RD SHARMA Solutions for Class 9 Maths Chapter 5 - Factorisation of Algebraic expressions

Chapter 5 - Factorisation of Algebraic Expressions Exercise 5.25

Question 1

The factors of $x^3 - 1 + y^3 + 3xy$ are
(a) $(x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$
(b) $(x + y + 1)(x^2 + y^2 + 1 - xy - x - y)$
(c) $(x - 1 + y)(x^2 - 1 - y^2 + x + y + xy)$
(d) $3(x + y - 1)(x^2 + y^2 - 1)$

Solution 1

By using identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

We can write,

$x^3 - 1 + y^3 + 3xy$

$= (x)^3 + (-1)^3 + (y)^3 - 3(-1)(x)(y)$

$= [x + (-1) + y][x^2 + (-1)^2 + y^2 - x(-1) - y(-1) - xy]$

$= (x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$

Hence, correct option is (a).

Question 2

The value of $\frac{(0.013)^3+(0.007)^3}{(0.013)^2-0.013\times0.007+(0.007)^2}$ is

(a) 0.006
(b) 0.02
(c) 0.0091
(d) 0.00185

Solution 2

By using identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, we have

$$\frac{(0.013)^3 + (0.007)^3}{(0.013)^2 - (0.013)(0.007) + (0.007)^2}$$

$$= \frac{\{(0.013) + (0.007)\}(0.013)^2 - (0.013)(0.007) + (0.007)^2}{(0.013)^2 - (0.013)(0.007) + (0.007)^2}$$

$= 0.013 + 0.007$

$= 0.020$

$= 0.02$

Hence, correct option is (b).

Question 3

The factors of $8a^3 + b^3 - 6ab + 1$ are
(a) $(2a + b - 1)(4a^2 + b^2 + 1 - 3ab - 2a)$
(b) $(2a - b + 1)(4a^2 + b^2 - 4ab + 1 - 2a + b)$
(c) $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$
(d) $(2a - 1 + b)(4a^2 + 1 - 4a - b - 2ab)$

Solution 3

We Know the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

So by using this identity, we can write given expression as

$(2a)^3 + (b)^3 + (1)^3 - 3(2a)(b)(1)$

$= (2a + b + 1)[(2a)^2 + b^2 + 1^2 - 2a \times b - b \times 1 - 2a \times 1]$

$= (2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a]$

Hence, correct option is (c).

Question 4

$(x + y)^3 - (x - y)^3$ can be factorized as

(a) $2y(3x^2 + y^2)$

(b) $2x(3x^2 + y^2)$

(c) $2y(3y^2 + x^2)$

(d) $2x(x^2 + 3y^2)$

Solution 4

We know the identity $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

Let $x + y = a$ and $x - y = b$

Then,

$a^3 - b^3$

$= (x + y)^3 - (x - y)^3$

$= [(\cancel{x} + y) - (\cancel{x} - y)][(x + y)^2 + (x - y)^2 + (x + y)(x - y)]$

$= 2y[x^2 + y^2 + \cancel{2}xy + x^2 + \cancel{y^2} - \cancel{2}xy + x^2 - \cancel{y^2}\}$

$= 2y(3x^2 + y^2)$

Hence, correct option is (a).

Question 5

The factors of $x^3 - x^2y - xy^2 + y^3$ are

(a) $(x + y)(x^2 - xy + y^2)$

(b) $(x + y)(x^2 + xy + y^2)$

(c) $(x + y)^2(x - y)$

(d) $(x - y)^2(x + y)$

Solution 5

$x^3 - x^2y - xy^2 + y^3 = x^3 + y^3 - xy(x + y)$

Now by identity $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$, we have

$$\begin{aligned} x^3 - x^2y - xy^2 + y^3 &= (x + y)(x^2 + y^2 - xy) - xy(x + y) \\ &= (x + y)(x^2 + y^2 - xy - xy) \\ &= (x + y)(x^2 + y^2 - 2xy) \\ &= (x + y)(x - y)^2 \end{aligned}$$

Hence, correct option is (d).

Question 6

The expression $(a - b)^3 + (b - c)^3 + (c - a)^3$ can be factorized as

(a) $(a - b)(b - c)(c - a)$

(b) $3(a - b)(b - c)(c - a)$

(c) $-3(a - b)(b - c)(c - a)$

(d) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Solution 6

By know that $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
If $a + b + c = 0$, then
$a^3 + b^3 + c^3 = 3abc$
In given expression,
let $a - b = A$, $b - c = B$, $c - a = C$
Now, $a - b + b - c + c - a = 0$
i.e. $A + B + C = 0$
$\Rightarrow A^3 + B^3 + C^3 = 3ABC$
$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$
Hence, correct option is (b).

Question 7

The value of $\dfrac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is
(a) 2
(b) 3
(c) 2.327
(d) 2.273

Solution 7

$$\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$$
$$\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.09 + 0.69}$$
$$= \frac{(2.3)^3 - (0.3)^3}{(2.3)^2 + (0.3)^2 + (2.3)(0.3)}$$
$$= \frac{(2.3 - 0.3)\{(2.3)^2 + (0.3)^2 + (2.3)(0.3)\}}{\left((2.3)^2 + (0.3)^2 + (2.3)(0.3)\right)}$$
$$= 2.3 - 0.3$$
$$= 2$$
Hence, correct option is (a).

Chapter 5 - Factorisation of Algebraic Expressions Exercise 5.26

Question 1

The factors of $a^2 - 1 - 2x - x^2$ are
(a) $(a - x + 1)(a - x - 1)$
(b) $(a + x - 1)(a - x + 1)$
(c) $(a + x + 1)(a - x - 1)$
(d) none of these

Solution 1

$a^2 - 1 - 2x - x^2$
$= a^2 - (1 + 2x + x^2)$
$= a^2 - (1 + x)^2$
$= [a - (1 + x)][a + (1 + x)]$
$= (a - x - 1)(a + x + 1)$
Hence, correct option is (c).

Question 2

The factors of $x^4 + x^2 + 25$ are
(a) $x^2 + 3x + 5)(x^2 - 3x + 5)$
(b) $(x^2 + 3x + 5)(x^2 + 3x - 5)$
(c) $(x^2 + x + 5)(x^2 - x + 5)$
(d) none of these

Solution 2

For making perfect square to x^4+x^2+25,
we add $+10x^2$ and $-10x^2$ to it.

$$\Rightarrow x^4+x^2+25$$
$$= x^4 + x^2 + 25 + 10x^2 - 10x^2$$
$$= [x^4 + 10x^2 + 25] - 9x^2$$
$$= (x^2 + 5)^2 - (3x)^2$$
$$= [(x^2 + 5) + 3x] [(x^2 + 5) - 3x]$$
$$= (x^2 + 3x + 5) (x^2 - 3x + 5)$$

Hence, correct option is (a).

Question 3

The factors of $x^2 + 4y^2 + 4y - 4xy - 2x - 8$ are
(a) $(x - 2y - 4)(x - 2y + 2)$
(b) $(x - y + 2)(x - 4y - 4)$
(c) $(x + 2y - 4)(x + 2y + 2)$
(d) none of these

Solution 3

$$x^2 + 4y^2 + 4y - 4xy - 2x - 8$$
$$= x^2 + (2y)^2 - 2 \times x(2y) + 4y - 2x - 8$$
$$= (x - 2y)^2 + 4y - 2x - 8 \quad ....(1)$$

Now making eq (1) a perfect sequare by adding 1 and -1

$$\begin{aligned}(x - 2y)^2 + 4y - 2x - 8 &= (x - 2y)^2 + 4y - 2x - 8 + 1 - 1\\ &= (x - 2y)^2 + (1)^2 - 2 \times (1) \times (x - 2y) - 9\\ &= (x - 2y - 1)^2 - (3)^2\\ &= [(x - 2y - 1) - 3] [x - 2y - 1 + 3]\\ &= (x - 2y - 4) (x - 2y + 2)\end{aligned}$$

Hence, correct Option is (a).

Question 4

The factors of $x^3 - 7x + 6$ are
(a) $x(x - 6)(x - 1)$
(b) $(x^2 - 6)(x - 1)$
(c) $(x + 1)(x + 2)(x - 3)$
(d) $(x - 1)(x + 3)(x - 2)$

Solution 4

$$x^3 - 7x + 6 = x^3 - 7x + 6 + 1 - 1 \text{ (by adding +1 \& -1 to R.H.S)}$$
$$= x^3 - 7x + 7 - 1$$
$$= (x^3 - 1) - 7(x - 1)$$

Now by identity $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$, *we get*

$$x^3 - 7x + 6 = (x^3 - 1) - 7(x - 1)$$
$$= (x - 1)(x^2 + x + 1) - 7(x - 1)$$
$$= (x - 1)(x^2 + x + 1 - 7)$$
$$= (x - 1)(x^2 + x - 6)$$
$$= (x - 1)(x + 3)(x - 2)$$

Hence, correct option is (d).

Question 5

The expression $x^4 + 4$ can be factorized as

(a) $(x^2 + 2x + 2)(x^2 - 2x + 2)$
(b) $(x^2 + 2x + 2)(x^2 + 2x - 2)$
(c) $(x^2 - 2x - 2)(x^2 - 2x + 2)$
(d) $(x^2 + 2)(x^2 - 2)$

Solution 5

$$x^4 + 4$$
$$= x^4 + 4 + 4x^2 - 4x^2$$
$$= (x^4 + 4x^2 + 4) - 4x^2$$
$$= (x^2 + 2)^2 - (2x)^2$$
$$= (x^2 + 2 - 2x)(x^2 + 2 + 2x)$$
$$= (x^2 + 2 - 2x)(x^2 + 2x + 2)$$

Hence, correct option is (a).

Question 6

If $3x = a + b + c$, then the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ is

(a) $a + b + c$
(b) $(a - b)(b - c)(c - a)$
(c) 0
(d) none of these

Solution 6

Question 7

If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^2$, then $k =$

(a) 1
(b) 2
(c) 4

(d) 8

Solution 7

Let $x + y = A$ and $x - y = B$

Now, $(A - B)^3 = A^3 - B^3 - 3AB(A - B)$

$\Rightarrow [(x + y) - (x - y)]^3 = (x + y)^3 - (x - y)^3 - 3(x + y)(x - y)\ [(x + y) - (x - y))$

$= (x + y)^3 - (x - y)^3 - 3(x^2 - y^2)(2y)$

$= (x + y)^3 - (x - y)^3 - 6y(x^2 - y^2)$

But, $(x + y)^3 - (x - y)^3 - 6y\,(x^2 - y^2) = ky^3$

$\Rightarrow [(x + y) - (x - y)]^3 = (2y)^3 = k8y^3$

$\Rightarrow (2y)^3 = ky^3$

$\Rightarrow 8y^3 = ky^3$

$\Rightarrow k = 8$

Hence, correct option is (d).

Question 8

If $x^3 - 3x^2 + 3x + 7 = (x + 1)(ax^2 + bx + c)$, then $a + b + c =$

(a) 4
(b) 12
(c) -10
(d) 3

Solution 8

$x^3 - 3x^2 + 3x + 7$

$= x^3 - 3x^2 + 3x - 1 + 1 + 7$ (by adding $+1$ & -1)

$= x^3 - 3x^2 + 3x + 1 + 6$

$= (x)^3 + (1)^3 - 3x^2 + 3x + 6$

$= (x + 1)(x^2 + 1 - x) - 3x^2 + 3x + 6$ {by identity $a^3 + b^3 = (a + b)\ (a^2 + b^2 - ab)$}

$= (x + 1)(x^2 - x + 1) - 3(x^2 - x - 2)$

$= (x + 1)(x^2 - x + 1) - 3(x + 1)(x - 2)$

$= (x + 1)[x^2 - x + 1 - 3(x - 2)]$

$= (x + 1)\ (x^2 - 4x + 7)$

$\Rightarrow ax^2 + bx + 6 = x^2 - 4x + 7$

$\Rightarrow a = 1,\ b = -4,\ c = 7$

$\Rightarrow a + b + c = 1 - 4 + 7 = 4$

Hence, correct option is (a).

Chapter 5 - Factorisation of Algebraic Expressions Exercise Ex. 5.1

Question 1

Factorize:

$x^3 + x - 3x^2 - 3$

Solution 1

$x^3 + x - 3x^2 - 3$

Taking x common in $\left(x^3 + x\right)$

$= x\left(x^2 + 1\right) - 3x^2 - 3$

Taking -3 common in $\left(-3x^2 - 3\right)$

$= x\left(x^2 + 1\right) - 3\left(x^2 + 1\right)$

Now, we take $\left(x^2 + 1\right)$ common

$= \left(x^2 + 1\right)\left(x - 3\right)$

$\therefore \quad x^3 + x - 3x^2 - 3 = \left(x^2 + 1\right)\left(x - 3\right)$

Question 2

Factorize:

$a\left(a + b\right)^3 - 3a^2b\left(a + b\right)$

Solution 2

$a\left(a + b\right)^3 - 3a^2b\left(a + b\right)$

Taking $\left(a + b\right)$ common in the two terms

$= \left(a + b\right)\left\{a\left(a + b\right)^2 - 3a^2b\right\}$

Now, using $\left(a + b\right)^2 = a^2 + b^2 + 2ab$

$= \left(a + b\right)\left\{a\left(a^2 + b^2 + 2ab\right) - 3a^2b\right\}$

$= \left(a + b\right)\left\{a^3 + ab^2 + 2a^2b - 3a^2b\right\}$

$= \left(a + b\right)\left\{a^3 + ab^2 - a^2b\right\}$

$= \left(a + b\right)a\left\{a^2 + b^2 - ab\right\}$

$= a\left(a + b\right)\left(a^2 + b^2 - ab\right)$

$\therefore \quad a\left(a + b\right)^3 - 3a^2b\left(a + b\right) = a\left(a + b\right)\left(a^2 + b^2 - ab\right)$

Question 3

Factorize:

$x\left(x^3 - y^3\right) + 3xy\left(x - y\right)$

Solution 3

$x\left(x^3 - y^3\right) + 3xy\left(x - y\right)$

Elaborating $x^3 - y^3$ using identity $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

$= x\left(x - y\right)\left(x^2 + xy + y^2\right) + 3xy\left(x - y\right)$

Taking common $x\left(x - y\right)$ in both the terms

$= x\left(x - y\right)\left\{x^2 + xy + y^2 + 3y\right\}$

$\therefore \quad x\left(x^3 - y^3\right) + 3xy\left(x - y\right) = x\left(x - y\right)\left(x^2 + xy + y^2 + 3y\right)$

Question 4

Factorize:

$a^2x^2 + \left(ax^2 + 1\right)x + a$

Solution 4

$a^2x^2 + \left(ax^2 + 1\right)x + a$

We multiply $x\left(ax^2 + 1\right) = ax^3 + x$

$= a^2x^2 + ax^3 + x + a$

Taking common ax^2 in $\left(a^2x^2 + ax^3\right)$ and 1 in $(x + a)$

$= ax^2\left(a + x\right) + 1\left(x + a\right)$

$= ax^2\left(a + x\right) + 1\left(a + x\right)$

Taking $\left(a + x\right)$ common in both the terms

$= \left(a + x\right)\left(ax^2 + 1\right)$

$\therefore \quad a^2x^2 + \left(ax^2 + 1\right)x + a = \left(a + x\right)\left(ax^2 + 1\right)$

Question 5

Factorize:

$x^2 + y - xy - x$

Solution 5

$x^2 + y - xy - x$

On rearranging

$x^2 - xy - x + y$

Taking x common in the $\left(x^2 + y\right)$ and -1 in $\left(-x + y\right)$

$= x\left(x - y\right) - 1\left(x - y\right)$

Taking $\left(x - y\right)$ common in both the terms

$= \left(x - y\right)\left(x - 1\right)$

$\therefore \; x^2 + y - xy - x = \left(x - y\right)\left(x - 1\right)$

Question 6

Factorize:

$x^3 - 2x^2y + 3xy^2 - 6y^3$

Solution 6

$x^3 - 2x^2y + 3xy^2 - 6y^3$

Taking x^2 common in $\left(x^3 - 2x^2y\right)$ and $+3y^2$ common in $\left(3xy^2 - 6y^3\right)$

$= x^2\left(x - 2y\right) + 3y^2\left(x - 2y\right)$

$= \left(x - 2y\right)\left(x^2 + 3y^2\right)$

$\therefore \; x^3 - 2x^2y + 3xy^2 - 6y^3 = \left(x - 2y\right)\left(x^2 + 3y^2\right)$

Question 7

Factorize:

$6ab - b^2 + 12ac - 2bc$

Solution 7

$6ab - b^2 + 12ac - 2bc$

Taking common b in $\left(6ab - b^2\right)$ and $2c$ in $\left(12ac - 2bc\right)$

$= b\left(6a - b\right) + 2c\left(6a - b\right)$

Taking $\left(6a - b\right)$ common in both terms

$= \left(6a - b\right)\left(b + 2c\right)$

$\therefore\ 6ab - b^2 + 12ac - 2bc = \left(6a - b\right)\left(b + 2c\right)$

Question 8

Factorize:

$x\left(x - 2\right)\left(x - 4\right) + 4x - 8$

Solution 8

$x\left(x - 2\right)\left(x - 4\right) + 4x - 8$

$= x\left(x - 2\right)\left(x - 4\right) + 4\left(x - 2\right)$

Taking $\left(x - 2\right)$ common in both terms

$= \left(x - 2\right)\left\{x\left(x - 4\right) + 4\right\}$

$= \left(x - 2\right)\left\{x^2 - 4x + 4\right\}$

Now splitting middle term of $x^2 - 4x + 4$

$= \left(x - 2\right)\left\{x^2 - 2x - 2x + 4\right\}$

$= \left(x - 2\right)\left\{x\left(x - 2\right) - 2\left(x - 2\right)\right\}$

$= \left(x - 2\right)\left\{\left(x - 2\right)\left(x - 2\right)\right\}$

$= \left(x - 2\right)\left(x - 2\right)\left(x - 2\right)$

$= \left(x - 2\right)^3$

$\therefore\ x\left(x - 2\right)\left(x - 4\right) + 4x - 8 = \left(x - 2\right)^3$

Question 9

Factorize:

$\left(a - b + c\right)^2 + \left(b - c + a\right)^2 + 2\left(a - b + c\right)\left(b - c + a\right)$

Solution 9

$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

Let $(a-b+c)=x$ *and* $(b-c+a)=y$

$=x^2+y^2+2xy$

Using identity $a^2+b^2+2ab=(a+b)^2$

$=(x+y)^2$

Now, substituting x and y

$=(a-b+c+b-c+a)^2$

Cancelling $-b, +b$ & $+c, -c$

$=(2a)^2$

$=4a^2$

$\therefore\ (a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)=4a^2$

Question 10

Factorize:

$a^2+2ab+b^2-c^2$

Solution 10

$a^2+2ab+b^2-c^2$

Using identity $a^2+2ab+b^2=(a+b)^2$

$=(a+b)^2-c^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a+b+c)(a+b-c)$

$\therefore\ a^2+2ab+b^2-c^2=(a+b+c)(a+b-c)$

Question 11

Factorize:

$a^2+4b^2-4ab-4c^2$

Solution 11

$a^2 + 4b^2 - 4ab - 4c^2$

On rearranging

$= a^2 - 4ab + 4b^2 - 4c^2$

$= (a)^2 - 2 \times a \times 2b + (2b)^2 - 4c^2$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$

$= (a - 2b)^2 - 4c^2$

$= (a - 2b)^2 - (2c)^2$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$= (a - 2b + 2c)(a - 2b - 2c)$

$\therefore\ a^2 + 4b^2 - 4ab - 4c^2 = (a - 2b + 2c)(a - 2b - 2c)$

Question 12

Factorize:

$x^2 - y^2 - 4xz + 4z^2$

Solution 12

$x^2 - y^2 - 4xz + 4z^2$

On rearranging the terms

$= x^2 - 4xz + 4z^2 - y^2$

$= (x)^2 - 2 \times x \times 2z + (2z)^2 - y^2$

Using identity $a^2 - 2ab + b^2 = (a - b)^2$

$= (x - 2z)^2 - y^2$

Using identity $a^2 - b^2 = (a + b)(a - b)$

$= (x - 2z + y)(x - 2z - y)$

$\therefore\ x^2 - y^2 - 4xz + 4z^2 = (x - 2z + y)(x - 2z - y)$

Question 13

Factorize:

$$2x^2 - \frac{5}{6}x + \frac{1}{12}$$

Solution 13

$$2x^2 - \frac{5}{6}x + \frac{1}{12}$$

Splitting the middle term,

$$= 2x^2 - \frac{x}{2} - \frac{x}{3} + \frac{1}{12} \qquad \left[\because -\frac{5}{6} = -\frac{1}{2} - \frac{1}{3} \text{ also } -\frac{1}{2} \times -\frac{1}{3} = 2 \times \frac{1}{12}\right]$$

$$= x\left(2x - \frac{1}{2}\right) - \frac{1}{6}\left(2x - \frac{1}{2}\right)$$

$$= \left(2x - \frac{1}{2}\right)\left(x - \frac{1}{6}\right)$$

$$\therefore \ 2x^2 - \frac{5}{6}x + \frac{1}{12} = \left(2x - \frac{1}{2}\right)\left(x - \frac{1}{6}\right)$$

Question 14

Factorize:

$$x^2 + \frac{12}{35}x + \frac{1}{35}$$

Solution 14

$$x^2 + \frac{12}{35}x + \frac{1}{35}$$

Splitting the middle term,

$$= x^2 + \frac{5}{35}x + \frac{7}{35}x + \frac{1}{35} \qquad \left[\because \frac{12}{35} = \frac{5}{35} + \frac{7}{35} \text{ and } \frac{5}{35} \times \frac{7}{35} = \frac{1}{35}\right]$$

$$= x^2 + \frac{x}{7} + \frac{x}{5} + \frac{1}{35}$$

$$= x\left(x + \frac{1}{7}\right) + \frac{1}{5}\left(x + \frac{1}{7}\right)$$

$$= \left(x + \frac{1}{7}\right)\left(x + \frac{1}{5}\right)$$

$$\therefore \ x^2 + \frac{12}{35}x + \frac{1}{35} = \left(x + \frac{1}{7}\right)\left(x + \frac{1}{5}\right)$$

Question 15

Factorize:

$21x^2 - 2x + \frac{1}{21}$

Solution 15

$21x^2 - 2x + \frac{1}{21}$

$= \left(\sqrt{21}x\right)^2 - 2 \times \sqrt{21}x \times \frac{1}{\sqrt{21}} + \left(\frac{1}{\sqrt{21}}\right)^2$

Using identity $a^2 - 2ab + b^2 = (a-b)^2$

$= \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$

$\therefore \; 21x^2 - 2x + \frac{1}{21} = \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$

Question 16

Give possible expressions for the length and breadth of the rectangle having

$35y^2 + 13y - 12$ as its area.

Solution 16

Area $= 35y^2 + 13y - 12$

Splitting the middle term,

Area $= 35y^2 + 28y - 15y - 12$
$= 7y(5y+4) - 3(5y+4)$

Area $= (5y+4)(7y-3)$

Also area of rectangle = Length $\times$ Breadth

$\therefore$ Possible length $= (5y+4)$ and breadth $= (7y-3)$
Or Possible length $= (7y-3)$ and breadth $= (5y+4)$

Question 17

What are the possible expressions for the dimensions of the cuboid whose volume is

$3x^2 - 12x$.

Solution 17

Here volume $= 3x^2 - 12x$

$= 3x\left(x-4\right)$

$= 3 \times x\left(x-4\right)$

Also volume = Length $\times$ Breadth $\times$ Height

$\therefore$ Possible expressions for dimensions of the cuboid are $= 3, x, \left(x-4\right)$

Question 18

Factorize:

$$\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6$$

Solution 18

$$\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6$$

$$= x^2+\frac{1}{x^2}-4x-\frac{4}{x}+4+2$$

$$= x^2+\frac{1}{x^2}+4+2-\frac{4}{x}-4x$$

$$= \left(x^2\right)+\left(\frac{1}{x}\right)^2+\left(-2\right)^2+2\times x\times\frac{1}{x}+2\times\frac{1}{x}\times\left(-2\right)+2\left(-2\right)x$$

Using identity

$$a^2+b^2+c^2+2ab+2bc+2ca=\left(a+b+c\right)^2$$

We get,

$$= \left[x+\frac{1}{x}+\left(-2\right)\right]^2$$

$$= \left[x+\frac{1}{x}-2\right]^2$$

$$= \left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$$

$$\therefore \left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$$

Question 19

Factorize:

$(x+2)(x^2+25)-10x^2-20x$

Solution 19

$(x+2)(x^2+25)-10x^2-20x$

$= (x+2)(x^2+25)-10x(x+2)$

Taking $(x+2)$ common in both terms

$= (x+2)(x^2+25-10x)$

$= (x+2)(x^2-10x+25)$

Splitting middle term of $x^2-10x+25$

$= (x+2)\{x^2-5x-5x+25\}$

$= (x+2)\{x(x-5)-5(x-5)\}$

$= (x+2)(x-5)(x-5)$

$\therefore\ (x+2)(x^2+25)-10x^2-20x = (x+2)(x-5)(x-5)$

Question 20

Factorize:

$2a^2+2\sqrt{6}ab+3b^2$

Solution 20

$2a^2+2\sqrt{6}ab+3b^2$

$= (\sqrt{2}a)^2+2\times\sqrt{2}a\times\sqrt{3}b+(\sqrt{3}b)^2$

Using identity $a^2+2ab+b^2=(a+b)^2$

$= (\sqrt{2}a+\sqrt{3}b)^2$

$= (\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

$\therefore\ 2a^2+2\sqrt{6}ab+3b^2 = (\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

Question 21

Factorize:

$a^2 + b^2 + 2(ab + bc + ca)$

Solution 21

$a^2 + b^2 + 2(ab + bc + ca)$

$= a^2 + b^2 + 2ab + 2bc + 2ca$

Using identity $a^2 + b^2 + 2ab = (a + b)^2$

We get,

$= (a + b)^2 + 2bc + 2ca$

$= (a + b)^2 + 2c(b + a)$

or $(a + b)^2 + 2c(a + b)$

Taking $(a + b)$ common

$= (a + b)(a + b + 2c)$

$\therefore\ a^2 + b^2 + 2(ab + bc + ca) = (a + b)(a + b + 2c)$

Question 22

Factorize:

$4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$

Solution 22

$$4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$$

Let $(x-y)=a,\ (x+y)=b$

$=4a^2-12ab+9b^2$

Splitting middle term $-12=-6-6$ also $4\times 9=-6\times -6$

$=4a^2-6ab-6ab+9b^2$

$=2a(2a-3b)-3b(2a-3b)$

$=(2a-3b)(2a-3b)$

$=(2a-3b)^2$

Substituting $a=x-y$ & $b=x+y$

$=[2(x-y)-3(x+y)]^2$

$=[2x-2y-3x-3y]^2$

$=[2x-3x-2y-3y]^2$

$=[-x-5y]^2$

$=[(-1)(x+5y)]^2$

$=(x+5y)^2 \qquad \left[\because (-1)^2=1\right]$

$\therefore\ 4(x-y)^2-12(x-y)(x+y)+9(x+y)^2=(x+5y)^2$

Question 23

Factorize:

$a^2-b^2+2bc-c^2$

Solution 23

$a^2 - b^2 + 2bc - c^2$

$= a^2 - \left(b^2 - 2bc + c^2\right)$

Using identity $a^2 - 2ab + b^2 = \left(a - b\right)^2$

$= a^2 - \left(b - c\right)^2$

Using identity $a^2 - b^2 = \left(a + b\right)\left(a - b\right)$

$= \left(a + b - c\right)\left(a - \left(b - c\right)\right)$
$= \left(a + b - c\right)\left(a - b + c\right)$

$\therefore \; a^2 - b^2 + 2bc - c^2 = \left(a + b - c\right)\left(a - b + c\right)$

Question 24

Factorize:

$xy^9 - yx^9$

Solution 24

$xy^9 - yx^9$

$= xy\left(y^8 - x^8\right)$

$= xy\left(\left(y^4\right)^2 - \left(x^4\right)^2\right)$

Using identity $a^2 - b^2 = (a+b)(a-b)$

$= xy\left(y^4 + x^4\right)\left(y^4 - x^4\right)$

$= xy\left(y^4 + x^4\right)\left(\left(y^2\right)^2 - \left(x^2\right)^2\right)$

Using identity $a^2 - b^2 = (a+b)(a-b)$

$= xy\left(y^4 + x^4\right)\left(y^2 + x^2\right)\left(y^2 - x^2\right)$

$= xy\left(y^4 + x^4\right)\left(y^2 + x^2\right)(y + x)(y - x)$

$= xy\left(x^4 + y^4\right)\left(x^2 + y^2\right)(x + y)(-1)(x - y)$

$\because \ (b - a) = -1(a - b)$

$= -xy\left(x^4 + y^4\right)\left(x^2 + y^2\right)(x + y)(x - y)$

$\therefore \ xy^9 - yx^9 = -xy\left(x^4 + y^4\right)\left(x^2 + y^2\right)(x + y)(x - y)$

Question 25

Factorize:

$x^4 + x^2y^2 + y^4$

Solution 25

$x^4 + x^2y^2 + y^4$

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$= x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2$
$= x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= \left(x^2\right)^2 + 2 \times x^2 \times y^2 + \left(y^2\right)^2 - \left(xy\right)^2$

Using identity $a^2 + 2ab + b^2 = \left(a + b\right)^2$

$= \left(x^2 + y^2\right)^2 - \left(xy\right)^2$

Using identity $a^2 - b^2 = \left(a + b\right)\left(a - b\right)$

$= \left(x^2 + y^2 + xy\right)\left(x^2 + y^2 - xy\right)$

$\therefore\ x^4 + x^2y^2 + y^4 = \left(x^2 + y^2 + xy\right)\left(x^2 + y^2 - xy\right)$

Question 26

Factorize:

$x^2 + 6\sqrt{2}x + 10$

Solution 26

$x^2 + 6\sqrt{2}x + 10$

Splitting middle term,

$= x^2 + 5\sqrt{2}x + \sqrt{2}x + 10 \qquad \left[\because 6\sqrt{2} = 5\sqrt{2} + \sqrt{2} \text{ and } 5\sqrt{2} \times \sqrt{2} = 10\right]$
$= x\left(x + 5\sqrt{2}\right) + \sqrt{2}\left(x + 5\sqrt{2}\right)$

$= \left(x + 5\sqrt{2}\right)\left(x + \sqrt{2}\right)$

$\therefore\ x^2 + 6\sqrt{2}x + 10 = \left(x + 5\sqrt{2}\right)\left(x + \sqrt{2}\right)$

Question 27

Factorize:

$x^2 - 2\sqrt{2}x - 30$

Solution 27

$x^2 - 2\sqrt{2}x - 30$

Splitting the middle term,

$= x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$ $\left[\because -2\sqrt{2} = -5\sqrt{2} + 3\sqrt{2} \text{ also } -5\sqrt{2} \times 3\sqrt{2} = -30\right]$

$= x\left(x - 5\sqrt{2}\right) + 3\sqrt{2}\left(x - 5\sqrt{2}\right)$

$= \left(x - 5\sqrt{2}\right)\left(x + 3\sqrt{2}\right)$

$\therefore\ x^2 - 2\sqrt{2}x - 30 = \left(x - 5\sqrt{2}\right)\left(x + 3\sqrt{2}\right)$

Question 28

Factorize:

$x^2 - \sqrt{3}x - 6$

Solution 28

$x^2 - \sqrt{3}x - 6$

Splitting the middle term,

$= x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$ $\left[\because -\sqrt{3} = -2\sqrt{3} + \sqrt{3} \text{ also } -2\sqrt{3} \times \sqrt{3} = -6\right]$

$= x\left(x - 2\sqrt{3}\right) + \sqrt{3}\left(x - 2\sqrt{3}\right)$

$= \left(x - 2\sqrt{3}\right)\left(x + \sqrt{3}\right)$

$\therefore\ x^2 - \sqrt{3}x - 6 = \left(x - 2\sqrt{3}\right)\left(x + \sqrt{3}\right)$

Question 29

Factorize:

$x^2 + 5\sqrt{5}x + 30$

Solution 29

$x^2 + 5\sqrt{5}x + 30$

Splitting the middle term,

$= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30$ $\left[\because 5\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} \text{ also } 2\sqrt{5} \times 3\sqrt{5} = 30\right]$

$= x\left(x + 2\sqrt{5}\right) + 3\sqrt{5}\left(x + 2\sqrt{5}\right)$

$= \left(x + 2\sqrt{5}\right)\left(x + 3\sqrt{5}\right)$

$\therefore\ x^2 + 5\sqrt{5}x + 30 = \left(x + 2\sqrt{5}\right)\left(x + 3\sqrt{5}\right)$

Question 30

Factorize:

$x^2 + 2\sqrt{3}x - 24$

Solution 30

$x^2 + 2\sqrt{3}x - 24$

Splitting the middle term,

$= x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$ $\left[\because 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3} \text{ also } 4\sqrt{3}\left(-2\sqrt{3}\right) = -24\right]$

$= x\left(x + 4\sqrt{3}\right) - 2\sqrt{3}\left(x + 4\sqrt{3}\right)$

$= \left(x + 4\sqrt{3}\right)\left(x - 2\sqrt{3}\right)$

$\therefore\ x^2 + 2\sqrt{3}x - 24 = \left(x + 4\sqrt{3}\right)\left(x - 2\sqrt{3}\right)$

Question 31

Factorize:

$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Solution 31

$5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Splitting the middle term,

$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$ $\quad\left[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5\sqrt{5} \times 3\sqrt{5}\right]$

$= 5x\left(\sqrt{5}x + 3\right) + \sqrt{5}\left(\sqrt{5}x + 3\right)$

$= \left(\sqrt{5}x + 3\right)\left(5x + \sqrt{5}\right)$

$\therefore \; 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = \left(\sqrt{5}x + 3\right)\left(5x + \sqrt{5}\right)$

Question 32

Factorize:

$2x^2 + 3\sqrt{5}x + 5$

Solution 32

$2x^2 + 3\sqrt{5}x + 5$

Splitting the middle term,

$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$ $\quad\left[\because 3\sqrt{5} = 2\sqrt{5} + \sqrt{5} \text{ also } 2\sqrt{5} \times \sqrt{5} = 2 \times 5\right]$

$= 2x\left(x + \sqrt{5}\right) + \sqrt{5}\left(x + \sqrt{5}\right)$

$= \left(x + \sqrt{5}\right)\left(2x + \sqrt{5}\right)$

$\therefore \; 2x^2 + 3\sqrt{5}x + 5 = \left(x + \sqrt{5}\right)\left(2x + \sqrt{5}\right)$

Question 33

Factorize:

$9\left(2a - b\right)^2 - 4\left(2a - b\right) - 13$

Solution 33

$9(2a-b)^2 - 4(2a-b) - 13$

Let $2a - b = x$

$= 9x^2 - 4x - 13$

Splitting the middle term,

$= 9x^2 - 13x + 9x - 13$

$= x(9x - 13) + 1(9x - 13)$

$= (9x - 13)(x + 1)$

substituting $x = 2a - b$

$= [9(2a - b) - 13](2a - b + 1)$

$= (18a - 9b - 13)(2a - b + 1)$

$\therefore \; 9(2a-b)^2 - 4(2a-b) - 13 = (18a - 9b - 13)(2a - b + 1)$

Question 34

Factorize:

$7(x-2y)^2 - 25(x-2y) + 12$

Solution 34

$7(x-2y)^2-25(x-2y)+12$

Let $x-2y=p$

$=7p^2-25p+12$

Splitting the middle term,

$=7p^2-21p-4p+12$

$=7p(p-3)-4(p-3)$

$=(p-3)(7p-4)$

substituting $p=x-2y$

$=(x-2y-3)(7(x-2y)-4)$

$=(x-2y-3)(7x-14y-4)$

$\therefore\ 7(x-2y)^2-25(x-2y)+12=(x-2y-3)(7x-14y-4)$

Question 35

Factorize:

$2(x+y)^2-9(x+y)-5$

Solution 35

$2(x+y)^2 - 9(x+y) - 5$

Let $x + y = z$

$= 2z^2 - 9z - 5$

Splitting middle term,

$= 2z^2 - 10z + z - 5$

$= 2z(z-5) + 1(z-5)$

$= (z-5)(2z+1)$

substituting $z = x + y$

$= (x+y-5)(2(x+y)+1)$

$= (x+y-5)(2x+2y+1)$

$\therefore\ 2(x+y)^2 - 9(x+y) - 5 = (x+y-5)(2x+2y+1)$

Chapter 5 - Factorisation of Algebraic Expressions Exercise Ex. 5.2

Question 1

Factorize:

$p^3 + 27$

Solution 1

$p^3 + 27$

$= p^3 + 3^3$

$= (p+3)(p^2 - 3p + 3^2)$ $\quad [\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (p+3)(p^2 - 3p + 9)$

$\therefore\ p^3 + 27 = (p+3)(p^2 - 3p + 9)$

Question 2

Factorize:

$y^3 + 125$

Solution 2

$y^3 + 125$

$= y^3 + 5^3$

$= (y+5)(y^2 - 5y + 5^2)$ $\quad [\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (y+5)(y^2 - 5y + 25)$

$\therefore\ y^3 + 125 = (y+5)(y^2 - 5y + 25)$

Question 3

Factorize:

$1 - 27a^3$

Solution 3

$1 - 27a^3$

$= (1)^3 - (3a)^3$

$= (1 - 3a)(1^2 + 1 \times 3a + (3a)^2)$ $\quad [\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= (1 - 3a)(1 + 3a + 9a^2)$

$\therefore\ 1 - 27a^3 = (1 - 3a)(1 + 3a + 9a^2)$

Question 4

Factorize:

$8x^3y^3 + 27a^3$

Solution 4

$8x^3y^3 + 27a^3$

$= (2xy)^3 + (3a)^3$

$= (2xy + 3a)((2xy)^2 - 2xy \times 3a + (3a)^2)$ $\quad [\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$

$\therefore\ 8x^3y^3 + 27a^3 = (2xy + 3a)(4x^2y^2 - 6xya + 9a^2)$

Question 5

Factorize:

$64a^3 - b^3$

Solution 5

$64a^3 - b^3$

$= (4a)^3 - b^3$

$= (4a - b)\left((4a)^2 + 4a \times b + b^2\right) \qquad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$

$= (4a - b)(16a^2 + 4ab + b^2)$

$\therefore \; 64a^3 - b^3 = (4a - b)(16a^2 + 4ab + b^2)$

Question 6

Factorize:

$\frac{x^3}{216} - 8y^3$

Solution 6

$\frac{x^3}{216} - 8y^3$

$= \left(\frac{x}{6}\right)^3 - (2y)^3$

$= \left(\frac{x}{6} - 2y\right)\left(\left(\frac{x}{6}\right)^2 + \frac{x}{6} \times 2y + (2y)^2\right) \qquad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$

$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

$\therefore \; \frac{x^3}{216} - 8y^3 = \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

Question 7

Factorize:

$10x^4y - 10xy^4$

Solution 7

$10x^4y - 10xy^4$

$= 10xy\left(x^3 - y^3\right)$

$= 10xy\left(x - y\right)\left(x^2 + xy + y^2\right)$ $\left[\because a^3 - b^3 = \left(a - b\right)\left(a^2 + ab + b^2\right)\right]$

$\therefore\ 10x^4y - 10xy^4 = 10xy\left(x - y\right)\left(x^2 + xy + y^2\right)$

Question 8

Factorize:

$54x^6y + 2x^3y^4$

Solution 8

$54x^6y + 2x^3y^4$

$= 2x^3y\left(27x^3 + y^3\right)$

$= 2x^3y\left(\left(3x\right)^3 + y^3\right)$

$= 2x^3y\left(3x + y\right)\left(\left(3x\right)^2 - 3x \times y + y^2\right)$ $\left[\because a^3 + b^3 = \left(a + b\right)\left(a^2 - ab + b^2\right)\right]$

$= 2x^3y\left(3x + y\right)\left(9x^2 - 3xy + y^2\right)$

$\therefore\ 54x^6y + 2x^3y^4 = 2x^3y\left(3x + y\right)\left(9x^2 - 3xy + y^2\right)$

Question 9

Factorize:

$32a^3 + 108b^3$

Solution 9

$32a^3 + 108b^3$

$= 4\left(8a^3 + 27b^3\right)$

$= 4\left((2a)^3 + (3b)^3\right)$ $\left[\text{Using } a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$

$= 4\left[(2a + 3b)\left((2a)^2 - 2a \times 3b + (3b)^2\right)\right]$

$= 4(2a + 3b)\left(4a^2 - 6ab + 9b^2\right)$

$\therefore\ 32a^3 + 108b^3 = 4(2a + 3b)\left(4a^2 - 6ab + 9b^2\right)$

Question 10

Factorize:

$(a - 2b)^3 - 512b^3$

Solution 10

$(a - 2b)^3 - 512b^3$

$= (a - 2b)^3 - (8b)^3$

$= (a - 2b - 8b)\left((a - 2b)^2 + (a - 2b)8b + (8b)^2\right)$ $\left[\because\ a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)\right]$

$= (a - 10b)\left(a^2 + 4b^2 - 4ab + 8b(a - 2b) + (8b)^2\right)$ $\left[\because\ (a - b)^2 = a^2 + b^2 - 2ab\right]$

$= (a - 10b)\left(a^2 + 4b^2 - 4ab + 8ab - 16b^2 + 64b^2\right)$

$= (a - 10b)\left(a^2 + 68b^2 - 16b^2 - 4ab + 8ab\right)$

$= (a - 10b)\left(a^2 + 52b^2 + 4ab\right)$

$\therefore\ (a - 2b)^3 - 512b^3 = (a - 10b)\left(a^2 + 4ab + 52b^2\right)$

Question 11

Factorize:

$8x^2y^3 - x^5$

Solution 11

$8x^2y^3 - x^5$

$= x^2\left(8y^3 - x^3\right)$

$= x^2\left(\left(2y\right)^3 - x^3\right)$

$= x^2\left(2y - x\right)\left(\left(2y\right)^2 + 2y\left(x\right) + x^2\right)$ $\quad\left[\because\ a^3 - b^3 = \left(a - b\right)\left(a^2 + ab + b^2\right)\right]$

$= x^2\left(2y - x\right)\left(4y^2 + 2xy + x^2\right)$

$\therefore\quad 8x^2y^3 - x^5 = x^2\left(2y - x\right)\left(4y^2 + 2xy + x^2\right)$

Question 12

Factorize:

$1029 - 3x^3$

Solution 12

$1029 - 3x^3$

$= 3\left(343 - x^3\right)$

$= 3\left(7^3 - x^3\right)$

$= 3\left(7 - x\right)\left(7^2 + 7 \times x + x^2\right)$ $\quad\left[\because\ a^3 - b^3 = \left(a - b\right)\left(a^2 + ab + b^2\right)\right]$

$= 3\left(7 - x\right)\left(49 + 7x + x^2\right)$

$\therefore\quad 1029 - 3x^3 = 3\left(7 - x\right)\left(49 + 7x + x^2\right)$

Question 13

Factorize:

$x^3y^3 + 1$

Solution 13

$x^3y^3 + 1$

$= \left(xy\right)^3 + 1^3$

$= \left(xy + 1\right)\left(\left(xy\right)^2 - xy \times 1 + 1^2\right)$ $\quad\left[\because\ a^3 + b^3 = \left(a + b\right)\left(a^2 - ab + b^2\right)\right]$

$= \left(xy + 1\right)\left(x^2y^2 - xy + 1\right)$

$\therefore\quad x^3y^3 + 1 = \left(xy + 1\right)\left(x^2y^2 - xy + 1\right)$

Question 14

Factorize:

$x^4y^4 - xy$

Solution 14

$x^4y^4 - xy$

$= xy\left(x^3y^3 - 1\right)$

$= xy\left(\left(xy\right)^3 - 1^3\right)$

$= xy\left(xy - 1\right)\left(\left(xy\right)^2 + \left(xy\right)1 + 1^2\right) \qquad \left[\because\ a^3 - b^3 = \left(a - b\right)\left(a^2 + ab + b^2\right)\right]$

$= xy\left(xy - 1\right)\left(x^2y^2 + xy + 1\right)$

$\therefore\ x^4y^4 - xy = xy\left(xy - 1\right)\left(x^2y^2 + xy + 1\right)$

Question 15

Factorize:

$a^3 + b^3 + a + b$

Solution 15

$a^3 + b^3 + a + b$

$= \left(a^3 + b^3\right) + 1\left(a + b\right)$

$= \left(a + b\right)\left(a^2 - ab + b^2\right) + 1\left(a + b\right)$

$= \left(a + b\right)\left(a^2 - ab + b^2 + 1\right)$

$\therefore\ a^3 + b^3 + a + b = \left(a + b\right)\left(a^2 - ab + b^2 + 1\right)$

Question 16

Simplify:

(i) $\dfrac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

(ii) $\dfrac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

(iii) $\dfrac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2}$

Solution 16

(i) $\dfrac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

$= \dfrac{173^3 + 127^3}{173^2 - 173 \times 127 + 127^2}$

$= \dfrac{(173 + 127)(173^2 - 173 \times 127 + 127^2)}{(173^2 - 173 \times 127 + 127^2)}$ $\left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$

$= (173 + 127) = 300$

(ii) $\dfrac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

$= \dfrac{155^3 - 55^3}{155^2 + 155 \times 55 + 55^2}$

$= \dfrac{(155 - 55)(155^2 + 155 \times 55 + 55^2)}{(155^2 + 155 \times 55 + 55^2)}$ $\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$

$= (155 - 55) = 100$

(iii) $\dfrac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2}$

$= \dfrac{1.2^3 - 0.2^3}{1.2^2 + 1.2 \times 0.2 + 0.2^2}$

$= \dfrac{(1.2 - 0.2)(1.2^2 + 1.2 \times 0.2 + 0.2^2)}{(1.2^2 + 1.2 \times 0.2 + 0.2^2)}$ $\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$

$= (1.2 - 0.2)$

$= 1$

Question 17

Factorize:

$(a + b)^3 - 8(a - b)^3$

Solution 17

$$(a+b)^3 - 8(a-b)^3$$

$$= (a+b)^3 - [2(a-b)]^3$$
$$= (a+b)^3 - (2a-2b)^3 \qquad \left[\text{Using } a^3 - b^3 = (a-b)(a^2+ab+b^2)\right]$$
$$= (a+b-(2a-2b))\left((a+b)^2 + (a+b)(2a-2b) + (2a-2b)^2\right)$$
$$= (a+b-2a+2b)\left(a^2+b^2+2ab+(a+b)(2a-2b)+(2a-2b)^2\right) \qquad \left[\because (a+b)^2 = a^2+b^2+2ab\right]$$
$$= (3b-a)\left(a^2+b^2+2ab+2a^2-2ab+2ab-2b^2+(2a-2b)^2\right)$$
$$= (3b-a)\left(3a^2+2ab-b^2+(2a-2b)^2\right)$$
$$= (3b-a)\left(3a^2+2ab-b^2+4a^2+4b^2-8ab\right) \qquad \left[\because (a-b)^2 = a^2+b^2-2ab\right]$$
$$= (3b-a)\left(3a^2+4a^2-b^2+4b^2+2ab-8ab\right)$$

$$= (3b-a)\left(7a^2+3b^2-6ab\right)$$

$$\therefore\ (a+b)^3 - 8(a-b)^3 = (-a+3b)\left(7a^2-6ab+3b^2\right)$$

Question 18

Factorize:

$$(x+2)^3 + (x-2)^3$$

Solution 18

$$(x+2)^3 + (x-2)^3$$

$$= (x+2+x-2)\left((x+2)^2 - (x+2)(x-2) + (x-2)^2\right) \qquad \left[\because\ a^3+b^3 = (a+b)(a^2-ab+b^2)\right]$$
$$= 2x\left(x^2+4x+4-(x+2)(x-2)+x^2-4x+4\right) \qquad \left[\because (a+b)^2 = a^2+2ab+b^2,\ (a-b)^2 = a^2\right.$$
$$= 2x\left(2x^2+8-(x^2-2^2)\right) \qquad \left[\because (a+b)(a-b) = a^2-b^2\right]$$
$$= 2x\left(2x^2+8-x^2+4\right)$$

$$= 2x\left(x^2+12\right)$$

$$\therefore\ (x+2)^3 + (x-2)^3 = 2x\left(x^2+12\right)$$

Question 19

Factorize:

$$x^6 + y^6$$

Solution 19

$x^6 + y^6$

$= (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)\left((x^2)^2 - x^2y^2 + (y^2)^2\right)$ $\left[\because\ a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$\therefore\ x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Question 20

Factorize:

$a^{12} + b^{12}$

Solution 20

$a^{12} + b^{12}$

$= (a^4)^3 + (b^4)^3$

$= (a^4 + b^4)\left((a^4)^2 - a^4 \times b^4 + (b^4)^2\right)$ $\left[\because\ a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$

$= (a^4 + b^4)(a^8 - a^4b^4 + b^8)$

$\therefore\ a^{12} + b^{12} = (a^4 + b^4)(a^8 - a^4b^4 + b^8)$

Question 21

Factorize:

$x^3 + 6x^2 + 12x + 16$

Solution 21

$x^3+6x^2+12x+16$

$= x^3+6x^2+12x+8+8$

$= x^3+3\times x^2\times 2+3\times x\times 2^2+2^3+8$

$= (x+2)^3+8$ $\left[\because\ a^3+3a^2b+3ab^2+b^3=(a+b)^3\right]$

$= (x+2)^3+2^3$

$= (x+2+2)\left((x+2)^2-2(x+2)+2^2\right)$ $\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$= (x+4)\left(x^2+4x+4-2x-4+4\right)$ $\left[\because (a+b)^2=a^2+2ab+b^2\right]$

$= (x+4)\left(x^2+2x+4\right)$

$\therefore\ x^3+6x^2+12x+16=(x+4)\left(x^2+2x+4\right)$

Question 22

Factorize:

$$a^3-\frac{1}{a^3}-2a+\frac{2}{a}$$

Solution 22

$a^3-\frac{1}{a^3}-2a+\frac{2}{a}$

$= \left(a^3-\frac{1}{a^3}\right)-2\left(a-\frac{1}{a}\right)$

$= \left(a^3-\left(\frac{1}{a}\right)^3\right)-2\left(a-\frac{1}{a}\right)$

$= \left(a-\frac{1}{a}\right)\left(a^2+a\times\frac{1}{a}+\left(\frac{1}{a}\right)^2\right)-2\left(a-\frac{1}{a}\right)$ $\left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$= \left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}\right)-2\left(a-\frac{1}{a}\right)$

$= \left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}-2\right)$

$= \left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$

$\therefore\ a^3-\frac{1}{a^3}-2a+\frac{2}{a}=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$

Question 23

Factorize:

$a^3+3a^2b+3ab^2+b^3-8$

Solution 23

$a^3+3a^2b+3ab^2+b^3-8$

$= (a+b)^3 - 8 \qquad \left[\because a^3+3a^2b+3ab^2+b^3 = (a+b)^3\right]$

=

$= (a+b)^3 - 2^3$

$= (a+b-2)\left((a+b)^2+(a+b)2+2^2\right)$

$= (a+b-2)\left(a^2+b^2+2ab+2a+2b+4\right)$

$\therefore \quad a^3+3a^2b+3ab^2+b^3-8 = (a+b-2)\left(a^2+b^2+2ab+2a+2b+4\right)$

Question 24

Factorize:

$8a^3-b^3-4ax+2bx$

Solution 24

$8a^3-b^3-4ax+2bx$

$= 8a^3-b^3-2x(2a-b)$

$= (2a)^3-b^3-2x(2a-b)$

$= (2a-b)\left((2a)^2+2a\times b+b^2\right)-2x(2a-b) \qquad \left[\because a^3-b^3 = (a-b)\left(a^2+ab+b^2\right)\right]$

$= (2a-b)\left(4a^2+2ab+b^2\right)-2x(2a-b)$

$= (2a-b)\left(4a^2+2ab+b^2-2x\right)$

$\therefore \quad 8a^3-b^3-4ax+2bx = (2a-b)\left(4a^2+2ab+b^2-2x\right)$

Chapter 5 - Factorisation of Algebraic Expressions Exercise Ex. 5.3

Question 1

Factorize:

$64a^3+125b^3+240a^2b+300ab^2$

Solution 1

$64a^3 + 125b^3 + 240a^2b + 300ab^2$

$= (4a)^3 + (5b)^3 + 3 \times (4a)^2 \times 5b + 3(4a)(5b)^2$

$= (4a + 5b)^3$ $\qquad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$

$= (4a + 5b)(4a + 5b)(4a + 5b)$

$\therefore \quad 64a^3 + 125b^3 + 240a^2b + 300ab^2 = (4a + 5b)(4a + 5b)(4a + 5b)$

Question 2

Factorize:

$125x^3 - 27y^3 - 225x^2y + 135xy^2$

Solution 2

$125x^3 - 27y^3 - 225x^2y + 135xy^2$

$= (5x)^3 - (3y)^3 - 3 \times (5x)^2 \times 3y + 3 \times (5x)(3y)^2$

$= (5x - 3y)^3$ $\qquad \left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3\right]$

$= (5x - 3y)(5x - 3y)(5x - 3y)$

$\therefore \quad 125x^3 - 27y^3 - 225x^2y + 135xy^2 = (5x - 3y)(5x - 3y)(5x - 3y)$

Question 3

Factorize:

$\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$

Solution 3

$$\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$$

$$= \left(\frac{2}{3}x\right)^3 + (1)^3 + 3 \times \left(\frac{2}{3}x\right)^2 \times 1 + 3(1)^2 \times \left(\frac{2}{3}x\right)$$

$$= \left(\frac{2}{3}x + 1\right)^3 \qquad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$$

$$= \left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)$$

$$\therefore \quad \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x = \left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)\left(\frac{2}{3}x + 1\right)$$

Question 4

Factorize:

$$8x^3 + 27y^3 + 36x^2y + 54xy^2$$

Solution 4

$$8x^3 + 27y^3 + 36x^2y + 54xy^2$$

$$= (2x)^3 + (3y)^3 + 3 \times (2x)^2 \times 3y + 3 \times (2x)(3y)^2$$

$$= (2x + 3y)^3 \qquad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$$

$$= (2x + 3y)(2x + 3y)(2x + 3y)$$

$$\therefore \quad 8x^3 + 27y^3 + 36x^2y + 54xy^2 = (2x + 3y)(2x + 3y)(2x + 3y)$$

Question 5

Factorize:

$$a^3 - 3a^2b + 3ab^2 - b^3 + 8$$

Solution 5

$a^3 - 3a^2b + 3ab^2 - b^3 + 8$

$= (a-b)^3 + 8$ $\left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3\right]$

$= (a-b)^3 + 2^3$

$= (a-b+2)\left((a-b)^2 - (a-b)2 + 2^2\right)$ $\left[\because a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$

$= (a-b+2)\left(a^2 + b^2 - 2ab - 2(a-b) + 4\right)$

$= (a-b+2)\left(a^2 + b^2 - 2ab - 2a + 2b + 4\right)$

$\therefore \quad a^3 - 3a^2b + 3ab^2 - b^3 + 8 = (a-b+2)\left(a^2 + b^2 - 2ab - 2a + 2b + 4\right)$

Question 6

Factorize:

$x^3 + 8y^3 + 6x^2y + 12xy^2$

Solution 6

$x^3 + 8y^3 + 6x^2y + 12xy^2$

$= (x)^3 + (2y)^3 + 3 \times x^2 \times 2y + 3 \times x \times (2y)^2$

$= (x+2y)^3$ $\left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$

$= (x+2y)(x+2y)(x+2y)$

$\therefore \quad x^3 + 8y^3 + 6x^2y + 12xy^2 = (x+2y)(x+2y)(x+2y)$

Question 7

Factorize:

$8x^3 + y^3 + 12x^2y + 6xy^2$

Solution 7

$8x^3 + y^3 + 12x^2y + 6xy^2$

$= (2x)^3 + y^3 + 3 \times (2x)^2 \times y + 3(2x) \times y^2$

$= (2x+y)^3$ $\left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$

$= (2x+y)(2x+y)(2x+y)$

$\therefore \quad 8x^3 + y^3 + 12x^2y + 6xy^2 = (2x+y)(2x+y)(2x+y)$

Question 8
Factorize:

$8a^3 + 27b^3 + 36a^2b + 54ab^2$

Solution 8
$8a^3 + 27b^3 + 36a^2b + 54ab^2$

$= (2a)^3 + (3b)^3 + 3 \times (2a)^2 \times 3b + 3 \times (2a)(3b)^2$
$= (2a + 3b)^3 \qquad \left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$

$= (2a + 3b)(2a + 3b)(2a + 3b)$

$\therefore \quad 8a^3 + 27b^3 + 36a^2b + 54ab^2 = (2a + 3b)(2a + 3b)(2a + 3b)$

Question 9
Factorize:

$8a^3 - 27b^3 - 36a^2b + 54ab^2$

Solution 9
$8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= (2a)^3 - (3b)^3 - 3 \times (2a)^2 3b + 3(2a)(3b)^2$
$= (2a - 3b)^3 \qquad \left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3\right]$

$= (2a - 3b)(2a - 3b)(2a - 3b)$

$\therefore \quad 8a^3 - 27b^3 - 36a^2b + 54ab^2 = (2a - 3b)(2a - 3b)(2a - 3b)$

Question 10
Factorize:

$x^3 - 12x(x - 4) - 64$

Solution 10

$x^3 - 12x(x-4) - 64$

$= x^3 - 12x^2 + 48x - 64$
$= (x)^3 - 3 \times x^2 \times 4 + 3 \times 4^2 \times x - 4^3$
$= (x-4)^3$ $\qquad \left[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3\right]$

$= (x-4)(x-4)(x-4)$

$\therefore \quad x^3 - 12x(x-4) - 64 = (x-4)(x-4)(x-4)$

Question 11
Factorize:

$a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

Solution 11
$a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

$= (ax)^3 - 3(ax)^2 \times b + 3(ax)b^2 - b^3$
$= (ax-b)^3$ $\qquad \left[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3\right]$

$= (ax-b)(ax-b)(ax-b)$

$\therefore \quad a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3 = (ax-b)(ax-b)(ax-b)$

Chapter 5 - Factorisation of Algebraic Expressions Exercise Ex. 5.4

Question 1
Factorize:

$a^3 + 8b^3 + 64c^3 - 24abc$

Solution 1

$a^3 + 8b^3 + 64c^3 - 24abc$

$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$

$= (a + 2b + 4c)\left(a^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a\right)$

$\left[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)\right]$

$= (a + 2b + 4c)\left(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac\right)$

$\therefore \quad a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)\left(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac\right)$

Question 2

Factorize:

$x^3 - 8y^3 + 27z^3 + 18xyz$

Solution 2

$x^3 - 8y^3 + 27z^3 + 18xyz$

$= x^3 + (-2y)^3 + (3z)^3 - 3 \times x \times (-2y)(3z)$

$= (x + (-2y) + 3z)\left(x^2 + (-2y)^2 + (3z)^2 - x(-2y) - (-2y)(3z) - 3z(x)\right)$

$\left[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)\right]$

$= (x - 2y + 3z)\left(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx\right)$

$\therefore \quad x^3 - 8y^3 + 27z^3 + 18xyz = (x - 2y + 3z)\left(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx\right)$

Question 3

Factorise : $27x^3 - y^3 - z^3 - 9xyz$

Solution 3

We Know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\left(x^2 + y^2 + z^2 - xy - yz - zx\right)$

$\therefore 27x^3 - y^3 - z^3 - 9xyz$

$= (3x)^3 + (-y)^3 + (-z)^3 - 3(3x)(-y)(-z)$

$= [3x + (-y) + (-z)][(3x)^2 + (-y)^2 + (-z)^2 - (3x)(-y) - (-y)(-z) - (-z)(3x)]$

$= (3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

Question 4

Factorize:

$\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

Solution 4

$\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

$= \left(\frac{x}{3}\right)^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3}(-y)(5z)$

$= \left(\frac{x}{3} + (-y) + 5z\right)\left(\left(\frac{x}{3}\right)^2 + (-y)^2 + (5z)^2 - \frac{x}{3}(-y) - (-y)5z - 5z\left(\frac{x}{3}\right)\right)$

$= \left(\frac{x}{3} - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5}{3}zx\right)$

$\therefore \quad \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{x}{3} - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5}{3}zx\right)$

Question 5

Factorize:

$8x^3 + 27y^3 - 216z^3 + 108xyz$

Solution 5

$8x^3 + 27y^3 - 216z^3 + 108xyz$

$= (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$

$= (2x + 3y + (-6z))\left((2x)^2 + (3y)^2 + (-6z)^2 - 2x \times 3y - 3y(-6z) - (-6z)2x\right)$

$= (2x + 3y - 6z)\left(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx\right)$

$\therefore \quad 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x + 3y - 6z)\left(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx\right)$

Question 6

Factorize:

$125 + 8x^3 - 27y^3 + 90xy$

Solution 6

$125+8x^3-27y^3+90xy$

$= 5^3+(2x)^3+(-3y)^3-3\times 5\times 2x\times(-3y)$

$= (5+2x+(-3y))(5^2+(2x)^2+(-3y)^2-5(2x)-2x(-3y)-(-3y)5)$

$= (5+2x-3y)(25+4x^2+9y^2-10x+6xy+15y)$

$\therefore\quad 125+8x^3-27y^3+90xy = (5+2x-3y)(25+4x^2+9y^2-10x+6xy+15y)$

Question 7

Factorize:

$8x^3-125y^3+180xy+216$

Solution 7

$8x^3-125y^3+180xy+216$

or, $8x^3-125y^3+216+180xy$

$= (2x)^3+(-5y)^3+6^3-3\times(2x)(-5y)(6)$

$= (2x+(-5y)+6)((2x)^2+(-5y)^2+6^2-2x(-5y)-(-5y)6-6(2x))$

$= (2x-5y+6)(4x^2+25y^2+36+10xy+30y-12x)$

$\therefore 8x^3-125y^3+180xy+216 = (2x-5y+6)(4x^2+25y^2+36+10xy+30y-12x)$

Question 8

Multiply:

(*i*) $x^2+y^2+z^2-xy+xz+yz$ *by* $x+y-z$

(*ii*) $x^2+4y^2+z^2+2xy+xz-2yz$ *by* $x-2y-z$

(*iii*) $x^2+4y^2+2xy-3x+6y+9$ *by* $x-2y+3$

(*iv*) $9x^2+25y^2+15xy+12x-20y+16$ *by* $3x-5y+4$

Solution 8

(i) $\left(x^2 + y^2 + z^2 - xy + xz + yz\right)$ by $\left(x + y - z\right)$

$= \left(x + y - z\right)\left(x^2 + y^2 + z^2 - xy + xz + yz\right)$

$= \left(x + y + (-z)\right)\left(x^2 + y^2 + (-z)^2 - xy - y\,(-z) - (-z)\,x\right)$

$= x^3 + y^3 + (-z)^3 - 3xy\,(-z)$ $\left[\because (a + b + c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right) = a^3 + b^3 + c^3 - 3abc\right]$

$= x^3 + y^3 - z^3 + 3xyz$

(ii) $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $\left(x - 2y - z\right)$

$= \left(x - 2y - z\right)\left(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz\right)$

$= \left(x + (-2y) + (-z)\right)\left(x^2 + (-2y)^2 + (-z)^2 - x\,(-2y) - (-2y)\,(-z) - (-z)\,x\right)$

$= x^3 + (-2y)^3 + (-z)^3 - 3 \times x\,(-2y)\,(-z)$ $\left[\because (a + b + c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right) = a^3 + b^3 + c^3 - 3abc\right.$

$= x^3 - 8y^3 - z^3 - 3 \times x \times 2yz$

$= x^3 - 8y^3 - z^3 - 6xyz$

(iii) $\left(x^2 + 4y^2 + 2xy - 3x + 6y + 9\right)$ by $\left(x - 2y + 3\right)$

$= \left(x - 2y + 3\right)\left(x^2 + 4y^2 + 9 + 2xy + 6y - 3x\right)$

$= \left(x + (-2y) + 3\right)\left(x^2 + (-2y)^2 + 3^2 - x\,(-2y) - (-2y)\,3 - 3x\right)$

$= x^3 + (-2y)^3 + 3^3 - 3 \times x\,(-2y)^3$ $\left[\because (a + b + c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right) = a^3 + b^3 + c^3 - 3abc\right]$

$= x^3 - 8y^3 + 27 + 18xy$

(iv) $\left(9x^2+25y^2+15xy+20y-12x+16\right)$ by $(3x-5y+4)$

$= (3x-5y+4)\left(9x^2+25y^2+15xy+20y-12x+16\right)$

$= \left(3x+(-5y)+4\right)\left\{(3x)^2+(-5y)^2+4^2-3x(-5y)-(-5y)4-4(3x)\right\}$

Using identity

$(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)=a^3+b^3+c^3-3abc$

Here, $a=3x$, $b=-5y$, $c=4$

$= (3x)^3+(-5y)^3+4^3-3(3x)(-5y)(4)$

$= 27x^3-125y^3+64+180xy$

$\therefore \quad (3x-5y+4)\left(9x^2+25y^2+15xy+20y-12x+16\right)=27x^3-125y^3+64+180xy$

Question 9

Factorize:

$(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Solution 9

$(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Let $(3x-2y)=a$, $(2y-4z)=b$, $(4z-3x)=c$

$\therefore \; a+b+c=3x-2y+2y-4z+4z-3x=0$

$\because \; a+b+c=0 \; \therefore \; a^3+b^3+c^3=3abc$

$\therefore \quad (3x-2y)^3+(2y-4z)^3+(4z-3x)^3=3(3x-2y)(2y-4z)(4z-3x)$

Question 10

Factorize:

$(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Solution 10

$(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Let $2x-3y=a,\ 4z-2x=b,\ 3y-4z=c$

$\therefore\ a+b+c=2x-3y+4z-2x+3y-4z=0$

$\because\ a+b+c=0\ \therefore\ a^3+b^3+c^3=3abc$

$\therefore\ (2x-3y)^3+(4z-2x)^3+(3y-4z)^3=3(2x-3y)(4z-2x)(3y-4z)$

Question 11

Factorize

$$\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$$

Solution 11

$$\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$$

Let $\left(\frac{x}{2}+y+\frac{z}{3}\right)=a,\left(\frac{x}{3}-\frac{2y}{3}+z\right)=b,\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)=c$

$$a+b+c=\frac{x}{2}+y+\frac{z}{3}+\frac{x}{3}-\frac{2y}{3}+z-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}$$

$$a+b+c=\left(\frac{x}{2}+\frac{x}{3}-\frac{5x}{6}\right)+\left(y-\frac{2y}{3}-\frac{y}{3}\right)+\left(\frac{z}{3}+z-\frac{4z}{3}\right)$$

$$a+b+c=\frac{3x}{6}+\frac{2x}{6}-\frac{5x}{6}+\frac{3y}{3}-\frac{2y}{3}-\frac{y}{3}+\frac{z}{3}+\frac{3z}{3}-\frac{4z}{3}$$

$$a+b+c=\frac{5x-5x}{6}+\frac{3y-3y}{3}+\frac{4z-4z}{3}$$

$a+b+c=0$

$\because a+b+c=0\qquad\therefore\quad a^3+b^3+c^3=3abc$

$$\therefore\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3=3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$$

Question 12

Factorize:

$(a-3b)^3+(3b-c)^3+(c-a)^3$

Solution 12

$$(a-3b)^3+(3b-c)^3+(c-a)^3$$

Let $(a-3b)=x, (3b-c)=y, (c-a)=z$

$$x+y+z=a-3b+3b-c+c-a=0$$

$\because x+y+z=0 \qquad \therefore x^3+y^3+z^3=3xyz$

$$\therefore (a-3b)^3+(3b-c)^3+(c-a)^3=3(a-3b)(3b-c)(c-a)$$

Question 13

Factorize:

$$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$$

Solution 13

$$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$$

$$=\left(\sqrt{2}a\right)^3+\left(\sqrt{3}b\right)^3+c^3-3\times\sqrt{2}a\times\sqrt{3}b\times c$$

$$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(\left(\sqrt{2}a\right)^2+\left(\sqrt{3}b\right)^2+c^2-\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)-\left(\sqrt{3}b\right)c-\left(\sqrt{2}a\right)c\right)$$

$$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$$

$$\therefore \quad 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$$

Question 14

Factorize:

$$3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$$

Solution 14

$$3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$$

$$=\left(\sqrt{3}a\right)^3+(-b)^3+\left(-\sqrt{5}c\right)^3-3\times\left(\sqrt{3}a\right)(-b)\left(-\sqrt{5}c\right)$$

$$=\left(\sqrt{3}a+(-b)+\left(-\sqrt{5}c\right)\right)\left(\left(\sqrt{3}a\right)^2+(-b)^2+\left(-\sqrt{5}c\right)^2-\sqrt{3}a(-b)-(-b)\left(-\sqrt{5}c\right)-\left(-\sqrt{5}c\right)\sqrt{3}a\right)$$

$$=\left(\sqrt{3}a-b-\sqrt{5}c\right)\left(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ac\right)$$

$$\therefore \quad 3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc=\left(\sqrt{3}a-b-\sqrt{5}c\right)\left(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ac\right)$$

Question 15

Factorize:

$$2\sqrt{2}a^3+16\sqrt{2}b^3+c^3-12abc$$

Solution 15

$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

$= \left(\sqrt{2}a\right)^3 + \left(2\sqrt{2}b\right)^3 + c^3 - 3 \times \sqrt{2}a \times 2\sqrt{2}b \times c$

$= \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(\left(\sqrt{2}a\right)^2 + \left(2\sqrt{2}b\right)^2 + c^2 - \left(\sqrt{2}a\right)\left(2\sqrt{2}b\right) - \left(2\sqrt{2}b\right)c - \left(\sqrt{2}a\right)c\right)$

$= \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac\right)$

$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac\right)$

Question 16

Find the value of $x^3 + y^3 - 12xy + 64$, when $x + y = -4$

Solution 16

$\because x + y = -4$

$\therefore x + y + 4 = 0 \qquad --(1)$

Now, $x^3 + y^3 - 12xy + 64$

$= x^3 + y^3 + 64 - 12xy$

$= (x)^3 + y^3 + 4^3 - 3 \times x \times y \times 4$

$= (x + y + 4)\left(x^2 + y^2 + 16 - xy - 4y - 4x\right)$

$= 0\left(x^2 + y^2 + 16 - xy - 4y - 4x\right) \qquad [\text{from } (1)]$

$= 0$

$\therefore \quad x^3 + y^3 - 12xy + 64 = 0$ when $x + y = -4$