RD SHARMA Solutions for Class 9 Maths Chapter 3 - Rationalisation

Chapter 3 - Rationalisation Exercise Ex. 3.2

Question 1

Simplify: $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\dfrac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

Solution 1

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$
$$=\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$
$$=\frac{\left(3\sqrt{2}-2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}+\frac{\sqrt{12}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$
$$=\frac{18-12\sqrt{6}+12}{18-12}+\frac{6+2\sqrt{6}}{3-2}$$
$$=\frac{30-12\sqrt{6}}{6}+6+2\sqrt{6}$$
$$=\frac{30-12\sqrt{6}+36+12\sqrt{6}}{6}$$
$$=\frac{66}{6}$$
$$=11$$

Question 2

Simplify: $\dfrac{7+3\sqrt{5}}{3+\sqrt{5}}-\dfrac{7-3\sqrt{5}}{3-\sqrt{5}}$

Solution 2

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{21-7\sqrt{5}+9\sqrt{5}-15}{(3)^2-\left(\sqrt{5}\right)^2} - \frac{21+7\sqrt{5}-9\sqrt{5}-15}{(3)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{6+2\sqrt{5}}{9-5} - \frac{6-2\sqrt{5}}{9-5}$$

$$= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$$

$$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$$

$$= \frac{4\sqrt{5}}{4}$$

$$= \sqrt{5}$$

Question 3

Rationalise the denominator of $\frac{3}{\sqrt{5}}$

Solution 3

We have,

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{3\sqrt{5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{3\sqrt{5}}{5}$$

$$\therefore \quad \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

Question 4

Rationalise the denominator of $\frac{3}{2\sqrt{5}}$

Solution 4

We have,

$$\frac{3}{2\sqrt{5}} = \frac{3}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{3\sqrt{5}}{2\left(\sqrt{5}\right)^2}$$

$$= \frac{3\sqrt{5}}{2 \times 5}$$

$$= \frac{3\sqrt{5}}{10}$$

$$\therefore \quad \frac{3}{2\sqrt{5}} = \frac{3\sqrt{5}}{10}$$

Question 5

Rationalise the denominator of $\frac{1}{\sqrt{12}}$

Solution 5

We have,

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{4 \times 3}} = \frac{1}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{3}}{6}$$

$$\therefore \quad \frac{1}{\sqrt{12}} = \frac{\sqrt{3}}{6}$$

Question 6

Rationalise the denominator of $\frac{\sqrt{2}}{\sqrt{5}}$

Solution 6

We have,

$$\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{2 \times 5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{10}}{5}$$

$$\therefore \quad \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$$

Question 7

Rationalise the denominator of $\frac{\sqrt{3}+1}{\sqrt{2}}$

Solution 7

We have,

$$\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{\left(\sqrt{3}+1\right)}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{3} \times \sqrt{2} + \sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{2}$$

$$\therefore \quad \frac{\sqrt{3}+1}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$$

Question 8

Rationalise the denominator of $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

Solution 8

We have,

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} = \frac{\left(\sqrt{2}+\sqrt{5}\right)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{2}\times\sqrt{3}+\sqrt{5}\times\sqrt{3}}{\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{15}}{3}$$

$$\therefore \qquad \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{6}+\sqrt{15}}{3}$$

Question 9

Rationalise the denominator of $\frac{3\sqrt{2}}{\sqrt{5}}$

Solution 9

We have,

$$\frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{3\sqrt{2\times 5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{3\sqrt{10}}{5}$$

$$\therefore \qquad \frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{10}}{5}$$

Question 10

Find the value of three places of decimals of $\frac{2}{\sqrt{3}}$. It is given that $\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution 10

We have,

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= \frac{2 \times 1.732}{3} \qquad \left[\because \sqrt{3} = 1.732\right]$$

$$= \frac{3.464}{3}$$

$$= 1.154$$

$$\therefore \quad \frac{2}{\sqrt{3}} = 1.154$$

Question 11

Find the value of three places of decimals of $\frac{3}{\sqrt{10}}$. It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution 11

We have,

$$\frac{3}{\sqrt{10}} = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{3\sqrt{10}}{\left(\sqrt{10}\right)^2}$$

$$= \frac{3\sqrt{10}}{10}$$

$$= \frac{3 \times 3.162}{10} \qquad \left[\because \sqrt{10} = 3.162\right]$$

$$= \frac{9.486}{10}$$

$$= 0.9486$$

$$\therefore \quad \frac{3}{\sqrt{10}} = 0.9486$$

Question 12

Find the value of three places of decimals of $\frac{\sqrt{5}+1}{\sqrt{2}}$. It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution 12

We have,

$$\frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{5}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{5\times 2}+\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{10}+\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{2}}{2}$$

$$= \frac{3.162+1.414}{2} \qquad \left[\because \sqrt{2} = 1.414, \text{ and } \sqrt{10} = 3.162\right]$$

$$= \frac{4.576}{2}$$

$$= 2.288$$

$$\therefore \quad \frac{\sqrt{5}+1}{\sqrt{2}} = 2.288$$

Question 13

Find the value of three places of decimals of $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$. It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution 13

We have,

$$\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{10 \times 2} + \sqrt{15 \times 2}}{\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{20}+\sqrt{30}}{2}$$

$$= \frac{\sqrt{2 \times 2 \times 5} + \sqrt{2 \times 3 \times 5}}{2}$$

$$= \frac{2\sqrt{5} + \sqrt{2} \times \sqrt{3} \times \sqrt{5}}{2}$$

$$= \frac{\sqrt{5}\left(2 + \sqrt{2} \times \sqrt{3}\right)}{2}$$

$$= \frac{(2.236)\left[2 + (1.414)(1.732)\right]}{2}$$

$$= \frac{(2.236)\left[2 + 2.449048\right]}{2}$$

$$= \frac{(2.236)(4.449048)}{2}$$

$$= \frac{9.948071328}{2}$$

$$= 4.974035664$$

$$\therefore \quad \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = 4.974$$

Question 14

Find the value of three places of decimals of $\frac{2+\sqrt{3}}{3}$. It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution 14

We have,

$$\frac{2+\sqrt{3}}{3} = \frac{2+1.732}{3} \qquad \left[\because \sqrt{3} = 1.732\right]$$

$$= \frac{3.732}{3}$$

$$= 1.244$$

$$\therefore \qquad \frac{2+\sqrt{3}}{3} = 1.244$$

Question 15

Find the value to three places of decimals of $\frac{\sqrt{2}-1}{\sqrt{5}}$. It is given that $\sqrt{2} = 1.414$, $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

Solution 15

We have,

$$\frac{\sqrt{2}-1}{\sqrt{5}} = \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{2}\times\sqrt{5}-\sqrt{5}}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{10}-\sqrt{5}}{5}$$

$$= \frac{3.162-2.236}{5}$$

$$= \frac{0.926}{5}$$

$$= 0.1852$$

$$\therefore \qquad \frac{\sqrt{2}-1}{\sqrt{5}} = 0.1852$$

Question 16

Express $\frac{1}{3+\sqrt{2}}$ with rational denominator.

Solution 16

We have,

$$\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)}$$

$$= \frac{3-\sqrt{2}}{\left(3\right)^2-\left(\sqrt{2}\right)^2} \qquad \left[\because \left(a+b\right)\left(a-b\right) = a^2-b^2\right]$$

$$= \frac{3-\sqrt{2}}{9-2}$$

$$= \frac{3-\sqrt{2}}{7}$$

$$\therefore \qquad \frac{1}{3+\sqrt{2}} = \frac{3-\sqrt{2}}{7}$$

Question 17

Express $\frac{1}{\sqrt{6}-\sqrt{5}}$ with rational denominator.

Solution 17

We have,

$$\frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} \qquad \left[\because \left(a-b\right)\left(a+b\right) = a^2-b^2\right]$$

$$= \frac{\sqrt{6}+\sqrt{5}}{6-5}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{1}$$

$$= \sqrt{6}+\sqrt{5}$$

$$\therefore \qquad \frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5}$$

Question 18

Express $\frac{16}{\sqrt{41}-5}$ with rational denominator.

Solution 18

We have,

$$\frac{16}{\sqrt{41}-5} = \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}-5\right)\left(\sqrt{41}+5\right)}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}\right)^2-25} \qquad \left[\because (a-b)(a+b) = a^2-b^2\right]$$

$$= \frac{16\left(\sqrt{41}+5\right)}{41-25}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{16}$$

$$= \sqrt{41}+5$$

$$\therefore \qquad \frac{16}{\sqrt{41}-5} = \sqrt{41}+5$$

Question 19

Express $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ with rational denominator.

Solution 19

We have,

$$\frac{30}{5\sqrt{3}-3\sqrt{5}} = \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{\left(5\sqrt{3}-3\sqrt{5}\right)\left(5\sqrt{3}+3\sqrt{5}\right)}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{\left(5\sqrt{3}\right)^2-\left(3\sqrt{5}\right)^2} \qquad \left[\because (a-b)(a+b) = a^2-b^2\right]$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{75-45}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{30}$$

$$= 5\sqrt{3}+3\sqrt{5}$$

$$\therefore \quad \frac{30}{5\sqrt{3}-3\sqrt{5}} = 5\sqrt{3}+3\sqrt{5}$$

Question 20

Express $\frac{1}{2\sqrt{5}-\sqrt{3}}$ with rational denominator.

Solution 20

We have,

$$\frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{\left(2\sqrt{5}-\sqrt{3}\right)\left(2\sqrt{5}+\sqrt{3}\right)}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{\left(2\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} \qquad \left[\because (a-b)(a+b) = a^2-b^2\right]$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{20-3}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{17}$$

$$\therefore \quad \frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{17}$$

Question 21

Express $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ with rational denominator.

Solution 21

We have,

$$\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} = \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{3}+1\right)\left(2\sqrt{2}+\sqrt{3}\right)}{\left(2\sqrt{2}-\sqrt{3}\right)\left(2\sqrt{2}+\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}\left(2\sqrt{2}+\sqrt{3}\right)+1\left(2\sqrt{2}+\sqrt{3}\right)}{\left(2\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{8-3}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

$$\therefore \quad \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} = \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

Question 22

Express $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ with rational denominator.

Solution 22

We have,

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$$

$$= \frac{\left(6-4\sqrt{2}\right)^2}{\left(6+4\sqrt{2}\right)\left(6-4\sqrt{2}\right)}$$

$$= \frac{(6)^2 - 2\times 6\times 4\sqrt{2} + \left(4\sqrt{2}\right)^2}{(6)^2 - \left(4\sqrt{2}\right)^2}$$

$$= \frac{36-48\sqrt{2}+32}{36-32}$$

$$= \frac{68-48\sqrt{2}}{4}$$

$$= \frac{4\left[17-12\sqrt{2}\right]}{4}$$

$$= 17-12\sqrt{2}$$

$$\therefore \quad \frac{6-4\sqrt{2}}{6+4\sqrt{2}} = 17-12\sqrt{2}$$

Question 23

Express $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ with rational denominator.

Solution 23

We have,

$$\frac{3\sqrt{2}+1}{2\sqrt{5}-3} = \frac{3\sqrt{2}+1}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3}$$

$$= \frac{\left(3\sqrt{2}+1\right)\left(2\sqrt{5}+3\right)}{\left(2\sqrt{5}-3\right)\left(2\sqrt{5}+3\right)}$$

$$= \frac{3\sqrt{2}\left(2\sqrt{5}+3\right)+1\left(2\sqrt{5}+3\right)}{\left(2\sqrt{5}\right)^2-\left(3\right)^2}$$

$$= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{20-9}$$

$$= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11}$$

$$\therefore \quad \frac{3\sqrt{2}+1}{2\sqrt{5}-3} = \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11}$$

Question 24

Express $\dfrac{b^2}{\sqrt{a^2+b^2}+a}$ with rational denominator.

Solution 24

We have,

$$\frac{b^2}{\sqrt{a^2+b^2}+a} = \frac{b^2}{\sqrt{a^2+b^2}+a} \times \frac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}-a}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{\left(\sqrt{a^2+b^2}+a\right)\left(\sqrt{a^2+b^2}-a\right)}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{\left(\sqrt{a^2+b^2}\right)^2-(a)^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{a^2+b^2-a^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{b^2}$$

$$= \sqrt{a^2+b^2}-a$$

$$\therefore \qquad \frac{b^2}{\sqrt{a^2+b^2}+a} = \sqrt{a^2+b^2}-a$$

Question 25

Rationalise the denominator and simplify:

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Solution 25

We have,

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$$

$$= \frac{\left(\sqrt{3}\right)^2 - 2 \times \sqrt{3} \times \sqrt{2} + \left(\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{3 - 2\sqrt{6} + 2}{3 - 2}$$

$$= \frac{5 - 2\sqrt{6}}{1}$$

$$= 5 - 2\sqrt{6}$$

$$\therefore \quad \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5 - 2\sqrt{6}$$

Question 26

Rationalise the denominator and simplify:

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

Solution 26

We have,

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{\left(5+2\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)}$$

$$= \frac{5\left(7-4\sqrt{3}\right)+2\sqrt{3}\left(7-4\sqrt{3}\right)}{\left(7\right)^2-\left(4\sqrt{3}\right)^2}$$

$$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}$$

$$= \frac{11-20\sqrt{3}+14\sqrt{3}}{1}$$

$$= 11-6\sqrt{3}$$

$$\therefore \quad \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = 11-6\sqrt{3}$$

Question 27

Rationalise the denominator and simplify:

$$\frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

Solution 27

We have,

$$\frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{\left(1+\sqrt{2}\right)\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

$$= \frac{1\left(3+2\sqrt{2}\right)+\sqrt{2}\left(3+2\sqrt{2}\right)}{\left(3\right)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3+2\sqrt{2}+3\sqrt{2}+4}{9-8}$$

$$= \frac{7+5\sqrt{2}}{1}$$

$$= 7+5\sqrt{2}$$

$$\therefore \quad \frac{1+\sqrt{2}}{3-2\sqrt{2}} = 7+5\sqrt{2}$$

Question 28

Rationalise the denominator and simplify:

$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

Solution 28

We have,

$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \times \frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}$$

$$= \frac{\left(2\sqrt{6}-\sqrt{5}\right)\left(3\sqrt{5}+2\sqrt{6}\right)}{\left(3\sqrt{5}-2\sqrt{6}\right)\left(3\sqrt{5}+2\sqrt{6}\right)}$$

$$= \frac{2\sqrt{6}\left(3\sqrt{5}+2\sqrt{6}\right)-\sqrt{5}\left(3\sqrt{5}+2\sqrt{6}\right)}{\left(3\sqrt{5}\right)^2-\left(2\sqrt{6}\right)^2}$$

$$= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{45-24}$$

$$= \frac{4\sqrt{30}+9}{21}$$

$$\therefore \quad \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{4\sqrt{30}+9}{21}$$

Question 29

Rationalise the denominator and simplify:

$$\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

Solution 29

We have,

$$\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{16\times 3}+\sqrt{9\times 2}}$$

$$= \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$$

$$= \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$= \frac{\left(4\sqrt{3}+5\sqrt{2}\right)\left(4\sqrt{3}-3\sqrt{2}\right)}{\left(4\sqrt{3}+3\sqrt{2}\right)\left(4\sqrt{3}-3\sqrt{2}\right)}$$

$$= \frac{4\sqrt{3}\left(4\sqrt{3}-3\sqrt{2}\right)+5\sqrt{2}\left(4\sqrt{3}-3\sqrt{2}\right)}{\left(4\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{48-18}$$

$$= \frac{18+8\sqrt{6}}{30}$$

$$= \frac{2\left[9+4\sqrt{6}\right]}{30}$$

$$= \frac{9+4\sqrt{6}}{15}$$

$$\therefore \quad \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{9+4\sqrt{6}}{15}$$

Question 30

Rationalise the denominator and simplify:

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

Solution 30

We have,

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \times \frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}$$

$$= \frac{\left(2\sqrt{3}-\sqrt{5}\right)\left(2\sqrt{2}-3\sqrt{3}\right)}{\left(2\sqrt{2}+3\sqrt{3}\right)\left(2\sqrt{2}-3\sqrt{3}\right)}$$

$$= \frac{2\sqrt{3}\left(2\sqrt{2}-3\sqrt{3}\right)-\sqrt{5}\left(2\sqrt{2}-3\sqrt{3}\right)}{\left(2\sqrt{2}\right)^2-\left(3\sqrt{3}\right)^2}$$

$$= \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{8-27}$$

$$= \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}$$

$$= \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

$$\therefore \quad \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

Question 31

Simplify:

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Solution 31

We have,

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}+\frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

$$=\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2+2\times\sqrt{5}\times\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2-2\times\sqrt{5}\times\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{5+3+2\sqrt{15}}{5-3}+\frac{5+3-2\sqrt{15}}{5-3}$$

$$=\frac{8+2\sqrt{15}}{2}+\frac{8-2\sqrt{15}}{2}$$

$$=\frac{8+2\sqrt{15}+8-2\sqrt{15}}{2}$$

$$=\frac{16}{2}=8$$

$$\therefore \quad \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=8$$

Question 32

Simplify:

$$\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$$

Solution 32

We have,

$$\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$=\frac{2-\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}+\frac{2\sqrt{5}+2\sqrt{3}}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}+\frac{2+\sqrt{5}}{\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right)}$$

$$=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}+\frac{2\sqrt{5}+2\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{2+\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}+\frac{2\sqrt{5}+2\sqrt{3}}{5-3}+\frac{2+\sqrt{5}}{4-5}$$

$$=\frac{2-\sqrt{3}}{1}+\frac{2\sqrt{5}+2\sqrt{3}}{2}+\frac{2+\sqrt{5}}{-1}$$

$$=2-\sqrt{3}+\frac{2\sqrt{5}+2\sqrt{3}}{2}-2-\sqrt{5}$$

$$=-\sqrt{5}-\sqrt{3}+\frac{2\sqrt{5}+2\sqrt{3}}{2}$$

$$=\frac{-2\sqrt{5}-2\sqrt{3}+2\sqrt{5}+2\sqrt{3}}{2}$$

$$=\frac{0}{2}=0$$

$$\therefore\qquad \frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}=0$$

Question 33

Simplify:

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

Solution 33

We have,

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

$$=\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{2\sqrt{5}-2\sqrt{3}}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}+\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{2\sqrt{5}-2\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{3\sqrt{5}-3\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{2\sqrt{5}-2\sqrt{3}}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}-\frac{3\sqrt{5}-3\sqrt{2}}{5-2}$$

$$=\frac{2\sqrt{5}-2\sqrt{3}}{2}+\sqrt{3}-\sqrt{2}-\frac{3\sqrt{5}-3\sqrt{2}}{3}$$

$$=\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}+\sqrt{3}-\sqrt{2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}$$

$$=\sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}$$

$$=0$$

$$\therefore \quad \frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}=0$$

Question 34

Determine rational numbers a and b if:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}=a-b\sqrt{3}$$

Solution 34

We have,

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$$= \frac{\left(\sqrt{3}\right)^2 + (1)^2 - 2 \times \sqrt{3} \times 1}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{3+1-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{2\left[2-\sqrt{3}\right]}{2}$$

$$\Rightarrow \quad \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} \qquad ---(1)$$

$$\therefore \quad \frac{\sqrt{3}-1}{\sqrt{3}+1} = a-b\sqrt{3} \qquad ---(2)$$

Using (1) and (2)

$$\therefore \quad a-b\sqrt{3} = 2-\sqrt{3}$$

$\Rightarrow \quad a = 2$ and $b = 1$

Question 35

Determine rational numbers a and b if:

$$\frac{4+\sqrt{2}}{2+\sqrt{2}} = a-\sqrt{b}$$

Solution 35

We have,

$$\frac{4+\sqrt{2}}{2+\sqrt{2}} = \frac{4+\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$= \frac{\left(4+\sqrt{2}\right)\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)}$$

$$= \frac{4\left(2-\sqrt{2}\right)+\sqrt{2}\left(2-\sqrt{2}\right)}{(2)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{8-4\sqrt{2}+2\sqrt{2}-2}{4-2}$$

$$= \frac{6-2\sqrt{2}}{2}$$

$$= \frac{2\left[3-\sqrt{2}\right]}{2}$$

$$\Rightarrow \quad \frac{4+\sqrt{2}}{2+\sqrt{2}} = 3-\sqrt{2}$$

but $\quad \dfrac{4+\sqrt{2}}{2+\sqrt{2}} = a-\sqrt{b}$

$$\Rightarrow \quad 3-\sqrt{2} = a-\sqrt{b}$$

$$\Rightarrow \quad a = 3 \text{ and } b = 2$$

Question 36

Determine rational numbers a and b if:

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$$

Solution 36

We have,

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{\left(3+\sqrt{2}\right)^2}{\left(3-\sqrt{2}\right)\left(3+\sqrt{2}\right)}$$

$$= \frac{(3)^2 + \left(\sqrt{2}\right)^2 + 2 \times 3 \times \sqrt{2}}{(3)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{9+2+6\sqrt{2}}{9-2}$$

$$= \frac{11+6\sqrt{2}}{7}$$

$$\Rightarrow \quad \frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{11}{7} + \frac{6\sqrt{2}}{7}$$

but $\quad \dfrac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

$$\Rightarrow \quad a + b\sqrt{2} = \frac{11}{7} + \frac{6\sqrt{2}}{7}$$

$$\Rightarrow \quad a = \frac{11}{7} \text{ and } b = \frac{6}{7}$$

Question 37

Determine rational numbers a and b if:

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$$

Solution 37

We have,

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = \frac{5+3\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{\left(5+3\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)}$$

$$= \frac{5\left(7-4\sqrt{3}\right)+3\sqrt{3}\left(7-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2}$$

$$= \frac{35-20\sqrt{3}+21\sqrt{3}-36}{49-48}$$

$$= \frac{-1+1\sqrt{3}}{1}$$

$$\Rightarrow \quad \frac{5+3\sqrt{3}}{7+4\sqrt{3}} = -1+\sqrt{3}$$

but $\quad \dfrac{5+3\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

$$\Rightarrow \quad a+b\sqrt{3} = -1+\sqrt{3}$$

$\Rightarrow \quad a = -1$ and $b = 1$

Question 38

Determine rational numbers a and b if:

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a-b\sqrt{77}$$

Solution 38

We have,

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}}$$

$$= \frac{\left(\sqrt{11}-\sqrt{7}\right)^2}{\left(\sqrt{11}+\sqrt{7}\right)\left(\sqrt{11}-\sqrt{7}\right)}$$

$$= \frac{\left(\sqrt{11}\right)^2 + \left(\sqrt{7}\right)^2 - 2 \times \sqrt{11} \times \sqrt{7}}{\left(\sqrt{11}\right)^2 - \left(\sqrt{7}\right)^2}$$

$$= \frac{11 + 7 - 2\sqrt{77}}{11 - 7}$$

$$= \frac{18 - 2\sqrt{77}}{4}$$

$$= \frac{2\left[9 - \sqrt{77}\right]}{4}$$

$$= \frac{9 - \sqrt{77}}{2}$$

$$\Rightarrow \quad \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{9}{2} - \frac{\sqrt{77}}{2}$$

but $\quad \dfrac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$

$$\Rightarrow \quad a - b\sqrt{77} = \frac{9}{2} - \frac{\sqrt{77}}{2}$$

$\Rightarrow \quad a = \dfrac{9}{2}$ and $b = \dfrac{1}{2}$

Question 39

Determine rational numbers a and b if:

$$\frac{4 + 3\sqrt{5}}{4 - 3\sqrt{5}} = a + b\sqrt{5}$$

Solution 39

We have,

$$\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}}$$

$$= \frac{\left(4+3\sqrt{5}\right)^2}{\left(4-3\sqrt{5}\right)\left(4+3\sqrt{5}\right)}$$

$$= \frac{(4)^2 + \left(3\sqrt{5}\right)^2 + 2\times 4\times 3\sqrt{5}}{(4)^2 - \left(3\sqrt{5}\right)^2}$$

$$= \frac{16+45+24\sqrt{5}}{16-45}$$

$$= \frac{61+24\sqrt{5}}{-29}$$

$$\Rightarrow \quad \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{-61}{29} + \left(-\frac{24}{29}\right)\sqrt{5}$$

but $\quad \dfrac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

$$\Rightarrow \quad a + b\sqrt{5} = \frac{-61}{29} + \left(-\frac{24}{29}\right)\sqrt{5}$$

$$\Rightarrow \quad a = \frac{-61}{29} \text{ and } b = -\frac{24}{29}$$

Question 40

Find the value of $\dfrac{6}{\sqrt{5}-\sqrt{3}}$, it being given that $\sqrt{3} = 1.732$ *and* $\sqrt{5} = 2.236$

Solution 40

We have,

$$\frac{6}{\sqrt{5}-\sqrt{3}} = \frac{6}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{6\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$$

$$= \frac{6\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{6\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$= \frac{6\left(\sqrt{5}+\sqrt{3}\right)}{2}$$

$$= 3\left(\sqrt{5}+\sqrt{3}\right)$$
$$= 3(2.236+1.732)$$
$$= 3(3.968)$$

$$= 11.904$$

$$\therefore \quad \frac{6}{\sqrt{5}-\sqrt{3}} = 11.904$$

Question 41

Find the of $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ to three places of decimals, it being given that $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.2360$, $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$.

Solution 41

We have,

$$\frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$= \frac{\left(3-\sqrt{5}\right)\left(3-2\sqrt{5}\right)}{\left(3+2\sqrt{5}\right)\left(3-2\sqrt{5}\right)}$$

$$= \frac{3\left(3-2\sqrt{5}\right)-\sqrt{5}\left(3-2\sqrt{5}\right)}{\left(3\right)^2-\left(2\sqrt{5}\right)^2}$$

$$= \frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20}$$

$$= \frac{19-9\sqrt{5}}{-11}$$

$$= \frac{9\sqrt{5}-19}{11}$$

$$= \frac{9\times 2.2360-19}{11} \qquad \left[\because \sqrt{5} = 2.2360\right]$$

$$= \frac{20.1240-19}{11}$$

$$= \frac{1.1240}{11}$$

$$= 0.102$$

$$\therefore \quad \frac{3-\sqrt{5}}{3+2\sqrt{5}} = 0.102$$

Question 42

Find the of $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ to three places of decimals, it being given that $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.2360$, $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$.

Solution 42

We have,

$$\frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{\left(1+\sqrt{2}\right)\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

$$= \frac{1\left(3+2\sqrt{2}\right)+\sqrt{2}\left(3+2\sqrt{2}\right)}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3+2\sqrt{2}+3\sqrt{2}+4}{9-8}$$

$$= 7+5\sqrt{2}$$
$$= 7+5\times 1.4142$$
$$= 7+7.0710$$
$$= 14.0710$$

$$\Rightarrow \quad \frac{1+\sqrt{2}}{3-2\sqrt{2}} = 14.0710$$

Question 43

If $x = 2+\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$

Solution 43

We have,

$x = 2+\sqrt{3}$

$$\therefore \quad \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$$

$$= \frac{2-\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1}$$

$$\Rightarrow \quad \frac{1}{x} = 2-\sqrt{3}$$

Now, $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left[x^2 - x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2\right]$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = \left(2+\sqrt{3}+2-\sqrt{3}\right)\left[\left(2+\sqrt{3}\right)^2 - 1 + \left(2-\sqrt{3}\right)^2\right]$$

$$= 4\left[(2)^2 + \left(\sqrt{3}\right)^2 + 2\times2\times\sqrt{3} - 1 + (2)^2 + \left(\sqrt{3}\right)^2 - 2\times2\times\sqrt{3}\right]$$

$$= 4\left[4+3+4\sqrt{3}-1+4+3-4\sqrt{3}\right]$$

$$= 4[13]$$

$$= 52$$

$$\Rightarrow \quad x^3 + \frac{1}{x^3} = 52$$

Question 44

If $x = 3+\sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$

Solution 44

We have,

$x = 3 + \sqrt{8}$

Now, $\frac{1}{x} = \frac{1}{3 + \sqrt{8}} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$

$= \frac{3 - \sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}$

$= \frac{3 - \sqrt{8}}{9 - 8} = 3 - \sqrt{8}$

$\Rightarrow \quad \frac{1}{x} = 3 - \sqrt{8}$

We know, $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x}$

$\Rightarrow \quad \left(3 + \sqrt{8} + 3 - \sqrt{8}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad (6)^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad 36 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 36 - 2 = 34$

Hence, $x^2 + \frac{1}{x^2} = 34$

Question 45

If $x = \frac{\sqrt{3} + 1}{2}$, find the value of $4x^3 + 2x^2 - 8x + 7$.

Solution 45

We have,

$$x = \frac{\sqrt{3}+1}{2}$$

$\Rightarrow \quad 2x = \sqrt{3} + 1$

$\Rightarrow \quad 2x - 1 = \sqrt{3}$

$\Rightarrow \quad (2x-1)^2 = \left(\sqrt{3}\right)^2$

$\Rightarrow \quad 4x^2 + 1 - 4x = 3$

$\Rightarrow \quad 4x^2 - 4x - 2 = 0$

$\therefore \quad 4x^3 + 2x^2 - 8x + 7$

$= x\left(4x^2 - 4x - 2\right) + \frac{3}{2}\left(4x^2 - 4x - 2\right) + 10$

$= x \times 0 + \frac{3}{2} \times 0 + 10$

$= 0 + 0 + 10$

$= 10$

$\Rightarrow \quad 4x^3 + 2x^2 - 8x + 7 = 10$

Chapter 3 - Rationalisation Exercise 3.16

Question 1

$\sqrt{10} \times \sqrt{15}$ is equal to

(a) $5\sqrt{6}$

(b) $6\sqrt{5}$

(c) $\sqrt{30}$

(d) $\sqrt{25}$

Solution 1

$10 = 5 \times 2$

$15 = 5 \times 3$

$\therefore \sqrt{10} \times \sqrt{15} = \sqrt{5 \times 2} \times \sqrt{5 \times 3}$

$= \sqrt{5} \times \sqrt{2} \times \sqrt{5} \times \sqrt{3}$

$= (\sqrt{5} \times \sqrt{5}) \times \sqrt{2} \times \sqrt{3}$

$= 5\sqrt{6}$

Hence, correct option is (a).

Question 2

$\sqrt[5]{6} \times \sqrt[5]{6}$ is equal to

(a) $\sqrt[5]{36}$

(b) $\sqrt[5]{6 \times 0}$

(c) $\sqrt[5]{6}$

(d) $\sqrt[5]{12}$

Solution 2

$\sqrt[5]{6} = (6)^{1/5}$

so $\sqrt[5]{6} \times \sqrt[5]{6} = (6)^{1/5} \times (6)^{1/5}$

$= (6 \times 6)^{1/5}$

$= (36)^{1/5}$

$= \sqrt[5]{36}$

Hence, correct option is (a).

Chapter 3 - Rationalisation Exercise 3.17

Question 1

The rationalisation factor of $\sqrt{3}$ is

(a) $-\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $2\sqrt{3}$

(d) $-2\sqrt{3}$

Solution 1

Rationalisation factor of any number like $\sqrt{a}$ is $\frac{1}{\sqrt{a}}$ or $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$.

So, Rationalisation factor of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$.

Hence, correct option is (b).

Question 2

The rationalisation factor of $2 + \sqrt{3}$ is

(a) $2-\sqrt{3}$

(b) $\sqrt{2}+3$

(c) $\sqrt{2}-3$

(d) $\sqrt{3}-2$

Solution 2

Rationalisation factor of any number $a \pm \sqrt{b}$ is $a \mp \sqrt{b}$.

So, Rationalisation factor of $2+\sqrt{3}$ will be $2 - \sqrt{3}$

Hence, correct option is (a).

Question 3

If $x = \sqrt{5}+2$, then $x - \frac{1}{x}$ equals

(a) $2\sqrt{5}$

(b) 4

(c) 2

(d) $\sqrt{5}$

Solution 3

$x = \sqrt{5}+2$

$$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2}\,\frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \frac{\sqrt{5}-2}{1} = \sqrt{5}-2$$

Now, $x-\frac{1}{x} = \sqrt{5}+2-(\sqrt{5}-2) = \sqrt{5}+2-\sqrt{5}+2 = 4$

Hence, correct option is (b).

Question 4

If $\frac{\sqrt{3}-1}{\sqrt{3}-1} = a-b\sqrt{3}$, then

(a) $a = 2, b = 1$
(b) $a = 2, b = -1$
(c) $a = -2, b = 1$
(d) $a = b = 1$

Solution 4

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Multiplying and dividing by the rationalisation factor of denominator, we get

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2}$$

$$= \frac{3-2\sqrt{3}+1}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{2\left(2-\sqrt{3}\right)}{2}$$

$$= 2-\sqrt{3}$$

Comparing with $a-b\sqrt{3}$, we get $a = 2$ and $b = 1$.
Hence, correct option is (a).

Question 5

The simplest rationalising factor of $\sqrt[3]{500}$ is

(a) $\sqrt[3]{2}$
(b) $\sqrt[3]{5}$
(c) $\sqrt{3}$
(d) none of these

Solution 5

$$\sqrt[3]{500} = (500)^{1/3} = \left(\frac{500 \times 2}{2}\right)^{1/3} = \left(\frac{1000}{2}\right)^{1/3} = (10^{\cancel{3}})^{1/\cancel{3}} \,.\, \frac{1}{2^{1/3}} \Rightarrow 10 \,.\, 2^{-1/3}$$

The simplest Rationalisation factor of $\sqrt[3]{500}$ after simplify it to $(10.2^{-1/3})$ is $2^{1/3}$ or $\sqrt[3]{2}$.
Hence, correct option is (a).

Question 6

The simplest Rationalising Factor of $\sqrt{3}+\sqrt{5}$ is

(a) $\sqrt{3}-5$

(b) $3-\sqrt{5}$

(c) $\sqrt{3}-\sqrt{5}$

(d) $\sqrt{3}+\sqrt{5}$

Solution 6

Rationalising factor of any number of kind $\sqrt{a}\pm\sqrt{b}$ is $\sqrt{a}\mp\sqrt{b}$

So, for given number $\sqrt{3}+\sqrt{5}$, Rationalising factor would be $\sqrt{3}-\sqrt{5}$.

Hence, correct option is (c).

Question 7

The simplest rationalising factor of $2\sqrt{5}-\sqrt{3}$ is

(a) $2\sqrt{5}+3$

(b) $2\sqrt{5}+\sqrt{3}$

(c) $\sqrt{5}+\sqrt{3}$

(d) $\sqrt{5}-\sqrt{3}$

Solution 7

Rationalising factor of any number of kind $a\sqrt{a}\pm\sqrt{b}$ is $\sqrt{a}\mp\sqrt{b}$

So, for given number $2\sqrt{5}-\sqrt{3}$, Rationalising factor would be $2\sqrt{5}+\sqrt{3}$.

Hence, correct option is (b).

Question 8

If $x=\dfrac{2}{3+\sqrt{7}}$, then $(x-3)^2=$

(a) 1

(b) 3

(c) 6

(d)7

Solution 8

$$x=\frac{2}{3+\sqrt{7}}=\frac{2}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}=\frac{2(3-\sqrt{7})}{(3)^2-(\sqrt{7})^2}=\frac{6-2\sqrt{7}}{9-7}=\frac{6-2\sqrt{7}}{2}=3-2\sqrt{7}$$

Now $(x-3)^2=(\cancel{3}-\sqrt{7}-\cancel{3})^2=(-\sqrt{7})^2=7$

Hence, correct option is (d).

Question 9

If $x = 7+4\sqrt{3}$ and $xy = 1$, then $\dfrac{1}{x^2}+\dfrac{1}{y^2}=$

(a) 64

(b) 134

(c) 194

(d) $\dfrac{1}{49}$

Solution 9

$x = 7+4\sqrt{3}$, $\quad xy = 1 \Rightarrow y = \frac{1}{x}$

$\therefore\ y = \frac{1}{7+4\sqrt{3}}$

$\therefore\ y = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{(7)^2-(4\sqrt{3})^2} = \frac{7-4\sqrt{3}}{49-48} = 7-4\sqrt{3}$

Now, $\frac{1}{x^2}+\frac{1}{y^2} = \frac{y^2+x^2}{x^2y^2} = \frac{x^2+y^2}{(xy)^2}$

$x^2 = (7+4\sqrt{3})^2 = 49+48+56\sqrt{3} = 97+56\sqrt{3}$

$y^2 = (7-4\sqrt{3})^2 = 49+48-56\sqrt{3} = 97-56\sqrt{3}$

$\therefore\ x^2+y^2 = 97+56\sqrt{3}+97-56\sqrt{3} = 194$

$xy = 1$

$\therefore\ \frac{x^2+y^2}{(xy)^2} = \frac{194}{(1)^2} = 194$

Hence, correct option is (c).

Question 10

If $x+\sqrt{15} = 4$, then $x+\frac{1}{x} =$

(a) 2
(b) 4
(c) 8
(d) 1

Solution 10

$x+\sqrt{15} = 4$

$\Rightarrow x = 4-\sqrt{15} \Rightarrow \frac{1}{x} = \frac{1}{4-\sqrt{15}}$

$\frac{1}{x} = \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}} = \frac{4+\sqrt{15}}{(4)^2-(\sqrt{15})^2} = \frac{4+\sqrt{15}}{16-15} = 4+\sqrt{15}$

Now, $x+\frac{1}{x} = 4-\sqrt{15}+4+\sqrt{15} = 8$

Hence, correct option is (c).

Question 11

if $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $x+y+xy =$

(a) 9
(b) 5
(c) 17
(d) 7

Solution 11

$x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$\therefore\ x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$

$y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\therefore\ y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{8-2\sqrt{15}}{2} = 4-\sqrt{15}$

$xy = (4+\sqrt{15})(4-\sqrt{15}) = 16-15 = 1$

Now, $x+y+xy = 4+\sqrt{15}+4-\sqrt{15}+1 = 4+4+1 = 9$

Hence, correct option is (a).

Question 12

If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $x^2+xy+y^2=$

(a) 101

(b) 99

(c) 98

(d) 102

Solution 12

$x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\therefore\ x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = 5-2\sqrt{6}$

$y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\therefore\ y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} = 5+2\sqrt{6}$

Now, x^2+xy+y^2

$= (5-2\sqrt{6})^2+(5-2\sqrt{6})(5+2\sqrt{6})+(5+2\sqrt{6})^2$

$= (25+24-20\sqrt{6})+(25-24)+(25+24+20\sqrt{6})$

$= 49-20\sqrt{6}+1+49+20\sqrt{6}$

$= 9$

Hence, correct option is (b).

Question 13

$\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(a) $3+2\sqrt{2}$

(b) $\frac{1}{3+2\sqrt{2}}$

(c) $3-2\sqrt{2}$

(d) $\frac{3}{2}-\sqrt{2}$

Solution 13

$$\frac{1}{\sqrt{9}-\sqrt{8}}$$

$$= \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}}$$

$$\frac{\sqrt{9}+\sqrt{8}}{(\sqrt{9})^2-(\sqrt{8})^2}$$

$$= \frac{\sqrt{9}+\sqrt{8}}{9-8}$$

$$= \sqrt{9}+\sqrt{8}$$

$$= 3+2\sqrt{2}$$

Hence, correct option is (a).

Question 14

The value of $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}+\sqrt{18}}$ is

(a) $\frac{4}{3}$

(b) 4

(c) 3

(d) $\frac{3}{4}$

Solution 14

$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$

$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$

$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

Now, $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}+\sqrt{18}} = \frac{4\sqrt{3}+4\sqrt{2}}{3\sqrt{3}+3\sqrt{2}}$

$$= \frac{4\ \cancel{(\sqrt{3}+\sqrt{2})}}{3\ \cancel{(\sqrt{3}+\sqrt{2})}}$$

$$= \frac{4}{3}$$

Hence, correct option is (a).

Chapter 3 - Rationalisation Exercise 3.18

Question 1

If $\frac{5-\sqrt{3}}{2+\sqrt{3}} = x+y\sqrt{3}$ then

(a) $x = 13,\ y = -7$

(b) $x = -13,\ y = 7$

(c) $x = -13,\ y = -7$

(d) $x = 13,\ y = 7$

Solution 1

$$\frac{5-\sqrt{3}}{2+\sqrt{3}}$$
$$= \frac{5-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$
$$= \frac{(5-\sqrt{3})(2-\sqrt{3})}{(2)^2-(\sqrt{3})^2}$$
$$= \frac{10-5\sqrt{3}-2\sqrt{3}+3}{4-3}$$
$$= \frac{13-7\sqrt{3}}{1}$$
$$= 13-7\sqrt{3}$$
$$\Rightarrow x = 13 \text{ and } y = -7$$
Hence, correct option is (a).

Question 2

If $x=\sqrt[3]{2+\sqrt{3}}$ then $x^3+\frac{1}{x^3}$ is

(a) 2
(b) 4
(c) 8
(d) 9

Solution 2

$$x = \sqrt[3]{2+\sqrt{3}} + (2+\sqrt{3})^{1/3}$$
$$x^3 = \{(2+\sqrt{3})^{1/\cancel{3}}\}^{\cancel{3}} = (2+\sqrt{3})$$
$$\Rightarrow \frac{1}{x^3} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

Now, $x^3+\frac{1}{x^3} = 2+\sqrt{3}+2-\sqrt{3} = 4$

Hence, correct option is (b).

Question 3

The value of $\sqrt{3-2\sqrt{2}}$ is

(a) $\sqrt{2}-1$
(b) $\sqrt{2}+1$
(c) $\sqrt{3}-\sqrt{2}$
(d) $\sqrt{3}+\sqrt{2}$

Solution 3

$$\sqrt{3-2\sqrt{2}}$$
$$=\sqrt{2+1-2\sqrt{2}}$$
$$=\sqrt{(\sqrt{2})^2+(1)^2-2(\sqrt{2})(1)}$$
$$=\sqrt{(\sqrt{2}-1)^2}$$
$$=\sqrt{2}-1$$
Hence, correct option is (a).

Question 4

The value of $\sqrt{5+2\sqrt{6}}$ is

(a) $\sqrt{3}-\sqrt{2}$
(b) $\sqrt{3}+\sqrt{2}$
(c) $\sqrt{5}+\sqrt{6}$
(d) none of these

Solution 4

$\sqrt{5+2\sqrt{6}}$

$=\sqrt{3+2\ +\ 2(\sqrt{3})(\sqrt{2})}$

$=\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2(\sqrt{3})(\sqrt{2})}$

$=\sqrt{(\sqrt{3}+\sqrt{2})^2}$

$=\sqrt{3}+\sqrt{2}$

Hence, correct option is (b).

Question 5

If $\sqrt{2}=\ 1.4142,$ then $\sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}-1}}$ is equal to

(a) 0.1718
(b) 5.8282
(c) 0.4142
(d) 2.4142

Solution 5

By Rationalising $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$, we get

$$\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1^2}=\frac{(\sqrt{2}-1)^2}{1}$$

$$\text{so}\ \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}=\sqrt{\frac{(\sqrt{2}-1)^2}{1}}=(\sqrt{2}-1)=1.4142-1=0.4142$$

Hence, correct option is (c).

Question 6

If $\sqrt{2}$ = 1.414, then value of $\sqrt{6}-\sqrt{3}$ upto three places of decimal is

(a) 0.235
(b) 0.717
(c) 1.414
(d) 0.471

Solution 6

$\sqrt{6}-\sqrt{3}$

$=\sqrt{3}(\sqrt{2}-1)$

Now, $\sqrt{3}=1.732$

$\sqrt{2}=1.414$

$\therefore\ \sqrt{6}-\sqrt{3}=1.732(1.414-1)=1.732\ (0.414)=\ 0.717$ (upto 3 decimal places)

Hence, correct option is (b).

Question 7

The positive square root of $7+\sqrt{48}$

(a) $7+2\sqrt{3}$
(b) $7+\sqrt{3}$
(c) $2+\sqrt{3}$
(d) $3+\sqrt{2}$

Solution 7

$\sqrt{7+\sqrt{48}}$

$=\sqrt{7+2\sqrt{12}}$

$=\sqrt{4\ +3\ +\ 2\sqrt{4}\times\sqrt{3}}$

$=\sqrt{(\sqrt{4})^2+(\sqrt{3})^2+\ 2\times\sqrt{4}\times\sqrt{3}}$

$=\sqrt{(\sqrt{4}+\sqrt{3})^2}$

$=\pm\ (\sqrt{4}+\sqrt{3})$

Positve value is $\sqrt{4}+\ \sqrt{3}=2+\sqrt{3}$

Hence, correct option is (c).

Question 8

If $x=\sqrt{6}+\sqrt{5}$, then $x^2+\frac{1}{x^2}-2=$

(a) $2\sqrt{6}$

(b) $2\sqrt{5}$

(c) 24

(d) 20

Solution 8

$x^2+\frac{1}{x^2}-2=\left(x-\frac{1}{x}\right)^2$

$x=\sqrt{6}+\sqrt{5}$

$\Rightarrow\frac{1}{x}=\frac{1}{\sqrt{6}+\sqrt{5}}=\frac{1}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{\sqrt{6}-\sqrt{5}}{1}=\sqrt{6}-\sqrt{5}$

Now,

$\left(x-\frac{1}{x}\right)^2=[\sqrt{6}+\sqrt{5}-(\sqrt{6}-\sqrt{5})]^2=(2\sqrt{5})^2=4\times5=20$

Hence, correct option is (d).

Question 9

If $\sqrt{13-a\sqrt{10}}=\sqrt{8}+\sqrt{5}$, then a =

(a) -5

(b) -6

(c) -4

(d) -2

Solution 9

$\sqrt{13-a\sqrt{10}}=\sqrt{8}+\sqrt{5}$

Squaring both sides, we get

$13-a\sqrt{10}=8+5+2\sqrt{40}$

$\Rightarrow\cancel{13}-a\sqrt{10}-\cancel{13}=2\times2\sqrt{10}$

$\Rightarrow-a\sqrt{10}=4\sqrt{10}$

$\Rightarrow a=-4$

Hence, correct option is (c).

Chapter 3 - Rationalisation Exercise Ex. 3.1

Question 1

Simplify

(i) $\sqrt[3]{4}\times\sqrt[3]{16}$ (ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

Solution 1

(i)
We have,

$$\sqrt[3]{4} \times \sqrt[3]{16} = (4)^{\frac{1}{3}} \times (16)^{\frac{1}{3}}$$
$$= (4 \times 16)^{\frac{1}{3}}$$
$$= (4 \times 4^2)^{\frac{1}{3}}$$
$$= (4^3)^{\frac{1}{3}}$$
$$= 4^{3 \times \frac{1}{3}}$$
$$= 4$$

$$\Rightarrow \quad \sqrt[3]{4} \times \sqrt[3]{16} = 4$$

(ii)
We have,

$$\frac{\sqrt[4]{1250}}{\sqrt[4]{2}} = \frac{(1250)^{\frac{1}{4}}}{(2)^{\frac{1}{4}}}$$
$$= \frac{(2 \times 5^4)^{\frac{1}{4}}}{(2)^{\frac{1}{4}}}$$
$$= \frac{2^{\frac{1}{4}} \times 5^{4 \times \frac{1}{4}}}{2^{\frac{1}{4}}}$$
$$= 5$$

$$\Rightarrow \quad \frac{\sqrt[4]{1250}}{\sqrt[4]{2}} = 5$$

Question 2
Simplify the following expressions:

(i) $(4 + \sqrt{7})(3 + \sqrt{2})$ (ii) $(3 + \sqrt{3})(5 - \sqrt{2})$ (iii) $(\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$

Solution 2

(i)
We have,

$\left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right)$
$= 4\left(3+\sqrt{2}\right)+\sqrt{7}\left(3+\sqrt{2}\right)$
$= 12+4\sqrt{2}+3\sqrt{7}+\sqrt{7\times 2}$

$= 12+4\sqrt{2}+3\sqrt{7}+\sqrt{14}$

$\therefore \quad \left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right) = 12+4\sqrt{2}+3\sqrt{7}+\sqrt{14}$

(ii)
We have,

$\left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right)$
$= 3\left(5-\sqrt{2}\right)+\sqrt{3}\left(5-\sqrt{2}\right)$

$= 15-3\sqrt{2}+5\sqrt{3}-\sqrt{6}$

$\therefore \quad \left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right) = 15-3\sqrt{2}+5\sqrt{3}-\sqrt{6}$

(iii)
We have,

$\left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right)$

$= \sqrt{5}\left(\sqrt{3}-\sqrt{5}\right)-2\left(\sqrt{3}-\sqrt{5}\right)$
$= \sqrt{15}-5-2\sqrt{3}+2\sqrt{5}$

$= \sqrt{15}-2\sqrt{3}+2\sqrt{5}-5$

$\therefore \quad \left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right) = \sqrt{15}-2\sqrt{3}+2\sqrt{5}-5$

Question 3

Simplify the following expressions:

(i) $\left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right)$

(ii) $\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right)$

(iii) $\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right)$

(iv) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(v) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Solution 3

(i)
We have,

$$\left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right)$$

$$= \left(11\right)^2 - \left(\sqrt{11}\right)^2 \qquad \left[\because a^2 - b^2 = (a+b)(a-b)\right]$$

$= 121 - 11$

$= 110$

$$\therefore \quad \left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right) = 110$$

(ii)
We have,

$$\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right)$$

$$= \left(5\right)^2 - \left(\sqrt{7}\right)^2 \qquad \left[\because (a+b)(a-b) = a^2 - b^2\right]$$

$= 25 - 7$

$= 18$

$$\therefore \quad \left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right) = 18$$

(iii)
We have,

$$\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right)$$

$$= \left(\sqrt{8}\right)^2 - \left(\sqrt{2}\right)^2 \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$= 8 - 2$

$= 6$

$$\therefore \quad \left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right) = 6$$

(iv) We have,

$$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$$

$$= \left(3\right)^2 - \left(\sqrt{3}\right)^2 \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

Question 4

Simplify the following expressions:

(i) $\left(\sqrt{3}+\sqrt{7}\right)^2$ (ii) $\left(\sqrt{5}-\sqrt{3}\right)^2$ (iii) $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

Solution 4

(i)

We have,

$\left(\sqrt{3}+\sqrt{7}\right)^2$

$= \left(\sqrt{3}\right)^2 + 2\times\sqrt{3}\times\sqrt{7} + \left(\sqrt{7}\right)^2 \qquad \left[\because (a+b)^2 = a^2+2ab+b^2\right]$

$= 3+2\sqrt{21}+7$

$= 10+2\sqrt{21}$

$\therefore \qquad \left(\sqrt{3}+\sqrt{7}\right)^2 = 10+2\sqrt{21}$

(ii)

We have,

$\left(\sqrt{5}-\sqrt{3}\right)^2$

$= \left(\sqrt{5}\right)^2 - 2\times\sqrt{5}\times\sqrt{3} + \left(\sqrt{3}\right)^2 \qquad \left[\because (a-b)^2 = a^2-2ab+b^2\right]$

$= 5-2\sqrt{15}+3$

$= 8-2\sqrt{15}$

$\therefore \qquad \left(\sqrt{5}-\sqrt{3}\right)^2 = 8-2\sqrt{15}$

(iii)

We have,

$\left(2\sqrt{5}+3\sqrt{2}\right)^2$

$= \left(2\sqrt{5}\right)^2 + \left(3\sqrt{2}\right)^2 + 2\times 2\sqrt{5}\times 3\sqrt{2}$

$= 20+18+12\sqrt{10}$

$= 38+12\sqrt{10}$

$\therefore \qquad \left(2\sqrt{5}+3\sqrt{2}\right)^2 = 38+12\sqrt{10}$