Chapter 14 – Co-ordinate Geometry has five exercises for which RD Sharma Solutions for Class 10 gives the correct solutions to these exercise problems. The main objective is to help students get the correct knowledge about problem-solving by introducing the right methods to solve them. This chapter consists of important concepts like:

- To find the distance between two points whose coordinates are given
- Find the coordinates of the point which divides the line segment joining two given points in a given ratio.
- The method of finding the area of a triangle in terms of the coordinates of its vertices.

Access the RD Sharma Solutions For Class 10 Chapter 14 – Co-ordinate Geometry

RD Sharma class 1o Chapter 14 Exercise 14.1 Page No: 14.4

1. On which axis do the following points lie?

(i) P (5, 0)

(ii) Q (0, -2)

(iii) R (-4, 0)

(iv) S (0, 5)

Solution:

(i) P (5, 0) lies on x – axis

(ii) Q (0, -2) lies on y – axis (negation half)

(iii) R (-4, 0) lies on x – axis (negative half)

(iv) S (0, 5) lies on y – axis

RD Sharma class 1o Chapter 14 Exercise 14.2 Page No: 14.15

1. Find the distance between the following pair of points:

(i) (- 6, 7) and (-1, -5)

(ii) (a + b, b + c) and (a -b, c – b)

(iii) (a sin α, – b cos α) and (- a cos α, b sin α)

(iv) (a, 0) and (0, b)

Solution:

(i) Let the given points be P (- 6, 7) and Q (- 1, – 5)

Here,

$x_1 = -6$, $y_1 = 7$ and

$x_2 = -1$, $y_2 = -5$

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ = $\sqrt{[-1-(-6)]^2 + (-5-7)^2}$

PQ = $\sqrt{(-1+6)^2 + (-5-7)^2}$

PQ =$\sqrt{(5)^2 + (-12)^2}$

PQ = $\sqrt{25+144}$

PQ = $\sqrt{(169)}$

PQ = 13

(ii) Let the given points be P (a + b, b + c) and Q (a – b, c – b)

Here,

x_1 = a + b, y_1 = b + c and

x_2 = a – b, y_2 = c – b

PQ = $\sqrt{x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ = $\sqrt{[a-b-(a+b)]^2 + (c-b-(b+c))^2}$

PQ = $\sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$

PQ = $\sqrt{(-2b)^2 + (-2b)^2}$

PQ = $\sqrt{4b^2 + 4b^2}$

PQ = $\sqrt{8b^2}$

PQ = $\sqrt{4 \times 2b^2}$

PQ = $2\sqrt{2b}$

$$PQ = \sqrt{x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[a - b - (a + b)]^2 + (c - b - (b + c))^2}$$

$$PQ = \sqrt{(a - b - a - b)^2 + (c - b - b - c)^2}$$

$$PQ = \sqrt{(-2b)^2 + (-2b)^2}$$

$$PQ = \sqrt{4b^2 + 4b^2}$$

$$PQ = \sqrt{8b^2}$$

$$PQ = \sqrt{4 \times 2b^2}$$

$$PQ = 2\sqrt{2b}$$

(iii) Let the given points be P(a sinα, – b cos α) and Q(-a cos α, b sin α) here

x_1 = a sin α, y_1 = – b cos α and

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(-a\, cos\, a - a\, sin\, a)^2 + [-b\, sin\, a - (-b\, cos\, a)]^2}$$

$$PQ = \sqrt{(-\mathrm{a} \cos \mathrm{a})^2 + (-\mathrm{a} \sin \mathrm{a})^2 + 2(-\mathrm{a} \cos \mathrm{a})(-\mathrm{a} \sin \mathrm{a}) + (\mathrm{b} \sin \mathrm{a})^2 + (-\mathrm{b} \cos \mathrm{a})^2 - 2(\mathrm{b} \sin \mathrm{a})(-}$$

$$PQ = \sqrt{\mathrm{a}^2 \cos^2 \mathrm{a} + \mathrm{a}^2 \sin^2 \mathrm{a} + 2\mathrm{a}^2 \cos \mathrm{a} \sin \mathrm{a} + \mathrm{b}^2 \sin^2 \mathrm{a} + \mathrm{b}^2 \sin \mathrm{a} \cos \mathrm{a}}$$

$$PQ = \sqrt{\mathrm{a}^2 (\cos^2 \mathrm{a} + \sin^2 \mathrm{a}) + 2\mathrm{a}^2 \cos \mathrm{a} \sin \mathrm{a} + \mathrm{b}^2 (\sin^2 \mathrm{a} + \cos^2 \mathrm{a}) + 2\mathrm{b}^2 \sin \mathrm{a} \cos \mathrm{a}}$$

$$PQ = \sqrt{\mathrm{a}^2 \times 1 + 2\mathrm{a}^2 \cos \mathrm{a} \sin \mathrm{a} + \mathrm{b}^2 \times 2\mathrm{b}^2 \sin \mathrm{a} \cos \mathrm{a}} \; [\therefore \sin^2 \mathrm{a} + \cos^2 \mathrm{a} = 1]$$

$$PQ = \sqrt{\mathrm{a}^2 + \mathrm{b}^2 + 2\mathrm{a}^2 \cos \mathrm{a} \sin \mathrm{a} + 2\mathrm{b}^2 \sin \mathrm{a} \cos \mathrm{a}}$$

$$PQ = \sqrt{(\mathrm{a}^2 + \mathrm{b}^2) + 2\cos \mathrm{a} \sin \mathrm{a}\, (\mathrm{a}^2 + \mathrm{b}^2)}$$

$$PQ = \sqrt{(\mathrm{a}^2 + \mathrm{b}^2)(1 + 2\cos \mathrm{a} \sin \mathrm{a})}$$

x_2– a cos α, y_2 = b sin α

(iv) Let the given points be P(a, 0) and Q (0, b)

Here,

x_1 = a, y_1 = 0, x_2 = 0, y_2 = b,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(0 - a)^2 + (b - 0)^2}$$

$$PQ = \sqrt{(-a)^2 + (b)^2}$$

$$PQ = \sqrt{a^2 + b^2}$$

2. Find the value of a when the distance between the points (3, a) and (4, 1) is √10.

Solution:

Let the given points be P (3, a) and Q(4, 1).

Here,

$$PQ = \sqrt{10}$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{(1)^2 + (1-a)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{1 + 1 + a^2 - 2a} \qquad [\therefore (a-b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow \sqrt{10} = \sqrt{2 + a^2 - 2a}$$

On squaring on both sides, we have

$$\Rightarrow (\sqrt{10})^2 = (\sqrt{2 + a^2 - 2a})^2$$

⇒ 10 = 2 + a^2 – 2a

⇒ a^2 – 2a + 2 – 10 = 0

⇒ a^2 – 2a – 8 = 0

By splitting the middle team,

⇒ a^2 – 4a + 2a – 8 = 0

$\Rightarrow a(a - 4) + 2(a - 4) = 0$

$\Rightarrow (a - 4)(a + 2) = 0$

$\Rightarrow a = 4, a = -2$

Thus, there are 2 possible values for a which are 4 and -2.

3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y), show that x + 3y = 0.

Solution:

Let the given points be P(2, 1) and Q(1,- 2) and R(x, y)

Also, PR = QR (given)

$$PR = \sqrt{(x-2)^2 + (y-1)^2}$$

$$\Rightarrow PR = \sqrt{(x^2+2)^2 - 2x \times 2 + y^2 + (1)^2 - 2 \times y \times 1}$$

$$\Rightarrow PR = \sqrt{x^2 + 4 - 4x + y^2 + 1 - 2y}$$

$$\Rightarrow PR = \sqrt{x^2 + 5 - 4x + y^2 - 2y}$$

$$QR = \sqrt{(x-1)^2 + (y+2)^2}$$

$$\Rightarrow QR = \sqrt{x^2 + 1 - 2x + y^2 + 4 + 4y}$$

$$\Rightarrow QR = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$$

But, PR = QR

$$\Rightarrow \sqrt{x^2 + 5 - 4x + y^2 - 2y} = \sqrt{x^2 + 5 - 2x + y^2 + 4y}$$

$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$

$\Rightarrow x^2 + 5 - 4x + y^2 - 2y = x^2 + 5 - 2x + y^2 + 4y$

$\Rightarrow – 4x + 2x – 2y – 4y = 0$

$\Rightarrow – 2x – 6y = 0$

$\Rightarrow – 2(x + 3y) = 0$

$\Rightarrow -2(x + 3y) = 0$

$\Rightarrow x + 3y = 0/-2$

$\Rightarrow x + 3y = 0$

- Hence Proved.

4. Find the value of x, y if the distances of the point (x, y) from (- 3, 0) as well as from (3, 0) are 4.

Solution:

Let the given points be P(x, y), Q(-3, 0) and R(3, 0)

$$PQ = \sqrt{(x+3)^2 + (y-0)^2}$$

$$\Rightarrow 4 = \sqrt{x^2 + 9 + 6x + y^2}$$

On squaring on both sides, we get

$$\Rightarrow (4)^2 = (\sqrt{x^2 + 9 + 6x + y^2})^2$$

$\Rightarrow 16 = x^2 + 9 + 6x + y^2$

$\Rightarrow x^2 + y^2 = 7 – 6x$ …… (1)

PR = $(\sqrt{(x-3)^2 + (y-0)^2})$

$\Rightarrow 4 = \sqrt{x^2 + 9 - 6x + y^2}$

On squaring on both sides,

$(4)^2 = (\sqrt{x^2 + 9 - 6x + y^2})^2$

$\Rightarrow 16 = x^2 + 9 - 6x + y^2$

$\Rightarrow x^2 + y^2 = 16 - 9 + 6x$

$\Rightarrow x^2 + y^2 = 7 + 6x$ .... (2)

Equating (1) and (2), we have

$7 - 6x = 7 + 6x$

$\Rightarrow 7 - 7 = 6x + 6x$

$\Rightarrow 0 = 12x$

$\Rightarrow x = 12$

Then, substituting the value of x = 0 in (2)

$x^2 + y^2 = 7 + 6x$

$0 + y^2 = 7 + 6 \times 0$

$y^2 = 7$

$y = + \sqrt{7}$

As y can have two values, the points are (12, √7) and (12, -√7).

5. The length of a line segment is of 10 units and the coordinates of one end-point are (2, -3). If the abscissa of the other end is 10, find the ordinate of the other end.

Solution:

Given,

Length of the line segment is 10 units.

Coordinates of one end-point are (2, -3) and the abscissa of the other end is 10.

So, let the ordinate of the other end be k.

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(10-2)^2 + (k+3)^2}$$

On squaring both sides, we get

$$100 = (10-2)^2 + (k+3)^2$$

$$100 = 64 + k^2 + 6k + 9$$

$$k^2 + 6k - 27 = 0$$

$$k^2 + 9k - 3k - 27 = 0$$

$$k(k+9) - 3(k+9) = 0$$

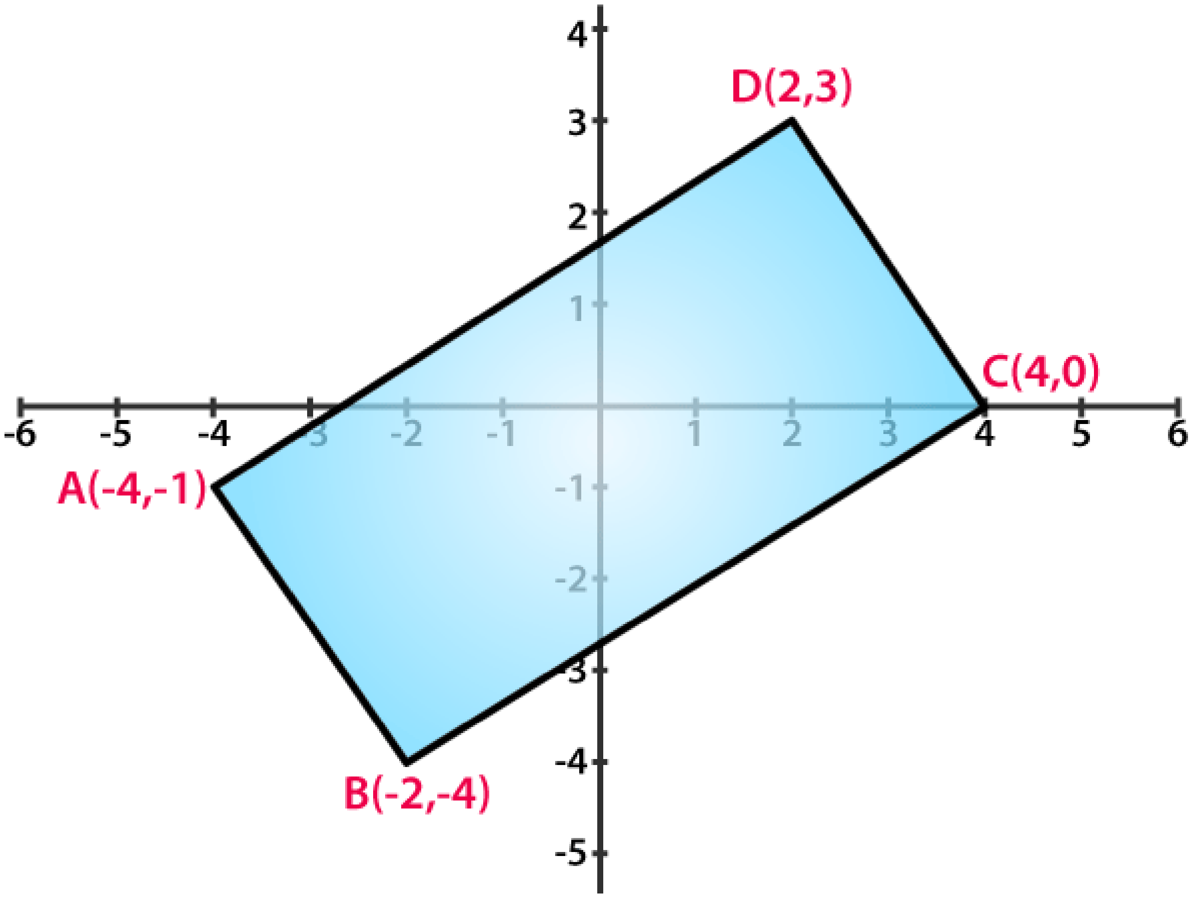
$$(k-3)(k+9) = 0$$

$$k = 3, k = -9$$

Therefore, the ordinates of the other end can be 3 or -9.

6. Show that the points A(- 4, -1), B(-2, – 4), C(4, 0) and D(2, 3) are the vertices points of a rectangle.

Solution:



v

Given: Points A(- 4, -1), B(-2, – 4), C(4, 0) and D(2, 3)

Required to prove: the points are the vertices points of a rectangle.

Vertices of rectangle ABCD are: A(- 4, -1), B(-2, – 4), C(4, 0) and D(2, 3)

We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(-2+4)^2 + (-4+1)^2} = \sqrt{4+9} = \sqrt{13}$ units

Length of side BC = $\sqrt{(4+2)^2 + (0+4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

Length of side CD = $\sqrt{(2-4)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$ units

Length of side AD = $\sqrt{(2+4)^2 + (3+1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

Finding the diagonals,

Length of diagonal BD = $\sqrt{(2+2)^2 + (3+4)^2} = \sqrt{16+49} = \sqrt{65}$ units

Length of diagonal AC = $\sqrt{(4+4)^2 + (0+1)^2} = \sqrt{64+1} = \sqrt{65}$ units

As the opposite sides are equal and also the diagonals are equal.

Therefore, the given points are the vertices of a rectangle.

- Hence Proved

7. Show that the points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram.

Solution:

Given: Points A (1,- 2), B (3, 6), C (5, 10) and D (3, 2)

Required to prove: the points are the vertices points of a parallelogram.

Vertices of a parallelogram ABCD are: A (1, -2), B (3, 6), C (5, 10) and D (3, 2)

We know that,

Length of a side = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of side AB = $\sqrt{(3-1)^2 + (6+2)^2} = \sqrt{(4+64)} = \sqrt{68}$ units

Length of side BC = $\sqrt{(5-3)^2 + (10-6)^2} = \sqrt{(4+16)} = \sqrt{20}$ units

Length of side CD = $\sqrt{(3-5)^2 + (2-10)^2} = \sqrt{(4+64)} = \sqrt{68}$ units

Length of side DA = $\sqrt{(3-1)^2 + (2+2)^2} = \sqrt{(4+16)} = \sqrt{20}$ units

Finding the diagonals,

Length of diagonal BD = $\sqrt{(3-3)^2 + (2-6)^2} = \sqrt{16} = 4$ units

Length of diagonal AC = $\sqrt{(5-1)^2 + (10+2)^2} = \sqrt{(16+144)} = \sqrt{160}$ units

It's seen that the opposite sides of the quadrilateral formed by the given four points are equal

i.e. (AB = CD) & (DA = BC)

Also, the diagonals BD & AC are found unequal.

Hence, the given points form a parallelogram.

- Hence Proved

8. Prove that the points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) are the vertices of a square.

Solution:

Given: Points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)

Required to prove: the points are the vertices points of a square.

Vertices of a square ABCD are: A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)

We know that,

$$\text{Length of a side} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Length of side AB} = \sqrt{(4-1)^2 + (2-7)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$$\text{Length of side BC} = \sqrt{(-1-4)^2 + (-2-1)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

$$\text{Length of side CD} = \sqrt{(-4+1)^2 + (4+1)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$$\text{Length of side DA} = \sqrt{(-4-1)^2 + (4-7)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

Finding the diagonals,

$$\text{Length of diagonal BD} = \sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{64+4} = \sqrt{68} \text{ units}$$

$$\text{Length of diagonal AC} = \sqrt{(-1-1)^2 + (-1-7)^2} = \sqrt{4+64} = \sqrt{68} \text{ units}$$

As the opposite sides are equal and also the diagonals are equal the given vertices are therefore the vertices of a square.

- Hence Proved

9. Prove that the points (3, 0), (6, 4) and (- 1, 3) are vertices of a right-angled isosceles triangle.

Solution:

Let the vertices of the triangle ABC be: A(3, 0), B(6, 4) and C (- 1, 3)

We know that,

$$\text{Length of a side} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Length of side AB} = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} \text{ units}$$

$$\text{Length of side BC} = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{49+1} = \sqrt{50} \text{ units}$$

$$\text{Length of side AC} = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} \text{ units}$$

It's seen that AB = AC, Thus, it's an isosceles triangle.

Verifying the Pythagoras theorem, we have

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

50 = 25 + 25

50 = 50

As $BC^2 = AB^2 + AC^2$

Therefore, the given vertices are of a right-angled isosceles triangle.

- Hence Proved

10. Prove that (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.

Solution:

From given,

Let consider the vertices of a triangle ABC as: A(2, -2), B(-2, 1) and C(5, 2)

We know that,

Length of a side = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(-2-2)^2+(1+2)^2} = \sqrt{16+9} = \sqrt{25}$ units

Length of side BC = $\sqrt{(5+2)^2+(2-1)^2} = \sqrt{49+1} = \sqrt{50}$ units

Length of side AC = $\sqrt{(5-2)^2+(2+2)^2} = \sqrt{9+16} = \sqrt{25}$ units

It's seen that AB = AC, thus the triangle is an isosceles triangle.

Verifying the Pythagoras theorem, we have

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

50 = 25 + 25

50 = 50

As $BC^2 = AB^2 + AC^2$

Therefore, the given triangle is right angled triangle.

Now,

Area of right angled triangle = $\frac{1}{2} base \times altitude$

Area of right angled triangle = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ square units

And, the length of hypotenuse (BC) = $\sqrt{50} = 5\sqrt{2}$ units

- Hence Proved

11. Prove that the points (2a, 4a), (2a, 6a) and (2a + √3a, 5a) are the vertices of an equilateral triangle.

Solution:

From given,

Let's consider the vertices of a triangle ABC as: A(2 a, 4 a), B(2 a, 6 a) and C(2a + √3a, 5a)

We know that,

Length of a side = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Length of side AB = $\sqrt{(2a-2a)^2+(6a-4a)^2} = \sqrt{(2a)^2}$ = 2a units

Length of side BC = $\sqrt{\left(2a+\sqrt{3}a-2a\right)^2+(5a-6a)^2}$ = $\sqrt{(4a^2)}$ = 2a units

Length of side AC = $\sqrt{\left(2a+\sqrt{3}a-2a\right)^2+(5a-4a)^2}$ = $\sqrt{(4a^2)}$ = 2a units

As all the sides are equal the triangle is an equilateral triangle.

Thus, the given vertices are of an equilateral triangle.

- Hence Proved

12. Prove that the points (2, 3), (-4, -6) and (1, 3/2) do not form a triangle.

Solution:

From given,

Let's consider the vertices of a triangle ABC as: A(2, 3), B(-4, -6) and C(1, 3/2)

We know that,

$$\text{Length of a side} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Length of side AB} = \sqrt{(-4-2)^2 + (-6-3)^2} = \sqrt{36+81} = \sqrt{117} \text{ units}$$

$$\text{Length of side BC} = \sqrt{(1+4)^2 + \left(\frac{3}{2}+6\right)^2} = \sqrt{25+56.25} = \sqrt{81.25} \text{ units}$$

$$\text{Length of side AC} = \sqrt{(1-2)^2 + \left(\frac{3}{2}-3\right)^2} = \sqrt{1+2.25} = \sqrt{2.25} \text{ units}$$

Thus, the given vertices do not form a triangle as the sum of two sides of a triangle is not greater than third side.

- Hence Proved

13. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right triangle ABC right angled at B. Find the values of a and hence the area of triangle ABC.

Solution:

Given,

A right triangle ABC, right angled at B.

Points A (2, 9), B (a, 5) and C (5, 5)

So, AC is the hypotenuse

Thus, from Pythagoras theorem we have

$AC^2 = AB^2 + BC^2$

$[(5 – 2)^2 + (5 – 9)^2] = [(a – 2)^2 + (5 – 9)^2] + [(5 – a)^2 + (5 – 5)^2]$ $[3^2 + (-4)^2] = [(a – 2)^2 + (-4)^2] + [(5 – a)^2 + 0]$
$9 + 16 = a^2 – 4a + 4 + 16 + 25 – 10a + a^2$

$2a^2 – 14a + 20 = 0$

$a^2 – 7a + 10 = 0$

$(a – 5)(a – 2) = 0$ [By factorization method]

So,

a = 5 or 2

Here, a = 5 is not possible as it coincides with point C. So, for a triangle to form the value of a = 2 is correct.

Thus, the coordinates of point B is (2, 5).

Now, the area of triangle ABC

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Delta = \frac{1}{2}[2(5-5) + 2(5-9) + 5(9-5)]$$

$$= \frac{1}{2}[2 \times 0 + 2(-4) + 5(4)]$$

$$= \frac{1}{2}(0 - 8 + 20) = \frac{1}{2} \times 12 = 6$$

Therefore, the area of triangle ABC is 6 sq. units

14. Show that the quadrilateral whose vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2) is a rhombus.

Solution:

Let A(2, -1), B(3 ,4), C(-2, 3) and D(-3, -2)

Then we have,

Length of AB = $\sqrt{[(3 - 2)^2 + (4 - (-1))^2]} = \sqrt{[(1)^2 + (5)^2]} = \sqrt{[1 + 25]} = \sqrt{26}$ units

Length of BC = $\sqrt{[(3 - (-2))^2 + (4 - 3)^2]} = \sqrt{[(5)^2 + (1)^2]} = \sqrt{[25 + 1]} = \sqrt{26}$ units

Length of CD = $\sqrt{[(-2 - (-3))^2 + (3 - 2)^2]} = \sqrt{[(-5)^2 + (1)^2]} = \sqrt{[25 + 1]} = \sqrt{26}$ units

Length of AD = $\sqrt{[(-3 - 2)^2 + (-2 - (-1))^2]} = \sqrt{[(-5)^2 + (-1)^2]} = \sqrt{[25 + 1]} = \sqrt{26}$ units

As AB = BC = CD = AD

We can say that,

Quadrilateral ABCD is a rhombus.

15. Two vertices of an isosceles triangle are (2, 0) and (2, 5). Find the third vertex if the length of the equal sides is 3.

Solution:

Let the third vertex be C (x, y)

And, given A (2, 0) & B (2, 5)

We have,

Length of AB = $\sqrt{[(2 - 2)^2 + (5 - 0)^2]} = \sqrt{[(0)^2 + (5)^2]} = \sqrt{[0 + 25]} = 5$ units

Length of BC = $\sqrt{[(x - 2)^2 + (y - 5)^2]} = \sqrt{[x^2 - 4x + 4 + y^2 - 10y + 25]}$

$= \sqrt{[x^2 - 4x + y^2 - 10y + 29]}$ units

Length of AC = $\sqrt{[(x - 2)^2 + (y - 0)^2]} = \sqrt{[x^2 - 4x + 4 + y^2]}$ units

Given that,

AC = BC = 3

So, $AC^2 = BC^2 = 9$

$x^2 - 4x + 4 + y^2 = x^2 - 4x + y^2 - 10y + 29$

$10y = 25$

$y = 25/10 = 2.5$

And,

$AC^2 = 9$

$x^2 - 4x + 4 + y^2 = 9$

$x^2 - 4x + 4 + (2.5)^2 = 9$

$x^2 - 4x + 4 + 6.25 = 9$

$x^2 - 4x + 1.25 = 0$

$D = (-4)^2 - 4 \times 1 \times 1.25 = 16 - 5 = 11$

So, the roots are

$x = -(-4) + \sqrt{11}/ 2 = (4 + 3.31)/ 2 = 3.65$

And,

$x = -(-4) - \sqrt{11}/ 2 = (4 - 3.31)/ 2 = 0.35$

Therefore, the third vertex can be C (3.65, 2.5) or (0.35, 2.5)

16. Which point on x – axis is equidistant from (5, 9) and (-4, 6)?

Solution:

Let A (5, 9) and B (-4, 6) be the given points

Let the point on x – axis equidistant from the above points be C(x, 0)

Now, we have

$AC = \sqrt{[(x - 5)^2 + (0 - 9)^2]} = \sqrt{[x^2 - 10x + 25 + 81]} = \sqrt{[x^2 - 10x + 106]}$

And,

$BC = \sqrt{[(x - (-4))^2 + (0 - 6)^2]} = \sqrt{[x^2 + 8x + 16 + 36]} = \sqrt{[x^2 + 8x + 52]}$

As AC = BC (given condition)

So, $AC^2 = BC^2$

$x^2 - 10x + 106 = x^2 + 8x + 52$

$18x = 54$

$x = 3$

Therefore, the point on the x-axis is (3, 0)

17. Prove that the points (-2, 5), (0, 1) and (2, -3) are collinear.

Solution:

Let A (-2, 5), B(0, 1) and C (2, -3) be the given points

So, we have

$AB = \sqrt{[(0 - (-2))^2 + (1 - 5)^2]} = \sqrt{[(2)^2 + (-4)^2]} = \sqrt{[4 + 16]} = \sqrt{20} = 2\sqrt{5}$ units

$BC = \sqrt{[(2 - 0)^2 + (-3 - 1)^2]} = \sqrt{[(2)^2 + (-4)^2]} = \sqrt{[4 + 16]} = \sqrt{20} = 2\sqrt{5}$ units

$AC = \sqrt{[(2 - (-2))^2 + (-3 - 5)^2]} = \sqrt{[(4)^2 + (-8)^2]} = \sqrt{[16 + 64]} = \sqrt{80} = 4\sqrt{5}$ units

Now, it's seen that

AB + BC = AC

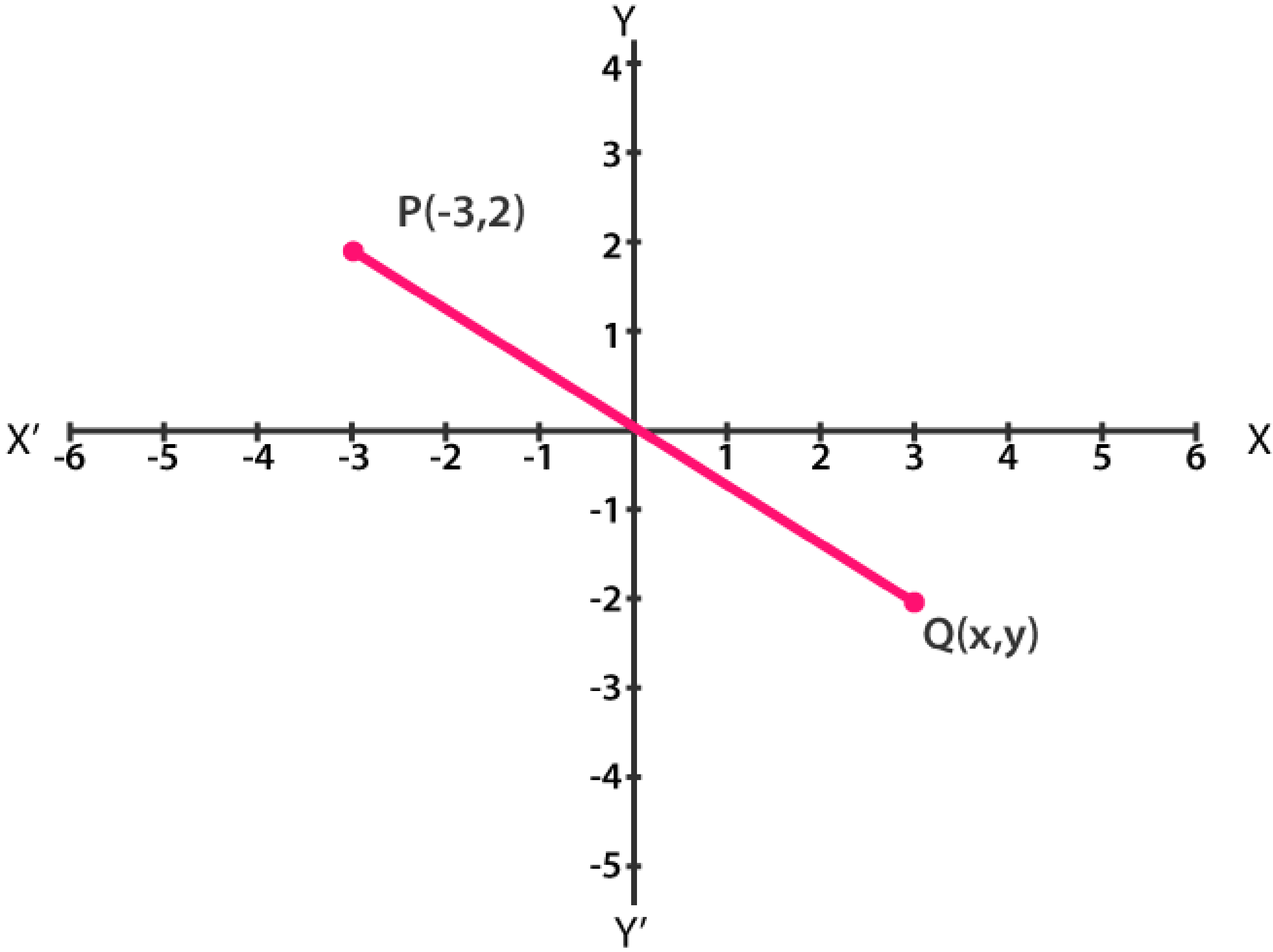
$2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}$

$4\sqrt{5} = 4\sqrt{5}$

Therefore, we can conclude that the given points (-2, 5), (0, 1) and (2, -3) are collinear

18. The coordinates of the point P are (-3, 2). Find the coordinates of the point Q which lies on the line joining P and origin such that OP = OQ.

Solution:



Let the coordinates of Q be taken as (x, y)

As Q lies on the line joining P and O(origin) with OP = OQ

Then, by mid-point theorem

$(x – 3)/2 = 0$

And,

$(y + 2)/ 2 = 0$

∴ $x = 3, y = -2$

Therefore, the coordinates of point Q are (3, -2)

19. Which point on the y-axis is equidistant from (2, 3) and (-4, 1)?

Solution:

Let A (2, 3) and B (-4, 1) be the given points

Let the point on y – axis equidistant from the above points be C (0, y)

Now, we have

$AC = \sqrt{[(0 – 2)^2 + (y – 3)^2]} = \sqrt{[y^2 – 6y + 9 + 4]} = \sqrt{[y^2 – 6y + 13]}$

And,

$BC = \sqrt{[(0 – (-4))^2 + (y – 1)^2]} = \sqrt{y^2 – 2y + 1 + 16]} = \sqrt{[y^2 – 2y + 17]}$

As AC = BC (given condition)

So, $AC^2 = BC^2$

$y^2 - 6y + 13 = y^2 - 2y + 17$

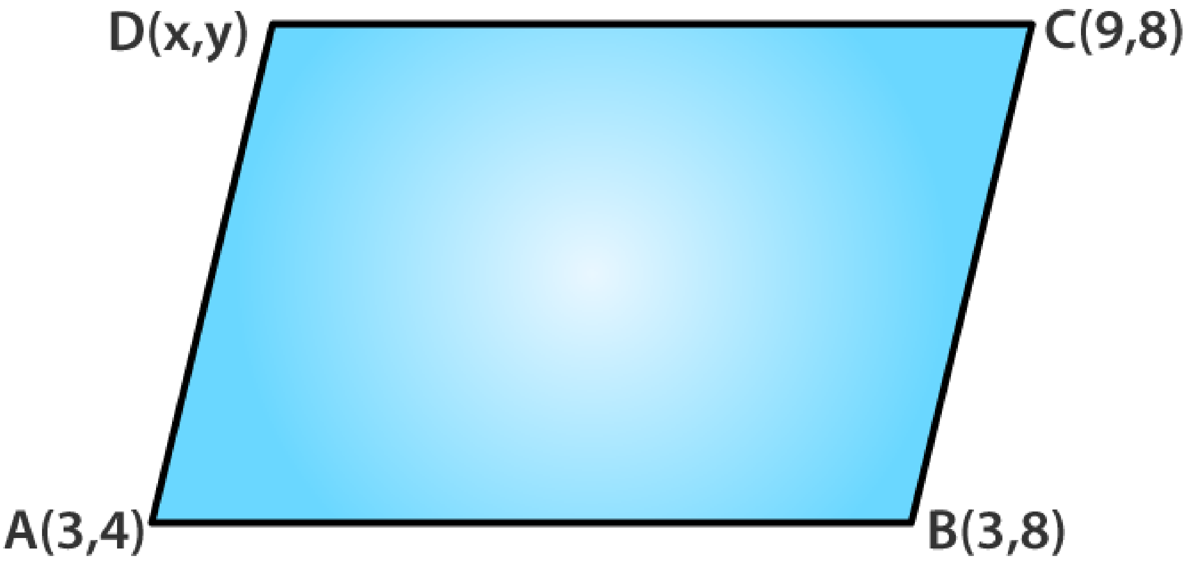
$-4y = 4$

$y = -1$

Therefore, the point on the y-axis is (0, -1)

20. The three vertices of a parallelogram are (3, 4), (3, 8) and (9, 8). Find the fourth vertex.

Solution:



Let A (3, 4), B (3, 8) and C (9, 8) be the given points.

And let the fourth vertex be D(x, y)

We know that,

In a parallelogram the diagonal bisect each other.

So, the mid-point of AC should be the same as the mid-point of BC

By mid-point theorem,

Mid-point of AC = (3 + 9/ 2), (4 + 8/ 2) = (6, 6)

Now,

The mid-point of BD = (3 + x/ 2, 8 + y/ 2)

And this point must be equal to (6, 6)

So, we have

(3 + x)/ 2 = 6 (8 + y)/ 2 = 6

3 + x = 12 8 + y = 12

x = 9 y = 4

Therefore, the fourth vertex is D (9, 4)

21. Find a point which is equidistant from the points A (-5, 4) and B (-1, 6). How many such points are there?

Solution:

Let P(x, y) be the equidistant point from points A (-5, 4) and B (-1, 6).

So, the mid-point can be the required point

(x, y) = ((-5 – 1/ 2), (4 + 6)/2)

(x , y) = (-6/2 , 10/2) = (-3, 5)

Thus, the required point is (-3, 5)

Now,

We also know that, AP = BP

So, $AP^2 = BP^2$

$(x + 5)^2 + (y – 4)^2 = (x + 1)^2 + (y – 6)^2$

$x^2 + 25 + 10 + y^2 – 8y + 16 = x^2 + 2x + 1 + y^2 – 12y + 36$

10x + 41 – 8y = 2x + 37 – 12y

8x + 4y + 4 = 0

2x + y + 1 = 0

Therefore, all the points which lie on the line 2x + y + 1 = 0 are equidistant from A and B.

22. The center of a circle is (2a, a – 7). Find the values of a if the circle passes through the point (11, -9) and has diameter 10√2 units.

Solution:

Given,

Diameter of the circle = 10√2 units

So, the radius = 5√2 units

Let the center of a circle be 0(2a, a-7) and the circle passes though the point P (11, -9).

Then, OP is the radius of the circle

OP = 5√2

$OP^2 = (5\sqrt{2}) = 50$

$(11- 2a)^2 + (-9 – a + 7)^2 = 50$

$121 – 44a + 4a^2 + 4 + a^2 + 4a = 50$

$5a^2 – 40a + 75 = 0$

$a^2 – 8a + 15 = 0$

(a – 5)(a – 3) = 0 [Factorisation method]

So, a = 5 or a = 3

23. Ayush starts walking from his house to office, Instead of going to the office directly, he goes to bank first, from there to his daughter’s school and then reaches the office. What is the extra distance travelled by Ayush in reaching the office? (Assume that all distance covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8) school at (13, 14) and office at (13, 26) and coordinates are in kilometer.

Solution:

The position of Ayush’s house is (2, 4) and the position of the bank is (5, 8).

So, the distance between the house and the bank,

$d_1 = \sqrt{[(5 – 2)^2 + (8 – 4)^2]} = \sqrt{[(3)^2 + (4)^2]} = \sqrt{[9 + 16]} = \sqrt{25} = 5$ km

The position of the bank is (5, 8) and the position of the school is (13, 14).

So, the distance between the bank and the school,

$d_2 = \sqrt{[(13 – 5)^2 + (14 – 8)^2]} = \sqrt{[(8)^2 + (6)^2]} = \sqrt{[64 + 36]} = \sqrt{100} = 10$ km

The position of the school is (13, 14) and the position of the office is (13, 26).

So, the distance between the school and the office,

$d_3 = \sqrt{[(13 – 13)^2 + (26 – 14)^2]} = \sqrt{[(0)^2 + (12)^2]} = \sqrt{144} = 12$ km

Let d be the total distance covered by Ayush

$d = d_1 + d_2 + d_3 = 5 + 10 + 12 = 27$ km

Let the D be the shortest distance from Ayush's house to the office,

$D = \sqrt{[(13 – 2)^2 + (26 – 4)^2]} = \sqrt{[(11)^2 + (22)^2]} = \sqrt{[121+ 484]} = \sqrt{605} = 24.6$ km

Thus, the extra distance covered by Ayush = d – D = 27 – 24.6 = 2.4 km

24. Find the value of k, if the point P(0, 2) is equidistant from (3, k) and (k, 5).

Solution:

Let the point P (0, 2) is equidistant from A (3, k) and B (k, 5)

So, PA = PB

$PA^2 = PB^2$

$(3 -0)^2 + (k -2)^2 = (k – 0)^2 + (5 – 2)^2$

$9 + k^2 + 4 – 4k – k^2 – 9 = 0$

$4 – 4k = 0$

$-4k = -4$

Therefore, the value of k = 1

25. If (-4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the (i) interior (ii) exterior of the triangle.

Solution:

Let B (-4, 3) and C (4, 3) be the given two vertices of the equilateral triangle.

Let A (x, y) be the third vertex.

Then, we have

AB = BC = AC

Let us consider the part AB = BC

$AB^2 = BC^2$

$(-4 – x)^2 + (3 – y)^2 = (4 + 4)^2 + (3 – 3)^2$

$16 + x^2 + 8x + 9 + y^2 – 6y = 64$

$x^2 + y^2 + 8x – 6y = 39$

Now, let us consider AB = AC

$AB^2 = AC^2$

$(-4 – x)^2 + (3 – y)^2 = (4 – x)^2 + (3 – y)^2$

$16 + x^2 + 8x + 9 + y^2 – 18y = 16 + x^2 – 8x + 9 + y^2 – 6y$

$16x = 0$

$x = 0$

Now, BC = AC

$BC^2 = AC^2$

$(4 + 4)^2 + (3 – 3)^2 = (4 – 0)^2 + (3 – y)^2$

$64 + 0 = 16 + 9 + y^2– 6y$

$64 = 16 + (3 – y)^2$

$(3 – y)^2 = 48$

$3 – y = \pm 4\sqrt{3}$

$y = 3 \pm 4\sqrt{3}$

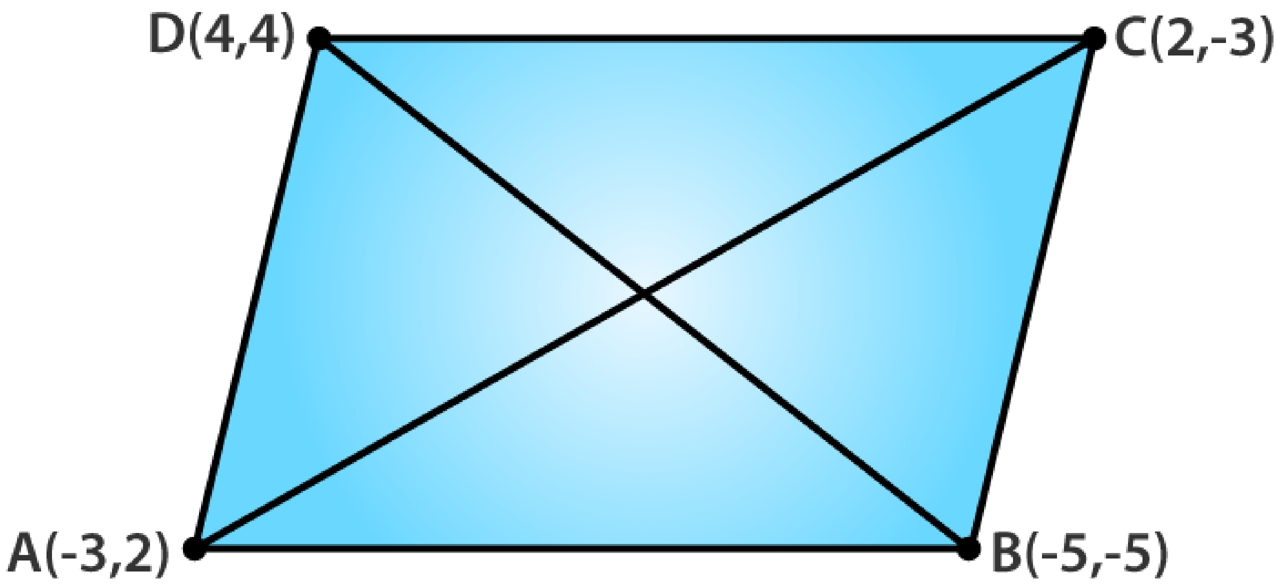
Therefore, the coordinates of the third vertex

(i) When origin lies in the interior of the triangle is $(0, 3 - 4\sqrt{3})$

(ii) When origin lies in the exterior of the triangle is $(0, 3 + 4\sqrt{3})$

26. Show that the points (-3, 2), (-5, -5), (2, -3) and (4, 4) are the vertices of a rhombus. Find the area of this rhombus.

Solution:



Let A(-3, 2), B(-5, -5), C(2, -3) and D(4, 4) be the given points.

Then we have,

$AB = \sqrt{[(-5 + 3)^2 + (-5 - 2)^2]} = \sqrt{[(2)^2 + (7)^2]} = \sqrt{[4 + 49]} = \sqrt{53}$ units

$BC = \sqrt{[(2 + 5)^2 + (-3 + 5)^2]} = \sqrt{[(7)^2 + (2)^2]} = \sqrt{[49 + 4]} = \sqrt{53}$ units

$CD = \sqrt{[(4 - 2)^2 + (4 + 3)^2]} = \sqrt{[(2)^2 + (7)^2]} = \sqrt{[4 + 49]} = \sqrt{53}$ units

$AD = \sqrt{[(4 + 3)^2 + (4 - 2)^2]} = \sqrt{[(7)^2 + (2)^2]} = \sqrt{[49 + 4]} = \sqrt{53}$ units

And the diagonals,

$AC = \sqrt{[(2 + 3)^2 + (-3 - 2)^2]} = \sqrt{[(5)^2 + (-5)^2]} = \sqrt{[25 + 25]} = 5\sqrt{2}$ units

$BD = \sqrt{[(4 + 5)^2 + (4 + 5)^2]} = \sqrt{[(9)^2 + (9)^2]} = \sqrt{[81 + 81]} = 9\sqrt{2}$ units

It's seen that,

As AB = BC = CD = AD and the diagonals AC ≠ BD

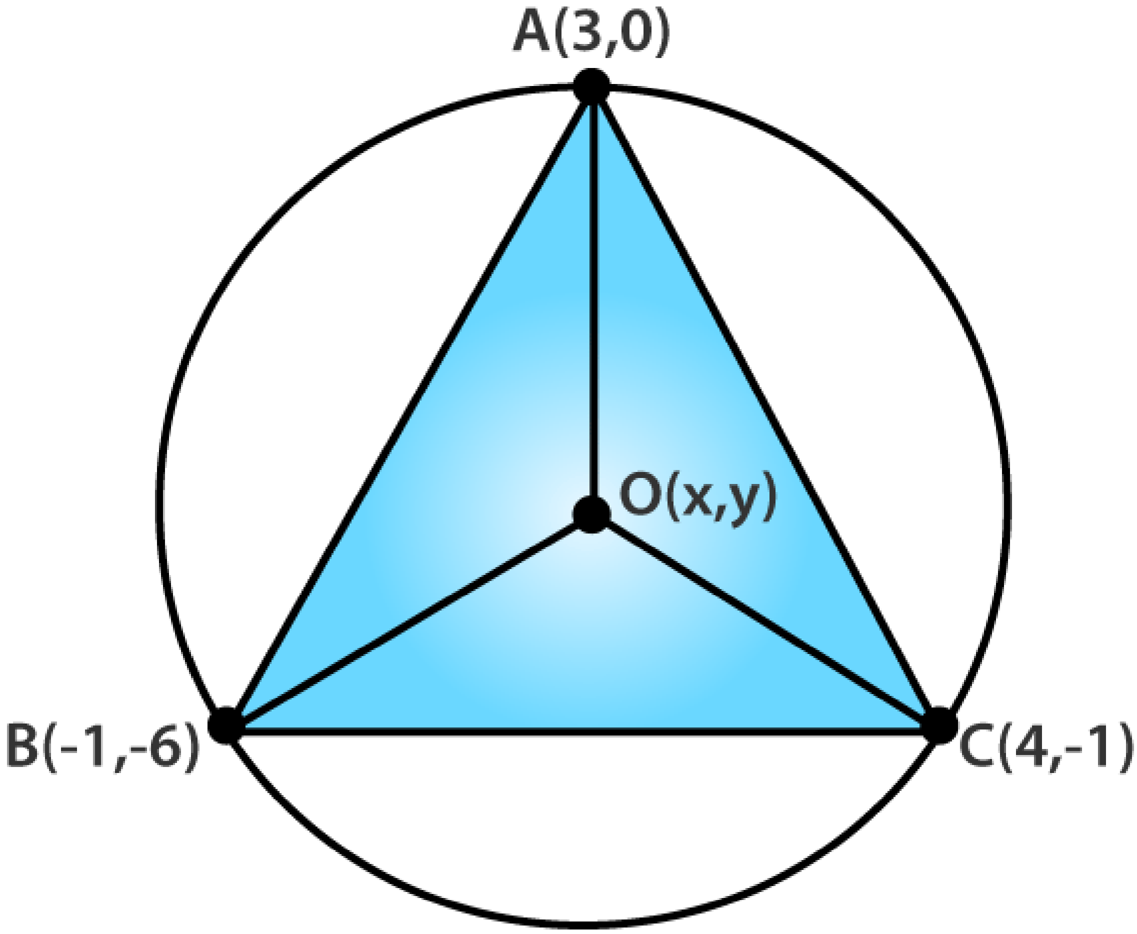
ABCD is a rhombus.

Now,

Area of rhombus ABCD = $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times 5\sqrt{2} \times 9\sqrt{2} = 45$ sq. units

27. Find the coordinates of the circumcenter of the triangle whose vertices are (3, 0), (-1, -6) and (4, -1). Also find the circumradius.

Solution:



Let A(3, 0), B(-1, -6) and C(4 , -1) be the given points.

Let O(x , y) be the circumcenter of the triangle

Then, OA = OB = OC

$OA^2 = OB^2$

$(x – 3)^2 + (y – 0)^2 = (x + 1)^2 + (y + 6)^2$

$x^2 + 9 – 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$

-8x -12y = 28

2x + 3y = -7 …..(i) [After simplification]

Again,

$OB^2 = OC^2$

$(x + 1)^2 + (y + 6)^2 = (x – 4)^2 + (y + 1)^2$

$x^2 + 2x + 1 + y^2 + 36 + 12y = x^2 + 16 – 8x + y^2 + 1 + 2y$

10x + 10y = -20

x + y = -2 …..(ii) [After simplification]

Hence, the circumcenter of the triangle is (1, -3)

Circumradius = distance from any of the given points (say B)

$=\sqrt{[(1 + 1)^2 + (-3 + 6)^2]} = \sqrt{(4 + 9)}$

$= \sqrt{13}$ units

28. Find a point on the x-axis which is equidistant from the points (7, 6) and (-3, 4).

Solution:

Let A(7, 6) and B(-3, 4) be the given points.

Let P(x, 0) be the point on the x-axis such that PA = PB

So, $PA^2 = PB^2$

$(x – 7)^2 + (0 – 6)^2 = (x + 3)^2 + (0 – 4)^2$

$x^2 + 49 – 14x + 36 = x^2 + 9 + 6x + 16$

-20x = -60

x = 3

Therefore, the point on x-axis is (3, 0).

RD Sharma class 1o Chapter 14 Exercise 14.3 Page No: 14.28

1. Find the coordinates of the point which divides the line segment joining (-1, 3) and (4, – 7) internally in the ratio 3: 4.

Solution:

Let P(x, y) be the required point.

By section formula, we know that the coordinates are

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$y = \frac{my_2 + ny_1}{m + n}$$

Here,

$x_1 = – 1$ $y_1 = 3$

$x_2 = 4$ $y_2 = -7$

m: n = 3: 4

Then,

$$x = \frac{3 \times 4 + 4 \times (-1)}{3 + 4} \times 3$$

$$x = \frac{3 \times 4 + 4 \times (-1)}{3 + 4}$$

$$x = \frac{12 - 4}{7}$$

$$x = \frac{8}{7}$$

$$y = \frac{3 \times (-7) + 4 \times 3}{3 + 4}$$

$$y = \frac{-21 + 12}{7}$$

$$y = \frac{-9}{7}$$

Therefore, the coordinates of P are (8/7, – 9/7)

2. Find the points of trisection of the line segment joining the points:

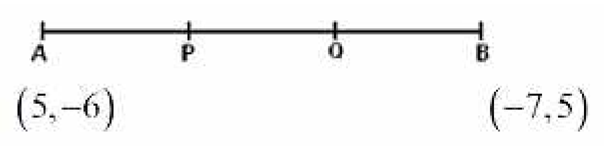
(i) (5, – 6) and (-7, 5)

(ii) (3, – 2) and (-3, – 4)

(iii) (2, – 2) and (-7, 4)

Solution:

(i) Let P and Q be the point of trisection of AB such that AP = PQ = QB



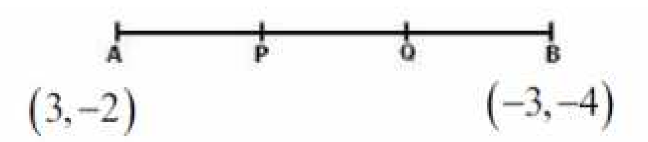
So, P divides AB internally in the ratio of 1: 2, thereby applying section formula, the coordinates of P will be

$$\left(\frac{2(-7) + (5)}{2 + 1}\right), \left(\frac{2(5) + 1(-6)}{2 + 1}\right) \text{ i.e., } \left(1, -\frac{7}{3}\right)$$

Now, Q also divides AB internally in the ratio of 2:1 so their coordinates will be

$$\left(\frac{2(-7) + 1(5)}{2 + 1}\right), \left(\frac{2(5) + 1(-6)}{2 + 1}\right) \text{ i.e., } \left(-3, \frac{4}{3}\right)$$

(ii) Let P and Q be the points of trisection of AB such that AP = PQ = QB



As, P divides AB internally in the ratio of 1: 2. Hence by applying section formula, the coordinates of P are

$$\left(\left(\frac{1(-3)+2(3)}{1+2}\right), \frac{1(-4)+(-2)}{1+2}\right) \text{ i.e., } \left(1, -\frac{8}{3}\right)$$

Now, Q also divides as internally in the ratio of 2: 1

So, the coordinates of Q are given by

$$\left(\left(\frac{2(-3)+1(3)}{2+1}\right), \frac{2(-4)+1(-2)}{2+1}\right) \text{ i.e., } \left(-1, \frac{-10}{3}\right)$$

(iii) Let P and Q be the points of trisection of AB such that AP = PQ = OQ

As, P divides AB internally in the ratio 1:2. So, the coordinates of P, by applying the section formula, are given by

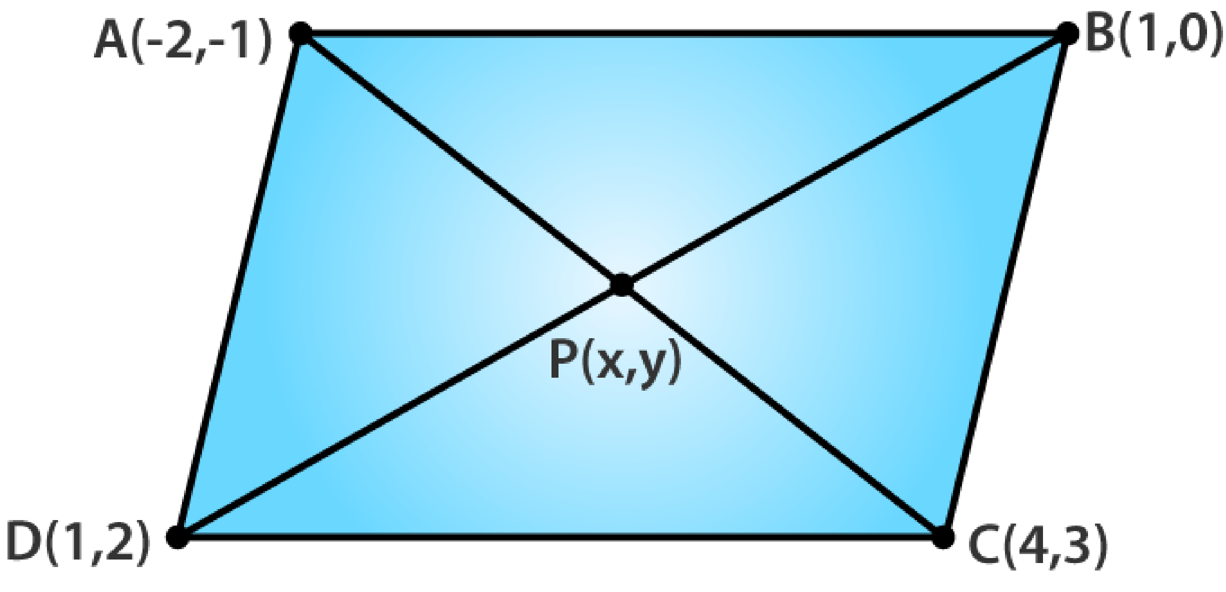
$$\left(\left(\frac{1(-7)+2(2)}{1+2}\right), \left(\frac{1(4)+2(-2)}{1+2}\right)\right), \text{i.e., } (-1, 0)$$

Now. Q also divides AB internally in the ration 2: 1. And the coordinates of Q are given by

$$\left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(2)}{2+1}\right), \text{i.e., } (-4, 2)$$

3. Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.

Solution:



Let A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) be the given points.

Let P(x, y) be the point of intersection of the diagonals of the parallelogram formed by the given points.

We know that, diagonals of a parallelogram bisect each other.

$$x = \frac{-2+4}{2}$$

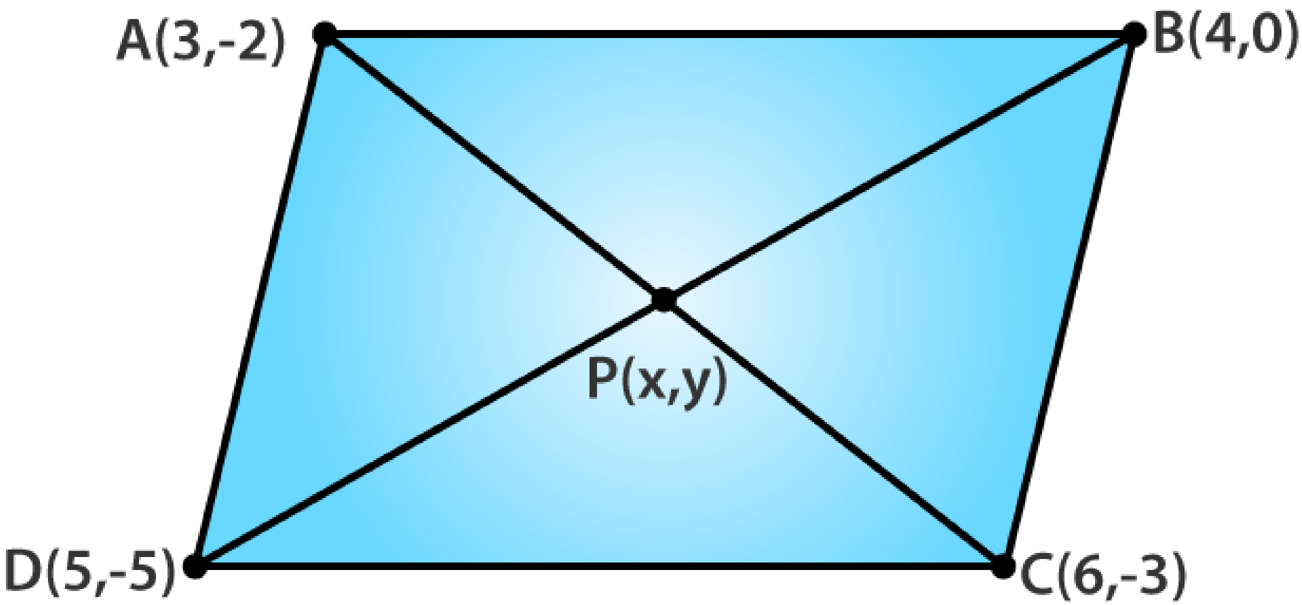
$$\Rightarrow x = \frac{2}{2} = 1$$

$$y = \frac{-1 + 3}{2} = \frac{2}{2} = 1$$

Therefore, the coordinates of P are (1, 1)

4. Prove that the points (3, 2), (4, 0), (6, -3) and (5, -5) are the vertices of a parallelogram.

Solution:



Let A(3, -2), B(4, 0), C(6, -3) and D(5, -5)

Let P(x, y) be the point of intersection of diagonals AC and BD of ABCD.

The mid-point of AC is given by,

$$x = \frac{3 + 6}{2} = \frac{9}{2}$$

$$y = \frac{-2-3}{2} = \frac{-5}{2}$$

$$\text{Mid} - \text{point of AC } \left(\frac{9}{5}, \frac{-5}{2}\right)$$

Again, the mid-point of BD is given by,

$$x = \frac{5+4}{2} = \frac{9}{2}$$

$$y = \frac{-5+0}{2} = \frac{-5}{2}$$

Thus, we can conclude that diagonals AC and BD bisect each other.

And, we know that diagonals of a parallelogram bisect each other

Therefore, ABCD is a parallelogram.

5. If P(9a – 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3 : 1, find the values of a and b.

Solution:

Given that, P(9a – 2, -b) divides the line segment joining A(3a + 1, -3) and B(8a, 5) in the ratio 3:1

Then, by section formula

Coordinates of P are

$$9a - 2 = \frac{3(8a) + 1(3a+1)}{3+1}$$

And,

$$-b = \frac{3(5) + 1(-3)}{3+1}$$

Solving for a, we have

(9a – 2) x 4 = 24a + 3a + 1

36a – 8 = 27a + 1

9a = 9

a = 1

Now, solving for b, we have

4 x –b = 15 – 3

-4b = 12

b = -3

Therefore, the values of a and b are 1 and -3 respectively.

6. If (a, b) is the mid-point of the line segment joining the points A (10, -6), B(k, 4) and a – 2b = 18, find the value of k and the distance AB.

Solution:

As (a, b) is the mid-point of the line segment A(10, -6) and B(k, 4)

So,

(a, b) = (10 + k / 2, -6 + 4/ 2)

a = (10 + k)/ 2 and b = -1

2a = 10 + k

k = 2a – 10

Given, a – 2b = 18

Using b = -1 in the above relation we get,

a – 2(-1) = 18

a = 18 – 2 = 16

So,

k = 2(16) – 10 = 32 – 10 = 22

Thus,

AB = $\sqrt{[(22 - 10)^2 + (4 + 6)^2]} = \sqrt{[(12)^2 + (10)^2]} = \sqrt{[144 + 100]} = 2\sqrt{61}$ units

7. Find the ratio in which the point (2, y) divides the line segment joining the points A(-2, 2) and B(3, 7). Also find the value of y.

Solution:

Let the point P(2, y) divide the line segment joining the points A(-2, 2) and B(3, 7) in the ratio k: 1

Then, the coordinates of P are given by

$$\left[\frac{3k + (-2) \times 1}{k + 1}, \frac{7k + 2 \times 1}{k + 1}\right]$$
$$= \left[\frac{3k - 2}{k + 1}, \frac{7k + 2}{k + 1}\right]$$

And, given the coordinates of P are (2, y)

So,

2 = (3k – 2)/ (k + 1) and y = (7k + 2)/ (k + 1)

Solving for k, we get

2(k + 1) = (3k – 2)

2k + 2 = 3k – 2

k = 4

Using k to find y, we have

y = (7(4) + 2)/ (4 + 1)

= (28 + 2)/5

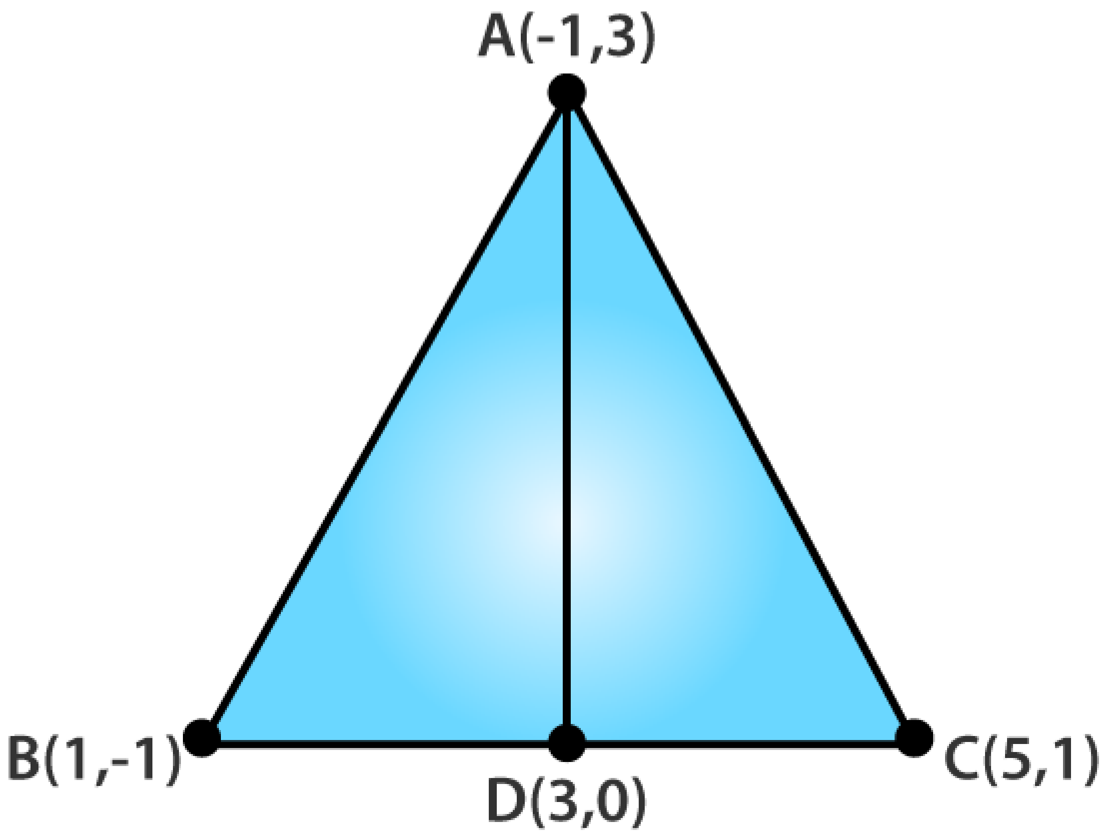
= 30/5

y = 6

Therefore, the ratio id 4: 1 and y = 6

8. If A(-1, 3), B(1, -1) and C(5, 1) are the vertices of a triangle ABC, find the length of median through A.

Solution:



Let AD be the median through A.

As, AD is the median, D is the mid-point of BC

So, the coordinates of D are (1 + 5/ 2, -1 + 1/ 2) = (3 , 0)

Therefore,

Length of median AD = $\sqrt{[(3 + 1)^2 + (0 - 3)^2]} = \sqrt{[(4)^2 + (-3)^2]} = \sqrt{[16 + 9]} = \sqrt{25}$ = 5 units

9. If the points P,Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B (7, 10) in 5 equal parts, find x, y and p.

Solution:

From question, we have

AP = PQ = QR = RS = SB

So, Q is the mid-point of A and S

Then,

x = (2 + 6)/ 2 = 8/2 = 4

7 = (y + p)/ 2

y + p = 14 ….. (1)

Now, since S divides QB in the ratio 2: 1

$$y = \frac{2\times10+1\times7}{2+1} \Rightarrow \frac{20+7}{3} = \frac{27}{3} = 9$$

From (*i*), $y + p = 14 \Rightarrow 9 + p = 14$

So, p = 14 – 9 = 5

Therefore, x = 4, y = 9 and p = 5

10. If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (-2, 3) and (5, 2) find the other vertices.

Solution:

Let A(1, 1) be the given vertex and D(-2, 3), E(5, 2) be the mid-points of AB and AC

Now, as D and E are the mid-points of AB and AC

$$\frac{x_1+1}{2} = -2, \quad \frac{y_1+1}{2} = 3$$

$$x_1 + 1 = -4 \qquad \Rightarrow y_1 + 1 = 6$$
$$x_1 = -5 \qquad \Rightarrow y_1 = 5$$

So, the coordinates of B are (-5, 5)

Again,

$$\frac{x_2+1}{2} = 5, \qquad \frac{y_2+1}{2} = 2$$

$$x_2 + 1 = 10 \qquad \Rightarrow y_2 + 1 = 4$$
$$x_2 = 9 \qquad \Rightarrow y_2 = 3$$

So, the coordinates of C are (9, 3)

Therefore, the other vertices of the triangle are (-5, 5) and (9, 3).

11. (i) In what ratio is the segment joining the points (-2, -3) and (3, 7) divides by the y-axis? Also, find the coordinates of the point of division.

Solution:

Let P(-2, -3) and Q(9, 3) be the given points.

Suppose y-axis divides PQ in the ratio k: 1 at R(0, y)

So, the coordinates of R are given by

$$\left[\frac{3k+(-2)\times1}{k+1}, \frac{7k+(-3)\times1}{k+1}\right]$$

Now, equating

$$\frac{3k+(-2)\times1}{k+1} = 0$$

3k – 2 = 0

k = 2/3

Therefore, the ratio is 2: 3

Putting k = 2/3 in the coordinates of R, we get

R (0, 1)

(ii) In what ratio is the line segment joining (-3, -1) and (-8, -9) divided at the point (-5, -21/5)?

Solution:

Let A(-3, -1) and B(-8, -9) be the given points.

And, let P be the point that divides AB in the ratio k: 1

So, the coordinates of P are given by

$$\left[\frac{-8k-3}{k+1}, \frac{-9k-1}{k+1}\right]$$

But, given the coordinates of P

On equating, we get

(-8k – 3)/ (k + 1) = -5

-8k – 3 = -5k – 5

3k = 2

k = 2/3

Thus, the point P divides AB in the ratio 2: 3

12. If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and 2x + 2y + 1 = 0 find the value of k.

Solution:

As (x , y) is the mid-point

x = (3 + k)/ 2 and y = (4 + 7)/ 2 = 11/2

Also,

Given that the mid-point lies on the line 2x + 2y + 1 = 0

2[(3 + k)/ 2] + 2(11/2) + 1 = 0

3 + k + 11 + 1 = 0

Thus, k = -15

13. Find the ratio in which the point P(3/4, 5/12) divides the line segments joining the point A(1/2, 3/2) and B(2, -5).

Solution:

Given,

Points A(1/2, 3/2) and B(2, -5)

Let the point P(3/4, 5/12) divide the line segment AB in the ratio k: 1

Then, we know that

P(3/4, 5/12) = (2k + ½)/ (k +1) , (2k + 3/2)/ (k + 1)

Now, equating the abscissa we get

¾ = (2k + ½)/ (k +1)

3(k + 1) = 4(2k + 1/2)

3k + 3 = 8k + 2

5k = 1

k = 1/5

Therefore, the ratio in which the point P(3/4, 5/12) divides is 1: 5

14. Find the ratio in which the line joining (-2, -3) and (5, 6) is divided by (i) x-axis (ii) y-axis. Also, find the coordinates of the point of division in each case.

Solution:

Let A(-2, -3) and B(5, 6) be the given points.

(i) Suppose x-axis divides AB in the ratio k: 1 at the point P

Then, the coordinates of the point of division are

$$\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$$

As, P lies in the x-axis, the y – coordinate is zero.

So,

6k – 3/ k + 1 = 0

6k – 3 = 0

k = ½

Thus, the required ratio is 1: 2

Using k in the coordinates of P

We get, P (1/3, 0)

(ii) Suppose y-axis divides AB in the ratio k: 1 at point Q

The, the coordinates of the point od division is given by

$$\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$$

As, Q lies on the y-axis, the x – ordinate is zero.

So,

5k – 2/ k + 1 = 0

5k – 2 = 0

k = 2/5

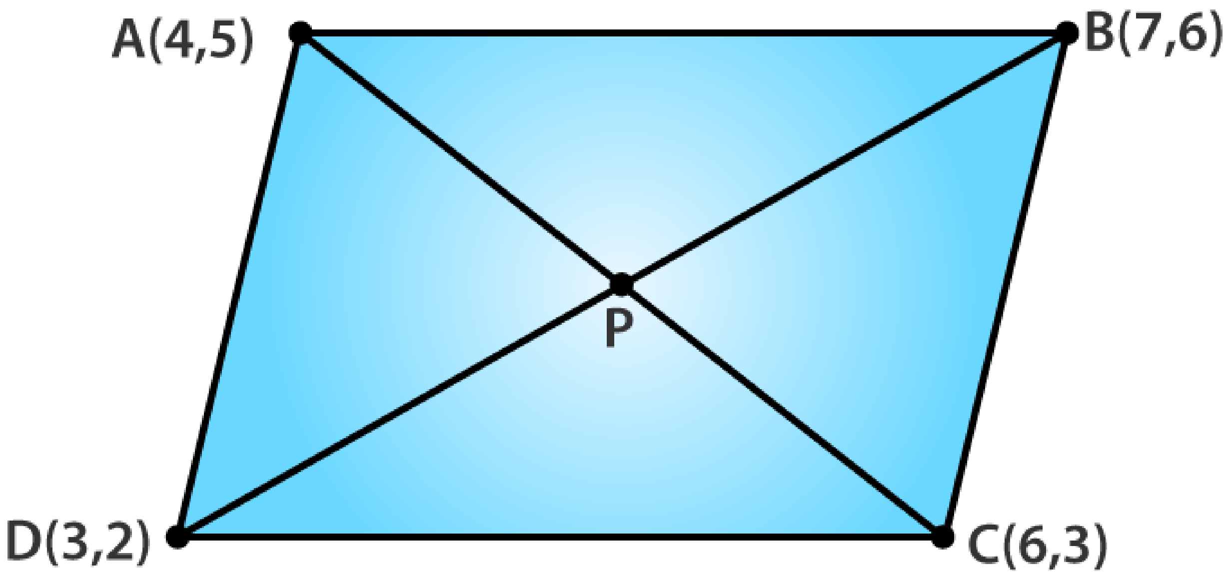
Thus, the required ratio is 2: 5

Using k in the coordinates of Q

We get, Q (0, -3/7)

15. Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle?

Solution:



Let A (4, 5), B(7, 6), C(6, 3) and D(3, 2) be the given points.

And, P be the point of intersection of AC and BD.

Coordinates of the mid-point of AC are (4+6/2 , 5+3/2) = (5, 4)

Coordinates of the mid-point of BD are (7+3/2 , 6+2/2) = (5, 4)

Thus, it's clearly seen that the mid-point of AC and BD are same.

So, ABCD is a parallelogram.

Now,

AC = $\sqrt{[(6 – 4)^2 + (3 – 5)^2]} = \sqrt{[(2)^2 + (-2)^2]} = \sqrt{[4 + 4]} = \sqrt{8}$ units

And,

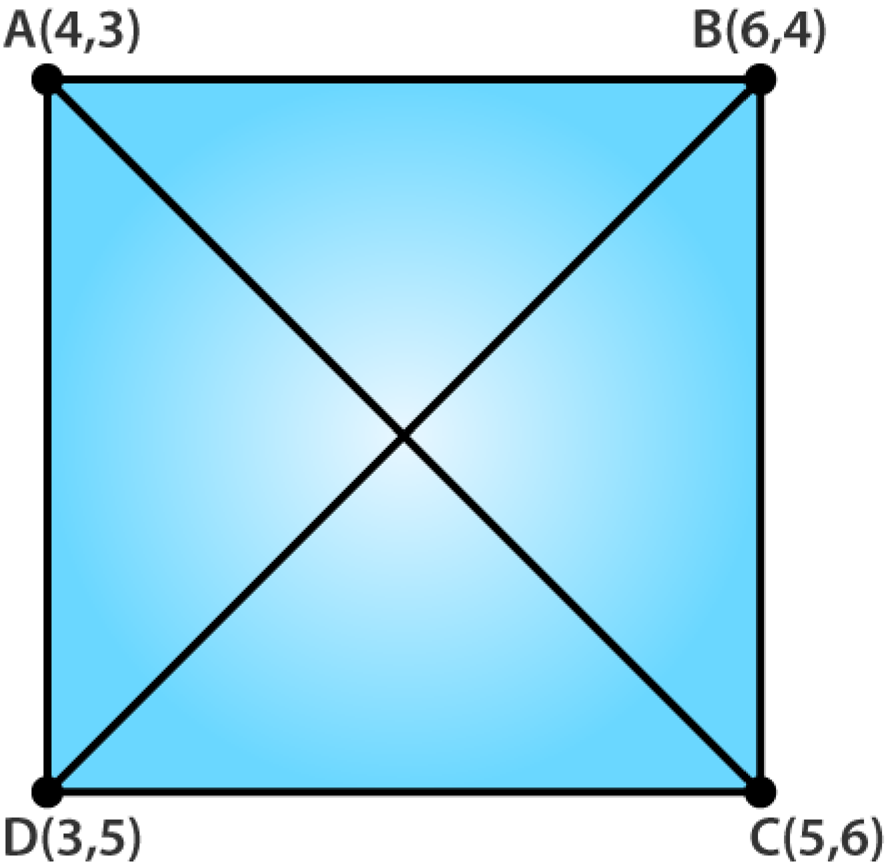
BD = $\sqrt{[(7 – 3)^2 + (6 – 2)^2]} = \sqrt{[(4)^2 + (4)^2]} = \sqrt{[16 + 16]} = \sqrt{32}$ units

Since, AC ≠ BD

Therefore, ABCD is not a rectangle.

16. Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.

Solution:



Let A(4,3) , B(6,4) , C(5,6) and D(3,5) be the given points.

The distance formula is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$AB = \sqrt{(4-6)^2 + (3-4)^2} = \sqrt{5}$$

$$BC = \sqrt{(6-5)^2 + (4-6)^2} = \sqrt{5}$$

$$CD = \sqrt{(5-3)^2 + (6-5)^2} = \sqrt{5}$$

$$AD = \sqrt{(4-3)^2 + (3-5)^2} = \sqrt{5}$$

It's seen that the length of all the sides are same.

Now, the length of diagonals are

$$AC = \sqrt{(4-5)^2 + (3-6)^2} = \sqrt{10}$$

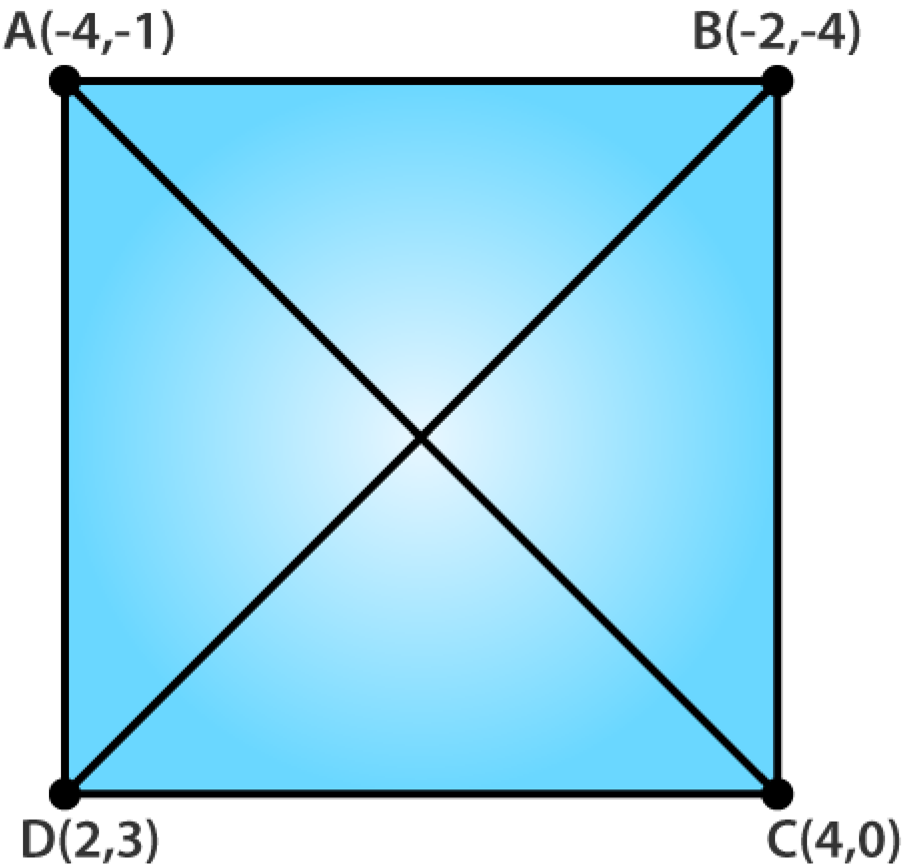
$$BD = \sqrt{(6-3)^2 + (4-5)^2} = \sqrt{10}$$

Also, the length of both the diagonals are same.

Therefore, we can conclude that the given points are the angular points of a square.

17. Prove that the points (-4, -1), (-2, -4), (4, 0) and (2, 3) are the vertices of a rectangle.

Solution:



Let A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3) be the given points.

Now,

Coordinates of the mid-point of AC are (-4 + 4/ 2, -1 + 0/2) = (0, -1/2)

Coordinates of the mid-point of BD are (-2 + 2/2, -4 + 3/2) = (0, -1/2)

Thus, it's seen that AC and BD have the same point.

And, we have diagonals

$$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$$

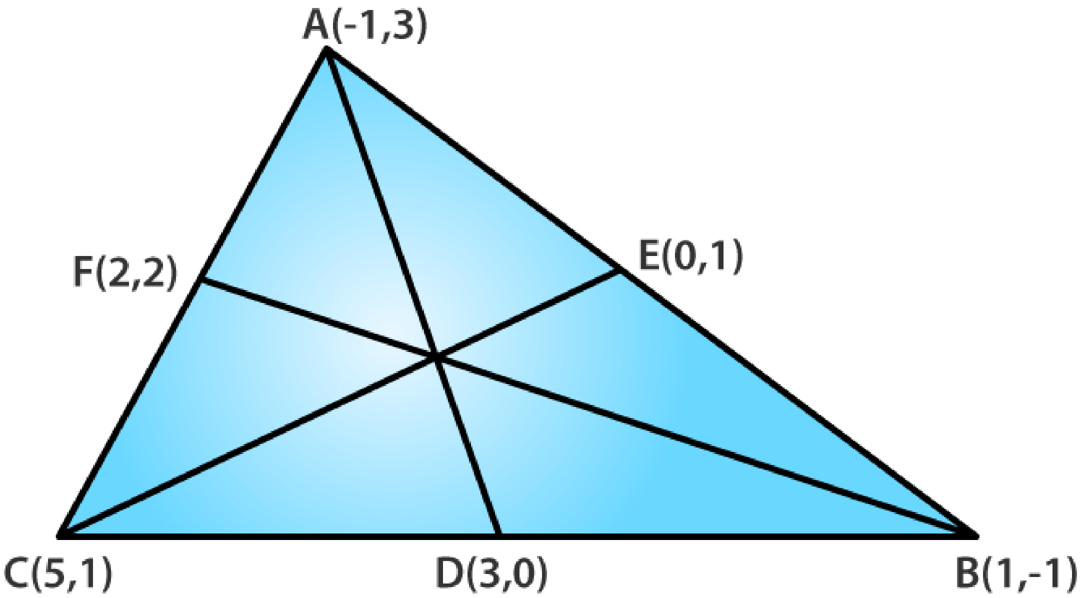
$$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$$

The length of diagonals are also same.

Therefore, the given points are the vertices of a rectangle.

18. Find the length of the medians of a triangle whose vertices are A(-1, 3), B(1, -1) and C(5, 1).

Solution:



Let AD, BF and CE be the medians of ΔABC

Coordinates of D are (5 + 1/ 2, 1 – 1/ 2) = (3, 0)

Coordinates of E are (-1 + 1/ 2, 3 – 1/ 2) = (0, 1)

Coordinates of F are (5 – 1/ 2, 1 + 3/ 2) = (2, 2)

Now,

Finding the length of the respectively medians:

$$\text{Length of } AD = \sqrt{(-1-3)^2 + (3-0)^2} = 5 \text{ units}$$

$$\text{Length of } BF = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10} \text{ units}$$

$$\text{Length of } CE = \sqrt{(5-0)^2 + (1-1)^2} = 5 \text{ units}$$

19. Find the ratio in which the line segment joining the points A (3, -3) and B (-2, 7) is divided by x- axis. Also, find the coordinates of the point of division.

Solution:

Let the point on the x-axis be (x, 0). [y – coordinate is zero]

And, let this point divides the line segment AB in the ratio of k : 1.

Now using the section formula for the y-coordinate, we have

0 = (7k – 3)/(k + 1)

7k – 3 = 0

k = 3/7

Therefore, the line segment AB is divided by x-axis in the ratio 3: 7

20. Find the ratio in which the point P(x, 2) divides the line segment joining the points A (12, 5) and B (4, -3). Also, find the value of x.

Solution:

Let P divide the line joining A and B and let it divide the segment in the ratio k: 1

Now, using the section formula for the y – coordinate we have

$2 = (-3k + 5)/ (k + 1)$

$2(k + 1) = -3k + 5$

$2k + 2 = -3k + 5$

$5k = 3$

$k = 3/5$

Thus, P divides the line segment AB in the ratio of 3: 5

Using value of k, we get the x – coordinate as

$x = 12 + 60/ 8 = 72/8 = 9$

Therefore, the coordinates of point P is (9, 2)

21. Find the ratio in which the point P(-1, y) lying on the line segment joining A(-3, 10) and B(6, -8) divides it. Also find the value of y.

Solution:

Let P divide A(-3, 10) and B(6, -8) in the ratio of k: 1

Given coordinates of P as (-1, y)

Now, using the section formula for x – coordinate we have

$-1 = 6k – 3/ k + 1$

$-(k + 1) = 6k – 3$

$7k = 2$

$k = 2/7$

Thus, the point P divides AB in the ratio of 2: 7

Using value of k, to find the y-coordinate we have

$y = (-8k + 10)/ (k + 1)$

$y = (-8(2/7) + 10)/ (2/7 + 1)$

$y = -16 + 70/ 2 + 7 = 54/9$

$y = 6$

Therefore, the y-coordinate of P is 6

22. Find the coordinates of a point A, where AB is the diameter of circle whose center is (2, -3) and B is (1, 4).

Solution:

Let the coordinates of point A be (x, y)

If AB is the diameter, then the center in the mid-point of the diameter

So,

$(2, -3) = (x + 1/ 2, y + 4/ 2)$

$2 = x + 1/2$ and $-3 = y + 4/ 2$

$4 = x + 1$ and $-6 = y + 4$

$x = 3$ and $y = -10$

Therefore, the coordinates of A are (3, -10)

23. If the points (-2, 1), (1, 0), (x , 3) and (1, y) form a parallelogram, find the values of x and y.

Solution:

Let A(-2, 1), B(1, 0), C(x , 3) and D(1, y) be the given points of the parallelogram.

We know that the diagonals of a parallelogram bisect each other.

So, the coordinates of mid-point of AC = Coordinates of mid-point of BD

$$\left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$$

$$\left(\frac{x-2}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

$$\frac{x-2}{2} = 1 \quad \text{and} \quad \frac{y}{2} = 1$$

$$x - 2 = 2 \qquad \Rightarrow y = 2$$

$$x = 4 \qquad \Rightarrow y = 2$$

Therefore, the value of x is 4 and the value of y is 2.

24. The points A(2, 0), B(9, 1), C(11, 6) and D(4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Solution:

Given points are A(2, 0), B(9, 1), C(11, 6) and D(4, 4).

Coordinates of mid-point of AC are (11+2/ 2, 6+0/ 2) = (13/2, 3)

Coordinates of mid-point of BD are (9+4/ 2, 1+4/ 2) = (13/2, 5/2)

As the coordinates of the mid-point of AC ≠ coordinates of mid-point of BD, ABCD is not even a parallelogram.

Therefore, ABCD cannot be a rhombus too.

25. In what ratio does the point (-4,6) divide the line segment joining the points A(-6,10) and B(3,-8)?

Solution:

Let the point (-4, 6) divide the line segment AB in the ratio k: 1.

So, using the section formula, we have

$$(-4, 6) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$$

$$-4 = \frac{3k-6}{k+1}$$

$$-4k - 4 = 3k - 6$$

$$7k = 2$$

$$k : 1 = 2 : 7$$

The same can be checked for the y-coordinate also.

Therefore, the ratio in which the point (-4, 6) divides the line segment AB is 2: 7

26. Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of division.

Solution:

Let P(5, -6) and Q(-1, -4) be the given points.

Let the y-axis divide the line segment PQ in the ratio k: 1

Then, by using section formula for the x-coordinate (as it’s zero) we have

$$\frac{-k+5}{k+1} = 0$$

$$-k + 5 = 0$$

$$k = 5$$

Thus, the ratio in which the y-axis divides the given 2 points is 5: 1

Now, for finding the coordinates of the point of division

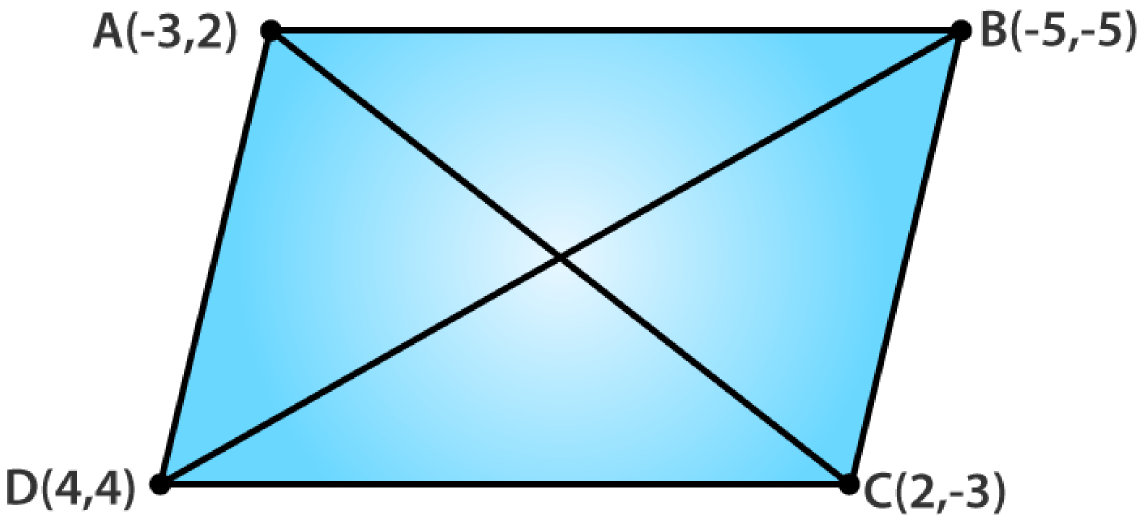
Putting k = 5, we get

$$\left(\frac{-5+5}{5+1}, \frac{-4\times5-6}{5+1}\right) = \left(0, \frac{-13}{3}\right)$$

Hence, the coordinates of the point of division are (0, -13/3)

27. Show that A(-3, 2), B(-5, 5), C(2, -3) and D(4, 4) are the vertices of a rhombus.

Solution:



Given points are A(-3, 2), B(-5, 5), C(2, -3) and D(4, 4)

Now,

Coordinates of the mid-point of AC are (-3+2/ 2, 2-3/ 2) = (-1/2, -1/2)

And,

Coordinates of mid-point of BD are (-5+4/ 2, -5+4/ 2) = (-1/2, -1/2)

Thus, the mid-point for both the diagonals are the same. So, ABCD is a parallelogram.

Next, the sides

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$
$$AB = \sqrt{4+49}$$
$$AB = \sqrt{53}$$

$$BC = \sqrt{(-5-2)^2 + (-5+3)^2}$$
$$BC = \sqrt{49+4}$$
$$BC = \sqrt{53}$$

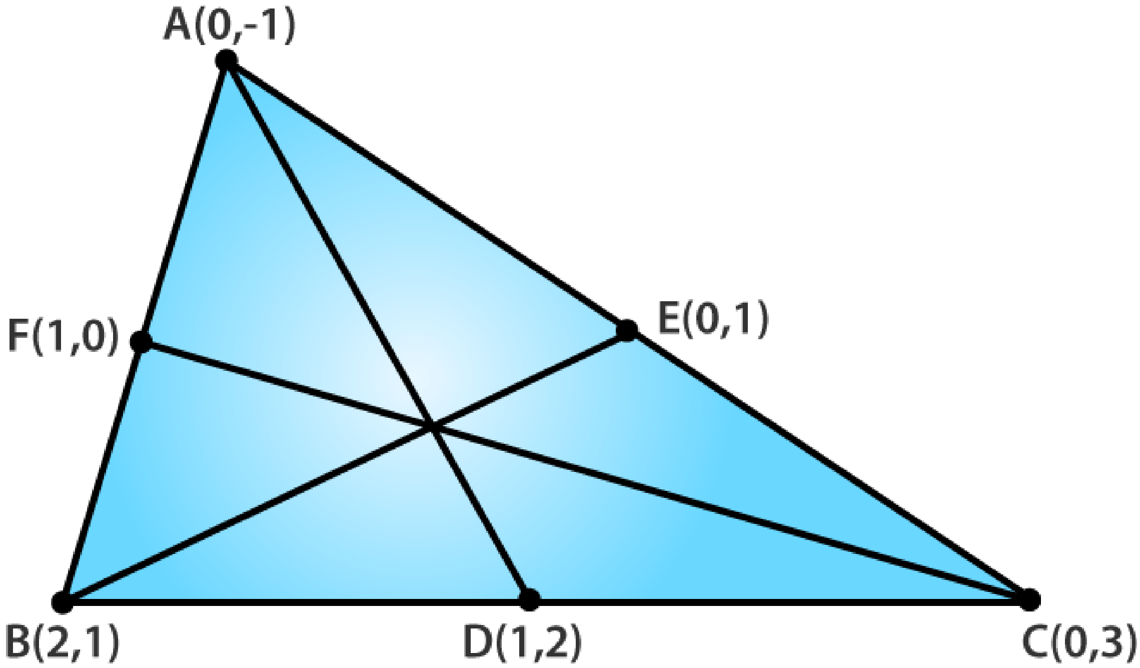
$$AB = BC$$

It's seen that ABCD is a parallelogram with adjacent sides equal.

Therefore, ABCD is a rhombus.

28. Find the lengths of the medians of a ΔABC having vertices at A(0, -1), B(2, 1) and C(0, 3).

Solution:



Let AD, BE and CF be the medians of ΔABC

Then,

Coordinates of D are (2+0/ 2, 1+3/ 2) = (1, 2)

Coordinates of E are (0/2, 3-1/ 2) = (0, 1)

Coordinates of F are (2+0/ 2, 1-1/ 2) = (1, 0)

Now, the length of the medians

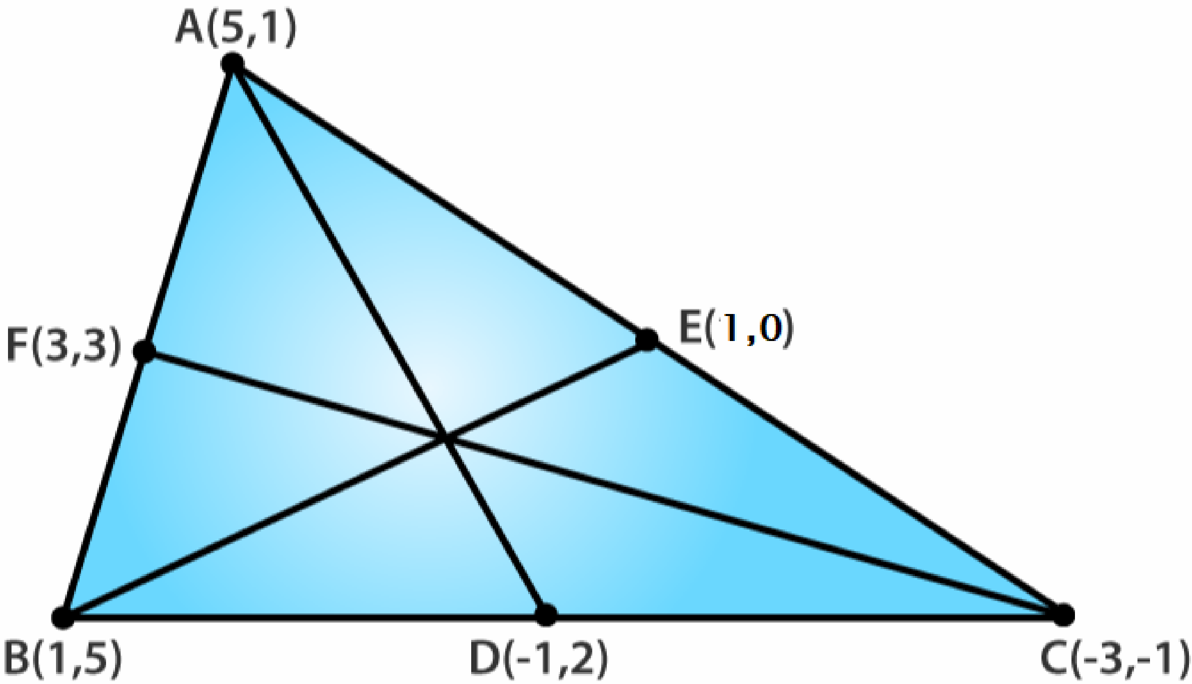
$$\text{Length of median } AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10} \text{ units}$$

$$\text{Length of median } BE = \sqrt{(2-0)^2 + (1-1)^2} = 2 \text{ units}$$

$$\text{Length of median } CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10} \text{ units}$$

29. Find the lengths of the median of a ΔABC having vertices at A(5, 1), B(1, 5) and C(-3, -1).

Solution:



Given vertices of ΔABC as A(5, 1), B(1, 5) and C(-3, -1).

Let AD, BE and CF be the medians

Coordinates of D are (1-3/ 2, 5-1/ 2) = (-1, 2)

Coordinates of E are (5-3/ 2, 1-1/2) = (1, 0)

Coordinates of F are (5+1/ 2, 1+5/ 2) = (3, 3)

Now, the length of the medians

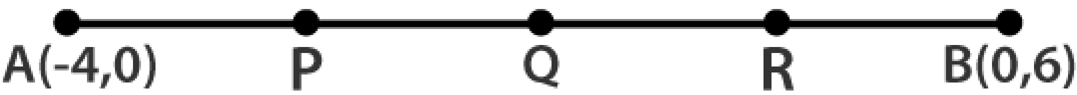
$$\text{Length of median } AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37} \text{ units}$$

$$\text{Length of median } BE = \sqrt{(1-1)^2 + (5-0)^2} = 5 \text{ units}$$

$$\text{Length of median } CF = \sqrt{(3+3)^2 + (3+1)^2} = 2\sqrt{13} = \sqrt{52} \text{ units}$$

30. Find the coordinates of the point which divide the line segment joining the points (-4, 0) and (0, 6) in four equal parts.

Solution:



Let A(-4, 0) and B(0, 6) be the given points

And, let P, Q and R be the points which divide AB is four equal points.

Now, we know that AP: PB = 1: 3

Using section formula the coordinates of P are

$$\left(\frac{1\times 0+3(-4)}{1+3}, \frac{1\times 6+3\times 0}{1+3}\right) = \left(-3, \frac{3}{2}\right)$$

And, it's seen that Q is the mid-point of AB

So, the coordinates of Q are

$$\left(\frac{-4+0}{2}, \frac{0+6}{2}\right) = (-2, 3)$$

Finally, the ratio of AR: BR is 3: 1

Then by using section formula the coordinates of R are

$$\left(\frac{3\times 0+1\times(-4)}{3+1}, \frac{3\times 6+1\times 0}{3+1}\right) = \left(-1, \frac{9}{2}\right)$$

RD Sharma class 1o Chapter 14 Exercise 14.4 Page No: 14.37

1. Find the centroid of the triangle whose vertices are:

(i) (1, 4), (-1, -1) and (3, -2) (ii) (-2, 3), (2, -1) and (4, 0)

Solution:

We know that the coordinates of the centroid of a triangle whose vertices are

(x_1, y_1), (x_2, y_2), (x_3, y_3) are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y+y_2+y_3}{3}\right)$$

(i) So, the coordinates of the centroid of a triangle whose vertices are

(1, 4), (-1, -1) and (3, -2) are

$$\left(\frac{1-1+3}{3}, \frac{4-1-2}{3}\right)$$

(1, 1/3)

Thus, centroid of the triangle is (1, 1/3)

(ii) So, the coordinates of the centroid of a triangle whose vertices are

(-2, 3), (2, -1) and (4, 0) are

$$\left(\frac{2-2+4}{3}, \frac{3-1+0}{3}\right)$$

(4/3, 2/3)

Thus, centroid of the triangle is (4/3, 2/3)

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

Solution:

Let the coordinates of the third vertex be (x, y)

Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$$

Given that the centroid for the triangle is at the origin (0, 0)

$$\therefore \frac{x+1+3}{3} = 0 \text{ and } \frac{y+2+5}{3} = 0$$

$\Rightarrow x + 4 = 0 \Rightarrow y + 7 = 0$

$\Rightarrow x = -4 \Rightarrow y = -7$

Therefore, the coordinates of the third vertex is (-4, -7)

3. Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

Solution:

Let the coordinates of the third vertex be (x, y)

Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x-3+0}{3}, \frac{y+1-2}{3}\right)$$

Given that the centroid for the triangle is at the origin (0, 0)

$$\therefore \frac{x-3+0}{3} = 0 \text{ and } \frac{y+1-2}{3} = 0$$

$\Rightarrow x - 3 = 0 \Rightarrow y - 1 = 0$

$\Rightarrow x = 3 \Rightarrow y = 1$

Therefore, the coordinates of the third vertex is (3, 1)

4. A(3, 2) and B(-2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates (5/3, -1/3). Find the coordinates of the third vertex C of the triangle.

Solution:

Let the coordinates of the third vertex C be (x, y)

Given, A(3, 2) and B(-2, 1) are two vertices of a triangle ABC

Then, we know that the coordinates of centroid of the triangle are

$$\left(\frac{x+3-2}{3}, \frac{y+2+1}{3}\right)$$

Given that the centroid for the triangle is **(**5/3, -1/3).

$\therefore \dfrac{x+3-2}{3} = 5/3$ and $\dfrac{y+2+1}{3} = -1/3$

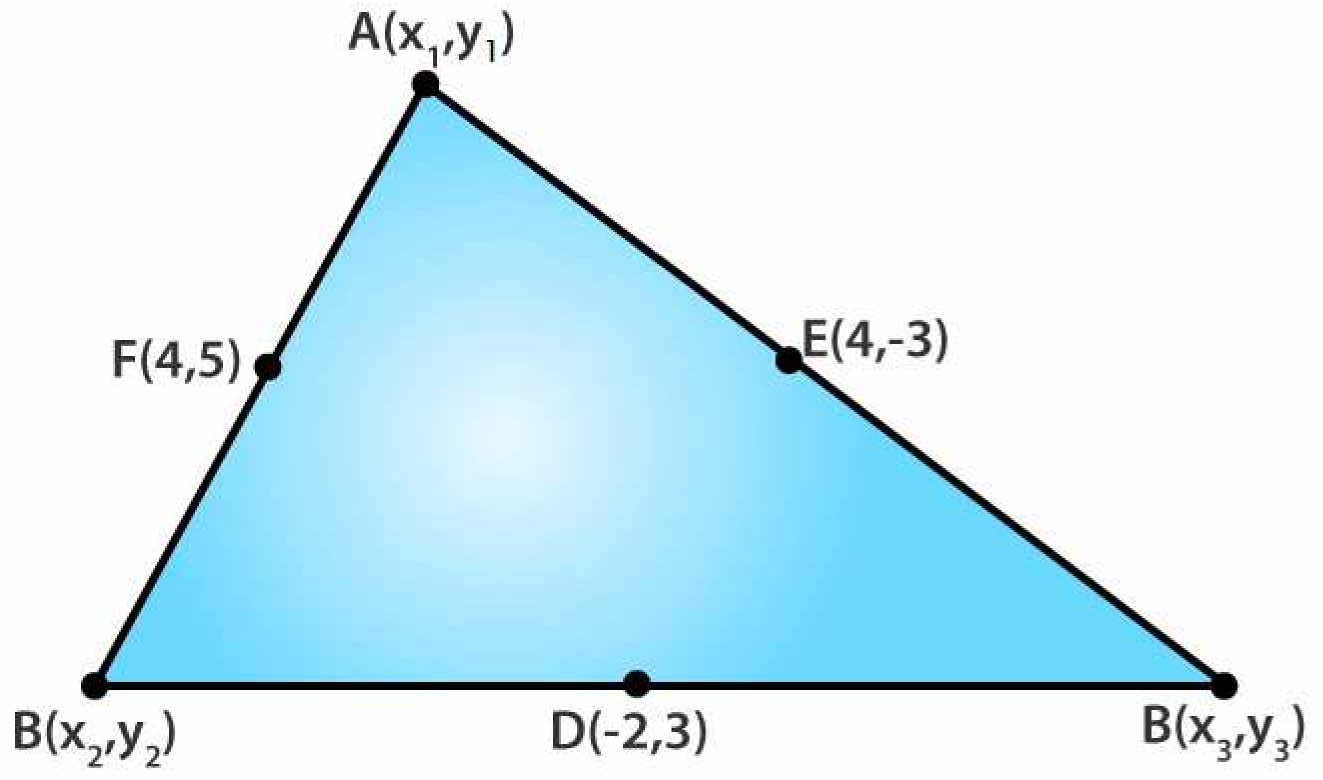
⇒ x + 1 = 5 ⇒ y + 3 = -1

⇒ x = 4 ⇒ y = -4

Therefore, the coordinates of the third vertex C is (4, -4)

5. If (-2, 3), (4, -3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.

Solution:



Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of triangle ABC.

Let D (-2, 3), E (4, -3) and F (4, 5) be the mid-points of sides BC, CA and AB respectively.

As D is the mid-point of BC

$\dfrac{x_2+x_3}{2} = -2$ and $\dfrac{y_2+y_3}{2} = 3$

$x_2+x_3 = -4$ and $y_2+y_3 = 6$ ……. (1)

Similarly E and F are the mid-points of AC and AB

$\dfrac{x_1+x_3}{2} = 4$ and $\dfrac{y_1+y_3}{2} = -3$

$x_1+x_3 = 8$ and $y_1+y_3 = -6$ …….. (2)

And,

$$\frac{x_1 + x_2}{2} = 4 \quad \text{and} \quad \frac{y_1 + y_2}{2} = 5$$

$$x_1 + x_2 = 8 \quad \text{and} \quad y_1 + y_2 = 10 \quad \text{...... (3)}$$

From (1), (2) and (3), we have

$$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -4 + 8 + 8 \text{ and}$$

$$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 6 - 6 + 10$$

$$2(x_1 + x_2 + x_3) = 12 \quad \text{and} \quad 2(y_1 + y_2 + y_3) = 10$$

$$x_1 + x_2 + x_3 = 6 \quad \text{and} \quad y_1 + y_2 + y_3 = 5 \quad \text{........ (4)}$$

Form (1) and (4), we get

$$x_1 - 4 = 6 \quad \text{and} \quad y_1 + 6 = 5$$

$$x_1 = 10 \quad \Rightarrow y_1 = -1$$

Thus, the coordinates of A are (10, -1)

From (2) and (4), we get

$$x_2 + 8 = 6 \quad \text{and} \quad y_2 - 6 = 5$$

$$x_2 = -2 \quad \Rightarrow y_2 = 11$$

Thus, the coordinates of B are (-2, 11)

From (3) and (4), we get

$$x_3 + 8 = 6 \quad \text{and} \quad y_3 + 10 = 5$$

$$x_3 = -2 \quad \Rightarrow y_3 = -5$$

Thus, the coordinates of C are (-2, -5)

Hence, the vertices of triangle ABC are A (10, -1), B (-2, 11) and C (-2, -5).

Therefore, the coordinates of the centroid of triangle ABC are

$$\left(\frac{10-2-2}{3}, \frac{-1+11-5}{3}\right) = \left(2, \frac{5}{3}\right)$$

RD Sharma class 1o Chapter 14 Exercise 14.5 Page No: 14.53

1. Find the area of a triangles whose vertices are

(i) (6, 3), (-3, 5) and (4, – 2)

(ii) [(at_1^2, at_1),(at_2^2, $2at_2$)(at_3^2, $2at_3$)]

(iii) (a, c + a), (a, c) and (-a, c – a)

Solution:

(i) Let A(6, 3), B(-3, 5) and C(4,-2) be the given points

We know that, area of a triangle is given by:

$1/2[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 + y_2)]$

Here,

$x_1 = 6$, $y_1 = 3$, $x_2 = -3$, $y_2 = 5$, $x_3 = 4$, $y_3 = -2$

So,

Area of ∆ABC = 1/2 [6(5+2)+(-3)(- 2 -3)+ 4(3 – 5)]

=1/2 [6 × 7- 3 × (– 5) + 4(– 2)]

= 1/2[42 +15 – 8]

= 49/2 sq. units

(ii) Let A = (x_1, y_1) = (at_1^2, $2at_1$), B = (x_2,y_2) = (at_2^2, $2at_2$), C= (x_3, y_3) = (at_3^2, $2at_3$) be the given points.

Then,

The area of ∆ABC is given by

$$= \frac{1}{2}\left[at_1^2(2at_2 - 2at_3) + at_2^2(2at_3 - 2at_1) + at_3^2(2at_1 - 2at_2)\right]$$

$$= \frac{1}{2}\left[at^2t_1^2t_2 - 2a^2t_1^2t_3 + 2a^2t_2^2t_3 - 2a^2t_2^2t_1 - 2a^2t_3^2t_2\right]$$

$$= \frac{1}{2} \times 2\left[a^2t_1^2(t_2 - t_3) + a^2t_2^2(t_3 - t_1) + a^2t_2^2(t_3 - t_1) + a^2 + t_3^2(t_1 - t_2)\right]$$

$$= a^2\left[t_1^2(t_2 - t_3) + t_2^2(t_3 - t_1) + t_3^2\ (t_1 - t_2)\right]$$

(iii) Let A = (x_1,y_1) = (a, c + a), B = (x_2, y_2) = (a, c) and C = (x_3, y_3) = (- a, c – a) be the given points

Then,

The area of ∆ABC is given by

= 1/2[a (– {c – a}) + a(c – a – (c + a)) +(– a)(c + a – a)]

= 1/2 [a(c – c + a) + a(c – a – c – a) – a(c + a – c)]

= 1/2[a × a + ax(– 2a) – a × a]

= 1/2[a^2 – $2a^2$ – a^2]

= 1/2×$(-2a)^2$

= – a^2

2. Find the area of the quadrilaterals, the coordinates of whose vertices are

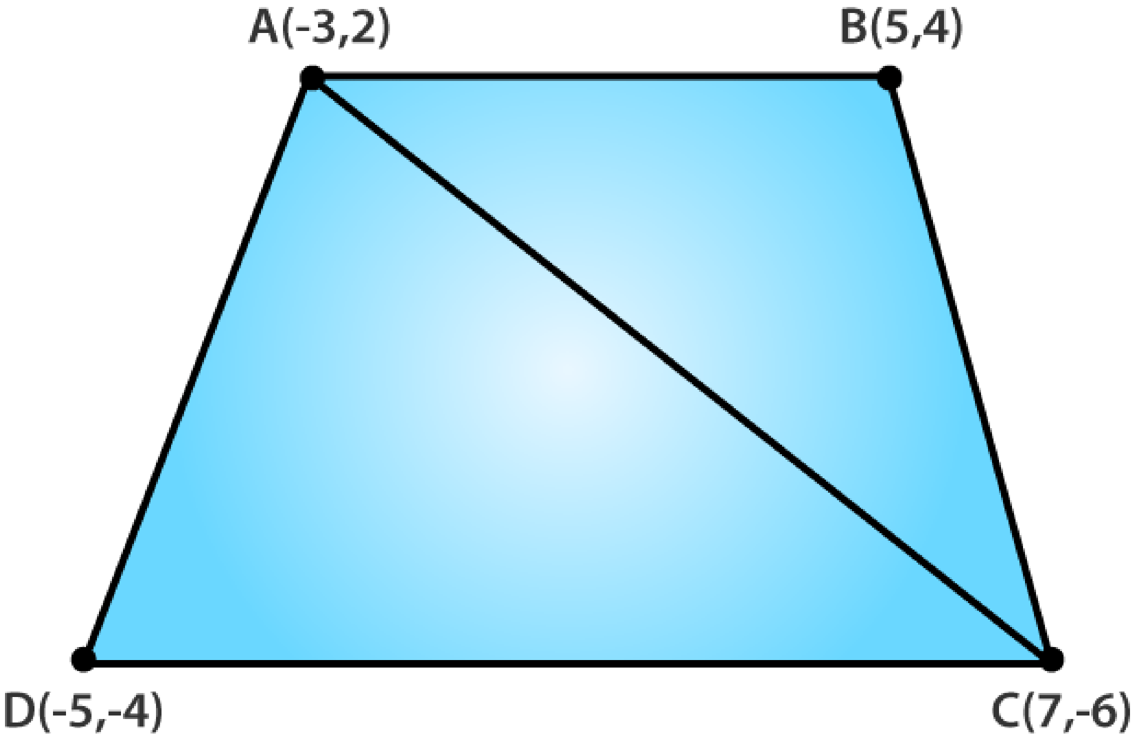
(i) (-3, 2), (5, 4), (7, -6) and (-5, – 4)

(ii) (1, 2), (6, 2), (5, 3) and (3, 4)

(iii) (-4, -2), (-3, -5), (3, -2), (2, 3)

Solution:

(i)



Let A(-3, 2), B(5, 4), C(7,- 6) and D (-5, – 4) be the given points.

Area of ∆ABC is given by

= 1/2[-3(4 + 6) + 5(- 6 – 2) + 7(2 – 4)]

= 1/2[-3×1 + 5×(-8) + 7(-2)]

= 1/2[- 30 – 40 -14]

= – 42

As the area cannot be negative,

The area of ∆ADC = 42 square units

Now, area of ∆ADC is given by

= 1/2[-3(– 6 + 4) + 7(- 4 – 2) + (- 5)(2 + 6)]

= 1/2[- 3(– 2) + 7(- 6) – 5 × 8]

= 1/2[6 – 42 – 40]

= 1/2 × – 76

= – 38

But, as the area cannot be negative,

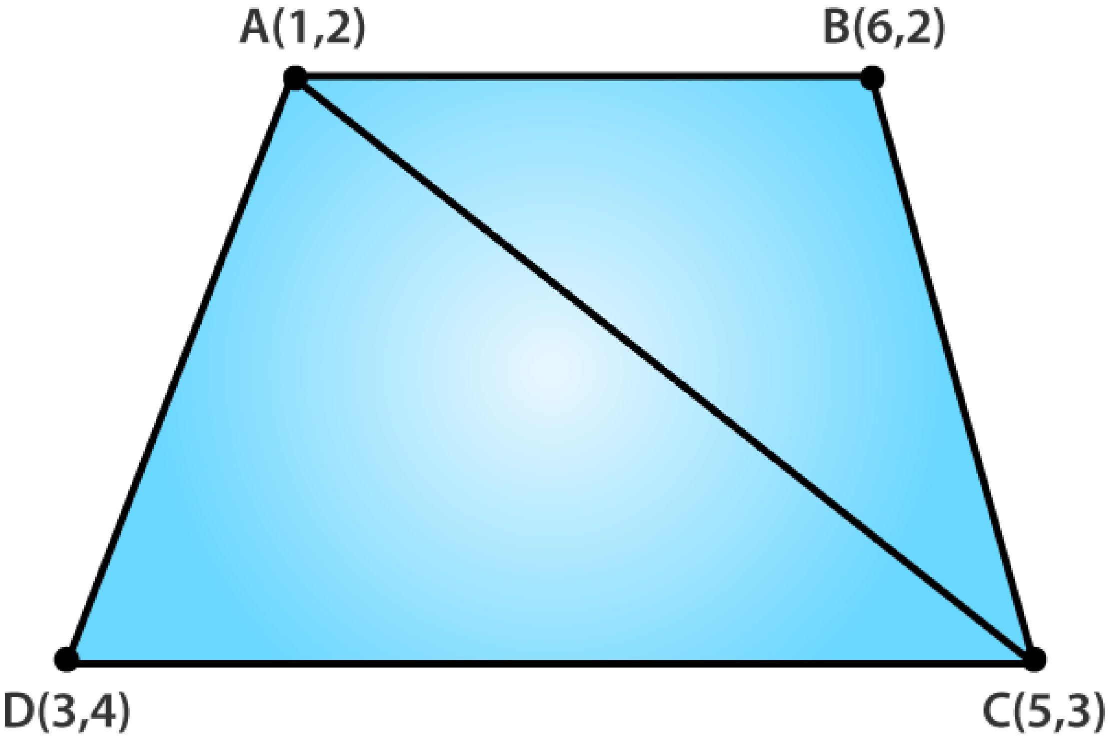
The area of ∆ADC = 38 square units

Thus, the area of quadrilateral ABCD = Ar. of ABC+ Ar. of ADC

= (42 + 38)

= 80 sq. units

(ii)



Let A(1, 2) , B (6, 2) , C (5, 3) and (3, 4) be the given points

Firstly, area of ∆ABC is given by

= 1/2[1(2 – 3) + 6(3 – 2) + 5(2 – 2)]

= 1/2[-1 + 6 × (1) + 0]

= 1/2[– 1 + 6]

= 5/2

Now, area of ∆ADC is given by

= 1/2[1(3 – 4) + 5(4 – 2) + 3(2 – 3)]

= 1/2[-1 × 5 × 2 + 3(-1)]

= 1/2[-1 + 10 – 3]

= 1/2[6]

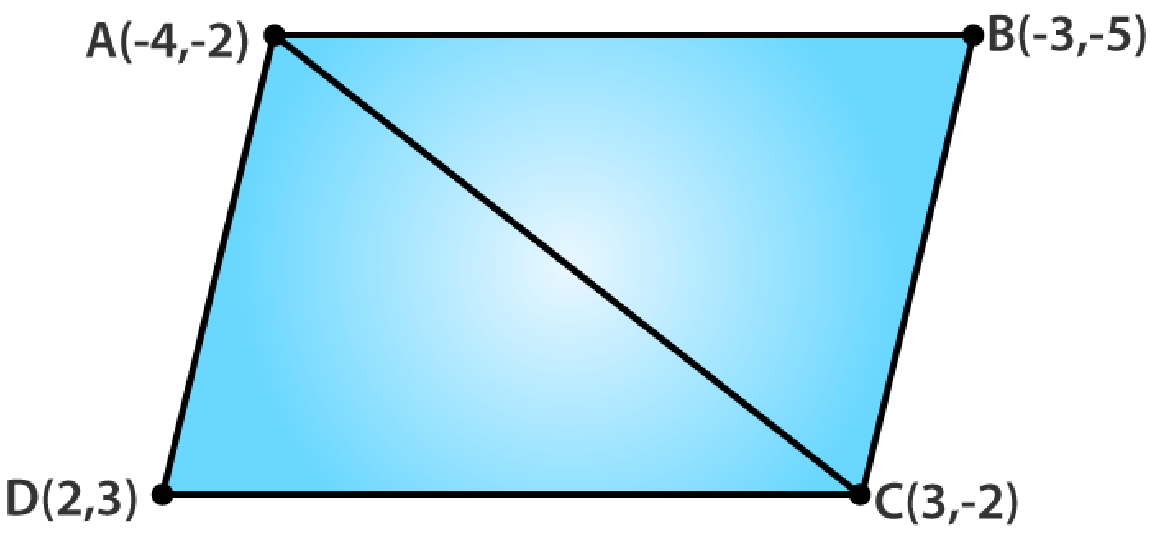
= 3

Thus, Area of quadrilateral ABCD = Area of ABC + Area of ADC

$$= \left(\frac{5}{2} + 3\right) \text{ sq. units}$$

$$= \frac{11}{2} \text{ sq. units}$$

(iii)



Let A (- 4, 2), B(– 3, – 5), C (3,- 2) and D(2, 3) be the given points

Firstly, area of ∆ABC is given by

= 1/2|(- 4)(- 5 + 2) – 3(-2 + 2) + 3(- 2 + 5)|

= 1/2|(-4)(-3) – 3(0) + 3(3)|

= 21/2

Now, the area of ∆ACD is given by

= 1/2|(– 4)(3 + 2) + 2(– 2 + 2) + 3(– 2 – 3)|

= 1/2|- 4(5) + 2(0) + 3(- 5)|= (- 35)/2

But, as the area can't negative,

The area of ∆ADC = 35/2

Thus, the area of quadrilateral (ABCD) = ar(∆ABC) + ar(∆ADC)

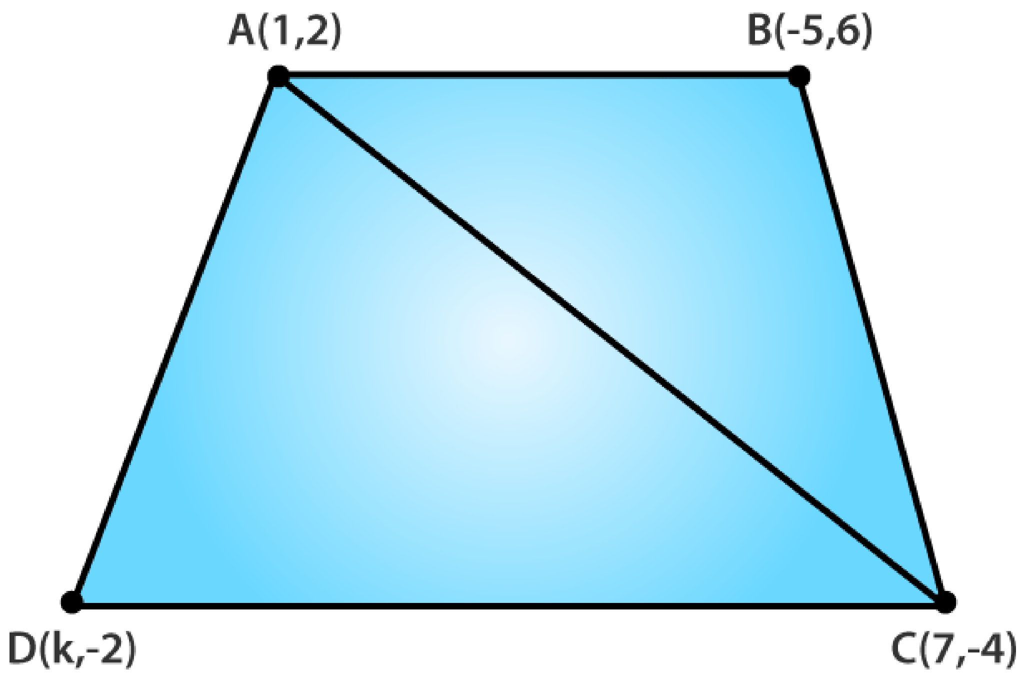
= 21/2 + 35/2

= 56/2

= 28 sq. units

3. The four vertices of a quadrilateral are (1, 2), (-5, 6), (7, -4) and (k, -2) taken in order. If the area of the quadrilateral is zero, find the value of k.

Solution:



Let A(1, 2), B(-5, 6), C(7, -4) and D(k, -2) be the given points

Firstly, area of ∆ABC is given by

= 1/2|(1)(6 + 4) – 5(-4 + 2) + 7(2 – 6)|

= 1/2|10 + 30 – 28|

= ½ x 12

= 6

Now, the area of ∆ACD is given by

= 1/2|(1)(-4 + 2) + 7(– 2 – 2) + k(2 + 4)|

= 1/2|- 2 + 7x(-4) + k(6)|

= (- 30 + 6k)/2

= -15 + 3k

= 3k – 15

Thus, the area of quadrilateral (ABCD) = ar(∆ABC) + ar(∆ADC)

= 6 + 3k – 15

= 3k – 9

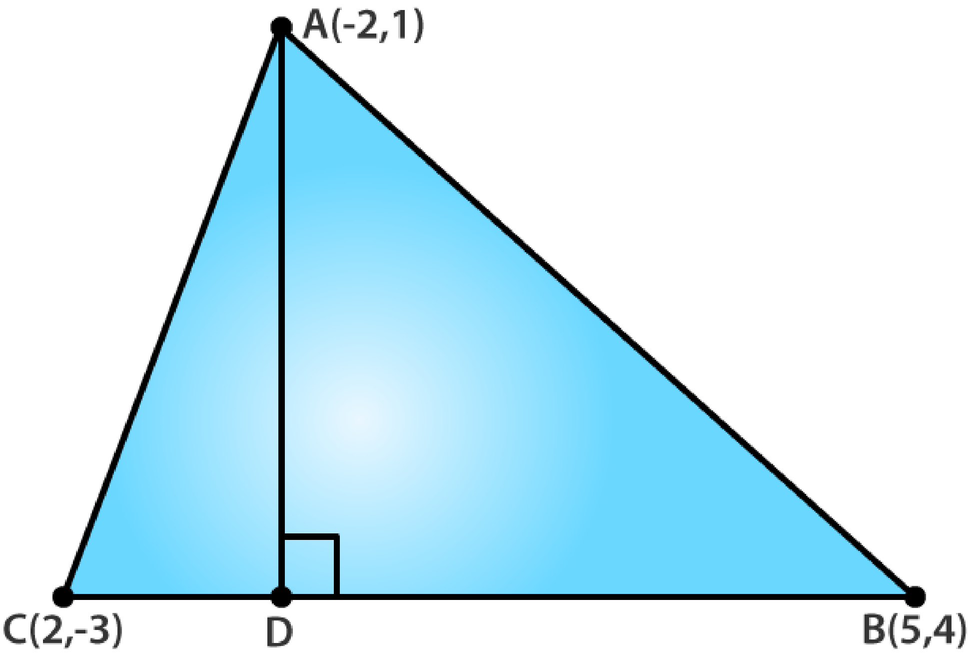
But, given area of quadrilateral is O.

So, 3k – 9 = 0

k = 9/3 = 3

4. The vertices of ΔABC are (-2, 1), (5, 4) and (2, -3) respectively. Find the area of the triangle and the length of the altitude through A.

Solution:



Let A(-2, 1), B(5, 4) and C(2, -3) be the vertices of ΔABC.

And let AD be the altitude through A.

Area of ΔABC is given by

= 1/2|(-2)(4 + 3) – 5(-3 – 1) + 2(1 – 4)|

= 1/2|-14 – 20 – 6|

= ½ x -40

= -20

But as the area cannot be negative,

The area of ΔABC = 20 sq. units

Now,

$$BC = \sqrt{(5-2)^2 + (4+3)^2}$$

$$BC = \sqrt{(3)^2 + (7)^2}$$

$$BC = \sqrt{58}$$

We know that, area of triangle

= ½ x Base x Altitude

20 = ½ x √58 x AD

AD = 40/ √58

Therefore, the altitude AD = 40/ √58

5. Show that the following sets of points are collinear.

(a) (2, 5), (4, 6) and (8, 8) (ii) (1, -1), (2, 1) and (4, 5)

Solution:

Condition: For the 3 points to be collinear the area of the triangle formed with the 3 points has to be zero.

(a) Let A(2, 5), B(4, 6) and C(8, 8) be the given points

Then, the area of ΔABC is given by

$$= \frac{1}{2}\left[2(6-8)+4(8-5)+8(5-6)\right]$$
$$= \frac{1}{2}\left[2\times(-2)+4\times3+8\times(-1)\right]$$
$$= \frac{1}{2}\left[-4+12-8\right]$$
$$= \frac{1}{2}\times0$$
$$= 0$$

Since, the area (ΔABC) = 0 the given points (2, 5), (4, 6) and (8, 8) are collinear.

(b) Let A(1, -1), B(2, 1) and C(4, 5) be the given points

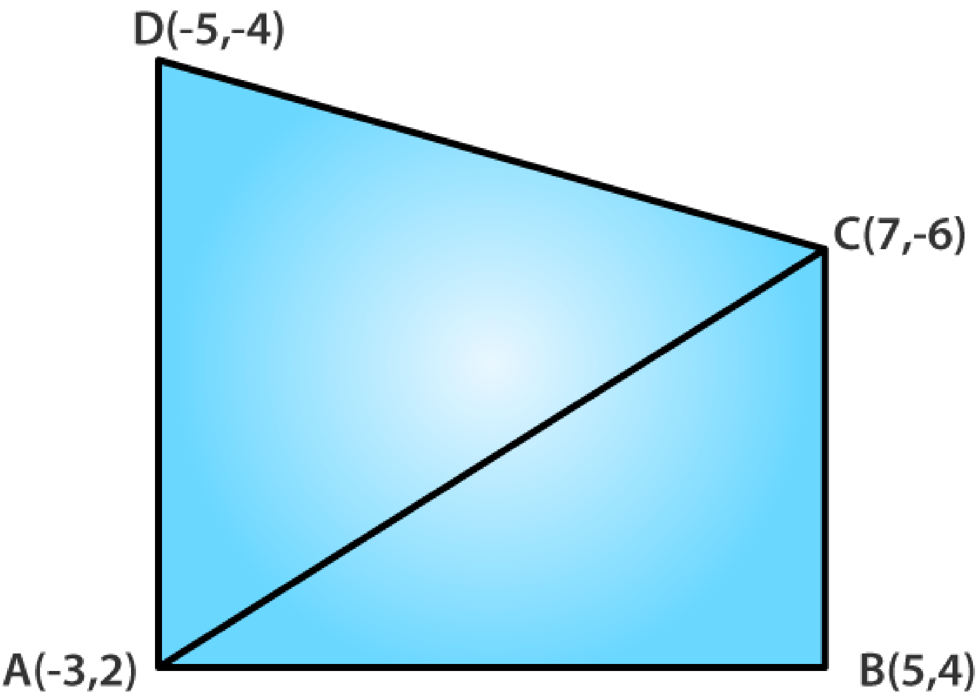
Then, the area of ΔABC is given by

$$= \frac{1}{2}\left[1(1-5)+2(5+1)+4(-1-1)\right]$$
$$= \frac{1}{2}\left[-4+12-8\right]$$
$$= \frac{1}{2}\times0$$
$$= 0$$

Since, the area (ΔABC) = 0 the given points (1, -1), (2, 1) and (4, 5) are collinear.

6. Find the area of a quadrilateral ABCD, the coordinates of whose vertices are A (-3, 2), B (5, 4), C (7, 6) and D (-5, -4).

Solution:



Let’s join AC. So, we have 2 triangles formed.

Now, the ar (ABCD) = Ar (ΔABC) + Ar (ΔACD)

Area of ΔABC is given by,

$$\frac{1}{2}\left|-3(4+6)+5(-6-2)+7(2-4)\right|$$

$$= \frac{1}{2}\left|-30-40-14\right|$$

$$= \frac{1}{2} \times 84$$

$$= 42 \text{ sq. units}$$

Next, the area of ΔACD is given by,

$$\frac{1}{2}\left|-3(-6+4)+7(-4-2)-5(2+6)\right|$$

$$= \frac{1}{2}\left|6-42-40\right|$$

$$= \frac{1}{2} \times 76$$

$$= 38 \text{ sq. units}$$

Thus, the area (ABCD) = 42 + 38 = 80 sq. units

7. In △ABC, the coordinates of vertex A are (0, -1) and D(1, 0) and E(0, 1) respectively the mid-points of

the sides AB and AC. If F is the mid-point of side BC, find the area of △DEF.

Solution:

Let B(a, b) and C(p, q) be the other two vertices of the △ABC

Now, we know that D is the mid-point of AB

So, coordinates of D = (0+a/ 2, -1+b/ 2)

(1, 0) = (a/2, b-1/2)

1 = a/2 and 0 = (b-1)/ 2

a = 2 and b = 1

Hence, the coordinates of B = (2, 1)

And, now

E is the mid-point of AC.

So, coordinates of E = (0+p/ 2, -1+q/ 2)

(0, 1) = (p/2 , (q -1)/ 2)

p/2 = 0 and 1 = (q – 1)/2

p = 0 and 2 = q -1

p = 0 and q = 3

Hence, the coordinates of C = (0, 3)

Again, F is the mid-point of BC

Coordinates of F = (2+0/ 2, 1+3/ 2) = (1, 2)

Thus, the area of △DEF is given by

$$= \frac{1}{2}\left|1(1-2)+0(2-0)+1(0-1)\right|$$
$$= \frac{1}{2}\left|-1+0-1\right|$$
$$= \frac{1}{2}\times 2$$
$$= 1 \text{ sq. unit}$$

8. Find the area of the triangle PQR with Q (3, 2) and the mid-points of the sides through Q being (2, -1) and (1, 2).

Solution:

Let the coordinates of P and R be (x_1, y_1) and (x_2, y_2) respectively.

And, let the points E and F be the mid-points of PQ and QR respectively.

$$\frac{x_1+3}{2} = 1, \frac{y_1+2}{2} = 2 \text{ and } \frac{x_2+3}{2} = 2, \frac{y_2+2}{2} = -1$$

$x_1 + 3 = 2$, $y_1 + 2 = 4$ and $x_2 + 3 = 4$, $y_2 + 2 = -2$

$x_1 = -1$, $y_1 = 2$ and $x_2 = 1$, $y_2 = -4$

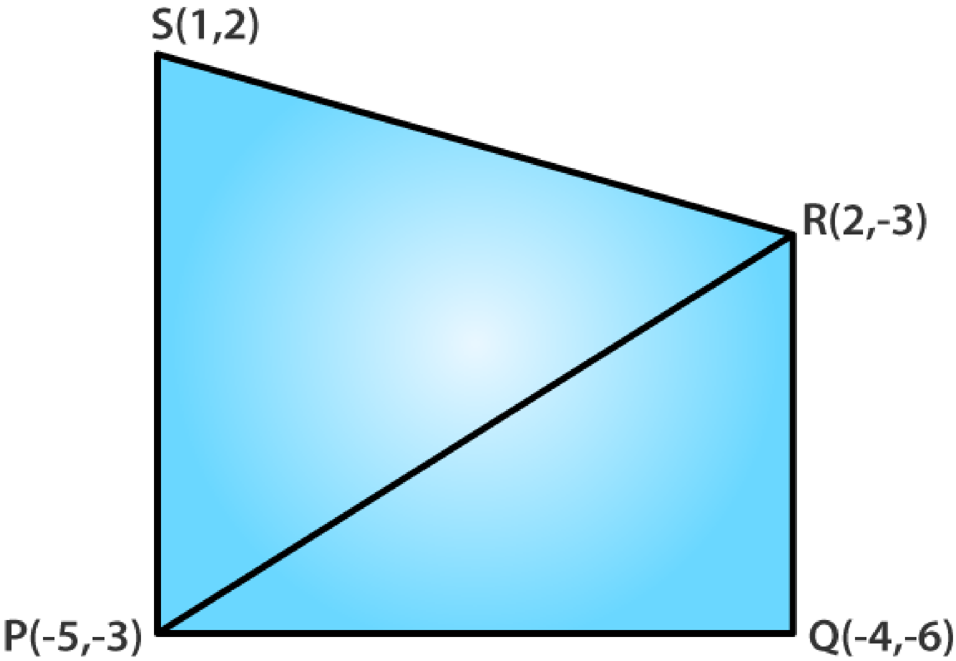
Hence, the coordinates of P and R are (-1, 2) and (1, 0) respectively.

Therefore, the area of △PQR is given by

$$= \frac{1}{2}\left|(3\times 2+(-1)\times(-4)+1\times 2)-(-1\times 2+1\times 2+3\times(-4))\right|$$
$$= \frac{1}{2}\left|(6+4+2)-(-2+2-12)\right|$$
$$= \frac{1}{2}\times\left|24\right|$$
$$= \frac{1}{2}\times 24$$
$$= 12 \text{ sq. units}$$

9. If P(-5, -3), Q(-4, -6), R(2, -3) and S(1, 2) are the vertices of a quadrilateral PQRS, find its area.

Solution:



First, let's join P and R.

Then,

Area of $\triangle$PSR is given by

$$= \frac{1}{2}|-5(2+3)+1(-3+3)+2(-3-2)|$$
$$= \frac{1}{2}|-5\times5+1\times0+2\times(-5)|$$
$$= \frac{1}{2}|-25+0-10|$$
$$= \frac{1}{2}|-35|$$
$$= \frac{35}{2}$$

And, now

Area of $\triangle$PQR is given by

$$= \frac{1}{2}|-5(-6+3)-4(-3+3)+2(-3+6)|$$
$$= \frac{1}{2}|-5\times(-3)-4\times 0+2\times 3|$$
$$= \frac{1}{2}|15+0+6|$$
$$= \frac{21}{2}$$

Thus,

Area of quad. PQRS = Area of △PSR + Area of △PQR

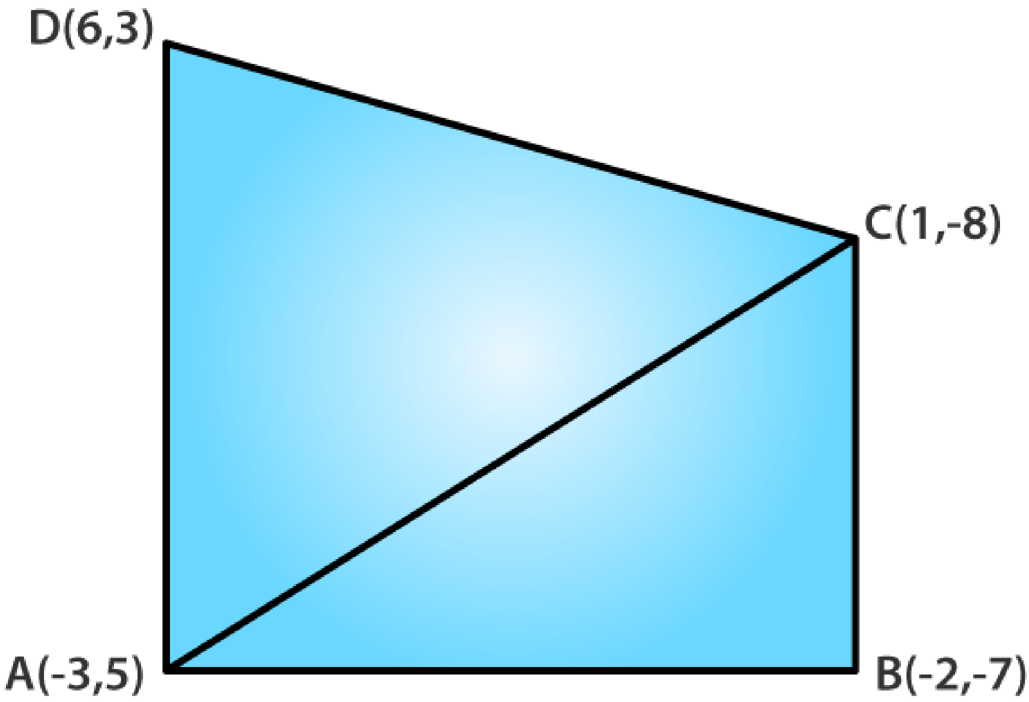
= 35/2 + 21/2

= 56/2

= 28 sq. units

10. If A (-3, 5), B(-2, -7), C(1, -8) and D(6, 3) are the vertices of a quadrilateral ABCD, find its area.

Solution:



Let's join A and C.

So, we get △ABC and △ADC

Hence,

The Area of quad. ABCD = Area of △ABC + Area of △ADC

$$= \frac{1}{2}|-3(-7-(-8))+(-2)(-8-5)+1(5-(-7))| + \frac{1}{2}|-3(3+8)+6(-8-5)+1(5-3)|$$
$$= \frac{1}{2}|-3(1)+(-2)(-13)+1(12)| + \frac{1}{2}|-3(11)+6(-13)+1(2)|$$
$$= \frac{1}{2}|-3+26+12| + \frac{1}{2}|-33-78+2|$$
$$= \frac{1}{2}|35| + \frac{1}{2}|-109|$$
$$= \frac{1}{2}\times 35 + \frac{1}{2}\times 109$$
$$= \frac{35+109}{2}$$
$$= \frac{144}{2}$$
$$= 72 \text{ sq. units}$$

Therefore, the area of the quadrilateral ABCD is 72 sq. units

11. For what value of a the points (a, 1), (1, -1) and (11, 4) are collinear?

Solution:

Let A (a, 1), B (1, -1) and C (11, 4) be the given points

Then the area of △ABC is given by,

$$= \frac{1}{2}\{a(-1-4)+1(4-1)+11(1+1)\}$$
$$= \frac{1}{2}\{-5a+3+22\}$$
$$= \frac{1}{2}\{-5a+25\}$$

We know that for the points to be collinear the area of △ABC has to be zero.

½(-5a + 25) = 0

5a = 25

∴ a = 5

12. Prove that the points (a, b), (a_1, b_1) and (a-a_1, b-b_1) are collinear if $ab_1 = a_1b$

Solution:

Let A (a, b), B (a_1, b_1) and C (a-a_1, b-b_1) be the given points.

So, the area of △ABC is given by,

$$= \frac{1}{2}\left\{a\left[b_1 - (b - b_1)\right] + a_1(b - b_1 - b) + (a - a_1)(b - b_1)\right\}$$
$$= \frac{1}{2}\left\{a(b_1 - b + b_1) + a_1(-b_1) + ab - ab_1 - a_1b + a_1b_1\right\}$$
$$= \frac{1}{2}\left\{ab_1 - ab + ab_1 - a_1b_1 + ab - ab_1 - a_1b + a_1b_1\right\}$$
$$= \frac{1}{2}\left\{ab_1 - a_1b\right\}$$

So, only if $ab_1 = a_1b$ the area becomes zero

△ABC = ½ (0) = 0

Therefore, the given points are collinear if $ab_1 = a_1b$

13. If the vertices of a triangle are (1,-3), (4,p) and (-9, 7) and its area is 15 sq. units, find the value (s) of p.

Solution:

Let A(1,-3), B(4,p) and C(-9, 7) be the vertices of △ABC

Area of △ABC = 15 sq. units

$$15 = \frac{1}{2}\left|1(p-7) + 4(7+3) - 9(-3-p)\right|$$
$$15 = \frac{1}{2}\left|p - 7 + 40 + 27 + 9p\right|$$
$$15 = \frac{1}{2}\left|10p + 60\right|$$
$$30 = 10p + 60 \text{ or } 30 = -10p - 60$$
$$10p = -30 \text{ or } 10p = -90$$
$$p = -3 \text{ or } p = -9$$

When modulus is removed, two cases arise:

$$15 = \frac{1}{2}|1(p-7)+4(7+3)-9(-3-p)|$$
$$15 = \frac{1}{2}|p-7+40+27+9p|$$
$$15 = \frac{1}{2}|10p+60|$$
$$30 = 10p+60 \text{ or } 30 = -10p-60$$
$$10p = -30 \text{ or } 10p = -90$$
$$p = -3 \text{ or } p = -9$$

14. If (x, y) be on the line joining the two points (1, -3) and (-4, 2). Prove that x + y + 2 = 0

Solution:

Let A (x, y), B (1, -3) and C (-4, 2) be the given points.

Area of △ABC is given by,

$$= \frac{1}{2}\left\{x(-3-2)+1(2-y)+(-4)(y+3)\right\}$$
$$= \frac{1}{2}\left\{-5x+2-y-4y-12\right\}$$
$$= \frac{1}{2}\left\{-5x-5y-10\right\}$$

As, the three points lie on the same line (that means they are collinear).

Then, the area of △ABC = 0

½ (-5x – 5y – 10) = 0

-5x – 5y – 10 = 0

-5(x + y + 2) = 0

x + y + 2 = 0

- Hence proved

15. Find the value of k if points (k, 3), (6, -2) and (-3, 4) are collinear.

Solution:

Let A (k, 3), B (6, -2) and C (-3, 4) be the given points.

Then, the area of △ABC is given by,

$$= \frac{1}{2}\{k(-2-4)+6(4-3)+(-3)(3+2)\}$$
$$= \frac{1}{2}\{-6k+6-15\}$$
$$= \frac{1}{2}\{-6k-9\}$$

As, the points are collinear.

Area of △ABC has to be zero.

½ x (-6k – 9) = 0

-6k – 9 = 0

k = -9/6

∴ k = -3/2

16. Find the value of k, if points A(7, -2), B(5, 1) and C(3, 2k) are collinear.

Solution:

Given,

Points A(7, -2), B(5, 1) and C(3, 2k)

Then, the area of △ABC is given by,

$$= \frac{1}{2}\{7(1-2k)+5(2k+2)+3(-2-1)\}$$
$$= \frac{1}{2}\{7-14k+10k+10-9\}$$
$$= \frac{1}{2}\{-4k+8\}$$

As, the points are collinear.

Area of △ABC has to be zero.

½ (-4k + 8) = 0

-4k + 8 = 0

-4k = -8

∴ k = 2