NCERT Solutions for Class 10 Maths Chapter 12 - Areas Related to Circles

Chapter 12 - Areas Related to Circles Exercise Ex. 12.1

Solution 1

Radius (r_1) of 1st circle = 19 cm
Radius (r_2) of 2nd circle = 9 cm
Let the radius of 3rd circle be r
Circumference of 1st circle = $2\pi r_1 = 2\pi(19) = 38\pi$ cm
Circumference of 2nd circle = $2\pi r_2 = 2\pi(9) = 18\pi$ cm
Circumference of 3rd circle = $2\pi r$
Given that
Circumference of 3rd circle = circumference of 1st circle + circumference of 2nd circle
$2\pi r = 38\pi + 18\pi = 56\pi$ cm

$$r = \frac{56\pi}{2\pi} = 28$$

So, radius of circle which has circumference equal to the sum of the circumference of given two circles is 28 cm.

Solution 2

Radius (r_1) of 1st circle = 8 cm
Radius (r_2) or 2nd circle = 6 cm
Let radius of 3rd circle be r

Area of 1st circle $= \pi r_1^2 = \pi(8)^2 = 64\pi$ cm²

Area of 2nd circle $= \pi r_2^2 = \pi(6)^2 = 36\pi$ cm²

Given that
Area of 3rd circle = area of 1st circle + area of 2nd circle

$$\pi r^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi r^2 = 64\pi + 36\pi$$

$$\pi r^2 = 100\pi$$

$$r^2 = 100 \text{ cm}^2$$

$$r = \pm 10$$

But radius cannot be negative, so radius of circle having area equal to the sum of the areas of the two circles is 10 cm.

Solution 3

Radius (r_1) of gold region (i.e. 1^{st} circle)

$$= \frac{21}{2} = 10.5 \text{ cm}$$

Given that each circle is 10.5 cm wider than previous circle.

So, radius (r_2) of 2^{nd} circle = 10.5 + 10.5

= 21 cm

Radius (r_3) of 3^{rd} circle = 21 + 10.5

= 31.5 cm

Radius (r_4) of 4^{th} circle = 31.5 + 10.5

= 42 cm

Radius (r_5) of 5^{th} circle = 42 + 10.5

= 52.5 cm

Area of golden region = area of 1^{st} circle $= \pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Area of Red region = area of 2^{nd} circle − area of 1^{st} circle

$= \pi r_2^2 - \pi r_1^2$

$= \pi(21)^2 - \pi(10.5)^2$

$= 441\pi - 110.25\pi = 330.75\pi$

$= 1039.5 \text{ cm}^2$

Area of blue region = area of 3^{rd} circle − area of 2^{nd} circle

$= \pi r_3^2 - \pi r_1^2$

$= \pi(31.5)^2 - \pi(21)^2$

$= 992.25\pi - 441\pi = 551.25\pi$

$= 1732.5 \text{ cm}^2$

Area of black region = area of 4^{th} circle − area of 3^{rd} circle

$= \pi r_4^2 - \pi r_3^2$

$= \pi(42)^2 - \pi(31.5)^2$

$= 1764\pi - 992.25\pi$

$= 771.75\pi = 2425.5 \text{ cm}^2$

Area of white region = area of 5^{th} circle − area of 4^{th} circle

$= \pi r_5^2 - \pi r_4^2$

$= \pi[(52.5)^2 - \pi(42)^2$

$= 2756.25\pi - 1764\pi$

$= 992.25\pi = 3118.5 \text{ cm}^2$

So areas of gold, red, blue, black, white regions are 346.5 cm^2, 1039.5 cm^2, 1732.5 cm^2, 2425.5 cm^2 and 3118.5 cm^2 respectively.

Solution 4

Diameter of wheel of car = 80 cm
Radius (r) of wheel of car = 40 cm
Circumference of wheel = $2\pi r$

$= 2\pi(40) = 80\pi$ cm

Speed of car = 66 km/hour

$$= \frac{66 \times 100000}{60} \text{ cm/min}$$

$= 110000$ cm/min

Distance travelled by car in 10 minutes
$= 110000 \times 10 = 1100000$ cm
Let the number of revolutions each wheel of car make is n.
$n \times$ distance travelled in 1 revolution (i.e. circumference) = distance traveled in 10 minutes.

$n \times 80\pi = 1100000$

$$n = \frac{1100000 \times 7}{80 \times 22}$$

$$= \frac{35000}{8} = 4375$$

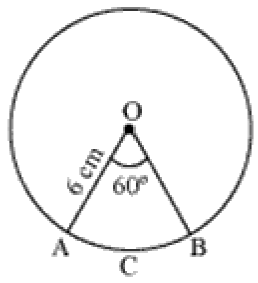
So each wheel of car will make 4375 revolutions.

Solution 5

Let the radius of the circle be r
Circumference of circle = $2\pi r$
Area of circle = πr^2
Given that circumference and area of the circle are equal.
So, $2\pi r = \pi r^2$
$2 = r$
Hence, the radius of the circle will be 2 units

Chapter 12 - Areas Related to Circles Exercise Ex. 12.2

Solution 1



Let OACB be a sector of circle making 60° angle at centre O of circle.

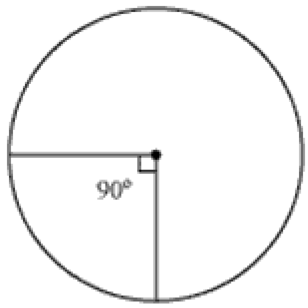
Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

So area of sector OACB $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$$

So area of sector of circle making 60 ° at centre of circle is $\frac{132}{7}$ cm²

Solution 2



Let radius of circle be r.
Circumference = 22 cm
$2\pi r = 22$

$$r = \frac{22}{2\pi}$$

$$= \frac{11}{\pi}$$

Quadrant of circle will subtend 90 ° angle at centre of circle.

So area of such quadrant of circle $= \frac{90^\circ}{360^\circ} \times \pi \times r^2$

$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

Solution 3

We know that in 1 hour (i.e. 60 minutes) minute hand rotates 360°.

So in 5 minutes, minute hand will rotate $= \frac{360°}{60} \times 5 = 30°$

So area swept by minute hand in 5 minutes will be the area of a sector of 30 ° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of sector of 30° $= \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14$

$$= \frac{22}{12} \times 2 \times 14$$

$$= \frac{11 \times 14}{3}$$

$$= \frac{154}{3} \text{ cm}^2$$

So area swept by minute hand in 5 minutes is $\frac{154}{3}$ cm².

Solution 4

Let AB be the chord of circle subtending 90 ° angle at centre O of circle.

(i) Area of minor sector OACB $= \frac{90°}{360°} \times \pi r^2$

$$= \frac{1}{4} \times 3.14 \times 10 \times 10$$

$$= \frac{314}{4} = 78.5 \text{ cm}^2$$

Area of $\Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10$

$$= 50 \text{ cm}^2$$

Area of minor segment ACB = Area of minor sector OACB – Area of ΔOAB

$$= 78.5 - 50 = 28.5 \text{ cm}^2$$

(ii) Area of major sector OADB $= \left(\frac{360° - 90°}{360°}\right) \times \pi r^2 = \left(\frac{270°}{360°}\right)\pi r^2$

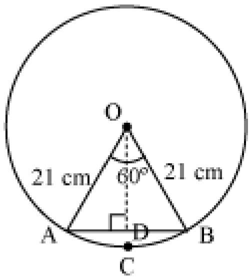
$$= \frac{3}{4} \times 3.14 \times 10 \times 10$$

$$= \frac{942}{4} \text{ cm}^2 = 235.5 \text{ cm}^2$$

Solution 5

Radius (r) of circle = 21 cm
Angle subtended by given arc = 60°

Length of an arc of a sector of angle $\theta = \frac{\theta}{360°} \times 2\pi r$



(i) Length of arc ACB $= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3$

$= 22$ cm

(ii) Area of sector OACB $= \frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

$= 231 \text{ cm}^2$

(iii) Now in ΔOAB

$\angle OAB = \angle OBA$ (as OA = OB)

$\angle OAB + \angle AOB + \angle OBA = 180°$

$2\angle OAB + 60° = 180°$

$\angle OAB = 60°$

So, ΔOAB is an equilateral triangle.

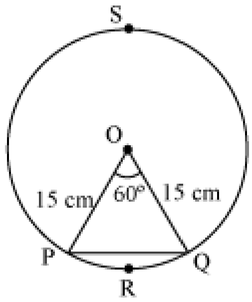
Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$

Area of segment ACB = Area of sector OACB − Area of ΔOAB

$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{ cm}^2$

Solution 6



Radius (r) of the circle = 15

Area of sector OPRQ = $\frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times 3.14 \times 15^2$

$= \frac{706.5}{6}$

= 117.75 cm^2

In ΔOPQ

$\angle OPQ = \angle OQP$ .... (Since OP = OQ)

$\angle OPQ + \angle OQP + \angle POQ = 180°$

$\therefore 2\angle OPQ = 120°$

$\therefore \angle OPQ = 60°$

ΔOPQ is an equilateral triangle.

Area of ΔOPQ =

$\frac{\sqrt{3}}{4} \times side^2 = \frac{\sqrt{3}}{4} \times 15^2 = \frac{225 \times 1.73}{4} = 97.3125\ cm^2$

Area of segment PRQ = Area of sector OPRQ –Area of ΔOPQ

= 117.75 – 97.3125

$= 20.4375$ cm^2

Area of major segment PSQ

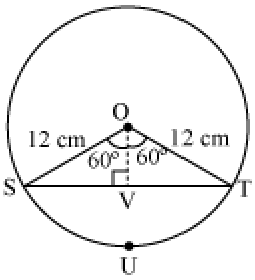
= Area of circle – Area of segment PRQ

$= 15^2\pi - 20.4375$

$= 3.14 \times 225 - 20.4375$

$= 686.0625$ cm^2

Solution 7



Draw a perpendicular OV on chord ST. It will bisect the chord ST.

SV = VT

In ΔOVS

$$\frac{OV}{OS} = \cos 60°$$

$$\frac{OV}{OS} = \frac{1}{2}$$

OV = 6

$$\frac{SV}{SO} = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$SV = 6\sqrt{3}$

ST = 2SV = $2 \times 6\sqrt{3} = 12\sqrt{3}$

Area of ΔOST = $\frac{1}{2} \times ST \times OV$

= $\frac{1}{2} \times 12\sqrt{3} \times 6$

= $36\sqrt{3}$

= 36 × 1.73

= 62.28

Area of sector OSUT = $\frac{120°}{360°} \times \pi \times 12^2$

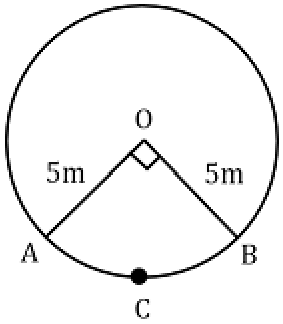
= 150.72

Area of segment SUT = Area of sector OSUT

= 150.72 – 62.28

= 88.44 cm^2

Solution 8



The horse can graze a sector of 90° in a circle of 5 m radius.

i. So area that can be grazed by horse = area of sector OACB

$= \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 5^2$

$= 19.63\ m^2$

ii. Area that can be grazed by the horse when the length of rope is 10 m long = $\frac{90°}{360°} \times \pi \times 10^2$

$= \frac{1}{4} \times 3.14 \times 100$

$= 78.5$

Change in grazing area = 78.5 – 19.63 = 58.87 cm^2

Solution 9

(i) Total length of wire required will be length of 5 diameters and circumference of brooch.

Radius of circle $= \frac{35}{2}$ mm

Circumference of brooch $= 2\pi r$

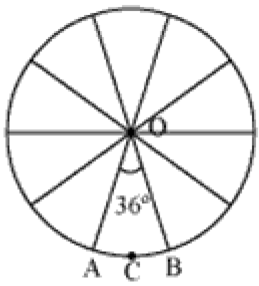
$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$$

$$= 110 \text{ mm}$$

Length of wire required $= 110 + 5 \times 35$

$$= 110 + 175 = 285 \text{ mm}$$

(ii) Each of 10 sectors of circle is subtending 36° at centre of circle.



So area of each sector $= \frac{36°}{360°} \times \pi r^2$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

$$= \frac{385}{4} \text{ mm}^2$$

Solution 10

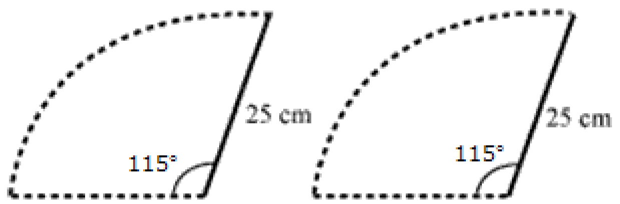
There are 8 ribs in umbrella. The area between two consecutive ribs is subtending an angle of $\frac{360°}{8} = 45°$ at centre of assumed flat circle.

So area between two consecutive ribs of circle $= \frac{45°}{360°} \times \pi r^2$

$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2$$

Solution 11



The figure shows that each blade of the wiper will sweep an area of a sector of $115°$ in a circle of 25 cm radius.

$$\text{Area of such sector} = \frac{115°}{360°} \times \pi \times (25)^2$$
$$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$$
$$= \frac{158125}{252} \text{ cm}^2$$

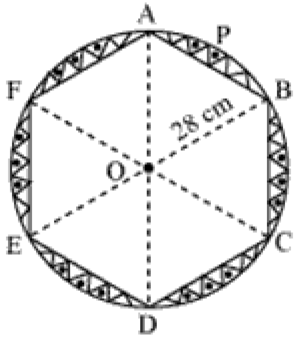
$$\text{Area swept by 2 blades} = 2 \times \frac{158125}{252}$$
$$= \frac{158125}{126} \text{ cm}^2$$

Solution 12

Lighthouse spreads light like a sector of angle 80° in a circle of 16.5 km radius

$$\text{Area of sector OACB} = \frac{80°}{360°} \times \pi r^2$$
$$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$$
$$= 189.97 \text{ km}^2$$

Solution 13



Designs are segments of circle.

Consider segment APB. Chord AB is a side of hexagon. Each chord will substitute $\frac{360^\circ}{6} = 60^\circ$ at centre of circle.

In ΔOAB

$\angle OAB = \angle OBA$ (as OA = OB)

$\angle AOB = 60^\circ$

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$

$2\angle OAB = 180^\circ - 60^\circ = 120^\circ$

$\angle OAB = 60^\circ$

So ΔOAB is an equilateral triangle

Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3}$ cm^2 = 333.2 cm^2.

Area of sector OAPB $= \frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$

$= \frac{1232}{3} = 410.6667 \text{cm}^2$

Area of segment APB = Area of sector OAPB − Area of ΔOAB

= 410.6667 − 333.2

= 77.4667 cm^2

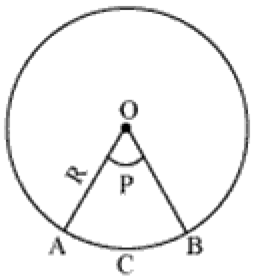
So, area of designs = 6 × 77.46 = 464.8 cm^2

Cost occurred in making 1 cm^2 designs = Rs.0.35

Cost occurred in making 464.8 cm^2 designs = 464.8×0.35 = 162.68

So, cost of making such designs is Rs.162.68.

Solution 14



We know that area of sector of angle $\theta = \frac{\theta}{360^\circ} \pi R^2$

Area of sector of angle P $= \frac{p}{360^\circ}(\pi R^2)$

$$= \left(\frac{p}{720^\circ}\right)(2\pi R^2)$$

Hence (d)

Chapter 12 - Areas Related to Circles Exercise Ex. 12.3

Solution 1

RQ is the diameter of circle, so $\angle RPQ$ will be 90°.
Now in ΔPQR by applying Pythagoras theorem
$RP^2 + PQ^2 = RQ^2$
$(7)^2 + (24)^2 = RQ^2$
$RQ = \sqrt{625} = 25$

Radius of circle $OR = \frac{RQ}{2} = \frac{25}{2}$

Since RQ is diameter of circle it divides circle in two equal parts.

Area of semicircle RPQOR $= \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2$$

$$= \frac{625}{8}\pi \text{ cm}^2$$

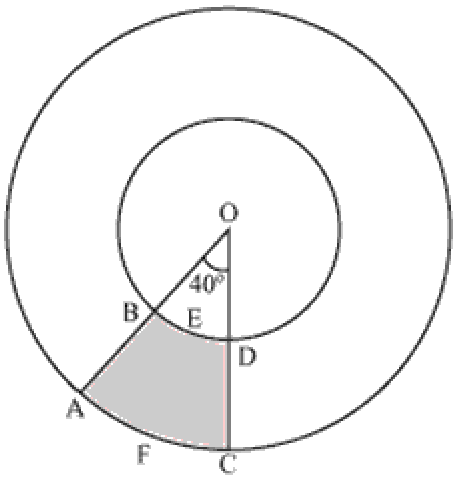
$$= \frac{6875}{28} \text{ cm}^2$$

$$\text{Area of } \Delta PQR = \frac{1}{2} \times PQ \times PR$$
$$= \frac{1}{2} \times 24 \times 7$$
$$= 84 \text{ cm}^2$$

Area of shaded region = area of semicircle RPQOR − area of ΔPQR

$$= \frac{6875}{28} - 84$$
$$= \frac{4523}{28} \text{ cm}^2$$

Solution 2

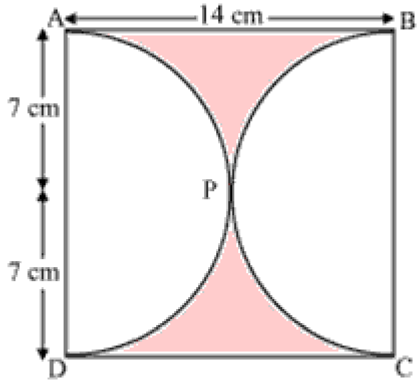


Radius of inner circle = 7 cm
Radius of outer circle = 14 cm
Area of shaded region
= area of sector OAFC − area of sector OBED

$$= \frac{40°}{360°} \times \pi(14)^2 - \frac{40°}{360°} \times \pi(7)^2$$
$$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9}$$
$$= \frac{154}{3} = 51.33 \text{ cm}^2$$

Solution 3

From the figure we see that radius of each semicircle is 7 cm



Area of each semicircle = $\frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$

$= 77\ cm^2$

Area of square ABCD = $(side)^2 = (14)^2$

$= 196\ cm^2$

Area of shaded region = area of square ABCD − area of semicircle APD − area of semicircle BPC

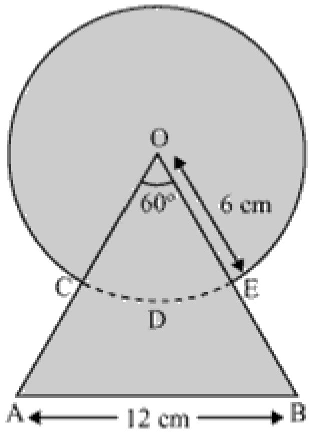
$= 196 - 77 - 77$

$= 196 - 154$

$= 42\ cm^2$

Solution 4

We know that each interior angle of an equilateral triangle is of 60°



So, area of sector OCDE $= \frac{60°}{360°}\pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}\ \text{cm}^2$

Area of $\Delta OAB = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3} \times 12 \times 12}{4}$

$= 36\sqrt{3}\ \text{cm}^2$

Area of circle $= \pi r^2$

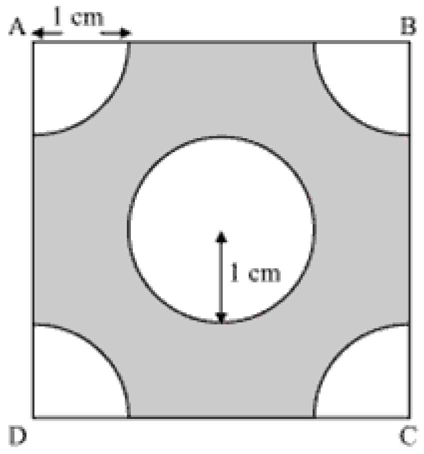
$= \frac{22}{7} \times 6 \times 6 = \frac{792}{7}\ \text{cm}^2$

Area of shaded region = area of ΔOAB + area of circle − area of sector OCDE

$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$

$= \left(36\sqrt{3} + \frac{660}{7}\right)\ \text{cm}^2$

Solution 5



Each quadrant is a sector of 90º in a circle of 1 cm radius.

So area of each quadrant $= \frac{90^\circ}{360^\circ}\pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (1)^2 = \frac{22}{28}\,\text{cm}^2$$

Area of square = (side)² = (4)² = 16 cm²

Area of circle = $\pi r^2 = \pi\,(1)^2$

$= \frac{22}{7}$ cm²

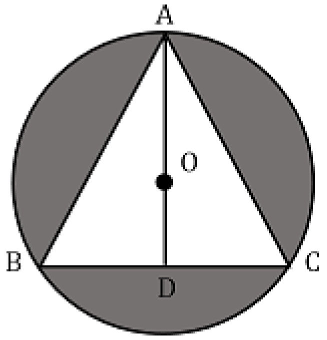
Area of shaded region = area of square − area of circle − 4 x area of quadrant

$= 16 - \frac{22}{7} - 4 \times \frac{22}{28}$

$= 16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7}$

$= \frac{112 - 44}{7} = \frac{68}{7}$ cm²

Solution 6



Radius (r) of circle = 32 cm

AD is the median of ΔABC

$$AO = \frac{2}{3}AD = 32$$

AD = 48 cm

In ΔABD

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\frac{3AB^2}{4} = (48)^2$$

$$AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}}$$

$$= 32\sqrt{3} \text{ cm}$$

$$\text{Area of equilateral triangle } \Delta ABC = \frac{\sqrt{3}}{4}\left(32\sqrt{3}\right)^2$$

$$= \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3 = 96 \times 8 \times \sqrt{3}$$

$$= 768\sqrt{3} \text{ cm}^2$$

$$\text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times (32)^2$$

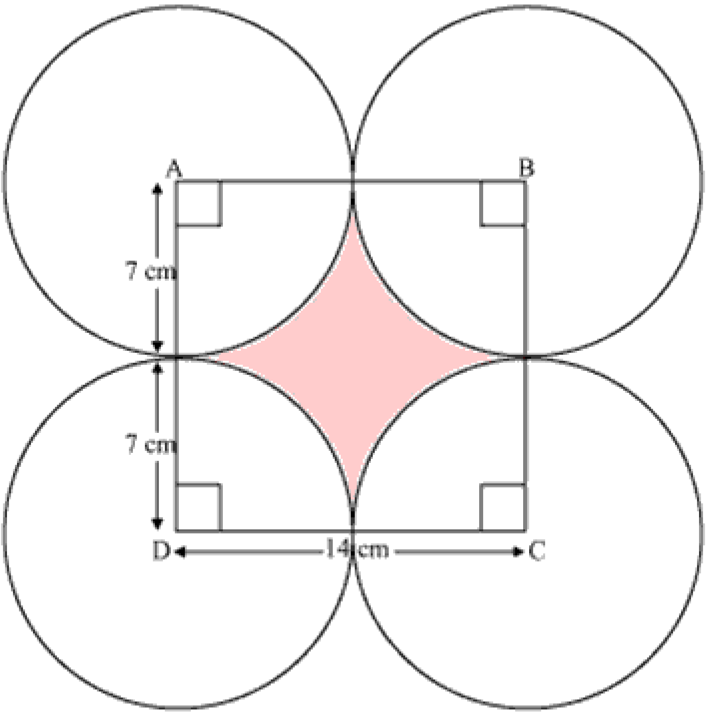
$$= \frac{22}{7} \times 1024$$

$$= \frac{22528}{7} \text{ cm}^2$$

$$\text{Area of design} = \text{area of circle} - \text{area of } \Delta ABC$$

$$= \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{ cm}^2$$

Solution 7



Area of each 4 sectors is equal to each other and is a sector of 90º in a circle of 7 cm radius.

Area of each sector $= \frac{90^\circ}{360^\circ} \times \pi(7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{2}$ cm^2

Area of square ABCD = $(\text{side})^2 = (14)^2$

$= 196$ cm^2

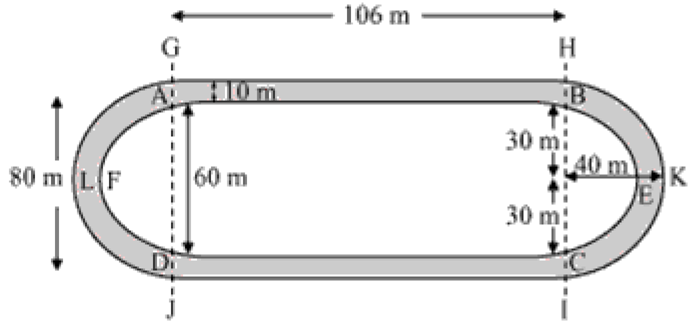
Area of shaded portion = area of square ABCD − 4 x area of each sector

$= 196 - 4 \times \frac{77}{2} = 196 - 154$

$= 42$ cm^2

So area of shaded portion is 42 cm^2

Solution 8



Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$= 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r$$

$$= 212 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30$$

$$= 212 + 2 \times \frac{22}{7} \times 30$$

$$= 212 + \frac{1320}{7}$$

$$= \frac{1484 + 1320}{7} = \frac{2804}{7} \text{ m}$$

Area of track = area of □GHIJ − area of □ABCD + area of semicircle HKI − area of semicircle BEC + area of semicircle GLJ − area of semicircle AFD

$$= 106 \times 80 - 106 \times 60 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2 + \frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2$$

$$= 106(80 - 60) + \frac{22}{7} \times (40)^2 - \frac{22}{7} \times (30)^2$$

$$= 106(20) + \frac{22}{7}\left[(40)^2 - (30)^2\right]$$

$$= 2120 + \frac{22}{7}(40 - 30)(40 + 30)$$

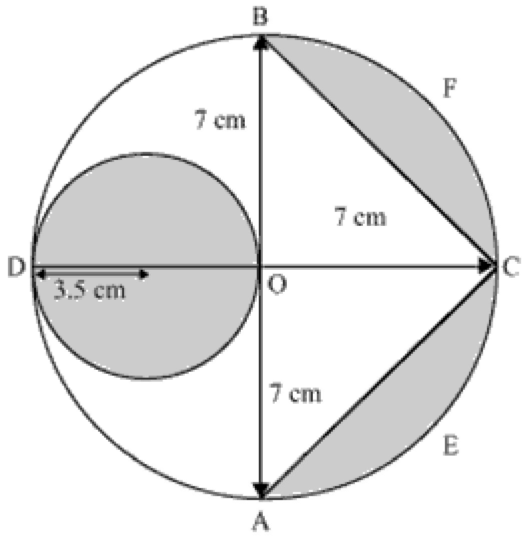
$$= 2120 + \left(\frac{22}{7}\right)(10)(70)$$

$$= 2120 + 2200$$

$$= 4320 \text{ m}^2$$

So area of track is 4320 m^2

Solution 9



Radius (r_1) of larger circle = 7 cm

Radius (r_2) of smaller circle $= \frac{7}{2}$ cm

Area of smaller circle $= \pi r_1^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2}$ cm^2

Area of semicircle AECFB of larger circle $= \frac{1}{2}\pi r_2^2$

$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$

$= 77$ cm^2

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 14 \times 7 = 49$ cm^2

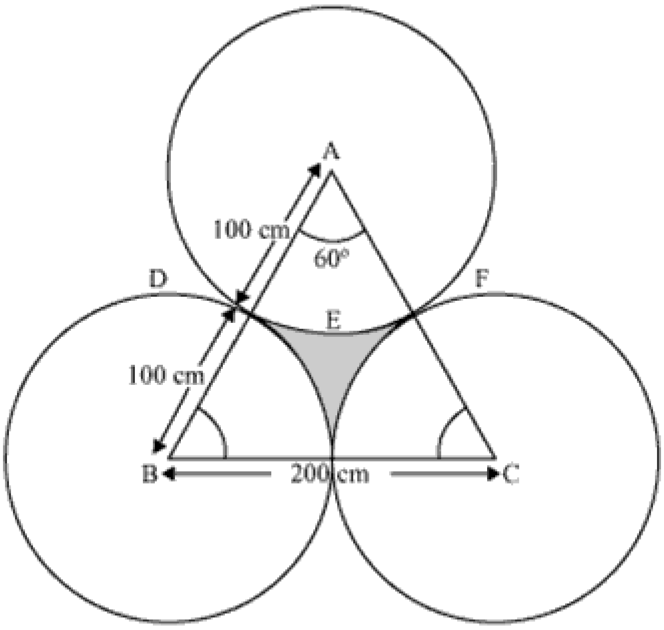
Area of shaded region = area of smaller circle + area of semicircle AECFB − area of ΔABC

$= \frac{77}{2} + 77 - 49$

$= 28 + \frac{77}{2} = 28 + 38.5$

$= 66.5$ cm^2

Solution 10



Let side of equilateral triangle be a
Area of equilateral triangle = 17320.5

$$\frac{\sqrt{3}}{4}(a)^2 = 17320.5$$
$$a^2 = 4 \times 10000$$
$$a = 200 \text{ cm}$$

Each sector is of 60°

So area of sector ADEF $= \frac{60^\circ}{360^\circ} \times \pi \times r^2$

$$= \frac{1}{6} \times \pi \times (100)^2$$
$$= \frac{3.14 \times 10000}{6}$$
$$= \frac{15700}{3}$$

Area of shaded region = area of equilateral triangle − 3 × area of each sector

$$= 17320.5 - 3 \times \frac{15700}{3}$$
$$= 17320.5 - 15700 = 1620.5 \text{ cm}^2$$

Solution 11

From the figure it is clear that side of square is 42 cm.

So area of square = $(\text{side})^2$

$= (42)^2$

$= 1764\ \text{cm}^2$

Area of each circle = πr^2

$$= \frac{22}{7} \times (7)^2 = 154\ \text{cm}^2$$

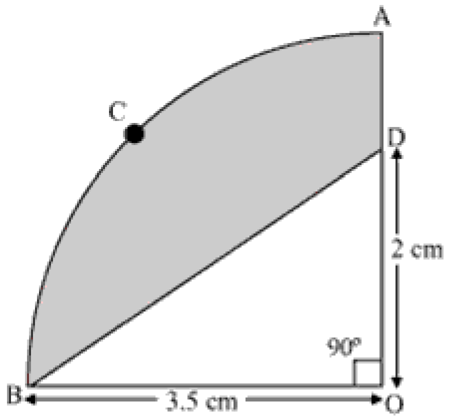
Area of 9 circles = 9×154

$= 1386\ \text{cm}^2$

Area of remaining portion of handkerchief = $1764 - 1386$

$= 378\ \text{cm}^2$

Solution 12



Since OACB is quadrant so it will subtend 90° angle at O.

Area of quadrant OACB $= \frac{90°}{360°} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8}\ \text{cm}^2$$

Area of ΔOBD $= \frac{1}{2} \times \text{OB} \times \text{OD}$

$$= \frac{1}{2} \times 3.5 \times 2$$

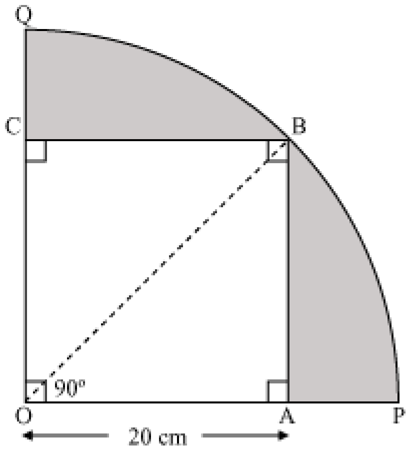
$$= 3.5\ \text{cm}^2$$

Area of shaded region = area of quadrant OACB − area of ΔOBD

$$= \frac{77}{8} - 3.5$$

$$= \frac{49}{8}\ \text{cm}^2$$

Solution 13



In ΔOAB

$OB^2 = OA^2 + AB^2$

$= (20)^2 + (20)^2$

$OB = 20\sqrt{2}$

Radius (r) of circle $= 20\sqrt{2}$ cm

Area of quadrant OPBQ $= \frac{90°}{360°} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$= \frac{1}{4} \times 3.14 \times 800$

$= 628\ \text{cm}^2$

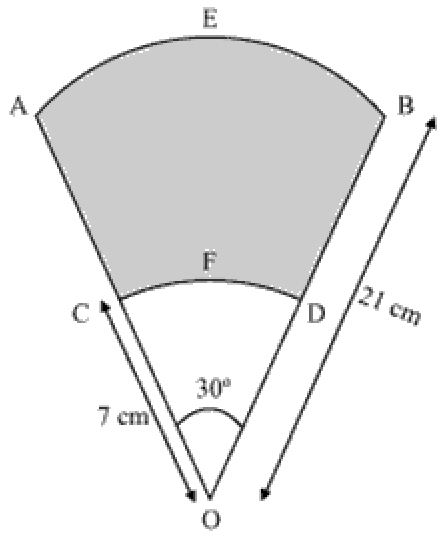
Area of □OABC $= (\text{side})^2 = (20)^2 = 400\ \text{cm}^2$

Area of shaded region = area of quadrant OPBQ − area of □OABC

$= 628 - 400$

$= 228\ \text{cm}^2$

Solution 14



Area of shaded region = area of sector OAEB − area of sector OCFD

$$= \frac{30^\circ}{360^\circ} \times \pi \times (21)^2 - \frac{30^\circ}{360^\circ} \times \pi \times (7)^2$$

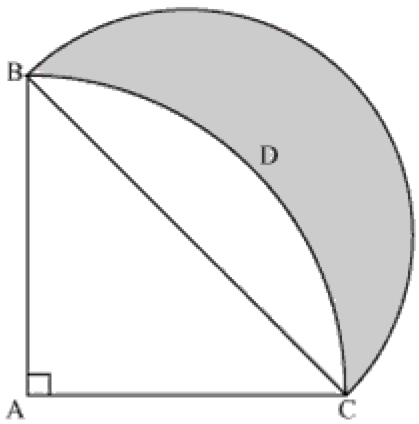
$$= \frac{1}{12} \times \pi\left[(21)^2 - (7)^2\right]$$

$$= \frac{1}{12} \times \frac{22}{7} \times \left[(21-7)(21+7)\right]$$

$$= \frac{22 \times 14 \times 28}{12 \times 7}$$

$$= \frac{308}{3}\ \text{cm}^2$$

Solution 15



As ABC is a quadrant of circle, $\angle BAC$ will be of 90º

In ΔABC

$BC^2 = AC^2 + AB^2$

$= (14)^2 + (14)^2$

$BC = 14\sqrt{2}$

Radius (r_1) of semicircle drawn on BC $= \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times 14 \times 14$

$= 98 \text{ cm}^2$

Area of sector ABDC $= \frac{90^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{ cm}^2$

Area of semicircle drawn on BC $= \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2$

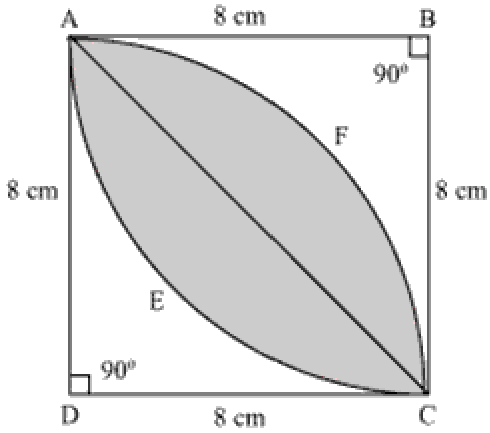
$= \frac{1}{2} \times \frac{22}{7} \times 98 = 154 \text{ cm}^2$

Area of shaded region = area of semicircle − (area of sector ABDC − area of ΔABC)

$= 154 - (154 - 98)$

$= 98 \text{ cm}^2$

Solution 16



The designed area is common region between two sectors BAEC and DAFC

$$\text{Area of sector BAEC} = \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (8)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 64$$

$$= \frac{22 \times 16}{7}$$

$$= \frac{352}{7} \text{ cm}^2$$

$$\text{Area of } \Delta BAC = \frac{1}{2} \times BA \times BC$$

$$= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$$

Area of designed portion = 2 x (area of segment AEC)

= 2 x (area of sector BAEC – area of ΔBAC)

$$= 2 \times \left(\frac{352}{7} - 32\right) = 2\left(\frac{352 - 224}{7}\right)$$

$$= \frac{2 \times 128}{7}$$

$$= \frac{256}{7} \text{ cm}^2$$