NCERT Solutions for Class 10 Maths Chapter 5 - Arithmetic Progressions

Chapter 5 - Arithmetic Progressions Exercise Ex. 5.1

Solution 1

(i) From the given data we see that
Taxi fare for 1^{st} km = 15
Taxi fare for first 2 kms = 15 + 8 = 23
Taxi fare for first 3 kms = 23 + 8 = 31
Taxi fare for first 4 kms = 31 + 8 = 39
Clearly 15, 23, 31, 39 ... forms an AP since every term is 8 more than the preceding term.

(ii) Let the initial volume of air in a cylinder be V liters. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time. In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}$th part of air will remain.

Therefore, volumes will be $V, \left(\frac{3V}{4}\right), \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V$..

Clearly, the adjacent terms of this sequence do not have constant difference between them. Therefore, this is not an AP

(iii) Cost of digging for first metre = 150
Cost of digging for first 2 metres = 150 + 50 = 200
Cost of digging for first 3 metres = 200 + 50 = 250
Cost of digging for first 4 metres = 250 + 50 = 300
Clearly, 150, 200, 250, 300 ... forms an AP because every term is 50 more than its preceding term.

(iv)We know that if Rs P is deposited at r % compound interest per annum for n years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after n years.
Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2, 10000\left(1+\frac{8}{100}\right)^3, 10000\left(1+\frac{8}{100}\right)^4, \ldots$$

Clearly, consecutive terms of this sequence, do not have the same difference between them. Therefore, this is not an AP

Concept Insight: To crack such problems read the question carefully, analyse and then write the data the initial value changes in a particular pattern. But the sequence represents an arithmetic progression if the difference of any two consecutive terms is constant.

Solution 2

(i) $a = 10, d = 10$
Let the series be a_1, a_2, a_3, a_4, a_5 ...
$a_1 = a = 10$
$a_2 = a_1 + d = 10 + 10 = 20$
$a_3 = a_2 + d = 20 + 10 = 30$
$a_4 = a_3 + d = 30 + 10 = 40$
$a_5 = a_4 + d = 40 + 10 = 50$
Therefore, the series will be 10, 20, 30, 40, 50 ...
First four terms of this AP will be 10, 20, 30, and 40.

(ii) $a = -2, d = 0$
Let the series be a_1, a_2, a_3, a_4 ...
$a_1 = a = -2$
$a_2 = a_1 + d = -2 + 0 = -2$
$a_3 = a_2 + d = -2 + 0 = -2$
$a_4 = a_3 + d = -2 + 0 = -2$
Therefore, the series will be −2, −2, −2, −2 ...
First four terms of this AP will be −2, −2, −2 and −2.

(iii) $a = 4, d = -3$
Let the series be a_1, a_2, a_3, a_4 ...
$a_1 = a = 4$
$a_2 = a_1 + d = 4 - 3 = 1$
$a_3 = a_2 + d = 1 - 3 = -2$
$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, −2 −5 ...
First four terms of this AP will be 4, 1, −2 and −5.

(iv) $a = -1, d = \frac{1}{2}$
Let the series be a_1, a_2, a_3, a_4 ...
$a_1 = a = -1$
$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$
$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$
$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$
Clearly, the series will be
$-1, -\frac{1}{2}, 0, \frac{1}{2}$ ....
First four terms of this AP will be $-1, -\frac{1}{2}, 0$ and $\frac{1}{2}$.

(v) $a = -1.25, d = -0.25$
Let the series be a_1, a_2, a_3, a_4 ...
$a_1 = a = -1.25$
$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$
$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$
$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$
Clearly, the series will be 1.25, −1.50, −1.75, −2.00 ....
First four terms of this AP will be −1.25, −1.50, −1.75 and −2.00.

Concept Insight: Remember the basic definition of an AP it can be generated given its first term and common difference by adding the common difference to the previous term i.e a,a+d,a+2d,.....a+(n-1)d

Solution 3

(i) 3, 1, −1, −3 ...
Here, first term , a = 3
Common difference, d = Second term − First term
= 1 − 3 = −2

(ii) −5, −1, 3, 7 ...
Here, first term , a = −5
Common difference, d = Second term − First term
= (−1) − (−5) = − 1 + 5 = 4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

Here, first term , $a = \frac{1}{3}$

Common difference, d = Second term − First term

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 ...
Here, first term , a = 0.6
Common difference, d = Second term − First term
= 1.7 − 0.6
= 1.1

Concept Insight: Remember the basic definition of an AP common difference is the difference of consecutive terms, other terms can be generated by following general pattern a,a+d,a+2d,.....a+(n-1)d

Solution 4

(i) 2, 4, 8, 16 …

Here

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

i.e., $a_{k+1} - a_k$ is not the same every time. Therefore, the given numbers are not in AP

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}$ …

Here

$a_2 - a = \frac{5}{2} - 2 = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, $d = \frac{1}{2}$ and the given numbers are in AP

Three more terms are

$a_5 = \frac{7}{2} + \frac{1}{2} = 4$

$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$

$a_7 = \frac{9}{2} + \frac{1}{2} = 5$

(iii) −1.2, −3.2, −5.2, −7.2 …

We have $a_2 - a_1 = (-3.2) - (-1.2) = -2$

$a_3 - a_2 = (-5.2) - (-3.2) = -2$

$a_4 - a_3 = (-7.2) - (-5.2) = -2$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -2$

The given numbers are in AP

Three more terms are

$a_5 = -7.2 - 2 = -9.2$

$a_6 = -9.2 - 2 = -11.2$

$a_7 = -11.2 - 2 = -13.2$

(iv) −10, −6, −2, 2 …

Here

$a_2 - a_1 = (-6) - (-10) = 4$

$a_3 - a_2 = (-2) - (-6) = 4$

$a_4 - a_3 = (2) - (-2) = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = 4$

The given numbers are in AP

Three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

Here

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = \sqrt{2}$

The given numbers are in AP

Three more terms are

$a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

$a_5 = -\frac{1}{2} - 0 = -\frac{1}{2}$

$a_6 = -\frac{1}{2} - 0 = -\frac{1}{2}$

$a_7 = -\frac{1}{2} - 0 = -\frac{1}{2}$

(ix) 1, 3, 9, 27 ...
Here
$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$
$a_4 - a_3 = 27 - 9 = 18$
i.e., $a_{k+1} - a_k$ is not the same every time.
Therefore, the given numbers are not in AP

(x) $a, 2a, 3a, 4a$ ...
Here
$a_2 - a_1 = 2a - a = a$
$a_3 - a_2 = 3a - 2a = a$
$a_4 - a_3 = 4a - 3a = a$
i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = a$
The given numbers are in AP
Three more terms are
$a_5 = 4a + a = 5a$
$a_6 = 5a + a = 6a$
$a_7 = 6a + a = 7a$

(xi) a, a^2, a^3, a^4 ...
Here
$a_2 - a_1 = a^2 - a = a(a - 1)$
$a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$
$a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$
i.e., $a_{k+1} - a_k$ is not the same every time.
Therefore, the given numbers are not in AP

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ...
Here
$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$
i.e., $a_{k+1} - a_k$ is same every time.
Therefore, the given numbers are in AP
And, $d = \sqrt{2}$
Three more terms are
$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$ ...
Here
$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3 \times 2} - \sqrt{3} = \sqrt{3}\left(\sqrt{2} - 1\right)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}\left(\sqrt{3} - \sqrt{2}\right)$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3 \times 3} = \sqrt{3}\left(2 - \sqrt{3}\right)$
i.e., $a_{k+1} - a_k$ is not the same every time.
Therefore, the given numbers are not in AP

(xiv) $1^2, 3^2, 5^2, 7^2$ ...
Or, 1, 9, 25, 49 .....

Chapter 5 - Arithmetic Progressions Exercise Ex. 5.2

Solution 1

I. $a = 7$, $d = 3$, $n = 8$, $a_n = ?$

We know that,

For an AP $a_n = a + (n - 1) d$

$= 7 + (8 - 1) 3$

$= 7 + (7) \times 3$

$= 7 + 21 = 28$

Hence, $a_n = 28$

II. Given that

$a = -18$, $n = 10$, $a_n = 0$, $d = ?$

We know that,

$a_n = a + (n - 1) d$

$0 = - 18 + (10 - 1) d$

$18 = 9d$

$$d = \frac{18}{9} = 2$$

Hence, common difference, $d = 2$

III. Given that

$d = -3$, $n = 18$, $a_n = -5$

We know that,

$a_n = a + (n - 1) d$

$-5 = a + (18 - 1) (-3)$

$-5 = a + (17) (-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, $a = 46$

IV. $a = -18.9$, $d = 2.5$, $a_n = 3.6$, $n = ?$

We know that,

$a_n = a + (n - 1) d$

$3.6 = - 18.9 + (n - 1) 2.5$

$3.6 + 18.9 = (n - 1) 2.5$

$22.5 = (n - 1) 2.5$

$$(n - 1) = \frac{22.5}{2.5}$$

$n - 1 = 9$

$n = 10$

Hence, $n = 10$

V. $a = 3.5$, $d = 0$, $n = 105$, $a_n = ?$

We know that,

$a_n = a + (n - 1) d$

$a_n = 3.5 + (105 - 1) 0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence, $a_n = 3.5$

Concept Insight: Three things determine an AP

First term, common difference and number of terms knowing these we can find the AP.

Solution 2

We have to find the 30th term and 11th term in I and II respectively.

I.

Given that

AP 10, 7, 4, ...

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10$

$= -3$

We know that, $a_n = a + (n - 1)\, d$

$a_{30} = 10 + (30 - 1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Hence, the correct answer is **C**.

II. Given that, AP $-3, -\frac{1}{2}, 2, \ldots$

First term $a = -3$

Common difference, $d = a_2 - a_1$

$= -\frac{1}{2} - (-3)$

$= -\frac{1}{2} + 3 = \frac{5}{2}$

We know that,

$a_n = a + (n - 1)\, d$

$a_{11} = -3 + (11 - 1)\left(\frac{5}{2}\right)$

$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$

$a_{11} = -3 + 25$

$a_{11} = 22$

Hence, the answer is **B**.

Concept Insight: Three things determine an AP

First term, common difference and number of terms knowing these any term of AP can be found.

Keep in mind that in place of the n^{th} term, put the value of the n^{th} term. And in place of 'n' put the number of that term

Solution 3

I. 2, □, 26

For this AP,

$a = 2$

$a_3 = 26$

We know that, $a_n = a + (n - 1)\, d$

$a_3 = 2 + (3 - 1)\, d$

$26 = 2 + 2d$

$24 = 2d$

$d = 12$

$a_2 = 2 + (2 - 1)\, 12$

$= 14$

Therefore, 14 is the missing term.

II. □, 13, □, 3

For this AP,

$a_2 = 13$ and

$a_4 = 3$

We know that, $a_n = a + (n - 1)\, d$

$a_2 = a + (2 - 1)\, d$

$13 = a + d \qquad \text{(I)}$

$a_4 = a + (4 - 1)\, d$

$3 = a + 3d \qquad \text{(II)}$

On subtracting (I) from (II), we obtain

$-10 = 2d$

$d = -5$

From equation (I), we obtain

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3 - 1)\,(-5)$

$\quad = 18 + 2\,(-5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

III. 5, □, □, $9\frac{1}{2}$

For this AP,

$a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that,

$a_n = a + (n - 1)d$

$a_4 = a + (4 - 1)d$

$\frac{19}{2} = 5 + 3d$

$\frac{19}{2} - 5 = 3d$

$\frac{9}{2} = 3d$

$d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. −4, □, □, □, □, 6

For this AP,

$a = -4$ and

$a_6 = 6$

We know that,

$a_n = a + (n - 1)\, d$

$a_6 = a + (6 - 1)\, d$

$6 = -4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2\,(2) = 0$

$a_4 = a + 3d = -4 + 3\,(2) = 2$

$a_5 = a + 4d = -4 + 4\,(2) = 4$

Solution 4

3, 8, 13, 18, …

For this AP,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this AP be 78.

$a_n = a + (n - 1)\,d$

$78 = 3 + (n - 1)\,5$

$75 = (n - 1)\,5$

$(n - 1) = 15$

$n = 16$

Hence, 16^{th} term of this AP is 78.

Concept Insight: Here nth term is given and n needs to be found substitute n^{th} term and 'n' at appropriate places.

Solution 5

I. 7, 13, 19, …, 205

For this AP,

$a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Let there are n terms in this AP

$a_n = 205$

We know that

$a_n = a + (n - 1)\,d$

Therefore, $205 = 7 + (n - 1)\,6$

$198 = (n - 1)\,6$

$33 = (n - 1)$

$n = 34$

Therefore, this given series has 34 terms in it.

II. $18,\ 15\frac{1}{2},\ 13, \ldots, -47$

For this AP,

$a = 18$

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$d = \frac{31 - 36}{2} = -\frac{5}{2}$

Let there are n terms in this AP

Therefore, $a_n = -47$ and we know that,

$a_n = a + (n - 1)\,d$

$-47 = 18 + (n - 1)\left(-\frac{5}{2}\right)$

$-47 - 18 = (n - 1)\left(-\frac{5}{2}\right)$

$-65 = (n - 1)\left(-\frac{5}{2}\right)$

$(n - 1) = \frac{-130}{-5}$

$(n - 1) = 26$

$n = 27$

Therefore, this given AP has 27 terms in it.

Concept Insight: Here nth term is given and n needs to be found substitute n^{th} term and 'n' at appropriate places.

Solution 6

For this AP,
$a = 11$
$d = a_2 - a_1 = 8 - 11 = -3$
Let -150 be the n^{th} term of this AP
We know that,

$$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-150 = 11 - 3n + 3$$
$$-164 = -3n$$
$$n = \frac{164}{3}$$

Clearly, n is not an integer.
Therefore, -150 is not a term of this AP

Concept Insight: In order to answer such questions main point to be kept in mind is the n the number of terms in a sequence can take only integral value. Here nth term is given and n needs to be found, substitute n^{th} term and 'n' at appropriate places.

Solution 7
Given that,
$a_{11} = 38$
$a_{16} = 73$
We know that,
$a_n = a + (n - 1) d$
$a_{11} = a + (11 - 1) d$
$38 = a + 10d$ (1)
Similarly,
$a_{16} = a + (16 - 1) d$
$73 = a + 15d$ (2)
On subtracting (1) from (2), we obtain
$35 = 5d$
$d = 7$
From equation (1),
$38 = a + 10 \times (7)$
$38 - 70 = a$
$a = -32$
$a_{31} = a + (31 - 1) d$
$= - 32 + 30 (7)$
$= - 32 + 210$
$= 178$
Hence, 31^{st} term is 178.

Concept Insight: Use the general term a + (n-1)d and solve the system of linear equations to get the values of a and d. Substitute n^{th} term and 'n' at appropriate places.

Solution 8

Given that,
$a_3 = 12$
$a_{50} = 106$
We know that,
$a_n = a + (n - 1)d$
$a_3 = a + (3 - 1)d$
$12 = a + 2d$ (I)
Similarly, $a_{50} = a + (50 - 1)d$
$106 = a + 49d$ (II)
On subtracting (I) from (II), we obtain
$94 = 47d$
$d = 2$
From equation (I), we obtain
$12 = a + 2(2)$
$a = 12 - 4 = 8$
$a_{29} = a + (29 - 1)d$
$a_{29} = 8 + (28)2$
$a_{29} = 8 + 56 = 64$
Therefore, 29^{th} term is 64.

Concept Insights: Here a and d can be determined by solving the system of equations formed using the general term a+(n-1)d substitute n^{th} term and 'n' at appropriate places.

Solution 9
Given that,
$a_3 = 4$
$a_9 = -8$
We know that,
$a_n = a + (n - 1)d$
$a_3 = a + (3 - 1)d$
$4 = a + 2d$ (I)
$a_9 = a + (9 - 1)d$
$-8 = a + 8d$ (II)
On subtracting equation (I) from (II), we obtain
$-12 = 6d$
$d = -2$
From equation (I), we obtain
$4 = a + 2(-2)$
$4 = a - 4$
$a = 8$
Let n^{th} term of this AP be zero.
$a_n = a + (n - 1)d$
$0 = 8 + (n - 1)(-2)$
$0 = 8 - 2n + 2$
$2n = 10$
$n = 5$
Hence, 5^{th} term of this AP is 0.

Solution 10
For an AP, $a_n = a + (n - 1)d$
$a_{17} = a + (17 - 1)d$
$a_{17} = a + 16d$
Similarly, $a_{10} = a + 9d$
It is given that
$a_{17} - a_{10} = 7$
$(a + 16d) - (a + 9d) = 7$
$7d = 7$
$d = 1$
Therefore, the common difference is 1.

Solution 11

Given AP is 3, 15, 27, 39, ...
$a = 3$
$d = a_2 - a_1 = 15 - 3 = 12$
$a_{54} = a + (54 - 1)\,d$
$= 3 + (53)\,(12)$
$= 3 + 636 = 639$
$132 + 639 = 771$
We have to find the term of this AP which is 771.
Let n^{th} term be 771.
$a_n = a + (n - 1)\,d$
$771 = 3 + (n - 1)\,12$
$768 = (n - 1)\,12$
$(n - 1) = 64$
$n = 65$
Therefore, 65^{th} term was 132 more than 54^{th} term.

Concept Insight: Add 132 to **54^{th} term** and equate it to the general term.
Remember n can take only integral values

Solution 12
Let the first term of these APs be a_1 and a_2 respectively and the common difference of these APs be d.
For first AP,
$a_{100} = a_1 + (100 - 1)\,d$
$= a_1 + 99d$
$a_{1000} = a_1 + (1000 - 1)\,d$
$a_{1000} = a_1 + 999d$
For second AP,
$a_{100} = a_2 + (100 - 1)\,d$
$= a_2 + 99d$
$a_{1000} = a_2 + (1000 - 1)\,d$
$= a_2 + 999d$
Given that, difference between
100^{th} term of these APs = 100
Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$
$a_1 - a_2 = 100$ (1)
Difference between 1000^{th} terms of these APs
$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$
From equation (1),

This difference, $a_1 - a_2 = 100$
Hence, the difference between 1000^{th} terms of these AP will be 100.

Concept Insight: There are two different APs in this questions denote there first terms with different variables.

Solution 13

First three-digit number that is divisible by 7 = 105
Next number = 105 + 7 = 112
Therefore, 105, 112, 119, …
All are three digit numbers which are divisible by 7 and thus, all these are terms of an AP having first term as 105 and common difference as 7.
The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5.
Clearly, 999 – 5 = 994 is the maximum possible three-digit number that is divisible by 7.
The series is as follows.
105, 112, 119, …, 994
Let 994 be the nth term of this AP
$a = 105$
$d = 7$
$a_n = 994$
$n = ?$
$a_n = a + (n - 1) d$
$994 = 105 + (n - 1) 7$
$889 = (n - 1) 7$
$(n - 1) = 127$
$n = 128$
Therefore, 128 three-digit numbers are divisible by 7.

Concepts Insights: The key step here is to determine the sequence.
Start with smallest and largest natural numbers and then use the divisibility test of 7.Again n the number of terms needs to be determined and the nth term is given.

Solution 14
First multiple of 4 that is greater than 10 is 12. Next will be 16.
Therefore, 12, 16, 20, 24, …
All these are divisible by 4 and thus, all these are terms of an AP with first term as 12 and common difference as 4.
When we divide 250 by 4, the remainder will be 2.
Therefore, 250 – 2 = 248 is divisible by 4.
The series is as follows.
12, 16, 20, 24, …, 248
Let 248 be the n^{th} term of this AP
$a = 12$
$d = 4$
$a_n = 248$
$a_n = a + (n-1)d$
$248 = 12 + (n-1)4$
$\frac{236}{4} = n - 1$
$59 = n - 1$
$n = 60$
Therefore, there are 60 multiples of 4 between 10 and 250.

Concept Insight: The key idea is to write the multiples of 4 and then find first and last multiples. Do not confuse it with factors of 4.

Solution 15

63, 65, 67, …
$a = 63$
$d = a_2 - a_1 = 65 - 63 = 2$
n^{th} term of this AP = $a_n = a + (n - 1)\, d$
$a_n = 63 + (n - 1)\, 2 = 63 + 2n - 2$
$a_n = 61 + 2n$ (1)
3, 10, 17, …
$a = 3$
$d = a_2 - a_1 = 10 - 3 = 7$
n^{th} term of this AP = $3 + (n - 1)\, 7$
$a_n = 3 + 7n - 7$
$a_n = 7n - 4$ (2)
It is given that, n^{th} term of these APs are equal to each other.
Equating both these equations, we obtain
$61 + 2n = 7n - 4$
$61 + 4 = 5n$
$5n = 65$
$n = 13$
Therefore, 13^{th} terms of both these APs are equal to each other.

Concept Insights: Equate the general term of two APs and solve the corresponding linear equation to obtain n.

Solution 16
$a_3 = 16$
$a + (3 - 1)\, d = 16$
$a + 2d = 16$ (1)
$a_7 - a_5 = 12$
$[a + (7 - 1)\, d] - [a + (5 - 1)\, d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12$
$d = 6$

From equation (1), we obtain
$a + 2\,(6) = 16$
$a + 12 = 16$
$a = 4$
Therefore, AP will be
4, 10, 16, 22, ……….

Concept Insights: Two equations in variables a and d can be obtained by using the value of third term and the difference between 7^{th} term and 5^{th} term.

Solution 17
Given AP is
3, 8, 13,….. …, 253
Common difference for this AP is 5.
this AP can be written in reverse order as
253, 248, 243, …, 13, 8, 5
For this AP,
$a = 253$
$d = 248 - 253 = -5$
$n = 20$
$a_{20} = a + (20 - 1)\, d$
$a_{20} = 253 + (19)\,(-5)$
$a_{20} = 253 - 95$
$a = 158$
Therefore, 20^{th} term from the last term is 158.

Concepts Insights: While finding the k^{th} term from the end of an AP , consider the AP. in the reverse order .i.e. the new AP has the last term of the original AP as its first term and the negative of the common difference of the original A.P. as its common difference. Then find the Kth term from the beginning of the new AP This will be the Kth term from the end of the original AP

Solution 18

We know that,

$a_n = a + (n - 1)d$

$a_4 = a + (4 - 1)d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$a + 5d = 12$ (1)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$ (2)

On subtracting equation (1) from (2), we obtain

$2d = 22 - 12$

$2d = 10$

$d = 5$

From equation (1), we obtain

$a + 5d = 12$

$a + 5(5) = 12$

$a + 25 = 12$

$a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Therefore, the first three terms of this AP are −13, −8, and −3.

Solution 19

It can be observed that the incomes that Subba Rao obtained in various years are in AP as every year, his salary is increased by Rs 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, …

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be Rs 7000.

Therefore, $a_n = a + (n - 1)d$

$7000 = 5000 + (n - 1)200$

$200(n - 1) = 2000$

$(n - 1) = 10$

$n = 11$

Therefore, in 11th year, his salary will be Rs 7000.

Concepts insight: Read the question carefully, analyse the information to identify the initial value and the pattern of change, then use the concept of general term of arithmetic progression. Again here number of term n needs to be found.

Solution 20

Given that,
$a = 5$
$d = 1.75$
$a_n = 20.75$
$n = ?$
$a_n = a + (n - 1) d$

$$20.75 = 5 + (n-1)1.75$$

$$15.75 = (n-1)1.75$$

$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$n - 1 = 9$
$n = 10$
Hence, n is 10.

Concepts insight: Read the question carefully, analyse the information to identify the initial value and the pattern of change , then use the concept of general term of arithmetic progression.

Chapter 5 - Arithmetic Progressions Exercise Ex. 5.3

Solution 1

(i) 2, 7, 12 ,…, to 10 terms

For this AP,

$a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$

$$= 5\left[4 + (9)\times(5)\right]$$

$$= 5 \times 49 = 245$$

(ii) –37, –33, –29 ,…, to 12 terms

For this AP,

$a = -37$

$d = a_2 - a_1 = (-33) - (-37)$

$= -33 + 37 = 4$

$n = 12$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(-37) + (12-1)4\right]$$

$$= 6\left[-74 + 11 \times 4\right]$$

$$= 6\left[-74 + 44\right]$$

$$= 6(-30) = -180$$

(iii) 0.6, 1.7, 2.8 ,…, to 100 terms

For this AP,

$a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

$n = 100$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(0.6) + (100-1)1.1\right]$$

$$= 50\left[1.2 + (99) \times (1.1)\right]$$

$$= 50\left[1.2 + 108.9\right]$$

$$= 50\left[110.1\right]$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots\ldots$, to 11 terms

For this AP,

$a = \frac{1}{15}$

$n = 11$

$$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15}$$

$$= \frac{5-4}{60} = \frac{1}{60}$$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$$

Concepts Insight: In order to find the sum of first n terms using $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, first term, common difference and the number of terms is needed. Common difference is the difference in consecutive terms of an AP.

Solution 2

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots\ldots + 84$

For this AP,

$a = 7$

$l = 84$

$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$

Let 84 be the n^{th} term of this AP

$l = a + (n - 1)d$

$84 = 7 + (n-1)\frac{7}{2}$

$77 = (n-1)\frac{7}{2}$

$22 = n - 1$

$n = 23$

We know that,

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{23}{2}[7+84]$

$= \frac{23 \times 91}{2} = \frac{2093}{2}$

$= 1046\frac{1}{2}$

(ii) $34 + 32 + 30 + \ldots\ldots\ldots + 10$

For this AP,

$a = 34$

$d = a_2 - a_1 = 32 - 34 = -2$

$l = 10$

Let 10 be the n^{th} term of this AP

$l = a + (n - 1)d$

$10 = 34 + (n - 1)(-2)$

$-24 = (n - 1)(-2)$

$12 = n - 1$

$n = 13$

$S_n = \frac{n}{2}(a+l)$

$= \frac{13}{2}(34+10)$

$= \frac{13 \times 44}{2} = 13 \times 22$

$= 286$

(iii) $(-5) + (-8) + (-11) + \ldots\ldots\ldots + (-230)$

For this AP,

$a = -5$

$l = -230$

$d = a_2 - a_1 = (-8) - (-5)$

$= -8 + 5 = -3$

Let -230 be the n^{th} term of this AP

$l = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

$n = 76$

And, $S_n = \frac{n}{2}(a+l)$

$= \frac{76}{2}[(-5)+(-230)]$

$= 38(-235)$

$= -8930$

Concept Insights: In order to find the sum of first n terms using $S_n = \frac{n}{2}(a+l)$, first term last term and the number of terms are required. Choose the formula based on the data given.

Solution 3

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$

As $a_n = a + (n - 1)d$,

$\therefore 50 = 5 + (n - 1)3$

$45 = (n - 1)3$

$15 = n - 1$

$n = 16$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{16} = \frac{16}{2}[5 + 50]$$

$$= 8 \times 55$$

$$= 440$$

(ii) Given that, $a = 7$, $a_{13} = 35$

As $a_n = a + (n - 1)\,d$,

$\therefore a_{13} = a + (13 - 1)\,d$

$35 = 7 + 12\,d$

$35 - 7 = 12d$

$28 = 12d$

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{13} = \frac{n}{2}[a + a_{13}]$$

$$= \frac{13}{2}[7 + 35]$$

$$= \frac{13 \times 42}{2} = 13 \times 21$$

$$= 273$$

(iii) Given that, $a_{12} = 37$, $d = 3$

As $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_n = \frac{12}{2}[4 + 37]$$

$$S_n = 6(41)$$

$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ (i)

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2a+(10-1)d\right]$$

$$125 = 5(2a+9d)$$

$$25 = 2a+9d \quad \text{(ii)}$$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d$ (iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v) Given that, $d = 5$, $S_9 = 75$

As $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$,

$$S_9 = \frac{9}{2}\left[2a+(9-1)5\right]$$

$$75 = \frac{9}{2}(2a+40)$$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$$a = \frac{-35}{3}$$

$a_n = a + (n - 1)d$

$a_9 = a + (9 - 1)(5)$

$$= \frac{-35}{3} + 8(5)$$

$$= \frac{-35}{3} + 40$$

$$= \frac{-35+120}{3} = \frac{85}{3}$$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$,

$90 = \frac{n}{2}\left[4+(n-1)8\right]$

$90 = n\ [2 + (n - 1)4]$

$90 = n\ [2 + 4n - 4]$

$90 = n\ (4n - 2) = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n\ (n - 5) + 18\ (n - 5) = 0$

$(n - 5)\ (4n + 18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$a_n = a + (n - 1)d$

$a_5 = 2 + (5 - 1)8$

$= 2 + (4)\ (8)$

$= 2 + 32 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

$S_n = \frac{n}{2}\left[a+a_n\right]$

$210 = \frac{n}{2}\left[8+62\right]$

$210 = \frac{n}{2}(70)$

$n = 6$

$a_n = a + (n - 1)d$

$62 = 8 + (6 - 1)d$

$62 - 8 = 5d$

$54 = 5d$

$d = \frac{54}{5}$

(viii) Given that, $a_n = 4$, $d = 2$, $S_n = -14$

$a_n = a + (n - 1)d$

$4 = a + (n - 1)2$

$4 = a + 2n - 2$

$a + 2n = 6$

$a = 6 - 2n$ (i)

$S_n = \frac{n}{2}\left[a+a_n\right]$

$-14 = \frac{n}{2}\left[a+4\right]$

$-28 = n\ (a + 4)$

$-28 = n\ (6 - 2n + 4)$ {From equation (i)}

$-28 = n\ (- 2n + 10)$

$-28 = - 2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$
$n^2 - 7n + 2n - 14 = 0$
$n(n - 7) + 2(n - 7) = 0$
$(n - 7)(n + 2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$
$n = 7$ or $n = -2$
However, n can take only positive integral values.
Therefore, $n = 7$
From equation (i), we obtain
$a = 6 - 2n$
$a = 6 - 2(7)$
$= 6 - 14$
$= -8$

(ix) Given that, $a = 3$, $n = 8$, $S = 192$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$192 = \frac{8}{2}\left[2 \times 3 + (8-1)d\right]$$

$192 = 4[6 + 7d]$
$48 = 6 + 7d$
$42 = 7d$
$d = 6$

(x) Given that, $l = 28$, $S = 144$ and there are total of 9 terms.

$$S_n = \frac{n}{2}(a + l)$$

$$144 = \frac{9}{2}(a + 28)$$

$(16) \times (2) = a + 28$
$32 = a + 28$
$a = 4$

Concepts Insight: In order to find the sum of first n terms either first term, common difference and the number of terms are required or the first term, last term and the number of terms are needed . Using the formula of sum of first n terms and nth term unknowns can be determined. Remember number of terms can take only positive integral value all other's can take any real value.

Solution 4

Let there be n terms of this AP
For this AP, $a = 9$
$d = a_2 - a_1 = 17 - 9 = 8$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$636 = \frac{n}{2}\left[2 \times a + (n-1)8\right]$$

$$636 = \frac{n}{2}\left[18 + (n-1)8\right]$$

$636 = n\ [9 + 4n - 4]$
$636 = n\ (4n + 5)$
$4n^2 + 5n - 636 = 0$
$4n^2 + 53n - 48n - 636 = 0$
$n\ (4n + 53) - 12\ (4n + 53) = 0$
$(4n + 53)\ (n - 12) = 0$
Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{-53}{4}$ or $n = 12$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

Concepts Insight: In order to find the sum of first n terms first term, common difference and the number of terms are required. Common difference is the difference of its consecutive terms. Using the formula of sum of first n terms and substituting Sn n can be determined.

Solution 5

Given that,
$a = 5$
$l = 45$
$S_n = 400$

$$S_n = \frac{n}{2}(a + l)$$

$$400 = \frac{n}{2}(5 + 45)$$

$$400 = \frac{n}{2}(50)$$

$n = 16$
$l = a + (n - 1)\ d$
$45 = 5 + (16 - 1)\ d$
$40 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Concept Insights: In order to find the sum of first n terms using $S_n = \frac{n}{2}(a+l)$, first term last term and the number of terms are required. Choose the formula based on the data given. Using the formula of sum of first n terms and substituting Sn n can be determined using last term d can be found.The key point to be noted here is to list what all needs to be found.

Solution 6

Given that,
$a = 17$
$l = 350$
$d = 9$
Let there be n terms in the AP
$l = a + (n - 1)d$
$350 = 17 + (n - 1)9$
$333 = (n - 1)9$
$(n - 1) = 37$
$n = 38$

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this AP contains 38 terms and the sum of the terms of this AP is 6973.

Solution 7

$d = 7$
$a_{22} = 149$
$S_{22} = ?$
$a_n = a + (n - 1)d$
$a_{22} = a + (22 - 1)d$
$149 = a + 21 \times 7$
$149 = a + 147$
$a = 2$

$$S_n = \frac{n}{2}(a + a_n)$$

$$S_{22} = \frac{22}{2}(2 + 149)$$

$$= 11(151) = 1661$$

Solution 8

Given that,
$a_2 = 14$
$a_3 = 18$
$d = a_3 - a_2 = 18 - 14 = 4$
$a_2 = a + d$
$14 = a + 4$
$a = 10$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)4]$$

$$= \frac{51}{2}[20 + (50)(4)]$$

$$= \frac{51(220)}{2} = 51(110)$$

$= 5610$

Solution 9

Given that,
$S_7 = 49$
$S_{17} = 289$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$$

$$49 = \frac{7}{2}(2a + 6d)$$

$7 = (a + 3d)$
$a + 3d = 7$ $\qquad (i)$

Similarly, $S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

$17 = (a + 8d)$
$a + 8d = 17$ $\qquad (ii)$
Subtracting equation (i) from equation (ii),
$5d = 10$
$d = 2$
From equation (i),
$a + 3(2) = 7$
$a + 6 = 7$
$a = 1$

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ &= \frac{n}{2}\left[2(1) + (n-1)(2)\right] \\ &= \frac{n}{2}(2 + 2n - 2) \\ &= \frac{n}{2}(2n) \end{aligned}$$

$= n^2$

Concept Insights: Two linear equations can be determined by using the sum of First 7 and 17 terms solving them gives a and d. Note that sum of n terms will involve the variable n.

Solution 10

(i)

$a_n = 3 + 4n$

$a_{k+1} = 3 + 4(k+1)$
$= 3 + 4k + 4$

Here $a_{k+1} - a_k = (3+4k+4) - (3+4k) = 4$ which is independent of k.

Therefore, this is an AP with common difference 4

Also, first term $= 7$.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7) + (15-1)4\right]$$

$$= \frac{15}{2}\left[(14) + 56\right]$$

$$= \frac{15}{2}(70)$$

$= 15 \times 35$
$= 525$

(ii)

$a_n = 9 - 5n$
Here $a_{k+1} - a_k = (9-5k-5)-(9-5k)$
$= -5$ which is independent of k.

Therefore, this is an AP with common difference as -5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(4) + (15-1)(-5)\right]$$

$$= \frac{15}{2}\left[8 + 14(-5)\right]$$

$$= \frac{15}{2}(8 - 70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

$= -465$

Solution 11

Given that,
$S_n = 4n - n^2$
First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$
Sum of first two terms = S_2
$= 4(2) - (2)^2 = 8 - 4 = 4$
Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$
$d = a_2 - a = 1 - 3 = -2$
$a_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$
Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$
$a_{10} = 5 - 2(10) = 5 - 20 = -15$
Hence, the sum of first two terms is 4. The second term is 1. 3^{rd}, 10^{th}, and n^{th} terms are −1, −15, and $5 - 2n$ respectively.

Concepts Insights: The key result to be used here is that S_n denotes sum to n terms so S_1 is the sum of first term or first term only S_2 represents sum of first and second term.

Solution 12
The positive integers that are divisible by 6 are
6, 12, 18, 24 ....
The AP whose first term is 6 and common difference is 6.
$a = 6$
$d = 6$
$S_{40} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

$= 20[12 + (39)(6)]$
$= 20(12 + 234)$
$= 20 \times 246$
$= 4920$

Concept Insights: The sequence can be obtained by writing first 40 multiples of 6. Remember that every multiple of 6 is divisible by 6 . 40 here indicates number of terms and not the last term.

Solution 13
Multiples of 8 are
8, 16, 24, 32...
These are in an AP, having first term as 8 and common difference as 8.
Therefore, $a = 8$
$d = 8$
$S_{15} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{15}{2}\left[2(8) + (15-1)8\right]$$

$$= \frac{15}{2}\left[16 + 14(8)\right]$$

$$= \frac{15}{2}(16 + 112)$$

$$= \frac{15(128)}{2} = 15 \times 64$$

$= 960$

Solution 14

The odd numbers between 0 and 50 are
1, 3, 5, 7, 9 … 49
Therefore, it can be observed that these odd numbers are in an AP
$a = 1$
$d = 2$
$l = 49$
$l = a + (n - 1)\,d$
$49 = 1 + (n - 1)2$
$48 = 2(n - 1)$
$n - 1 = 24$
$n = 25$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{25} = \frac{25}{2}(1+49)$$

$$= \frac{25(50)}{2} = (25)(25)$$

$= 625$

Solution 15
Penalties are in an AP having first term as 200 and common difference as 50.
$a = 200$
$d = 50$
Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$$= \frac{30}{2}\left[2(200)+(30-1)50\right]$$

= 15 [400 + 1450]
= 15 (1850)
= 27750
Therefore, the contractor has to pay Rs 27750 as penalty.
Concept Insight: Read the question carefully, analyse the give data. Initial value is given which increases by a fixed amount. Apply properties of arithmetic progression to solve it.

Solution 16
Let the cost of 1st prize be P.
Cost of 2nd prize = $P - 20$
And cost of 3rd prize = $P - 40$
Cost of the prizes are forming an AP having common difference −20 and first term as P.
$a = P$
$d = -20$
Given that, $S_7 = 700$

$$\frac{7}{2}\left[2a+(7-1)\,d\right] = 700$$

$$\frac{\left[2a+(6)(-20)\right]}{2} = 100$$

$a + 3(-20) = 100$
$a - 60 = 100$
$a = 160$
Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.
Concept Insight: Read the question carefully, analyze the give data. Initial value is given which increases by a fixed amount. Apply properties of arithmetic progression to solve it.

Solution 17

Number of trees planted by the students forms an AP.
1, 2, 3, 4, 5………………..12
First term, $a = 1$
Common difference, $d = 2 - 1 = 1$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(1) + (12-1)(1)\right]$$

= 6 (2 + 11)
= 6 (13)
= 78
Therefore, number of trees planted by 1 section of the classes = 78
Number of trees planted by 3 sections of the classes = 3 × 78 = 234
Therefore, 234 trees will be planted by the students.

Concept Insight: Read the question carefully, analyze the give data. Initial value is given which increases by a fixed amount. Apply properties of arithmetic progression to solve it. Multiply the sum by 3 to obtain complete answer.

Solution 18
Semi-perimeter of circle = πr

$I_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$I_2 = \pi(1) = \pi$ cm

$I_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

Therefore, I_1, I_2, I_3 ,i.e. the lengths of the semi-circles forms an AP,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots\ldots$$

$$a = \frac{\pi}{2}$$

$$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$S_{13} = ?$
We know that the sum of n terms of an a AP is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$$

$$= \left(\frac{13}{2}\right)(7\pi)$$

$$= \frac{91\pi}{2}$$

$$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$$

= 143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Concept Insight: Read the question carefully, analyze the give data. Initial value is given which increases by a fixed amount. Apply properties of arithmetic progression to solve it. Multiply the sum by 3 to obtain complete answer.

Solution 19

Here the numbers of logs in each rows are in an AP
20, 19, 18...
For this AP,
$a = 20$
$d = a_2 - a_1 = 19 - 20 = -1$
Let a total of 200 logs be placed in n rows.
$S_n = 200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2(20) + (n-1)(-1)\right]$$

$400 = n(40 - n + 1)$
$400 = n(41 - n)$
$400 = 41n - n^2$
$n^2 - 41n + 400 = 0$
$n^2 - 16n - 25n + 400 = 0$
$n(n - 16) - 25(n - 16) = 0$
$(n - 16)(n - 25) = 0$
Either $(n - 16) = 0$ or $n - 25 = 0$
$n = 16$ or $n = 25$
$a_n = a + (n - 1)d$
$a_{16} = 20 + (16 - 1)(-1)$
$a_{16} = 20 - 15$
$a_{16} = 5$
Similarly,
$a_{25} = 20 + (25 - 1)(-1)$
$a_{25} = 20 - 24$
$= -4$
Clearly, the number of logs in 16^{th} row is 5. However, the number of logs in 25^{th} row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16^{th} row is 5.

Concept Insight: Read the question carefully, analyze the give data. Initial value is given which increases by a fixed amount. Apply properties of arithmetic progression to solve it. Multiply the sum by 3 to obtain complete answer.

Solution 20

In a potato race, a bucket is placed at the starting point, which is 5m from the first potato and other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line.

So, we get the series as
5, 8, 11, 14, ...
Here, a= 5 and d = 8 - 5 = 11 - 8= 3
the difference between the two consecutive terms are same.
So, this is an arithmetic Progression.
According to the condition, we get the series as
5 + 5 , 8 + 8, 11 + 11, ...
10, 16, 22, ...
Here a = 10 and d = 16 - 10 = 6
the difference between the two consecutive terms are same.
So, this is an arithmetic Progression.
The total distance the competitor has to run is given by,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{10}{2}\left[2 \times 10 + (10-1)6\right]$$
$$= 5(20 + 54)$$
$$= 5 \times 74$$
$$= 370$$

Therefore the total distance the competitor has to run is 370 m.

Chapter 5 - Arithmetic Progressions Exercise Ex. 5.4

Solution 1

Given AP is 121, 117, 113 ...

$a = 121$

$d = 117 - 121 = -4$

$a_n = a + (n - 1)\, d$

$= 121 + (n - 1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have to find the first negative term of this AP

Therefore, $a_n < 0$

$125 - 4n < 0$

$125 < 4n$

$n > \frac{125}{4}$

$n > 31.25$

Therefore, 32^{nd} term will be the first negative term of this AP

Concept Insight: Three things determine an AP
First term, common difference and number of terms knowing these any term of AP can be found.
Use nth term and solve inequality remember n is always an integer.

Solution 2

We know that,

$a_n = a + (n - 1)\,d$

$a_3 = a + (3 - 1)\,d$

$a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d$ (i)

Also, it is given that $(a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

From equation (*i*),

$(3-4d+2d)\times(3-4d+6d)=8$

$(3-2d)\times(3+2d)=8$

$9-4d^2=8$

$4d^2=9-8=1$

$d^2=\frac{1}{4}$

$d=\pm\frac{1}{2}$

$d=\frac{1}{2}$ or $-\frac{1}{2}$

From equation (*i*),

$\left(\text{When d is } \frac{1}{2}\right)$

$a=3-4d$

$a=3-4\left(\frac{1}{2}\right)$

$=3-2=1$

$\left(\text{When d is } -\frac{1}{2}\right)$

$a=3-4\left(-\frac{1}{2}\right)$

$a=3+2=5$

$S_n=\frac{n}{2}\left[2a(n-1)\,d\right]$

$\left(\text{When ais1 and d is } \frac{1}{2}\right)$

$S_{16}=\frac{16}{2}\left[2\,(1)+(16-1)\left(\frac{1}{2}\right)\right]$

$=8\left[2+\frac{15}{2}\right]$

$=4\,(19)=76$

$\left(\text{When a is 5 and d is } -\frac{1}{2}\right)$

$S_{16}=\frac{16}{2}\left[2\,(5)+(16-1)\left(-\frac{1}{2}\right)\right]$

$=8\left[10+(15)\left(-\frac{1}{2}\right)\right]$

$=8\left(\frac{5}{2}\right)$

$=20$

Solution 3

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$$\therefore \text{Total number of rungs} = \frac{2\frac{1}{2}\times 100}{25} + 1 = \frac{250}{25} + 1 = 11$$

Now, as the lengths of the rungs decrease uniformly, they will be in an AP

The length of the wood required for the rungs equals the sum of all the terms of this AP

First term, $a = 45$

Last term, $l = 25$

$n = 11$

$$S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{10} = \frac{11}{2}(45+25) = \frac{11}{2}(70) = 385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Solution 4

The number of houses was 1, 2, 3, ... 49.

It can be observed that the number of houses are in an AP having a as 1 and d also as 1.

Let us assume that the number of x^{th} house was like this.

We know that,

$$\text{Sum of n terms in an A.P} = \frac{n}{2}[2a + (n-1)d]$$

Sum of number of houses preceding x^{th} house = S_{x-1}

$$= \frac{(x-1)}{2}[2a+(x-1-1)d]$$

$$= \frac{x-1}{2}[2(1)+(x-2)(1)]$$

$$= \frac{x-1}{2}[2+x-2]$$

$$= \frac{(x)(x-1)}{2}$$

Sum of number of houses preceding x^{th} house = $S_{49} - S_x$

$$= \frac{49}{2}[2(1)+(49-1)(1)] - \frac{x}{2}[2(1)+(x-1)(1)]$$

$$= \frac{49}{2}(2+49-1) - \frac{x}{2}(2+x-1)$$

$$= \left(\frac{49}{2}\right)(50) - \frac{x}{2}(x+1)$$

$$= 25(49) - \frac{x(x+1)}{2}$$

It is given that these sums are equal to each other.

$$\frac{x(x-1)}{2} = 25(49) - x\left(\frac{x+1}{2}\right)$$

$$\frac{x^2}{2} - \frac{x}{2} = 1225 - \frac{x^2}{2} - \frac{x}{2}$$

$$x^2 = 1225$$

$$x = \pm 35$$

However, the house numbers are positive integers.

The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

Solution 5

From the figure, we see that

1^{st} step is $\frac{1}{2}$m wide,

2^{nd} step is 1 m wide,

3^{rd} step is $\frac{3}{2}$m wide.

Therefore, the width of each step is increasing by $\frac{1}{2}$m each time whereas their height $\frac{1}{4}$m and length 50 m remains the same.

Therefore, the widths of these steps are

$\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

Volume of concrete in 1^{st} step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2^{nd} step $= \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete in 3^{rd} step $= \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an AP

$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$

$a = \frac{25}{4}$

$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$

and $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\frac{25}{4}\right]$

$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$

$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$

$= \frac{15}{2}(100) = 750$

Volume of concrete required to build the terrace is 750 m^3.