RD SHARMA Solutions for Class 9 Maths Chapter 12 - Congruent Triangles

Chapter 12 - Congruent Triangles Exercise 12.85

Question 1

If $\triangle ABC \cong \triangle LKM$, then side of $\triangle LKM$ equal to side AC of $\triangle ABC$ is

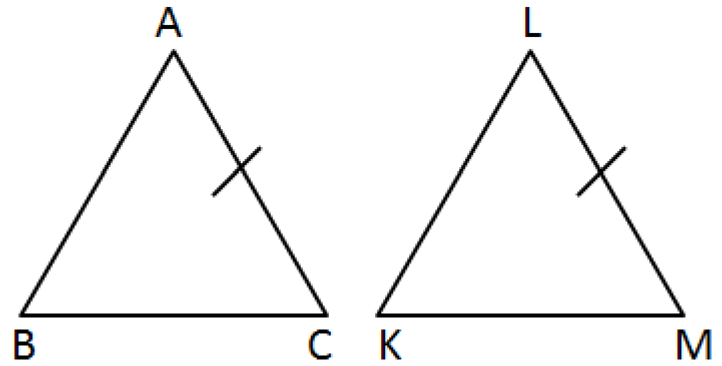
(a) LK

(b) KM

(c) LM

(d) None of these

Solution 1



If $\triangle ABC \cong \triangle LKM$, then from figure $AC = LM$.

Hence, correct option is (c).

Question 2

If $\triangle ABC \cong \triangle ACB$, then $\triangle ABC$ is isosceles with

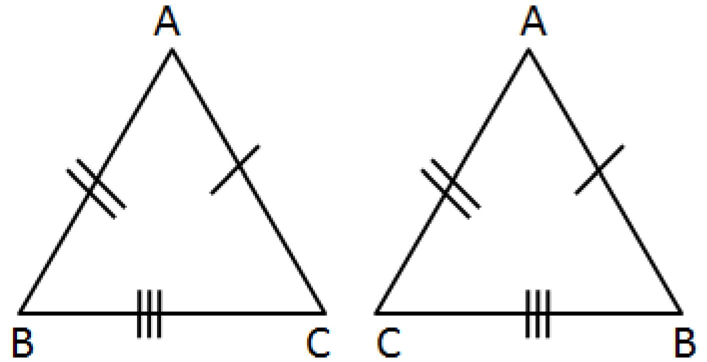
(a) $AB = AC$

(b) $AB = BC$

(c) $AC = BC$

(d) None of these

Solution 2

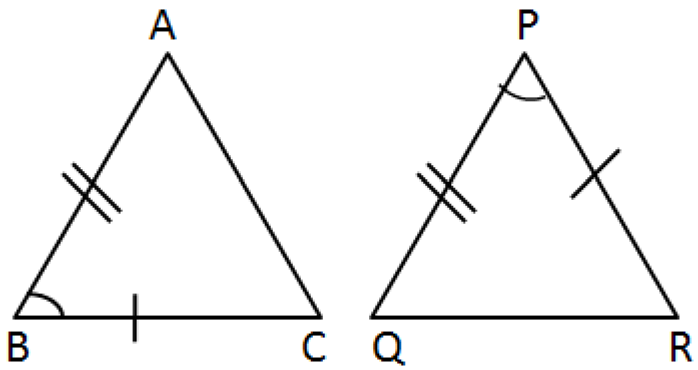


$\triangle ABC \cong \triangle ACB$
$\Rightarrow AB = AC$
or
$AC = AB$
So, in $\triangle ABC$ is isosceles with $AB = AC$.
Hence, correct option is (a).

Question 3
If $\triangle ABC \cong \triangle PQR$ and $\triangle ABC$ is not congruent to $\triangle RPQ$, then which of the following is not true:
(a) $BC = PQ$
(b) $AC = PR$
(c) $AB = PQ$
(d) $QR = BC$

Solution 3
$\triangle ABC \cong \triangle PQR$
$\Rightarrow AB = PR, AC = PR, BC = QR$
$\triangle ABC \ncong \triangle RQP$
$\Rightarrow AB \neq QR, AC \neq RP, BC \neq PQ$
So, option (a) is not true.
Hence, correct option is (a).

Question 4
In triangles ABC and PQR three equality relations between some parts are as follows :
$AB = QP$, $\angle B = \angle P$ and $BC = PR$
State which of the congruence conditions applies :
(a) SAS
(b) ASA
(c) SSS
(d) RHS

Solution 4


From given conditions, we have
$AB = PQ$
$BC = PR$
And the angle between these sides are also equal
i.e. $\angle B = \angle P$
So SAS property.
Hence, correct option is (a).

Question 5

In triangles ABC and PQR, if $\angle A = \angle R$, $\angle B = \angle P$ and $AB = RP$, then which one of the following congruence conditions applies :

(a) SAS

(b) ASA

(c) SSS

(d) RHS

Solution 5

From given conditions,

$\angle B = \angle P$

$\angle A = \angle R$

And the side containing then is also equal

i.e. $AB = PR$

So ASA property.

Hence, correct option is (b).

Question 6

If $\triangle PQR \cong \triangle EFD$, then $ED =$

(a) PQ

(b) QR

(c) PR

(d) None of these

Solution 6

$\triangle PQR \cong \triangle EFD$,

$\Rightarrow ED = PR$ (congruent sides of congruent triangles)

Hence, correct option is (c).

Question 7

If $\triangle PQR \cong \triangle EFD$, then $\angle E =$

(a) $\angle P$

(b) $\angle Q$

(c) $\angle R$

(d) None of these

Solution 7

$\triangle PQR \cong \triangle EFD$,

$\Rightarrow \angle E = \angle P$ (congruent angles of congruent triangles)

Hence, correct option is (a).

Question 8

In a $\triangle ABC$, if $AB = AC$ and BC is produced to D such that $\angle ACD = 100°$, then $\angle A =$

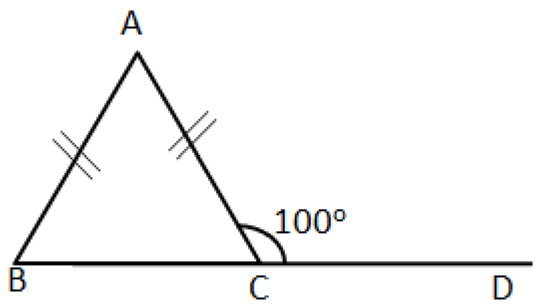
(a) 20°

(b) 40°

(c) 60°

(d) 80°

Solution 8



AB = AC

$\Rightarrow \angle ABC = \angle ACB$ (Isoscles $\triangle$ Property)

$\angle ACB = 180° - 100° = 80°$

$\Rightarrow \angle ABC = \angle ACB = 80°$

$\angle A = 180° - \angle ACB - \angle ABC = 180° - 80° - 80° = 20°$

Hence, correct option is (a).

Question 9

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, then the measure of vertex angle of the triangle is

(a) 100°

(b) 120°

(c) 110°

(d) 130°

Solution 9

Let $\triangle ABC$ be an isosceles triangle with

vertex angle $= \angle A$ and base angles $= \angle B$ and $\angle C$

Now, $\angle A = 2(\angle B + \angle C)$

$\Rightarrow \frac{\angle A}{2} = \angle B + \angle C$ ....(1)

Also in $\triangle ABC$,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + (\angle B + \angle C) = 180$

$\Rightarrow \angle A + \frac{\angle A}{2} = 180°$ ....[From (1)]

$\Rightarrow \frac{3}{2}\angle A = 180°$

$\Rightarrow \angle A = \frac{180° \times 2}{3}$

$\Rightarrow \angle A = 120°$

Hence, correct option is (b).

Question 10

Which of the following is not a criterion for congruence of triangles?

(a) SAS

(b) SSA

(c) ASA

(d) SSS

Solution 10

If two triangles have two congruent sides and a congruent non – included angle, then $\triangle$s are not necessarily congruent. This is why there is no 'Side Side angle' i.e. SSA postulate.

Hence, correct option is (b).

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Question 11

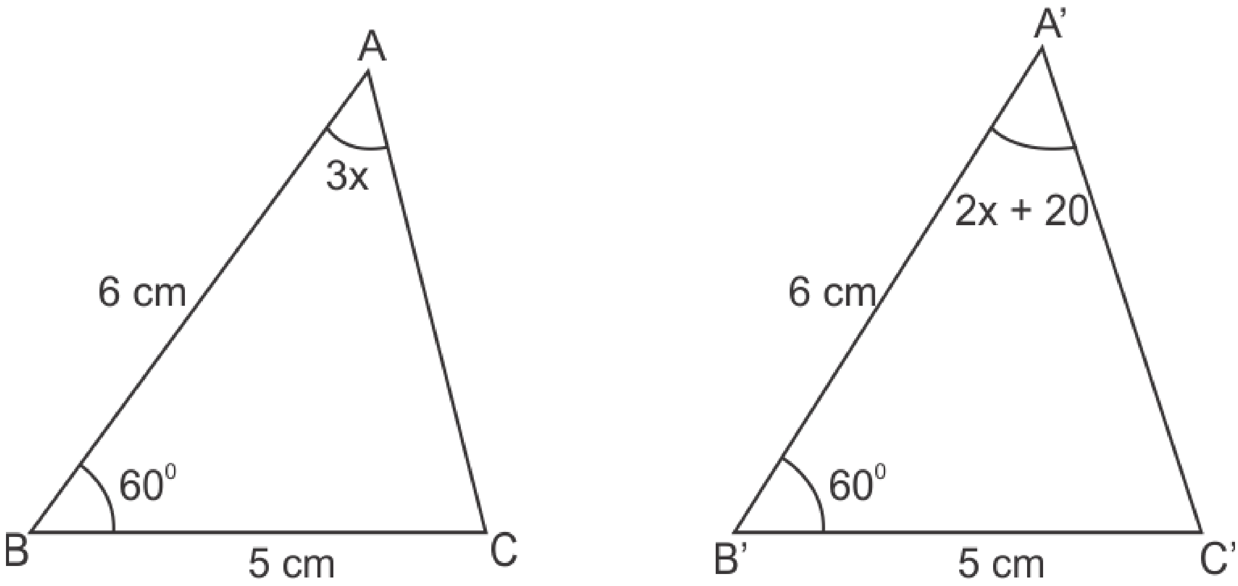
In the figure, the measure of $\angle B'A'C'$ is

(a) 50°

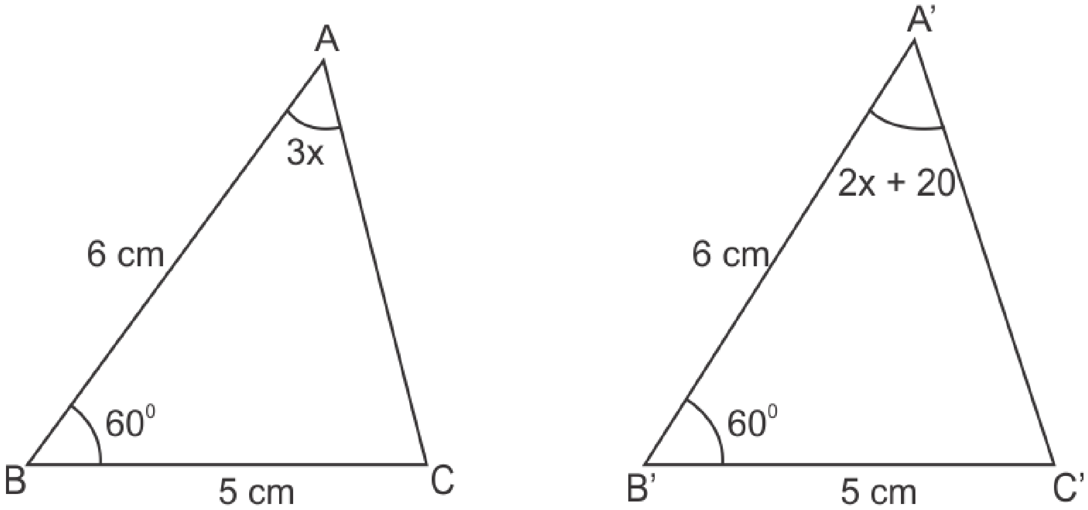
(b) 60°

(c) 70°

(d) 80°



Solution 11



In $\triangle ABC$ and $\triangle A'B'C'$,

$AB = A'B'$

$BC = B'C'$

$\angle ABC = \angle A'B'C'$

So $\triangle ABC \cong \triangle A'B'C'$ by SAS creterion

$\Rightarrow \angle BAC = \angle B'A'C'$

$\Rightarrow 3x = 2x + 20$

$\Rightarrow x = 20°$

$\Rightarrow 2x + 20 = 2 \times 20 + 20 = 60° = \angle B'A'C'$

Hence, correct option is (b).

Question 12

If ABC and DEF are two triangles such that $\triangle ABC \cong \triangle FDE$ and $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$. Then, which of the following is true?

(a) $DF = 5$ cm, $\angle F = 60°$

(b) $DE = 5$ cm, $\angle E = 60°$

(c) $DF = 5$ cm, $\angle E = 60°$

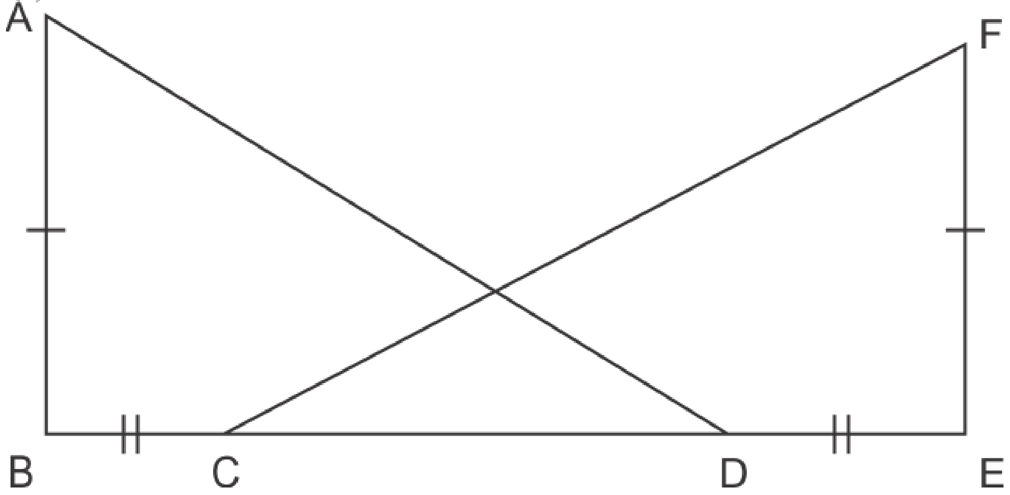
(d) $DE = 5$ cm, $\angle D = 40°$

Solution 12

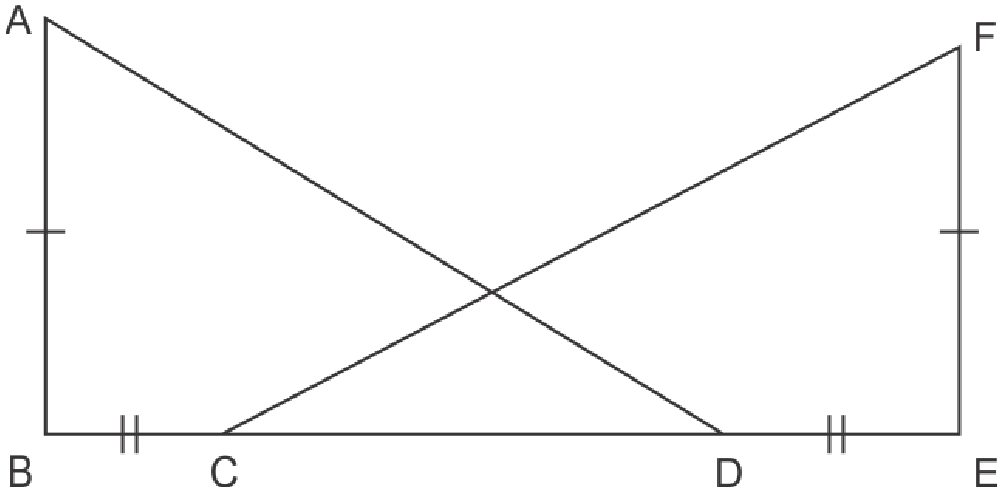
In $\triangle ABC$,
$\angle C = 180° - \angle A + \angle B = 180° - 80° - 40° = 60°$
$\triangle ABC \cong \triangle FDE$
$\Rightarrow AB = FD = 5$ cm
$\Rightarrow \angle B = \angle D = 40°$
$\Rightarrow \angle A = \angle F = 80°$
$\Rightarrow \angle C = \angle E = 60°$
$\Rightarrow DF = FD = 5$cm and $\angle E = 60°$
Hence, correct option is (c).

Question 13

In the figure, $AB \perp BE$ and $FE \perp BE$. If $BC = DE$ and $AB = EF$, then $\triangle ABD$ is congruent to
(a) $\triangle EFC$
(b) $\triangle ECF$
(c) $\triangle CEF$
(d) $\triangle FEC$



Solution 13

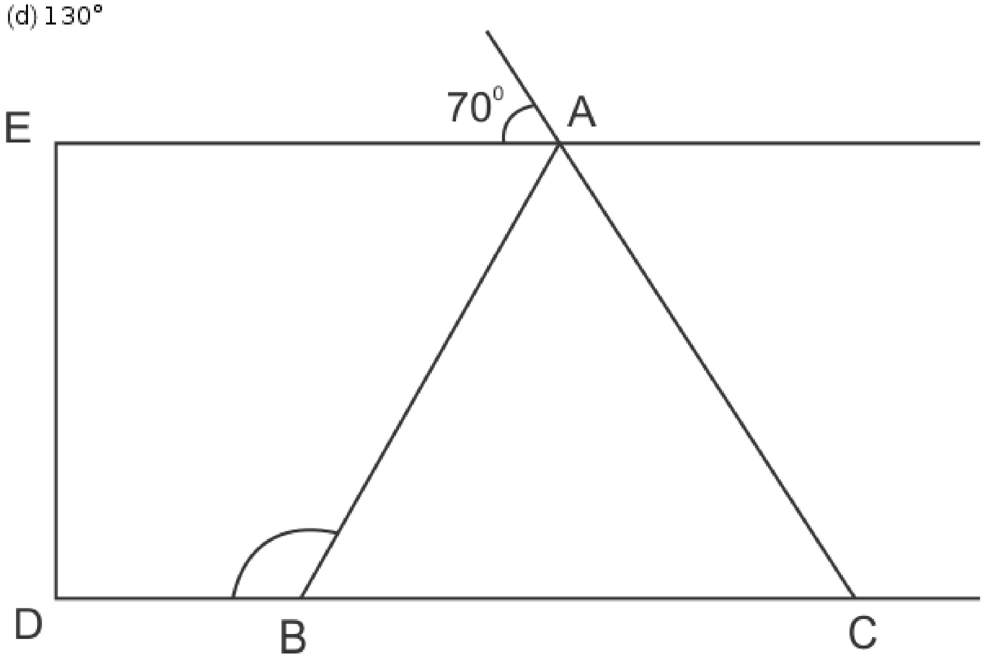


$AB = EF$
$BC = DE$
$BC + CD = DE + CD$ (adding CD both sides)
$BC + CD = BD,\ DE + CD = CE$
So $BD = CE$
Now Consider $\triangle ABD$, & $\triangle FEC$
$AB = FE$
$BD = EC$
$\angle ABD = \angle FEC = 90°$
So $\triangle ABD \cong \triangle FEC$ by SAS creterion.
Hence, correct option is (d).

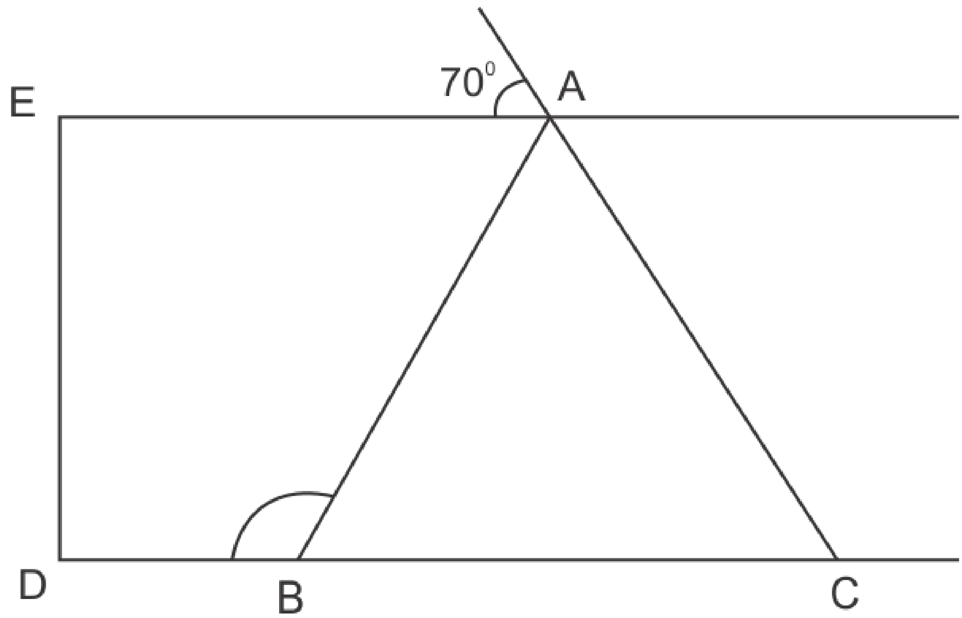
Question 14

In figure, if $AE \parallel DC$ and $AB = AC$, the value of $\angle ABD$ is

(a) 70°
(b) 110°
(c) 120°
(d) 130°



Solution 14



If AE || DC and AC is transversal,

then $\angle FAC = 70°$ (Opposite angles)

Also $\angle FAC = \angle ACB = 70°$ (Alternate angles)

Since $AB = AC$, $\triangle ABC$ is isosceles.

So $\angle ABC = \angle ACB$

$\Rightarrow \angle ABC = 70°$

Now $\angle ABD = 180° - \angle ABC = 180° - 70° = 110°$

Hence, correct option is (b).

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Question 15

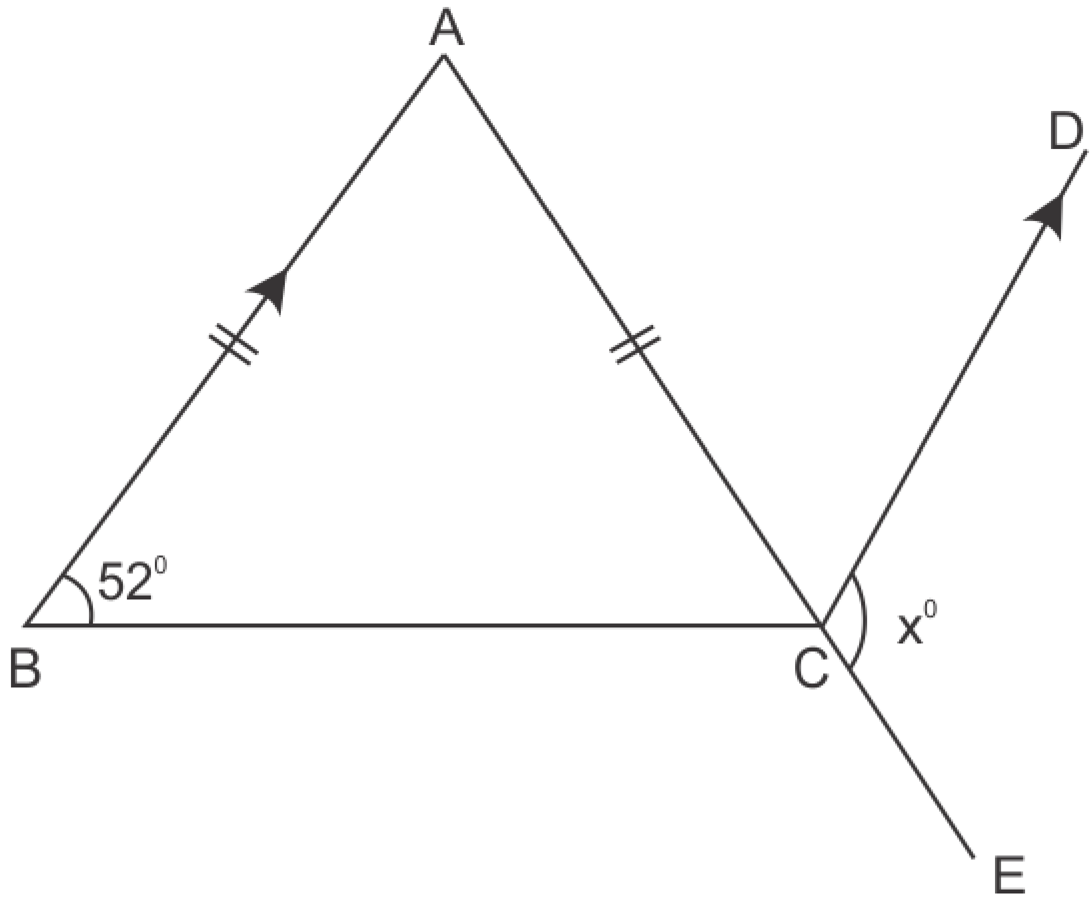
In the figure, ABC is an isosceles triangle whose side AC is produced to E. Through C, CD is drawn parallel to BA. The value of x is

(a) 52°

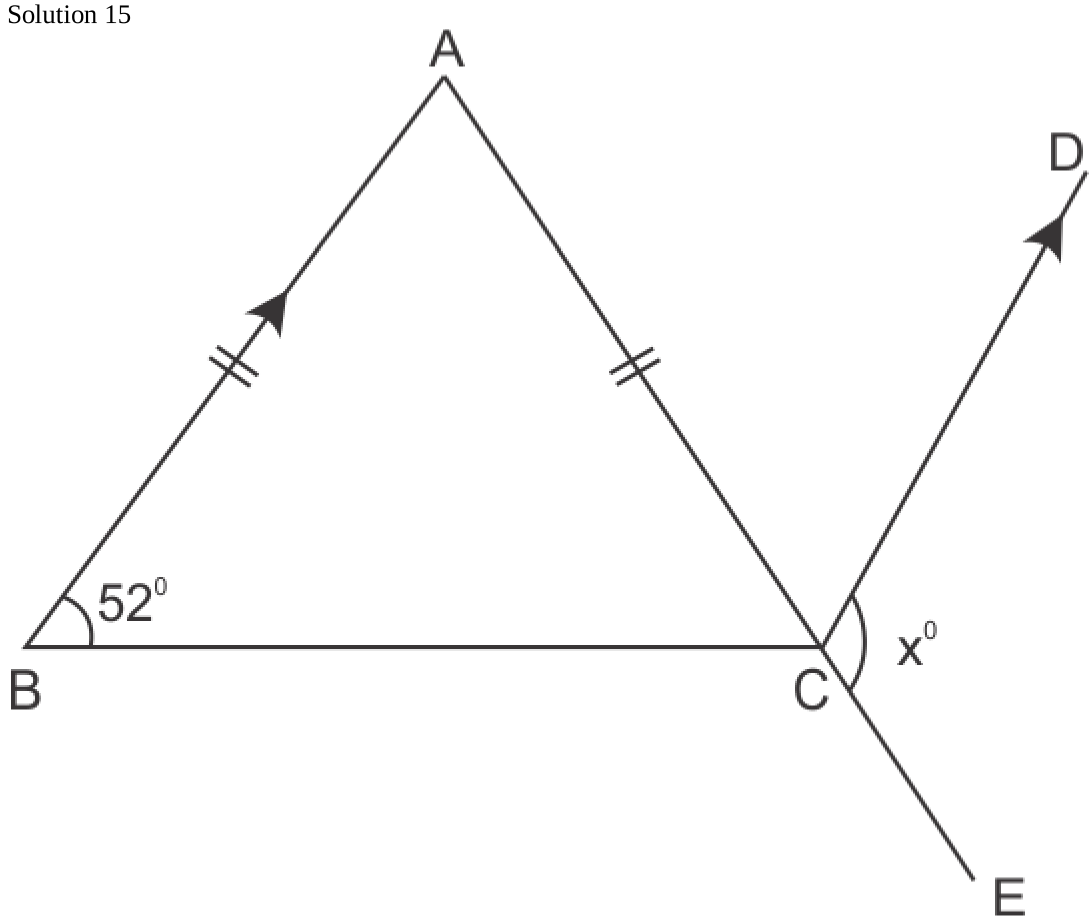
(b) 76°

(c) 156°

(d) 104°

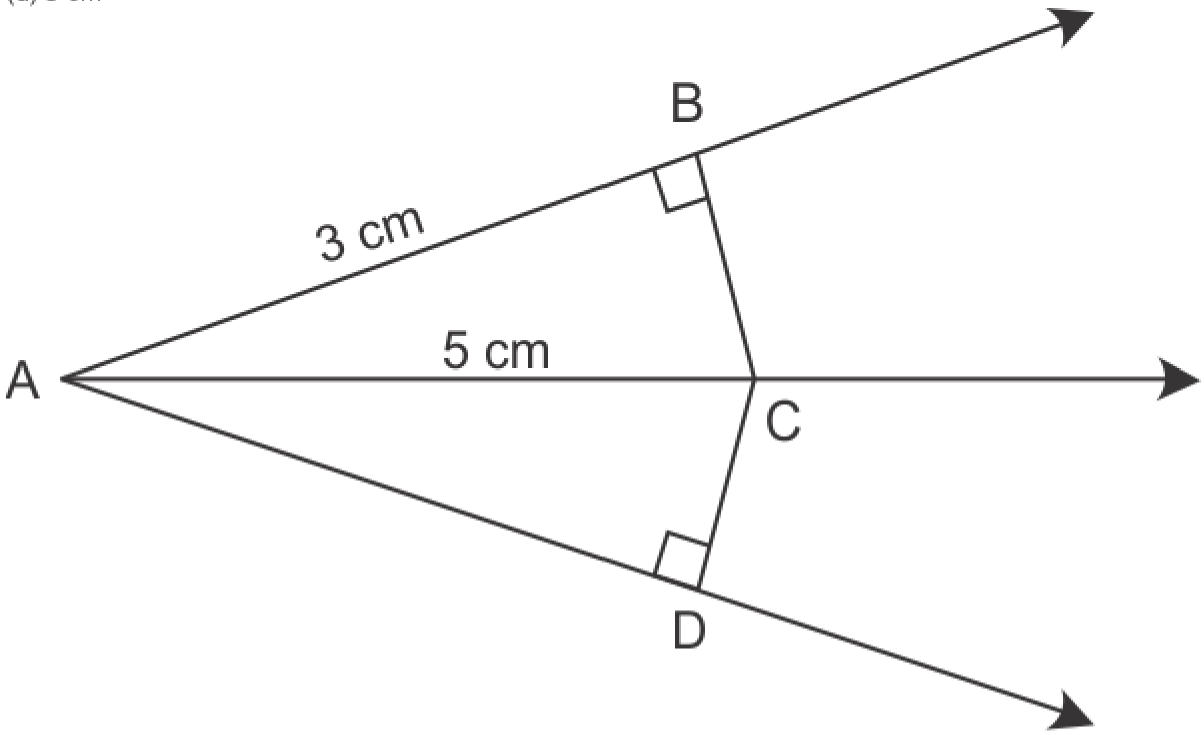


Solution 15

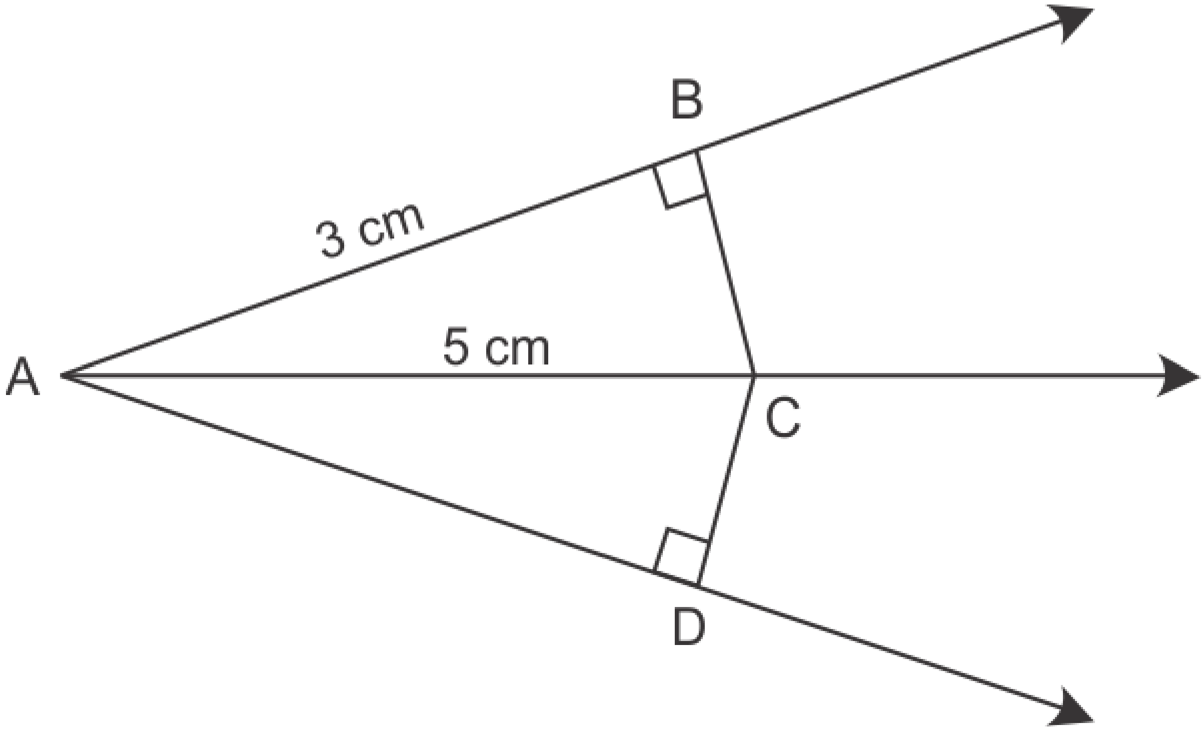


$\triangle ABC$ is isosceles
$\angle ABC = \angle ACB = 52°$
then $\angle BAC = 180° - 52° - 52° = 76°$
If $AB \parallel CD$, AC is transversal
then $\angle BAC = \angle ACD$ (Alternate angles)
$\Rightarrow \angle ACD = 76°$
Now from figure,
$\angle ACD + x° = 180°$
$\Rightarrow x° = 180° - 76°$
$\Rightarrow x° = 104°$
Hence, correct option is (d).

Question 16
In figure, if AC is bisector of $\angle BAD$ such that $AB = 3$ cm and $AC = 5$ cm, then $CD =$
(a) 2 cm
(b) 3 cm
(c) 4 cm
(d) 5 cm



Solution 16



Consider $\triangle ABC$ and $\triangle ADC$

$\angle ABC = \angle ADC = 90°$

$\angle BAC = \angle CAD$ (AC is bisector of $\angle A$)

Also if two angles are equal, then the third angle will also be equal.

$\Rightarrow \angle BCA = \angle DCA$

Now, $AC = AC$ (common)

So by ASA property, $\triangle ABC \cong \triangle ADC$

$\Rightarrow BC = CD$

And, $BC = \sqrt{AC^2 - AB^2} = \sqrt{25 - 9} = 4$ cm

$\Rightarrow CD = 4$ cm

Hence, correct option is (c).

Question 17

D, E, F are the mid – points of the sides BC, CA and AB respectively of $\triangle ABC$. Then $\triangle DEF$ is congruent to triangle

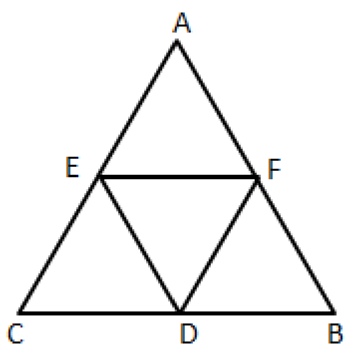
(a) ABC

(b) AEF

(c) BFD, CDE

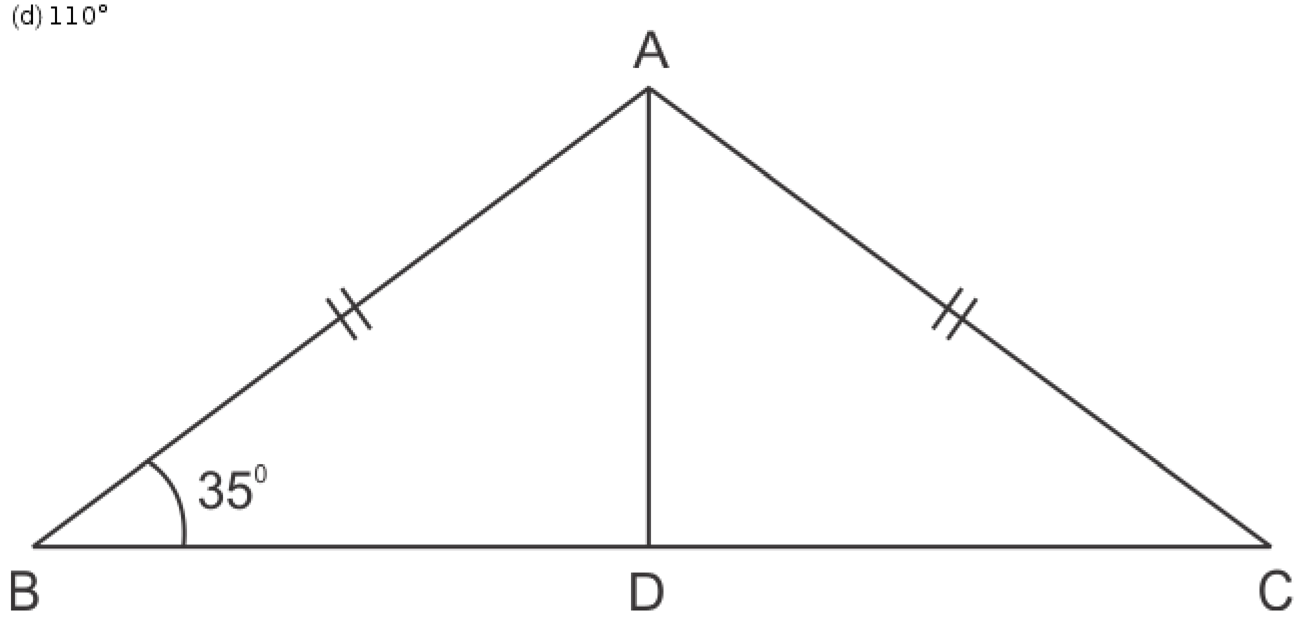
(d) AFE, FBD, EDC

Solution 17

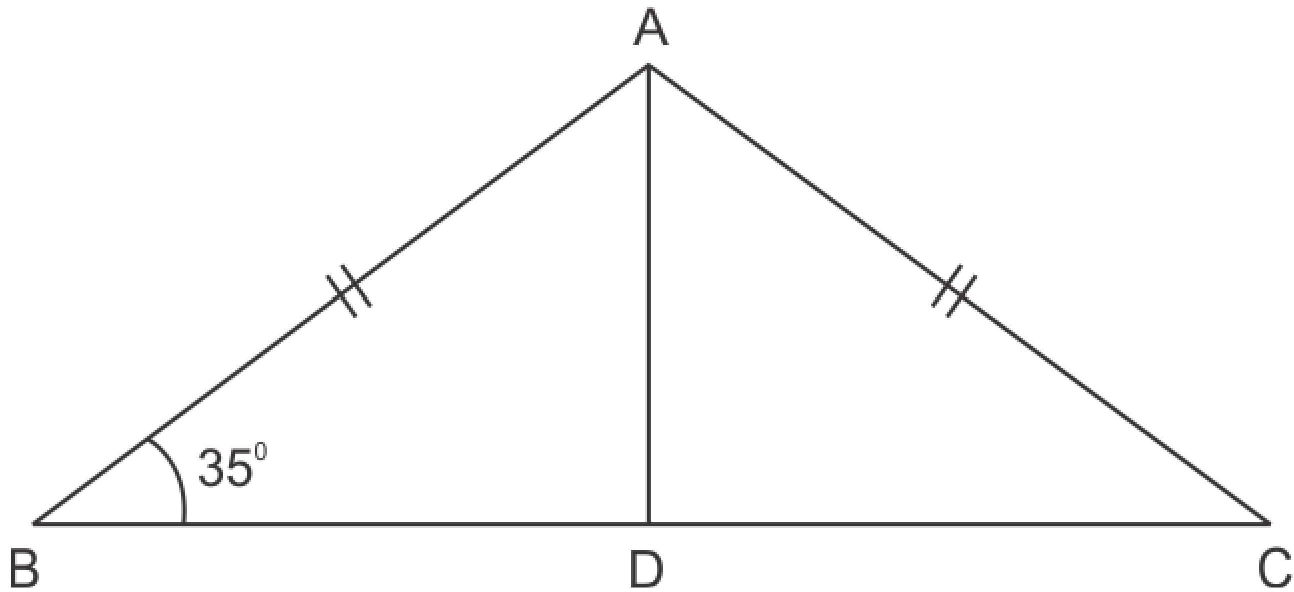


In any triangle, a line joining the mid – points of any two sides is parallel to the third side.
$\Rightarrow EF \parallel BC \; EF \parallel DC$ and BD
Similiarly $DF \parallel AC$.
$\Rightarrow DF \parallel AE$ and EC
Also $DE \parallel AB$.
$\Rightarrow DE \parallel AF$ and BF
From this information it is clear that EFDC, EFBD, EAFD
are the parallelogram by property.
Now consider one parallelogram EFDC
Consider $\triangle DEF$ and $\triangle EDC$
$DE = ED$ (Common)
$\angle DEF = \angle EDC$
$\angle EDF = \angle DEC$ (ASA property)
$\Rightarrow \triangle DEF \cong \triangle EDC$
Similiarly in Parallelogram EAFD,
$\triangle DEF \cong \triangle AFE$
And in Parallelogram EFBD
$\triangle DEF \cong \triangle FBD$
Hence, corect option is (d).
Note: Option (d) modified.

Question 18
ABC is an isosceles triangle such that $AB = AC$ and AD is the medium to base BC. Then, $\angle BAD =$
(a) 55°
(b) 70°
(c) 35°
(d) 110°



Solution 18



If AD is the median, then D is the mid – point of BC.

$\Rightarrow BD = DC$

So consider $\triangle ADB$ and $\triangle ADC$

$AD = AD$ (Common)

$DB = DC$

$BA = CA$

So by SSS, $\triangle ADB \cong \triangle ADC$

Now $\angle B = \angle C = 35°$

$\Rightarrow \angle BAD = \angle DAC$

So in $\triangle ABC$,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2\angle BAD + 35° + 35° = 180°$

$\Rightarrow 2\angle BAD = 110°$

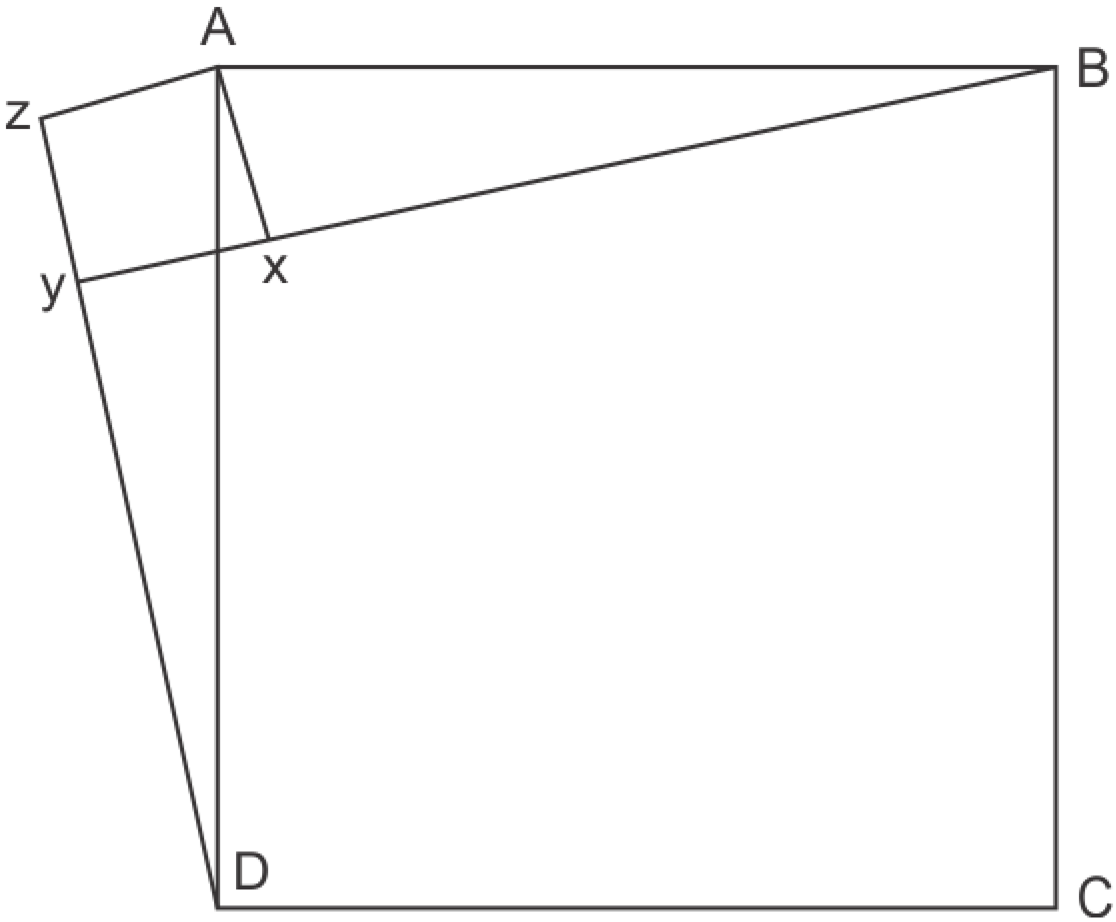
$\Rightarrow \angle BAD = 55°$

Hence, correct option is (a).

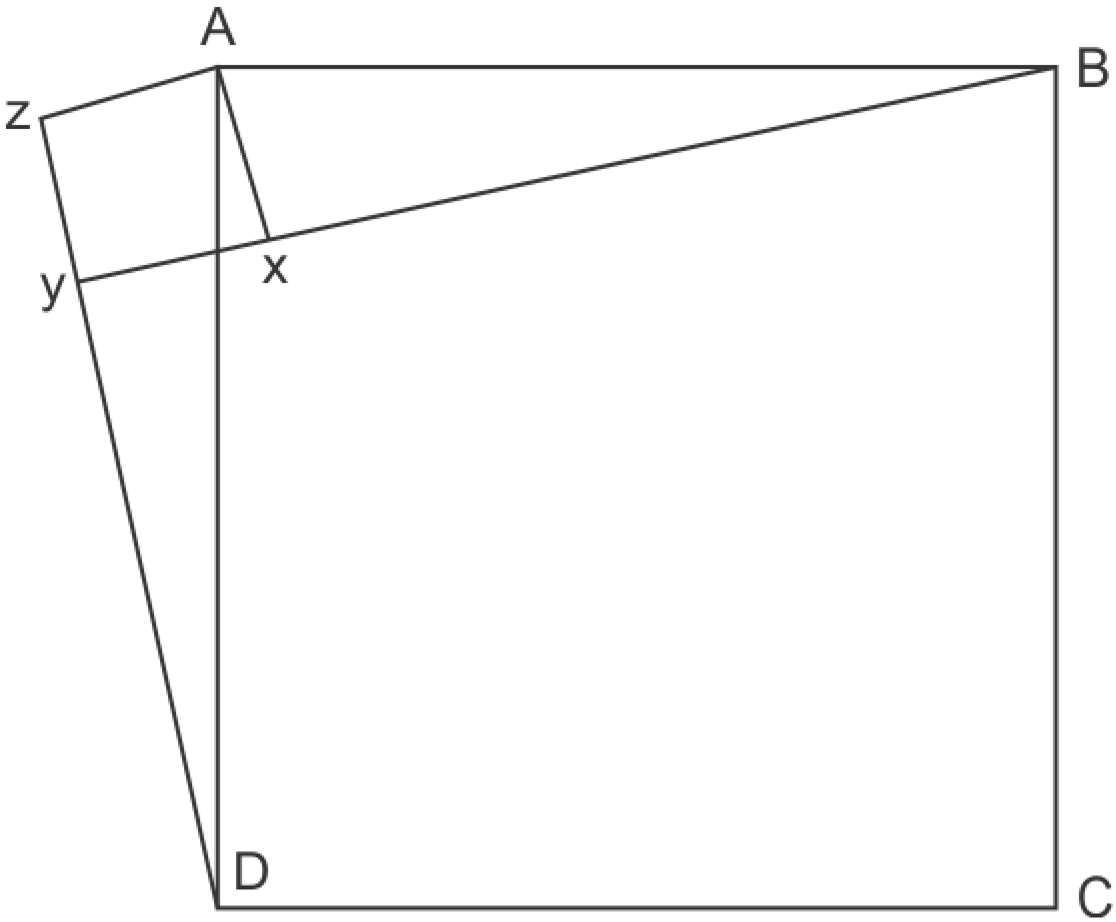
Question 19

In figure, X is a point in the interior of square ABCD. AXYZ is also a square. If DY = 3 cm and AZ = 2 cm, then BY =

(a) 5 cm
(b) 6 cm
(c) 7 cm
(d) 8 cm



Solution 19



Consider $\triangle AZD$ and $\triangle AXB$

$AZ = AX = 2$ cm (AXYZ is a square)

$\angle AZD = \angle AXB = 90°$

$AD = AB$ (ABCD is a square)

So by RHS creterion, $\triangle AZD \cong \triangle AXB$

$\Rightarrow ZD = XB$

Now, $ZD = ZY + DY$

$= 2 \text{ cm} + 3 \text{ cm}$ $(ZY = AZ = 2 \text{ cm})$

$= 5$ cm

$\Rightarrow XB = 5$ cm

$\Rightarrow BY = YX + XB = 2 \text{ cm} + 5 \text{ cm} = 7$ cm

Hence, correct option is (c).

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Question 20

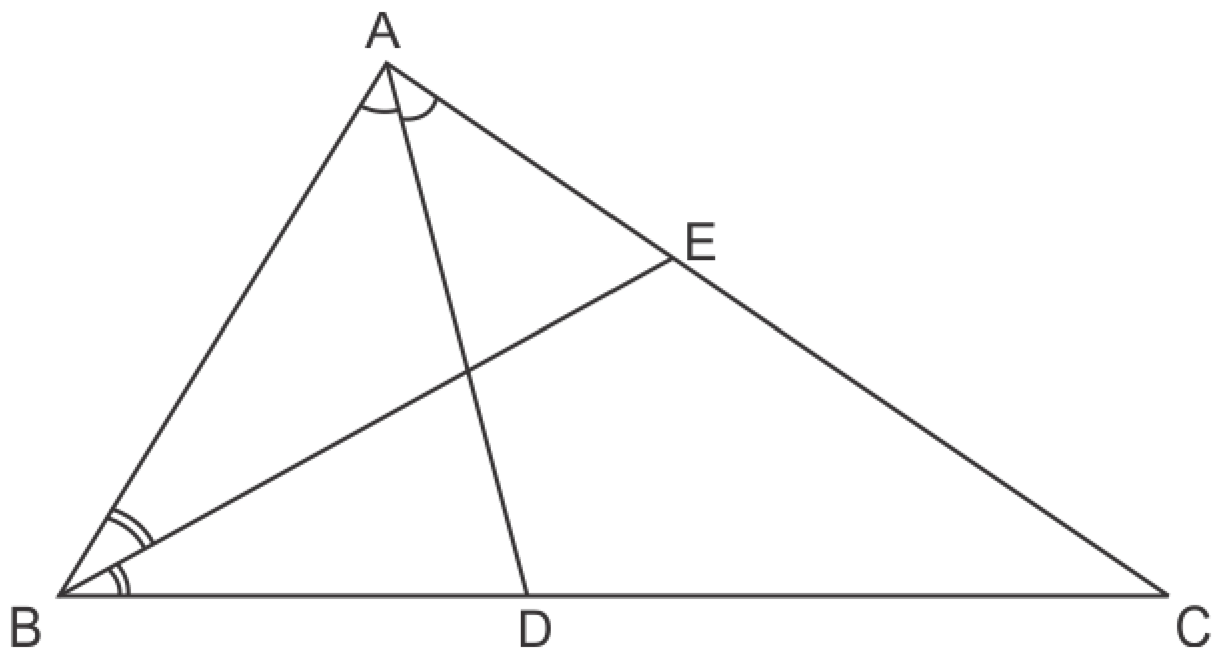
In figure, ABC is a triangle in which $\angle B = 2\angle C$. D is a point on side BC such that AD bisects $\angle BAC$ and $AB = CD$. BE is the bisector of $\angle B$. The measure of $\angle BAC$ is

(a) 72°

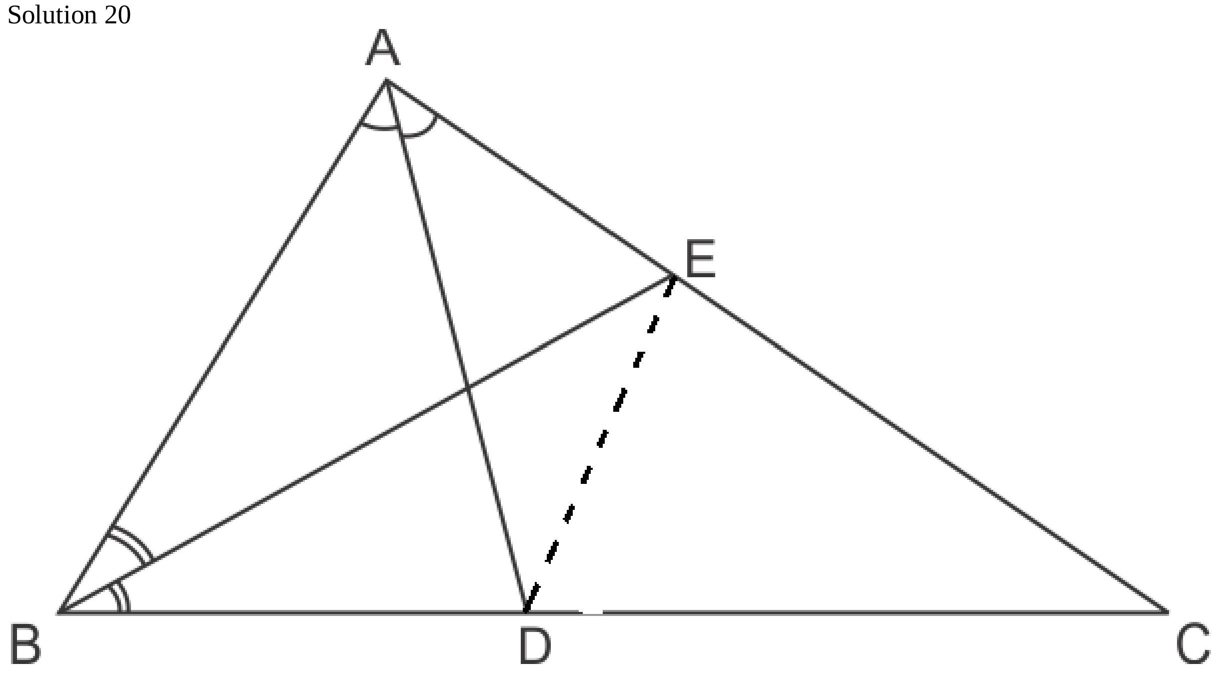
(b) 73°

(c) 74°

(d) 95°

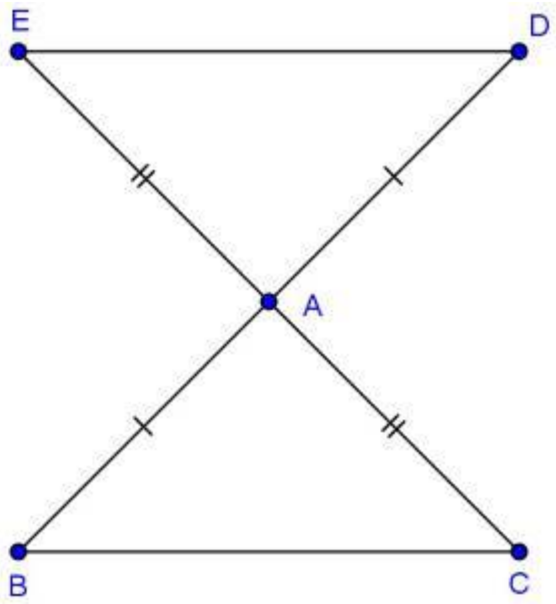


Solution 20



$\angle ABE = \angle EBC$ (BE is bisector of $\angle B$)
and $\angle C = \frac{\angle B}{2}$
$\Rightarrow \angle EBC = \angle ECB$
So $\triangle EBC$ is isosceles triangle.
$\Rightarrow EB = EC$ ....(1)
Now Consider $\triangle ABE$ and $\triangle DCE$
$AB = DC$ (Given)
$BE = CE$ [From (1)]
$\angle ABE = \angle DCE$ (From above data)
So $\triangle ABE \cong \triangle DCE$ by SAS property
$\Rightarrow AE = DE$
$\angle BAE = \angle CDE = \angle A$
Now consider $\triangle AED$,
$AE = DE$ (above proved)
$\Rightarrow \triangle AED$ is isosceles triangl
$\Rightarrow \angle EAD = \angle EDA = \frac{\angle A}{2}$ (AD is Bisector of $\angle A$) ....(2)
Now, consider $\triangle ABC$,
$\angle A + \angle B + C = 180°$
$\Rightarrow \angle A + 2\angle C + \angle C = 180°$ $(\angle B = 2\angle C)$
$\Rightarrow \angle A + 3\angle C = 180°$ ....(3)
Consider $\triangle ADC$,
$\Rightarrow \frac{\angle A}{2} + \angle ADC + \angle C = 180°$
$\Rightarrow \frac{\angle A}{2} + (\angle EDA + \angle CDE) + \angle C = 180°$
$\Rightarrow \frac{\angle A}{2} + \frac{\angle A}{2} + \angle A + \angle C = 180°$
$\Rightarrow \angle A + \angle A + \angle C = 180°$
$\Rightarrow 2\angle A + \angle C = 180°$ ....(4)
Right hand side of equations (3) and (4) are equal, hence Left hand side.
$\Rightarrow \angle A + 3\angle C = 2\angle A + \angle C$
$\Rightarrow \angle A = 2\angle C$
Substituting in equation (3),
$2\angle C + 3\angle C = 180°$
$\Rightarrow 5\angle C = 180°$
$\Rightarrow \angle C = 36°$
$\Rightarrow \angle A = 2 \times 36° = 72°$
Hence, correct option is (a).

Chapter 12 - Congruent Triangles Exercise Ex. 12.1
Question 1
In fig., the sides BA and CA have been produced such that BA = AD and CA = AE.



Solution 1

In $\triangle ADE$ and $\triangle ABC$

$AD = AB$ [given]

$AE = AC$ [given]

$\angle DAE = \angle BAC$ [vertically opposite angles]

So, by S.A.S congruence criterion

$\triangle ABC \cong \triangle ADE$

$\therefore \angle EDA = \angle CBA$ [c.p.c.t]

and $\angle DEA = \angle BCA$ [c.p.c.t]

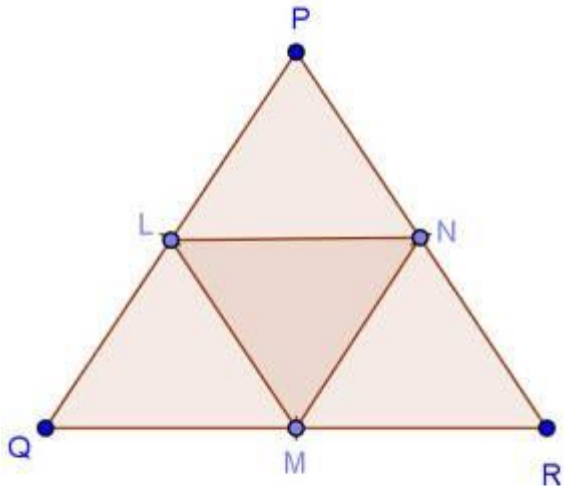
But they are also alternate angles

$\therefore DE \parallel BC$

Question 2

In a $\triangle PQR$, if $PQ = QR$ and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that $LN = MN$.

Solution 2



In $\triangle PQR$

$\because PQ = QR$

$\therefore \angle QPR = \angle PRQ$ [Angle opposite to equal sides are equal]

$In\ \triangle PNL$ and $\triangle RNM$

$\angle LPN = \angle MRN$ $[\because \angle QPR = \angle PRQ]$

$Also, PQ = QR$

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}QR$

$\Rightarrow PL = MR$

$PN = NR$ [given]

By SSS congurence criterion

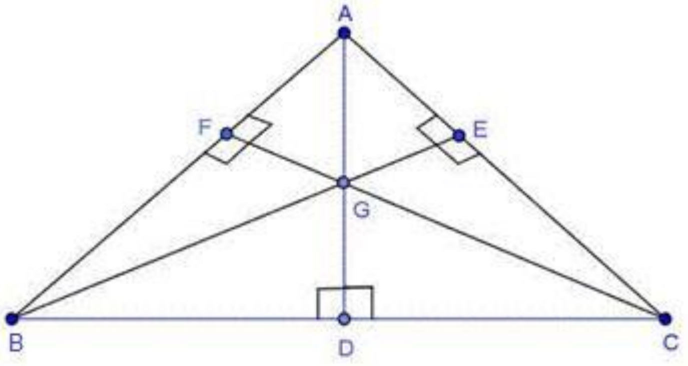
$\triangle PNL \cong \triangle RNM$

$\therefore LN = MN$ $[c.p.c.t]$

Question 3

Prove that the medians of an equilateral triangle are equal.

Solution 3



In ΔCBF and ΔBCE

$\angle B = \angle C = 60°$ [Angles of an equilateral triangle]
$BC = BC$ [common]
$BF = EC$ $\left[\because AB = AC \therefore \frac{1}{2}AB = \frac{1}{2}AC\right]$

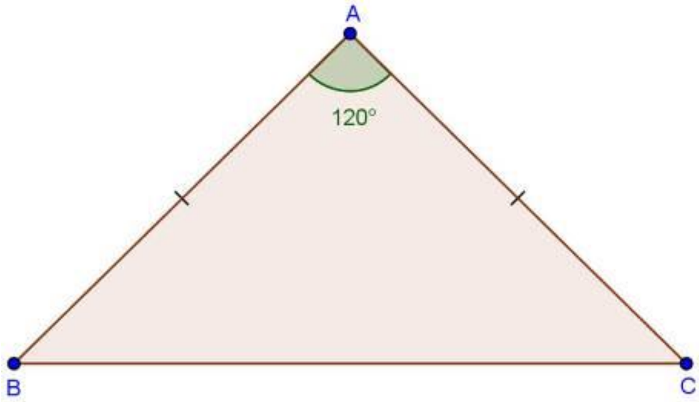
By SAS, $\Delta CBF \cong \Delta BCE$
$\therefore BE = CF$ [$c.p.c.t$]

Similarly AD = BE
$\therefore AD = BE = CF.$
Hence proved.

Question 4

In a Δ ABC, if $\angle A = 120°$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Solution 4



$In \triangle ABC$

$\because AB = AC$

$\Rightarrow \angle B = \angle C$ [Angle opposite to equal sides are equal]

Now in $\triangle ABC$

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle B + \angle C = 180^\circ - 120^\circ = 60^\circ$

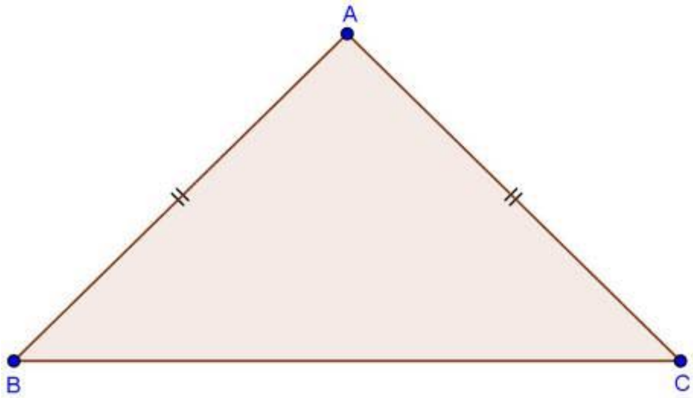
$\Rightarrow 2\angle B = 60^\circ$

$\Rightarrow \angle B = \angle C = 30^\circ$

Question 5

In a $\triangle ABC$, if $AB = AC$ and $\angle B = 70^\circ$, find $\angle A$.

Solution 5



$In \triangle ABC$

$\because AB = AC$

$\Rightarrow \angle B = \angle C$ [Angle opposite to equal sides are equal]

$\Rightarrow \angle B = \angle C = 70^\circ$ [given]

$\therefore \angle A = 180^\circ - \angle B - \angle C$ [Angle sum property]

$= 180^\circ - 70^\circ - 70^\circ$

$= 40^\circ$

Question 6

The vertical angle of an isosceles triangle is 100º. Find its base angles.

Solution 6

Given, ABC is an isosceles triangle with AB = AC and $\angle A = 100°$

Since, AB = AC, $\angle B = \angle C$.

Using angle sum property, we have:

$\angle A + \angle B + \angle C = 180°$

$100° + 2\angle B = 180°$

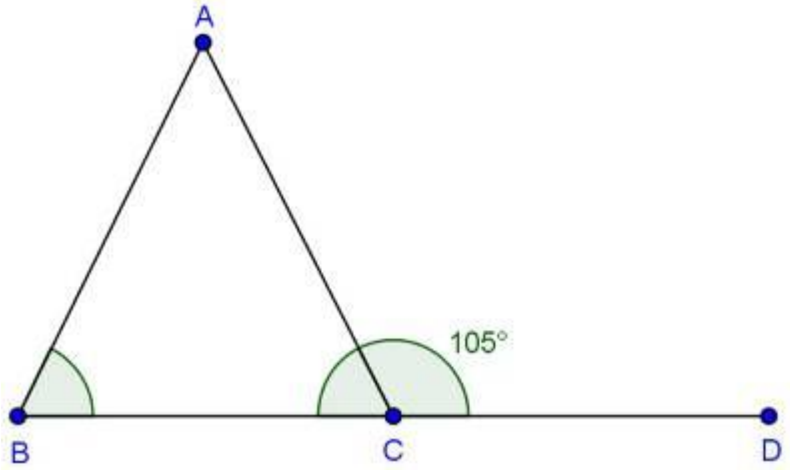
$2\angle B = 180° - 100° = 80°$

$\angle B = 40°$

Thus, the base angles of the isosceles triangle are 40° each.

Question 7

In fig., AB = Ac and $\angle$ACD = 105°, find $\angle$BAC.



Solution 7

Since $AB = AC$

$\therefore \angle ABC = \angle ACB$

Now, $\angle ACB = 180° - 105°$ [linear pair]

$\Rightarrow \angle ACB = 75°$

$\therefore \angle ABC = 75°$

$\therefore \angle BAC = 180° - \angle ABC - \angle ACB$ [Angle sum property of $\triangle$]

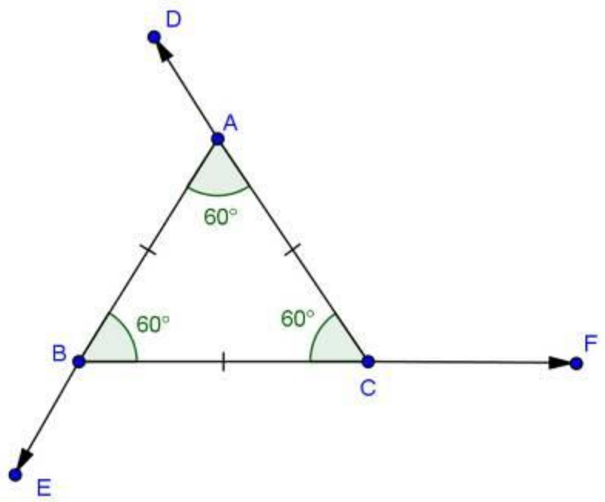
$= 180° - 75° - 75°$

$= 30°$

Question 8

Find the measure of each exterior angle of an equilateral triangle.

Solution 8



$\angle ACF = \angle ABC + \angle BAC$ $\qquad$ [$\because$ Exterior angle = sum of opposite interior angles]

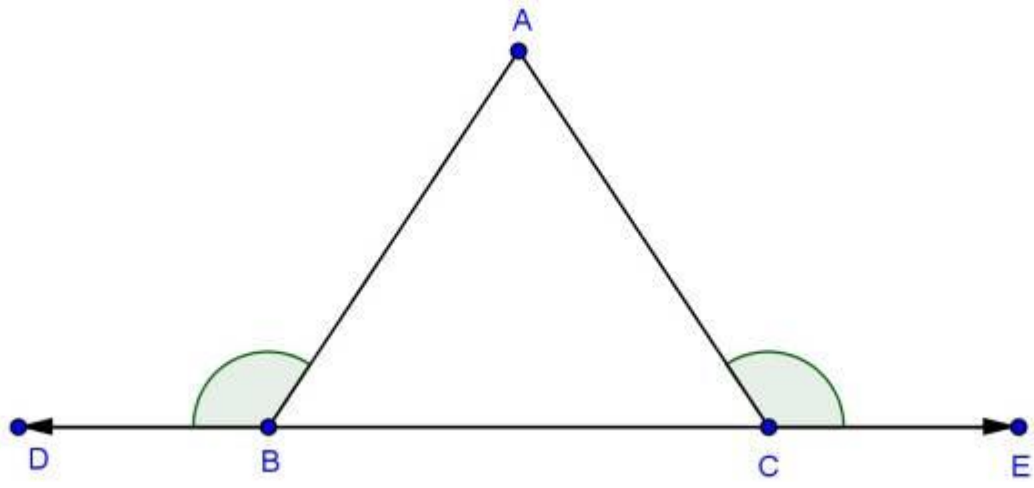
$\Rightarrow \angle ACF = 60^\circ + 60^\circ$
$\qquad = 120^\circ$

Similarly, $\angle BAD = 120^\circ$
and $\angle CBF = 120^\circ$

Question 9

If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Solution 9



$\angle DBA = \angle ACB + \angle A \quad --- (1)$ $\qquad$ [$\because$ Exterior angle = sum of opposite interior angles]

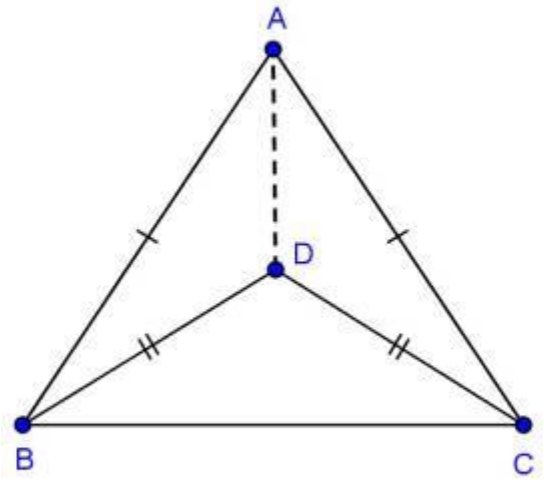
$\angle ACE = \angle ABC + \angle A \quad --- (2)$ $\qquad$ [$\because$ Exterior angle = sum of opposite interior angles]

But $\angle ACB = \angle ABC \; (\because AB = AC)$

$\therefore$ From (1) and (2)
$\angle DBA = \angle ACE$.

Question 10

In fig., AB =AC and DB = DC, find the ratio ∠ABD = ∠ACD.



Solution 10

Construction: AD is joined

Now in $\triangle ADB$ and $\triangle ADC$

$AD = AD$ [common]

$AB = AC$ [given]

$DB = DC$ [given]

$\therefore$ By SSS congurence criterion $\triangle ADB \cong \triangle ADC$

$\therefore \angle ABD = \angle ACD$ [c.p.c.t]

$$\therefore \frac{\angle ABD}{\angle ACD} = \frac{1}{1} = 1:1$$

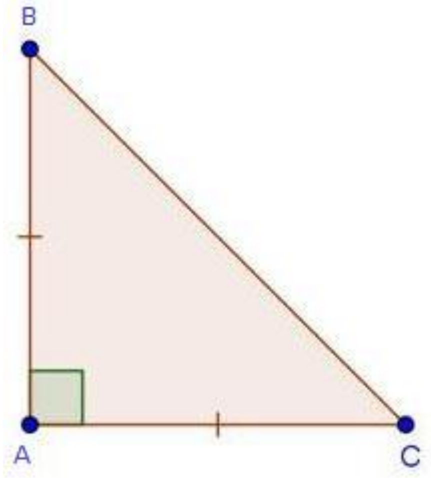
Question 11

Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

ABC is a right-angled triangle in which ∠A = 90º and AB = AC. Find ∠B and ∠C.

Solution 11



In ΔABC

$\angle BAC = 90°$ [given]

and $AB = AC$ [given]

$\angle ABC = \angle ACB$

Again in ΔABC

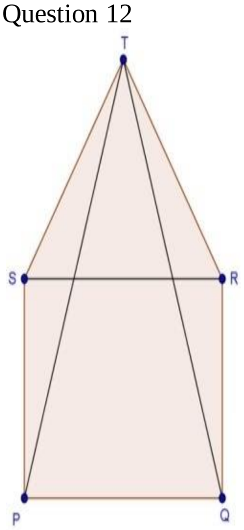
$\angle BAC + \angle ACB + \angle ABC = 180°$ [Angle sum property]

$\Rightarrow 2\angle ABC = 180° - 90° = 90°$

$\Rightarrow \angle ABC = 45°$

$\Rightarrow \angle ABC = \angle ACB = 45°$

Question 12



In the figure, $PQRS$ is a square and SRT is an equilateral triangle. Prove that

(i) PT= QT

(ii) $\angle TQR = 15°$

Solution 12

Since PQRS is a square and Δ SRT is an equilateral triangle

$\therefore \angle PSR = 90°$ and $\angle TSR = 60°$

$\Rightarrow \angle PSR + \angle TSR = 90° + 60°$

$\Rightarrow \angle PST = 150°$

Similarly, $\angle QRT = 150°$

In ΔPST and ΔQRT

$ST = TR$ [given]

$PS = QR$ [given]

$\angle PST = \angle QRT = 150°$ [shown above]

$\therefore$ By SAS congruence rule, $\Delta PST \cong \Delta QRT$

$\therefore PT = QT$ [c.p.c.t]

Now in ΔTRQ

$TR = RQ$ [given]

$\therefore \angle TQR = \angle RTQ$

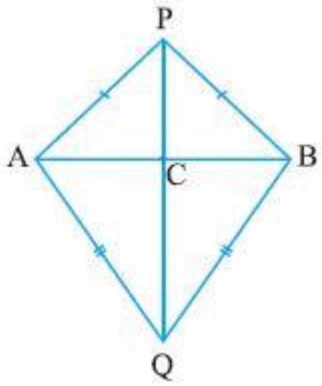
$\therefore \angle RTQ + \angle RQT + \angle TRQ = 180°$ [angle sum property]

$\Rightarrow 2\angle TQR + 150° = 180°$

$\Rightarrow \angle TQR = 15°$

Question 13

AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See fig.). Show that the line PQ is perpendicular bisector of AB.



Solution 13

In ΔPAQ and ΔPBQ

$AP = BP$ (Given)

$AQ = BQ$ (Given)

$PQ = PQ$ (Common)

So, $\Delta PAQ \cong \Delta PBQ$ (SSS rule)

Therefore, $\angle APQ = \angle BPQ$ (CPCT).

Now let us consider ΔPAC and ΔPBC.

You have: $AP = BP$ (Given)

$\angle APC = \angle BPC$ ($\angle APQ = \angle BPQ$ proved above)

$PC = PC$ (Common)

So, $\Delta PAC \cong \Delta PBC$ (SAS rule)

Therefore, $AC = BC$ (CPCT) (1)

$\angle ACP = \angle BCP$ (CPCT)

and $\angle ACP + \angle BCP = 180°$ (Linear pair)

So, $2\angle ACP = 180°$

Or, $\angle ACP = 90°$ (2)

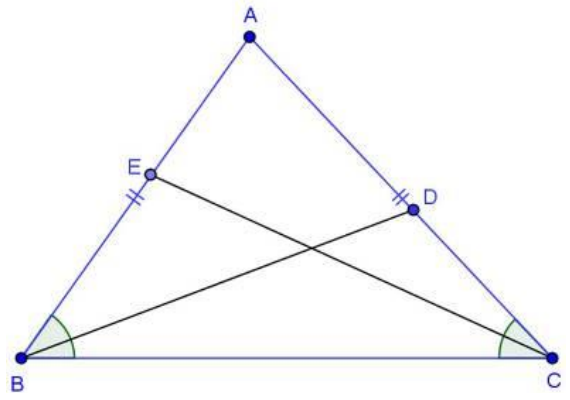
From (1) and (2), you can easily conclude that PQ is the perpendicular bisector of AB.

Chapter 12 - Congruent Triangles Exercise Ex. 12.2

Question 1

BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles ΔABC with $AB = AC$. Prove that $BD = CE$.

Solution 1



In $\triangle ABC$

$\because AB = AC$

$\therefore \angle ABC = \angle ACB$ [Angle opposite to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle DBC = \angle ECB$ [BD and CE bisects $\angle B$ and $\angle C$]

Now In $\triangle DBC$ and $\triangle ECB$

$\angle DBC = \angle ECB$ [proved earlier]

$\angle B = \angle C$ [given]

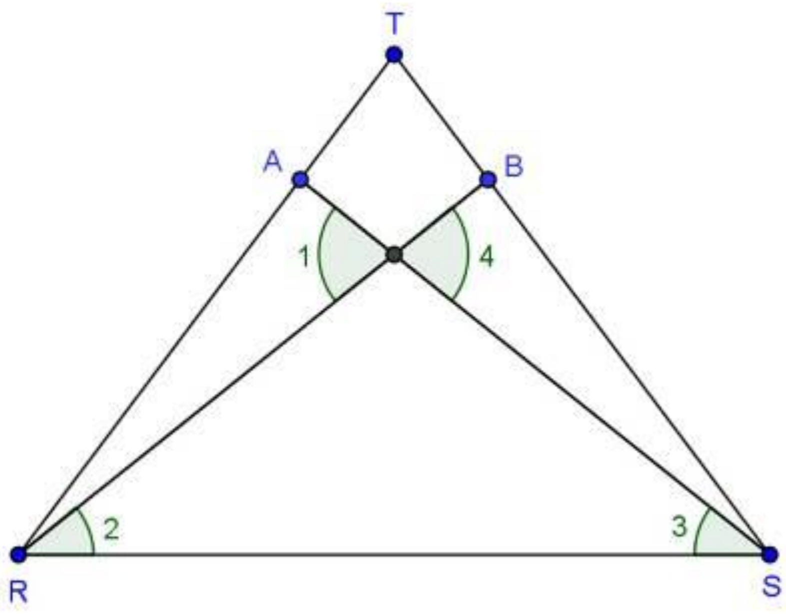
$BC = BC$ [common]

By ASA congurence criterion $\triangle DBC \cong \triangle ECB$

$\therefore BD = CE$ [$c.p.c.t$]

Question 2

In fig., it is given RT = TS, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$ prove that $\Delta RBT \cong \Delta SAT$.



Solution 2

Here, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$ [∵ Exterior angle = sum of opposite interior angles]

$\angle 1 = \angle 4$ [vertically opposite angles]

$\therefore 2\angle 2 = 2\angle 3$

$\Rightarrow \angle 2 = \angle 3$

Now $RT = TS$ [given]

$\Rightarrow \angle TRS = \angle TSR$ [Angle opposite to equal sides are equal]

$\therefore \angle TRS - \angle 2 = \angle TSR - \angle 3$

$\Rightarrow \angle TRB = \angle TSA$

Now in $\triangle RBT$ and $\triangle SAT$

$\angle T = \angle T$ [common]

$\angle TRB = \angle TSA$ [proved earlier]

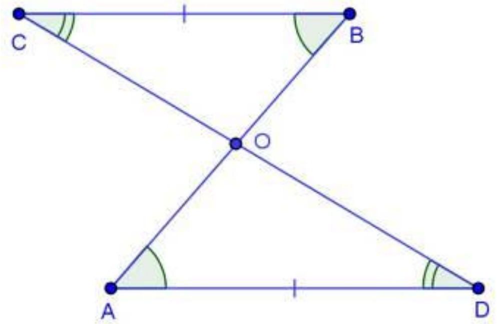
$RT = TS$ [given]

By ASA congruence criterion $\triangle RBT \cong \triangle SAT$

Question 3

Two lines AB and CD intersect at O such that BC is equal and parallel to AD.
Prove that the lines AB and CD bisect at O.

Solution 3



In $\triangle$AOD and $\triangle$BOC

$\angle$BCO = $\angle$ADO [alternate angles]

$\angle$DAO = $\angle$CBO [alternate angles]

BC = AD [given]

By ASA congruence criterion $\triangle AOD \cong \triangle BOC$

$\therefore$ BO = OA [c.p.c.t]

OC = OD [c.p.c.t]

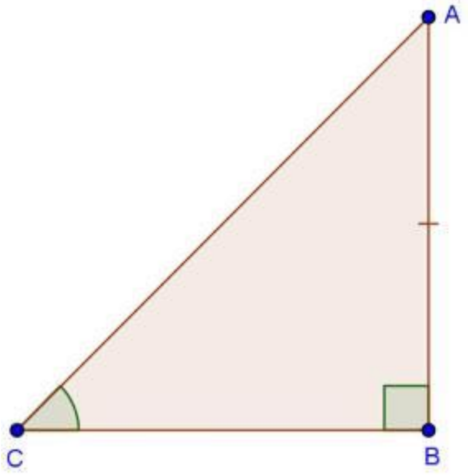
Therefore, AB and CD bisect at O.

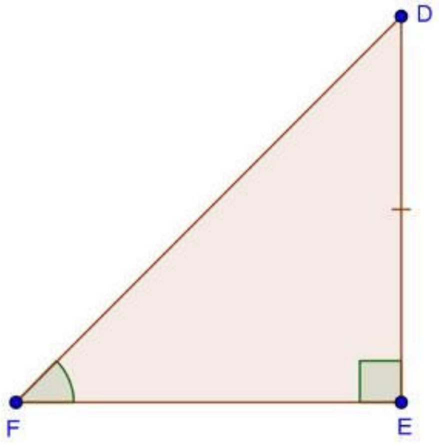
Chapter 12 - Congruent Triangles Exercise Ex. 12.3

Question 1

In two right triangles one side and acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution 1





Let ABC and DEF be two right triangles.

Since $\angle C = \angle F$ [given]

$\angle B = \angle E = 90^\circ$

$\therefore \angle A = \angle D$

Now in $\triangle ABC$ and $\triangle DEF$

$\angle A = \angle D$ [proved earlier]

$AB = DE$ [given]

$\angle B = \angle E = 90^\circ$

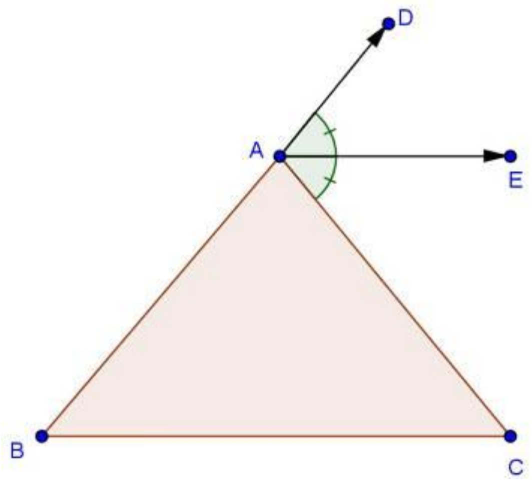
By ASA congurence criterion $\triangle ABC \cong \triangle DEF$

Question 2

If the bisector of the exterior vertical angle of a triangle be parallel to the base.

Show that the triangle is isosceles.

Solution 2



Given AE || BC
and AE is the bisector of $\angle DAC$
$\therefore \angle DAE = \angle EAC$ [given] $---(1)$

$\because AE \parallel BC$
$\therefore \angle EAC = \angle ACB$ [alternate angles] $---(2)$

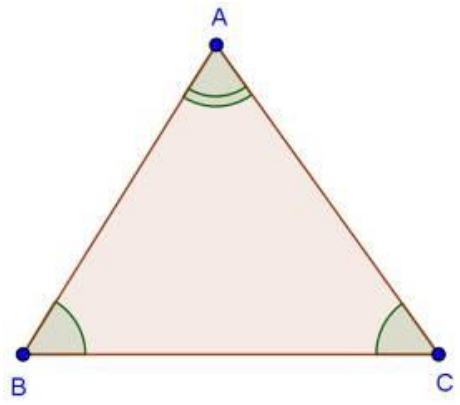
and $\angle DAE = \angle ABC$ [corresponding angles] $---(3)$

From (1), (2) and (3)
$\angle ABC = \angle ACB$
Hence $AB = AC$

$\therefore$ Triangle ABC is isosceles.

Question 3
In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Solution 3



$\angle BAC = 2(\angle ABC + \angle ACB)$ [given]

$\therefore \angle ABC + \angle ACB = \frac{1}{2}\angle BAC$

Now in $\triangle ABC$

$\angle ABC + \angle BAC + \angle ACB = 180°$

$\Rightarrow \angle BAC + \frac{1}{2}\angle BAC = 180°$

$\Rightarrow \frac{3}{2}\angle BAC = 180°$

$\Rightarrow \angle BAC = 120°$

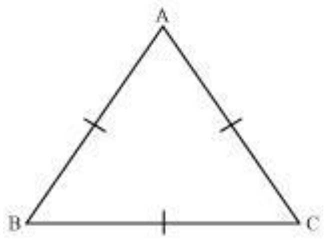
Also, $\angle ABC + \angle ACB = 60°$ $[\because \angle ABC = \angle ACB]$

$\Rightarrow \angle ABC = \angle ACB = \frac{60°}{2} = 30°$

Question 4

Show that the angles of an equilateral triangle are 60º each.

Solution 4



Let us consider that ABC is an equilateral triangle.
So, AB = BC = AC
Now, AB = AC

$\Rightarrow \angle C = \angle B$ (angles opposite to equal sides of a triangle are equal)

We also have
AC = BC

$\Rightarrow \angle B = \angle A$ (angles opposite to equal sides of a triangle are equal)

So, we have

$\angle A = \angle B = \angle C$

Now, in ΔABC

$\angle A + \angle B + \angle C = 180^o$

$\Rightarrow \angle A + \angle A + \angle A = 180^o$

$\Rightarrow 3\angle A = 180^o$

$\Rightarrow \angle A = 60^o$

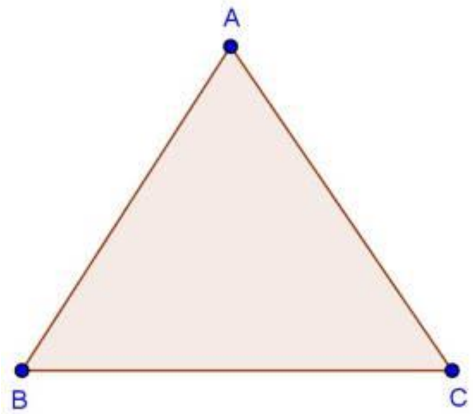
$\Rightarrow \angle A = \angle B = \angle C = 60^o$

Hence, in an equilateral triangle all interior angles are of 60º.

Question 5

Angles A, B, C of a triangle ABC are equal to each other. Prove that $\triangle ABC$ is equilateral.

Solution 5



Given $\angle A = \angle B = \angle C$

Now in $\triangle ABC$

$\angle A + \angle B + \angle C = 180°$ (sum of angles of a $\triangle$)

$\Rightarrow \angle A + \angle A + \angle A = 180°$

$\Rightarrow 3\angle A = 180°$

$\Rightarrow \angle A = \angle B = \angle C = 60°$

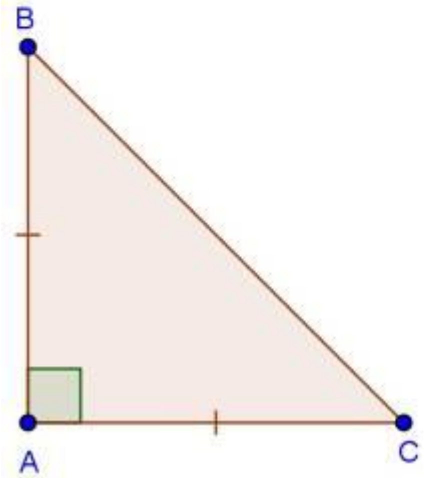
Hence $\triangle ABC$ is an equilateral $\triangle$.

Question 6

ABC is a right angled triangle in which $\angle A = 90°$ and $AB = AC$.

Find $\angle B$ and $\angle C$.

Solution 6



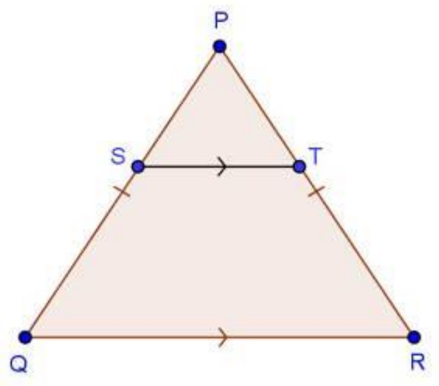
$In \triangle ABC$
$\because AB = AC$
$\Rightarrow \angle B = \angle C$

$\text{Now} \angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow 90^\circ + \angle B + \angle C = 180^\circ$
$\Rightarrow \angle B + \angle C = 90^\circ$
$\Rightarrow 2\angle B = 90^\circ$ $[\because \angle B = \angle C]$
$\Rightarrow \angle B = 45^\circ$
$\Rightarrow \angle B = \angle C = 45^\circ$

Question 7

PQR is a triangle in which $PQ = PR$ and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that $PS = PT$.

Solution 7



In $\triangle PQR$

$\because ST \parallel QR$

$\Rightarrow \angle PST = \angle PQR$ [corresponding angles]

and $\angle PTS = \angle PRQ$ [corresponding angles]

But $\angle PQR = \angle PRQ$ [given]

$\therefore \angle PST = \angle PTS$

In $\triangle PST$

$\because \angle PST = \angle PTS$

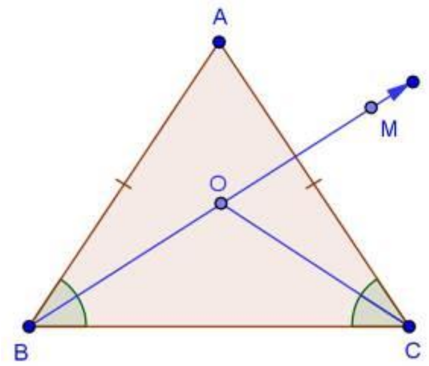
$\Rightarrow PS = PT$ [Sides opposite to equal angles are equal]

Hence $PS = PT$

Question 8

In a $\triangle ABC$, it is given that $AB = AC$ and the bisectors of $\angle B$ and $\angle C$ intersect at O. If M is a point on BO produced, prove that $\angle MOC = \angle ABC$.

Solution 8



$In \triangle ABC$

$\angle MOC = \angle OBC + \angle OCB$ [$\because$ Exterior angle = sum of opposite interior angles]

$= \frac{1}{2} \times [2\angle OBC + 2\angle OCB]$

$= \frac{1}{2}[\angle ABC + \angle ACB]$

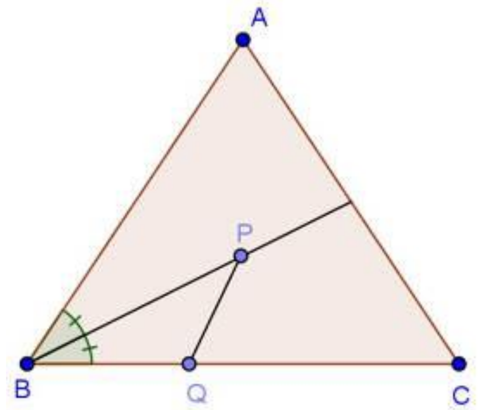
$= \frac{1}{2}[2\angle ABC]$

Hence, $\angle MOC = \angle ABC$

Question 9

P is a point on the bisector of an $\angle ABC$. If the line through P parallel to AB meets BC at Q, prove that $\triangle BPQ$ is isosceles.

Solution 9



$\angle ABP = \angle PBQ$ [BP bisects $\angle ABC$] ---(1)

$\angle ABP = \angle BPQ$ [Alternate angles] ---(2)

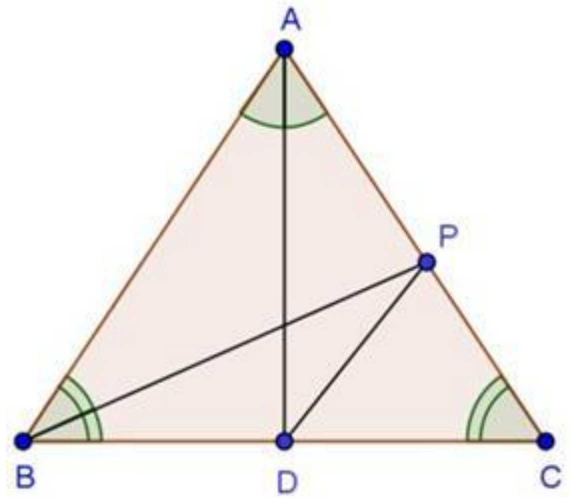
From (1) and (2)
$\angle PBQ = \angle BPQ$

$Now, In \triangle BPQ$
$\Rightarrow \angle PBQ = \angle BPQ$
$\Rightarrow BQ = QP$ [Sides opposite to equal angles are equal]

Hence $\triangle BPQ$ is an isosceles triangle.

Question 10
ABC is a triangle in which $\angle B = 2\angle C$. D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$.
Prove that $\angle BAC = 72°$.

Solution 10


A
P
B
D
C

Construction: Draw BP, the bisector of $\angle ABC$. Join PD.

Let $\angle BAD = \angle CAD = x$
Then, $\angle BAC = 2x$ ...(i)

Suppose $\angle ACB = y$, then $\angle ABC = 2y$. [Given]

Since, BP is the bisector of $\angle ABC = 2y$
$\therefore \quad \angle CBP = y$

In ΔBPC, we have
$\angle CBP = \angle BCP = y$
$\Rightarrow BP = CP$ [Sides opposite equal angles are equal]

Now in ΔABP and ΔDCP
$\angle ABP = \angle DCP = y$
$AB = CD$ (given)
and $BP = CP$ (proved above)

Using SAS congruence rule,
$\Delta ABP \cong \Delta DCP$
$\therefore \angle BAP = \angle CDP$ (c.p.c.t)

And, $AP = DP$ (c.p.c.t)

$\Rightarrow \angle CDP = 2x$ and $\angle ADP = \angle DAP = x$

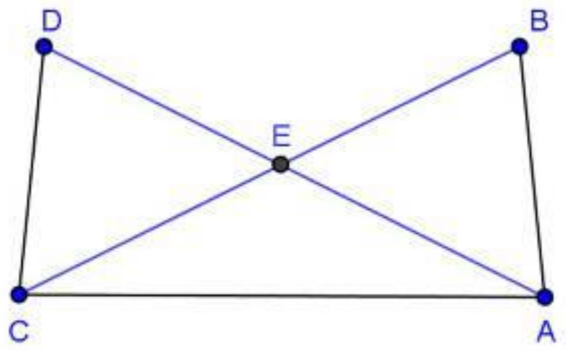
In ΔABD, we have
$\angle ADC = \angle ABD = x + 2x = 2y + x$
$\Rightarrow x = y$

In ΔABC, we have
$\angle A + \angle B + \angle C = 180°$
$2x + 2y + y = 180°$
$\Rightarrow 5x = 180°$
$\Rightarrow x = 36°$

$\therefore \angle A = 2x = 72°$

Chapter 12 - Congruent Triangles Exercise Ex. 12.4
Question 1

In fig., it is given that Ab = CD and AD = BC. prove that ΔADC $\cong$ ΔCBA



Solution 1

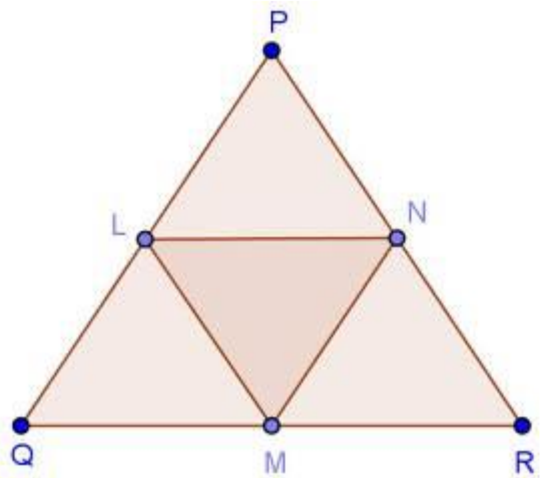
In $\triangle ADC$ and $\triangle CBA$

$AB = CD$	(given)
$AC = AC$	(common)
$AD = BC$	(given)

By SSS congurence criterion $\triangle ADC \cong \triangle CBA$

Question 2

In a $\triangle PQR$, if $PQ = QR$ and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that $LN = MN$.



Solution 2

In $\triangle PNL$ and $\triangle RNM$

$\frac{1}{2}PQ = \frac{1}{2}QR$

$\Rightarrow PL = MR$

$PN = NR$ $\quad$ $(\because N$ is the mid-point of $PR)$

$\angle LPN = \angle MRN$ $\quad$ $(\because QP = QR)$

$\therefore$ From SAS $\triangle PNL \cong \triangle RNM$

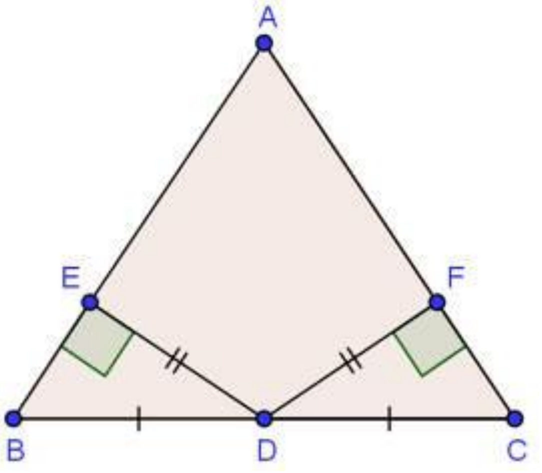
$\therefore LN = NM$ $\quad$ $(c.p.c.t)$

Chapter 12 - Congruent Triangles Exercise Ex. 12.5

Question 1

ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Solution 1



In $\triangle BDE$ and $\triangle CDF$

$\angle BED = \angle DFC = 90^\circ$ $\quad$ [given]

$DE = DF$ $\quad$ [given]

$BD = DC$ $\quad$ [D is the midpoint]

By RHS cogurence criterion $\triangle BDE \cong \triangle CDF$

$\Rightarrow \angle B = \angle C$ $\quad$ $[c.p.c.t]$

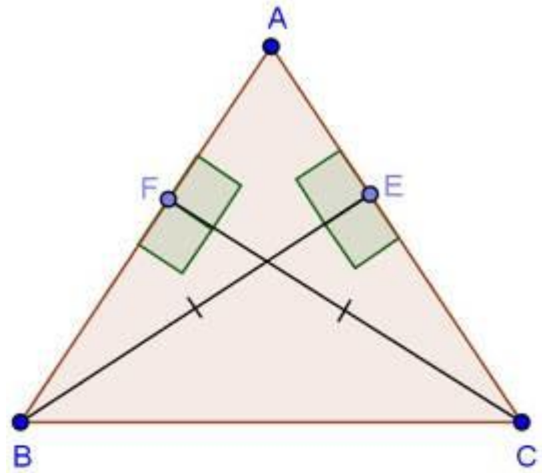
$\Rightarrow AB = AC$ $\quad$ [Sides opposite to equal angles are equal]

Hence $\triangle ABC$ is isosceles.

Question 2

ABC is a triangle in which BE and CF are respectively, the perpendiculars to the sides AC and AB. If $BE = CF$, prove that $\triangle ABC$ is isosceles.

Solution 2



In $\triangle BEC$ and $\triangle CFB$

$BC = BC$	[common hypotenuse]
$\angle BFC = \angle CEB = 90^\circ$	[given]
$BE = CF$	[given]

By RHS congurence criterion $\triangle BEC \cong \triangle CFB$

$\therefore \angle B = \angle C$ [$c.p.c.t$]

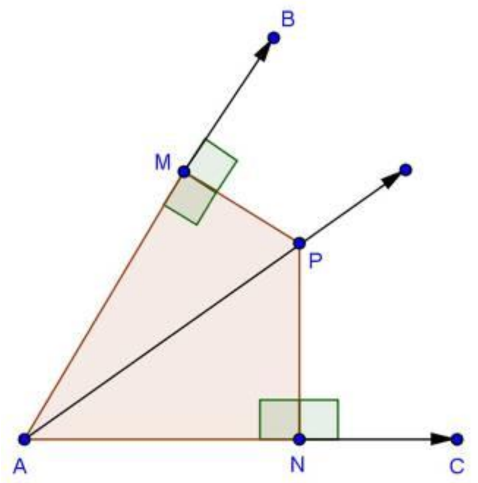
$\Rightarrow AB = AC$

$\Rightarrow \triangle ABC$ is isosceles.

Question 3

If perpendicular from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

Solution 3



Here $PM = PN$

and $\angle PMA = \angle PNA = 90^\circ$

In $\triangle APM$ and $\triangle APN$

$AP = AP$ [common]

$PN = PM$ [given]

$\angle PMA = \angle PNA = 90^\circ$ [given]

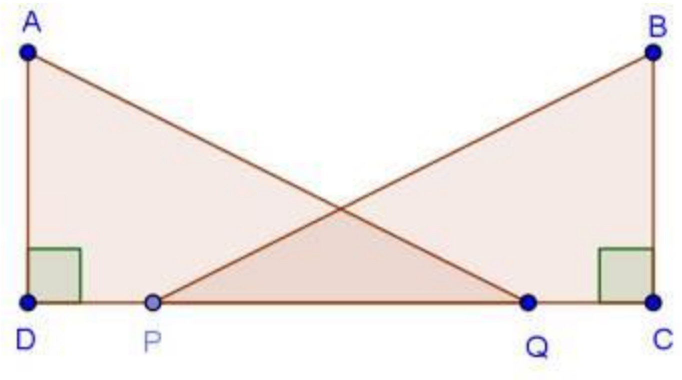
By RHS congruence criterion $\Delta APM \cong \Delta APN$

$\therefore \angle MAP = \angle NAP$ [c.p.c.t]

Hence, AP is the bisector of $\angle BAC$.

Question 4

In fig., AD $\perp$ CD and CB $\perp$ CD. If AQ = BP an DP = CQ, prove that $\angle$DAQ = $\angle$CBP.



Solution 4

In $\triangle DAQ$ and $\triangle CBP$

$\angle ADQ = \angle BCP = 90^\circ$
$DP = CQ$ [given]
$\Rightarrow DP + PQ = CQ + PQ$
$\Rightarrow DQ = CP$
$AQ = BP$ [given]

By RHS congurence criterion $\triangle DAQ \cong \triangle CBP$
$\therefore \angle DAQ = \angle CBP$ [c.p.c.t]

Question 5
Which of the following statements are True (T) and which are False (f):

(i) Sides opposite to equal angles of a triangle may be unequal.

(ii) Angles opposite to equal sides of a triangle are equal.

(iii) The measure of each angle of an equilaterial triangle is 60º.

(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isoscles.

(v) The bisectors of two equal angles of a traingle are equal.

(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.

(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.

(viii) If any two sides of a right triangle are respectively equal to two sides of other right triagnle, then the two triangles are congruent.

(ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Solution 5
(i) False

(ii) True

(iii) True

(iv) False

(v) True

(vi) False

(vii) False

(viii) False

(ix) True

Question 6

Fill in the blanks in the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are _____.

(ii) Angle opposite to equal sides of a triangle are _____.

(iii) In an equilateral triangle all angles are _____.

(iv) In a $\triangle ABC$ if $\angle A = \angle C$, then AB = ___.

(v) If altitudes CE and BF of a triangle ABC are equal, then AB = ___.

(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is _______ CE.

(vii) In right triangles ABC and DEF, if hypotenuse $AB = EF$ and side $AC = DE$, then $\Delta ABC \cong \Delta$_____.

Solution 6

(i) equal

(ii) equal

(iii) equal

(iv) BC

(v) AC

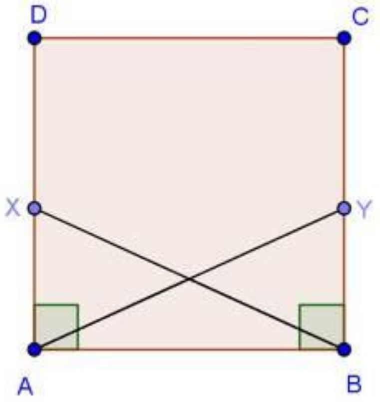
(vi) equal to

(vii) EFD

Question 7

$ABCD$ is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$.
Prove that $BY = AX$ and $\angle BAY = \angle ABX$.

Solution 7



In $\triangle ABX$ and $\triangle BAY$

$AY = BX$ [given]

$AB = AB$ [common]

$\angle BAX = \angle ABY = 90°$ [given]

By RHS congurence criterion $\triangle ABX \cong \triangle BAY$

$\therefore AY = BX$ [$c.p.c.t$]

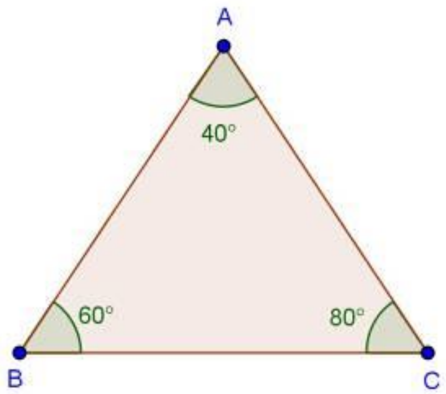
$\angle BAY = \angle ABX$ [$c.p.c.t$]

Chapter 12 - Congruent Triangles Exercise Ex. 12.6

Question 1

In $\triangle ABC$, if $\angle A = 40°$ and $\angle B = 60°$. Determine the longest and shortest sides of the triangle.

Solution 1



$\because \angle A = 40°$ and $\angle B = 60°$

$\therefore \angle C = 180° - \angle A - \angle B$ [Angle sum property of $\triangle$]

$= 180° - 40° - 60°$

$= 80°$

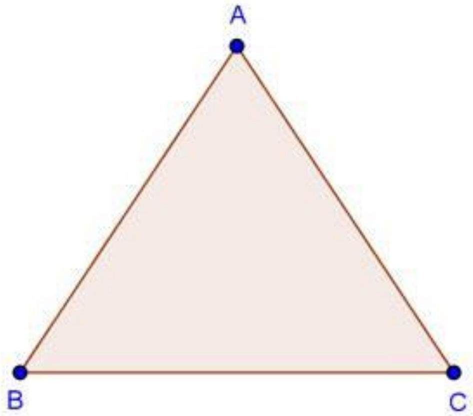
$\therefore$ Longest side = AB [greatest angle has longest side opposite to it]

Shortest side = BC [smallest angle has smallest side opposite to it]

Question 2

In a $\triangle ABC$, if $\angle B = \angle C = 45°$, Which the longest side.

Solution 2



$\because \angle B = \angle C = 45°$

$\therefore \angle A = 180° - \angle B - \angle C$ [Angle sum property of $\triangle$]

$= 180° - 45° - 45°$

$= 90°$

$\therefore$ Longest side $= BC$ [as it has greatest angle opposite to it]

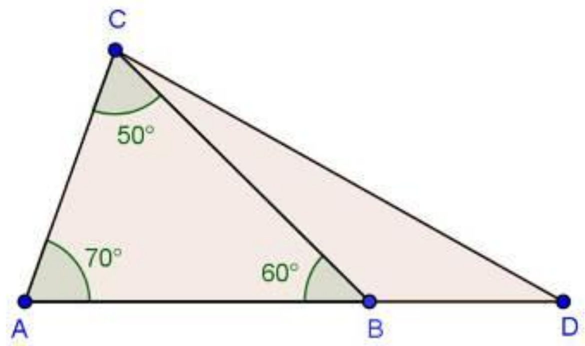
Question 3

In $\triangle ABC$, side AB is produced to D so that $BD = BC$. If $\angle B = 60°$ and $\angle A = 70°$, prove that:

(i) $AD > CD$

(ii) $AD > AC$

Solution 3



(i)$\angle A = 70^\circ, \angle B = 60^\circ$

$\therefore \angle C = 180^\circ - \angle A - \angle B$ [Angle sum property of $\triangle$]

$= 180^\circ - 70^\circ - 60^\circ$

$= 50^\circ$

$\angle CBD = 180^\circ - 60^\circ = 120^\circ$ [linear pair]

$\therefore \angle BCD = \angle BDC = 30^\circ$

$\therefore \angle ACD = 50^\circ + 30^\circ = 80^\circ$

$\angle CAD = 70^\circ$

$\therefore \angle ACD > \angle CAD$

$\Rightarrow AD > CD$

(ii)$\angle ACD = 80^\circ$

$\angle ABC = 60^\circ$

$\therefore \angle ACD > \angle ABC$

$\Rightarrow AD > AC$

Question 4

Is it possible to draw a triangle with sides of length 2cm, 3cm and 7 cm?

Solution 4

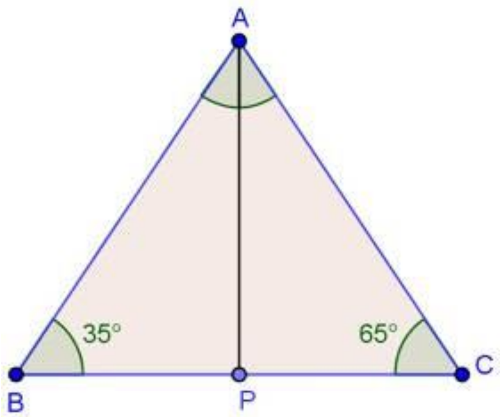
Here, 2 + 3 < 7

Hence, it is not possible because triangle can be drawn only if the sum of any two sides is greater than third side.

Question 5

In a $\triangle ABC$, $\angle B = 35^\circ$, $\angle C = 65^\circ$ and the bisector of $\angle BAC$ meets BC in P. Arrange AP, BP and CP in descending order.

Solution 5



Let $\angle BAP = \angle CAP = x$

$\therefore \angle BAC = 2x$

Now, in ΔABC

$\Rightarrow \angle BAC + \angle ABC + \angle ACB = 180°$ [sum of all angles of a Δ]

$\Rightarrow 2x + 35° + 65° = 180°$

$\Rightarrow 2x = 80°$

$\Rightarrow x = 40°$

In ΔACP, we have

$\angle ACP > \angle CAP$

$\Rightarrow AP > CP \quad ---(1)$

and in ΔABP, we have

$\angle BAP > \angle ABP$

$\Rightarrow BP > AP \quad ---(2)$

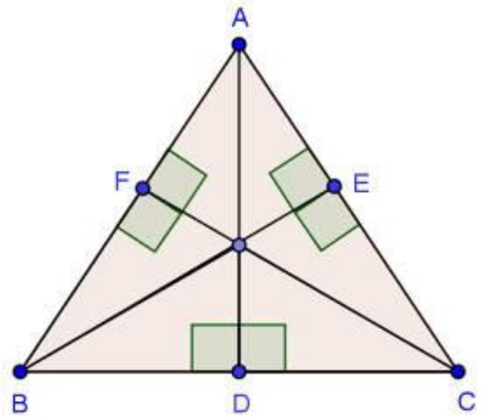
From (1) and (2)

$BP > AP > CP$

Question 6

Prove that the perimeter of a triangle is greater than the sum of its altitudes.

Solution 6



To prove : $AD + BE + CF < AB + BC + AC$

Since perpendicular is the shortest of all the line segment from a point not lying on it.

Now We have $AD \perp BC$

$\Rightarrow AD < BC \quad ---(1)$

also $BE \perp AC$

$\Rightarrow BE < AC \quad ---(2)$

and $CF \perp AB$

$\Rightarrow CF < AB \quad ---(3)$

Now adding (1), (2) and (3)

We get

$AD + BE + CE < AB + BC + CA$

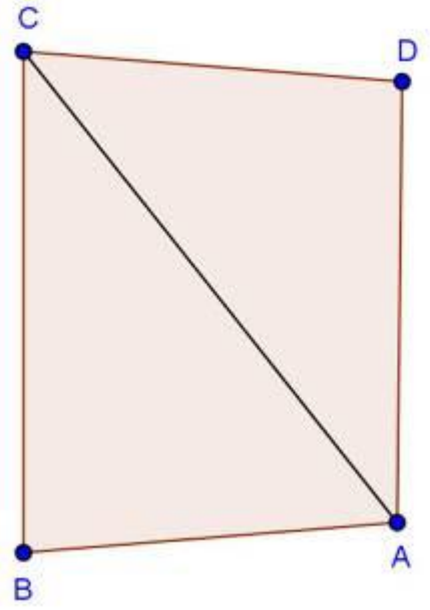
Hence, proved.

Question 7

In fig., prove that:

i. CD + DA + AB + BC > 2AC

ii. CD + DA + AB > BC



Solution 7

Since the sum of any two sides of a triangle is greater than the third side. Therefore,
In $\triangle ADC$, we have

$AD + DC > AC$ --- (1)

In $\triangle ABC$
$AB + BC > AC$ --- (2)

Adding (1) and (2)
$AB + BC + CD + DA > 2AC.$

In $\triangle ACD$

$CD + DA > AC$
$\Rightarrow CD + DA + AB > AC + AB$ --- (3)

Now in $\triangle ABC$
$AC + AB > BC$ --- (4)

$\therefore$ From (3) and (4) we have
$CD + DA + AB > BC$

Question 8

Which of the following statements are true (T) and which are false (F)?

(i) Sum of the three sides of a triangle is less than the sum of its three altitudes.

(ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.

(iii) Sum of any two sides of a triangle is greater than the third side.

(iv) Difference of any two sides of a triangle is equal to the third side.

(v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.

(vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Solution 8

(i) False

(ii) True

(iii) True

(iv) False

(v) True

(vi) True

Question 9

Fill the blanks to make the following statements true.

(i) In the right triangle the hypotenuse is the _______ side.

(ii) The sum of of three altitudes of a triangle is ___ than its perimeter.

(iii) Sum of any two sides of a triangle is _______ than third side.

(iv) If two angles of a triangle are unequal, then the smaller angle has the _______ side opposite to it.

(v) Difference of any two sides of a triangle is ___ than the third side.

(vi) If two sides of a triangle are unequal, then the larger side has _______ angle opposite to it.

Solution 9

(i) largest

(ii) less

(iii) greater

(iv) smaller

(v) less

(vi) greater

Question 10

O is any point in the interior of ΔABC. Prove that:

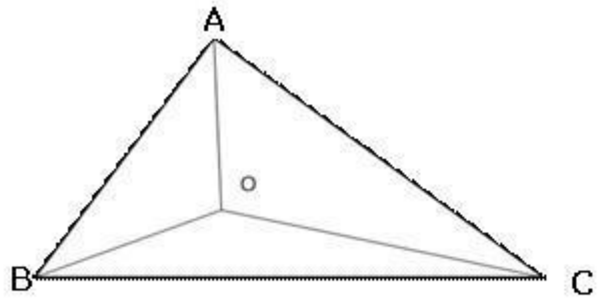
(i) $AB + AC > OB + OC$

(ii) $AB + BC + CA > OA + OB + OC$

(iii) $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

Solution 10

Given: ABC is a triangle and O is a point inside it.



To Prove :

(i) AB + AC > OB +OC

(ii)AB + BC + CA > OA + OB + OC

(iii) OA + OB + OC > $\frac{1}{2}$(AB + BC + CA)

Proof:

(i) In ΔABC,

AB + AC > BC ....(i)

And in ΔOBC,

OB + OC > BC ....(ii)

Subtracting (i) from (i) we get

(AB + AC) - (OB + OC) > (BC-BC)

i.e. AB + AC > OB + OC

(ii) AB + AC > OB + OC [proved in (i)]

Similarly, AB + BC > OA + OC

And AC + BC > OA + OB

Adding both sides of these three inequalities, we get

(AB + AC) + (AC + BC) + (AB + BC) > OB + OC + OA + OB + OA + OC

i.e. 2(AB + BC + AC) > 2(OA + OB + OC)

Therefore, AB + BC + AC > OA + OB + OC

(iii) In ΔOAB

OA + OB > AB ...(i)

In ΔOBC,

OB + OC > BC ...(ii)

And, in ΔOCA,

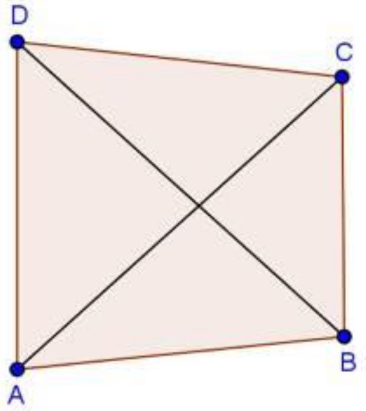
OC + OA > CA ...(iii)

Adding (i), (ii) and (iii) we get

Question 11

Prove that in a quadrilateral the sum of all the sides is greater than the sum of its diagonals.

Solution 11



Diagonal AC and BD is joined.

Since sum of any two sides of a triangle is greater than the third side.

Therefore,

In $\triangle ABC$

$AB + BC > AC \qquad --- (1)$

In $\triangle ACD$

$AD + DC > AC \qquad --- (2)$

In $\triangle ABD$

$AB + AD > BD \qquad --- (3)$

and in $\triangle BCD$

$BC + CD > BD \qquad --- (4)$

adding (1), (2), (3) and (4) we get

$2AD + 2DC + 2AB + 2BC > 2AC + 2BD$

$\Rightarrow AB + BC + CD + DA > AC + BD$

Hence proved.