RD SHARMA Solutions for Class 12-science Maths Chapter 27 - Direction Cosines and Direction Ratios

Chapter 27 - Direction Cosines and Direction Ratios Exercise Ex. 27.1

Question 1

If a line makes angles of 90°, 60° and 30° with the positive direction of x,y and z-axis respectively, find its direction cosines.

Solution 1

Let l, m and n be the direction cosines of a line.
$l = \cos 90° = 0$

$m = \cos 60° = \frac{1}{2}$

$n = \cos 30° = \frac{\sqrt{3}}{2}$

$\therefore$ The direction cosines of the line are $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$.

Question 2

If a line has direction ratios 2, -1, -2, determine its direction cosines.

Solution 2

Let the direction cosines of the line be l, m, n.
Here,
$a = 2, b = -1, c = -2$ are the direction ratios of the line.

$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$

$l = \frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, m = \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, n = \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$

$l = \frac{2}{\sqrt{9}}, m = \frac{-1}{\sqrt{9}}, n = \frac{-2}{\sqrt{9}}$

$l = \frac{2}{3}, m = -\frac{1}{3}, n = -\frac{2}{3}$

$\therefore$ The direction ratios of the line are $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$.

Question 3

Find the direction cosines of the line passing through two points (-2, 4, -5) and (1, 2, 3).

Solution 3

The direction ratios of the line joining $(-2, 4, -5)$ and $(1, 2, 3)$ are,
$(1+2, 2-4, 3+5) = (3, -2, 8)$
Here, $a = 3, b = -2, c = 8$
Direction cosines are

$$\frac{3}{\sqrt{3^2+(-2)^2+8^2}}, \frac{-2}{\sqrt{3^2+(-2)^2+8^2}}, \frac{8}{\sqrt{3^2+(-2)^2+8^2}}$$

$$= \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

Question 4

Using direction ratios show that the points *A* (2, 3, -4), *B* (1, -2, 3) and *C* (3, 8, -11) are collinear.

Solution 4

Here $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$.
Direction ratios of $AB = (1-2, -2-3, 3+4) = (-1, -5, 7)$
Direction ratios of $BC = (3-1, 8+2, -11-3) = (2, 10, -14)$

Here, the respective direction cosines of AB and AC,

$$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14}$$ are proportional.

Also, B is the common point between the two lines,
$\therefore$ The points $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$ are collinear.

Question 5

Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 1, 2) and (-5, -5, -2)

Solution 5

$A(3,5,-4), B(-1,1,2)$ and $C(-5,-5,-2)$

The direction ratios of the side $AB = (-1-3, 1-5, 2+4)$

$= (-4, -4, 6)$

Direction cosines of AB will be

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{6}{\sqrt{(-4)^2+(-4)^2+6^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-4}{\sqrt{68}}, \frac{6}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of the side $BC = (-5+1, -5-1, -2-2)$

$= (-4, -6, -4)$

Direction cosines of BC will be

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-6}{\sqrt{68}}, \frac{-4}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

The direction ratios of the side $AC = (-5-3, -5-5, -2+4)$

$= (-8, -10, 2)$

Direction cosines of AC will be

$$\frac{-8}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{-10}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{2}{\sqrt{(-8)^2+(-10)^2+2^2}}$$

$$= \frac{-8}{\sqrt{168}}, \frac{-10}{\sqrt{168}}, \frac{2}{\sqrt{168}}$$

$$= \frac{-4}{\sqrt{42}}, \frac{-5}{\sqrt{42}}, \frac{1}{\sqrt{42}}$$

Question 6

Find the angle between the vectors with direction ratios 1, –2, 1 and 4, 3, 2.

Solution 6

Let, θ be the angle between the vectors with direction ratios a, b, c and a_2, b_2, c_2 then.

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(1)(4) + (-2)(3) + (1)(2)}{\sqrt{(1)^2 + (-2)^2 + (1)^2}\sqrt{(4)^2 + (3)^2 + (2)^2}}$$

$$= \frac{4 - 6 + 2}{\sqrt{1 + 4 + 1}\sqrt{16 + 9 + 4}}$$

$$= \frac{6 - 6}{\sqrt{6}\sqrt{29}}$$

$$= \frac{0}{\sqrt{174}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Question 7

Find the angle between the vectors whose direction cosines are proportional to 2, 3, -6 and 3, -4, 5.

Solution 7

Here, given that the direction cosines of the vectors are proportional to 2, 3, – 6 and 3, – 4, 5.

Therefore, 2, 3, – 6 and 3, – 4, 5 are the direction ratios of two vectors.

Let, θ be the angle between two vectors having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(2)(3) + (3)(-4) + (-6)(5)}{\sqrt{(2)^2 + (3)^2 + (-6)^2}\sqrt{(3)^2 + (-4)^2 + (5)^2}}$$

$$\cos\theta = \frac{6 - 12 - 30}{\sqrt{4 + 9 + 36}\sqrt{9 + 16 + 25}}$$

$$= \frac{6 - 42}{\sqrt{49}\sqrt{50}}$$

$$= \frac{-36 \times \sqrt{2}}{7 \times 5 \times \sqrt{2} \times \sqrt{2}} \qquad \text{(Rationalizing the denominator)}$$

$$= \frac{-36\sqrt{2}}{70}$$

$$\cos\theta = \frac{-18\sqrt{2}}{35}$$

$$\theta = \cos^{-1}\left(\frac{-18\sqrt{2}}{35}\right)$$

Question 8

Find the acute angle between the lines whose direction ratios are 2:3:6 and 1 : 2 : 2.

Solution 8

The vectors, represented by these are

$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$
and $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

Let, θ be the angle between the lines,
then,

$$\cos\theta = \frac{\vec{a}\times\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$$

$$= \frac{\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)}{\sqrt{(2)^2 + (3)^2 + (6)^2}\sqrt{(1)^2 + (2)^2 + (2)^2}}$$

$$= \frac{(2)(1) + (3)(2) + (6)(2)}{\sqrt{4 + 9 + 36}\sqrt{1 + 4 + 4}}$$

$$= \frac{2 + 6 + 12}{\sqrt{49}\sqrt{9}}$$

$$= \frac{20}{7 \times 3}$$

$$\cos\theta = \frac{20}{21}$$

$$\theta = \cos^{-1}\left(\frac{20}{21}\right)$$

Angle between the lines $= \cos^{-1}\left(\frac{20}{21}\right)$

Question 9

Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

Solution 9

We have, $(2, 3, 4), (-1, -2, 1)$ and $(5, 8, 7)$

Let the points are A, B, C respectively.

Position vector of $A = 2\hat{i} + 3\hat{j} + 4\hat{k}$
Position vector of $B = -\hat{i} - 2\hat{j} + \hat{k}$
Position vector of $C = 5\hat{i} + 8\hat{j} + 7\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(-\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$
$= -\hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = -3\hat{i} - 5\hat{j} - 3\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(5\hat{i} + 8\hat{j} + 7\hat{k}\right) - \left(-\hat{i} - 2\hat{j} + \hat{k}\right)$
$= 5\hat{i} + 8\hat{j} + 7\hat{k} + \hat{i} + 2\hat{j} - \hat{k}$

$\overrightarrow{BC} = 6\hat{i} + 10\hat{j} + 6\hat{k}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$\overrightarrow{BC} = -2\ \overrightarrow{AB}$

So, $\overrightarrow{BC}$ is parallel to $\overrightarrow{AB}$ but $\vec{B}$ is the common vector,

Hence, A, B, C are collinear

Question 10

Show that the line through points $(4, 7, 8)$ and $(2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1)$ and $(1, 2, 5)$.

Solution 10

line throught points $(4, 7, 8)$ *and* $(2, 3, 4)$

$$\frac{x-4}{2}=\frac{y-7}{4}=\frac{z-8}{4} \rightarrow \frac{x-4}{1}=\frac{y-7}{2}=\frac{z-8}{2}$$

line through the points $(-1, -2, 1)$ and $(1, 2, 5)$

$$\frac{x+1}{-2}=\frac{y+2}{-4}=\frac{z-1}{-4} \rightarrow \frac{x+1}{1}=\frac{y+2}{2}=\frac{z-1}{2}$$

the direction ratios are same for both the lines

$\therefore$ they are parallel to each other

Question 11

Show that the line through the points $(1, -1, 2)$ and $(3, 4, -2)$ is perpendicular to the line through the $(0, 3, 2)$ and $(3, 5, 6)$.

Solution 11

Given,

$A(1, -1, 2)$ and $B(3, 4, -2)$
$C(0, 3, 2)$ and $D(3, 5, 6)$

Direction ratios of line AB

$a_1 = 2, \qquad b_1 = 5, \qquad c_1 = -4$

Direction ratios of line CD

$a_2 = 3, \qquad b_2 = 2, \qquad c_2 = 4$

We know that, lines are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\begin{aligned} LHS &= (2)(3) + (5)(2) + (-4)(4) \\ &6 + 10 - 16 \\ &16 - 16 \\ &0 \end{aligned}$$

$\therefore LHS = RHS$

Lines are perpendicular

Question 12

Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1)$ and $(4, 3, -1)$.

Solution 12

Here,

$A(0, 0, 0)$ and $B(2, 1, 1)$
$C(3, 5, -1)$ and $D(4, 3, -1)$

Direction ratios of line AB
$a_1 = 2,\quad b_1 = 1,\quad c_1 = 1$

Direction ratios of line CD
$a_2 = 1,\quad b_2 = -2,\quad c_2 = 0$

Now,

$a_1a_2 + b_1b_2 + c_1c_2$
$= (2)(1) + (1)(-2) + (1)(0)$
$= 2 - 2 + 0$

$= 0$

Since, $a_1a_2 + b_1b_2 + c_1c_2 = 0$, lines are perpendicular

Question 13

Find the angle between the lines whose direction ratios are proportional to a, b, c and $b - c, c - a, a - b$.

Solution 13

Given, that the direction ratios of lines are proportional to a, b, c and $b-c$, $c-a$, $a-b$.

Let, $\vec{x}$ and $\vec{y}$ be the vector parallel to these lines respectively, so

$\vec{x} = a\hat{i} + b\hat{j} + c\hat{k}$

And, $\vec{y} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$

Let, θ be the angle between $\vec{x}$ and $\vec{y}$, so,

$$\cos\theta = \frac{\vec{x} \times \vec{y}}{|\vec{x}||\vec{y}|}$$

$$= \frac{\left(a\hat{i} + b\hat{j} + c\hat{k}\right)\left[(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}\right]}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}$$

$$= \frac{(a)(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{b^2+c^2-2bc+c^2+a^2-2ca+a^2+b^2-2ab}}$$

$$\cos\theta = \frac{ab-ac+bc-ba+ca-bc}{\sqrt{a^2+b^2+c^2}\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}}$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Angle between the lines $= \frac{\pi}{2}$

Question 14

If the coordinates of the points A, B, C, D are $(1, 2, 3)(4, 5, 7), (-4, 3, -6)$ and $(2, 9, 2)$ then find the angle between AB and CD.

Solution 14

Here we have,

$A(1, 2, 3)$, $B(4,5,7)$, $C(-4, 3, -6)$ $D(2, 9, 2)$

Direction ratios of AB

$a_1 = 3,\quad b_1 = 3,\quad c_1 = 4$

Direction ratios of CD

$a_2 = 6,\quad b_2 = 6,\quad c_2 = 8$

Let, θ be the angle between AB and CD, so,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{(3)(6) + (3)(6) + (4)(8)}{\sqrt{(3)^2 + (3)^2 + (4)^2}\sqrt{(6)^2 + (6)^2 + (8)^2}}$$

$$= \frac{18 + 18 + 32}{\sqrt{9 + 9 + 16}\sqrt{36 + 36 + 64}}$$

$$= \frac{68}{\sqrt{34}\sqrt{136}}$$

$$= \frac{68}{\sqrt{34}\ 2\sqrt{34}}$$

$$= \frac{68}{34 \times 2}$$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\theta = 0^\circ$

Therefore,

Angle between AB and $CD = 0^\circ$

Question 15

Find the direction cosines of the lines, connected by the relations: $l + m + n = 0$ and $2lm + 2\ln - mn = 0$.

Solution 15

The given equations are

$$2lm + 2ln - mn = 0$$

$$l + m + n = 0$$

$$\to l = -(m+n) \text{...........}(1)$$

$$2l(m+n) = mn \to l = \frac{mn}{2(m+n)} \text{..............}(2)$$

put $l = -(m+n)$ in (2)

$$\to -(m+n) = \frac{mn}{2(m+n)} \to -2(m+n)^2 = mn$$

$$\to -2(m^2+n^2+2mn) = mn \to (m^2+n^2+2mn) = -\frac{mn}{2}$$

$$\to \left(m^2+n^2+2mn+\frac{mn}{2}\right) = 0 \to \left(m^2+n^2+\frac{5mn}{2}\right) = 0$$

$$\to (2m^2+2n^2+5mn) = 0 \to (2m+n)(m+2n) = 0$$

$$\to m = -\frac{n}{2} \to l = -\left(n - \frac{n}{2}\right) = -\frac{n}{2}$$

$$\to m = -2n \to l = -(-2n+n) = n$$

Thus the direction ratios of two lines are proportional to $-\frac{n}{2}, -\frac{n}{2}, n$

and $n, -2n, n$

$i.e. -\frac{1}{2}, -\frac{1}{2}, 1$ and $1, -2, 1$

Hence the direction cosines are

$-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ and $1, -2, 1$

Question 16(i)

Find the angle between the lines whose direction cosines are given by the equations

$l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$

Solution 16(i)

Given that, $l + m + n = 0 \qquad ---(\text{i})$

$l^2 + m^2 - n^2 = 0 \qquad ---(\text{ii})$

From equation (i),

$l = -(m+n)$

Put the value of l in equation (ii),

$$\left[-(m+n)\right]^2 + m^2 - n^2 = 0$$
$$(m+n)^2 + m^2 - n^2 = 0$$
$$m^2 + n^2 + 2mn + m^2 - n^2 = 0$$
$$2m^2 + 2mn = 0$$
$$2m(m+n) = 0$$

$m = 0,\ m + n = 0$

$m = -n$ and $m = 0$

Put the value of $m = -n$ in equation (i)

$l = -(-n+n)$

$l = 0$

Again put the value of $m = 0$ in equation (i)

$l = -(m+n)$

$= -(0+n)$

$l = -n$

Thus the direction ratios are proportional to

$0, -n, n$ and $-n, 0, n$

$\Rightarrow \qquad 0, -1, 1$ and $-1, 0, 1$

So, vectors parallel to these lines are

$\vec{a} = 0 \times \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 0 \times \hat{j} + \hat{k}$ respectively.

Let, θ be the angle between the $\vec{a}$ and $\vec{b}$

So, $\cos\theta = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a} = 0\times\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 0\times\hat{j} + \hat{k}$ respectively.

$$\cos\theta = \frac{\left(0\times\hat{i} - \hat{j} + \hat{k}\right)\times\left(-\hat{i} + 0\times\hat{j} + \hat{k}\right)}{\sqrt{0^2 + (-1)^2 + (1)^2}\sqrt{(-1)^2 + (0)^2 + (1)^2}}$$

$$= \frac{(0)(-1) + (-1)(0) + (1)(1)}{\sqrt{1+1}\sqrt{1+1}}$$

$$= \frac{0+0+1}{\sqrt{2}\times\sqrt{2}}$$

$$= \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

So, angle between the lines = $\dfrac{\pi}{3}$

Question 16(ii)

Find the angle between the lines whose direction cosines are given by the equations

$2l - m + 2n = 0$ and $mn + nl + lm = 0$

Solution 16(ii)

Given that,

$$2l - m + 2n = 0 \qquad ---(i)$$
$$mn + nl + lm = 0 \qquad ---(ii)$$

From equation (i),

$2l - m + 2n = 0$
$m = 2l + 2n$

Put the value of m in equation (ii),

$mn + nl + lm = 0$
$(2l + 2n)n + nl + l(2l + 2n) = 0$
$2\ln + 2n^2 + nl + 2l^2 + 2\ln = 0$
$2l^2 + 5\ln + 2n^2 = 0$
$2l^2 + 4\ln + \ln + 2n^2 = 0$
$2l(l + 2n) + n(l + 2n) = 0$
$(l + 2n)(2l + n) = 0$
$l + 2n = 0 \quad \text{or} \quad 2l + n = 0$
$l = -2n \quad \text{or} \quad l = -\frac{n}{2}$

Put the value of $l = -2n$ in equation (i)

$2l - m + 2n = 0$
$2(-2n) - m + 2n = 0$
$-4n - m + 2n = 0$
$-2n - m = 0$
$-2n = m$

$m = -2n$

Again, put the value of $l = -\frac{1}{2}n$ in equation (i)

$2l - m + 2n = 0$
$2\left(-\frac{1}{2}n\right) - m + 2n = 0$
$-n - m + 2n = 0$
$-m + n = 0$
$-m = -n$

$m = n$

So, direction cosines of the lines are given by,

$$-2n,\ -2n,\ n \quad \text{or} \quad -\frac{1}{2}n,\ n,\ n$$

$$-2,\ -2,\ 1 \quad \text{or} \quad -\frac{1}{2},\ 1,\ 1$$

So, vectors parallel to these lines

$\vec{a} = -2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = -\frac{1}{2}\hat{i} + \hat{j} + \hat{k}$ respectively.

Let, θ be the angle between $\vec{a}$ and $\vec{b}$,

$$\cos\theta = \frac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)\times\left(-\frac{1}{2}\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{(-2)^2 + (-2)^2 + (1)^2}\sqrt{\left(-\frac{1}{2}\right)^2 + (1)^2 + (1)^2}}$$

$$= \frac{(-2)\left(-\frac{1}{2}\right) + (-2)(1) + (1)(1)}{\sqrt{4+4+1}\sqrt{\frac{1}{4}+1+1}}$$

$$= \frac{1-2+1}{\sqrt{9}\sqrt{\frac{9}{4}}}$$

$$\cos\theta = \frac{0}{3\times\frac{3}{2}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Angle between the lines $= \frac{\pi}{2}$

Question 16(iii)

Find the angle between the lines whose direction cosines are given by the equations

$l + 2m + 3n = 0$ and $3lm - 4ln + mn = 0$

Solution 16(iii)

Here,

$l+2m+3n=0 \qquad ---(\text{i})$

$3lm-4\ln+mn=0 \qquad ---(\text{ii})$

From equation (i),

$l+2m+3n=0$

$l=-2m-3n$

Put the value of l in equation (ii),

$3lm-4\ln+mn=0$

$3(-2m-3n)m-4(-2m-3n)n+mn=0$

$-6m^2-9nm+8mn+12n^2+mn=0$

$-6m^2+12n^2=0$

$-6m^2=-12n^2$

$m^2=2n^2$

$m=\pm\sqrt{2n^2}$

$m=n\sqrt{2}$ or $m=-n\sqrt{2}$

Put $m=n\sqrt{2}$ in equation (i)

$l+2m+3n=0$

$l+2\left(n\sqrt{2}\right)+3n=0$

$l+n\left(2\sqrt{2}+3\right)=0$

$l=-\left(2\sqrt{2}+3\right)n$

Again, m $=-\sqrt{2}n$ in equation (i)

$l+2m+3n=0$

$l+2\left(-\sqrt{2}n\right)+3n=0$

$l-2\sqrt{2}n+3n=0$

$l+n\left(-2\sqrt{2}+3\right)=0$

$l=\left(2\sqrt{2}-3\right)n$

Thus, direction cosines of the lines are given by,

$$-\left(2\sqrt{2}+3\right)n,\ \sqrt{2}n,\ n \quad or \quad \left(2\sqrt{2}-3\right)n,\ -\sqrt{2}n,\ n$$
$$-\left(2\sqrt{2}+3\right),\ \sqrt{2},\ 1 \quad or \quad \left(2\sqrt{2}-3\right),\ -\sqrt{2},\ 1$$

So, vectors parallel to these lines are

$$\vec{a} = -\left(2\sqrt{2}+3\right)\hat{i} + \sqrt{2}\hat{j} + \hat{k} \text{ and } \vec{b} = \left(2\sqrt{2}-3\right)\hat{i} - \sqrt{2}\hat{j} + \hat{k}$$

Let, θ be the angle between the lines,
then,

$$\cos\theta = \frac{\vec{a} \times \vec{b}}{\left|\vec{a}\right| \times \left|\vec{b}\right|}$$

$$= \frac{-\left(2\sqrt{2}+3\right)\times\left(2\sqrt{2}-3\right)+\left(\sqrt{2}\right)\times\left(-\sqrt{2}\right)+(1)(1)}{\sqrt{\left(2\sqrt{2}+3\right)^2+\left(\sqrt{2}\right)^2+(1)^2}\sqrt{\left(2\sqrt{2}-3\right)^2+\left(-\sqrt{2}\right)^2+(1)^2}}$$

$$= \frac{-(8-9)-2+1}{\sqrt{8+9+12\sqrt{2}+2+1}\sqrt{8+9-12\sqrt{2}+2+1}}$$

$$= \frac{-(-1)-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$
$$= \frac{1-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$

$$\cos\theta = 0$$
$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Angle between the lines $= \frac{\pi}{2}$

Question 16(iv)

Find the angle between the lines whose direction cosines are given by equations
2l + 2m - n = 0, mn + ln + lm = 0

Solution 16(iv)

The given equations are,

$2l + 2m - n = 0.........(i)$

$mn + ln + lm = 0.........(ii)$

From(i), we get $n = 2l + 2m$.

Putting $n = 2l + 2m$ in (ii), we get

$m(2l + 2m) + l(2l + 2m) + lm = 0$

$\Rightarrow 2lm + 2m^2 + 2l^2 + 2ml + lm = 0$

$\Rightarrow 2m^2 + 5lm + \ + 2l^2 = 0$

$\Rightarrow 2m^2 + 4lm + lm + 2l^2 = 0$

$\Rightarrow (2m + l)(m + 2l) = 0$

$\Rightarrow m = -\frac{l}{2}$ or $m = -2l$

By putting $m = -\frac{l}{2}$ in (i) we get $n = l$

By putting $m = -2l$ in (i) we get $n = -2l$

So direction ratios of two lines are proportional to

$l, -\frac{l}{2}, l$ and $l, -2l, -2l$ or, $1, -\frac{1}{2}, 1$ and $1, -2, -2$

So, vectors parallel to these lines are

$\vec{a} = \hat{i} - \frac{1}{2}\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$

If θ is the angle between the lines, then θ is also the angle between $\vec{a}$ and $\vec{b}$.

$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|} = \frac{1 + 1 - 2}{\sqrt{1 + \frac{1}{4} + 1}\sqrt{1 + 4 + 9}} = 0$$

$$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

Chapter 27 - Direction Cosines and Direction Ratios Exercise MCQ

Question 1

For every point P(x, y, z) on the xy-plane,

a. x = 0
b. y = 0
c. z = 0
d. x = y = z = 0

Solution 1
Correct option: (c)
Every point on xy-plane z co-ordinate is always zero.

Question 2
For every point P(x, y, z) on the x-axis (except the origin),

a. $x = 0, y = 0, z \neq 0$
b. $x = 0, z = 0, y \neq 0$
c. $y = 0, z = 0, x \neq 0$
d. $x = y = z = 0$

Solution 2
Correct option: (c)
Point (x, y, z) is on the x-axis. Hence, y and z co-ordinate will be zero except the origin.

Question 3
A rectangular parallelopiped is formed by planes drawn through the points (5, 7, 9) and (2, 3, 7) parallel to the coordinate planes. The length of an edge of this rectangular parallelopiped is

a. 2
b. 3
c. 4
d. all of these

Solution 3
Correct option: (d)
Coordinates of the points given are diagonally opposite vertices of a parallelepiped. Hence, edges of parallelepiped can be 5-2, 7-3, 9-7 ⇒3,4,2.

Question 4
A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelopiped is

a. 7
b. $\sqrt{38}$
c. $\sqrt{155}$
d. none of these

Solution 4
Correct option : (a)

$$\text{Edges of parallelopiped are } 5-2,\ 9-3,\ 7-5 \Rightarrow 3,\ 6,\ 2$$

$$\text{Length of the diagonal } = \sqrt{9+36+4}$$

$$\text{Length of the diagonal } = 7$$

Question 5
The xy-plane divides the line joining the points (-1, 3, 4) and (2, -5, 6)

a. internally in the ratio 2 : 3
b. externally in the ratio 2 : 3
c. internally in the ratio 3 : 2
d. externally in the ratio 3 : 2

Solution 5

Correct option: (b)

Let, xy plane divides given points in the ratio of $t : 1$.

$\Rightarrow$ point of intersection of planes $= \left(\frac{-1+2t}{t+1}, \frac{3-5t}{t+1}, \frac{4+6t}{t+1}\right)$

Plane is on xy axis. Hence, z – coordinate is always zero.

$$\frac{4+6t}{t+1} = 0$$

$$4+6t = 0$$

$$t = \frac{-2}{3}$$

$\Rightarrow$ xy plane divides given points externally in the ratio $2 : 3$

Question 6

If the x-coordinate of a point P on the join of Q (2, 2, 1) and R (5, 1, -2) is 4, then its z-coordinate is

a. 2
b. 1
c. -1
d. -2

Solution 6

Correct option: (c)

Suppose P divides the line QR in the ratio $t : 1$.

Using section formula

$$\left(\frac{5t+2}{t+1}, \frac{2+t}{t+1}, \frac{1-2t}{t+1}\right)$$

Given that x – coordinate is 4.

$$\Rightarrow \frac{5t+2}{t+1} = 4$$

$$\Rightarrow 5t+2 = 4t+4$$

$$\Rightarrow t = 2$$

Hence, z – coordinate will be

$$\frac{1-2t}{t+1} = \frac{1-4}{2+1}$$

$$z = -1$$

Question 7

The distance of the point P(a, b, c) from the x-axis is

a. $\sqrt{b^2+c^2}$
b. $\sqrt{a^2+c^2}$
c. $\sqrt{a^2+b^2}$
d. none of these

Solution 7

Correct option: (a)

Coordinate of a point on x – axis $(a, 0, 0)$

The distance of the point $P(a, b, c)$ from x – axis

$$= \sqrt{(a-a)^2 + b^2 + c^2}$$

$$= \sqrt{b^2 + c^2}$$

Question 8

Ratio in which the xy-plane divides the join of (1, 2, 3) and (4, 2, 1) is

a. 3 : 4 internally
b. 3 : 1 externally
c. 1 : 2 internally
d. 2 : 1 externally

Solution 8

Correct option: (b)

Let P divides given points in the ratio $t : 1$.

$$\Rightarrow \text{coordinates of } P = \left(\frac{4t+1}{t+1}, \frac{2t+2}{t+1}, \frac{t+3}{t+1}\right)$$

Plane is on xy axis $\Rightarrow z = 0$

$$\Rightarrow \frac{t+3}{t+1} = 0$$

$$\Rightarrow t = -3$$

Point divides externally the given points in the ratio $3 : 1$.

Question 9

If P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) are collinear, then R divides PQ in the ratio

a. 3 : 2 internally
b. 3 : 2 externally
c. 2 : 1 internally
d. 2 : 1 externally

Solution 9

Correct option: (b)

Let R divides PQ in the ratio t : 1.

$$(9, 8, -10) = \left(\frac{5t+3}{t+1}, \frac{4t+2}{t+1}, \frac{-6t-4}{t+1}\right)$$

$$\Rightarrow \frac{5t+3}{t+1} = 9$$

$$\Rightarrow 5t + 3 = 9t + 9$$

$$\Rightarrow 4t = -6$$

$$\Rightarrow t = \frac{-3}{2}$$

⇒ R divides PQ in the ratio 3 : 2 externally.

Question 10

A(3, 2, 0), B(5, 3, 2) and C(-9, 6, -3) are the vertices of a triangle ABC. If the bisector of ∠BAC meets BC at D, then coordinates of D are

a. (19/8, 57/16, 17/16)
b. (-19/8, 57/16, 17/16)
c. (19/8, -57/16, 17/16)
d. none of these

Solution 10

Correct option: (a)

As bisector ∠BAC meets BC at D.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

Given that points A(3, 2, 0), B(5, 3, 2), C(−9, 6, −3)

Using distance formula,

AB = 3, AC = 13

$$\Rightarrow \frac{BD}{DC} = \frac{3}{13}$$

D divides BC in the ratio 3 : 13 internally.

Using section formula,

$$D = \left(\frac{-9 \times 3 + 5 \times 13}{16}, \frac{6 \times 3 + 13 \times 3}{16}, \frac{-3 \times 3 + 13 \times 2}{16}\right)$$

$$D = \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$$

Question 11

If O is the origin, OP = 3 with direction ratios proportional to -1, 2, -2 then the coordinates of P are

a. (-1, 2, -2)
b. (1, 2, 2)
c. (-1/9, 2/9, -2/9)
d. (3, 6, -9)

Solution 11

Correct option: (a)

Directions of OP from the origin

(-1,2,-2)=(x,y,z)

Question 12
The angle between the two diagonals of a cube is

a. $30°$
b. $45°$
c. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
d. $\cos^{-1}\left(\frac{1}{3}\right)$

Solution 12
Correct option: (d)
The angle between the two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

Question 13
If a line makes angles α, β, γ, δ with four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is equal to

a. $\frac{1}{3}$
b. $\frac{2}{3}$
c. $\frac{4}{3}$
d. $\frac{8}{3}$

Solution 13
Correct option: (c)
A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube then
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

Chapter 27 - Direction Cosines and Direction Ratios Exercise Ex. 27VSAQ

Question 1
Define direction cosines of a directed line.

Solution 1

If a directed line segment OP makes angles α, β, γ with OX, OY and OZ respectively, then

$\cos\alpha$, $\cos\beta$, $\cos\gamma$ are known as the direction cosines of OP and are generally denoted by l, m, n. Thus direction cosines of line OP are given by

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

Question 2

What are the direction cosines of X-axis?

Solution 2

We know that, if α, β, γ are the angles that a line makes with the positive sides of coordinate axes respectively, then direction cosines of the line are given by

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

We know that, X-axis makes angles $0°, 90°, 90°$ with coordinate axes respectively, so

$\alpha = 0°$, $\beta = 90°$, $\gamma = 90°$

$\Rightarrow \quad l = \cos 0°, \quad m = \cos 90°, \quad n = \cos 90°$
$\Rightarrow \quad l = 1, \quad m = 0, \quad n = 0$

So,
Direction cosines of x-axis are given by 1, 0, 0.

Question 3

What are the direction cosines of Y-axis?

Solution 3

We know that, if α, β, γ are the angles that a line makes with the positive sides of coordinate axes respectively, then direction cosines of the line are given by

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

We know that, Y-axis makes angles $90°, 0°, 90°$ with coordinate axes respectively, so

$\alpha = 90°$, $\beta = 0°$, $\gamma = 90°$

$\Rightarrow \quad l = \cos 90°, \quad m = \cos 0°, \quad n = \cos 90°$
$\Rightarrow \quad l = 0, \quad m = 1, \quad n = 0$

So,
Direction cosines of y-axis are given by 0, 1, 0.

Question 4

What are the direction cosines of Z-axis?

Solution 4

We know that, if α, β, γ are the angles that a line makes with the positive sides of coordinate axes respectively, then direction cosines of the line are given by

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

We know that, Z-axis makes angles $90°, 90°, 0°$ with coordinate axes respectively, so

$\alpha = 90°$, $\beta = 90°$, $\gamma = 0°$

$\Rightarrow \quad l = \cos 90°, \quad m = \cos 90°, \quad n = \cos 0°$

$\Rightarrow \quad l = 0, \quad m = 0, \quad n = 1$

So,

Direction cosines of z-axis are given by 0, 0, 1.

Question 5

Write the distances of the point $(7, -2, 3)$ from XY, YZ and XZ -planes.

Solution 5

We know that, point $P(x, y, z)$ gives us

The distance of P from XY-plane = z units
The distance of P from YZ-plane = x units
The distance of P from ZX-plane = y units.

Here, The point $P(7, -2, 3)$, so,

Distance of P from XY-plane = 3 units
Distance of P from YZ-plane = 7 units
Distance of P from ZX-plane = 2 units.

Question 6

Write the distance of the point $(3, -5, 12)$ from X-axis?

Solution 6

We know that, distance of a point $(x,\ y,\ z)$ from x-axis is gives by

$$OP = \sqrt{y^2 + z^2}$$

Therefore, Distance of the point $p\,(3,\ -5,\ 12)$ from x-axis $= \sqrt{(-5)^2 + (12)^2}$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

Required distance = 13 Units.

Question 7

Write the ratio in which YZ-plane divides the segment joining $P\,(-2,\ 5,\ 9)$ and $Q\,(3,\ -2,\ 4)$.

Solution 7

Suppose line segment joining $P\,(-2,\ 5,\ 9)$ and $Q\,(3,\ -2,\ 4)$ is divided YZ-plane in k:1, so using section formula

$$\text{Coordinate of } R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

$$(0, y, z) = \left(\frac{3k-2}{k+1}, \frac{-2k+5}{k+1}, \frac{4k+9}{k+1}\right)$$

$$\Rightarrow \quad \frac{3k-2}{k+1} = 0$$

$$\Rightarrow \quad 3k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{3}$$

Thus, YZ-plane divides PQ is 2:3 internally

Question 8

A line makes an angle of 60° with each of X-axis and Y-axis. Find the acute angle made by the line with Z-axis.

Solution 8

Let, the line makes an angle of Q with positive side of Z-axis.

We know that, if α, β, γ are angles that a line makes with the positive axes then direction cosines of the line are

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ and $l^2 + m^2 + n^2 = 1$

Here, $\alpha = 60^\circ$, $\beta = 60^\circ$, $\gamma = \theta$

$\Rightarrow \quad l = \cos 60^\circ, \; m = \cos 60^\circ, \; n = \cos\theta$

$\Rightarrow \quad l = \frac{1}{2}, \; m = \frac{1}{2}, \; n = \cos\theta$

$$l^2 + m^2 + n^2 = 1$$
$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (\cos\theta)^2 = 1$$
$$\frac{1}{4} + \frac{1}{4} + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \frac{1}{4} - \frac{1}{4}$$
$$\cos^2\theta = \frac{4-1-1}{4}$$
$$\cos^2\theta = \frac{1}{2}$$
$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}} \quad or \quad \cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) or \quad \theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{4} \quad or \quad \theta = \pi - \frac{\pi}{4}$

$\Rightarrow \quad \theta = \frac{\pi}{4} \quad or \quad \theta = \frac{3\pi}{4}$

But θ is an acute angle (Given), so

$\theta = \frac{\pi}{4}$

Required angle $= \frac{\pi}{4}$

Question 9

If a line makes angles α, β and γ with the coordinate axes, find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

Solution 9

We know that, if α, β, γ are the angles made by a line with coordinate axes,
Then,

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 1 \qquad ---(1)$$

Now,

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma$$

$$= \left(2\cos^2\alpha - 1\right) + \left(2\cos^2\beta - 1\right) + \left(2\cos^2\gamma - 1\right) \qquad \left[\text{Since } \cos 2x = 2\cos^2 x - 1\right]$$
$$= 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma - 3$$
$$= 2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right) - 3$$
$$= 2(1) - 3 \qquad \left[\text{Using equation } (1)\right]$$
$$= -1$$

So,
$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

Question 10

Write the ratio in which the line segment joining (a, b, c) and $(-a, -c, -b)$ is civided by the xy-plane.

Solution 10

Let the line segment joining $P(a, b, c)$ and $Q(-a, -c, -b)$ is divided by the xy-plane in k: 1, so

Using section formula

$$\text{Coordinate of } R = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

$$(x, y, 0) = \left(\frac{-ka + a}{k+1}, \frac{-kc + b}{k+1}, \frac{-kb + c}{k+1}\right)$$
$$\Rightarrow \quad \frac{-kb + c}{k+1} = 0$$
$$\Rightarrow \quad -kb + c = 0$$

$$\Rightarrow \quad k = \frac{c}{b}$$

So, xy-plane divides PQ is c:b internally

Question 11

Write the inclination of a line with Z-axis, if its direction ratios are proportional to 0, 1, – 1.

Solution 11

Given, direction ratios of the line are 0,1, – 1

Direction cosines of the line are given by

$$= \frac{0}{\sqrt{(0)^2 + (1)^2 + (-1)^2}}, \frac{1}{\sqrt{(0)^2 + (1)^2 + (-1)^2}}, \frac{-1}{\sqrt{(0)^2 + (1)^2 + (-1)^2}}$$

$$= 0, \frac{1}{\sqrt{1+1}}, \frac{-1}{\sqrt{1+1}}$$

$$= 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$$

We know that, if α, β, γ are the angles that line makes with the coordinate axes respectively, then direction cosines of the line are given by

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

So,

$$\cos\gamma = -\frac{1}{\sqrt{2}}$$

$$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \pi - \frac{\pi}{4}$$

$$\gamma = \frac{3\pi}{4}$$

Question 12

Write the angle between the lines whose direction ratios are proportional to 1, – 2,1 and 4, 3, 2.

Solution 12

We know that, if a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two lines, and θ be the angle between lines, then, angle between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = 1$
$a_2 = 4$, $b_2 = 3$, $c_2 = 2$, so,

$$\cos\theta = \frac{(1)(4) + (-2)(3) + (1)(2)}{\sqrt{(1)^2 + (-2)^2 + (1)^2}\sqrt{(4)^2 + (3)^2 + (2)^2}}$$

$$= \frac{4 - 6 + 2}{\sqrt{1 + 4 + 1}\sqrt{16 + 9 + 4}}$$

$$= \frac{0}{\sqrt{6}\sqrt{29}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Angle between given lines is $\frac{\pi}{2}$

Question 13

Write the distance of the point $P(x, y, z)$ from XOY plane.

Solution 13

Distance of the point $P(x, y, z)$ from XOY plane = $|z|$ unit.

Because the point P may be on the positive side of z-axis or negative side of Z-axis.

Question 14

Write the coordinates of the projection of the point $P(x, y, z)$ on XOZ plane.

Solution 14

Projection vector of point $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ on $\vec{b}$ is given by $\left\{\frac{\vec{a} \times \vec{b}}{|\vec{b}|^2}\right\}\vec{b}$

Given, point is $P(x, y, z)$

$\Rightarrow \quad \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Any point on XOZ plane is given by

$\vec{b} = x\hat{i} + 0 \times \hat{j} + z \times \hat{k}$

So, projection vector of $\vec{a}$ on XOZ-plane is given by $= \left\{\frac{\vec{a} \times \vec{b}}{|\vec{b}|^2}\right\}\vec{b}$

$$= \left[\frac{(x\hat{i} + y\hat{j} + z\hat{k})(x\hat{i} + 0\hat{j} + z\hat{k})}{\left(\sqrt{(x)^2 + (0)^2 + (z)^2}\right)^2}\right](x\hat{i} + 0 \times \hat{j} + z\hat{k})$$

$$= \left[\frac{(x)(x) + (y)(0) + (z)(z)}{x^2 + z^2}\right](x\hat{i} + 0\hat{j} + z\hat{k})$$

$$= \left(\frac{x^2 + z^2}{x^2 + z^2}\right)(x\hat{i} + 0 \times \hat{j} + z\hat{k})$$

$$= x\hat{i} + 0 \times \hat{j} + z\hat{k}$$

So, coordinate of the projection of point $P(x, y, z)$ on XOZ-plane is given by $(x, 0, z)$.

Question 15

Write the coordinates of the projection of the point $P(2, -3, 5)$ on Y-axis.

Solution 15

We know that, coordinate of projections of the point $P(2, -3, 0)$

Since, any point on Y-axis is given by $(0, y, 0)$.

So the projection of the point P on $y-axis$ is $(0,-3,0)$

Question 16

Find the distance of the point $P(2, 3, 4)$ from the x-axis.

Solution 16

We know that, distance of point $P(x, y, z)$ from x-axis is given by $\sqrt{y^2 + z^2}$

So, distance of point $P(2, 3, 4)$ from x-axis = $\sqrt{(3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

= 5 units.

Required distance = 5 units.

Question 17

If a line has direction ratios proportional to 2, -1, -2, then what are its direction cosines?

Solution 17

direction ratios $\rightarrow (2,-1,-2)$

$$\sqrt{(2)^2+(-1)^2+(-2)^2} = \sqrt{9} = 3$$

direction cosines $\rightarrow \left(\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

Question 18

Write direction cosines of a line parallel to -axis.

Solution 18

$\frac{x}{0} = \frac{y}{0} = \frac{z-z_1}{1} \rightarrow$ equation of $z-axis$

equation of line parallel to z-axis will be

$$\frac{x-x_2}{0} = \frac{y-y_2}{0} = \frac{z-z_2}{1}$$

direction cosines will be $(0,0,1)$

Question 19

If a unit vector $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with θ.

Solution 19

The direction cosines of the vector are

$$l = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$n = \cos\theta$$

We know,

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{1}{2} - \frac{1}{4}$$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

Question 20

Write the distance of a point *P* (a, b, c) from x-axis.

Solution 20

Any point on the x − axis can be written as (a, 0, 0)

∴ Distance of the point from the x − axis

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(a - a)^2 + (b - 0)^2 + (c - 0)^2}$

$= \sqrt{0^2 + b^2 + c^2}$

$= \sqrt{b^2 + c^2}$

∴ Distance of the point from the x − axis is $\sqrt{b^2 + c^2}$.