NCERT Solutions for Class 9 Maths Chapter 12 - Heron's Formula

Chapter 12 - Heron's Formula Exercise Ex. 12.1

Solution 1

Side of traffic signal board = a

Perimeter of traffic signal board = 3 a

$$2s = 3a \Rightarrow s = \frac{3}{2}a$$

By Heron's formula

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of given triangle} = \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$

$$= \sqrt{\left(\frac{3}{2}a\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$$

$$= \frac{\sqrt{3}}{4}a^2 \qquad \ldots (1)$$

Perimeter of traffic signal board = 180 cm

$$(a) = \frac{180}{3}\text{ cm} = 60\text{ cm}$$

Side of traffic signal board

$$= \frac{\sqrt{3}}{4}(60\text{ cm})^2$$

Using equation (1), area of traffic of signal board

$$= \left[\frac{3600}{4}\sqrt{3}\right]\text{cm}^2 = 900\sqrt{3}\text{ cm}^2$$

Solution 2

We may observe that sides of triangle a, b, c are of 122 m, 22 m, and 120 m respectively

Perimeter of triangle = (122 + 22 + 120) m

2s = 264 m

s = 132 m

By Heron's formula

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of given triangle} = \left[\sqrt{132(132-122)(132-22)(132-120)}\right]\text{m}^2$$

$$= \left[\sqrt{132(10)(110)(12)}\right]\text{m}^2 = 1320\text{ m}^2$$

Rent of 1 m^2 area per year = Rs.5000

Rent of 1 m^2 area per month = Rs $\frac{5000}{12}$

$$= \text{Rs.}\left[\frac{5000}{12}\cdot 3\cdot 1320\right]$$

Rent of 1320 m^2 area for 3 months

= Rs.(5000 330) = Rs.1650000

So, company had to pay Rs.1650000.

Solution 3

We may observe that the area to be painted in colour is a triangle, having its sides as 11 m, 6 m, and 15 m.

Perimeter of such triangle = (11 + 6 + 15) m

2 s = 32 m

s = 16 m

By Heron's formula

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{16(16-11)(16-6)(16-15)}\right] m^2$$
$$= \left(\sqrt{16\times5\times10\times1}\right) m^2$$
$$= 20\sqrt{2}\ m^2$$

So, the area painted in colour is $20\sqrt{2}\ m^2$.

Solution 4

Let third side of triangle be x.

Perimeter of given triangle = 42 cm

18 cm + 10 cm + x = 42

x = 14 cm

$$s = \frac{\text{perimeter}}{2} = \frac{42\text{ cm}}{2} = 21\text{ cm}$$

By Heron's formula

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{Area of given triangle} = \left(\sqrt{21(21-18)(21-10)(21-14)}\right) cm^2$$
$$= \left(\sqrt{21(3)(11)(7)}\right) cm^2$$
$$= 21\sqrt{11}\ cm^2$$

Solution 5

Let the common ratio between the sides of given triangle be x.

So, side of triangle will be 12x, 17x, and 25x.

Perimeter of this triangle = 540 cm

12x + 17x + 25x = 540 cm

54x = 540 cm

x = 10 cm

Sides of triangle will be 120 cm, 170 cm, and 250 cm.

$$s = \frac{\text{perimeter of tringle}}{2} = \frac{540\text{ cm}}{2} = 270\text{ cm}$$

By Heron's formula

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{270(270-120)(270-170)(270-250)}\right] cm^2$$
$$= \left[\sqrt{270\times150\times100\times20}\right] cm^2$$
$$= 9000\ cm^2$$

So, area of this triangle will be 9000 cm^2.

Solution 6

Let third side of this triangle be x.

Perimeter of triangle = 30 cm

12 cm + 12 cm + x = 30 cm

x = 6 cm

$$s = \frac{\text{perimeter of triangle}}{2} = \frac{30\text{ cm}}{2} = 15\text{ cm}$$

By Heron's formula

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{15(15-12)(15-12)(15-6)}\right] cm^2$$
$$= \left[\sqrt{15(3)(3)(9)}\right] cm^2$$
$$= 9\sqrt{15}\ cm^2$$

Chapter 12 - Heron's Formula Exercise Ex. 12.2

Solution 1

Let us join BD.

In $\triangle$BCD applying Pythagoras theorem

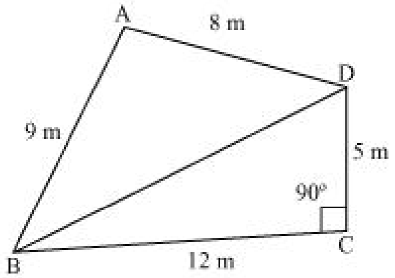
$BD^2 = BC^2 + CD^2$

$= (12)^2 + (5)^2$

$= 144 + 25$

$BD^2 = 169$

$BD = 13$ m



Area of $\triangle$BCD

$$= \frac{1}{2} \cdot BC \cdot CD = \left[\frac{1}{2} \cdot 12 \cdot 5\right] m^2 = 30\ m^2$$

For $\triangle$ABD

$$s = \frac{\text{perimeter}}{2} = \frac{(9+8+13)\ cm}{2} = 15\ cm$$

By Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Area of triangle

$$= \left[\sqrt{15(15-9)(15-8)(15-13)}\right] m^2$$

$$= \left(\sqrt{15 \times 6 \times 7 \times 2}\right) m^2$$

$$= 6\sqrt{35}\ m^2$$

$$= (6 \times 5.916) m^2$$

$$= 35.496\ m^2$$

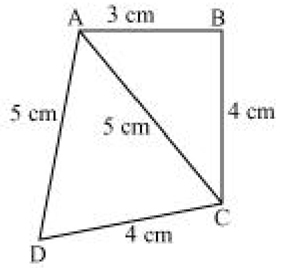
Area of park = Area of $\triangle$ABD + Area of $\triangle$BCD

$= 35.496 + 30\ m^2$

$= 65.496\ m^2$

$= 65.5\ m^2$ (approximately)

Solution 2



For $\triangle$ABC

$AC^2 = AB^2 + BC^2$

$(5)^2 = (3)^2 + (4)^2$

So, $\triangle$ABC is a right angle triangle, right angled at point B.

Area of $\triangle$ABC $= \frac{1}{2} \cdot AB \cdot BC = \frac{1}{2} \cdot 3 \cdot 4 = 6 \text{ cm}^2$

For $\triangle$DAC

Perimeter = 2s = DA + AC + CD = (5 + 5 + 4) cm = 14 cm

s = 7 cm

By Heron's formula

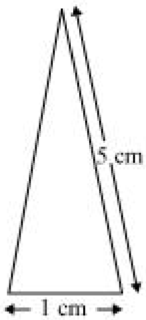
Area of triangle $\sqrt{s(s-a)(s-b)(s-c)} \text{ cm}^2$

$$\text{Area of } \Delta ADC = \left[\sqrt{7(7-5)(7-5)(7-4)}\right] \text{cm}^2$$
$$= \left(\sqrt{7 \times 2 \times 2 \times 3}\right) \text{cm}^2$$
$$= 2\sqrt{21} \text{ cm}^2$$
$$= (2 \times 4.583) \text{cm}^2$$
$$= 9.166 \text{ cm}^2$$

Area of ABCD = Area of $\triangle$ABC + Area of $\triangle$ACD

= (6 + 9.166) cm^2 = 15.166 cm^2 = 15.2 cm^2 (approximately)

Solution 3



For triangle I

This triangle is a isosceles triangle.

Perimeter = 2s = (5 + 5 + 1) cm = 11cm

$$s = \frac{11}{2} \text{ cm} = 5.5 \text{ cm}$$

$$= \left[\sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}\right] \text{cm}^2$$
$$= \left[\sqrt{(5.5)(0.5)(0.5)(4.5)}\right] \text{cm}^2$$
$$= 0.75\sqrt{11} \text{ cm}^2$$
$$= (0.75 \times 3.317) \text{cm}^2$$
$$= 2.488 \text{ cm}^2 \quad \text{(approximately)}$$

For quadrilateral II

This quadrilateral is a rectangle.

Area = $l \times b$ = (6.5 $\times$ 1) cm^2 = 6.5 cm^2

For quadrilateral III

This quadrilateral is a trapezium.

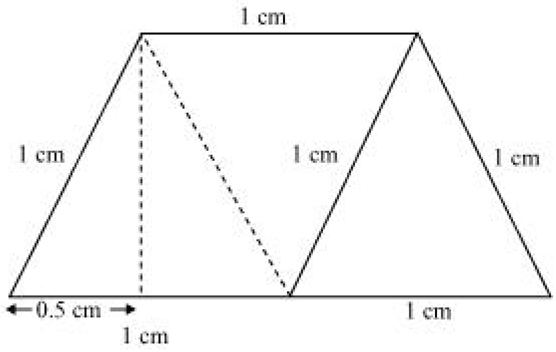
Perpendicular height of parallelogram $= \left[\sqrt{(1)^2 - (0.5)^2}\right] \text{ cm}$

$$= \sqrt{0.75}\ \text{cm} = 0.866\ \text{cm}$$

Area = Area of parallelogram + Area of equilateral triangle

$$= (0.866)1 + \frac{\sqrt{3}}{4}(1)^2$$

= 0.866 + 0.433 = 1.299 cm2



Area of triangle (iv) = Area of triangle in (v)

$$= \left(\frac{1}{2} \times 1.5 \times 6\right) \text{cm}^2 = 4.5\ \text{cm}^2$$

Total area of the paper used = 2.488 + 6.5 + 1.299 + 4.5 2
= 19.287 cm^2

Solution 4

For triangle

Perimeter of triangle = (26 + 28 + 30) cm = 84 cm

2s = 84 cm

s = 42 cm

By Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Area of triangle

$$= \left[\sqrt{42(42-26)(42-28)(42-30)}\right] \text{cm}^2$$

Area of triangle

$$= \left[\sqrt{42(16)(14)(12)}\right] \text{cm}^2 = 336\ \text{cm}^2$$

Let height of parallelogram be h

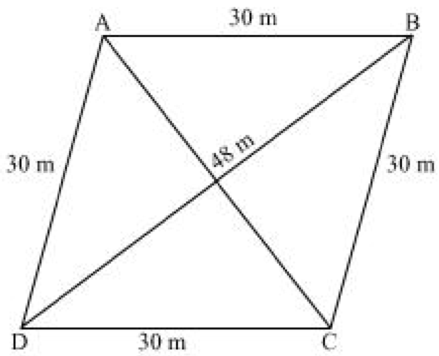
Area of parallelogram = Area of triangle

h 28 cm = 336 cm^2

h = 12 cm

So, height of the parallelogram is 12 cm.

Solution 5



Let ABCD be a rhombus shaped field.

For $\triangle$ BCD

$$s = \frac{(48+30+30)\text{m}}{2} = 54 \text{ m}$$

Semi perimeter,

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Area of triangle

$$= [\sqrt{54(54-48)(54-30)(54-30)}]$$

Therefore area of $\triangle$ BCD

$$= \sqrt{54(6)(24)(24)} = 3 \cdot 6 \cdot 24 = 432 \text{ m}^2$$

Area of field = 2 $\cdot$ Area of $\triangle$ BCD

= (2 $\cdot$ 432) m^2 = 864 m^2

Area for grazing for 1 cow= $\frac{864}{18}$ = 48 m^2

Each cow will be getting 48 m^2 area of grass field.

Solution 6

For each triangular piece

$$s = \frac{(20 + 50 + 50) \text{ cm}}{2} = 60 \text{ cm}$$

Semi perimeter

By Heron's formula

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Area of triangle

$$\text{Area of each triangular piece} = \left[\sqrt{60(60-50)(60-50)(60-20)}\right]\text{cm}^2$$

$$= \left[\sqrt{60(10)(10)(40)}\right]\text{cm}^2 = 200\sqrt{6} \text{ cm}^2$$

Since, there are 5 triangular pieces made of two different colours cloth.

$$= \left(5 \times 200\sqrt{6}\right)\text{cm}^2$$

So, area of each cloth required

$$= 1000\sqrt{6} \text{ cm}^2$$

Solution 7

We know that

Area of square $\frac{1}{2}$ (diagonal)2

$$= \frac{1}{2}(32 \text{ cm})^2 = 512 \text{ cm}^2$$

Area of given kite

Area of 1st shade = Area of 2nd shade

$$= \frac{512 \text{ cm}^2}{2} = 256 \text{ cm}^2$$

So, area of paper required in each shape = 256 cm2.

For IIIrd triangle

$$s = \frac{(6+6+8)\text{cm}}{2} = 10 \text{ cm}$$

Semi perimeter

By Heron's formula

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Area of IIIrd triangle $= \sqrt{10(10-6)(10-6)(10-8)}$

$$= \left(\sqrt{10\times4\times4\times2}\right)\text{cm}^2$$

$$= \left(4\times2\sqrt{5}\right)\text{cm}^2$$

$$= 8\sqrt{5}\ \text{cm}^2$$

$$= (8\times2.24)\,\text{cm}^2$$

$$= 17.92\ \text{cm}^2$$

Area of paper required for IIIrd shade = 17.92 cm^2

Solution 8

We may observe that
Semi perimeter of each triangular shaped tile

$$s = \frac{(35+28+9)\,\text{cm}}{2} = 36\ \text{cm}$$

By Heron's formula

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Area of each tile $= \left[\sqrt{36(36-35)(36-28)(36-9)}\right]\text{cm}^2$

$$= \left[\sqrt{36\times1\times8\times27}\right]\text{cm}^2$$

$$= 36\sqrt{6}\ \text{cm}^2$$

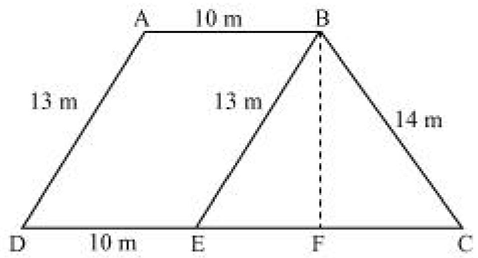
= (36 2.45) cm^2
=88.2 cm^2

Area of 16 tiles = (16 88.2) cm^2= 1411.2 cm^2

Cost of polishing per cm^2 area = 50 p

Cost of polishing 1411.2 cm^2 area = Rs. (1411.2 0.50) = Rs.705.60
So, it will cost Rs.705.60 while polishing all the tiles.

Solution 9



Draw a line BE parallel to AD and draw a perpendicular BF on CD.

Now we may observe that ABED is a parallelogram.
BE = AD = 13 m
ED = AB = 10 m
EC = 25 - ED = 15 m

For $\triangle$ BEC

Semi perimeter $$s = \frac{(13+14+15)\,\text{m}}{2} = 21\ \text{m}$$

By Heron's formula

Area of triangle $$= \sqrt{s(s-a)(s-b)(s-c)}$$

Area of $\triangle$ BEC $$= \left[\sqrt{21(21-13)(21-14)(21-15)}\right]\text{m}^2$$

$$= \left[\sqrt{21(8)(7)(6)}\right]$$ $\text{m}^2 = 84\ \text{m}^2$

Area of $\triangle$ BEC $$= \frac{1}{2}\times \text{CE} \times \text{BF}$$

$$84\ \text{cm}^2 = \frac{1}{2}\times 15\ \text{cm} \times \text{BF}$$

$$\text{BF} = \left(\frac{168}{15}\right)\ \text{cm} = 11.2\ \text{cm}$$

Area of ABED = BF DE = 11.2 10

= 112 m^2

Area of field = 84 + 112

= 196 m^2