NCERT Solutions for Class 10 Maths Chapter 10 - Circles

Chapter 10 - Circles Exercise Ex. 10.1

Solution 1

A circle can have infinite tangents.

Solution 2

(i) one

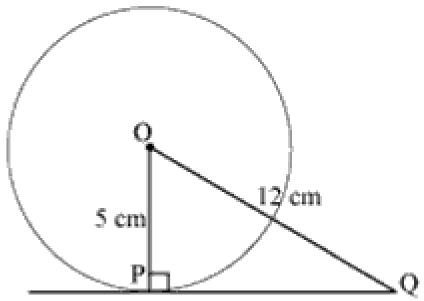
(ii) secant

(iii) two

(iv) point of contact

Solution 3

Radius is perpendicular to the tangent at the point of contact. So, $OP \perp PQ$.



Now, applying Pythagoras theorem in ΔOPQ,

$OP^2 + PQ^2 = OQ^2$

$5^2 + PQ^2 = 12^2$

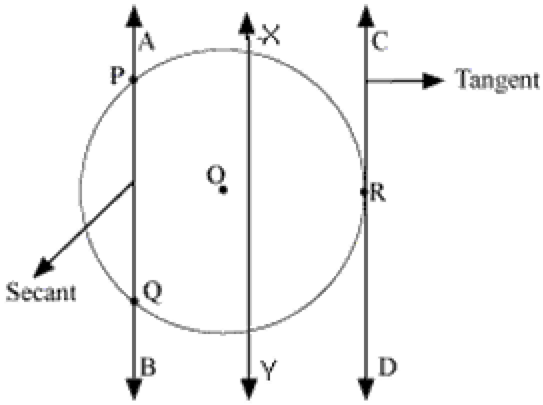
$PQ^2 = 144 - 25$

$PQ = \sqrt{119}$ cm

Hence, (D) is the correct answer.

Concept insight: To answer such type of problems, remember to use the result that the radius is perpendicular to the tangent at the point of contact and then make use of Pythagoras theorem in the right triangle.

Solution 4

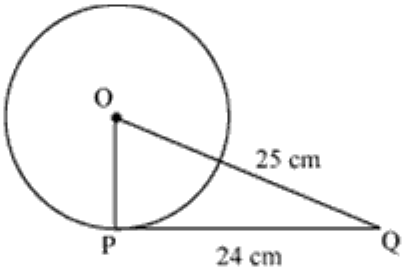


AB and CD are two lines parallel to the line XY. Line AB is intersecting the circle at exactly two points, P and Q. So, line AB is secant of this circle. The line CD is intersecting the circle at exactly one point, R. So, line CD is tangent to circle.

Concept: To draw the two lines, remember that a tangent touches the circle at one point only and a secant intersects the circle at two distinct points.

Chapter 10 - Circles Exercise Ex. 10.2

Solution 1



Let O be the center of the circle
OQ = 25cm and PQ=24 cm

Since, radius is perpendicular to tangent at the point of contact, $OP \perp PQ$.

Applying Pythagoras theorem in Δ OPQ,
$OP^2 + PQ^2 = OQ^2$
$OP^2 + 24^2 = 25^2$
$OP^2 = 625 - 576$
$OP^2 = 49$
$OP = 7$
Thus, the radius of the circle is 7 cm.
Hence, alternative (a) is correct.

Concept insight: To answer such type of problems, remember to use the result that the radius is perpendicular to the tangent at the point of contact and then make use of Pythagoras theorem in the right triangle.

Solution 2

Since, radius is perpendicular to tangent at the point of contact, $OP \perp TP$ and $OQ \perp TQ$.

$\angle OPT = 90^\circ$

$\angle OQT = 90^\circ$

In the quadrilateral POQT,

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$

$90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\therefore \angle PTQ = 70^\circ$

Hence, alternative (B) is correct.

Concept insight: Here, TP and TQ are tangents to the circle and OP and OQ are the radii of the circle. So, here the result, radius is perpendicular to tangent at the point of contact, will be used. Also, remember that the sum of all the interior angles of a quadrilateral is 360°.

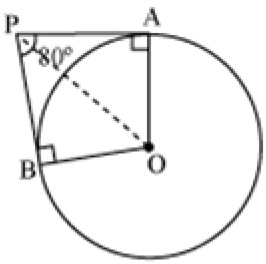
Solution 3

Radius drawn to these tangents will be perpendicular to the tangents.

Therefore, $OA \perp PA$ and $OB \perp PB$

$\angle OBP = 90^\circ$

$\angle OAP = 90^\circ$



In the quadrilateral AOBP,

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$

$90^\circ + 80^\circ + 90^\circ + \angle BOA = 360^\circ$

$\angle BOA = 100^\circ$

In ΔOPB and ΔOPA,

AP = BP (tangents drawn from an external point are equal in length)

OA = OB (radius of circle)

OP = OP (common side)

$\therefore \Delta OPB \cong \Delta OPA$

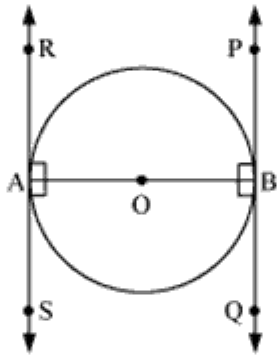
$\therefore \angle POB = \angle POA$

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Concept insight: Here, PA and PB are tangents to the circle and OA and OB are the radii of the circle. So, here the result, radius is perpendicular to tangent at the point of contact, will be used. Also, remember that the sum of all the interior angles of a quadrilateral is 360°. The key step in this question is to use the congruency of triangles OPB and OPA obtained by joining OP.

Solution 4



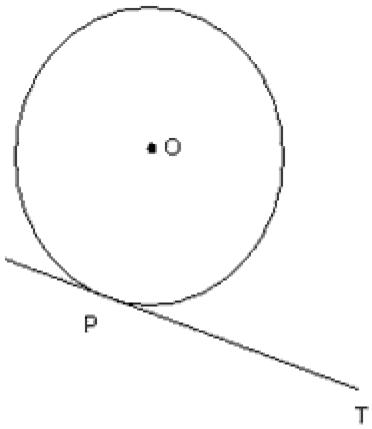
Let AB be a diameter of circle. Two tangents PQ and RS are drawn at the end points of the diameter AB.

It is known that the radius is perpendicular to tangent at the point of contact.
Therefore, $\angle OAR = 90°$, $\angle OAS = 90°$, $\angle OBP = 90°$ and $\angle OBQ = 90°$

$\angle OAR = \angle OBQ$ (alternate interior angles)
$\angle OAS = \angle OBP$ (alternate interior angles)
Since, alternate interior angles are equal, lines PQ and RS will be parallel.

Concept insight: Here, the result that the radius is perpendicular to tangent at the point of contact, can be used. Then, the criteria of two lines to be parallel will be used here.

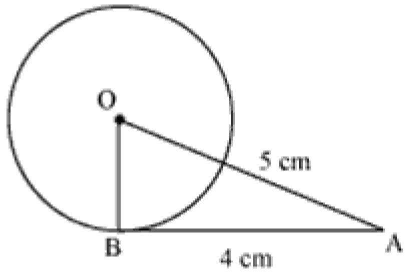
Solution 5



Let P be the point of contact and PT be the tangent at the point P on the circle with centre O.
Since OP is radius of the circle and PT is a tangent at P, $OP \perp PT$.

Thus, the perpendicular at the point of contact to the tangent passes through the centre.

Solution 6

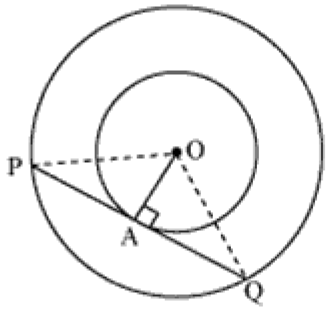


AB is a tangent drawn to the circle, with centre O, from point A.
OA = 5cm and AB = 4 cm

Since, radius is perpendicular at the point of contact, OB $\perp$ AB.

Applying Pythagoras theorem in ΔABO,
$AB^2 + BO^2 = OA^2$
$4^2 + BO^2 = 5^2$
$BO^2 = 9$
$BO = 3$
Hence, the radius of the circle is 3 cm.

Solution 7



Let two concentric circles be centered at point O. Let PQ be the chord of the larger circle which touches the smaller circle at point A. So, PQ is tangent to smaller circle.
Since, OA is radius of circle, $OA \perp PQ$

Applying Pythagoras theorem in ΔOAP,
$OA^2 + AP^2 = OP^2$
$3^2 + AP^2 = 5^2$
$AP^2 = 16$
$AP = 4$ cm

In ΔOPQ, as $OA \perp PQ$, $AP = AQ$
(Perpendicular from center of circle bisects the chord)
$\therefore PQ = 2AP = 2 \times 4$ cm $= 8$ cm
So, length of chord of larger circle is 8cm.

Concept insight: Here, PQ is a chord of the larger circle and it touches the smaller circle at a point. So, it will act as a tangent to the smaller circle. Then, by applying the result that the radius is perpendicular to the tangent at the point of contact, a right traingle will be obtained in which Pythagoras theorem will be applied. Then, to find the length of the chord PQ, a result can be used, which states that, perpendicular drawn from center of a circle bisects the chord.

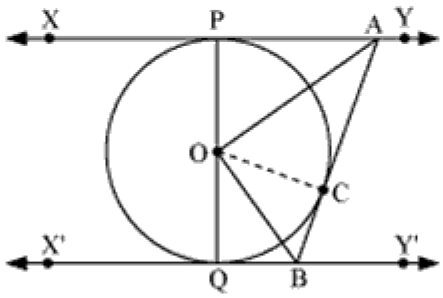
Solution 8

It can be observed that:
DR = DS (tangents from point D)
CR = CQ (tangents from point C)
BP = BQ (tangents from point B)
AP = AS (tangents from point A)

Adding the above four equations,
DR + CR + BP + AP = DS + CQ + BQ + AS
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)
CD + AB = AD + BC

Concept insight: AP and AS; BP and BQ; CR and CQ; DR and DS are the pair tangents drawn to the circle drawn from the external points A, B, C, D respectively. So, the result that the tangents drawn from an external point are equal, will be applied here.

Solution 9



Join OC.
In ΔOPA and ΔOCA,

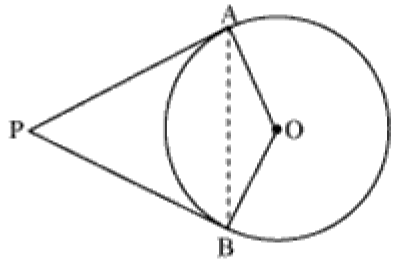
OP = OC	(Radius of the same circle)
AP = AC	(tangents from point A)
AO = AO	(common)
$\Delta OPA \cong \Delta OCA$	(SSS congruence rule)
$\therefore \angle POA = \angle COA$	... (1)

Similarly $\Delta OQB \cong \Delta OCB$	
$\therefore \angle QOB = \angle COB$	... (2)

Since POQ is a diameter of circle we can say it is a straight line.
So, $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$
Now from equations (1) and (2),
$2\angle COA + 2\angle COB = 180^\circ$
$(\angle COA + \angle COB) = 90^\circ$
$\angle AOB = 90^\circ$

Concept insight: Here, the key idea is to establish the equality of $\angle POA$ and $\angle COA$ by proving the congruency of ΔOPA and ΔOCA and the equality of $\angle QOB$ and $\angle COB$ by proving the congruency of ΔOQB and ΔOCB.

Solution 10



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to circle which are touching circle at point A and B respectively.
AB is the line segment, joining point of contacts A and B together such that it subtends $\angle AOB$ at center O of circle.

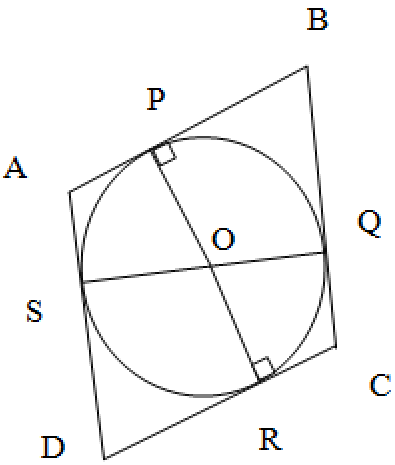
As the radius is perpendicular to the tangent at the point of contact, $\angle OAP = 90°$.
Similarly, $\angle OBP = 90°$

In quadrilateral OAPB,
$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$
$90° + \angle APB + 90° + \angle BOA = 360°$
$\angle APB + \angle BOA = 180°$

Hence, the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Solution 11

Since, ABCD is a parallelogram,
AB = CD (i)
BC = AD (ii)



Now, it can be observed that:
DR = DS (tangents on circle from point D)
CR = CQ (tangents on circle from point C)
BP = BQ (tangents on circle from point B)

AP = AS (tangents on circle from point A)

Adding all the above four equations,

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC (iii)

From equation (i) (ii) and (ii):

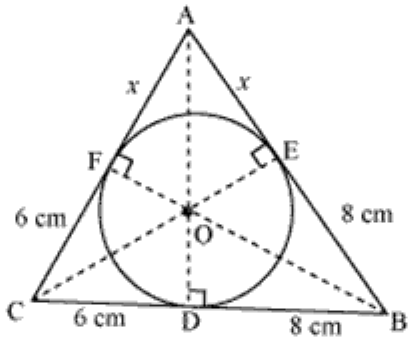
2AB = 2BC

AB = BC

AB = BC = CD = DA

Hence, ABCD is a rhombus.

Solution 12



Let given circle touches the sides AB and AC of the triangle at point E and F respectively and the length of line segment AF be x.

Now, it can be observed that:

CF = CD = 6cm (tangents from point C)

BE = BD = 8cm (tangents from point B)

AE = AF = x (tangents from point A)

AB = AE + EB = x + 8

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = 6 + x

2s = AB + BC + CA = x + 8 + 14 + 6 + x = 28 + 2x

s = 14 + x

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{(14+x)\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$$
$$= \sqrt{(14+x)(x)(8)(6)}$$
$$= 4\sqrt{3(14x+x^2)}$$

$$\text{Area of } \Delta OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$$
$$\text{Area of } \Delta OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$$
$$\text{Area of } \Delta OAB = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$$
$$4\sqrt{3(14x+x^2)} = 56 + 4x$$
$$\sqrt{3(14x+x^2)} = 14 + x$$
$$3(14x+x^2) = (14+x)^2$$
$$42x + 3x^2 = 196 + x^2 + 28x$$

$$2x^2 + 14x - 196 = 0$$
$$x^2 + 7x - 98 = 0$$
$$x^2 + 14x - 7x - 98 = 0$$
$$x(x+14) - 7(x+14) = 0$$
$$(x+14)(x-7) = 0$$

$\therefore\ x = -14$ or 7

But $x = -14$ is not possible as length of sides will be negative.

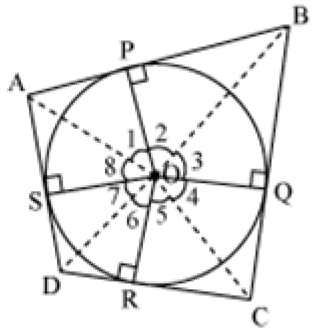
So, $x = 7$

Hence AB = $x + 8 = 7 + 8 = 15$ cm

CA = $6 + x = 6 + 7 = 13$ cm

Concept insight: Carefully analyse the given conditions. We have to find the sides AB and AC. For that a composition of results will be used. By using the result that tangents drawn from an external point are equal in length, a part of the length of the two sides can be found. Then the formula of area of a triangle will be used.

Solution 13



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S.
Join the vertices of the ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS	(tangents from same point)
OP = OS	(radius of circle)
OA = OA	(common)
So, $\Delta OAP \cong \Delta OAS$	(SSS congruence rule)

$\therefore \angle POA = \angle SOA$
$\angle 1 = \angle 8$

Similarly,
$\angle 2 = \angle 3$
$\angle 4 = \angle 5$
$\angle 6 = \angle 7$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$
$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$
$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$
$\angle AOB + \angle COD = 180°$

Similarly, $\angle BOC + \angle DOA = 180°$
Hence opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Concept insight: Here, it is given that a quadrilateral is circumscribing a circle so draw the diagram carefully. From the figure, you can observe pairs of tangents drawn from external points, so the result that tangents from an external point are equal in length will be applied. Then, the fact that the angle at a point is 360° will be used.