NCERT Solutions for Class 12-science Maths Chapter 13 - Probability

Chapter 13 - Probability Exercise Ex. 13.1

Solution 1

It is given that P(E) = 0.6, P(F) = 0.3, and P(E ∩ F) = 0.2

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$\Rightarrow P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

Solution 2

It is given that P(B) = 0.5 and P(A ∩ B) = 0.32

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{16}{25}$$

Solution 3

It is given that P(A) = 0.8, P(B) = 0.5, and P(B|A) = 0.4

(i) P (B|A) = 0.4

$$\therefore \frac{P(A \cap B)}{P(A)} = 0.4$$

$$\Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$$

$$\Rightarrow P(A \cap B) = 0.32$$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$$

(iii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.8 + 0.5 - 0.32 = 0.98$$

Solution 4

It is given that, $2P(A)=P(B)=\frac{5}{13}$

$$\Rightarrow P(A)=\frac{5}{26} \text{ and } P(B)=\frac{5}{13}$$

$$P(A|B)=\frac{2}{5}$$
$$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{2}{5}$$
$$\Rightarrow P(A\cap B)=\frac{2}{5}\times P(B)=\frac{2}{5}\times\frac{5}{13}=\frac{2}{13}$$

It is known that, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow P(A\cup B)=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}$$
$$\Rightarrow P(A\cup B)=\frac{5+10-4}{26}$$
$$\Rightarrow P(A\cup B)=\frac{11}{26}$$

Solution 5

It is given that $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$, and $P(A\cup B)=\frac{7}{11}$

(i) $P(A\cup B)=\frac{7}{11}$

$$\therefore P(A)+P(B)-P(A\cap B)=\frac{7}{11}$$
$$\Rightarrow \frac{6}{11}+\frac{5}{11}-P(A\cap B)=\frac{7}{11}$$
$$\Rightarrow P(A\cap B)=\frac{11}{11}-\frac{7}{11}=\frac{4}{11}$$

(ii) It is known that, $P(A|B)=\frac{P(A\cap B)}{P(B)}$

$$\Rightarrow P(A|B)=\frac{\frac{4}{11}}{\frac{5}{11}}=\frac{4}{5}$$

(iii) It is known that, $P(B|A)=\frac{P(A\cap B)}{P(A)}$

$$\Rightarrow P(B|A)=\frac{\frac{4}{11}}{\frac{6}{11}}=\frac{4}{6}=\frac{2}{3}$$

Solution 6

If a coin is tossed three times, then the sample space S is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that the sample space has 8 elements.

(i) E = {HHH, HTH, THH, TTH}

F = {HHH, HHT}

$\therefore E \cap F$ = {HHH}

$$P(F)=\frac{2}{8}=\frac{1}{4} \text{ and } P(E\cap F)=\frac{1}{8}$$

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{8}}{\frac{1}{4}}=\frac{4}{8}=\frac{1}{2}$$

(ii) E = {HHH, HHT, HTH, THH}

F = {HHT, HTH, HTT, THH, THT, TTH, TTT}

$\therefore E \cap F$ = {HHT, HTH, THH}

Clearly, $P(E\cap F)=\frac{3}{8}$ and $P(F)=\frac{7}{8}$

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{3}{8}}{\frac{7}{8}}=\frac{3}{7}$$

(iii) E = {HHH, HHT, HTT, HTH, THH, THT, TTH}

F = {HHT, HTT, HTH, THH, THT, TTH, TTT}

$$\therefore E\cap F=\{HHT, HTT, HTH, THH, THT, TTH\}$$

$$P(F)=\frac{7}{8} \text{ and } P(E\cap F)=\frac{6}{8}$$

$$\text{Therefore, } P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{6}{8}}{\frac{7}{8}}=\frac{6}{7}$$

Solution 7

If two coins are tossed once, then the sample space S is

S = {HH, HT, TH, TT}

(i) E = {HT, TH}

F = {HT, TH}

$\therefore E \cap F = \{HT, TH\}$

$P(F) = \frac{2}{4} = \frac{1}{2}$

$P(E \cap F) = \frac{2}{4} = \frac{1}{2}$

$\therefore\ P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

(ii) E = {HH}

F = {TT}

$E \cap F = \Phi$

$P\ (F) = \frac{1}{4}$ and $P\ (E \cap F) = 0$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{\frac{1}{4}} = 0$

Solution 8

If a die is thrown three times, then the number of elements in the sample space will be $6 \times 6 \times 6 = 216$

$$E = \left\{ \begin{array}{l} (1,1,4),(1,2,4),...(1,6,4) \\ (2,1,4),(2,2,4),...(2,6,4) \\ (3,1,4),(3,2,4),...(3,6,4) \\ (4,1,4),(4,2,4),...(4,6,4) \\ (5,1,4),(5,2,4),...(5,6,4) \\ (6,1,4),(6,2,4),...(6,6,4) \end{array} \right\}$$

$$F = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

$$\therefore E \cap F = \{(6,5,4)\}$$

$$P(F) = \frac{6}{216} \text{ and } P(E \cap F) = \frac{1}{216}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Solution 9

If mother (M), father (F), and son (S) line up for the family picture, then the sample space will be

S = {MFS, MSF, FMS, FSM, SMF, SFM}

E = {MFS, FMS, SMF, SFM}

F = {MFS, SFM}

E ∩ F = {MFS, SFM}

$$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

Solution 10

Let the first observation be from the black die and second from the red die.

When two dice (one black and another red) are rolled, the sample space S has 6 × 6 = 36 number of elements.

1. Let

A: Obtaining a sum greater than 9

= {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

B: Black die results in a 5.

= {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

A ∩ B = {(5, 5), (5, 6)}

The conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5, is given by P (A|B).

$$\therefore P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{2}{6} = \frac{1}{3}$$

(b) E: Sum of the observations is 8.

= {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

F: Red die resulted in a number less than 4.

$$= \left\{\begin{matrix}(1,1),(1,2),(1,3),(2,1),(2,2),(2,3), \\ (3,1),(3,2),(3,3),(4,1),(4,2),(4,3), \\ (5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\end{matrix}\right\} \quad \begin{matrix}\therefore E\cap F = \{(5,3),(6,2)\} \\ P(F) = \frac{18}{36} \text{ and } P(E\cap F) = \frac{2}{36}\end{matrix}$$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4, is given by P (E|F).

$$\text{Therefore, } P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

Solution 11

When a fair die is rolled, the sample space S will be

S = {1, 2, 3, 4, 5, 6}

It is given that E = {1, 3, 5}, F = {2, 3}, and G = {2, 3, 4, 5}

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$P(G) = \frac{4}{6} = \frac{2}{3}$$

(i) E ∩ F = {3}

$$\therefore P(E \cap F) = \frac{1}{6}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

(ii) E ∩ G = {3, 5}

$$\therefore P(E \cap G) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(G|E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

(iii) E ∪ F = {1, 2, 3, 5}

(E ∪ F) ∩ G = {1, 2, 3, 5} ∩{2, 3, 4, 5} = {2, 3, 5}

E ∩ F = {3}

(E ∩ F) ∩ G = {3}∩{2, 3, 4, 5} = {3}

$$\therefore P(E \cup G) = \frac{4}{6} = \frac{2}{3}$$

$$P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{6}$$

$$P((E \cap F) \cap G) = \frac{1}{6}$$

$$\therefore P((E \cup F) | G)) = \frac{P((E \cup F) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$P((E \cap F) | G) = \frac{P((E \cap G) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$$

$$\therefore P(E \cup G) = \frac{4}{6} = \frac{2}{3}$$

$$P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{6}$$

$$P((E \cap F) \cap G) = \frac{1}{6}$$

$$\therefore P((E \cup F) | G)) = \frac{P((E \cup F) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$P((E \cap F) | G) = \frac{P((E \cap G) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$$

Solution 12

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be

S = {(b, b), (b, g), (g, b), (g, g)}

Let A be the event that both children are girls.

$$\therefore \mathrm{A} = \{(g,\ g)\}$$

(i) Let B be the event that the youngest child is a girl.

$$\therefore \mathrm{B} = [(b, g), (g, g)]$$
$$\Rightarrow \mathrm{A} \cap \mathrm{B} = \{(g,\ g)\}$$
$$\therefore \mathrm{P}(\mathrm{B}) = \frac{2}{4} = \frac{1}{2}$$
$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by P (A|B).

$$\mathrm{P}(\mathrm{A} \mid \mathrm{B}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

(ii) Let C be the event that at least one child is a girl.

$$\therefore \mathrm{C} = \{(b,\ g), (g,\ b), (g,\ g)\}$$
$$\Rightarrow \mathrm{A} \cap \mathrm{C} = \{g,\ g\}$$

$$\Rightarrow \mathrm{P}(\mathrm{C}) = \frac{3}{4}$$
$$\mathrm{P}(\mathrm{A} \cap \mathrm{C}) = \frac{1}{4}$$

The conditional probability that both are girls, given that at least one child is a girl, is given by P(A|C).

$$\text{Therefore, } \mathrm{P}(\mathrm{A} \mid \mathrm{C}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{C})}{\mathrm{P}(\mathrm{C})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Solution 13

The given data can be tabulated as

	True/False	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us denote E = easy questions, M = multiple choice questions, D = difficult questions, and T = True/False questions

Total number of questions = 1400

Total number of multiple choice questions = 900

Therefore, probability of selecting an easy multiple choice question is

$P(E \cap M) = \frac{500}{1400} = \frac{5}{14}$

Probability of selecting a multiple choice question, P (M), is

$\frac{900}{1400} = \frac{9}{14}$

P (E|M) represents the probability that a randomly selected question will be an easy question, given that it is a multiple choice question.

$$P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}$$

Therefore, the required probability is $\frac{5}{9}$.

Solution 14

When dice is thrown, number of observations in the sample space = 6 × 6 = 36

Let A be the event that the sum of the numbers on the dice is 4 and B be the event that the two numbers appearing on throwing the two dice are different.

$$\therefore A = \{(1,\ 3),(2,\ 2),(3,\ 1)\}$$

$$B = \left\{\begin{array}{l} (1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),\ (2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),\ (3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),\ (4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5) \end{array}\right\}$$

$$A \cap B = \{(1,3),(3,1)\}$$

$$\therefore P(B) = \frac{30}{36} = \frac{5}{6} \text{ and } P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Let P (A|B) represent the probability that the sum of the numbers on the dice is 4, given that the two numbers appearing on throwing the two dice are different.

$$\therefore P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{15}$$

Therefore, the required probability is $\frac{1}{15}$.

Solution 15

The outcomes of the given experiment can be represented by the following tree diagram.

The sample space of the experiment is,

$$S=\begin{Bmatrix}(1,\mathrm{H}),(1,\mathrm{T}),(2,\mathrm{H}),(2,\mathrm{T}),(3,1)(3,2),(3,3),(3,4),(3,5),(3,6),\\(4,\mathrm{H}),(4,\mathrm{T}),(5,\mathrm{H}),(5,\mathrm{T}),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

Let A be the event that the coin shows a tail and B be the event that at least one die shows 3.

$$\therefore \mathrm{A}=\{(1,\mathrm{T}),(2,\mathrm{T}),(4,\mathrm{T}),(5,\mathrm{T})\}$$
$$\mathrm{B}=\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(6,3)\}$$
$$\Rightarrow \mathrm{A}\cap\mathrm{B}=\phi$$
$$\therefore \mathrm{P}(\mathrm{A}\cap\mathrm{B})=0$$
$$\text{Then, } \mathrm{P}(\mathrm{B})=\mathrm{P}(\{3,1\})+\mathrm{P}(\{3,2\})+\mathrm{P}(\{3,3\})+\mathrm{P}(\{3,4\})+\mathrm{P}(\{3,5\})+\mathrm{P}(\{3,6\})+\mathrm{P}(\{6,3\})$$
$$=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}$$
$$=\frac{7}{36}$$

Probability of the event that the coin shows a tail, given that at least one die shows 3, is given by P(A|B).

Therefore,

$$\mathrm{P}(\mathrm{A}\mid\mathrm{B})=\frac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{\mathrm{P}(\mathrm{B})}=\frac{0}{\frac{7}{36}}=0$$

Solution 16

It is given that $\mathrm{P}(\mathrm{A})=\frac{1}{2}$ and $\mathrm{P}(\mathrm{B})=0$

$$\mathrm{P}(\mathrm{A}\mid\mathrm{B})=\frac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{\mathrm{P}(\mathrm{B})}=\frac{\mathrm{P}(\mathrm{A}\cap\mathrm{B})}{0}$$

Therefore, P (A|B) is not defined.

Thus, the correct answer is C.

Solution 17

It is given that, P(A|B) = P(B|A)

$$\Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)}$$

P (A) = P (B)

Thus, the correct answer is D.

Chapter 13 - Probability Exercise Ex. 13.2

Solution 1

It is given that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$

A and B are independent events. Therefore,

$$P(A\cap B) = P(A)\cdot P(B) = \frac{3}{5}\cdot\frac{1}{5} = \frac{3}{25}$$

Solution 2

There are 26 black cards in a deck of 52 cards.

Let P (A) be the probability of getting a black card in the first draw.

$$\therefore P(A) = \frac{26}{52} = \frac{1}{2}$$

Let P (B) be the probability of getting a black card on the second draw.

Since the card is not replaced,

$$\therefore P(B) = \frac{25}{51}$$

Thus, probability of getting both the cards black $= \frac{1}{2}\times\frac{25}{51} = \frac{25}{102}$

Solution 3

Let A, B, and C be the respective events that the first, second, and third drawn orange is good.

Therefore, probability that first drawn orange is good, P (A) $=\frac{12}{15}$

The oranges are not replaced.

Therefore, probability of getting second orange good, P (B) $= \frac{11}{14}$

Similarly, probability of getting third orange good, P(C) $=\frac{10}{13}$

The box is approved for sale, if all the three oranges are good.

Thus, probability of getting all the oranges good $=\frac{12}{15}\times\frac{11}{14}\times\frac{10}{13}=\frac{44}{91}$

Therefore, the probability that the box is approved for sale is $\frac{44}{91}$.

Solution 4

If a fair coin and an unbiased die are tossed, then the sample space S is given by,

$$S = \begin{Bmatrix} (H,1),(H,2),(H,3),(H,4),(H,5),(H,6), \\ (T,1),(T,2),(T,3),(T,4),(T,5),(T,6) \end{Bmatrix}$$

Let A: Head appears on the coin

$$A = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\}$$

$$\Rightarrow P(A) = \frac{6}{12} = \frac{1}{2}$$

B: 3 on die $= \{(H,3),\ (T,3)\}$

$$P(B) = \frac{2}{12} = \frac{1}{6}$$

$$A \cap B = \{(H,3)\}$$

$$P(A \cap B) = \frac{1}{12}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{6} = P(A \cap B)$$

Therefore, A and B are independent events.

Solution 5

When a die is thrown, the sample space (S) is

S = {1, 2, 3, 4, 5, 6}

Let A: the number is even = {2, 4, 6}

$$P(A)=\frac{3}{6}=\frac{1}{2}$$

B: the number is red = {1, 2, 3}

$$P(B)=\frac{3}{6}=\frac{1}{2}$$

A ∩ B = {2}

$$P(AB)=P(A\cap B)=\frac{1}{6}$$

$$P(A)\cdot P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\neq\frac{1}{6}$$

$$\Rightarrow P(A)\cdot P(B)\neq P(AB)$$

Therefore, A and B are not independent.

Solution 6

It is given that $P(E)=\frac{3}{5}$, $P(F)=\frac{3}{10}$, and $P(EF)=P(E\cap F)=\frac{1}{5}$

$$P(E)\cdot P(F)=\frac{3}{5}\cdot\frac{3}{10}=\frac{9}{50}\neq\frac{1}{5}$$
$$\Rightarrow P(E)\cdot P(F)\neq P(EF)$$

Therefore, E and F are not independent.

Solution 7

It is given that $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{5}$, and $P(B)=p$

(i) When A and B are mutually exclusive, A ∩ B = Φ

P (A ∩ B) = 0

It is known that, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-0$$
$$\Rightarrow p=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$$

(ii) When A and B are independent, $P(A\cap B)=P(A)\cdot P(B)=\frac{1}{2}p$

It is known that, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \frac{3}{5}=\frac{1}{2}+p-\frac{1}{2}p$$
$$\Rightarrow \frac{3}{5}=\frac{1}{2}+\frac{p}{2}$$
$$\Rightarrow \frac{p}{2}=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$$
$$\Rightarrow p=\frac{2}{10}=\frac{1}{5}$$

Solution 8

It is given that P (A) = 0.3 and P (B) = 0.4

(i) If A and B are independent events, then

$$P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 0.12$$

(ii) It is known that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$$

(iii) It is known that, $P(A | B) = \dfrac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A | B) = \frac{0.12}{0.4} = 0.3$$

(iv) It is known that, $P(B | A) = \dfrac{P(A \cap B)}{P(A)}$

$$\Rightarrow P(B | A) = \frac{0.12}{0.3} = 0.4$$

Solution 9

It is given that, $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$

P(not on A and not on B) = $P(A' \cap B')$

P(not on A and not on B) = $P((A \cup B))'$ $\left[A' \cap B' = (A \cup B)'\right]$

$$= 1 - P(A \cup B)$$
$$= 1 - \left[P(A) + P(B) - P(A \cap B)\right]$$
$$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right]$$
$$= 1 - \frac{5}{8}$$
$$= \frac{3}{8}$$

Solution 10

It is given that $P(A)=\frac{1}{2}$, $P(B)=\frac{7}{12}$, and $P(\text{not A or not B})=\frac{1}{4}$

$$\Rightarrow P(A' \cup B')=\frac{1}{4}$$

$$\Rightarrow P\left((A \cap B)'\right)=\frac{1}{4} \qquad \left[A' \cup B' = (A \cap B)'\right]$$

$$\Rightarrow 1-P(A \cap B)=\frac{1}{4}$$

$$\Rightarrow P(A \cap B)=\frac{3}{4} \qquad ...(1)$$

However, $P(A)\cdot P(B)=\frac{1}{2}\cdot\frac{7}{12}=\frac{7}{24}$ ...(2)

Here, $\frac{3}{4}\neq\frac{7}{24}$

$\therefore P(A \cap B)\neq P(A)\cdot P(B)$

Therefore, A and B are independent events.

Solution 11

It is given that P (A) = 0.3 and P (B) = 0.6

Also, A and B are independent events.

(i) $\therefore P(A \text{ and } B) = P(A) \cdot P(B)$

$\Rightarrow P(A \cap B) = 0.3 \times 0.6 = 0.18$

(ii) P (A and not B) = $P(A \cap B')$

$= P(A) - P(A \cap B)$
$= 0.3 - 0.18$
$= 0.12$

(iii) P (A or B) = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.6 - 0.18$
$= 0.72$

(iv) P (neither A nor B) = $P(A' \cap B')$

$= P((A \cup B)')$
$= 1 - P(A \cup B)$
$= 1 - 0.72$
$= 0.28$

Solution 12

Probability of getting an odd number in a single throw of a die = $\frac{3}{6} = \frac{1}{2}$

Similarly, probability of getting an even number = $\frac{3}{6} = \frac{1}{2}$

Probability of getting an even number three times = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Therefore, probability of getting an odd number at least once

= 1 − Probability of getting an odd number in none of the throws

= 1 − Probability of getting an even number thrice

$= 1 - \frac{1}{8}$
$= \frac{7}{8}$

Solution 13

Total number of balls = 18

Number of red balls = 8

Number of black balls = 10

(i) Probability of getting a red ball in the first draw $= \frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

Probability of getting a red ball in the second draw $= \frac{8}{18} = \frac{4}{9}$

Therefore, probability of getting both the balls red $= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

(ii) Probability of getting first ball black $= \frac{10}{18} = \frac{5}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as red $= \frac{8}{18} = \frac{4}{9}$

Therefore, probability of getting first ball as black and second ball as red $= \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

(iii) Probability of getting first ball as red = $\frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

Probability of getting second ball as black = $\frac{10}{18} = \frac{5}{9}$

Therefore, probability of getting first ball as black and second ball as red =
$\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

Therefore, probability that one of them is black and other is red

= Probability of getting first ball black and second as red + Probability of getting first ball red and second ball black

$$= \frac{20}{81} + \frac{20}{81}$$
$$= \frac{40}{81}$$

Solution 14

Probability of solving the problem by A, P (A) = $\frac{1}{2}$

Probability of solving the problem by B, P (B) = $\frac{1}{3}$

Since the problem is solved independently by A and B,

$$\therefore P(AB) = P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$
$$P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

i. Probability that the problem is solved = P(A $\cup$ B)

Probability of solving the problem by A, P (A) $= \frac{1}{2}$

Probability of solving the problem by B, P (B) $= \frac{1}{3}$

Since the problem is solved independently by A and B,

$$\therefore P(AB) = P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$
$$P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) Probability that exactly one of them solves the problem is given by,
$$P(A).\, P(B') + P(B) \cdot P(A')$$

$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$$
$$= \frac{1}{3} + \frac{1}{6}$$
$$= \frac{1}{2}$$

Solution 15

(i) In a deck of 52 cards, 13 cards are spades and 4 cards are aces.

P(E) = P(the card drawn is a spade) = $\frac{13}{52} = \frac{1}{4}$

P(F) = P(the card drawn is an ace) = $\frac{4}{52} = \frac{1}{13}$

In the deck of cards, only 1 card is an ace of spades.

P(EF) = P(the card drawn is spade and an ace) = $\frac{1}{52}$

P(E) × P(F) = $\frac{1}{4} \cdot \frac{1}{13} = \frac{1}{52} = P(EF)$

P(E) × P(F) = P(EF)

Therefore, the events E and F are independent.

(ii) In a deck of 52 cards, 26 cards are black and 4 cards are kings.

P(E) = P(the card drawn is black) = $\frac{26}{52} = \frac{1}{2}$

P(F) = P(the card drawn is a king) = $\frac{4}{52} = \frac{1}{13}$

In the pack of 52 cards, 2 cards are black as well as kings.

P (EF) = P(the card drawn is a black king) = $\frac{2}{52}=\frac{1}{26}$

P(E) × P(F) = $\frac{1}{2}\cdot\frac{1}{13}=\frac{1}{26}=P(EF)$

Therefore, the given events E and F are independent.

(iii) In a deck of 52 cards, 4 cards are kings, 4 cards are queens, and 4 cards are jacks.

P(E) = P(the card drawn is a king or a queen) = $\frac{8}{52}=\frac{2}{13}$

P(F) = P(the card drawn is a queen or a jack) = $\frac{8}{52}=\frac{2}{13}$

There are 4 cards which are king or queen and queen or jack.

P(EF) = P(the card drawn is a king or a queen, or queen or a jack)

$= \frac{4}{52}=\frac{1}{13}$

P(E) × P(F) = $\frac{2}{13}\cdot\frac{2}{13}=\frac{4}{169}\neq\frac{1}{13}$

$\Rightarrow P(E)\cdot P(F)\neq P(EF)$

Therefore, the given events E and F are not independent.

Solution 16

Let H denote the students who read Hindi newspaper and E denote the students who read English newspaper.

It is given that,

$$P(H) = 60\% = \frac{6}{10} = \frac{3}{5}$$

$$P(E) = 40\% = \frac{40}{100} = \frac{2}{5}$$

$$P(H \cap E) = 20\% = \frac{20}{100} = \frac{1}{5}$$

i. Probability that a student reads Hindi or English newspaper is,

$$\begin{aligned}(H \cup E)' &= 1 - P(H \cup E) \\ &= 1 - \{P(H) + P(E) - P(H \cap E)\} \\ &= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right) \\ &= 1 - \frac{4}{5} \\ &= \frac{1}{5}\end{aligned}$$

(ii) Probability that a randomly chosen student reads English newspaper, if she reads Hindi news paper, is given by P (E|H).

$$P(E|H) = \frac{P(E \cap H)}{P(H)}$$

$$= \frac{\frac{1}{5}}{\frac{3}{5}}$$

$$= \frac{1}{3}$$

(iii) Probability that a randomly chosen student reads Hindi newspaper, if she reads English newspaper, is given by P (H|E).

$$P(H|E) = \frac{P(H \cap E)}{P(E)}$$

$$= \frac{\frac{1}{5}}{\frac{2}{5}}$$

$$= \frac{1}{2}$$

Solution 17

When two dice are rolled, the number of outcomes is 36.

The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

E = {(2, 2)}

$$\Rightarrow P(E) = \frac{1}{36}$$

Therefore, the correct answer is D.

Solution 18

Two events A and B are said to be independent, if P(AB) = P(A) × P(B)

Consider the result given in alternative **B**.

$$\begin{aligned}
&P(A'B') = [1 - P(A)][1 - P(B)] \\
&\Rightarrow P(A' \cap B') = 1 - P(A) - P(B) + P(A) \cdot P(B) \\
&\Rightarrow 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A) \cdot P(B) \\
&\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B) \\
&\Rightarrow P(A) + P(B) - P(AB) = P(A) + P(B) - P(A) \cdot P(B) \\
&\Rightarrow P(AB) = P(A) \cdot P(B)
\end{aligned}$$

This implies that A and B are independent, if $P(A'B') = [1 - P(A)][1 - P(B)]$

Distracter Rationale

A. Let P (A) = m, P (B) = n, $0 < m, n < 1$

A and B are mutually exclusive.

$$\begin{aligned}
&\therefore A \cap B = \phi \\
&\Rightarrow P(AB) = 0 \\
&\text{However, } P(A) \cdot P(B) = mn \neq 0 \\
&\therefore P(A) \cdot P(B) \neq P(AB)
\end{aligned}$$

C. Let A: Event of getting an odd number on throw of a die = {1, 3, 5}

$$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$$

B: Event of getting an even number on throw of a die = {2, 4, 6}

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

Here, $A \cap B = \phi$

$$\therefore P(AB) = 0$$

$$P(A) \cdot P(B) = \frac{1}{4} \neq 0$$

$$\Rightarrow P(A) \cdot P(B) \neq P(AB)$$

D. From the above example, it can be seen that,

$$P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$$

However, it cannot be inferred that A and B are independent.

Thus, the correct answer is B.

Chapter 13 - Probability Exercise Ex. 13.3

Solution 1

The urn contains 5 red and 5 black balls.

Let a red ball be drawn in the first attempt.

$\therefore$ P (drawing a red ball) $=\frac{5}{10}=\frac{1}{2}$

If two red balls are added to the urn, then the urn contains 7 red and 5 black balls.

P (drawing a red ball) $=\frac{7}{12}$

Let a black ball be drawn in the first attempt.

$\therefore$ P (drawing a black ball in the first attempt) $=\frac{5}{10}=\frac{1}{2}$

If two black balls are added to the urn, then the urn contains 5 red and 7 black balls.

P (drawing a red ball) $=\frac{5}{12}$

Therefore, probability of drawing second ball as red is

$$\frac{1}{2}\times\frac{7}{12}+\frac{1}{2}\times\frac{5}{12}=\frac{1}{2}\left(\frac{7}{12}+\frac{5}{12}\right)=\frac{1}{2}\times1=\frac{1}{2}$$

Solution 2

Let E_1 and E_2 be the events of selecting first bag and second bag respectively.

$$\therefore P(E_1) = P(E_2) = \frac{1}{2}$$

Let A be the event of getting a red ball.

$$\Rightarrow P(A|E_1) = P(\text{drawing a red ball from first bag}) = \frac{4}{8} = \frac{1}{2}$$

$$\Rightarrow P(A|E_2) = P(\text{drawing a red ball from second bag}) = \frac{2}{8} = \frac{1}{4}$$

The probability of drawing a ball from the first bag, given that it is red, is given by P $(E_2|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{8}}$$

$$= \frac{2}{3}$$

Solution 3

Let E_1 and E_2 be the events that the student is a hostler and a day scholar respectively and A be the event that the chosen student gets grade A.

$$\therefore P(E_1) = 60\% = \frac{60}{100} = 0.6$$

$$P(E_2) = 40\% = \frac{40}{100} = 0.4$$

$$P(A|E_1) = P(\text{student getting an A grade is a hostler}) = 30\% = 0.3$$

$$P(A|E_2) = P(\text{student getting an A grade is a day scholar }) = 20\% = 0.2$$

The probability that a randomly chosen student is a hostler, given that he has an A grade, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)} \\ &= \frac{0.6\times 0.3}{0.6\times 0.3+0.4\times 0.2} \\ &= \frac{0.18}{0.26} \\ &= \frac{18}{26} \\ &= \frac{9}{13} \end{aligned}$$

Solution 4

Let E_1 and E_2 be the respective events that the student knows the answer and he guesses the answer.

Let A be the event that the answer is correct.

$$\therefore P(E_1) = \frac{3}{4}$$

$$P(E_2) = \frac{1}{4}$$

The probability that the student answered correctly, given that he knows the answer, is 1.

$P(A|E_1) = 1$

Probability that the student answered correctly, given that he guessed, is $\frac{1}{4}$.

$$\therefore P(A|E_2) = \frac{1}{4}$$

The probability that the student knows the answer, given that he answered it correctly, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{3}{4}\cdot 1}{\frac{3}{4}\cdot 1 + \frac{1}{4}\cdot\frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$= \frac{\frac{3}{4}}{\frac{13}{16}}$$

$$= \frac{12}{13}$$

Solution 5

Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.

Since E_1 and E_2 are events complimentary to each other,

$P(E_1) + P(E_2) = 1$

$P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

Let A be the event that the blood test result is positive.

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

$$P(A|E_1) = P(\text{result is positive given the person has disease}) = 99\% = 0.99$$

$$P(A|E_2) = P(\text{result is positive given that the person has no disease}) = 0.5\% = 0.005$$

Probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{0.001\times 0.99}{0.001\times 0.99 + 0.999\times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995}$$

$$= \frac{0.00099}{0.005985}$$

$$= \frac{990}{5985}$$

$$= \frac{110}{665}$$

$$= \frac{22}{133}$$

Solution 6

Let E_1, E_2, and E_3 be the respective events of choosing a two headed coin, a biased coin, and an unbiased coin.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads.

A two-headed coin will always show heads.

$$\therefore P(A|E_1) = P(\text{coin showing heads, given that it is a two-headed coin}) = 1$$

Probability of heads coming up, given that it is a biased coin= 75%

$$\therefore P(A|E_2) = P(\text{coin showing heads, given that it is a biased coin}) = \frac{75}{100} = \frac{3}{4}$$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$$\therefore P(A|E_3) = P(\text{coin showing heads, given that it is an unbiased coin}) = \frac{1}{2}$$

The probability that the coin is two-headed, given that it shows heads, is given by

$P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)}$$

$$= \frac{1}{\frac{9}{4}}$$

$$= \frac{4}{9}$$

Solution 7

Let E_1, E_2, and E_3 be the respective events that the driver is a scooter driver, a car driver, and a truck driver.

Let A be the event that the person meets with an accident.

There are 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers.

Total number of drivers = 2000 + 4000 + 6000 = 12000

P (E_1) = P (driver is a scooter driver) $= \frac{2000}{12000} = \frac{1}{6}$

P (E_2) = P (driver is a car driver) $= \frac{4000}{12000} = \frac{1}{3}$

P (E_3) = P (driver is a truck driver) $= \frac{6000}{12000} = \frac{1}{2}$

$$P(A|E_1) = P(\text{scooter driver met with an accident}) = 0.01 = \frac{1}{100}$$

$$P(A|E_2) = P(\text{car driver met with an accident}) = 0.03 = \frac{3}{100}$$

$$P(A|E_3) = P(\text{truck driver met with an accident}) = 0.15 = \frac{15}{100}$$

The probability that the driver is a scooter driver, given that he met with an accident, is given by P ($E_1|A$).

By using Bayes' theorem, we obtain

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$=\frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{6}\cdot\frac{1}{100}+\frac{1}{3}\cdot\frac{3}{100}+\frac{1}{2}\cdot\frac{15}{100}}$$

$$=\frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{100}\left(\frac{1}{6}+1+\frac{15}{2}\right)}$$

$$=\frac{\frac{1}{6}}{\frac{104}{12}}$$

$$=\frac{1}{6}\times\frac{12}{104}$$

$$=\frac{1}{52}$$

Solution 8

Let E_1 and E_2 be the respective events of items produced by machines A and B. Let X be the event that the produced item was found to be defective.

Probability of items produced by machine A, P (E_1) $=60\%=\frac{3}{5}$

Probability of items produced by machine B, P (E_2) $=40\%=\frac{2}{5}$

Probability that machine A produced defective items, P ($X|E_1$) $=2\%=\frac{2}{100}$

Probability that machine B produced defective items, P ($X|E_2$) $=1\%=\frac{1}{100}$

The probability that the randomly selected item was from machine B, given that it is defective, is given by P ($E_2|X$).

By using Bayes' theorem, we obtain

$$P(E_2|X) = \frac{P(E_2)\cdot P(X|E_2)}{P(E_1)\cdot P(X|E_1) + P(E_2)\cdot P(X|E_2)}$$

$$= \frac{\frac{2}{5}\cdot\frac{1}{100}}{\frac{3}{5}\cdot\frac{2}{100} + \frac{2}{5}\cdot\frac{1}{100}}$$

$$= \frac{\frac{2}{500}}{\frac{6}{500} + \frac{2}{500}}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

Solution 9

Let E_1 and E_2 be the respective events that the first group and the second group win the competition. Let A be the event of introducing a new product.

$P(E_1)$ = Probability that the first group wins the competition = 0.6

$P(E_2)$ = Probability that the second group wins the competition = 0.4

$P(A|E_1)$ = Probability of introducing a new product if the first group wins = 0.7

$P(A|E_2)$ = Probability of introducing a new product if the second group wins = 0.3

The probability that the new product is introduced by the second group is given by

$P(E_2|A)$.

By using Bayes' theorem, we obtain

$$\begin{aligned}
P(E_2|A) &= \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)} \\
&= \frac{0.4\times 0.3}{0.6\times 0.7+0.4\times 0.3} \\
&= \frac{0.12}{0.42+0.12} \\
&= \frac{0.12}{0.54} \\
&= \frac{12}{54} \\
&= \frac{2}{9}
\end{aligned}$$

Solution 10

Let E_1 be the event that the outcome on the die is 5 or 6 and E_2 be the event that the outcome on the die is 1, 2, 3, or 4.

$$\therefore P(E_1)=\frac{2}{6}=\frac{1}{3} \text{ and } P(E_2)=\frac{4}{6}=\frac{2}{3}$$

Let A be the event of getting exactly one head.

P $(A|E_1)$ = Probability of getting exactly one head by tossing the coin three times if she gets 5 or 6 $=\frac{3}{8}$

P $(A|E_2)$ = Probability of getting exactly one head in a single throw of coin if she gets 1, 2, 3, or 4 $=\frac{1}{2}$

The probability that the girl threw 1, 2, 3, or 4 with the die, if she obtained exactly one head, is given by P $(E_2|A)$.

By using Bayes' theorem, we obtain

$$P(E_2|A)=\frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{3}{8}+\frac{2}{3}\cdot\frac{1}{2}}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}\left(\frac{3}{8}+1\right)}$$

$$=\frac{1}{\frac{11}{8}}$$

$$=\frac{8}{11}$$

Solution 11

Let E_1, E_2, and E_3 be the respective events of the time consumed by machines A, B, and C for the job.

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}$$
$$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$$
$$P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing defective items.

$$P(X|E_1) = 1\% = \frac{1}{100}$$
$$P(X|E_2) = 5\% = \frac{5}{100}$$
$$P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by P (E_1|A).

By using Bayes' theorem, we obtain

$$P(E_1|X) = \frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1) + P(E_2)\cdot P(X|E_2) + P(E_3)\cdot P(X|E_3)}$$
$$= \frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{2}\cdot\frac{1}{100} + \frac{3}{10}\cdot\frac{5}{100} + \frac{1}{5}\cdot\frac{7}{100}}$$
$$= \frac{\frac{1}{100}\cdot\frac{1}{2}}{\frac{1}{100}\left(\frac{1}{2} + \frac{3}{2} + \frac{7}{5}\right)}$$
$$= \frac{\frac{1}{2}}{\frac{17}{5}}$$
$$= \frac{5}{34}$$

Solution 12

Let E_1 and E_2 be the respective events of choosing a diamond card and a card which is not diamond.

Let A denote the lost card.

Out of 52 cards, 13 cards are diamond and 39 cards are not diamond.

$$\therefore P(E_1) = \frac{13}{52} = \frac{1}{4}$$
$$P(E_2) = \frac{39}{52} = \frac{3}{4}$$

When one diamond card is lost, there are 12 diamond cards out of 51 cards.

Two cards can be drawn out of 12 diamond cards in ${}^{12}C_2$ ways.

Similarly, 2 diamond cards can be drawn out of 51 cards in ${}^{51}C_2$ ways. The probability of getting two cards, when one diamond card is lost, is given by P $(A|E_1)$.

$$P(A|E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12!}{2!\times 10!} \times \frac{2!\times 49!}{51!} = \frac{11\times 12}{50\times 51} = \frac{22}{425}$$

When the lost card is not a diamond, there are 13 diamond cards out of 51 cards.

Two cards can be drawn out of 13 diamond cards in ${}^{13}C_2$ ways whereas 2 cards can be drawn out of 51 cards in ${}^{51}C_2$ ways.

The probability of getting two cards, when one card is lost which is not diamond, is given by P $(A|E_2)$.

$$P(A|E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13!}{2!\times 11!} \times \frac{2!\times 49!}{51!} = \frac{12\times 13}{50\times 51} = \frac{26}{425}$$

The probability that the lost card is diamond is given by P $(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{4}\cdot\frac{22}{425}}{\frac{1}{4}\cdot\frac{22}{425} + \frac{3}{4}\cdot\frac{26}{425}}$$

$$= \frac{\frac{1}{425}\left(\frac{22}{4}\right)}{\frac{1}{425}\left(\frac{22}{4} + \frac{26\times 3}{4}\right)}$$

$$= \frac{\frac{11}{2}}{25}$$

$$= \frac{11}{50}$$

Solution 13

Let E_1 and E_2 be the events such that

E_1: A speaks truth

E_2: A speaks false

Let X be the event that a head appears.

$$P(E_1) = \frac{4}{5}$$

$$\therefore P(E_2) = 1 - P(E_1) = 1 - \frac{4}{5} = \frac{1}{5}$$

If a coin is tossed, then it may result in either head (H) or tail (T).

The probability of getting a head is $\frac{1}{2}$ whether A speaks truth or not.

$$\therefore P(X|E_1) = P(X|E_2) = \frac{1}{2}$$

The probability that there is actually a head is given by P $(E_1|X)$.

$$P(E_1|X)=\frac{P(E_1)\cdot P(X|E_1)}{P(E_1)\cdot P(X|E_1)+P(E_2)\cdot P(X|E_2)}$$

$$=\frac{\frac{4}{5}\cdot\frac{1}{2}}{\frac{4}{5}\cdot\frac{1}{2}+\frac{1}{5}\cdot\frac{1}{2}}$$

$$=\frac{\frac{1}{2}\cdot\frac{4}{5}}{\frac{1}{2}\left(\frac{4}{5}+\frac{1}{5}\right)}$$

$$=\frac{\frac{4}{5}}{1}$$

$$=\frac{4}{5}$$

Therefore, the correct answer is A.

Solution 14

If $A \subset B$, then $A \cap B = A$

$P(A \cap B) = P(A)$

Also, $P(A) < P(B)$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}\neq\frac{P(B)}{P(A)}\quad \dots (1)$$

Thus, $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}\quad \dots (2)$

It is known that, $P(B) \leq 1$

$$\Rightarrow \frac{1}{P(B)} \geq 1$$

$$\Rightarrow \frac{P(A)}{P(B)} \geq P(A)$$

From (2), we obtain

$$\Rightarrow P(A|B) \geq P(A) \qquad \dots(3)$$

$\therefore P(A|B)$ is not less than $P(A)$.

Thus, from (3), it can be concluded that the relation given in alternative C is correct.

Chapter 13 - Probability Exercise Ex. 13.4

Solution 1

It is known that the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities = 0.4 + 0.4 + 0.2 = 1

Therefore, the given table is a probability distribution of random variables.

(ii) It can be seen that for X = 3, P (X) = −0.1

It is known that probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities = 0.6 + 0.1 + 0.2 = 0.9 ≠ 1

Therefore, the given table is not a probability distribution of random variables.

(iv) Sum of the probabilities = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 ≠ 1

Therefore, the given table is not a probability distribution of random variables.

Solution 2

The two balls selected can be represented as BB, BR, RB, RR, where B represents a black ball and R represents a red ball.

X represents the number of black balls.

X (BB) = 2

X (BR) = 1

X (RB) = 1

X (RR) = 0

Therefore, the possible values of X are 0, 1, and 2.

Yes, X is a random variable.

Solution 3

A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

X (6 H, 0T) $=|6-0|=6$

X (5 H, 1 T) $=|5-1|=4$

X (4 H, 2 T) $=|4-2|=2$

X (3 H, 3 T) $=|3-3|=0$

X (2 H, 4 T) $=|2-4|=2$

X (1 H, 5 T) $=|1-5|=4$

X (0H, 6 T) $=|0-6|=6$

Thus, the possible values of X are 6, 4, 2, and 0.

Solution 4

(i) When one coin is tossed twice, the sample space is

{HH, HT, TH, TT}

Let X represent the number of heads.

X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

P (X = 0) = P (TT) $= \frac{1}{4}$

P (X = 1) = P (HT) + P (TH) $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

P (X = 2) = P (HH) $= \frac{1}{4}$

Thus, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) When three coins are tossed simultaneously, the sample space is
$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let X represent the number of tails.

It can be seen that X can take the value of 0, 1, 2, or 3.

P (X = 0) = P (HHH) = $\frac{1}{8}$

P (X = 1) = P (HHT) + P (HTH) + P (THH) = $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

P (X = 2) = P (HTT) + P (THT) + P (TTH) = $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

P (X = 3) = P (TTT) = $\frac{1}{8}$

Thus, the probability distribution is as follows.

X	0	1	2	3
P (X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) When a coin is tossed four times, the sample space is

$$S=\left\{\begin{array}{l}HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT\end{array}\right\}$$

Let X be the random variable, which represents the number of heads.

It can be seen that X can take the value of 0, 1, 2, 3, or 4.

P (X = 0) = P (TTTT) = $\frac{1}{16}$

P (X = 1) = P (TTTH) + P (TTHT) + P (THTT) + P (HTTT)

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

P (X = 2) = P (HHTT) + P (THHT) + P (TTHH) + P (HTTH) + P (HTHT)

+ P (THTH)

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{6}{16}=\frac{3}{8}$

P (X = 3) = P (HHHT) + P (HHTH) + P (HTHH) + P (THHH)

$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{4}{16}=\frac{1}{4}$

P (X = 4) = P (HHHH) = $\frac{1}{16}$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Solution 5

When a die is tossed two times, we obtain $(6 \times 6) = 36$ number of observations.

Let X be the random variable, which represents the number of successes.

i. Here, success refers to the number greater than 4.

P (X = 0) = P (number less than or equal to 4 on both the tosses) = $\frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$

P (X = 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$$= \frac{4}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{2}{6} = \frac{4}{9}$$

P (X = 2) = P (number greater than 4 on both the tosses)

$$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Thus, the probability distribution is as follows.

X	1	1	2
P (X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die.

P (Y = 0) = P (six does not appear on any of the dice) $= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P (Y = 1) = P (six appears on at least one of the dice) = $\frac{11}{36}$

Thus, the required probability distribution is as follows.

Y	0	1
P (Y)	$\frac{25}{36}$	$\frac{11}{36}$

Solution 6

It is given that out of 30 bulbs, 6 are defective.

Number of non-defective bulbs = 30 − 6 = 24

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

∴ P (X = 0) = P (4 non-defective and 0 defective) = ${}^4C_0 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{256}{625}$

P (X = 1) = P (3 non-defective and 1 defective) $= {}^4C_1 \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (2 non-defective and 2 defective) $= {}^4C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (1 non-defective and 3 defective) $= {}^4C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right) = \frac{16}{625}$

P (X = 4) = P (0 non-defective and 4 defective) $= {}^4C_4 \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Solution 7

Let the probability of getting a tail in the biased coin be x.

P (T) = x

P (H) = $3x$

For a biased coin, P (T) + P (H) = 1

$\Rightarrow x + 3x = 1$

$\Rightarrow 4x = 1$

$\Rightarrow x = \frac{1}{4}$

$\therefore \mathrm{P}(\mathrm{T}) = \frac{1}{4}$ and $\mathrm{P}(\mathrm{H}) = \frac{3}{4}$

When the coin is tossed twice, the sample space is {HH, TT, HT, TH}.

Let X be the random variable representing the number of tails.

P (X = 0) = P (no tail) = P (H) × P (H) $= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P (X = 1) = P (one tail) = P (HT) + P (TH)

$= \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4}$

$= \frac{3}{16} + \frac{3}{16}$

$= \frac{3}{8}$

P (X = 2) = P (two tails) = P (TT) $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Solution 8

(i) It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\therefore 0+k+2k+2k+3k+k^2+2k^2+\left(7k^2+k\right)=1$$

$$\Rightarrow 10k^2+9k-1=0$$

$$\Rightarrow \left(10k-1\right)\left(k+1\right)=0$$

$$\Rightarrow k=-1,\frac{1}{10}$$

k = − 1 is not possible as the probability of an event is never negative.

$$\Rightarrow k=\frac{1}{10}$$

(ii) P (X < 3) = P (X = 0) + P (X = 1) + P (X = 2)

$$=0+k+2k$$

$$=3k$$

$$=3\times\frac{1}{10}$$

$$=\frac{3}{10}$$

(iii) P (X > 6) = P (X = 7)

$$=7k^2+k$$

$$=7\times\left(\frac{1}{10}\right)^2+\frac{1}{10}$$

$$=\frac{7}{100}+\frac{1}{10}$$

$$=\frac{17}{100}$$

(iv) P (0 < X < 3) = P (X = 1) + P (X = 2)

$$=k+2k$$

$$=3k$$

$$=3\times\frac{1}{10}$$

$$=\frac{3}{10}$$

Solution 9

(a) It is known that the sum of probabilities of a probability distribution of random variables is one.

$k + 2k + 3k + 0 = 1$

$6k = 1$

$k = \frac{1}{6}$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$= k + 2k$

$= 3k$

$= \frac{3}{6}$

$= \frac{1}{2}$

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= k + 2k + 3k$$

$$= 6k$$

$$= \frac{6}{6}$$

$$= 1$$

$$P(X \ge 2) = P(X = 2) + P(X > 2)$$

$$= 3k + 0$$

$$= 3k$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Solution 10

Let X denote the success of getting heads.

Therefore, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that X can take the value of 0, 1, 2, or 3.

$$\begin{aligned}\therefore P(X=0) &= P(TTT)\\ &= P(T)\cdot P(T)\cdot P(T)\\ &= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{1}{8}\end{aligned}$$

P (X = 1) = P (HHT) + P (HTH) + P (THH)

$$\begin{aligned}&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{3}{8}\end{aligned}$$

P(X = 2) = P (HHT) + P (HTH) + P (THH)

$$\begin{aligned}&= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{3}{8}\end{aligned}$$

$$\therefore P(X=3) = P(HHH)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of X E(X), μ $= \sum X_i P(X_i)$

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{3}{8} + \frac{3}{4} + \frac{3}{8}$$

$$= \frac{3}{2}$$

$$= 1.5$$

Solution 11

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

P (X = 0) = P (not getting six on any of the dice) = $\frac{25}{36}$

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die)

$$= 2\left(\frac{1}{6}\times\frac{5}{6}\right) = \frac{10}{36}$$

P (X = 2) = P (six on both the dice) = $\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X = E(X) = $\sum X_i P(X_i)$

$$= 0\times\frac{25}{36} + 1\times\frac{10}{36} + 2\times\frac{1}{36}$$
$$= \frac{1}{3}$$

Solution 12

The two positive integers can be selected from the first six positive integers without replacement in 6 × 5 = 30 ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6.

For X = 2, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X=2)=\frac{2}{30}=\frac{1}{15}$$

For X = 3, the possible observations are (1, 3), (2, 3), (3, 1), and (3, 2).

$$\therefore P(X=3)=\frac{4}{30}=\frac{2}{15}$$

For X = 4, the possible observations are (1, 4), (2, 4), (3, 4), (4, 3), (4, 2), and (4, 1).

$$\therefore P(X=4)=\frac{6}{30}=\frac{1}{5}$$

For X = 5, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2), and (5, 1).

$$\therefore P(X=5)=\frac{8}{30}=\frac{4}{15}$$

For X = 6, the possible observations are (1, 6), (2, 6) , (3, 6), (4, 6), (5, 6), (6,5) (6, 4), (6, 3), (6, 2), and (6, 1).

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

$$\begin{aligned}\text{Then, } E(X) &= \sum X_i P(X_i)\\ &= 2\cdot\frac{1}{15}+3\cdot\frac{2}{15}+4\cdot\frac{1}{5}+5\cdot\frac{4}{15}+6\cdot\frac{1}{3}\\ &= \frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2\\ &= \frac{70}{15}\\ &= \frac{14}{3}\end{aligned}$$

Solution 13

When two fair dice are rolled, $6 \times 6 = 36$ observations are obtained.

$P(X = 2) = P(1, 1) = \frac{1}{36}$

$P(X = 3) = P(1, 2) + P(2, 1) = \frac{2}{36} = \frac{1}{18}$

$P(X = 4) = P(1, 3) + P(2, 2) + P(3, 1) = \frac{3}{36} = \frac{1}{12}$

$P(X = 5) = P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1) = \frac{4}{36} = \frac{1}{9}$

$P(X = 6) = P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1) = \frac{5}{36}$

$P(X = 7) = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) = \frac{6}{36} = \frac{1}{6}$

$P(X = 8) = P(2, 6) + P(3, 5) + P(4, 4) + P(5, 3) + P(6, 2) = \frac{5}{36}$

$P(X = 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3) = \frac{4}{36} = \frac{1}{9}$

$P(X = 10) = P(4, 6) + P(5, 5) + P(6, 4) = \frac{3}{36} = \frac{1}{12}$

P(X = 11) = P(5, 6) + P(6, 5) = $\frac{2}{36}=\frac{1}{18}$

P(X = 12) = P(6, 6) = $\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

$$\begin{aligned}\text{Then, } E(X) &= \sum X_i \cdot P(X_i)\\ &= 2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}\\ &\quad+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}\\ &= \frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{5}{6}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}\\ &= 7\end{aligned}$$

$$\begin{aligned}E(X^2) &= \sum X_i^2 \cdot P(X_i)\\ &= 4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}\\ &\quad+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}\\ &= \frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4\\ &= \frac{987}{18}=\frac{329}{6}=54.833\end{aligned}$$

$$\begin{aligned}\text{Then, } \mathrm{Var}(X) &= E(X^2)-[E(X)]^2\\ &= 54.833-(7)^2\\ &= 54.833-49\\ &= 5.833\end{aligned}$$

$$\begin{aligned}\therefore \text{Standard deviation} &= \sqrt{\mathrm{Var}(X)}\\ &= \sqrt{5.833}\\ &= 2.415\end{aligned}$$

Solution 14

There are 15 students in the class. Each student has the same chance to be chosen.

Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X = 14) = \frac{2}{15}$, $P(X = 15) = \frac{1}{15}$, $P(X = 16) = \frac{2}{15}$, $P(X = 16) = \frac{3}{15}$,

$P(X = 18) = \frac{1}{15}$, $P(X = 19) = \frac{2}{15}$, $P(X = 20) = \frac{3}{15}$, $P(X = 21) = \frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
P	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of X = E(X)

$$= \sum X_i P(X_i)$$

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21)$$

$$= \frac{263}{15}$$

$$= 17.53$$

E(X^2) $= \sum X_i^2 P(X_i)$

$$= (14)^2 \cdot \frac{2}{15} + (15)^2 \cdot \frac{1}{15} + (16)^2 \cdot \frac{2}{15} + (17)^2 \cdot \frac{3}{15} +$$

$$(18)^2 \cdot \frac{1}{15} + (19)^2 \cdot \frac{2}{15} + (20)^2 \cdot \frac{3}{15} + (21)^2 \cdot \frac{1}{15}$$

$$= \frac{1}{15} \cdot (392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441)$$

$$= \frac{4683}{15}$$

$$= 312.2$$

$$\therefore \text{Variance}(X) = E(X^2) - [E(X)]^2$$

$$= 312.2 - \left(\frac{263}{15}\right)^2$$

$$= 312.2 - 307.4177$$

$$= 4.7823$$

$$\approx 4.78$$

$$\text{Standard derivation} = \sqrt{\text{Variance}(X)}$$

$$= \sqrt{4.78}$$

$$= 2.186 \approx 2.19$$

Solution 15

It is given that P(X = 0) = 30% = $\frac{30}{100} = 0.3$

$$P(X = 1) = 70\% = \frac{70}{100} = 0.7$$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

$$\begin{aligned}\text{Then, } E(X) &= \sum X_i P(X_i)\\ &= 0\times 0.3 + 1\times 0.7\\ &= 0.7\end{aligned}$$

$$\begin{aligned}E(X^2) &= \sum X_i^2 P(X_i)\\ &= 0^2\times 0.3 + (1)^2\times 0.7\\ &= 0.7\end{aligned}$$

It is known that, Var (X) = $E(X^2) - [E(X)]^2$

= 0.7 − $(0.7)^2$

= 0.7 − 0.49

= 0.21

Solution 16

Let X be the random variable representing a number on the die.

The total number of observations is six.

$$\therefore P(X=1)=\frac{3}{6}=\frac{1}{2}$$

$$P(X=2)=\frac{2}{6}=\frac{1}{3}$$

$$P(X=5)=\frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Mean = E(X) = $\sum p_i x_i$

$$=\frac{1}{2}\times 1+\frac{1}{3}\times 2+\frac{1}{6}\cdot 5$$

$$=\frac{1}{2}+\frac{2}{3}+\frac{5}{6}$$

$$=\frac{3+4+5}{6}$$

$$=\frac{12}{6}$$

$$=2$$

The correct answer is B.

Solution 17

Let X denote the number of aces obtained. Therefore, X can take any of the values of 0, 1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

P (X = 0) = P (0 ace and 2 non-ace cards) = $\frac{{}^4C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{1128}{1326}$

P (X = 1) = P (1 ace and 1 non-ace cards) = $\frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{192}{1326}$

P (X = 2) = P (2 ace and 0 non- ace cards) = $\frac{{}^4C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{6}{1326}$

Thus, the probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, E(X) = $\sum p_i x_i$

$$= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326}$$

$$= \frac{204}{1326}$$

$$= \frac{2}{13}$$

Therefore, the correct answer is D.

Chapter 13 - Probability Exercise Ex. 13.5

Solution 1

The repeated tosses of a die are Bernoulli trials. Let X denote the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p = \frac{3}{6} = \frac{1}{2}$

$$\therefore q = 1 - p = \frac{1}{2}$$

X has a binomial distribution.

Therefore, P (X = x) = ${}^{n}\mathrm{C}_{n-x} q^{n-x} p^{x}$, where $n = 0, 1, 2 \ldots n$

$$= {}^{6}\mathrm{C}_{x} \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^{x}$$
$$= {}^{6}\mathrm{C}_{x} \left(\frac{1}{2}\right)^{6}$$

(i) P (5 successes) = P (X = 5)

$$= {}^{6}\mathrm{C}_{5} \left(\frac{1}{2}\right)^{6}$$
$$= 6 \cdot \frac{1}{64}$$
$$= \frac{3}{32}$$

(ii) P(at least 5 successes) = P(X ≥ 5)

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{1}{2}\right)^6 + {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= 6 \cdot \frac{1}{64} + 1 \cdot \frac{1}{64}$$

$$= \frac{7}{64}$$

(iii) P (at most 5 successes) = P(X ≤ 5)

$$= 1 - P(X > 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

Solution 2

Probability of getting doublets in a single throw of the pair of dice is

$$p=\frac{6}{36}=\frac{1}{6}$$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has the binomial distribution with $n = 4$, $p=\frac{1}{6}$, and $q=\frac{5}{6}$

$$\therefore \mathrm{P}(\mathrm{X}=x)={}^{n}\mathrm{C}_{x}q^{n-x}p^{x}, \text{ where } x=0, 1, 2, 3 \ldots n$$

$$={}^{4}\mathrm{C}_{x}\left(\frac{5}{6}\right)^{4-x}\cdot\left(\frac{1}{6}\right)^{x}$$

$$={}^{4}\mathrm{C}_{x}\cdot\frac{5^{4-x}}{6^{4}}$$

P (2 successes) = P (X = 2)

$$={}^{4}\mathrm{C}_{2}\cdot\frac{5^{4-2}}{6^{4}}$$

$$=6\cdot\frac{25}{1296}$$

$$=\frac{25}{216}$$

Solution 3

Let X denote the number of defective items in a sample of 10 items drawn successively. Since the drawing is done with replacement, the trials are Bernoulli trials.

$$\Rightarrow p = \frac{5}{100} = \frac{1}{20}$$

$$\therefore q = 1 - \frac{1}{20} = \frac{19}{20}$$

X has a binomial distribution with n = 10 and $p = \frac{1}{20}$

P(X = x) = ${}^{n}C_x q^{n-x} p^x$, where $x = 0, 1, 2 \ldots n$

$$= {}^{10}C_x \left(\frac{19}{20}\right)^{10-x} \cdot \left(\frac{1}{20}\right)^x$$

P (not more than 1 defective item) = P (X ≤ 1)

$$= P(X=0) + P(X=1)$$

$$= {}^{10}C_0 \left(\frac{19}{20}\right)^{10} \cdot \left(\frac{1}{20}\right)^0 + {}^{10}C_1 \left(\frac{19}{20}\right)^9 \cdot \left(\frac{1}{20}\right)^1$$

$$= \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^9 \cdot \left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20}\right)^9 \cdot \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \left(\frac{19}{20}\right)^9 \cdot \left(\frac{29}{20}\right)$$

$$= \left(\frac{29}{20}\right) \cdot \left(\frac{19}{20}\right)^9$$

Solution 4

Let X represent the number of spade cards among the five cards drawn. Since the drawing of card is with replacement, the trials are Bernoulli trials.

In a well shuffled deck of 52 cards, there are 13 spade cards.

$$\Rightarrow p = \frac{13}{52} = \frac{1}{4}$$
$$\therefore q = 1 - \frac{1}{4} = \frac{3}{4}$$

X has a binomial distribution with n = 5 and $p = \frac{1}{4}$

$$\begin{aligned} P(X = x) &= {}^{n}C_{x} q^{n-x} p^{x}, \text{ where } x = 0, 1, \ldots n \\ &= {}^{5}C_{x} \left(\frac{3}{4}\right)^{5-x} \left(\frac{1}{4}\right)^{x} \end{aligned}$$

(i) P (all five cards are spades) = P(X = 5)

$$= {}^{5}C_{5} \left(\frac{3}{4}\right)^{0} \cdot \left(\frac{1}{4}\right)^{5}$$
$$= 1 \cdot \frac{1}{1024}$$
$$= \frac{1}{1024}$$

(ii) P (only 3 cards are spades) = P(X = 3)

$$= {}^{5}C_{3} \cdot \left(\frac{3}{4}\right)^{2} \cdot \left(\frac{1}{4}\right)^{3}$$
$$= 10 \cdot \frac{9}{16} \cdot \frac{1}{64}$$
$$= \frac{45}{512}$$

(iii) P (none is a spade) = P(X = 0)

$$= {}^{5}C_{0} \cdot \left(\frac{3}{4}\right)^{5} \cdot \left(\frac{1}{4}\right)^{0}$$
$$= 1 \cdot \frac{243}{1024}$$
$$= \frac{243}{1024}$$

Solution 5

Let X represent the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.

It is given that, $p = 0.05$

$$\therefore q = 1 - p = 1 - 0.05 = 0.95$$

X has a binomial distribution with $n = 5$ and $p = 0.05$

$$\therefore \mathrm{P}(\mathrm{X} = x) = {}^{n}\mathrm{C}_{x} q^{n-x} p^{x}, \text{ where } x = 1, 2, \dots n$$
$$= {}^{5}\mathrm{C}_{x} (0.95)^{5-x} \cdot (0.05)^{x}$$

(i) P (none) = P(X = 0)

$$= {}^{5}\mathrm{C}_{0} (0.95)^{5} \cdot (0.05)^{0}$$
$$= 1 \times (0.95)^{5}$$
$$= (0.95)^{5}$$

(ii) P (not more than one) = P(X ≤ 1)

$$= \mathrm{P}(\mathrm{X} = 0) + \mathrm{P}(\mathrm{X} = 1)$$
$$= {}^{5}\mathrm{C}_{0} (0.95)^{5} \times (0.05)^{0} + {}^{5}\mathrm{C}_{1} (0.95)^{4} \times (0.05)^{1}$$
$$= 1 \times (0.95)^{5} + 5 \times (0.95)^{4} \times (0.05)$$
$$= (0.95)^{5} + (0.25)(0.95)^{4}$$
$$= (0.95)^{4} [0.95 + 0.25]$$
$$= (0.95)^{4} \times 1.2$$

(iii) P (more than 1) = P(X > 1)

$$= 1 - \mathrm{P}(\mathrm{X} \le 1)$$
$$= 1 - \mathrm{P}(\text{not more than } 1)$$
$$= 1 - (0.95)^{4} \times 1.2$$

(iv) P (at least one) = P(X ≥ 1)

$$= 1 - \mathrm{P}(\mathrm{X} < 1)$$
$$= 1 - \mathrm{P}(\mathrm{X} = 0)$$
$$= 1 - {}^{5}\mathrm{C}_{0} (0.95)^{5} \times (0.05)^{0}$$
$$= 1 - 1 \times (0.95)^{5}$$
$$= 1 - (0.95)^{5}$$

Solution 6

Let X denote the number of balls marked with the digit 0 among the 4 balls drawn.

Since the balls are drawn with replacement, the trials are Bernoulli trials.

X has a binomial distribution with $n = 4$ and $p = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore \mathrm{P}(\mathrm{X} = x) = {}^{n}\mathrm{C}_{x} q^{n-x} \cdot p^{x}, x = 1, 2, \ldots n$$

$$= {}^{4}\mathrm{C}_{x} \left(\frac{9}{10}\right)^{4-x} \cdot \left(\frac{1}{10}\right)^{x}$$

P (none marked with 0) = P (X = 0)

$$= {}^{4}\mathrm{C}_{0} \left(\frac{9}{10}\right)^{4} \cdot \left(\frac{1}{10}\right)^{0}$$

$$= 1 \cdot \left(\frac{9}{10}\right)^{4}$$

$$= \left(\frac{9}{10}\right)^{4}$$

Solution 7

The repeated tosses of a coin are Bernoulli trails. Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

X has a binomial distribution with $n = 20$ and $p = \frac{1}{2}$

$$\begin{aligned}\therefore \mathrm{P}(\mathrm{X}=x) &= {}^{n}\mathrm{C}_{x} q^{n-x} p^{x}, \text{ where } x = 0, 1, 2, \ldots n \\ &= {}^{20}\mathrm{C}_{x}\left(\frac{1}{2}\right)^{20-x}\cdot\left(\frac{1}{2}\right)^{x} \\ &= {}^{20}\mathrm{C}_{x}\left(\frac{1}{2}\right)^{20}\end{aligned}$$

P (at least 12 questions answered correctly) = P(X ≥ 12)

$$\begin{aligned}&= \mathrm{P}(\mathrm{X}=12)+\mathrm{P}(\mathrm{X}=13)+\ldots+\mathrm{P}(\mathrm{X}=20) \\ &= {}^{20}\mathrm{C}_{12}\left(\frac{1}{2}\right)^{20} + {}^{20}\mathrm{C}_{13}\left(\frac{1}{2}\right)^{20} + \ldots + {}^{20}\mathrm{C}_{20}\left(\frac{1}{2}\right)^{20} \\ &= \left(\frac{1}{2}\right)^{20}\cdot\left[{}^{20}\mathrm{C}_{12} + {}^{20}\mathrm{C}_{13} + \ldots + {}^{20}\mathrm{C}_{20}\right]\end{aligned}$$

Solution 8

X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{Then, } P(X = x) = {}^{n}C_{x} q^{n-x} p^{x}$$

$$= {}^{6}C_{x} \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^{x}$$

$$= {}^{6}C_{x} \left(\frac{1}{2}\right)^{6}$$

It can be seen that P(X = x) will be maximum, if ${}^{6}C_{x}$ will be maximum.

$$\text{Then, } {}^{6}C_{0} = {}^{6}C_{6} = \frac{6!}{0! \cdot 6!} = 1$$

$${}^{6}C_{1} = {}^{6}C_{5} = \frac{6!}{1! \cdot 5!} = 6$$

$${}^{6}C_{2} = {}^{6}C_{4} = \frac{6!}{2! \cdot 4!} = 15$$

$${}^{6}C_{3} = \frac{6!}{3! \cdot 3!} = 20$$

The value of ${}^{6}C_{3}$ is maximum. Therefore, for $x = 3$, P(X = x) is maximum.

Thus, X = 3 is the most likely outcome.

Solution 9

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

$$\therefore \mathrm{P}(\mathrm{X} = x) = {}^{n}\mathrm{C}_{x} q^{n-x} p^{x}$$
$$= {}^{5}\mathrm{C}_{x} \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^{x}$$

P (guessing more than 4 correct answers) = P(X ≥ 4)

$$= \mathrm{P}(\mathrm{X} = 4) + \mathrm{P}(\mathrm{X} = 5)$$
$$= {}^{5}\mathrm{C}_{4} \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^{4} + {}^{5}\mathrm{C}_{5} \left(\frac{1}{3}\right)^{5}$$
$$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243}$$
$$= \frac{10}{243} + \frac{1}{243}$$
$$= \frac{11}{243}$$

Solution 10

Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with n = 50 and $p=\frac{1}{100}$

$$\therefore q=1-p=1-\frac{1}{100}=\frac{99}{100}$$

$$\therefore \mathrm{P}(\mathrm{X}=x)={}^{n}\mathrm{C}_{x}q^{n-x}p^{x}={}^{50}\mathrm{C}_{x}\left(\frac{99}{100}\right)^{50-x}\cdot\left(\frac{1}{100}\right)^{x}$$

(a) P (winning at least once) = P (X ≥ 1)

$$=1-\mathrm{P}(\mathrm{X}<1)$$
$$=1-\mathrm{P}(\mathrm{X}=0)$$
$$=1-{}^{50}\mathrm{C}_{0}\left(\frac{99}{100}\right)^{50}$$
$$=1-1\cdot\left(\frac{99}{100}\right)^{50}$$
$$=1-\left(\frac{99}{100}\right)^{50}$$

(b) P (winning exactly once) = P(X = 1)

$$= {}^{50}C_1\left(\frac{99}{100}\right)^{49}\cdot\left(\frac{1}{100}\right)^{1}$$
$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$
$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P (at least twice) = P(X ≥ 2)

$$= 1 - P(X < 2)$$
$$= 1 - P(X \le 1)$$
$$= 1 - \left[P(X = 0) + P(X = 1)\right]$$
$$= 1 - P(X = 0) - P(X = 1)$$
$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2}\cdot\left(\frac{99}{100}\right)^{49}$$
$$= 1 - \left(\frac{99}{100}\right)^{49}\cdot\left[\frac{99}{100} + \frac{1}{2}\right]$$
$$= 1 - \left(\frac{99}{100}\right)^{49}\cdot\left(\frac{149}{100}\right)$$
$$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$$

Solution 11

The repeated tossing of a die are Bernoulli trials. Let X represent the number of times of getting 5 in 7 throws of the die.

Probability of getting 5 in a single throw of the die, $p = \frac{1}{6}$

$$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

Clearly, X has the probability distribution with $n = 7$ and $p = \frac{1}{6}$

$$\therefore P(X = x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{7}C_{x} \left(\frac{5}{6}\right)^{7-x} \cdot \left(\frac{1}{6}\right)^{x}$$

P (getting 5 exactly twice) = P(X = 2)

$$= {}^{7}C_{2} \left(\frac{5}{6}\right)^{5} \cdot \left(\frac{1}{6}\right)^{2}$$

$$= 21 \cdot \left(\frac{5}{6}\right)^{5} \cdot \frac{1}{36}$$

$$= \left(\frac{7}{12}\right)\left(\frac{5}{6}\right)^{5}$$

Solution 12

The repeated tossing of the die are Bernoulli trials. Let X represent the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die, $p=\frac{1}{6}$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has a binomial distribution with n = 6

$$\therefore \mathrm{P}(\mathrm{X}=x)={}^{n}\mathrm{C}_{x}q^{n-x}p^{x}={}^{6}\mathrm{C}_{x}\left(\frac{5}{6}\right)^{6-x}\cdot\left(\frac{1}{6}\right)^{x}$$

P (at most 2 sixes) = P(X ≤ 2)

$$=\mathrm{P}(\mathrm{X}=0)+\mathrm{P}(\mathrm{X}=1)+\mathrm{P}(\mathrm{X}=2)$$

$$={}^{6}\mathrm{C}_{0}\left(\frac{5}{6}\right)^{6}+{}^{6}\mathrm{C}_{1}\cdot\left(\frac{5}{6}\right)^{5}\cdot\left(\frac{1}{6}\right)+{}^{6}\mathrm{C}_{2}\left(\frac{5}{6}\right)^{4}\cdot\left(\frac{1}{6}\right)^{2}$$

$$=1\cdot\left(\frac{5}{6}\right)^{6}+6\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^{5}+15.\frac{1}{36}\cdot\left(\frac{5}{6}\right)^{4}$$

$$=\left(\frac{5}{6}\right)^{6}+\left(\frac{5}{6}\right)^{5}+\frac{5}{12}\cdot\left(\frac{5}{6}\right)^{4}$$

$$=\left(\frac{5}{6}\right)^{4}\left[\left(\frac{5}{6}\right)^{2}+\left(\frac{5}{6}\right)+\left(\frac{5}{12}\right)\right]$$

$$=\left(\frac{5}{6}\right)^{4}\cdot\left[\frac{25}{36}+\frac{5}{6}+\frac{5}{12}\right]$$

$$=\left(\frac{5}{6}\right)^{4}\cdot\left[\frac{25+30+15}{36}\right]$$

$$=\frac{70}{36}\cdot\left(\frac{5}{6}\right)^{4}$$

$$=\frac{35}{18}\cdot\left(\frac{5}{6}\right)^{4}$$

Solution 13

The repeated selections of articles in a random sample space are Bernoulli trails. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly, X has a binomial distribution with n = 12 and p = 10% $= \frac{10}{100} = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore \mathrm{P}(\mathrm{X} = x) = {}^{n}\mathrm{C}_{x} q^{n-x} p^{x} = {}^{12}\mathrm{C}_{x}\left(\frac{9}{10}\right)^{12-x} \cdot \left(\frac{1}{10}\right)^{x}$$

P (selecting 9 defective articles) $= {}^{12}C_{9}\left(\frac{9}{10}\right)^{3}\left(\frac{1}{10}\right)^{9}$

$$= 220 \cdot \frac{9^3}{10^3} \cdot \frac{1}{10^9}$$
$$= \frac{22 \times 9^3}{10^{11}}$$

Solution 14

The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denote the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb, $p = \frac{10}{100} = \frac{1}{10}$

$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

Clearly, X has a binomial distribution with n = 5 and $p = \frac{1}{10}$

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x \left(\frac{9}{10}\right)^{5-x} \left(\frac{1}{10}\right)^x$

P (none of the bulbs is defective) = P(X = 0)

$= {}^5C_0 \cdot \left(\frac{9}{10}\right)^5$

$= 1 \cdot \left(\frac{9}{10}\right)^5$

$= \left(\frac{9}{10}\right)^5$

The correct answer is C.

Solution 15

The repeated selection of students who are swimmers are Bernoulli trials. Let X denote the number of students, out of 5 students, who are swimmers.

Probability of students who are not swimmers, $q = \frac{1}{5}$

$$\therefore p = 1 - q = 1 - \frac{1}{5} = \frac{4}{5}$$

Clearly, X has a binomial distribution with n = 5 and $p = \frac{4}{5}$

$$P(X = x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{5}C_{x} \left(\frac{1}{5}\right)^{5-x} \cdot \left(\frac{4}{5}\right)^{x}$$

P (four students are swimmers) = P(X = 4) $= {}^{5}C_{4} \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^{4}$

Therefore, the correct answer is A.

Chapter 13 - Probability Exercise Misc. Ex.

Solution 1

It is given that, P (A) ≠ 0

(i) A is a subset of B.

$$\Rightarrow A \cap B = A$$
$$\therefore P(A \cap B) = P(B \cap A) = P(A)$$
$$\therefore P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

(ii) $A \cap B = \phi$

$$\Rightarrow P(A \cap B) = 0$$
$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

Solution 2

If a couple has two children, then the sample space is

S = {(b, b), (b, g), (g, b), (g, g)}

(i) Let E and F respectively denote the events that both children are males and at least one of the children is a male.

$$\therefore E \cap F = \{(b,\ b)\} \Rightarrow P(E \cap F) = \frac{1}{4}$$

$$P(E) = \frac{1}{4}$$

$$P(F) = \frac{3}{4}$$

$$\Rightarrow\ P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

(ii) Let A and B respectively denote the events that both children are females and the elder child is a female.

$$A = \{(g,\ g)\} \Rightarrow P(A) = \frac{1}{4}$$

$$B = \{(g,\ b),(g,\ g)\} \Rightarrow P(B) = \frac{2}{4}$$

$$A \cap B = \{(g,\ g)\} \Rightarrow P(A \cap B) = \frac{1}{4}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

Solution 3

It is given that 5% of men and 0.25% of women have grey hair.

Therefore, percentage of people with grey hair = (5 + 0.25) % = 5.25%

Probability that the selected haired person is a male $= \frac{5}{5.25} = \frac{20}{21}$

Solution 4

A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed.

$$\therefore p = \mathrm{P}(\text{right-handed}) = \frac{9}{10}$$

$$q = \mathrm{P}(\text{left-handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

= 1 − P (more than 6 are right-handed)

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

Solution 5

Total number of balls in the urn = 25

Balls bearing mark 'X' = 10

Balls bearing mark 'Y' = 15

p = P (ball bearing mark 'X') = $\frac{10}{25} = \frac{2}{5}$

q = P (ball bearing mark 'Y') = $\frac{15}{25} = \frac{3}{5}$

Six balls are drawn with replacement. Therefore, the number of trials are Bernoulli trials.

Let Z be the random variable that represents the number of balls with 'Y' mark on them in the trials.

Clearly, Z has a binomial distribution with n = 6 and p = $\frac{2}{5}$.

P (Z = z) = ${}^{n}C_{z} p^{n-z} q^{z}$

(i) P (all will bear 'X' mark) = P (Z = 0) = ${}^{6}C_{0}\left(\frac{2}{5}\right)^{6} = \left(\frac{2}{5}\right)^{6}$

(ii) P (not more than 2 bear 'Y' mark) = P (Z ≤ 2)

= P (Z = 0) + P (Z = 1) + P (Z = 2)

$$= {}^6C_0 (p)^6 (q)^0 + {}^6C_1 (p)^5 (q)^1 + {}^6C_2 (p)^4 (q)^2$$

$$= \left(\frac{2}{5}\right)^6 + 6\left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right) + 15\left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2$$

$$= \left(\frac{2}{5}\right)^4 \left[\left(\frac{2}{5}\right)^2 + 6\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + 15\left(\frac{3}{5}\right)^2\right]$$

$$= \left(\frac{2}{5}\right)^4 \left[\frac{4}{25} + \frac{36}{25} + \frac{135}{25}\right]$$

$$= \left(\frac{2}{5}\right)^4 \left[\frac{175}{25}\right]$$

$$= 7\left(\frac{2}{5}\right)^4$$

(iii) P (at least one ball bears 'Y' mark) = P (Z ≥ 1) = 1 − P (Z = 0)

$$= 1 - \left(\frac{2}{5}\right)^6$$

(iv) P (equal number of balls with 'X' mark and 'Y' mark) = P (Z = 3)

$$= {}^6C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3$$

$$= \frac{20 \times 8 \times 27}{15625}$$

$$= \frac{864}{3125}$$

Solution 6

Let p and q respectively be the probabilities that the player will clear and knock down the hurdle.

$$\therefore p = \frac{5}{6}$$

$$\Rightarrow q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$$

Let X be the random variable that represents the number of times the player will knock down the hurdle.

Therefore, by binomial distribution, we obtain

P (X = x) = ${}^{n}C_{x} p^{n-x} q^{x}$

P (player knocking down less than 2 hurdles) = P (X < 2)

= P (X = 0) + P (X = 1)

$$= {}^{10}C_{0}(q)^{0}(p)^{10} + {}^{10}C_{1}(q)(p)^{9}$$

$$= \left(\frac{5}{6}\right)^{10} + 10 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{9}$$

$$= \left(\frac{5}{6}\right)^{9}\left[\frac{5}{6} + \frac{10}{6}\right]$$

$$= \frac{5}{2}\left(\frac{5}{6}\right)^{9}$$

$$= \frac{(5)^{10}}{2 \times (6)^{9}}$$

Solution 7

The probability of getting a six in a throw of die is $\frac{1}{6}$ and not getting a six is $\frac{5}{6}$.

Let $p = \frac{1}{6}$ and $q = \frac{5}{6}$

The probability that the 2 sixes come in the first five throws of the die is

$$^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{10\times(5)^3}{(6)^5}$$

Probability that third six comes in the sixth throw = $\frac{10\times(5)^3}{(6)^5}\times\frac{1}{6}$

$$= \frac{10\times125}{(6)^6}$$

$$= \frac{10\times125}{46656}$$

$$= \frac{625}{23328}$$

Solution 8

In a leap year, there are 366 days i.e., 52 weeks and 2 days.

In 52 weeks, there are 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be

Monday and Tuesday

Tuesday and Wednesday

Wednesday and Thursday

Thursday and Friday

Friday and Saturday

Saturday and Sunday

Sunday and Monday

Total number of cases = 7

Favourable cases = 2

Probability that a leap year will have 53 Tuesdays = $\frac{2}{7}$

Solution 9

The probability of success is twice the probability of failure.

Let the probability of failure be x.

Probability of success = $2x$

$$x + 2x = 1$$
$$\Rightarrow 3x = 1$$
$$\Rightarrow x = \frac{1}{3}$$
$$\therefore 2x = \frac{2}{3}$$

Let $p = \frac{1}{3}$ and $q = \frac{2}{3}$

Let X be the random variable that represents the number of successes in six trials.

By binomial distribution, we obtain

P (X = x) = ${}^{n}C_{x} p^{n-x} q^{x}$

Probability of at least 4 successes = P (X ≥ 4)

= P (X = 4) + P (X = 5) + P (X = 6)

$$= {}^{6}C_{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{2} + {}^{6}C_{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right) + {}^{6}C_{6}\left(\frac{2}{3}\right)^{6}$$
$$= \frac{15(2)^{4}}{3^{6}} + \frac{6(2)^{5}}{3^{6}} + \frac{(2)^{6}}{3^{6}}$$
$$= \frac{(2)^{4}}{(3)^{6}}\left[15 + 12 + 4\right]$$
$$= \frac{31 \times 2^{4}}{(3)^{6}}$$
$$= \frac{31}{9}\left(\frac{2}{3}\right)^{4}$$

Solution 10

Let the man toss the coin n times. The n tosses are n Bernoulli trials.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

$\Rightarrow p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

$$\therefore P(X = x) = {}^{n}C_{x} p^{n-x} q^{x} = {}^{n}C_{x} \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^{x} = {}^{n}C_{x} \left(\frac{1}{2}\right)^{n}$$

It is given that,

P (getting at least one head) > $\frac{90}{100}$

P ($x \geq 1$) > 0.9

1 − P (x = 0) > 0.9

$$1 - {}^{n}C_{0} \cdot \frac{1}{2^{n}} > 0.9$$

$${}^{n}C_{0} \cdot \frac{1}{2^{n}} < 0.1$$

$$\frac{1}{2^{n}} < 0.1$$

$$2^{n} > \frac{1}{0.1}$$

$$2^{n} > 10 \qquad ...(1)$$

The minimum value of n that satisfies the given inequality is 4.

Thus, the man should toss the coin 4 or more than 4 times.

Solution 11

In a throw of a die, the probability of getting a six is $\frac{1}{6}$ and the probability of not getting a 6 is $\frac{5}{6}$.

Three cases can occur.

i. If he gets a six in the first throw, then the required probability is $\frac{1}{6}$.

Amount he will receive = Re 1

ii. If he does not get a six in the first throw and gets a six in the second throw, then probability $=\left(\frac{5}{6}\times\frac{1}{6}\right)=\frac{5}{36}$

Amount he will receive = −Re 1 + Re 1 = 0

iii. If he does not get a six in the first two throws and gets a six in the third throw, then probability $=\left(\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right)=\frac{25}{216}$

Amount he will receive = −Re 1 − Re 1 + Re 1 = −1

Expected value he can win $=\frac{1}{6}(1)+\left(\frac{5}{6}\times\frac{1}{6}\right)(0)+\left[\left(\frac{5}{6}\right)^2\times\frac{1}{6}\right](-1)$

$$=\frac{1}{6}-\frac{25}{216}$$

$$=\frac{36-25}{216}=\frac{11}{216}$$

Solution 12

Let R be the event of drawing the red marble.

Let E_A, E_B, and E_C respectively denote the events of selecting the box A, B, and C.

Total number of marbles = 40

Number of red marbles = 15

$$\therefore P(R)=\frac{15}{40}=\frac{3}{8}$$

Probability of drawing the red marble from box A is given by P (E_A|R).

$$\therefore P(E_A|R)=\frac{P(E_A\cap R)}{P(R)}=\frac{\frac{1}{40}}{\frac{3}{8}}=\frac{1}{15}$$

Probability that the red marble is from box B is P (E_B|R).

$$\Rightarrow P(E_B|R)=\frac{P(E_B\cap R)}{P(R)}=\frac{\frac{6}{40}}{\frac{3}{8}}=\frac{2}{5}$$

Probability that the red marble is from box C is P (E_C|R).

$$\Rightarrow P(E_C|R)=\frac{P(E_C\cap R)}{P(R)}=\frac{\frac{8}{40}}{\frac{3}{8}}=\frac{8}{15}$$

Solution 13

Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$$\therefore P(A)=0.40$$

$$P(E_1)=P(E_2)=\frac{1}{2}$$

Probability of having heart attack if he is treated with mediation then mediation reduce the risk by 30%.
Hence there is a risk of 70%.

$P(A|E_1) = 0.40 \times 0.70 = 0.28$
Probability of having heart attack if he is treated with drugs then mediation reduce the risk by 25%.
Hence there is a risk of 75%.
$P(A|E_2) = 0.40 \times 0.75 = 0.30$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$$

$$= \frac{14}{29}$$

Solution 14

The total number of determinants of second order with each element being 0 or 1 is $(2)^4 = 16$

The value of determinant is positive in the following cases. $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$ $\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$\Rightarrow$ Required probability $= \frac{3}{16}$

Solution 15

Let the event in which A fails and B fails be denoted by E_A and E_B.

$P(E_A) = 0.2$

$P(E_A \cap E_B) = 0.15$

$P(\text{B fails alone}) = P(E_B) - P(E_A \cap E_B)$

$0.15 = P(E_B) - 0.15$

$P(E_B) = 0.3$

(i) $P(E_A|E_B) = \frac{P(E_A \cap E_B)}{P(E_B)} = \frac{0.15}{0.3} = 0.5$

(ii) $P(\text{A fails alone}) = P(E_A) - P(E_A \cap E_B)$

$= 0.2 - 0.15$

$= 0.05$

Solution 16

Let E_1 and E_2 respectively denote the events that a red ball is transferred from bag I to II and a black ball is transferred from bag I to II.

$P(E_1) = \frac{3}{7}$ and $P(E_2) = \frac{4}{7}$

Let A be the event that the ball drawn is red.

When a red ball is transferred from bag I to II,

$P(A|E_1) = \frac{5}{10} = \frac{1}{2}$

When a black ball is transferred from bag I to II,

$P(A|E_2) = \frac{4}{10} = \frac{2}{5}$

$$\therefore P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{4}{7} \times \frac{2}{5}}{\frac{3}{7} \times \frac{1}{2} + \frac{4}{7} \times \frac{2}{5}}$$

$$= \frac{16}{31}$$

Solution 17

P (A) ≠ 0 and $P(B|A)=1$

$$P(B|A)=\frac{P(B\cap A)}{P(A)}$$

$$1=\frac{P(B\cap A)}{P(A)}$$

$$P(A)=P(B\cap A)$$

$$\Rightarrow A\subset B$$

Thus, the correct answer is A.

Solution 18

$$P(A|B)>P(A)$$

$$\Rightarrow \frac{P(A\cap B)}{P(B)}>P(A)$$

$$\Rightarrow P(A\cap B)>P(A)\cdot P(B)$$

$$\Rightarrow \frac{P(A\cap B)}{P(A)}>P(B)$$

$$\Rightarrow P(B|A)>P(B)$$

Thus, the correct answer is C.

Solution 19

$$P(A)+P(B)-P(A\text{ and }B)=P(A)$$

$$\Rightarrow P(A)+P(B)-P(A\cap B)=P(A)$$

$$\Rightarrow P(B)-P(A\cap B)=0$$

$$\Rightarrow P(A\cap B)=P(B)$$

$$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B)}{P(B)}=1$$

Thus, the correct answer is B.