

Access answers to RD Sharma Solutions for Class 11 Maths Chapter 32
– Statistics

EXERCISE 32.1 PAGE NO: 32.6

1. Calculate the mean deviation about the median of the following observation :

(i) 3011, 2780, 3020, 2354, 3541, 4150, 5000

(ii) 38, 70, 48, 34, 42, 55, 63, 46, 54, 44

(iii) 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

(iv) 22, 24, 30, 27, 29, 31, 25, 28, 41, 42

(v) 38, 70, 48, 34, 63, 42, 55, 44, 53, 47

Solution:

(i) 3011, 2780, 3020, 2354, 3541, 4150, 5000

To calculate the Median (M), let us arrange the numbers in ascending order.

Median is the middle number of all the observation.

2354, 2780, 3011, 3020, 3541, 4150, 5000

So, Median = 3020 and $n = 7$

By using the formula to calculate Mean Deviation,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

| x_i | $ d_i = x_i - 3020 $ |
|-------|------------------------|
| 3011 | 9 |
| 2780 | 240 |
| 3020 | 0 |
| 2354 | 666 |
| 3541 | 521 |
| 4150 | 1130 |
| 5000 | 1980 |
| Total | 4546 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= 1/7 \times 4546$$

$$= 649.42$$

∴ The Mean Deviation is 649.42.

(ii) 38, 70, 48, 34, 42, 55, 63, 46, 54, 44

To calculate the Median (M), let us arrange the numbers in ascending order.

Median is the middle number of all the observation.

34, 38, 42, 44, 46, 48, 54, 55, 63, 70

Here the Number of observations are Even then Median = $(46+48)/2 = 47$

Median = 47 and n = 10

By using the formula to calculate Mean Deviation,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

| x_i | $ d_i = x_i - 47 $ |
|-------|----------------------|
| 38 | 9 |
| 70 | 23 |
| 48 | 1 |
| 34 | 13 |
| 42 | 5 |
| 55 | 8 |
| 63 | 16 |
| 46 | 1 |
| 54 | 7 |
| 44 | 3 |
| Total | 86 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= 1/10 \times 86$$

$$= 8.6$$

∴ The Mean Deviation is 8.6.

(iii) 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

To calculate the Median (M), let us arrange the numbers in ascending order.

Median is the middle number of all the observation.

30, 34, 38, 40, 42, 44, 50, 51, 60, 66

Here the Number of observations are Even then Median = $(42+44)/2 = 43$

Median = 43 and $n = 10$

By using the formula to calculate Mean Deviation,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

| x_i | $ d_i = x_i - 43 $ |
|-------|----------------------|
| 30 | 13 |
| 34 | 9 |
| 38 | 5 |
| 40 | 3 |
| 42 | 1 |
| 44 | 1 |
| 50 | 7 |
| 51 | 8 |
| 60 | 17 |
| 66 | 23 |
| Total | 87 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 87$$

$$= 8.7$$

\therefore The Mean Deviation is 8.7.

(iv) 22, 24, 30, 27, 29, 31, 25, 28, 41, 42

To calculate the Median (M), let us arrange the numbers in ascending order.

Median is the middle number of all the observation.

22, 24, 25, 27, 28, 29, 30, 31, 41, 42

Here the Number of observations are Even then Median = $(28+29)/2 = 28.5$

Median = 28.5 and $n = 10$

By using the formula to calculate Mean Deviation,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

| x_i | $ d_i = x_i - 28.5 $ |
|-------|------------------------|
| 22 | 6.5 |
| 24 | 4.5 |
| 30 | 1.5 |
| 27 | 1.5 |
| 29 | 0.5 |
| 31 | 2.5 |
| 25 | 3.5 |
| 28 | 0.5 |
| 41 | 12.5 |
| 42 | 13.5 |
| Total | 47 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 47$$

$$= 4.7$$

\therefore The Mean Deviation is 4.7.

(v) 38, 70, 48, 34, 63, 42, 55, 44, 53, 47

To calculate the Median (M), let us arrange the numbers in ascending order.

Median is the middle number of all the observation.

34, 38, 43, 44, 47, 48, 53, 55, 63, 70

Here the Number of observations are Even then Median = $(47+48)/2 = 47.5$

Median = 47.5 and $n = 10$

By using the formula to calculate Mean Deviation,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

| x_i | $ d_i = x_i - 47.5 $ |
|-------|------------------------|
| 38 | 9.5 |
| 70 | 22.5 |
| 48 | 0.5 |
| 34 | 13.5 |
| 63 | 15.5 |
| 42 | 5.5 |
| 55 | 7.5 |
| 44 | 3.5 |
| 53 | 5.5 |
| 47 | 0.5 |
| Total | 84 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 84$$

$$= 8.4$$

\therefore The Mean Deviation is 8.4.

2. Calculate the mean deviation from the mean for the following data :

(i) 4, 7, 8, 9, 10, 12, 13, 17

(ii) 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

(iii) 38, 70, 48, 40, 42, 55, 63, 46, 54, 44

(iv) 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

(v) 57, 64, 43, 67, 49, 59, 44, 47, 61, 59

Solution:

(i) 4, 7, 8, 9, 10, 12, 13, 17

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - x|$$

So, let 'x' be the mean of the given observation.

$$x = [4 + 7 + 8 + 9 + 10 + 12 + 13 + 17]/8$$

$$= 80/8$$

$$= 10$$

Number of observations, 'n' = 8

| x_i | $ d_i = x_i - 10 $ |
|-------|----------------------|
| 4 | 6 |
| 7 | 3 |
| 8 | 2 |
| 9 | 1 |
| 10 | 0 |
| 12 | 2 |
| 13 | 3 |
| 17 | 7 |
| Total | 24 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{8} \times 24$$

$$= 3$$

∴ The Mean Deviation is 3.

(ii) 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - x|$$

So, let 'x' be the mean of the given observation.

$$x = [13 + 17 + 16 + 14 + 11 + 13 + 10 + 16 + 11 + 18 + 12 + 17]/12$$

$$= 168/12$$

$$= 14$$

Number of observations, 'n' = 12

| x_i | $ d_i = x_i - 14 $ |
|-------|----------------------|
| 13 | 1 |
| 17 | 3 |
| 16 | 2 |
| 14 | 0 |
| 11 | 3 |
| 13 | 1 |
| 10 | 4 |
| 16 | 2 |
| 11 | 3 |
| 18 | 4 |
| 12 | 2 |
| 17 | 3 |
| Total | 28 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{12} \times 28$$

$$= 2.33$$

∴ The Mean Deviation is 2.33.

(iii) 38, 70, 48, 40, 42, 55, 63, 46, 54, 44

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - x|$$

So, let 'x' be the mean of the given observation.

$$x = [38 + 70 + 48 + 40 + 42 + 55 + 63 + 46 + 54 + 44]/10$$

$$= 500/10$$

$$= 50$$

Number of observations, 'n' = 10

| x_i | $ d_i = x_i - 50 $ |
|-------|----------------------|
| 38 | 12 |
| 70 | 20 |
| 48 | 2 |
| 40 | 10 |
| 42 | 8 |
| 55 | 5 |
| 63 | 13 |
| 46 | 4 |
| 54 | 4 |
| 44 | 6 |
| Total | 84 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 84$$

$$= 8.4$$

\therefore The Mean Deviation is 8.4.

(iv) 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - x|$$

So, let 'x' be the mean of the given observation.

$$x = [36 + 72 + 46 + 42 + 60 + 45 + 53 + 46 + 51 + 49]/10$$

$$= 500/10$$

$$= 50$$

Number of observations, 'n' = 10

| x_i | $ d_i = x_i - 50 $ |
|-------|----------------------|
| 36 | 14 |
| 72 | 22 |
| 46 | 4 |
| 42 | 8 |
| 60 | 10 |
| 45 | 5 |
| 53 | 3 |
| 46 | 4 |
| 51 | 1 |
| 49 | 1 |
| Total | 72 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 72$$

$$= 7.2$$

∴ The Mean Deviation is 7.2.

(v) 57, 64, 43, 67, 49, 59, 44, 47, 61, 59

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

Where, $|d_i| = |x_i - x|$

So, let 'x' be the mean of the given observation.

$$x = \frac{[57 + 64 + 43 + 67 + 49 + 59 + 44 + 47 + 61 + 59]}{10}$$

$$= \frac{550}{10}$$

$$= 55$$

Number of observations, 'n' = 10

| x_i | $ d_i = x_i - 55 $ |
|-------|----------------------|
| 57 | 2 |

| | |
|-------|----|
| 64 | 9 |
| 43 | 12 |
| 67 | 12 |
| 49 | 6 |
| 59 | 4 |
| 44 | 11 |
| 47 | 8 |
| 61 | 6 |
| 59 | 4 |
| Total | 74 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 74$$

$$= 7.4$$

∴ The Mean Deviation is 7.4.

3. Calculate the mean deviation of the following income groups of five and seven members from their medians:

| I Income in ₹ | II Income in ₹ |
|--------------------------------|---------------------------------|
| 4000 | 3800 |
| 4200 | 4000 |
| 4400 | 4200 |
| 4600 | 4400 |
| 4800 | 4600 |
| | 4800 |
| | 5800 |

Solution:

Let us calculate the mean deviation for the first data set.

Since the data is arranged in ascending order,

4000, 4200, 4400, 4600, 4800

Median = 4400

Total observations = 5

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - M|$$

| x_i | $ d_i = x_i - 4400 $ |
|-------|------------------------|
| 4000 | 400 |
| 4200 | 200 |
| 4400 | 0 |
| 4600 | 200 |
| 4800 | 400 |
| Total | 1200 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{5} \times 1200$$

$$= 240$$

Let us calculate the mean deviation for the second data set.

Since the data is arranged in ascending order,

3800, 4000, 4200, 4400, 4600, 4800, 5800

Median = 4400

Total observations = 7

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - M|$$

| x_i | $ d_i = x_i - 4400 $ |
|-------|------------------------|
| 3800 | 600 |
| 4000 | 400 |

| | |
|-------|------|
| 4200 | 200 |
| 4400 | 0 |
| 4600 | 200 |
| 4800 | 400 |
| 5800 | 1400 |
| Total | 3200 |

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 &= \frac{1}{7} \times 3200 \\
 &= 457.14
 \end{aligned}$$

∴ The Mean Deviation of set 1 is 240 and set 2 is 457.14

4. The lengths (in cm) of 10 rods in a shop are given below:

40.0, 52.3, 55.2, 72.9, 52.8, 79.0, 32.5, 15.2, 27.9, 30.2

(i) Find the mean deviation from the median.

(ii) Find the mean deviation from the mean also.

Solution:

(i) Find the mean deviation from the median

Let us arrange the data in ascending order,

15.2, 27.9, 30.2, 32.5, 40.0, 52.3, 52.8, 55.2, 72.9, 79.0

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - M|$$

The number of observations are Even then Median = $(40+52.3)/2 = 46.15$

Median = 46.15

Number of observations, 'n' = 10

| | |
|-------|-------------------------|
| x_i | $ d_i = x_i - 46.15 $ |
| 40.0 | 6.15 |
| 52.3 | 6.15 |
| 55.2 | 9.05 |

| | |
|-------|-------|
| 72.9 | 26.75 |
| 52.8 | 6.65 |
| 79.0 | 32.85 |
| 32.5 | 13.65 |
| 15.2 | 30.95 |
| 27.9 | 19.25 |
| 30.2 | 15.95 |
| Total | 167.4 |

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 &= \frac{1}{10} \times 167.4 \\
 &= 16.74
 \end{aligned}$$

∴ The Mean Deviation is 16.74.

(ii) Find the mean deviation from the mean also.

We know that,

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 \text{Where, } |d_i| &= |x_i - \bar{x}|
 \end{aligned}$$

So, let 'x' be the mean of the given observation.

$$\begin{aligned}
 \bar{x} &= [40.0 + 52.3 + 55.2 + 72.9 + 52.8 + 79.0 + 32.5 + 15.2 + 27.9 + 30.2] / 10 \\
 &= 458 / 10 \\
 &= 45.8
 \end{aligned}$$

Number of observations, 'n' = 10

| x_i | $ d_i = x_i - 45.8 $ |
|-------|------------------------|
| 40.0 | 5.8 |
| 52.3 | 6.5 |
| 55.2 | 9.4 |
| 72.9 | 27.1 |

| | |
|-------|-------|
| 52.8 | 7 |
| 79.0 | 33.2 |
| 32.5 | 13.3 |
| 15.2 | 30.6 |
| 27.9 | 17.9 |
| 30.2 | 15.6 |
| Total | 166.4 |

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 &= \frac{1}{10} \times 166.4 \\
 &= 16.64
 \end{aligned}$$

∴ The Mean Deviation is 16.64

5. In question 1(iii), (iv), (v) find the number of observations lying between $\bar{X} - M.D$ and $\bar{X} + M.D$, where M.D. is the mean deviation from the mean.

Solution:

(iii) 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

We know that,

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 \text{Where, } |d_i| &= |x_i - \bar{x}|
 \end{aligned}$$

So, let 'x' be the mean of the given observation.

$$\begin{aligned}
 x &= [34 + 66 + 30 + 38 + 44 + 50 + 40 + 60 + 42 + 51]/10 \\
 &= 455/10 \\
 &= 45.5
 \end{aligned}$$

Number of observations, 'n' = 10

| x_i | $ d_i = x_i - 45.5 $ |
|-------|------------------------|
| 34 | 11.5 |
| 66 | 20.5 |
| 30 | 15.5 |
| 38 | 7.5 |

| | |
|-------|------|
| 44 | 1.5 |
| 50 | 4.5 |
| 40 | 5.5 |
| 60 | 14.5 |
| 42 | 3.5 |
| 51 | 5.5 |
| Total | 90 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 90$$

$$= 9$$

Now

$$X^{----} - M.D = 45.5 - 9 = 36.5$$

$$X^{----} + M.D = 45.5 + 9 = 54.5$$

So, There are total 6 observation between $X^{----} - M.D$ and $X^{----} + M.D$
(iv) 22, 24, 30, 27, 29, 31, 25, 28, 41, 42

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - x|$$

So, let 'x' be the mean of the given observation.

$$x = \frac{[22 + 24 + 30 + 27 + 29 + 31 + 25 + 28 + 41 + 42]}{10}$$

$$= \frac{299}{10}$$

$$= 29.9$$

Number of observations, 'n' = 10

| x_i | $ d_i = x_i - 29.9 $ |
|-------|------------------------|
| 22 | 7.9 |
| 24 | 5.9 |
| 30 | 0.1 |
| 27 | 2.9 |

| | |
|-------|------|
| 29 | 0.9 |
| 31 | 1.1 |
| 25 | 4.9 |
| 28 | 1.9 |
| 41 | 11.1 |
| 42 | 12.1 |
| Total | 48.8 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{10} \times 48.8$$

$$= 4.88$$

Now

$$\bar{X} - M.D. = 29.9 - 4.88 = 25.02$$

$$\bar{X} + M.D. = 29.9 + 4.88 = 34.78$$

So, There are total 5 observation between $\bar{X} - M.D.$ and $\bar{X} + M.D.$

(v) 38, 70, 48, 34, 63, 42, 55, 44, 53, 47

We know that,

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$\text{Where, } |d_i| = |x_i - \bar{x}|$$

So, let 'x' be the mean of the given observation.

$$x = \frac{[38 + 70 + 48 + 34 + 63 + 42 + 55 + 44 + 53 + 47]}{10}$$

$$= \frac{494}{10}$$

$$= 49.4$$

Number of observations, 'n' = 10

| | |
|-------|------------------------|
| x_i | $ d_i = x_i - 49.4 $ |
| 38 | 11.4 |
| 70 | 20.6 |

| | |
|-------|------|
| 48 | 1.4 |
| 34 | 15.4 |
| 63 | 13.6 |
| 42 | 7.4 |
| 55 | 5.6 |
| 44 | 5.4 |
| 53 | 3.6 |
| 47 | 2.4 |
| Total | 86.8 |

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 &= \frac{1}{10} \times 86.8 \\
 &= 8.68
 \end{aligned}$$

Now

$$\bar{X} - M.D. = 49.4 - 8.68 = 40.72$$

$$\bar{X} + M.D. = 49.4 + 8.68 = 58.08$$

So, There are total 6 observation between $\bar{X} - M.D.$ and $\bar{X} + M.D.$

EXERCISE 32.2 PAGE NO: 32.11

1. Calculate the mean deviation from the median of the following frequency distribution:

| | | | | | | | | | |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Heights in inches | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| No. of students | 15 | 20 | 32 | 35 | 35 | 22 | 20 | 10 | 8 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

We know, Median is the Middle term,

So, Median = 61

Let x_i = Heights in inches

And, f_i = Number of students

| x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ $= x_i - 61 $ | $f_i d_i $ |
|-------|---------|----------------------|---------------------------------------|-------------|
| 58 | 15 | 15 | 3 | 45 |
| 59 | 20 | 35 | 2 | 40 |
| 60 | 32 | 67 | 1 | 32 |
| 61 | 35 | 102 | 0 | 0 |
| 62 | 35 | 137 | 1 | 35 |
| 63 | 22 | 159 | 2 | 44 |
| 64 | 20 | 179 | 3 | 60 |
| 65 | 10 | 189 | 4 | 40 |
| 66 | 8 | 197 | 5 | 40 |
| | N = 197 | | | Total = 336 |

N=197

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 &= \frac{1}{197} \times 336 \\
 &= 1.70
 \end{aligned}$$

∴ The mean deviation is 1.70.

2. The number of telephone calls received at an exchange in 245 successive 2-minute intervals is shown in the following frequency distribution:

| Number of calls | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----------------|----|----|----|----|----|----|----|----|
| Frequency | 14 | 21 | 25 | 43 | 51 | 40 | 39 | 12 |

Compute the mean deviation about the median.

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

We know, Median is the even term, $(3+5)/2 = 4$

So, Median = 8

Let x_i = Number of calls

And, f_i = Frequency

| x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ $= x_i - 61 $ | $f_i d_i $ |
|-------|-------------|----------------------|---------------------------------------|-------------|
| 0 | 14 | 14 | 4 | 56 |
| 1 | 21 | 35 | 3 | 63 |
| 2 | 25 | 60 | 2 | 50 |
| 3 | 43 | 103 | 1 | 43 |
| 4 | 51 | 154 | 0 | 0 |
| 5 | 40 | 194 | 1 | 40 |
| 6 | 39 | 233 | 2 | 78 |
| 7 | 12 | 245 | 3 | 36 |
| | | | | Total = 366 |
| | Total = 245 | | | |

$N = 245$

$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$

$= \frac{1}{245} \times 366$

$= 1.49$

\therefore The mean deviation is 1.49.

3. Calculate the mean deviation about the median of the following frequency distribution:

| | | | | | | | |
|-------|---|---|---|----|----|----|----|
| x_i | 5 | 7 | 9 | 11 | 13 | 15 | 17 |
| f_i | 2 | 4 | 6 | 8 | 10 | 12 | 8 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

We know, $N = 50$

Median = $(50)/2 = 25$

So, the median Corresponding to 25 is 13

| x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ $= x_i - 61 $ | $f_i d_i $ |
|-------|------------|----------------------|---------------------------------------|-------------|
| 5 | 2 | 2 | 8 | 16 |
| 7 | 4 | 6 | 6 | 24 |
| 9 | 6 | 12 | 4 | 24 |
| 11 | 8 | 20 | 2 | 16 |
| 13 | 10 | 30 | 0 | 0 |
| 15 | 12 | 42 | 2 | 24 |
| 17 | 8 | 50 | 4 | 32 |
| | Total = 50 | | | Total = 136 |

$N = 50$

$$MD = \frac{1}{N} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{50} \times 136$$

$$= 2.72$$

\therefore The mean deviation is 2.72.

4. Find the mean deviation from the mean for the following data:

(i)

| | | | | | | |
|-------|---|---|---|----|----|----|
| x_i | 5 | 7 | 9 | 10 | 12 | 15 |
| f_i | 8 | 6 | 2 | 2 | 2 | 6 |

Solution:

To find the mean deviation from the mean, firstly let us calculate the mean.

By using the formula,

$$Mean = \frac{\sum f_i x_i}{f_i}$$

| x_i | f_i | Cumulative Frequency ($x_i f_i$) | $ d_i = x_i - \text{Mean} $ | $f_i d_i $ |
|-------|---------------|---------------------------------------|-------------------------------|---------------|
| 5 | 8 | 40 | 4 | 32 |
| 7 | 6 | 42 | 2 | 12 |
| 9 | 2 | 18 | 0 | 0 |
| 10 | 2 | 20 | 1 | 2 |
| 12 | 2 | 24 | 3 | 6 |
| 15 | 6 | 90 | 6 | 36 |
| | Total = 26 | Total = 234 | | Total = 88 |

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

$$= 234/26$$

$$= 9$$

$$\text{Mean deviation} = \frac{\sum f_i |d_i|}{f_i}$$

$$= 88/26$$

$$= 3.3$$

\therefore The mean deviation is 3.3

(ii)

| x_i | 5 | 10 | 15 | 20 | 25 |
|-------|---|----|----|----|----|
| f_i | 7 | 4 | 6 | 3 | 5 |

Solution:

To find the mean deviation from the mean, firstly let us calculate the mean.

By using the formula,

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

| x_i | f_i | Cumulative Frequency ($x_i f_i$) | $ d_i = x_i - \text{Mean} $ | $f_i d_i $ |
|-------|---------------|---------------------------------------|-------------------------------|----------------|
| 5 | 7 | 35 | 9 | 63 |
| 10 | 4 | 40 | 4 | 16 |
| 15 | 6 | 90 | 1 | 6 |
| 20 | 3 | 60 | 6 | 18 |
| 25 | 5 | 125 | 11 | 55 |
| | | | | |
| | Total = 25 | Total = 350 | | Total = 158 |

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

$$= 350/25$$

$$= 14$$

$$\text{Mean deviation} = \frac{\sum f_i |d_i|}{f_i}$$

$$= 158/25$$

$$= 6.32$$

\therefore The mean deviation is 6.32

(iii)

| x_i | 10 | 30 | 50 | 70 | 90 |
|-------|----|----|----|----|----|
| f_i | 4 | 24 | 28 | 16 | 8 |

Solution:

To find the mean deviation from the mean, firstly let us calculate the mean.

By using the formula,

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

| x_i | f_i | Cumulative Frequency | $ d_i = x_i -$ | $f_i d_i $ |
|-------|-------|----------------------|------------------|-------------|
|-------|-------|----------------------|------------------|-------------|

| | | $(x_i f_i)$ | Mean | |
|----|------------|--------------|------|--------------|
| 10 | 4 | 40 | 40 | 160 |
| 30 | 24 | 720 | 20 | 480 |
| 50 | 28 | 1400 | 0 | 0 |
| 70 | 16 | 1120 | 20 | 320 |
| 90 | 8 | 720 | 40 | 320 |
| | | | | |
| | Total = 80 | Total = 4000 | | Total = 1280 |

$$Mean = \frac{\sum f_i x_i}{f_i}$$

$$= 4000/80$$

$$= 50$$

$$Mean deviation = \frac{\sum f_i |d_i|}{f_i}$$

$$= 1280/80$$

$$= 16$$

∴ The mean deviation is 16

5. Find the mean deviation from the median for the following data :

(i)

| | | | | |
|-------------------------|-----------|-----------|-----------|-----------|
| x_i | 15 | 21 | 27 | 30 |
| f_i | 3 | 5 | 6 | 7 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

We know, $N = 21$

$$Median = (21)/2 = 10.5$$

So, the median Corresponding to 10.5 is 27

| x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|-------|------------|----------------------|---------------------|-------------|
| 15 | 3 | 3 | 15 | 45 |
| 21 | 5 | 8 | 9 | 45 |
| 27 | 6 | 14 | 3 | 18 |
| 30 | 7 | 21 | 0 | 0 |
| | | | | |
| | Total = 21 | Total = 46 | | Total = 108 |

$$N = 21$$

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{21} \times 108$$

$$= 5.14$$

∴ The mean deviation is 5.14

(ii)

| x_i | 74 | 89 | 42 | 54 | 91 | 94 | 35 |
|-------|----|----|----|----|----|----|----|
| f_i | 20 | 12 | 2 | 4 | 5 | 3 | 4 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

We know, $N = 50$

$$\text{Median} = \frac{(50)}{2} = 25$$

So, the median Corresponding to 25 is 74

| x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|-------|-------|----------------------|---------------------|-------------|
| 74 | 20 | 4 | 39 | 156 |
| 89 | 12 | 6 | 32 | 64 |
| 42 | 2 | 10 | 20 | 80 |
| 54 | 4 | 30 | 0 | 0 |
| 91 | 5 | 42 | 15 | 180 |

| | | | | |
|----|------------|-------------|----|-------------|
| 94 | 3 | 47 | 17 | 85 |
| 35 | 4 | 50 | 20 | 60 |
| | Total = 50 | Total = 189 | | Total = 625 |

$$N = 50$$

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{50} \times 625$$

$$= 12.5$$

∴ The mean deviation is 12.5

(iii)

| | | | | | |
|------------------------|-----------|-----------|-----------|-----------|-----------|
| Marks obtained | 10 | 11 | 12 | 14 | 15 |
| No. of students | 2 | 3 | 8 | 3 | 4 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

We know, $N = 20$

$$\text{Median} = \frac{(20)}{2} = 10$$

So, the median Corresponding to 10 is 12

| x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|-------|------------|----------------------|---------------------|-------------|
| 10 | 2 | 2 | 2 | 4 |
| 11 | 3 | 5 | 1 | 3 |
| 12 | 8 | 13 | 0 | 0 |
| 14 | 3 | 16 | 2 | 6 |
| 15 | 4 | 20 | 3 | 12 |
| | Total = 20 | | | Total = 25 |

$$N = 20$$

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{20} \times 25$$

$$= 1.25$$

∴ The mean deviation is 1.25

EXERCISE 32.3 PAGE NO: 32.16

1. Compute the mean deviation from the median of the following distribution:

| Class | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|-----------|------|-------|-------|-------|-------|
| Frequency | 5 | 10 | 20 | 5 | 10 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

Median is the middle term of the X_i ,

Here, the middle term is 25

So, Median = 25

| Class Interval | x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|----------------|-------|------------|----------------------|---------------------|-------------|
| 0-10 | 5 | 5 | 5 | 20 | 100 |
| 10-20 | 15 | 10 | 15 | 10 | 100 |
| 20-30 | 25 | 20 | 35 | 0 | 0 |
| 30-40 | 35 | 5 | 91 | 10 | 50 |
| 40-50 | 45 | 10 | 101 | 20 | 200 |
| | | Total = 50 | | | Total = 450 |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{50} \times 450$$

$$= 9$$

∴ The mean deviation is 9

2. Find the mean deviation from the mean for the following data:

(i)

| Classes | 0-100 | 100-200 | 200-300 | 300-400 | 400-500 | 500-600 | 600-700 | 700-800 |
|-------------|-------|---------|---------|---------|---------|---------|---------|---------|
| Frequencies | 4 | 8 | 9 | 10 | 7 | 5 | 4 | 3 |

Solution:

To find the mean deviation from the mean, firstly let us calculate the mean.

By using the formula,

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

$$= 17900/50$$

$$= 358$$

| Class Interval | x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|----------------|-------|------------|----------------------|---------------------|--------------|
| 0-100 | 50 | 4 | 200 | 308 | 1232 |
| 100-200 | 150 | 8 | 1200 | 208 | 1664 |
| 200-300 | 250 | 9 | 2250 | 108 | 972 |
| 300-400 | 350 | 10 | 3500 | 8 | 80 |
| 400-500 | 450 | 7 | 3150 | 92 | 644 |
| 500-600 | 550 | 5 | 2750 | 192 | 960 |
| 600-700 | 650 | 4 | 2600 | 292 | 1168 |
| 700-800 | 750 | 3 | 2250 | 392 | 1176 |
| | | Total = 50 | Total = 17900 | | Total = 7896 |

$$N = 50$$

$$MD = \frac{1}{N} \sum_{i=1}^n |d_i|$$

$$= 1/50 \times 7896$$

$$= 157.92$$

∴ The mean deviation is 157.92

(ii)

| Classes | 95-105 | 105-115 | 115-125 | 125-135 | 135-145 | 145-155 |
|-------------|--------|---------|---------|---------|---------|---------|
| Frequencies | 9 | 13 | 16 | 26 | 30 | 12 |

Solution:

To find the mean deviation from the mean, firstly let us calculate the mean.

By using the formula,

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

$$= 13630/106$$

$$= 128.58$$

| Class Interval | x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|----------------|-------|---------|----------------------|---------------------|----------------|
| 95-105 | 100 | 9 | 900 | 28.58 | 257.22 |
| 105-115 | 110 | 13 | 1430 | 18.58 | 241.54 |
| 115-125 | 120 | 16 | 1920 | 8.58 | 137.28 |
| 125-135 | 130 | 26 | 3380 | 1.42 | 36.92 |
| 135-145 | 140 | 30 | 4200 | 11.42 | 342.6 |
| 145-155 | 150 | 12 | 1800 | 21.42 | 257.04 |
| | | N = 106 | Total = 13630 | | Total = 1272.6 |

$$N = 106$$

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{106} \times 1272.6$$

$$= 12.005$$

∴ The mean deviation is 12.005

3. Compute mean deviation from mean of the following distribution:

| Marks | 10- | 20- | 30- | 40- | 50- | 60- | 70- | 80- |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|
|-------|-----|-----|-----|-----|-----|-----|-----|-----|

| | | | | | | | | |
|------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| No. of students | 8 | 10 | 15 | 25 | 20 | 18 | 9 | 5 |

Solution:

To find the mean deviation from the mean, firstly let us calculate the mean.

By using the formula,

$$\text{Mean} = \frac{\sum f_i x_i}{f_i}$$

$$= 5390/110$$

$$= 49$$

| Class Interval | x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|----------------|-------|---------|----------------------|---------------------|--------------|
| 10-20 | 15 | 8 | 120 | 34 | 272 |
| 20-30 | 25 | 10 | 250 | 24 | 240 |
| 30-40 | 35 | 15 | 525 | 14 | 210 |
| 40-50 | 45 | 25 | 1125 | 4 | 100 |
| 50-60 | 55 | 20 | 1100 | 6 | 120 |
| 60-70 | 65 | 18 | 1170 | 16 | 288 |
| 70-80 | 75 | 9 | 675 | 26 | 234 |
| 80-90 | 85 | 5 | 425 | 36 | 180 |
| | | N = 110 | Total = 5390 | | Total = 1644 |

$$N = 110$$

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{1}{110} \times 1644$$

$$= 14.94$$

∴ The mean deviation is 14.94

4. The age distribution of 100 life-insurance policy holders is as follows:

| Age (on nearest birthday) | 17-19.5 | 20-25.5 | 26-35.5 | 36-40.5 | 41-50.5 | 51-55.5 | 56-60.5 | 61-70.5 |
|---------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| No. of persons | 5 | 16 | 12 | 26 | 14 | 12 | 6 | 5 |

Calculate the mean deviation from the median age.

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

$$N = 96$$

$$\text{So, } N/2 = 96/2 = 48$$

The cumulative frequency just greater than 48 is 59, and the corresponding value of x is 38.25

$$\text{So, Median} = 38.25$$

| Class Interval | x_i | f_i | Cumulative Frequency | $ d_i = x_i - M $ | $f_i d_i $ |
|----------------|-------|------------|----------------------|---------------------|---------------|
| 17-19.5 | 18.25 | 5 | 5 | 20 | 100 |
| 20-25.5 | 22.75 | 16 | 21 | 15.5 | 248 |
| 26-35.5 | 30.75 | 12 | 33 | 7.5 | 90 |
| 36-40.5 | 38.25 | 26 | 59 | 0 | 0 |
| 41-50.5 | 45.75 | 14 | 73 | 7.5 | 105 |
| 51-55.5 | 53.25 | 12 | 85 | 15 | 180 |
| 56-60.5 | 58.25 | 6 | 91 | 20 | 120 |
| 61-70.5 | 65.75 | 5 | 96 | 27.5 | 137.5 |
| | | Total = 96 | | | Total = 980.5 |

$$N = 96$$

$$\begin{aligned}
 MD &= \frac{1}{n} \sum_{i=1}^n |d_i| \\
 &= \frac{1}{96} \times 980.5 \\
 &= 10.21
 \end{aligned}$$

∴ The mean deviation is 10.21

5. Find the mean deviation from the mean and from a median of the following distribution:

| Marks | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|-----------------|------|-------|-------|-------|-------|
| No. of students | 5 | 8 | 15 | 16 | 6 |

Solution:

To find the mean deviation from the median, firstly let us calculate the median.

$$N = 50$$

$$\text{So, } N/2 = 50/2 = 25$$

The cumulative frequency just greater than 25 is 28, and the corresponding value of x is 28

$$\text{So, Median} = 28$$

By using the formula to calculate Mean,

$$\begin{aligned}
 \text{Mean} &= \frac{\sum f_i x_i}{\sum f_i} \\
 &= 1350/50 \\
 &= 27
 \end{aligned}$$

| Class Interval | x_i | f_i | Cumulative Frequency | $ d_i = x_i - \text{Median} $ | $f_i d_i $ | $F_i X_i$ | $ X_i - \text{Mean} $ | $F_i X_i - \text{Mean} $ |
|----------------|-------|-------|----------------------|---------------------------------|-------------|-----------|-----------------------|---------------------------|
| 0-10 | 5 | 5 | 5 | 23 | 115 | 25 | 22 | 110 |
| 10-20 | 15 | 8 | 13 | 13 | 104 | 120 | 12 | 96 |
| 20-30 | 25 | 15 | 28 | 3 | 45 | 375 | 2 | 30 |
| 30-40 | 35 | 16 | 44 | 7 | 112 | 560 | 8 | 128 |
| 40-50 | 45 | 6 | 50 | 17 | 102 | 270 | 18 | 108 |

| | | | | | | | | |
|--|--|--------------|--|--|-------------------|--------------------|--|----------------|
| | | N = 50 | | | Total = 478 | Total = 1350 | | Total = 472 |
|--|--|--------------|--|--|-------------------|--------------------|--|----------------|

Mean deviation from Median = $478/50 = 9.56$

And, Mean deviation from Median = $472/50 = 9.44$

∴ The Mean Deviation from the median is 9.56 and from mean is 9.44.

EXERCISE 32.4 PAGE NO: 32.28

1. Find the mean, variance and standard deviation for the following data:

(i) 2, 4, 5, 6, 8, 17

Let Mean be,

$$\bar{X} = \frac{2+4+5+6+8+17}{6}$$

$$\bar{X} = \frac{42}{6} = 7$$

| X_i | $(x_i - \bar{X}) = (x_i - 7)$ | $(x_i - 7)^2$ |
|-------|-------------------------------|--|
| 2 | -3 | 25 |
| 4 | -3 | 9 |
| 5 | -2 | 4 |
| 6 | -1 | 1 |
| 8 | 1 | 1 |
| 17 | 10 | 100 |
| | | $\sum_{i=1}^6 (x_i - \bar{X})^2 = 140$ |

$N = 6$

$$\begin{aligned} \text{Variance (X)} &= \frac{1}{n} \sum_{i=1}^6 (x_i - \bar{X})^2 \\ &= 140/6 \\ &= 23.33 \end{aligned}$$

Variance = 23.33

Standard deviation = $\sqrt{\text{Var}(X)}$

$$\sigma = \sqrt{23.33}$$

Standard deviation = 4.83

(ii) 6, 7, 10, 12, 13, 4, 8, 12

Let Mean be,

$$\bar{X} = \frac{6+7+10+12+13+4+8+12}{8}$$

$$\bar{X} = \frac{72}{8} = 9$$

| X_i | $(x_i - \bar{X}) = (x_i - 9)$ | $(x_i - \bar{X})^2$ |
|-------|-------------------------------|---------------------------------------|
| 6 | -3 | 9 |
| 7 | -2 | 4 |
| 10 | 1 | 1 |
| 12 | 3 | 9 |
| 13 | 4 | 16 |
| 4 | -5 | 25 |
| 12 | 3 | 9 |
| | | $\sum_{i=1}^8 (x_i - \bar{X})^2 = 74$ |

$$N = 8$$

$$\begin{aligned} \text{Variance (X)} &= \frac{1}{n} \sum_{i=1}^8 (x_i - \bar{X})^2 \\ &= 74/8 \\ &= 9.25 \end{aligned}$$

$$\text{Variance} = 9.25$$

$$\text{Standard deviation} = \sqrt{\text{Var}(X)}$$

$$\sigma = \sqrt{9.25}$$

$$\text{Standard deviation} = 3.04$$

2. The variance of 20 observations is 4. If each observation is multiplied by 2, find the variance of the resulting observations.

Solution:

Let Assume, $x_1, x_2, x_3, \dots, x_{20}$ be the given observations.

Given: Variance(X) = 5

$$X = \frac{1}{n} \times \sum (x_i - \bar{X})^2$$

Now, Let u_1, u_2, \dots, u_{20} be the new observation,

When we multiply the new observation by 2, then

$$U_i = 2x_i \text{ (for } i=1, 2, 3, \dots, 20) \dots (i)$$

Now,

Mean:

$$\begin{aligned} \bar{U} &= \frac{\sum_{i=1}^{20} u_i}{n} \\ &= \frac{\sum_{i=1}^{20} 2x_i}{20} \end{aligned}$$

$$\text{Mean} = 2\bar{X}$$

$$\text{Since, } u_i - \bar{U} = 2x_i - 2\bar{X}$$

$$= 2(x_i - \bar{X})$$

$$\text{Now, } (u_i - \bar{U})^2 = (2(x_i - \bar{X}))^2$$

$$4(x_i - \bar{X})^2$$

Comparing both the observations

$$\frac{\sum_{i=1}^{20} (u_i - \bar{U})^2}{20} = \frac{\sum_{i=1}^{20} 4(x_i - \bar{X})^2}{20}$$

$$= 4 \times \frac{\sum_{i=1}^{20} (x_i - \bar{X})^2}{20}$$

$$\text{Variance (U)} = 4 \times \text{Variance (X)}$$

$$= 4 \times 5$$

$$= 20$$

\therefore The variance of new observations is 20.

3. The variance of 15 observations is 4. If each observation is increased by 9, find the variance of the resulting observations.

Solution:

Let Assume, $x_1, x_2, x_3, \dots, x_{15}$ be the given observations.

Given: Variance (X) = 4

$$X = \frac{1}{n} \times \sum (x_i - \bar{X})^2$$

Now, Let u_1, u_2, \dots, u_{20} be the new observation,

When new observation increase by 9, then

$$U_i = x_i + 9 \text{ (for } i=1, 2, 3, \dots, 20) \dots (i)$$

Now,

$$\bar{U} = \frac{1}{n} \sum_{i=1}^{15} u_i$$

$$= \frac{1}{15} \sum_{i=1}^{15} (x_i + 9)$$

$$= \frac{1}{15} \sum_{i=1}^{15} x_i + \frac{9 \times 15}{15}$$

$$\bar{U} = 9 + \bar{X}$$

$$u_i - \bar{U} = (x_i + 9) - (9 + \bar{X})$$

$$u_i - \bar{U} = x_i - \bar{X}$$

$$\frac{\sum_{i=1}^{15} (u_i - \bar{U})^2}{15} = \frac{\sum_{i=1}^{15} 4(x_i - \bar{X})^2}{15}$$

$$= \frac{\sum_{i=1}^{15} (u_i - \bar{U})^2}{15} = 4$$

Variance (U) = 4

∴ The variance of new observations is 4.

4. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, find the other two observations.

Solution:

Let x and y be the other two observation. And Mean is 4.4

$$\text{Let Mean} = \frac{1+2+6+x+y}{5} = 4.4$$

$$\Rightarrow 9 + x + y = 22$$

$$x + y = 13 \dots\dots (1)$$

Now, Let Variance (X) is the variance of this observation which is to be 8.24

If \bar{X} is the mean than we get,

$$8.24 = \frac{1}{5} (1^2 + 2^2 + 6^2 + x^2 + y^2) - (\bar{x})^2$$

$$8.24 = \frac{1}{5} (1^2 + 2^2 + 6^2 + x^2 + y^2) - (4.4)^2$$

$$8.24 = \frac{1}{5} (41 + x^2 + y^2) - 19.36$$

$$x^2 + y^2 = 97 \dots\dots (2)$$

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

By substituting the value we get,

$$13^2 + (x - y)^2 = 2 \times 97$$

$$(x - y)^2 = 194 - 169$$

$$(x - y)^2 = 25$$

$$x - y = \pm 5 \dots\dots (3)$$

On solving equations (1) and (3) we get,

$$2x = 18$$

$$x = 9 \text{ and } y = 4$$

∴ The other two observations are 9 and 4.

5. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:

Let Assume, $x_1, x_2, x_3, \dots, x_6$ be the given observations.

Given: Variance (X) = 8

$N = 6$ and $\sigma = 4$ (SD)

$$\bar{X} = \frac{1}{n} \times \sum x_i$$

$$8 = \frac{1}{6} \times \sum_{i=1}^6 x_i$$

Now, Let u_1, u_2, \dots, u_6 be the new observation,

When we multiply the new observation by 3, then

$$U_i = 3x_i \text{ (for } i = 1, 2, 3, \dots, 6) \dots\dots (1)$$

Now,

$$\begin{aligned}\bar{U} &= \frac{1}{n} \sum_{i=1}^6 u_i \\ &= \frac{1}{6} \sum_{i=1}^6 (3x_i) \\ &= 3 \times \frac{1}{6} \sum_{i=1}^6 (x_i)\end{aligned}$$

$$\begin{aligned}\bar{U} &= 3\bar{X} \\ &= 3 \times 8 = 24\end{aligned}$$

$$U = 24$$

So, the Mean of new observation is 24

Now,

Standard Deviation $\sigma_x = 4$

$$\sigma_x^2 = \text{Variance } X$$

Since, Variance (X) = 16

$$\begin{aligned}\text{Variance } (U) &= \frac{1}{6} \sum_{i=1}^6 (3x_i - 3\bar{X})^2 \\ &= 3^2 \times \frac{1}{6} \times \sum (x_i - \bar{X})^2 \\ &= 9 \times 16\end{aligned}$$

$$\sigma_u^2 = \text{Variance } (U)$$

$$\sigma_u^2 = 144$$

$$\sigma = 12$$

\therefore The mean of new observation is 24 and Standard deviation of new observation is 12.

6. The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:

Let x and y be the other two observation. And Mean is 9

$$\text{Let Mean} = \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$x + y = 12 \dots\dots (1)$$

Now, let Variance (X) be the variance of this observation which is to be 9.25

If \bar{X} is the mean than we get,

$$9.25 = \frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (\bar{X})^2$$

$$9.25 = \frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (9)^2$$

$$642 + x^2 + y^2 = 722$$

$$x^2 + y^2 = 80 \dots\dots (2)$$

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

By substituting the value we get,

$$12^2 + (x - y)^2 = 2 \times 80$$

$$(x - y)^2 = 160 - 144$$

$$(x - y)^2 = 14$$

$$x - y = \pm 4 \dots\dots (3)$$

On solving equations (1) and (3) we get,

$$x = 8, 4 \text{ and } y = 4, 8$$

\therefore The other two observations are 8 and 4.

EXERCISE 32.5 PAGE NO: 32.37

1. Find the standard deviation for the following distribution:

| | | | | | | | |
|----|-----|------|------|------|------|------|------|
| x: | 4.5 | 14.5 | 24.5 | 34.5 | 44.5 | 54.5 | 64.5 |
| f: | 1 | 5 | 12 | 22 | 17 | 9 | 4 |

Solution:

By using the formula for standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

So,

$$\text{Mean} = \frac{4.5+14.5+24+34.5+44.4+54.5+64.5}{7} = 34.4$$

| X_i | F_i | $d_i = (x_i - \text{mean})$ | $u_i = \frac{x_i - \text{mean}}{10}$ | $f_i u_i$ | U_i^2 | $f_i u_i^2$ |
|-------|-----------------|-----------------------------|--------------------------------------|---------------------|---------|------------------------|
| 4.5 | 1 | -30 | -3 | -3 | 9 | 9 |
| 14.5 | 5 | -20 | -2 | -10 | 4 | 20 |
| 24 | 12 | -10 | -1 | -12 | 1 | 12 |
| 34.5 | 22 | 0 | 0 | 0 | 0 | 0 |
| 44.5 | 17 | 10 | 1 | 17 | 1 | 17 |
| 54.5 | 9 | 20 | 2 | 18 | 4 | 36 |
| 64.5 | 4 | 30 | 3 | 12 | 9 | 36 |
| | $\sum f_i = 70$ | | | $\sum u_i f_i = 22$ | | $\sum u_i^2 f_i = 130$ |

Now,

$$N = 70, \sum u_i f_i = 22, \sum u_i^2 f_i = 130$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 10^2 \left[\frac{1}{70} \times 130 - \left(\frac{1}{70} \times 22 \right)^2 \right]$$

$$= 100 \left[\frac{130}{70} - \left(\frac{22}{70} \right)^2 \right]$$

$$= 100 \left[\frac{13}{7} - \frac{121}{1225} \right]$$

$$= 100 [1.857 - 0.0987]$$

$$= 100 [1.7583]$$

$$\text{Var}(X) = 175.83$$

$$\text{Standard Deviation, } \sigma = \sqrt{\text{Var}(X)}$$

$$= \sqrt{175.83}$$

$$= 13.26$$

\therefore The standard deviation is 13.26

2. Table below shows the frequency f with which 'x' alpha particles were radiated from a diskette

| | | | | | | | | | | | | | |
|-----------|-----------|------------|------------|------------|------------|------------|------------|------------|-----------|-----------|-----------|-----------|-----------|
| x: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| f: | 51 | 203 | 383 | 525 | 532 | 408 | 273 | 139 | 43 | 27 | 10 | 4 | 2 |

Calculate the mean and variance.

Solution:

By using the formula to find mean,

$$\text{Mean} = \frac{\sum f_i x_i}{N} = \frac{10078}{2600} = 3.88$$

| X_i | F_i | $F_i X_i$ | $(X_i - \bar{X})$ | $(X_i - \bar{X})^2$ | $F_i (X_i - \bar{X})^2$ |
|-------|----------|------------------------|-------------------|---------------------|---|
| 0 | 51 | 0 | -3.88 | 15.05 | 767.55 |
| 1 | 203 | 203 | -2.88 | 8.29 | 1682.87 |
| 2 | 383 | 766 | -1.88 | 3.53 | 1351.99 |
| 3 | 525 | 1575 | -0.88 | 0.77 | 404.25 |
| 4 | 532 | 2128 | 0.12 | 0.014 | 7.448 |
| 5 | 408 | 2040 | 1.12 | 1.25 | 510 |
| 6 | 273 | 1638 | 2.12 | 4.49 | 1225.77 |
| 7 | 139 | 973 | 3.12 | 9.73 | 1352.47 |
| 8 | 42 | 344 | 4.12 | 16.97 | 729.71 |
| 9 | 27 | 243 | 5.12 | 26.21 | 707.67 |
| 10 | 10 | 100 | 6.12 | 37.45 | 374.5 |
| 11 | 4 | 44 | 7.12 | 50.69 | 202.76 |
| 12 | 2 | 24 | 8.12 | 65.93 | 131.86 |
| | $N=2600$ | $\sum f_i x_i = 10078$ | | | $\sum f_i (x_i - \bar{X})^2 = 9448.848$ |

Now,

$N = 2600$

$$\text{Variance}(X) = \frac{\sum f_i (x_i - \bar{X})^2}{N}$$

$$\sigma^2 = \frac{9448.848}{2600} = 3.63$$

\therefore The mean is 3.88 and variance is 3.63

3. Find the mean, and standard deviation for the following data:

(i)

| | | | | | | |
|------------------------------------|-----------|-----------|-----------|-----------|-----------|------------|
| Year render: | 10 | 20 | 30 | 40 | 50 | 60 |
| No. of persons (cumulative) | 15 | 32 | 51 | 78 | 97 | 109 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| X_i | F_i | f_i | $u_i = \frac{x_i - \text{mean}}{10}$ | $f_i u_i$ | U_i^2 | $f_i u_i^2$ |
|-------|-------|------------------|--------------------------------------|-----------------------|---------|--------------------------|
| 10 | 15 | 15 | -2.5 | -37.5 | 6.25 | 93.75 |
| 20 | 32 | 17 | -1.5 | -25.5 | 2.25 | 38.25 |
| 30 | 51 | 19 | -0.5 | -9.5 | 0.25 | 4.75 |
| 40 | 78 | 27 | 0.5 | 13.5 | 0.25 | 6.75 |
| 50 | 97 | 19 | 1.5 | 28.5 | 2.25 | 42.75 |
| 60 | 109 | 12 | 2.5 | 30 | 6.25 | 75 |
| | | $\sum f_i = 109$ | | $\sum u_i f_i = -0.5$ | | $\sum u_i^2 f_i = 261.2$ |

Now,

$$N = 109, \sum u_i f_i = -0.5, \sum u_i^2 f_i = 261.2$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\bar{X} = 35 + 10 \left(\frac{-0.5}{109} \right)$$

$$\bar{X} = 34.96$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\begin{aligned} \text{Var}(X) &= 100 \left[\frac{261.25}{109} - \frac{0.25}{11881} \right] \\ &= 100 \times 2.396 \end{aligned}$$

$$\text{Variance} = 239.6$$

$$\begin{aligned} \text{Standard Deviation, } \sigma &= \sqrt{239.6} \\ &= 15.47 \text{ years} \end{aligned}$$

\therefore The standard deviation is 15.47

(ii)

| | | | | | | | | | | | | | | | |
|-------------------|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| Marks: | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Frequency: | 1 | 6 | 6 | 8 | 8 | 2 | 2 | 3 | 0 | 2 | 1 | 0 | 0 | 0 | 1 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| x_i | f_i | $f_i x_i$ | $f_i x_i^2$ |
|-------|-------|-----------|-------------|
| 2 | 1 | 2 | 4 |
| 3 | 6 | 18 | 54 |
| 4 | 6 | 24 | 96 |
| 5 | 8 | 40 | 200 |
| 6 | 8 | 48 | 288 |
| 7 | 2 | 14 | 98 |
| 8 | 2 | 16 | 128 |
| 9 | 3 | 27 | 243 |
| 10 | 0 | 0 | 0 |
| 11 | 2 | 22 | 242 |
| 12 | 1 | 12 | 144 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 1 | 16 | 256 |
| | N=40 | Total=239 | Total=1753 |

Now,

$$N = 40, \sum x_i f_i = 239, \sum x_i^2 f_i = 1753$$

$$\text{Mean, } \bar{X} = \left(\frac{\sum x_i f_i}{N} \right)$$

$$\bar{X} = \frac{239}{40}$$

$$= 5.975$$

$$\text{Var}(X) = \frac{1753}{40} - (5.97)^2$$

$$\text{Variance} = 8.12$$

$$\text{Standard Deviation, } \sigma = \sqrt{8.12}$$

$$= 2.85 \text{ years}$$

\therefore The standard deviation is 2.85

4. Find the standard deviation for the following data:

(i)

| | | | | | |
|-----------|---|----|----|----|----|
| x: | 3 | 8 | 13 | 18 | 23 |
| f: | 7 | 10 | 15 | 10 | 6 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| X_i | F_i | $F_i X_i$ | $(x_i - \bar{X})$ | $(x_i - \bar{X})^2$ | $(x_i - \bar{X})^2 f$ |
|-------|-----------------|----------------------|-------------------|---------------------|--------------------------------------|
| 3 | 7 | 21 | -9.79 | 95.84 | 670.88 |
| 8 | 10 | 80 | -4.79 | 22.94 | 229.4 |
| 13 | 15 | 195 | 0.21 | 0.04 | 0.6 |
| 18 | 10 | 180 | 5.21 | 27.14 | 271.4 |
| 23 | 6 | 138 | 10.21 | 104.24 | 625.44 |
| | $\sum f_i = 48$ | $\sum f_i x_i = 614$ | | | $\sum (x_i - \bar{X})^2 f = 1797.32$ |

Now, $N = 48$

$$\text{Var}(X) = \frac{\sum (x_i - \bar{X})^2 f}{\sum f_i}$$

$$\text{Var}(X) = \frac{1797.32}{48}$$

Variance = 37.44

$$\text{Standard Deviation, } \sigma = \sqrt{37.44}$$

$$= 6.12$$

\therefore The standard deviation is 6.12

(ii)

| | | | | | | |
|-----------|----------|----------|-----------|-----------|-----------|----------|
| x: | 2 | 3 | 4 | 5 | 6 | 7 |
| f: | 4 | 9 | 16 | 14 | 11 | 6 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| x_i | f_i | $f_i x_i$ | $f_i x_i^2$ |
|-------|--------|----------------|-------------|
| 2 | 4 | 8 | 16 |
| 3 | 9 | 27 | 81 |
| 4 | 16 | 64 | 256 |
| 5 | 14 | 70 | 350 |
| 6 | 11 | 66 | 396 |
| 7 | 6 | 42 | 294 |
| | $N=60$ | Total = 277 | Total=1393 |

Now,

$$N = 60, \sum x_i f_i = 277, \sum x_i^2 f_i = 1393$$

$$\text{Mean, } \bar{X} = \left(\frac{\sum x_i f_i}{N} \right)$$

$$\bar{X} = \frac{277}{60}$$
$$= 4.62$$

$$\text{Var}(X) = \frac{1393}{60} - (4.62)^2$$

$$\text{Variance} = 1.88$$

$$\text{Standard Deviation, } \sigma = \sqrt{1.88}$$
$$= 1.37$$

\therefore The standard deviation is 1.37

EXERCISE 32.6 PAGE NO: 32.41

1. Calculate the mean and S.D. for the following data:

| Expenditure (in ₹): | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---------------------|------|-------|-------|-------|-------|
| Frequency: | 14 | 13 | 27 | 21 | 15 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| Expenditure | Mid Point(X_i) | F_i | $F_i X_i$ | $(x_i - \bar{X})$ | $(x_i - \bar{X})^2$ | $(x_i - \bar{X})^2 f$ |
|-------------|--------------------|--------------------|--------------------------|-------------------|---------------------|---|
| 0-10 | 5 | 14 | 70 | -21.1 | 445.21 | 6233.94 |
| 10-20 | 15 | 13 | 195 | -11.1 | 123.21 | 1601.1 |
| 20-30 | 25 | 27 | 675 | -1.1 | 1.21 | 34.67 |
| 30-40 | 35 | 21 | 735 | 8.9 | 79.21 | 1663.41 |
| 40-50 | 45 | 15 | 675 | 18.9 | 357.21 | 53.58 |
| | | $\sum f_i$ = 90 | $\sum f_i x_i$ = 2350 | | | $\sum (x_i - \bar{X})^2 f$ = 1797.32 |

Now,

$$\text{Mean, } \bar{X} = \frac{\sum f_i x_i}{f_i}$$

$$\bar{X} = \frac{2350}{90}$$

$$= 26.11$$

$$\text{Var}(X) = \frac{14891.9}{90}$$

$$\text{Variance} = 165.47$$

$$\text{Standard Deviation, } \sigma = \sqrt{165.47}$$

$$= 12.86$$

\therefore The standard deviation is 12.86

2. Calculate the standard deviation for the following data:

| | | | | | | | |
|-------------------|-------------|--------------|--------------|---------------|----------------|----------------|----------------|
| Class: | 0-30 | 30-60 | 60-90 | 90-120 | 120-150 | 150-180 | 180-210 |
| Frequency: | 9 | 17 | 43 | 82 | 81 | 44 | 24 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| Class | F_i | x_i | $u_i = \frac{x_i - \text{mean}}{30}$ | $f_i u_i$ | U_i^2 | $f_i u_i^2$ |
|---------|-------|------------------|--------------------------------------|----------------------|---------|------------------------|
| 0-30 | 9 | 15 | -3 | -27 | 9 | 81 |
| 30-60 | 17 | 45 | -2 | -34 | 4 | 68 |
| 60-90 | 43 | 75 | -1 | -43 | 1 | 43 |
| 90-120 | 82 | 105 | 0 | 0 | 0 | 0 |
| 120-150 | 81 | 135 | 1 | 81 | 1 | 81 |
| 150-180 | 44 | 165 | 2 | 88 | 4 | 176 |
| 180-210 | 24 | 195 | 3 | 72 | 9 | 216 |
| | | $\sum f_i = 300$ | | $\sum u_i f_i = 137$ | | $\sum u_i^2 f_i = 665$ |

Now,

$$N = 300, \sum u_i f_i = 137, \sum u_i^2 f_i = 665$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\begin{aligned} \bar{X} &= 105 + 30 \left(\frac{137}{300} \right) \\ &= 118.7 \end{aligned}$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\begin{aligned} \text{Var}(X) &= \frac{900}{90000} [300 \times 665 - 18769] \\ &= \frac{1}{100} [199500 - 18769] \end{aligned}$$

$$\text{Variance} = 1807.31$$

$$\begin{aligned} \text{Standard Deviation, } \sigma &= \sqrt{1807.31} \\ &= 42.51 \end{aligned}$$

\therefore The standard deviation is 42.51

3. Calculate the A.M. and S.D. for the following distribution:

| | | | | | | | | |
|-------------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Frequency: | 18 | 16 | 15 | 12 | 10 | 5 | 2 | 1 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| Class | F_i | x_i | $u_i = \frac{x_i - \text{mean}}{10}$ | $f_i u_i$ | $f_i u_i^2$ |
|-------|-----------------|-------|--------------------------------------|----------------------|------------------------|
| 0-10 | 18 | 5 | -3 | -54 | 162 |
| 10-20 | 16 | 15 | -2 | -32 | 64 |
| 20-30 | 15 | 25 | -1 | -15 | 15 |
| 30-40 | 12 | 35 | 0 | 0 | 0 |
| 40-50 | 10 | 45 | 1 | 10 | 10 |
| 50-60 | 5 | 55 | 2 | 10 | 20 |
| 60-70 | 2 | 65 | 3 | 6 | 18 |
| 70-80 | 1 | 75 | 4 | 4 | 16 |
| | $\sum f_i = 79$ | | | $\sum u_i f_i = -71$ | $\sum u_i^2 f_i = 305$ |

Now,

$$N = 79, \sum u_i f_i = -71, \sum u_i^2 f_i = 305$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\begin{aligned} \bar{X} &= 35 + 10 \left(\frac{-71}{79} \right) \\ &= 26.01 \end{aligned}$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 100 \left[\frac{305}{79} - \frac{5041}{6241} \right]$$

$$\text{Variance} = 305.20$$

$$\begin{aligned} \text{Standard Deviation, } \sigma &= \sqrt{305.20} \\ &= 17.47 \end{aligned}$$

\therefore The standard deviation is 17.47

4. A student obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later found that one observation was wrongly copied as 50, the correct figure is 40. Find the correct mean and S.D.

Solution:

Given: Uncorrected mean is 40 and corrected SD is 5.1 and $N = 100$

Here, $\bar{x} = 40$, $\sigma = 5.1$ and $n = 100$

Then, $\sum x_o = 4000$

The corrected sum of observation, $\sum x_n = 4000 - 50 + 40$

$$\sum x_n = 3990$$

So,

$$\begin{aligned}\bar{x}_n &= \frac{\sum x_n}{n} \\ &= 3990/100 \\ &= 39.90\end{aligned}$$

Now,

Given Incorrect SD = 5.1

$$\sigma = 5.1$$

$$\sum (x_i - \bar{x}_o)^2 = 2601$$

$$\sum (x_i - \bar{x}_o)^2 = 2601 - 100 + 0.01 = 2501.1$$

$$\text{Corrected SD, } \sigma_n = \sqrt{\frac{\sum (x_i - \bar{x}_o)^2}{n}}$$

$$\begin{aligned}\sigma_n &= \sqrt{\frac{2501.01}{100}} \\ &= 5\end{aligned}$$

\therefore Correct mean is 39.9 and correct SD is 5

5. Calculate the mean, median and standard deviation of the following distribution

| Class-interval | 31-35 | 36-40 | 41-45 | 46-50 | 51-55 | 56-60 | 61-65 | 66-70 |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Frequency: | 2 | 3 | 8 | 12 | 16 | 5 | 2 | 3 |

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

| Class | F_i | x_i | $u_i = \frac{x_i - \text{mean}}{4}$ | $f_i u_i$ | $f_i u_i^2$ |
|-------|-----------------|-------|-------------------------------------|----------------------|------------------------|
| 31-35 | 2 | 33 | -4 | -8 | 32 |
| 36-40 | 3 | 38 | -3 | -9 | 27 |
| 41-45 | 8 | 43 | -2 | -16 | 32 |
| 46-50 | 12 | 48 | -1 | -12 | 12 |
| 51-55 | 16 | 53 | 0 | 0 | 0 |
| 56-60 | 5 | 58 | 1 | 5 | 5 |
| 61-65 | 2 | 63 | 2 | 4 | 8 |
| 66-70 | 2 | 68 | 3 | 6 | 18 |
| | $\sum f_i = 50$ | | | $\sum u_i f_i = -30$ | $\sum u_i^2 f_i = 134$ |

Now,

$$N = 50, \sum u_i f_i = -30, \sum u_i^2 f_i = 134$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\bar{X} = 53 + 5 \left(-\frac{30}{50} \right)$$

$$= 50$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 25 \left[\frac{134}{50} - \frac{9}{25} \right]$$

$$\text{Variance} = 58$$

$$\text{Standard Deviation, } \sigma = \sqrt{58}$$

$$= 7.62$$

\therefore The standard deviation is 7.62

EXERCISE 32.7 PAGE NO: 32.47

1. Two plants A and B of a factory show the following results about the number of workers and the wages paid to them

| | Plant A | Plant B |
|---------------------------------------|---------|---------|
| No. of workers | 5000 | 6000 |
| Average monthly wages | ₹2500 | ₹2500 |
| The variance of distribution of wages | 81 | 100 |

In which plant A or B is there greater variability in individual wages?

Solution:

Variation of the distribution of wages in plant A ($\sigma^2 = 18$)

So, Standard deviation of the distribution A ($\sigma = 9$)

Similarly, the Variation of the distribution of wages in plant B ($\sigma^2 = 100$)

So, Standard deviation of the distribution B ($\sigma = 10$)

And, Average monthly wages in both the plants is 2500,

Since, the plant with a greater value of SD will have more variability in salary.

\therefore Plant B has more variability in individual wages than plant A

2. The means and standard deviations of heights and weights of 50 students in a class are as follows:

| | Weights | Heights |
|---------------------------|----------------|------------------|
| Mean | 63.2 kg | 63.2 inch |
| Standard deviation | 5.6 kg | 11.5 inch |

Which shows more variability, heights or weights?

Solution:

Given: The mean and SD is given of 50 students.
 Let us find which shows more variability, height and weight.
 By using the formulas,

$$\text{Coefficient of variations} = \frac{\text{SD}}{\text{Mean}} \times 100$$

$$\begin{aligned} \text{Coefficient of variations in weights} &= \frac{\text{SD}}{\text{Mean}} \times 100 \\ &= \frac{5.6}{63.2} \times 100 = 8.86 \end{aligned}$$

$$\begin{aligned} \text{The coefficient of variations in heights} &= \frac{\text{SD}}{\text{Mean}} \times 100 \\ &= \frac{11.5}{63.2} \times 100 = 18.19 \end{aligned}$$

As results clearly show that coefficient of variations in heights is greater than coefficient of variations in weights.

∴ Heights will show more variability than weights

3. The coefficient of variation of two distribution are 60% and 70%, and their standard deviations are 21 and 16 respectively. What is their arithmetic means?

Solution:

Here, the Coefficient of variation for the first distribution is 60

And, Coefficient of variation for the first distribution is 70

SD (σ_1) = 21 and SD (σ_2) = 16

We know that, Coefficients of variation = $\frac{\text{SD}}{\text{Mean}} \times 100$

So,

$$\text{Mean, } \bar{X} = \frac{\text{SD}}{\text{CV}} \times 100$$

For first distribution

$$\begin{aligned} \bar{X} &= \frac{21}{60} \times 100 \\ &= 35 \end{aligned}$$

For the second distribution

$$\bar{X} = \frac{16}{70} \times 100$$

$$= 22.86$$

∴ Means are 35 and 22.86

4. Calculate coefficient of variation from the following data:

| | | | | | | |
|-------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Income (in ₹): | 1000-1700 | 1700-2400 | 2400-3100 | 3100-3800 | 3800-4500 | 4500-5200 |
| No. of families: | 12 | 18 | 20 | 25 | 35 | 10 |

Solution:

Let us find the standard deviation of the frequency:

| Class | F _i | x _i | $u_i = \frac{x_i - \text{mean}}{700}$ | f _i u _i | f _i u _i ² |
|-----------|------------------|----------------|---------------------------------------|-------------------------------|--|
| 1000-1700 | 12 | 1350 | -2 | -24 | 48 |
| 1700-2400 | 18 | 2050 | -1 | -18 | 18 |
| 2400-3100 | 20 | 2750 | 0 | 0 | 0 |
| 3100-3800 | 25 | 3450 | 1 | 25 | 25 |
| 3800-4500 | 35 | 4150 | 2 | 70 | 140 |
| 4500-5200 | 10 | 4850 | 3 | 30 | 90 |
| | $\sum f_i = 120$ | | | $\sum u_i f_i = 83$ | $\sum u_i^2 f_i = 321$ |

Now,

$$N = 120, \sum u_i^2 f_i = 321$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\begin{aligned} \bar{X} &= 2750 + 700 \left(\frac{83}{120} \right) \\ &= 3234.17 \end{aligned}$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 490000 \left[\left(\frac{321}{120} \right) - \left(\frac{83}{120} \right)^2 \right]$$

$$\text{Variance} = 1076332.64$$

$$\begin{aligned} \text{Standard Deviation, } \sigma &= \sqrt{1076332.64} \\ &= 1037.47 \end{aligned}$$

$$\begin{aligned} \text{Coefficients of variation} &= \frac{1037.46}{3234.17} \times 100 \\ &= 32.08 \end{aligned}$$

∴ The coefficient variation is 32.08

5. An analysis of the weekly wages paid to workers in two firms A and B, belonging to the same industry gives the following results:

| | Firm A | Firm B |
|---|--------|--------|
| No. of wage earners | 586 | 648 |
| Average weekly wages | ₹52.5 | ₹47.5 |
| The variance of the distribution of wages | 100 | 121 |

- (i) Which firm A or B pays out the larger amount as weekly wages?
(ii) Which firm A or B has greater variability in individual wages?

Solution:

$$(i) \text{ Average weekly wages} = \frac{\text{Total weekly wages}}{\text{No. of workers}}$$

$$\text{Total weekly wages} = (\text{Average weekly wages}) \times (\text{No. of workers})$$

$$\text{Total weekly wages of Firm A} = 52.5 \times 586 = \text{Rs } 30765$$

$$\text{Total weekly wages of Firm B} = 47.5 \times 648 = \text{Rs } 30780$$

Firm B pays a larger amount as Firm A

(ii) Here, SD (firm A) 10 and SD (Firm B) = 11

$$\begin{aligned} \text{Coefficient variance (Firm A)} &= \frac{10}{52.5} \times 100 \\ &= 19.04 \end{aligned}$$

$$\begin{aligned} \text{Coefficient variance (Firm B)} &= \frac{11}{47.5} \times 100 \\ &= 23.15 \end{aligned}$$

∴ Coefficient variance of firm B is greater than that of firm A, Firm B has greater variability in individual wages.

6. The following are some particulars of the distribution of weights of boys and girls in a class:

| | Boys | Girls |
|-------------|-------|-------|
| Number | 100 | 50 |
| Mean weight | 60 kg | 45 kg |
| Variance | 9 | 4 |

Which of the distributions is more variable?

Solution:

Given: SD (Boys) is 3 and SD (girls) = 2

$$\text{Coefficient variability} = \frac{\text{SD}}{\text{Mean}} \times 100$$

$$\begin{aligned}\text{Coefficient variance (Boys)} &= \frac{3}{60} \times 100 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{Coefficient variance (Girls)} &= \frac{2}{45} \times 100 \\ &= 4.4\end{aligned}$$

\therefore Coefficient variance of Boys is greater than Coefficient variance of girls, and then the distribution of weights of boys is more variable than that of girls.