an arc cutting OC produced at E. Point E so obtained represent $\sqrt{3.5}$.

In order to represent $\sqrt{9.4}$ on number line, we follow the following steps:

(1) Mark a point F on the line drawn such that $AF = 9.4$ cm.

(2) Mark a point G on AF produced such that $FG = 1$ unit.

(3)Find mid-point of AG. Let the mid-point be O_1.

(4) Taking O_1 as the centre and $O_1A = O_1G$ as radius draw a semi-circle. Also, draw a line passing through F perpendicular to O_1F. Suppose it cuts the semi-circle at H.

(5)Taking F as the centre and FH as radius draw an arc cutting O_1G produced at I. Point I so obtained represents $\sqrt{9.4}$.

In order to represent $\sqrt{10.5}$ on number line, we follow the following steps:

(1) Mark a point J on the line such that $AJ = 10.5$ cm.

(2) Mark a point K on AJ produced such that $JK = 1$ unit.

(3)Find mid-point of AK. Let the mid-point be O_2.

(4) Taking O_2 as the centre and $O_2A = O_2K$ as radius draw a semi-circle. Also, draw a line passing through J perpendicular to O_2J. Suppose it cuts the semi-circle at L.

(5)Taking J as the centre and JL as radius draw an arc cutting O_2K produced at M.
Point M so obtained represents $\sqrt{10.5}$.

Chapter 1 - Number Systems Exercise Ex. 1.6

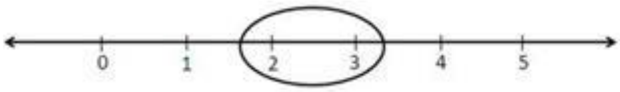
Question 1

Visualise 2.665 on the number line, Using successive magnification.

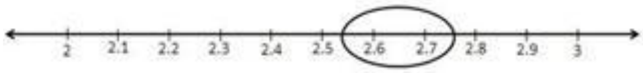
Solution 1

The following steps for successive magnification to visualise 2.665 are:

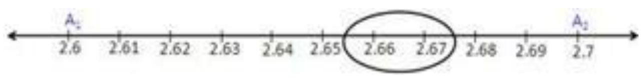
(1) We observe that 2.665 is located somewhere between 2 and 3 on the number line.
So, let us look at the portion of the number line between 2 and 3.



(2) We divide this portion into 10 equal parts and mark each point of division.
The first mark to the right of 2 will represent 2.1, the next 2.2 and soon. Again we observe that 2.665 lies between 2.6 and 2.7.

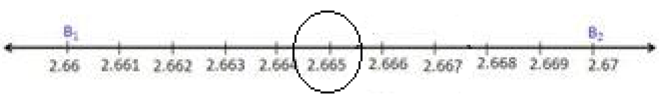


(3) We mark these points A_1 and A_2 respectively. The first mark on the right side of A_1, will represent 2.61, the number 2.62, and soon. We observe 2.665 lies between 2.66 and 2.67.



(4) Let us mark 2.66 as B_1 and 2.67 as B_2. Again divide the B_1B_2 into ten equal parts. The first mark on the right side of B_1 will represent 2.661, then next 2.662, and so on.

Clearly, fifth point will represent 2.665.

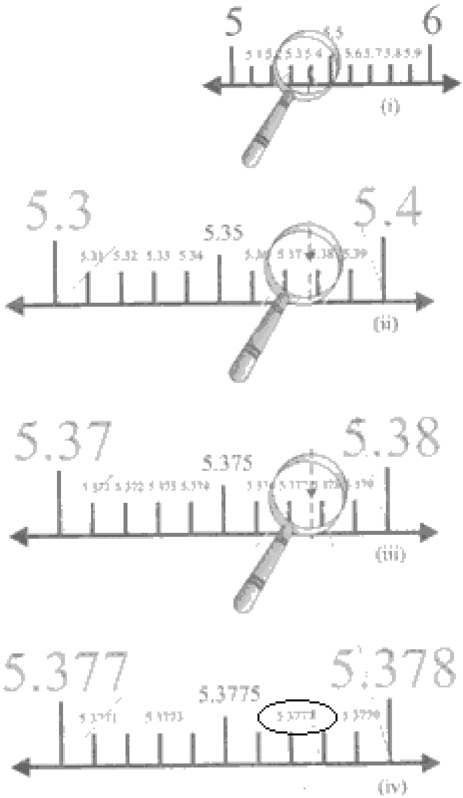


Question 2

Visualize the representation of $5.3\overline{7}$ on the number line upto 5 decimal places, that is, up to 5.37777.

Solution 2

Once again we proceed by successive magnification, and successively decrease the lengths of the portions of the number line in which $5.3\overline{7}$ is located. First, we see that $5.3\overline{7}$ is located between 5 and 6. In the next step, we locate $5.3\overline{7}$ between 5.3 and 5.4. To get a more accurate visualization of the representation, we divide this portion of the number line into 10 equal parts and use a magnifying glass to visualize that $5.3\overline{7}$ lies between 5.37 and 5.38. To visualize $5.3\overline{7}$ more accurately, we again divide the portion between 5.37 and 5.38 into ten equal parts and use a magnifying glass to visualize that $5.3\overline{7}$ lies between 5.377 and 5.378. Now to visualize $5.3\overline{7}$ still more accurately, we divide the portion between 5.377 and 5.378 into 10 equal parts, and visualize the representation of $5.3\overline{7}$ as in fig.,(iv) . Notice that $5.3\overline{7}$ is located closer to 5.3778 than to 5.3777(iv)



PROMOTED CONTENT