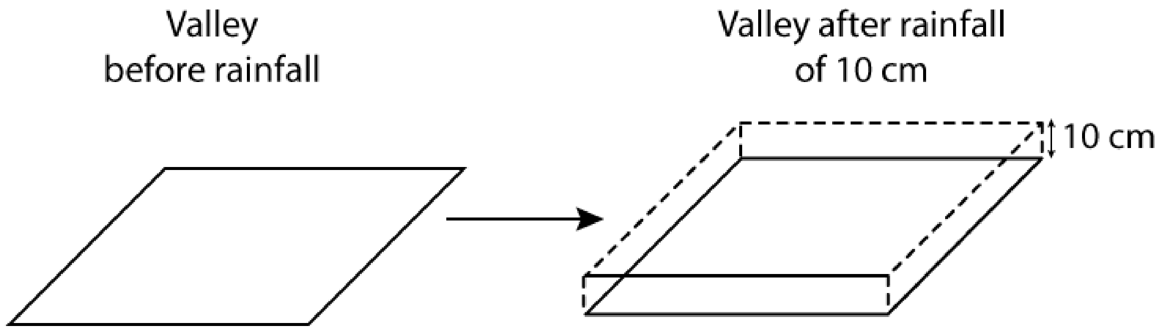
NCERT Solutions for Class 10 Maths Chapter 13 - Surface Areas and Volumes

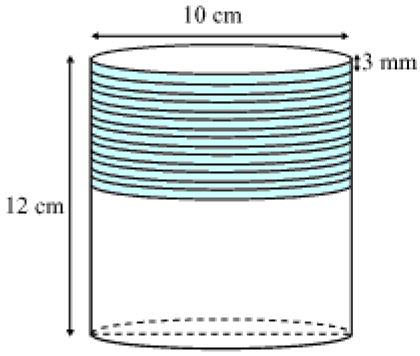
Chapter 13 - Surface Areas and Volumes Exercise Ex. 13.5
Solution 4



Rainfall in the valley = 10 cm = 0.1 m
Area of the valley = 7280 km^2 = 7280000000 m^2
Volume of rain water = 7280 × 10 × 1/1,00,000 = 0.7280 km^3
Volume of three rivers = 3 × 1072 × 75/1000 × 3/1000 = 0.7236 km

Hence, the two are approximately equivalent.

Solution 1



1 round of wire will cover 3 mm height of cylinder.

$$\text{Number of rounds} = \frac{\text{Height of cylinder}}{\text{Diameter of wire}}$$

$$= \frac{12}{0.3} = 40 \text{ rounds}$$

Length of wire required in 1 round = circumference of base of cylinder

$= 2\pi r = 2\pi \times 5 = 10\pi$

Length of wire in 40 rounds = $40 \times 10\pi$

$$= \frac{400 \times 22}{7} = \frac{8800}{7}$$

= 1257.14 cm = 12.57 m

Radius of wire $= \frac{0.3}{2} = 0.15$ cm

Volume of wire = Area of cross section of wire × length of wire

$= \pi(0.15)^2 \times 1257.14$

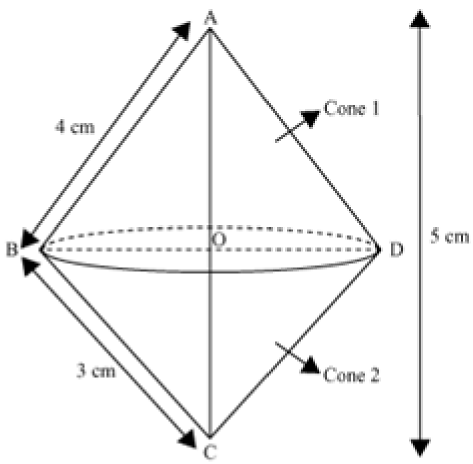
= 88.89 cm^3

Mass = Volume × Density

= 88.89 × 8.88

= 789.41 gm

Solution 2



Double cone so formed by revolving this right angle triangle ABC about its hypotenuse is shown in figure.

Hypotenuse AC $= \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5\text{cm}$

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 4 \times 3$

$\frac{1}{2} \times 5 \times OB = \frac{12}{2}$

$OB = \frac{12}{5} = 2.4 \text{ cm}$

Volume of double cone = Volume of cone 1 + Volume of cone 2

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3}\pi r^2 (h_1 + h_2) = \frac{1}{3}\pi r^2 (OA + OC)$

$= \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 (5)$

$= 30.14 \text{ cm}^3$

Surface area of double cone = surface area of cone 1 + surface area of cone 2

$= \pi r l_1 + \pi r l_2$

$= \pi r [4 + 3] = \frac{22}{7} \times 2.4 \times 7$

$= 52.8 \text{ cm}^2$

Solution 3

Volume of cistern $= 150 \times 120 \times 110$

$= 1980000 \text{ cm}^3$

Volume to be filled in cistern $= 1980000 - 129600$

$= 1850400 \text{ cm}^3$

Let n numbers of porous bricks were placed in cistern.

So volume of n bricks $= n \times 22.5 \times 7.5 \times 6.5$

$= 1096.875n$

As each bricks absorbs one-seventeenth of its volume, so volume absorbed by there bricks $= \frac{n}{17}(1096.875)$

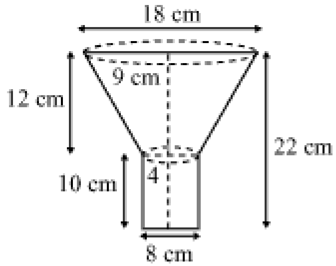
$1850400 + \frac{n}{17}(1096.875) = (1096.875)n$

$1850400 = \frac{16n}{17}(1096.875)$

$n = 1792.41$

So, 1792 bricks were placed in the cistern.

Solution 5



Radius (r_1) of upper circular end of frustum part $= \frac{18}{2} = 9$ cm

Radius (r_2) of lower circular end of frustum part = radius of circular end of cylindrical part $= \frac{8}{2} = 4$ cm

Height (h_1) of frustum part = 22 – 10 = 12 cm

Height (h_2) of cylindrical part = 10 cm

Slant height (l) of frustum part

$= \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(9-4)^2 + (12)^2} = 13$ cm

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

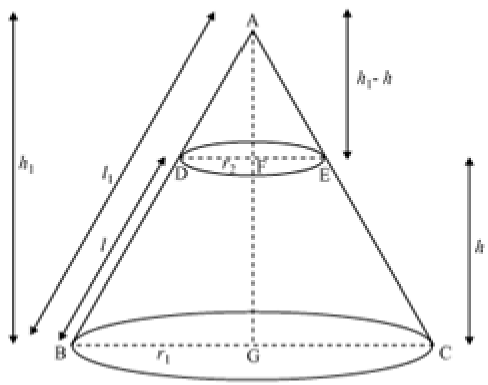
$= \pi(r_1 + r_2)l + 2\pi r_2 h_2$

$= \frac{22}{7} \times (9+4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$

$= \frac{22}{7}[169 + 80] = \frac{22 \times 249}{7}$

$= 782\frac{4}{7}$ cm^2

Solution 6



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base.
In ΔABG and ADF
As $\Delta ABG \sim \Delta ADF$

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$\frac{1 - l}{l_1} = \frac{r_2}{r_1}$$

$$\frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\frac{l_1}{l} = \frac{r_1}{r_1 - r_2}$$

CSA of frustum DECB = CSA of cone ABC − CSA cone ADE

$$= \pi r_1 l_1 - \pi r_2 (l_1 - l)$$

$$= \pi r_1 \left(\frac{l r_1}{r_1 - r_2} \right) - \pi r_2 \left[\frac{r_1 l}{r_1 - r_2} - l \right]$$

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \pi r_2 \left(\frac{r_1 l - r_1 l + r_2 l}{r_1 - r_2} \right)$$

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \frac{\pi r_2^2 l}{r_1 - r_2}$$

$$= \pi l \left[\frac{r_1^2 - r_2^2}{r_1 - r_2} \right]$$

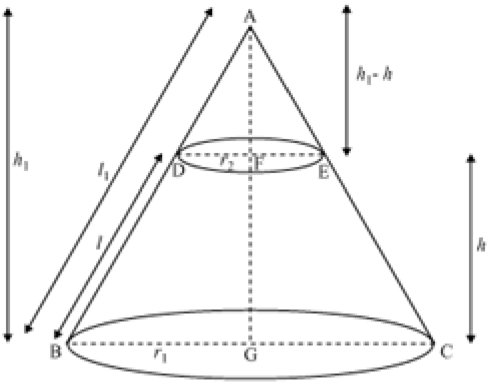
CSA of frustum = $\pi (r_1 + r_2) l$

Total surface are of frustum = CSA of frustum + area of upper circular end + area of lower circular end

$$= \pi (r_1 + r_2) l + \pi r_2^2 + \pi r_1^2$$

$$= \pi \left[(r_1 + r_2) l + r_1^2 + r_2^2 \right]$$

Solution 7



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base.
In ΔABG and ADF
$\Delta ABG \sim \Delta ADF$

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

Volume of frustum of cone = Volume of cone ABC − Volume of cone ADE

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$= \frac{\pi}{3}\left[r_1^2 h_1 - r_2^2 (h_1 - h)\right]$$

$$= \frac{\pi}{3}\left[r_1^2\left(\frac{hr_1}{r_1 - r_2}\right) - r_2^2\left(\frac{hr_1}{r_1 - r_2} - h\right)\right]$$

$$= \frac{\pi}{3}\left[\left(\frac{hr_1^3}{r_1 - r_2}\right) - r_2^2\left(\frac{hr_1 - hr_1 + hr_2}{r_1 - r_2}\right)\right]$$

$$= \frac{\pi}{3}\left[\frac{hr_1^3}{r_1 - r_2} - \frac{hr_2^3}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3}h\left[\frac{r_1^3 - r_2^3}{r_1 - r_2}\right]$$

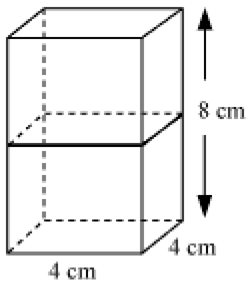
$$= \frac{\pi}{3}h\left[\frac{(r_1 - r_2)(r_1^2 + r_2^2 + r_1 r_2)}{r_1 - r_2}\right]$$

$$= \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$$

Chapter 13 - Surface Areas and Volumes Exercise Ex. 13.1

Solution 1

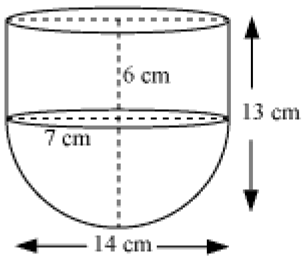
Given that
Volume of cubes = 64 cm^3
$(Edge)^3 = 64$
Edge = 4



If cubes are joined end to end, dimensions of resulting cuboid will be 4 cm, 4 cm, 8 cm.

$$\begin{aligned}\therefore \text{surface area and cuboids} &= 2(lb + bh + lh)\\ &= 2(4\times 4 + 4\times 8 + 4\times 8)\\ &= 2(16 + 32 + 32)\\ &= 2(16 + 64)\\ &= 2\times 80 = 160\ cm^2\end{aligned}$$

Solution 2



radius (r) of cylindrical part and hemispherical part =7 cm
Height of hemispherical part = radius = 7 cm.
Height of cylindrical part (h) = 13 −7 = 6 cm
Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part

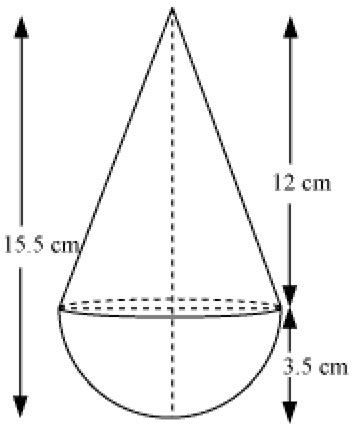
$$= 2\pi rh + 2\pi r^2$$

$$\text{Inner surface area of vessel} = 2\times\frac{22}{7}\times 7\times 6 + 2\times\frac{22}{7}\times 7\times 7$$

$$= 44(6+7) = 44\times 13$$

$$= 572\ cm^2$$

Solution 3



Radius of conical part and hemispherical part = 3.5 cm

Height of hemispherical part = radius (r) = 3.5 = $\frac{7}{2}$ cm.

Height of conical part (h) = 15.5 –3.5 = 12 cm

Slant height (l) of conical part = $\sqrt{r^2+h^2}$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4}+144} = \sqrt{\frac{49+576}{4}}$$

$$= \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

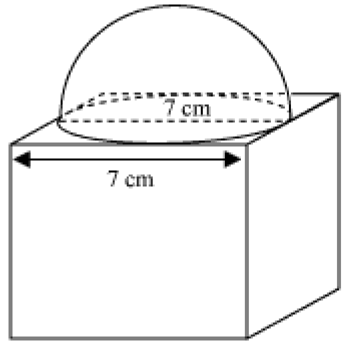
Total surface area of toy = $\pi rl + 2\pi r^2$

$$= \frac{22}{7}\times\frac{7}{2}\times\frac{25}{2} + 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$$= 137.5 + 77 = 214.5 cm^2$$

Concept Insight: Drawing a correct figure with appropriate labels and dimensions written on it is very important not only it will help you to understand the problem easily but also give an idea to examiner

Solution 4



From the figure it the greatest diameter possible for such hemisphere is equal to cube's edge i.e. 7cm.

Radius (r) of hemispherical part = $\frac{7}{2}$ = 3.5cm.

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part – area of base of hemispherical part

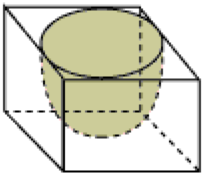
$$= 6\ (\text{edge})^2 + 2\pi r^2 - \pi r^2 = 6\ (\text{edge})^2 + \pi r^2$$

Total surface area of solid = $6(7)^2 + \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$

$$= 294 + 38.5 = 332.5\ cm^2$$

Concept Insights: Remember to consider the visible surface for calculating the surface area

Solution 5



Diameter of hemisphere = edge of cube = l

Radius of hemisphere = $\frac{l}{2}$

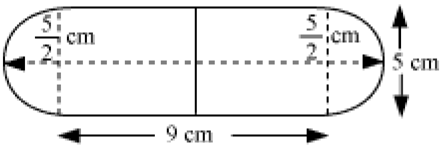
Total surface area of solid = Surface area of cubical part + CSA of hemispherical
part – area of base of hemispherical part

$= 6\ (\text{edge})^2 + 2\pi r^2 - \pi r^2 = 6\ (\text{edge})^2 + \pi r^2$

$$\text{Total surface area of solid } = 6l^2 + \pi \times \left(\frac{l}{2}\right)^2$$

$$= \left(6l^2 + \frac{\pi l^2}{4}\right)\text{unit}^2$$

Solution 6



Radius (r) of cylindrical part = radius (r) of hemispherical part

$$= \frac{\text{diameter of the capsule}}{2} = \frac{5}{2}$$

Length of cylindrical part (h) = length of the entire capsule – 2 × r

= 14 – 5 = 9cm

Surface area of capsule = 2 × CSA of hemispherical part + CSA of cylindrical part.

$$= 2 \times 2\pi r^2 + 2\pi rh$$

$$= 4\pi\left(\frac{5}{2}\right)^2 + 2\pi\left(\frac{5}{2}\right)(9)$$

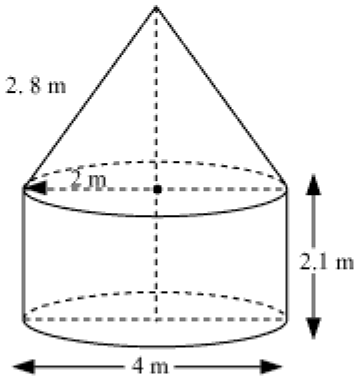
$$= 25\pi + 45\pi$$

$$= 70\pi \text{ mm}^2$$

$$= 70 \times \frac{22}{7}$$

$$= 220 \text{ mm}^2$$

Solution 7



Given that

Height (h) of the cylindrical part = 2.1m

Diameter of the cylindrical part = 4m

So, radius of the cylindrical part = 2m

Slant height (l) of conical part = 2.8m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi[2.8 + 2 \times 2.1] = 2\pi[2.8 + 4.2] = 2 \times \frac{22}{7} \times 7$

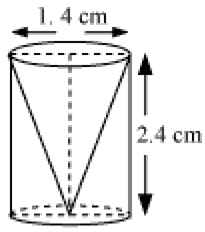
$= 44m^2$

Cost of $1m^2$ canvas = Rs.500

Cost of $44m^2$ canvas = 44 × 500 = 22000

So, it will cost Rs.22000 for making such a tent.

Solution 8



Given that

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4cm

Diameter of the cylindrical part = 1.4cm

So, radius (r) of the cylindrical part = 0.7cm

Slant height (l) of conical part = $\sqrt{r^2 + h^2}$

$= \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$

$= \sqrt{6.25} = 2.5$

Total surface area of remaining solid will be =

= CSA of cylindrical part + CSA of conical part + area of cylindrical base

$= 2\pi rh + \pi rl + \pi r^2$

$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$

$= 4.4 \times 2.4 + 2.2 \times 2.5 + 2.2 \times 0.7$

$= 10.56 + 5.50 + 1.54 = 17.60\ cm^2$

Clearly total surface area of the remaining solid to the nearest cm^2 is 18 cm^2.

Solution 9

Given that

Radius (r) of cylindrical part = radius (r) of hemispherical part = 3.5cm

Height of cylindrical part (h) = 10cm

Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$

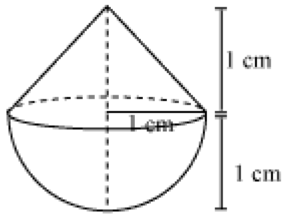
$= 70\pi + 49\pi$

$= 119\pi$

$= 17 \times 22 = 374 \text{ cm}^2$

Chapter 13 - Surface Areas and Volumes Exercise Ex. 13.2

Solution 1



Given that

Height (h) of conical part = radius(r) of conical part = 1cm

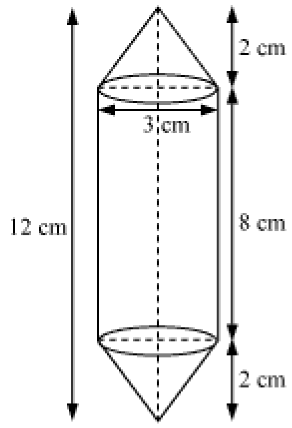
Radius(r) of hemispherical part = radius of conical part (r) = 1cm

Volume of solid = volume of conical part + volume of hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi(1)^2(1) + \frac{2}{3}\pi(1) = \frac{\pi}{3} + \frac{2\pi}{3} = \pi \text{ cm}^3$$

Solution 2



From the figure Height (h_1) of each conical part = 2cm

Height (h_2) of cylindrical part = 12 – 2 × height of conical part

= 12 – 2 × 2 = 8cm

Radius (r) of cylindrical part = radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model = volume of cylinder + 2 × volume of cones

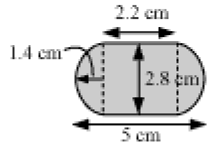
$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$$

$$= \pi\left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 (2)$$

$$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$$

$$= 18\pi + 3\pi = 21\pi = 66 \text{ cm}^2$$

Solution 3



Radius (r) of cylindrical part = radius (r) of hemispherical part = $\frac{2.8}{2} = 1.4$ cm

Length of each hemispherical part = radius of hemispherical part = 1.4cm

Length (h) of cylindrical part = 5 – 2 × length of hemispherical part

= 5 – 2 ×1.4 = 2.2cm

Volume of one gulab jamun = volume of cylindrical part + 2 × volume of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi (1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 13.55 + 11.50 = 25.05 \text{ cm}^3$$

Volume of 45 gulab jamuns = $45 \times 25.05 = 1{,}127.25$ cm^3

Volume of sugar syrup = 30% of volume

$$= \frac{30}{100} \times 1{,}127.25$$

$$\approx 338 \text{ cm}^3$$

Solution 4

Depth (h) of each conical depression = 1.4cm

Radius (r) of each conical depression = 0.5cm

Volume of wood = volume of cuboid – 4 × volume of cones

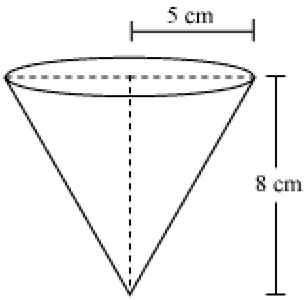
$$= lbh - 4 \times \frac{1}{3}\pi r^2 h$$

$$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 1.4$$

$$= 525 - 1.466$$

$$= 523.53 \text{ cm}^3$$

Solution 5



Height (h) of conical vessel = 8cm

Radius (r_1) of conical vessel = 5cm

Radius (r_2) of lead shots = 0.5cm

Let n lead shots were dropped in the vessel

Volume of water spilled = Volume of dropped lead shots

$$\frac{1}{4}\times \text{volume of cone} = n\times\frac{4}{3}\pi r_2^3$$

$$\frac{1}{4}\times\frac{1}{3}\pi r_1^2 h = n\times\frac{4}{3}\pi r_2^3$$

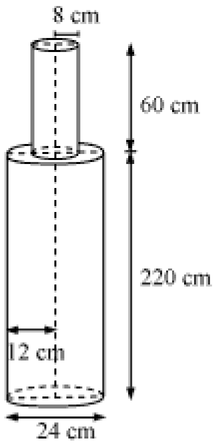
$$r_1^2 h = n\times 16 r_2^3$$

$$5^2\times 8 = n\times 16\times(0.5)^3$$

$$n = \frac{25\times 8}{16\times\left(\frac{1}{2}\right)^3} = 100$$

Hence number of lead shots dropped in the vessel are 100.

Solution 6



From the figure we have

Height (h_1) of larger cylinder = 220cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12cm

Height (h_2) of smaller cylinder = 60cm

Radius (r_2) of larger cylinder = 8cm

Total volume of pole = volume of larger cylinder + volume of smaller cylinder

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$

$$= \pi(12)^2\times 220 + \pi(8)^2\times 60$$

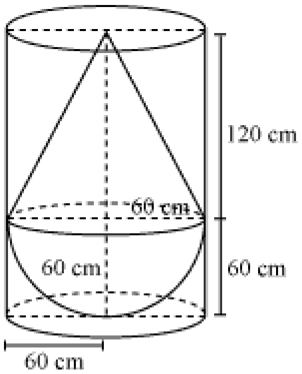
$$= \pi[144\times 220 + 64\times 60]$$

$$= 35520\times 3.14 = 1{,}11{,}532.8\ \text{cm}^3$$

Mass of 1 cm^3 iron = 8gm

Mass of 111532.8 cm^3 iron = 111532.8 × 8 = 892262.4 gm = 892.262 kg.

Solution 7



Radius (r) of hemispherical part = radius (r) of conical part = 60cm

Height (h_2) of conical part of solid = 120cm

Height (h_1) of cylinder = 180cm

Radius (r) of cylinder = 60cm

Volume of water left = volume of cylinder – volume of solid

= volume of cylinder – (volume of cone + volume of hemisphere)

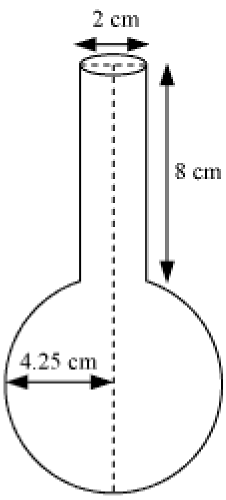
$$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

$$= \pi(60)^2(180) - \left(\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right)$$

$$= \pi(60)^2\left[(180) - (40+40)\right]$$

$$= \pi(3,600)(100) = 3,60,000\pi \text{ cm}^3 = 1131428.57\text{cm}^3 = 1.131\text{m}^3$$

Solution 8



Height (h) of cylindrical part = 8cm

Radius (r_2) of cylindrical part = $\frac{2}{2} = 1$ cm

Radius (r_1) of hemispherical part = 8.5/2

Volume of vessel = volume of sphere + volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$$

$$= 346.82 \text{ cm}^3$$

Hence she is wrong.

Chapter 13 - Surface Areas and Volumes Exercise Ex. 13.3

Solution 1

Radius (r_1) of sphere = 4.2

Radius (r_2) of cylinder = 6

Let height of cylinder be h. The object formed by recasting the sphere will be same in volume.

So, volume of sphere = volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{4}{3}\pi(4.2)^3 = \pi(6)^2 h$$

$$\frac{4}{3}\times\frac{4.2\times4.2\times4.2}{36} = h$$

$$h = (1.4)^3 = 2.74 \text{ cm}$$

Hence, the height of cylinder so formed will be 2.74 cm.

Solution 2

Radius (r_1) of 1st sphere = 6

Radius (r_2) of 2nd sphere = 8

Radius (r_3) of 3rd sphere = 10

Let radius of resulting sphere be r

The object formed by recasting these spheres will be same in volume to the sum of volumes of these spheres.

Volume of 3 spheres = volume of resulting sphere

$$\frac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3}\pi r^3$$

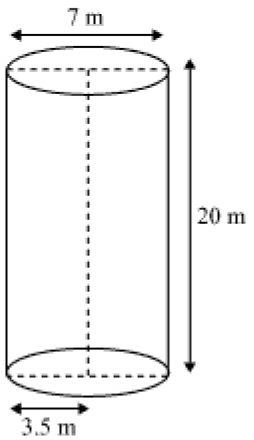
$$\frac{4}{3}\pi\left[6^3 + 8^3 + 10^3\right] = \frac{4}{3}\pi r^3$$

$$r^3 = 216 + 512 + 1,000 = 1,728$$

$$r = 12 \text{ cm.}$$

So, radius of sphere so formed will be 12cm.

Solution 3



The shape of well will be cylindrical.

Depth (h) of well = 20m

Radius (r) of circular end of well = $\frac{7}{2}$ m

Area of platform = length × breadth = 22 × 14 m^2.

Volume of soil dug from well will be equal to the volume of soil scattered on platform.

Volume of soil from well = volume of soil used to make such platform

$\pi \times r^2 \times h$ = Area of platform × Height of platform

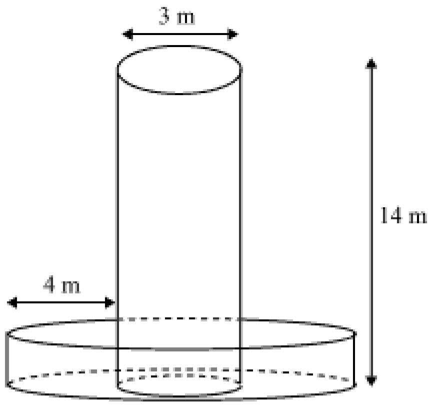
$\pi \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times h$

$\therefore h = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22 \times 14} = \frac{5}{2}$ m

$\therefore$ Height of platform = 2.5 m

So, height of such platform will be 2.5m.

Solution 4



The shape of well will be cylindrical.

Depth (h_1) of well = 14m

Radius (r_1) of circular end of well = $\frac{3}{2}$ m

Width of embankment = 4m

From the figure embankment will be in a cylindrical shape having outer radius (r_2) as $4+\frac{3}{2}=\frac{11}{2}$ m and inner radius (r_1) as $\frac{3}{2}$ m .

Let height of embankment be h_2.

Volume of soil dug from well = volume of earth used to form embankment

$$\pi\times r_1^2\times h_1=\pi\times\left(r_2^2-r_1^2\right)\times h_2$$

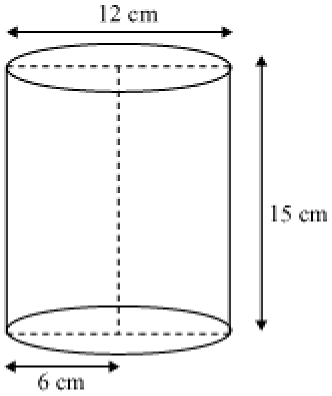
$$\pi\times\left(\frac{3}{2}\right)^2\times 14=\pi\times\left[\left(\frac{11}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]\times h$$

$$\frac{9}{4}\times 14=\frac{112}{4}\times h$$

$$h=\frac{9}{8}=1.125\text{ m}$$

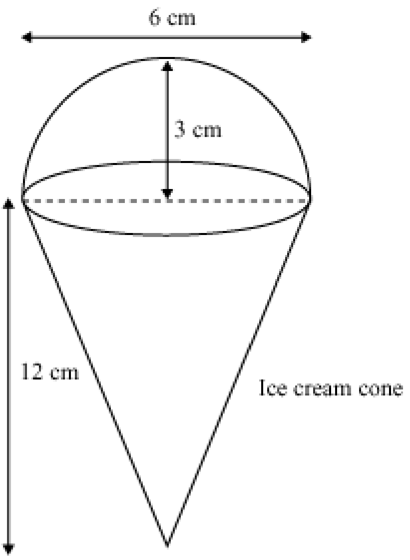
So, height of embankment will be 1.125 m.

Solution 5



Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2}$ = 6 cm



Radius (r_2) of circular end of ice cream cone = $\frac{6}{2}$ = 3 cm

Height (h_2) of conical part of ice cream cone = 12 cm
Let n ice cream cones be filled with the ice cream of container
Volume of ice cream in cylinder = n × volume of 1 ice-cream cone

$$\pi \times r_1^2 \times h_1 = n \times \left[\frac{1}{3}\pi \times r_2^2 \times h_2 + \frac{2}{3}\pi \times r_2^3\right]$$

$$\pi \times 6^2 \times 15 = n \times \left[\frac{1}{3}\pi \times 3^2 \times 12 + \frac{2}{3}\pi \times 3^3\right]$$

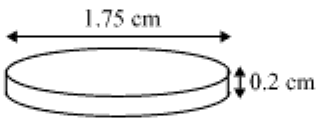
$$36 \times 15 = n[36 + 18]$$

$$n = \frac{36 \times 15}{54}$$

$$n = 10$$

So, number of ice-cream cones, filled with the ice cream of container is 10.

Solution 6



Coins are cylindrical in shape
Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\frac{1.75}{2}$ = 0.875 cm

Let n coins were melted to form the required cuboids.
Volume of n coins = Volume of cuboids

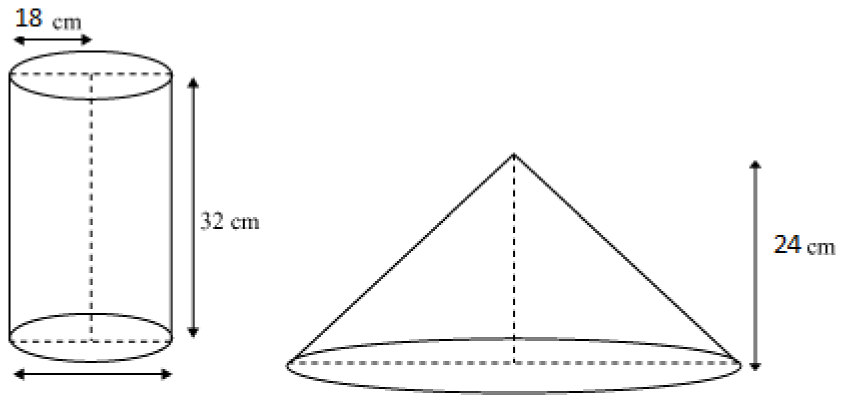
$$n \times \pi \times r^2 \times h_1 = l \times b \times h$$

$$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$$

$$n = \frac{5.5 \times 10 \times 3.5 \times 7}{(0.875)^2 \times 0.2 \times 22} = 400$$

So, number of coins melted to form such a cuboid is 400.

Solution 7



Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let radius of circular end of conical heap be r_2.

Volume of sand in the cylindrical bucket will be equal to the volume of sand in conical heap

Volume of sand in the cylindrical bucket = volume of sand in conical heap

$$\pi \times r_1^2 \times h_1 = \frac{1}{3}\pi \times r_2^2 \times h_2$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$r_2 = 18 \times 2 = 36$ cm

Slant height $= \sqrt{36^2 + 24^2} = \sqrt{12^2 \times (3^2 + 2^2)} = 12\sqrt{13}$ cm

So, radius and slant height of conical heap are 36 cm and $12\sqrt{13}$ cm respectively.

Solution 8

Consider an area of cross section of canal be ABCD.

Area of cross section = $6 \times 1.5 = 9$ m^2

Speed of water = 10 km/h = $\frac{10000}{60}$ meter/min

Volume of water that flows in 1 minute from canal = $9 \times \frac{10000}{60} = 1500 m^3$

Volume of water that flows in 30 minutes from canal = $30 \times 1500 = 45000$ m^3

Let irrigated area be A. Volume of water irrigated in the required area will be equal to the volume of water flowed in 10 minutes from canal.

Volume of water that flows in 30 minutes from canal = Volume of water irrigated in the required area

$$45000 = \frac{A \times 8}{100}$$

A = 562500 m^2.

So, area irrigated in 30 minutes is 562500 m^2.

Solution 9

Consider an area of cross section of pipe as shown in figure

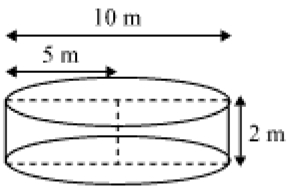
Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1$ m

Area of cross section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi m^2$

Speed of water = 3 km/h = $\frac{3000}{60}$ = 50 meter/min

Volume of water that flows in 1 minute from pipe = $50 \times 0.01\pi = 0.5\pi\ m^3$

Volume of water that flows in t minutes from pipe = $t \times 0.5\pi\ m^3$



Radius (r_2) of circular end of cylindrical tank = $\frac{10}{2} = 5$ m

Depth (h_2) of cylindrical tank = 2 m

Let in t minutes the tank will be filled completely.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

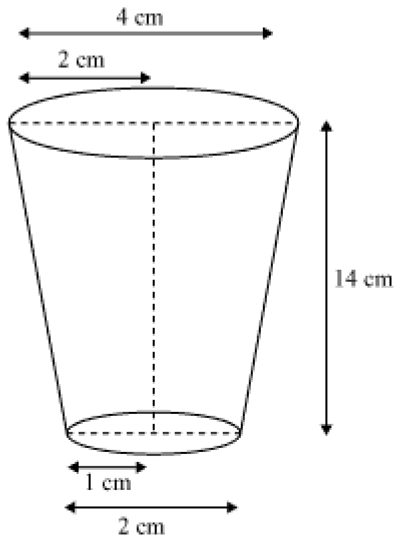
$t \times 0.5 = 5^2 \times 2$

t = 100

So, cylindrical tank will be filled in 100 minutes.

Chapter 13 - Surface Areas and Volumes Exercise Ex. 13.4

Solution 1



Radius (r_1) of upper base of glass $= \frac{4}{2} = 2$ cm

Radius (r_2) of lower base of glass $= \frac{2}{2} = 1$ cm

Capacity of glass = Volume of frustum of cone

$= \frac{1}{3}\pi h\ \left(r_1^2 + r_2^2 + r_1 r_2\right)$

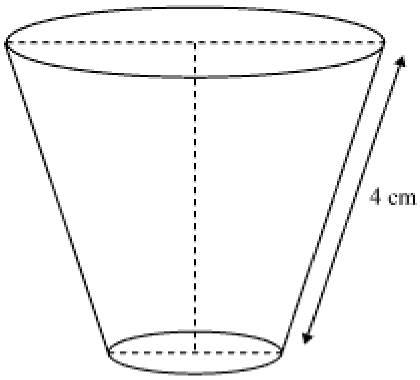
$= \frac{1}{3}\pi h\left[(2)^2 + (1)^2 + (2)(1)\right]$

$= \frac{1}{3} \times \frac{22}{7} \times 14\left[4 + 1 + 2\right]$

$= \frac{308}{3} = 102\frac{2}{3}\ \text{cm}^3$

So, the capacity of the glass is $102\frac{2}{3}\ \text{cm}^3$.

Solution 2



Perimeter of upper circular end of frustum = 18

$2\pi r_1 = 18$

$r_1 = \frac{9}{\pi}$

Perimeter of lower end of frustum = 6 cm.

$2\pi r_2 = 6$

$r_2 = \frac{3}{\pi}$

Slant height (l) of frustum = 4

CSA of frustum = $\pi (r_1 + r_2) l$

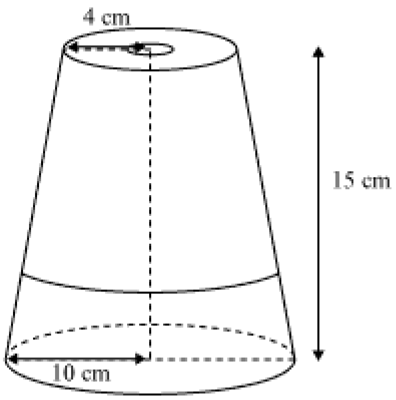
$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right)4$

$= 12 \times 4$

$= 48 \text{ cm}^2$

So, the curved surface area of the frustum is 48 cm^2.

Solution 3



Radius (r_2) at upper circular end = 4 cm

Radius (r_1) at lower circular end = 10 cm

Slant height (l) of frustum = 15 cm

Area of material used for making the fez = CSA of frustum + area of upper circular end

$= \pi (r_1 + r_2) l + \pi r_2^2$

$= \pi (10 + 4)\, 15 + \pi (4)^2$

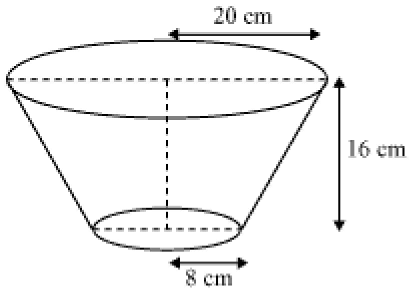
$= \pi (14)\, 15 + 16\pi$

$= 210\pi + 16\pi = \frac{226 \times 22}{7}$

$= 710\frac{2}{7} \text{ cm}^2$

So, the area of material used for making it is $710\frac{2}{7} \text{ cm}^2$.

Solution 4



Radius (r_1) of upper end of container = 20 cm

Radius (r_2) of lower end of container = 8 cm

Height (h) of container = 16 cm

Slant height (l) of frustum = $\sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(20-8)^2 + (16)^2}$

$= \sqrt{(12)^2 + (16)^2} = \sqrt{144 + 256}$

$= 20$ cm

Capacity of container = Volume of frustum

$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 16 \times \left[(20)^2 + (8)^2 + (20)(8)\right]$

$= \frac{1}{3} \times 3.14 \times 16\,(400 + 64 + 160)$

$= \frac{1}{3} \times 3.14 \times 16 \times 624 = 10449.92 \text{ cm}^3$

$= 10.45$ lit.

Cost of 1 litre milk = Rs.20

Cost of 10.45 litre milk = 10.45×20

= Rs.209

Area of metal sheet used to make the container

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi\ (20 + 8)\ 20 + \pi\ (8)^2$

$= 560\ \pi + 64\ \pi = 624\ \pi\ \text{cm}^2$

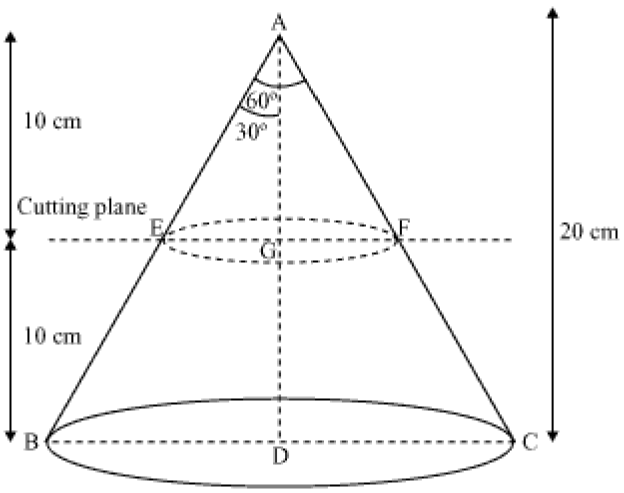
Cost of 100 cm^2 metal sheet = Rs.8

Cost of 624 π cm^2 metal sheet = $\frac{624 \times 3.14 \times 8}{100}$

$= 156.75$

So, the cost of the milk which can completely fill the container is Rs.209 and the cost of metal sheet used to make the container is Rs.156.75.

Solution 5



In ΔAEG

$$\frac{EG}{AG} = \tan 30°$$

$$EG = \frac{10}{\sqrt{3}} \text{ cm} = \frac{10\sqrt{3}}{3}$$

In ΔABD

$$\frac{BD}{AD} = \tan 30°$$

$$BD = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{ cm}$$

Radius (r_1) of upper end of frustum $= \frac{10\sqrt{3}}{3}$ cm

Radius (r_2) of lower end of container $= \frac{20\sqrt{3}}{3}$ cm

Height (h) of container = 10 cm

Volume of frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \frac{1}{3} \times \pi \times 10\left[\left(\frac{10\sqrt{3}}{3}\right)^2 + \left(\frac{20\sqrt{3}}{3}\right)^2 + \frac{\left(10\sqrt{3}\right)\left(20\sqrt{3}\right)}{3 \times 3}\right]$$

$$= \frac{10}{3}\pi\left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right]$$

$$= \frac{10}{3} \times \frac{22}{7} \times \frac{700}{3} = \frac{22000}{9} \text{ cm}^3$$

Radius (r) of wire $= \frac{1}{16} \times \frac{1}{2} = \frac{1}{32}$ cm

Let length of wire be l.

Volume of wire = Area of cross section × length

$= (\pi r^2)\ (l)$

$= \pi\left(\frac{1}{32}\right)^2 \times l$

Volume of frustum = Volume of wire

$$\frac{22000}{9} = \frac{22}{7} \times \left(\frac{1}{32}\right)^2 \times l$$

$$\frac{7000}{9} \times 1024 = l$$

$l = 796444.44$ cm

$= 7964.44$ meter