Access answers to RD Sharma Solutions for Class 11 Maths Chapter 14 – Quadratic Equations

EXERCISE 14.1 PAGE NO: 14.5

Solve the following quadratic equations by factorization method only:

1. $x^2 + 1 = 0$

Solution:

Given: $x^2 + 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$x^2 - i^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(x + i)(x - i) = 0$

$x + i = 0$ or $x - i = 0$

$x = -i$ or $x = i$

∴ The roots of the given equation are i, -i

2. $9x^2 + 4 = 0$

Solution:

Given: $9x^2 + 4 = 0$

$9x^2 + 4 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

So,

$9x^2 + 4(-i^2) = 0$

$9x^2 - 4i^2 = 0$

$(3x)^2 - (2i)^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b)(a - b)$]
$(3x + 2i)(3x - 2i) = 0$

$3x + 2i = 0$ or $3x - 2i = 0$

$3x = -2i$ or $3x = 2i$

$x = -2i/3$ or $x = 2i/3$

∴ The roots of the given equation are 2i/3, -2i/3

3. $x^2 + 2x + 5 = 0$

Solution:

Given: $x^2 + 2x + 5 = 0$

$x^2 + 2x + 1 + 4 = 0$

$x^2 + 2(x)(1) + 1^2 + 4 = 0$

$(x + 1)^2 + 4 = 0$ [since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x + 1)^2 + 4 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x + 1)^2 + 4(-i^2) = 0$

$(x + 1)^2 - 4i^2 = 0$

$(x + 1)^2 – (2i)^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b) (a – b)$]
$(x + 1 + 2i)(x + 1 – 2i) = 0$

$x + 1 + 2i = 0$ or $x + 1 – 2i = 0$

$x = –1 – 2i$ or $x = –1 + 2i$

∴ The roots of the given equation are -1+2i, -1-2i

4. $4x^2 – 12x + 25 = 0$

Solution:

Given: $4x^2 – 12x + 25 = 0$

$4x^2 – 12x + 9 + 16 = 0$

$(2x)^2 – 2(2x)(3) + 3^2 + 16 = 0$

$(2x – 3)^2 + 16 = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(2x – 3)^2 + 16 \times 1 = 0$

We know, $i^2 = –1 \Rightarrow 1 = –i^2$

By substituting $1 = –i^2$ in the above equation, we get

$(2x – 3)^2 + 16(–i^2) = 0$

$(2x – 3)^2 – 16i^2 = 0$

$(2x – 3)^2 – (4i)^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b) (a – b)$]
$(2x – 3 + 4i) (2x – 3 – 4i) = 0$

$2x – 3 + 4i = 0$ or $2x – 3 – 4i = 0$

$2x = 3 – 4i$ or $2x = 3 + 4i$

$x = 3/2 – 2i$ or $x = 3/2 + 2i$

∴ The roots of the given equation are 3/2 + 2i, 3/2 – 2i

5. $x^2 + x + 1 = 0$

Solution:

Given: $x^2 + x + 1 = 0$

$x^2 + x + ¼ + ¾ = 0$

$x^2 + 2 (x) (1/2) + (1/2)^2 + ¾ = 0$

$(x + 1/2)^2 + ¾ = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x + 1/2)^2 + ¾ \times 1 = 0$

We know, $i^2 = –1 \Rightarrow 1 = –i^2$

By substituting $1 = –i^2$ in the above equation, we get

$(x + ½)^2 + ¾ (-1)^2 = 0$

$(x + ½)^2 + ¾ i^2 = 0$

$(x + ½)^2 – (\sqrt{3}i/2)^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b) (a – b)$]
$(x + ½ + \sqrt{3}i/2) (x + ½ – \sqrt{3}i/2) = 0$

$(x + ½ + \sqrt{3}i/2) = 0$ or $(x + ½ – \sqrt{3}i/2) = 0$

$x = -1/2 – \sqrt{3}i/2$ or $x = -1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $-1/2 + \sqrt{3}i/2$, $-1/2 – \sqrt{3}i/2$

6. $4x^2 + 1 = 0$

Solution:

Given: $4x^2 + 1 = 0$

We know, $i^2 = –1 \Rightarrow 1 = –i^2$

By substituting $1 = –i^2$ in the above equation, we get

$4x^2 – i^2 = 0$

$(2x)^2 – i^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b)(a – b)$]
$(2x + i)(2x – i) = 0$

$2x + i = 0$ or $2x – i = 0$

$2x = –i$ or $2x = i$

$x = -i/2$ or $x = i/2$

∴ The roots of the given equation are $i/2$, $-i/2$

7. $x^2 – 4x + 7 = 0$

Solution:

Given: $x^2 – 4x + 7 = 0$

$x^2 – 4x + 4 + 3 = 0$

$x^2 – 2(x)(2) + 2^2 + 3 = 0$

$(x – 2)^2 + 3 = 0$ [Since, $(a – b)^2 = a^2 – 2ab + b^2$]

$(x – 2)^2 + 3 \times 1 = 0$

We know, $i^2 = –1 \Rightarrow 1 = –i^2$

By substituting $1 = –i^2$ in the above equation, we get

$(x – 2)^2 + 3(–i^2) = 0$

$(x – 2)^2 – 3i^2 = 0$

$(x – 2)^2 – (\sqrt{3}i)^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b)(a – b)$]
$(x – 2 + \sqrt{3}i)(x – 2 – \sqrt{3}i) = 0$

$(x – 2 + \sqrt{3}i) = 0$ or $(x – 2 – \sqrt{3}i) = 0$

$x = 2 – \sqrt{3}i$ or $x = 2 + \sqrt{3}i$

$x = 2 \pm \sqrt{3}i$

∴ The roots of the given equation are $2 \pm \sqrt{3}i$

8. $x^2 + 2x + 2 = 0$

Solution:

Given: $x^2 + 2x + 2 = 0$

$x^2 + 2x + 1 + 1 = 0$

$x^2 + 2(x)(1) + 1^2 + 1 = 0$

$(x + 1)^2 + 1 = 0$ [∵ $(a + b)^2 = a^2 + 2ab + b^2$]

We know, $i^2 = –1 \Rightarrow 1 = –i^2$

By substituting $1 = –i^2$ in the above equation, we get

$(x + 1)^2 + (–i^2) = 0$

$(x + 1)^2 – i^2 = 0$

$(x + 1)^2 – (i)^2 = 0$

[By using the formula, $a^2 – b^2 = (a + b)(a – b)$]
$(x + 1 + i)(x + 1 – i) = 0$

$x + 1 + i = 0$ or $x + 1 – i = 0$

$x = –1 – i$ or $x = –1 + i$

$x = -1 \pm i$

∴ The roots of the given equation are $-1 \pm i$

9. $5x^2 – 6x + 2 = 0$

Solution:

Given: $5x^2 – 6x + 2 = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm\sqrt{(b^2 – 4ac)})/2a$

Here, $a = 5$, $b = -6$, $c = 2$

So,

$x = (-(-6) \pm\sqrt{(-6^2 – 4 (5)(2))})/ 2(5)$

$= (6 \pm \sqrt{(36-40)})/10$

$= (6 \pm \sqrt{(-4)})/10$

$= (6 \pm \sqrt{4(-1)})/10$

We have $i^2 = –1$

By substituting $–1 = i^2$ in the above equation, we get

$x = (6 \pm \sqrt{4i^2})/10$

$= (6 \pm 2i)/10$

$= 2(3\pm i)/10$

$= (3\pm i)/5$

$x = 3/5 \pm i/5$

∴ The roots of the given equation are $3/5 \pm i/5$

10. $21x^2 + 9x + 1 = 0$

Solution:

Given: $21x^2 + 9x + 1 = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm\sqrt{(b^2 – 4ac)})/2a$

Here, $a = 21$, $b = 9$, $c = 1$

So,

$x = (-9 \pm\sqrt{(9^2 – 4 (21)(1))})/ 2(21)$

$= (-9 \pm \sqrt{(81-84)})/42$

$= (-9 \pm \sqrt{(-3)})/42$

$= (-9 \pm \sqrt{3(-1)})/42$

We have $i^2 = –1$

By substituting $–1 = i^2$ in the above equation, we get

$x = (-9 \pm \sqrt{3i^2})/42$

$= (-9 \pm \sqrt{(\sqrt{3}i)^2}/42$

$= (-9 \pm \sqrt{3}i)/42$

$= -9/42 \pm \sqrt{3}i/42$

$= -3/14 \pm \sqrt{3}i/42$

∴ The roots of the given equation are $-3/14 \pm \sqrt{3}i/42$

11. $x^2 – x + 1 = 0$

Solution:

Given: $x^2 - x + 1 = 0$

$x^2 - x + 1/4 + 3/4 = 0$

$x^2 - 2 (x) (1/2) + (1/2)^2 + 3/4 = 0$

$(x - 1/2)^2 + 3/4 = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x - 1/2)^2 + 3/4 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x - ½)^2 + ¾ (-1)^2 = 0$

$(x - ½)^2 + ¾ (-i)^2 = 0$

$(x - ½)^2 - (\sqrt{3}i/2)^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b) (a - b)$]
$(x - ½ + \sqrt{3}i/2) (x - ½ - \sqrt{3}i/2) = 0$

$(x - ½ + \sqrt{3}i/2) = 0$ or $(x - ½ - \sqrt{3}i/2) = 0$

$x = 1/2 - \sqrt{3}i/2$ or $x = 1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $1/2 + \sqrt{3}i/2$, $1/2 - \sqrt{3}i/2$

12. $x^2 + x + 1 = 0$

Solution:

Given: $x^2 + x + 1 = 0$

$x^2 + x + 1/4 + 3/4 = 0$

$x^2 + 2 (x) (1/2) + (1/2)^2 + 3/4 = 0$

$(x + 1/2)^2 + 3/4 = 0$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$(x + 1/2)^2 + 3/4 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$(x + ½)^2 + ¾ (-1)^2 = 0$

$(x + ½)^2 + ¾ i^2 = 0$

$(x + ½)^2 - (\sqrt{3}i/2)^2 = 0$

[By using the formula, $a^2 - b^2 = (a + b) (a - b)$]
$(x + ½ + \sqrt{3}i/2) (x + ½ - \sqrt{3}i/2) = 0$

$(x + ½ + \sqrt{3}i/2) = 0$ or $(x + ½ - \sqrt{3}i/2) = 0$

$x = -1/2 - \sqrt{3}i/2$ or $x = -1/2 + \sqrt{3}i/2$

∴ The roots of the given equation are $-1/2 + \sqrt{3}i/2$, $-1/2 - \sqrt{3}i/2$

13. $17x^2 - 8x + 1 = 0$

Solution:

Given: $17x^2 - 8x + 1 = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm\sqrt{(b^2 - 4ac)})/2a$

Here, $a = 17$, $b = -8$, $c = 1$

So,

$x = (-(-8) \pm\sqrt{(-8^2 - 4 (17)(1))})/ 2(17)$

$= (8 \pm \sqrt{(64-68)})/34$

$= (8 \pm \sqrt{(-4)})/34$

$= (8 \pm \sqrt{4(-1)})/34$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$x = (8 \pm \sqrt{(2i)^2})/34$

$= (8 \pm 2i)/34$

$= 2(4 \pm i)/34$

$= (4 \pm i)/17$

$x = 4/17 \pm i/17$

∴ The roots of the given equation are $4/17 \pm i/17$

EXERCISE 14.2 PAGE NO: 14.13

1. Solving the following quadratic equations by factorization method:

(i) $x^2 + 10ix - 21 = 0$

(ii) $x^2 + (1 - 2i)x - 2i = 0$

(iii) $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

(iv) $6x^2 - 17ix - 12 = 0$

Solution:

(i) $x^2 + 10ix - 21 = 0$

Given: $x^2 + 10ix - 21 = 0$

$x^2 + 10ix - 21 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$x^2 + 10ix - 21(-i^2) = 0$

$x^2 + 10ix + 21i^2 = 0$

$x^2 + 3ix + 7ix + 21i^2 = 0$

$x(x + 3i) + 7i(x + 3i) = 0$

$(x + 3i)(x + 7i) = 0$

$x + 3i = 0$ or $x + 7i = 0$

$x = -3i$ or $-7i$

∴ The roots of the given equation are $-3i, -7i$

(ii) $x^2 + (1 - 2i)x - 2i = 0$

Given: $x^2 + (1 - 2i)x - 2i = 0$

$x^2 + x - 2ix - 2i = 0$

$x(x + 1) - 2i(x + 1) = 0$

$(x + 1)(x - 2i) = 0$

$x + 1 = 0$ or $x - 2i = 0$

$x = -1$ or $2i$

∴ The roots of the given equation are $-1, 2i$

(iii) $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

Given: $x^2 - (2\sqrt{3} + 3i)x + 6\sqrt{3}i = 0$

$x^2 - (2\sqrt{3}x + 3ix) + 6\sqrt{3}i = 0$

$x^2 - 2\sqrt{3}x - 3ix + 6\sqrt{3}i = 0$

$x(x - 2\sqrt{3}) - 3i(x - 2\sqrt{3}) = 0$

$(x - 2\sqrt{3})(x - 3i) = 0$

$(x - 2\sqrt{3}) = 0$ or $(x - 3i) = 0$

$x = 2\sqrt{3}$ or $x = 3i$

∴ The roots of the given equation are $2\sqrt{3}$, $3i$

(iv) $6x^2 - 17ix - 12 = 0$

Given: $6x^2 - 17ix - 12 = 0$

$6x^2 - 17ix - 12 \times 1 = 0$

We know, $i^2 = -1 \Rightarrow 1 = -i^2$

By substituting $1 = -i^2$ in the above equation, we get

$6x^2 - 17ix - 12(-i^2) = 0$

$6x^2 - 17ix + 12i^2 = 0$

$6x^2 - 9ix - 8ix + 12i^2 = 0$

$3x(2x - 3i) - 4i(2x - 3i) = 0$

$(2x - 3i)(3x - 4i) = 0$

$2x - 3i = 0$ or $3x - 4i = 0$

$2x = 3i$ or $3x = 4i$

$x = 3i/2$ or $x = 4i/3$

∴ The roots of the given equation are $3i/2$, $4i/3$

2. Solve the following quadratic equations:

(i) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

(ii) $x^2 - (5 - i)x + (18 + i) = 0$

(iii) $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

(iv) $x^2 - (2 + i)x - (1 - 7i) = 0$

(v) $ix^2 - 4x - 4i = 0$

(vi) $x^2 + 4ix - 4 = 0$

(vii) $2x^2 + \sqrt{15}ix - i = 0$

(viii) $x^2 - x + (1 + i) = 0$

(ix) $ix^2 - x + 12i = 0$

(x) $x^2 - (3\sqrt{2} - 2i)x - \sqrt{2}i = 0$

(xi) $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

(xii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Solution:

(i) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Given: $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

$x^2 - (3\sqrt{2}x + 2ix) + 6\sqrt{2}i = 0$

$x^2 - 3\sqrt{2}x - 2ix + 6\sqrt{2}i = 0$

$x(x - 3\sqrt{2}) - 2i(x - 3\sqrt{2}) = 0$

$(x - 3\sqrt{2})(x - 2i) = 0$

$(x - 3\sqrt{2}) = 0$ or $(x - 2i) = 0$

$x = 3\sqrt{2}$ or $x = 2i$

∴ The roots of the given equation are 3√2, 2i

(ii) $x^2 – (5 – i) x + (18 + i) = 0$

Given: $x^2 – (5 – i) x + (18 + i) = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm\sqrt{(b^2 – 4ac)})/2a$

Here, a = 1, b = -(5-i), c = (18+i)

So,

$$x = \frac{-(-(5-i)) \pm \sqrt{(-(5-i))^2 - 4(1)(18+i)}}{2(1)}$$

$$= \frac{(5-i) \pm \sqrt{(5-i)^2 - 4(18+i)}}{2}$$

$$= \frac{(5-i) \pm \sqrt{25 - 10i + i^2 - 72 - 4i}}{2}$$

$$= \frac{(5-i) \pm \sqrt{-47 - 14i + i^2}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{(5-i) \pm \sqrt{-47 - 14i + (-1)}}{2}$$

$$= \frac{(5-i) \pm \sqrt{-48 - 14i}}{2}$$

$$= \frac{(5-i) \pm \sqrt{(-1)(48 + 14i)}}{2}$$

$$= \frac{(5-i) \pm \sqrt{i^2(48 + 14i)}}{2}$$

$$= \frac{(5-i) \pm i\sqrt{48 + 14i}}{2}$$

We can write 48 + 14i = 49 – 1 + 14i

So,

$48 + 14i = 49 + i^2 + 14i$ [∵ $i^2 = –1$]

$= 7^2 + i^2 + 2(7)(i)$

$= (7 + i)^2$ [Since, $(a + b)^2 = a^2 + b^2 + 2ab$]

By using the result $48 + 14i = (7 + i)^2$, we get

$$= \frac{(5-\mathrm{i}) \pm \mathrm{i}\sqrt{(7+\mathrm{i})^2}}{2}$$

$$= \frac{(5-\mathrm{i}) \pm \mathrm{i}(7+\mathrm{i})}{2}$$

$$= \frac{(5-\mathrm{i}) + \mathrm{i}(7+\mathrm{i})}{2} \text{ or } \frac{(5-\mathrm{i}) - \mathrm{i}(7+\mathrm{i})}{2}$$

$$= \frac{5-\mathrm{i}+7\mathrm{i}+\mathrm{i}^2}{2} \text{ or } \frac{5-\mathrm{i}-7\mathrm{i}-\mathrm{i}^2}{2}$$

$$= \frac{5+6\mathrm{i}+(-1)}{2} \text{ or } \frac{5-8\mathrm{i}-(-1)}{2}$$

$$= \frac{5+6\mathrm{i}-1}{2} \text{ or } \frac{5-8\mathrm{i}+1}{2}$$

$$= \frac{4+6\mathrm{i}}{2} \text{ or } \frac{6-8\mathrm{i}}{2}$$

$$= \frac{2(2+3\mathrm{i})}{2} \text{ or } \frac{2(3-4\mathrm{i})}{2}$$

x = 2 + 3i or 3 – 4i

∴ The roots of the given equation are 3 – 4i, 2 + 3i

(iii) $(2 + i)x^2 – (5- i)x + 2 (1 – i) = 0$

Given: $(2 + i)x^2 – (5- i)x + 2 (1 – i) = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm\sqrt{(b^2 – 4ac)})/2a$

Here, a = (2+i), b = -(5-i), c = 2(1-i)

So,

$$x = \frac{-\left(-(5-\mathrm{i})\right) \pm \sqrt{\left(-(5-\mathrm{i})\right)^2 - 4(2+\mathrm{i})\left(2(1-\mathrm{i})\right)}}{2(2+\mathrm{i})}$$

$$= \frac{(5-\mathrm{i}) \pm \sqrt{(5-\mathrm{i})^2 - 8(2+\mathrm{i})(1-\mathrm{i})}}{2(2+\mathrm{i})}$$

$$= \frac{(5-\mathrm{i}) \pm \sqrt{25-10\mathrm{i}+\mathrm{i}^2 - 8(2-2\mathrm{i}+\mathrm{i}-\mathrm{i}^2)}}{2(2+\mathrm{i})}$$

We have $i^2 = –1$

By substituting $–1 = i^2$ in the above equation, we get

$$= \frac{(5-i) \pm \sqrt{25 - 10i + (-1) - 8\big(2 - i - (-1)\big)}}{2(2+i)}$$

$$= \frac{(5-i) \pm \sqrt{24 - 10i - 8(3-i)}}{2(2+i)}$$

$$= \frac{(5-i) \pm \sqrt{24 - 10i - 24 + 8i}}{2(2+i)}$$

$$= \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)}$$

We can write $-2i = -2i + 1 - 1$

$-2i = -2i + 1 + i^2$ [Since, $i^2 = -1$]

$= 1 - 2i + i^2$

$= 1^2 - 2(1)(i) + i^2$

$= (1 - i)^2$ [By using the formula, $(a - b)^2 = a^2 - 2ab + b^2$]

By using the result $-2i = (1 - i)^2$, we get

$$x = \frac{(5-i) \pm \sqrt{(1-i)^2}}{2(2+i)}$$

$$= \frac{(5-i) \pm (1-i)}{2(2+i)}$$

$$= \frac{(5-i) + (1-i)}{2(2+i)} \text{ or } \frac{(5-i) - (1-i)}{2(2+i)}$$

$$= \frac{5 - i + 1 - i}{2(2+i)} \text{ or } \frac{5 - i - 1 + i}{2(2+i)}$$

$$= \frac{6 - 2i}{2(2+i)} \text{ or } \frac{4}{2(2+i)}$$

$$= \frac{3 - i}{2 + i} \text{ or } \frac{2}{2 + i}$$

Let us multiply and divide by (2 - i), we get

$$= \frac{3 - i}{2 + i} \times \frac{2 - i}{2 - i} \text{ or } \frac{2}{2 + i} \times \frac{2 - i}{2 - i}$$

$$= \frac{(3-i)(2-i)}{(2+i)(2-i)} \text{ or } \frac{2(2-i)}{(2+i)(2-i)}$$

$$= \frac{6 - 3i - 2i + i^2}{2^2 - i^2} \text{ or } \frac{4 - 2i}{2^2 - i^2}$$

$$= \frac{6-5i+(-1)}{4-(-1)} \text{ or } \frac{4-2i}{4-(-1)}$$

$$= \frac{5-5i}{4+1} \text{ or } \frac{4-2i}{4+1}$$

$$= \frac{5(1-i)}{5} \text{ or } \frac{4-2i}{5}$$

x = (1 – i) or 4/5 – 2i/5

∴ The roots of the given equation are (1 – i), 4/5 – 2i/5

(iv) $x^2 - (2 + i)x - (1 - 7i) = 0$

Given: $x^2 - (2 + i)x - (1 - 7i) = 0$

We shall apply discriminant rule,

Where, x = (-b ±√(b^2 – 4ac))/2a

Here, a = 1, b = -(2+i), c = -(1-7i)

So,

$$x = \frac{-(-(2+i)) \pm \sqrt{(-(2+i))^2 - 4(1)(-(1-7i))}}{2(1)}$$

$$= \frac{(2+i) \pm \sqrt{(2+i)^2 + 4(1-7i)}}{2}$$

$$= \frac{(2+i) \pm \sqrt{4+4i+i^2+4-28i}}{2}$$

$$= \frac{(2+i) \pm \sqrt{8-24i+i^2}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{(2+i) \pm \sqrt{8-24i+(-1)}}{2}$$

$$= \frac{(2+i) \pm \sqrt{7-24i}}{2}$$

We can write $7 - 24i = 16 - 9 - 24i$

$7 - 24i = 16 + 9(-1) - 24i$

$= 16 + 9i^2 - 24i$ [$\because i^2 = -1$]

$= 4^2 + (3i)^2 - 2(4)(3i)$

$= (4 - 3i)^2$ [$\because (a - b)^2 = a^2 - b^2 + 2ab$]

By using the result $7 - 24i = (4 - 3i)^2$, we get

$$x = \frac{(2+i) \pm \sqrt{(4-3i)^2}}{2}$$

We can write 7 – 24i = 16 – 9 – 24i

7 – 24i = 16 + 9(–1) – 24i

= 16 + 9i^2 – 24i [∵ i^2 = –1]

= 4^2 + $(3i)^2$ – 2(4) (3i)

= $(4 – 3i)^2$ [∵ $(a – b)^2 = a^2 – b^2 + 2ab$]

By using the result 7 – 24i = $(4 – 3i)^2$, we get

$$x = \frac{(2+i) \pm \sqrt{(4-3i)^2}}{2}$$

$$= \frac{(2+i) \pm (4-3i)}{2}$$

$$= \frac{(2+i) + (4-3i)}{2} \text{ or } \frac{(2+i) - (4-3i)}{2}$$

$$= \frac{2+i+4-3i}{2} \text{ or } \frac{2+i-4+3i}{2}$$

$$= \frac{6-2i}{2} \text{ or } \frac{-2+4i}{2}$$

$$= \frac{2(3-i)}{2} \text{ or } \frac{2(-1+2i)}{2}$$

x = 3 – i or -1 + 2i

∴ The roots of the given equation are (-1 + 2i), (3 – i)

(v) ix^2 – 4x – 4i = 0

Given: ix^2 – 4x – 4i = 0

ix^2 + 4x(–1) – 4i = 0 [We know, i^2 = –1]

So by substituting –1 = i^2 in the above equation, we get

ix^2 + 4xi^2 – 4i = 0

i(x^2 + 4ix – 4) = 0

x^2 + 4ix – 4 = 0

x^2 + 4ix + 4(–1) = 0

x^2 + 4ix + 4i^2 = 0 [Since, i^2 = –1]

x^2 + 2ix + 2ix + 4i^2 = 0

x(x + 2i) + 2i(x + 2i) = 0

(x + 2i) (x + 2i) = 0

$(x + 2i)^2$ = 0

x + 2i = 0

x = –2i, -2i

∴ The roots of the given equation are –2i, –2i

(vi) x^2 + 4ix – 4 = 0

Given: x^2 + 4ix – 4 = 0

x^2 + 4ix + 4(–1) = 0 [We know, i^2 = –1]

So by substituting –1 = i^2 in the above equation, we get

x^2 + 4ix + 4i^2 = 0

$x^2 + 2ix + 2ix + 4i^2 = 0$

$x(x + 2i) + 2i(x + 2i) = 0$

$(x + 2i)(x + 2i) = 0$

$(x + 2i)^2 = 0$

$x + 2i = 0$

$x = -2i, -2i$

∴ The roots of the given equation are $-2i, -2i$

(vii) $2x^2 + \sqrt{15}ix - i = 0$

Given: $2x^2 + \sqrt{15}ix - i = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$

Here, $a = 2$, $b = \sqrt{15}i$, $c = -i$

So,

$$x = \frac{-(\sqrt{15}i) \pm \sqrt{(\sqrt{15}i)^2 - 4(2)(-i)}}{2(2)}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{15i^2 + 8i}}{4}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{-\sqrt{15}i \pm \sqrt{15(-1) + 8i}}{4}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{8i - 15}}{4}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{(-1)(15 - 8i)}}{4}$$

$$= \frac{-\sqrt{15}i \pm \sqrt{i^2(15 - 8i)}}{4}$$

$$= \frac{-\sqrt{15}i \pm i\sqrt{15 - 8i}}{4}$$

We can write $15 - 8i = 16 - 1 - 8i$

$15 - 8i = 16 + (-1) - 8i$

$= 16 + i^2 - 8i$ [$\because i^2 = -1$]

$= 4^2 + (i)^2 - 2(4)(i)$

$= (4 - i)^2$ [Since, $(a - b)^2 = a^2 - b^2 + 2ab$]

By using the result $15 - 8i = (4 - i)^2$, we get

$$x = \frac{-\sqrt{15}i \pm i\sqrt{(4-i)^2}}{4}$$

$$= \frac{-\sqrt{15}i \pm i(4-i)}{4}$$

$$= \frac{-\sqrt{15}i + i(4-i)}{4} \text{ or } \frac{-\sqrt{15}i - i(4-i)}{4}$$

$$= \frac{-\sqrt{15}i + 4i - i^2}{4} \text{ or } \frac{-\sqrt{15}i - 4i + i^2}{4}$$

$$= \frac{-\sqrt{15}i+4i-(-1)}{4} \text{ or } \frac{-\sqrt{15}i-4i+(-1)}{4}$$

$$= \frac{-\sqrt{15}i + 4i + 1}{4} \text{ or } \frac{-\sqrt{15}i - 4i - 1}{4}$$

$$= \frac{1 + \left(4 - \sqrt{15}\right)i}{4} \text{ or } \frac{-1 - \left(4 + \sqrt{15}\right)i}{4}$$

$$x = \frac{1}{4} + \left(\frac{4 - \sqrt{15}}{4}\right)i \text{ or } -\frac{1}{4} - \left(\frac{4 + \sqrt{15}}{4}\right)i$$

∴ The roots of the given equation are [1+ (4 – √15)i/4] , [-1 -(4 + √15)i/4]

(viii) $x^2 - x + (1 + i) = 0$

Given: $x^2 - x + (1 + i) = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$

Here, a = 1, b = -1, c = (1+i)

So,

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1+i)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 - 4(1+i)}}{2}$$

$$= \frac{1 \pm \sqrt{1 - 4 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{1 - 4 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{-3 - 4i}}{2}$$

$$= \frac{1 \pm \sqrt{(-1)(3 + 4i)}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{1 \pm \sqrt{i^2(3 + 4i)}}{2}$$

$$= \frac{1 \pm i\sqrt{3 + 4i}}{2}$$

We can write 3 + 4i = 4 – 1 + 4i

$3 + 4i = 4 + i^2 + 4i$ [$\because i^2 = -1$]

$= 2^2 + i^2 + 2(2)(i)$

$= (2 + i)^2$ [Since, $(a + b)^2 = a^2 + b^2 + 2ab$]

By using the result $3 + 4i = (2 + i)^2$, we get

$$x = \frac{1 \pm i\sqrt{(2 - i)^2}}{2}$$

$$= \frac{1 \pm i(2 + i)}{2}$$

$$= \frac{1 + i(2 + i)}{2} \text{ or } \frac{1 - i(2 + i)}{2}$$

$$= \frac{1 + 2i + i^2}{2} \text{ or } \frac{1 - 2i - i^2}{2}$$

$$= \frac{1+2i+(-1)}{2} \text{ or } \frac{1-2i-(-1)}{2}$$

$$= \frac{1 + 2i - 1}{2} \text{ or } \frac{1 - 2i + 1}{2}$$

x = 2i/2 or (2 – 2i)/2

x = i or 2(1-i)/2

x = i or (1 – i)

∴ The roots of the given equation are (1-i), i

(ix) $ix^2 – x + 12i = 0$

Given: $ix^2 – x + 12i = 0$

$ix^2 + x(–1) + 12i = 0$ [We know, $i^2 = –1$]

so by substituting $–1 = i^2$ in the above equation, we get

$ix^2 + xi^2 + 12i = 0$

$i(x^2 + ix + 12) = 0$

$x^2 + ix + 12 = 0$

$x^2 + ix – 12(–1) = 0$

$x^2 + ix – 12i^2 = 0$ [Since, $i^2 = –1$]

$x^2 – 3ix + 4ix – 12i^2 = 0$

$x(x – 3i) + 4i(x – 3i) = 0$

$(x – 3i) (x + 4i) = 0$

$x – 3i = 0$ or $x + 4i = 0$

$x = 3i$ or $–4i$

∴ The roots of the given equation are -4i, 3i

(x) $x^2 – (3\sqrt{2} – 2i)x – \sqrt{2}i = 0$

Given: $x^2 – (3\sqrt{2} – 2i)x – \sqrt{2}i = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm \sqrt{(b^2 – 4ac)})/2a$

Here, $a = 1$, $b = -(3\sqrt{2} – 2i)$, $c = –\sqrt{2}i$

So,

$$x = \frac{-\left(-(3\sqrt{2}-2i)\right) \pm \sqrt{\left(-(3\sqrt{2}-2i)\right)^2 - 4(1)(-\sqrt{2}i)}}{2(1)}$$

$$= \frac{(3\sqrt{2}-2i) \pm \sqrt{(3\sqrt{2}-2i)^2 + 4\sqrt{2}i}}{2}$$

$$= \frac{(3\sqrt{2}-2i) \pm \sqrt{18 - 12\sqrt{2}i + 4i^2 + 4\sqrt{2}i}}{2}$$

$$= \frac{(3\sqrt{2}-2i) \pm \sqrt{18 - 8\sqrt{2}i + 4i^2}}{2}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{(3\sqrt{2}-2i) \pm \sqrt{18 - 8\sqrt{2}i + 4(-1)}}{2}$$

$$= \frac{(3\sqrt{2}-2i) \pm \sqrt{18 - 8\sqrt{2}i - 4}}{2}$$

$$= \frac{(3\sqrt{2}-2i) \pm \sqrt{14 - 8\sqrt{2}i}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{18 - 8\sqrt{2}i - 4}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm \sqrt{14 - 8\sqrt{2}i}}{2}$$

We can write $14 - 8\sqrt{2}i = 16 - 2 - 8\sqrt{2}i$

$14 - 8\sqrt{2}i = 16 + 2(-1) - 8\sqrt{2}i$

$= 16 + 2i^2 - 8\sqrt{2}i$ [Since, $i^2 = -1$]

$= 4^2 + (\sqrt{2}i)^2 - 2(4)(\sqrt{2}i)$

$= (4 - \sqrt{2}i)^2$ [By using the formula, $(a - b)^2 = a^2 - 2ab + b^2$]

By using the result $14 - 8\sqrt{2}i = (4 - \sqrt{2}i)^2$, we get

$$x = \frac{(3\sqrt{2} - 2i) \pm \sqrt{(4 - \sqrt{2}i)^2}}{2}$$

$$= \frac{(3\sqrt{2} - 2i) \pm (4 - \sqrt{2}i)}{2}$$

$$= \frac{3\sqrt{2} - 2i}{2} \pm \frac{4 - \sqrt{2}i}{2}$$

∴ The roots of the given equation are $\frac{3\sqrt{2} - 2i}{2} \pm \frac{4 - \sqrt{2}i}{2}$

(xi) $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

Given: $x^2 - (\sqrt{2} + i)x + \sqrt{2}i = 0$

$x^2 - (\sqrt{2}x + ix) + \sqrt{2}i = 0$

$x^2 - \sqrt{2}x - ix + \sqrt{2}i = 0$

$x(x - \sqrt{2}) - i(x - \sqrt{2}) = 0$

$(x - \sqrt{2})(x - i) = 0$

$(x - \sqrt{2}) = 0$ or $(x - i) = 0$

$x = \sqrt{2}$ or $x = i$

∴ The roots of the given equation are i, $\sqrt{2}$

(xii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$

Given: $2x^2 - (3 + 7i)x + (9i - 3) = 0$

We shall apply discriminant rule,

Where, $x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$

Here, $a = 2$, $b = -(3 + 7i)$, $c = (9i - 3)$

So,

$$x = \frac{-(-(3+7i)) \pm \sqrt{(-(3+7i))^2 - 4(2)(9i-3)}}{2(2)}$$

$$= \frac{(3+7i) \pm \sqrt{(3+7i)^2 - 8(9i-3)}}{4}$$

$$= \frac{(3+7i) \pm \sqrt{9 + 42i + 49i^2 - 72i + 24}}{4}$$

$$= \frac{(3+7i) \pm \sqrt{33 - 30i + 49i^2}}{4}$$

We have $i^2 = -1$

By substituting $-1 = i^2$ in the above equation, we get

$$= \frac{(3+7i) \pm \sqrt{33 - 30i + 49(-1)}}{4}$$

$$= \frac{(3+7i) \pm \sqrt{33 - 30i - 49}}{4}$$

$$= \frac{(3+7i) \pm \sqrt{-16 - 30i}}{4}$$

$$= \frac{(3+7i) \pm \sqrt{(-1)(16+30i)}}{4}$$

$$= \frac{(3+7i) \pm \sqrt{i^2(16+30i)}}{4}$$

$$= \frac{(3+7i) \pm i\sqrt{16+30i}}{4}$$

We can write $16 + 30i = 25 - 9 + 30i$

$16 + 30i = 25 + 9(-1) + 30i$

$= 25 + 9i^2 + 30i$ [$\because i^2 = -1$]

$= 5^2 + (3i)^2 + 2(5)(3i)$

$= (5 + 3i)^2$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

By using the result $16 + 30i = (5 + 3i)^2$, we get

$$x = \frac{(3+7i) \pm i\sqrt{(5+3i)^2}}{4}$$

$$= \frac{(3+7i) \pm i(5+3i)}{4}$$

$$= \frac{(3+7i)+i(5+3i)}{4} \text{ or } \frac{(3+7i)-i(5+3i)}{4}$$

$$= \frac{3+7i+5i+3i^2}{4} \text{ or } \frac{3+7i-5i-3i^2}{4}$$

$$= \frac{3+12i+3i^2}{4} \text{ or } \frac{3+2i-3i^2}{4}$$

$$= \frac{3+12i+3(-1)}{4} \text{ or } \frac{3+2i-3(-1)}{4}$$

$$= \frac{3+12i-3}{4} \text{ or } \frac{3+2i+3}{4}$$

$$= \frac{12}{4}i \text{ or } \frac{6+2i}{4}$$

$$= 3i \text{ or } \frac{6}{4}+\frac{2}{4}i$$

$$x = 3i \text{ or } \frac{3}{2}+\frac{1}{2}i$$

$= 3i$ or $(3 + i)/2$

∴ The roots of the given equation are $(3 + i)/2$, $3i$