Access answers to Maths RD Sharma Solutions For Class 12 Chapter 2 – Function

Exercise 2.1 Page No: 2.31

1. Give an example of a function

(i) Which is one-one but not onto.

(ii) Which is not one-one but onto.

(iii) Which is neither one-one nor onto.

Solution:

(i) Let f: Z → Z given by f(x) = 3x + 2

Let us check one-one condition on f(x) = 3x + 2

Injectivity:

Let x and y be any two elements in the domain (Z), such that f(x) = f(y).

f (x) = f(y)

⇒ 3x + 2 =3y + 2

⇒ 3x = 3y

⇒ x = y

⇒ f(x) = f(y)

⇒ x = y

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain (Z), such that f(x) = y for some element x in Z(domain).

Let f(x) = y

⇒ 3x + 2 = y

⇒ 3x = y – 2

⇒ x = (y – 2)/3. It may not be in the domain (Z)

Because if we take y = 3,

x = (y – 2)/3 = (3-2)/3 = 1/3 ∉ domain Z.

So, for every element in the co domain there need not be any element in the domain such that f(x) = y.

Thus, f is not onto.

(ii) Example for the function which is not one-one but onto

Let f: Z → N ∪ {0} given by f(x) = |x|

Injectivity:

Let x and y be any two elements in the domain (Z),

Such that f(x) = f(y).

⇒ |x| = |y|

⇒ x = ± y

So, different elements of domain f may give the same image.

So, f is not one-one.

Surjectivity:

Let y be any element in the co domain (Z), such that f(x) = y for some element x in Z (domain).

f(x) = y

⇒ |x| = y

⇒ x = ± y

Which is an element in Z (domain).

So, for every element in the co-domain, there exists a pre-image in the domain.

Thus, f is onto.

(iii) Example for the function which is neither one-one nor onto.

Let f: Z → Z given by f(x) = $2x^2 + 1$

Injectivity:

Let x and y be any two elements in the domain (Z), such that f(x) = f(y).

f(x) = f(y)

⇒ $2x^2+1 = 2y^2+1$

⇒ $2x^2 = 2y^2$

⇒ $x^2 = y^2$

$\Rightarrow x = \pm y$

So, different elements of domain f may give the same image.

Thus, f is not one-one.

Surjectivity:

Let y be any element in the co-domain (Z), such that f(x) = y for some element x in Z (domain).

$f(x) = y$

$\Rightarrow 2x^2+1=y$

$\Rightarrow 2x^2= y - 1$

$\Rightarrow x^2 = (y-1)/2$

$\Rightarrow x = \sqrt{((y-1)/2)} \notin Z$ always.

For example, if we take, y = 4,

$x = \pm \sqrt{((y-1)/2)}$

$= \pm \sqrt{((4-1)/2)}$

$= \pm \sqrt{(3/2)} \notin Z$

So, x may not be in Z (domain).

Thus, f is not onto.

2. Which of the following functions from A to B are one-one and onto?

(i) $f_1 = \{(1, 3), (2, 5), (3, 7)\}$; A = {1, 2, 3}, B = {3, 5, 7}

(ii) $f_2 = \{(2, a), (3, b), (4, c)\}$; A = {2, 3, 4}, B = {a, b, c}

(iii) $f_3 = \{(a, x), (b, x), (c, z), (d, z)\}$; A = {a, b, c, d,}, B = {x, y, z}.

Solution:

(i) Consider $f_1 = \{(1, 3), (2, 5), (3, 7)\}$; A = {1, 2, 3}, B = {3, 5, 7}

Injectivity:

$f_1(1) = 3$

$f_1(2) = 5$

$f_1(3) = 7$

⇒ Every element of A has different images in B.

So, f_1 is one-one.
Surjectivity:

Co-domain of f_1 = {3, 5, 7}

Range of f_1 =set of images = {3, 5, 7}
⇒ Co-domain = range

So, f_1 is onto.
(ii) Consider f_2 = {(2, a), (3, b), (4, c)}; A = {2, 3, 4}, B = {a, b, c}
f_2 = {(2, a), (3, b), (4, c)}; A = {2, 3, 4}, B = {a, b, c}
Injectivity:

f_2 (2) = a

f_2 (3) = b

f_2 (4) = c
⇒ Every element of A has different images in B.

So, f_2 is one-one.
Surjectivity:

Co-domain of f_2 = {a, b, c}
Range of f_2 = set of images = {a, b, c}
⇒ Co-domain = range
So, f_2 is onto.
(iii) Consider f_3 = {(a, x), (b, x), (c, z), (d, z)} ; A = {a, b, c, d,}, B = {x, y, z}
Injectivity:

f_3 (a) = x

f_3 (b) = x

f_3 (c) = z

f_3 (d) = z

⇒ a and b have the same image x.

Also c and d have the same image z

So, f_3 is not one-one.

Surjectivity:

Co-domain of f_1 ={x, y, z}

Range of f_1 =set of images = {x, z}

So, the co-domain is not same as the range.

So, f_3 is not onto.

3. Prove that the function f: N → N, defined by $f(x) = x^2 + x + 1$, is one-one but not onto

Solution:

Given f: N → N, defined by $f(x) = x^2 + x + 1$

Now we have to prove that given function is one-one

Injectivity:

Let x and y be any two elements in the domain (N), such that f(x) = f(y).

⇒ $x^2 + x + 1 = y^2 + y + 1$

⇒ $(x^2 – y^2) + (x – y) = 0$ `

⇒ (x + y) (x- y) + (x – y) = 0

⇒ (x – y) (x + y + 1) = 0

⇒ x – y = 0 [x + y + 1 cannot be zero because x and y are natural numbers

⇒ x = y

So, f is one-one.

Surjectivity:

When x = 1

$x^2 + x + 1 = 1 + 1 + 1 = 3$

⇒ x + x +1 ≥ 3, for every x in N.

⇒ f(x) will not assume the values 1 and 2.

So, f is not onto.

4. Let A = {−1, 0, 1} and f = {(x, x^2) : x ∈ A}. Show that f : A → A is neither one-one nor onto.

Solution:

Given A = {−1, 0, 1} and f = {(x, x^2): x ∈ A}

Also given that, f(x) = x^2

Now we have to prove that given function neither one-one or nor onto.

Injectivity:

Let x = 1

Therefore f(1) = 1^2=1 and

f(-1)=$(-1)^2$=1

⇒ 1 and -1 have the same images.

So, f is not one-one.

Surjectivity:

Co-domain of f = {-1, 0, 1}

f(1) = 1^2 = 1,

f(-1) = $(-1)^2$ = 1 and

f(0) = 0

⇒ Range of f = {0, 1}

So, both are not same.

Hence, f is not onto

5. Classify the following function as injection, surjection or bijection:

(i) f: N → N given by f(x) = x^2

(ii) f: Z → Z given by f(x) = x^2

(iii) f: N → N given by f(x) = x^3

(iv) f: Z → Z given by f(x) = x^3

(v) f: R → R, defined by f(x) = |x|

(vi) f: Z → Z, defined by f(x) = x^2 + x

(vii) f: Z → Z, defined by f(x) = x − 5

(viii) f: R → R, defined by f(x) = sin x

(ix) f: R → R, defined by f(x) = x^3 + 1

(x) f: R → R, defined by f(x) = x^3 − x

(xi) f: R → R, defined by f(x) = $\sin^2 x + \cos^2 x$

(xii) f: Q − {3} → Q, defined by f (x) = (2x +3)/(x-3)

(xiii) f: Q → Q, defined by f(x) = x^3 + 1

(xiv) f: R → R, defined by f(x) = $5x^3$ + 4

(xv) f: R → R, defined by f(x) = $5x^3$ + 4

(xvi) f: R → R, defined by f(x) = 1 + x^2

(xvii) f: R → R, defined by f(x) = x/(x^2 + 1)

Solution:

(i) Given f: N → N, given by f(x) = x^2

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (N), such that f(x) = f(y).

f(x) = f(y)

$x^2 = y^2$

x = y (We do not get ± because x and y are in N that is natural numbers)

So, f is an injection.

Surjection condition:

Let y be any element in the co-domain (N), such that f(x) = y for some element x in N (domain).

f(x) = y

x^2= y

x = √y, which may not be in N.

For example, if y = 3,

x = √3 is not in N.

So, f is not a surjection.

Also f is not a bijection.

(ii) Given f: Z → Z, given by $f(x) = x^2$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (Z), such that f(x) = f(y).

$f(x) = f(y)$

$x^2 = y^2$

$x = \pm y$

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (Z), such that f(x) = y for some element x in Z (domain).

$f(x) = y$

$x^2 = y$

$x = \pm \sqrt{y}$ which may not be in Z.

For example, if y = 3,

$x = \pm \sqrt{3}$ is not in Z.

So, f is not a surjection.

Also f is not bijection.

(iii) Given f: N → N given by $f(x) = x^3$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (N), such that f(x) = f(y).

$f(x) = f(y)$

$x^3 = y^3$

$x = y$

So, f is an injection

Surjection condition:

Let y be any element in the co-domain (N), such that f(x) = y for some element x in N (domain).

f(x) = y

x^3= y

x = $\sqrt[3]{y}$ which may not be in N.

For example, if y = 3,

X = $\sqrt[3]{3}$ is not in N.

So, f is not a surjection and f is not a bijection.

(iv) Given f: Z → Z given by f(x) = x^3

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (Z), such that f(x) = f(y)

f(x) = f(y)

$x^3 = y^3$

x = y

So, f is an injection.

Surjection condition:

Let y be any element in the co-domain (Z), such that f(x) = y for some element x in Z (domain).

f(x) = y

x^3 = y

x = $\sqrt[3]{y}$ which may not be in Z.

For example, if y = 3,

x = $\sqrt[3]{3}$ is not in Z.

So, f is not a surjection and f is not a bijection.

(v) Given f: R → R, defined by f(x) = |x|

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (R), such that f(x) = f(y)

f(x) = f(y)

$|x|=|y|$

$x = \pm y$

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

$f(x) = y$

$|x|=y$

$x = \pm y \in Z$

So, f is a surjection and f is not a bijection.

(vi) Given f: Z → Z, defined by $f(x) = x^2 + x$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (Z), such that f(x) = f(y).

$f(x) = f(y)$

$x^2 + x = y^2 + y$

Here, we cannot say that x = y.

For example, x = 2 and y = – 3

Then,

$x^2 + x = 2^2 + 2 = 6$

$y^2 + y = (-3)^2 - 3 = 6$

So, we have two numbers 2 and -
3 in the domain Z whose image is same as 6.

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (Z),

such that f(x) = y for some element x in Z (domain).

$f(x) = y$

$x^2 + x = y$

Here, we cannot say $x \in Z$.

For example, $y = -4$.

$x^2 + x = -4$

$x^2 + x + 4 = 0$

$x = (-1 \pm \sqrt{-5})/2 = (-1 \pm i\sqrt{5})/2$ which is not in Z.

So, f is not a surjection and f is not a bijection.

(vii) Given f: Z → Z, defined by $f(x) = x - 5$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (Z), such that f(x) = f(y).

$f(x) = f(y)$

$x - 5 = y - 5$

$x = y$

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (Z), such that f(x) = y for some element x in Z (domain).

$f(x) = y$

$x - 5 = y$

$x = y + 5$, which is in Z.

So, f is a surjection and f is a bijection

(viii) Given f: R → R, defined by $f(x) = \sin x$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

$f(x) = f(y)$

$\sin x = \sin y$

Here, x may not be equal to y because $\sin 0 = \sin \pi$.

So, 0 and π have the same image 0.

So, f is not an injection.

Surjection test:

Range of f = [-1, 1]

Co-domain of f = R

Both are not same.

So, f is not a surjection and f is not a bijection.

(ix) Given f: R → R, defined by $f(x) = x^3 + 1$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

$f(x) = f(y)$

$x^3+1 = y^3+ 1$

$x^3= y^3$

$x = y$

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

$f(x) = y$

$x^3+1=y$

$x = \sqrt[3]{(y – 1)} \in R$

So, f is a surjection.

So, f is a bijection.

(x) Given f: R → R, defined by $f(x) = x^3 - x$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

f(x) = f(y)

$x^3 - x = y^3 - y$

Here, we cannot say x = y.

For example, x = 1 and y = -1

$x^3 - x = 1 - 1 = 0$

$y^3 - y = (-1)^3 - (-1) - 1 + 1 = 0$

So, 1 and -1 have the same image 0.

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

f(x) = y

$x^3 - x = y$

By observation we can say that there exist some x in R, such that $x^3 - x = y$.

So, f is a surjection and f is not a bijection.

(xi) Given f: R → R, defined by $f(x) = \sin^2 x + \cos^2 x$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

$f(x) = \sin^2 x + \cos^2 x$

We know that $\sin^2 x + \cos^2 x = 1$

So, f(x) = 1 for every x in R.

So, for all elements in the domain, the image is 1.

So, f is not an injection.

Surjection condition:

Range of f = {1}

Co-domain of f = R

Both are not same.

So, f is not a surjection and f is not a bijection.

(xii) Given f: Q − {3} → Q, defined by f (x) = (2x +3)/(x-3)

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (Q − {3}), such that f(x) = f(y).

$f(x) = f(y)$

$(2x + 3)/(x – 3) = (2y + 3)/(y – 3)$

$(2x + 3) (y − 3) = (2y + 3) (x − 3)$

$2xy − 6x + 3y − 9 = 2xy − 6y + 3x − 9$

$9x = 9y$

$x = y$

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (Q − {3}), such that f(x) = y for some element x in Q (domain).

$f(x) = y$

$(2x + 3)/(x – 3) = y$

$2x + 3 = x y − 3y$

$2x – x y = −3y − 3$

$x (2−y) = −3 (y + 1)$

$x = (3(y + 1))/(y – 1)$ which is not defined at y = 2.

So, f is not a surjection and f is not a bijection.

(xiii) Given f: Q → Q, defined by $f(x) = x^3 + 1$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (Q), such that f(x) = f(y).

$f(x) = f(y)$

$x^3 + 1 = y^3 + 1$

$x^3 = y^3$

x = y

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (Q), such that f(x) = y for some element x in Q (domain).

f(x) = y

$x^3 + 1 = y$

$x = \sqrt[3]{(y-1)}$, which may not be in Q.

For example, if y= 8,

$x^3 + 1 = 8$

$x^3 = 7$

$x = \sqrt[3]{7}$, which is not in Q.

So, f is not a surjection and f is not a bijection.

(xiv) Given f: R → R, defined by $f(x) = 5x^3 + 4$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection test:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

f(x) = f(y)

$5x^3 + 4 = 5y^3 + 4$

$5x^3 = 5y^3$

$x^3 = y^3$

x = y

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

f(x) = y

$5x^3 + 4 = y$

$5x^3 + 4 = y$

$X^3 = (y – 4)/5 \in R$

So, f is a surjection and f is a bijection.

(xv) Given f: R → R, defined by $f(x) = 5x^3 + 4$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

f(x) = f(y)

$5x^3 + 4 = 5y^3 + 4$

$5x^3 = 5y^3$

$x^3 = y^3$

x = y

So, f is an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

f(x) = y

$5x^3 + 4 = y$

$5x^3 + 4 = y$

$X^3 = (y – 4)/5 \in R$

So, f is a surjection and f is a bijection.

(xvi) Given f: R → R, defined by $f(x) = 1 + x^2$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

f(x) = f(y)

$1 + x^2 = 1 + y^2$

$x^2 = y^2$

$x = \pm y$

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

$f(x) = y$

$1 + x^2 = y$

$x^2 = y - 1$

$x = \pm \sqrt{-1} = \pm i$` is not in R.

So, f is not a surjection and f is not a bijection.

(xvii) Given f: R → R, defined by $f(x) = x/(x^2 + 1)$

Now we have to check for the given function is injection, surjection and bijection condition.

Injection condition:

Let x and y be any two elements in the domain (R), such that f(x) = f(y).

$f(x) = f(y)$

$x /(x^2 + 1) = y /(y^2 + 1)$

$x y^2 + x = x^2 y + y$

$xy^2 - x^2 y + x - y = 0$

$-x y (-y + x) + 1 (x - y) = 0$

$(x - y) (1 - x y) = 0$

$x = y$ or $x = 1/y$

So, f is not an injection.

Surjection test:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain).

$f(x) = y$

$x /(x^2 + 1) = y$

$y x^2 - x + y = 0$

$x ((-1) \pm \sqrt{(1-4x^2)})/(2y)$ if $y \neq 0$

$= (1 \pm \sqrt{(1-4y^2)})/ (2y)$, which may not be in R

For example, if y=1, then

$(1 \pm \sqrt{(1-4)}) / (2y) = (1 \pm i\sqrt{3})/2$, which is not in R

So, f is not surjection and f is not bijection.

6. If f: A → B is an injection, such that range of f = {a}, determine the number of elements in A.

Solution:

Given f: A → B is an injection

And also given that range of f = {a}

So, the number of images of f = 1

Since, f is an injection, there will be exactly one image for each element of f .

So, number of elements in A = 1.

7. Show that the function f: R − {3} → R − {2} given by f(x) = (x-2)/(x-3) is a bijection.

Solution:

Given that f: R − {3} → R − {2} given by f (x) = (x-2)/(x-3)

Now we have to show that the given function is one-one and on-to

Injectivity:

Let x and y be any two elements in the domain (R − {3}), such that f(x) = f(y).

f(x) = f(y)

$\Rightarrow (x - 2)/(x - 3) = (y - 2)/(y - 3)$

$\Rightarrow (x - 2)(y - 3) = (y - 2)(x - 3)$

$\Rightarrow x y - 3x - 2y + \ = x y = 3y - 2x + 6$

$\Rightarrow x = y$

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain (R − {2}), such that f(x) = y for some element x in R − {3} (domain).

f(x) = y

$\Rightarrow (x - 2)/(x - 3) = y$

$\Rightarrow x - 2 = x y - 3y$

⇒ x y – x = 3y – 2

⇒ x (y – 1) = 3y – 2

⇒ x = (3y – 2)/ (y – 1), which is in R – {3}

So, for every element in the co-domain, there exists some pre-image in the domain.

⇒ f is onto.

Since, f is both one-one and onto, it is a bijection.

8. Let *A* = [-1, 1]. Then, discuss whether the following function from *A* to itself is one-one, onto or bijective:

(i) f (x) = x/2

(ii) g (x) = |x|

(iii) h (x) = x^2

Solution:

(i) Given f: A → A, given by f (x) = x/2

Now we have to show that the given function is one-one and on-to

Injection test:

Let x and y be any two elements in the domain (A), such that f(x) = f(y).

f(x) = f(y)

x/2 = y/2

x = y

So, f is one-one.

Surjection test:

Let y be any element in the co-domain (A), such that f(x) = y for some element x in A (domain)

f(x) = y

x/2 = y

x = 2y, which may not be in A.

For example, if y = 1, then

x = 2, which is not in A.

So, f is not onto.

So, f is not bijective.

(ii) Given f: A → A, given by g (x) = |x|

Now we have to show that the given function is one-one and on-to

Injection test:

Let x and y be any two elements in the domain (A), such that f(x) = f(y).

f(x) = f(y)

|x| = |y|

x = ± y

So, f is not one-one.

Surjection test:

For y = -1, there is no value of x in A.

So, f is not onto.

So, f is not bijective.

(iii) Given f: A → A, given by h (x) = x^2

Now we have to show that the given function is one-one and on-to

Injection test:

Let x and y be any two elements in the domain (A), such that f(x) = f(y).

f(x) = f(y)

$x^2 = y^2$

x = ±y

So, f is not one-one.

Surjection test:

For y = – 1, there is no value of x in A.

So, f is not onto.

So, f is not bijective.

9. Are the following set of ordered pair of a function? If so, examine whether the mapping is injective or surjective:

(i) {(x, y): x is a person, y is the mother of x}

(ii) {(a, b): a is a person, b is an ancestor of a}

Solution:

Let f = {(x, y): x is a person, y is the mother of x}

As, for each element x in domain set, there is a unique related element y in co-domain set.

So, f is the function.

Injection test:

As, y can be mother of two or more persons

So, f is not injective.

Surjection test:

For every mother y defined by (x, y), there exists a person x for whom y is mother.

So, f is surjective.

Therefore, f is surjective function.

(ii) Let g = {(a, b): a is a person, b is an ancestor of a}

Since, the ordered map (a, b) does not map 'a' – a person to a living person.

So, g is not a function.

10. Let *A* = {1, 2, 3}. Write all one-one from *A* to itself.

Solution:

Given A = {1, 2, 3}

Number of elements in A = 3

Number of one-one functions = number of ways of arranging 3 elements = 3! = 6

(i) {(1, 1), (2, 2), (3, 3)}

(ii) {(1, 1), (2, 3), (3, 2)}

(iii) {(1, 2), (2, 2), (3, 3)}

(iv) {(1, 2), (2, 1), (3, 3)}

(v) {(1, 3), (2, 2), (3, 1)}

(vi) {(1, 3), (2, 1), (3,2)}

11. If f: R → R be the function defined by f(x) = $4x^3 + 7$, show that f is a bijection.

Solution:

Given f: R → R is a function defined by f(x) = $4x^3 + 7$

Injectivity:

Let x and y be any two elements in the domain (R), such that f(x) = f(y)

⇒ $4x^3 + 7 = 4y^3 + 7$

⇒ $4x^3 = 4y^3$

⇒ $x^3 = y^3$

⇒ x = y

So, f is one-one.

Surjectivity:

Let y be any element in the co-domain (R), such that f(x) = y for some element x in R (domain)

f(x) = y

⇒ $4x^3 + 7 = y$

⇒ $4x^3 = y - 7$

⇒ $x^3 = (y – 7)/4$

⇒ x = $\sqrt[3]{}$(y-7)/4 in R

So, for every element in the co-domain, there exists some pre-image in the domain. f is onto.

Since, f is both one-to-one and onto, it is a bijection.

Exercise 2.2 Page No: 2.46

1. Find gof and fog when f: R → R and g : R → R is defined by

(i) $f(x) = 2x + 3$ and $g(x) = x^2 + 5$.

(ii) $f(x) = 2x + x^2$ and $g(x) = x^3$

(iii) $f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

(iv) $f(x) = x$ and $g(x) = |x|$

(v) $f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

(vi) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

Solution:

(i) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

Also given that $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

Now, $(gof)(x) = g(f(x))$

$= g(2x + 3)$

$= (2x + 3)^2 + 5$

$= 4x^2 + 9 + 12x + 5$

$= 4x^2 + 12x + 14$

Now, $(fog)(x) = f(g(x))$

$= f(x^2 + 5)$

$= 2(x^2 + 5) + 3$

$= 2x^2 + 10 + 3$

$= 2x^2 + 13$

(ii) Given, f: R → R and g: R → R

so, gof: R → R and fog: R → R

$f(x) = 2x + x^2$ and $g(x) = x^3$

$(gof)(x) = g(f(x))$

$= g\ (2x+x^2)$

$= (2x+x^2)^3$

Now, (fog) (x) = f (g (x))

$= f\ (x^3)$

$= 2\ (x^3) + (x^3)^2$

$= 2x^3 + x^6$

(iii) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

$f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

(gof) (x) = g (f (x))

$= g\ (x^2 + 8)$

$= 3\ (x^2+8)^3 + 1$

Now, (fog) (x) = f (g (x))

$= f\ (3x^3 + 1)$

$= (3x^3+1)^2 + 8$

$= 9x^6 + 6x^3 + 1 + 8$

$= 9x^6 + 6x^3 + 9$

(iv) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

f(x) = x and g(x) = |x|

(gof) (x) = g (f (x))

= g (x)

= |x|

Now (fog) (x) = f (g (x))

= f (|x|)

= |x|

(v) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

$f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

(gof) (x) = g (f(x))

$= g\ (x^2 + 2x - 3)$

$= 3\ (x^2 + 2x - 3) - 4$

$= 3x^2 + 6x - 9 - 4$

$= 3x^2 + 6x - 13$

Now, (fog) (x) = f (g (x))

$= f\ (3x - 4)$

$= (3x - 4)^2 + 2\ (3x - 4) - 3$

$= 9x^2 + 16 - 24x + 6x - 8 - 3$

$= 9x^2 - 18x + 5$

(vi) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

$f(x) = 8x^3$ and $g(x) = x^{1/3}$

(gof) (x) = g (f (x))

$= g\ (8x^3)$

$= (8x^3)^{1/3}$

$= [(2x)^3]^{1/3}$

$= 2x$

Now, (fog) (x) = f (g (x))

$= f\ (x^{1/3})$

$= 8\ (x^{1/3})^3$

$= 8x$

2. Let f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3) (4, 9) (5, 9)}. Show that gof and fog are both defined. Also, find fog and gof.

Solution:

Given f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3) (4, 9) (5, 9)}

f : {3, 9, 12} → {1, 3, 4} and g : {1, 3, 4, 5} → {3, 9}

Co-domain of f is a subset of the domain of g.

So, gof exists and gof: {3, 9, 12} → {3, 9}

(gof) (3) = g (f (3)) = g (1) = 3

(gof) (9) = g (f (9)) = g (3) = 3

(gof) (12) = g (f (12)) = g (4) = 9

⇒ gof = {(3, 3), (9, 3), (12, 9)}

Co-domain of g is a subset of the domain of f.

So, fog exists and fog: {1, 3, 4, 5} → {3, 9, 12}

(fog) (1) = f (g (1)) = f (3) = 1

(fog) (3) = f (g (3)) = f (3) = 1

(fog) (4) = f (g (4)) = f (9) = 3

(fog) (5) = f (g (5)) = f (9) = 3

⇒ fog = {(1, 1), (3, 1), (4, 3), (5, 3)}

3. Let f = {(1, −1), (4, −2), (9, −3), (16, 4)} and g = {(−1, −2), (−2, −4), (−3, −6), (4, 8)}. Show that gof is defined while fog is not defined. Also, find gof.

Solution:

Given f = {(1, −1), (4, −2), (9, −3), (16, 4)} and g = {(−1, −2), (−2, −4), (−3, −6), (4, 8)}

f: {1, 4, 9, 16} → {-1, -2, -3, 4} and g: {-1, -2, -3, 4} → {-2, -4, -6, 8}

Co-domain of f = domain of g

So, gof exists and gof: {1, 4, 9, 16} → {-2, -4, -6, 8}

(gof) (1) = g (f (1)) = g (−1) = −2

(gof) (4) = g (f (4)) = g (−2) = −4

(gof) (9) = g (f (9)) = g (−3) = −6

(gof) (16) = g (f (16)) = g (4) = 8

So, gof = {(1, −2), (4, −4), (9, −6), (16, 8)}

But the co-domain of g is not same as the domain of f.

So, fog does not exist.

4. Let A = {a, b, c}, B = {u v, w} and let f and g be two functions from A to B and from B to A, respectively, defined as: f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.
Show that f and g both are bijections and find fog and gof.

Solution:

Given f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.

Also given that A = {a, b, c}, B = {u v, w}
Now we have to show f and g both are bijective.
Consider f = {(a, v), (b, u), (c, w)} and f: A → B

Injectivity of f: No two elements of A have the same image in B.

So, f is one-one.

Surjectivity of f: Co-domain of f = {u v, w}

Range of f = {u v, w}

Both are same.

So, f is onto.

Hence, f is a bijection.
Now consider g = {(u, b), (v, a), (w, c)} and g: B → A

Injectivity of g: No two elements of B have the same image in A.

So, g is one-one.

Surjectivity of g: Co-domain of g = {a, b, c}

Range of g = {a, b, c}

Both are the same.

So, g is onto.

Hence, g is a bijection.
Now we have to find fog,

we know that Co-domain of g is same as the domain of f.

So, fog exists and fog: {u v, w} → {u v, w}
(fog) (u) = f (g (u)) = f (b) = u
(fog) (v) = f (g (v)) = f (a) = v

(fog) (w) = f (g (w)) = f (c) = w

So, fog = {(u, u), (v, v), (w, w)}

Now we have to find gof,

Co-domain of f is same as the domain of g.

So, fog exists and gof: {a, b, c} → {a, b, c}

(gof) (a) = g (f (a)) = g (v) = a

(gof) (b) = g (f (b)) = g (u) = b

(gof) (c) = g (f (c)) = g (w) = c

So, gof = {(a, a), (b, b), (c, c)}

5. Find fog (2) and gof (1) when f: R → R; f(x) = x^2 + 8 and g: R → R; g(x) = $3x^3$ + 1.

Solution:

Given f: R → R; f(x) = $x^2 + 8$ and g: R → R; g(x) = $3x^3 + 1$.

Consider (fog) (2) = f (g (2))

= f $(3 \times 2^3 + 1)$

= f$(3 \times 8 + 1)$

= f (25)

= $25^2 + 8$

= 633

(gof) (1) = g (f (1))

= g $(1^2 + 8)$

= g (9)

= $3 \times 9^3 + 1$

= 2188

6. Let R^+ be the set of all non-negative real numbers.
If f: $R^+ \rightarrow R^+$ and g : $R^+ \rightarrow R^+$ are defined as f(x)=x^2 and g(x)=+ $\sqrt{x}$, find fog and gof. Are they equal functions.

Solution:

Given f: $R^+ \rightarrow R^+$ and g: $R^+ \rightarrow R^+$

So, fog: $R^+ \rightarrow R^+$ and gof: $R^+ \rightarrow R^+$

Domains of fog and gof are the same.

Now we have to find fog and gof also we have to check whether they are equal or not,

Consider (fog) (x) = f (g (x))

= f (√x)

= $\sqrt{x}^2$

= x

Now consider (gof) (x) = g (f (x))

= g (x^2)

= $\sqrt{x^2}$

= x

So, (fog) (x) = (gof) (x), ∀x ∈ R^+

Hence, fog = gof

7. Let f: R → R and g: R → R be defined by f(x) = x^2 and g(x) = x + 1. Show that fog ≠ gof.

Solution:

Given f: R → R and g: R → R.

So, the domains of f and g are the same.

Consider (fog) (x) = f (g (x))

= f (x + 1) = $(x + 1)^2$

= x^2 + 1 + 2

Again consider (gof) (x) = g (f (x))

= g (x^2) = x^2 + 1

So, fog ≠ gof

Exercise 2.3 Page No: 2.54

1. Find fog and gof, if

(i) f (x) = e^x, g (x) = $\log_e$ x

(ii) f (x) = x^2, g (x) = cos x

(iii) f (x) = |x|, g (x) = sin x

(iv) f (x) = x+1, g(x) = e^x

(v) f (x) = $\sin^{-1}$ x, g(x) = x^2

(vi) f (x) = x+1, g (x) = sin x

(vii) f(x)= x + 1, g (x) = 2x + 3

(viii) f(x) = c, c ∈ R, g(x) = sin x^2

(ix) f(x) = x^2 + 2 , g (x) = 1 − 1/ (1-x)

Solution:

(i) Given f (x) = e^x, g(x) = $\log_e$ x

Let f: R → (0, ∞); and g: (0, ∞) → R

Now we have to calculate fog,

Clearly, the range of g is a subset of the domain of f.

fog: (0, ∞) → R

(fog) (x) = f (g (x))

= f ($\log_e$ x)

= $\log_e e^x$

= x

Now we have to calculate gof,

Clearly, the range of f is a subset of the domain of g.

⇒ fog: R→ R

(gof) (x) = g (f (x))

= g (e^x)

= $\log_e e^x$

= x

(ii) f (x) = x^2, g(x) = cos x

f: R→ [0, ∞) ; g: R→[−1, 1]

Now we have to calculate fog,

Clearly, the range of g is not a subset of the domain of f.

⇒ Domain (fog) = {x: x ∈ domain of g and g (x) ∈ domain of f}

⇒ Domain (fog) = x: x ∈ R and cos x ∈ R}

⇒ Domain of (fog) = R

(fog): R→ R

(fog) (x) = f (g (x))

= f (cos x)

= $\cos^2 x$

Now we have to calculate gof,

Clearly, the range of f is a subset of the domain of g.

⇒ fog: R→R

(gof) (x) = g (f (x))

= g (x^2)

= cos x^2

(iii) Given f (x) = |x|, g(x) = sin x

f: R → (0, ∞) ; g : R→[−1, 1]

Now we have to calculate fog,

Clearly, the range of g is a subset of the domain of f.

⇒ fog: R→R

(fog) (x) = f (g (x))

= f (sin x)

= |sin x|

Now we have to calculate gof,

Clearly, the range of f is a subset of the domain of g.

⇒ fog : R→ R

(gof) (x) = g (f (x))

= g (|x|)

= sin |x|

(iv) Given f (x) = x + 1, g(x) = e^x

f: R→R ; g: R → [1, ∞)

Now we have calculate fog:

Clearly, range of g is a subset of domain of f.

⇒ fog: R→R

(fog) (x) = f (g (x))

= f (e^x)

= $e^x + 1$

Now we have to compute gof,

Clearly, range of f is a subset of domain of g.

⇒ fog: R→R

(gof) (x) = g (f (x))

= g (x+1)

= e^{x+1}

(v) Given f (x) = $\sin^{-1} x$, g(x) = x^2

f: [−1,1]→ [(-π)/2 ,π/2]; g : R → [0, ∞)

Now we have to compute fog:

Clearly, the range of g is not a subset of the domain of f.

Domain (fog) = {x: x ∈ domain of g and g (x) ∈ domain of f}

Domain (fog) = {x: x ∈ R and x^2 ∈ [−1, 1]}

Domain (fog) = {x: x ∈ R and x ∈ [−1, 1]}

Domain of (fog) = [−1, 1]

fog: [−1,1] → R

(fog) (x) = f (g (x))

= f (x^2)

= $\sin^{-1} (x^2)$

Now we have to compute gof:

Clearly, the range of f is a subset of the domain of g.

fog: [−1, 1] → R

(gof) (x) = g (f (x))

= g ($\sin^{-1} x$)

= $(\sin^{-1} x)^2$

(vi) Given f(x) = x+1, g(x) = sin x

f: R→R ; g: R→[−1, 1]

Now we have to compute fog

Set of the domain of f.

⇒ fog: R→ R

(fog) (x) = f (g (x))

= f (sin x)

= sin x + 1

Now we have to compute gof,

Clearly, the range of f is a subset of the domain of g.

⇒ fog: R → R

(gof) (x) = g (f (x))

= g (x+1)

= sin (x+1)

(vii) Given f (x) = x+1, g (x) = 2x + 3

f: R→R ; g: R → R

Now we have to compute fog

Clearly, the range of g is a subset of the domain of f.

⇒ fog: R→ R

(fog) (x) = f (g (x))

= f (2x+3)

= 2x + 3 + 1

= 2x + 4

Now we have to compute gof

Clearly, the range of f is a subset of the domain of g.

⇒ fog: R → R

(gof) (x) = g (f (x))

= g (x+1)

= 2 (x + 1) + 3

= 2x + 5

(viii) Given f (x) = c, g (x) = sin x^2

f: R → {c} ; g: R→ [0, 1]

Now we have to compute fog

Clearly, the range of g is a subset of the domain of f.

fog: R→R

(fog) (x) = f (g (x))

= f (sin x^2)

= c

Now we have to compute gof,

Clearly, the range of f is a subset of the domain of g.

⇒ fog: $R \rightarrow R$

(gof) (x) = g (f (x))

= g (c)

= sin c^2

(ix) Given f (x) = x^2+ 2 and g (x) = 1 – 1 / (1 – x)

f: $R \rightarrow [2, \infty)$

For domain of g: $1 - x \neq 0$

⇒ $x \neq 1$

⇒ Domain of g = $R - \{1\}$

g (x)= 1 – 1/ (1 – x) = (1 – x – 1)/ (1 – x) = (-x)/(1 – x)

For range of g

y = (- x)/ (1 – x)

⇒ y – x y = − x

⇒ y = x y − x

⇒ y = x (y−1)

⇒ x = y/(y – 1)

Range of g = $R - \{1\}$

So, g: $R - \{1\} \rightarrow R - \{1\}$

Now we have to compute fog

Clearly, the range of g is a subset of the domain of f.

⇒ fog: $R - \{1\} \rightarrow R$

(fog) (x) = f (g (x))

= f ((-x)/ (x – 1))

= $((-x)/ (x-1))^2 + 2$

= $(x^2 + 2x^2 + 2 - 4x) / (1 - x)^2$

`= $(3x^2 - 4x + 2)/ (1 - x)^2$

Now we have to compute gof

Clearly, the range of f is a subset of the domain of g.

⇒ gof: $R \rightarrow R$

$(gof)(x) = g(f(x))$

$= g(x^2 + 2)$

$= 1 – 1 / (1 – (x^2 + 2))$

$= – 1/ (1 – (x^2 + 2))$

$= (x^2 + 2)/ (x^2 + 1)$

2. Let $f(x) = x^2 + x + 1$ and $g(x) = \sin x$. Show that fog ≠ gof.

Solution:

Given $f(x) = x^2 + x + 1$ and $g(x) = \sin x$

Now we have to prove fog ≠ gof

$(fog)(x) = f(g(x))$

$= f(\sin x)$

$= \sin^2 x + \sin x + 1$

And $(gof)(x) = g(f(x))$

$= g(x^2 + x + 1)$

$= \sin(x^2 + x + 1)$

So, fog ≠ gof.

3. If $f(x) = |x|$, prove that fof = f.

Solution:

Given $f(x) = |x|$,

Now we have to prove that fof = f.

Consider $(fof)(x) = f(f(x))$

$= f(|x|)$

$= ||x||$

$= |x|$

$= f(x)$

So,

$(fof)(x) = f(x), \forall x \in R$

Hence, fof = f

4. If $f(x) = 2x + 5$ and $g(x) = x^2 + 1$ be two real functions, then describe each of the following functions:
(i) fog
(ii) gof

(iii) fof
(iv) f^2
Also, show that fof ≠ f^2

Solution:

f(x) and g(x) are polynomials.

⇒ f: R → R and g: R → R.

So, fog: R → R and gof: R → R.

(i) (fog) (x) = f (g (x))

= f (x^2 + 1)

= 2 (x^2 + 1) + 5

=$2x^2$ + 2 + 5

= $2x^2$ +7

(ii) (gof) (x) = g (f (x))

= g (2x +5)

= g $(2x + 5)^2$ + 1

= $4x^2$ + 20x + 26

(iii) (fof) (x) = f (f (x))

= f (2x +5)

= 2 (2x + 5) + 5

= 4x + 10 + 5

= 4x + 15

(iv) f^2 (x) = f (x) x f (x)

= (2x + 5) (2x + 5)

= $(2x + 5)^2$

= $4x^2$ + 20x +25

5. If f(x) = sin x and g(x) = 2x be two real functions, then describe gof and fog. Are these equal functions?

Solution:

Given f(x) = sin x and g(x) = 2x

We know that

f: R→ [−1, 1] and g: R→ R

Clearly, the range of f is a subset of the domain of g.

gof: $R \rightarrow R$

(gof) (x) = g (f (x))

= g (sin x)

= 2 sin x

Clearly, the range of g is a subset of the domain of f.

fog: $R \rightarrow R$

So, (fog) (x) = f (g (x))

= f (2x)

= sin (2x)

Clearly, fog ≠ gof

Hence they are not equal functions.

6. Let f, g, h be real functions given by f(x) = sin x, g (x) = 2x and h (x) = cos x. Prove that fog = go (f h).

Solution:

Given that f(x) = sin x, g (x) = 2x and h (x) = cos x

We know that f: $R \rightarrow [-1, 1]$ and g: $R \rightarrow R$

Clearly, the range of g is a subset of the domain of f.

fog: $R \rightarrow R$

Now, (f h) (x) = f (x) h (x) = (sin x) (cos x) = ½ sin (2x)

Domain of f h is R.

Since range of sin x is [-1, 1], $-1 \leq \sin 2x \leq 1$

$\Rightarrow -1/2 \leq \sin x/2 \leq 1/2$

Range of f h = [-1/2, 1/2]

So, (f h): $R \rightarrow [(-1)/2, 1/2]$

Clearly, range of f h is a subset of g.

$\Rightarrow$ go (f h): $R \rightarrow R$

$\Rightarrow$ Domains of fog and go (f h) are the same.

So, (fog) (x) = f (g (x))

= f (2x)

= sin (2x)

And (go (f h)) (x) = g ((f h) (x))

= g (sin x cos x)

= 2sin x cos x

= sin (2x)

⇒ (fog) (x) = (go (f h)) (x), ∀x ∈ R

Hence, fog = go (f h)

Exercise 2.4 Page No: 2.68

1. State with reason whether the following functions have inverse:
(i) f: {1, 2, 3, 4} → {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g: {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) h: {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Solution:

(i) Given f: {1, 2, 3, 4} → {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

We have:

f (1) = f (2) = f (3) = f (4) = 10

⇒ f is not one-one.

⇒ f is not a bijection.

So, f does not have an inverse.

(ii) Given g: {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

from the question it is clear that g (5) = g (7) = 4

⇒ f is not one-one.

⇒ f is not a bijection.

So, f does not have an inverse.

(iii) Given h: {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Here, different elements of the domain have different images in the co-domain.

⇒ h is one-one.

Also, each element in the co-domain has a pre-image in the domain.

⇒ h is onto.

⇒ h is a bijection.

Therefore h inverse exists.

⇒ h has an inverse and it is given by

h^{-1} = {(7, 2), (9, 3), (11, 4), (13, 5)}

2. Find f^{-1} if it exists: f: A → B, where

(i) A = {0, −1, −3, 2}; B = {−9, −3, 0, 6} and f(x) = 3 x.

(ii) A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} and f(x) = x^2

Solution:

(i) Given A = {0, −1, −3, 2}; B = {−9, −3, 0, 6} and f(x) = 3 x.

So, f = {(0, 0), (-1, -3), (-3, -9), (2, 6)}

Here, different elements of the domain have different images in the co-domain.

Clearly, this is one-one.

Range of f = Range of f = B

so, f is a bijection and,

Thus, f^{-1} exists.

Hence, f^{-1}= {(0, 0), (-3, -1), (-9, -3), (6, 2)}

(ii) Given A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} and f(x) = x^2

So, f = {(1, 1), (3, 9), (5, 25), (7, 49), (9, 81)}

Here, different elements of the domain have different images in the co-domain.

Clearly, f is one-one.

But this is not onto because the element 0 in the co-domain (B) has no pre-image in the domain (A)

⇒ f is not a bijection.

So, f^{-1}does not exist.

3. Consider f: {1, 2, 3} → {a, b, c} and g: {a, b, c} → {apple, ball, cat} defined as f (1) = a, f (2) = b, f (3) = c, g (a) = apple, g (b) = ball and g (c) = cat. Show that f, g and gof are invertible.
Find f^{-1}, g^{-1} and $gof^{-1}$and show that $(gof)^{-1} = f^{-1} o\ g^{-1}$

Solution:

Given f = {(1, a), (2, b), (c , 3)} and g = {(a , apple) , (b , ball) , (c , cat)}
Clearly , f and g are bijections.

So, f and g are invertible.

Now,

f^{-1} = {(a ,1) , (b , 2) , (3,c)} and g^{-1} = {(apple, a), (ball , b), (cat , c)}

So, f^{-1} o g^{-1}= {apple, 1), (ball, 2), (cat, 3)}……… (1)

f: {1,2,3,} → {a, b, c} and g: {a, b, c} → {apple, ball, cat}

So, gof: {1, 2, 3} → {apple, ball, cat}

⇒ (gof) (1) = g (f (1)) = g (a) = apple

(gof) (2) = g (f (2))

= g (b)

= ball,

And (gof) (3) = g (f (3))

= g (c)

= cat

∴ gof = {(1, apple), (2, ball), (3, cat)}

Clearly, gof is a bijection.

So, gof is invertible.

$(gof)^{-1}$ = {(apple, 1), (ball, 2), (cat, 3)}……. (2)

Form (1) and (2), we get

$(gof)^{-1} = f^{-1}$ o g^{-1}

4. Let A = {1, 2, 3, 4}; B = {3, 5, 7, 9}; C = {7, 23, 47, 79} and f: A → B, g: B → C be defined as f(x) = 2x + 1 and g(x) = $x^2 - 2$. Express $(gof)^{-1}$ and f^{-1} og^{-1} as the sets of ordered pairs and verify that $(gof)^{-1} = f^{-1}$ og^{-1}.

Solution:

Given that f (x) = 2x + 1

⇒ f= {(1, 2(1) + 1), (2, 2(2) + 1), (3, 2(3) + 1), (4, 2(4) + 1)}

= {(1, 3), (2, 5), (3, 7), (4, 9)}

Also given that g(x) = $x^2 - 2$

⇒ g = {(3, 32−2), (5, 52−2), (7, 72−2), (9, 92−2)}

= {(3, 7), (5, 23), (7, 47), (9, 79)}

Clearly f and g are bijections and, hence, f^{-1}: B→ A and g^{-1}: C→ B exist.

So, f^{-1}= {(3, 1), (5, 2), (7, 3), (9, 4)}

And g^{-1}= {(7, 3), (23, 5), (47, 7), (79, 9)}

Now, (f^{-1} o g^{-1}): C→ A

f^{-1} o g^{-1} = {(7, 1), (23, 2), (47, 3), (79, 4)}……….(1)

Also, f: A→B and g: B → C,

⇒ gof: A → C, $(gof)^{-1}$: C→ A

So, f^{-1} o $g^{-1}$and $(gof)^{-1}$ have same domains.

(gof) (x) = g (f (x))

=g (2x + 1)

=$(2x + 1)^2 - 2$

⇒ (gof) (x) = $4x^2 + 4x + 1 - 2$

⇒ (gof) (x) = $4x^2 + 4x - 1$

Then, (gof) (1) = g (f (1))

= 4 + 4 − 1

=7,

(gof) (2) = g (f (2))

= 4 + 4 – 1 = 23,

(gof) (3) = g (f (3))

= 4 + 4 – 1 = 47 and

(gof) (4) = g (f (4))

= 4 + 4 − 1 = 79

So, gof = {(1, 7), (2, 23), (3, 47), (4, 79)}

⇒ (gof) – 1 = {(7, 1), (23, 2), (47, 3), (79, 4)}…… (2)

From (1) and (2), we get:

$(gof)^{-1} = f^{-1} \text{ o } g^{-1}$

5. Show that the function f: Q → Q, defined by f(x) = 3x + 5, is invertible. Also, find f^{-1}

Solution:

Given function f: Q → Q, defined by f(x) = 3x + 5

Now we have to show that the given function is invertible.

Injection of f:

Let x and y be two elements of the domain (Q),

Such that f(x) = f(y)

⇒ 3x + 5 = 3y + 5

⇒ 3x = 3y

⇒ x = y

so, f is one-one.

Surjection of f:

Let y be in the co-domain (Q),

Such that f(x) = y

⇒ 3x +5 = y

⇒ 3x = y – 5

⇒ x = (y -5)/3 in (domain)

⇒ f is onto.

So, f is a bijection and, hence, it is invertible.

Now we have to find f^{-1}:

Let $f^{-1}(x)$ = y…… (1)

⇒ x = f(y)

⇒ x = 3y + 5

⇒ x −5 = 3y

⇒ y = (x – 5)/3

Now substituting these values in 1 we get

So, $f^{-1}(x) = (x – 5)/3$

6. Consider f: R → R given by f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

Solution:

Given f: R → R given by f(x) = 4x + 3

Now we have to show that the given function is invertible.

Consider injection of f:

Let x and y be two elements of domain (R),

Such that f(x) = f(y)

⇒ 4x + 3 = 4y + 3

⇒ 4x = 4y

⇒ x = y

So, f is one-one.

Now surjection of f:

Let y be in the co-domain (R),

Such that f(x) = y.

⇒ 4x + 3 = y

⇒ 4x = y -3

⇒ x = (y-3)/4 in R (domain)

⇒ f is onto.

So, f is a bijection and, hence, is invertible.

Now we have to find f^{-1}

Let $f^{-1}(x) = y$……. (1)

⇒ x = f (y)

⇒ x = 4y + 3

⇒ x − 3 = 4y

⇒ y = (x -3)/4

Now substituting these values in 1 we get

So, $f^{-1}(x) = (x-3)/4$

7. Consider f: $R \to R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1} of f given by $f^{-1}(x) = \sqrt{(x-4)}$ where R^+ is the set of all non-negative real numbers.

Solution:

Given f: $R \to R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$.

Now we have to show that f is invertible,

Consider injection of f:

Let x and y be two elements of the domain (Q),

Such that f(x) = f(y)

$\Rightarrow x^2 + 4 = y^2 + 4$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y$ (as co-domain as R+)

So, f is one-one

Now surjection of f:

Let y be in the co-domain (Q),

Such that f(x) = y

$\Rightarrow x^2 + 4 = y$

$\Rightarrow x^2 = y – 4$

$\Rightarrow x = \sqrt{(y-4)}$ in R

$\Rightarrow$ f is onto.

So, f is a bijection and, hence, it is invertible.

Now we have to finding f^{-1}:

Let $f^{-1}(x) = y$…… (1)

$\Rightarrow x = f(y)$

$\Rightarrow x = y^2 + 4$

$\Rightarrow x - 4 = y^2$

$\Rightarrow y = \sqrt{(x-4)}$

So, $f^{-1}(x) = \sqrt{(x-4)}$

Now substituting these values in 1 we get, So, $f^{-1}(x) = \sqrt{(x-4)}$

8. If $f(x) = (4x + 3)/(6x – 4)$, $x \neq (2/3)$ show that $fof(x) = x$, for all $x \neq (2/3)$. What is the inverse of f?

Solution:

It is given that $f(x) = (4x + 3)/(6x – 4)$, $x \neq 2/3$

Now we have to show $fof(x) = x$

$(fof)(x) = f(f(x))$

$= f((4x+ 3)/(6x – 4))$

$= (4((4x + 3)/(6x -4)) + 3)/(6((4x +3)/(6x – 4)) – 4)$

$= (16x + 12 + 18x – 12)/(24x + 18 – 24x + 16)$

$= (34x)/(34)$

$= x$

Therefore $fof(x) = x$ for all $x \neq 2/3$

=> $fof = 1$

Hence, the given function f is invertible and the inverse of f is f itself.

9. Consider $f: R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with

$f^{-1}(x) = (\sqrt{(x +6)}-1)/3$

Solution:

Given $f: R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x – 5$

We have to show that f is invertible.

Injectivity of f:

Let x and y be two elements of domain (R^+),

Such that $f(x) = f(y)$

$\Rightarrow 9x^2 + 6x – 5 = 9y^2 + 6y - 5$

$\Rightarrow 9x^2 + 6x = 9y^2 + 6y$

$\Rightarrow x = y$ (As, $x, y \in R^+$)

So, f is one-one.

Surjectivity of f:

Let y is in the co domain (Q)

Such that $f(x) = y$

$\Rightarrow 9x^2 + 6x - 5 = y$

$\Rightarrow 9x^2 + 6x = y + 5$

$\Rightarrow 9x^2 + 6x +1 = y + 6$ (By adding 1 on both sides)

$\Rightarrow (3x + 1)^2 = y + 6$

$\Rightarrow 3x + 1 =\sqrt{(y + 6)}$

$\Rightarrow 3x = \sqrt{(y + 6)} - 1$

$\Rightarrow x = (\sqrt{(y + 6)}-1)/3$ in R^+ (domain)

f is onto.

So, f is a bijection and hence, it is invertible.

Now we have to find f^{-1}

Let f−1(x) = y….. (1)

$\Rightarrow x = f (y)$

$\Rightarrow x = 9y^2 + 6y - 5$

$\Rightarrow x + 5 = 9y^2 + 6y$

$\Rightarrow x + 6 = 9y^2+ 6y + 1$ (adding 1 on both sides)

$\Rightarrow x + 6 = (3y + 1)^2$

$\Rightarrow 3y + 1 = \sqrt{(x + 6)}$

$\Rightarrow 3y =\sqrt{(x +6)} -1$

$\Rightarrow y = (\sqrt{(x+6)}-1)/3$

Now substituting these values in 1 we get,

So, $f^{-1}(x) = (\sqrt{(x-6)}-1)/3$

10. If f: R → R be defined by f(x) = $x^3 - 3$, then prove that f^{-1} exists and find a formula for f^{-1}. Hence, find f^{-1} (24) and f^{-1} (5).

Solution:

Given f: R → R be defined by f(x) = $x^3 - 3$

Now we have to prove that f^{-1} exists

Injectivity of f:

Let x and y be two elements in domain (R),

Such that, $x^3 - 3 = y^3 - 3$

$\Rightarrow x^3 = y^3$

$\Rightarrow x = y$

So, f is one-one.

Surjectivity of f:

Let y be in the co-domain (R)

Such that f(x) = y

$\Rightarrow x^3 - 3 = y$

$\Rightarrow x^3 = y + 3$

$\Rightarrow x = \sqrt[3]{(y+3)}$ in R

$\Rightarrow$ f is onto.

So, f is a bijection and, hence, it is invertible.

Finding f^{-1}:

Let $f^{-1}(x) = y$…….. (1)

$\Rightarrow x= f(y)$

$\Rightarrow x = y^3 - 3$

$\Rightarrow x + 3 = y^3$

$\Rightarrow y = \sqrt[3]{(x + 3)} = f^{-1}(x)$ [from (1)]

So, $f^{-1}(x) = \sqrt[3]{(x + 3)}$

Now, $f^{-1}(24) = \sqrt[3]{(24 + 3)}$

$= \sqrt[3]{27}$

$= \sqrt[3]{3^3}$

$= 3$

And $f^{-1}(5) = \sqrt[3]{(5 + 3)}$

$= \sqrt[3]{8}$

$= \sqrt[3]{2^3}$

$= 2$

11. A function f: R → R is defined as $f(x) = x^3 + 4$. Is it a bijection or not? In case it is a bijection, find $f^{-1}(3)$.

Solution:

Given that f: R → R is defined as $f(x) = x^3 + 4$

Injectivity of f:

Let x and y be two elements of domain (R),
Such that f (x) = f (y)
$\Rightarrow x^3 + 4 = y^3 + 4$
$\Rightarrow x^3 = y^3$
$\Rightarrow x = y$
So, f is one-one.
Surjectivity of f:

Let y be in the co-domain (R),
Such that f(x) = y.
$\Rightarrow x^2 + 4 = y$
$\Rightarrow x^3 = y – 4$
$\Rightarrow x = \sqrt[3]{(y – 4)}$ in R (domain)
$\Rightarrow$ f is onto.

So, f is a bijection and, hence, is invertible.
Finding f^{-1}:
Let $f^{-1}(x) = y$…… (1)
$\Rightarrow x = f(y)$
$\Rightarrow x = y^3 + 4$
$\Rightarrow x - 4 = y^3$
$\Rightarrow y = \sqrt[3]{(x-4)}$
So, $f^{-1}(x) = \sqrt[3]{(x-4)}$ [from (1)]
$f^{-1}(3) = \sqrt[3]{(3 – 4)}$
$= \sqrt[3]{-1}$
$= -1$