Access answers to Maths RD Sharma Solutions For Class 12 Chapter 4 – Inverse Trigonometric Functions

Exercise 4.1 Page No: 4.6

1. Find the principal value of the following:

$(i)\ sin^{-1}(-\sqrt{\frac{3}{2}})$

$(ii)\ sin^{-1}(\cos\frac{2\pi}{3})$

$(iii)\ \sin^{-1}(\frac{\sqrt{3}-1}{2\sqrt{2}})$

$(iv)\ \sin^{-1}(\frac{\sqrt{3}+1}{2\sqrt{2}})$

$(v)\ \sin^{-1}(\cos\frac{3\pi}{4})$

$(vi)\ \sin^{-1}(\tan\frac{5\pi}{4})$

Solution:

$(i) Let\sin^{-1}(\frac{-\sqrt{3}}{2}) = y$

$Then\ siny = (\frac{-\sqrt{3}}{2})$

$= -\sin(\frac{\pi}{3})$

$= sin(-\frac{\pi}{3})$

$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1}\ is\ \left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$And - sin\frac{\pi}{3} = sin(\frac{-\pi}{3})$

$Therefore\ principal\ value\ of\sin^{-1}(\frac{-\sqrt{3}}{2}) = \frac{-\pi}{3}$

$$(ii)\, Let\, \sin^{-1}(\cos\frac{2\pi}{3}) = y$$

$$Then\ \ sin y = \cos(\frac{2\pi}{3})$$

$$= -\sin(\frac{\pi}{2} + \frac{\pi}{6})$$

$$= -sin(\frac{\pi}{6})$$

$$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1} is \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$And\ \ -sin(\frac{\pi}{6}) = cos(\frac{2\pi}{3})$$

$$Therefore\ principal\ value\ of\ \sin^{-1}(\cos\frac{2\pi}{3})\ is \frac{-\pi}{6}$$

(iii) Given functions can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the values,

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

Taking LCM and cross multiplying we get,

$$= \frac{\pi}{12}$$

(iv) The given question can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the corresponding values we get

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{7\pi}{12}$$

(v) Let

$\sin^{-1}\left(\cos\frac{3\pi}{4}\right) = y$

Then above equation can be written as

$\sin y = \cos\frac{3\pi}{4} = -\sin\left(\pi - \frac{3\pi}{4}\right) = -\sin(\frac{\pi}{4})$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore above equation becomes,

$-\sin\left(\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$ is $-\frac{\pi}{4}$

(vi) Let

$y = \sin^{-1}\left(\tan\frac{5\pi}{4}\right)$

Therefore above equation can be written as

$\text{Sin } y = \left(\tan\frac{5\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 = \sin\left(\frac{\pi}{2}\right)$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{\pi}{2}\right) = \tan\frac{5\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$ is $\frac{\pi}{2}$.

2.

(i) $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}}$

(ii) $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

Solution:

(i) The given question can be written as,

$$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\left(2 \times \frac{1}{\sqrt{2}}\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}(1)$$

By substituting the corresponding values, we get

$$= \frac{\pi}{6} - \frac{\pi}{2}$$

$$= -\frac{\pi}{3}$$

(ii) Given question can be written as

We know that $\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \cos(\pi/3)$

$$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$$

Now substituting the values we get,

$$= \sin^{-1}\left\{\frac{\sqrt{3}}{2}\right\}$$

$$= \frac{\pi}{6}$$

Exercise 4.2 Page No: 4.10

1. Find the domain of definition of $f(x) = \cos^{-1}(x^2 - 4)$

Solution:

Given $f(x) = \cos^{-1}(x^2 - 4)$

We know that domain of $\cos^{-1}(x^2 - 4)$ lies in the interval [-1, 1]

Therefore, we can write as

$-1 \leq x^2 - 4 \leq 1$

$4 - 1 \leq x^2 \leq 1 + 4$

$3 \leq x^2 \leq 5$

$\pm\sqrt{3} \leq x \leq \pm\sqrt{5}$

$-\sqrt{5} \leq x \leq -\sqrt{3}$ and $\sqrt{3} \leq x \leq \sqrt{5}$

Therefore domain of $\cos^{-1}(x^2 - 4)$ is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

2. Find the domain of $f(x) = \cos^{-1} 2x + \sin^{-1} x$.

Solution:

Given that $f(x) = \cos^{-1} 2x + \sin^{-1} x$.

Now we have to find the domain of f(x),

We know that domain of $\cos^{-1} x$ lies in the interval [-1, 1]

Also know that domain of $\sin^{-1} x$ lies I the interval [-1, 1]

Therefore, the domain of $\cos^{-1}(2x)$ lies in the interval [-1, 1]

Hence we can write as,

$-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

Hence domain $\cos^{-1}(2x) + \sin^{-1} x$ lies in the interval $[-\frac{1}{2}, \frac{1}{2}]$

Exercise 4.3 Page No: 4.14

1. Find the principal value of each of the following:

(i) $\tan^{-1}(1/\sqrt{3})$

(ii) $\tan^{-1}(-1/\sqrt{3})$

(iii) $\tan^{-1}(\cos(\pi/2))$

(iv) $\tan^{-1}(2\cos(2\pi/3))$

Solution:

(i) Given $\tan^{-1}(1/\sqrt{3})$

We know that for any $x \in R$, $\tan^{-1}$ represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x.

So, $\tan^{-1}(1/\sqrt{3})$ = an angle in $(-\pi/2, \pi/2)$ whose tangent is $(1/\sqrt{3})$

But we know that the value is equal to $\pi/6$

Therefore $\tan^{-1} (1/\sqrt{3}) = \pi/6$

Hence the principal value of $\tan^{-1} (1/\sqrt{3}) = \pi/6$

(ii) Given $\tan^{-1} (-1/\sqrt{3})$

We know that for any $x \in R$, $\tan^{-1}$ represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x.

So, $\tan^{-1} (1-/\sqrt{3})$ = an angle in $(-\pi/2, \pi/2)$ whose tangent is $(1/\sqrt{3})$

But we know that the value is equal to $-\pi/6$

Therefore $\tan^{-1} (-1/\sqrt{3}) = -\pi/6$

Hence the principal value of $\tan^{-1} (-1/\sqrt{3}) = -\pi/6$

(iii) Given that $\tan^{-1} (\cos (\pi/2))$

But we know that $\cos (\pi/2) = 0$

We know that for any $x \in R$, $\tan^{-1}$ represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x.

Therefore $\tan^{-1} (0) = 0$

Hence the principal value of $\tan^{-1} (\cos (\pi/2)$ is 0.

(iv) Given that $\tan^{-1} (2 \cos (2\pi/3))$

But we know that $\cos \pi/3 = -1$

Therefore $\tan^{-1} (2 \cos (2\pi/3)) = \tan^{-1} (2 \times -½)$

$= \tan^{-1}(-1)$

$= -\pi/4$

Hence the principal value of $\tan^{-1} (2 \cos (2\pi/3))$ is $-\pi/4$

Exercise 4.4 Page No: 4.18

1. Find the principal value of each of the following:

(i) $\sec^{-1} (-\sqrt{2})$

(ii) $\sec^{-1} (2)$

(iii) $\sec^{-1} (2 \sin (3\pi/4))$

(iv) $\sec^{-1} (2 \tan (3\pi/4))$

Solution:

(i) Given $\sec^{-1} (-\sqrt{2})$

Now let $y = \sec^{-1} (-\sqrt{2})$

Sec y = $-\sqrt{2}$

We know that sec $\pi/4 = \sqrt{2}$

Therefore – sec $(\pi/4) = \sqrt{2}$

= sec $(\pi - \pi/4)$

= sec $(3\pi/4)$

Thus the range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\pi/2\}$

And sec $(3\pi/4) = -\sqrt{2}$

Hence the principal value of $\sec^{-1}(-\sqrt{2})$ is $3\pi/4$

(ii) Given $\sec^{-1}(2)$

Let $y = \sec^{-1}(2)$

Sec y = 2

= Sec $\pi/3$

Therefore the range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\pi/2\}$ and sec $\pi/3 = 2$

Thus the principal value of $\sec^{-1}(2)$ is $\pi/3$

(iii) Given $\sec^{-1}(2 \sin(3\pi/4))$

But we know that $\sin(3\pi/4) = 1/\sqrt{2}$

Therefore $2 \sin(3\pi/4) = 2 \times 1/\sqrt{2}$

$2 \sin(3\pi/4) = \sqrt{2}$

Therefore by substituting above values in $\sec^{-1}(2 \sin(3\pi/4))$, we get

$\text{Sec}^{-1}(\sqrt{2})$

Let $\text{Sec}^{-1}(\sqrt{2}) = y$

Sec y = $\sqrt{2}$

Sec $(\pi/4) = \sqrt{2}$

Therefore range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\pi/2\}$ and sec $(\pi/4) = \sqrt{2}$

Thus the principal value of $\sec^{-1}(2 \sin(3\pi/4))$ is $\pi/4$.

(iv) Given $\sec^{-1}(2 \tan(3\pi/4))$

But we know that $\tan(3\pi/4) = -1$

Therefore, $2 \tan(3\pi/4) = 2 \times -1$

$2 \tan(3\pi/4) = -2$

By substituting these values in $\sec^{-1}(2 \tan(3\pi/4))$, we get

Sec^{-1} (-2)

Now let y = Sec^{-1} (-2)

Sec y = – 2

– sec (π/3) = 2

= sec (π – π/3)

= sec (2π/3)

Therefore the range of principal value of sec^{-1} is [0, π] – {π/2} and sec (2π/3) = -2

Thus the principal value of sec^{-1} (2 tan (3π/4)) is (2π/3)

Exercise 4.5 Page No: 4.21

1. Find the principal values of each of the following:

(i) $cosec^{-1}$ (-√2)

(ii) $cosec^{-1}$ (-2)

(iii) $cosec^{-1}$ (2/√3)

(iv) $cosec^{-1}$ (2 cos (2π/3))

Solution:

(i) Given $cosec^{-1}$ (-√2)

Let y = $cosec^{-1}$ (-√2)

Cosec y = -√2

– Cosec y = √2

– Cosec (π/4) = √2

– Cosec (π/4) = cosec (-π/4) [since –cosec θ = cosec (-θ)]

The range of principal value of $cosec^{-1}$ [-π/2, π/2] – {0} and cosec (-π/4) = – √2

Cosec (-π/4) = – √2

Therefore the principal value of $cosec^{-1}$ (-√2) is – π/4

(ii) Given $cosec^{-1}$ (-2)

Let y = $cosec^{-1}$ (-2)

Cosec y = -2

– Cosec y = 2

– Cosec (π/6) = 2

– Cosec (π/6) = cosec (-π/6) [since –cosec θ = cosec (-θ)]

The range of principal value of cosec^{-1} [-π/2, π/2] – {0} and cosec (-π/6) = – 2

Cosec (-π/6) = – 2

Therefore the principal value of cosec^{-1} (-2) is – π/6

(iii) Given cosec^{-1} (2/√3)

Let y = cosec^{-1} (2/√3)

Cosec y = (2/√3)

Cosec (π/3) = (2/√3)

Therefore range of principal value of cosec^{-1} is [-π/2, π/2] – {0} and cosec (π/3) = (2/√3)

Thus, the principal value of cosec^{-1} (2/√3) is π/3

(iv) Given cosec^{-1} (2 cos (2π/3))

But we know that cos (2π/3) = – ½

Therefore 2 cos (2π/3) = 2 × – ½

2 cos (2π/3) = -1

By substituting these values in cosec^{-1} (2 cos (2π/3)) we get,

Cosec^{-1} (-1)

Let y = cosec^{-1} (-1)

– Cosec y = 1

– Cosec (π/2) = cosec (-π/2) [since –cosec θ = cosec (-θ)]

The range of principal value of cosec^{-1} [-π/2, π/2] – {0} and cosec (-π/2) = – 1

Cosec (-π/2) = – 1

Therefore the principal value of cosec^{-1} (2 cos (2π/3)) is – π/2

Exercise 4.6 Page No: 4.24

1. Find the principal values of each of the following:

(i) $\cot^{-1}(-\sqrt{3})$

(ii) $\text{Cot}^{-1}(\sqrt{3})$

(iii) $\cot^{-1}(-1/\sqrt{3})$

(iv) $\cot^{-1}(\tan 3\pi/4)$

Solution:

(i) Given $\cot^{-1}(-\sqrt{3})$

Let $y = \cot^{-1}(-\sqrt{3})$

$-\text{Cot}(\pi/6) = \sqrt{3}$

$= \text{Cot}(\pi - \pi/6)$

$= \cot(5\pi/6)$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$ and $\cot(5\pi/6) = -\sqrt{3}$

Thus, the principal value of $\cot^{-1}(-\sqrt{3})$ is $5\pi/6$

(ii) Given $\text{Cot}^{-1}(\sqrt{3})$

Let $y = \cot^{-1}(\sqrt{3})$

$\text{Cot}(\pi/6) = \sqrt{3}$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$ and

Thus, the principal value of $\cot^{-1}(\sqrt{3})$ is $\pi/6$

(iii) Given $\cot^{-1}(-1/\sqrt{3})$

Let $y = \cot^{-1}(-1/\sqrt{3})$

$\text{Cot } y = (-1/\sqrt{3})$

$-\text{Cot}(\pi/3) = 1/\sqrt{3}$

$= \text{Cot}(\pi - \pi/3)$

$= \cot(2\pi/3)$

The range of principal value of $\cot^{-1}(0, \pi)$ and $\cot(2\pi/3) = -1/\sqrt{3}$

Therefore the principal value of $\cot^{-1}(-1/\sqrt{3})$ is $2\pi/3$

(iv) Given $\cot^{-1}(\tan 3\pi/4)$

But we know that $\tan 3\pi/4 = -1$

By substituting this value in $\cot^{-1}(\tan 3\pi/4)$ we get

$\text{Cot}^{-1}(-1)$

Now, let $y = \cot^{-1}(-1)$

$\text{Cot } y = (-1)$

$-\text{Cot}(\pi/4) = 1$

$= \text{Cot}(\pi - \pi/4)$

= cot (3π/4)

The range of principal value of $\cot^{-1}(0, \pi)$ and cot (3π/4) = – 1

Therefore the principal value of $\cot^{-1}(\tan 3\pi/4)$ is 3π/4

Exercise 4.7 Page No: 4.42

1. Evaluate each of the following:

(i) $\sin^{-1}(\sin \pi/6)$

(ii) $\sin^{-1}(\sin 7\pi/6)$

(iii) $\sin^{-1}(\sin 5\pi/6)$

(iv) $\sin^{-1}(\sin 13\pi/7)$

(v) $\sin^{-1}(\sin 17\pi/8)$

(vi) $\sin^{-1}\{(\sin - 17\pi/8)\}$

(vii) $\sin^{-1}(\sin 3)$

(viii) $\sin^{-1}(\sin 4)$

(ix) $\sin^{-1}(\sin 12)$

(x) $\sin^{-1}(\sin 2)$

Solution:

(i) Given $\sin^{-1}(\sin \pi/6)$

We know that the value of sin π/6 is ½

By substituting this value in $\sin^{-1}(\sin \pi/6)$

We get, $\sin^{-1}(1/2)$

Now let y = $\sin^{-1}(1/2)$

Sin (π/6) = ½

The range of principal value of $\sin^{-1}(-\pi/2, \pi/2)$ and sin (π/6) = ½

Therefore $\sin^{-1}(\sin \pi/6) = \pi/6$

(ii) Given $\sin^{-1}(\sin 7\pi/6)$

But we know that sin 7π/6 = – ½

By substituting this in $\sin^{-1}(\sin 7\pi/6)$ we get,

$\text{Sin}^{-1}(-1/2)$

Now let y = $\sin^{-1}(-1/2)$

– Sin y = ½

– Sin (π/6) = ½

– Sin (π/6) = sin (- π/6)

The range of principal value of $\sin^{-1}(-\pi/2, \pi/2)$ and sin (- π/6) = – ½

Therefore $\sin^{-1}(\sin 7\pi/6) = -\pi/6$

(iii) Given $\sin^{-1}(\sin 5\pi/6)$

We know that the value of sin 5π/6 is ½

By substituting this value in $\sin^{-1}(\sin \pi/6)$

We get, $\sin^{-1} (1/2)$

Now let $y = \sin^{-1} (1/2)$

Sin (π/6) = ½

The range of principal value of $\sin^{-1}(-\pi/2, \pi/2)$ and sin (π/6) = ½

Therefore $\sin^{-1}(\sin 5\pi/6) = \pi/6$

(iv) Given $\sin^{-1}(\sin 13\pi/7)$

Given question can be written as sin (2π – π/7)

Sin (2π – π/7) can be written as sin (π/7) [since sin (2π – θ) = sin (-θ)]

By substituting these values in $\sin^{-1}(\sin 13\pi/7)$ we get $\sin^{-1}(\sin - \pi/7)$

As $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$

Therefore $\sin^{-1}(\sin 13\pi/7) = -\pi/7$

(v) Given $\sin^{-1}(\sin 17\pi/8)$

Given question can be written as sin (2π + π/8)

Sin (2π + π/8) can be written as sin (π/8)

By substituting these values in $\sin^{-1}(\sin 17\pi/8)$ we get $\sin^{-1}(\sin \pi/8)$

As $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$

Therefore $\sin^{-1}(\sin 17\pi/8) = \pi/8$

(vi) Given $\sin^{-1}\{(\sin - 17\pi/8)\}$

But we know that – sin θ = sin (-θ)

Therefore (sin -17π/8) = – sin 17π/8

– Sin 17π/8 = – sin (2π + π/8) [since sin (2π – θ) = sin (θ)]

It can also be written as – sin (π/8)

– Sin (π/8) = sin (-π/8) [since – sin θ = sin (-θ)]

By substituting these values in $\sin^{-1}\{(\sin - 17\pi/8)\}$ we get,

$\text{Sin}^{-1}(\sin - \pi/8)$

As $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$

Therefore $\sin^{-1}(\sin -\pi/8) = -\pi/8$

(vii) Given $\sin^{-1}(\sin 3)$

We know that $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$ which is approximately equal to $[-1.57, 1.57]$

But here $x = 3$, which does not lie on the above range,

Therefore we know that $\sin(\pi - x) = \sin(x)$

Hence $\sin(\pi - 3) = \sin(3)$ also $\pi - 3 \in [-\pi/2, \pi/2]$

$\text{Sin}^{-1}(\sin 3) = \pi - 3$

(viii) Given $\sin^{-1}(\sin 4)$

We know that $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$ which is approximately equal to $[-1.57, 1.57]$

But here $x = 4$, which does not lie on the above range,

Therefore we know that $\sin(\pi - x) = \sin(x)$

Hence $\sin(\pi - 4) = \sin(4)$ also $\pi - 4 \in [-\pi/2, \pi/2]$

$\text{Sin}^{-1}(\sin 4) = \pi - 4$

(ix) Given $\sin^{-1}(\sin 12)$

We know that $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$ which is approximately equal to $[-1.57, 1.57]$

But here $x = 12$, which does not lie on the above range,

Therefore we know that $\sin(2n\pi - x) = \sin(-x)$

Hence $\sin(2n\pi - 12) = \sin(-12)$

Here $n = 2$ also $12 - 4\pi \in [-\pi/2, \pi/2]$

$\text{Sin}^{-1}(\sin 12) = 12 - 4\pi$

(x) Given $\sin^{-1}(\sin 2)$

We know that $\sin^{-1}(\sin x) = x$ with $x \in [-\pi/2, \pi/2]$ which is approximately equal to $[-1.57, 1.57]$

But here $x = 2$, which does not lie on the above range,

Therefore we know that $\sin(\pi - x) = \sin(x)$

Hence $\sin(\pi - 2) = \sin(2)$ also $\pi - 2 \in [-\pi/2, \pi/2]$

$Sin^{-1}(\sin 2) = \pi - 2$

2. Evaluate each of the following:

(i) $\cos^{-1}\{\cos(-\pi/4)\}$

(ii) $\cos^{-1}(\cos 5\pi/4)$

(iii) $\cos^{-1}(\cos 4\pi/3)$

(iv) $\cos^{-1}(\cos 13\pi/6)$

(v) $\cos^{-1}(\cos 3)$

(vi) $\cos^{-1}(\cos 4)$

(vii) $\cos^{-1}(\cos 5)$

(viii) $\cos^{-1}(\cos 12)$

Solution:

(i) Given $\cos^{-1}\{\cos(-\pi/4)\}$

We know that $\cos(-\pi/4) = \cos(\pi/4)$ [since $\cos(-\theta) = \cos\theta$

Also know that $\cos(\pi/4) = 1/\sqrt{2}$

By substituting these values in $\cos^{-1}\{\cos(-\pi/4)\}$ we get,

$Cos^{-1}(1/\sqrt{2})$

Now let $y = \cos^{-1}(1/\sqrt{2})$

Therefore $\cos y = 1/\sqrt{2}$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos(\pi/4) = 1/\sqrt{2}$

Therefore $\cos^{-1}\{\cos(-\pi/4)\} = \pi/4$

(ii) Given $\cos^{-1}(\cos 5\pi/4)$

But we know that $\cos(5\pi/4) = -1/\sqrt{2}$

By substituting these values in $\cos^{-1}\{\cos(5\pi/4)\}$ we get,

$Cos^{-1}(-1/\sqrt{2})$

Now let $y = \cos^{-1}(-1/\sqrt{2})$

Therefore $\cos y = -1/\sqrt{2}$

$- Cos(\pi/4) = 1/\sqrt{2}$

$Cos(\pi - \pi/4) = -1/\sqrt{2}$

$Cos(3\pi/4) = -1/\sqrt{2}$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos(3\pi/4) = -1/\sqrt{2}$

Therefore $\cos^{-1}\{\cos(5\pi/4)\} = 3\pi/4$

(iii) Given $\cos^{-1}(\cos 4\pi/3)$

But we know that $\cos (4\pi/3) = -1/2$

By substituting these values in $\cos^{-1}\{\cos (4\pi/3)\}$ we get,

$\text{Cos}^{-1}(-1/2)$

Now let $y = \cos^{-1}(-1/2)$

Therefore $\cos y = -1/2$

$-\text{Cos} (\pi/3) = 1/2$

$\text{Cos} (\pi - \pi/3) = -1/2$

$\text{Cos} (2\pi/3) = -1/2$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos (2\pi/3) = -1/2$

Therefore $\cos^{-1}\{\cos (4\pi/3)\} = 2\pi/3$

(iv) Given $\cos^{-1}(\cos 13\pi/6)$

But we know that $\cos (13\pi/6) = \sqrt{3}/2$

By substituting these values in $\cos^{-1}\{\cos (13\pi/6)\}$ we get,

$\text{Cos}^{-1}(\sqrt{3}/2)$

Now let $y = \cos^{-1}(\sqrt{3}/2)$

Therefore $\cos y = \sqrt{3}/2$

$\text{Cos} (\pi/6) = \sqrt{3}/2$

Hence range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos (\pi/6) = \sqrt{3}/2$

Therefore $\cos^{-1}\{\cos (13\pi/6)\} = \pi/6$

(v) Given $\cos^{-1}(\cos 3)$

We know that $\cos^{-1}(\cos \theta) = \theta$ if $0 \le \theta \le \pi$

Therefore by applying this in given question we get,

$\text{Cos}^{-1}(\cos 3) = 3, 3 \in [0, \pi]$

(vi) Given $\cos^{-1}(\cos 4)$

We have $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi] \approx [0, 3.14]$

And here $x = 4$ which does not lie in the above range.

We know that $\cos (2\pi - x) = \cos(x)$

Thus, $\cos (2\pi - 4) = \cos (4)$ so $2\pi-4$ belongs in $[0, \pi]$

Hence $\cos^{-1}(\cos 4) = 2\pi - 4$

(vii) Given $\cos^{-1}(\cos 5)$

We have $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi] \approx [0, 3.14]$

And here x = 5 which does not lie in the above range.

We know that $\cos(2\pi - x) = \cos(x)$

Thus, $\cos(2\pi - 5) = \cos(5)$ so $2\pi - 5$ belongs in $[0, \pi]$

Hence $\cos^{-1}(\cos 5) = 2\pi - 5$

(viii) Given $\cos^{-1}(\cos 12)$

$\text{Cos}^{-1}(\cos x) = x$ if $x \in [0, \pi] \approx [0, 3.14]$

And here x = 12 which does not lie in the above range.

We know $\cos(2n\pi - x) = \cos(x)$

$\text{Cos}(2n\pi - 12) = \cos(12)$

Here n = 2.

Also $4\pi - 12$ belongs in $[0, \pi]$

$\therefore \cos^{-1}(\cos 12) = 4\pi - 12$

3. Evaluate each of the following:

(i) $\tan^{-1}(\tan \pi/3)$

(ii) $\tan^{-1}(\tan 6\pi/7)$

(iii) $\tan^{-1}(\tan 7\pi/6)$

(iv) $\tan^{-1}(\tan 9\pi/4)$

(v) $\tan^{-1}(\tan 1)$

(vi) $\tan^{-1}(\tan 2)$

(vii) $\tan^{-1}(\tan 4)$

(viii) $\tan^{-1}(\tan 12)$

Solution:

(i) Given $\tan^{-1}(\tan \pi/3)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

By applying this condition in the given question we get,

$\text{Tan}^{-1}(\tan \pi/3) = \pi/3$

(ii) Given $\tan^{-1}(\tan 6\pi/7)$

We know that $\tan 6\pi/7$ can be written as $(\pi - \pi/7)$

$\text{Tan}(\pi - \pi/7) = -\tan \pi/7$

We know that $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

$\text{Tan}^{-1}(\tan 6\pi/7) = -\pi/7$

(iii) Given $\tan^{-1}(\tan 7\pi/6)$

We know that $\tan 7\pi/6 = 1/\sqrt{3}$

By substituting this value in $\tan^{-1}(\tan 7\pi/6)$ we get,

$\text{Tan}^{-1}(1/\sqrt{3})$

Now let $\tan^{-1}(1/\sqrt{3}) = y$

Tan $y = 1/\sqrt{3}$

$\text{Tan}(\pi/6) = 1/\sqrt{3}$

The range of the principal value of $\tan^{-1}$ is $(-\pi/2, \pi/2)$ and $\tan(\pi/6) = 1/\sqrt{3}$

Therefore $\tan^{-1}(\tan 7\pi/6) = \pi/6$

(iv) Given $\tan^{-1}(\tan 9\pi/4)$

We know that $\tan 9\pi/4 = 1$

By substituting this value in $\tan^{-1}(\tan 9\pi/4)$ we get,

$\text{Tan}^{-1}(1)$

Now let $\tan^{-1}(1) = y$

Tan $y = 1$

$\text{Tan}(\pi/4) = 1$

The range of the principal value of $\tan^{-1}$ is $(-\pi/2, \pi/2)$ and $\tan(\pi/4) = 1$

Therefore $\tan^{-1}(\tan 9\pi/4) = \pi/4$

(v) Given $\tan^{-1}(\tan 1)$

But we have $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

By substituting this condition in given question

$\text{Tan}^{-1}(\tan 1) = 1$

(vi) Given $\tan^{-1}(\tan 2)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

But here $x = 2$ which does not belongs to above range

We also have $\tan(\pi - \theta) = -\tan(\theta)$

Therefore $\tan(\theta - \pi) = \tan(\theta)$

$\text{Tan}(2 - \pi) = \tan(2)$

Now $2 - \pi$ is in the given range

Hence $\tan^{-1}(\tan 2) = 2 - \pi$

(vii) Given $\tan^{-1}(\tan 4)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

But here x = 4 which does not belongs to above range

We also have $\tan(\pi - \theta) = -\tan(\theta)$

Therefore $\tan(\theta - \pi) = \tan(\theta)$

$\text{Tan}(4 - \pi) = \tan(4)$

Now $4 - \pi$ is in the given range

Hence $\tan^{-1}(\tan 2) = 4 - \pi$

(viii) Given $\tan^{-1}(\tan 12)$

As $\tan^{-1}(\tan x) = x$ if $x \in [-\pi/2, \pi/2]$

But here x = 12 which does not belongs to above range

We know that $\tan(n\pi - \theta) = -\tan(\theta)$

$\text{Tan}(\theta - 2n\pi) = \tan(\theta)$

Here n = 4

$\text{Tan}(12 - 4\pi) = \tan(12)$

Now $12 - 4\pi$ is in the given range

$\therefore \tan^{-1}(\tan 12) = 12 - 4\pi$.

Exercise 4.8 Page No: 4.54

1. Evaluate each of the following:

(i) $\sin(\sin^{-1} 7/25)$

(ii) $\text{Sin}(\cos^{-1} 5/13)$

(iii) $\text{Sin}(\tan^{-1} 24/7)$

(iv) $\text{Sin}(\sec^{-1} 17/8)$

(v) $\text{Cosec}(\cos^{-1} 8/17)$

(vi) $\text{Sec}(\sin^{-1} 12/13)$

(vii) $\text{Tan}(\cos^{-1} 8/17)$

(viii) $\cot(\cos^{-1} 3/5)$

(ix) $\text{Cos}(\tan^{-1} 24/7)$

Solution:

(i) Given sin ($\sin^{-1}$ 7/25)

Now let y = $\sin^{-1}$ 7/25

Sin y = 7/25 where y ∈ [0, π/2]

Substituting these values in sin ($\sin^{-1}$ 7/25) we get

Sin ($\sin^{-1}$ 7/25) = 7/25

(ii) Given Sin ($\cos^{-1}$ 5/13)

Let $\cos^{-1}\frac{5}{13} = y$

$\Rightarrow \cos y = \frac{5}{13}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find

$$\sin\left(\cos^{-1}\frac{5}{13}\right) = \sin y$$

We know that $\sin^2\theta + \cos^2\theta = 1$

By substituting this trigonometric identity we get

$\Rightarrow \sin y = \pm\sqrt{1-\cos^2 y}$

Where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sin y = \sqrt{1-\cos^2 y}$

Now by substituting cos y value we get

$\Rightarrow \sin y = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$\Rightarrow \sin y = \sqrt{1-\frac{25}{169}}$

$\Rightarrow \sin y = \sqrt{\frac{144}{169}}$

$\Rightarrow \sin y = \frac{12}{13} \Rightarrow \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right] = \frac{12}{13}$

(iii) Given Sin ($\tan^{-1}$ 24/7)

Let $\tan^{-1}\frac{24}{7} = y$

$\Rightarrow \tan y = \frac{24}{7}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find

$$\sin\left(\tan^{-1}\frac{24}{7}\right) = \sin y$$

We know that $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow 1 + \cot^2 y = \text{cosec}^2 y$

Now substituting this trigonometric identity we get,

$$\Rightarrow 1 + \left(\frac{7}{24}\right)^2 = \text{cosec}^2 y$$

$$\Rightarrow 1 + \frac{49}{576} = \frac{1}{\sin^2 y}$$

On rearranging we get,

$$\Rightarrow \sin^2 y = \frac{576}{625}$$

$\Rightarrow \sin y = \frac{24}{25}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

$$\Rightarrow \sin\left(\tan^{-1}\frac{24}{7}\right) = \frac{24}{25}$$

(iv) Given Sin ($\sec^{-1}$ 17/8)

Let $\sec^{-1}\frac{17}{8} = y$

$\Rightarrow \sec y = \frac{17}{8}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have find

$$\sin\left(\sec^{-1}\frac{17}{8}\right) = \sin y$$

We know that, $\cos y = \dfrac{1}{\sec y}$

$\Rightarrow \cos y = \dfrac{8}{17}$

Now, $\sin y = \sqrt{1-\cos^2 y}$ where $y \in \left[0, \dfrac{\pi}{2}\right]$

By substituting, cos y value we get,

$\Rightarrow \sin y = \sqrt{1-\left(\dfrac{8}{17}\right)^2}$

$\Rightarrow \sin y = \sqrt{\dfrac{225}{289}}$

$\Rightarrow \sin y = \dfrac{15}{17}$

$\Rightarrow \sin\left(\sec^{-1}\dfrac{17}{8}\right) = \dfrac{15}{17}$

(v) Given Cosec ($\cos^{-1}$ 8/17)

Let $\cos^{-1}\dfrac{3}{5} = y$

$\Rightarrow \cos y = \dfrac{3}{5}$ Where $y \in \left[0, \dfrac{\pi}{2}\right]$

Now we have to find

$$\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right) = \operatorname{cosec} y$$

We know that $\sin^2\theta + \cos^2\theta = 1$

On rearranging and substituting we get,

$\Rightarrow \sin y = \sqrt{1-\cos^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now substituting the value of cos y we get

$$\Rightarrow \sin y = \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin y = \sqrt{\frac{16}{25}}$$

$$\Rightarrow \sin y = \frac{4}{5}$$

$$\Rightarrow \operatorname{cosec} y = \frac{5}{4}$$

$$\Rightarrow \operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right) = \frac{5}{4}$$

(vi) Given Sec $(\sin^{-1} 12/13)$

Let $\sin^{-1}\frac{12}{13} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$$\Rightarrow \sin y = \frac{12}{13}$$

Now we have to find

$$\sec\left(\sin^{-1}\frac{12}{13}\right) = \sec y$$

We know that $\sin^2\theta + \cos^2\theta = 1$

According to this identity cos y can be written as

$\Rightarrow \cos y = \sqrt{1-\sin^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now substituting the value of sin y we get,

$$\Rightarrow \cos y = \sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos y = \sqrt{1-\frac{144}{169}}$$

$$\Rightarrow \cos y = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos y = \frac{5}{13}$$

$$\Rightarrow \sec y = \frac{1}{\cos y}$$

$$\Rightarrow \sec y = \frac{13}{5}$$

$$\Rightarrow \sec\left(\sin^{-1}\frac{12}{13}\right) = \frac{13}{5}$$

(vii) Given Tan ($\cos^{-1}$ 8/17)

Let $\cos^{-1}\frac{8}{17} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \frac{8}{17}$

Now we have to find

$$\tan\left(\cos^{-1}\frac{8}{17}\right) = \tan y$$

We know that $1+\tan^2\theta = \sec^2\theta$

Rearranging and substituting the value of tan y we get,

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

We have sec y = 1/cos y

$$\Rightarrow \tan y = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$$

$$\Rightarrow \tan y = \sqrt{\left(\frac{17}{8}\right)^2 - 1}$$

$$\Rightarrow \tan y = \sqrt{\frac{289}{64} - 1}$$

$$\Rightarrow \tan y = \sqrt{\frac{225}{64}}$$

$$\Rightarrow \tan y = \frac{15}{8}$$

$$\Rightarrow \tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{15}{8}$$

(viii) Given cot ($\cos^{-1}$ 3/5)

Let $\cos^{-1}\frac{3}{5} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \frac{3}{5}$

Now we have to find

$\cot\left(\cos^{-1}\frac{3}{5}\right) = \cot y$

We know that $1+\tan^2\theta = \sec^2\theta$

Rearranging and substituting the value of tan y we get,

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

We have sec y = 1/cos y, on substitution we get,

$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{5}{3}\right)^2 - 1}$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\frac{16}{9}}$

$\Rightarrow \cot y = \frac{3}{4}$

$\Rightarrow \cot\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{4}$

(ix) Given Cos ($\tan^{-1}$ 24/7)

Let $\tan^{-1}\frac{24}{7} = y$

$\Rightarrow \tan y = \frac{24}{7}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

Now we have to find,

$\cos\left(\tan^{-1}\frac{24}{7}\right) = \cos y$

We know that $1+\tan^2\theta = \sec^2\theta$

$\Rightarrow 1+\tan^2 y = \sec^2 y$

On rearranging and substituting the value of sec y we get,

$\Rightarrow \sec y = \sqrt{1+\tan^2 y}$ Where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \sec y = \sqrt{1+\left(\frac{24}{7}\right)^2}$

$\Rightarrow \sec y = \sqrt{\frac{625}{49}}$

$\Rightarrow \sec y = \frac{25}{7}$

.

$\Rightarrow \cos y = \frac{1}{\sec y}$

$\Rightarrow \cos y = \frac{7}{25}$

$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right) = \frac{7}{25}$

Exercise 4.9 Page No: 4.58

1. Evaluate:

(i) Cos {sin^{-1} (-7/25)}

(ii) Sec {cot^{-1} (-5/12)}

(iii) Cot {sec^{-1} (-13/5)}

Solution:

(i) Given Cos {sin^{-1} (-7/25)}

Let $\sin^{-1}\left(-\frac{7}{25}\right) = x$ Where $x \in \left[-\frac{\pi}{2}, 0\right]$

$\Rightarrow \sin x = -\frac{7}{25}$

Now we have to find

$$\cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right] = \cos x$$

We know that $\sin^2 x + \cos^2 x = 1$

On rearranging and substituting we get,

$\Rightarrow \cos x = \sqrt{1 - \sin^2 x}$ Since $x \in \left[-\frac{\pi}{2}, 0\right]$

$\Rightarrow \cos x = \sqrt{1 - \frac{49}{625}}$

$\Rightarrow \cos x = \sqrt{\frac{576}{625}}$

$\Rightarrow \cos x = \frac{24}{25}$

$\Rightarrow \cos\left[\sin^{-1}\left(-\frac{7}{25}\right)\right] = \frac{24}{25}$

(ii) Given Sec {cot^{-1} (-5/12)}

Let $\cot^{-1}\left(-\frac{5}{12}\right) = x$ where $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \cot x = -\frac{5}{12}$

Now we have to find,

$\sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right] = \sec x$

We know that $1 + \tan^2 x = \sec^2 x$

On rearranging, we get

$\Rightarrow 1 + \frac{1}{\cot^2 x} = \sec^2 x$

Substituting these values we get,

$\Rightarrow \sec x = -\sqrt{1 + \frac{1}{\cot^2 x}}$ Since $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \sec x = -\sqrt{1 + \left(\frac{12}{5}\right)^2}$

$\Rightarrow \sec x = -\frac{13}{5}$

$\Rightarrow \sec\left[\cot^{-1}\left(-\frac{5}{12}\right)\right] = -\frac{13}{5}$

(iii) Given Cot {sec^{-1} (-13/5)}

Let $\sec^{-1}\left(-\frac{13}{5}\right)=x$ where $x\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow \sec x=-\frac{13}{5}$

Now we have find,

$$\cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right]=\cot x$$

We know that $1+\tan^2 x=\sec^2 x$

On rearranging, we get

$\Rightarrow \tan x=-\sqrt{\sec^2 x-1}$

Now substitute the value of sec x, we get

$\Rightarrow \tan x=-\sqrt{\left(-\frac{13}{5}\right)^2-1}$

$\Rightarrow \tan x=-\frac{12}{5}$

$\Rightarrow \cot x=-\frac{5}{12}$

$\Rightarrow \cot\left[\sec^{-1}\left(-\frac{13}{5}\right)\right]=-\frac{5}{12}$

Exercise 4.10 Page No: 4.66

1. Evaluate:

(i) Cot ($\sin^{-1}$ (3/4) + $\sec^{-1}$ (4/3))

(ii) Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x < 0

(iii) Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x > 0

(iv) Cot ($\tan^{-1}$ a + $\cot^{-1}$ a)

(v) Cos ($\sec^{-1}$ x + cosec^{-1} x), |x| ≥ 1

Solution:

(i) Given Cot ($\sin^{-1}$ (3/4) + $\sec^{-1}$ (4/3))

$$= \cot\left(\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{3}{4}\right)$$

$$\left(\because \sec^{-1} x = \cos^{-1}\frac{1}{x}\right)$$

We have

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

By substituting these values in given questions, we get

$$= \cot\frac{\pi}{2}$$

= 0

(ii) Given Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x < 0

$$= \sin\left(\tan^{-1} x + (\cot^{-1} x - \pi)\right) \left(\because \tan^{-1}\theta = \cot^{-1}\frac{1}{\theta} - \pi \qquad \text{for } x < 0\right)$$

$= \sin\left(\frac{\pi}{2} - \pi\right)\left(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right)$

On simplifying, we get

$= \sin\left(-\frac{\pi}{2}\right)$

We know that sin (- θ) = – sin θ

$= -\sin\frac{\pi}{2} = -1$

(iii) Given Sin ($\tan^{-1}$ x + $\tan^{-1}$ 1/x) for x > 0

$= \sin\left(\tan^{-1} x + \cot^{-1} x\right)\left(\because \tan^{-1}\theta = \cot^{-1}\frac{1}{\theta} \qquad \text{for } x > 0\right)$

Again we know that,

$\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}$

Now by substituting above identity in given question we get,

$= \sin\frac{\pi}{2}$

= 1

(iv) Given Cot ($\tan^{-1}$ a + $\cot^{-1}$ a)

We know that,

$\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}$

Now by substituting above identity in given question we get,

$= \cot\left(\frac{\pi}{2}\right)$

= 0

(v) Given Cos (sec^{-1} x + $cosec^{-1}$ x), |x| ≥ 1

We know that

$$\sec^{-1}\theta = \cos^{-1}\frac{1}{\theta}$$

Again we have

$$\operatorname{cosec}^{-1}\theta = \sin^{-1}\frac{1}{\theta}$$

By substituting these values in given question we get,

$$= \cos\left(\cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}\right)$$

We know that from the identities,

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

Now by substituting we get,

$$= \cos\frac{\pi}{2}$$

2. If $\cos^{-1}$ x + $\cos^{-1}$ y = π/4, find the value of $\sin^{-1}$ x + $\sin^{-1}$ y.

Solution:

Given $\cos^{-1}$ x + $\cos^{-1}$ y = π/4

We know that

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

Now substituting above identity in given question we get,

$$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}y\right) = \frac{\pi}{4}$$

Adding and simplifying we get,

$$\Rightarrow \pi - \left(\sin^{-1}x + \sin^{-1}y\right) = \frac{\pi}{4}$$

On rearranging,

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \pi - \frac{\pi}{4}$$

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{3\pi}{4}$$

3. If $\sin^{-1} x + \sin^{-1} y = \pi/3$ and $\cos^{-1} x - \cos^{-1} y = \pi/6$, find the values of x and y.

Solution:

Given $\sin^{-1} x + \sin^{-1} y = \pi/3$ ……. Equation (i)

And $\cos^{-1} x - \cos^{-1} y = \pi/6$ ……… Equation (ii)

Subtracting Equation (ii) from Equation (i), we get

$$\Rightarrow (\sin^{-1}x - \cos^{-1}x) + (\sin^{-1}y + \cos^{-1}y) = \frac{\pi}{3} - \frac{\pi}{6}$$

We know that,

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

By substituting above identity, we get

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + \left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$

Replacing $\sin^{-1}$ x by $\pi/2 - \cos^{-1}$ x and rearranging we get,

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = -\frac{\pi}{3}$$

Now by adding,

$$\Rightarrow 2\cos^{-1} x = \frac{5\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{5\pi}{12}$$

$$\Rightarrow x = \cos\left(\frac{5\pi}{12}\right)$$

$$\Rightarrow x = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

We know that Cos (A + B) = Cos A. Cos B – Sin A. Sin B, substituting this we get,

$$\Rightarrow x = \cos\frac{\pi}{4}.\cos\frac{\pi}{6} - \sin\frac{\pi}{4}.\sin\frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Now, putting the value of $\cos^{-1} x$ in equation (ii)

$\Rightarrow \frac{5\pi}{12} - \cos^{-1} y = \frac{\pi}{6}$

$\Rightarrow \cos^{-1} y = \frac{\pi}{4}$

$\Rightarrow y = \frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}}$ And $y = \frac{1}{\sqrt{2}}$

4. If cot ($\cos^{-1}$ 3/5 + $\sin^{-1}$ x) = 0, find the value of x.

Solution:

Given cot ($\cos^{-1}$ 3/5 + $\sin^{-1}$ x) = 0

On rearranging we get,

($\cos^{-1}$ 3/5 + $\sin^{-1}$ x) = $\cot^{-1}$ (0)

(Cos^{-1} 3/5 + $\sin^{-1}$ x) = π/2

We know that $\cos^{-1}$ x + $\sin^{-1}$ x = π/2

Then $\sin^{-1}$ x = π/2 – $\cos^{-1}$ x

Substituting the above in ($\cos^{-1}$ 3/5 + $\sin^{-1}$ x) = π/2 we get,

(Cos^{-1} 3/5 + π/2 – $\cos^{-1}$ x) = π/2

Now on rearranging we get,

(Cos^{-1} 3/5 – $\cos^{-1}$ x) = π/2 – π/2

(Cos^{-1} 3/5 – $\cos^{-1}$ x) = 0

Therefore Cos^{-1} 3/5 = $\cos^{-1}$ x

On comparing the above equation we get,

$x = 3/5$

5. If $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17\pi^2/36$, find x.

Solution:

Given $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17\pi^2/36$

We know that $\cos^{-1} x + \sin^{-1} x = \pi/2$

Then $\cos^{-1} x = \pi/2 – \sin^{-1} x$

Substituting this in $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17\pi^2/36$ we get

$(\sin^{-1} x)^2 + (\pi/2 – \sin^{-1} x)^2 = 17\pi^2/36$

Let $y = \sin^{-1} x$

$y^2 + ((\pi/2) – y)^2 = 17\pi^2/36$

$y^2 + \pi^2/4 – y^2 – 2y((\pi/2) – y) = 17\pi^2/36$

$\pi^2/4 – \pi y + 2y^2 = 17\pi^2/36$

On rearranging and simplifying, we get

$2y^2 – \pi y + 2/9\,\pi^2 = 0$

$18y^2 – 9\pi y + 2\pi^2 = 0$

$18y^2 – 12\pi y + 3\pi y + 2\pi^2 = 0$

$6y(3y – 2\pi) + \pi(3y – 2\pi) = 0$

Now, $(3y – 2\pi) = 0$ and $(6y + \pi) = 0$

Therefore $y = 2\pi/3$ and $y = – \pi/6$

Now substituting $y = – \pi/6$ in $y = \sin^{-1} x$ we get

$\sin^{-1} x = – \pi/6$

$x = \sin(- \pi/6)$

$x = -1/2$

Now substituting $y = -2\pi/3$ in $y = \sin^{-1} x$ we get

$x = \sin(2\pi/3)$

$x = \sqrt{3}/2$

Now substituting $x = \sqrt{3}/2$ in $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17\pi^2/36$ we get,

$= \pi/3 + \pi/6$

$= \pi/2$ which is not equal to $17\pi^2/36$

So we have to neglect this root.

Now substituting $x = -1/2$ in $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17\pi^2/36$ we get,

$= \pi^2/36 + 4\ \pi^2/9$

$= 17\ \pi^2/36$

Hence x = -1/2.

Exercise 4.11 Page No: 4.82

1. Prove the following results:

(i) $\tan^{-1}(1/7) + \tan^{-1}(1/13) = \tan^{-1}(2/9)$

(ii) $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$

(iii) $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \sin^{-1}(1/\sqrt{5})$

Solution:

(i) Given $\tan^{-1}(1/7) + \tan^{-1}(1/13) = \tan^{-1}(2/9)$

Consider LHS

$$\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

According to the formula, we can write as

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}}\right)$$

$$= \tan^{-1}\left(\frac{20}{90}\right)$$

$$= \tan^{-1}\left(\frac{2}{9}\right)$$

= RHS

Hence, the proof.

(ii) Given $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$

Consider LHS

$\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16})$

We know that, Formula

$$\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Now, by substituting the formula we get,

$$= \tan^{-1}\left(\frac{\frac{12}{13}}{\sqrt{1-\left(\frac{12}{13}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right) + \tan^{-1}(\frac{63}{16})$$

$$= \tan^{-1}(\frac{12}{5}) + \tan^{-1}(\frac{3}{4}) + \tan^{-1}(\frac{63}{16})$$

Again we know that,

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}$$

Again by substituting, we get

$$= \pi + \tan^{-1}(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}) + \tan^{-1}(\frac{63}{16})$$

$$= \pi + \tan^{-1}(-\frac{63}{16}) + \tan^{-1}(\frac{63}{16})$$

We know that,

$$\tan^{-1}(-x) = -\tan^{-1}x$$

$$= \pi - \tan^{-1}(-\frac{63}{16}) + \tan^{-1}(\frac{63}{16})$$

$= \pi$

So, $\sin^{-1}(\frac{12}{13}) + \cos^{-1}\frac{4}{5} + \tan^{-1}(\frac{63}{16}) = \pi$

Hence, the proof.

(iii) Given $\tan^{-1}(1/4) + \tan^{-1}(2/9) = \sin^{-1}(1/\sqrt{5})$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$$

We know that,

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\frac{x+y}{1-xy}$$

By substituting this formula we get,

$$= \tan^{-1}\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}$$

$$= \tan^{-1}\frac{\frac{17}{36}}{\frac{34}{36}}$$

$$= \tan^{-1}\frac{\frac{17}{36}}{\frac{34}{36}}$$

$$= \tan^{-1}\frac{1}{2}$$

Now let, $\tan\theta = \frac{1}{2}$

Therefore, $\sin\theta = \frac{1}{\sqrt{5}}$

So, $\theta = \sin^{-1}\frac{1}{\sqrt{5}}$

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \text{RHS}$$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Hence, Proved.

2. Find the value of $\tan^{-1}(x/y) - \tan^{-1}\{(x-y)/(x+y)\}$

Solution:

Given $\tan^{-1}(x/y) - \tan^{-1}\{(x-y)/(x+y)\}$

We know that,

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

Now by substituting the formula, we get

$$= \tan^{-1} \frac{\frac{x}{y} - \left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y} \times \left(\frac{x-y}{x+y}\right)}$$

$$= \tan^{-1} \frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}$$

$$= \tan^{-1} \frac{x^2+y^2}{x^2+y^2}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

So,

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$$

Exercise 4.12 Page No: 4.89

1. Evaluate: Cos (sin $^{-1}$ 3/5 + sin^{-1} 5/13)

Solution:

Given Cos (sin $^{-1}$ 3/5 + sin^{-1} 5/13)

We know that,

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

By substituting this formula we get,

$$= \cos\left(\sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]\right)$$

$$= \cos\left(\sin^{-1}\left[\frac{3}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{4}{5}\right]\right)$$

$$= \cos\left(\sin^{-1}\left[\frac{56}{65}\right]\right)$$

Again, we know that

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$$

Now substituting, we get

$$= \cos\left(\cos^{-1}\sqrt{1-\left(\frac{56}{65}\right)^2}\right)$$

$$= \cos\left(\cos^{-1}\sqrt{\frac{33}{65}}\right)$$

$$= \frac{33}{65}$$

Hence, $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \frac{33}{65}$

Exercise 4.13 Page No: 4.92

1. If $\cos^{-1}(x/2) + \cos^{-1}(y/3) = \alpha$, then prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$

Solution:

Given $\cos^{-1}(x/2) + \cos^{-1}(y/3) = \alpha$

We know that,

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$$

Now by substituting, we get

$$\Rightarrow \cos^{-1}\left[\frac{x}{2}\times\frac{y}{3} - \sqrt{1-\left(\frac{x}{2}\right)^2}\sqrt{1-\left(\frac{y}{3}\right)^2}\right] = \alpha$$

$$\Rightarrow \left[\frac{xy}{6} - \frac{\sqrt{4-x^2}}{2}\times\frac{\sqrt{9-y^2}}{3}\right] = \cos\alpha$$

$$\Rightarrow xy - \sqrt{4-x^2}\times\sqrt{9-y^2} = 6\cos\alpha$$

$$\Rightarrow xy - 6\cos\alpha = \sqrt{4-x^2}\sqrt{9-y^2}$$

On squaring both the sides we get

$$\Rightarrow (xy - 6\cos\alpha)^2 = (4-x^2)(9-y^2)$$

$$\Rightarrow x^2y^2 + 36\cos^2\alpha - 12xy\cos\alpha = 36 - 9x^2 - 4y^2 + x^2y^2$$

$$\Rightarrow 9x^2 + 4y^2 - 36 + 36\cos^2\alpha - 12xy\cos\alpha = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\alpha - 36(1-\cos^2\alpha) = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\alpha - 36\sin^2\alpha = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy\cos\alpha = 36\sin^2\alpha$$

Hence the proof.

2. Solve the equation: $\cos^{-1}(a/x) - \cos^{-1}(b/x) = \cos^{-1}(1/b) - \cos^{-1}(1/a)$

Solution:

Given $\cos^{-1}(a/x) - \cos^{-1}(b/x) = \cos^{-1}(1/b) - \cos^{-1}(1/a)$

$\Rightarrow \cos^{-1}\frac{a}{x} + \cos^{-1}\frac{1}{a} = \cos^{-1}\frac{1}{b} + \cos^{-1}\frac{b}{x}$

We know that,

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

By substituting this formula we get,

$\Rightarrow \cos^{-1}\left[\frac{1}{x} - \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2}\right] = \cos^{-1}\left[\frac{1}{x} - \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}\right]$

$\Rightarrow \frac{1}{x} - \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2} = \frac{1}{x} - \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}$

$\Rightarrow \sqrt{1-\left(\frac{a}{x}\right)^2}\sqrt{1-\left(\frac{1}{a}\right)^2} = \sqrt{1-\left(\frac{b}{x}\right)^2}\sqrt{1-\left(\frac{1}{b}\right)^2}$

Squaring on both the sides, we get

$\Rightarrow \left(1-\left(\frac{a}{x}\right)^2\right)\left(1-\left(\frac{1}{a}\right)^2\right) = \left(1-\left(\frac{b}{x}\right)^2\right)\left(1-\left(\frac{1}{b}\right)^2\right)$

$\Rightarrow 1 - \left(\frac{a}{x}\right)^2 - \left(\frac{1}{a}\right)^2 + \left(\frac{1}{x}\right)^2 = 1 - \left(\frac{b}{x}\right)^2 - \left(\frac{1}{b}\right)^2 + \left(\frac{1}{x}\right)^2$

$\Rightarrow \left(\frac{b}{x}\right)^2 - \left(\frac{a}{x}\right)^2 = \left(\frac{1}{a}\right)^2 - \left(\frac{1}{b}\right)^2$

On simplifying, we get

$\Rightarrow (b^2 - a^2)\, a^2b^2 = x^2(b^2 - a^2)$

$\Rightarrow x^2 = a^2b^2$

$\Rightarrow x = a\,b$

Exercise 4.14 Page No: 4.115

1. Evaluate the following:

(i) tan {2 tan^{-1} (1/5) – π/4}

(ii) Tan {1/2 sin^{-1} (3/4)}

(iii) Sin {1/2 cos^{-1} (4/5)}

(iv) Sin (2 tan $^{-1}$ 2/3) + cos (tan^{-1} √3)

Solution:

(i) Given tan {2 tan^{-1} (1/5) – π/4}

We know that,

$$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{if } |x| < 1$$

And $\frac{\pi}{4}$ can be written as $\tan^{-1}(1)$

Now substituting these values we get,

$$= \tan\left\{\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right) - \tan^{-1}1\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}1\right\}$$

Again we know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Now substituting this formula, we get

$$= \tan\left\{\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}\right)\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{-7}{17}\right)\right\}$$

$$= -\frac{7}{17}$$

(ii) Given tan {1/2 sin^{-1} (3/4)}

Let $\frac{1}{2}\sin^{-1}\frac{3}{4} = \text{t}$

Therefore,

$\Rightarrow \sin^{-1}\frac{3}{4} = 2\text{t}$

$\Rightarrow \sin 2\text{t} = \frac{3}{4}$

Now, by Pythagoras theorem, we have

$\Rightarrow \sin 2\text{t} = \frac{3}{4} = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\Rightarrow \cos 2\text{t} = \frac{\sqrt{4^2 - 3^2}}{4} = \frac{\text{Base}}{\text{hypotenuse}}$

$\Rightarrow \cos 2\text{t} = \frac{\sqrt{7}}{4}$

By considering, given question

$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\}$

$= \tan(\text{t})$

We know that,

$$\tan(\text{x}) = \sqrt{\frac{1 - \cos 2\text{x}}{1 + \cos 2\text{x}}}$$

$= \sqrt{\frac{1 - \cos 2\text{t}}{1 + \cos 2\text{t}}}$

$= \sqrt{\frac{1 - \frac{\sqrt{7}}{4}}{1 + \frac{\sqrt{7}}{4}}}$

$= \sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}}$

Now by rationalizing the denominator, we get

$= \sqrt{\frac{(4-\sqrt{7})(4-\sqrt{7})}{(4+\sqrt{7})(4-\sqrt{7})}}$

$= \sqrt{\frac{(4-\sqrt{7})^2}{9}}$

$= \frac{4-\sqrt{7}}{3}$

Hence

$$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\} = \frac{4-\sqrt{7}}{3}$$

(iii) Given sin {1/2 $\cos^{-1}$ (4/5)}

We know that

$$\cos^{-1}x = 2\sin^{-1}\left(\pm\sqrt{\frac{1-x}{2}}\right)$$

Now by substituting this formula we get,

$$\sin\left(\frac{1}{2}2\sin^{-1}\left(\pm\sqrt{\frac{1-\frac{4}{5}}{2}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\pm\sqrt{\frac{1}{2\times5}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\pm\frac{1}{\sqrt{10}}\right)\right)$$

As we know that

$$\sin(\sin^{-1}x) = x \text{ as } n \in [-1, 1]$$

$$= \pm\frac{1}{\sqrt{10}}$$

Hence, $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \pm\frac{1}{\sqrt{10}}$

(iv) Given Sin (2 tan $^{-1}$ 2/3) + cos (tan^{-1} √3)

We know that

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}(x);$$

$$\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \tan^{-1}(x);$$

Now by substituting these formulae we get,

$$= \sin\left(\sin^{-1}\left(\frac{2\times\frac{2}{3}}{1+\frac{4}{9}}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+3}}\right)\right)$$

$= \sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{12}{13} + \frac{1}{2}$

$= \frac{37}{26}$

Hence,

$$\sin\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \frac{37}{26}$$

2. Prove the following results:

(i) 2 $\sin^{-1}$ (3/5) = $\tan^{-1}$ (24/7)

(ii) $\tan^{-1}$ ¼ + $\tan^{-1}$ (2/9) = ½ $\cos^{-1}$ (3/5) = ½ $\sin^{-1}$ (4/5)

(iii) $\tan^{-1}$ (2/3) = ½ $\tan^{-1}$ (12/5)

(iv) $\tan^{-1}$ (1/7) + 2 $\tan^{-1}$ (1/3) = π/4

(v) $\sin^{-1}$ (4/5) + 2 $\tan^{-1}$ (1/3) = π/2

(vi) 2 $\sin^{-1}$ (3/5) – $\tan^{-1}$ (17/31) = π/4

(vii) 2 $\tan^{-1}$ (1/5) + $\tan^{-1}$ (1/8) = $\tan^{-1}$ (4/7)

(viii) 2 $\tan^{-1}$ (3/4) – $\tan^{-1}$ (17/31) = π/4

(ix) 2 $\tan^{-1}$ (1/2) + $\tan^{-1}$ (1/7) = $\tan^{-1}$ (31/17)

(x) 4 $\tan^{-1}$(1/5) – $\tan^{-1}$(1/239) = π/4

Solution:

(i) Given 2 $\sin^{-1}$ (3/5) = $\tan^{-1}$ (24/7)

Consider LHS

$2\sin^{-1}\frac{3}{5}$

We know that

$$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Now by substituting the above formula we get,

$= 2 \times \tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}}\right)$

$= 2 \times \tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)$

$= 2 \times \tan^{-1}\left(\frac{3}{4}\right)$

Again we know that

$$2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{if } |x| < 1$$

Therefore,

$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right)$

= RHS

So, $2\sin^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{24}{7}\right)$

Hence the proof.

(ii) Given $\tan^{-1} \frac{1}{4} + \tan^{-1} (2/9) = \frac{1}{2} \cos^{-1} (3/5) = \frac{1}{2} \sin^{-1} (4/5)$

Consider LHS

$= \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$

We know that

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Now by substituting this formula, we get

$$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$$

$$= \tan^{-1}\left(\frac{17}{34}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

Multiplying and dividing by 2

$$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{1}{2}\right)\right\}$$

Again we know that

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{\frac{3}{4}}{\frac{5}{4}}\right)$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$$

= RHS

So, $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

Now,

$= \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

We know that,

$= \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$

By substituting this, we get

$= \frac{1}{2}\sin^{-1}\sqrt{1-\frac{9}{25}}$

$= \frac{1}{2}\sin^{-1}\sqrt{\frac{16}{25}}$

$= \frac{1}{2}\sin^{-1}\frac{4}{5}$

= RHS

So, $\tan^{-1}(\frac{1}{4}) + \tan^{-1}(\frac{2}{9}) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right) = \frac{1}{2}\sin^{-1}\frac{4}{5}$

Hence the proof.

(iii) Given $\tan^{-1}(2/3) = ½ \tan^{-1}(12/5)$

Consider LHS

$= \tan^{-1}(\frac{2}{3})$

Now, Multiplying and dividing by 2, we get

$= \frac{1}{2}\left\{2\tan^{-1}\left(\frac{2}{3}\right)\right\}$

We know that

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

By substituting the above formula we get

$= \frac{1}{2}\tan^{-1}\left(\frac{2\times\frac{2}{3}}{1-\frac{4}{9}}\right)$

$= \frac{1}{2}\tan^{-1}\left(\frac{\frac{4}{3}}{\frac{5}{9}}\right)$

$= \frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)$

= RHS

So, $tan^{-1}(\frac{2}{3}) = \frac{1}{2}tan^{-1}\left(\frac{12}{5}\right)$

Hence the proof.

(iv) Given $\tan^{-1}(1/7) + 2\tan^{-1}(1/3) = \pi/4$

Consider LHS

$= \tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3})$

We know that,

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

By substituting the above formula we get,

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}})$

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{\frac{2}{3}}{\frac{8}{9}})$

$= \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{3}{4})$

Again we know that

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$$

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}\right)$

$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So, $\tan^{-1}(\frac{1}{7}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

Hence the proof.

(v) Given $\sin^{-1}(4/5) + 2\tan^{-1}(1/3) = \pi/2$

Consider LHS

$= \sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3})$

We know that,

$\sin^{-1}(x) = \tan^{-1}(\frac{x}{\sqrt{1-x^2}})$

And, $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Now by substituting the formula we get,

$= \tan^{-1}(\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}}}) + \tan^{-1}(\frac{2\times\frac{1}{3}}{1-\frac{1}{9}})$

$= \tan^{-1}(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}) + \tan^{-1}(\frac{\frac{2}{3}}{\frac{8}{9}})$

$= \tan^{-1}(\frac{4}{3}) + \tan^{-1}(\frac{3}{4})$

We know that,

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\times\frac{3}{4}}\right)$

$= \tan^{-1}\left(\frac{\frac{25}{12}}{0}\right)$

$= \tan^{-1}(\infty)$

$= \frac{\pi}{2}$

= RHS

So, $\sin^{-1}(\frac{4}{5}) + 2\tan^{-1}(\frac{1}{3}) = \frac{\pi}{2}$

Hence Proved

(vi) Given 2 $\sin^{-1}$ (3/5) – $\tan^{-1}$ (17/31) = π/4

Consider LHS

$$= 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

We know that

$$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

According to the formula we have,

$$= 2\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{16}{25}}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

Again we know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

By substituting this formula, we get

$$= \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

Again we have,

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x - y}{1 + xy}$$

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}}\right)$

$= \tan^{-1}\left(\frac{\frac{744 - 119}{217}}{\frac{217 + 408}{217}}\right)$

$= \tan^{-1}\left(\frac{625}{625}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$ = RHS

So, $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Hence the proof.

(vii) Given 2 $\tan^{-1}$ (1/5) + $\tan^{-1}$ (1/8) = $\tan^{-1}$ (4/7)

Consider LHS

$= 2 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

We know that

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$= \tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$= \tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

Again from the formula we have,

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$$

$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{1}{8}}{1-\frac{5}{12}\times\frac{1}{8}}\right)$

$= \tan^{-1}\left(\frac{\frac{10+3}{24}}{\frac{96-5}{96}}\right)$

$= \tan^{-1}\left(\frac{13}{24}\times\frac{96}{91}\right)$

$= \tan^{-1}\left(\frac{4}{7}\right)$

= RHS

So, $2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{4}{7}\right)$

Hence the proof.

(viii) Given $2\tan^{-1}(3/4) - \tan^{-1}(17/31) = \pi/4$

Consider LHS

$= 2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31})$

We know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$= \tan^{-1}(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}) - \tan^{-1}(\frac{17}{31})$

$= \tan^{-1}(\frac{3}{2}\times\frac{16}{7}) - \tan^{-1}(\frac{17}{31})$

$= \tan^{-1}(\frac{24}{7}) - \tan^{-1}(\frac{17}{31})$

We know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Again by substituting the formula we get,

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$

$= \tan^{-1}\left(\frac{\frac{744-119}{217}}{\frac{217+408}{217}}\right)$

$= \tan^{-1}\left(\frac{625}{625}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So, $2\tan^{-1}(\frac{3}{4}) - \tan^{-1}(\frac{17}{31}) = \frac{\pi}{4}$

Hence the proof.

(ix) Given 2 tan^{-1} (1/2) + tan^{-1} (1/7) = tan^{-1} (31/17)

Consider LHS

$= 2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7})$

We know that,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$= \tan^{-1}(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}) + \tan^{-1}(\frac{1}{7})$

$= \tan^{-1}(\frac{\frac{2}{2}}{\frac{3}{4}}) + \tan^{-1}(\frac{1}{7})$

$= \tan^{-1}(\frac{4}{3}) + \tan^{-1}(\frac{1}{7})$

Again by using the formula, we can write as

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{1}{7}\times\frac{4}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{31}{21}}{\frac{17}{21}}\right)$

$= \tan^{-1}\left(\frac{31}{17}\right)$

= RHS

So, $2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$

Hence the proof.

(x) Given 4 $\tan^{-1}(1/5) - \tan^{-1}(1/239) = \pi/4$

Consider LHS

$= 4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239})$

We know that,

$$4\tan^{-1}x = \tan^{-1}\left(\frac{4x - 4x^3}{1 - 6x^2 + x^4}\right)$$

Now by substituting the formula, we get

$= \tan^{-1}\left(\frac{4\times\frac{1}{5}-4\left(\frac{1}{5}\right)^3}{1-6\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^4}\right) - \tan^{-1}(\frac{1}{239})$

$= \tan^{-1}(\frac{120}{119}) - \tan^{-1}(\frac{1}{239})$

Again we know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x - y}{1 + xy}$$

$= \tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{239}}{1-\frac{120}{119}\times\frac{1}{239}}\right)$

$= \tan^{-1}\left(\frac{120\times239-119}{119\times239+120}\right)$

$= \tan^{-1}\left(\frac{28561}{28561}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

= RHS

So,

$$4\tan^{-1}(\frac{1}{5}) - \tan^{-1}(\frac{1}{239}) = \frac{\pi}{4}$$

Hence the proof.

3. If $\sin^{-1}(2a/1 + a^2) - \cos^{-1}(1 - b^2/1 + b^2) = \tan^{-1}(2x/1 - x^2)$, then prove that $x = (a - b)/(1 + ab)$

Solution:

Given $\sin^{-1}(2a/1 + a^2) - \cos^{-1}(1 - b^2/1 + b^2) = \tan^{-1}(2x/1 - x^2)$

Consider,

$$\Rightarrow \sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

We know that,

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by applying these formulae in given equation we get,

$$\Rightarrow 2\tan^{-1}(a) - 2\tan^{-1}(b) = 2\tan^{-1}(x)$$

$$\Rightarrow 2(\tan^{-1}(a) - \tan^{-1}(b)) = 2\tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(a) - \tan^{-1}(b) = \tan^{-1}(x)$$

Again we know that,

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Now by substituting this in above equation we get,

$$\Rightarrow \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1}(x)$$

On comparing we get,

$$\Rightarrow x = \frac{a-b}{1+ab}$$

Hence the proof.

4. Prove that:

(i) $\tan^{-1}\{(1 - x^2)/ 2x)\} + \cot^{-1}\{(1 - x^2)/ 2x)\} = \pi/2$

(ii) $\sin \{\tan^{-1} (1 - x^2)/ 2x) + \cos^{-1} (1 - x^2)/ (1 + x^2)\} = 1$

Solution:

(i) Given $\tan^{-1}\{(1 - x^2)/ 2x)\} + \cot^{-1}\{(1 - x^2)/ 2x)\} = \pi/2$

Consider LHS

$$= \tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x}$$

We know that,

$$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$$

Now by applying the above formula we get,

$= \tan^{-1}\left(\frac{1-x^2}{2x}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Again we know

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

By substituting this we get,

$= \tan^{-1}\left(\frac{\left(\frac{1-x^2}{2x}\right)+\left(\frac{2x}{1-x^2}\right)}{1-\left(\frac{1-x^2}{2x}\right)\times\left(\frac{2x}{1-x^2}\right)}\right)$

$= \tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right)$

$= \tan^{-1}\left(\frac{1+x^4+2x^2}{0}\right)$

$= \tan^{-1}(\infty)$

$= \frac{\pi}{2}$ = RHS

$$\tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x} = \frac{\pi}{2}$$

(ii) Given sin {$\tan^{-1}(1 – x^2)/ 2x) + \cos^{-1}(1 – x^2)/ (1 + x^2)$}

Consider LHS

$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}\right)$

We know that,

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Now by applying the formula in above question we get,

$$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + 2\tan^{-1}x\right)$$

Again, we have

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Now by substituting the formula we get,

$$= \sin\left(\tan^{-1}\frac{1-x^2}{2x} + \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$$

Again we know that,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Now by applying the formula,

$$= \sin\left(\tan^{-1}\left(\frac{\frac{1-x^2}{2x}+\left(\frac{2x}{1-x^2}\right)}{1-\frac{1-x^2}{2x}\times\left(\frac{2x}{1-x^2}\right)}\right)\right)$$

$$= \sin\left(\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}}\right)\right)$$

$$= \sin\left(\tan^{-1}\left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{0}\right)\right)$$

$$= \sin(\tan^{-1}(\infty))$$

$= \sin\left(\frac{\pi}{2}\right)$

= 1

= RHS

So,

$$\sin^{-1}\left(\tan^{-1}\frac{1-x^2}{2x} + \cos^{-1}\frac{1-x^2}{1+x^2}\right) = 1$$

Hence the proof.

5. If $\sin^{-1}(2a/ 1+ a^2) + \sin^{-1}(2b/ 1+ b^2) = 2 \tan^{-1} x$, prove that x = (a + b/ 1 – a b)

Solution:

Given $\sin^{-1}(2a/ 1+ a^2) + \sin^{-1}(2b/ 1+ b^2) = 2 \tan^{-1} x$

Consider

$$\sin^{-1}(\frac{2a}{1+a^2}) + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}(x)$$

We know that,

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Now by applying the above formula we get,

$$\Rightarrow 2\tan^{-1}(a) + 2\tan^{-1}(b) = 2\tan^{-1}(x)$$

$$\Rightarrow 2(\tan^{-1}(a) + \tan^{-1}(b)) = 2\tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}(x)$$

Again we have,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Now by substituting, we get

$$\Rightarrow \tan^{-1}(\frac{a+b}{1-ab}) = \tan^{-1}(x)$$

On comparing we get,

$$\Rightarrow x = \frac{a+b}{1-ab}$$

Hence the proof.