

Access RD Sharma Solutions for Class 11 Maths Chapter 26 – Ellipse

EXERCISE 26.1 PAGE NO: 26.22

1. Find the equation of the ellipse whose focus is (1, -2), the directrix $3x - 2y + 5 = 0$ and eccentricity equal to $1/2$.

Solution:

Given:

Focus = (1, -2)

Directrix = $3x - 2y + 5 = 0$

Eccentricity = $1/2$

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

$$SP = ePM$$

$$SP^2 = e^2PM^2$$

$$(x - 1)^2 + (y - (-2))^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|3x - 2y + 5|}{\sqrt{3^2 + (-2)^2}}\right)^2$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = \frac{1}{4} \times \frac{(3x - 2y + 5)^2}{9 + 4}$$

$$x^2 + y^2 - 2x + 4y + 5 = \frac{1}{52} \times (9x^2 + 4y^2 + 25 - 12xy - 20y + 30x)$$

Upon cross multiplying, we get

$$52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 - 12xy - 20y + 30x + 25$$

$$43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$$

\therefore The equation of the ellipse is $43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$

2. Find the equation of the ellipse in the following cases:

(i) focus is (0, 1), directrix is $x + y = 0$ and $e = 1/2$.

(ii) focus is (-1, 1), directrix is $x - y + 3 = 0$ and $e = 1/2$.

(iii) focus is (-2, 3), directrix is $2x + 3y + 4 = 0$ and $e = 4/5$.

(iv) focus is (1, 2), directrix is $3x + 4y - 7 = 0$ and $e = \frac{1}{2}$.

Solution:

(i) focus is (0, 1), directrix is $x + y = 0$ and $e = \frac{1}{2}$

Given:

Focus is (0, 1)

Directrix is $x + y = 0$

$e = \frac{1}{2}$

Let $P(x, y)$ be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

$$SP = ePM$$

$$SP^2 = e^2PM^2$$

$$(x - 0)^2 + (y - 1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x + y|}{\sqrt{1^2 + 1^2}}\right)^2$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x + y|)^2}{1 + 1}$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{8} \times (x^2 + y^2 + 2xy)$$

Upon cross multiplying, we get

$$8x^2 + 8y^2 - 16y + 8 = x^2 + y^2 + 2xy$$

$$7x^2 + 7y^2 - 2xy - 16y + 8 = 0$$

\therefore The equation of the ellipse is $7x^2 + 7y^2 - 2xy - 16y + 8 = 0$

(ii) focus is (- 1, 1), directrix is $x - y + 3 = 0$ and $e = \frac{1}{2}$

Given:

Focus is (- 1, 1)

Directrix is $x - y + 3 = 0$

$e = \frac{1}{2}$

Let $P(x, y)$ be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

$$SP = ePM$$

$$SP^2 = e^2PM^2$$

$$(x - (-1))^2 + (y - 1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x-y+3|}{\sqrt{1^2+1^2}}\right)^2$$

$$x^2 + 2x + 1 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x-y+3|)^2}{1+1}$$

$$x^2 + y^2 + 2x - 2y + 2 = \frac{1}{8} \times (x^2 + y^2 + 9 - 2xy - 6y + 6x)$$

Upon cross multiplying, we get

$$8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + y^2 - 2xy + 6x - 6y + 9$$

$$7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$$

\therefore The equation of the ellipse is $7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$

(iii) focus is $(-2, 3)$, directrix is $2x + 3y + 4 = 0$ and $e = 4/5$

Focus is $(-2, 3)$

Directrix is $2x + 3y + 4 = 0$

$$e = 4/5$$

Let $P(x, y)$ be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

$$SP = ePM$$

$$SP^2 = e^2PM^2$$

$$(x - (-2))^2 + (y - 3)^2 = \left(\frac{4}{5}\right)^2 \left(\frac{|2x + 3y + 4|}{\sqrt{2^2 + 3^2}}\right)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = \frac{16}{25} \times \frac{(|2x + 3y + 4|)^2}{4 + 9}$$

$$x^2 + y^2 + 4x - 6y + 13 = (16/325) \times (4x^2 + 9y^2 + 16 + 12xy + 16x + 24y)$$

Upon cross multiplying, we get

$$325x^2 + 325y^2 + 1300x - 1950y + 4225 = 64x^2 + 144y^2 + 192xy + 256x + 384y + 256$$

$$261x^2 + 181y^2 - 192xy + 1044x - 2334y + 3969 = 0$$

∴ The equation of the ellipse is $261x^2 + 181y^2 - 192xy + 1044x - 2334y + 3969 = 0$

(iv) focus is (1, 2), directrix is $3x + 4y - 7 = 0$ and $e = \frac{1}{2}$.

Given:

focus is (1, 2)

directrix is $3x + 4y - 7 = 0$

$e = \frac{1}{2}$.

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

We also know that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

$$SP = ePM$$

$$SP^2 = e^2PM^2$$

$$(x - 1)^2 + (y - 2)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|3x + 4y - 5|}{\sqrt{3^2 + 4^2}}\right)^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = \frac{1}{4} \times \frac{(|3x + 4y - 5|)^2}{9 + 16}$$

$$x^2 + y^2 - 2x - 4y + 5 = \frac{1}{100} \times (9x^2 + 16y^2 + 25 + 24xy - 30x - 40y)$$

Upon cross multiplying, we get

$$100x^2 + 100y^2 - 200x - 400y + 500 = 9x^2 + 16y^2 + 24xy - 30x - 40y + 25$$

$$91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$$

∴ The equation of the ellipse is $91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$

3. Find the eccentricity, coordinates of foci, length of the latus – rectum of the following ellipse:

(i) $4x^2 + 9y^2 = 1$

(ii) $5x^2 + 4y^2 = 1$

(iii) $4x^2 + 3y^2 = 1$

(iv) $25x^2 + 16y^2 = 1600$

(v) $9x^2 + 25y^2 = 225$

Solution:

(i) $4x^2 + 9y^2 = 1$

Given:

The equation of ellipse $\Rightarrow 4x^2 + 9y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = \frac{1}{4}$, $b^2 = \frac{1}{9}$

$$= \sqrt{\frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{5}{36}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

Length of latus rectum $= 2b^2/a$

$$= [2 (1/9)] / (1/2)$$

$$= 4/9$$

Coordinates of foci ($\pm ae, 0$)

$$\begin{aligned}\text{foci} &= \left(\pm \frac{1}{2} \times \frac{\sqrt{5}}{3}, 0 \right) \\ &= \left(\pm \frac{\sqrt{5}}{6}, 0 \right)\end{aligned}$$

\therefore The eccentricity is $\frac{\sqrt{5}}{3}$, foci are $\left(\pm \frac{\sqrt{5}}{6}, 0 \right)$ and length of the latus rectum is $\frac{4}{9}$.

(ii) $5x^2 + 4y^2 = 1$

Given:

The equation of ellipse $\Rightarrow 5x^2 + 4y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 1/5$ and $b^2 = 1/4$

$$\begin{aligned}e &= \sqrt{\frac{\frac{1}{4} - \frac{1}{5}}{\frac{1}{4}}} \\ &= \sqrt{\frac{\frac{1}{20}}{\frac{1}{4}}} \\ &= \sqrt{\frac{1}{5}}\end{aligned}$$

Length of latus rectum $= 2b^2/a$

$$= [2(1/5)] / (1/2)$$

$$= 4/5$$

Coordinates of foci ($\pm ae, 0$)

$$\begin{aligned}\text{foci} &= \left(0, \pm \frac{1}{2} \times \sqrt{\frac{1}{5}} \right) \\ &= \left(0, \pm \frac{1}{2\sqrt{5}} \right)\end{aligned}$$

\therefore The eccentricity is $\sqrt{\frac{1}{5}}$, foci are $(0, \pm \frac{1}{2\sqrt{5}})$ and length of the latus rectum is $\frac{4}{5}$.

(iii) $4x^2 + 3y^2 = 1$

Given:

The equation of ellipse $\Rightarrow 4x^2 + 3y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{3}} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 1/4$ and $b^2 = 1/3$

$$\begin{aligned} e &= \sqrt{1 - \frac{\frac{1}{3}}{\frac{1}{4}}} \\ &= \sqrt{\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}}} \\ &= \sqrt{\frac{\frac{1}{12}}{\frac{1}{3}}} \\ &= \sqrt{\frac{1}{4}} \\ &= \frac{1}{2} \\ &= \sqrt{\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}}} \\ &= \sqrt{\frac{\frac{1}{12}}{\frac{1}{3}}} \\ &= \sqrt{\frac{1}{4}} \\ &= \frac{1}{2} \end{aligned}$$

$$\text{Length of latus rectum} = 2b^2/a$$

$$= [2(1/4)] / (1/\sqrt{3})$$

$$= \sqrt{3}/2$$

Coordinates of foci ($\pm ae$, 0)

$$\text{foci} = \left(0, \pm \frac{1}{\sqrt{3}} \times \frac{1}{2}\right)$$

$$= \left(0, \pm \frac{1}{2\sqrt{3}}\right)$$

\therefore The eccentricity is $\frac{\sqrt{3}}{2}$, foci are $\left(0, \pm \frac{1}{2\sqrt{3}}\right)$ and length of the latus rectum is $\frac{\sqrt{3}}{2}$.

(iv) $25x^2 + 16y^2 = 1600$

Given:

The equation of ellipse $\Rightarrow 25x^2 + 16y^2 = 1600$

This equation can be expressed as

$$\frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$$

$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 64$ and $b^2 = 100$

$$e = \sqrt{1 - \frac{64}{100}}$$

$$= \sqrt{\frac{100 - 64}{100}}$$

$$= \sqrt{\frac{36}{100}}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

$$\begin{aligned}
 &= \sqrt{\frac{36}{100}} \\
 &= \frac{6}{10} \\
 &= \frac{3}{5}
 \end{aligned}$$

Length of latus rectum = $2b^2/a$

$$= [2(64)] / (100)$$

$$= 32/25$$

Coordinates of foci ($\pm ae, 0$)

$$\begin{aligned}
 \text{foci} &= \left(0, \pm 10 \times \frac{3}{5}\right) \\
 &= (0, \pm 6)
 \end{aligned}$$

\therefore The eccentricity is $\frac{3}{5}$, foci are $(0, \pm 6)$ and length of the latus rectum is $\frac{32}{25}$.

$$(v) \ 9x^2 + 25y^2 = 225$$

Given:

The equation of ellipse $\Rightarrow 9x^2 + 25y^2 = 225$

This equation can be expressed as

$$\begin{aligned}
 \frac{9x^2}{225} + \frac{25y^2}{225} &= 1 \\
 \frac{x^2}{25} + \frac{y^2}{9} &= 1
 \end{aligned}$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 25$ and $b^2 = 9$

$$\begin{aligned}
 e &= \sqrt{\frac{25-9}{25}} \\
 &= \sqrt{\frac{16}{25}} \\
 &= \frac{4}{5}
 \end{aligned}$$

Length of latus rectum = $2b^2/a$

$$= [2(9)] / (5)$$

$$= 18/5$$

Coordinates of foci ($\pm ae, 0$)

$$\begin{aligned}\text{foci} &= \left(\pm 5 \times \frac{4}{5}, 0 \right) \\ &= (\pm 4, 0)\end{aligned}$$

\therefore The eccentricity is $\frac{4}{5}$, foci are $(\pm 4, 0)$ and length of the latus rectum is $\frac{18}{5}$.

4. Find the equation to the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (-3, 1) and has eccentricity $\sqrt{2/5}$.

Solution:

Given:

The point (-3, 1)

Eccentricity = $\sqrt{2/5}$

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (1)$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\sqrt{\frac{2}{5}} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{2}{5} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{5}$$

$$b^2 = \frac{3a^2}{5} \dots (2)$$

Now let us substitute equation (2) in equation (1), we get

$$\frac{x^2}{a^2} + \frac{y^2}{\frac{3a^2}{5}} = 1$$

$$\frac{x^2}{a^2} + \frac{5y^2}{3a^2} = 1$$

$$3x^2 + 5y^2 = 3a^2$$

It is given that the curve passes through the point (-3, 1).

So by substituting the point in the curve we get,

$$3(-3)^2 + 5(1)^2 = 3a^2$$

$$3(9) + 5 = 3a^2$$

$$32 = 3a^2$$

$$a^2 = 32/3$$

From equation (2)

$$b^2 = 3a^2/5$$

$$= 3(32/3) / 5$$

$$= 32/5$$

So now, the equation of the ellipse is given as:

$$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$$

$$\frac{3x^2}{32} + \frac{5y^2}{32} = 1$$

$$3x^2 + 5y^2 = 32$$

∴ The equation of the ellipse is $3x^2 + 5y^2 = 32$.

5. Find the equation of the ellipse in the following cases:

(i) eccentricity $e = \frac{1}{2}$ and foci $(\pm 2, 0)$

(ii) eccentricity $e = \frac{2}{3}$ and length of latus – rectum = 5

(iii) eccentricity $e = \frac{1}{2}$ and semi – major axis = 4

(iv) eccentricity $e = \frac{1}{2}$ and major axis = 12

(v) The ellipse passes through $(1, 4)$ and $(-6, 1)$

Solution:

(i) Eccentricity $e = \frac{1}{2}$ and foci $(\pm 2, 0)$

Given:

Eccentricity $e = \frac{1}{2}$

Foci $(\pm 2, 0)$

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{2} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{4}$$

$$b^2 = \frac{3a^2}{4}$$

$$b^2 = 3a^2/4$$

It is given that foci $(\pm 2, 0) \Rightarrow$ foci $= (\pm ae, 0)$

Where, $ae = 2$

$$a(1/2) = 2$$

$$a = 4$$

$$a^2 = 16$$

We know $b^2 = 3a^2/4$

$$b^2 = 3(16)/4$$

$$= 12$$

So the equation of the ellipse can be given as

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\frac{3x^2 + 4y^2}{48} = 1$$

$$3x^2 + 4y^2 = 48$$

\therefore The equation of the ellipse is $3x^2 + 4y^2 = 48$

(ii) eccentricity $e = 2/3$ and length of latus rectum $= 5$

Given:

Eccentricity $e = 2/3$

Length of latus – rectum $= 5$

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{2}{3} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{4}{9} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{5}{9}$$

$$b^2 = \frac{5a^2}{9}$$

By using the formula, length of the latus rectum is $2b^2/a$

$$\frac{2b^2}{a} = 5$$

$$b^2 = \frac{5a}{2}$$

Since, $b^2 = 5a^2/9$

$$\frac{5a^2}{9} = \frac{5a}{2}$$

$$\frac{a}{9} = \frac{1}{2}$$

$$a = \frac{9}{2}$$

$$a^2 = \frac{81}{4}$$

Now, substituting the value of a^2 , we get

$$b^2 = \frac{5\left(\frac{81}{4}\right)}{9}$$

$$b^2 = \frac{45}{4}$$

So the equation of the ellipse can be given as

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

$$\frac{(20x^2 + 36y^2)}{405} = 1$$

$$20x^2 + 36y^2 = 405$$

∴ The equation of the ellipse is $20x^2 + 36y^2 = 405$.

(iii) eccentricity $e = \frac{1}{2}$ and semi – major axis = 4

Given:

Eccentricity $e = \frac{1}{2}$

Semi – major axis = 4

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{2} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{4}$$

$$b^2 = \frac{3a^2}{4}$$

$$\frac{1}{2} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{4}$$

$$b^2 = \frac{3a^2}{4}$$

It is given that the length of the semi – major axis is a

$$a = 4$$

$$a^2 = 16$$

We know, $b^2 = 3a^2/4$

$$b^2 = 3(16)/4$$

$$= 12$$

So the equation of the ellipse can be given as

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\frac{3x^2 + 4y^2}{48} = 1$$

$$3x^2 + 4y^2 = 48$$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 48$.

(iv) eccentricity $e = \frac{1}{2}$ and major axis = 12

Given:

Eccentricity $e = \frac{1}{2}$

Major axis = 12

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{2} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{4}$$

$$b^2 = \frac{3a^2}{4}$$

$$b^2 = 3a^2/4$$

It is given that length of major axis is $2a$.

$$2a = 12$$

$$a = 6$$

$$a^2 = 36$$

So, by substituting the value of a^2 , we get

$$b^2 = 3(36)/4$$

$$= 27$$

So the equation of the ellipse can be given as

$$\frac{x^2}{36} + \frac{y^2}{27} = 1$$

$$\frac{3x^2 + 4y^2}{108} = 1$$

$$3x^2 + 4y^2 = 108$$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 108$.

(v) The ellipse passes through (1, 4) and (-6, 1)

Given:

The points (1, 4) and (-6, 1)

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (1)$$

Let us substitute the point (1, 4) in equation (1), we get

$$\begin{aligned}\frac{1^2}{a^2} + \frac{4^2}{b^2} &= 1 \\ \frac{1}{a^2} + \frac{16}{b^2} &= 1 \\ \frac{b^2 + 16a^2}{a^2b^2} &= 1\end{aligned}$$

$$b^2 + 16a^2 = a^2b^2 \dots (2)$$

Let us substitute the point (-6, 1) in equation (1), we get

$$\begin{aligned}\frac{(-6)^2}{a^2} + \frac{1^2}{b^2} &= 1 \\ \frac{36}{a^2} + \frac{1}{b^2} &= 1 \\ \frac{36b^2 + a^2}{a^2b^2} &= 1\end{aligned}$$

$$a^2 + 36b^2 = a^2b^2 \dots (3)$$

Let us multiply equation (3) by 16 and subtract with equation (2), we get

$$(16a^2 + 576b^2) - (b^2 + 16a^2) = (16a^2b^2 - a^2b^2)$$

$$575b^2 = 15a^2b^2$$

$$15a^2 = 575$$

$$a^2 = 575/15$$

$$= 115/3$$

So from equation (2),

$$b^2 + 16\left(\frac{115}{3}\right) = b^2\left(\frac{115}{3}\right)$$

$$b^2\left(\frac{112}{3}\right) = \frac{1840}{3}$$

$$b^2 = \frac{115}{7}$$

So the equation of the ellipse can be given as

$$\frac{x^2}{\frac{115}{3}} + \frac{y^2}{\frac{115}{7}} = 1$$

$$\frac{3x^2}{115} + \frac{7y^2}{115} = 1$$

$$3x^2 + 7y^2 = 115$$

∴ The equation of the ellipse is $3x^2 + 7y^2 = 115$.

6. Find the equation of the ellipse whose foci are (4, 0) and (- 4, 0), eccentricity = 1/3.

Solution:

Given:

Foci are (4, 0) (- 4, 0)

Eccentricity = 1/3.

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{3} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{9} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{8}{9}$$

$$b^2 = \frac{8a^2}{9}$$

$$\frac{b^2}{a^2} = \frac{8}{9}$$

$$b^2 = \frac{8a^2}{9}$$

It is given that foci = (4, 0) (- 4, 0) => foci = ($\pm ae$, 0)

Where, $ae = 4$

$$a(1/3) = 4$$

$$a = 12$$

$$a^2 = 144$$

By substituting the value of a^2 , we get

$$b^2 = 8a^2/9$$

$$b^2 = 8(144)/9$$

$$= 128$$

So the equation of the ellipse can be given as

$$\frac{x^2}{144} + \frac{y^2}{128} = 1$$

$$\therefore \text{The equation of the ellipse is } \frac{x^2}{144} + \frac{y^2}{128} = 1$$

7. Find the equation of the ellipse in the standard form whose minor axis is equal to the distance between foci and whose latus – rectum is 10.

Solution:

Given:

Minor axis is equal to the distance between foci and whose latus – rectum is 10.

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We know that length of the minor axis is $2b$ and distance between the foci is $2ae$.

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$2b = 2ae$$

$$b = ae$$

$$b = a \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$b^2 = a^2 - b^2$$

$$a^2 = 2b^2 \dots (1)$$

We know that the length of the latus rectum is $2b^2/a$

It is given that length of the latus rectum = 10

So by equating, we get

$$2b^2/a = 10$$

$$a^2/a = 10 \text{ [Since, } a^2 = 2b^2]$$

$$a = 10$$

$$a^2 = 100$$

Now, by substituting the value of a^2 we get

$$2b^2/a = 10$$

$$2b^2/10 = 10$$

$$2b^2 = 10(10)$$

$$b^2 = 100/2$$

$$= 50$$

So the equation of the ellipse can be given as

$$\frac{x^2}{100} + \frac{y^2}{50} = 1$$

$$\frac{x^2 + 2y^2}{100} = 1$$

$$x^2 + 2y^2 = 100$$

\therefore The equation of the ellipse is $x^2 + 2y^2 = 100$.

8. Find the equation of the ellipse whose centre is (-2, 3) and whose semi – axis are 3 and 2 when the major axis is (i) parallel to x – axis (ii) parallel to the y – axis.

Solution:

Given:

Centre = (-2, 3)

Semi – axis are 3 and 2

(i) When major axis is parallel to x-axis

Now let us find the equation to the ellipse.

We know that the equation of the ellipse with centre (p, q) is given by

$$\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$$

Since major axis is parallel to x – axis

So, $a = 3$ and $b = 2$.

$$a^2 = 9$$

$$b^2 = 4$$

So the equation of the ellipse can be given as

$$\frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$$
$$\frac{4(x+2)^2 + 9(y-3)^2}{36} = 1$$

$$4(x^2 + 4x + 4) + 9(y^2 - 6y + 9) = 36$$

$$4x^2 + 16x + 16 + 9y^2 - 54y + 81 = 36$$

$$4x^2 + 9y^2 + 16x - 54y + 61 = 0$$

\therefore The equation of the ellipse is $4x^2 + 9y^2 + 16x - 54y + 61 = 0$.

(ii) When major axis is parallel to y-axis

Now let us find the equation to the ellipse.

We know that the equation of the ellipse with centre (p, q) is given by

$$\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$$

Since major axis is parallel to y – axis

So, $a = 2$ and $b = 3$.

$$a^2 = 4$$

$$b^2 = 9$$

So the equation of the ellipse can be given as

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$$
$$\frac{9(x+2)^2 + 4(y-3)^2}{36} = 1$$

$$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = 36$$

$$9x^2 + 36x + 36 + 4y^2 - 24y + 36 = 36$$

$$9x^2 + 4y^2 + 36x - 24y + 36 = 0$$

\therefore The equation of the ellipse is $9x^2 + 4y^2 + 36x - 24y + 36 = 0$.

9. Find the eccentricity of an ellipse whose latus – rectum is

(i) Half of its minor axis

(ii) Half of its major axis

Solution:

Given:

We need to find the eccentricity of an ellipse.

(i) If latus – rectum is half of its minor axis

We know that the length of the semi – minor axis is b and the length of the latus – rectum is $2b^2/a$.

$$2b^2/a = b$$

$$a = 2b \dots (1)$$

By using the formula,

We know that eccentricity of an ellipse is given as

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

From equation (1)

$$\begin{aligned} e &= \sqrt{\frac{a^2 - b^2}{a^2}} \\ &= \sqrt{\frac{(2b)^2 - b^2}{(2b)^2}} \\ &= \sqrt{\frac{4b^2 - b^2}{4b^2}} \\ &= \sqrt{\frac{3}{4}} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

(ii) If latus – rectum is half of its major axis

We know that the length of the semi – major axis is a and the length of the latus – rectum is $2b^2/a$.

$$2b^2/a$$

$$a^2 = 2b^2 \dots (1)$$

By using the formula,

We know that eccentricity of an ellipse is given as

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

From equation (1)

$$\begin{aligned} e &= \sqrt{\frac{a^2 - b^2}{a^2}} \\ &= \sqrt{\frac{2b^2 - b^2}{2b^2}} \\ &= \sqrt{\frac{b^2}{2b^2}} \\ &= \sqrt{\frac{1}{2}} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$