

Access Answers of Maths NCERT Class 9

Chapter 9 – Areas of Parallelograms and Triangles

Class 9 Maths Chapter 9 Exercise 9.1 Page: 155

1. Which of the following figures lie on the same base and in-between the same parallels. In such a case, write the common base and the two parallels.

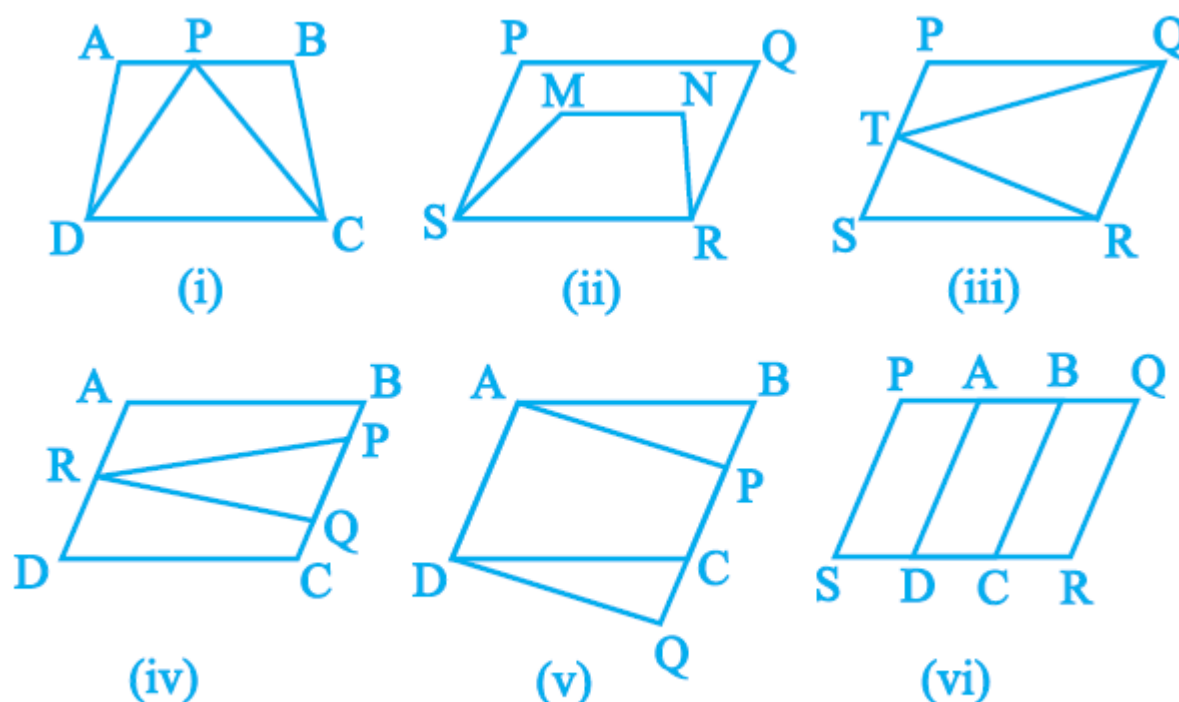


Fig. 9.8

Solution:

1. Trapezium ABCD and $\triangle PDC$ lie on the same DC and in-between the same parallel lines AB and DC.
2. Parallelogram PQRS and trapezium SMNR lie on the same base SR but not in-between the same parallel lines.
3. Parallelogram PQRS and $\triangle TRQ$ lie on the same base QR and in-between the same parallel lines QR and PS.
4. Parallelogram ABCD and $\triangle PQR$ do not lie on the same base but in-between the same parallel lines BC and AD.
5. Quadrilateral ABQD and trapezium APCD lie on the same base AD and in-between the same parallel lines AD and BQ.

6. Parallelogram PQRS and parallelogram ABCD do not lie on the same base SR but in-between the same parallel lines SR and PQ.

Exercise 9.2 Page: 159

1. In Fig. 9.15, ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.

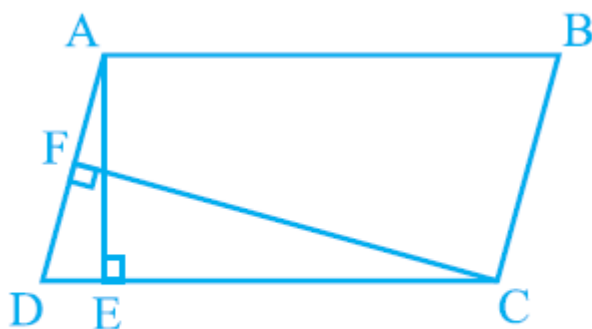


Fig. 9.15

Solution:

Given,

$AB = CD = 16$ cm (Opposite sides of a parallelogram)

$CF = 10$ cm and $AE = 8$ cm

Now,

Area of parallelogram = Base \times Altitude

$= CD \times AE = AD \times CF$

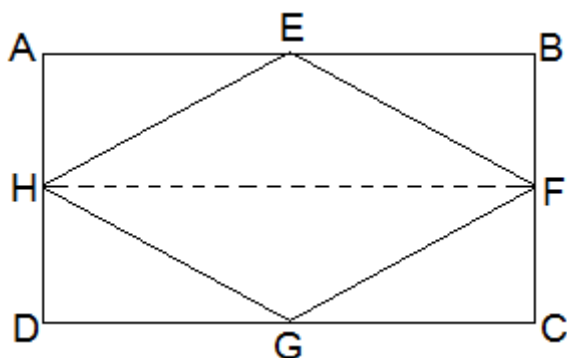
$\Rightarrow 16 \times 8 = AD \times 10$

$\Rightarrow AD = \frac{128}{10} = 12.8$ cm

$\Rightarrow AD = 12.8$ cm

2. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that $\text{ar} (EFGH) = \frac{1}{2} \text{ar}(ABCD)$.

Solution:



Given,

E, F, G and H are the mid-points of the sides of a parallelogram ABCD, respectively.

To Prove,

$$\text{ar}(\text{EFGH}) = \frac{1}{2} \text{ar}(\text{ABCD})$$

Construction,

H and F are joined.

Proof,

$AD \parallel BC$ and $AD = BC$ (Opposite sides of a parallelogram)

$$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$$

Also,

$AH \parallel BF$ and $DH \parallel CF$

$\Rightarrow AH = BF$ and $DH = CF$ (H and F are mid points)

\therefore , ABFH and HFCD are parallelograms.

Now,

We know that, $\triangle EFH$ and parallelogram ABFH, both lie on the same FH the common base and in-between the same parallel lines AB and HF.

$$\therefore \text{area of } \triangle EFH = \frac{1}{2} \text{area of ABFH} \text{ — (i)}$$

$$\text{And, area of } \triangle GHF = \frac{1}{2} \text{area of HFCD} \text{ — (ii)}$$

Adding (i) and (ii),

$$\text{area of } \triangle EFH + \text{area of } \triangle GHF = \frac{1}{2} \text{area of ABFH} + \frac{1}{2} \text{area of HFCD}$$

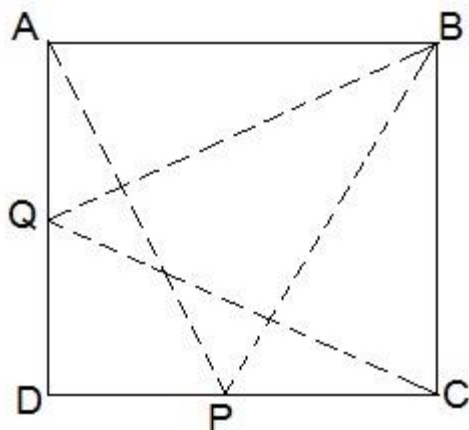
$$\Rightarrow \text{area of EFGH} = \text{area of ABFH}$$

$$\Rightarrow \text{ar}(\text{EFGH}) = \frac{1}{2} \text{ar}(\text{ABCD})$$

3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD.

Show that $\text{ar}(\text{APB}) = \text{ar}(\text{BQC})$.

Solution:



$\triangle APB$ and parallelogram $ABCD$ lie on the same base AB and in-between same parallel AB and DC .

,

$$\text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\text{parallelogram } ABCD) \text{ — (i)}$$

Similarly,

$$\text{ar}(\triangle BQC) = \frac{1}{2} \text{ar}(\text{parallelogram } ABCD) \text{ — (ii)}$$

From (i) and (ii),

$$\text{we have } \text{ar}(\triangle APB) = \text{ar}(\triangle BQC)$$

4. In Fig. 9.16, P is a point in the interior of a parallelogram $ABCD$. Show that

$$\text{(i) } \text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(ABCD)$$

$$\text{(ii) } \text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$$

[Hint : Through P , draw a line parallel to AB .]

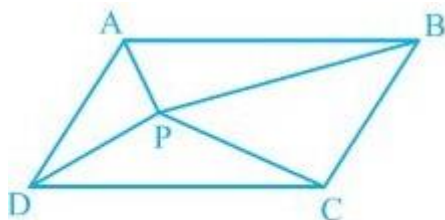
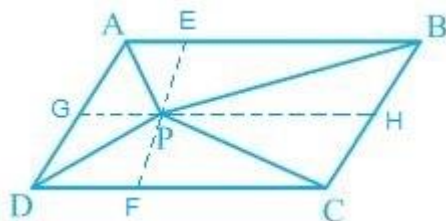


Fig. 9.16

Solution:



(i) A line GH is drawn parallel to AB passing through P .

In a parallelogram,

$AB \parallel GH$ (by construction) — (i)

\therefore ,

$AD \parallel BC \Rightarrow AG \parallel BH$ — (ii)

From equations (i) and (ii),

ABHG is a parallelogram.

Now,

$\triangle APB$ and parallelogram ABHG are lying on the same base AB and in-between the same parallel lines AB and GH.

$\therefore \text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\text{ABHG})$ — (iii)

also,

$\triangle PCD$ and parallelogram CDGH are lying on the same base CD and in-between the same parallel lines CD and GH.

$\therefore \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{CDGH})$ — (iv)

Adding equations (iii) and (iv),

$$\text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \{\text{ar}(\text{ABHG}) + \text{ar}(\text{CDGH})\}$$

$$\Rightarrow \text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{ABCD})$$

(ii) A line EF is drawn parallel to AD passing through P.

In the parallelogram,

$AD \parallel EF$ (by construction) — (i)

\therefore ,

$AB \parallel CD \Rightarrow AE \parallel DF$ — (ii)

From equations (i) and (ii),

AEDF is a parallelogram.

Now,

$\triangle APD$ and parallelogram AEFD are lying on the same base AD and in-between the same parallel lines AD and EF.

$\therefore \text{ar}(\triangle APD) = \frac{1}{2} \text{ar}(\text{AEFD})$ — (iii)

also,

$\triangle PBC$ and parallelogram BCFE are lying on the same base BC and in-between the same parallel lines BC and EF.

$\therefore \text{ar}(\triangle PBC) = \frac{1}{2} \text{ar}(\text{BCFE})$ — (iv)

Adding equations (iii) and (iv),

$$\text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \frac{1}{2} \{\text{ar}(\text{AEFD}) + \text{ar}(\text{BCFE})\}$$

$$\Rightarrow \text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$$

5. In Fig. 9.17, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

$$\text{ar (PQRS)} = \text{ar (ABRS)}$$

$$\text{ar (AXS)} = \frac{1}{2} \text{ar (PQRS)}$$

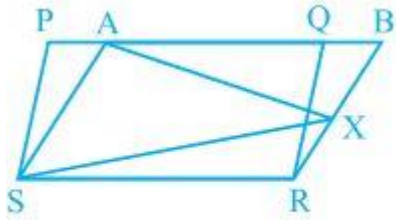


Fig. 9.17

Solution:

(i) Parallelogram PQRS and ABRS lie on the same base SR and in-between the same parallel lines SR and PB.

$$\therefore \text{ar(PQRS)} = \text{ar(ABRS)} \text{ — (i)}$$

(ii) Δ AXS and parallelogram ABRS are lying on the same base AS and in-between the same parallel lines AS and BR.

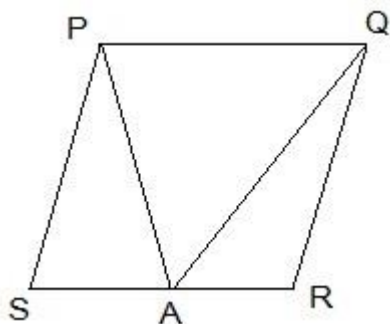
$$\therefore \text{ar}(\Delta\text{AXS}) = \frac{1}{2} \text{ar(ABRS)} \text{ — (ii)}$$

From (i) and (ii), we find that,

$$\text{ar}(\Delta\text{AXS}) = \frac{1}{2} \text{ar(PQRS)}$$

6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Solution:



The field is divided into three parts each in triangular shape.

Let, Δ PSA, Δ PAQ and Δ QAR be the triangles.

$$\text{Area of } \Delta\text{PSA} + \Delta\text{PAQ} + \Delta\text{QAR} = \text{Area of PQRS} \text{ — (i)}$$

$$\text{Area of } \Delta\text{PAQ} = \frac{1}{2} \text{area of PQRS} \text{ — (ii)}$$

Here, the triangle and parallelogram are on the same base and in-between the same parallel lines.

From (i) and (ii),

$$\text{Area of } \triangle PSA + \text{Area of } \triangle QAR = \frac{1}{2} \times \text{area of PQRS} \text{ — (iii)}$$

From (ii) and (iii), we can conclude that,

The farmer must sow wheat or pulses in $\triangle PAQ$ or either in both $\triangle PSA$ and $\triangle QAR$.

Exercise 9.3 Page: 162

1. In Fig.9.23, E is any point on median AD of a $\triangle ABC$. Show that $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$.

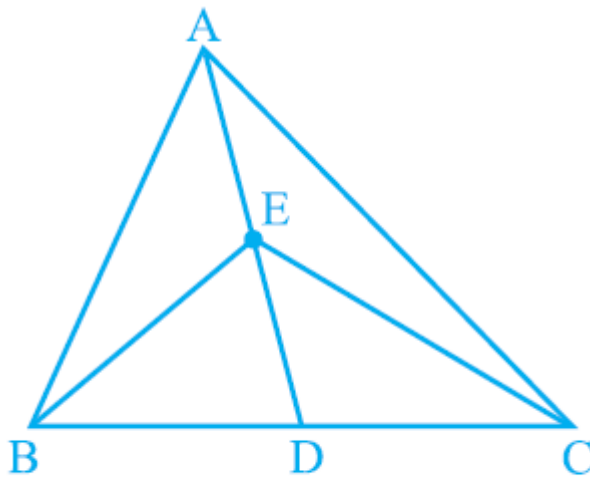


Fig. 9.23

Solution:

Given,

AD is median of $\triangle ABC$. \therefore , it will divide $\triangle ABC$ into two triangles of equal area.

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD) \text{ — (i)}$$

also,

ED is the median of $\triangle ABC$.

$$\therefore \text{ar}(\triangle EBD) = \text{ar}(\triangle ECD) \text{ — (ii)}$$

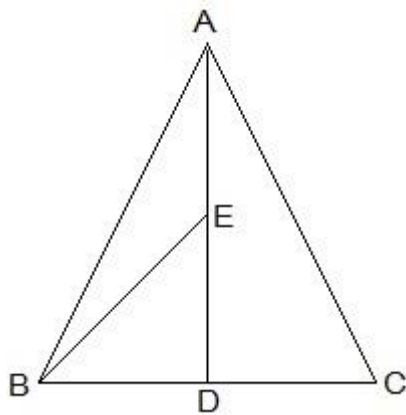
Subtracting (ii) from (i),

$$\text{ar}(\triangle ABD) - \text{ar}(\triangle EBD) = \text{ar}(\triangle ACD) - \text{ar}(\triangle ECD)$$

$$\Rightarrow \text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$$

2. In a triangle ABC, E is the mid-point of median AD. Show that $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$.

Solution:



$$\text{ar}(\triangle BED) = \left(\frac{1}{2}\right) \times BD \times DE$$

Since, E is the mid-point of AD,

$$AE = DE$$

Since, AD is the median on side BC of triangle ABC,

$$BD = DC$$

,

$$DE = \left(\frac{1}{2}\right)AD \text{ — (i)}$$

$$BD = \left(\frac{1}{2}\right)BC \text{ — (ii)}$$

From (i) and (ii), we get,

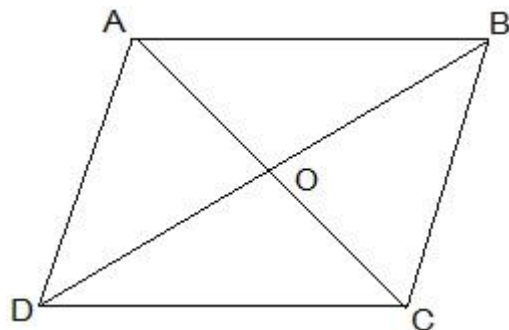
$$\text{ar}(\triangle BED) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) BC \times \left(\frac{1}{2}\right)AD$$

$$\Rightarrow \text{ar}(\triangle BED) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$$

3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution:



O is the mid point of AC and BD. (diagonals bisect each other)

In $\triangle ABC$, BO is the median.

$$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) \text{ — (i)}$$

also,

In $\triangle BCD$, CO is the median.

$$\therefore \text{ar}(\triangle BOC) = \text{ar}(\triangle COD) \text{ — (ii)}$$

In $\triangle ACD$, OD is the median.

$$\therefore \text{ar}(\triangle AOD) = \text{ar}(\triangle COD) \text{ — (iii)}$$

In $\triangle ABD$, AO is the median.

$$\therefore \text{ar}(\triangle AOD) = \text{ar}(\triangle AOB) \text{ — (iv)}$$

From equations (i), (ii), (iii) and (iv), we get,

$$\text{ar}(\triangle BOC) = \text{ar}(\triangle COD) = \text{ar}(\triangle AOD) = \text{ar}(\triangle AOB)$$

Hence, we get, the diagonals of a parallelogram divide it into four triangles of equal area.

4. In Fig. 9.24, $\triangle ABC$ and $\triangle ABD$ are two triangles on the same base AB . If line-segment CD is bisected by AB at O , show that: $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$.

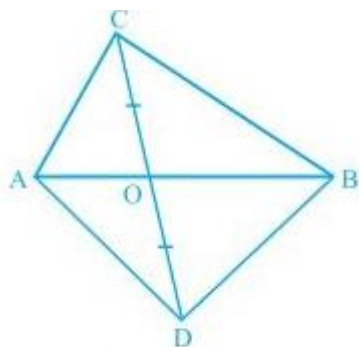


Fig. 9.24

Solution:

In $\triangle ABC$, AO is the median. (CD is bisected by AB at O)

$$\therefore \text{ar}(\triangle AOC) = \text{ar}(\triangle AOD) \text{ — (i)}$$

also,

$\triangle BCD$, BO is the median. (CD is bisected by AB at O)

$$\therefore \text{ar}(\triangle BOC) = \text{ar}(\triangle BOD) \text{ — (ii)}$$

Adding (i) and (ii),

We get,

$$\text{ar}(\triangle AOC) + \text{ar}(\triangle BOC) = \text{ar}(\triangle AOD) + \text{ar}(\triangle BOD)$$

$$\Rightarrow \text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$$

5. D , E and F are respectively the mid-points of the sides BC , CA and AB of a $\triangle ABC$.

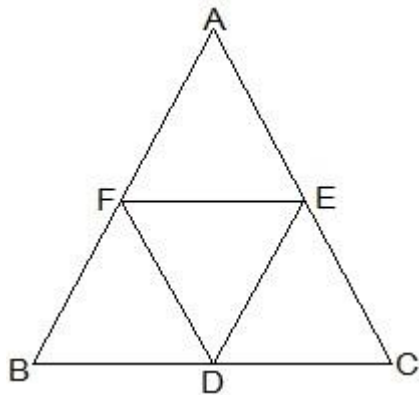
Show that

(i) $BDEF$ is a parallelogram.

(ii) $\text{ar}(\text{DEF}) = \frac{1}{4} \text{ar}(\text{ABC})$

(iii) $\text{ar}(\text{BDEF}) = \frac{1}{2} \text{ar}(\text{ABC})$

Solution:



(i) In $\triangle ABC$,

$EF \parallel BC$ and $EF = \frac{1}{2} BC$ (by mid point theorem)

also,

$BD = \frac{1}{2} BC$ (D is the mid point)

So, $BD = EF$

also,

BF and DE are parallel and equal to each other.

\therefore , the pair opposite sides are equal in length and parallel to each other.

\therefore BDEF is a parallelogram.

(ii) Proceeding from the result of (i),

BDEF, DCEF, AFDE are parallelograms.

Diagonal of a parallelogram divides it into two triangles of equal area.

$\therefore \text{ar}(\triangle BFD) = \text{ar}(\triangle DEF)$ (For parallelogram BDEF) — (i)

also,

$\text{ar}(\triangle AFE) = \text{ar}(\triangle DEF)$ (For parallelogram DCEF) — (ii)

$\text{ar}(\triangle CDE) = \text{ar}(\triangle DEF)$ (For parallelogram AFDE) — (iii)

From (i), (ii) and (iii)

$$\text{ar}(\triangle BFD) = \text{ar}(\triangle AFE) = \text{ar}(\triangle CDE) = \text{ar}(\triangle DEF)$$

$$\Rightarrow \text{ar}(\triangle BFD) + \text{ar}(\triangle AFE) + \text{ar}(\triangle CDE) + \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC)$$

$$\Rightarrow 4 \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\text{DEF}) = \frac{1}{4} \text{ar}(\text{ABC})$$

(iii) Area (parallelogram BDEF) = $\text{ar}(\triangle DEF) + \text{ar}(\triangle BDE)$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = \text{ar}(\triangle DEF) + \text{ar}(\triangle DEF)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = 2 \times \text{ar}(\triangle DEF)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = 2 \times \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\text{parallelogram BDEF}) = \frac{1}{2} \text{ar}(\triangle ABC)$$

6. In Fig. 9.25, diagonals AC and BD of quadrilateral ABCD intersect at O such that $OB = OD$.

If $AB = CD$, then show that:

$$\text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$$

$$\text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$$

$DA \parallel CB$ or ABCD is a parallelogram.

[Hint : From D and B, draw perpendiculars to AC.]

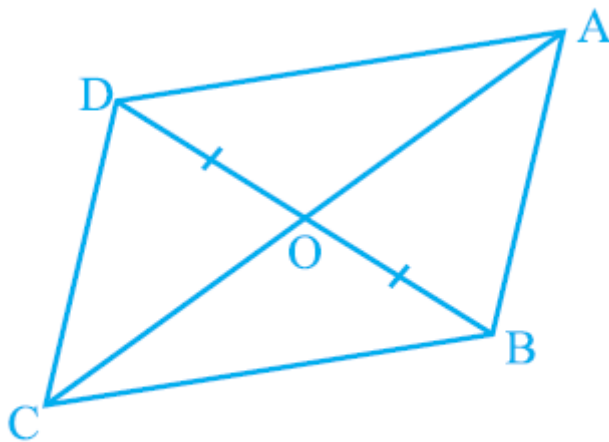


Fig. 9.25

Solution:

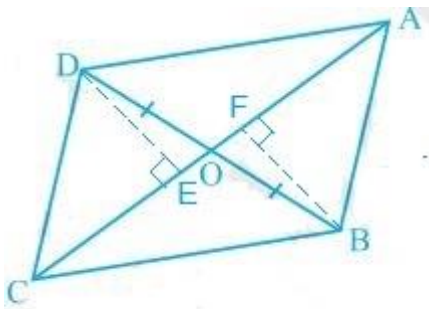


Fig. 9.25

Given,

$$OB = OD \text{ and } AB = CD$$

Construction,

$DE \perp AC$ and $BF \perp AC$ are drawn.

Proof:

(i) In $\triangle DOE$ and $\triangle BOF$,

$\angle DEO = \angle BFO$ (Perpendiculars)

$\angle DOE = \angle BOF$ (Vertically opposite angles)

$OD = OB$ (Given)

$\therefore \triangle DOE \cong \triangle BOF$ by AAS congruence condition.

$\therefore DE = BF$ (By CPCT) — (i)

also, $\text{ar}(\triangle DOE) = \text{ar}(\triangle BOF)$ (Congruent triangles) — (ii)

Now,

In $\triangle DEC$ and $\triangle BFA$,

$\angle DEC = \angle BFA$ (Perpendiculars)

$CD = AB$ (Given)

$DE = BF$ (From i)

$\therefore \triangle DEC \cong \triangle BFA$ by RHS congruence condition.

$\therefore \text{ar}(\triangle DEC) = \text{ar}(\triangle BFA)$ (Congruent triangles) — (iii)

Adding (ii) and (iii),

$\text{ar}(\triangle DOE) + \text{ar}(\triangle DEC) = \text{ar}(\triangle BOF) + \text{ar}(\triangle BFA)$

$\Rightarrow \text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$

(ii) $\text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$

Adding $\text{ar}(\triangle OCB)$ in LHS and RHS, we get,

$\Rightarrow \text{ar}(\triangle DOC) + \text{ar}(\triangle OCB) = \text{ar}(\triangle AOB) + \text{ar}(\triangle OCB)$

$\Rightarrow \text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$

(iii) When two triangles have same base and equal areas, the triangles will be in between the same parallel lines

$\text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$

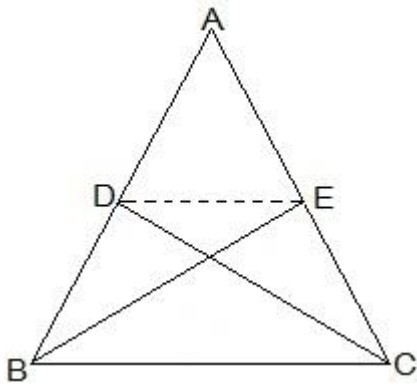
$DA \parallel BC$ — (iv)

For quadrilateral ABCD, one pair of opposite sides are equal ($AB = CD$) and other pair of opposite sides are parallel.

\therefore ABCD is parallelogram.

7. D and E are points on sides AB and AC respectively of $\triangle ABC$ such that $\text{ar}(\triangle DBC) = \text{ar}(\triangle EBC)$. Prove that $DE \parallel BC$.

Solution:



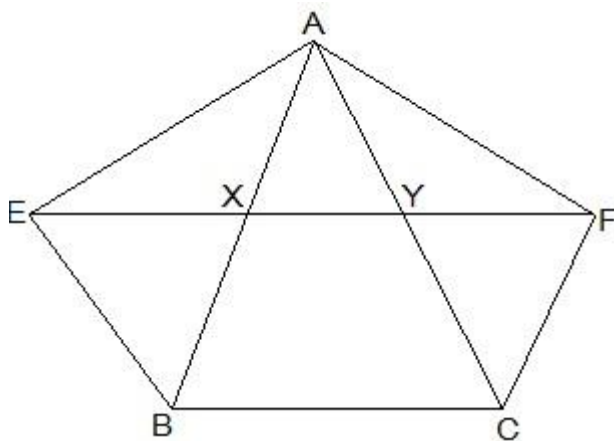
$\triangle DBC$ and $\triangle ECB$ are on the same base BC and also having equal areas.

\therefore , they will lie between the same parallel lines.

\therefore , $DE \parallel BC$.

8. XY is a line parallel to side BC of a triangle ABC . If $BE \parallel AC$ and $CF \parallel AB$ meet XY at E and F respectively, show that $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$

Solution:



Given,

$XY \parallel BC$, $BE \parallel AC$ and $CF \parallel AB$

To show,

$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$

Proof:

$BCYE$ is a \parallel gm as $\triangle ABE$ and \parallel gm $BCYE$ are on the same base BE and between the same parallel lines BE and AC .

\therefore , $\text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(BCYE) \dots (1)$

Now,

$CF \parallel AB$ and $XY \parallel BC$

$\Rightarrow CF \parallel AB$ and $XF \parallel BC$

\Rightarrow BCFX is a || gm

As $\triangle ACF$ and || gm BCFX are on the same base CF and in-between the same parallel AB and FC .

$$\therefore, \text{ar}(\triangle ACF) = \frac{1}{2} \text{ar}(\text{BCFX}) \dots (2)$$

But,

||gm BCFX and || gm BCYE are on the same base BC and between the same parallels BC and EF.

$$\therefore, \text{ar}(\text{BCFX}) = \text{ar}(\text{BCYE}) \dots (3)$$

From (1) , (2) and (3) , we get

$$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$$

$$\Rightarrow \text{ar}(\text{BEYC}) = \text{ar}(\text{BXFC})$$

As the parallelograms are on the same base BC and in-between the same parallels EF and BC—(iii)

Also,

$\triangle AEB$ and ||gm BEYC are on the same base BE and in-between the same parallels BE and AC.

$$\Rightarrow \text{ar}(\triangle AEB) = \frac{1}{2} \text{ar}(\text{BEYC}) \text{ — (iv)}$$

Similarly,

$\triangle ACF$ and || gm BXFC on the same base CF and between the same parallels CF and AB.

$$\Rightarrow \text{ar}(\triangle ACF) = \frac{1}{2} \text{ar}(\text{BXFC}) \text{ — (v)}$$

From (iii), (iv) and (v),

$$\text{ar}(\triangle AEB) = \text{ar}(\triangle ACF)$$

9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Fig. 9.26). Show that $\text{ar}(\text{ABCD}) = \text{ar}(\text{PBQR})$.

[Hint : Join AC and PQ. Now compare $\text{ar}(\triangle ACQ)$ and $\text{ar}(\triangle APQ)$.]

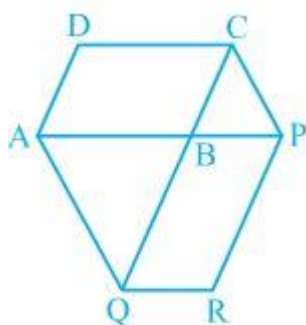
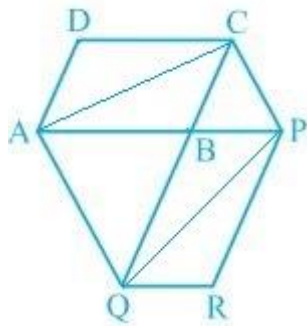


Fig. 9.26

Solution:



AC and PQ are joined.

$\text{ar}(\triangle ACQ) = \text{ar}(\triangle APQ)$ (On the same base AQ and between the same parallel lines AQ and CP)

$$\Rightarrow \text{ar}(\triangle ACQ) - \text{ar}(\triangle ABQ) = \text{ar}(\triangle APQ) - \text{ar}(\triangle ABQ)$$

$$\Rightarrow \text{ar}(\triangle ABC) = \frac{1}{2} \text{ar}(\triangle QBP) \text{ — (i)}$$

AC and QP are diagonals ABCD and PBQR.

$$\therefore, \text{ar}(\triangle ABC) = \frac{1}{2} \text{ar}(\triangle ABCD) \text{ — (ii)}$$

$$\text{ar}(\triangle QBP) = \frac{1}{2} \text{ar}(\triangle PBQR) \text{ — (iii)}$$

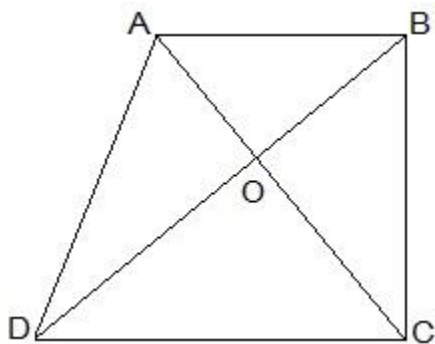
From (ii) and (iii),

$$\frac{1}{2} \text{ar}(\triangle ABCD) = \frac{1}{2} \text{ar}(\triangle PBQR)$$

$$\Rightarrow \text{ar}(\triangle ABCD) = \text{ar}(\triangle PBQR)$$

10. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at O. Prove that $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$.

Solution:



$\triangle DAC$ and $\triangle DBC$ lie on the same base DC and between the same parallels AB and CD.

$$\text{ar}(\triangle DAC) = \text{ar}(\triangle DBC)$$

$$\Rightarrow \text{ar}(\triangle DAC) - \text{ar}(\triangle DOC) = \text{ar}(\triangle DBC) - \text{ar}(\triangle DOC)$$

$$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

11. In Fig. 9.27, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

Show that

- (i) $\text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$
- (ii) $\text{ar}(AEDF) = \text{ar}(ABCDE)$

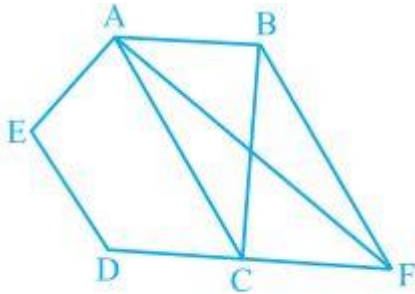


Fig. 9.27

Solution:

(i) $\triangle ACB$ and $\triangle ACF$ lie on the same base AC and between the same parallels AC and BF.

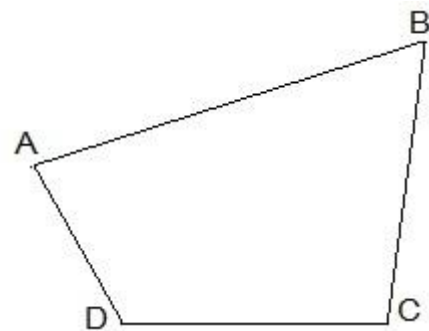
$$\therefore \text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$$

(ii) $\text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$

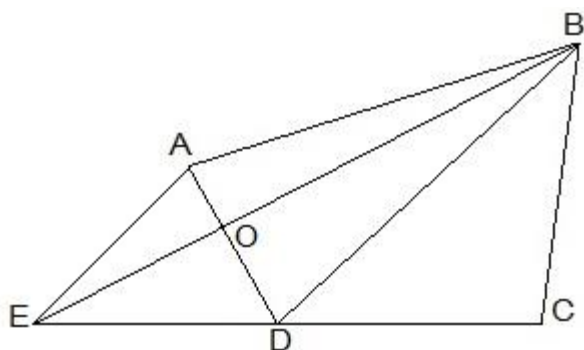
$$\Rightarrow \text{ar}(\triangle ACB) + \text{ar}(\triangle ACDE) = \text{ar}(\triangle ACF) + \text{ar}(\triangle ACDE)$$

$$\Rightarrow \text{ar}(ABCDE) = \text{ar}(AEDF)$$

12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.



Let ABCD be the plot of the land of the shape of a quadrilateral.



To Construct,

Join the diagonal BD.

Draw AE parallel to BD.

Join BE, that intersected AD at O.

We get,

$\triangle BCE$ is the shape of the original field

$\triangle AOB$ is the area for constructing health centre.

$\triangle DEO$ is the land joined to the plot.

To prove:

$$\text{ar}(\triangle DEO) = \text{ar}(\triangle AOB)$$

Proof:

$\triangle DEB$ and $\triangle DAB$ lie on the same base BD, in-between the same parallels BD and AE.

$$\text{ar}(\triangle DEB) = \text{ar}(\triangle DAB)$$

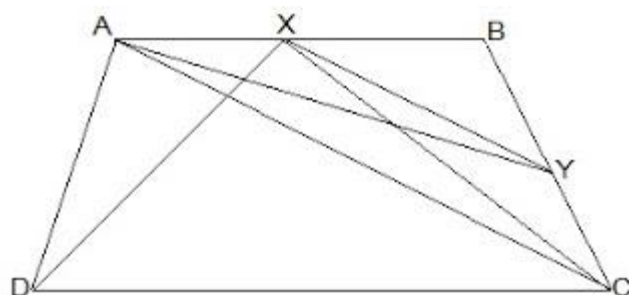
$$\Rightarrow \text{ar}(\triangle DEB) - \text{ar}(\triangle DOB) = \text{ar}(\triangle DAB) - \text{ar}(\triangle DOB)$$

$$\Rightarrow \text{ar}(\triangle DEO) = \text{ar}(\triangle AOB)$$

13. ABCD is a trapezium with $AB \parallel DC$. A line parallel to AC intersects AB at X and BC at Y. Prove that $\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$.

[Hint : Join CX.]

Solution:



Given,

ABCD is a trapezium with $AB \parallel DC$.

$XY \parallel AC$

Construction,

Join CX

To Prove,

$$\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$$

Proof:

$\text{ar}(\triangle ADX) = \text{ar}(\triangle AXC)$ — (i) (Since they are on the same base AX and in-between the same parallels AB and CD)

also,

$\text{ar}(\triangle AXC) = \text{ar}(\triangle ACY)$ — (ii) (Since they are on the same base AC and in-between the same parallels XY and AC.)

(i) and (ii),

$$\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$$

14. In Fig.9.28, $AP \parallel BQ \parallel CR$. Prove that $\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$.

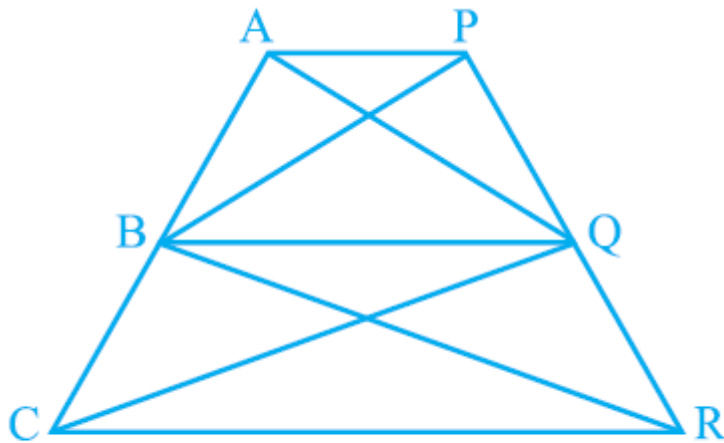


Fig. 9.28

Solution:

Given,

$$AP \parallel BQ \parallel CR$$

To Prove,

$$\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$$

Proof:

$\text{ar}(\triangle AQB) = \text{ar}(\triangle PBQ)$ — (i) (Since they are on the same base BQ and between the same parallels AP and BQ.)

also,

$\text{ar}(\triangle BQC) = \text{ar}(\triangle BQR)$ — (ii) (Since they are on the same base BQ and between the same parallels BQ and CR.)

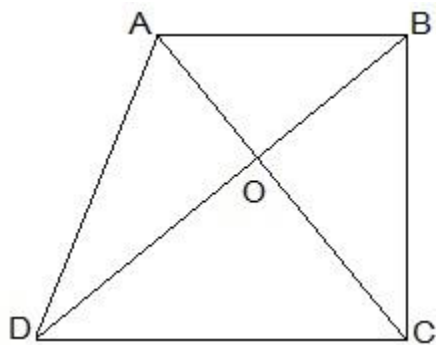
Adding (i) and (ii),

$$\text{ar}(\triangle AQB) + \text{ar}(\triangle BQC) = \text{ar}(\triangle PBQ) + \text{ar}(\triangle BQR)$$

$$\Rightarrow \text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$$

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$. Prove that ABCD is a trapezium.

Solution:



Given,

$$\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

To Prove,

ABCD is a trapezium.

Proof:

$$\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

$$\Rightarrow \text{ar}(\triangle AOD) + \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) + \text{ar}(\triangle AOB)$$

$$\Rightarrow \text{ar}(\triangle ADB) = \text{ar}(\triangle ACB)$$

Areas of $\triangle ADB$ and $\triangle ACB$ are equal. \therefore , they must lying between the same parallel lines.

$$\therefore, AB \parallel CD$$

\therefore , ABCD is a trapezium.

16. In Fig.9.29, $\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC)$ and $\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$. Show that both the quadrilaterals ABCD and DCPR are trapeziums.

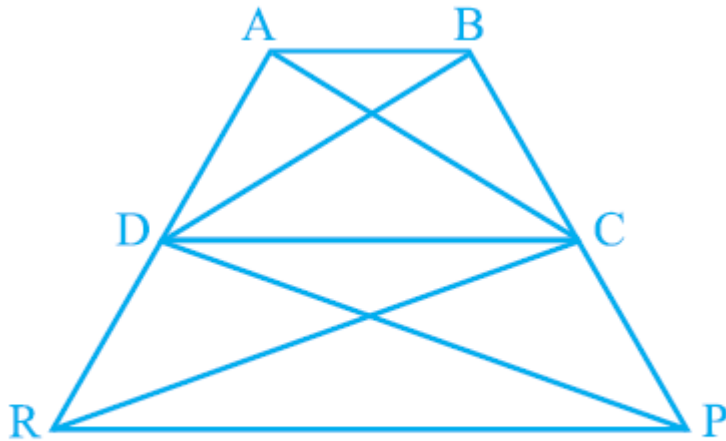


Fig. 9.29

Solution:

Given,

$$\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC)$$

$$\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$$

To Prove,

ABCD and DCPR are trapeziums.

Proof:

$$\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$$

$$\Rightarrow \text{ar}(\triangle BDP) - \text{ar}(\triangle DPC) = \text{ar}(\triangle DRC)$$

$$\Rightarrow \text{ar}(\triangle BDC) = \text{ar}(\triangle ADC)$$

$$\text{ar}(\triangle BDC) = \text{ar}(\triangle ADC).$$

\therefore , $\text{ar}(\triangle BDC)$ and $\text{ar}(\triangle ADC)$ are lying in-between the same parallel lines.

\therefore , $AB \parallel CD$

ABCD is a trapezium.

Similarly,

$$\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC).$$

\therefore , $\text{ar}(\triangle DRC)$ and $\text{ar}(\triangle DPC)$ are lying in-between the same parallel lines.

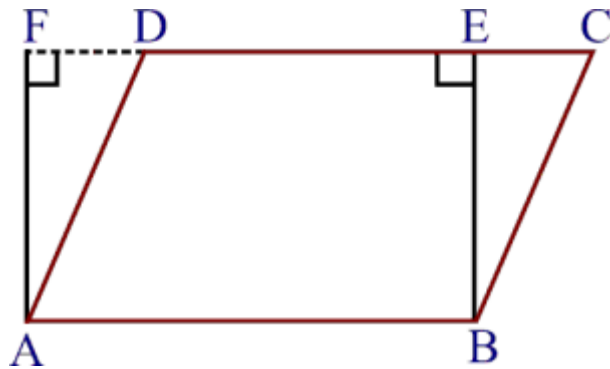
\therefore , $DC \parallel PR$

\therefore , DCPR is a trapezium.

Exercise 9.4(Optional)* Page: 164

1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Solution:



Given,

\parallel gm ABCD and a rectangle ABEF have the same base AB and equal areas.

To prove,

Perimeter of \parallel gm ABCD is greater than the perimeter of rectangle ABEF.

Proof,

We know that, the opposite sides of a \parallel gm and rectangle are equal.

$AB = DC$ [As ABCD is a \parallel gm]

and, $AB = EF$ [As ABEF is a rectangle]

$DC = EF \dots (i)$

Adding AB on both sides, we get,

$\Rightarrow AB + DC = AB + EF \dots (ii)$

We know that, the perpendicular segment is the shortest of all the segments that can be drawn to a given line from a point not lying on it.

$BE < BC$ and $AF < AD$

$\Rightarrow BC > BE$ and $AD > AF$

$\Rightarrow BC + AD > BE + AF \dots (iii)$

Adding (ii) and (iii), we get

$AB + DC + BC + AD > AB + EF + BE + AF$

$\Rightarrow AB + BC + CD + DA > AB + BE + EF + FA$

\Rightarrow perimeter of \parallel gm ABCD $>$ perimeter of rectangle ABEF.

, the perimeter of the parallelogram is greater than that of the rectangle.

Hence Proved.

2. In Fig. 9.30, D and E are two points on BC such that $BD = DE = EC$.

Show that $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$.

Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?

[Remark: Note that by taking $BD = DE = EC$, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide DABC into n triangles of equal areas.]

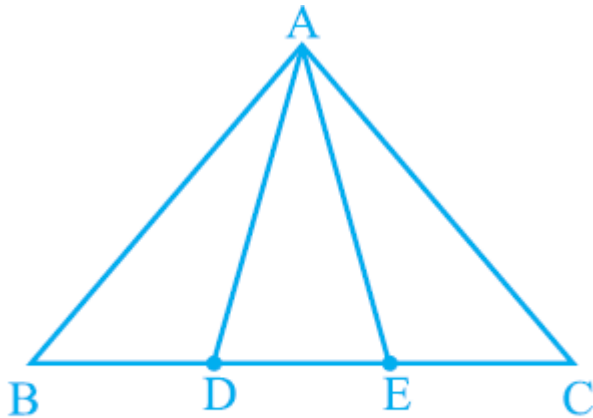


Fig. 9.30

Solution:

Given,

$$BD = DE = EC$$

To prove,

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$$

Proof,

In $(\triangle ABE)$, AD is median [since, $BD = DE$, given]

We know that, the median of a triangle divides it into two parts of equal areas

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle AED) \text{ ---(i)}$$

Similarly,

In $(\triangle ADC)$, AE is median [since, $DE = EC$, given]

$$\text{ar}(\triangle ADE) = \text{ar}(\triangle AEC) \text{ ---(ii)}$$

From the equation (i) and (ii), we get

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$$

3. In Fig. 9.31, ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$.

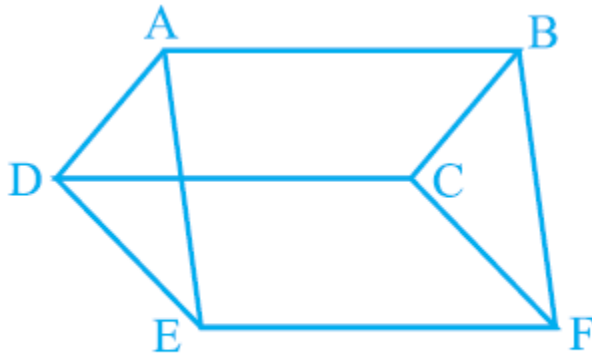


Fig. 9.31

Solution:

Given,

ABCD, DCFE and ABFE are parallelograms

To prove,

$$\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$$

Proof,

In $\triangle ADE$ and $\triangle BCF$,

$AD = BC$ [Since, they are the opposite sides of the parallelogram ABCD]

$DE = CF$ [Since, they are the opposite sides of the parallelogram DCFE]

$AE = BF$ [Since, they are the opposite sides of the parallelogram ABFE]

$\triangle ADE = \triangle BCF$ [Using SSS Congruence theorem]

$$\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF) \text{ [By CPCT]}$$

4. In Fig. 9.32, ABCD is a parallelogram and BC is produced to a point Q such that $AD = CQ$. If AQ intersect DC at P, show that $\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$.

[Hint : Join AC.]

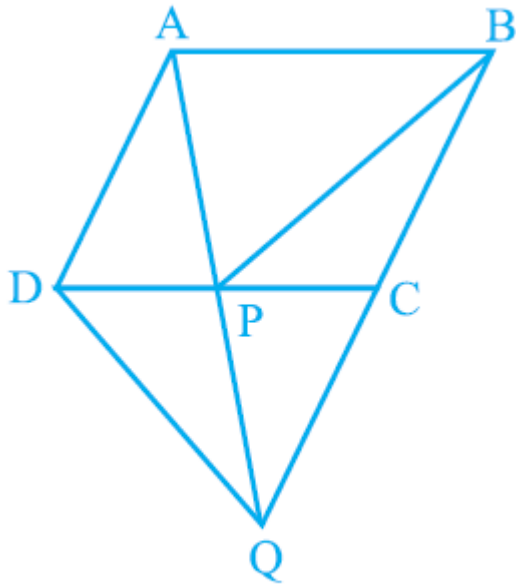


Fig. 9.32

Solution:

Given:

ABCD is a parallelogram

$AD = CQ$

To prove:

$\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$

Proof:

In $\triangle ADP$ and $\triangle QCP$,

$\angle APD = \angle QPC$ [Vertically Opposite Angles]

$\angle ADP = \angle QCP$ [Alternate Angles]

$AD = CQ$ [given]

, $\triangle ADO = \triangle ACD$ [AAS congruency]

, $DP = CP$ [CPCT]

In $\triangle CDQ$, QP is median. [Since, $DP = CP$]

Since, median of a triangle divides it into two parts of equal areas.

, $\text{ar}(\triangle DPQ) = \text{ar}(\triangle QPC)$ —(i)

In $\triangle PBQ$, PC is median. [Since, $AD = CQ$ and $AD = BC \Rightarrow BC = QC$]

Since, median of a triangle divides it into two parts of equal areas.

, $\text{ar}(\triangle QPC) = \text{ar}(\triangle BPC)$ —(ii)

From the equation (i) and (ii), we get

$$\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$$

5. In Fig.9.33, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that:

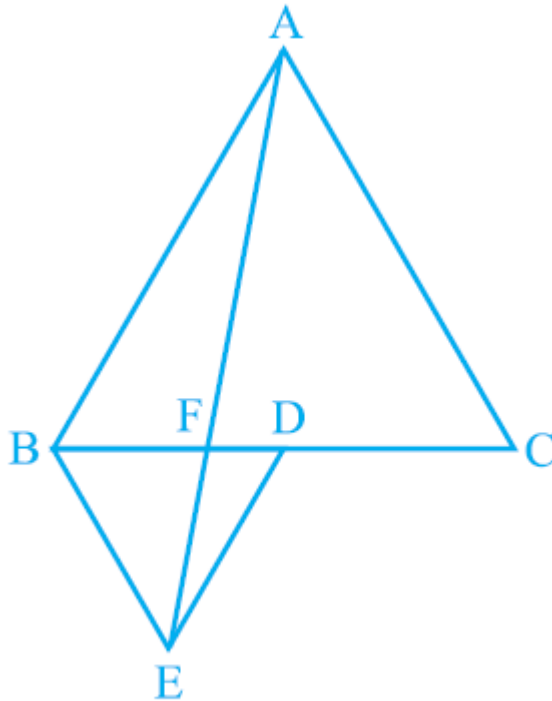


Fig. 9.33

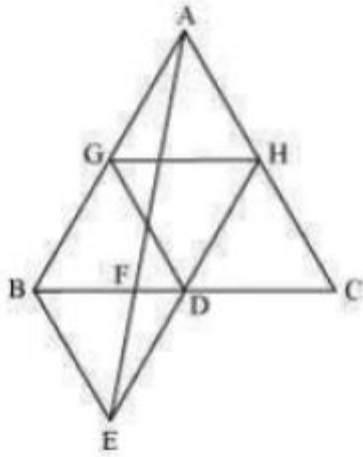
- (i) $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$
- (ii) $\text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle BAE)$
- (iii) $\text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC)$
- (iv) $\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$
- (v) $\text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$
- (vi) $\text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$

Solution:

(i) Assume that G and H are the mid-points of the sides AB and AC respectively.

Join the mid-points with line-segment GH. Here, GH is parallel to third side.

, BC will be half of the length of BC by mid-point theorem.



$\therefore GH = \frac{1}{2}BC$ and $GH \parallel BD$

$\therefore GH = BD = DC$ and $GH \parallel BD$ (Since, D is the mid-point of BC)

Similarly,

$GD = HC = HA$

$HD = AG = BG$

, $\triangle ABC$ is divided into 4 equal equilateral triangles $\triangle BGD$, $\triangle AGH$, $\triangle DHC$ and $\triangle GHD$

We can say that,

$\triangle BGD = \frac{1}{4}\triangle ABC$

Considering, $\triangle BDG$ and $\triangle BDE$

$BD = BD$ (Common base)

Since both triangles are equilateral triangle, we can say that,

$BG = BE$

$DG = DE$

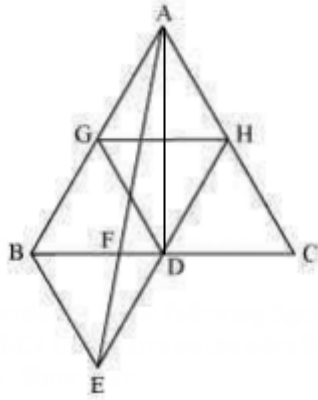
, $\triangle BDG \cong \triangle BDE$ [By SSS congruency]

, $\text{area}(\triangle BDG) = \text{area}(\triangle BDE)$

$\text{ar}(\triangle BDE) = \frac{1}{4}\text{ar}(\triangle ABC)$

Hence proved

(ii) $\text{ar}(\triangle BDE) = \text{ar}(\triangle AED)$ (Common base DE and $DE \parallel AB$)



$$\text{ar}(\triangle BDE) - \text{ar}(\triangle FED) = \text{ar}(\triangle AED) - \text{ar}(\triangle FED)$$

$$\text{ar}(\triangle BEF) = \text{ar}(\triangle AFD) \dots (i)$$

Now,

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABF) + \text{ar}(\triangle AFD)$$

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABF) + \text{ar}(\triangle BEF) \text{ [From equation (i)]}$$

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABE) \dots (ii)$$

AD is the median of $\triangle ABC$.

$$\text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC)$$

$$= \frac{4}{2} \text{ar}(\triangle BDE)$$

$$= 2 \text{ar}(\triangle BDE) \dots (iii)$$

From (ii) and (iii), we obtain

$$2 \text{ar}(\triangle BDE) = \text{ar}(\triangle ABE)$$

$$\text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle ABE)$$

Hence proved

$$(iii) \text{ar}(\triangle ABE) = \text{ar}(\triangle BEC) \text{ [Common base BE and } BE \parallel AC]$$

$$\text{ar}(\triangle ABF) + \text{ar}(\triangle BEF) = \text{ar}(\triangle BEC)$$

From eqⁿ (i), we get,

$$\text{ar}(\triangle ABF) + \text{ar}(\triangle AFD) = \text{ar}(\triangle BEC)$$

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle BEC)$$

$$(1/2) \text{ar}(\triangle ABC) = \text{ar}(\triangle BEC)$$

$$\text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC)$$

Hence proved

(iv) $\triangle BDE$ and $\triangle AED$ lie on the same base (DE) and are in-between the parallel lines DE and AB.

$$\therefore \text{ar}(\triangle BDE) = \text{ar}(\triangle AED)$$

Subtracting $\text{ar}(\triangle FED)$ from L.H.S and R.H.S,

We get,

$$\therefore \text{ar}(\triangle BDE) - \text{ar}(\triangle FED) = \text{ar}(\triangle AED) - \text{ar}(\triangle FED)$$

$$\therefore \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$$

Hence proved

(v) Assume that h is the height of vertex E , corresponding to the side BD in $\triangle BDE$.

Also assume that H is the height of vertex A , corresponding to the side BC in $\triangle ABC$.

While solving Question (i),

We saw that,

$$\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

While solving Question (iv),

We saw that,

$$\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD).$$

$$\therefore \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$$

$$= 2 \text{ar}(\triangle FED)$$

$$\text{Hence, } \text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$$

Hence proved

$$\text{(vi) } \text{ar}(\triangle AFC) = \text{ar}(\triangle AFD) + \text{ar}(\triangle ADC)$$

$$= 2 \text{ar}(\triangle FED) + \frac{1}{2} \text{ar}(\triangle ABC) \text{ [using (v)]}$$

$$= 2 \text{ar}(\triangle FED) + \frac{1}{2} [4 \text{ar}(\triangle BDE)] \text{ [Using result of Question (i)]}$$

$$= 2 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle BDE)$$

Since, $\triangle BDE$ and $\triangle AED$ are on the same base and between same parallels

$$= 2 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle AED)$$

$$= 2 \text{ar}(\triangle FED) + 2 [\text{ar}(\triangle AFD) + \text{ar}(\triangle FED)]$$

$$= 2 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle AFD) + 2 \text{ar}(\triangle FED) \text{ [From question (viii)]}$$

$$= 4 \text{ar}(\triangle FED) + 2 \text{ar}(\triangle AFD)$$

$$\Rightarrow \text{ar}(\triangle AFC) = 8 \text{ar}(\triangle FED)$$

$$\Rightarrow \text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$$

Hence proved

6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

$$\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC).$$

[Hint : From A and C, draw perpendiculars to BD.]

Solution:

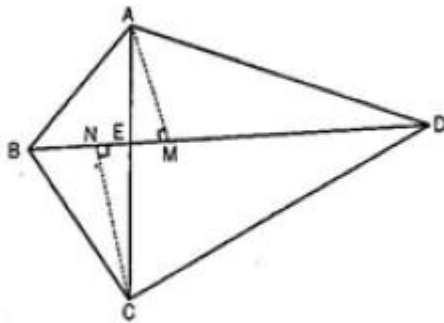
Given:

The diagonal AC and BD of the quadrilateral ABCD, intersect each other at point E.

Construction:

From A, draw AM perpendicular to BD

From C, draw CN perpendicular to BD



To Prove,

$$ar(\Delta AED) \times ar(\Delta BEC) = ar(\Delta ABE) \times ar(\Delta CDE)$$

Proof,

$$ar(\Delta ABE) = \frac{1}{2} \times BE \times AM \dots\dots\dots i$$

$$ar(\Delta AED) = \frac{1}{2} \times DE \times AM \dots\dots\dots ii$$

Dividing eq. ii by i , we get,

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{\frac{1}{2} \times DE \times AM}{\frac{1}{2} \times BE \times AM} \Rightarrow \frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{DE}{BE} \dots\dots\dots iii$$

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{DE}{BE} \Rightarrow ar(\Delta AED) \times BE = ar(\Delta ABE) \times DE \dots\dots\dots iii$$

Similarly,

$$\frac{ar(\Delta CDE)}{ar(\Delta BEC)} = \frac{DE}{BE} \Rightarrow ar(\Delta CDE) \times BE = ar(\Delta BEC) \times DE \dots\dots\dots iv$$

From eq. iii and iv , we get

$$\begin{aligned} ar(\Delta AED) \times BE &= ar(\Delta ABE) \times DE \\ ar(\Delta CDE) \times BE &= ar(\Delta BEC) \times DE \\ ar(\Delta AED) \times ar(\Delta BEC) &= ar(\Delta ABE) \times ar(\Delta CDE) \end{aligned}$$

Hence proved.

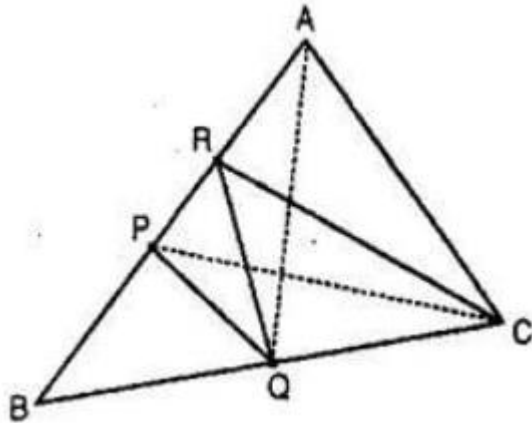
7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that:

(i) $\text{ar}(\text{PRQ}) = \frac{1}{2} \text{ar}(\text{ARC})$

(ii) $\text{ar}(\text{RQC}) = \frac{3}{8} \text{ar}(\text{ABC})$

(iii) $\text{ar}(\text{PBQ}) = \text{ar}(\text{ARC})$

Solution:



(i) We know that, median divides the triangle into two triangles of equal area,

PC is the median of ABC.

$$\text{ar}(\triangle BPC) = \text{ar}(\triangle APC) \dots\dots\dots(i)$$

RC is the median of APC.

$$\text{ar}(\triangle ARC) = \frac{1}{2} \text{ar}(\triangle APC) \dots\dots\dots(ii)$$

PQ is the median of BPC.

$$\text{ar}(\triangle PQC) = \frac{1}{2} \text{ar}(\triangle BPC) \dots\dots\dots(iii)$$

From eq. (i) and (iii), we get,

$$\text{ar}(\triangle PQC) = \frac{1}{2} \text{ar}(\triangle APC) \dots\dots\dots(iv)$$

From eq. (ii) and (iv), we get,

$$\text{ar}(\triangle PQC) = \text{ar}(\triangle ARC) \dots\dots\dots(v)$$

P and Q are the mid-points of AB and BC respectively [given]

$PQ \parallel AC$

$$\text{and, } PA = \frac{1}{2} AC$$

Since, triangles between same parallel are equal in area, we get,

$$\text{ar}(\triangle APQ) = \text{ar}(\triangle PQC) \dots\dots\dots(vi)$$

From eq. (v) and (vi), we obtain,

$$\text{ar}(\triangle APQ) = \text{ar}(\triangle ARC) \dots\dots\dots(vii)$$

R is the mid-point of AP.

RQ is the median of APQ.

$$\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta APQ) \dots\dots\dots(\text{viii})$$

From (vii) and (viii), we get,

$$\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta ARC)$$

Hence Proved.

(ii) PQ is the median of BPC

$$\text{ar}(\Delta PQC) = \frac{1}{2} \text{ar}(\Delta BPC)$$

$$= \frac{1}{2} \times \frac{1}{2} \text{ar}(\Delta ABC)$$

$$= \frac{1}{4} \text{ar}(\Delta ABC) \dots\dots\dots(\text{ix})$$

Also,

$$\text{ar}(\Delta PRC) = \frac{1}{2} \text{ar}(\Delta APC) \text{ [From (iv)]}$$

$$\text{ar}(\Delta PRC) = \frac{1}{2} \times \frac{1}{2} \text{ar}(\Delta ABC)$$

$$= \frac{1}{4} \text{ar}(\Delta ABC) \dots\dots\dots(\text{x})$$

Add eq. (ix) and (x), we get,

$$\text{ar}(\Delta PQC) + \text{ar}(\Delta PRC) = \frac{1}{4} \times \frac{1}{4} \text{ar}(\Delta ABC)$$

$$\text{ar}(\text{quad. PQCR}) = \frac{1}{4} \text{ar}(\Delta ABC) \dots\dots\dots(\text{xi})$$

Subtracting $\text{ar}(\Delta PRQ)$ from L.H.S and R.H.S,

$$\text{ar}(\text{quad. PQCR}) - \text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta ABC) - \text{ar}(\Delta PRQ)$$

$$\text{ar}(\Delta RQC) = \frac{1}{2} \text{ar}(\Delta ABC) - \frac{1}{2} \text{ar}(\Delta ARC) \text{ [From result (i)]}$$

$$\text{ar}(\Delta ARC) = \frac{1}{2} \text{ar}(\Delta ABC) - \frac{1}{2} \times \frac{1}{2} \text{ar}(\Delta APC)$$

$$\text{ar}(\Delta RQC) = \frac{1}{2} \text{ar}(\Delta ABC) - \frac{1}{4} \text{ar}(\Delta APC)$$

$$\text{ar}(\Delta RQC) = \frac{1}{2} \text{ar}(\Delta ABC) - \left(\frac{1}{4} \times \frac{1}{2} \text{ar}(\Delta ABC) \right) \text{ [As, PC is median of } \Delta ABC]$$

$$\text{ar}(\Delta RQC) = \frac{1}{2} \text{ar}(\Delta ABC) - \frac{1}{8} \text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta RQC) = \left(\frac{1}{2} - \frac{1}{8} \right) \text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta RQC) = \frac{3}{8} \text{ar}(\Delta ABC)$$

$$\text{(iii) ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta ARC) \text{ [From result (i)]}$$

$$2 \text{ar}(\Delta PRQ) = \text{ar}(\Delta ARC) \dots\dots\dots(\text{xii})$$

$$\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta APQ) \text{ [RQ is the median of APQ]} \dots\dots\dots(\text{xiii})$$

But, we know that,

$\text{ar}(\Delta APQ) = \text{ar}(\Delta PQC)$ [From the reason mentioned in eq. (vi)]
(xiv)

From eq. (xiii) and (xiv), we get,

$$\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta PQC) \text{(xv)}$$

At the same time,

$$\text{ar}(\Delta BPQ) = \text{ar}(\Delta PQC) \text{ [PQ is the median of BPC](xvi)}$$

From eq. (xv) and (xvi), we get,

$$\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta BPQ) \text{(xvii)}$$

From eq. (xii) and (xvii), we get,

$$2 \times \frac{1}{2} \text{ar}(\Delta BPQ) = \text{ar}(\Delta ARC)$$

$$\Rightarrow \text{ar}(\Delta BPQ) = \text{ar}(\Delta ARC)$$

Hence Proved.

8. In Fig. 9.34, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX \perp DE meets BC at Y. Show that:

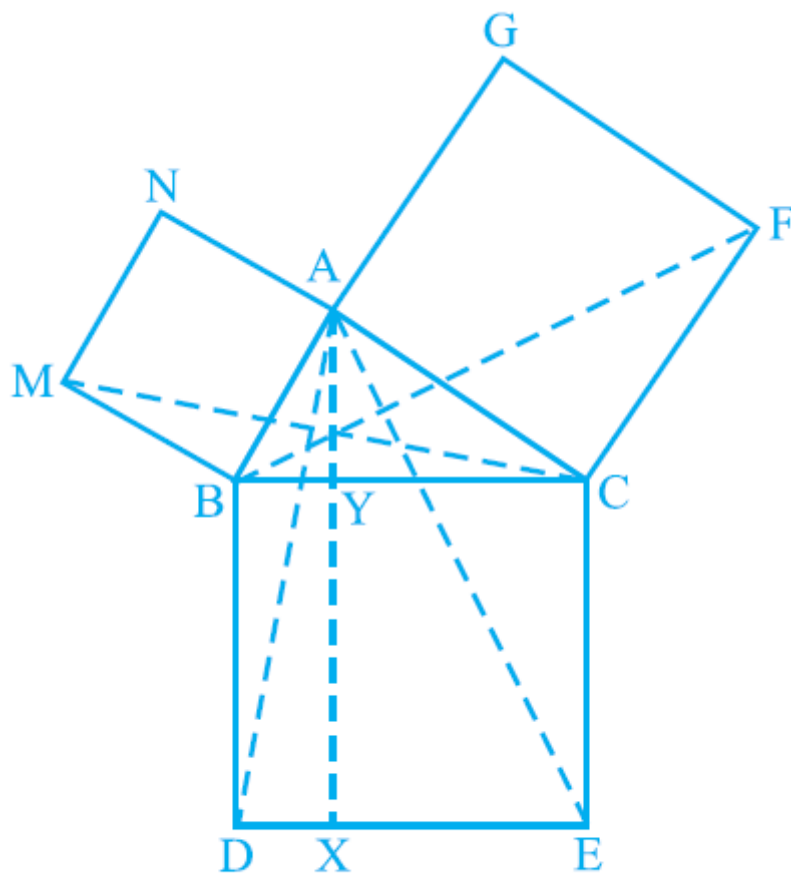


Fig. 9.34

- (i) $\triangle MBC \cong \triangle ABD$
- (ii) $\text{ar}(\text{BYXD}) = 2\text{ar}(\text{MBC})$
- (iii) $\text{ar}(\text{BYXD}) = \text{ar}(\text{ABMN})$
- (iv) $\triangle FCB \cong \triangle ACE$
- (v) $\text{ar}(\text{CYXE}) = 2\text{ar}(\text{FCB})$
- (vi) $\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})$
- (vii) $\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})$

Note : Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in Class X.

Solution:

(i) We know that each angle of a square is 90° . Hence, $\angle ABM = \angle DBC = 90^\circ$

$$\therefore \angle ABM + \angle ABC = \angle DBC + \angle ABC$$

$$\therefore \angle MBC = \angle ABD$$

In $\triangle MBC$ and $\triangle ABD$,

$$\angle MBC = \angle ABD \text{ (Proved above)}$$

$$MB = AB \text{ (Sides of square ABMN)}$$

$$BC = BD \text{ (Sides of square BCED)}$$

$$\therefore \triangle MBC \cong \triangle ABD \text{ (SAS congruency)}$$

(ii) We have

$$\triangle MBC \cong \triangle ABD$$

$$\therefore \text{ar}(\triangle MBC) = \text{ar}(\triangle ABD) \dots (i)$$

It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)

$\therefore BD \parallel AX$ (Two lines perpendicular to same line are parallel to each other)

$\triangle ABD$ and parallelogram BYXD are on the same base BD and between the same parallels BD and AX .

$$\text{Area}(\triangle YXD) = 2 \text{Area}(\triangle MBC) \text{ [From equation (i)]} \dots (ii)$$

(iii) $\triangle MBC$ and parallelogram ABMN are lying on the same base MB and between same parallels MB and NC .

$$2 \text{ar}(\triangle MBC) = \text{ar}(\text{ABMN})$$

$$\text{ar}(\triangle YXD) = \text{ar}(\text{ABMN}) \text{ [From equation (ii)]} \dots (iii)$$

(iv) We know that each angle of a square is 90° .

$$\therefore \angle FCA = \angle BCE = 90^\circ$$

$$\therefore \angle FCA + \angle ACB = \angle BCE + \angle ACB$$

$$\therefore \angle FCB = \angle ACE$$

In $\triangle FCB$ and $\triangle ACE$,

$$\angle FCB = \angle ACE$$

$$FC = AC \text{ (Sides of square ACFG)}$$

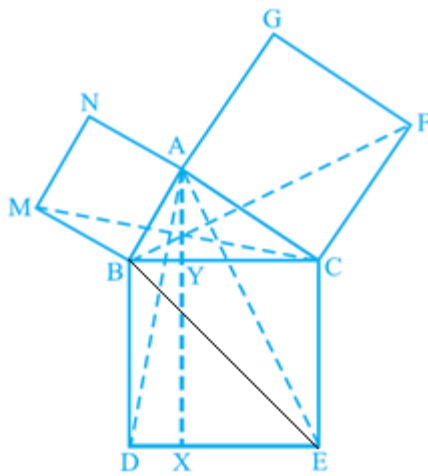
$$CB = CE \text{ (Sides of square BCED)}$$

$$\triangle FCB \cong \triangle ACE \text{ (SAS congruency)}$$

(v) $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC) [given]

Hence,

$CE \parallel AX$ (Two lines perpendicular to the same line are parallel to each other)



Consider $\triangle ACE$ and parallelogram $CYXE$

$\triangle ACE$ and parallelogram $CYXE$ are on the same base CE and between the same parallels CE and AX .

$$\therefore \text{ar}(\triangle YXE) = 2 \text{ ar}(\triangle ACE) \dots \text{(iv)}$$

We had proved that

$$\therefore \triangle FCB \cong \triangle ACE$$

$$\text{ar}(\triangle FCB) \cong \text{ar}(\triangle ACE) \dots \text{(v)}$$

From equations (iv) and (v), we get

$$\text{ar}(CYXE) = 2 \text{ ar}(\triangle FCB) \dots \text{(vi)}$$

(vi) Consider $\triangle FCB$ and parallelogram $ACFG$

$\triangle FCB$ and parallelogram $ACFG$ are lying on the same base CF and between the same parallels CF and BG .

$$\therefore \text{ar}(ACFG) = 2 \text{ ar}(\triangle FCB)$$

$$\therefore \text{ar}(ACFG) = \text{ar}(CYXE) \text{ [From equation (vi)]} \dots \text{(vii)}$$

(vii) From the figure, we can observe that

$$\text{ar}(\triangle CED) = \text{ar}(\triangle YXD) + \text{ar}(CYXE)$$

$$\therefore \text{ar}(\triangle CED) = \text{ar}(\triangle BMN) + \text{ar}(\triangle ACFG) \text{ [From equations (iii) and (vii)].}$$