Access answers to Maths RD Sharma Solutions For Class 12 Chapter 13 – Derivatives as a Rate Measurer

Exercise 13.1 Page No: 13.4

1. Find the rate of change of the total surface area of a cylinder of radius r and height h, when the radius varies.

Solution:

We know that total Surface Area of Cylinder = $2 \pi r^2 + 2 \pi r h$

Given that radius of the cylinder varies.

Therefore, we need to find $\frac{dS}{dr}$

Where S = Surface Area of Cylinder and r = radius of Cylinder.

$$\frac{dS}{dr} = 4 \pi r + 2 \pi h$$

Hence, Rate of change of total surface area of the cylinder when the radius is varying is given by ($4 \pi r + 2 \pi h$).

2. Find the rate of change of the volume of a sphere with respect to its diameter.

Solution:

We know that the volume of a Sphere = $\frac{1}{6} \pi D^3$

Where D = diameter of the Sphere

We need to find, $\frac{dV}{dD}$ where V = Volume of the sphere and D = Diameter of the Sphere.

$$\frac{dV}{dD} = \frac{\pi D^2}{2}$$

Hence, Rate of change of volume of sphere with respect to the diameter of the sphere is given by $\frac{\pi D^2}{2}$.

3. Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm.

Solution:

We know that volume of Sphere = $\frac{4}{3}\pi r^3$

And surface Area of Sphere = $4\pi r^2$

We need to find, $\frac{dV}{dS}$ where V = Volume of the Sphere and S = Surface Area of the Sphere.

$$\frac{dV}{dS} = \frac{dV}{dr} \times \frac{dr}{dS}$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dS}{dr} = 8\pi r$$

$$\frac{dV}{dS} = \frac{4\pi r^2}{8\pi r}$$

$$\frac{dV}{dS} = \frac{r}{2}$$

$$\left(\frac{dV}{dS}\right)_{\text{at } r=2} = \frac{2}{2} = 1 \text{ cm}$$

4. Find the rate of change of the area of a circular disc with respect to its circumference when the radius is 3 cm.

Solution:

We know that area of a Circular disc = $\pi r^2$and circumference of a Circular disc = $2\pi r$

Where r = radius of Circular Disc.

Now we have to find $\frac{dA}{dC}$ where A = Area of Circular disc and C = Circumference of the Circular disk.

$$\frac{dA}{dC}=\frac{dA}{dr}\times\frac{dr}{dC}$$

$$\frac{dA}{dr}=2\,\pi\,r$$

$$\frac{dC}{dr}=2\,\pi$$

$$\frac{dA}{dC}=\frac{2\pi r}{2\pi}=r$$

$$\left(\frac{dA}{dr}\right)_{\text{at }x=3}=3\text{ cm}$$

5. Find the rate of change of the volume of a cone with respect to the radius of its base.

Solution:

We know that the volume of Cone = $\frac{1}{3}\pi r^2 h$

Where r = radius of the cone

h = height of the cone

We have to find, $\frac{dV}{dr}$ where V = Volume of cone and r = radius of the cone.

$$\frac{dV}{dr}=\frac{2}{3}\pi rh$$

Exercise 13.2 Page No: 13.19

1. The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long?

Solution:

Given the side of a square sheet is increasing at the rate of 4 cm per minute.

To find rate of area increasing when the side is 8 cm long

Let the side of the given square sheet be x cm at any instant time.

Then according to the given question, we can write as

Rate of side of the sheet increasing is, $\frac{dx}{dt} = 4 \text{ cm/min} \ldots \text{(i)}$

Then the area of the square sheet at any time t will be

$A = x^2 \text{ cm}^2$.

Applying derivative with respect to time on both sides we get,

$$\frac{dA}{dt} = \frac{d(x^2)}{dt}$$

$$\Rightarrow \frac{dA}{dt} = 2x\frac{dx}{dt}$$

$\Rightarrow \frac{dA}{dt} = 2x \times 4 = 8x \ldots\ldots \text{(ii)}$ [From equation (i)]

So when the side is 8cm long, the rate of area increasing will become

$\Rightarrow \frac{dA}{dt} = 8 \times 8$ [From equation (ii)]

$$\Rightarrow \frac{dA}{dt} = 64\text{cm}^2/\text{min}$$

Hence the area is increasing at the rate of $64\text{cm}^2/\text{min}$ when the side is 8 cm long

2. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 1 cm long?

Solution:

Given the edge of a variable cube is increasing at the rate of 3 cm per second.

To find rate of volume of the cube increasing when the edge is 1 cm long

Let the edge of the given cube be x cm at any instant time.

Then according to the given question we can write as

Rate of edge of the cube increasing is, $\frac{dx}{dt} = 3 \text{ cm/sec} \ldots (i)$

Then the volume of the cube at any time t will be

$V = x^3 \text{ cm}^3$.

Applying derivative with respect to time on both sides we get,

$$\frac{dV}{dt} = \frac{d(x^3)}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 3x^2 \times 3 = 9x^2 \ldots\ldots (ii)$$ [From equation (i)]

When the edge of the cube is 1cm long the rate of volume increasing becomes

$$\Rightarrow \frac{dV}{dt} = 9 \times (1)^2 = 9\text{cm}^3/\text{sec}$$

Hence the volume of the cube increasing at the rate of 9cm^3/sec when the edge of the cube is 1 cm long

3. The side of a square is increasing at the rate of 0.2 cm/sec. Find the rate of increase of the perimeter of the square.

Solution:

Given the side of a square is increasing at the rate of 0.2 cm/sec.

To find rate of increase of the perimeter of the square

Let the edge of the given cube be x cm at any instant time.

Then according to the given question, we can write as

Rate of side of the square increasing is, $\frac{dx}{dt} = 0.2 \text{ cm/sec} \ldots (i)$

Then the perimeter of the square at any time t will be

Then the perimeter of the square at any time t will be

P = 4x cm.

Applying derivative with respect to time on both sides we get,

$$\frac{dP}{dt} = \frac{d(4x)}{dt}$$

$$\Rightarrow \frac{dP}{dt} = 4\frac{dx}{dt}$$

$\Rightarrow \frac{dP}{dt} = 4 \times 0.2 = 0.8 cm/sec$ [From equation (i)]

Hence the rate of increase of the perimeter of the square will be 0.8cm/sec

4. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

Solution:

Given the radius of a circle is increasing at the rate of 0.7 cm/sec.

To find rate of increase of its circumference

Let the radius of the given circle be r cm at any instant time.

Then according to the given question, we can write as

Rate of radius of a circle is increasing is, $\frac{dr}{dt} = 0.7\ cm/sec \ldots (i)$

Then the circumference of the circle at any time t will be

C = 2πr cm.

Applying derivative with respect to time on both sides we get,

$$\frac{dC}{dt} = \frac{d(2\pi r)}{dt}$$

$$\Rightarrow \frac{dC}{dt} = 2\pi\frac{dr}{dt}$$

$\Rightarrow \frac{dC}{dt} = 2\pi \times 0.7 = 1.4\pi\ cm/sec$ [From equation (i)]

Hence the rate of increase of the circle's circumference will be 1.4 π cm/sec

Hence the rate of increase of the circle’s circumference will be 1.4 π cm/sec

5. The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/sec. Find the rate of increase of its surface area, when the radius is 7 cm.

Solution:

Given the radius of a spherical soap bubble is increasing at the rate of 0.2 cm/sec.

To find rate of increase of its surface area, when the radius is 7 cm

Let the radius of the given spherical soap bubble be r cm at any instant time.

Then according to the given question we can write as

Rate of radius of the spherical soap bubble is increasing

Rate of radius of the spherical soap bubble is increasing

is, $\frac{dr}{dt} = 0.2\ \text{cm/sec} \ldots \text{(i)}$

Then the surface area of the spherical soap bubble at any time t will be

$S = 4\pi r^2\ cm^2$.

Applying derivative with respect to time on both sides we get,

$$\frac{dS}{dt} = \frac{d(4\pi r^2)}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 4\pi\frac{d(r^2)}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 4\pi \times 2r\frac{dr}{dt}$$

$\Rightarrow \frac{dS}{dt} = 4\pi \times 2r \times 0.2 = 1.6\pi r \ldots \ldots \text{(ii)}$ [From equation (i)]

So when the radius is 7cm, the rate of surface area will become,

$$\Rightarrow \frac{dS}{dt} = 1.6\pi \times (7)$$

$$\Rightarrow \frac{dS}{dt} = 11.2\pi\ \text{cm}^2/\text{sec}$$

Hence the rate of increase of its surface area, when the radius is 7 cm is 11.2 π cm^2/sec

Hence the rate of increase of its surface area, when the radius is 7 cm is 11.2 π cm^2/sec

6. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

Solution:

Given spherical balloon inflated by pumping in 900 cubic centimetres of gas per second

To find the rate at which the radius of the balloon is increasing when the radius is 15 cm

Let the radius of the given spherical balloon be r cm and let V be the volume of the spherical balloon at any instant time

Then according to the given question,

As the balloon is inflated by pumping 900 cubic centimetres of gas per second hence the rate of volume of the spherical balloon increases by

$$\frac{dV}{dt} = 900\ \text{cm}^3/\text{sec} \ldots \text{(i)}$$

We know volume of the spherical balloon is $V = \frac{4}{3}\pi r^3$.

Applying derivative with respect to time on both sides we get,

$$\frac{dV}{dt} = \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \frac{d(r^3)}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$$

$$\Rightarrow 900 = 4\pi r^2 \frac{dr}{dt} \text{ [From equation (i)]}$$

So when the radius is 15cm, the above equation becomes,

$$\Rightarrow 900 = 4\pi \times (15)^2 \frac{dr}{dt}$$

$$\Rightarrow 900 = 900\pi \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{\pi}\text{cm/sec}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{\pi}\text{cm/sec}$$

Hence the rate at which the radius of the balloon is increasing when the radius is 15 cm will be $\frac{1}{\pi}$ cm/sec.

7. The radius of an air bubble is increasing at the rate of 0.5 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Solution:

Given radius of an air bubble is increasing at the rate of 0.5 cm/sec

To find the rate at which the volume of the bubble increasing when the radius is 1 cm

Let the radius of the given air bubble be r cm and let V be the volume of the air bubble at any instant time

Then according to the given question,

Rate of increase in the radius of the air bubble is, $\frac{dr}{dt} = 0.5\ \mathrm{cm/sec} \ldots \text{(i)}$

We know volume of the air bubble is $V = \frac{4}{3}\pi r^3$.

Applying derivative with respect to time on both sides we get,

$$\frac{dV}{dt} = \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi\frac{d(r^3)}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2\frac{dr}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \times 0.5$$

So when the radius is 1cm, the above equation becomes,

$$\Rightarrow \frac{dV}{dt} = 4\pi \times (1)^2 \times 0.5$$

$$\Rightarrow \frac{dV}{dt} = 2\pi\ \mathrm{cm^3/sec}$$

Hence the rate at which the volume of the air bubble is increasing when the radius is 1 cm will be 2^{π} $\mathrm{cm^3/sec}$.

$$\Rightarrow \frac{dV}{dt} = 2\pi \text{ cm}^3/\text{sec}$$

Hence the rate at which the volume of the air bubble is increasing when the radius is 1 cm will be 2π cm^3/sec.

8. A man 2 metres high walks at a uniform speed of 5 km/hr. away from a lamp – post 6 metres high. Find the rate at which the length of his shadow increases.

Solution:

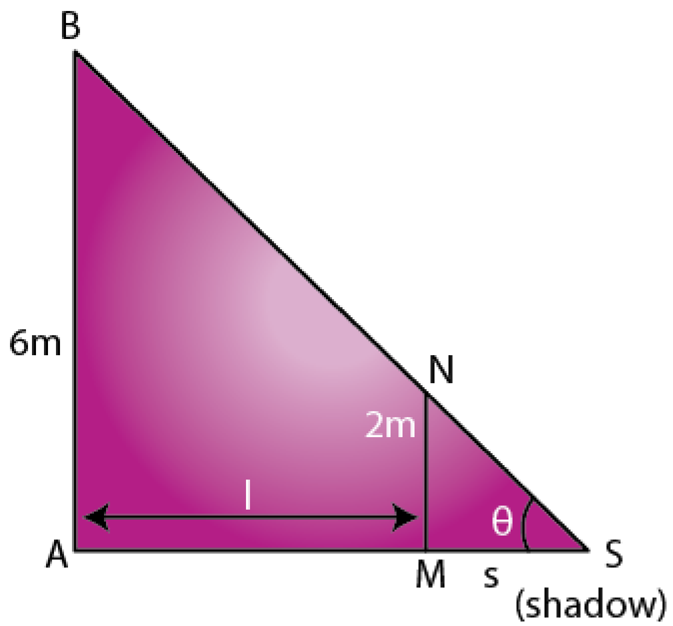
Given a man 2 metres high walks at a uniform speed of 5 km/hr. away from a lamp – post 6 metres high

To find the rate at which the length of his shadow increases

Let AB be the lamp post and let MN be the man of height 2m.

Let AL = l meter and MS be the shadow of the man

Let length of the shadow MS = s (as shown in the below figure)



Given man walks at the speed of 5 km/hr.

$$\therefore \frac{dl}{dt} = 5\frac{km}{h} \ldots.(i)$$

So the rate at which the length of the man's shadow increases will be $\frac{ds}{dt}$

Consider ΔASB

$$\tan\theta = \frac{AB}{AS}$$

$$\Rightarrow \tan\theta = \frac{6}{l + s} \ldots.(ii)$$

Now consider ΔMSN, we get

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$$\tan\theta = \frac{MN}{MS}$$

$$\Rightarrow \tan\theta = \frac{2}{s} \dots.(iii)$$

So from equation (ii) and (iii),

$$\frac{6}{l+s} = \frac{2}{s}$$

$$\Rightarrow 6s = 2(l+s)$$

$$\Rightarrow 6s - 2s = 2l$$

$$\Rightarrow l = 2s$$

Applying derivative with respect to time on both sides we get,

$$\frac{dl}{dt} = \frac{d(2s)}{dt}$$

$$\Rightarrow \frac{dl}{dt} = 2\frac{ds}{dt}$$

$$\Rightarrow 5 = 2\frac{ds}{dt}$$ [From equation (i)]

$$\Rightarrow \frac{ds}{dt} = \frac{5}{2} = 2.5 km/hr$$

Hence the rate at which the length of his shadow increases by 2.5 km/hr. and it is independent to the current distance of the man from the base of the light.

9. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/sec. At the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

Solution:

Given a stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/sec.

To find the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing

Let r be the radius of the circle and A be the area of the circle

When stone is dropped into the lake waves moves in circle at speed of 4cm/sec. i.e., radius of the circle increases at a rate of 4cm/sec

$$\frac{dr}{dt} = 4\text{cm/sec} \ldots \ldots \text{(i)}$$

We know that area of the circle is πr^2

Now,

$$\frac{dA}{dt} = \frac{d(\pi r^2)}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \pi \frac{d(r^2)}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \pi \times 2r \frac{dr}{dt}$$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \times 4 \ldots \ldots .. \text{(ii)}$$

So when the radius of the circular wave is 10 cm, the above equation becomes,

$$\Rightarrow \frac{dA}{dt} = 2\pi \times 10 \times 4$$

$$\Rightarrow \frac{dA}{dt} = 80\pi \text{ cm}^2 \backslash \text{sec}$$

Hence the enclosed area is increasing at the rate of 80 π cm^2/sec

10. A man 160 cm tall walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1m/sec. How fast is the length of his shadow increasing when he is 1m away from the pole?

Solution:

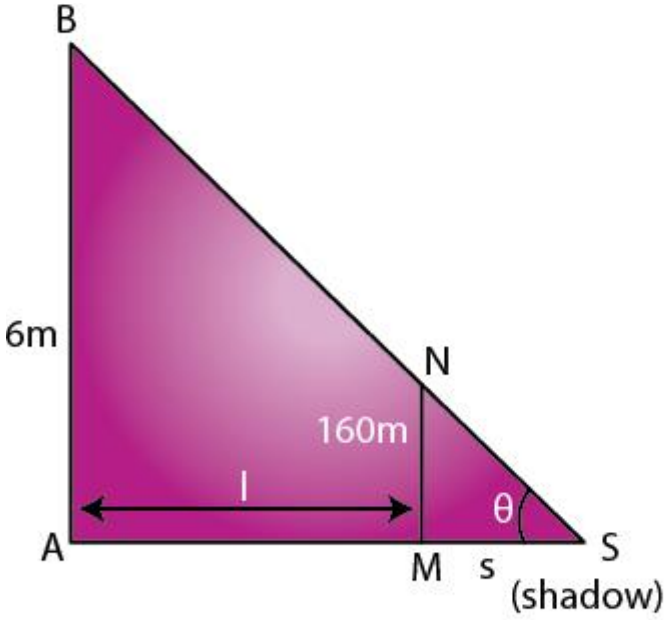
Given a man 160cm tall walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1m/sec

To find the rate at which the length of his shadow increases when he is 1m away from the pole

Let AB be the lamp post and let MN be the man of height 160cm or 1.6m.

Let AL = l meter and MS be the shadow of the man

Let length of the shadow MS = s (as shown in the below figure)



Given man walks at the speed of 1.1 m/sec

$$\therefore \frac{dl}{dt} = 1.1 \text{m/sec} \ldots. (i)$$

So the rate at which the length of the man's shadow increases will be $\frac{ds}{dt}$

Consider ΔASB

$$\tan\theta = \frac{AB}{AS}$$

$$\Rightarrow \tan\theta = \frac{6}{l + s} \ldots. (ii)$$

Now consider ΔMSN, we get

$$\tan\theta = \frac{MN}{MS}$$

$$\Rightarrow \tan\theta = \frac{1.6}{s} \ldots. (iii)$$

So from equation (ii) and (iii),

$$\frac{6}{l + s} = \frac{1.6}{s}$$

$$\Rightarrow 6s = 1.6(l + s)$$

$$\Rightarrow 6s - 1.6s = 1.6l$$

$$\Rightarrow l = 2.75s$$

$$\frac{6}{l + s} = \frac{1.6}{s}$$

$$\Rightarrow 6s = 1.6(l + s)$$

$$\Rightarrow 6s - 1.6s = 1.6l$$

$$\Rightarrow l = 2.75s$$

Applying derivative with respect to time on both sides we get,

$$\frac{dl}{dt} = \frac{d(2.75s)}{dt}$$

$$\Rightarrow \frac{dl}{dt} = 2.75\frac{ds}{dt}$$

$\Rightarrow 1.1 = 2.75\frac{ds}{dt}$ [From equation (i)]

$$\Rightarrow \frac{ds}{dt} = 0.4\ m/sec$$

Hence the rate at which the length of his shadow increases by 0.4 m/sec, and it is independent to the current distance of the man from the base of the light.

11. A man 180 cm tall walks at a rate of 2 m/sec. away, from a source of light that is 9 m above the ground. How fast is the length of his shadow increasing when he is 3 m away from the base of light?

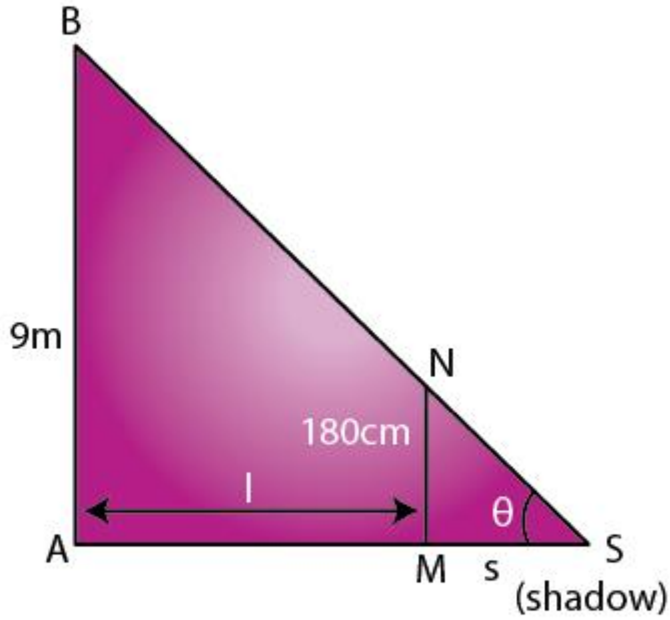
Solution:

Given a man 180cm tall walks at a rate of 2 m/sec away; from a source of light that is 9 m above the ground

To find the rate at which the length of his shadow increases when he is 3m away from the pole

Let AB be the lamp post and let MN be the man of height 180cm or 1.8m.

Let AL = l meter and MS be the shadow of the man



Let length of the shadow MS = s (as shown in the below figure)

Given man walks at the speed of 2m/sec

$$\therefore \frac{dl}{dt} = 2m/sec \ldots .(i)$$

So the rate at which the length of the man's shadow increases will be $\frac{ds}{dt}$

Consider ΔASB

$$\tan\theta = \frac{AB}{AS}$$

$$\Rightarrow \tan\theta = \frac{9}{l + s} \ldots .(ii)$$

Now consider ΔMSN, we get

$$\tan\theta = \frac{MN}{MS}$$

$$\Rightarrow \tan\theta = \frac{2}{s} \ldots .(iii)$$

So from equation (ii) and (iii),

$$\frac{9}{l + s} = \frac{2}{s}$$

$$\Rightarrow 9s = 2(l + s)$$

$$\Rightarrow 9s - 2s = 2l$$

$$\Rightarrow l = 3.5s$$

Applying derivative with respect to time on both sides we get,

$$\frac{dl}{dt} = \frac{d(3.5s)}{dt}$$

$$\Rightarrow \frac{dl}{dt} = 3.5\frac{ds}{dt}$$

$\Rightarrow 2 = 3.5\frac{ds}{dt}$ [From equation (i)]

ds/dt = 0.57m/sec

Hence the rate at which the length of his shadow increases by 0.57 m/sec, and it is independent to the current distance of the man from the base of the light.

ds/dt = 0.57m/sec

Hence the rate at which the length of his shadow increases by 0.57 m/sec, and it is independent to the current distance of the man from the base of the light.

12. A ladder 13 m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec. How fast the angle θ between the ladder and the ground is is changing when the foot of the ladder is 12 m away from the wall.

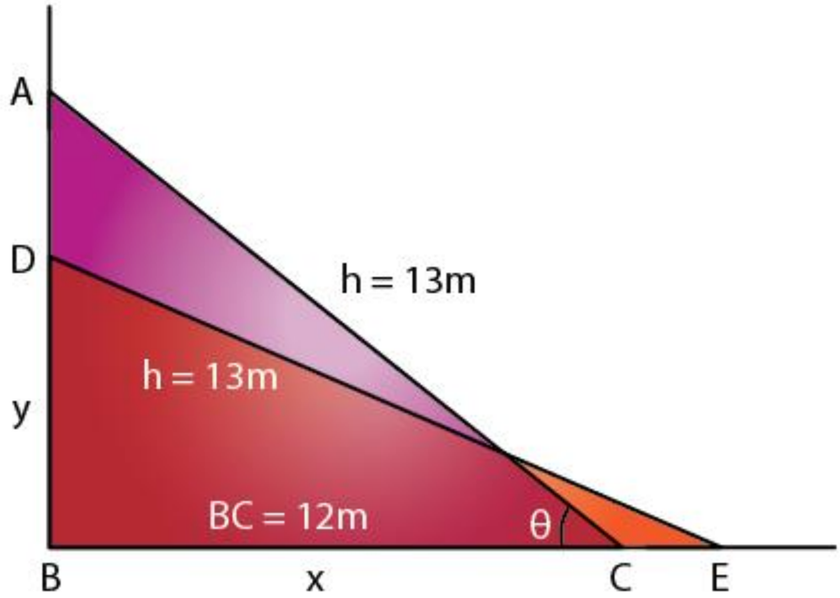
Solution:

Given a ladder 13 m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec

To find how fast is the angle θ between the ladder and the ground is changing when the foot of the ladder is 12 m away from the wall

Let AC be the position of the ladder initially, then AC = 13m.

DE be the position of the ladder after being pulled at the rate of 1.5m/sec, then DE = 13m as shown in the below figure.



So it is given that foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec

$\therefore \frac{dx}{dt} = 1.5\text{m/sec}$ .... (i)

Consider ΔABC, it is right angled triangle, so by applying the Pythagoras theorem, we get

$AB^2 + BC^2 = AC^2$

$\Rightarrow y^2 + x^2 = h^2$.......... (ii)

$\Rightarrow y^2 + (12)^2 = (13)^2$

$\Rightarrow y^2 = 169 - 144 = 25$

$\Rightarrow y = 5$

And in same triangle,

$$\sec\theta = \frac{AC}{BC} = \frac{13}{12} \ldots.(iii)$$

Now differentiate equation (ii) with respect to time, we get

$$\frac{d(y^2 + x^2)}{dt} = \frac{d(h^2)}{dt}$$

$$\Rightarrow \frac{d(y^2)}{dt} + \frac{d(x^2)}{dt} = \frac{d(h^2)}{dt}$$

$$\Rightarrow 2y\frac{dy}{dt} + 2x\frac{dx}{dt} = 2h\frac{dh}{dt}$$

Now substituting the values of x, y, h and $\frac{dx}{dt}$, we get

$$\Rightarrow 2(5)\frac{dy}{dt} + 2(12)(1.5) = 2(13)\frac{dh}{dt}$$

The value of h is always constant as the ladder is not increasing or decreasing in size, hence the above equation becomes,

$$\Rightarrow 10\frac{dy}{dt} + 24(1.5) = 2(13)(0)$$

$$\Rightarrow 10\frac{dy}{dt} = -36$$

$$\Rightarrow \frac{dy}{dt} = -3.6 \ldots \ldots.(iv)$$

And considering the same triangle,

$$\tan\theta = \frac{AB}{BC} = \frac{y}{x}$$

Differentiating the above equation with respect to time we get

$$\frac{d(\tan\theta)}{dt} = \frac{d\left(\frac{y}{x}\right)}{dt}$$

By quotient rule,

$$\Rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt}-y\frac{dx}{dt}}{x^2}$$

Substituting the values of sec θ, x, y, $\frac{dy}{dt}$ and $\frac{dx}{dt}$ the above equation becomes,

$$\Rightarrow \left(\frac{13}{12}\right)^2\frac{d\theta}{dt} = \frac{12\times(-3.6)-5\times1.5}{(12)^2}$$

$$\Rightarrow \left(\frac{169}{144}\right)\frac{d\theta}{dt} = \frac{-43.2-7.5}{144}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{-50.7}{144}\times\frac{144}{169}$$

$$\Rightarrow \frac{d\theta}{dt} = -0.3\text{rad/sec}$$

Hence the angle θ between the ladder and the ground is changing at the rate of - 0.3 rad/sec when the foot of the ladder is 12 m away from the wall.

Hence the angle θ between the ladder and the ground is changing at the rate of 0.3 rad/sec (because angle cannot be negative) when the foot of the ladder is 12 m away from the wall.

13. A particle moves along the curve $y = x^2 + 2x$. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate?

Solution:

Given a particle moves along the curve $y = x^2 + 2x$.

To find the points at which the curve are the x and y coordinates of the particle changing at the same rate

Equation of curve is $y = x^2 + 2x$

Differentiating the above equation with respect to x, we get

$dy/dx = [d(x^2 + 2x)]/dx$

$$\Rightarrow \frac{dy}{dx} = \frac{d(x^2)}{dx} + \frac{d(2x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2x\frac{dx}{dx} + 2\frac{dx}{dx} = 2x + 2 \ldots\ldots\ldots (i)$$

When x and y coordinates of the particle are changing at the same rate, we get

$$\frac{dy}{dt} = \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dt}{dt}$$

$$\Rightarrow \frac{dy}{dx} = 1$$

Now substitute the value from equation (i), we get

$2x + 2 = 1 \Rightarrow 2x = - 1$

$$\Rightarrow x = -\frac{1}{2}$$

Substitute this value of x in the given equation of curve, we get

$y = x^2 + 2x$

$$\Rightarrow y = \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$$

$$\Rightarrow y = \frac{1}{4} - 1$$

$$\Rightarrow y = -\frac{3}{4}$$

Hence the points at which the curve are the x and y coordinates of the particle changing at the same rate is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

14. If $y = 7x - x^3$ and x increases at the rate of 4 units per second, how fast is the slope of the curve changing when x = 2?

Solution:

Given equation of curve $y = 7x - x^3$ and x increases at the rate of 4 units per second.

To find how fast is the slope of the curve changing when x = 2

Equation of curve is $y = 7x - x^3$

Differentiating the above equation with respect to x, we get slope of the curve

$$\frac{dy}{dx} = \frac{d(7x - x^3)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(7x)}{dx} - \frac{d(x^3)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 7\frac{dx}{dx} - 3x^2\frac{dx}{dx} = 7 - 3x^2 \ldots\ldots\ldots(i)$$

Let m be the slope of the given curve then the above equation becomes,

$m = 7 - 3x^2$......... (ii)

And it is given x increases at the rate of 4 units per second, so

$$\frac{dx}{dt} = 4 \text{ units/sec} \ldots\ldots.(iii)$$

Now differentiating the equation of slope i.e., equation (ii) we get

$$\frac{dm}{dt} = \frac{d(7 - 3x^2)}{dt}$$

$$\Rightarrow \frac{dm}{dt} = \frac{d(7)}{dt} - \frac{d(3x^2)}{dt}$$

$$\Rightarrow \frac{dm}{dt} = 0 - (3 \times 2x)\frac{dx}{dt}$$

$$\Rightarrow \frac{dm}{dt} = -6x\frac{dx}{dt} = -6x \times 4 \ldots..(iv)$$

When x = 2, equation (iv) becomes,

$$\Rightarrow \frac{dm}{dt} = -6x \times 4 = -6 \times 2 \times 4 = -48$$

The slope cannot be negative,

Hence the slope of the curve is changing at the rate of 48 units/sec when $x = 2$

15. A particle moves along the curve $y = x^3$. Find the points on the curve at which the y – coordinate changes three times more rapidly than the x – coordinate.

Solution:

Given a particle moves along the curve $y = x^3$.

To find the points on the curve at which the y – coordinate changes three times more rapidly than the x – coordinate

Equation of curve is $y = x^3$

Differentiating the above equation with respect to t, we get

$$\frac{dy}{dt} = \frac{d(x^3)}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{d(x^3)}{dt}$$

$$\Rightarrow \frac{dy}{dt} = 3x^2\frac{dx}{dt} \ldots\ldots\ldots (i)$$

When y - coordinate changes three times more rapidly than the x - coordinate, that is

$$\frac{dy}{dt} = 3\frac{dx}{dt} \ldots\ldots (ii)$$

Equating equation (i) and equation (ii), we get

$$3x^2\frac{dx}{dt} = 3\frac{dx}{dt}$$

$$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

When $x = 1$, $y = x^3 = (1)^3 \Rightarrow y = 1$

When $x = -1$, $y = x^3 = (-1)^3 \Rightarrow y = -1$

Hence the points on the curve at which the y – coordinate changes three times more rapidly than the x – coordinate are $(1, 1)$ and $(-1, -1)$.