RD SHARMA Solutions for Class 9 Maths Chapter 6 - Factorisation of Polynomials

Chapter 6 - Factorisation of Polynomials Exercise 6.34

Question 1

If $x - 2$ is a factor of $x^2 + 3ax - 2a$, then a =

(a) 2

(b) -2

(c) 1

(d) -1

Solution 1

Let $p(x) = x^2 + 3ax - 2a$ be the given polynomial.

$x - 2$ is a factor of $p(x)$.

Thus, $p(2) = 0$

$(2)^2 + 3a \times 2 - 2a = 0$

$4 + 4a = 0$

$a = -1$

Hence, correct option is (d).

Question 2

If $x^3 + 6x^2 + 4x + k$ is exactly divisible by $x + 2$, then k =

(a) -6

(b) -7

(c) -8

(d) -10

Solution 2

Since, $p(x) = x^3 + 6x^2 + 4x + k$ is exactly divisible by $x + 2$,

$(x + 2)$ is a factor of $p(x)$.

So, $p(-2) = 0$

i.e $(-2)^3 + 6(-2)^2 + 4(-2) + k = 0$

$-8 + 24 - 8 + k = 0$

$24 - 16 + k = 0$

$8 + k = 0$

$k = -8$

Hence, correct option is (c).

Question 3

If $x - a$ is a factor of $x^3 - 3x^2a + 2a^2x + b$, then the value of b is

(a) 0

(b) 2

(c) 1

(d) 3

Solution 3

Let $p(x) = x^3 - 3x^2a + 2a^2x + b$

$(x - a)$ is a factor of $p(x)$.

So, $p(a) = 0$

$a^3 - 3a^2.a + 2a^2.a + b = 0$

$a^3 - 3a^3 + 2a^3 + b = 0$

$3a^3 - 3a^3 + b = 0$

$b = 0$

Hence, correct option is (a).

Question 4
If $x^{140} + 2x^{151} + k$ is divisible by $x + 1$, then the value of k is
(a) 1
(b) -3
(c) 2
(d) -2

Solution 4
Let $p(x) = x^{140} + 2x^{151} + k$
Since p(x) is divisible by (x + 1),
(x + 1) is a factor of p(x).
So, $p(-1) = 0$
$(-1)^{140} + 2(-1)^{151} + k = 0$
$1 + 2(-1) + k = 0$
$1 - 2 + k = 0$
$k - 1 = 0$
$k = 1$
Hence, correct option is (a).

Question 5
If $x + 2$ is a factor of $x^2 + mx + 14$, then m =
(a) 7
(b) 2
(c) 9
(d) 14

Solution 5
If $x + 2$ is a factor of $x^2 + mx + 14$,
then at $x = -2$,
$x^2 + mx + 14 = 0$
i.e. $(-2)^2 + m(-2) + 14 = 0$
$4 - 2m + 14 = 0$
$2m = 18$
$m = 9$
Hence, correct option is (c).

Question 6
If $x - 3$ is a factor of $x^2 - ax - 15$, then a =
(a) -2
(b) 5
(c) -5
(d) 3

Solution 6
$x - 3$ is a factor of $x^2 - ax - 15$,
then at $x = 3$,
$x^2 - ax - 15 = 0$
i.e. $(3)^2 - a(3) - 15 = 0$
$9 - 3a - 15 = 0$
$a = -2$
Hence, correct option is (a).

Question 7

If $x^{51} + 51$ is divided by $x + 1$, the remainder is
(a) 0
(b) 1
(c) 49
(d) 50

Solution 7
When a polynomial p(x) is divided by q(x) i.e. $(x \pm \alpha)$ then $p(\mp \alpha)$ is the remainder.
If $x \pm \alpha$ is the factor of polynomial, then remainder is '0'.
So if $x^{51} + 51$ is divided by $x + 1$,
remainder $= (-1)^{51} + 51 = -1 + 51 = 50$
Hence, correct option is (d).

Question 8
If x + 1 is a factor of the polynomial $2x^2 + kx$, then k =
(a) -2
(b) -3
(c) 4
(d) 2

Solution 8
x + 1 is a factor of $p(x) = 2x^2 + kx$
Then, p(-1) = 0
i.e. $2(-1)^2 + k(-1) = 0$
2 - k = 0
k = 2
Hence, correct option is (d).

Question 9
If $x + \alpha$ is a factor of $x^4 - a^2x^2 + 3x - 6a$, then a =
(a) 0
(b) -1
(c) 1
(d) 2

Solution 9
$x + a$ is a factor of polynomial, $p(x) = x^4 - a^2x^2 + 3x - 6a$
Then, at $x = -a$, $p(x) = 0$
$\Rightarrow (-a)^4 - a^2(-a)^2 + 3(-a) - 6a = 0$
$\Rightarrow \cancel{a^4} - \cancel{a^4} - 3a - 6a = 0$
$\Rightarrow -9a = 0$
$\Rightarrow a = 0$
Hence, correct option is (a).

Question 10
The value of k for which x - 1 is a factor of $4x^3 + 3x^2 - 4x + k$, is
(a) 3
(b) 1
(c) -2
(d) -3

Solution 10

Let $p(x) = 4x^3 + 3x^2 - 4x + k$
Now, if $(x - 1)$ is a facor of $p(x)$, then at $x = 1$, $p(x) = 0$
So, $p(1) = 0$
$\Rightarrow 4(1)^3 + 3(1)^2 - 4(1) + k = 0$
$\Rightarrow \cancel{4} + 3 - \cancel{4} + k = 0$
$\Rightarrow k = -3$
Hence, correct option is (d).

Question 11
If x + 2 and x - 1 are the factors of $x^3 + 10x^2 + mx + n$, then the values of m and n are respectively
(a) 5 and -3
(b) 17 and -8
(c) 7 and -18
(d) 23 and -19

Solution 11
If (x + 2) and (x - 1) are factors of polynomial $x^3 + 10x^2 + mx + n$,
then x = -2, x = +1 will satisfy the polynomial.
Let $p(x) = x^3 + 10x^2 + mx + n$
Then, p(-2) = 0
$(-2)^3 + 10(-2)^2 + m(-2) + n = 0$
-8 + 40 - 2m + n = 0
32 - 2m + n = 0 ....(1)
And, p(1) = 0
$(1)^3 + 10(1)^2 + m(1) + n = 0$
1 + 10 + m + n = 0
11 + m + n = 0 ....(2)
Substracting equation (1) from equation (2), we get
-21 + 3m = 0
3m = 21
m = 7
Substituting m = 7 in equation (2),
11 + 7 + n = 0
18 + n = 0
n = -18
Hence, correct option is (c).

Chapter 6 - Factorisation of Polynomials Exercise 6.35

Question 1
(x + 1) is a factor of $x^n + 1$ only if
(a) n is an odd integer
(b) n is an even integer
(c) n is a negative integer
(d) n is a positive integer

Solution 1
If x + 1 is a factor of $x^n + 1$,
then, at x = -1, $x^n + 1 = 0$
$(-1)^n + 1 = 0$
$(-1)^n = -1$
$(-1)^n$ will be equal to -1 if and only if n is an odd integer.

If n is even, then $(-1)^n = 1$
So, n should be an odd integer.
Hence, correct option is (a).

Question 2
If $x^2 + x + 1$ is a factor of the polynomial $3x^3 + 8x^2 + 8x + 3 + 5k$, then the value of k is
(a) 0
(b) 2/5
(c) 5/2
(d) -1

Solution 2
Let $p(x) = 3x^3 + 8x^2 + 8x + 3 + 5k$ and $q(x) = x^2 + x + 1$
Now, if q(x) is a factor of p(x), then arranging p(x) in order to have q(x) in common,
$p(x) = 3x^3 + 3x^2 + 3x + 5x^2 + 5x + 3 + 2 - 2 + 5k$ [adding +2, −2 in p(x)]
$= 3x(x^2 + x + 1) + 5(x^2 + x + 1) + 5k - 2$
$p(x) = (x^2 + x + 1)(3x + 5) + 5k - 2$ ....(1)
From equation (1), we can see if we divide p(x) by q(x),
then quotient will be $(3x + 5)$ and remainder will be $(5k - 2)$
But q (x) is a factor of p(x).
So, remainder $= 0 \Rightarrow 5k - 2 = 0 \Rightarrow k = \frac{2}{5}$
Hence, correct option is (b).

Question 3
If $(3x - 1)^7 = a_7x^7 + a_6x^6 + a_5x^5 + ... + a_1x + a_0$, then $a_7 + a_6 + a_5 + ... + a_1 + a_0 =$
(a) 0
(b) 1
(c) 128
(d) 64

Solution 3
Correct option (c)
$(3x - 1)^7 = a_7x^7 + a_6x^6 + ... + a_1x + a_0$ ....(1)
Putting x = 1 in equation (1), we have
$[3(1) - 1]^7 = a_7 + a_6 + ... + a_1 + a_0$
So, $a_7 + a_6 + a_5 + ... + a_1 + a_0 = 2^7 = 128$
Hence, correct option is (c).

Question 4
If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, then
(a) $p = r$
(b) $p + r = 0$
(c) $2p + r = 0$
(d) $p + 2r = 0$

Solution 4

Let $f(x) = px^2 + 5x + r$

Now, if $x - 2$ and $x - \frac{1}{2}$ are factors of $f(x)$,

then at $x = 2$ and $x = \frac{1}{2}$, $f(x) = 0$.

So, $f(2) = 0$, $f\left(\frac{1}{2}\right) = 0$

$\Rightarrow p(2)^2 + 5(2) + r = 0$ and $p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$

$\Rightarrow 4p + r + 10 = 0$ ....(1) and $4r + p + 10 = 0$ ....(2)

Subtracting equation (2) from (1), we have

$3p - 3r = 0$

$\Rightarrow p = r$

Hence, correct option is (a).

Question 5

If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, then

(a) $a + c + e = b + d$

(b) $a + b + e = c + d$

(c) $a + b + c = d + e$

(d) $b + c + d = a + e$

Solution 5

If $x^2 - 1$ is factor of $p(x) = ax^4 + bx^3 + cx^2 + dx + e$,

then $(x - 1)$ and $(x + 1)$ will also be factors of $p(x)$.

Because $x^2 - 1 = (x - 1)(x + 1)$

Then, at $x = 1$ and $x = -1$, $p(x) = 0$

$\Rightarrow p(1) = 0$ and $p(-1) = 0$

$\Rightarrow a + b + c + d + e = 0$ ....(1) and $a - b + c - d + e = 0$ ....(2)

Adding equations (1) and (2),

$2a + 2c + 2e = 0$

$\Rightarrow a + c + e = 0$ ....(3)

Substracting equation (2) from (1)

$2b + 2d = 0$

$\Rightarrow b + d = 0$ ....(4)

From equations (3) and (4), we get

$a + c + e = b + d$

Hence, correct option is (a).

Question 6

Let $f(x)$ be a polynomial such that $f\left(-\frac{1}{2}\right) = 0$, then a factor of $f(x)$ is

(a) $2x - 1$

(b) $2x + 1$

(c) $x - 1$

(d) $x + 1$

Solution 6

If f(x) is a polynomial and $f(\alpha) = 0$, then $(x - \alpha)$ is a factor of f(x) or vice versa if $(x - \alpha)$ is a factor of f(x) then $f(\alpha) = 0$.

Now, $f\left(\frac{-1}{2}\right) = 0$

So, at $x = \frac{-1}{2}$, $f(x) = 0$

or at $2x = -1$, $f(x) = 0$

or at $2x + 1 = 0$, $f(x) = 0$

$\Rightarrow (2x + 1)$ is a factor of f(x)

Hence, correct option is (b).

Question 7

When $x^3 - 2x^2 + ax - b$ is divided by $x^2 - 2x - 3$, then remainder is x - 6. The values of a and b are respectively

(a) -2, -6

(b) 2 and -6

(c) -2 and 6

(d) 2 and 6

Solution 7

Let $p(x) = x^3 - 2x^2 + ax - b$, $r(x) = x - 6$ and $q(x) = x^2 - 2x - 3$

Then q(x) is a factor of [p(x) - r(x)]

{because if p(x) is divided by q(x), remainder is r(x). So, [p(x) - r(x)] will be exactly divided by q(x)}

Now, $q(x) = x^2 - 2x - 3 = (x - 3)(x + 1)$

If q(x) is a factor of [p(x) - r(x)] then (x - 3) and (x + 1) are also factors of [p(x) - r(x)]

So, at x = 3 and x = -1, p(x) - r(x) will be zero.

Now p(3) - r(3) = 0

i.e. $(3)^3 - 2(3)^2 + a(3) - b - (3 - 6) = 0$

i.e. 27 - 18 + 3a - b + 3 = 0

i.e. 3a - b + 12 = 0 ....(1)

And, p(-1) - r(-1) = 0

i.e. $(-1)^3 - 2(-1)^2 + a(-1) - b - (-1 - 6) = 0$

i.e. -1 - 2 - a - b + 7 = 0

i.e -a - b + 4 = 0 ....(2)

Subtracting equation (2) from equation (1), we get

4a + 8 = 0

a = -2

From (2), -(-2) - b + 4 = 0

b = 6

Hence, correct option is (c).

Question 8

One factor of $x^4 + x^2 - 20$ is $x^2 + 5$. The other factor is

(a) $x^2 - 4$

(b) x - 4

(c) $x^2 - 5$

(d) x + 2

Solution 8

$x^4 + x^2 - 20$

$= x^4 + 5x^2 - 4x^2 - 20$

$=x^2(x^2 + 5) - 4(x^2 + 5)$

$= (x^2 + 5)(x^2 - 4)$
So, other factor is $x^2 - 4$.
Hence, correct option is (a).

Question 9
If (x - 1) is a factor of polynomial f(x) but not of g(x), then it must be a factor of
(a) f(x) g(x)
(b) -f(x) + g(x)
(c) f(x) - g(x)
(d) {f(x) + g(x)} g(x)

Solution 9
If $x - 1$ is a factor of $f(x)$ then definitely $f(1) = 0$
And, $x - 1$ is not a factor of $g(x)$, then $g(1) \neq 0$
So, at $x = 1$
option (a) $f(1)\,g(1) = 0 \times g(1) = 0$
option (b) $-f(1) + g(1) = 0 + g(1) = g(1) \neq 0$
option (c) $f(1) - g(1) = 0 - g(1) = -g(1) \neq 0$
option (d) $\{f(1) + g(1)\}\,g(1) = \{0 + g(1)\}\,g(1) = \{g(1)\}^2 \neq 0$
So at $x = 1$ only, $f(x)\,g(x) = 0$
Thus, $(x - 1)$ is factor of $f(x)\,g(x)$ too.
Hence, correct option is (a).

Chapter 6 - Factorisation of Polynomials Exercise Ex. 6.1

Question 1
Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 15$
(ii) $y^2 + 2\sqrt{3}$
(iii) $3\sqrt{x} + \sqrt{2}x$
(iv) $x - \frac{4}{x}$
(v) $x^{12} + y^3 + t^{50}$

Solution 1

(i) $3x^2 - 4x + 15$ is a polynomial because exponent of variable x is a positive integer.

(ii) $y^2 + 2\sqrt{3}$, this expression is a polynomial in one variable y.

(iii) $3\sqrt{x} + \sqrt{2}x$

Here the exponent of variable t in a is $\frac{1}{2}$, which is not a positive integer.

So this expression is not a polynomial.

(iv) $x - \frac{4}{x}$ is not a polynomial as the exponent of x in $\frac{4}{x}$ is not a positive integer.

(v) $x^{12} + y^3 + t^{50}$, this expression is a polynomial in 3 variables x, y and t.

Question 2

Write the coefficient of x^2 in each of the following:

(i) $17 - 2x + 7x^2$

(ii) $9 - 12x + x^3$

(iii) $\frac{\pi}{6}x^2 - 3x + 4$

(iv) $\sqrt{3}x - 7$

Solution 2

(i) The coefficient of x^2 in $17 - 2x + 7x^2$ is 7.

(ii) The coefficient of x^2 in $9 - 12x + x^3$ is 0.

(iii) The coefficient of x^2 in $\frac{\pi}{6}x^2 - 3x + 4$ is $\frac{\pi}{6}$

(iv) The coefficient of x^2 in $\sqrt{3}x - 7$ is 0.

Question 3

Write the degrees of each of the following expression:

(i) $7x^3 + 4x^2 - 3x + 12$

(ii) $12 - x + 2x^3$

(iii) $5y - \sqrt{2}$

(iv) 7

(v) 0

Solution 3

(i) The degree of $7x^3 + 4x^2 - 3x + 12$ is 3.

(ii) The degree of $12 - x + 2x^3$ is 3.

(iii) The degree of $5y - \sqrt{2}$ is 1.

(iv) The degree of 7 is 0.

(v) The degree of 0 is undefined.

Question 4

Classify the following polynomial as linear, quadratic, cubic and bi-quadratic polynomials.

(i) $x + x^2 + 4$

(ii) $3x - 2$

(iii) $2x + x^2$

(iv) $3y$

(v) $t^2 + 1$

(vi) $7t^4 + 4t^3 + 3t - 2$

Solution 4

(i) $x + x^2 + 4$ is a quadratic polynomial.

(ii) $3x - 2$ is a linear polynomial.

(iii) $2x + x^2$ is a quadratic polynomial.

(iv) $3y$ is a linear polynomial.

(v) $t^2 + 1$ is a quadratic polynomial.

(vi) $7t^4 + 4t^3 + 3t - 2$ is a bi-quadratic polynomial.

Question 5

Classify the following polynomial as polynomials in one-variable, two variables etc.

(i) $x^2 - xy + 7y^2$

(ii) $x^2 - 2tx + 7t^2 - x + t$

(iii) $t^3 - 3t^2 + 4t - 5$

(iv) $xy + yz + zx$

Solution 5

(i) $x^2 - xy + 7y^2$ is a polynomial in two variable.

(ii) $x^2 - 2tx + 7t^2 - x + t$ is a polynomial in two variable.

(iii) $t^3 - 3t^2 + 4t - 5$ is a polynomial in one variable.

(iv) $xy + yz + zx$ is a polynomial in three variable.

Question 6

Identify polynomials in the following:

(i) $f(x) = 4x^3 - x^2 - 3x + 7$

(ii) $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$

(iii) $p(x) = \frac{2}{3}x^2 - \frac{7x}{4} + 9$

(iv) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$

(v) $h(x) = x^4 - x^{3/2} + x - 1$

(vi) $f(x) = 2 + \frac{3}{x} + 4x$

Solution 6

(i) $f(x) = 4x^3 - x^2 - 3x + 7$ is a polynomial.

(ii) $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$ is not a polynomial.

(iii) $p(x) = \frac{2}{3}x^2 - \frac{7x}{4} + 9$ is a polynomial.

(iv) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$ is not a polynomial.

(v) $h(x) = x^4 - x^{3/2} + x - 1$ is not a polynomial.

(vi) $f(x) = 2 + \frac{3}{x} + 4x$ is not a polynomial.

Question 7

Identify constant, linear, quadratic and cubic polynomials from the following polynomials:

(i) $f(x) = 0$

(ii) $g(x) = 2x^3 - 7x + 4$

(iii) $h(x) = -3x + \frac{1}{2}$

(iv) $p(x) = 2x^2 - x + 4$

(v) $q(x) = 4x + 3$

(vi) $r(x) = 3x^3 = 4x^2 + 5x - 7$

Solution 7

(i) $f(x) = 0$ is a constant polynomial.

(ii) $g(x) = 2x^3 - 7x + 4$ is a cubic polynomial.

(iii) $h(x) = -3x + \frac{1}{2}$ is a linear polynomial.

(iv) $p(x) = 2x^2 - x + 4$ is a quadratic polynomial.

(v) $q(x) = 4x + 3$ is a linear polynomial.

(vi) $h(x) = 3x^3 + 4x^2 + 5x - 7$ is a cubic polynomial.

Question 8
Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution 8
Degree of a polynomial is the highest power of variable in the polynomial.
Binomial has two terms in it. So binomial of degree 35 can be written as $x^{35} + 7$.
Monomial has only one term in it. So monomial of degree 100 can be written as $7x^{100}$.

Concept Insight: Mono, bi and tri means one, two and three respectively. So, monomial is a polynomial having one term similarly for binomials and trinomials. Degree is the highest exponent of variable. The answer is not unique in such problems . Remember that the terms are always separated by +ve or -ve sign and not with .

Chapter 6 - Factorisation of Polynomials Exercise Ex. 6.2
Question 1
If $f(x) = 2x^3 - 13x^2 + 17x + 12$, find:

(i) f(2)

(ii) f(-3)

(iii) f(0)

Solution 1
(i)

$f(x) = 2x^3 - 13x^2 + 17x + 12$

$f(2) = 2(2)^3 - 13(2)^2 + 17(2) + 12$

$= 16 - 52 + 34 + 12$

$= 10$

(ii)

$f(-3) = 2(-3)^3 - 13(-3)^2 + 17(-3) + 12$

$= -54 - 117 - 51 + 12$

$= - 210$

(iii)

$f(0) = 2(0)^3 - 13(0)^2 + 17(0) + 12$

$= 0 - 0 + 0 + 12$

=12

Question 2

Verify whether the indicated numbers are zeroes of the polynomials corresponding to them in the following cases:

(i) $f(x) = 3x + 1, x = -\frac{1}{3}$

(ii) $f(x) = x^2 - 1, x = 1, -1$

(iii) $g(x) = 3x^2 - 2, x = \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$

(iv) $p(x) = x^3 - 6x^2 + 11x - 6, x = 1,2,3$

(v) $f(x) = 5x - \pi, x = \frac{4}{5}$

(vi) $f(x) = x^2, x = 0$

(vii) $f(x) = lx + m, x = -\frac{m}{l}$

(viii) $f(x) = 2x + 1, x = \frac{1}{2}$

Solution 2

(i) If $x = \frac{-1}{3}$ is a zero of given polynomial f(x) = 3x + 1, then

$f\left(-\frac{1}{3}\right)$ should be 0.

Now, $f\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1 = -1 + 1 = 0$

So, $x = \frac{-1}{3}$ is a zero of given polynomial

(ii) If x = 1 and x = - 1 are zeroes of polynomial $f(x) = x^2 - 1$ then f(1) and f(- 1) should be 0

Now $f(1) = (1)^2 - 1 = 0$

$f(-1) = (-1)^2 - 1 = 0$

Hence x = 1 and - 1 are zeroes of given polynomial

(iii) If $x = \frac{2}{\sqrt{3}}$ and $x = -\frac{2}{\sqrt{3}}$ are zeroes of polynomial $g(x) = 3x^2 - 2$ then

$$g\left(\frac{2}{\sqrt{3}}\right) \text{and } g\left(-\frac{2}{\sqrt{3}}\right) \text{should be } 0$$

$$\text{Now, } g\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2 = 3\left(\frac{4}{3}\right) - 2 = 4 - 2 = 2$$

$$g\left(-\frac{2}{\sqrt{3}}\right) = 3\left(-\frac{2}{\sqrt{3}}\right)^2 - 2 = 3\left(\frac{4}{3}\right) - 2 = 4 - 2 = 2$$

Hence, $x = \frac{2}{\sqrt{3}}$ and $x = -\frac{2}{\sqrt{3}}$ are not the zeroes of given polynomial.

(iv) If x = 1 and x = 2 and x = 3 are zeroes of polynomial $p(x)$, then

p(1) an[illegible] p(3) [illegible] uld be 0.

Now, $p(1) =$ $- 6 + 11 - 6 = 0$

$p(2) = 8 - 24$ $- 22 - 6 = 0$

$p(3) = 27 - 5$ $+ 33 - 6 = 0$

So, x = 1 and x = 2 and x = 3 are zeroes of given polynomial.

(v) If $x = \frac{4}{5}$ is a z ro of polynomial f(x) = 5x - π ,

then $f\left(\frac{4}{5}\right)$ sh uld be 0

Now, $f\left(\frac{4}{5}\right) =$ $\left(\frac{4}{5}\right) - \pi = 4 - \pi$

As $f\left(\frac{4}{5}\right) \neq 0$

Question 3

If $x = 2$ is a root of the polynomial $f(x) = 2x^2 - 3x + 7a$, find the value of a.

Solution 3

$\because x = 2$ is a root of the polynomial $f(x) = 2x^2 - 3x + 7a$

$\therefore f(2) = 0$

$f(2) = 2(2)^2 - 3(2) + 7a = 0$

$\Rightarrow 2(4) - 3(2) + 7a = 0$

$\Rightarrow 8 - 6 + 7a = 0$

$\Rightarrow 2 + 7a = 0$

$\Rightarrow 7a = -2$

$\Rightarrow a = \frac{-2}{7}$

$\therefore a = \frac{-2}{7}$

Question 4

If $x = -\frac{1}{2}$ is a zero of the polynomial $p(x) = 8x^3 - ax^2 - x + 2$, find the value of a.

Solution 4

Since, $x = -\frac{1}{2}$ is a zero of $p(x) = 8x^3 - ax^2 - x + 2$

$\therefore p\left(-\frac{1}{2}\right) = 0$

$\Rightarrow 8\left(-\frac{1}{2}\right)^3 - a\left(-\frac{1}{2}\right)^2 - \left(\frac{-1}{2}\right) + 2$

$\Rightarrow 8\left(\frac{-1}{8}\right) - a\left(\frac{1}{4}\right) + \frac{1}{2} + 2 = 0$

$\Rightarrow -1 - \frac{a}{4} + \frac{1}{2} + 2 = 0$

$\Rightarrow -1 + \frac{1}{2} + 2 = \frac{a}{4}$

$\Rightarrow 1 + \frac{1}{2} = \frac{a}{4}$

$\Rightarrow \frac{3}{2} = \frac{a}{4}$

$\Rightarrow \frac{3 \times 4}{2} = a$

$\Rightarrow a = 6$

$\therefore a = 6$

Question 5

If $x = 0$ and $x = -1$ are the roots of the polynomial $f(x) = 2x^3 - 3x^2 + ax + b$, find the value of a and b.

Solution 5

Since $x = 0$ is a root of $f(x)$

$\therefore f(0) = 0$

$f(0) = 2(0)^3 - 3(0)^2 + a(0) + b = 0$

$\Rightarrow b = 0 \qquad (1)$

Since $x = -1$ is a root of $f(x)$

$\therefore f(-1) = 0$

$f(-1) = 2(-1)^3 - 3(-1)^2 + a(-1) + b = 0$

$\Rightarrow 2(-1) - 3(1) - a + b = 0$

$\Rightarrow -2 - 3 - a + b = 0$

$\Rightarrow -5 - a + b = 0$

$\Rightarrow -5 - a = 0 \qquad [\text{substituting } b = 0]$

$\Rightarrow -5 = a$

$\therefore a = -5, b = 0$

Question 6

Find the integral roots of the polynomial $f(x) = x^3 + 6x^2 + 11x + 6$

Solution 6

Clearly, $f(x)$ is a polynomial with integer coefficients and the coefficient of the highest degree term is 1.
Therefore, integer roots of $f(x)$ can be the integer factor of 6, which are $\pm 1, \pm 2, \pm 3, \pm 6$.

$f(1) = 1^3 + 6(1)^2 + 11(1) + 6 \neq 0$
$\therefore x = 1$ is not a root of f(x)

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$
$= -1 + 6 - 11 + 6$
$= 0$
$\therefore x = -1$ is a root of f(x)

$f(2) = 2^3 + 6(2)^2 + 11(2) + 6 \neq 0$
$\therefore x = 2$ is not a root of f(x)

$f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6$
$= -8 + 24 - 22 + 6$
$= 0$
$\therefore x = -2$ is a root of f(x)

$f(3) = 3^3 + 6(3)^2 + 11(3) + 6 \neq 0$
$\therefore x = 3$ is not a root of f(x)

$f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6$
$= -27 + 6(9) - 33 + 6$
$= -27 + 54 - 33 + 6$
$= 0$
$\therefore x = -3$ is a root of f(x)

$f(6) = 6^3 + 6(6)^2 + 11(6) + 6 \neq 0$
$\therefore x = 6$ is not a root of f(x)

$f(-6) = (-6)^3 + 6(-6)^2 + 11(-6) + 6 \neq 0$
$\therefore x = -6$ is not a root of f(x)

Hence integral roots of $f(x)$ are $-1, -2$ & -3

Question 7

Find rational roots of the polynomial $f(x) = 2x^3 + x^2 - 7x - 6$.

Solution 7

Let $f(x) = 2x^3 + x^2 - 7x - 6$

Clearly, f(x) is a cubic polynomial with integer coefficients. If $\frac{b}{c}$ is a rational roots in lowest terms, then the value of b are limited to the factors of 6 which are ± 1, ± 2.

Hence, the possible rational roots of f(x) are ± 11 ± 2, ± 3, ± 6, $\frac{1}{2}$, $\pm \frac{3}{2}$

We observe that

$f(2) = 2 \times 8 + 4 - 7 \times 2 - 6 = 16 + 4 - 14 - 6 = 0$

$f(-1) = 2 \times -1 + 1 - 7 \times -1 - 6 = -2 + 1 + 7 - 6 = 0$

and $f\left(-\frac{3}{2}\right) = 2 \times$

$$-\frac{27}{8}+\frac{9}{4}-7\times-\frac{3}{2}-6=-\frac{18}{4}+\frac{21}{2}-6=-\frac{9}{2}+\frac{21}{2}-6=\frac{12}{2}-6=0$$

Hence, 2, -1 and $-\frac{3}{2}$ are rational roots of f(x)

Chapter 6 - Factorisation of Polynomials Exercise Ex. 6.3

Question 1

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$f(x) = x^3 + 4x^2 - 3x + 10, g(x) = x + 4$

Solution 1

Let $g(x) = 0$

$\therefore x + 4 = 0$
$\therefore x = -4$

Applying Remainder theorem

$f(x) = x^3 + 4x^2 - 3x + 10$

$f(x) = (-4)^3 + 4(-4)^2 - 3(-4) + 10$

$\quad = -64 + 64 + 12 + 10 = 22$

$\therefore$ Remainder = 22

Verification

$$\begin{array}{r|l}
 & x^2 - 3 \\
\hline
x + 4 & x^3 + 4x^2 - 3x + 10 \\
 & \underline{\overset{-}{x^3} \overset{-}{+ 4x^2}} \\
 & \qquad\quad -3x + 10 \\
 & \qquad\quad \underline{\overset{+}{-3x} \overset{+}{- 12}} \\
 & \qquad\qquad\quad \underline{22}
\end{array}$$

Hence verified.

Question 2

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7, g(x) = x - 1$

Solution 2

Let $g(x) = 0$

$\Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

Applying Remainder theorem

$f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$

$f(1) = 4(1)^4 - 3(1)^3 - 2(1)^2 + 1 - 7$

$\quad = 4 - 3 - 2 + 1 - 7$

$\quad = 5 - 12 = -7$

$\therefore$ Remainder $= -7$

Verification:

$$
\begin{array}{r|l}
 & 4x^3 + x^2 - x \\
\hline
x - 1 & 4x^4 - 3x^3 - 2x^2 + x - 7 \\
 & \underline{\underset{-}{4x^4} \underset{+}{- 4x^3}} \\
 & \quad x^3 - 2x^2 + x - 7 \\
 & \quad \underline{\underset{-}{x^3} \underset{+}{- x^2}} \\
 & \qquad - x^2 + x - 7 \\
 & \qquad \underline{\underset{+}{- x^2} \underset{-}{+ x}} \\
 & \qquad\qquad - 7
\end{array}
$$

Hence verified

Question 3

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2, g(x) = x + 2$

Solution 3

$\text{Let } g(x) = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

$f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$

Applying Remainder theorem

$$
\begin{aligned}
f(-2) &= 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2 \\
&= 2(16) - 6(-8) + 2(4) + 2 + 2 \\
&= 32 + 48 + 8 + 2 + 2 \\
&= 44 + 48 = 92
\end{aligned}
$$

$\therefore$ Remainder = 92

Verification:

$$
\begin{array}{r|l}
 & 2x^3 - 10x^2 + 22x - 45 \\
\hline
x + 2 & 2x^4 - 6x^3 + 2x^2 - x + 2 \\
 & 2x^4 + 4x^3 \\
 & \underline{- \quad -} \\
 & -10x^3 + 2x^2 - x + 2 \\
 & -10x^3 - 20x^2 \\
 & \underline{+ \quad +} \\
 & 22x^2 - x + 2 \\
 & 22x^2 + 44x \\
 & \underline{- \quad -} \\
 & -45x + 2 \\
 & -45x - 90 \\
 & \underline{\quad +} \\
 & 92
\end{array}
$$

Hence verified.

Question 4

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$f(x) = 4x^3 - 12x^2 + 14x - 3, g(x) = 2x - 1$

Solution 4

Let $g(x) = 0$

$\Rightarrow 2x - 1 = 0$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

Applying Remainder theorem

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$

$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 14 \times \frac{1}{2} - 3$$

$$= \frac{1}{2} - 3 + 7 - 3$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

$\therefore$ Remainder $= \frac{3}{2}$

Verification:

$$
\begin{array}{r}
2x^2 - 5x + \frac{9}{2} \\
2x - 1\,\big)\overline{\,4x^3 - 12x^2 + 14x - 3} \\
\underset{-}{4x^3} \underset{+}{-\,2x^2} \qquad\qquad\quad \\
\hline
-10x^2 + 14x - 3 \\
\underset{+}{-\,10x^2} \underset{-}{+\,5x} \qquad \\
\hline
9x - 3 \\
\underset{-}{9x} \underset{+}{-} \frac{9}{2} \\
\hline
\frac{3}{2}
\end{array}
$$

Hence verified.

Question 5

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$f(x) = x^3 - 6x^2 + 2x - 4, g(x) = 1 - 2x$

Solution 5

Let $g(x) = 0$

$\Rightarrow 1 - 2x = 0$

$\Rightarrow 2x = 1$

$\Rightarrow x = \dfrac{1}{2}$

Applying Remainder theorem

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - \frac{6}{4} + 1 - 4$$

$$= \frac{1}{8} - \frac{6}{4} - 3$$

$$= \frac{1 - 12 - 24}{8} = \frac{-35}{8}$$

Verification:

$$-\frac{x^2}{2} + \frac{11x}{4} + \frac{3}{8}$$

$$-2x + 1 \overline{\smash{\big)}\, x^3 - 6x^2 + 2x - 4}$$

$$\underline{\underset{-}{x^3} \underset{+}{-} \frac{x^2}{2}}$$

$$\frac{-11x^2}{2} + 2x - 4$$

$$\underline{\underset{+}{}\frac{-11x^2}{2} \underset{-}{+} \frac{11x}{4}}$$

$$\frac{-3x}{4} - 4$$

$$\underline{\underset{+}{}\frac{-3x}{4} \underset{-}{+} \frac{3}{8}}$$

$$-\frac{35}{8}$$

Hence verified.

Question 6

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem. [illegible]

[illegible]

Solution 6

Let $g(x) = 0$
$x - 2 = 0$
$x = 2$

Applying Remainder theorem

$$
\begin{aligned}
f(2) &= 2^4 - 3(2)^2 + 4 \\
&= 16 - 3(4) + 4 \\
&= 16 - 12 + 4 \\
&= 20 - 12 = 8
\end{aligned}
$$

$\therefore$ Remainder = 8

Verification:

$$
\begin{array}{r|l}
 & x^3 + 2x^2 + x + 2 \\
\hline
x - 2 & x^4 - 3x^2 + 4 \\
 & \underline{\overset{-}{x^4} \quad \overset{+}{-2x^3}} \\
 & 2x^3 - 3x^2 + 4 \\
 & \underline{\overset{-}{2x^3} \overset{+}{-4x^2}} \\
 & x^2 + 4 \\
 & \underline{\overset{-}{x^2} \quad \overset{+}{-2x}} \\
 & 2x + 4 \\
 & \underline{\overset{-}{2x} \overset{+}{-4}} \\
 & 8
\end{array}
$$

Hence verified.

Question 7

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$f(x) = 9x^3 - 3x^2 + x - 5, g(x) = x - \frac{2}{3}$

Solution 7

Let $g(x) = 0$

$$\Rightarrow x - \frac{2}{3} = 0$$

$$\Rightarrow x = \frac{2}{3}$$

Applying Remainder theorem

$$f\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right)^3 - 3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) - 5$$

$$= 9 \times \frac{8}{27} - 3 \times \frac{4}{9} + \frac{2}{3} - 5$$

$$= \frac{8}{3} - \frac{4}{3} + \frac{2}{3} - 5$$

$$= \frac{8 - 4 + 2 - 15}{3} = \frac{-9}{3} = -3$$

Verification:

$$\begin{array}{r|l} & 9x^2 + 3x + 3 \\ \hline x - \frac{2}{3} & 9x^3 - 3x^2 + x - 5 \\ & \underline{\overset{-}{}9x^3 \overset{+}{-} 6x^2} \\ & 3x^2 + x - 5 \\ & \underline{\overset{-}{}3x^2 \overset{+}{-} 2x} \\ & 3x - 5 \\ & \underline{\overset{-}{}3x \overset{+}{-} 2} \\ & -3 \end{array}$$

Hence verified.

Question 8

If $f(x)$ is divided by $g(x)$, find remainder using remainder theorem and verify the result by actual division.

$$f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27},\ g(x) = x + \frac{2}{3}$$

Solution 8

Let $g(x) = 0$

$\Rightarrow x + \frac{2}{3} = 0$

$\Rightarrow x = \frac{-2}{3}$

Applying Remainder theorem

$$f\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right)^4 + 2\left(\frac{-2}{3}\right)^3 - \frac{1}{3}\left(\frac{-2}{3}\right)^2 - \frac{1}{9}\left(\frac{-2}{3}\right) + \frac{2}{27}$$

$$= 3 \times \frac{16}{81} + 2\left(\frac{-8}{27}\right) - \frac{1}{3}\left(\frac{4}{9}\right) + \frac{2}{27} + \frac{2}{27}$$

$$= \frac{16}{27} - \frac{16}{27} - \frac{4}{27} + \frac{4}{27} = 0$$

Verification:

$$\begin{array}{r|l}
 & 3x^3 - \frac{x}{3} + \frac{1}{9} \\ \hline
x + \frac{2}{3} & 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27} \\
 & \underline{\overset{-}{}3x^4 \underset{-}{+} 2x^3} \\
 & \quad \frac{-x^2}{3} - \frac{x}{9} + \frac{2}{27} \\
 & \quad \underline{\frac{-x^2}{3} - \frac{2x}{9}} \quad (+,\ +) \\
 & \qquad \frac{x}{9} + \frac{2}{27} \\
 & \qquad \underline{\frac{x}{9} + \frac{2}{27}} \quad (-,\ -) \\
 & \qquad\quad 0
\end{array}$$

Hence verified.

Question 9

If the polymial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by $x - 2$, find the value of a

Solution 9

$$\text{Let } p(x) = 2x^3 + ax^2 + 3x - 5$$
$$f(x) = x^3 + x^2 - 4x + a$$
$$g(x) = x - 2$$

$$\text{Let } g(x) = 0$$
$$\Rightarrow x - 2 = 0$$
$$\Rightarrow x = 2$$

$$p(2) = 2(2)^3 + a(2)^2 + 3(2) - 5$$
$$= 16 + 4a + 6 - 5$$
$$= 17 + 4a$$

$$f(2) = 2^3 + 2^2 - 4(2) + a$$
$$= 8 + 4 - 8 + a$$
$$= 4 + a$$

$\because p(x)$ and $f(x)$ leaves the same remainder when divided by $g(x)$

$$\therefore p(2) = f(2)$$
$$\Rightarrow 17 + 4a = 4 + a$$
$$\Rightarrow 4a - a = 4 - 17$$
$$\Rightarrow 3a = -13$$
$$\Rightarrow a = \frac{-13}{3}$$

Question 10

If the polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$, when divided by (x-2) leave the same remainder, find the value of a.

Solution 10

Let $p(x) = ax^3 + 3x^2 - 13$ and $q(x)$ [illegible]

Question 11

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i)	$x + 1$	(ii)	$x - \frac{1}{2}$	(iii)	x
(iv)	$x + \pi$	(v)	$5 + 2x$		

Solution 11

Let $p(x) = x^3 + 3x^2 + 3x + 1$.

(i) $x + 1$

Zero of $x + 1$ is -1.

$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = 0$

So, the remainder is 0.

(ii) $x - \frac{1}{2}$

Zero of $x-\frac{1}{2}$ is $\frac{1}{2}$

$$p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$$
$$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$$
$$=\frac{1+6+12+8}{8}$$
$$=\frac{27}{8}$$

So, the remainder is $\frac{27}{8}$

(iii) x

Zero of x is 0.

$p(0) = (0)^3 + 3(0)^2 + 3(0) + 1 = 1$

So, the remainder is 1.

(iv) $x + \pi$

Zero of $x + \pi$ is: $x + \pi = 0 \Rightarrow x = -\pi$

$$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$$
$$= -\pi^3 + 3\pi^2 - 3\pi + 1$$

So, the remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$

(v) $5 + 2x$

Zero of $5+2x$ is:

$5 + 2x = 0 \Rightarrow 2x = -5$

i.e. $x = -\frac{5}{2}$

$$p\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$$
$$=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$$
$$=\frac{-125+150-60+8}{}$$

Question 12

The polynomials ax^3+3x^2-3 and $2x^3-5x+a$ when divided by $(x-4)$ leave the remainders R_1 and R_2 respectively. Find the values of a in each of the following cases, if

(i) $R_1 = R_2$

(ii) $R_1 + R_2 = 0$

(iii) $2R_1 - R_2 = 0$

Solution 12

Let $p(x) = ax^3+3x^2-3$ and $q(x) = 2x^3-5x+a$ be the given polynomials.

Now, R_1 = remainder when $p(x)$ is divided by $x-4$

$\Rightarrow R_1 = p(4)$

$\Rightarrow R_1 = a(4)^3 + 3(4)^2 - 3$

$\Rightarrow R_1 = 64a + 48 - 3$

$\Rightarrow R_1 = 64a + 45$

And

$\Rightarrow R_2$ = remainder when $q(x)$ is divided by $x-4$

$\Rightarrow R_2 = 2(4)^3 - 5(4) + a$

$\Rightarrow R_2 = 128 - 20 + a$

$\Rightarrow R_2 = 108 + a$

(i) $R_1 = R_2$

Substituting the values, we get,

$64a + 45 = 108 + a$

$64a - a = 108 - 45$

$63a = 63$

$a = 1$

(ii) $R_1 + R_2 = 0$

Substituting the values, we get,

$64a + 45 + 108 + a = 0$

$65a + 153 = 0$

$65a = -153$

$a = -\frac{153}{65}$

(iii) $2R_1 - R_2 = 0$

Substituting the values, we get,

$2(64a + 45) - (108 + a) = 0$

$128a + 90 - 108 - a = 0$

$127a - 18 = 0$

$127a = 18$

$a = \frac{18}{127}$

Chapter 6 - Factorisation of Polynomials Exercise Ex. 6.4

Question 1

Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = x^3 - 6x^2 + 11x - 6$

$g(x) = x - 3$

Solution 1

Let $g(x) = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$

$$\begin{aligned} f(3) &= 3^3 - 6(3)^2 + 11(3) - 6 \\ &= 27 - 6(9) + 33 - 6 \\ &= 27 - 54 + 33 - 6 \\ &= 60 - 60 = 0 \end{aligned}$$

$\because f(3) = 0$,by factor theorem $x - 3$ is a factor of $f(x)$

Question 2

Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$

$g(x) = x + 5$

Solution 2

Let $g(x) = 0$

$\Rightarrow x + 5 = 0$

$\Rightarrow x = -5$

Now,

$$\begin{aligned} f(-5) &= 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10 \\ &= 3(625) + 17(-125) + 9(25) + 35 - 10 \\ &= 1875 - 2125 + 225 + 35 - 10 \\ &= 2135 - 2135 = 0 \end{aligned}$$

$\because f(-5) = 0$, by factor theorem, $x + 5$ is a factor of $f(x)$.

Question 3

Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15$

$g(x) = x + 3$

Solution 3

Let $g(x) = 0$
$\therefore x + 3 = 0$
$\Rightarrow x = -3$

Now,
$f(-3) = (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15$
$= -243 + 243 + 27 - 27 - 15 + 15 = 0$

$\because f(-3) = 0$, by factor theorem, $x + 3$ is a factor of $f(x)$.

Question 4
Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = x^3 - 6x^2 - 19x + 84$
$g(x) = x - 7$

Solution 4
Let $g(x) = 0$
$\therefore x - 7 = 0$
$\Rightarrow x = 7$

$f(7) = 7^3 - 6(7)^2 - 19(7) + 84$
$= 343 - 6(49) - 19(7) + 84$
$= 343 - 294 - 133 + 84$
$= 427 - 427 = 0$

$\because f(7) = 0$, by factor theorem, $x - 7$ is a factor of $f(x)$.

Question 5
Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = 3x^3 + x^2 - 20x + 12$
$g(x) = 3x - 2$

Solution 5

Let $g(x) = 0$

$\Rightarrow 3x - 2 = 0$

$\Rightarrow x = \frac{2}{3}$

$$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20\left(\frac{2}{3}\right) + 12$$
$$= \frac{24}{27} + \frac{4}{9} - \frac{40}{3} + 12$$
$$= \frac{24 + 12 - 360 + 324}{27}$$
$$= \frac{360 - 360}{27}$$
$$= \frac{0}{27}$$
$$= 0$$

$\because f\left(\frac{2}{3}\right) = 0$, by factor theorem, $3x - 2$ is a factor of $f(x)$.

Question 6

Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = 2x^3 - 9x^2 + x + 12$

$g(x) = 3 - 2x$

Solution 6

Let $g(x) = 0$

$\Rightarrow 3 - 2x = 0$

$\Rightarrow 3 = 2x$

$\Rightarrow x = \frac{3}{2}$

Now,

$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^{3} - \left(\frac{3}{2}\right)^{2} + \left(\frac{3}{2}\right) + 12$

Question 7

Using factor theorem determine if $g(x)$ is a factor of $f(x)$.

$f(x) = x^3 - 6x^2 + 11x - 6$

$g(x) = x^2 - 3x + 2$

Solution 7

$$g(x) = x^2 - 3x + 2$$
$$= x^2 - 2x - x + 2$$
$$= x(x-2) - 1(x-2)$$
$$= (x-2)(x-1)$$

Let $g(x) = 0$
$\Rightarrow (x-2)(x-1) = 0$

If $x - 2 = 0 \Rightarrow x = 2$
If $x - 1 = 0 \Rightarrow x = 1$

$$f(2) = 2^3 - 6(2)^2 + 11(2) - 6$$
$$= 8 - 24 + 22 - 6$$
$$= 30 - 30$$
$$= 0$$

$$f(1) = 1^3 - 6(1)^2 + 11(1) - 6$$
$$= 1 - 6 + 11 - 6$$
$$= 0$$

$\because f(2) = 0$ and $f(1) = 0$, by factor theorem, $(x-2)$ *and* $(x-1)$ both are factors of $f(x)$
Hence, $(x-2)(x-1)$ is a factor of $f(x)$

Question 8
Show that $(x-2)$, $(x+3)$ and $(x-4)$ are factors of $x^3 - 3x^2 - 10x + 24$

Solution 8

$f(x) = x^3 - 3x^2 - 10x + 24$

Let, $x - 2 = 0$
$\Rightarrow x = 2$

$$\begin{aligned} f(2) &= 2^3 - 3(2)^2 - 10(2) + 24 \\ &= 8 - 12 - 20 + 24 \\ &= 32 - 32 = 0 \end{aligned}$$

Let, $x + 3 = 0$
$\Rightarrow x = -3$

$$\begin{aligned} f(-3) &= (-3)^3 - 3(-3)^2 - 10(-3) + 24 \\ &= -27 - 3(9) + 30 + 24 \\ &= -27 - 27 + 30 + 24 \\ &= -54 + 54 = 0 \end{aligned}$$

Let, $x - 4 = 0$
$\Rightarrow x = 4$

$$\begin{aligned} f(4) &= 4^3 - 3(4)^2 - 10(4) + 24 \\ &= 64 - 3(16) - 40 + 24 \\ &= 64 - 48 - 40 + 24 \\ &= 88 - 88 = 0 \end{aligned}$$

$\because f(2) = 0, f(-3) = 0, f(4) = 0$
$\therefore$ By factor theorem $(x - 2)$, $(x + 3)$ & $(x - 4)$ are factors of $x^3 - 3x^2 - 10x + 24$

Question 9
Show that $(x + 4)$, $(x - 3)$ and $(x - 7)$ are factors of $f(x) = x^3 - 6x^2 - 19x + 84$

Solution 9

Let $x + 4 = 0$

$x = -4$

Question 10

For what value of a is $(x - 5)$ a factor of $x^3 - 3x^2 + ax - 10$

Solution 10

Let, $x - 5 = 0$
$x = 5$

$\because (x-5)$ is a factor of $x^3 - 3x^2 + ax - 10$
$\therefore f(5) = 0$

$f(5) = 5^3 - 3(5)^2 + a(5) - 10 = 0$
$\Rightarrow 125 - 3 \times 25 + 5a - 10 = 0$
$\Rightarrow 125 - 85 + 5a = 0$
$\Rightarrow 40 + 5a = 0$
$\Rightarrow 5a = -40$
$\Rightarrow a = \frac{-40}{5} = -8$

Question 11
Find the value of a such that $(x-4)$ is a factor of $5x^3 - 7x^2 - ax - 28$

Solution 11
Let $g(x) = x - 4, f(x) = 5x^3 - 7x^2 - ax - 28$

Let $g(x) = 0$
$\Rightarrow x - 4 = 0$
$\Rightarrow x = 4$, since $(x-4)$ is a factor of $f(x) \therefore f(4) = 0$

$f(4) = 5(4)^3 - 7(4)^2 - a(4) - 28 = 0$
$\Rightarrow 5(64) - 7(16) - 4a - 28 = 0$
$\Rightarrow 320 - 112 - 4a - 28 = 0$
$\Rightarrow 180 - 4a = 0$
$\Rightarrow 4a = 180$
$\Rightarrow a = \frac{180}{4} = 45$

Question 12
Find the value of a, if $(x+2)$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Solution 12

Let $g(x) = x + 2$, and $f(x) = 4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Let $g(x) = 0$
$\Rightarrow x + 2 = 0$
$\Rightarrow x = -2$,

$\because g(x)$ is a factor of $f(x)$
$\therefore f(-2) = 0$

$f(-2) = 4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$
$\Rightarrow 4(16) + 2(-8) - 3(4) + 8(-2) + 5a = 0$
$\Rightarrow 64 - 16 - 12 - 16 + 5a = 0$
$\Rightarrow 20 + 5a = 0$
$\Rightarrow 5a = -20$
$\Rightarrow a = \dfrac{-20}{5} = -4$

$\therefore a = -4$

Question 13

Find the value of k, if $(x - 3)$ is a factor of $k^2x^3 - kx^2 + 3kx - k$

Solution 13

Let $g(x) = x - 3, f(x) = k^2x^3 - kx^2 + 3kx - k$

Let $g(x) = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3,$

Since $(x - 3)$ is a factor of $f(x)$

$\therefore f(3) = 0$

$f(3) = k^2 3^3 - k3^2 + 3k(3) - k = 0$

$\Rightarrow 27k^2 - 9k + 9k - k = 0$

$\Rightarrow 27k^2 - k = 0$

$\Rightarrow k(27k - 1) = 0$

$\therefore k = 0, \quad 27k - 1 = 0$

$27k = 1$

$k = \frac{1}{27}$

Hence $k = 0, k = \frac{1}{27}$

Question 14

Find the value of a and b, if $x^2 - 4$ is a factor of $ax^4 + 2x^3 - 3x^2 + bx - 4$

Solution 14

Let $g(x) = x^2 - 4, f(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$

Let , $g(x) = 0$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow \quad x^2 = 4$

$x = \pm 2$

Since $\left(x^2 - 4\right)$ is a factor of $f(x)$

$\therefore f(2) = 0$ and $f(-2) = 0$

$f(2) = a(2)^4 + 2(2)^3 - 3(2)^2 + b(2) - 4 = 0$

$\Rightarrow 16a + 16 - 12 + 2b - 4 = 0$

$\Rightarrow 16a + 2b = 0$

$\Rightarrow 16a = -2b$

$\Rightarrow a = \dfrac{-2b}{16} = \dfrac{-b}{8} \qquad --(1)$

Also $f(-2) = 0$

$f(-2) = a(-2)^4 + 2(-2)^3 - 3(-2)^2 + b(-2) - 4 = 0$

$\Rightarrow 16a - 16 - 12 - 2b - 4 = 0$

$\Rightarrow 16a - 2b - 32 = 0$

$\Rightarrow 16a = 2b + 32$

$\Rightarrow a = \dfrac{2b + 32}{16} \qquad --(2)$

Equating equations (1) and (2)

$\Rightarrow \dfrac{-b}{8} = \dfrac{2b + 32}{16}$

$\Rightarrow \dfrac{-16b}{8} = 2b + 32$

$\Rightarrow -2b = 2b + 32$

$\Rightarrow -4b = 32$

$\Rightarrow b = -8$

Substituting $b = -8$ in equation (1)

$a = \dfrac{-b}{8} = \dfrac{-(-8)}{8} = \dfrac{8}{8} = 1$

$\therefore a = 1, b = -8$

Question 15

Find α and β, if $x + 1$ and $x + 2$ are factors of $x^3 + 3x^2 - 2\alpha x + \beta$

Solution 15

Let $x + 1 = 0$

Question 16

If $x - 2$ is a factor of each of the following two polynomials, find the values of a in each case

(i) $x^3 - 2ax^2 + ax - 1$ (ii) $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Solution 16

(i) $x^3 - 2ax^2 + ax - 1$

Let, $x - 2 = 0$

$\therefore x = 2$

$\because x - 2$ is a factor of $p(x) = x^3 - 2ax^2 + ax - 1$

$\therefore p(2) = 0$

$p(2) = 2^3 - 2a(2)^2 + 2a - 1 = 0$

$\Rightarrow 8 - 8a + 2a - 1 = 0$

$\Rightarrow 7 - 6a = 0$

$\Rightarrow 6a = 7$

$\Rightarrow a = \dfrac{7}{6}$

(ii) $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Let, $x - 2 = 0$

$\therefore x = 2$

$\because x - 2$ is a factor of $p(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

$\therefore p(2) = 0$

$p(2) = 2^5 - 3(2)^4 - a(2)^3 + 3 \times a \times 2^2 + 2 \times 2 \times a + 4 = 0$

$\Rightarrow 32 - 48 - 8a + 12a + 4a + 4 = 0$

$\Rightarrow 8a - 12 = 0$

$\Rightarrow 8a = 12$

$\Rightarrow a = \dfrac{12}{8} = \dfrac{3}{2}$

Question 17

In each of the following two polynomials, find the value of a, if $(x - a)$ is a factor:

(i) $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$ (ii) $x^5 - a^2x^3 + 2x + a + 1$

Solution 17

(i) $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$

Let $x - a = 0 \Rightarrow x = a$
$\because (x - a)$ is a factor of the polymial $f(x)$
$\therefore f(a) = 0$

$f(a) = 0 \Rightarrow a^6 - a \times a^5 + a^4 - a \times a^3 + 3a - a + 2 = 0$
$\Rightarrow a^6 - a^6 + a^4 - a^4 + 3a - a + 2 = 0$
$\Rightarrow 2a + 2 = 0$
$\Rightarrow 2a = -2$
$\Rightarrow a = -1$

(ii) $x^5 - a^2x^3 + 2x + a + 1$

Let $x - a = 0 \Rightarrow x = a$
$\because (x - a)$ is a factor of $f(x) = x^5 - a^2x^3 + 2x + a + 1$
$\therefore f(a) = 0$

$f(a) \Rightarrow a^5 - a^2 \times a^3 + 2a + a + 1 = 0$
$\Rightarrow a^5 - a^5 + 3a + 1 = 0$
$\Rightarrow 3a = -1$
$\Rightarrow a = -\frac{1}{3}$

Question 18

In each of the following two polynomials, find the value of a if $x + a$ is a factor.

(i) $x^3 + ax^2 - 2x + a + 4$ (ii) $x^4 - a^2x^2 + 3x - a$

Solution 18

(i) $x^3 + ax^2 - 2x + a + 4$

Let $x + a = 0$
$\Rightarrow x = -a$

$\because (x + a)$ is a factor of $f(x) = x^3 + ax^2 - 2x + a + 4$
$\therefore p(-a) = 0$

$p(-a) = (-a)^3 + a(-a)^2 - 2(-a) + a + 4 = 0$
$\Rightarrow -a^3 + a^3 + 2a + a + 4 = 0$
$\Rightarrow 3a + 4 = 0$
$\Rightarrow 3a = -4$
$\Rightarrow a = \dfrac{-4}{3}$

(ii) $x^4 - a^2x^2 + 3x - a$

Let $x + a = 0$
$\Rightarrow x = -a$

$\because (x + a)$ is a factor $f(x) = x^4 - a^2x^2 + 3x - a$
$\therefore f(-a) = 0$

$f(-a) = (-a)^4 - a^2(-a)^2 + 3(-a) - a = 0$
$\Rightarrow a^4 - a^4 - 3a - a = 0$
$\Rightarrow -4a = 0$
$\Rightarrow a = 0$

Question 19

Find the value of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $x^2 - 1$

Solution 19

Let $g(x) = x^2 - 1, p(x) = x^4 + px^3 + 2x^2 - 3x + q$

Let $g(x) = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm\sqrt{1} = \pm 1$

$\because g(x)$ is a factor of $p(x)$

$\therefore p(1) = 0, p(-1) = 0$

$p(1) = 1^4 + p(1)^3 + 2(1)^2 - 3(1) + q = 0$

$\Rightarrow 1 + p + 2 - 3 + q = 0$

$\Rightarrow p + q = 0$

$\Rightarrow p = -q \quad ---(1)$

$p(-1) = (-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q = 0$

$\Rightarrow 1 + p(-1) + 2 + 3 + q = 0$

$\Rightarrow 6 - p + q = 0$

$\Rightarrow p = 6 + q \quad ---(2)$

Equating (1) & (2)

$-q = 6 + q$

$\Rightarrow -q - q = 6$

$\Rightarrow -2q = 6$

$\Rightarrow q = \frac{-6}{2} = -3$

$\therefore p = 6 + (-3) = 6 - 3 = 3 \quad [\text{from } (2)]$

$\therefore p = 3, q = -3$

Question 20

Find the value of a and b if $(x+1)$ and $(x-1)$ are factors of $x^4 + ax^3 - 3x^2 + 2x + b$

Solution 20

Let, $x + 1 = 0$

$x = -1$

$\because (x+1)$ is a factor of $f(x) = x^4 + ax^3 - 3x^2 + 2x + b$

$\therefore f(-1) = 0$

$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + 2(-1) + b = 0$

$\Rightarrow 1 - a - 3 - 2 + b = 0$

$\Rightarrow -4 - a + b = 0$

$\Rightarrow b = 4 + a \qquad ---(1)$

Let, $x - 1 = 0$

$x = 1$

$\because (x-1)$ is a factor of $f(x)$

$\therefore f(1) = 0$

$f(1) = 1^4 + a(1)^3 - 3(1)^2 + 2(1) + b = 0$

$\Rightarrow 1 + a - 3 + 2 + b = 0$

$\Rightarrow a + b = 0$

$\Rightarrow b = -a \qquad ---(2)$

Equating (1) & (2)

$4 + a = -a$

$\Rightarrow 4 = -2a$

$\Rightarrow a = -2$

$\therefore b = -a = -(-2) = 2$ from (2)

$\therefore a = -2, b = 2$

Question 21

If $x^3 + ax^2 - bx + 10$ is divisible by $x^2 - 3x + 2$, find a and b

Solution 21

Let, $p(x) = x^3 + ax^2 - bx + 10$

$g(x) = x^2 - 3x + 2$

Let $g(x) = 0$

$\Rightarrow x^2 - 3x + 2 = 0$
$\Rightarrow x^2 - 2x - x + 2 = 0$
$\Rightarrow x(x - 2) - 1(x - 2) = 0$
$\Rightarrow x - 2 = 0, x - 1 = 0$
$\therefore x = 2, x = 1$

Since $x^2 - 3x + 2$ is a factor of $p(x)$
$\therefore (x - 2)(x - 1)$ is a factor of $p(x)$

Hence, $p(2) = 0, p(1) = 0$
$p(2) = 2^3 + a(2)^2 - b(2) + 10 = 0$
$\Rightarrow 8 + 4a - 2b + 10 = 0$
$\Rightarrow 4a - 2b + 18 = 0$
$\Rightarrow 4a = -18 + 2b$
$\Rightarrow a = \dfrac{-18 + 2b}{4} = \dfrac{2(-9 + b)}{4} = \dfrac{-9 + b}{2} \quad -(1)$

$p(1) = 1^3 + a(1)^2 - b(1) + 10 = 0$
$\Rightarrow a - b + 11 = 0$
$\Rightarrow a = b - 11 \quad -(2)$

Equating (1) & (2)
$\dfrac{-9 + b}{2} = b - 11$
$\Rightarrow -9 + b = 2b - 22$
$\Rightarrow -9 + 22 = 2b - b$
$\Rightarrow 13 = b$

Subsitituting $b = 13$ in equation (2)
$a = b - 11 = 13 - 11 = 2$

$\therefore a = 2, b = 13$

Question 22

If both $x + 1$ and $x - 1$ are factors of $ax^3 + x^2 - 2x + b$, find a and b

Solution 22

Let, $x + 1 = 0$

$x = -1$

$\because (x + 1)$ is a factor of $p(x) = ax^3 + x^2 - 2x + b$

$\therefore p(-1) = 0$

$p(-1) = a(-1)^3 + (-1)^2 - 2(-1) + b = 0$

$\Rightarrow -a + 1 + 2 + b = 0$

$\Rightarrow -a + 3 + b = 0$

$\Rightarrow a = 3 + b \quad --- (1)$

Let, $x - 1 = 0$

$x = 1$

$\because (x - 1)$ is a factor of $p(x)$

$\therefore p(1) = 0$

$p(1) = a(1)^3 + 1^2 - 2(1) + b = 0$

$\Rightarrow a + 1 - 2 + b = 0$

$\Rightarrow a = -b + 1 \quad --- (2)$

Equating (1) & (2)

$\Rightarrow 3 + b = -b + 1$

$\Rightarrow b + b = 1 - 3$

$\Rightarrow 2b = -2$

$\Rightarrow b = -1$

Substituting $b = -1$ in equation (2)

$a = -(-1) + 1 = 1 + 1 = 2$

$\therefore a = 2, b = -1$

Question 23

What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisible by $x^2 + x - 6$

Solution 23

Let $p(x) = x^3 - 3x^2 - 12x + 19$ and $q(x) = x^2 + x - 6$

When $p(x)$ is divided by $q(x)$ then the remainder is a linear equation, $r(x)$.
Let $r(x) = ax + b$ when added to $p(x)$ we get an expression which is divisible by $q(x)$

$$f(x) = p(x) + r(x)$$
$$\begin{aligned} f(x) &= x^3 - 3x^2 - 12x + 19 + ax + b \\ &= x^3 - 3x^2 - 12x + ax + 19 + b \\ &= x^3 - 3x^2 - x(12 - a) + 19 + b \end{aligned}$$

Now, $q(x) = x^2 + x - 6 = x^2 + 3x - 2x - 6 = (x + 3)(x - 2)$

Also, $f(x)$ is divisible by $q(x)$
$\therefore f(-3) = 0, f(2) = 0$

$$f(-3) = (-3)^3 - 3(-3)^2 - 12(-3) + 19 + a(-3) + b = 0$$
$$\Rightarrow -27 - 27 + 36 + 19 - 3a + b = 0$$
$$\Rightarrow b = 3a + 27 + 27 - 36 - 19$$
$$\Rightarrow b = 3a - 1 \quad -(1)$$

$$f(2) = 2^3 - 3(2)^2 - 12(2) + 19 + 2a + b = 0$$
$$\Rightarrow 8 - 12 - 24 + 19 + 2a + b = 0$$
$$\Rightarrow b = 9 - 2a \qquad -(2)$$

Equating (1) & (2)
$$3a - 1 = 9 - 2a$$
$$\Rightarrow 5a = 10$$
$$\Rightarrow a = 2$$
$\therefore b = 3a - 1 = 3(2) - 1 = 5$ *(substituting* $b = 5$ *, in equation 1)*
Hence , $ax + b = 2x + 5$

$\therefore$ The number that should be added to $x^3 - 3x^2 - 12x + 19$ to make it perfectly divisible by $x^2 + x - 6$ is $2x + 5$

Question 24

What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$?

Solution 24

$$
\begin{array}{r|l}
 & x - 7 \\
\hline
x^2 + x - 12 & x^3 - 6x^2 - 15x + 80 \\
 & x^3 + x^2 - 12x \\
 & - \quad - \quad + \\
\hline
 & -7x^2 - 3x + 80 \\
 & -7x^2 - 7x + 84 \\
 & + \quad + \quad - \\
\hline
\end{array}
$$

$\text{Remainder} = 4x - 4$

$\therefore$ $4x - 4$ should be subtracted from $x^3 - 6x^2 - 15x + 80$
so that the result is exactly divisible by $x^2 + x - 12$

Question 25

What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$?

Solution 25

We know that, Dividend = Divisor $\times$ Quotient + Remainder

Dividend = $3x^3 + x^2 - 22x + 9$

Divisor = $3x^2 + 7x - 6$

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
3x^2 + 7x - 6 & 3x^3 + x^2 - 22x + 9 \\
 & 3x^3 + 7x^2 - 6x \\
 & - \quad - \quad + \\
\hline
 & -6x^2 - 16x + 9 \\
 & -6x^2 - 14x + 12 \\
 & + \quad + \quad - \\
\hline
 & -2x - 3 \\
\hline
\end{array}
$$

$\text{Remainder} = -2x - 3$

So, $-(-2x - 3) = 2x + 3$ should be added to $3x^3 + x^2 - 22x + 9$ to make it exactly divisible by $3x^2 + 7x - 6$.

Chapter 6 - Factorisation of Polynomials Exercise Ex. 6.5

Question 1

Using factor theorem, factorize the following polynomial:

$f(x) = x^3 + 6x^2 + 11x + 6$

Solution 1

Let $x = 1$

$f(1) = 1^3 + 6(1)^2 + 11(1) + 6 \neq 0$

Let $x = -1$

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6 = 12 - 12 = 0$

$\therefore x = -1$ is a solution

$\Rightarrow x + 1 = 0$

i.e $(x+1)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
\hline
x+1 & x^3 + 6x^2 + 11x + 6 \\
 & \underline{\overset{-}{x^3} \overset{-}{+} x^2} \\
 & \quad 5x^2 + 11x \\
 & \quad \underline{\overset{-}{5x^2} \overset{-}{+} 5x} \\
 & \qquad 6x + 6 \\
 & \qquad \underline{\overset{-}{6x} \overset{-}{+} 6} \\
 & \qquad\quad 0
\end{array}
$$

By division algorithm

$$
\begin{aligned}
x^3 + 6x^2 + 11x + 6 &= (x+1)(x^2 + 5x + 6) \\
&= (x+1)(x^2 + 2x + 3x + 6) \\
&= (x+1)(x(x+2) + 3(x+2)) \\
&= (x+1)(x+2)(x+3)
\end{aligned}
$$

$\therefore x^3 + 6x^2 + 11x + 6 = (x+1)(x+2)(x+3)$

Question 2

Using factor theorem, factorize the following polynomial:

$f(x) = x^3 + 2x^2 - x - 2$

Solution 2

For $x = 1$

$f(1) = 1^3 + 2(1)^2 - 1 - 2 = 0$

$\because x = 1$ is a solution

$\Rightarrow x - 1 = 0$

i.e., $(x - 1)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x - 1 & x^3 + 2x^2 - x - 2 \\
 & \underline{\overset{-}{x^3} \overset{+}{- x^2}} \\
 & \quad 3x^2 - x \\
 & \quad \underline{\overset{-}{3x^2} \overset{+}{- 3x}} \\
 & \qquad 2x - 2 \\
 & \qquad \underline{\overset{-}{2x} \overset{+}{- 2}} \\
 & \qquad\quad 0
\end{array}
$$

By division algorithm

$$
\begin{aligned}
x^3 + 2x^2 - x - 2 &= (x - 1)(x^2 + 3x + 2) \\
&= (x - 1)(x^2 + 2x + x + 2) \\
&= (x - 1)(x(x + 2) + 1(x + 2)) \\
&= (x - 1)(x + 2)(x + 1)
\end{aligned}
$$

$\therefore x^3 + 2x^2 - x - 2 = (x - 1)(x + 2)(x + 1)$

Question 3

Using factor theorem, factorize the following polynomial:

$f(x) = x^3 - 6x^2 + 3x + 10$

Solution 3

For $x = 2$

$f(2) = 2^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0$

$\because x = 2$ is a solution of $f(x)$

i.e., $x - 2$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 - 4x - 5 \\ \hline x - 2 & x^3 - 6x^2 + 3x + 10 \\ & \underline{\overset{-}{x^3} \overset{+}{-} 2x^2} \\ & \quad - 4x^2 + 3x \\ & \quad \underline{\overset{+}{-} 4x^2 \overset{-}{+} 8x} \\ & \qquad - 5x + 10 \\ & \qquad \underline{\overset{+}{-} 5x \overset{-}{+} 10} \\ & \qquad\quad 0 \end{array}$$

By division algorithm

$$\begin{aligned} x^3 - 6x^2 + 3x + 10 &= (x - 2)(x^2 - 4x - 5) \\ &= (x - 2)(x^2 - 5x + x - 5) \\ &= (x - 2)(x(x - 5) + 1(x - 5)) \\ &= (x - 2)(x - 5)(x + 1) \end{aligned}$$

$\therefore x^3 - 6x^2 + 3x + 10 = (x - 2)(x - 5)(x + 1)$

Question 4

Using factor theorem, factorize: $x^4 - 7x^3 + 9x^2 + 7x - 10$

Solution 4

Let $f(x) = x^4 - 7x^3 + 9x^2 + 7x - 10$

The factors of constant term in f(x) are $\pm 1, \pm 2, \pm 5$ and ± 10

We have,

$f(1) = 1 - 7 + 9 + 7 - 10 = 0$

$\Rightarrow$ (x - 1) is a factor of f(x)

$f(-1) = 1 + 7 + 9 - 7 - 10 = 0$

$\Rightarrow$ (x + 1) is a factor of f(x)

$f(2) = 16 - 56 + 36 + 14 - 10 = 0$

$\Rightarrow$ (x - 2) is a factor of f(x)

$f(-2) = 16 + 56 - 36 - 14 - 10 = 12$

$\Rightarrow$ (x + 2) is not a factor of f(x)

$f(5) = 625 - 875 + 225 + 35 - 10 = 0$

$\Rightarrow$ (x - 5) is a factor of f(x)

Since f(x) is a polynomial of degree 4. So, it cannot have more than 4 linear factors. Thus, factors of f(x) are (x-1), (x+1), (x-2) and (x-5).

Therefore,

$f(x) = k(x-1)(x+1)(x-2)(x-5)$

$x^4 - 7x^3 + 9x^2 + 7x - 10 = k(x-1)(x+1)(x-2)(x-5)$ ....(i)

Putting x = 0 on both sides, we get,

$-10 = k(-1)(1)(-2)(-5)$

$-10 = -10k$

$k=1$

Substituting k = 1 in (i), we get,

$x^4 - 7x^3 + 9x^2 + 7x - 10 = (x-1)(x+1)(x-2)(x-5)$

Question 5

Using factor theorem, factorize: $3x^3 - x^2 - 3x + 1$

Solution 5

Let $f(x) = 3x^3 - x^2 - 3x + 1$

The factor of the coefficient of x^3 is 3. So, the possible rational roots of $f(x)$ are ± 1, and $\pm \frac{1}{3}$

We have,

$f(1) = 3 - 1 - 3 + 1 = 0$

$\Rightarrow$ $(x - 1)$ is a factor of $f(x)$

$f(-1) = -3 - 1 + 3 + 1 = 0$

$\Rightarrow$ $(x + 1)$ is a factor of $f(x)$

So, $(x-1)$ and $(x+1)$ are factors of $f(x)$

$\Rightarrow$ $(x-1)(x+1)$ is also a factor of $f(x)$

$\Rightarrow$ $x^2 - 1$ is a factor of $f(x)$.

Let us now divide $f(x) = 3x^3 - x^2 - 3x + 1$ by $x^2 - 1$ to get the other factors of $f(x)$.
By long division, we have:

$$\begin{array}{r|l} & 3x - 1 \\ \hline x^2 - 1 & 3x^3 - x^2 - 3x + 1 \\ & 3x^3 \qquad - 3x \\ & - \qquad\quad + \\ \hline & \quad -x^2 \qquad + 1 \\ & \quad -x^2 \qquad + 1 \\ & \quad + \qquad\quad - \\ \hline & \qquad 0 \end{array}$$

Therefore,

$3x^3 - x^2 - 3x + 1 = (x^2 - 1)(3x - 1)$

Now,

$(x^2 - 1) = (x-1)(x+1)$

Hence,

$3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$

Question 6

Using factor theorem, factorize each of the following polynomials:

$x^3 - 23x^2 + 142x - 120$

Solution 6

Let $p(x) = x^3 = 23x^2 + 142x - 120$

We shall now look for all the factors of -120. Some of these are 1, ±2, ± 3 ,

±4, ± 5, ± 6, ± 8, ± 12, ± 15, ± 20, ± 24, ± 30, ± 60.

By trial, we find that p(1) = 0. So x - 1 is a factor of p(x).

Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$

$= (x - 1)(x^2 - 22x + 120)$ [Taking (x - 1) common]

We could have also got this by dividing p(x) by x - 1.

Now $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have:

$X^2 - 22x + 120 = x^2 - 12x - 10x + 120$

$= x(x - 12) - 10(x - 12)$

$= (x - 12)(x - 10)$

So, $x^3 - 23x^2 - 142x - 120 = (x - 1)(x - 10)(x - 12)$

Question 7

Using factor theorem, factorize: $y^3 - 7y + 6$

Solution 7

Let $f(y) = y^3 - 7y + 6$

The factors of constant term in f(y) are $\pm 1, \pm 2, \pm 3$ and ± 6

We have,

$f(1) = 1 - 7 + 6 = 0$

$\Rightarrow$ (y - 1) is a factor of f(y)

$f(-1) = -1 + 7 + 6 = 12$

$\Rightarrow$ (y + 1) is not a factor of f(y)

$f(2) = 8 - 14 + 6 = 0$

$\Rightarrow$ (y - 2) is a factor of f(y)

$f(-2) = -8 + 14 + 6 = 12$

$\Rightarrow$ (y + 2) is not a factor of f(y)

$f(3) = 27 - 21 + 6 = 12$

$\Rightarrow$ (y - 3) is not a factor of f(y)

$f(-3) = -27 + 21 + 6 = 0$

$\Rightarrow$ (y + 3) is a factor of f(y)

Since f(y) is a polynomial of degree 3. So, it cannot have more than 3 linear factors. Thus, factors of f(y) are (y-1), (y-2) and (y + 3).

Therefore,

$f(y) = k(y-1)(y-2)(y+3)$

$y^3 - 7y + 6 = k(y-1)(y-2)(y+3)$ ....(i)

Putting y = 0 on both sides, we get,

$6 = k(-1)(-2)(3)$

$6 = 6k$

$k=1$

Substituting k = 1 in (i), we get,

$y^3 - 7y + 6 = (y-1)(y-2)(y+3)$

Question 8

Using factor theorem, factorize: $x^3 - 10x^2 - 53x - 42$

Solution 8

Let $f(x) = x^3 - 10x^2 - 53x - 42$

The factors of constant term in $f(x)$ are $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21$ and ± 42

We have,

$f(1) = 1 - 10 - 53 - 42 = -104$

$\Rightarrow$ $(x - 1)$ is not a factor of $f(x)$

$f(-1) = -1 - 10 + 53 - 42 = 0$

$\Rightarrow$ $(x + 1)$ is a factor of $f(x)$

$f(2) = 8 - 40 - 106 - 42 = -180$

$\Rightarrow$ $(x - 2)$ is not a factor of $f(x)$

$f(-2) = -8 - 40 + 106 - 42 = 16$

$\Rightarrow$ $(x + 2)$ is not a factor of $f(x)$

$f(3) = 27 - 90 - 159 - 42 = -264$

$\Rightarrow$ $(x - 3)$ is not a factor of $f(x)$

$f(-3) = -27 - 90 + 159 - 42 = 0$

$\Rightarrow$ $(x + 3)$ is a factor of $f(x)$

Similarly, it can be verified that $(x - 14)$ is a factor of $f(x)$.

$f(14) = 2744 - 1960 - 742 - 42 = 0$

$\Rightarrow$ $(x - 14)$ is a factor of $f(x)$

Since $f(x)$ is a polynomial of degree 3. So, it cannot have more than 3 linear factors. Thus, factors of $f(x)$ are $(x+1)$, $(x+3)$ and $(x-14)$.

Therefore,

$f(x) = k(x+1)(x+3)(x-14)$

$x^3 - 10x^2 - 53x - 42 = k(x+1)(x+3)(x-14)$.......(i)

Putting $x = 0$ on both sides

$-42 = k(1)(3)(-14)$

$-42 = -42k$

$k=1$

Substituting $k = 1$ in (i), we get

$x^3 - 10x^2 - 53x - 42 = (x+1)(x+3)(x-14)$

Question 9

Using factor theorem, factorize: $y^3 - 2y^2 - 29y - 42$

Solution 9

Let $f(y) = y^3 - 2y^2 - 29y - 42$

The factors of constant term in f(y) are $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21$ and ± 42

We have,

$f(1) = 1 - 2 - 29 - 42 = -72$

$\Rightarrow$ (y - 1) is not a factor of f(y)

$f(-1) = -1 - 2 + 29 - 42 = -16$

$\Rightarrow$ (y + 1) is not a factor of f(y)

$f(2) = 8 - 8 - 58 - 42 = -100$

$\Rightarrow$ (y - 2) is not a factor of f(y)

$f(-2) = -8 - 8 + 58 - 42 = 0$

$\Rightarrow$ (y + 2) is a factor of f(y)

$f(3) = 27 - 18 - 87 - 42 = -120$

$\Rightarrow$ (y - 3) is not a factor of f(y)

$f(-3) = -27 - 18 + 87 - 42 = 0$

$\Rightarrow$ (y + 3) is a factor of f(y)

Similarly, it can be verified that (y - 7) is a factor of f(y).

$f(7) = 343 - 98 - 203 - 42 = 0$

$\Rightarrow$ (y - 7) is a factor of f(y)

Since f(y) is a polynomial of degree 3. So, it cannot have more than 3 linear factors. Thus, factors of f(y) are (y+2), (y+3) and (y-7).

Therefore,

$f(y) = k(y+2)(y+3)(y-7)$

$y^3 - 2y^2 - 29y - 42 = k(y+2)(y+3)(y-7)$.......(i)

Putting y = 0 on both sides

$-42 = k(2)(3)(-7)$

$-42 = -42k$

$k=1$

Substituting k = 1 in (i), we get,

$y^3 - 2y^2 - 29y - 42 = (y+2)(y+3)(y-7)$

Question 10

Using factor theorem, factorize: $2y^3 - 5y^2 - 19y + 42$

Solution 10

Let $f(y) = 2y^3 - 5y^2 - 19y + 42$

The factors of constant term in $f(y)$ are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21$ and ± 42.

The coefficient of y^3 is 2. Hence, possible rational roots of $f(y)$ are:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}$

We have,

$f(1) = 2 - 5 - 19 + 42 = 20$

$\Rightarrow$ $(y - 1)$ is not a factor of $f(y)$

$f(-1) = -2 - 5 + 19 + 42 = 54$

$\Rightarrow$ $(y + 1)$ is not a factor of $f(y)$

$f(2) = 16 - 20 - 38 + 42 = 0$

$\Rightarrow$ $(y - 2)$ is a factor of $f(y)$

$f(-2) = -16 - 20 + 38 + 42 = 44$

$\Rightarrow$ $(y + 2)$ is not a factor of $f(y)$

$f(3) = 54 - 45 - 57 + 42 = -6$

$\Rightarrow$ $(y - 3)$ is not a factor of $f(y)$

$f(-3) = -54 - 45 + 57 + 42 = 0$

$\Rightarrow$ $(y + 3)$ is a factor of $f(y)$

So, $(y-2)$ and $(y+3)$ are factors of $f(y)$

$\Rightarrow$ $(y-2)(y+3)$ is also a factor of $f(y)$

$\Rightarrow y^2 + y - 6$ is a factor of $f(y)$.

Let us now divide $f(y) = 2y^3 - 5y^2 - 19y + 42$ by $y^2 + y - 6$ to get the other factors of $f(y)$. By long division, we have:

$$
\begin{array}{r|l}
 & 2y - 7 \\
\hline
y^2 + y - 6 & 2y^3 - 5y^2 - 19y + 42 \\
 & 2y^3 + 2y^2 - 12y \\
 & - \quad - \quad + \\
\hline
 & -7y^2 - 7y + 42 \\
 & -7y^2 - 7y + 42 \\
 & + \quad + \quad - \\
\hline
 & 0
\end{array}
$$

Therefore,

$2y^3 - 5y^2 - 19y + 42 = (y^2 + y - 6)(2y-7)$

$\Rightarrow 2y^3 - 5y^2 - 19y + 42 = (y-2)(y+3)(2y-7)$

Question 11

$x^3 + 13x^2 + 32x + 20$

Solution 11

Let $p(x) = x^3 + 13x^2 + 32x + 20$
The factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5$ ... ...
By hit and trial method
$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$
$= -1 + 13 - 32 + 20$
$= 33 - 33 = 0$
As p(-1) is zero, so x + 1 is a factor of this polynomial p(x).

Let us find the quotient while dividing $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$
By long division

$$\begin{array}{r} x^2+12x+20 \\ x+1\overline{)\,x^3+13x^2+32x+20} \\ x^3+\ \ x^2 \qquad\qquad \\ -\quad\ - \qquad\qquad \\ \hline 12x^2+32x \qquad \\ 12x^2+12x \qquad \\ -\quad\ - \qquad \\ \hline 20x+20 \\ 20x+20 \\ -\quad - \\ \hline 0 \\ \hline \end{array}$$

We know that

Dividend = Divisor $\times$ Quotient + Remainder
$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$
$= (x + 1)(x^2 + 10x + 2x + 20)$
$= (x + 1)[x(x + 10) + 2(x + 10)]$
$= (x + 1)(x + 10)(x + 2)$
$= (x + 1)(x + 2)(x + 10)$

Question 12

Factorise:

$x^3 - 3x^2 - 9x - 5$

Solution 12

Let $p(x) = x^3 - 3x^2 - 9x - 5$
Factors of 5 are $\pm 1, \pm 5$.
By hit and trial method
$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$
$= -1 - 3 + 9 - 5 = 0$
So x + 1 is a factor of this polynomial
Let us find the quotient while dividing $x^3 + 3x^2 - 9x - 5$ by x + 1
By long division

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
\hline
x+1 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 + \ \ x^2 \\
 & - \quad\ - \\
\hline
 & \quad -4x^2 - 9x - 5 \\
 & \quad -4x^2 - 4x \\
 & \quad + \quad\ + \\
\hline
 & \qquad\quad -5x - 5 \\
 & \qquad\quad -5x - 5 \\
 & \qquad\quad + \quad + \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}
$$

Now, Dividend = Divisor $\times$ Quotient + Remainder

$\therefore$ $x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) + 0$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)[(x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

$= (x - 5)(x + 1)(x + 1)$

Question 13

Factorise:

$2y^3 + y^2 - 2y - 1$

Solution 13

Let $p(y) = 2y^3 + y^2 - 2y - 1$

By hit and trial method

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

So, $y - 1$ is a factor of this polynomial

By long division method,

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad\ + \\
\hline
 & \qquad 3y^2 - 2y - 1 \\
 & \qquad 3y^2 - 3y \\
 & \qquad - \quad\ + \\
\hline
 & \qquad\qquad\quad y - 1 \\
 & \qquad\qquad\quad y - 1 \\
\hline
 & \qquad\qquad\quad\ \ 0 \\
\hline
\end{array}
$$

$p(y) = 2y^3 + y^2 - 2y - 1$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1) (2y^2 + 2y + y + 1)$
$= (y - 1) [2y (y + 1) + 1 (y + 1)]$
$= (y - 1) (y + 1) (2y + 1)$

Question 14
Using factor theorem, factorize: $x^3 - 2x^2 - x + 2$

Solution 14
Let $f(x) = x^3 - 2x^2 - x + 2$

The factors of constant term in f(x) are $\pm 1, \pm 2$

We have,

$f(1) = 1 - 2 - 1 + 2 = 0$

$\Rightarrow$ (x - 1) is a factor of f(x)

$f(-1) = -1 - 2 + 1 + 2 = 0$

$\Rightarrow$ (x + 1) is a factor of f(x)

$f(2) = 8 - 8 - 2 + 2 = 0$

$\Rightarrow$ (x - 2) is a factor of f(x)

Since f(x) is a polynomial of degree 3. So, it cannot have more than 3 linear factors. Thus, factors of f(x) are (x-1), (x+1) and (x-2).

Therefore,

$f(x) = k(x-1)(x+1)(x-2)$

$x^3 - 2x^2 - x + 2 = k(x-1)(x+1)(x-2)$.......(i)

Putting x = 0 on both sides, we get,

$2 = k(-1)(1)(-2)$

$2 = 2k$

$k=1$

Substituting k = 1 in (i), we get,

$x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2)$

Question 15
Factorize the following polynomial:

(i) $x^3 + 13x^2 + 31x - 45$ given that $x + 9$ is a factor.
(ii) $4x^3 + 20x^2 + 33x + 18$ given that $2x + 3$ is a factor.

Solution 15

(i)

Let us divide $x^3 + 13x^2 + 31x - 45$ by $x + 9$ to get other factors.

$$
\begin{array}{r}
x^2 + 4x - 5 \\
x + 9 \overline{\smash{\big)}\, x^3 + 13x^2 + 31x - 45} \\
x^3 + 9x^2 \qquad\qquad\quad \\
- \quad - \qquad\qquad\quad \\
\hline
4x^2 + 31x \qquad \\
4x^2 + 36x \qquad \\
- \quad - \qquad \\
\hline
-5x - 45 \\
-5x - 45 \\
+ \quad + \\
\hline
0
\end{array}
$$

$f(x) = x^3 + 13x^2 + 31x - 45$

$\Rightarrow f(x) = (x^2+4x-5)\ (x+9)$

$\Rightarrow f(x) = (x^2+5x-x-5)(x+9)$

$\Rightarrow f(x) = \{x(x+5)-1(x+5)\}(x+9)$

$\Rightarrow f(x) = (x-1)(x+5)(x+9)$

(ii)

Let us divide $4x^3 + 20x^2 + 33x + 18$ by $2x + 3$ to get other factors.

$$
\begin{array}{r}
2x^2 + 7x + 6 \\
2x + 3 \overline{\smash{\big)}\, 4x^3 + 20x^2 + 33x + 18} \\
4x^3 + 6x^2 \qquad\qquad\quad \\
- \quad - \qquad\qquad\quad \\
\hline
14x^2 + 33x \qquad \\
14x^2 + 21x \qquad \\
- \quad - \qquad \\
\hline
12x + 18 \\
12x + 18 \\
- \quad - \\
\hline
0
\end{array}
$$

$f(x) = 4x^3 + 20x^2 + 33x + 18$

$\Rightarrow f(x) = (2x^2+7x+6)\ (2x+3)$

$\Rightarrow f(x) = (2x^2+4x+3x+6)(2x+3)$

$\Rightarrow f(x) = \{2x(x+2)+3(x+2)\}(2x+3)$

$\Rightarrow f(x) = (x+2)(2x+3)(2x+3)$

$\Rightarrow f(x) = (x+2)(2x+3)^2$

Question 16
Using factor theorem, factroize : $x^4 - 2x^3 - 7x^2 + 8x + 12$

Solution 16
Let $f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$

The factors of constant term in f(x) are $\pm1, \pm2, \pm3, \pm4, \pm6$ and ± 12

We have,

$f(1) = 1 - 2 - 7 + 8 + 12 = 12$

$\Rightarrow$ (x - 1) is not a factor of f(x)

$f(-1) = 1 + 2 - 7 - 8 + 12 = 0$

$\Rightarrow$ (x + 1) is a factor of f(x)

$f(2) = 16 - 16 - 28 + 16 + 12 = 0$

$\Rightarrow$ (x - 2) is a factor of f(x)

$f(-2) = 16 + 16 - 28 -16 + 12 = 0$

$\Rightarrow$ (x + 2) is a factor of f(x)

$f(3) = 81 - 54 - 63 + 24 + 12 = 0$

$\Rightarrow$ (x - 3) is a factor of f(x)

Since f(x) is a polynomial of degree 4. So, it cannot have more than 4 linear factors. Thus, factors of f(x) are (x+1), (x-2), (x+2) and (x-3).

Therefore,

$f(x) = k(x+1)(x+2)(x-2)(x-3)$

$x^4 - 2x^3 - 7x^2 + 8x + 12= k(x+1)(x+2)(x-2)(x-3)$ ....(i)

Putting x = 0 on both sides, we get,

$12 = k(1)(2)(-2)(-3)$

$12 = 12k$

$k=1$

Substituting k = 1 in (i), we get,

$x^4 - 2x^3 - 7x^2 + 8x + 12 = (x+1)(x+2)(x-2)(x-3)$

Question 17
Using factor theorem, factroize : $x^4 + 10x^3 + 35x^2 + 50x + 24$

Solution 17

Let $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$

The factors of constant term in $f(x)$ are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12$ and ± 24

We have,

$f(1) = 1 + 10 + 35 + 50 + 24 = 120$

$\Rightarrow$ $(x - 1)$ is not a factor of $f(x)$

$f(-1) = 1 - 10 + 35 - 50 + 24 = 0$

$\Rightarrow$ $(x + 1)$ is a factor of $f(x)$

$f(2) = 16 + 80 + 140 + 100 + 24 = 360$

$\Rightarrow$ $(x - 2)$ is not a factor of $f(x)$

$f(-2) = 16 - 80 + 140 - 100 + 24 = 0$

$\Rightarrow$ $(x + 2)$ is a factor of $f(x)$

$f(3) = 81 + 270 + 315 + 150 + 24 = 840$

$\Rightarrow$ $(x - 3)$ is not a factor of $f(x)$

$f(-3) = 81 - 270 + 315 - 150 + 24 = 0$

$\Rightarrow$ $(x + 3)$ is a factor of $f(x)$

$f(4) = 256 + 640 + 560 + 200 + 24 = 1680$

$\Rightarrow$ $(x - 4)$ is not a factor of $f(x)$

$f(-4) = 256 - 640 + 560 - 200 + 24 = 0$

$\Rightarrow$ $(x + 4)$ is a factor of $f(x)$

Since $f(x)$ is a polynomial of degree 4. So, it cannot have more than 4 linear factors. Thus, factors of $f(x)$ are $(x+1)$, $(x+2)$, $(x+3)$ and $(x+4)$.

Therefore,

$f(x) = k(x+1)(x+2)(x+3)(x+4)$

$x^4 + 10x^3 + 35x^2 + 50x + 24 = k(x+1)(x+2)(x+3)(x+4)$ ....(i)

Putting $x = 0$ on both sides, we get,

$24 = k(1)(2)(3)(4)$

$24 = 24k$

$k=1$

Substituting $k = 1$ in (i), we get,

$x^4 + 10x^3 + 35x^2 + 50x + 24 = (x+1)(x+2)(x+3)(x+4)$

Question 18

Using factor theorem, factorize : $2x^4 - 7x^3 - 13x^2 + 63x - 45$

Solution 18

Let $f(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

The factors of constant term in $f(x)$ are $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15$ and ± 45. The factor of coefficient of x^4 is 2. Hence, possible rational roots of $f(x)$ are:

$$\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2} \text{ and } \pm \frac{45}{2}$$

We have,

$f(1) = 2 - 7 - 13 + 63 - 45 = 0$

$\Rightarrow$ $(x - 1)$ is a factor of $f(x)$

$f(-1) = 2 + 7 - 13 - 63 - 45 = -112$

$\Rightarrow$ $(x + 1)$ is not a factor of $f(x)$

$f(3) = 162 - 189 - 117 + 189 - 45 = 0$

$\Rightarrow$ $(x - 3)$ is a factor of $f(x)$

So, $(x-1)$ and $(x-3)$ are factors of $f(x)$

$\Rightarrow$ $(x-1)(x-3)$ is also a factor of $f(x)$

$\Rightarrow x^2 - 4x + 3$ is a factor of $f(x)$.

Let us now divide $f(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$ by $x^2 - 4x + 3$ to get the other factors of $f(x)$. By long division, we have:

$$\begin{array}{r|l}
 & 2x^2 + x - 15 \\
x^2 - 4x + 3 & 2x^4 - 7x^3 - 13x^2 + 63x - 45 \\
 & 2x^4 - 8x^3 + 6x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3 - 19x^2 + 63x \\
 & x^3 - 4x^2 + 3x \\
 & - \quad + \quad - \\
\hline
 & -15x^2 + 60x - 45 \\
 & -15x^2 + 60x - 45 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}$$

Therefore,

$2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x^2 - 4x + 3)(2x^2 + x - 15)$

$\Rightarrow 2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x-1)(x-3)(2x^2 + x - 15)$

Now,

$(2x^2 + x - 15) = 2x(x+3) - 5(x+3) = (x+3)(2x-5)$

Hence,

$2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x-1)(x-3)(x+3)(2x-5)$