Access answers to RD Sharma Solutions for Class 11 Maths Chapter 12 – Mathematical Induction

EXERCISE 12.1 PAGE NO: 12.3

1. If P (n) is the statement "n (n + 1) is even", then what is P (3)?

Solution:

Given:

P (n) = n (n + 1) is even.

So,

P (3) = 3 (3 + 1)

= 3 (4)

= 12

Hence, P (3) = 12, P (3) is also even.

2. If P (n) is the statement "n^3 + n is divisible by 3", prove that P (3) is true but P (4) is not true.

Solution:

Given:

P (n) = n^3 + n is divisible by 3

We have P (n) = n^3 + n

So,

P (3) = 3^3 + 3

= 27 + 3

= 30

P (3) = 30, So it is divisible by 3

Now, let's check with P (4)

P (4) = 4^3 + 4

= 64 + 4

= 68

P (4) = 68, so it is not divisible by 3

Hence, P (3) is true and P (4) is not true.

3. If P (n) is the statement "$2^n \geq 3n$", and if P (r) is true, prove that P (r + 1) is true.

Solution:

Given:

P (n) = "$2^n \geq 3n$" and p(r) is true.

We have, P (n) = $2^n \geq 3n$

Since, P (r) is true

So,

$2^r \geq 3r$

Now, let's multiply both sides by 2

$2 \times 2^r \geq 3r \times 2$

$2^{r+1} \geq 6r$

$2^{r+1} \geq 3r + 3r$ [since $3r > 3 = 3r + 3r \geq 3 + 3r$]

$\therefore 2^{r+1} \geq 3(r + 1)$

Hence, P (r + 1) is true.

4. If P (n) is the statement "$n^2 + n$" is even", and if P (r) is true, then P (r + 1) is true

Solution:

Given:

P (n) = $n^2 + n$ is even and P (r) is true, then $r^2 + r$ is even

Let us consider $r^2 + r = 2k$ … (i)

Now, $(r + 1)^2 + (r + 1)$

$r^2 + 1 + 2r + r + 1$

$(r^2 + r) + 2r + 2$

$2k + 2r + 2$ [from equation (i)]

$2(k + r + 1)$

2μ

∴ $(r + 1)^2 + (r + 1)$ is Even.

Hence, P (r + 1) is true.

5. Given an example of a statement P (n) such that it is true for all n ϵ N.

Solution:

Let us consider

P (n) = 1 + 2 + 3 + – – – – – – + n = n(n+1)/2

So,

P (n) is true for all natural numbers.

Hence, P (n) is true for all n ∈ N.

6. If P (n) is the statement "$n^2 – n + 41$ is prime", prove that P (1), P (2) and P (3) are true. Prove also that P (41) is not true.

Solution:

Given:

$P(n) = n^2 – n + 41$ is prime.

$P(n) = n^2 – n + 41$

P (1) = 1 – 1 + 41

= 41

P (1) is Prime.

Similarly,

$P(2) = 2^2 – 2 + 41$

= 4 – 2 + 41

= 43

P (2) is prime.

Similarly,

$P (3) = 3^2 – 3 + 41$

$= 9 – 3 + 41$

$= 47$

P (3) is prime

Now,

$P (41) = (41)^2 – 41 + 41$

$= 1681$

P (41) is not prime

Hence, P (1), P(2), P (3) are true but P (41) is not true.

EXERCISE 12.2 PAGE NO: 12.27

Prove the following by the principle of mathematical induction:

1. 1 + 2 + 3 + … + n = n (n +1)/2 i.e., the sum of the first n natural numbers is n (n + 1)/2.

Solution:

Let us consider $P (n) = 1 + 2 + 3 + \ldots + n = n (n +1)/2$

For, $n = 1$

LHS of $P (n) = 1$

RHS of $P (n) = 1 (1+1)/2 = 1$

So, LHS = RHS

Since, P (n) is true for $n = 1$

Let us consider P (n) be the true for $n = k$, so

$1 + 2 + 3 + \ldots + k = k (k+1)/2$ … (i)

Now,

$(1 + 2 + 3 + \ldots + k) + (k + 1) = k (k+1)/2 + (k+1)$

$= (k + 1) (k/2 + 1)$

$= [(k + 1) (k + 2)] / 2$

$= [(k+1) [(k+1) + 1]] / 2$

P (n) is true for $n = k + 1$

P (n) is true for all $n \in N$

So, by the principle of Mathematical Induction

Hence, P (n) = 1 + 2 + 3 + ….. + n = n (n +1)/2 is true for all n ∈ N.

2. $1^2 + 2^2 + 3^2 + \ldots + n^2 = [n (n+1) (2n+1)]/6$

Solution:

Let us consider P (n) = $1^2 + 2^2 + 3^2 + \ldots + n^2 = [n (n+1) (2n+1)]/6$

For, n = 1

P (1) = [1 (1+1) (2+1)]/6

1 = 1

P (n) is true for n = 1

Let P (n) is true for n = k, so

P (k): $1^2 + 2^2 + 3^2 + \ldots + k^2 = [k (k+1) (2k+1)]/6$

Let's check for P (n) = k + 1, so

P (k) = $1^2 + 2^2 + 3^2 + ------ + k^2 + (k + 1)^2 = [k + 1 (k+2) (2k+3)] /6$

$= 1^2 + 2^2 + 3^2 + ------ + k^2 + (k + 1)^2$

$= [k + 1 (k+2) (2k+3)] /6 + (k + 1)^2$

$= (k +1) [(2k^2 + k)/6 + (k + 1)/1]$

$= (k +1) [2k^2 + k + 6k + 6]/6$

$= (k +1) [2k^2 + 7k + 6]/6$

$= (k +1) [2k^2 + 4k + 3k + 6]/6$

$= (k +1) [2k(k + 2) + 3(k + 2)]/6$

$= [(k +1) (2k + 3) (k + 2)] / 6$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

3. $1 + 3 + 3^2 + \ldots + 3^{n-1} = (3^n – 1)/2$

Solution:

Let P (n) = $1 + 3 + 3^2 + ----- + 3^{n-1} = (3^n – 1)/2$

Now, For n = 1

P (1) = 1 = $(3^1 – 1)/2$ = 2/2 =1

P (n) is true for n = 1

Now, let's check for P (n) is true for n = k

P (k) = 1 + 3 + 3^2 + – – – – – + 3^{k-1} = $(3^k – 1)/2$ … (i)

Now, we have to show P (n) is true for n = k + 1

P (k + 1) = 1 + 3 + 3^2 + – – – – – + 3^k = $(3^{k+1} – 1)/2$

Then, {1 + 3 + 3^2 + – – – – – + 3^{k-1}} + 3^{k+1-1}

= (3k – 1)/2 + 3^k using equation (i)

= (3k – 1 + 2×3^k)/2

= (3×3 k – 1)/2

= $(3^{k+1} – 1)/2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

4. 1/1.2 + 1/2.3 + 1/3.4 + … + 1/n(n+1) = n/(n+1)

Solution:

Let P (n) = 1/1.2 + 1/2.3 + 1/3.4 + … + 1/n(n+1) = n/(n+1)

For, n = 1

P (n) = 1/1.2 = 1/1+1

1/2 = 1/2

P (n) is true for n = 1

Let's check for P (n) is true for n = k,

1/1.2 + 1/2.3 + 1/3.4 + … + 1/k(k+1) + k/(k+1) (k+2) = (k+1)/(k+2)

Then,

1/1.2 + 1/2.3 + 1/3.4 + … + 1/k(k+1) + k/(k+1) (k+2)

= 1/(k+1)/(k+2) + k/(k+1)

= 1/(k+1) [k(k+2)+1]/(k+2)

$= 1/(k+1)\ [k^2 + 2k + 1]/(k+2)$

$=1/(k+1)\ [(k+1)\ (k+1)]/(k+2)$

$= (k+1) / (k+2)$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

5. 1 + 3 + 5 + … + (2n – 1) = n^2 i.e., the sum of first n odd natural numbers is n^2.

Solution:

Let P (n): $1 + 3 + 5 + … + (2n – 1) = n^2$

Let us check P (n) is true for n = 1

P (1) = $1 =1^2$

1 = 1

P (n) is true for n = 1

Now, Let's check P (n) is true for n = k

P (k) = $1 + 3 + 5 + … + (2k – 1) = k^2$ … (i)

We have to show that

$1 + 3 + 5 + … + (2k – 1) + 2(k + 1) – 1 = (k + 1)^2$

Now,

$1 + 3 + 5 + … + (2k – 1) + 2(k + 1) – 1$

$= k^2 + (2k + 1)$

$= k^2 + 2k + 1$

$= (k + 1)^2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

6. 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3n-1) (3n+2) = n/(6n+4)

Solution:

Let P (n) = 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3n-1) (3n+2) = n/(6n+4)

Let us check P (n) is true for n = 1

P (1): 1/2.5 = 1/6.1+4 => 1/10 = 1/10

P (1) is true.

Now,

Let us check for P (k) is true, and have to prove that P (k + 1) is true.

P (k): 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3k-1) (3k+2) = k/(6k+4)

P (k +1): 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3k-1)(3k+2) + 1/(3k+3-1)(3k+3+2)

: k/(6k+4) + 1/(3k+2)(3k+5)

: [k(3k+5)+2] / [2(3k+2)(3k+5)]

: (k+1) / (6(k+1)+4)

P (k + 1) is true.

Hence proved by mathematical induction.

7. 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3n-2)(3n+1) = n/3n+1

Solution:

Let P (n) = 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3n-2)(3n+1) = n/3n+1

Let us check for n = 1,

P (1): 1/1.4 = 1/4

1/4 = 1/4

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3k-2)(3k+1) = k/3k+1 … (i)

So,

[1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3k-2)(3k+1)]+ 1/(3k+1)(3k+4) = k/(3k+1) + 1/(3k+1)(3k+4)

= 1/(3k+1) [k/1 + 1/(3k+4)]

= 1/(3k+1) [k(3k+4)+1]/(3k+4)

= 1/(3k+1) [$3k^2$ + 4k + 1]/ (3k+4)

= 1/(3k+1) [$3k^2$ + 3k+k+1]/(3k+4)

= [3k(k+1) + (k+1)] / [(3k+4) (3k+1)]

= [(3k+1)(k+1)] / [(3k+4) (3k+1)]

= (k+1) / (3k+4)

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

8. 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2n+1)(2n+3) = n/3(2n+3)

Solution:

Let P (n) = 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2n+1)(2n+3) = n/3(2n+3)

Let us check for n = 1,

P (1): 1/3.5 = 1/3(2.1+3)

: 1/15 = 1/15

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) = k/3(2k+3) … (i)

So,

1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) + 1/[2(k+1)+1][2(k+1)+3]

1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) + 1/(2k+3)(2k+5)

Now substituting the value of P (k) we get,

= k/3(2k+3) + 1/(2k+3)(2k+5)

= [k(2k+5)+3] / [3(2k+3)(2k+5)]

= (k+1) / [3(2(k+1)+3)]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

9. 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4n-1)(4n+3) = n/3(4n+3)

Solution:

Let P (n) = 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4n-1)(4n+3) = n/3(4n+3)

Let us check for n = 1,

P (1): 1/3.7 = 1/(4.1-1)(4+3)

: 1/21 = 1/21

P (n) is true for n =1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4k-1)(4k+3) = k/3(4k+3) …. (i)

So,

1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4k-1)(4k+3) + 1/(4k+3)(4k+7)

Substituting the value of P (k) we get,

= k/(4k+3) + 1/(4k+3)(4k+7)

= 1/(4k+3) [k(4k+7)+3] / [3(4k+7)]

= 1/(4k+3) [4k^2 + 7k +3]/ [3(4k+7)]

= 1/(4k+3) [4k^2 + 3k+4k+3] / [3(4k+7)]

= 1/(4k+3) [4k(k+1)+3(k+1)]/ [3(4k+7)]

= 1/(4k+3) [(4k+3)(k+1)] / [3(4k+7)]

= (k+1) / [3(4k+7)]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

10. 1.2 + 2.2^2 + 3.2^3 + … + n.2^n = (n–1) 2^{n+1} + 2

Solution:

Let P (n) = 1.2 + 2.2^2 + 3.2^3 + … + n.2^n = (n–1) 2^{n+1} + 2

Let us check for n = 1,

$P(1): 1.2 = 0.2^0 + 2$

$: 2 = 2$

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

$P(k): 1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k = (k-1)\, 2^{k+1} + 2$ …. (i)

So,

$\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\} + (k + 1)2^{k+1}$

Now, substituting the value of P (k) we get,

$= [(k - 1)2^{k+1} + 2] + (k + 1)2^{k+1}$ using equation (i)

$= (k - 1)2^{k+1} + 2 + (k + 1)2^{k+1}$

$= 2^{k+1}(k - 1 + k + 1) + 2$

$= 2^{k+1} \times 2k + 2$

$= k \times 2^{k+2} + 2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

11. 2 + 5 + 8 + 11 + … + (3n – 1) = 1/2 n (3n + 1)

Solution:

Let P (n) = 2 + 5 + 8 + 11 + … + (3n – 1) = 1/2 n (3n + 1)

Let us check for n = 1,

$P(1): 2 = 1/2 \times 1 \times 4$

$: 2 = 2$

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 2 + 5 + 8 + 11 + … + (3k – 1) = 1/2 k (3k + 1) … (i)

So,

2 + 5 + 8 + 11 + … + (3k – 1) + (3k + 2)

Now, substituting the value of P (k) we get,

$= 1/2 \times k (3k + 1) + (3k + 2)$ by using equation (i)

$= [3k^2 + k + 2 (3k + 2)] / 2$

$= [3k^2 + k + 6k + 2] / 2$

$= [3k^2 + 7k + 2] / 2$

$= [3k^2 + 4k + 3k + 2] / 2$

$= [3k (k+1) + 4(k+1)] / 2$

$= [(k+1) (3k+4)] /2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

12. 1.3 + 2.4 + 3.5 + … + n. (n+2) = 1/6 n (n+1) (2n+7)

Solution:

Let P (n): 1.3 + 2.4 + 3.5 + … + n. (n+2) = 1/6 n (n+1) (2n+7)

Let us check for n = 1,

P (1): $1.3 = 1/6 \times 1 \times 2 \times 9$

: 3 = 3

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.3 + 2.4 + 3.5 + … + k. (k+2) = 1/6 k (k+1) (2k+7) … (i)

So,

1.3 + 2.4 + 3.5 + … + k. (k+2) + (k+1) (k+3)

Now, substituting the value of P (k) we get,

$= 1/6\ k (k+1) (2k+7) + (k+1) (k+3)$ by using equation (i)

$= (k+1) [\{k(2k+7)/6\} + \{(k+3)/1\}]$

$= (k+1) [(2k^2 + 7k + 6k + 18)] / 6$

$= (k+1) [2k^2 + 13k + 18] / 6$

$= (k+1) [2k^2 + 9k + 4k + 18] / 6$

$= (k+1) [2k(k+2) + 9(k+2)] / 6$

= (k+1) [(2k+9) (k+2)] / 6

= 1/6 (k+1) (k+2) (2k+9)

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

13. 1.3 + 3.5 + 5.7 + … + (2n – 1) (2n + 1) = n($4n^2$ + 6n – 1)/3

Solution:

Let P (n): 1.3 + 3.5 + 5.7 + … + (2n – 1) (2n + 1) = n($4n^2$ + 6n – 1)/3

Let us check for n = 1,

P (1): (2.1 – 1) (2.1 + 1) = 1(4.1^2 + 6.1 -1)/3

: 1×3 = 1(4+6-1)/3

: 3 = 3

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) = k($4k^2$ + 6k – 1)/3 … (i)

So,

1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) + (2k + 1) (2k + 3)

Now, substituting the value of P (k) we get,

= k($4k^2$ + 6k – 1)/3 + (2k + 1) (2k + 3) by using equation (i)

= [k($4k^2$ + 6k-1) + 3 ($4k^2$ + 6k + 2k + 3)] / 3

= [$4k^3$ + $6k^2$ – k + $12k^2$ + 18k + 6k + 9] /3

= [$4k^3$ + $18k^2$ + 23k + 9] /3

= [$4k^3$ + $4k^2$ + $14k^2$ + 14k +9k + 9] /3

= [(k+1) ($4k^2$ + 8k +4 + 6k + 6 – 1)] / 3

= [(k+1) 4[$(k+1)^2$ + 6(k+1) -1]] /3

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

14. $1.2 + 2.3 + 3.4 + \ldots + n(n+1) = [n (n+1) (n+2)] / 3$

Solution:

Let P (n): $1.2 + 2.3 + 3.4 + \ldots + n(n+1) = [n (n+1) (n+2)] / 3$

Let us check for n = 1,

P (1): $1(1+1) = [1(1+1) (1+2)] /3$

: 2 = 2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $1.2 + 2.3 + 3.4 + \ldots + k(k+1) = [k (k+1) (k+2)] / 3$ … (i)

So,

$1.2 + 2.3 + 3.4 + \ldots + k(k+1) + (k+1) (k+2)$

Now, substituting the value of P (k) we get,

$= [k (k+1) (k+2)] / 3 + (k+1) (k+2)$ by using equation (i)

$= (k+2) (k+1) [k/2 + 1]$

$= [(k+1) (k+2) (k+3)] /3$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

15. $1/2 + 1/4 + 1/8 + \ldots + 1/2^n = 1 – 1/2^n$

Solution:

Let P (n): $1/2 + 1/4 + 1/8 + \ldots + 1/2^n = 1 – 1/2^n$

Let us check for n = 1,

P (1): $1/2^1 = 1 – 1/2^1$

: 1/2 = 1/2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

Let P (k): $1/2 + 1/4 + 1/8 + \ldots + 1/2^k = 1 – 1/2^k$ … (i)

So,

$1/2 + 1/4 + 1/8 + … + 1/2^k + 1/2^{k+1}$

Now, substituting the value of P (k) we get,

$= 1 – 1/2^k + 1/2^{k+1}$ by using equation (i)

$= 1 – ((2-1)/2^{k+1})$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

16. $1^2 + 3^2 + 5^2 + … + (2n – 1)^2 = 1/3\ n (4n^2 – 1)$

Solution:

Let P (n): $1^2 + 3^2 + 5^2 + … + (2n – 1)^2 = 1/3\ n (4n^2 – 1)$

Let us check for n = 1,

P (1): $(2.1 – 1)^2 = 1/3 × 1 × (4 – 1)$

: 1 = 1

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $1^2 + 3^2 + 5^2 + … + (2k – 1)^2 = 1/3\ k (4k^2 – 1)$ … (i)

So,

$1^2 + 3^2 + 5^2 + … + (2k – 1)^2 + (2k + 1)^2$

Now, substituting the value of P (k) we get,

$= 1/3\ k (4k^2 – 1) + (2k + 1)^2$ by using equation (i)

$= 1/3\ k (2k + 1) (2k – 1) + (2k + 1)^2$

$= (2k + 1) [\{k(2k-1)/3\} + (2k+1)]$

$= (2k + 1) [2k^2 – k + 3(2k+1)] / 3$

$= (2k + 1) [2k^2 – k + 6k + 3] / 3$

$= [(2k+1) 2k^2 + 5k + 3] /3$

$= [(2k+1) (2k(k+1)) + 3 (k+1)] /3$

$= [(2k+1) (2k+3) (k+1)] /3$

$= (k+2)/2 [4k^2 + 6k + 2k + 3]$

$= (k+2)/2\ [4k^2 + 8k – 1]$

$= (k+2)/2\ [4(k+1)^2 – 1]$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

17. $a + ar + ar^2 + … + ar^{n – 1} = a [(r^n – 1)/(r – 1)], r \neq 1$

Solution:

Let P (n): $a + ar + ar^2 + … + ar^{n – 1} = a [(r^n – 1)/(r – 1)]$

Let us check for n = 1,

P (1): $a = a (r^1 – 1)/(r-1)$

: a = a

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $a + ar + ar^2 + … + ar^{k – 1} = a [(r^k – 1)/(r – 1)]$ … (i)

So,

$a + ar + ar^2 + … + ar^{k – 1} + ar^k$

Now, substituting the value of P (k) we get,

$= a [(r^k – 1)/(r – 1)] + ar^k$ by using equation (i)

$= a[r^k – 1 + r^k(r-1)] / (r-1)$

$= a[r^k – 1 + r^{k+1} – r^{-k}] / (r-1)$

$= a[r^{k+1} – 1] / (r-1)$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

18. a + (a + d) + (a + 2d) + … + (a + (n-1)d) = n/2 [2a + (n-1)d]

Solution:

Let P (n): a + (a + d) + (a + 2d) + … + (a + (n-1)d) = n/2 [2a + (n-1)d]

Let us check for n = 1,

P (1): a = ½ [2a + (1-1)d]

: a = a

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): a + (a + d) + (a + 2d) + … + (a + (k-1)d) = k/2 [2a + (k-1)d] … (i)

So,

a + (a + d) + (a + 2d) + … + (a + (k-1)d) + (a + (k)d)

Now, substituting the value of P (k) we get,

= k/2 [2a + (k-1)d] + (a + kd) by using equation (i)

= [2ka + k(k-1)d + 2(a+kd)] / 2

= [2ka + k^2d – kd + 2a + 2kd] / 2

= [2ka + 2a + k^2d + kd] / 2

= [2a(k+1) + d(k^2 + k)] / 2

= (k+1)/2 [2a + kd]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

19. 5^{2n} – 1 is divisible by 24 for all n ϵ N

Solution:

Let P (n): 5^{2n} – 1 is divisible by 24

Let us check for n = 1,

P (1): 5^2 – 1 = 25 – 1 = 24

P (n) is true for n = 1. Where, P (n) is divisible by 24

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 5^{2k} – 1 is divisible by 24

: 5^{2k} – 1 = 24λ … (i)

We have to prove,

$5^{2k + 1}$ – 1 is divisible by 24

$5^{2(k + 1)} – 1 = 24μ$

So,

$= 5^{2(k + 1)} – 1$

$= 5^{2k}.5^2 – 1$

$= 25.5^{2k} – 1$

= 25.(24λ + 1) – 1 by using equation (1)

= 25.24λ + 24

= 24λ

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

20. 3^{2n} + 7 is divisible by 8 for all n ϵ N

Solution:

Let P (n): 3^{2n} + 7 is divisible by 8

Let us check for n = 1,

P (1): $3^2 + 7 = 9 + 7 = 16$

P (n) is true for n = 1. Where, P (n) is divisible by 8

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 3^{2k} + 7 is divisible by 8

: $3^{2k} + 7 = 8λ$

: $3^{2k} = 8λ – 7$ … (i)

We have to prove,

$3^{2(k + 1)}$ + 7 is divisible by 8

$3^{2k + 2} + 7 = 8μ$

So,

$= 3^{2(k + 1)} + 7$

$= 3^{2k}.3^2 + 7$

$= 9.3^{2k} + 7$

$= 9.(8\lambda – 7) + 7$ by using equation (i)

$= 72\lambda – 63 + 7$

$= 72\lambda – 56$

$= 8(9\lambda – 7)$

$= 8\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

21. $5^{2n + 2} – 24n – 25$ is divisible by 576 for all n ϵ N

Solution:

Let P (n): $5^{2n + 2} – 24n – 25$ is divisible by 576

Let us check for n = 1,

P (1): $5^{2.1+2} – 24.1 – 25$

: 625 – 49

: 576

P (n) is true for n = 1. Where, P (n) is divisible by 576

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $5^{2k + 2} – 24k – 25$ is divisible by 576

: $5^{2k + 2} – 24k – 25 = 576\lambda$ …. (i)

We have to prove,

$5^{2k + 4} – 24(k + 1) – 25$ is divisible by 576

$5^{(2k + 2) + 2} – 24(k + 1) – 25 = 576\mu$

So,

$= 5^{(2k + 2) + 2} – 24(k + 1) – 25$

$= 5^{(2k + 2)}.5^2 – 24k – 24– 25$

$= (576\lambda + 24k + 25)25 – 24k– 49$ by using equation (i)

$= 25. 576\lambda + 576k + 576$

$= 576(25\lambda + k + 1)$

$= 576\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all $n \in N$.

22. $3^{2n+2} - 8n - 9$ is divisible by 8 for all n ϵ N

Solution:

Let P (n): $3^{2n+2} - 8n - 9$ is divisible by 8

Let us check for n = 1,

P (1): $3^{2.1+2} - 8.1 - 9$

: 81 – 17

: 64

P (n) is true for n = 1. Where, P (n) is divisible by 8

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $3^{2k+2} - 8k - 9$ is divisible by 8

: $3^{2k+2} - 8k - 9 = 8\lambda$ … (i)

We have to prove,

$3^{2k+4} - 8(k+1) - 9$ is divisible by 8

$3^{(2k+2)+2} - 8(k+1) - 9 = 8\mu$

So,

$= 3^{2(k+1)}.3^2 - 8(k+1) - 9$

$= (8\lambda + 8k + 9)9 - 8k - 8 - 9$

$= 72\lambda + 72k + 81 - 8k - 17$ using equation (1)

$= 72\lambda + 64k + 64$

$= 8(9\lambda + 8k + 8)$

$= 8\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all $n \in N$.

23. $(ab)^n = a^n b^n$ for all n ϵ N

Solution:

Let P (n): $(ab)^n = a^n b^n$

Let us check for n = 1,

P (1): $(ab)^1 = a^1 b^1$

: ab = ab

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $(ab)^k = a^k b^k$ … (i)

We have to prove,

$(ab)^{k+1} = a^{k+1}.b^{k+1}$

So,

$= (ab)^{k+1}$

$= (ab)^k (ab)$

$= (a^k b^k) (ab)$ using equation (1)

$= (a^{k+1}) (b^{k+1})$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

24. n (n + 1) (n + 5) is a multiple of 3 for all n ϵ N.

Solution:

Let P (n): n (n + 1) (n + 5) is a multiple of 3

Let us check for n = 1,

P (1): 1 (1 + 1) (1 + 5)

: 2 × 6

: 12

P (n) is true for n = 1. Where, P (n) is a multiple of 3

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): k (k + 1) (k + 5) is a multiple of 3

: $k(k + 1) (k + 5) = 3\lambda$ … (i)

We have to prove,

$(k + 1)[(k + 1) + 1][(k + 1) + 5]$ is a multiple of 3

$(k + 1)[(k + 1) + 1][(k + 1) + 5] = 3\mu$

So,

$= (k + 1) [(k + 1) + 1] [(k + 1) + 5]$

$= (k + 1) (k + 2) [(k + 1) + 5]$

$= [k (k + 1) + 2(k + 1)] [(k + 5) + 1]$

$= k (k + 1) (k + 5) + k(k + 1) + 2(k + 1) (k + 5) + 2(k + 1)$

$= 3\lambda + k^2 + k + 2(k^2 + 6k + 5) + 2k + 2$

$= 3\lambda + k^2 + k + 2k^2 + 12k + 10 + 2k + 2$

$= 3\lambda + 3k^2 + 15k + 12$

$= 3(\lambda + k^2 + 5k + 4)$

$= 3\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all $n \in N$.

25. $7^{2n} + 2^{3n-3}. 3n - 1$ is divisible by 25 for all n ϵ N

Solution:

Let P (n): $7^{2n} + 2^{3n-3}. 3n - 1$ is divisible by 25

Let us check for n = 1,

P (1): $7^2 + 2^0.3^0$

: $49 + 1$

: 50

P (n) is true for n = 1. Where, P (n) is divisible by 25

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $7^{2k} + 2^{3k-3}. 3k - 1$ is divisible by 25

: $7^{2k} + 2^{3k-3}. 3^{k-1} = 25\lambda$ … (i)

We have to prove that:

$7^{2k + 1} + 2^{3k}. 3^k$ is divisible by 25

$7^{2k + 2} + 2^{3k}. 3^k = 25\mu$

So,

$= 7^{2(k + 1)} + 2^{3k}. 3^k$

$= 7^{2k}.7^1 + 2^{3k}. 3^k$

$= (25\lambda – 2^{3k – 3}. 3^{k – 1}) 49 + 2^{3k}. 3k$ by using equation (i)

$= 25\lambda. 49 – 2^{3k}/8. 3^k/3. 49 + 2^{3k}. 3^k$

$= 24×25×49\lambda – 2^{3k} . 3^k . 49 + 24 . 2^{3k}.3^k$

$= 24×25×49\lambda – 25 . 2^{3k}. 3^k$

$= 25(24 . 49\lambda – 2^{3k}. 3^k)$

$= 25\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.