Access answers to RD Sharma Solutions for Class 11 Maths Chapter 11 – Trigonometric Equations

1. Find the general solutions of the following equations:

(i) sin x = 1/2

(ii) cos x = – √3/2

(iii) cosex x = – √2

(iv) sec x = √2

(v) tan x = -1/√3

(vi) √3 sec x = 2

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) sin x = 1/2

We know sin 30° = sin π/6 = ½

So,

Sin x = sin π/6

∴ the general solution is

x = nπ + $(-1)^n$ π/6, where n ∈ Z. [since, sin x = sin A => x = nπ + $(-1)^n$ A]

(ii) cos x = – √3/2

We know, cos 150° = (- √3/2) = cos 5π/6

So,

Cos x = cos 5π/6

∴ the general solution is

x = 2nπ ± 5π/6, where n ϵ Z.

(iii) cosex x = – √2

Let us simplify,

$1/\sin x = -\sqrt{2}$ [since, $\operatorname{cosec} x = 1/\sin x$]

$\sin x = -1/\sqrt{2}$

$= \sin [\pi + \pi/4]$

$= \sin 5\pi/4$ or $\sin (-\pi/4)$

∴ the general solution is

$x = n\pi + (-1)^{n+1} \pi/4$, where $n \in Z$.

(iv) $\sec x = \sqrt{2}$

Let us simplify,

$1/\cos x = \sqrt{2}$ [since, $\sec x = 1/\cos x$]

$\cos x = 1/\sqrt{2}$

$= \cos \pi/4$

∴ the general solution is

$x = 2n\pi \pm \pi/4$, where $n \in Z$.

(v) $\tan x = -1/\sqrt{3}$

Let us simplify,

$\tan x = -1/\sqrt{3}$

$\tan x = \tan (\pi/6)$

$= \tan (-\pi/6)$ [since, $\tan (-x) = -\tan x$]

∴ the general solution is

$x = n\pi + (-\pi/6)$, where $n \in Z$.

or $x = n\pi - \pi/6$, where $n \in Z$.

(vi) $\sqrt{3} \sec x = 2$

Let us simplify,

$\sec x = 2/\sqrt{3}$

$1/\cos x = 2/\sqrt{3}$

$\cos x = \sqrt{3}/2$

$= \cos (\pi/6)$

∴ the general solution is

$x = 2n\pi \pm \pi/6$, where $n \in Z$.

2. Find the general solutions of the following equations:

(i) sin 2x = √3/2

(ii) cos 3x = 1/2

(iii) sin 9x = sin x

(iv) sin 2x = cos 3x

(v) tan x + cot 2x = 0

(vi) tan 3x = cot x

(vii) tan 2x tan x = 1

(viii) tan mx + cot nx = 0

(ix) tan px = cot qx

(x) sin 2x + cos x = 0

(xi) sin x = tan x

(xii) sin 3x + cos 2x = 0

Solution:

The general solution of any trigonometric equation is given as:

$\sin x = \sin y$, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

$\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in Z$.

$\tan x = \tan y$, implies $x = n\pi + y$, where $n \in Z$.

(i) $\sin 2x = \sqrt{3}/2$

Let us simplify,

$\sin 2x = \sqrt{3}/2$

$= \sin(\pi/3)$

∴ the general solution is

$2x = n\pi + (-1)^n \pi/3$, where $n \in Z$.

$x = n\pi/2 + (-1)^n \pi/6$, where $n \in Z$.

(ii) $\cos 3x = 1/2$

Let us simplify,

cos 3x = 1/2

= cos (π/3)

∴ the general solution is

$3x = 2n\pi \pm \pi/3$, where $n \in Z$.

$x = 2n\pi/3 \pm \pi/9$, where $n \in Z$.

(iii) sin 9x = sin x

Let us simplify,

Sin 9x – sin x = 0

Using transformation formula,

Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

So,

= 2 cos (9x+x)/2 sin (9x-x)/2

=> cos 5x sin 4x = 0

Cos 5x = 0 or sin 4x = 0

Let us verify both the expressions,

Cos 5x = 0

Cos 5x = cos π/2

5x = (2n + 1)π/2

x = (2n + 1)π/10, where $n \in Z$.

sin 4x = 0

sin 4x = sin 0

4x = nπ

x = nπ/4, where $n \in Z$.

∴ the general solution is

x = (2n + 1)π/10 or nπ/4, where $n \in Z$.

(iv) sin 2x = cos 3x

Let us simplify,

sin 2x = cos 3x

cos (π/2 – 2x) = cos 3x [since, sin A = cos (π/2 – A)]

π/2 – 2x = 2nπ ± 3x

π/2 – 2x = 2nπ + 3x [or] π/2 – 2x = 2nπ – 3x

5x = π/2 + 2nπ [or] x = 2nπ – π/2

5x = π/2 (1 + 4n) [or] x = π/2 (4n – 1)

x = π/10 (1 + 4n) [or] x = π/2 (4n – 1)

∴ the general solution is

x = π/10 (4n + 1) [or] x = π/2 (4n – 1), where n ϵ Z.

(v) tan x + cot 2x = 0

Let us simplify,

tan x = – cot 2x

tan x = – tan (π/2 – 2x) [since, cot A = tan (π/2 – A)]

tan x = tan (2x – π/2) [since, – tan A = tan -A]

x = nπ + 2x – π/2

x = nπ – π/2

∴ the general solution is

x = nπ – π/2, where n ϵ Z.

(vi) tan 3x = cot x

Let us simplify,

tan 3x = cot x

tan 3x = tan (π/2 – x) [since, cot A = tan (π/2 – A)]

3x = nπ + π/2 – x

4x = nπ + π/2

x = nπ/4 + π/8

∴ the general solution is

x = nπ/4 + π/8, where n ϵ Z.

(vii) tan 2x tan x = 1

Let us simplify,

$\tan 2x \tan x = 1$

$\tan 2x = 1/\tan x$

$= \cot x$

$\tan 2x = \tan (\pi/2 – x)$ [since, $\cot A = \tan (\pi/2 – A)$]

$2x = n\pi + \pi/2 – x$

$3x = n\pi + \pi/2$

$x = n\pi/3 + \pi/6$

∴ the general solution is

$x = n\pi/3 + \pi/6$, where $n \in Z$.

(viii) $\tan mx + \cot nx = 0$

Let us simplify,

$\tan mx + \cot nx = 0$

$\tan mx = – \cot nx$

$= – \tan (\pi/2 – nx)$ [since, $\cot A = \tan (\pi/2 – A)$]

$\tan mx = \tan (nx + \pi/2)$ [since, $– \tan A = \tan -A$]

$mx = k\pi + nx + \pi/2$

$(m – n) x = k\pi + \pi/2$

$(m – n) x = \pi (2k + 1)/2$

$x = \pi (2k + 1)/2(m – n)$

∴ the general solution is

$x = \pi (2k + 1)/2(m – n)$, where $m, n, k \in Z$.

(ix) $\tan px = \cot qx$

Let us simplify,

$\tan px = \cot qx$

$\tan px = \tan (\pi/2 – qx)$ [since, $\cot A = \tan (\pi/2 – A)$]

$px = n\pi \pm (\pi/2 – qx)$

$(p + q) x = n\pi + \pi/2$

$x = n\pi/(p+q) + \pi/2(p+q)$

$= \pi\ (2n + 1)/\ 2(p+q)$

∴ the general solution is

$x = \pi\ (2n + 1)/\ 2(p+q)$, where $n \in Z$.

(x) $\sin 2x + \cos x = 0$

Let us simplify,

$\sin 2x + \cos x = 0$

$\cos x = -\sin 2x$

$\cos x = -\cos (\pi/2 - 2x)$ [since, $\sin A = \cos (\pi/2 - A)$]

$= \cos (\pi - (\pi/2 - 2x))$ [since, $-\cos A = \cos (\pi - A)$]

$= \cos (\pi/2 + 2x)$

$x = 2n\pi \pm (\pi/2 + 2x)$

So,

$x = 2n\pi + (\pi/2 + 2x)$ [or] $x = 2n\pi - (\pi/2 + 2x)$

$x = -\pi/2 - 2n\pi$ [or] $3x = 2n\pi - \pi/2$

$x = -\pi/2\ (1 + 4n)$ [or] $x = \pi/6\ (4n - 1)$

∴ the general solution is

$x = -\pi/2\ (1 + 4n)$, where $n \in Z$. [or] $x = \pi/6\ (4n - 1)$

$x = \pi/2\ (4n - 1)$, where $n \in Z$. [or] $x = \pi/6\ (4n - 1)$, where $n \in Z$.

(xi) $\sin x = \tan x$

Let us simplify,

$\sin x = \tan x$

$\sin x = \sin x/\cos x$

$\sin x \cos x = \sin x$

$\sin x (\cos x - 1) = 0$

So,

$\text{Sin } x = 0$ or $\cos x - 1 = 0$

$\text{Sin } x = \sin 0$ [or] $\cos x = 1$

Sin x = sin 0 [or] cos x = cos 0

x = nπ [or] x = 2mπ

∴ the general solution is

x = nπ [or] 2mπ, where n, m ϵ Z.

(xii) sin 3x + cos 2x = 0

Let us simplify,

sin 3x + cos 2x = 0

cos 2x = – sin 3x

cos 2x = – cos (π/2 – 3x) [since, sin A = cos (π/2 – A)]

cos 2x = cos (π – (π/2 – 3x)) [since, -cos A = cos (π – A)]

cos 2x = cos (π/2 + 3x)

2x = 2nπ ± (π/2 + 3x)

So,

2x = 2nπ + (π/2 + 3x) [or] 2x = 2nπ – (π/2 + 3x)

x = -π/2 – 2nπ [or] 5x = 2nπ – π/2

x = -π/2 (1 + 4n) [or] x = π/10 (4n – 1)

x = – π/2 (4n + 1) [or] π/10 (4n – 1)

∴ the general solution is

x = – π/2 (4n + 1) [or] π/10 (4n – 1)

x = π/2 (4n – 1) [or] π/10 (4n – 1), where n ϵ Z.

3. Solve the following equations:

(i) $\sin^2 x$ – cos x = 1/4

(ii) 2 $\cos^2 x$ – 5 cos x + 2 = 0

(iii) 2 $\sin^2 x$ + √3 cos x + 1 = 0

(iv) 4 $\sin^2 x$ – 8 cos x + 1 = 0

(v) $\tan^2 x$ + (1 – √3) tan x – √3 = 0

(vi) 3 $\cos^2 x$ – 2√3 sin x cos x – 3 $\sin^2 x$ = 0

(vii) cos 4x = cos 2x

Solution:

The general solution of any trigonometric equation is given as:

$\sin x = \sin y$, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

$\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in Z$.

$\tan x = \tan y$, implies $x = n\pi + y$, where $n \in Z$.

(i) $\sin^2 x - \cos x = 1/4$

Let us simplify,

$\sin^2 x - \cos x = ¼$

$1 - \cos^2 x - \cos x = 1/4$ [since, $\sin^2 x = 1 - \cos^2 x$]

$4 - 4\cos^2 x - 4\cos x = 1$

$4\cos^2 x + 4\cos x - 3 = 0$

Let cos x be 'k'

So,

$4k^2 + 4k - 3 = 0$

$4k^2 - 2k + 6k - 3 = 0$

$2k(2k - 1) + 3(2k - 1) = 0$

$(2k - 1) + (2k + 3) = 0$

$(2k - 1) = 0$ or $(2k + 3) = 0$

$k = 1/2$ or $k = -3/2$

$\cos x = 1/2$ or $\cos x = -3/2$

we shall consider only $\cos x = 1/2$. $\cos x = -3/2$ is not possible.

so,

$\cos x = \cos 60^o = \cos \pi/3$

$x = 2n\pi \pm \pi/3$

∴ the general solution is

$x = 2n\pi \pm \pi/3$, where $n \in Z$.

(ii) $2\cos^2 x - 5\cos x + 2 = 0$

Let us simplify,

$2 \cos^2 x – 5 \cos x + 2 = 0$

Let cos x be 'k'

$2k^2 – 5k + 2 = 0$

$2k^2 – 4k – k +2 = 0$

$2k(k – 2) -1(k -2) = 0$

$(k – 2) (2k – 1) = 0$

$k = 2$ or $k = 1/2$

$\cos x = 2$ or $\cos x = 1/2$

we shall consider only $\cos x = 1/2$. $\cos x = 2$ is not possible.

so,

$\cos x = \cos 60^o = \cos \pi/3$

$x = 2n\pi \pm \pi/3$

∴ the general solution is

$x = 2n\pi \pm \pi/3$, where $n \in Z$.

(iii) $2 \sin^2 x + \sqrt{3} \cos x + 1 = 0$

Let us simplify,

$2 \sin^2 x + \sqrt{3} \cos x + 1 = 0$

$2 (1 – \cos^2 x) + \sqrt{3} \cos x + 1 = 0$ [since, $\sin^2 x = 1 – \cos^2 x$]

$2 – 2 \cos^2 x + \sqrt{3} \cos x + 1 = 0$

$2 \cos^2 x – \sqrt{3} \cos x – 3 = 0$

Let cos x be 'k'

$2k^2 – \sqrt{3} k – 3 = 0$

$2k^2 -2\sqrt{3} k + \sqrt{3} k – 3 = 0$

$2k(k – \sqrt{3}) +\sqrt{3}(k – \sqrt{3}) = 0$

$(2k + \sqrt{3}) (k – \sqrt{3}) = 0$

$k = \sqrt{3}$ or $k = -\sqrt{3}/2$

$\cos x = \sqrt{3}$ or $\cos x = -\sqrt{3}/2$

we shall consider only $\cos x = -\sqrt{3}/2$. $\cos x = \sqrt{3}$ is not possible.

so,

$\cos x = -\sqrt{3}/2$

$\cos x = \cos 150° = \cos 5\pi/6$

$x = 2n\pi \pm 5\pi/6$, where $n \in Z$.

(iv) $4 \sin^2 x - 8 \cos x + 1 = 0$

Let us simplify,

$4 \sin^2 x - 8 \cos x + 1 = 0$

$4 (1 - \cos^2 x) - 8 \cos x + 1 = 0$ [since, $\sin^2 x = 1 - \cos^2 x$]

$4 - 4 \cos^2 x - 8 \cos x + 1 = 0$

$4 \cos^2 x + 8 \cos x - 5 = 0$

Let cos x be 'k'

$4k^2 + 8k - 5 = 0$

$4k^2 - 2k + 10k - 5 = 0$

$2k(2k - 1) + 5(2k - 1) = 0$

$(2k + 5)(2k - 1) = 0$

$k = -5/2 = -2.5$ or $k = 1/2$

$\cos x = -2.5$ or $\cos x = 1/2$

we shall consider only $\cos x = 1/2$. $\cos x = -2.5$ is not possible.

so,

$\cos x = \cos 60^o = \cos \pi/3$

$x = 2n\pi \pm \pi/3$

∴ the general solution is

$x = 2n\pi \pm \pi/3$, where $n \in Z$.

(v) $\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$

Let us simplify,

$\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$

$\tan^2 x + \tan x - \sqrt{3} \tan x - \sqrt{3} = 0$

$\tan x (\tan x + 1) - \sqrt{3} (\tan x + 1) = 0$

$(\tan x + 1)(\tan x – \sqrt{3}) = 0$

$\tan x = -1$ or $\tan x = \sqrt{3}$

As, $\tan x \in (-\infty, \infty)$ so both values are valid and acceptable.

$\tan x = \tan(-\pi/4)$ or $\tan x = \tan(\pi/3)$

$x = m\pi – \pi/4$ or $x = n\pi + \pi/3$

∴ the general solution is

$x = m\pi – \pi/4$ or $n\pi + \pi/3$, where $m, n \in Z$.

(vi) $3 \cos^2 x – 2\sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$

Let us simplify,

$3 \cos^2 x – 2\sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$

$3 \cos^2 x – 3\sqrt{3} \sin x \cos x + \sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$

$3 \cos x (\cos x – \sqrt{3}\sin x) + \sqrt{3} \sin x (\cos x – \sqrt{3} \sin x) = 0$

$\sqrt{3} (\cos x – \sqrt{3} \sin x) (\sqrt{3} \cos x + \sin x) = 0$

$\cos x – \sqrt{3} \sin x = 0$ or $\sin x + \sqrt{3} \cos x = 0$

$\cos x = \sqrt{3} \sin x$ or $\sin x = -\sqrt{3} \cos x$

$\tan x = 1/\sqrt{3}$ or $\tan x = -\sqrt{3}$

As, $\tan x \in (-\infty, \infty)$ so both values are valid and acceptable.

$\tan x = \tan(\pi/6)$ or $\tan x = \tan(-\pi/3)$

$x = m\pi + \pi/6$ or $x = n\pi – \pi/3$

∴ the general solution is

$x = m\pi + \pi/6$ or $n\pi – \pi/3$, where $m, n \in Z$.

(vii) $\cos 4x = \cos 2x$

Let us simplify,

$\cos 4x = \cos 2x$

$4x = 2n\pi \pm 2x$

So,

$4x = 2n\pi + 2x$ [or] $4x = 2n\pi – 2x$

$2x = 2n\pi$ [or] $6x = 2n\pi$

$x = n\pi$ [or] $x = n\pi/3$

∴ the general solution is

$x = n\pi$ [or] $n\pi/3$, where $n \in Z$.

4. Solve the following equations:

(i) $\cos x + \cos 2x + \cos 3x = 0$

(ii) $\cos x + \cos 3x - \cos 2x = 0$

(iii) $\sin x + \sin 5x = \sin 3x$

(iv) $\cos x \cos 2x \cos 3x = 1/4$

(v) $\cos x + \sin x = \cos 2x + \sin 2x$

(vi) $\sin x + \sin 2x + \sin 3x = 0$

(vii) $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

(viii) $\sin 3x - \sin x = 4 \cos^2 x - 2$

(ix) $\sin 2x - \sin 4x + \sin 6x = 0$

Solution:

The general solution of any trigonometric equation is given as:

$\sin x = \sin y$, implies $x = n\pi + (-1)^n y$, where $n \in Z$.

$\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in Z$.

$\tan x = \tan y$, implies $x = n\pi + y$, where $n \in Z$.

(i) $\cos x + \cos 2x + \cos 3x = 0$

Let us simplify,

$\cos x + \cos 2x + \cos 3x = 0$

we shall rearrange and use transformation formula

$\cos 2x + (\cos x + \cos 3x) = 0$

by using the formula, $\cos A + \cos B = 2 \cos (A+B)/2 \cos (A-B)/2$

$\cos 2x + 2 \cos (3x+x)/2 \cos (3x-x)/2 = 0$

$\cos 2x + 2\cos 2x \cos x = 0$

$\cos 2x (1 + 2 \cos x) = 0$

$\cos 2x = 0$ or $1 + 2\cos x = 0$

$\cos 2x = \cos 0$ or $\cos x = -1/2$

$\cos 2x = \cos \pi/2$ or $\cos x = \cos (\pi - \pi/3)$

$\cos 2x = \cos \pi/2$ or $\cos x = \cos (2\pi/3)$

$2x = (2n + 1) \pi/2$ or $x = 2m\pi \pm 2\pi/3$

$x = (2n + 1) \pi/4$ or $x = 2m\pi \pm 2\pi/3$

∴ the general solution is

$x = (2n + 1) \pi/4$ or $2m\pi \pm 2\pi/3$, where $m, n \in Z$.

(ii) $\cos x + \cos 3x - \cos 2x = 0$

Let us simplify,

$\cos x + \cos 3x - \cos 2x = 0$

we shall rearrange and use transformation formula

$\cos x - \cos 2x + \cos 3x = 0$

$- \cos 2x + (\cos x + \cos 3x) = 0$

By using the formula, $\cos A + \cos B = 2 \cos (A+B)/2 \cos (A-B)/2$

$- \cos 2x + 2 \cos (3x+x)/2 \cos (3x-x)/2 = 0$

$- \cos 2x + 2\cos 2x \cos x = 0$

$\cos 2x (-1 + 2 \cos x) = 0$

$\cos 2x = 0$ or $-1 + 2\cos x = 0$

$\cos 2x = \cos 0$ or $\cos x = 1/2$

$\cos 2x = \cos \pi/2$ or $\cos x = \cos (\pi/3)$

$2x = (2n + 1) \pi/2$ or $x = 2m\pi \pm \pi/3$

$x = (2n + 1) \pi/4$ or $x = 2m\pi \pm \pi/3$

∴ the general solution is

$x = (2n + 1) \pi/4$ or $2m\pi \pm \pi/3$, where $m, n \in Z$.

(iii) $\sin x + \sin 5x = \sin 3x$

Let us simplify,

$\sin x + \sin 5x = \sin 3x$

$\sin x + \sin 5x – \sin 3x = 0$

we shall rearrange and use transformation formula

$– \sin 3x + \sin x + \sin 5x = 0$

$– \sin 3x + (\sin 5x + \sin x) = 0$

By using the formula, $\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

$– \sin 3x + 2 \sin (5x+x)/2 \cos (5x-x)/2 = 0$

$2\sin 3x \cos 2x – \sin 3x = 0$

$\sin 3x (2\cos 2x – 1) = 0$

$\sin 3x = 0$ or $2\cos 2x – 1 = 0$

$\sin 3x = \sin 0$ or $\cos 2x = 1/2$

$\sin 3x = \sin 0$ or $\cos 2x = \cos \pi/3$

$3x = n\pi$ or $2x = 2m\pi \pm \pi/3$

$x = n\pi/3$ or $x = m\pi \pm \pi/6$

∴ the general solution is

$x = n\pi/3$ or $m\pi \pm \pi/6$, where $m, n \in Z$.

(iv) $\cos x \cos 2x \cos 3x = 1/4$

Let us simplify,

$\cos x \cos 2x \cos 3x = 1/4$

$4 \cos x \cos 2x \cos 3x – 1 = 0$

By using the formula,

$2 \cos A \cos B = \cos (A + B) + \cos (A – B)$

$2(2\cos x \cos 3x) \cos 2x – 1 = 0$

$2(\cos 4x + \cos 2x) \cos 2x – 1 = 0$

$2(2\cos^2 2x – 1 + \cos 2x) \cos 2x – 1 = 0$ [using $\cos 2A = 2\cos^2 A – 1$]

$4\cos^3 2x – 2\cos 2x + 2\cos^2 2x – 1 = 0$

$2\cos^2 2x (2\cos 2x + 1) -1(2\cos 2x + 1) = 0$

$(2\cos^2 2x – 1)(2 \cos 2x + 1) = 0$

So,

$2\cos 2x + 1 = 0$ or $(2\cos^2 2x – 1) = 0$

$\cos 2x = -1/2$ or $\cos 4x = 0$ [using $\cos 2θ = 2\cos^2θ – 1$]

$\cos 2x = \cos (π – π/3)$ or $\cos 4x = \cos π/2$

$\cos 2x = \cos 2π/3$ or $\cos 4x = \cos π/2$

$2x = 2mπ ± 2π/3$ or $4x = (2n + 1) π/2$

$x = mπ ± π/3$ or $x = (2n + 1) π/8$

∴ the general solution is

$x = mπ ± π/3$ or $(2n + 1) π/8$, where $m, n \in Z$.

(v) $\cos x + \sin x = \cos 2x + \sin 2x$

Let us simplify,

$\cos x + \sin x = \cos 2x + \sin 2x$

upon rearranging we get,

$\cos x – \cos 2x = \sin 2x – \sin x$

By using the formula,

$\sin A – \sin B = 2 \cos (A+B)/2 \sin (A-B)/2$

$\cos A – \cos B = – 2 \sin (A+B)/2 \sin (A-B)/2$

So,

$-2 \sin (2x+x)/2 \sin (2x-x)/2 = 2 \cos (2x+x)/2 \sin (2x-x)/2$

$2 \sin 3x/2 \sin x/2 = 2 \cos 3x/2 \sin x/2$

$\text{Sin } x/2 (\sin 3x/2 – \cos 3x/2) = 0$

So,

$\text{Sin } x/2 = 0$ or $\sin 3x/2 = \cos 3x/2$

$\text{Sin } x/2 = \sin mπ$ or $\sin 3x/2 / \cos 3x/2 = 0$

$\text{Sin } x/2 = \sin mπ$ or $\tan 3x/2 = 1$

$\text{Sin } x/2 = \sin mπ$ or $\tan 3x/2 = \tan π/4$

$x/2 = mπ$ or $3x/2 = nπ + π/4$

$x = 2m\pi$ or $x = 2n\pi/3 + \pi/6$

∴ the general solution is

$x = 2m\pi$ or $2n\pi/3 + \pi/6$, where $m, n \in Z$.

(vi) $\sin x + \sin 2x + \sin 3x = 0$

Let us simplify,

$\sin x + \sin 2x + \sin 3x = 0$

we shall rearrange and use transformation formula

$\sin 2x + \sin x + \sin 3x = 0$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

So,

$\text{Sin } 2x + 2 \sin (3x+x)/2 \cos (3x-x)/2 = 0$

$\text{Sin } 2x + 2\sin 2x \cos x = 0$

$\text{Sin } 2x (2 \cos x + 1) = 0$

$\text{Sin } 2x = 0$ or $2\cos x + 1 = 0$

$\text{Sin } 2x = \sin 0$ or $\cos x = -1/2$

$\text{Sin } 2x = \sin 0$ or $\cos x = \cos (\pi - \pi/3)$

$\text{Sin } 2x = \sin 0$ or $\cos x = \cos 2\pi/3$

$2x = n\pi$ or $x = 2m\pi \pm 2\pi/3$

$x = n\pi/2$ or $x = 2m\pi \pm 2\pi/3$

∴ the general solution is

$x = n\pi/2$ or $2m\pi \pm 2\pi/3$, where $m, n \in Z$.

(vii) $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

Let us simplify,

$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

we shall rearrange and use transformation formula

$\sin x + \sin 3x + \sin 2x + \sin 4x = 0$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

So,

$2 \sin (3x+x)/2 \cos (3x-x)/2 + 2 \sin (4x+2x)/2 \cos (4x-2x)/2 = 0$

$2 \sin 2x \cos x + 2 \sin 3x \cos x = 0$

$2\cos x (\sin 2x + \sin 3x) = 0$

Again by using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

we get,

$2\cos x (2 \sin (3x+2x)/2 \cos (3x-2x)/2) = 0$

$2\cos x (2 \sin 5x/2 \cos x/2) = 0$

$4 \cos x \sin 5x/2 \cos x/2 = 0$

So,

$\cos x = 0$ or $\sin 5x/2 = 0$ or $\cos x/2 = 0$

$\cos x = \cos 0$ or $\sin 5x/2 = \sin 0$ or $\cos x/2 = \cos 0$

$\cos x = \cos \pi/2$ or $\sin 5x/2 = k\pi$ or $\cos x/2 = \cos (2p + 1) \pi/2$

$x = (2n + 1) \pi/2$ or $5x/2 = k\pi$ or $x/2 = (2p + 1) \pi/2$

$x = (2n + 1) \pi/2$ or $x = 2k\pi/5$ or $x = (2p + 1)$

$x = n\pi + \pi/2$ or $x = 2k\pi/5$ or $x = (2p + 1)$

∴ the general solution is

$x = n\pi + \pi/2$ or $x = 2k\pi/5$ or $x = (2p + 1)$, where $n, k, p \in Z$.

(viii) $\sin 3x – \sin x = 4 \cos^2 x – 2$

Let us simplify,

$\sin 3x – \sin x = 4 \cos^2 x – 2$

$\sin 3x – \sin x = 2(2 \cos^2 x – 1)$

$\sin 3x – \sin x = 2 \cos 2x$ [since, $\cos 2A = 2\cos^2 A – 1$]

By using the formula,

$\sin A – \sin B = 2 \cos (A+B)/2 \sin (A-B)/2$

So,

$2 \cos (3x+x)/2 \sin (3x-x)/2 = 2 \cos 2x$

$2 \cos 2x \sin x – 2 \cos 2x = 0$

$2 \cos 2x (\sin x – 1) = 0$

Then,

$2 \cos 2x = 0$ or $\sin x – 1 = 0$

$\text{Cos } 2x = 0$ or $\sin x = 1$

$\text{Cos } 2x = \cos 0$ or $\sin x = \sin 1$

$\text{Cos } 2x = \cos 0$ or $\sin x = \sin \pi/2$

$2x = (2n + 1) \pi/2$ or $x = m\pi + (-1)^m \pi/2$

$x = (2n + 1) \pi/4$ or $x = m\pi + (-1)^m \pi/2$

∴ the general solution is

$x = (2n + 1) \pi/4$ or $m\pi + (-1)^m \pi/2$, where $m, n \in Z$.

(ix) $\sin 2x – \sin 4x + \sin 6x = 0$

Let us simplify,

$\sin 2x – \sin 4x + \sin 6x = 0$

we shall rearrange and use transformation formula

$– \sin 4x + \sin 6x + \sin 2x = 0$

By using the formula,

$\sin A + \sin B = 2 \sin (A+B)/2 \cos (A-B)/2$

we get,

$– \sin 4x + 2 \sin (6x+2x)/2 \cos (6x-2x)/2 = 0$

$– \sin 4x + 2 \sin 4x \cos 2x = 0$

$\text{Sin } 4x (2 \cos 2x – 1) = 0$

So,

$\text{Sin } 4x = 0$ or $2 \cos 2x – 1 = 0$

$\text{Sin } 4x = \sin 0$ or $\cos 2x = 1/2$

$\text{Sin } 4x = \sin 0$ or $\cos 2x = \pi/3$

$4x = n\pi$ or $2x = 2m\pi \pm \pi/3$

$x = n\pi/4$ or $x = m\pi \pm \pi/6$

∴ the general solution is

$x = n\pi/4$ or $m\pi \pm \pi/6$, where m, n ϵ Z.

5. Solve the following equations:

(i) tan x + tan 2x + tan 3x = 0

(ii) tan x + tan 2x = tan 3x

(iii) tan 3x + tan x = 2 tan 2x

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) tan x + tan 2x + tan 3x = 0

Let us simplify,

tan x + tan 2x + tan 3x = 0

tan x + tan 2x + tan (x + 2x) = 0

By using the formula,

tan (A+B) = [tan A + tan B] / [1 – tan A tan B]

So,

tan x + tan 2x + [[tan x + tan 2x]/[1- tan x tan 2x]] = 0

(tan x + tan 2x) (1 + 1/(1- tan x tan 2x)) = 0

(tan x + tan 2x) ([2 – tan x tan 2x] / [1 – tan x tan 2x]) = 0

Then,

(tan x + tan 2x) = 0 or ([2 – tan x tan 2x] / [1 – tan x tan 2x]) = 0

(tan x + tan 2x) = 0 or [2 – tan x tan 2x] = 0

tan x = tan (-2x) or tan x tan 2x = 2

x = nπ + (-2x) or tax x [2tan x/(1 – $\tan^2$ x)] = 2 [Using, tan 2x = 2 tan x / 1-$\tan^2$ x]

$3x = n\pi$ or $2\tan^2 x / (1-\tan^2 x) = 2$

$3x = n\pi$ or $2\tan^2 x = 2(1 - \tan^2 x)$

$3x = n\pi$ or $2\tan^2 x = 2 - 2\tan^2 x$

$3x = n\pi$ or $4\tan^2 x = 2$

$x = n\pi/3$ or $\tan^2 x = 2/4$

$x = n\pi/3$ or $\tan^2 x = 1/2$

$x = n\pi/3$ or $\tan x = 1/\sqrt{2}$

$x = n\pi/3$ or $x = \tan \alpha$ [let $1/\sqrt{2}$ be 'α']

$x = n\pi/3$ or $x = m\pi + \alpha$

∴ the general solution is

$x = n\pi/3$ or $m\pi + \alpha$, where $\alpha = \tan^{-1} 1/\sqrt{2}$, $m, n \in Z$.

(ii) $\tan x + \tan 2x = \tan 3x$

Let us simplify,

$\tan x + \tan 2x = \tan 3x$

$\tan x + \tan 2x - \tan 3x = 0$

$\tan x + \tan 2x - \tan (x + 2x) = 0$

By using the formula,

$\tan (A+B) = [\tan A + \tan B] / [1 - \tan A \tan B]$

So,

$\tan x + \tan 2x - [[\tan x + \tan 2x]/[1 - \tan x \tan 2x]] = 0$

$(\tan x + \tan 2x) (1 - 1/(1 - \tan x \tan 2x)) = 0$

$(\tan x + \tan 2x) ([- \tan x \tan 2x] / [1 - \tan x \tan 2x]) = 0$

Then,

$(\tan x + \tan 2x) = 0$ or $([- \tan x \tan 2x] / [1 - \tan x \tan 2x]) = 0$

$(\tan x + \tan 2x) = 0$ or $[- \tan x \tan 2x] = 0$

$\tan x = \tan (-2x)$ or $-\tan x \tan 2x = 0$

$\tan x = \tan (-2x)$ or $2\tan^2 x / (1 - \tan^2 x) = 0$ [Using, $\tan 2x = 2 \tan x / 1-\tan^2 x$]

$x = n\pi + (-2x)$ or $x = m\pi + 0$

$3x = n\pi$ or $x = m\pi$

$x = n\pi/3$ or $x = m\pi$

∴ the general solution is

$x = n\pi/3$ or $m\pi$, where $m, n \in Z$.

(iii) $\tan 3x + \tan x = 2 \tan 2x$

Let us simplify,

$\tan 3x + \tan x = 2 \tan 2x$

$\tan 3x + \tan x = \tan 2x + \tan 2x$

upon rearranging we get,

$\tan 3x – \tan 2x = \tan 2x – \tan x$

By using the formula,

$\tan (A-B) = [\tan A – \tan B] / [1 + \tan A \tan B]$

so,

$[(\tan 3x – \tan 2x) (1+\tan 3x \tan 2x)] / [1 + \tan 3x \tan 2x] = [(\tan 2x-\tan x) (1+\tan x \tan 2x)] / [1 + \tan 2x \tan x]$

$\tan (3x – 2x) (1 + \tan 3x \tan 2x) = \tan (2x – x) (1 + \tan x \tan 2x)$

$\tan x [1 + \tan 3x \tan 2x – 1 – \tan 2x \tan x] = 0$

$\tan x \tan 2x (\tan 3x – \tan x) = 0$

so,

$\tan x = 0$ or $\tan 2x = 0$ or $(\tan 3x – \tan x) = 0$

$\tan x = 0$ or $\tan 2x = 0$ or $\tan 3x = \tan x$

$x = n\pi$ or $2x = m\pi$ or $3x = k\pi + x$

$x = n\pi$ or $x = m\pi/2$ or $2x = k\pi$

$x = n\pi$ or $x = m\pi/2$ or $x = k\pi/2$

∴ the general solution is

$x = n\pi$ or $m\pi/2$ or $k\pi/2$, where, $m, n, k \in Z$.