Exercise 6.1 Page No: 6.12

1. Find the values of each of the following:

(i) 13^2

(ii) 7^3

(iii) 3^4

Solution:

(i) Given 13^2

$13^2 = 13 \times 13 = 169$

(ii) Given 7^3

$7^3 = 7 \times 7 \times 7 = 343$

(iii) Given 3^4

$3^4 = 3 \times 3 \times 3 \times 3$

$= 81$

2. Find the value of each of the following:

(i) $(-7)^2$

(ii) $(-3)^4$

(iii) $(-5)^5$

Solution:

(i) Given $(-7)^2$

We know that $(-a)^{\text{even number}} =$ positive number

$(-a)^{\text{odd number}} =$ negative number

We have, $(-7)^2 = (-7) \times (-7)$

$= 49$

(ii) Given $(-3)^4$

We know that $(-a)^{\text{even number}} =$ positive number

$(-a)^{\text{odd number}} =$ negative number

We have, $(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$

$= 81$

(iii) Given $(-5)^5$

We know that $(-a)^{\text{even number}} =$ positive number

$(-a)^{\text{odd number}} =$ negative number

We have, $(-5)^5 = (-5) \times (-5) \times (-5) \times (-5) \times (-5)$

$= -3125$

3. Simplify:

(i) 3 × 10^2

(ii) 2^2 × 5^3

(iii) 3^3 × 5^2

Solution:

(i) Given 3 × 10^2

3 × 10^2 = 3 × 10 × 10

= 3 × 100

= 300

(ii) Given 2^2 × 5^3

2^2 × 5^3 = 2 × 2 × 5 × 5 × 5

= 4 × 125

= 500

(iii) Given 3^3 × 5^2

3^3 × 5^2 = 3 × 3 × 3 × 5 × 5

= 27 × 25

= 675

4. Simply:

(i) 3^2 × 10^4

(ii) 2^4 × 3^2

(iii) 5^2 × 3^4

Solution:

(i) Given 3^2 × 10^4

3^2 × 10^4 = 3 × 3 × 10 × 10 × 10 × 10

= 9 × 10000

= 90000

(ii) Given 2^4 × 3^2

2^4 × 3^2 = 2 × 2 × 2 × 2 × 3 × 3

= 16 × 9

= 144

(iii) Given 5^2 × 3^4

5^2 × 3^4 = 5 × 5 × 3 × 3 × 3 × 3

= 25 × 81

= 2025

5. Simplify:

(i) (-2) × (-3)^3

(ii) (-3)^2 × (-5)^3

(iii) (-2)^5 × (-10)^2

Solution:

(i) Given $(-2) \times (-3)^3$

$(-2) \times (-3)^3 = (-2) \times (-3) \times (-3) \times (-3)$

$= (-2) \times (-27)$

$= 54$

(ii) Given $(-3)^2 \times (-5)^3$

$(-3)^2 \times (-5)^3 = (-3) \times (-3) \times (-5) \times (-5) \times (-5)$

$= 9 \times (-125)$

$= -1125$

(iii) Given $(-2)^5 \times (-10)^2$

$(-2)^5 \times (-10)^2 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-10) \times (-10)$

$= (-32) \times 100$

$= -3200$

6. Simplify:

(i) $(3/4)^2$

(ii) $(-2/3)^4$

(iii) $(-4/5)^5$

Solution:

(i) Given $(3/4)^2$

$(3/4)^2 = (3/4) \times (3/4)$

$= (9/16)$

(ii) Given $(-2/3)^4$

$(-2/3)^4 = (-2/3) \times (-2/3) \times (-2/3) \times (-2/3)$

$= (16/81)$

(iii) Given $(-4/5)^5$

$(-4/5)^5 = (-4/5) \times (-4/5) \times (-4/5) \times (-4/5) \times (-4/5)$

$= (-1024/3125)$

7. Identify the greater number in each of the following:

(i) 2^5 or 5^2

(ii) 3^4 or 4^3

(iii) 3^5 or 5^3

Solution:

(i) Given 2^5 or 5^2

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$= 32$

$5^2 = 5 \times 5$

$= 25$

Therefore, $2^5 > 5^2$

(ii) Given 3^4 or 4^3

$3^4 = 3 \times 3 \times 3 \times 3$

$= 81$

$4^3 = 4 \times 4 \times 4$

$= 64$

Therefore, $3^4 > 4^3$

(iii) Given 3^5 or 5^3

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

$= 243$

$5^3 = 5 \times 5 \times 5$

$= 125$

Therefore, $3^5 > 5^3$

8. Express each of the following in exponential form:

(i) (-5) × (-5) × (-5)

(ii) (-5/7) × (-5/7) × (-5/7) × (-5/7)

(iii) (4/3) × (4/3) × (4/3) × (4/3) × (4/3)

Solution:

(i) Given (-5) × (-5) × (-5)

Exponential form of (-5) × (-5) × (-5) = $(-5)^3$

(ii) Given (-5/7) × (-5/7) × (-5/7) × (-5/7)

Exponential form of (-5/7) × (-5/7) × (-5/7) × (-5/7) = $(-5/7)^4$

(iii) Given (4/3) × (4/3) × (4/3) × (4/3) × (4/3)

Exponential form of (4/3) × (4/3) × (4/3) × (4/3) × (4/3) = $(4/3)^5$

9. Express each of the following in exponential form:

(i) x × x × x × x × x × x × a × a × b × b × b

(ii) (-2) × (-2) × (-2) × (-2) × a × a × a

(iii) (-2/3) × (-2/3) × x × x × x × x

Solution:

(i) Given x × x × x × x × x × x × a × a × b × b × b

Exponential form of x × x × x × x × x × x × a × a × b × b × b = $x^4 a^2 b^3$

(ii) Given (-2) × (-2) × (-2) × (-2) × a × a × a

Exponential form of (-2) × (-2) × (-2) × (-2) × a × a × a = $(-2)^4 a^3$

(iii) Given (-2/3) × (-2/3) × x × x × x × x

Exponential form of (-2/3) × (-2/3) × x × x × x × x = $(-2/3)^2 x^3$

10. Express each of the following numbers in exponential form:

(i) 512

(ii) 625

(iii) 729

Solution:

(i) Given 512

Prime factorization of 512 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

= 2^9

(ii) Given 625

Prime factorization of 625 = 5 x 5 x 5 x 5

= 5^4

(iii) Given 729

Prime factorization of 729 = 3 x 3 x 3 x 3 x 3 x 3

= 3^6

11. Express each of the following numbers as a product of powers of their prime factors:
(i) 36
(ii) 675
(iii) 392

Solution:

(i) Given 36

Prime factorization of 36 = 2 x 2 x 3 x 3

= 2^2 x 3^2

(ii) Given 675

Prime factorization of 675 = 3 x 3 x 3 x 5 x 5

= 3^3 x 5^2

(iii) Given 392

Prime factorization of 392 = 2 x 2 x 2 x 7 x 7

= 2^3 x 7^2

12. Express each of the following numbers as a product of powers of their prime factors:
(i) 450
(ii) 2800
(iii) 24000

Solution:

(i) Given 450

Prime factorization of 450 = 2 x 3 x 3 x 5 x 5

= 2 x 3^2 x 5^2

(ii) Given 2800

Prime factorization of 2800 = 2 x 2 x 2 x 2 x 5 x 5 x 7

= 2^4 x 5^2 x 7

(iii) Given 24000

Prime factorization of $24000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

$= 2^6 \times 3 \times 5^3$

13. Express each of the following as a rational number of the form (p/q):

(i) $(3/7)^2$

(ii) $(7/9)^3$

(iii) $(-2/3)^4$

Solution:

(i) Given $(3/7)^2$

$(3/7)^2 = (3/7) \times (3/7)$

$= (9/49)$

(ii) Given $(7/9)^3$

$(7/9)^3 = (7/9) \times (7/9) \times (7/9)$

$= (343/729)$

(iii) Given $(-2/3)^4$

$(-2/3)^4 = (-2/3) \times (-2/3) \times (-2/3) \times (-2/3)$

$= ((16/81)$

14. Express each of the following rational numbers in power notation:

(i) (49/64)

(ii) (- 64/125)

(iii) (-12/16)

Solution:

(i) Given (49/64)

We know that $7^2 = 49$ and $8^2 = 64$

Therefore $(49/64) = (7/8)^2$

(ii) Given (- 64/125)

We know that $4^3 = 64$ and $5^3 = 125$

Therefore $(- 64/125) = (- 4/5)^3$

(iii) Given (-1/216)

We know that $1^3 = 1$ and $6^3 = 216$

Therefore $-1/216) = - (1/6)^3$

15. Find the value of the following:

(i) $(-1/2)^2 \times 2^3 \times (3/4)^2$

(ii) $(-3/5)^4 \times (4/9)^4 \times (-15/18)^2$

Solution:

(i) Given $(-1/2)^2 \times 2^3 \times (3/4)^2$

$(-1/2)^2 \times 2^3 \times (3/4)^2 = 1/4 \times 8 \times 9/16$

$= 9/8$

(ii) Given $(-3/5)^4 \times (4/9)^4 \times (-15/18)^2$

$(-3/5)^4 \times (4/9)^4 \times (-15/18)^2 = (81/625) \times (256/6561) \times (225/324)$

$= (64/18225)$

16. If a = 2 and b= 3, the find the values of each of the following:

(i) $(a + b)^a$

(ii) $(a\,b)^b$

(iii) $(b/a)^b$

(iv) $((a/b) + (b/a))^a$

Solution:

(i) Consider $(a + b)^a$

Given a = 2 and b= 3

$(a + b)^a = (2 + 3)^2$

$= (5)^2$

$= 25$

(ii) Given a = 2 and b = 3

Consider, $(a\,b)^b = (2 \times 3)^3$

$= (6)^3$

$= 216$

(iii) Given a =2 and b = 3

Consider, $(b/a)^b = (3/2)^3$

$= 27/8$

(iv) Given a = 2 and b = 3

Consider, $((a/b) + (b/a))^a = ((2/3) + (3/2))^2$

$= (4/9) + (9/4)$

LCM of 9 and 6 is 36

$= 169/36$

Exercise 6.2 Page No: 6.28

1. Using laws of exponents, simplify and write the answer in exponential form

(i) $2^3 \times 2^4 \times 2^5$

(ii) $5^{12} \div 5^3$

(iii) $(7^2)^3$

(iv) $(3^2)^5 \div 3^4$

(v) $3^7 \times 2^7$

(vi) $(5^{21} \div 5^{13}) \times 5^7$

Solution:

(i) Given $2^3 \times 2^4 \times 2^5$

We know that first law of exponents states that $a^m \times a^n \times a^p = a^{(m+n+p)}$

Therefore above equation can be written as $2^3 \times 2^4 \times 2^5 = 2^{(3+4+5)}$

$= 2^{12}$

(ii) Given $5^{12} \div 5^3$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

Therefore given question can be written as $5^{12} \div 5^3 = 5^{12-3} = 5^9$

(iii) Given $(7^2)^3$

According to the law of exponents we have $(a^m)^n = a^{mn}$

Therefore given question can be written as $(7^2)^3 = 7^6$

(iv) Given $(3^2)^5 \div 3^4$

According to the law of exponents we have $(a^m)^n = a^{mn}$

Therefore $(3^2)^5 \div 3^4 = 3^{10} \div 3^4$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

$3^{10} \div 3^4 = 3^{(10-4)} = 3^6$

(v) Given $3^7 \times 2^7$

We know that law of exponents states that $a^m \times b^m = (a \times b)^m$

$3^7 \times 2^7 = (3 \times 2)^7 = 6^7$

(vi) Given $(5^{21} \div 5^{13}) \times 5^7$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

$= 5^{(21-13)} \times 5^7$

$= 5^8 \times 5^7$

According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$

$= 5^{(8+7)} = 5^{15}$

2. Simplify and express each of the following in exponential form:

(i) $\{(2^3)^4 \times 2^8\} \div 2^{12}$

(ii) $(8^2 \times 8^4) \div 8^3$

(iii) $(5^7/5^2) \times 5^3$

(iv) $(5^4 \times x^{10}y^5)/ (5^4 \times x^7y^4)$

Solution:

(i) Given $\{(2^3)^4 \times 2^8\} \div 2^{12}$

$\{(2^3)^4 \times 2^8\} \div 2^{12} = \{2^{12} \times 2^8\} \div 2^{12}$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 2^{(12+8)} \div 2^{12}$ [According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= 2^{20} \div 2^{12}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 2^{(20-12)}$

$= 2^8$

(ii) Given $(8^2 \times 8^4) \div 8^3$

$(8^2 \times 8^4) \div 8^3$ [According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= 8^{(2+4)} \div 8^3$

$= 8^6 \div 8^3$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 8^{(6-3)} = 8^3 = (2^3)^3 = 2^9$

(iii) Given $(5^7/5^2) \times 5^3$

$= 5^{(7-2)} \times 5^3$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^5 \times 5^3$ [According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= 5^{(5+3)} = 5^8$

(iv) Given $(5^4 \times x^{10}y^5)/(5^4 \times x^7y^4)$

$= (5^{4-4} \times x^{10-7}y^{5-4})$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^0 x^3 y^1$ [since $5^0 = 1$]

$= 1x^3y$

3. Simplify and express each of the following in exponential form:

(i) $\{(3^2)^3 \times 2^6\} \times 5^6$

(ii) $(x/y)^{12} \times y^{24} \times (2^3)^4$

(iii) $(5/2)^6 \times (5/2)^2$

(iv) $(2/3)^5 \times (3/5)^5$

Solution:

(i) Given $\{(3^2)^3 \times 2^6\} \times 5^6$

$= \{3^6 \times 2^6\} \times 5^6$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 6^6 \times 5^6$ [since law of exponents states that $a^m \times b^m = (a \times b)^m$]

$= 30^6$

(ii) Given $(x/y)^{12} \times y^{24} \times (2^3)^4$

$= (x^{12}/y^{12}) \times y^{24} \times 2^{12}$

$= x^{12} \times y^{24-12} \times 2^{12}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= x^{12} \times y^{12} \times 2^{12}$

$= (2xy)^{12}$

(iii) Given $(5/2)^6 \times (5/2)^2$

$= (5/2)^{6+2}$ [According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= (5/2)^8$

(iv) Given $(2/3)^5 \times (3/5)^5$

$= (2/5)^5$ [since law of exponents states that $a^m \times b^m = (a \times b)^m$]

4. Write $9 \times 9 \times 9 \times 9 \times 9$ in exponential form with base 3.

Solution:

Given $9 \times 9 \times 9 \times 9 \times 9 = (9)^5 = (3^2)^5$

$= 3^{10}$

5. Simplify and write each of the following in exponential form:

(i) $(25)^3 \div 5^3$

(ii) $(81)^5 \div (3^2)^5$

(iii) $9^8 \times (x^2)^5 / (27)^4 \times (x^3)^2$

(iv) $3^2 \times 7^8 \times 13^6 / 21^2 \times 91^3$

Solution:

(i) Given $(25)^3 \div 5^3$

$= (5^2)^3 \div 5^3$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 5^6 \div 5^3$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^{6-3}$

$= 5^3$

(ii) Given $(81)^5 \div (3^2)^5$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= (81)^5 \div 3^{10}$ [$81 = 3^4$]

$= (3^4)^5 \div 3^{10}$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 3^{20} \div 3^{10}$

$= 3^{20-10}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 3^{10}$

(iii) Given $9^8 \times (x^2)^5 / (27)^4 \times (x^3)^2$

$= (3^2)^8 \times (x^2)^5 / (3^3)^4 \times (x^3)^2$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 3^{16} \times x^{10} / 3^{12} \times x^6$

$= 3^{16-12} \times x^{10-6}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 3^4 \times x^4$

$= (3x)^4$

(iv) Given $(3^2 \times 7^8 \times 13^6) / (21^2 \times 91^3)$

$= (3^2 \times 7^2 7^8 \times 13^6) / (21^2 \times 13^3 \times 7^3)$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= (21^2 \times 7^2 \times 13^6) / (21^2 \times 13^3 \times 7^3)$

$= (7^6 \times 13^6) / (13^3 \times 7^3)$

$= 91^6 / 91^3$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 91^{6-3}$

$= 91^3$

6. Simplify:

(i) $(3^5)^{11} \times (3^{15})^4 - (3^5)^{18} \times (3^5)^5$

(ii) $(16 \times 2^{n+1} - 4 \times 2^n)/(16 \times 2^{n+2} - 2 \times 2^{n+2})$

(iii) $(10 \times 5^{n+1} + 25 \times 5^n)/(3 \times 5^{n+2} + 10 \times 5^{n+1})$

(iv) $(16)^7 \times (25)^5 \times (81)^3/(15)^7 \times (24)^5 \times (80)^3$

Solution:

(i) Given $(3^5)^{11} \times (3^{15})^4 - (3^5)^{18} \times (3^5)^5$

$= (3)^{55} \times (3)^{60} - (3)^{90} \times (3)^{25}$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 3^{55+60} - 3^{90+25}$

$= 3^{115} - 3^{115}$

$= 0$

(ii) Given $(16 \times 2^{n+1} - 4 \times 2^n)/(16 \times 2^{n+2} - 2 \times 2^{n+2})$

$= (2^4 \times 2^{(n+1)} - 2^2 \times 2^n)/(2^4 \times 2^{(n+2)} - 2^{2+1} \times 2^2)$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 2^2 \times 2^{(n+3-2n)}/)2^2 \times 2^{(n+4-2n+1)}$

$= 2^n \times 2^3 - 2^n/ 2^n \times 2^4 - 2^n \times 2$

$= 2^n(2^3 - 1)/ 2^n(2^4 - 1)$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 8 -1 /16 -2$

$= 7/14$

$= (1/2)$

(iii) Given $(10 \times 5^{n+1} + 25 \times 5^n)/(3 \times 5^{n+2} + 10 \times 5^{n+1})$

$= (10 \times 5^{n+1} + 5^2 \times 5^n)/(3 \times 5^{n+2} + (2 \times 5) \times 5^{n+1})$

$= (10 \times 5^{n+1} + 5 \times 5^{n+1})/(3 \times 5^{n+2} + (2 \times 5) \times 5^{n+1})$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 5^{n+1} (10+5)/ 5^{n+1} (10+15)$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 15/25$

$= (3/5)$

(iv) Given $(16)^7 \times (25)^5 \times (81)^3/(15)^7 \times (24)^5 \times (80)^3$

$= (16)^7 \times (5^2)^5 \times (3^4)^3/(3 \times 5)^7 \times (3 \times 8)^5 \times (16 \times 5)^3$

$= (16)^7 \times (5^2)^5 \times (3^4)^3/3^7 \times 5^7 \times 3^5 \times 8^5 \times 16^3 \times 5^3$

$= (16)^7/ 8^5 \times 16^3$

$= (16)^4/8^5$

$= (2 \times 8)^4/8^5$

$= 2^4/8$

$= (16/8)$

$= 2$

7. Find the values of n in each of the following:

(i) $5^{2n} \times 5^3 = 5^{11}$

(ii) $9 \times 3^n = 3^7$

(iii) $8 \times 2^{n+2} = 32$

(iv) $7^{2n+1} \div 49 = 7^3$

(v) $(3/2)^4 \times (3/2)^5 = (3/2)^{2n+1}$

(vi) $(2/3)^{10} \times \{(3/2)^2\}^5 = (2/3)^{2n-2}$

Solution:

(i) Given $5^{2n} \times 5^3 = 5^{11}$

$= 5^{2n+3} = 5^{11}$

On equating the coefficients, we get

$2n + 3 = 11$

$\Rightarrow 2n = 11 - 3$

$\Rightarrow 2n = 8$

$\Rightarrow n = (8/2)$

$\Rightarrow n = 4$

(ii) Given $9 \times 3^n = 3^7$

$= (3)^2 \times 3^n = 3^7$

$= (3)^{2+n} = 3^7$

On equating the coefficients, we get

$2 + n = 7$

$\Rightarrow n = 7 - 2 = 5$

(iii) Given $8 \times 2^{n+2} = 32$

$= (2)^3 \times 2^{n+2} = (2)^5$ [since $2^3 = 8$ and $2^5 = 32$]
$= (2)^{3+n+2} = (2)^5$

On equating the coefficients, we get

$3 + n + 2 = 5$

$\Rightarrow n + 5 = 5$

$\Rightarrow n = 5 - 5$

$\Rightarrow n = 0$

(iv) Given $7^{2n+1} \div 49 = 7^3$

$= 7^{2n+1} \div 7^2 = 7^3$ [since $49 = 7^2$]

$= 7^{2n+1-2} = 7^3$

$= 7^{2n-1} = 7^3$

On equating the coefficients, we get

$2n - 1 = 3$

$\Rightarrow 2n = 3 + 1$

$\Rightarrow 2n = 4$

$\Rightarrow n = 4/2 = 2$

(v) Given $(3/2)^4 \times (3/2)^5 = (3/2)^{2n+1}$

$= (3/2)^{4+5} = (3/2)^{2n+1}$

$= (3/2)^9 = (3/2)^{2n+1}$

On equating the coefficients, we get

$2n + 1 = 9$

$\Rightarrow 2n = 9 - 1$

$\Rightarrow 2n = 8$

$\Rightarrow n = 8/2 = 4$

(vi) Given $(2/3)^{10} \times \{(3/2)^2\}^5 = (2/3)^{2n-2}$

$= (2/3)^{10} \times (3/2)^{10} = (2/3)^{2n-2}$

$= 2^{10} \times 3^{10}/3^{10} \times 2^{10} = (2/3)^{2n-2}$

$= 1 = (2/3)^{2n-2}$

$= (2/3)^0 = (2/3)^{2n-2}$

On equating the coefficients, we get

$0 = 2n - 2$

$2n - 2 = 0$

$2n = 2$

$n = 1$

8. If $(9^n \times 3^2 \times 3^n - (27)^n)/ (3^3)^5 \times 2^3 = (1/27)$, find the value of n.

Solution:

Given $(9^n \times 3^2 \times 3^n - (27)^n)/ (3^3)^5 \times 2^3 = (1/27)$

$= (3^2)^n \times 3^3 \times 3^n - (3^3)^n/ (3^{15} \times 2^3) = (1/27)$

$= 3^{(2n+2+n)} - (3^3)^n/ (3^{15} \times 2^3) = (1/27)$

$= 3^{(3n+2)} - (3^3)^n/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times 3^2 - 3^{3n}/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times (3^2 - 1)/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times (9 - 1)/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times (8)/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n} \times 2^3/ (3^{15} \times 2^3) = (1/27)$

$= 3^{3n}/3^{15} = (1/27)$

$= 3^{3n-15} = (1/27)$

$= 3^{3n-15} = (1/3^3)$

$= 3^{3n-15} = 3^{-3}$

On equating the coefficients, we get

$3n - 15 = -3$

$\Rightarrow$ 3n = -3 + 15

$\Rightarrow$ 3n = 12

$\Rightarrow$ n = 12/3 = 4

Exercise 6.3 Page No: 6.30

Express the following numbers in the standard form:
(i) 3908.78
(ii) 5,00,00,000
(iii) 3,18,65,00,000
(iv) 846 × 10^7
(v) 723 × 10^9

Solution:

(i) Given 3908.78

3908.78 = 3.90878 x 10^3 [since the decimal point is moved 3 places to the left]

(ii) Given 5,00,00,000

5,00,00,000 = 5,00,00,000.00 = 5 x 10^7 [since the decimal point is moved 7 places to the left]

(iii) Given 3,18,65,00,000

3,18,65,00,000 = 3,18,65,00,000.00

= 3.1865 x 10^9 [since the decimal point is moved 9 places to the left]

(iv) Given 846 × 10^7

846 × 10^7 = 8.46 x 10^2 x 10 [since the decimal point is moved 2 places to the left]
= 8.46 x 10^9 [since a^m x a^n = a^{m+n}]

(v) Given 723 × 10^9

723 × 10^9 = 7.23 x 10^2 x 10^9 [since the decimal point is moved 2 places to the left]
= 7.23 x 10^{11} [since a^m x a^n = a^{m+n}]

2. Write the following numbers in the usual form:
(i) 4.83 × 10^7
(ii) 3.21 × 10^5
(iii) 3.5 × 10^3

Solution:

(i) Given 4.83 × 10^7

4.83 × 10^7 = 483 × 10^{7-2} [since the decimal point is moved two places to the right]

= 483 × 10^5

= 4, 83, 00,000

(ii) Given 3.21 × 10^5

3.21 × 10^5 = 321 x 10^{5-2} [since the decimal point is moved two places to the right]
= 321 x 10^3

= 3, 21,000

(iii) Given 3.5×10^3

$3.5 \times 10^3 = 35 \times 10^{3-1}$ [since the decimal point is moved one place to the right]

$= 35 \times 10^2$

$= 3,500$

3. Express the numbers appearing in the following statements in the standard form:

(i) The distance between the Earth and the Moon is 384,000,000 meters.
(ii) Diameter of the Earth is 1, 27, 56,000 meters.
(iii) Diameter of the Sun is 1,400,000,000 meters.
(iv) The universe is estimated to be about 12,000,000,000 years old.

Solution:

(i) Given the distance between the Earth and the Moon is 384,000,000 meters.

The distance between the Earth and the Moon is 3.84×10^8 meters.

[Since the decimal point is moved 8 places to the left.]

(ii) Given diameter of the Earth is 1, 27, 56,000 meters.

The diameter of the Earth is 1.2756×10^7 meters.

[Since the decimal point is moved 7 places to the left.]

(iii) Given diameter of the Sun is 1,400,000,000 meters.

The diameter of the Sun is 1.4×10^9 meters.

[Since the decimal point is moved 9 places to the left.]

(iv) Given the universe is estimated to be about 12,000,000,000 years old.

The universe is estimated to be about 1.2×10^{10} years old.

[Since the decimal point is moved 10 places to the left.]

EVER

Exercise 6.4 Page No: 6.31

1. Write the following numbers in the expanded exponential forms:
(i) 20068
(ii) 420719
(iii) 7805192
(iv) 5004132
(v) 927303

Solution:

(i) Given 20068

$20068 = 2 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

(ii) Given 420719

$420719 = 4 \times 10^5 + 2 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 9 \times 10^0$

(iii) Given 7805192

$7805192 = 7 \times 10^6 + 8 \times 10^5 + 0 \times 10^4 + 5 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$

(iv) Given 5004132

$5004132 = 5 \times 10^6 + 0 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 1 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$

(v) Given 927303

$927303 = 9 \times 10^5 + 2 \times 10^4 + 7 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$

2. Find the number from each of the following expanded forms:
(i) $7 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$
(ii) $5 \times 10^5 + 4 \times 10^4 + 2 \times 10^3 + 3 \times 10^0$
(iii) $9 \times 10^5 + 5 \times 10^2 + 3 \times 10^1$
(iv) $3 \times 10^4 + 4 \times 10^2 + 5 \times 10^0$

Solution:

(i) Given $7 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$

 = 7 x 10000 + 6 x 1000 + 0 x 100 + 4 x 10 + 5 x 1

= 70000 + 6000 + 0 + 40 + 5

= 76045

(ii) Given $5 \times 10^5 + 4 \times 10^4 + 2 \times 10^3 + 3 \times 10^0$

= 5 x 100000 + 4 x 10000 + 2 x 1000 + 3 x 1

= 500000 + 40000 + 2000 + 3

= 542003

(iii) Given $9 \times 10^5 + 5 \times 10^2 + 3 \times 10^1$

= 9 x 100000 + 5 x 100 + 3 x 10

= 900000 + 500 + 30

= 900530

(iv) Given $3 \times 10^4 + 4 \times 10^2 + 5 \times 10^0$

= 3 x 10000 + 4 x 100 + 5 x 1

= 30000 + 400 + 5

= 30405