Solutions for Class 11 Maths Chapter 11

EXERCISE 11.1 PAGE NO: 241

In each of the following Exercise 1 to 5, find the equation of the circle with

1. Centre (0, 2) and radius 2

Solution:

Given:

Centre (0, 2) and radius 2

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (0, 2) and radius (r) = 2

The equation of the circle is

$(x - 0)^2 + (y - 2)^2 = 2^2$

$x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

∴ The equation of the circle is $x^2 + y^2 - 4y = 0$

2. Centre (–2, 3) and radius 4

Solution:

Given:

Centre (-2, 3) and radius 4

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (-2, 3) and radius (r) = 4

The equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

$x^2 + y^2 + 4x - 6y - 3 = 0$

∴ The equation of the circle is $x^2 + y^2 + 4x - 6y - 3 = 0$

3. Centre (1/2, 1/4) and radius (1/12)

Solution:

Given:

Centre (1/2, 1/4) and radius 1/12

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (1/2, 1/4) and radius (r) = 1/12

The equation of the circle is

$(x - 1/2)^2 + (y - 1/4)^2 = (1/12)^2$

$x^2 - x + ¼ + y^2 - y/2 + 1/16 = 1/144$

$x^2 - x + ¼ + y^2 - y/2 + 1/16 = 1/144$

$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0$

$144x^2 - 144x + 144y^2 - 72y + 44 = 0$

$36x^2 + 36x + 36y^2 - 18y + 11 = 0$

$36x^2 + 36y^2 - 36x - 18y + 11= 0$

∴ The equation of the circle is $36x^2 + 36y^2 - 36x - 18y + 11= 0$

4. Centre (1, 1) and radius √2

Solution:

Given:

Centre (1, 1) and radius √2

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (1, 1) and radius (r) = √2

The equation of the circle is

$(x-1)^2 + (y-1)^2 = (\sqrt{2})^2$

$x^2 - 2x + 1 + y^2 -2y + 1 = 2$

$x^2 + y^2 - 2x -2y = 0$

∴ The equation of the circle is $x^2 + y^2 - 2x -2y = 0$

5. Centre (–a, –b) and radius $\sqrt{(a^2 - b^2)}$

Solution:

Given:

Centre (-a, -b) and radius $\sqrt{(a^2 - b^2)}$

Let us consider the equation of a circle with centre (h, k) and

Radius r is given as $(x - h)^2 + (y - k)^2 = r^2$

So, centre (h, k) = (-a, -b) and radius (r) = $\sqrt{(a^2 – b^2)}$

The equation of the circle is

$(x + a)^2 + (y + b)^2 = (\sqrt{(a^2 – b^2)}^2)$

$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 – b^2$

$x^2 + y^2 +2ax + 2by + 2b^2 = 0$

∴ The equation of the circle is $x^2 + y^2 +2ax + 2by + 2b^2 = 0$

In each of the following Exercise 6 to 9, find the centre and radius of the circles.

6. $(x + 5)^2 + (y – 3)^2 = 36$

Solution:

Given:

The equation of the given circle is $(x + 5)^2 + (y – 3)^2 = 36$

$(x – (-5))^2 + (y – 3)^2 = 6^2$ [which is of the form $(x – h)^2 + (y – k)^2 = r^2$]

Where, h = -5, k = 3 and r = 6

∴ The centre of the given circle is (-5, 3) and its radius is 6.

7. $x^2 + y^2 – 4x – 8y – 45 = 0$

Solution:

Given:

The equation of the given circle is $x^2 + y^2 – 4x – 8y – 45 = 0$.

$x^2 + y^2 – 4x – 8y – 45 = 0$

$(x^2 – 4x) + (y^2 -8y) = 45$

$(x^2 – 2(x) (2) + 2^2) + (y^2 – 2(y) (4) + 4^2) – 4 – 16 = 45$

$(x – 2)^2 + (y – 4)^2 = 65$

$(x – 2)^2 + (y – 4)^2 = (\sqrt{65})^2$ [which is form $(x-h)^2 +(y-k)^2 = r^2$]

Where h = 2, K = 4 and r = $\sqrt{65}$

∴ The centre of the given circle is (2, 4) and its radius is $\sqrt{65}$.

8. $x^2 + y^2 – 8x + 10y – 12 = 0$

Solution:

Given:

The equation of the given circle is $x^2 + y^2 -8x + 10y -12 = 0$.

$x^2 + y^2 – 8x + 10y – 12 = 0$

$(x^2 – 8x) + (y^2 + 10y) = 12$

$(x^2 – 2(x) (4) + 4^2) + (y^2 – 2(y) (5) + 5^2) – 16 – 25 = 12$

$(x – 4)^2 + (y + 5)^2 = 53$

$(x – 4)^2 + (y – (-5))^2 = (\sqrt{53})^2$ [which is form $(x-h)^2 +(y-k)^2 = r^2$]

Where h = 4, K= -5 and r = $\sqrt{53}$

∴ The centre of the given circle is (4, -5) and its radius is $\sqrt{53}$.

9. $2x^2 + 2y^2 – x = 0$

Solution:

The equation of the given of the circle is $2x^2 + 2y^2 –x = 0$.

$2x^2 + 2y^2 –x = 0$

$(2x^2 + x) + 2y^2 = 0$

$(x^2 – 2 (x) (1/4) + (1/4)^2) + y^2 – (1/4)^2 = 0$

$(x – 1/4)^2 + (y – 0)^2 = (1/4)^2$ [which is form $(x-h)^2 +(y-k)^2 = r^2$]

Where, h = ¼, K = 0, and r = ¼

∴ The center of the given circle is (1/4, 0) and its radius is 1/4.

10. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line 4x + y = 16.

Solution:

Let us consider the equation of the required circle be $(x – h)^2+ (y – k)^2 = r^2$

We know that the circle passes through points (4,1) and (6,5)

So,

$(4 – h)^2 + (1 – k)^2 = r^2$ ……………..(1)

$(6– h)^2+ (5 – k)^2 = r^2$ ………………(2)

Since, the centre (h, k) of the circle lies on line 4x + y = 16,

4h + k =16………………… (3)

From the equation (1) and (2), we obtain

$(4 – h)^2+ (1 – k)^2 =(6 – h)^2 + (5 – k)^2$

$16 – 8h + h^2 +1 -2k +k^2 = 36 -12h +h^2+15 – 10k + k^2$

$16 – 8h + 1 - 2k + 12h - 25 - 10k$

$4h + 8k = 44$

$h + 2k = 11$……………. (4)

On solving equations (3) and (4), we obtain h=3 and k= 4.

On substituting the values of h and k in equation (1), we obtain

$(4 – 3)^2 + (1 – 4)^2 = r^2$

$(1)^2 + (-3)^2 = r^2$

$1+9 = r^2$

$r = \sqrt{10}$

so now, $(x – 3)^2 + (y – 4)^2 = (\sqrt{10})^2$

$x^2 – 6x + 9 + y^2 – 8y + 16 = 10$

$x^2 + y^2 – 6x – 8y + 15 = 0$

∴ The equation of the required circle is $x^2 + y^2 – 6x – 8y + 15 = 0$

11. Find the equation of the circle passing through the points (2, 3) and (–1, 1) and whose centre is on the line x – 3y – 11 = 0.

Solution:

Let us consider the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$

We know that the circle passes through points (2,3) and (-1,1).

$(2 – h)^2 + (3 – k)^2 = r^2$ ……………..(1)

$(-1 – h)^2 + (1 – k)^2 = r^2$ ………………(2)

Since, the centre (h, k) of the circle lies on line x – 3y – 11= 0,

$h – 3k = 11$………………… (3)

From the equation (1) and (2), we obtain

$(2 – h)^2 + (3 – k)^2 = (-1 – h)^2 + (1 – k)^2$

$4 – 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 – 2k + k^2$

$4 – 4h + 9 - 6k = 1 + 2h + 1 - 2k$

$6h + 4k = 11$……………. (4)

Now let us multiply equation (3) by 6 and subtract it from equation (4) to get,

$6h + 4k – 6(h-3k) = 11 – 66$

$6h + 4k – 6h + 18k = 11 – 66$

$22 k = – 55$

$K = -5/2$

Substitute this value of K in equation (4) to get,

$6h + 4(-5/2) = 11$

$6h – 10 = 11$

$6h = 21$

$h = 21/6$

$h = 7/2$

We obtain $h = 7/2$and $k = -5/2$

On substituting the values of h and k in equation (1), we get

$(2 – 7/2)^2 + (3 + 5/2)^2 = r^2$

$[(4-7)/2]^2 + [(6+5)/2]^2 = r^2$

$(-3/2)^2 + (11/2)^2 = r^2$

$9/4 + 121/4 = r^2$

$130/4 = r^2$

The equation of the required circle is

$(x – 7/2)^2 + (y + 5/2)^2 = 130/4$

$[(2x-7)/2]^2 + [(2y+5)/2]^2 = 130/4$

$4x^2 -28x + 49 +4y^2 + 20y + 25 =130$

$4x^2 +4y^2 -28x + 20y – 56 = 0$

$4(x^2 +y^2 -7x + 5y – 14) = 0$

$x^2 + y^2 – 7x + 5y – 14 = 0$

∴ The equation of the required circle is $x^2 + y^2 – 7x + 5y – 14 = 0$

12. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Solution:

Let us consider the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$

We know that the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.

So now, the equation of the circle is $(x – h)^2 + y^2 = 25$.

It is given that the circle passes through the point (2, 3) so the point will satisfy the equation of the circle.

$(2 – h)^2 + 3^2 = 2$

$(2 – h)^2 = 25-9$

$(2 – h)^2 = 16$

$2 – h = \pm \sqrt{16} = \pm 4$

If $2-h = 4$, then $h = -2$

If $2-h = -4$, then $h = 6$

Then, when $h = -2$, the equation of the circle becomes

$(x – 2)^2 + y^2 = 25$

$x^2 -12x + 36 + y^2 = 25$

$x^2 + y^2 – 4x + 21 = 0$

When $h = 6$, the equation of the circle becomes

$(x – 6)^2 + y^2 = 25$

$x^2 -12x + 36 + y^2 = 25$

$x^2 + y^2 -12x + 11 = 0$

∴ The equation of the required circle is $x^2 + y^2 – 4x + 21 = 0$ and $x^2 + y^2 -12x + 11 = 0$

13. Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.

Solution:

Let us consider the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$

We know that the circle passes through (0, 0),

So, $(0 – h)^2 + (0 – k)^2 = r^2$

$h^2 + k^2 = r^2$

Now, The equation of the circle is $(x – h)^2 + (y – k)^2 = h^2 + k^2$.

It is given that the circle intercepts a and b on the coordinate axes.

i.e., the circle passes through points (a, 0) and (0, b).

So, $(a – h)^2 + (0 – k)^2 = h^2 + k^2$……………..(1)

$(0 – h)^2+ (b– k)^2 =h^2 +k^2$………………(2)

From equation (1), we obtain

$a^2 – 2ah + h^2 +k^2 = h^2 +k^2$

$a^2 – 2ah = 0$

$a(a – 2h) =0$

$a = 0$ or $(a -2h) = 0$

However, $a \neq 0$; hence, $(a -2h) = 0$

$h = a/2$

From equation (2), we obtain

$h^2 – 2bk + k^2 + b^2= h^2 +k^2$

$b^2 – 2bk = 0$

$b(b– 2k) = 0$

$b= 0$ or $(b-2k) =0$

However, $a \neq 0$; hence, $(b -2k) = 0$

$k =b/2$

So, the equation is

$(x – a/2)^2 + (y – b/4)^2 = (a/2)^2 + (b/2)^2$

$[(2x-a)/2]^2 + [(2y+b)/2]^2 = (a^2 + b^2)/4$

$4x^2 – 4ax + a^2 +4y^2 – 4by + b^2 = a^2 + b^2$

$4x^2 + 4y^2 -4ax – 4by = 0$

$4(x^2 +y^2 -7x + 5y – 14) = 0$

$x^2 + y^2 – ax – by = 0$

∴ The equation of the required circle is $x^2 + y^2 – ax – by = 0$

14. Find the equation of a circle with centre (2,2) and passes through the point (4,5).

Solution:

Given:

The centre of the circle is given as (h, k) = (2,2)

We know that the circle passes through point (4,5), the radius (r) of the circle is the distance between the points (2,2) and (4,5).

$r = \sqrt{[(2-4)^2 + (2-5)^2]}$

$= \sqrt{[(-2)^2 + (-3)^2]}$

$= \sqrt{[4+9]}$

$= \sqrt{13}$

The equation of the circle is given as

$(x - h)^2 + (y - k)^2 = r^2$

$(x - h)^2 + (y - k)^2 = (\sqrt{13})^2$

$(x - 2)^2 + (y - 2)^2 = (\sqrt{13})^2$

$x^2 - 4x + 4 + y^2 - 4y + 4 = 13$

$x^2 + y^2 - 4x - 4y = 5$

∴ The equation of the required circle is $x^2 + y^2 - 4x - 4y = 5$

15. Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Solution:

Given:

The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$(x - 0)^2 + (y - 0)^2 = 5^2$ [which is of the form $(x - h)^2 + (y - k)^2 = r^2$]

Where, h = 0, k = 0 and r = 5.

So the distance between point (-2.5, 3.5) and the centre (0,0) is

$\sqrt{[(-2.5 - 0)^2 + (-3.5 - 0)^2]}$

$\sqrt{(6.25 + 12.25)}$

$\sqrt{18.5}$

4.3 [which is < 5]

Since, the distance between point (-2.5, -3.5) and the centre (0, 0) of the circle is less than the radius of the circle, point (-2.5, -3.5) lies inside the circle.

EXERCISE 11.2 PAGE NO: 246

In each of the following Exercises 1 to 6, find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

1. $y^2 = 12x$

Solution:

Given:

The equation is $y^2 = 12x$

Here we know that the coefficient of x is positive.

So, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we get,

$4a = 12$

$a = 3$

Thus, the co-ordinates of the focus = (a, 0) = (3, 0)

Since, the given equation involves y^2, the axis of the parabola is the x-axis.

∴ The equation of directrix, x = -a, then,

$x + 3 = 0$

Length of latus rectum = 4a = 4 × 3 = 12

2. $x^2 = 6y$

Solution:

Given:

The equation is $y^2 = 6y$

Here we know that the coefficient of y is positive.

So, the parabola opens upwards.

On comparing this equation with $y^2 = 4ay$, we get,

$4a = 6$

$a = 6/4$

$= 3/2$

Thus, the co-ordinates of the focus = (0,a) = (0, 3/2)

Since, the given equation involves x^2, the axis of the parabola is the y-axis.

∴ The equation of directrix, $y = -a$, then,
$y = -3/2$
Length of latus rectum = $4a = 4(3/2) = 6$

3. $y^2 = -8x$

Solution:

Given:
The equation is $y^2 = -8x$
Here we know that the coefficient of x is negative.
So, the parabola open towards the left.
On comparing this equation with $y^2 = -4ax$, we get,
$-4a = -8$
$a = -8/-4 = 2$
Thus, co-ordinates of the focus = $(-a,0) = (-2, 0)$
Since, the given equation involves y^2, the axis of the parabola is the x-axis.
∴ Equation of directrix, $x = a$, then,
$y = 2$
Length of latus rectum = $4a = 4(2) = 8$

4. $x^2 = -16y$

Solution:

Given:
The equation is $x^2 = -16y$
Here we know that the coefficient of y is negative.
So, the parabola opens downwards.
On comparing this equation with $x^2 = -4ay$, we get,
$-4a = -16$
$a = -16/-4$
$= 4$
Thus, co-ordinates of the focus = $(0,-a) = (0,-4)$
Since, the given equation involves x^2, the axis of the parabola is the y-axis.

∴ The equation of directrix, x =a, then,

y = 4

Length of latus rectum = 4a = 4(4) = 16

5. $y^2 = 10x$

Solution:

Given:

The equation is $y^2 = 10x$

Here we know that the coefficient of x is positive.

So, the parabola open towards the right.

On comparing this equation with $y^2 = -4ax$, we get,

4a = 10

a = 10/4 = 5/2

Thus, co-ordinates of the focus = (a,0) = (5/2, 0)

Since, the given equation involves y^2, the axis of the parabola is the x-axis.

∴ The equation of directrix, x =-a, then,

x = – 5/2

Length of latus rectum = 4a = 4(5/2) = 10

6. $x^2 = – 9y$

Solution:

Given:

The equation is $x^2 = -9y$

Here we know that the coefficient of y is negative.

So, the parabola open downwards.

On comparing this equation with $x^2 = -4ay$, we get,

-4a = -9

a = -9/-4 = 9/4

Thus, co-ordinates of the focus = (0,-a) = (0, 9/4)

Since, the given equation involves x^2, the axis of the parabola is the y-axis.

∴ The equation of directrix, y = a, then,

$x = 9/4$

Length of latus rectum = 4a = 4(9/4) = 9

In each of the Exercises 7 to 12, find the equation of the parabola that satisfies the given conditions:

7. Focus (6,0); directrix x = – 6

Solution:

Given:

Focus (6,0) and directrix x = -6

We know that the focus lies on the x–axis is the axis of the parabola.

So, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

It is also seen that the directrix, x = -6 is to the left of the y-axis,

While the focus (6, 0) is to the right of the y –axis.

Hence, the parabola is of the form $y^2 = 4ax$.

Here, a = 6

∴ The equation of the parabola is $y^2 = 24x$.

8. Focus (0,–3); directrix y = 3

Solution:

Given:

Focus (0, -3) and directrix y = 3

We know that the focus lies on the y–axis, the y-axis is the axis of the parabola.

So, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

It is also seen that the directrix, y = 3 is above the x- axis,

While the focus (0,-3) is below the x-axis.

Hence, the parabola is of the form $x^2 = -4ay$.

Here, a = 3

∴ The equation of the parabola is $x^2 = -12y$.

9. Vertex (0, 0); focus (3, 0)

Solution:

Given:

Vertex (0, 0) and focus (3, 0)

We know that the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis. [x-axis is the axis of the parabola.]

The equation of the parabola is of the form $y^2 = 4ax$.

Since, the focus is (3, 0), a = 3

∴ The equation of the parabola is $y^2 = 4 \times 3 \times x$,

$y^2 = 12x$

10. Vertex (0, 0); focus (–2, 0)

Solution:

Given:

Vertex (0, 0) and focus (-2, 0)

We know that the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis. [x-axis is the axis of the parabola.]

The equation of the parabola is of the form $y^2=-4ax$.

Since, the focus is (-2, 0), a = 2

∴ The equation of the parabola is $y^2 = -4 \times 2 \times x$,

$y^2 = -8x$

11. Vertex (0, 0) passing through (2, 3) and axis is along x-axis.

Solution:

We know that the vertex is (0, 0) and the axis of the parabola is the x-axis

The equation of the parabola is either of the from $y^2 = 4ax$ or $y^2 = -4ax$.

Given that the parabola passes through point (2, 3), which lies in the first quadrant.

So, the equation of the parabola is of the form $y^2 = 4ax$, while point (2, 3) must satisfy the equation $y^2 = 4ax$.

Then,

$3^2 = 4a(2)$

$3^2 = 8a$

$9 = 8a$

$a = 9/8$

Thus, the equation of the parabola is

$y^2 = 4 (9/8)x$

$= 9x/2$

$2y^2 = 9x$

∴ The equation of the parabola is $2y^2 = 9x$

12. Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis.

Solution:

We know that the vertex is (0, 0) and the parabola is symmetric about the y-axis.

The equation of the parabola is either of the from $x^2 = 4ay$ or $x^2 = -4ay$.

Given that the parabola passes through point (5, 2), which lies in the first quadrant.

So, the equation of the parabola is of the form $x^2 = 4ay$, while point (5, 2) must satisfy the equation $x^2 = 4ay$.

Then,

$5^2 = 4a(2)$

$25 = 8a$

$a = 25/8$

Thus, the equation of the parabola is

$x^2 = 4 (25/8)y$

$x^2 = 25y/2$

$2x^2 = 25y$

∴ The equation of the parabola is $2x^2 = 25y$

EXERCISE 11.3 PAGE NO: 255

In each of the Exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

1. $x^2/36 + y^2/16 = 1$

Solution:

Given:

The equation is $x^2/36 + y^2/16 = 1$

Here, the denominator of $x^2/36$ is greater than the denominator of $y^2/16$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 6$ and $b = 4$.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(36-16)}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Then,

The coordinates of the foci are $(2\sqrt{5}, 0)$ and $(-6, 0)$.

The coordinates of the vertices are $(6, 0)$ and $(-6, 0)$

Length of major axis $= 2a = 2\ (6) = 12$

Length of minor axis $= 2b = 2\ (4) = 8$

Eccentricity, $e^{c/a} = 2\sqrt{5}/6 = \sqrt{5}/3$

Length of latus rectum $= 2b^2/a = (2\times16)/6 = 16/3$

2. $x^2/4 + y^2/25 = 1$

Solution:

Given:

The equation is $x^2/4 + y^2/25 = 1$

Here, the denominator of $y^2/25$ is greater than the denominator of $x^2/4$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 5$ and $b = 2$.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(25-4)}$

$= \sqrt{21}$

Then,

The coordinates of the foci are $(0, \sqrt{21})$ and $(0, -\sqrt{21})$.

The coordinates of the vertices are (0, 5) and (0, -5)

Length of major axis = 2a = 2 (5) = 10

Length of minor axis = 2b = 2 (2) = 4

Eccentricity, $e^{c/a} = \sqrt{21}/5$

Length of latus rectum = $2b^2/a = (2\times 2^2)/5 = (2\times 4)/5 = 8/5$

3. $x^2/16 + y^2/9 = 1$

Solution:

Given:

The equation is $x^2/16 + y^2/9 = 1$ or $x^2/4^2 + y^2/3^2 = 1$

Here, the denominator of $x^2/16$ is greater than the denominator of $y^2/9$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

$a = 4$ and $b = 3$.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(16-9)}$

$= \sqrt{7}$

Then,

The coordinates of the foci are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$.

The coordinates of the vertices are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$

Length of major axis = 2a = 2 (4) = 8

Length of minor axis = 2b = 2 (3) = 6

Eccentricity, $e^{c/a} = \sqrt{7}/4$

Length of latus rectum = $2b^2/a = (2\times3^2)/4 = (2\times9)/4 = 18/4 = 9/2$

4. $x^2/25 + y^2/100 = 1$

Solution:

Given:

The equation is $x^2/25 + y^2/100 = 1$

Here, the denominator of $y^2/100$ is greater than the denominator of $x^2/25$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

b = 5 and a =10.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(100-25)}$

$= \sqrt{75}$

$= 5\sqrt{3}$

Then,

The coordinates of the foci are $(0, 5\sqrt{3})$ and $(0, -5\sqrt{3})$.

The coordinates of the vertices are $(0, \sqrt{10})$ and $(0, -\sqrt{10})$

Length of major axis = 2a = 2 (10) = 20

Length of minor axis = 2b = 2 (5) = 10

Eccentricity, $e^{c/a} = 5\sqrt{3}/10 = \sqrt{3}/2$

Length of latus rectum = $2b^2/a = (2\times5^2)/10 = (2\times25)/10 = 5$

5. $x^2/49 + y^2/36 = 1$

Solution:

Given:

The equation is $x^2/49 + y^2/36 = 1$

Here, the denominator of $x^2/49$ is greater than the denominator of $y^2/36$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

b = 6 and a =7

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(49-36)}$

$= \sqrt{13}$

Then,

The coordinates of the foci are $(\sqrt{13}, 0)$ and $(-\sqrt{3}, 0)$.

The coordinates of the vertices are (7, 0) and (-7, 0)

Length of major axis = 2a = 2 (7) = 14

Length of minor axis = 2b = 2 (6) = 12

Eccentricity, $e^{c/a} = \sqrt{13}/7$

Length of latus rectum = $2b^2/a = (2\times6^2)/7 = (2\times36)/7 = 72/7$

6. $x^2/100 + y^2/400 = 1$

Solution:

Given:

The equation is $x^2/100 + y^2/400 = 1$

Here, the denominator of $y^2/400$ is greater than the denominator of $x^2/100$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

b = 10 and a =20.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(400-100)}$

$= \sqrt{300}$

$= 10\sqrt{3}$

Then,

The coordinates of the foci are $(0, 10\sqrt{3})$ and $(0, -10\sqrt{3})$.

The coordinates of the vertices are (0, 20) and (0, -20)

Length of major axis = 2a = 2 (20) = 40

Length of minor axis = 2b = 2 (10) = 20

Eccentricity, $e^{c/a}$ = $10\sqrt{3}/20 = \sqrt{3}/2$

Length of latus rectum = $2b^2/a = (2\times10^2)/20 = (2\times100)/20 = 10$

7. $36x^2 + 4y^2 = 144$

Solution:

Given:

The equation is $36x^2 + 4y^2 = 144$ or $x^2/4 + y^2/36 = 1$ or $x^2/2^2 + y^2/6^2 = 1$

Here, the denominator of $y^2/6^2$ is greater than the denominator of $x^2/2^2$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

b = 2 and a = 6.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(36-4)}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Then,

The coordinates of the foci are $(0, 4\sqrt{2})$ and $(0, -4\sqrt{2})$.

The coordinates of the vertices are (0, 6) and (0, -6)

Length of major axis = 2a = 2 (6) = 12

Length of minor axis = 2b = 2 (2) = 4

Eccentricity, $e^{c/a}$ = $4\sqrt{2}/6 = 2\sqrt{2}/3$

Length of latus rectum = $2b^2/a = (2\times2^2)/6 = (2\times4)/6 = 4/3$

8. $16x^2 + y^2 = 16$

Solution:

Given:

The equation is $16x^2 + y^2 = 16$ or $x^2/1 + y^2/16 = 1$ or $x^2/1^2 + y^2/4^2 = 1$

Here, the denominator of $y^2/4^2$ is greater than the denominator of $x^2/1^2$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

b =1 and a =4.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(16-1)}$

$= \sqrt{15}$

Then,

The coordinates of the foci are $(0, \sqrt{15})$ and $(0, -\sqrt{15})$.

The coordinates of the vertices are (0, 4) and (0, -4)

Length of major axis = 2a = 2 (4) = 8

Length of minor axis = 2b = 2 (1) = 2

Eccentricity, $e^{c/a} = \sqrt{15}/4$

Length of latus rectum = $2b^2/a = (2\times1^2)/4 = 2/4 = ½$

9. $4x^2 + 9y^2 = 36$

Solution:

Given:

The equation is $4x^2 + 9y^2 = 36$ or $x^2/9 + y^2/4 = 1$ or $x^2/3^2 + y^2/2^2 = 1$

Here, the denominator of $x^2/3^2$ is greater than the denominator of $y^2/2^2$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

a =3 and b =2.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(9-4)}$

$= \sqrt{5}$

Then,

The coordinates of the foci are $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.

The coordinates of the vertices are (3, 0) and (-3, 0)

Length of major axis = 2a = 2 (3) = 6

Length of minor axis = 2b = 2 (2) = 4

Eccentricity, $e^{c/a}$ = √5/3

Length of latus rectum = $2b^2/a$ = $(2\times2^2)/3$ = (2×4)/3 = 8/3

In each of the following Exercises 10 to 20, find the equation for the ellipse that satisfies the given conditions:

10. Vertices (± 5, 0), foci (± 4, 0)

Solution:

Given:

Vertices (± 5, 0) and foci (± 4, 0)

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, a = 5 and c = 4.

It is known that $a^2 = b^2 + c^2$.

So, $5^2 = b^2 + 4^2$

$25 = b^2 + 16$

$b^2 = 25 – 16$

b = √9

= 3

∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2 = 1$ or $x^2/25 + y^2/9 = 1$

11. Vertices (0, ± 13), foci (0, ± 5)

Solution:

Given:

Vertices (0, ± 13) and foci (0, ± 5)

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, a =13 and c = 5.

It is known that $a^2 = b^2 + c^2$.

$13^2 = b^2+5^2$

$169 = b^2 + 15$

$b^2 = 169 - 125$

$b = \sqrt{144}$

$= 12$

∴ The equation of the ellipse is $x^2/12^2 + y^2/13^2 = 1$ or $x^2/144 + y^2/169 = 1$

12. Vertices (± 6, 0), foci (± 4, 0)

Solution:

Given:

Vertices (± 6, 0) and foci (± 4, 0)

Here, the vertices are on the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, a = 6 and c = 4.

It is known that $a^2 = b^2 + c^2$.

$6^2 = b^2+4^2$

$36 = b^2 + 16$

$b^2 = 36 - 16$

$b = \sqrt{20}$

∴ The equation of the ellipse is $x^2/6^2 + y^2/(\sqrt{20})^2 = 1$ or $x^2/36 + y^2/20 = 1$

13. Ends of major axis (± 3, 0), ends of minor axis (0, ±2)

Solution:

Given:

Ends of major axis (± 3, 0) and ends of minor axis (0, ±2)

Here, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, a = 3 and b = 2.

∴ The equation for the ellipse $x^2/3^2 + y^2/2^2 = 1$ or $x^2/9 + y^2/4 = 1$

14. Ends of major axis (0, ±√5), ends of minor axis (±1, 0)

Solution:

Given:

Ends of major axis $(0, \pm\sqrt{5})$ and ends of minor axis $(\pm 1, 0)$

Here, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $a = \sqrt{5}$ and $b = 1$.

∴ The equation for the ellipse $x^2/1^2 + y^2/(\sqrt{5})^2 = 1$ or $x^2/1 + y^2/5 = 1$

15. Length of major axis 26, foci (±5, 0)

Solution:

Given:

Length of major axis is 26 and foci (±5, 0)

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, $2a = 26$

$a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$.

$13^2 = b^2+4^2$

$169 = b^2 + 25$

$b^2 = 169 – 25$

$b = \sqrt{144}$

$= 12$

∴ The equation of the ellipse is $x^2/13^2 + y^2/12^2 = 1$ or $x^2/169 + y^2/144 = 1$

16. Length of minor axis 16, foci (0, ±6).

Solution:

Given:

Length of minor axis is 16 and foci (0, ±6).

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, 2b =16

b = 8 and c = 6.

It is known that $a^2 = b^2 + c^2$.

$a^2 = 8^2 + 6^2$

= 64 + 36

=100

b = √100

= 10

∴ The equation of the ellipse is $x^2/8^2 + y^2/10^2 =1$ or $x^2/64 + y^2/100 = 1$

17. Foci (±3, 0), a = 4

Solution:

Given:

Foci (±3, 0) and a = 4

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

Then, c = 3 and a = 4.

It is known that $a^2 = b^2 + c^2$.

$a^2 = 8^2 + 6^2$

= 64 + 36

= 100

$16 = b^2 + 9$

$b^2 = 16 – 9$

= 7

∴ The equation of the ellipse is $x^2/16 + y^2/7 = 1$

18. b = 3, c = 4, centre at the origin; foci on the x axis.

Solution:

Given:

$b = 3$, $c = 4$, centre at the origin and foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where ‘a’ is the semi-major axis.

Then, $b = 3$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$a^2 = 3^2 + 4^2$

$= 9 + 16$

$=25$

$a = \sqrt{25}$

$= 5$

∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2$ or $x^2/25 + y^2/9 = 1$

19. Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Solution:

Given:

Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Since the centre is at (0, 0) and the major axis is on the y- axis, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where ‘a’ is the semi-major axis.

The ellipse passes through points (3, 2) and (1, 6).

So, by putting the values $x = 3$ and $y = 2$, we get,

$3^2/b^2 + 2^2/a^2 = 1$

$9/b^2 + 4/a^2$…. (1)

And by putting the values $x = 1$ and $y = 6$, we get,

$1^1/b^2 + 6^2/a^2 = 1$

$1/b^2 + 36/a^2 = 1$ …. (2)

On solving equation (1) and (2), we get

$b^2 = 10$ and $a^2 = 40$.

∴ The equation of the ellipse is $x^2/10 + y^2/40 = 1$ or $4x^2 + y^2 = 40$

20. Major axis on the x-axis and passes through the points (4,3) and (6,2).

Solution:

Given:

Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Since the major axis is on the x-axis, the equation of the ellipse will be the form

$x^2/a^2 + y^2/b^2 = 1$…. (1) [Where 'a' is the semi-major axis.]

The ellipse passes through points (4, 3) and (6, 2).

So by putting the values x = 4 and y = 3 in equation (1), we get,

$16/a^2 + 9/b^2 = 1$ …. (2)

Putting, x = 6 and y = 2 in equation (1), we get,

$36/a^2 + 4/b^2 = 1$ …. (3)

From equation (2)

$16/a^2 = 1 – 9/b^2$

$1/a^2 = (1/16 (1 – 9/b^2))$ …. (4)

Substituting the value of $1/a^2$ in equation (3) we get,

$36/a^2 + 4/b^2 = 1$

$36(1/a^2) + 4/b^2 = 1$

$36[1/16 (1 – 9/b^2)] + 4/b^2 = 1$

$36/16 (1 – 9/b^2) + 4/b^2 = 1$

$9/4 (1 – 9/b^2) + 4/b^2 = 1$

$9/4 – 81/4b^2 + 4/b^2 = 1$

$-81/4b^2 + 4/b^2 = 1 – 9/4$

$(-81+16)/4b^2 = (4-9)/4$

$-65/4b^2 = -5/4$

$-5/4(13/b^2) = -5/4$

$13/b^2 = 1$

$1/b^2 = 1/13$

$b^2 = 13$

Now substitute the value of b^2 in equation (4) we get,

$1/a^2 = 1/16(1 – 9/b^2)$

$= 1/16(1 – 9/13)$

$= 1/16((13-9)/13)$

$= 1/16(4/13)$

$= 1/52$

$a^2 = 52$

Equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$

By substituting the values of a^2 and b^2 in above equation we get,

$x^2/52 + y^2/13 = 1$

EXERCISE 11.4 PAGE NO: 262

In each of the Exercises 1 to 6, find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

1. $x^2/16 – y^2/9 = 1$

Solution:

Given:

The equation is $x^2/16 – y^2/9 = 1$ or $x^2/4^2 – y^2/3^2 = 1$

On comparing this equation with the standard equation of hyperbola $x^2/a^2 – y^2/b^2 = 1$,

We get $a = 4$ and $b = 3$,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 4^2 + 3^2$

$= \sqrt{25}$

$c = 5$

Then,

The coordinates of the foci are (±5, 0).

The coordinates of the vertices are (±4, 0).

Eccentricity, $e = c/a = 5/4$

Length of latus rectum $= 2b^2/a = (2 \times 3^2)/4 = (2\times9)/4 = 18/4 = 9/2$

2. $y^2/9 – x^2/27 = 1$

Solution:

Given:

The equation is $y^2/9 – x^2/27 = 1$ or $y^2/3^2 – x^2/27^2 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 – x^2/b^2 = 1$,

We get $a = 3$ and $b = \sqrt{27}$,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 3^2 + (\sqrt{27})^2$

$= 9 + 27$

$c^2 = 36$

$c = \sqrt{36}$

$= 6$

Then,

The coordinates of the foci are (0, 6) and (0, -6).

The coordinates of the vertices are (0, 3) and (0, – 3).

Eccentricity, $e = c/a = 6/3 = 2$

Length of latus rectum $= 2b^2/a = (2 \times 27)/3 = (54)/3 = 18$

3. $9y^2 – 4x^2 = 36$

Solution:

Given:

The equation is $9y^2 – 4x^2 = 36$ or $y^2/4 – x^2/9 = 1$ or $y^2/2^2 – x^2/3^2 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 – x^2/b^2 = 1$,

We get $a = 2$ and $b = 3$,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 4 + 9$

$c^2 = 13$

$c = \sqrt{13}$

Then,

The coordinates of the foci are $(0, \sqrt{13})$ and $(0, -\sqrt{13})$.

The coordinates of the vertices are (0, 2) and (0, – 2).

Eccentricity, $e = c/a = \sqrt{13}/2$

Length of latus rectum $= 2b^2/a = (2 \times 3^2)/2 = (2\times9)/2 = 18/2 = 9$

4. $16x^2 - 9y^2 = 576$

Solution:

Given:

The equation is $16x^2 - 9y^2 = 576$

Let us divide the whole equation by 576, we get

$16x^2/576 - 9y^2/576 = 576/576$

$x^2/36 - y^2/64 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 - x^2/b^2 = 1$,

We get a = 6 and b = 8,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 36 + 64$

$c^2 = \sqrt{100}$

$c = 10$

Then,

The coordinates of the foci are (10, 0) and (-10, 0).

The coordinates of the vertices are (6, 0) and (-6, 0).

Eccentricity, $e = c/a = 10/6 = 5/3$

Length of latus rectum $= 2b^2/a = (2 \times 8^2)/6 = (2\times64)/6 = 64/3$

5. $5y^2 - 9x^2 = 36$

Solution:

Given:

The equation is $5y^2 – 9x^2 = 36$

Let us divide the whole equation by 36, we get

$5y^2/36 – 9x^2/36 = 36/36$

$y^2/(36/5) – x^2/4 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 – x^2/b^2 = 1$,

We get $a = 6/\sqrt{5}$ and $b = 2$,

It is known that, $a^2 + b^2 = c^2$

So,

$c^2 = 36/5 + 4$

$c^2 = 56/5$

$c = \sqrt{(56/5)}$

$= 2\sqrt{14}/\sqrt{5}$

Then,

The coordinates of the foci are $(0, 2\sqrt{14}/\sqrt{5})$ and $(0, – 2\sqrt{14}/\sqrt{5})$.

The coordinates of the vertices are $(0, 6/\sqrt{5})$ and $(0, -6/\sqrt{5})$.

Eccentricity, $e = c/a = (2\sqrt{14}/\sqrt{5}) / (6/\sqrt{5}) = \sqrt{14}/3$

Length of latus rectum $= 2b^2/a = (2 \times 2^2)/6/\sqrt{5} = (2\times4)/6/\sqrt{5} = 4/\sqrt{5}/3$

6. $49y^2 – 16x^2 = 784$.

Solution:

Given:

The equation is $49y^2 – 16x^2 = 784$.

Let us divide the whole equation by 784, we get

$49y^2/784 – 16x^2/784 = 784/784$

$y^2/16 – x^2/49 = 1$

On comparing this equation with the standard equation of hyperbola $y^2/a^2 – x^2/b^2 = 1$,

We get $a = 4$ and $b = 7$,

It is know that, $a^2 + b^2 = c^2$

So,

$c^2 = 16 + 49$

$c^2 = 65$

$c = \sqrt{65}$

Then,

The coordinates of the foci are $(0, \sqrt{65})$ and $(0, -\sqrt{65})$.

The coordinates of the vertices are (0, 4) and (0, -4).

Eccentricity, $e = c/a = \sqrt{65}/4$

Length of latus rectum $= 2b^2/a = (2 \times 7^2)/4 = (2\times49)/4 = 49/2$

In each Exercises 7 to 15, find the equations of the hyperbola satisfying the given conditions

7. Vertices (±2, 0), foci (±3, 0)

Solution:

Given:

Vertices (±2, 0) and foci (±3, 0)

Here, the vertices are on the x-axis.

So, the equation of the hyperbola is of the form $x^2/a^2 - y^2/b^2 = 1$

Since, the vertices are (±2, 0), so, a = 2

Since, the foci are (±3, 0), so, c = 3

It is know that, $a^2 + b^2 = c^2$

So, $2^2 + b^2 = 3^2$

$b^2 = 9 - 4 = 5$

∴ The equation of the hyperbola is $x^2/4 - y^2/5 = 1$

8. Vertices (0, ± 5), foci (0, ± 8)

Solution:

Given:

Vertices (0, ± 5) and foci (0, ± 8)

Here, the vertices are on the y-axis.

So, the equation of the hyperbola is of the form $y^2/a^2 - x^2/b^2 = 1$

Since, the vertices are (0, ±5), so, a = 5

Since, the foci are (0, ±8), so, c = 8

It is know that, $a^2 + b^2 = c^2$

So, $5^2 + b^2 = 8^2$

$b^2 = 64 – 25 = 39$

∴ The equation of the hyperbola is $y^2/25 – x^2/39 = 1$

9. Vertices (0, ± 3), foci (0, ± 5)

Solution:

Given:

Vertices (0, ± 3) and foci (0, ± 5)

Here, the vertices are on the y-axis.

So, the equation of the hyperbola is of the form $y^2/a^2 – x^2/b^2 = 1$

Since, the vertices are (0, ±3), so, a = 3

Since, the foci are (0, ±5), so, c = 5

It is known that, $a^2 + b^2 = c^2$

So, $3^2 + b^2 = 5^2$

$b^2 = 25 – 9 = 16$

∴ The equation of the hyperbola is $y^2/9 – x^2/16 = 1$

10. Foci (±5, 0), the transverse axis is of length 8.

Solution:

Given:

Foci (±5, 0) and the transverse axis is of length 8.

Here, the foci are on x-axis.

The equation of the hyperbola is of the form $x^2/a^2 – y^2/b^2 = 1$

Since, the foci are (±5, 0), so, c = 5

Since, the length of the transverse axis is 8,

2a = 8

a = 8/2

= 4

It is known that, $a^2 + b^2 = c^2$

$4^2 + b^2 = 5^2$

$b^2 = 25 – 16$

= 9

∴ The equation of the hyperbola is $x^2/16 – y^2/9 = 1$

11. Foci (0, ±13), the conjugate axis is of length 24.

Solution:

Given:

Foci (0, ±13) and the conjugate axis is of length 24.

Here, the foci are on y-axis.

The equation of the hyperbola is of the form $y^2/a^2 – x^2/b^2 = 1$

Since, the foci are (0, ±13), so, c = 13

Since, the length of the conjugate axis is 24,

2b = 24

b = 24/2

= 12

It is known that, $a^2 + b^2 = c^2$

$a^2 + 12^2 = 13^2$

$a^2 = 169 – 144$

= 25

∴ The equation of the hyperbola is $y^2/25 – x^2/144 = 1$

12. Foci (± 3√5, 0), the latus rectum is of length 8.

Solution:

Given:

Foci (± 3√5, 0) and the latus rectum is of length 8.

Here, the foci are on x-axis.

The equation of the hyperbola is of the form $x^2/a^2 – y^2/b^2 = 1$

Since, the foci are (± 3√5, 0), so, c = ± 3√5

Length of latus rectum is 8

$2b^2/a = 8$

$2b^2 = 8a$

$b^2 = 8a/2$

= 4a

It is known that, $a^2 + b^2 = c^2$

$a^2 + 4a = 45$
$a^2 + 4a – 45 = 0$
$a^2 + 9a – 5a – 45 = 0$
$(a + 9) (a -5) = 0$
$a = -9$ or 5
Since, a is non – negative, $a = 5$
So, $b^2 = 4a$
$= 4 × 5$
$= 20$
∴ The equation of the hyperbola is $x^2/25 – y^2/20 = 1$
13. Foci (± 4, 0), the latus rectum is of length 12
Solution:
Given:
Foci (± 4, 0) and the latus rectum is of length 12
Here, the foci are on x-axis.
The equation of the hyperbola is of the form $x^2/a^2 – y^2/b^2 = 1$
Since, the foci are (± 4, 0), so, $c = 4$
Length of latus rectum is 12
$2b^2/a = 12$
$2b^2 = 12a$
$b^2 = 12a/2$
$= 6a$
It is known that, $a^2 + b^2 = c^2$
$a^2 + 6a = 16$
$a^2 + 6a – 16 = 0$
$a^2 + 8a – 2a – 16 = 0$
$(a + 8) (a – 2) = 0$
$a = -8$ or 2
Since, a is non – negative, $a = 2$
So, $b^2 = 6a$
$= 6 × 2$

= 12

∴ The equation of the hyperbola is $x^2/4 - y^2/12 = 1$

14. Vertices (±7, 0), e = 4/3

Solution:

Given:

Vertices (±7, 0) and e = 4/3

Here, the vertices are on the x- axis

The equation of the hyperbola is of the form $x^2/a^2 - y^2/b^2 = 1$

Since, the vertices are (± 7, 0), so, a = 7

It is given that e = 4/3

c/a = 4/3

3c = 4a

Substitute the value of a, we get

3c = 4(7)

c = 28/3

It is known that, $a^2 + b^2 = c^2$

$7^2 + b^2 = (28/3)^2$

$b^2 = 784/9 - 49$

= (784 – 441)/9

= 343/9

∴ The equation of the hyperbola is $x^2/49 - 9y^2/343 = 1$

15. Foci (0, ±√10), passing through (2, 3)

Solution:

Given:

Foci (0, ±√10) and passing through (2, 3)

Here, the foci are on y-axis.

The equation of the hyperbola is of the form $y^2/a^2 - x^2/b^2 = 1$

Since, the foci are (±√10, 0), so, c = √10

It is known that, $a^2 + b^2 = c^2$

$b^2 = 10 - a^2$ ………….. (1)

It is given that the hyperbola passes through point (2, 3)

So, $9/a^2 – 4/b^2 = 1$ … (2)

From equations (1) and (2), we get,

$9/a^2 – 4/(10-a^2)^2 = 1$

$9(10 – a^2) – 4a^2 = a^2(10 –a^2)$

$90 – 9a^2 – 4a^2 = 10a^2 – a^4$

$a^4 – 23a^2 + 90 = 0$

$a^4 – 18a^2 – 5a^2 + 90 = 0$

$a^2(a^2 -18) -5(a^2 -18) = 0$

$(a^2 – 18) (a^2 -5) = 0$

$a^2 = 18$ or 5

In hyperbola, $c > a$ i.e., $c^2 > a^2$

So, $a^2 = 5$

$b^2 = 10 – a^2$

$= 10 – 5$

$= 5$

∴ The equation of the hyperbola is $y^2/5 – x^2/5 = 1$

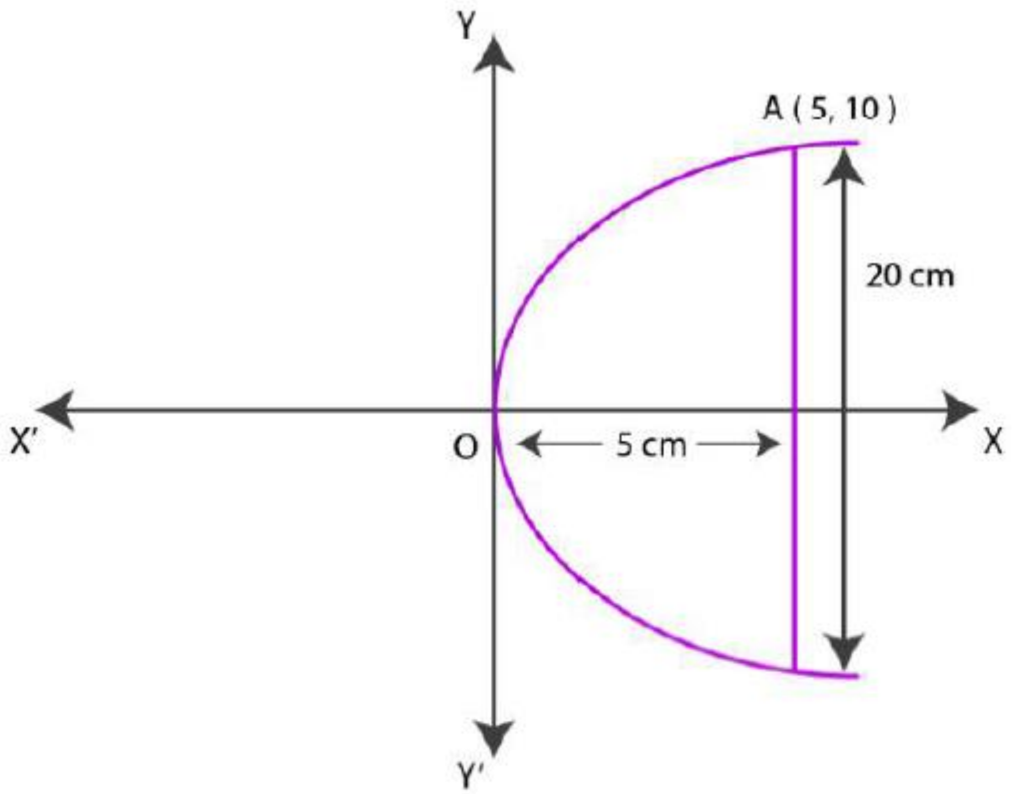
Miscellaneous EXERCISE PAGE NO: 264

1. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

Solution:

We know that the origin of the coordinate plane is taken at the vertex of the parabolic reflector, where the axis of the reflector is along the positive x – axis.

Diagrammatic representation is as follows:



We know that the equation of the parabola is of the form $y^2 = 4ax$ (as it is opening to the right)

Since, the parabola passes through point A(10, 5),

$y^2 = 4ax$

$10^2 = 4a(5)$

$100 = 20a$

$a = 100/20$

$= 5$

The focus of the parabola is $(a, 0) = (5, 0)$, which is the mid – point of the diameter.

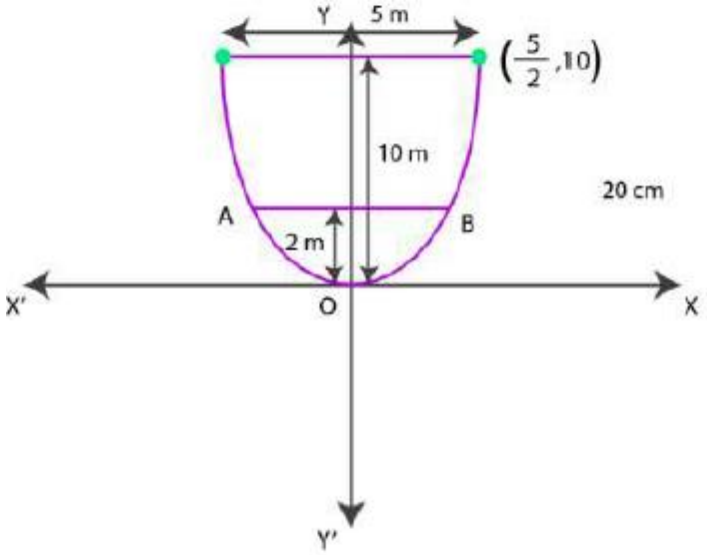
Hence, the focus of the reflector is at the mid-point of the diameter.

2. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

Solution:

We know that the origin of the coordinate plane is taken at the vertex of the arch, where its vertical axis is along the negative y –axis.

Diagrammatic representation is as follows:



The equation of the parabola is of the form $x^2 = -4ay$ (as it is opening downwards).

It is given that at base arch is 10m high and 5m wide.

So, y = -10 and x = 5/2 from the above figure.

It is clear that the parabola passes through point (5/2, -10)

So, $y^2 = 4ax$

$(5/2)^2 = -4a(-10)$

$4a = 25/(4\times10)$

$= 5/8$

we know the arch is in the form of a parabola whose equation is $x^2 = -5/8y$

We need to find width, when height = 2m.

To find x, when y = -2.

When, y = -2,

$x^2 = – 5/8 (-2)$

$= 5/4$

$x = \sqrt{(5/4)}$

$= \sqrt{5}/2$

$AB = 2 \times \sqrt{5}/2m$

$= \sqrt{5}m$

= 2.23m (approx.)

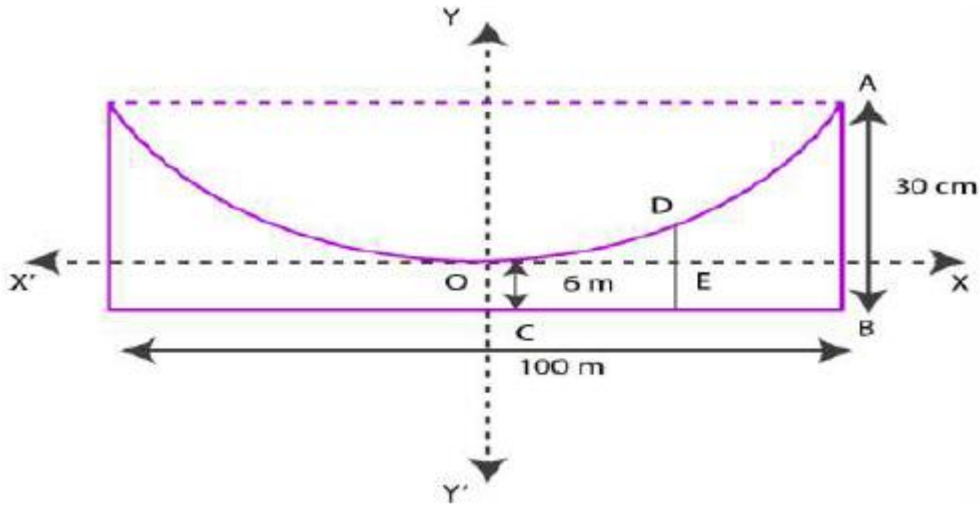
Hence, when the arch is 2m from the vertex of the parabola, its width is approximately 2.23m.

3. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m.
Find the length of a supporting wire attached to the roadway 18 m from the middle.

Solution:

We know that the vertex is at the lowest point of the cable. The origin of the coordinate plane is taken as the vertex of the parabola, while its vertical axis is taken along the positive y – axis.

Diagrammatic representation is as follows:



Here, AB and OC are the longest and the shortest wires, respectively, attached to the cable.

DF is the supporting wire attached to the roadways, 18m from the middle.

So, AB = 30m, OC = 6m, and BC = 50m.

The equation of the parabola is of the from $x^2 = 4ay$ (as it is opening upwards).

The coordinates of point A are (50, 30 -6) = (50, 24)

Since, A(50, 24) is a point on the parabola.

$y^2 = 4ax$

$(50)^2 = 4a(24)$

$a = (50×50)/(4×24)$

$= 625/24$

Equation of the parabola, $x^2 = 4ay = 4×(625/24)y$ or $6x^2 = 625y$

The x – coordinate of point D is 18.

Hence, at x = 18,

$6(18)^2 = 625y$

$y = (6×18×18)/625$

= 3.11(approx.)

Thus, DE = 3.11 m

DF = DE +EF = 3.11m +6m = 9.11m

Hence, the length of the supporting wire attached to the roadway 18m from the middle is approximately 9.11m.

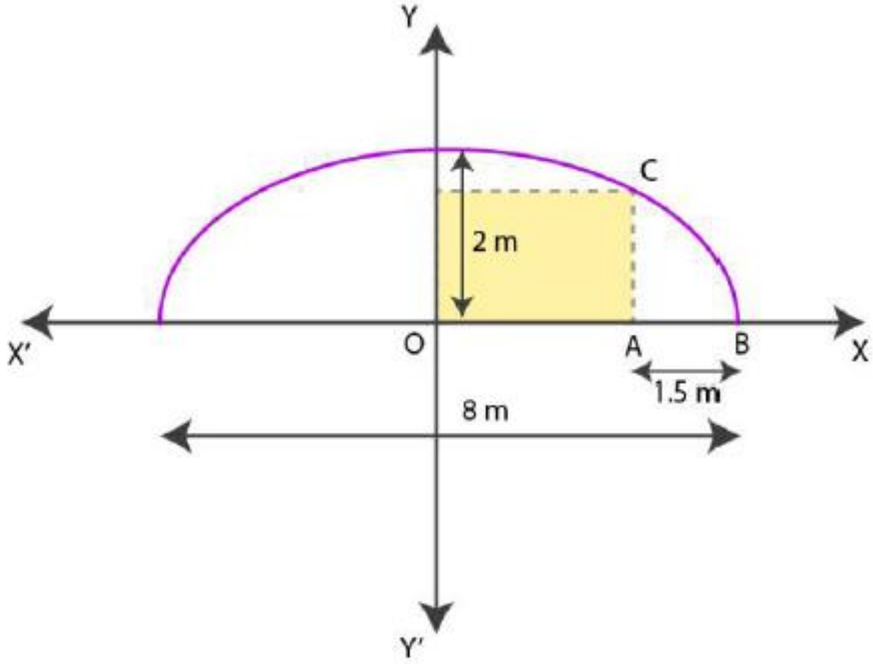
4. An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Solution:

Since, the height and width of the arc from the centre is 2m and 8m respectively, it is clear that the length of the major axis is 8m, while the length of the semi- minor axis is 2m.

The origin of the coordinate plane is taken as the centre of the ellipse, while the major axis is taken along the x-axis.

Hence, Diagrammatic representation of semi- ellipse is as follows:



The equation of the semi – ellipse will be of the from $x^2/16 + y^2/4 = 1$, $y \geq 0$ … (1

Let A be a point on the major axis such that AB = 1.5m.

Now draw AC ⊥ OB.

OA = (4 – 1.5)m = 2.5m

The x – coordinate of point C is 2.5

On substituting the value of x with 2.5 in equation (1), we get,

$(2.5)^2/16 + y^2/4 = 1$

$6.25/16 + y^2/4 = 1$

$y^2 = 4 (1 – 6.25/16)$

$= 4 (9.75/16)$

$= 2.4375$

$y = 1.56$ (approx.)

So, AC = 1.56m

Hence, the height of the arch at a point 1.5m from one end is approximately 1.56m.

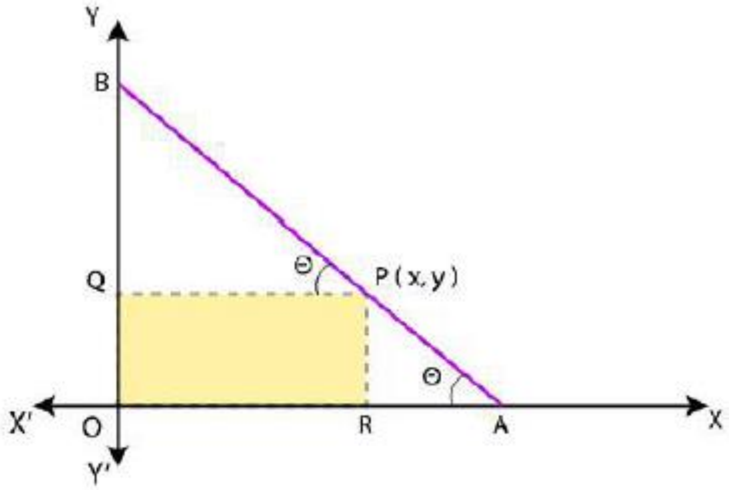
5. A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis.

Solution:

Let AB be the rod making an angle ϴ with OX and P(x,y) be the point on it such that

AP = 3cm.

Diagrammatic representation is as follows:



Then, PB = AB – AP = (12 – 3) cm = 9cm [AB = 12cm]

From P, draw PQ ⊥ OY and PR ⊥ OX.

In ΔPBQ, cos θ = PQ/PB = x/9

Sin θ = PR/PA = y/3

we know that, $\sin^2 \theta + \cos^2 \theta = 1$,

So,

$(y/3)^2 + (x/9)^2 = 1$ or

$x^2/81 + y^2/9 = 1$

Hence, the equation of the locus of point P on the rod is $x^2/81 + y^2/9 = 1$

6. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.

Solution:

The given parabola is $x^2 = 12y$.

On comparing this equation with $x^2 = 4ay$, we get,

4a = 12

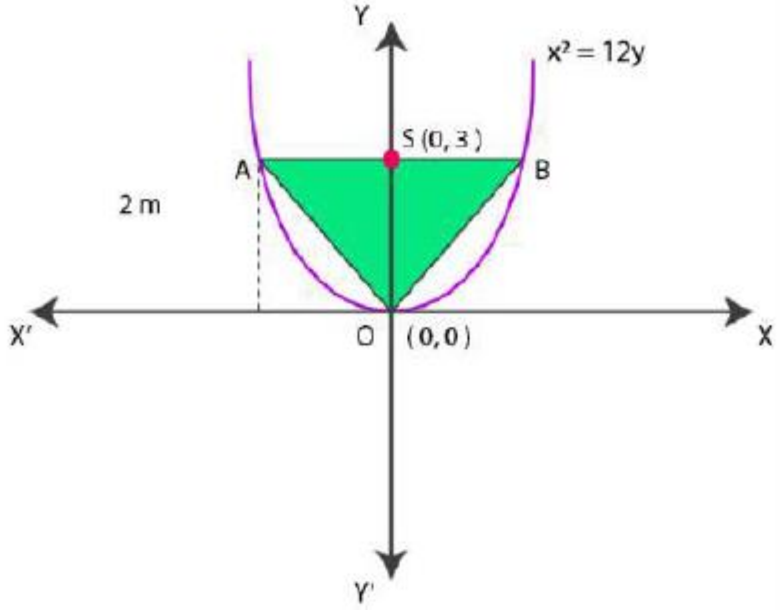
a = 12/4

= 3

The coordinates of foci are S(0,a) = S(0,3).

Now let AB be the latus rectum of the given parabola.

The given parabola can be roughly drawn as



At y = 3, $x^2 = 12(3)$

$x^2 = 36$

x = ±6

So, the coordinates of A are (-6, 3), while the coordinates of B are (6, 3)

Then, the vertices of ΔOAB are O(0,0), A (-6,3) and B(6,3).

By using the formula,

Area of ΔOAB = ½ [0(3-3) + (-6)(3-0) + 6(0-3)] $unit^2$

= ½ [(-6) (3) + 6 (-3)] $unit^2$

= ½ [-18-18] $unit^2$

= ½ [-36] $unit^2$

= 18 $unit^2$

∴ Area of ΔOAB is 18 $unit^2$

7. A man running a racecourse notes that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.
Find the equation of the posts traced by the man.

Solution:

Let A and B be the positions of the two flag posts and P(x, y) be the position of the man.

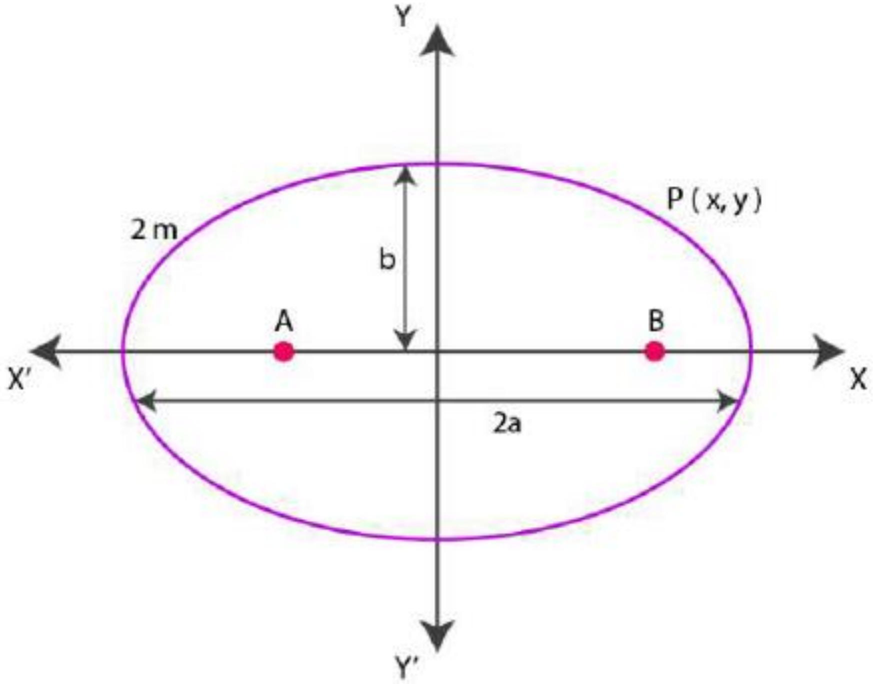
So, PA + PB = 10.

We know that if a point moves in plane in such a way that the sum of its distance from two fixed point is constant, then the path is an ellipse and this constant value is equal to the length of the major axis of the ellipse.

Then, the path described by the man is an ellipse where the length of the major axis is 10m, while points A and B are the foci.

Now let us take the origin of the coordinate plane as the centre of the ellipse, and taking the major axis along the x- axis,

The diagrammatic representation of the ellipse is as follows:



The equation of the ellipse is in the form of $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.

So, 2a = 10

a = 10/2

= 5

Distance between the foci, 2c = 8

c = 8/2

= 4

By using the relation, $c = \sqrt{(a^2 - b^2)}$, we get,

$4 = \sqrt{(25 - b^2)}$

$16 = 25 - b^2$

$b^2 = 25 - 1$

= 9

b = 3

Hence, equation of the path traced by the man is $x^2/25 + y^2/9 = 1$

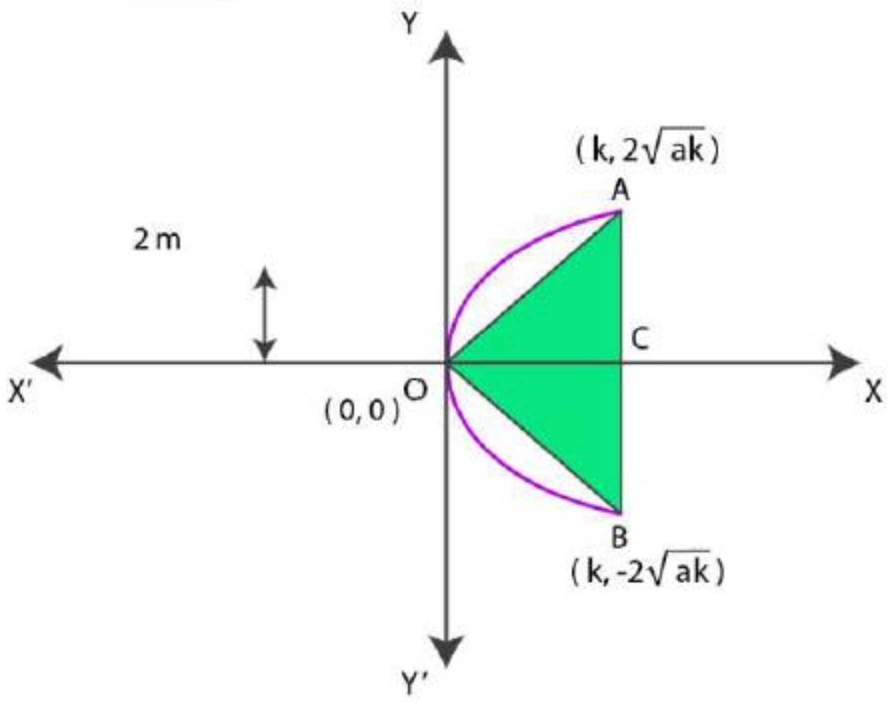
8. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

Solution:

Let us consider OAB be the equilateral triangle inscribed in parabola $y^2 = 4ax$.

Let AB intersect the x – axis at point C.

Diagrammatic representation of the ellipse is as follows:



Now let OC = k

From the equation of the given parabola, we have,

So, $y^2 = 4ak$

$y = \pm 2\sqrt{ak}$

The coordinates of points A and B are $(k, 2\sqrt{ak})$, and $(k, -2\sqrt{ak})$

$AB = CA + CB$

$= 2\sqrt{ak} + 2\sqrt{ak}$

$= 4\sqrt{ak}$

Since, OAB is an equilateral triangle, $OA^2 = AB^2$.

Then,

$k^2 + (2\sqrt{ak})^2 = (4\sqrt{ak})^2$

$k^2 + 4ak = 16ak$

$k^2 = 12ak$

$k = 12a$

Thus, $AB = 4\sqrt{ak} = 4\sqrt{(a \times 12a)}$

$= 4\sqrt{12a^2}$

$= 4\sqrt{(4a \times 3a)}$

$= 4(2)\sqrt{3}a$

$= 8\sqrt{3}a$

Hence, the side of the equilateral triangle inscribed in parabola $y^2 = 4ax$ is $8\sqrt{3}a$.