RD SHARMA Solutions for Class 12-science Maths Chapter 30 - Linear programming

Chapter 30 - Linear programming Exercise Ex. 30.1

Question 1

A small manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry, then sent to the machince shop for finishing. The number of man-hours of labour required in each shop for the production of each unit of A and B, and the number of man-hours the firm has available per week are as follows:

Gadget	Foundry	Machine-shop
A	**10**	**5**
B	**6**	**4**
Firm's capacity per week	1000	600

The profit on the sale of A is Rs 30 per unit as compared with Rs 20 per unit of B. The problem is to determine the weekly production of gadgets A and B, so that the total profit is maximized. Formulate this problem as a LPP.

Solution 1

The given data may be put in the following tabular form:-

Gadget	Foundry	Machine-shop	Profit
A	**10**	**5**	**Rs 30**
B	**6**	**4**	**Rs 20**
Firm's capacity per week	1000	600	

Let required weekly production of gadgets A and B be x and y respectively.

Given that, profit on each gadget A is Rs 30
So, profit on x gadget of type $A = 30x$
Profit on each gadget of type B = Rs 20
So, profit on y gadget of type $B = 20y$

Let Z denote the total profit, so

$$Z = 30x + 20y$$

Given, production of one gadget A requires 10 hours per week for foundry and gadget B requires 6 hours per week for foundry.

So, x units of gadget A requires $10x$ hours per week and y units of gadget B requires by hours per week, But the maximum capacity of foundry per week is 1000 hours, so

$$10x + 6y \leq 1000$$

This is first constraint.

Given, production of one unit gadget A requires 5 hours per week of machine shop and production of one unit of gadget B requires 4 hours per week of machine shop.

So, x units of gadget A requires $5x$ hours per week and y units of gadget B requires $4y$ hours per week, but the maximum capacity of machine shop is 600 hours per week
So, $5x + 4y \leq 600$

This is second constraint.
Hence, mathematical formulation of LPP is:
Find x and y which
Maximize $Z = 30x + 2y$

Subject to constraints,

$$10x + 6y \leq 1000$$
$$5x + 4y \leq 600$$

And, $x, y \geq 0$ [Since production cannot be less than zero]

Question 2

A company is making two products A and B. The cost of producing one unit of products A and B are Rs 60 and Rs 80 respectively. As per the agreement, the company has to supply at least 200 units of product B to its regular customers. One unit of product A requires one machine hour whereas product B has machine hours available abundantly within the company. Total machine hours available for product A are 400 hours. One unit of each product A and B requires one labour hour each and total of 500 labour hours are available. The company wants to minimize the cost of production by satisfying the given requirements. Formulate the problem as a LPP.

Solution 2

The given information can be written in tabular form as below:

Product	Machine hours	Labour hours	Profit
A	**1**	**1**	**Rs 60**
B	**–**	**1**	**Rs 80**
Total capacity	400 for A	500	
Minimum supply of product B is 200 units.			

Let production of product A be x units and production of product B be y units.

Given, profit on one unit of product A = Rs 60
So, profit on x unit of product A = Rs $60x$
Given, profit on one unit of product B = Rs 80
So, profit on y units of product B = Rs $80y$

Let Z denote the total profit, so

$$Z = 60x + 80y$$

Given, minimum supply of product B is 200
So, $y \geq 200$ (First constraint)

Given that, production of one unit of product A requires 1 hour of machince hours, so x units of product A requires x hours but given total machine hours available for product A is 400 hours, so

$$x \leq 400 \qquad \text{(Second constraint)}$$

Given, each unit of product A and B requires one hour of labour hour, so x units of product A require x hours and y units of product B require y hours of labour hours but total labour hours available are 500, so

$$x + y \leq 500 \qquad \text{(Third constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

Minimize $Z = 60x + 80y$

Subject to constraints,

$$y \geq 200$$
$$x \leq 400$$
$$x + y \leq 500$$
$$x, y \geq 0 \qquad \text{[Since production of product cannot be less than zero]}$$

Question 3

A firm manufactures 3 products A, B and C. The profits are Rs 3, Rs 2 and Rs 4 respectively. The firm has 2 machines and below is the required processing time in minutes for each machine on each product:

Machine	Products		
	A	B	C
M_1	4	3	5
M_2	2	2	4

Machines M_1 and M_2 have 200 and 2500 machine minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up a LPP to maximize the profit.

Solution 3

Product	Machine (M_1)	Machine (M_2)	Profit
A	**4**	**2**	**3**
B	**3**	**2**	**2**
C	**5**	**4**	**4**
Capacity maximum	2000	2500	

Let required production of product A, B and C be x, y and z units respectively.

Given, profit on one unit of product A, B and C are Rs 3, Rs 2, Rs 4, so
Profit on x unit of A, y unit of B and z unit of C are given by Rs. $3x$, Rs $2y$, Rs $4z$.

Let U be the total profit, so

$$U = 3x + 2y + 4z$$

Given, one unit of product A, B and C requires 4,3 and 5 minutes on machine M_1. So, x units of product A, y units of B and z units of product C need $4x$, $3y$ and $5z$ minutes on machine M_1 is 2000 minutes, so

$$4x + 3y + 5z \le 200 \quad \text{(First constraint)}$$

Given, one unit of product A, B and C requires 2,2 and 4 minutes on machine M_2. So, x units of A, y units of B and z units of C require $2x$, $2y$ and $4z$ minutes on machine M_2 is 2500 minutes, so

$$2x + 2y + 4z \le 2500 \quad \text{(Second constraint)}$$

Also, given that firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's.

$$\begin{aligned} &100 \le x \le 150 \\ &y \ge 200 \qquad \text{(Other constraints)} \\ &z \ge 50 \end{aligned}$$

Hence, mathematical formulation of LPP is :-
Find x, y and z which

$$\text{maximize } U = 3x + 2y + 4z$$

Subject to constraints,

$$\begin{aligned} &4x + 3y + 5z \le 2000 \\ &2x + 2y + 4z \le 2500 \\ &100 \le x \le 150 \\ &y \ge 200 \\ &z \ge 50 \end{aligned}$$

And, $x, y, z \ge 0$ [Since, x, y, z are non-negative]

Question 4

A firm manufactures two types of products A and B and sells them at a profit of Rs 2 on type A and Rs 3 on type B. Each product is processed on two machines M_1 and M_2. Type A requires one minute of processing time on M_1 and two minutes of M_2; type B requires one minute on M_1 and one minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for 10 hours during any working day. Formulate the problem as a LPP.

Solution 4

Given information can be written in tabular form as below:

Product	M_1	M_2	Profit
A	**1**	**2**	**2**
B	**1**	**1**	**3**
Capacity	6 hours 40 min =400 min.	10 hours =600 min.	

Let required production of product A be x units and product B be y units.

Given, profit on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B will be Rs $2x$ and Rs $3y$ respectively.

Let total profit be Z, so

$$Z = 2x + 3y$$

Given, production of one unit of product A and B require 1 and 1 minute on machine M_1 respectively, so production of x units of product A and y units of product B require x minutes and y minutes on machine M_1 but total time available on machine M_1 is 600 minutes, so

$$x + y \le 400 \quad \text{(First constraint)}$$

Given, production of one unit of product A and B require 2 minutes and 1 minutes on machine M_2 respectively. So production of x units of product A and y units of product B require $2x$ minutes and y minutes respectively on machine M_2 but machine M_2 is available for 600 minutes, so

$$2x + y \le 600 \quad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is:-

Find x and y which

$$\text{maximize } Z = 2x + 3y$$

Subject to constraints,

$$x + y \le 400$$

$$2x + y \le 600$$

and, $x, y \ge 0$ [Since production of product can not be less than zero]

Question 5

A rubber company is engaged in producing three types of tyres *A*,*B* and *C*. Each type requires processing in two plants, Plant I and Plant II. The capacities of the two plants, in number of tyres per day, are as follows:

Plant	*A*	*B*	*C*
I	50	100	100
II	60	60	200

The monthly demand for tyre *A*,*B* and *C* is 2500, 3000 and 7000 respectively. If plant I costs Rs 2500 per day, and plant II costs Rs 3500 per day to operate, how many days should each be run per month to minimize cost while meeting the demand? Formulate the problem as LPP.

Solution 5

Given information can be tabulated as:-

Plant	*A*	*B*	*C*	Cost
I	**50**	**100**	**100**	**2500**
II	**60**	**60**	**200**	**3500**
Monthly demand	2500	3000	7000	

Let plant I requires x days and plant II requires y days per month to minimize cost.

Given, plant I and II costs Rs 2500 perday and Rs 3500 perday respectively, so cost to run plant I and II is Rs $2500x$ and Rs $3500y$ per month.

Let Z be the total cost per month, so

$$Z = 2500x + 3500y$$

Given, production of tyre A from plant I and II is 50 and 60 respectively, so production of tyre A from plant I and II will be $50x$ and $60y$ respectively per month but the maximum demand of tyre A is 2500 per month so,

$$50x + 60y \geq 2500 \qquad [\text{First constraint}]$$

Given, production of tyre B from plant I and II is 100 and 60 respectively, so production of tyre B from plant I and II will be $100x$ and $60y$ per month respectively but the maximum demand of tyre B is 3000 per month, so

$$100x + 60y \geq 3000 \qquad [\text{Second constraint}]$$

Given, production of tyre C from plant I and II is 100 and 200 respectively. So production of tyre B from plant I and II will be $100x$ and $200y$ per month respectively but the maximum demand of tyre C is 7000 per day, so

$$100x + 200y \geq 7000 \qquad [\text{Third constraint}]$$

Hence, mathematical formulation of LPP is..

Find x and y which

Minimize $Z = 2500x + 3500y$

Subject to constraint,

$$50x + 60y \geq 2500$$
$$100x + 60y \geq 3000$$
$$100x + 200y \geq 7000$$

And, $x, y \geq 0$ [Since number of days can not be less than zero]

Question 6

A company sells two different products A and B. The two products are produced in a common production process and are sold in two different markets. The production process has a total capacity of 45000 man-hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The market has been surveyed and company officials feel that the maximum number of units of A that can be sold is 7000 and that of B is 10,000. If the profit is Rs 60 per unit for the product A and Rs 40 per unit for the product B, how many units of each product should be sold to maximize profit? Formulate the problem as LPP.

Solution 6

Given information can be tabulated as below:-

Product	Man hours	Maximum demand	Profit
A	**5**	**7000**	**60**
B	**3**	**10000**	**40**
Total capacity	45000		

Let required production of product A be x units and production of product B be y units.

Given, profits on one unit of product A and B are Rs 60 and Rs 40 respectively, so profits on x units of product A and y units of product B are Rs $60x$ and Rs $40y$.

Let Z be the total profit, so

$$Z = 60x + 40y$$

Given, production of one unit of product A and B require 5 hours and 3 hours respectively man hours, so x unit of product A and y units of product B require $5x$ hours and $3y$ hours of man hours respectively but total man hours available are 45000 hours, so

$$5x + 3y \le 45000 \qquad \text{(First constraint)}$$

Given, demand for product A is maximum 7000, so

$$x \le 7000 \qquad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is,

Find x and y which

$$\text{maximize } Z = 60x + 40y$$

Subject to constraints,

$$5x + 3y \le 45000$$
$$x \le 7000$$
$$y \le 10000$$

$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Question 7

To maintain his health a person must fulfill certain minimum daily requirements for several kinds of nutrients. Assuming that there are only three kinds of nutrients- calcium, protein and calories and the person's diet consists of only two food items, I and II, whose price and nutrient contents are shown in the table below:

	Food I (per *lb*)	Food II (per *lb*)	Minimum daily requirement for the nutrient
Calcium	**10**	**4**	**20**
Protein	**5**	**5**	**20**
Calories	**2**	**6**	**13**
Price (Rs)	**0.60**	**1.00**	

What combination of two food items will satisfy the daily requirement and entail the least cost? Formulate this as a LPP.

Solution 7

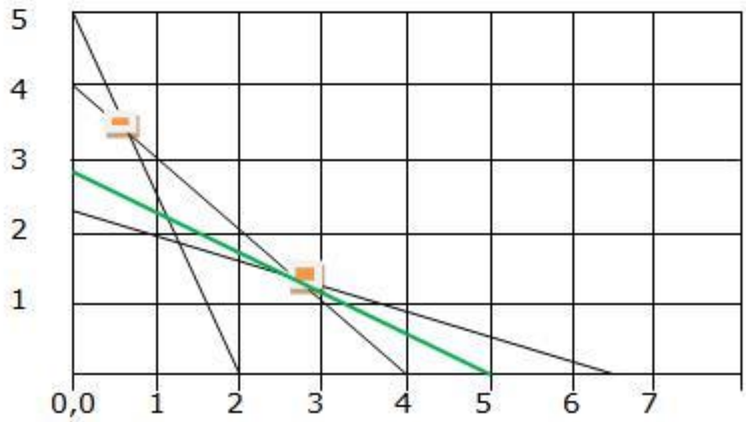
Let x and y be the packets of 25 gm of Food I and Food II purchased. Let Z be the price paid. Obviously price has to be minimized.

Take a mass balance on the nutrients from Food I and II,

Calcium	$10x + 4y \geq 20$	
	$5x + 2y \geq 10$	........(i)
Protein	$5x + 5\ y \geq 20$	
	$x + y \ \geq\ 4$	........(ii)
Calories	$2x + 6y \geq 13$	......(iii)

These become the constraints for the cost function, Z to be minimized ie., $0.6x + y = Z$, given cost of Food I is Rs 0.6/- and Rs 1/- per lb

From (i), (ii) & (iii) we get points on the X & Y-axis as [0, 5] & [2, 0] ; [0, 4] & [4, 0] ; [0, 13/6] & [6.5, 0]
Plotting these



When Z has an optimal value (maximum or minimum), where the variables *x* and *y* are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Here the feasible region is the unbounded region A-B-C-D

Computing the value of Z at the corner points of the feasible region ABHG

Point	Corner point	Value of Z = 0.6x + y
A	2, 5	6.2
B	0.67, 3.33	3.73
C	2.75, 1.25	2.9
D	6.5, 2.16	6.06

The smallest value of Z is 2.9 at the point (2.75, 1.25).
We cannot say that the minimum value of Z is 2.9 as the feasible region is unbounded.

Therefore, we have to draw the graph of the inequality
$0.6x + y < 2.9$

Plotting this to see if the resulting line (in green) has any point common with the feasible region. Since there are no common points this is the minimum value of the function Z and the mix is

Food I = 2.75 lb; Food II = 1.25 lb; Price = Rs 2.9

Question 8

A manufacturer can produce two products, *A* and *B*, during a given time period. Each of these products requires four different manufacturing operations: grinding, turning, assembling and testing. The manufacturing requirements in hours per unit of products *A* and *B* are given below.

	A	*B*
Grinding	1	2
Turning	3	1
Assembling	6	3
Testing	5	4

The available capacities of these operations in hours for the given time period are: grinding 30; turning 60, assembling 200; testing 200. The contribution to profit is Rs 2 for each unit of *A* and Rs 3 for each unit of *B*. The firm can sell all that it produces at the prevailing market price. Determine the optimum amount of *A* and *B* to produce during the given time period. Formulate this as a LPP.

Solution 8

Given information can be tabulated as:-

Product	Grinding	Turning	Assembling	Testing	Profit
A *B*	1 2	3 1	6 3	5 4	2 3
Maximum capacity	30 hours	60 hours	200 hours	200 hours	

Let required production of product A and B be x and y respectively

Given, profits on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B are given by $2x$ and $3y$ respectively. Let Z be total profit, so

$$Z = 2x + 3y$$

Given, production of 1 unit of product A and B require 1 hour and 2 hours of grinding respectively, so, production of x units of product A and y units of of product B require x hours and $2y$ hours of grinding respectively but maximum time available for grinding is 3 hours, so

$$x + 2y \leq 30 \quad \text{(First constraint)}$$

Given, production of 1 unit of product A and B require 3 hours and 1 hours of turning respectively, so x units of product A and y units of of product B require $3x$ hours and y hours of turning respectively but total time available for turning is 60 hours, so

$$3x + y \leq 60 \quad \text{(Second constraint)}$$

Given, production of 1 unit of product A and B require 6 hour and 3 hours of assembling respectively, so productinon of x units of product A and y units of product B require $6x$ hours and $3y$ hours of assembling respectively but total time available for assembling is 200 hours, so

$$6x + 3y \leq 200 \quad \text{(Third constraint)}$$

Given, production of 1 unit of product A and B require 5 hours and 4 hours of testing respectively, so productinon of x units of product A and y units of product B require $5x$ hours and $4y$ hours of testing respectively but total time available for testing is 200 hours, so

$$5x + 4y \leq 200 \quad \text{(Fourth constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

maximize $Z = 2x + 3y$

Subject to constraints,

$$x + 2y \leq 30$$
$$3x + y \leq 60$$
$$6x + 3y \leq 200$$
$$5x + 4y \leq 200$$

and, $x, y \geq 0$ [Since production can not be negative]

Question 9

Vitamins A and B are found in two different foods F_1 and F_2. One unit of food F_1 contains 2 units of vitamin A and 3 units of vitamin B. One unit of food F_2 contains 4 units of vitamin A and 2 units of vitamin B. One unit of food F_1 and F_2 cost Rs 5 and 2.5 respectively. The minimum daily requirements for a person of vitamin minimum requirement of vitamin A and B is not harmful, find out the optimum mixture of food F_1 and F_2 at the minimum cost which meets the daily minimum requirement of vitamin A and B. Formulate this as a LPP.

Solution 9

Given information can be tabulated as below:

Foods	Vitamin A	Vitamin B	Cost
F_1	**2**	**3**	**5**
F_2	**4**	**2**	**2.5**
Minimum daily requirement	40	50	

Let required quantity of food F_1 be x units and quantity of food F_2 be y units.

Given, costs of one unit of food F_1 and F_2 are Rs 5 and Rs 2.5 respectively, so costs of x units of food F_1 and y units of food F_2 are Rs $5x$ and Rs $2.5y$ respectively.
Let Z be the total cost, so

$$Z = 5x + 2.5y$$

Given, one unit of food F_1 and food F_2 contain 2 and 4 units of vitamin A respectively, so x unit of Food F_1 and y units of food F_2 contain $2x$ and $4y$ units of vitamin A respectively, but minimum requirement of vitamin A is 40 unit, so

$$2x + 4y \geq 40 \qquad \text{(First constraint)}$$

Given, one unit of food F_1 and food F_2 contain 3 and 2 units of vitamin B respectively, so x unit of Food F_1 and y units of food F_2 contain $3x$ and $2y$ units of vitamin B respectively, but minimum daily requirement of vitamin B is 40 unit, so

$$3x + 2y \geq 50 \qquad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

$$\text{Minimize } Z = 5x + 2.5y$$

Subject to constraint,

$$2x + 4y \geq 40$$
$$3x + 2y \geq 50$$

$$x, y \geq 0 \qquad \left[\text{Since requirement of food } F_1 \text{ and } F_2 \text{ can not be less than zero.}\right]$$

Question 10

An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop *A*, which performs the basic assembly operation, must work 5 man-days on each truck but only 2 man-days on each automobile. Shop *B*, which performs finishing operations, must work 3 man-days for each automobile or truck that it produces. Because of men and machine limitations, shop *A* has 180 man-days per week available while shop *B* has 135 man-days per week. If the manufacturer makes a profit of Rs 30000 on each truck and Rs 2000 on each automobile, how many of each should he produce to maximize his profit? Formulate this as a LPP.

Solution 10

Let the number of automobiles produced be x and let the number of trucks produced be y.

Let Z be the profit function to be maximized.
Z = 2,000x + 30,000y

The constraints are on the man hours worked

Shop A	$2x + 5y \le 180$	(i)	assembly
Shop B	$3x + 3y \le 135$	(ii)	finishing
	$x \ge 0$; $y \ge 0$		

Corner points can be obtained from
$2x + 5y = 180 \Rightarrow x=0;\ y=36$ and $x=90;\ y=0$
$3x + 3y \le 135 \Rightarrow x=0;\ y=45$ and $x=45;\ y=0$
Solving (i) & (ii) gives x = 15 & y = 30

Corner point	Value of Z =2,000x + 30,000y
0,0	0
0, 36	10,80,000
15, 30	9,30,000
45, 0	90,000

0 automobiles and 36 trucks will give max profit of 10,80,000/-

Question 11

Two tailors *A* and *B* earn Rs 150 and Rs 200 per day respectively. *A* can stitch 6 shirts and 4 pants per day while *B* can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Solution 11

	Taylor A		Taylor B	Limit
Variable	x		y	
Shirts	6x	+	10y	≥ 60
Pants	4x	+	4y	≥ 32
Earn Rs.	150	+	200	Z

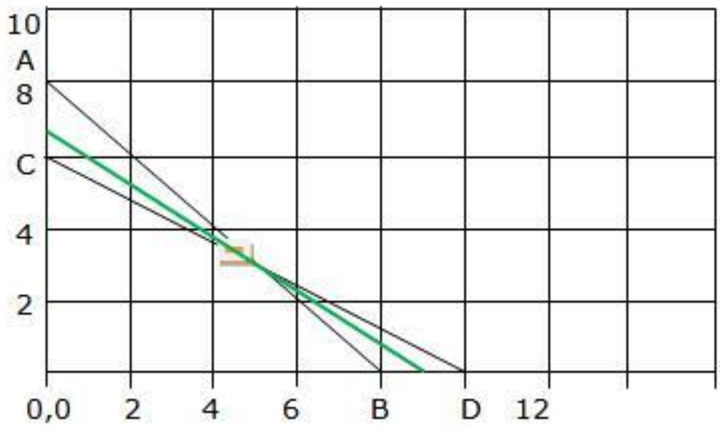
The above LPP can be presented in a table above.

To minimize labour cost means to assume minimize the earnings i.e. Min Z = 150x + 200y
s.t. the constraints

$x \geq 0$; $y \geq 0$	at least 1 shirt & pant is required
$6x + 10y \geq 60$	require at least 60 shirts
$4x + 4y \geq 32$	require at least 32 pants

Solving the above inequalities as equations we get,
x = 5 and y = 3

other corner points obtained are [0, 6] & [10, 0]
[0, 8] & [8,0]



The feasible region is the open unbounded region A-E-D

Point E(5, 3) may not be the minimal value. So, plot $150x + 200y < 1350$ to see if there is a common region with A-E-D

The green line has no common point , therefore

Corner point	Value of Z = 150x + 200y
0,8	0
10, 0	1500
5, 3	1350

Stitching 5 shirts and 3 pants minimizes labour cost to Rs.1350/-

Question 12

An airline agrees to charter planes for a group. The group needs at least 160 first class seats and at least 300 tourist class seats. The airline must use at least two of its model 314 planes which have 20 first class and 30 tourist class seats. The airline will also use some of its model 535 planes which have 20 first class seats and 60 tourist class seats. Each flight of a model 314 plane costs the company Rs 1 lakh, and each flight of a model 535 plane costs Rs 1.5 lakh. How many of each type of plane should be used to minimize the flight cost? Formulate this as a LPP.

Given information can be tabulated as below

Plane	First class	Tourist class	Cost
Model 314	**20**	**30**	**100000**
Model 335	**20**	**60**	**150000**
Requirement minimum	160 seats	300 seats	

Solution 12

	Model 314		Model 535	Limit
Variable	x		y	
F class	20x	+	20y	≥ 160
T class	30x	+	60y	≥ 300
Cost	1.x lakh	+	1.5y lakh	Z

The above LPP can be presented in a table above.

The flight cost is to be minimized i.e, Min $Z = x + 1.5y$
s.t. the constraints

$x \geq 2$	at least 2 planes of model 314 must be used
$y \geq 0$	at least 1 plane of model 535 must be used
$20x + 20y \geq 160$	require at least 160 F class seats
$30x + 60y \geq 300$	require at least 300 T class seats

Solving the above inequalities as equations we get,
When x=0, y=8 and when y=0, x=8
When x=0, y=5 and when y=0, x=10

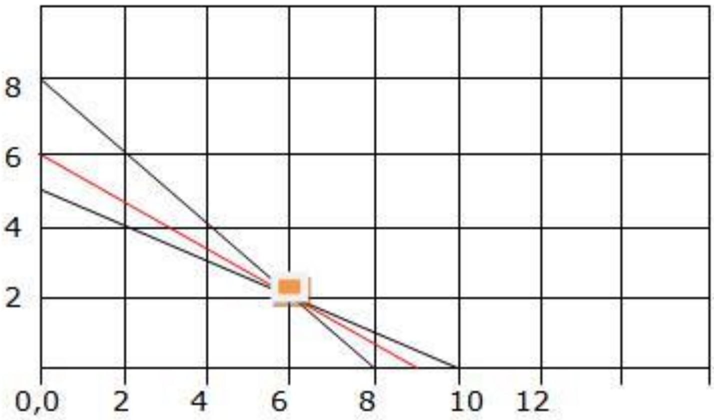
We get an unbounded region 8-E-10 as a feasible solution. Plotting the corner points and evaluating we have,

Corner point	Value of Z = x + 1.5y
10, 0	10
0, 8	12
6, 2	9

Since we obtained an unbounded region as the feasible solution a plot of Z (x+1.5 < 9) is plotted.

Since there are no common points point E is the point that gives a minimum value.

Using 6 planes of model 314 & 2 of model 535 gives minimum cost of 9 lakh rupees.



Question 13

Amit's mathematics teacher has given him three very long lists of problems with the instruction to submit not more than 100 of them (correctly solved) for credit. The problem in the first set are worth 5 points each, those in the second set are worth 4 points each, and those in the third set are worth 6 points each. Amit knows from experience that he requires on the average 3 minutes to solve a 5 point problem, 2 minutes to solve a 4 point problem, and 4 minutes to solve a 6 point problem. Because he has other subjects to worry about, he can not afford to debote more than $3\frac{1}{2}$ hours altogether to his mathematics assignment. Moreover, the first two sets of problems involve numerical calculations and he knows that he cannot stand more than $2\frac{1}{2}$ hours work on this type of problem Under these circumstances, how many problems in each of these categories shall he do in order to get maximum possible credit for his efforts? Formulate this as a LPP.

Solution 13

Given information can be tabulated as below

Sets	Time requirement	Points
I	**3**	**5**
II	**2**	
III	**4**	**6**

Time for all three sets $= 3\frac{1}{2}$ hours

Time for Set I and Set II $= 2\frac{1}{2}$ hours

Number of questions maximum 100

Let he should x, y, z questions from set I, II and III respectively.

Given, each question from set I, II, III earn 5, 4, 6 points respectively, so x questions of set I, y questions of set II and z questions of set III earn $5x$, $4y$ and $6z$ points, let total point credit be U

So, $U = 5x + 4y + 6z$

Given, each question of set I, II and III require 3,2 and 4 minutes respectively, so x questions of set I, y questions of set II and z questions of set III require $3x$, $2y$ and $4z$ mimutes respectively but given that total time to devote in all three sets is $3\frac{1}{2}$ hours $= 210$ minutes and first two sets is $2\frac{1}{2}$ hours $= 150$ minutes

So,

$$3x + 2y + 4z \le 210 \quad \text{(First constraint)}$$

$$3x + 2y \le 150 \quad \text{(Second constraint)}$$

Given, total number of questions cannot exceed 100

So, $x + y + z \le 100$ (Third constraint)

Hence, mathematical formulation of LPP is

Find x and y which

maximize $U = 5x + 4y + 6z$

Subject to constraint,

$$3x + 2y + 4z \le 210$$

$$3x + 2y \le 150$$

$$x + y + z \le 100$$

$$x, y, z \ge 0$$

[Since number of questions to solve from each set cannot be less than zero]

Question 14

A farmer has a 100 acre farm. He can sell the tomatoes, lettuce, or radishes he can raise. The price he can obtain is Rs 1 per kilogram for tomatoes, Rs 0.75 a head for lettuce and Rs 2 per kilogram for radishes. The average yield per acre is 2000 kgs for radishes, 3000 heads of lettuce and 1000 kilograms of radishes. Fertilizer is available at Rs 0.50 per kg and the amount required per acre is 100 kgs each for tomatoes and lettuce and 50 kilograms for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs 20 per man-day. Formulate this problem as a LPP to maximize the farmer's total profit.

Solution 14

Given information can be tabulated as below

Product	Yield	Cultivation	Price	Fertilizers
Tomatoes	2000 kg	5 days	1	100 kg
Lettuce	3000 kg	6 days	0.75	100 kg
Radishes	1000 kg	5 days	2	50 kg
Average 2000 kg/per acre Total land = 100 Acre Cost g fertilizers = Rs 0.50 per kg. A total of 400 days of cultivation labour with Rs 20 per day				

Let required quantity of field for tomatoes, lettuce and radishes be x, y and z Acre respectively.

Given, costs of cultivation and harveshing of tomatoes, lettuce and radishes are 5×20 = Rs 100, 6×20 = Rs 120, 5×20 = Rs 100 respectively per acre. Cost of fertilizers for tomatoes, lettuce and radishes 100×0.50 = Rs 50, 100×0.50 = Rs 50 and 50×0.50 = Rs 25 respectively per acre.

So, total costs of production of tomatoes, lettuce and radishes are Rs $100 + 50$ = Rs $150x$, Rs $120 + 50$ = Rs $170y$ and radishes are Rs $100 + 25$ = Rs $125z$ respectively total selling price of tomatoes, lettuce and radishes, according to yield are 2000×1 = Rs $2000x$, 3000×0.75 = Rs $2250y$ and 100×2 = Rs $2000z$ respectively.

Let U be the total profit,
So,

$$U = (2000x - 150x) + (2250y - 170y) + (2000z - 125z)$$
$$U = 1850x + 2080y + 1875z$$

Given, farmer has 100 acre form
So, $x + y + z \le 100$ (First constraint)

Number of cultivation and harvesting days are 400
So, $5x + 6y + 5z \le 400$

Hence, mathematical formulation of LPP is
Find x, y, z which
maximize $U = 1850x + 2080y + 1875z$

Subject to constraint,

$$x + y + z \le 100$$
$$5x + 6y + 5z \le 400$$

$$x, y, z \ge 0$$

[Since from used for cultivation cannot be less than zero.]

Question 15

A firm manufactures two products, each of which must be processed through two departments, 1 and 2. The hourly requirements per unit for each product in each department, the weekly capacities in each department, selling price per unit, labour cost per unit, and raw material cost per unit are summarized as follows:

	Product *A*	Product *B*	Weekly capacity
Department 1	3	2	130
Department 2	4	6	260
Selling price per unit	Rs 25	Rs 30	
Labour cost per unit	Rs 16	Rs 20	
Raw material cost per unit	Rs 14	Rs 4	

The problem is to determine the number of units to produce each product so as to maximize total contribution to profit. Formulate this as a LPP.

Solution 15

Given information can be tabulated as below:

Product	Department 1	Department 2	Selling price	Labour cost	Raw material cost
A	**3**	**4**	**25**	**16**	**4**
B	**2**	**6**	**30**	**20**	**4**
Capacity	130	260			

Let the required product of product A and B be x and y units respectively.

Given, labour cost and raw material cost of one unit of product A is Rs 16 and Rs 4, so total cost of product A is Rs 16 + Rs 4 = Rs 20
And given selling price of 1 unit of product A is Rs 25,
So, profit on one unit of product

$A = 25 - 20 = \text{Rs } 5$

Again, given labour cost and raw material cost of one unit of product B is Rs 20 and Rs 4
So, that cost of product B is Rs 20 + Rs 4 = Rs 24
And given selling price of 1 unit of product B is Rs 30
So, profit on one unit of product $B = 30 - 24 = \text{Rs } 6$

Hence, profits on x unit of product A and y units of product B are Rs $5x$ and Rs $6y$ respectively.
Let Z be the total profit , so $Z = 5x + 6y$

Given, production of one unit of product A and B need to process for 3 and 4 hours respectively in department 1, so production of x units of product A and y units of product B need to process for $3x$ and $4y$ hours respectively in Department 1. But total capacity of Department 1 is 130 hour ,
So, $3x + 2y \le 130$ (First constraint)

Given, production of one unit of product A and B need to process for 4 and 6 hours respectively in department 2, so production of x units of product A and y units of product B need to process for $4x$ and $6y$ hours respectively in Department 2 but total capacity of Department 2 is 260 hours
So, $4x + 6y \le 260$ (Second constraint)

Hence, mathematical formulation of LPP is,
Find x and y which
Maximize $Z = 5x + 6y$

Subject to constraint,

$3x + 2y \le 130$

$4x + 6y \le 260$

$x, y \ge 0$ [Since production cannot be less than zero]

Chapter 30 - Linear programming Exercise Ex. 30.2
Question 1

Solve the linear programming problem by graphical method.

Maximize $Z = 5x + 3y$

Subject to

$$3x + 5y \leq 15$$

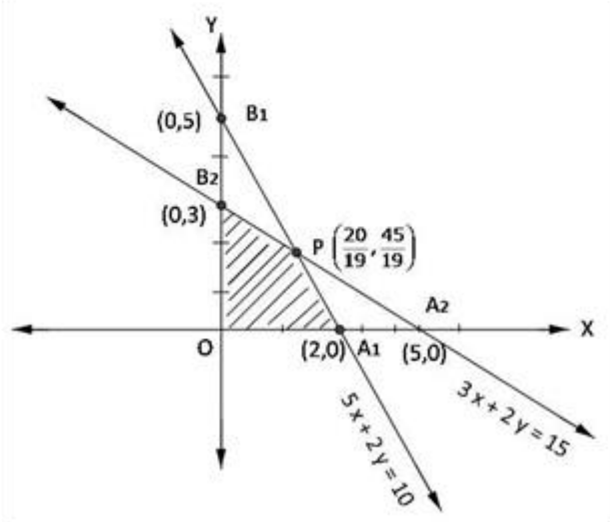
$$5x + 2y \leq 10$$

$$x, y \geq 0$$

Solution 1

Converting the given inequations into equations, we get

$3x + 5y = 15,\ 5x + 2y = 10,\ x = 0,\ y = 0$



Region represented by $5x + 2y \le 10$: The line meets coordinate axes at $A_1(2,0)$ and $B_1(0,5)$ respectively. Join these points to obtain the line $5x + 2y = 10$, clearly, $(0,0)$ satisfies the in eqation $5x + 2y \le 10$, so, the region in xy-plane that contains the origin represents the solution set if the given in equation.

Region represented by $3x + 5y \le 10$: The line meets coordinate axes at $A_2(5,0)$ and $B_2(0,3)$ respectively. Join these points to obtain the line $3x + 5y = 15$, clearly, $(0,0)$ satisfies the in eqation $3x + 5y \le 15$, so, the region in xy-plane contains the origin represents the solution set if the given in equation.

Region represented by $x \ge 0$, $y \ge 0$: It clearly represents first quadrant of xy-plane. Common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $0(0,0)$, $A(2,0)$, $P\left(\frac{20}{19}, \frac{45}{19}\right)$, $B_2(0,3)$.

The value of $Z = 5x + 3y$ at

$0(0,0)$	$= 5 \times + 3 \times 0$
$A(2,0)$	$= 5 \times 2 + 3 \times 0 = 10$
$P\left(\frac{20}{19}, \frac{45}{19}\right)$	$= 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$
$B_2(0,3)$	$= 5 \times 0 + 3 \times 3 = 9$

Clearly, Z is maximum at $P\left(\frac{20}{19}, \frac{45}{19}\right)$

So, $x = \frac{20}{19}$, $y = \frac{45}{19}$, maximum $Z = \frac{235}{19}$

Question 2

Solve the linear programming problem by graphical method.

Maximize $Z = 9x + 3y$

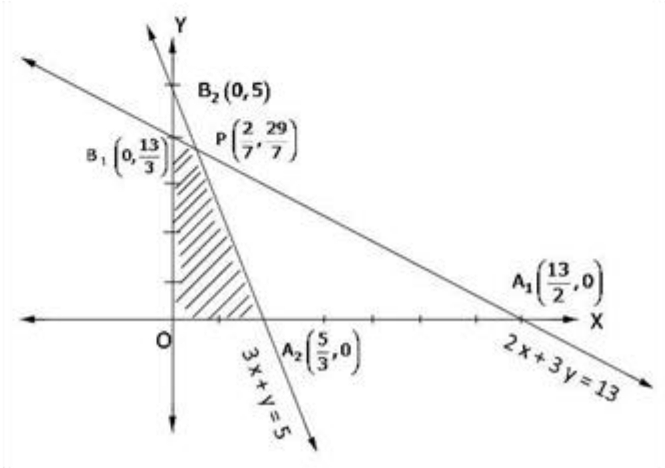
Subject to

$$2x + 3y \le 13$$
$$3x + y \le 5$$
$$x, y \ge 0$$

Solution 2

Converting the given inequations into equations, we get

$2x + 3y = 13$, $3x + y = 5$, and $x = 0$, $y = 0$



Region represented by $2x+3y \le 13$: The line meets coordinate axes at $A_1\left(\frac{13}{2},0\right)$ and $B_1\left(0,\frac{13}{3}\right)$respectively. Join these points to obtain the line $2x+3y=13$, clearly, $(0,0)$ satisfies the in eqation $2x+3y \le 13$, so, the region in xy-plane that contains origin represents the solution set of $2x+3y \le 13$.

Region represented by $3x+y \le 5$: The line meets coordinate axes at $A_2\left(\frac{5}{3},0\right)$ and $B_2(0,5)$ respectively. Join these points to obtain the line $3x+y=5$, clearly, $(0,0)$ satisfies the in eqation $3x+y \le 5$, so, the region in xy-plane that contains origin represents the solution set of $3x+y \le 5$.

Region represented by $x, y \ge 0$: It clearly represent first quadrant of xy-plane. The common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $0(0,0)$, $A\left(\frac{5}{3},0\right)$, $P\left(\frac{2}{7},\frac{29}{7}\right)$, $B_2\left(0,\frac{13}{3}\right)$.

The value of $Z = 9x+3y$ at

$0(0,0)$	$=9(0)+3(0)=0$
$A_1\left(\frac{5}{3},0\right)$	$=9\left(\frac{5}{3}\right)+3(0)=15$
$P\left(\frac{2}{7},\frac{29}{7}\right)$	$=9\left(\frac{2}{7}\right)+3\left(\frac{29}{7}\right)=15$
$B_2\left(0,\frac{13}{3}\right)$	$=9(0)+3\left(\frac{13}{3}\right)=13$

Clearly, Z is maximum at at every point on the line joining A_1 and P, so

$x=\frac{5}{3}$ or $\frac{2}{7}$, $y=0$ or $\frac{29}{7}$

and maximum $Z=15$.

Question 3

Solve the linear programming problem by graphical method.

Minimize $Z = 18x+10y$

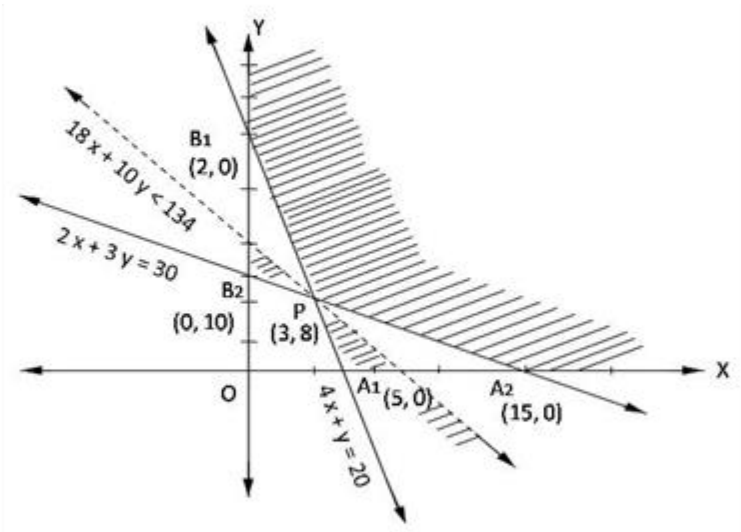
Subject to

$$4x+y \ge 20$$
$$2x+3y \ge 30$$
$$x, y \ge 0$$

Solution 3

Converting given inequations into equations as

$$4x + y = 20,\ 2x + 3y = 30,\ x = 0,\ y = 0$$



Region represented by in equation $4x + y \geq 20$: The line $4x + y = 20$ meets the coordinate axes at $A_1 (5,0)$ and $B_1 (0,20)$. Joining A_1B_1 we get $4x + y = 20$. Clearly, $(0,0)$, also does not satisfies the in eqation, so the region does not containing the origin represents the in equality $4x + y \geq 20$ in the xy-plane.

Region represented by in equation $2x + 3y \geq 30$: The line $2x + 3y = 30$ meets the coordinate axes at $A_2 (15,0)$ and $B_2 (0,20)$. Obtain line $2x + 3y = 30$ by joining A_2 and B_2. Clearly, $(0,0)$, does not satisfies the in eqation $2x + 3y \geq 30$, so the region does not containing the origin represents the in equality $2x + 3y \geq 30$ in the xy-plane.

Region represented by $x, y \geq 0$: $x, y \geq 0$ represents the first quadrant of xy-plane.

The shaded region is the feasible region with corner points $A_2 (15,0)$, $P (3,8)$, $B_1 (0,20)$ where P is obtained by solving $2x + 3y = 30$ and $4x + y = 20$ simultaneously.

The value of $Z = 18x + 10y$ at

$A_2 (15,0)$	$= 18(15) + 10(8) = 270$
$P (3,8)$	$= 18(3) + 10(8) = 134$
$B_1 (0,20)$	$= 18(0) + 10(20) = 200$

Clearly, Z is manimum at $x = 3$ and $y = 8$. The minimum value of Z is 134.

We observe that open half plane represented by $18x + 10y < 134$ does not have points in common with the solution region. So Z has

Minimum value $= 134$ at $x = 3$, $y = 8$

Question 4

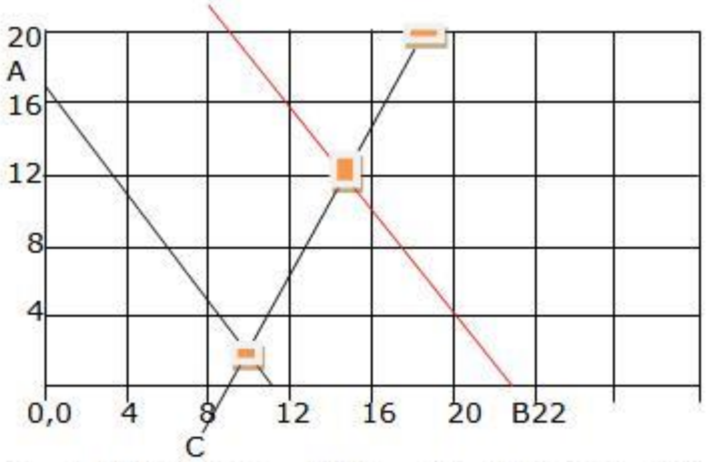
Solve the linear programming problem by graphical method.

Maximize $Z = 50x + 30y$

Subject to

$$2x + y \geq 18$$
$$3x + 2y \leq 34$$
$$x, y \geq 0$$

Solution 4



2x-y $\geq$ 18 ; when x = 12, y = 6 & when y=0, x=9
3x+2y $\leq$ 34 ; when x = 0, y = 17 & when y=0, x=34/3

Plotting these points gives line AB and CD
The feasible area is the unbounded area D-E-12

Corner point	Value of Z = 50x + 30y
10, 2	560
11.3, 17	1076.66

The maximize value of Z = 50x+30y, occurs at x = 34/3, y = 17

Since we have an unbounded region as the feasible area plot 50x + 30y > 1076.66

Since the region D-F-B has common points with region D-E-12 the problem has no optimal maximum value.

Question 5

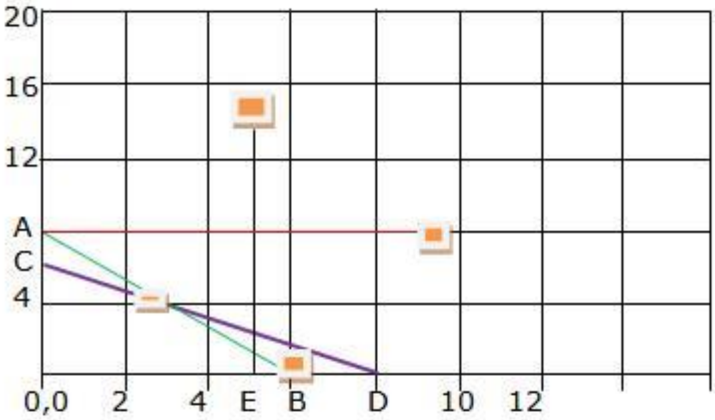
Solve the linear programming problem by graphical method.

Maximize $Z = 4x + 3y$

Subject to

$$3x + 4y \leq 24$$
$$8x + 6y \leq 48$$
$$x \leq 5$$
$$y \leq 6$$
$$x, y \geq 0$$

Solution 5



$3x+4y \le 24$; when $x = 0$, $y = 6$ & when $y=0$, $x=8$, line AB

$8x+6y \le 48$; when $x = 0$, $y = 8$ & when $y=0$, $x=6$, line CD

Plotting $x \le 5$ gives line EF; Plotting $y \le 6$ gives line AG
The feasible area is 0,0-C-H-G-E

Corner point	Value of Z = 4x + 3y
0, 0	0
0, 6	18
3.4, 3.4	24
5, 1	23
5,0	20

The maximum of $Z = 4x+3y$, occurs at $x = 3.4$, $y = 3.4$

Question 6

Solve the linear programming problem by graphical method.

Maximize $Z = 15x + 10y$

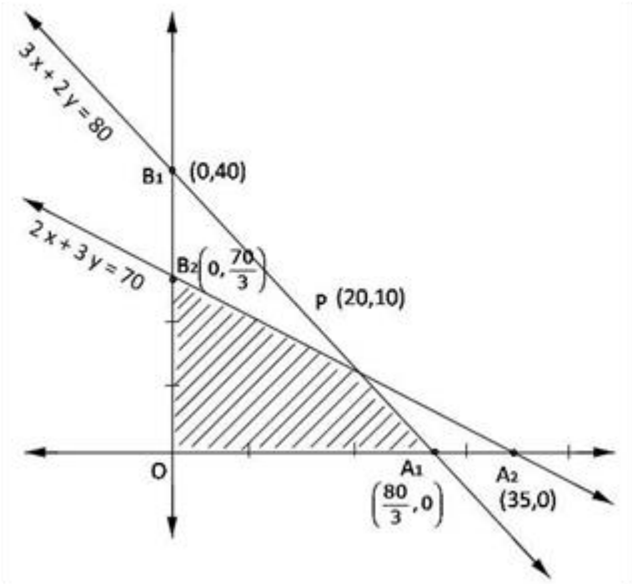
Subject to

$$3x + 2y \le 80$$
$$2x + 3y \le 70$$
$$x, y \ge 0$$

Solution 6

Converting the inequations into equations as

$$3x + 2y = 80,\ 2x + 3y = 70,\ x = y = 0$$



Region represented by $3x + 2y \le 80$: Line $3x + 2y = 80$ meets coordinate axes at $A_1\left(\frac{80}{3}, 0\right)$ and $B_1(0, 40)$, clearly, $(0,0)$ satisfies the $3x + 2y \le 80$, so, region containing the origin represents by $3x + 2y \le 80$ in xy-plane

Region represented by $2x + 3y \le 70$: Line $2x + 3y = 70$ meets the coordinate axes at $A_2(35,0)$ and $B_2\left(0, \frac{70}{3}\right)$, clearly, $(0,0)$ satisfies the $2x + 3y \le 70$ so, the region containing the origin represents by $2x + 3y \le 70$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

So, shaded area OA_1PB_2 represents the feasible region.

Coordinate of $P(20,10)$ can be obtained by solving $3x + 2y = 80$ and $2x + 3y = 70$

Now, the value of $Z = 15x + 10y$ at

$$
\begin{aligned}
&O(0,0) && = 15(0) + 10(0) = 0 \\
&A_1\left(\frac{80}{3}, 0\right) && = 15\left(\frac{80}{3}\right) + 10(0) = 400 \\
&P(20,10) && = 15(20) + 10(10) = 400 \\
&B_2\left(0, \frac{70}{3}\right) && = 15(0) + 10\left(\frac{70}{3}\right) = \frac{700}{3}
\end{aligned}
$$

So, maximum $Z = 400$ is on each and every point on the line joining A_1P, so we can have,

maximum $Z = 400$ at $x = \frac{80}{3}$ and $y = 0$

maximum $Z = 400$ at $x = 20$ and $y = 10$

Question 7

Solve the linear programming problem by graphical method.

Maximize $Z = 10x + 6y$

Subject to

$3x + y \le 12$

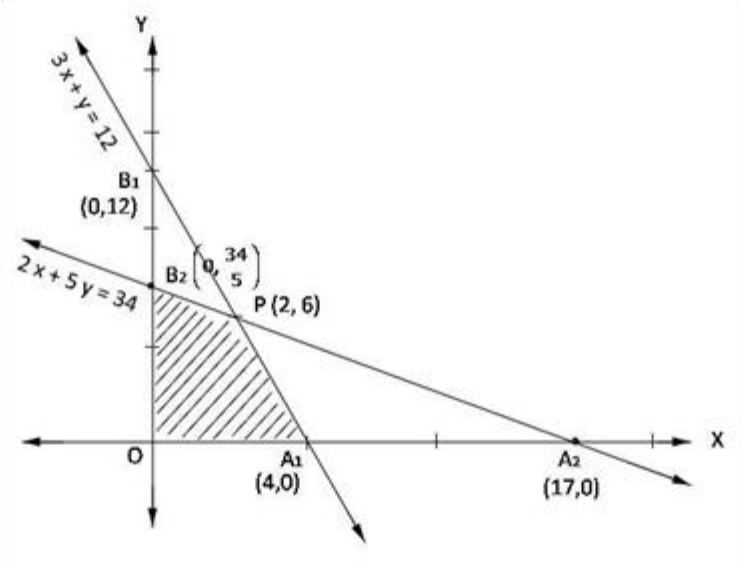
$2x + 5y \le 34$

$x, y \ge 0\ x, y \ge 0$

Solution 7

Converting the given inequations into equations

$$3x + y = 12,\ 2x + 5y = 34,\ x = y = 0$$



Region represented by $3x + y \leq 12$: Line $3x + y = 12$ meets the coordinate axes at $A_1(4,0)$ and $B_1(0,12)$, clearly, $(0,0)$ satisfies $3x + y \leq 12$, so, region containing origin is represented by $3x + y \leq 12$ in xy-plane

Region represented by $2x + 5y \leq 34$: Line $2x + y = 34$ meets coordinate axes at $A_2(17,0)$ and $B_2\left(0, \frac{34}{5}\right)$, clearly, $(0,0)$ satisfies the $2x + 5y \leq 34$ so, region containing origin represents $2x + 5y \leq 34$ in xy-plane

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane

Therefore, shaded area OA_1PB_2 is the feasible region.

The coordinate of $P(2,6)$ is obtained by solving $2x + 5y = 34$ and $3x + y = 12$

The value of $Z = 10x + 6y$ at

$O(0,0) = 10(0) + 6(0) = 0$

$A_1(4,0) = 10(4) + 6(0) = 40$

$P(2,6) = 10(2) + 6(6) = 56$

$B_2\left(0, \frac{34}{5}\right) = 10(0) + 6\left(\frac{34}{5}\right) = \frac{204}{5} = 40\frac{4}{5}$

Hence, maximum $Z = 56$ at $x = 2,\ y = 6$

Question 8

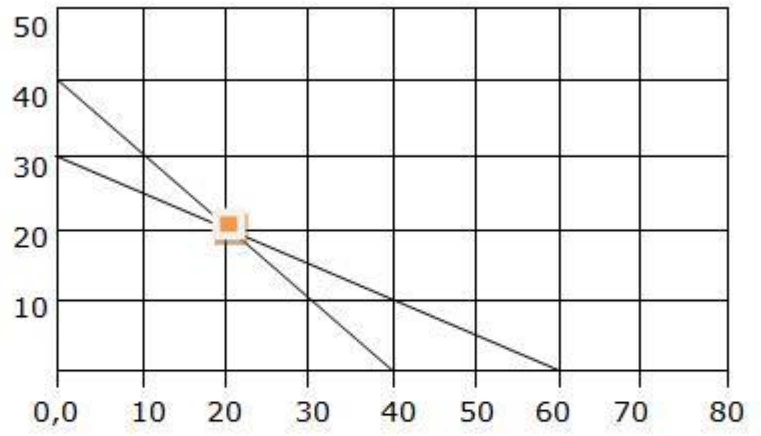
Solve the linear programming problem by graphical method.

Maximize $Z = 3x + 4y$

Subject to

$$2x + 2y \le 80$$
$$2x + 4y \le 120$$

Solution 8



2x+2y≤80; when x=0, y=40 and when y=0, x=40
2x+4y≤120; when x=0, y=30 and when y=0, x=60

The intersection of the two plotted lines gives (20, 20)
Feasible area is 30-C-40

Corner point	Value of Z = 3x + 4y
0, 0	0
0, 30	120
20, 20	140
40, 0	120

The maxima is obtained at x=20, y=20 and is 140

Question 9

Solve the linear programming problem by graphical method.

Maximize $Z = 7x + 10y$

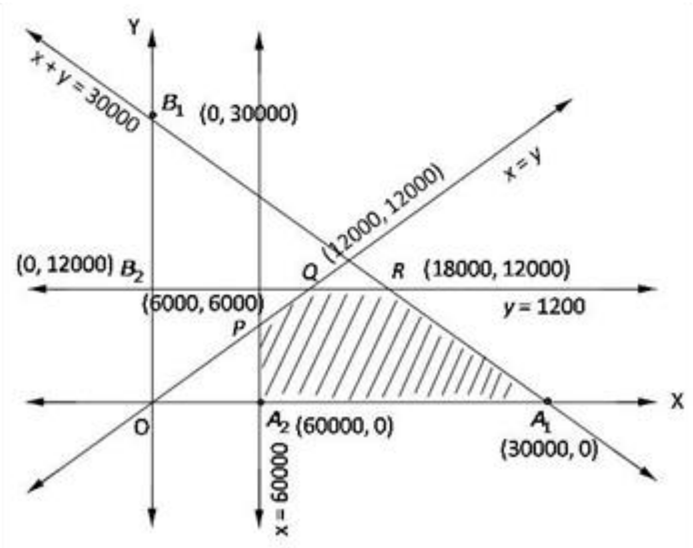
Subject to

$$x + y \le 30000$$
$$y \le 12000$$
$$x \ge 6000$$
$$x \ge y$$
$$x, y \ge 0$$

Solution 9

Converting the given inequations into equations,

$x + y = 30000$, $y = 12000$, $x = 6000$, $x = y$, $x = y = 0$



Region represented by $x + y \le 30000$: Line $x + y = 30000$ meets the coordinate axes at $A_1(30000,0)$ and $B_1(0,30000)$, clearly $(0,0)$ satisfies $x + y \le 30000$, so, region containing the origin represents $x + y \le 30000$ in xy-plane

Region represented by $y \le 12000$: Line $y = 12000$ is parallel to x -axis and meets y -axis at $B_2(0,12000)$. Clearly $(0,0)$ satisfies $y \le 12000$, so, region containing origin represents $y \le 12000$ in xy -plane.

Region represented by $x \le 6000$: Line $x = 6000$ is parallel to y -axis and meets x -axis at $A_2(6000,0)$. Clearly $(0,0)$ satisfies $x \le 6000$, so, region containing origin represents $x \le 6000$ in xy -plane.

Region represented by $x \ge y$: Line $x = y$ passes through origin and point $Q(12000,12000)$. Clearly, $A_2(6000,0)$ satisfies $x \ge y$, so, region containing $A_2(6000,0)$ represents $x \ge y$ in xy -plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy -plane.

Shaded region A_2A_1QP represents the feasible region.

Coordinates of $R(18000,12000)$ is obtained by solving $x + y = 30000$ and $y = 12000$, $Q(12000,12000)$ is obtained by solving $x = y$ and $y = 12000$, $P(6000,6000)$ is obtained by solving $x = y$ and $x = 6000$.

The value of $Z = 7x + 10y$ at

$A_2(6000,0)$	$= 7(6000) + 10(0) = 42000$
$A_1(30000,0)$	$= 7(30000) + 10(0) = 210000$
$R(18000,12000)$	$= 7(18000) + 10(12000) = 246000$
$Q(12000,12000)$	$= 7(12000) + 10(12000) = 204000$
$P(6000,6000)$	$= 7(6000) + 10(6000) = 102000$

So, maximum $Z = 246000$ at $x = 18000$, $y = 12000$

Question 10

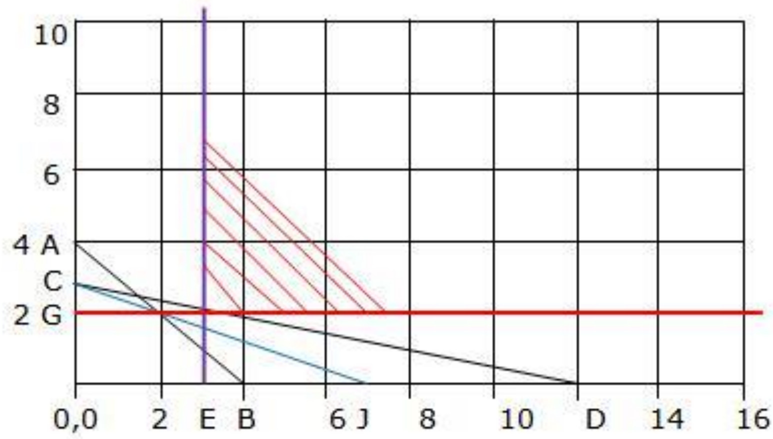
Solve the linear programming problem by graphical method.

Minimize $Z = 2x + 4y$

Subject to

$$x + y \ge 8$$
$$x + 4y \ge 12$$
$$x \ge 3,\ y \ge 2$$

Solution 10



$2x+2y \geq 8$;When $x=0$, $y=4$ & when $y=0$, $x=4$ line AB
$x+4y \geq 12$; When $x=0$, $y=3$ & when $y=0$, $x=12$ line CD
$x \geq 3$, $y \geq 2$ are the lines parallel to Y-axis and X-axis resp.

The diverging shaded area in red lines is the area of feasible solution. This area is unbounded.
$Z = 2x+4y$ @ (3,2) = 14.

Plot $2x+4y > 14$ line CJ to see if there is any common region. There is no common region so there is no optimal solution.

Question 11

Solve the linear programming problem by graphical method.

Minimize $Z = 5x + 3y$

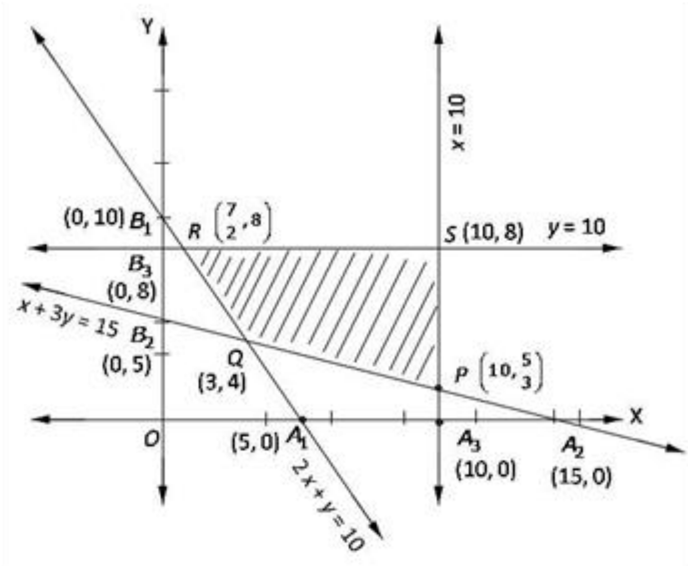
Subject to

$$
\begin{aligned}
2x + y &\geq 10 \\
x + 3y &\geq 15 \\
x &\leq 10 \\
y &\leq 8 \\
x, y &\geq 0
\end{aligned}
$$

Solution 11

Converting the given inequations into equations,

$$2x + y = 10,\ x + 3y = 15,\ x = 10,\ y = 8,\ x = y = 0$$



Region represented by $2x+y \geq 10$: Line $2x+y=10$ meets coordinate axes at $A_1(5,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $2x+y \geq 10$, so, region not containing origin represents $2x+y \geq 10$ in xy - plane.

Region represented by $x+3y \geq 15$: Line $x+3y=15$ meets coordinate axes at $A_2(15,0)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $x+3y \geq 15$, so, region not containing origin represents $x+3y \geq 15$ in xy - plane.

Region represented by $x \leq 10$: Line $x=10$ is parallel to y-axis and meet x-axis at $A_3(10,0)$. Clearly $(0,0)$ satisfies $x \leq 10$, so region containing origin represent $x \leq 10$ in xy-plane.

Region represented by $y \leq 8$: Line $y=8$ is parallel to x-axis and meet y-axis at $B_3(0,8)$, clearly $(0,0)$ satisfies $y \leq 8$, so region containing origin represent $y \leq 8$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

Shaded region $QPSR$ is the feasible region. $Q(3,4)$ is obtained by solving $2x+y=10$ and $x+3y=15$, $P\left(10,\frac{5}{3}\right)$ is obtained by solving $x+3y=15$ and $x=10$, $R\left(\frac{7}{2},8\right)$ is obtained by $2x+y=10$ and $y=8$.

The value of $Z=5x+3y$ at

$$P\left(10,\frac{5}{3}\right)=5(10)+3\left(\frac{5}{3}\right)=55$$
$$Q(3,4)=5(3)+3(4)=27$$
$$R\left(\frac{7}{2},8\right)=5\left(\frac{7}{2}\right)+3(8)=\frac{83}{2}=41\frac{1}{2}$$
$$S(10,8)=5(10)+3(8)=74$$

So,

Minimum $Z=27$ at $x=3$, $y=4$

Question 12

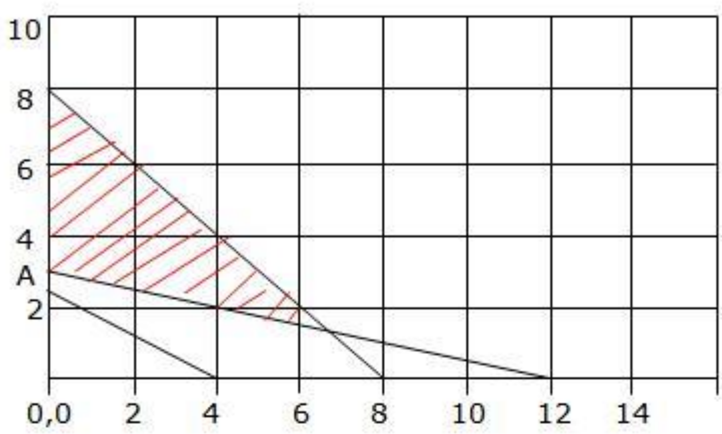
Solve the linear programming problem by graphical method.

Minimize $Z=30x+20y$

Subject to

$$x+y \leq 8$$
$$x+4y \geq 12$$
$$5x+8y=20$$
$$x, y \geq 0$$

Solution 12



x + y≤8 ; when x=0, y=8 & when y=0, x=8, line 8-8
x + 4y≥12; when x=0, y=3 & when y=0, x=12 line A-12
5x+8y=20; when x=0, y=5/2 & when y=0, x=4

The shaded area in red is the area of feasible solution.

Corner point	Value of Z = 30x + 20y
0, 3	60
0, 8	160
6.66, 1.33	226.66

The maxima is obtained at x=6.66, y=1.33 and is 226.66

Question 13 Old

Solve the linear programming problem by graphical method.

Maximize $Z = 4x + 3y$

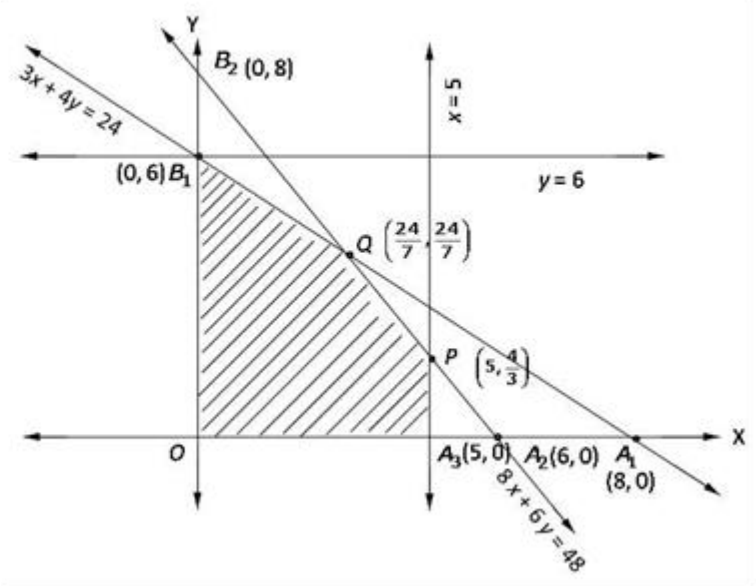
Subject to

$$3x + 4y \le 24$$
$$8x + 6y \le 48$$
$$x \le 5$$
$$y \le 6$$
$$x, y \ge 0$$

Solution 13 Old

Converting the given inequations into equations,

$$3x + 4y = 24,\ 8x + 6y = 48,\ x = 5,\ y = 6,\ x = y = 0$$



Region represented by $3x + 4y \le 24$: Line $3x + 4y = 24$ meets coordinate axes at $A_1(8, 0)$ and $B_1(0, 6)$, clearly $(0,0)$ satisfies $3x + 4y \le 24$, so region containing origin represents $3x + 4y \le 24$ in xy -plane.

Region represented by $8x + 6y \le 48$: Line $8x + 6y = 48$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,8)$. Clearly, $(0,0)$ satisfies $8x + 6y \le 48$, so region containing origin represents $8x + 6y \le 48$ in xy -plane.

Region represented $x \le 5$: Line $x = 5$ is parallel to y-axis and meets x-axis at $A_3(5, 0)$. Clearly $(0,0)$ satisfies $x \le 5$, so region containing origin represent $x \le 5$ in xy-plane.

Region represented by $y \le 6$: Line $y = 6$ is parallel to x-axis and meets y-axis at $B_1(0, 6)$. Clearly $(0,0)$ satisfies $y \le 6$, so, region containing origin represents $y \le 6$ in xy-plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy-plane.

So, shaded region QA_3PQB represents feasible region.

Coordinate of $P\left(5, \frac{4}{3}\right)$ is obtained by solving $8x + 6y = 48$ and $x = 5$, coordinate of $Q\left(\frac{24}{7}, \frac{24}{7}\right)$ is obtained by solving $3x + 4y = 24$ and $8x + 6y = 48$.

The value of $Z = 4x + 3y$ at

$$
\begin{array}{ll}
0(0, 0) & = 4(0) + 3(0) = 0 \\
A_3(5, 0) & = 4(5) + 3(0) = 20 \\
P\left(5, \frac{4}{3}\right) & = 4(5) + 3\left(\frac{4}{3}\right) = 24 \\
Q\left(\frac{24}{7}, \frac{24}{7}\right) & = 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = 24 \\
B_1(0, 6) & = 4(0) + 3(6) = 18
\end{array}
$$

So, maximum $Z = 24$ at $x = 5$, $y = \frac{4}{3}$ or $x = \frac{24}{7}$, $y = \frac{24}{7}$ or at every point joining PQ.

Question 14

Solve the linear programming problem by graphical method.

Minimize $Z = x - 5y + 20$

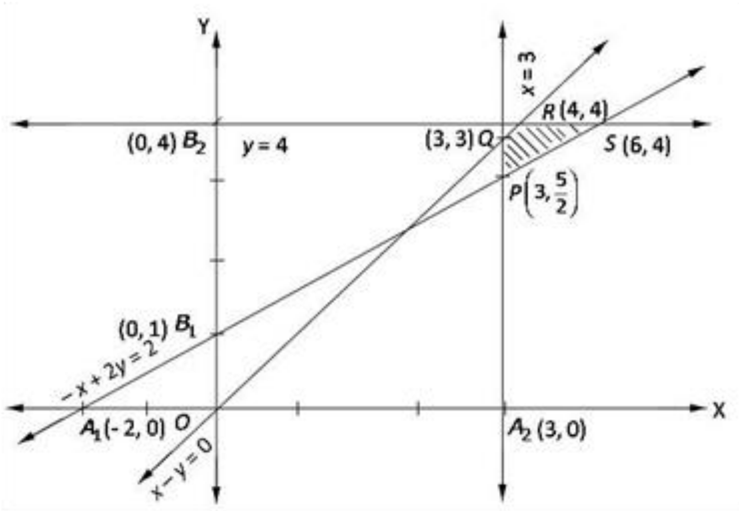
Subject to

$$\begin{aligned} x - y &\geq 0 \\ -x + 2y &\geq 2 \\ x &\geq 3 \\ y &\leq 4 \\ x, y &\geq 0 \end{aligned}$$

Solution 14

Converting the given inequations into equations,

$$x - y = 0,\ -x + 2y = 2,\ x = 3,\ y = 4,\ x = y = 0$$



Region represented by $x - y \geq 0$: $x - y = 0$ is a line passing through origin and $R(4, 4)$. Clearly, $(3,0)$ satisfies $x - y \geq 0$, so, region containing $(3,0)$ represents $x - y \geq 0$ in xy-plane.

Region represented by $-x + 2y \geq 2$: Line $-x + 2y = 2$ meets coordinate axes at $A_1(-2,0)$ and $B_1(0,1)$. Clearly, $(0,0)$ does not satisfy $-x + 2y \geq 2$, so, region not containing origin represents $-x + 2y \geq 2$ in xy -plane.

Region represented $x \geq 3$: Line $x = 3$ is parallel to y-axis and meets x-axis at $A_2(3, 0)$. Clearly, $(0,0)$ does not satisfy $x \geq 3$, so region not containing origin represent $x \geq 3$ in xy-plane.

Region represented by $y \leq 4$: Line $y = 4$ is parallel to x-axis and meets y-axis at $B_2(0, 4)$. Clearly $(0,0)$ satisfies $y \leq 4$, so region containing origin represents $y \leq 4$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

So, shaded region $PQRS$ represents feasible region.

The coordinate of $P\left(3, \frac{5}{2}\right)$ is obtained by solving $x = 3$ and $-x + 2y = 2$, $Q(3,3)$ by solving $x = 3$ and $x - y = 0$, $R(4, 4)$ by solving $x = 4$ and $x - y = 0$, $S(6, 4)$ by solving $y = 4$ and $-x + 2y = 2$

The value of $Z = x - 5y + 20$ at

$$P\left(3, \frac{5}{2}\right) = 3 - 5\left(\frac{5}{2}\right) + 20 = \frac{21}{2} = 11\frac{1}{2}$$

$$Q(3,3) = 3 - 5(3) + 20 = 8$$

$$R(4, 4) = 4 - 5(4) + 20 = 4$$

$$S(6, 4) = 6 - 5(4) + 20 = 6$$

Hence,

Minimum $Z = 4$ at $x = 4$ and $y = 4$

Question 15

Solve the linear programming problem by graphical method.

Maximize $Z = 3x + 5y$

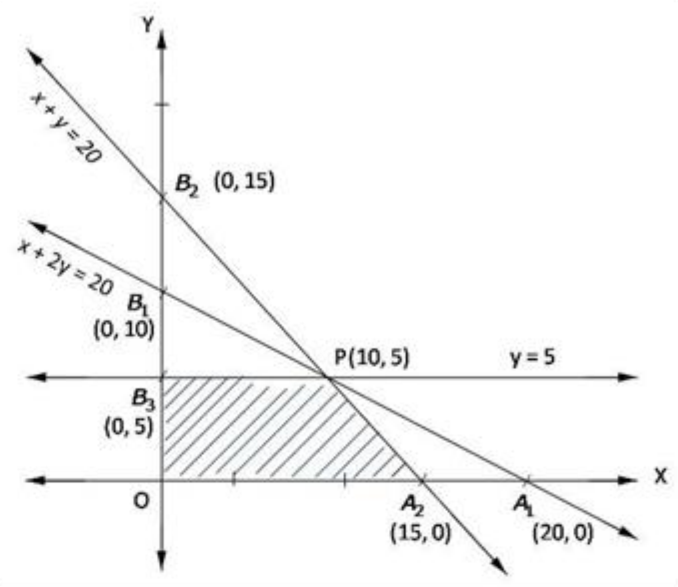
Subject to

$$x + 2y \leq 20$$
$$x + y \leq 15$$
$$y \leq 5$$
$$x, y \geq 0$$

Solution 15

Converting the given inequations into equations:-

$x + 2y = 20,\ x + y = 15,\ y = 5,\ x = y = 0$



Region represented by $x + 2y \le 20$: Line $x + 2y = 20$ meets coordinate axes at $A_1(20, 0)$ and $B_1(0, 10)$, clearly, $(0,0)$ satisfies $x + 2y \le 20$, so region containing origin represents $x + 2y \le 20$ in xy -plane.

Region represented by $x + y \le 15$: Line $x + y = 15$ meets coordinate axes at $A_2(15, 0)$ and $B_2(0, 15)$, clearly, $(0,0)$ satisfies $x + y \le 15$, so region containing origin represents $x + y \le 15$ in xy -plane.

Region represented by $y \le 5$: Line $y = 5$ is parallel to x-axis and meets at $B_3(0, 5)$ on y-axis. Clearly $(0,0)$ satisfies $y \le 5$, so region containing origin represents $y \le 5$ in xy-plane.

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane.

So, shaded region OA_2PB_3 represents the feasible region.

Coordinate of $P(10, 5)$ is obtained by solving $x + 2y = 20$ and $y = 5$.

The value of $Z = 3x + 5y$ at

$O(0,0) = 3(0) + 5(0) = 0$

$A_2(15,0) = 3(15) + 5(0) = 45$

$P(10,5) = 3(10) + 5(5) = 55$

$B_3(0,5) = 3(0) + 5(5) = 25$

Hence, maximum $Z = 55$ at $x = 10$ and $y = 5$

Question 16

Solve the linear programming problem by graphical method.

Minimize $Z = 3x_1 + 5x_2$

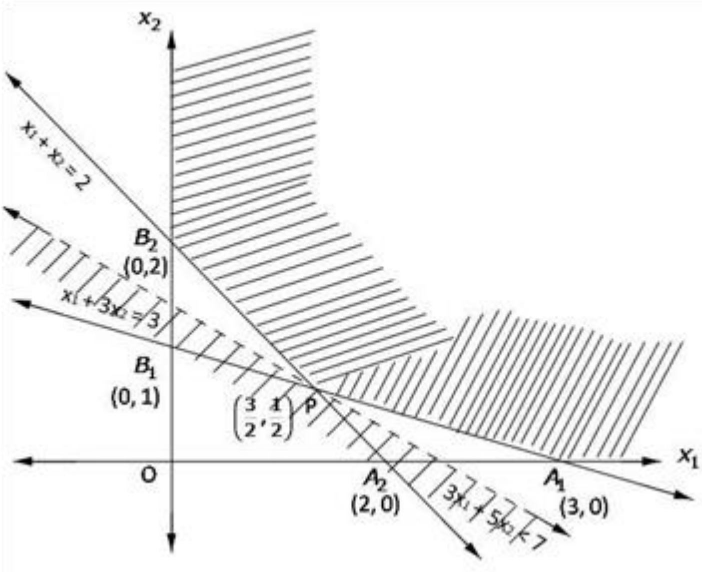
Subject to

$$x_1 + 3x_2 \geq 3$$
$$x_1 + x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

Solution 16

Converting the given inequations into equations,

$$x_1 + 3x_2 = 3,\ x_1 + x_2 = 2,\ x_1 = x_2 = 0$$



Region represented by $x_1 + 3x_2 \ge 3$: Line $x_1 + 3x_2 = 3$ meets the coordinate axes at $A_1(3,0)$ and $B_1(0,1)$, clearly, $(0,0)$ does not satisfy $x_1 + 3x_2 \ge 3$, so, region not containing $(3,0)$ represents $x_1 + 3x_2 \ge 3$ in x_1x_2-plane.

Region represented by $x_1 + x_2 \ge 2$: Line $x_1 + x_2 = 2$ meets the coordinate axes at $A_2(2,0)$ and $B_2(0,2)$, clearly, $(0,0)$ does not satisfy $x_1 + x_2 \ge 2$, so, region not containing origin represents $x_1 + x_2 \ge 2$ in x_1x_2-plane.

Region represented $x_1, x_2 \ge 0$: It represents the first quadrant in x_1x_2-plane.

The unbounded shaded region with corner points $A_1(3,0)$, $B_2(0,2)$, and $P\left(\frac{3}{2},\frac{1}{2}\right)$.

$P\left(\frac{3}{2},\frac{1}{2}\right)$ is obtained by $x_1 + x_2 = 2$ and $x_1 + 3x_2 = 3$.

The value of $Z = 3x_1 + 5x_2$ at

$$A_1(3,0) \quad = 3(3) + 5(0) = 9$$
$$P\left(\frac{3}{2},\frac{1}{2}\right) \quad = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = 7$$
$$B_2(0,2) \quad = 3(0) + 5(2) = 10$$

The smallest value of $Z = 7$,
region has no point in common, so smallest value is the minimum value.

Hence, minimum $Z = 7$ at $x = \frac{3}{2}$ and $y = \frac{1}{2}$

Question 17

Solve the linear programming problem by graphical method.

Maximize $Z = 2x + 3y$

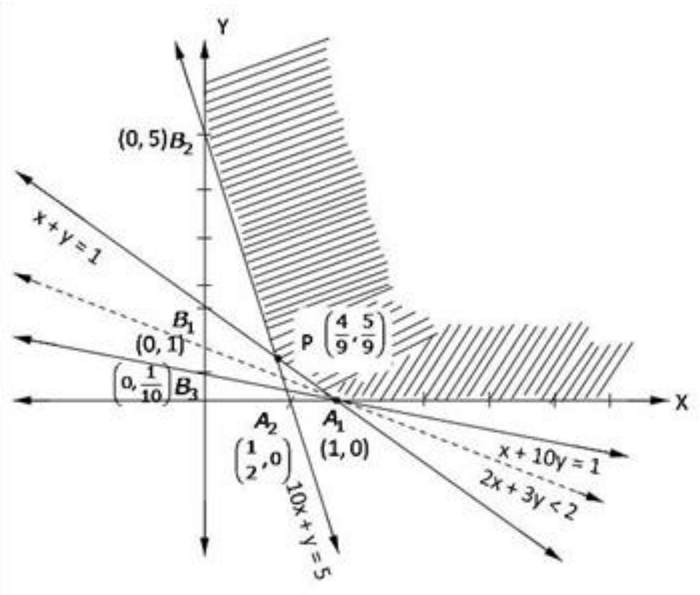
Subject to

$$\begin{aligned} x + y &\geq 1 \\ 10x + y &\geq 5 \\ x + 10y &\geq 1 \\ x, y &\geq 0 \end{aligned}$$

Solution 17

Converting the given inequations into equations

$$x + y = 1,\ 10x + y = 5,\ x + 10y = 1,\ x = y = 0$$



Region represented by $x + y \geq 1$: Line $x + y = 1$ meets coordinate axes at $A_1(1,0)$ and $B_1(0,1)$, dearly, $(0,0)$ does not satisfy $x + y \geq 1$, so region not containing origin represents $x + y \geq 1$ in xy -plane.

Region represented by $10x + y \geq 5$: Line $10x + y = 5$ meets coordinate axes at $A_2\left(\frac{1}{2}, 0\right)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $10x + y \geq 5$, so region not containing origin represents $10x + y \geq 5$ in xy -plane.

Region represented by $x + 10y \geq 1$: Line $x + 10y = 1$ meets coordinate axes $A_1(1,0)$ and $B_3\left(0, \frac{1}{10}\right)$. Clearly, $(0,0)$ does not satisfy $x + 10y \geq 1$, so, region not containing origin represents $x + 10y \geq 1$ in xy-plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

So, unbounded shaded represents feasible region. Its corner points are $A_1(1,0)$, $P\left(\frac{4}{9}, \frac{5}{9}\right)$ and $B_2(0,5)$.

The coordinate of $P\left(\frac{4}{9}, \frac{5}{9}\right)$ is obtained by solving $10x + y = 5$ and $x + y = 1$.

The value of $Z = 2x + 3y$ at

$$\begin{aligned}
A_1(1,0) \quad &= 2(1) + 3(0) = 2 \\
P\left(\frac{4}{9}, \frac{5}{9}\right) \quad &= 2\left(\frac{4}{8}\right) + 3\left(\frac{5}{9}\right) = \frac{23}{9} = 2\frac{5}{9} \\
B_2(0,5) \quad &= 2(0) + 3(5) = 15
\end{aligned}$$

The smallest value of Z is 2. Now, open half plane $2x + 3y < 2$ has no point in common with feasible region so, smallest value of Z is the minimum value.

Hence, maximum $Z = 2$ at $x = 1$ and $y = 0$

Question 18

Solve the linear programming problem by graphical method.

Maximize $Z = -x_1 + 2x_2$

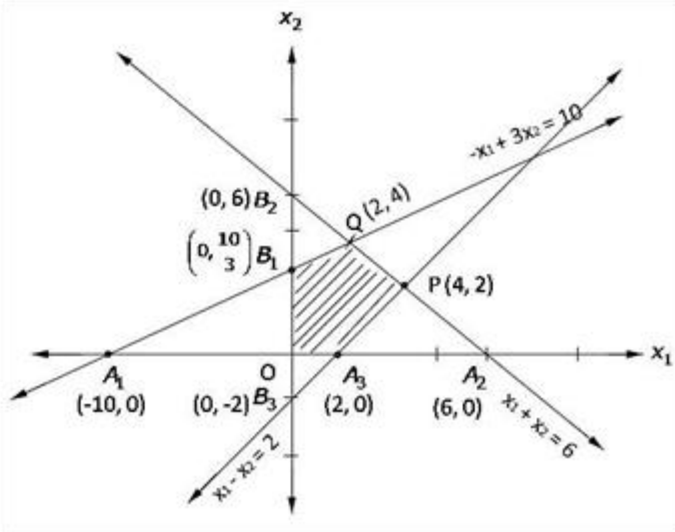
Subject to

$$\begin{aligned}
-x_1 + 3x_2 &\leq 10 \\
x_1 + x_2 &\leq 6 \\
x_1 - x_2 &\leq 2 \\
x_1, x_2 &\geq 0
\end{aligned}$$

Solution 18

Converting the given inequations into equations,

$$-x_1 + 3x_2 = 10,\ x_1 + x_2 = 6,\ x_1 = x_2 = 2,\ x_1 = x_2 = 0$$



Region represented by $-x_1 + 3x_2 \le 10$: Line $-x_1 + 3x_2 = 10$ meets coordinate axes at $A_1(-10,0)$ and $B_1\left(0,\frac{10}{3}\right)$, clearly, $(0,0)$ satisfies $-x_1 + 3x_2 \le 10$, so region containing origin represents $-x_1 + 3x_2 \le 10$ in x_1x_2 -plane.

Region represented by $x_1 + x_2 \le 6$: Line $x_1 + x_2 = 6$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,6)$. Clearly, $(0,0)$ satisfies $x_1 + x_2 \le 6$, so region containing origin represents $x_1 + x_2 \le 6$ in x_1x_2 -plane.

Region represented by $x_1 - x_2 \le 2$: Line $x_1 - x_2 = 2$ meets coordinate axes at $A_3(2,0)$ and $B_3(0,-2)$. Clearly, $(0,0)$ satisfies $x_1 - x_2 \le 2$, so, region containing origin represents $x_1 - x_2 \le 2$ in x_1x_2 -plane.

Region represented $x_1, x_2 \ge 0$: It represents first quadrant in x_1x_2-plane.

So, shaded region OA_3PQB, represents feasible region.

Coordinate of $P(4,2)$ is obtained by solving $x_1 + x_2 = 6$ and $x_1 - x_2 = 2$, $Q(2,4)$ by solving $x_1 + x_2 = 6$ and $-x_1 + 3x_2 = 10$

The value of $Z = -x_1 + 2x_2$ at

$$
\begin{aligned}
&O(0,0) && = -(0) + 2(0) = 0 \\
&A_3(2,0) && = -(2) + 2(0) = -2 \\
&P(4,2) && = -(4) + 2(2) = 0 \\
&Q(2,4) && = -(2) + 2(4) = 6 \\
&B_1\left(0,\frac{10}{3}\right) && = -(0) + 2\left(\frac{10}{3}\right) = \frac{20}{3} = 6\frac{2}{3}
\end{aligned}
$$

Hence, maximum $Z = \frac{20}{3}$ at $x = 0$ and $y = \frac{10}{3}$

Question 19

Solve the linear programming problem by graphical method.

Maximize $Z = x + y$

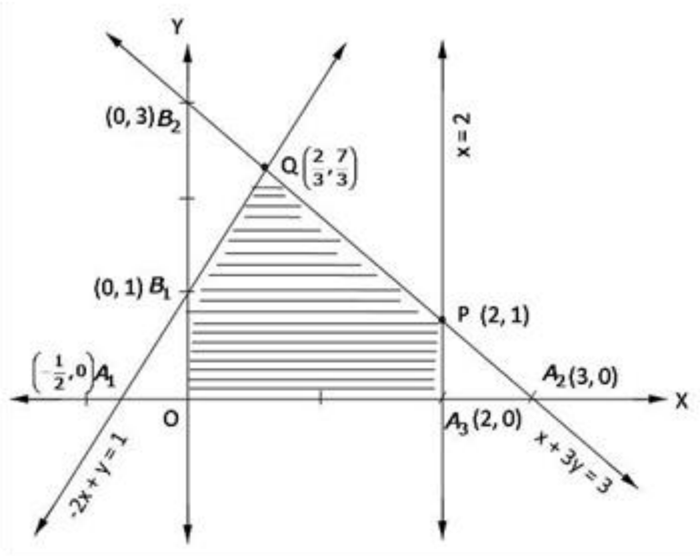
Subject to

$$
\begin{aligned}
-2x + y &\le 1 \\
x &\le 2 \\
x + y &\le 3 \\
x, y &\ge 0
\end{aligned}
$$

Solution 19

Converting the given inequations into equations,

$$-2x + y = 1,\ x = 2,\ x + y = 3,\ x = y = 0$$



Region represented by $-2x + y \le 1$: Line $-2x + y = 1$ meets coordinate axes at $A_1\left(\frac{-1}{2}, 0\right)$ and $B_1(0,1)$, clearly, $(0,0)$ satisfies $-2x + y \le 1$, so region containing origin represents $-2x + y \le 1$ in xy-plane.

Region represented by $x \le 2$: Line $x = 2$ is parallel to y-axis and meets x-axis at $A_3(2,0)$. Clearly, $(0,0)$ satisfies $x \le 2$, so region containing origin represents $x \le 2$ in xy-plane.

Region represented by $x + y \le 3$: Line $x + y = 3$ meets coordinate axes at $A_2(3,0)$ and $B_2(0,3)$. Clearly, $(0,0)$ satisfies $x + y \le 3$, so region containing origin represents $x + y \le 3$ in xy-plane.

Region represented by $x, y \ge 0$: It represents first quadrant in xy-plane.

So, shaded region OA_3PQB, represents the feasible region.

Coordinates of $P(2,1)$ is obtained by solving $x + y = 3$ and $x = 2$, $Q\left(\frac{2}{3}, \frac{7}{3}\right)$ by solving $-2x + y = 1$ and $x + y = 3$.

The value of $Z = x + y$ at

$$O(0,0) = 0 + 0 = 0$$
$$A_3(2,0) = 2 + 0 = 2$$
$$P(2,1) = 2 + 1 = 2$$
$$Q\left(\frac{2}{3}, \frac{7}{3}\right) = \frac{2}{3} + \frac{7}{3} = 3$$
$$B_1(0,1) = 0 + 1 = 1$$

So, maximum $Z = 3$ is at every point on the line joining PQ.

Hence, maximum $Z = 3$ at $x = 2$ and $y = 1$ Or $x = \frac{2}{3}$ and $y = \frac{7}{3}$

Question 20

Solve the linear programming problem by graphical method.

Maximize $Z = 3x_1 + 4x_2$, if possible,

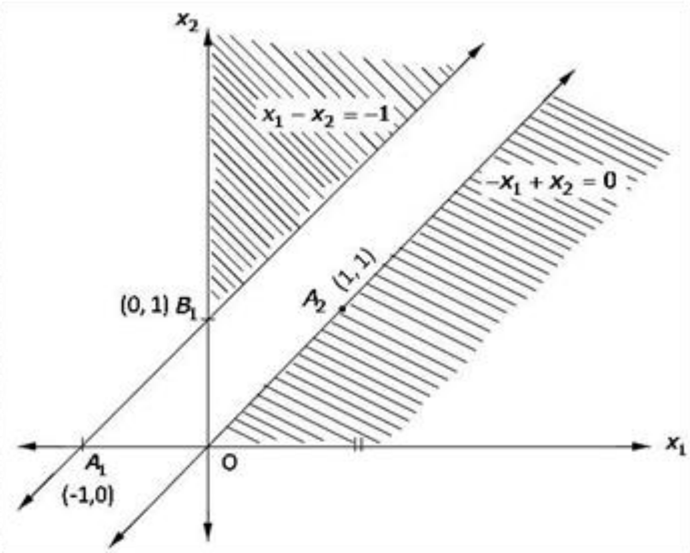
Subject to the constraints

$$x_1 - x_2 \le -1$$
$$-x_1 + x_2 \le 0$$
$$x_1, x_2 \ge 0$$

Solution 20

Converting the given inequations into equations,

$$x_1 - x_2 = -1,\ -x_1 + x_2 = 0,\ x_1 = x_2 = 0$$



Region represented by $x_1 - x_2 \leq -1$: Line $x_1 - x_2 = -1$ meets coordinate axes at $A_1\left(-1, 0\right)$ and $B_1\left(0, 1\right)$, clearly, $\left(0,0\right)$ does not satisfy $x_1 - x_2 \leq -1$, so region not containing origin represents $x_1 - x_2 \leq -1$ in x_1x_2 -plane.

Region represented by $-x_1 + x_2 \leq 0$: Line $-x_1 + x_2 = 0$ passes through origin and $A_2\left(1,1\right)$. Clearly, $\left(0,0\right)$ does not satisfy $-x_1 + x_2 \leq 0$, so, region not containing $\left(0,1\right)$ represents $-x_1 + x_2 \leq 0$ in x_1x_2 -plane.

Since, there is not common shaded region represented by $x_1 - x_2 \leq -1$ and $-x_1 + x_2 \leq 0$ which can form feasible region.

Hence, maximum $Z = 3x_1 + 4x_2$ does not exists.

Question 21

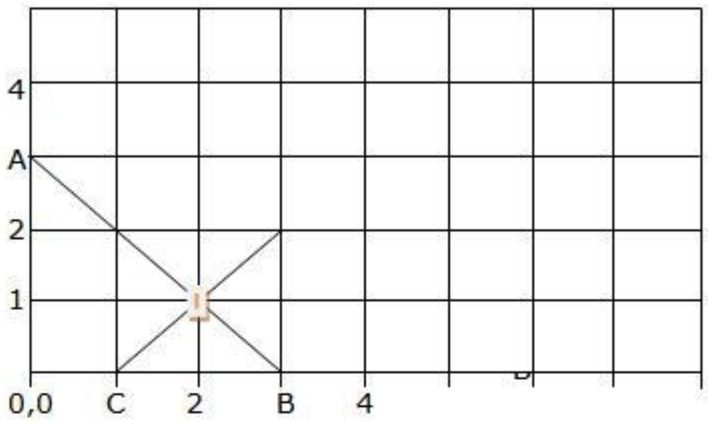
Solve the linear programming problem by graphical method.

Maximize $Z = 3x + 3y$, if possible,

Subject to the constraints

$$x - y \leq 1$$
$$x + y \geq 3$$
$$x, y \geq 0$$

Solution 21



x-y≤1; when x = 0, y=1 & when y=0, x=2
x + y≥3; when x = 0, y=3 & when y=0, x=3, line AB
a unbounded region A-C-D is obtained using the constraints.

Corner point	Value of Z = 3x + 3y
0, 3	9
2, 1	9

So an optimal solution does not exist.

Question 22

Show the solution one of the following inequalities on a graph paper:

$5x + y \geq 10$

$x + y \geq 6$

$x + 4y \geq 12$

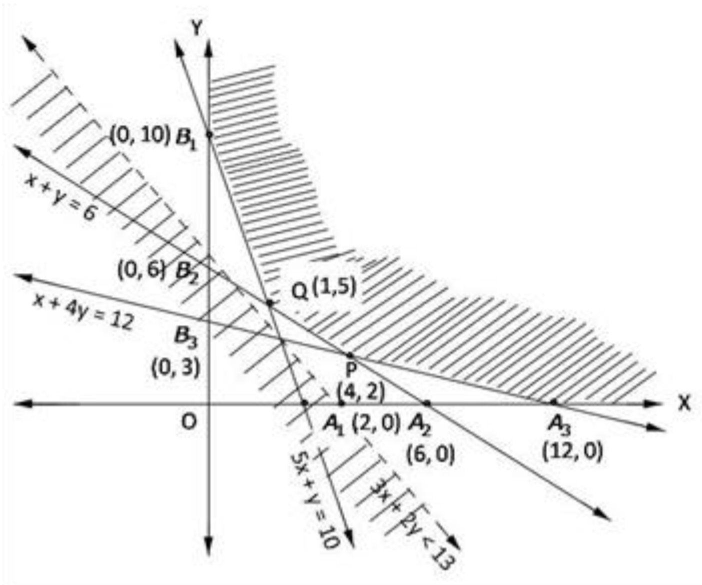
$x \geq 0, y \geq 0.$

Find x and y for which $3x + 2y$ is minimum subject to these inequalities. Use a graphical method.

Solution 22

Converting the given inequations into equations

$5x + y = 10,\ x + y = 6,\ x + 4y = 12,\ x = y = 0$



Region represented by $5x + y \geq 10$: Line $5x + y = 10$ meets coordinate axes at $A_1(2,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $5x + y \geq 10$, so region not containing origin represents $5x + y \geq 10$ in xy -plane.

Region represented by $x + y \geq 6$: Line $x + y = 6$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,6)$. Clearly, $(0,0)$ does not satisfy $x + y \geq 6$, so region not containing origin represents $x + y \geq 6$ in xy -plane.

Region represented by $x + 4y \geq 12$: Line $x + 4y = 12$ meets coordinate axes at $A_3(12,0)$ and $B_3(0,3)$. Clearly, $(0,0)$ does not satisfy $x + 4y \geq 12$, so, region not containing origin $x + 4y \geq 12$ in xy -plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

The unbounded shaded region with corner points $A_3(12,0)$, $P(4,2)$, $Q(1,5)$, $B_1(0,10)$ represents feasible region. Point P is obtained by solving $x + 4y = 12$ and $x + y = 6$, Q by solving $x + y = 6$ and $5x + y = 10$.

The value of $Z = 3x + 2y$ at

$A_3(12,0) \quad = 3(12) + 2(0) = 36$

$P(4,2) \quad = 3(4) + 2(2) = 16$

$Q(1,5) \quad = 3(1) + 2(5) = 13$

$B(0,10) \quad = 3(0) + 2(10) = 20$

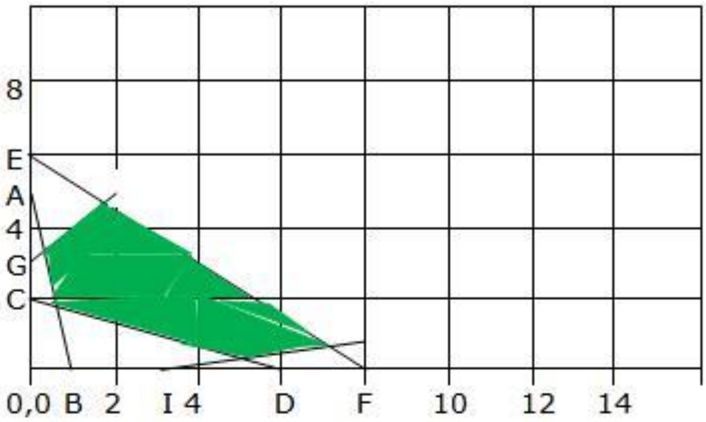
Smallest value of $Z = 13$, Now open half plane $3x + 2y < 13$ has no point in common with feasible region, so, smallest value is the minimum value of Z, Hence

Minimum $Z = 13$ at $x = 1$, $y = 5$

Question 23

Find the maximum and minimum value of $2x + y$ subject to the constraints:
$x + 3y \geq 6$, $x - 3y \leq 3$, $3x + 4t \leq 24$, $-3x + 2y \leq 6$, $5x + y \geq 5$, $x, y \geq 0$.

Solution 23



$x+3y \geq 6$; or $y=-0.333x+2$; when $x=0$, $y=2$ & when $y=0$, $x=6$; line CD
$x-3y \leq 3$; or $y=0.333x-1$; when $x=0$, $y=-1$ & when $y=0$, $x=3$; line IJ
$3x+4y \leq 24$; or $y=-0.75x+6$; when $x=0$, $y=6$ & when $y=0$, $x=8$; line EF
$-3x+2y \leq 6$; or $y= 1.5x+3$; when $x=0$, $y=3$ & when $y=0$, $x=-2$;line GH
$5x+y \geq 5$; or $y=-5x+5$; when $x=0$, $y=5$ & when $y=0$, $x=1$; line AB

The feasible area is shaded in green

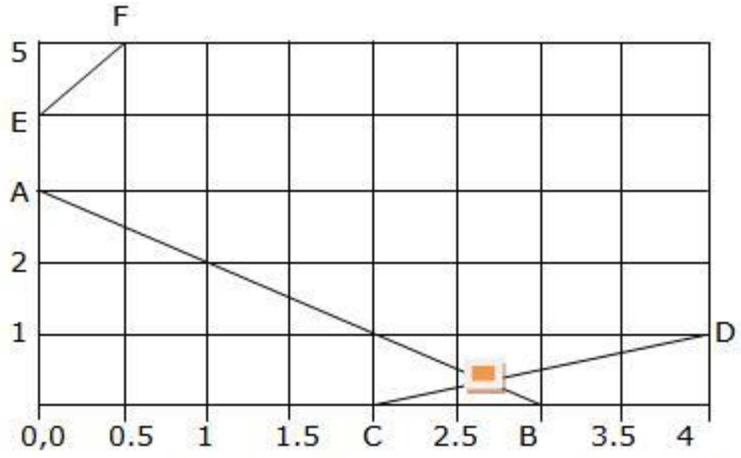
Corner point	Value of Z = 2x + y	
4.5, 0.5		9.5
0.64, 1.78		3.07
6.46, 1.15	Maximum	14.07
1.33, 5		7.6667
0.30, 3.46		4.0769

Maximum value is 14.07 at the point (6.46, 1.15)
Minimum value is 3.07 at the point (0.64, 1.78)

Question 24

Find the minimum value of $3x + 5y$ subject to the constraints
$-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x, y \geq 0$.

Solution 24



-2x+y≤4; or y=2x+4; when x=0, y=4 & when y=0, x=-2 line EF

x+y≥3; or y=-x+3; when x=0, y=3 & when y=0, x=3; line AB

x-2y≤2; or y=0.5x-1; when x=0, y=-1 & when y=0,

x=2 line CD

The feasible solution is the unbounded area with F-E-A-G-D

Corner point	Value of Z = 3x + 5y	
(2.67, 0.33)	Minimum	9.66
(0, 3)		15
(0, 4)		20

To check whether it is the minimal value plot the objective function with a value less than 9.66 or y=-0.6x-1.932

it can be seen that the values of x and y are always negative. So there is no optimal solution.

Question 25

Solve the following linear programming problem graphically:

Maximize $Z = 60x + 15y$

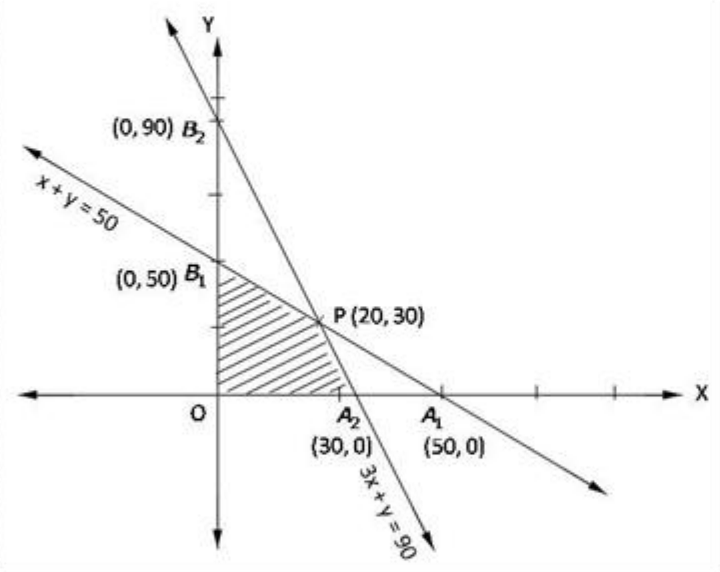
Subject to constraints

$$x + y \le 50$$
$$3x + y \le 90$$
$$x, y \ge 0$$

Solution 25

Converting the given inequations into equations,

$x + y = 50,\ 3x + y = 90,\ \ x = y = 0$



Region represented by $x + y \le 50$: Line $x + y = 50$ meets coordinate axes at $A_1(50, 0)$ and $B_1(0, 50)$. Clearly, $(0, 0)$ satisfies $x + y \le 50$, so, region containing origin represents $x + y \le 50$ in xy -plane.

Region represented by $3x + y \le 90$: Line $3x + y = 90$ meets coordinate axes at $A_2(30, 0)$ and $B_2(0, 90)$. Clearly, $(0, 0)$ satisfies $3x + y \le 90$, so, region containing origin represents $3x + y \le 90$ in xy -plane.

Region represented by $x, y \ge 0$: It represents first quadrant in xy-plane.

Shaded region OA_2PB_1 represents the feasible region. $P(20, 30)$ can be obtained by solving $x + y = 50$ and $3x + y = 90$.

The value of $Z = 60x + 15y$ at

$O(0,0) \quad = 60(0) + 15(0) = 0$

$A_2(30,0) \quad = 60(30) + 15(0) = 1800$

$P(20,30) \quad = 60(20) + 15(30) = 1650$

$B_1(0,50) \quad = 60(0) + 15(50) = 750$

Hence,

maximum Z is 1800 at $x = 30$ and $y = 0$.

Question 26

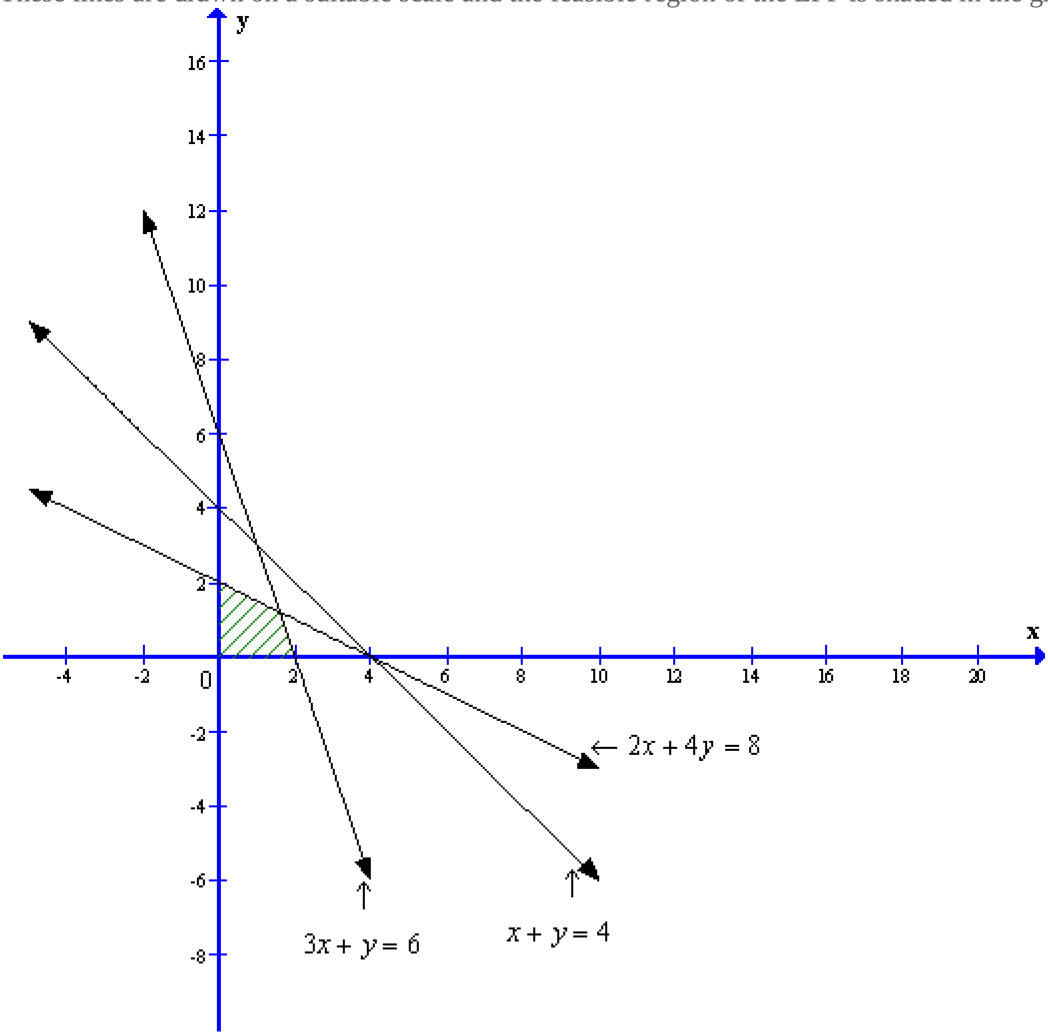
Find graphically, the maximum value of z = 2x + 5y, subject to constraints given below:

2x + 4y £ 8

3x + y £ 6
X + y £ 4
X ³ 0, y ³ 0

Solution 26

Converting the inequations into equations, we obtain the lines
2x + 4y = 8, 3x + y = 6, x + y = 4, x = 0, y = 0.
These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in the graph.



From the graph we can see the corner points as (0, 2) and (2, 0).

Now solving the equations 3x + y = 6 and 2x + 4y = 8 we get the values of x and y as $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Substituting $x = \frac{8}{5}$ and $y = \frac{6}{5}$ in Z = 2x + 5y we get,

$$Z = 2\left(\frac{8}{5}\right) + 5\left(\frac{6}{5}\right)$$

$$Z = \frac{46}{5}$$

Hence maximum value of Z is $\frac{46}{5}$ at $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Chapter 30 - Linear programming Exercise Ex. 30.3

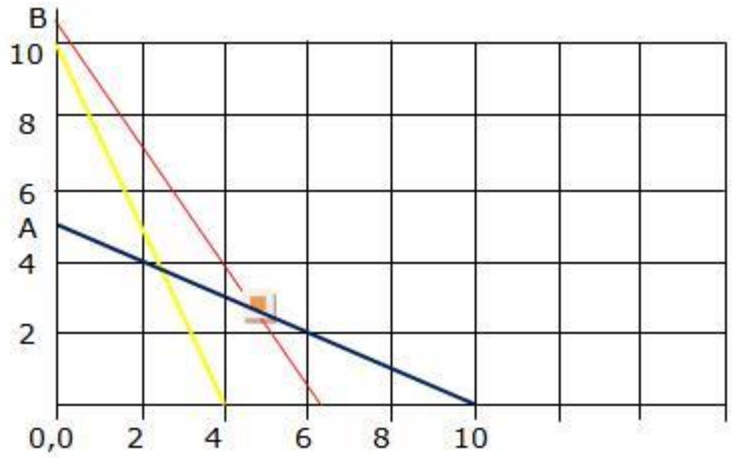
Question 1

A diet of two foods F_1 and F_2 contains nutrients thiamine, phosphorous and iron. The amount of each nutrient in each of the food (in milligrams per 25 gms) is given in the following table:

Food / Nutrients	F_1	F_2
Thiamine	**0.25**	**0.10**
Phosphorous	**0.75**	**1.50**
Iron	**1.60**	**0.80**

The minimim requirement of the nutrients in the diet are 1.00 mg of thiamine, 7.50 mg of phosphorous and 10.00 mg of iron. The cost of F_1 is 20 paise per 25 gms while the cost of F_2 is 15 paise per 25 gms. Find the minimim cost of diet.

Solution 1



Let x and y be the No. of 25 gm packets of foods F_1 and F_2

Minimum cost of diet $Z = 0.20x + 0.15y$

The constraints are

$0.25x + 0.1y \geq 1$; when x=0, y=10 & y=0, x=4 10-4

$0.75x + 1.5y \geq 7.5$; when x=0, y=5 & y=0, x=10 A-10

$1.6x + 0.8y \quad \geq 10$; when x=0, y=25/2 & y=0, x=25/4

The feasible region is the open region B-E-10

The minimum cost of the diet can be checked by finding the value of Z at corner points B, E & 10

Corner point	Value of Z = 20x + 15y
0, 12.5	187.5
10, 0	200
5, 2.5	137.5

Since the feasible region is an open region so we plot $20x + 15y < 137.5$, to check whether the resulting open half plane has any point common with the feasible region. Since it has common points $Z = 20\,x + 15\,y$

There is no optimal minimum value subject to the given constraints.

Question 2

A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods *A* and *B*, are available at a cost of Rs 4 and Rs 3 per unit respectively. If one unit of *A* contains 200 units of vitamin, 1 units of mineral and 40 calories and one unit of food B contains 100 units of vitamin, 2 units of minerals and 40 calories, find what combination of foods should be used to have the least cost?

Solution 2

Let required quantity of food A and B be x and y units respectively.

Costs of one unit of food A and B are Rs 4 and Rs 3 per unit respectively, so, costs of x unit of food A and y unit of food B are $4x$ and $3y$ respectively. Let Z be minimum total cost, so

$$Z = 4x + 3y$$

Since one unit of food A and B contain 200 and 100 units of vitamin respectively. So, x units of food A and y units of food B contain $200x$ and $100y$ units of vitamin but minimum requirement of vitamin is 4000 units, so

$$200x + 100y \ge 4000$$

$$\Rightarrow \quad 2x + y \ge 40 \quad (\text{first constraint})$$

Since one unit of food A and B contain 1 unit and 2 unit of minerals, so x units of food A and y units of food B contain x and $2y$ units of minerals respectively but minimum requirement of minerals is 50 units, so

$$x + 2y \ge 50 \quad (\text{second constraint})$$

Since one unit of food A and B contain 40 calories each, so x units of food A and y units of food B contain $40x$ and $40y$ calories respectively but minimum requirement of calories is 1400, so

$$40x + 40y \ge 1400$$

$$\Rightarrow \quad 2x + 2y \ge 70$$

$$\Rightarrow \quad x + y \ge 35 \quad (\text{third constraint})$$

So, mathematical formulation of LPP is find x and y which

minimize $Z = 4x + 3y$

Subject to constraint,

$$2x + y \ge 40$$

$$x + 2y \ge 50$$

$$x + y \ge 35$$

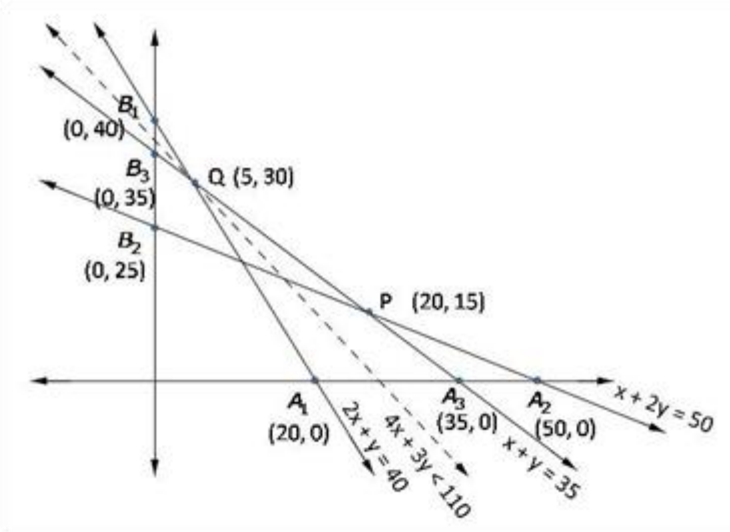
$$x, y \ge 0 \quad [\text{Since quantity of food can not be less than zero}]$$

Region $2x + y \ge 40$: Line $2x + y = 40$ meets axes at $A_1(20,0)$, $B_1(0,40)$ region not containing origin represents $2x + y \ge 40$ as $(0,0)$ does not satisfy $2x + y \ge 40$.

Region $x + 2y \ge 50$: Line $x + 2y = 50$ meets axes at $A_2(50,0)$, $B_2(0,25)$. Region not containing origin represents $x + 2y \ge 50$ as $(0,0)$ does not satisfy $x + 2y \ge 50$.

Region $x + y \geq 35$: Line $x + y = 35$ meets axes at $A_3(35,0)$, $B_3(0,35)$. Region not containing origin represents $x + y \geq 35$ as $(0,0)$ does not satisfy $x + y \geq 35$.

Region $x, y \geq 0$: It represent first quadrant in xy-plane.



Unbounded shaded region A_2PQB_1 represents feasible region with corner points $A_2(50,0)$, $P(20,15)$, $Q(5,30)$, $B_1(0,40)$

The value of $Z = 4x + 3y$ at

$A_2(50,0) \quad = 4(50) + 3(0) = 2000$

$P(20,15) \quad = 4(20) + 3(15) = 125$

$Q(5,30) \quad = 4(5) + 3(30) = 110$

$B_1(0,40) \quad = 4(0) + 3(40) = 110$

Smallest value of $Z = 110$

Open half plane $4x + 3y < 110$ has no point in common with feasible region, so, smallest value is the minimum value.

Hence,

quantity of food $A = x = 5$ unit

quantity of food $B = y = 30$ unit

minimum cost $=$ Rs 110

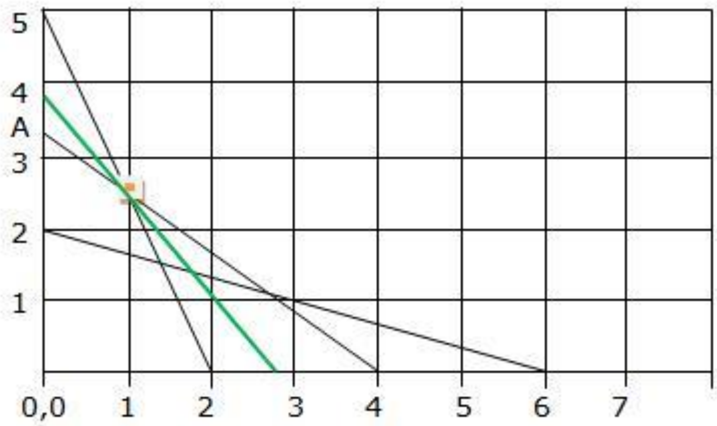
Question 3

To mantain one's health, a person must fulfil certain minimum daily requirements for the following three nutrients-calcium, protien and calories. This diet consists of only items I and II whose prices and nutrient contents are shown below:

	Food I	Food II	Minimum daily Requirement
Calcium	10	4	20
Protein	5	6	20
Calories	2	6	12
Price	Re 0.60 per unit	Re 1.00 per unit	

Find the combination of food items so that the cost may be minimum.

Solution 3



Let x & y be the units of Food I and Food II resptly.

The objective function is to minimize the function
$Z = 0.6x + y$ such that
$10x + 4y \geq 20$ requirement of calcium, line 5-2
$5x + 6y \geq 20$ requirement of protein, line A-4
$2x + 6y \geq 12$ requirement of calories, line 2-6
These when plotted give 5-F-E-6 an open unbounded region.

The function $20x + 15y < 57.5$ needs to be plotted to check if there are any common points. The green line shows that there are no common points. So

Corner point	Value of Z = 0.6x +y
0, 5	5
F(1, 2.5)	3.1
E(2.67, 1.11)	2.71
6, 0	3.6

The minimum cost occurs when Food I is 1 unit and Food II is 2.5 units. Since it is an unbounded region plotting Z<3.1 gives the green line which has no common points, so (1, 2.5) can be said to be a minimum point.

Question 4

A hospital dietician wishes to find the cheapest combination of two foods, *A* and *B*, that containst at least 0.5 milligram of thiamin and at least 600 calories. Each unit of *A* contains 0.12 milligram of thiamin and 100 calories, while each unit of *B* contains 0.10 milligram of thiamin and 150 calories. If each food costs 10 paise per unit,how many units of each should be combined at a minimum cost?

Solution 4

Let required quantity of food A and food B be x and y units.

Given, costs of one unit of food A and B are 10 paise per unit each, so costs of x unit of food A and y unit of food B are $10x$ and $10y$ respectively, let Z be total cost of foods, so

$$Z = 10x + 10y$$

Since one unit of food A and B contain 0.12 mg and 0.10 mg of Thiamin respectively, so, x units of food A and y units of food B contain $0.12x$ mg and $0.10y$ mg of Thiamin respectively but minimum requirement of Thiamin is 0.4 mg, so

$$0.12x + 0.10y \geq 0.5$$

$$\Rightarrow \quad 12x + 10y \geq 50$$

$$\Rightarrow \quad 6x + 5y \geq 25 \qquad \text{(first constraint)}$$

Since one unit of food A and B contain 100 and 150 Calories respectively, so x units of food A and y units of food B contain $100x$ and $150y$ units of Calories but minimum requirement of Calories is 600, so

$$100x + 150y \geq 600$$

$$\Rightarrow \quad 2x + 3y \geq 12 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

minimize $Z = 10x + 10y$

Subject to constraint,

$$6x + 5y \geq 25$$

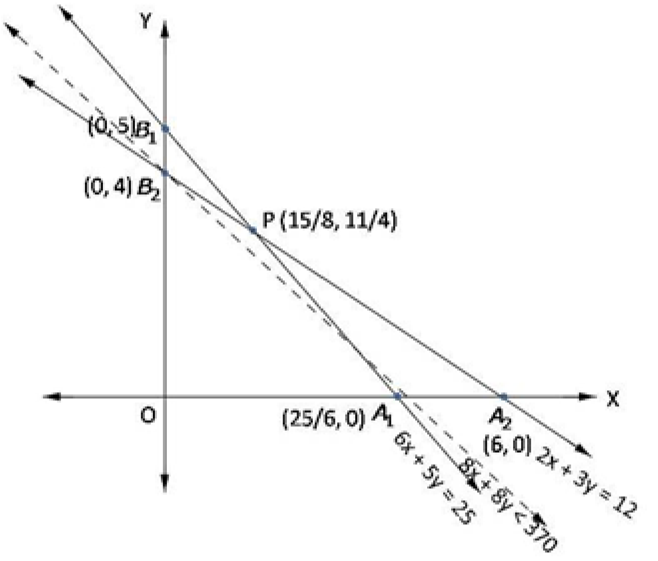
$$2x + 3y \geq 12$$

$$x, y \geq 0 \qquad \left[\text{Since quantity of food } A \text{ and } B \text{ can not be less than zero}\right]$$

Region $6x + 5y \geq 25$: $6x + 5y = 25$ meets axes at $A_1\left(\frac{25}{6}, 0\right)$, $B_1(0,5)$. Region not containing origin represents $6x + 5y \geq 25$ as $(0,0)$ does not satisfy $6x + 5y \geq 25$.

Region $2x + 3y \geq 12$: Line $2x + 3y = 12$ meets axes at $A_2(6,0)$, $B_2(0,4)$. Region not containing origin represents $2x + 3y \geq 12$ as $(0,0)$ does not satisfy $2x + 3y \geq 12$.

Region $x, y \geq 0$ represent first quadrant in xy-plane.



Unbounded shaded region A_2PB_1 represents feasible region with corner points $A_2(6,0)$, $P\left(\frac{15}{8},\frac{11}{4}\right)$, $B_1(0,5)$

The value of $Z = 10x + 10y$ at

$$A_2(6,0) \quad = 10(6) + 10(0) = 60$$
$$P\left(\frac{15}{8},\frac{11}{4}\right) \quad = 10\left(\frac{15}{8}\right) + 10\left(\frac{11}{4}\right) = \frac{370}{8} = 46\frac{1}{4}$$
$$B_1(0,5) \quad = 10(0) + 10(5) = 50$$

Smallest value of Z is $46\frac{1}{4}$.

Now open half plane $10x + 10y < \frac{370}{8}$

$\Rightarrow$ $8x + 8y < 370$ has no point in common with feasible region, so smallest value is the minimum value.

Hence,

Required quantity of food $A = \frac{15}{8}$ units, food $B = \frac{11}{4}$ units

minimum cost = Rs 46.25

$$\Rightarrow \frac{15}{8} = 1.875 \textit{ units of food A and } \frac{11}{4} = 2.75 \textit{ units of B}$$

Question 5

A dietcian mixes together two kinds of food in such a way that the mixture contains at least 6 units of vitamin *A*, 7 unit of vitamin *B*, 11 units of vitamin *C*, and 9 units of vitamin *D*. The vitamin contents of 1 kg of food *X* and 1 kg of food *Y* are given below:

	Vitamin *A*	**Vitamin *B***	**Vitamin *C***	**Vitamin *D***
Food *X*	**1**	**1**	**1**	**2**
Food *Y*	**2**	**1**	**3**	**1**

One kg of food X costs Rs 5, whereas one kg of food Y costs Rs 8. Find the least cost of the mixture which will produce the desired diet.

Solution 5

Let required quantity of food X and food Y be x kg and y kg.

Since costs of food X and Y are Rs 5 and Rs 8 per kg., So, costs of food X and food Y are Rs. $5x$ and Rs. $8y$ respectively. Let Z be the total cost of food, then

$$Z = 5x + 8y$$

Since one kg of food X and Y contain 1 and 2 unit of vitamin A, so, x kg of food X and y kg of food Y contain x and $2y$ units of vitamin A respectively but minimum requirement of vitamin A is 6 units, so

$$x + 2y \geq 6 \qquad \text{(first constraint)}$$

Since one kg of food X and Y contain 1 unit of vitamin B each, so x kg of food X and y kg of food Y contain x and y units of vitamin B but minimum requirement of vitamin B is 7 units, so

$$x + y \geq 7 \qquad \text{(second constraint)}$$

Since one kg of food X and food Y contain 1 unit and 3 units of vitamin C respectively, so x kg of food X and y kg of food Y contain x and $3y$ units of vitamin C respectively but minimum requirement of vitamin C is 11 units, so

$$x + 3y \geq 11 \qquad \text{(third constraint)}$$

Since 1 kg of food X and food Y contain 2 units and 1 unit of vitamin D respectively, so, x kg of food X and y kg of food Y contain $2x$ and y units of vitamin D respectively but minimum requirement of vitamin D is 9 units, so

$$2x + y \geq 9 \qquad \text{(fourth constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

minimize $Z = 5x + 8y$

Subject to constraints,

$$x + 2y \geq 6$$
$$x + y \geq 7$$
$$x + 3y \geq 11$$
$$2x + y \geq 9$$
$$x, y \geq 0 \qquad \left[\text{Since quantity of food } X \text{ and } Y \text{ can not be less than zero}\right]$$

Region $x + 2y \geq 6$: Line $x + 2y = 6$ meets axes at $A_1(6,0)$, $B_1(0,3)$. Region not containing origin represents $x + 2y \geq 6$ as $(0,0)$ does not satisfy $x + 2y \geq 6$.

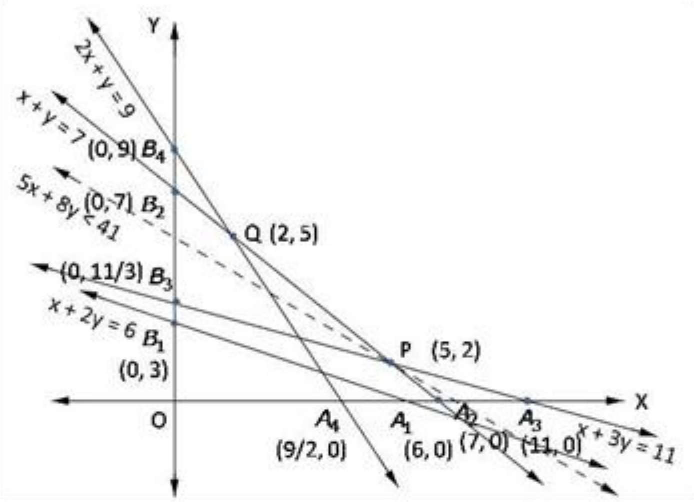
Region $x + y \geq 7$: Line $x + y = 7$ meets axes at $A_2(7,0)$, $B_2(0,7)$ respectively. Region not containing origin represents $x + y \geq 7$ as $(0,0)$ does not satisfy $x + y \geq 7$.

Region $x + 3y \geq 11$: Line $x + 3y = 11$ meets axes at $A_3(11, 0)$, $B_3\left(0, \frac{11}{3}\right)$ respectively.

Region not containing origin represents $x + 3y \geq 11$ as $(0,0)$ does not satisfy $x + 3y \geq 11$.

Region $2x + y \geq 9$: Line $2x + y = 9$ meets axes at $A_4\left(\frac{9}{2}, 0\right)$, $B_4(0,9)$ respectively. Region not containing origin represents $2x + y \geq 9$ as $(0,0)$ does not satisfy $2x + y \geq 9$.

Region $x, y \geq 0$ it represent first quadrant.



Unbounded shaded region $A_2 PQ\ B_4$ is the feasible region with corner points $A_3(11, 0)$, $P(5,2)$, $Q(2,5)$, $B_4(0,9)$

The value of $Z = 5x + 8y$ at

$A_3(11, 0)$	$= 5(11) + 8(0) = 55$
$P(5, 2)$	$= 5(5) + 8(2) = 41$
$Q(2,5)$	$= 5(2) + 8(5) = 50$
$B_4(0, 9)$	$= 5(0) + 8(9) = 72$

Smallest value of Z is 41.

Now open half plane $5x + 8y < 41$ has no point is common with feasible region, os, smallest value of is the minimum value.
hence

last cost of mixture= Rs 41

Question 6

A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available.Food F_1 costs Rs 4 per unit and F_2 costs Rs 6 per units one unit of food F_1 contains 3 units of vitamin A and and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minium cost for diet that consits of mixture of foods and also meets the mineral nutritional requiredments.

Solution 6

Let quantity of food F_1 and F_2 be x and y units.
respectively.
Given, costs of one unit of food F_1 and F_2 be Rs 4 and Rs 6 per unit, So, costs of X unit of food $F_1$and Y units of food F_2 be $4x$ and $6y$ respectively,
Let Z be the total cost , so

$$Z = 4x + 8y$$

Since one unit of food F_1 and F_2 contain 3 and 6 unit of vitamin A respectively, so,
x units of food F_1 and y units of food F_2 contain $3x$ and $6y$ units of vitamin A respectively, but minimum requirement
of vitamin A is 80 units, so

$$3x + 6y \geq 80 \qquad \text{(first constraint)}$$

Since one unit of food F_1 and F_2 contain 4 unit and 3 unit of mineral,
so x unit of food F_1 and y unit of food F_2 contain $4x$ and $3y$ units of mineral respectively but minimum requirement of minerals be 100 units, so

$$4x + 3y \geq 100$$

$$\Rightarrow \qquad 4x + 3y \geq 100 \qquad \text{(second constraint)}$$

mathematical formulation of LPP is, Find x and y which minimum

$$Z = 4x + 6y$$

Subject to constraints,

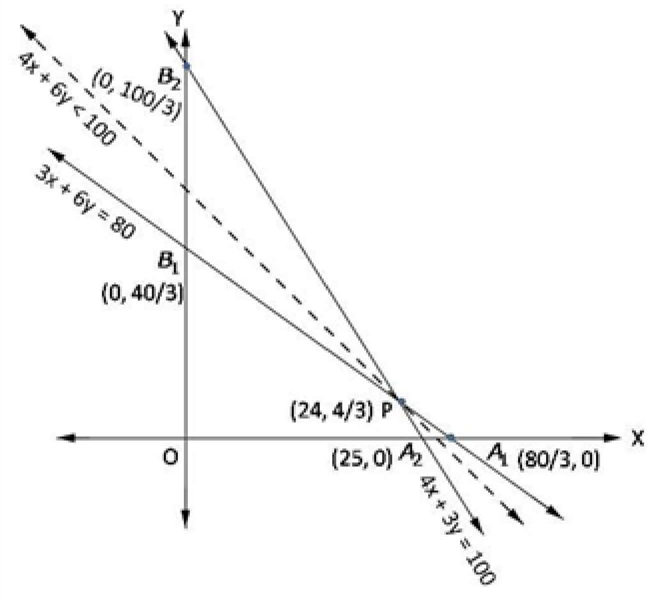
$$3x + 6y \geq 80$$
$$4x + 3y \geq 100$$
$$x, y \geq 0$$

$$\left[\text{since quantity of food can not be less than zero}\right]$$

Region $3x + 6y \geq 80$: line $3x + 6y = 80$ meets axes at $A_1\left(\frac{80}{3}, 0\right), B_1\left(0, \frac{40}{3}\right)$ respectively. Region not containing origin represents $3x + 6y \geq 80$ as $(0,0)$ does not satisfy $3x + 6y \geq 80$.

Region $4x + 3y \geq 100$ line $3x + 6y = 100$ meets axes at $A_2(25, 0), B_2\left(0, \frac{100}{3}\right)$ respectively. Region not containing origin represents $4x + 3y \geq 100$ as $(0,0)$ does not satisfy $4x + 3y \geq 100$.

Region $x, y \geq 0$ represents first quadrant



Unbounded shaded region $A_1 P\ B_2$ represents feasible region with corner points $A_1\left(\frac{80}{3}, 0\right)$, $P\left(24, \frac{4}{3}\right)$, $B_2\left(0, \frac{100}{3}\right)$.

The value of $Z = 4x + 6y$ at

$$A_1\left(\frac{80}{3}, 0\right) \quad = 4\left(\frac{80}{3}\right) + 6(0) = \frac{320}{3}$$

$$P\left(24, \frac{4}{3}\right) \quad = 4(24) + 6\left(\frac{4}{3}\right) = 104$$

$$B_2\left(0, \frac{100}{3}\right) \quad = 4(0) + 6\left(\frac{100}{3}\right) = 200$$

Smallest value of Z is 104 .Now open half plane $4x + 6y < 104$ has no point in common with feasible region so, smallest value is minimum value.

Hence,

Minimum cost of mixture = Rs 104

Question 7

Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protien and at least 36 milligrams of iron.Knowing that bran contains 80 grams of protien and 40 milligrams of protien of iron per kilogram, and that rice contains 100 grams of protien and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran cost Rs 5 per kg and rice costs Rs 4 per kg.

Solution 7

Let required quantity of bran and rice be x kg and y kg.
Given, costs of one kg of bran and rice are Rs 5 and Rs 4 per kg, So, costs of X unit of bran and Y kg of rice are $5x$ and Rs $4y$ respectively,
Let total cost of bran and rice be Z, so,

$$Z = 5x + 4y$$

Since one kg of bran and rice contain 80 and 100 mg of protien, so,
x kg of bran and y kg of rice contain $80x$ and $100y$ grms of protien respectively, but minimum requirement of protien for kelloggs is 88 gms, so

$$80x + 100y \geq 88$$

$$\Rightarrow \quad 20x + 25y \geq 22 \qquad \text{(first constraint)}$$

Since one kg of bran and rice contain 40 mg and 30 mg of iron, so,
x kg of bran and y kg of rice contain $40x$ and $30y$ mg of iron respectively, but minimum requirement of iron is 36 mg for kelloggs, so

$$40x + 30y \geq 36 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 5x + 4y$$

subject to constraints,

$$20x + 25y \geq 22$$
$$40x + 30y \geq 36$$
$$x, y \geq 0$$

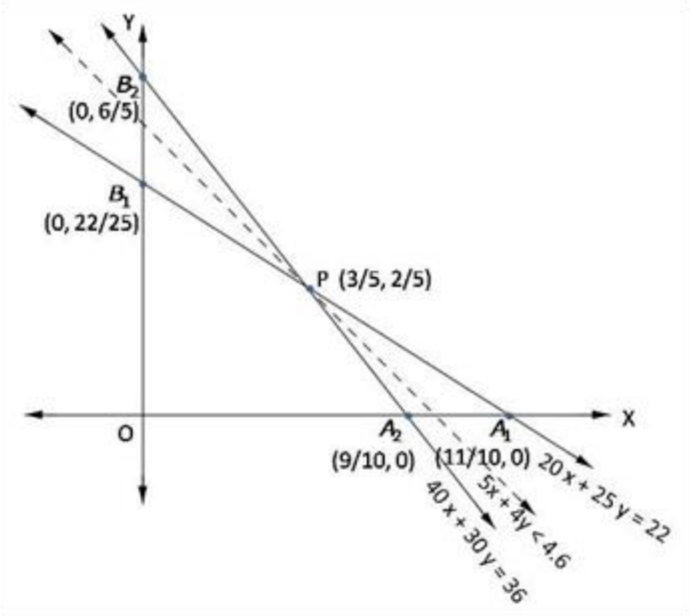
[Since quantity of bran and rice can not be less than zero]

Region $20x + 25y \geq 22$: line $20x + 25y = 22$ meets axes at $A_1\left(\frac{11}{10}, 0\right), B_1\left(0, \frac{22}{25}\right)$ respectively. Region not containing origin represents $20x + 25y \geq 22$ as $(0,0)$ does not satisfy $20x + 25y \geq 22$.

Region $40x + 30y \geq 36$ line $40x + 30y = 36$ meets axes at $A_2\left(\frac{9}{10}, 0\right), B_2\left(0, \frac{6}{5}\right)$. Region not containing origin represents $40x + 30y \geq 36$ as $(0,0)$ does not satisfy $40x + 30y \geq 36$.

Region $x, y \geq 0$: it represent first quadrant is xy – plane.

Unbounded shaded region $A_1 P B_2$ represents feasible region with corner points $A_1\left(\frac{11}{10}, 0\right)$, $P\left(\frac{3}{5}, \frac{2}{5}\right), B_2\left(0, \frac{6}{5}\right)$.



The value of $Z = 5x + 4y$ at

$$A_1\left(\frac{11}{10}, 0\right) \quad = 5\left(\frac{11}{10}\right) + 4(0) = 5.5$$

$$P\left(\frac{3}{5}, \frac{2}{5}\right) \quad = 5\left(\frac{3}{5}\right) + 4\left(\frac{2}{5}\right) = 4.6$$

$$B_2\left(0, \frac{6}{5}\right) \quad = 5(0) + 4\left(\frac{6}{5}\right) = 4.8$$

Smallest value of Z is 4.6. Now open half plane $5x + 4y < 4.6$ has no point in common with feasible region so, smallest value z is the minimum value.

Hence

Minimum cost of mixture = Rs 4.6

Question 8

A wholesale dealer deals in two kinds, A and B (say) of mixture of nuts. Each kg. of mixture A contains 60 grams of almonds, 30 grams of cashew nuts and 30 grams of hazel nuts. Each kg. of mixture B contains 30 grams of almonds, 60 grams of cashew nuts and 180 grams of hazel nuts. The remander of both mixture is per nuts. The dealer is contemplating to use mixtures A and B to make a bag which will contain at least 240 grams of almonds, 300 grams of cashew nuts and 540 grams of hazel nuts.

Mixture A costs Rs 8 per kg. and mixture B costs Rs 12 per kg. Assuming that mixtures A and B are uniform, use graphicall method to determine the number of kg. of each mixture which he should use to minimise the cost of the bag.

Solution 8

Let required number of bag A and bag B be x and y respectively.

Since, costs of each bag A and bag B are Rs 8 and Rs 12 per kg., So,
cost of x number of bag A and y number of bag B are Rs $8x$ and Rs $12y$ respectively,
Let Z be total cost of bags, so,

$$Z = 8x + 12y$$

Since, each bag A and B contain 60 and 30 gms. of almonds respectively. so,
x bags of A and y bags of B contain $60x$ and $30y$ gms. of almonds respectively but,
mixtures should contain at least 240 gms almonds, so,

$$60x + 30y \geq 240$$

$$\Rightarrow \quad 2x + y \geq 8 \qquad \text{(first constraint)}$$

Since, each bag A and B contain 30 and 60 gms. of cashew nuts respectively. so,
x bags of A and y bags of B contain $30x$ and $60y$ gms. of cashew nuts respectively but,
mixtures should contain at least 300 gms of cashew nuts, so,

$$30x + 60y \geq 300$$

$$\Rightarrow \quad x + 2y \geq 10 \qquad \text{(second constraint)}$$

Since, each bag A and B contain 30 and 180 gms. of hazel nuts respectively. so,
x bags of A and y bags of B contain $30x$ and $180y$ gms. of hazel nuts respectively but,
mixtures should contain at least 540 gms of hazel nuts, so,

$$30x + 180y \geq 540$$

$$\Rightarrow \quad x + 6y \geq 18 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 8x + 12y$$

subject to constraints,

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x + 6y \geq 18$$
$$x, y \geq 0$$

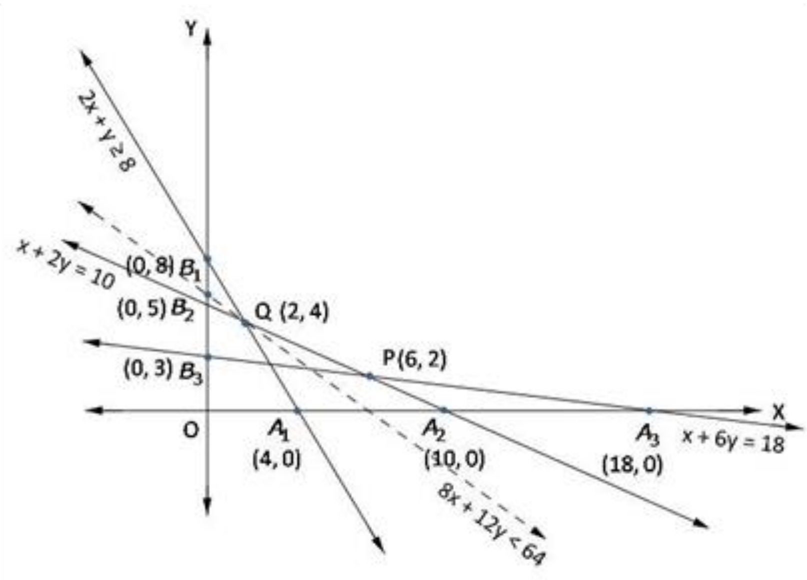
[Since quantity of bags can not be less than zero]

Region $2x + y \geq 8$: line $2x + y = 8$ meets axes at $A_1(4,0)$, $B_1(0,8)$ respectively. Region not containing origin represents $2x + y \geq 8$ as $(0,0)$ does not satisfy $2x + y \geq 8$.

Region $x + 2y \geq 10$: line $x + 2y = 10$ meets axes at $A_2(10,0)$, $B_2(0,5)$ respectively. Region not containing origin represents $2x + y \geq 10$ as $(0,0)$ does not satisfy $x + 2y \geq 10$

Region $x + 6y \geq 18$: line $x + 6y = 18$ meets axes at $A_3(18, 0)$, $B_3(0,3)$ respectively. Region not containing origin represents $x + 6y \geq 8$ as $(0,0)$ does not satisfy $x + 6y \geq 8$

Region $x, y \geq 0$: it represents first quadrant.



Unbouded shaded region A_3PQB_1 is feasible region with corner point $A_3(18,0)$, $P(6,2)$ $Q(2,4)$, $B_1(0,8)$. P is obtained by solving $x + 6y = 18$ and $x + 2y = 10$, Q is obtained by solving $2x + y = 8$ and $x + 2y = 10$

The value of $z = 8x + 12y$ at

$A_3(18,0)$	$= 8(18) + 12(0) = 144$
$P(6,2)$	$= 8(6) + 12(2) = 72$
$Q(2,4)$	$= 8(2) + 12(4) = 64$
$B_1(0,8)$	$= 8(0) + 12(8) = 96$

Smallest value of Z is 64, open half plane $8x + 12y \geq 64$ has no point is common with feasible region, so, smallest value is the minimum value

Minimum cost = Rs64
quantity of mixture A = 2 kg.
quantity of mixture B = 4kg

Question 9

One kind of cake requires 300 gm of flour and 15 gm of fat, another kind of cake requires 150 gm of flour and 30 gm of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 gm of fat, assuming that there is no shortage of the other ingradients used in making the cakes. Make it as an LPP and solve it graphically.

Solution 9

Let required number of cakes of type A and B are x and y respectively.

Let Z be total number of cakes ,so,

$$Z = x + y$$

Since one unit of cake of type A and B contain 300 gm and 150 gm flour respectively, so, x unit of cake of type A and y units of cake of type B require $300x$ and $150y$ gms of flour respectivley, but maximum flour available is $7.5 \times 1000 = 7500$ gm ,so

$$300x + 150y \le 7500$$

$$\Rightarrow \quad 2x + y \le 50 \qquad \text{(first constraint)}$$

Since one unit of cake of type A and B contain 15 and 30 gm fat respectively, so, x unit of cake of type A and y units of cake of type B contain $15x$ and $30y$ gms of fat respectivley, but maximum fat available is 600 gm ,so

$$15x + 30y \le 600$$

$$\Rightarrow \quad x + 2y \le 40 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$2x + y \le 50$$

$$x + 2y \le 40$$

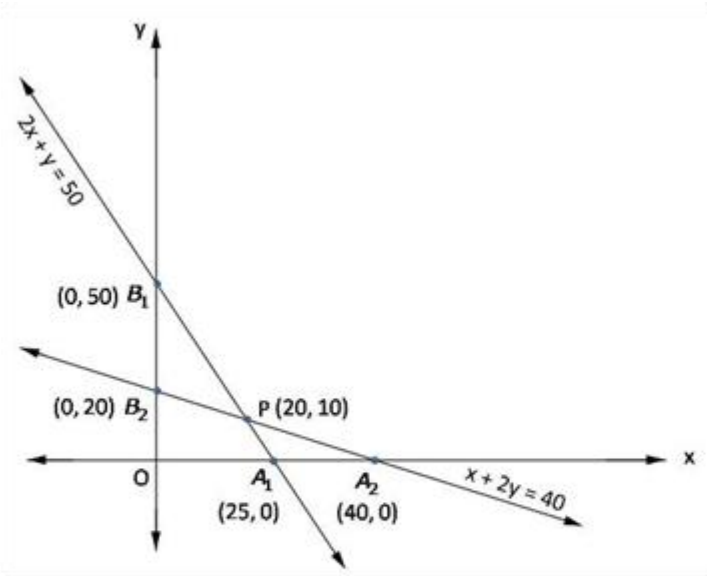
$$x, y \ge 0 \qquad [\text{Since number of cakes can not be less than zero}]$$

Region $2x + y \le 50$: line $2x + y = 50$ meets axes at $A_1(25,0)$, $B_1(0,50)$ respectively.
Region containing origin represents $2x + y \le 50$ as $(0,0)$ satisfies $2x + y \le 50$.

Region $x + 2y \le 40$: line $x + 2y = 40$ meets axes at $A_2(40,0)$, $B_2(0,20)$ respectively.
Region containing origin represents $x + 2y \le 40$ as $(0,0)$ satisfies $x + 2y \le 40$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(20,10)$ is obtained by solving $x + 2y = 40$ and $2x + y = 50$



The value of $Z = x + y$ at

$O(0,0) \quad = 0 + 0 = 0$

$A_1(25,0) \quad = 25 + 0 = 25$

$P(20,10) \quad = 20 + 10 = 30$

$B_2(0,20) \quad = 0 + 20 = 20$

maximum $Z = 30$ at $x = 20$, $y = 10$

Number of books of type $A = 20$, type $B = 10$

Question 10

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. food p costs Rs. 60 kg and Food Q costs Rs. 80 kg. Food P contains 3 units / kg of Vitamin A and 5 units/ kg of Vitamin B while food Q contains 4 unit / kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

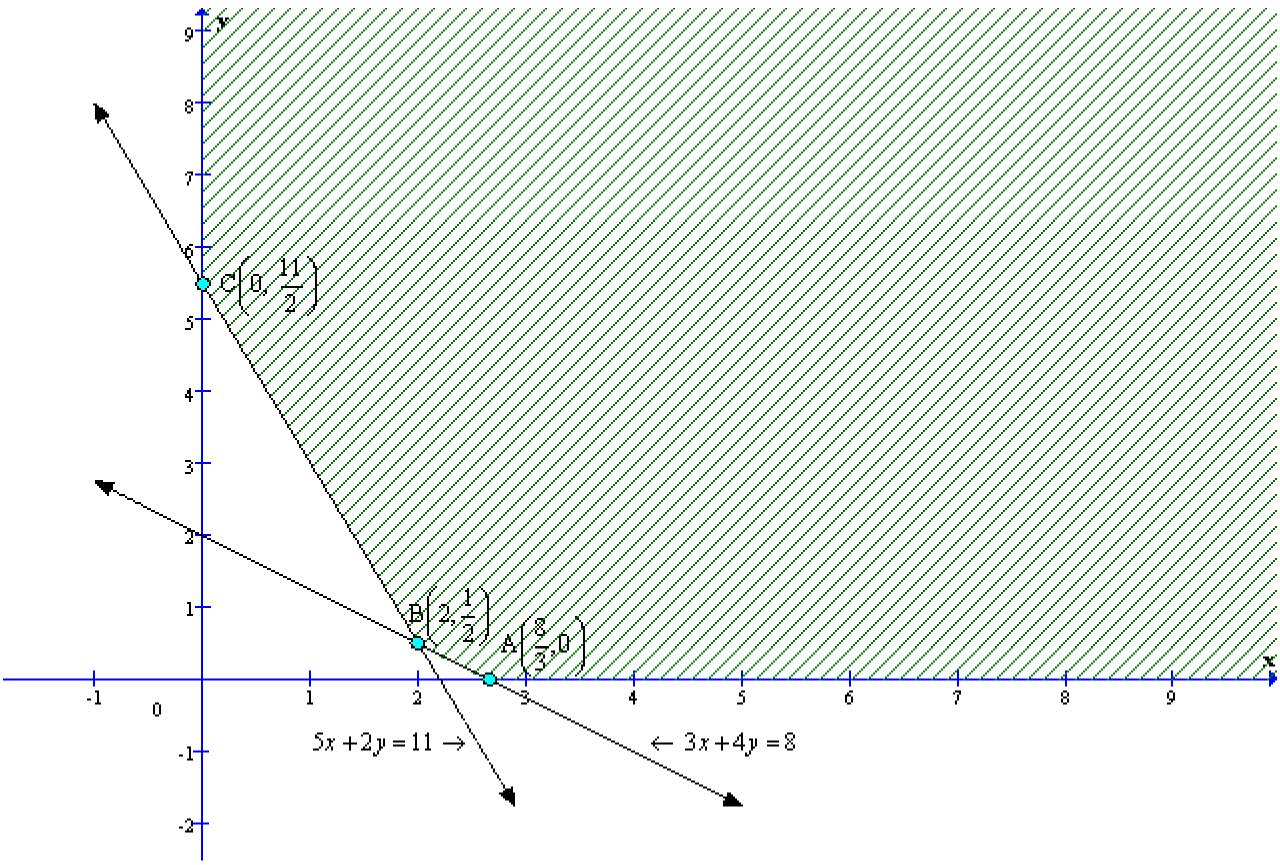
Solution 10

Let x kg of food P and y kg of food Q are mixed together to make the mixture.

Then the mathematical model of the LPP is as follows:
Minimize $Z = 60x + 80y$
Subject to $3x + 4y \geq 8$,
$5x + 2y \geq 11$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$3x + 4y = 8$,
$5x + 2y = 11$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{8}{3}, 0\right), B\left(2, \frac{1}{2}\right)$ and $C\left(0, \frac{11}{2}\right)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 60x + 80y$
$A\left(\frac{8}{3}, 0\right)$	$Z = 160$
$B\left(2, \frac{1}{2}\right)$	$Z = 160$
$C\left(0, \frac{11}{2}\right)$	$Z = 440$

The minimum value of the mixture is Rs. 160 at all points on the line segment joining points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$.

Question 11

One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

Solution 11

Let x be the number of one kind of cake and
y be the number of second kind of cakes that are made.

Then the mathematical model of the LPP is as follows:

Maximize $Z = x + y$

Subject to $200x + 100y \leq 5000$,

$25x + 50y \leq 1000$

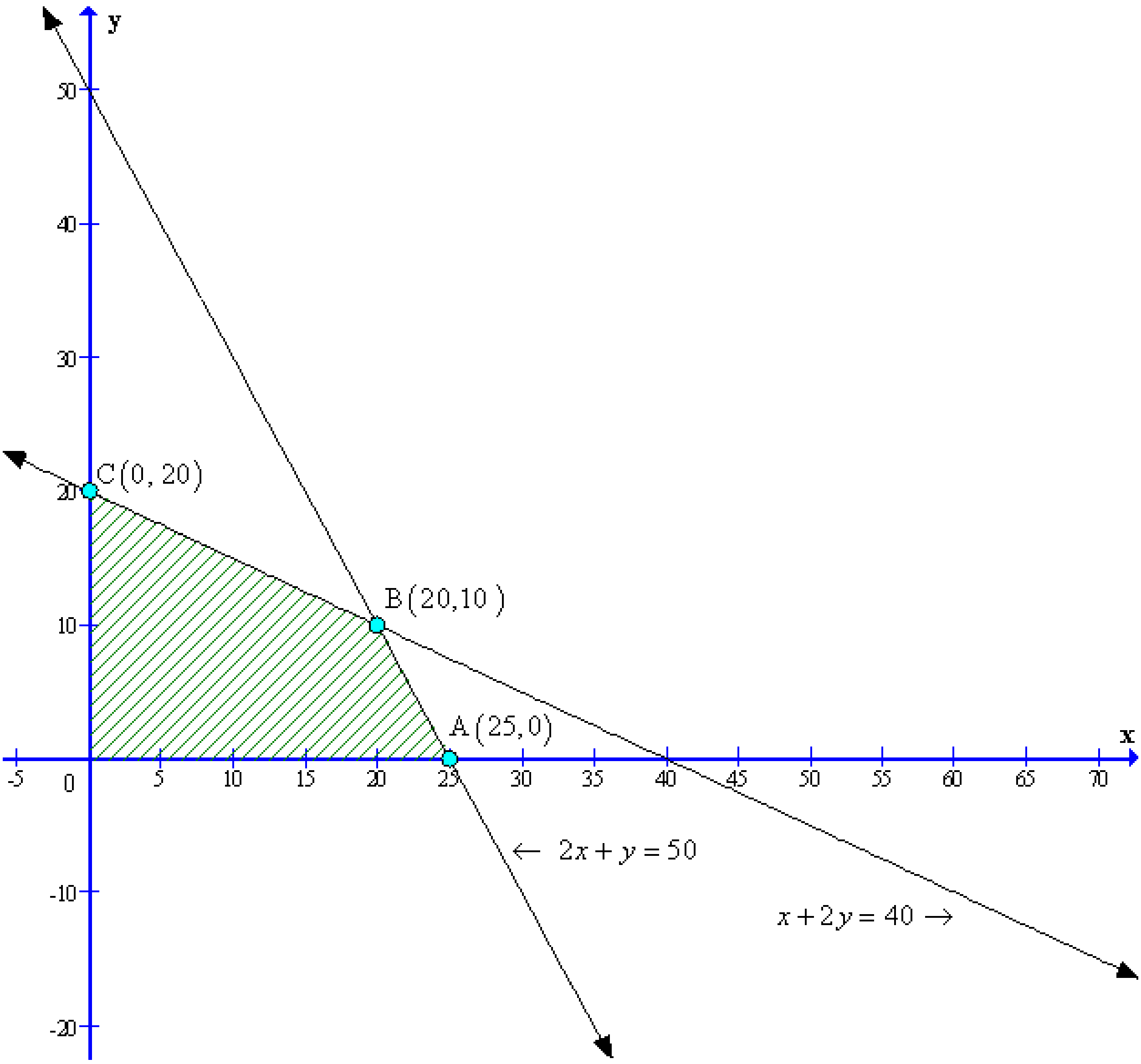
and $x \geq 0,\ y \geq 0$

To solve the LPP we draw the lines,

$2x + y = 50$,

$x + 2y = 40$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(25, 0), B(20,10) and C(0,20).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = x + y
A(25, 0)	Z = 25
B(20, 10)	Z = 30
C(0, 20)	Z = 20

The maximum of 30 cakes can be made.

Question 12

A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240units of calcium, atleasy 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

Solution 12

Let x be the number of packets of food P

y be the number of packets of food Q used to minimize vitamin A.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 6x + 3y$

Subject to $12x + 3y \geq 240$,

$4x + 20y \geq 460$

$6x + 4y \leq 300$

and $x \geq 0,\ y \geq 0$

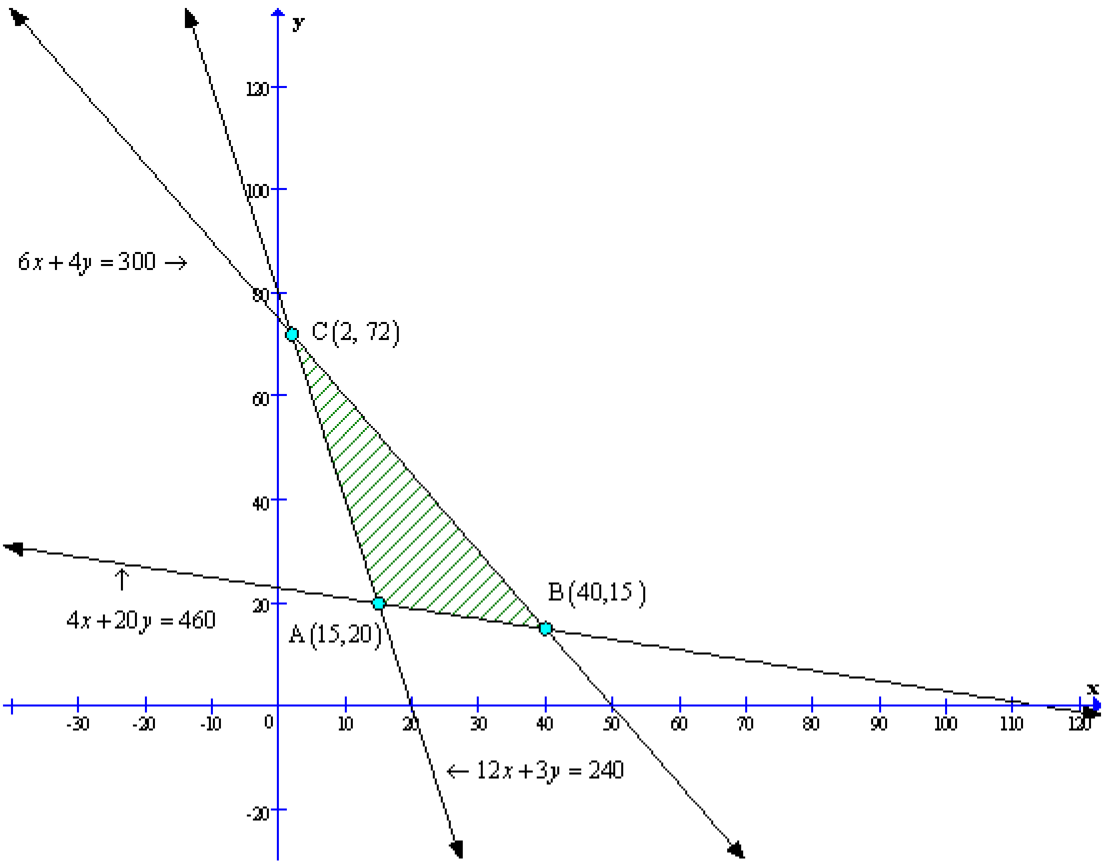
To solve the LPP we draw the lines,

$12x + 3y = 240$,

$4x + 20y = 460$,

$6x + 4y = 300$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(15, 20), B(40, 15) and C(2, 72).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 6x + 3y$
A(15, 20)	Z = 150
B(40, 15)	Z = 285
C(2, 72)	Z = 228

15 packets of food P and 20 packets of food Q should be used to minimise the amount of vitamin A.
The minimum amount of vitamin A is 150 units.

Question 13

A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs. 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs. 200 per bag contains 1.5 unit of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture pre bag?

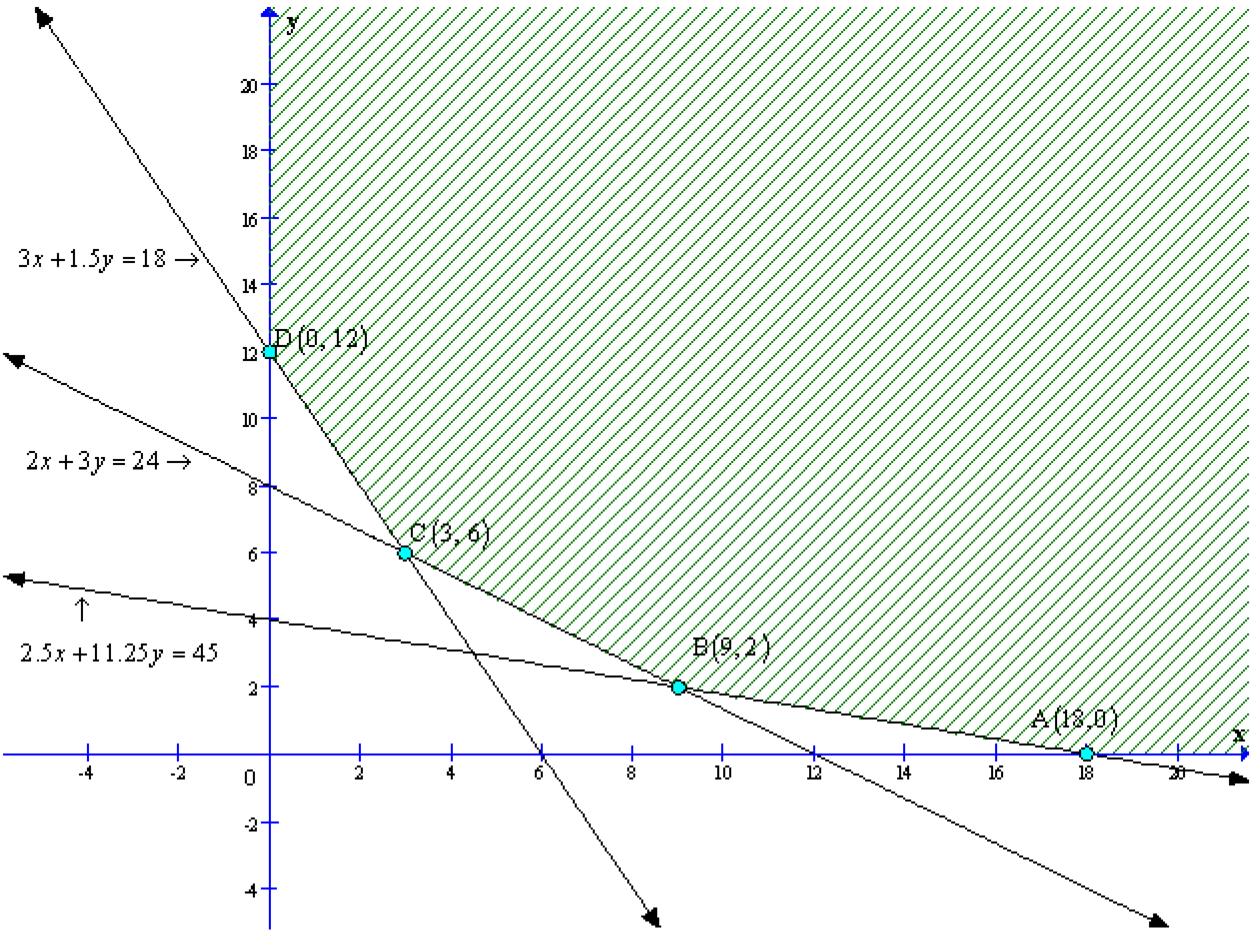
Solution 13

Let x be the number of bags of brand P
y be the number of bags of brand Q.

Then the mathematical model of the LPP is as follows:
Minimize $Z = 250x + 200y$
Subject to $3x + 1.5y \geq 18$,
$2.5x + 11.25y \geq 45$
$2x + 3y \geq 24$
and $x \geq 0,\ y \geq 0$

To solve the LPP we draw the lines,
$3x + 1.5y = 18$,
$2.5x + 11.25y = 45$
$2x + 3y = 24$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(18, 0), B(9,2), C(3, 6) and D(0, 12).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 250x + 200y
A(18, 0)	Z = 4500
B(9, 2)	Z = 2650
C(3, 6)	Z = 1950
D(0,12)	Z = 2400

3 bags of brand P and 6 bags of brand Q should be mixed in order to prepare the mixture having a minumum cost per bag.

Mimum cost of the mixture per bag is = $\frac{1950}{9}$ = Rs. 216.67.

Note: Answer given in the book is incorrect.

Question 14

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs. 16 and one kg of food costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

Solution 14

Let x be the amount of food X and y be the amount of food Y that is to be mixed which will produce the required diet.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 16x + 20y$

Subject to $x + 2y \geq 10$,

$2x + 2y \geq 12$

$3x + y \geq 8$

and $x \geq 0$, $y \geq 0$

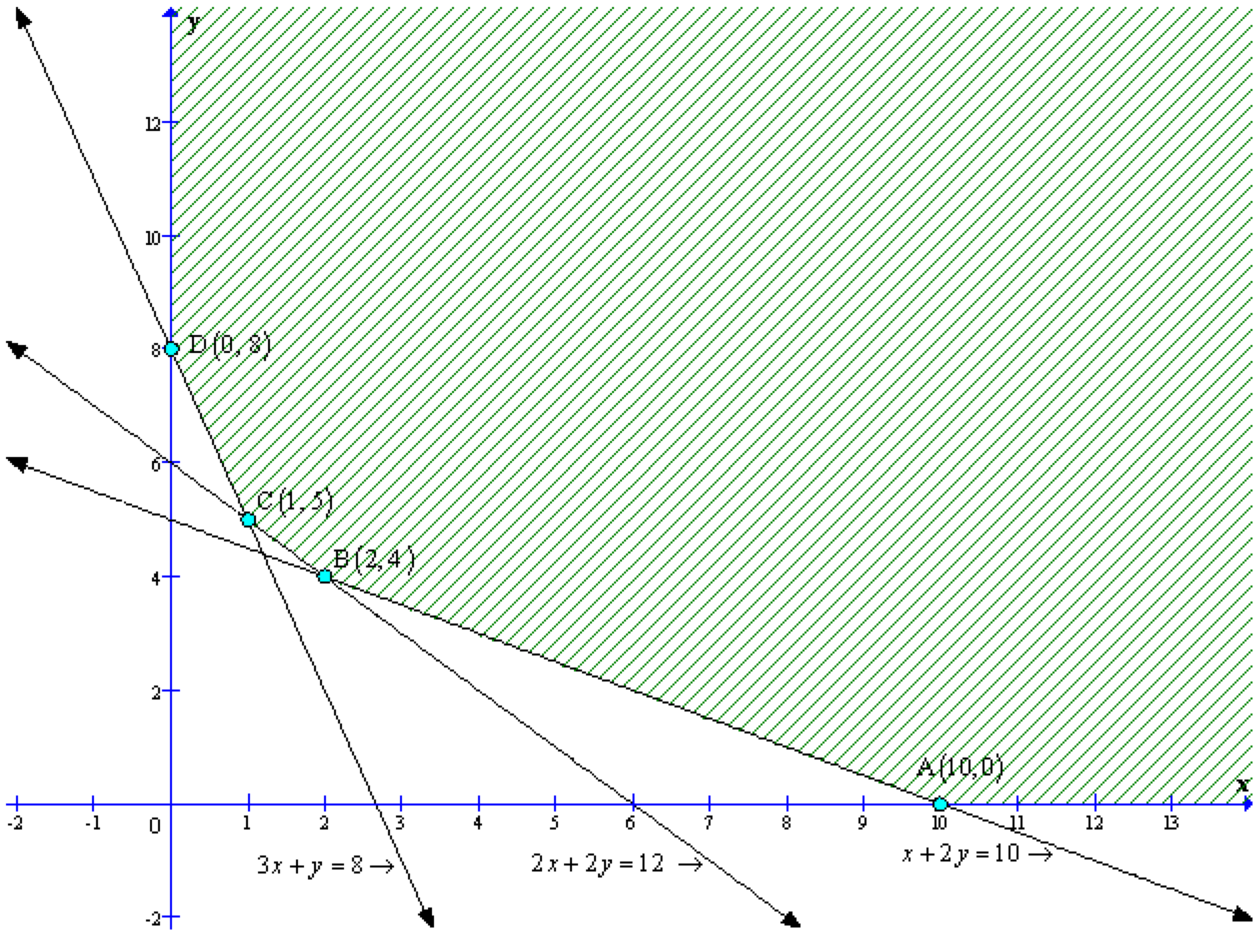
To solve the LPP we draw the lines,

$x + 2y = 10$,

$2x + 2y = 12$

$3x + y = 8$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(10, 0), B(2, 4), C(1, 5) and D(0, 8).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 16x + 20y
A(10, 0)	Z = 160
B(2, 4)	Z = 112
C(1, 5)	Z = 116
D(0, 8)	Z = 160

2 kg of food X and 4 kg of food y will be required to mimimize the cost of the diet. The least cost of the mixture is Rs. 112.

Question 15

A fruit grower can use two types of fertilizer in his garden, brand P and Q. The amounts (in kg) of nitrogen, acid potash and chlorine in a bag of each brand are given in the Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

Kg per bag

	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

Solution 15

Let x bags of fertilizer P and y bags of fertilizer Q used in the garden to minimize the usage of nitrogen.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 3x + 3.5y$

Subject to $x + 2y \geq 240$,

$3x + 1.5y \geq 270$

$1.5x + 2y \leq 310$

and $x \geq 0$, $y \geq 0$

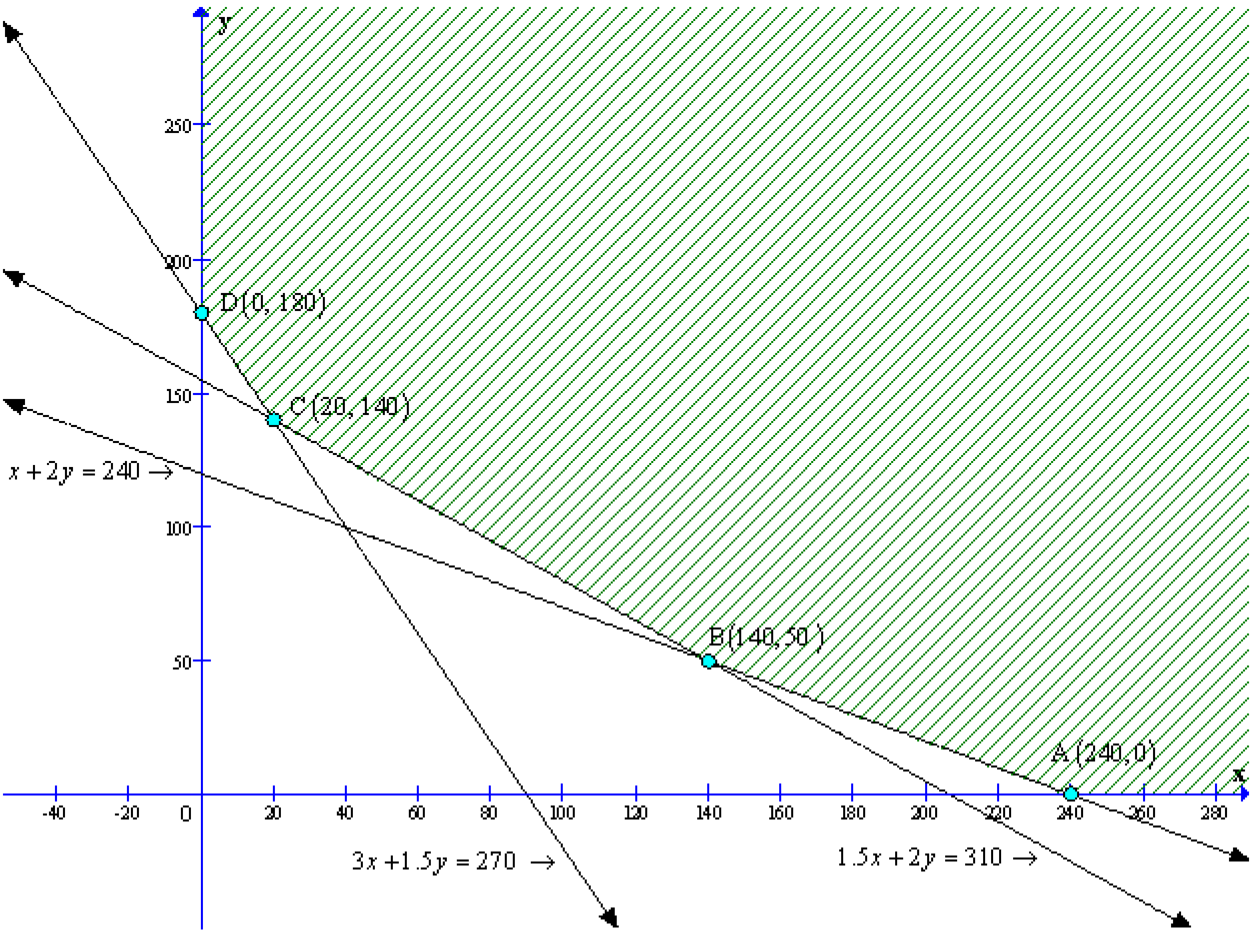
To solve the LPP we draw the lines,

$x + 2y = 240$,

$3x + 1.5y = 270$

$1.5x + 2y = 310$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 100), B(140, 50) and C(20, 140).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 3x + 3.5y
A(40, 100)	Z = 470
B(140, 50)	Z = 595
C(20, 140)	Z = 550

40 bags of brand P and 100 bags of brand Q should be used to minimize the amount of nitrogen added to the garden.
The minimum amount of nitrogen added in the garden is 470kg.

Chapter 30 - Linear programming Exercise Ex. 30.4

Question 1

If a young man drives his vehicle at 25km/hr, he has to spend Rs 2per km on petrol. if he drives it as a fast of 40 km/hr, the petrol cost increase to Rs 5 per km. He has Rs 100 to speed on petrol and travel a maximum distance in one hour time with less polution . Express this problem as an LPP and solve it graphically. What value do you find hear?

Solution 1

Let he drives x km at a speed f 25 km/hr and y km at a speed of 40 km/hr.
Let Z be total distance travelled by him, so,

$$Z = x + y$$

Since he spend Rs 2 per km on petrol when speed is 25 km/hr and Rs 5 per km on petrol when speed is 40 km/hr, so, expence on x km and y km are Rs $2x$ and Rs $5y$ respectivley, but he has only Rs 100.,so

$$2x + 5y \le 100 \qquad \text{(first constraint)}$$

$$\text{Time taken to travel } x \text{ km} = \frac{\text{Distance}}{\text{speed}}$$

$$= \frac{x}{25} \text{ hr}$$

$$\text{Time taken to travel } y \text{ km} = \frac{y}{40} \text{ hr}$$

Given he has 1 hr to travel, so

$$\frac{x}{25} + \frac{y}{40} \le 1$$

$$\Rightarrow \quad 40x + 25y \le 1000$$

$$\Rightarrow \quad 8x + 5y \le 200 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = x + y$$

Subject to constriants,

$$2x + 5y \le 100$$

$$8x + 5y \le 200$$

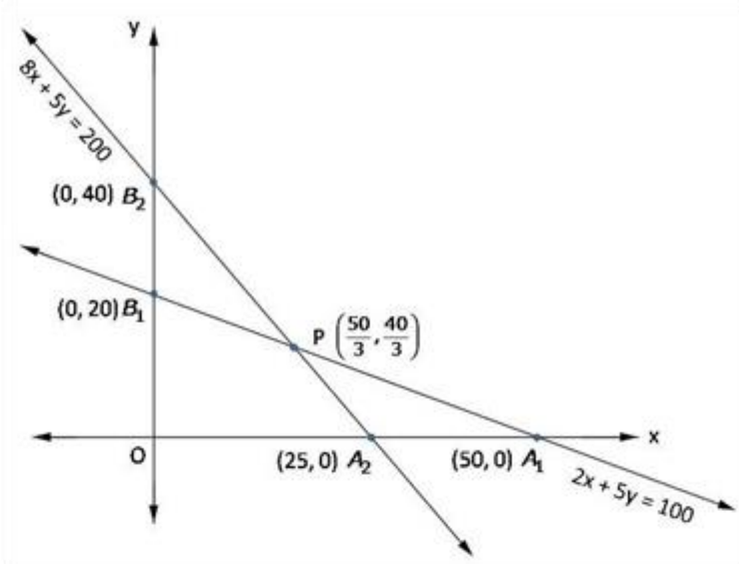
$$x, y \ge 0 \qquad [\text{Since distances can not be less than zero}]$$

Region $2x + 5y \le 100$: line $2x + 5y = 100$ meets axes at $A_1(50,0)$, $B_1(0,20)$ respectively.
Region containing origin represents $2x + 5y \le 100$ as $(0,0)$ satisfies $2x + 5y \le 100$.

Region $8x + 5y \le 200$: line $8x + 5y = 200$ meets axes at $A_2(25,0)$, $B_2(0,40)$ respectively.
Region containing origin represents $8x + 5y \le 200$ as $(0,0)$ satisfies $8x + 5y \le 200$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P\left(\frac{50}{3}, \frac{40}{3}\right)$ is obtained by solving $8x + 5y = 200$, $2x + 5y = 100$



The value of $Z = x + y$ at

$O(0,0) = 0 + 0 = 0$

$A_2(25,0) = 25 + 0 = 25$

$P\left(\frac{50}{3}, \frac{40}{3}\right) = \frac{50}{3} + \frac{40}{3} = 30$

$B_1(0,20) = 0 + 20 = 20$

maximum $Z = 30$ at $x = \frac{50}{3}$, $y = \frac{40}{3}$

Distance travelled at speed of 25 km/hr = $\frac{50}{3}$ km

and at speed of 40 km/hr = $\frac{40}{3}$ km

maximum distance = 30 km.

Question 2

A manufacturer has three machines installed in his factory. Machines I and II are capable of being operated for at most 12 hours wheareas Machine III must operate at least for 5 hours a day.He produces only two items, each requiring the use of three machines. The number of hours required for producing one unit each of the items on the three machines is given in the following table:

Item	*Number of hours required by the machine*		
	I	II	III
A	1	2	1
B	2	1	$\frac{5}{4}$

He makes a profit of Rs 6.00 on item *A* and Rs 4.00 on item *B*. Assuming that he can sell all that he produces, how many of each item should he produce so as to maximize his profit? Determine his maximum progit. Formulate this LPP mathematically and then solve it.

Solution 2

Let required quantity of items A and B.

Given, profits on one item A and B are Rs 6 and Rs 4 respectively So, profits on X items of type A and Y items of type B are $6x$ and Rs $4y$ respectively,
Let total profit be z, so,

$$Z = 6x + 4y$$

Given, machine I works 1 hour and 2 hours on item A and B respectively, so,
x number of item A and y number of item B need x hour and $2y$ hours on machine I respectively, but machine I works at most 12 hours, so

$$x + 2y \geq 12 \qquad \text{(first constraint)}$$

Given, machine II works 2 hours and 1 hours on item A and B respectively, so,
x number of item A and y number of item B need $2x$ hours and y hour on machine II , but machine II works maximum 12 hours, so

$$2x + y \geq 12 \qquad \text{(second constraint)}$$

Given, machine III works 1 hour and $\frac{5}{4}$ hour on one item A and B respectively, so,
x number of item A and y number of item B need x hour and $\frac{5}{4}y$ hours respectively on machine III , but machine III works at least 5 hours, so

$$x + \frac{5}{4}y \geq 5$$

$$4x + 5y \geq 20 \quad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$z = 6x + 4y$$

subject to constraints,

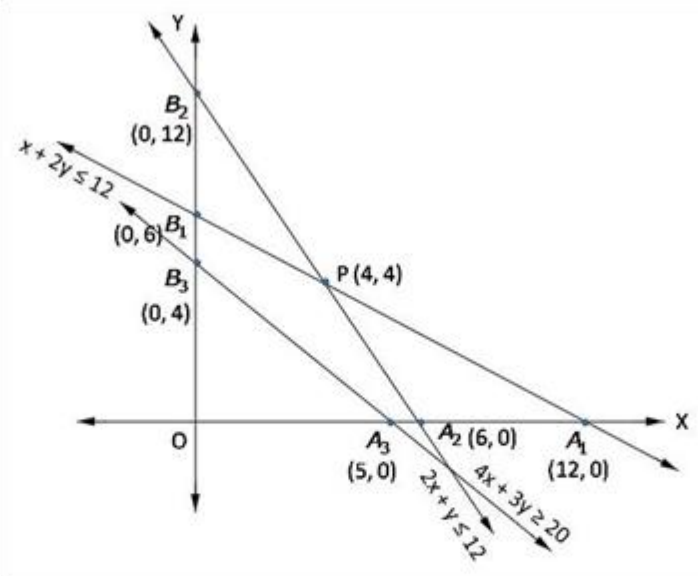
$$x + 2y \geq 12$$
$$2x + y \geq 12$$
$$4x + 5y \geq 20$$
$$x, y \geq 0$$

[Since number of item A and B not be less than zero]

Region $x + 2y \geq 12$: line $x + 2y = 12$ meets axes at $A_1(12, 0), B_1(0,6)$ respectively. Region containing origin represents $x + 2y \geq 12$ as $(0,0)$ satisfies $2x + y \geq 12$.

Region $4x + 5y \geq 20$: line $4x + 5y = 20$ meets axes at $A_3(5,0), B_3(0, 4)$ respectively. Region not containing origin represents $4x + 5y \geq 20$ as $(0,0)$ does not satisfy $4x + 5y \geq 20$.

Region $x, y \geq 0$: it represent first quadrant.



Shaded region $A_2A_3PB_3B_1$ represents feasible region.

The value of $Z = 6x + 4y$ at

$A_2(6,0)$	$= 6(6) + 4(0) = 36$
$A_3(5,0)$	$= 6(5) + 4(0) = 30$
$B_3(0,4)$	$= 6(0) + 4(4) = 16$
$B_2(0,6)$	$= 6(0) + 4(6) = 24$
$P(4,4)$	$= 6(4) + 4(4) = 40$

Hence, Z is maximum at $x = 4, Y = 4$

Required number of product $A = 4$, product $B = 4$

Miximum profit = Rs 40

Question 3

Two tailors, A and B earn Rs 15 and Rs 20 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minmum labour cost?

Solution 3

Suppose tailor A and B work for x and y days respectively.

Since, tailor A and B earn Rs 15 and Rs 20 respectively So, tailor A and B earn is X and Y days Rs $15x$ and $20y$ respectively, let Z denote maximum profit that gives minimum labour cost, so,

$$Z = 15x + 20y$$

Since, Tailor A and B stitch 6 and 10 shirts respectively in a day, so,
tailor A can stitch 6x and B can stitch $10y$ shirts in x and y days respectively, but it is desired to produce 60 shirts at least, so

$$6x + 10y \geq 60$$
$$3x + 5y \geq 30 \qquad \text{(first constraint)}$$

Since, Tailor A and B stitch 4 pants per day each, so,
tailor A can stitch 4x and B can stitch $4y$ pants in x and y days respectively, but it is desired to produce at least 32 pants, so

$$4x + 4y \geq 32$$
$$x + y \geq 8 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 15x + 20y$$

subject to constraints,

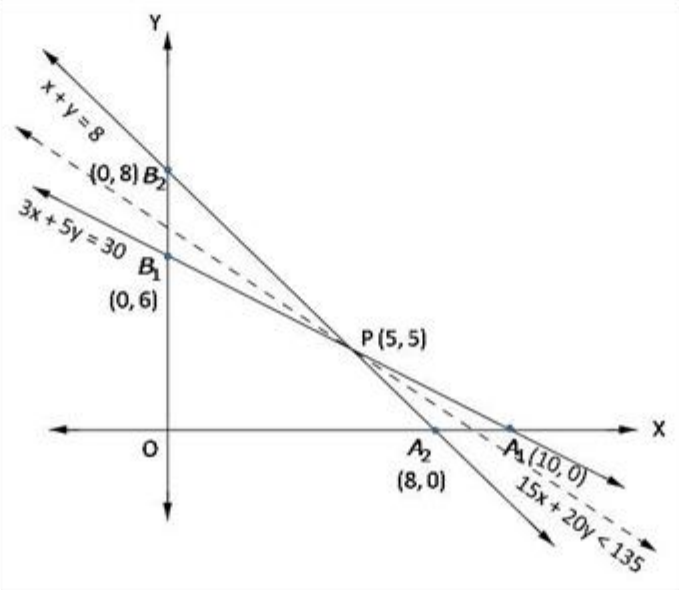
$$3x + 5y \geq 30$$
$$x + y \geq 8$$
$$x, y \geq 0$$
$$\left[\text{Since x and y not be less than zero}\right]$$

Region $3x + 5y \geq 30$: line $3x + 5y = 30$ meets axes at $A_1(10,0), B_1(0,6)$ respectively. Region not containing origin represents $3x + 5y \geq 30$ as $(0,0)$ does not satisfy $3x + 5y \geq 30$.

Region $x + y \geq 8$: line $x + y = 8$ meets axes at $A_2(8,0), B_2(0,8)$ respectively. Region not containing origin represents $x + y \geq 8$ as $(0,0)$ does not satisfy $x + y \geq 8$.

Region $x, y \geq 8$: it represent first quadrant.

Unbounded shaded region $A_1 P B_2$ represents feasible region with corner points $A_1(10,0), P(5,3), B_2(0,8)$.



The value of $Z = 15x + 20y$ at

$A_1\left(10,0\right) \quad = 15\left(10\right) + 20\left(0\right) = 150$

$P\left(5,3\right) \quad = 15\left(5\right) + 20\left(3\right) = 135$

$B_2\left(0,8\right) \quad = 15\left(0\right) + 20\left(8\right) = 160$

Smallest value of Z is 135 ,Now open half plane $15x + 20y < 135$ has no point in common with feasible region, so smallest value is the minimum value. So,

$Z = 135$, at $x = 5, y = 3$

Tailor A should work for 5 days and B should work for 3 days

Question 4

A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

Solution 4

Let the factory manufacture x screws of type A and y screws of type B on each day. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Screw A	**Screw B**	**Availability**
Automatic Machine (min)	4	6	$4 \times 60 = 120$
Hand Operated Machine (min)	6	3	$4 \times 60 = 120$

The profit on a package of screws A is Rs 7 and on the package of screws B is Rs 10. Therefore, the constraints are

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 7x + 10y$ ... (1)

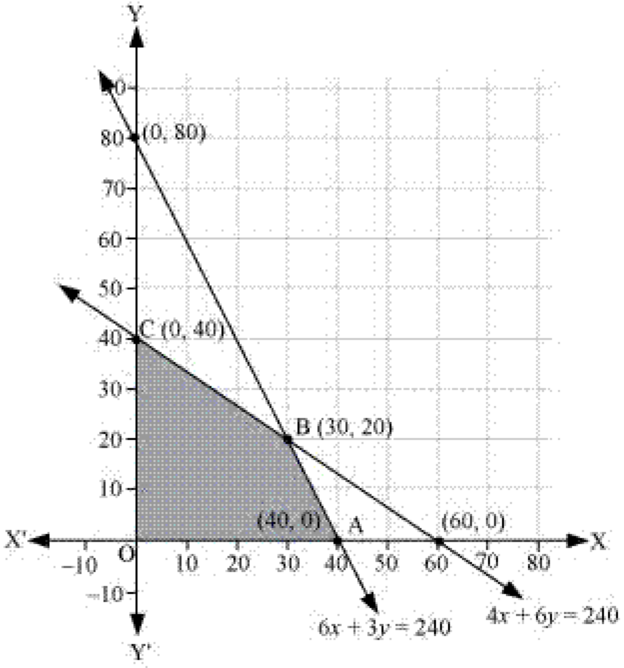
subject to the constraints,

$4x + 6y \leq 240$ ... (2)

$6x + 3y \leq 240$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is



The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

Corner point	$Z = 7x + 10y$	
A(40, 0)	280	
B(30, 20)	410	→ Maximum
C(0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

Question 5

A company produces two types of leather belts, say type *A* and *B*. Belt *A* is a superior quality and belt *B* is of a lower quality. Profits on each type of belt are Rs 2 and Rs 1.50 per belt, respectively. Each belt of type *A* requires twice as much time as required by a belt of type *B*. If all belts were of type *B*, the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belt per day (both *A* and *B* combined). Belt *A* requires a fancy buckle and only 400 fancy buckles are available for this per day. For belt of type *B*, only 700 buckles are available per day.

Solution 5

Let required number of belt A and B be x and y.
Given, profit on belt A and B be Rs 2 and Rs 1.50 per belt, So, profit on x belt of type A and Y belt fo type B be 2x and $1.5y$ respectively,
Let Z be total profit, so,

$$Z = 2x + 1.5y$$

Since, each belt of type A requires twice as much time as belt B. Let each belt B require 1 hour to make, so, A requires 2 hours. For x and y belts of type A and B. It required 2x and y hours to make but total time available is equal to procduction 1000 belt B that is 1000 hours, so,

$$2x + y \le 1000 \qquad \text{(first constraint)}$$

Given supply of leather only for 800 belts per day (both A and B combined), so

$$x + y \le 800 \qquad \text{(second constraint)}$$

Buckels available for A is only 400 and for B only 700, so,

$$x \le 400 \qquad \text{(third constraint)}$$

$$y \le 700 \qquad \text{(fourth constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 2x + 1.5y$$

subject to constraints,

$$2x + y \le 1000$$
$$x + y \le 800$$
$$x \le 400$$
$$y \le 700$$
$$x, y \ge 0$$

[Since number of belt can not be less than zero]

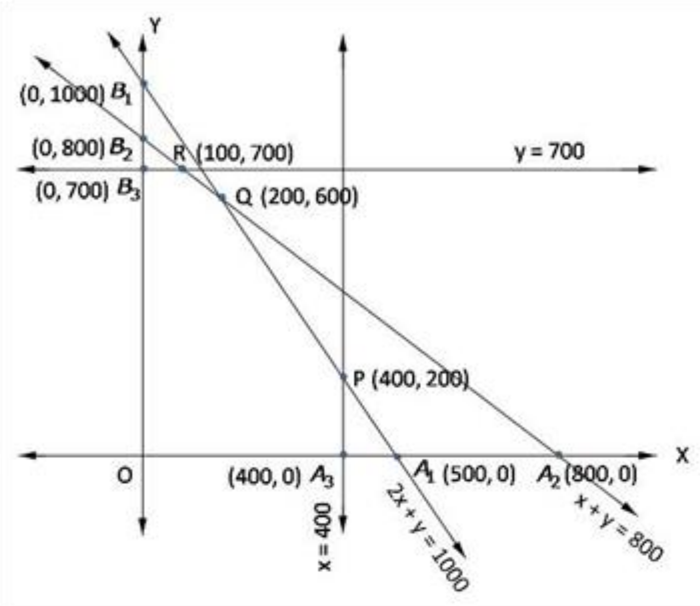
Region $2x + y \le 1000$: line $2x + y = 1000$ meets axes at $A_1(500,0), B_1(0,1000)$ respectively. Region containing origin represents $2x + y \le 1000$ as $(0,0)$ satisfies $2x + y \le 1000$.

Region $x + y \le 800$: line $x + y = 800$ meets axes at $A_2(800,0), B_2(0,800)$ respectively. Region containing origin represents $x + y \le 800$ as $(0,0)$ satisfies $x + y \le 800$.

Region Region $x \leq 400$: line $x = 400$ meets axes is parallel to y axis and meet x – axis at $A_3(400,0)$. Region containing origin represents $x \leq 400$ as $(0,0)$ satisfies $x \leq 400$.

Region Region $y \leq 400$: line $y = 700$ is parallel to x– axis and meet y – axis at $B_3(0,700)$. Region containing origin represents $y \leq 700$ as $(0,0)$ satisfies $y \leq 700$.

Region $x, y \geq 0$: it represent first quadrant.



Shaded region OA_3PQRB_3 is feasible region, P is points of intersections of $2x + y = 1000$ and $x = 400$, Q is the point of intersection of $x + y = 800$ and $2x + y = 1000$, R is not point of intersection of $y = 700$, $x + y = 800$.

The value of $Z = 2x + 1.5y$ at

$O(0,0)$	$= 2(0) + 1.5(0) = 0$
$A_3(400,0)$	$= 2(400) + 1.5(0) = 800$
$P(400,200)$	$= 2(400) + 1.5(200) = 1100$
$Q(200,600)$	$= 2(200) + 1.5(600) = 1300$
$R(100,700)$	$= 2(100) + 1.5(700) = 1250$
$B_3(0,700)$	$= 2(0) + 1.5(700) = 1050$

Therefore, maximum $Z = 1300$, at $x = 200, y = 600$

Required number belt $A = 200$,belt $B = 600$
maximum profit = Rs 1300

Question 6

A small manufacturer has employed 5 skilled men and 10 semi-skilled men and makes an article in two qualities deluxe model and an ordinary model. The making of a deluxe model requires two hrs. Work by a skilled man and 2 hrs. work by a semi-killed man. The ordinary model requires 1 hr by a skilled man and 3 hrs. by a semi-killed man.By union rules no man may work more than 8 hrs per day. The manufaturers clear profit on deluxe model is Rs 15 and on an ordinary model is Rs 10. How many of each type should be made in order to maximize his total daily profit.

Solution 6

Let required number of deluxe model and ordinary model be x and y respectively.

Since, profits on each model of deluxe and ordinary type model are Rs 15 and Rs 10 respectively. So, profits on x deluxe models and y ordinary models are $15x$ and $10y$

Let Z be total profit, then,

$$Z = 15x + 10y$$

Since, each deluxe and ordinary model require 2 and 1 hour of skilled men,so, x deluxe and y ordinary models required 2x and y hours of skilled men but time available by skilled men is $5 \times 8 = 40$ hours, So,

$$2x + y \le 40 \qquad \text{(first constraint)}$$

Since, each deluxe and ordinary model require 2 and 3 hours of semi-skilled men,so, x deluxe and y ordinary models require 2x and 3y hours of semi-skilled men respectively but total time available by semi-skilled men is $10 \times 8 = 80$ hours, So,

$$2x + 3y \le 80 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 15x + 10y$$

subject to constraints,

$$2x + y \le 40$$
$$2x + 3y \le 80$$
$$x, y \ge 0$$

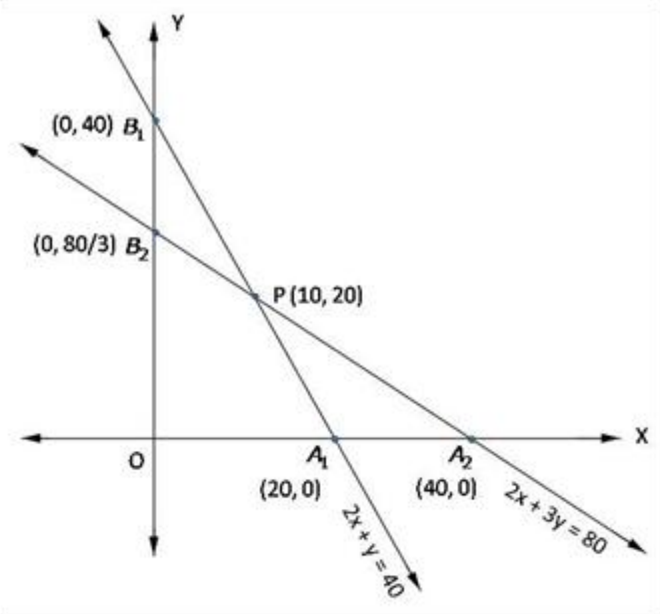
[Since number of deluxe and ordinary models can not be less than zero]

Region $2x + y \le 40$: line $2x + y = 40$ meets axes at $A_1(20,0), B_1(0,40)$ respectively. Region containing origin represents $2x + y \le 40$ as $(0,0)$ satisfies $2x + y \le 40$.

Region $2x + 3y \le 80$: line $2x + 3y = 80$ meets axes at $A_2(40,0), B_2\left(0,\frac{80}{3}\right)$ respectively. Region containing origin represents $2x + 3y \le 80$.

Region $x, y \ge 0$: it represent first quadrant.

Shaded region OA_1PB_2 represents feasible region, $P(10,20)$ is points of intersections of $2x + y = 40$ $2x + 3y = 80$.



The value of $Z = 15x + 10y$ at

$O(0,0)$	$= 15(0) + 10(0) = 0$
$A_1(20,0)$	$= 15(20) + 10(0) = 300$
$P(10,20)$	$= 15(10) + 10(20) = 350$
$B_2\left(0, \frac{80}{3}\right)$	$= 15(0) + 10\left(\frac{80}{3}\right) = \frac{800}{3}$

Therefore, maximum $Z = 350$, at $x = 10, y = 20$

Required number deluxe model = 10
number of ordinary model = 600
maximum profit =Rs 350

Question 7

A manufaturer makes two types A and B of tea-cups. There machines are needed for the manufaturer and the time in minute required for each cup on the machines is given below:

	Machines		
	I	II	III
A	12	18	6
B	6	0	**9**

Each machine is available for a maximum of 6 hours per day. If the profit on each cup *A* is 75 paise and that on each cup *B* is 50 paise, show that 15 tea-cups of type *A* and 30 of type *B* should be manufatured in a day to get the maximum profit.

Solution 7

Let required number of tea-cups of type A and B are x and y respectively.

Since, profits on each tea-cups of type A and B are 75 paise and 50 paise So, profits on x tea-cups of type A and y tea-cups of type B are $75x$ and $50y$ respectively, Let total profit on tea-cups be Z, so,

$$Z = 75x + 50y$$

Since, each tea-cup of type A and B require to work machine I for 12 and 6 minutes respectively so, x tea cups of type B require to work on machine I for $12x$ and $6y$ minutes respectively .
Total time available on machine I is $6 \times 60 = 360$ minutes. so,

$$12x + 6y \geq 360 \qquad (\text{first constraint})$$

Since, each tea-cup of type A and B require to work machine II for 18 and 0 minutes respectively so, x tea cups of type A and y tea cups of B require to work on machine II for $18x$ and $0y$ minutes respe
but Total time available on machine II is $6 \times 60 = 360$ minutes. so,

$$18x + 0y \geq 360 \qquad (\text{second constraint})$$

$$x \leq 20$$

Since, each tea-cup of type A and B require to work machine III for 6 and 9 minutes respectively so, x tea cups of type A and y tea cups of B require to work on machine III for $6x$ and $9y$ minutes respe
Total time available on machine III is $6 \times 60 = 360$ minutes. so,

$$6x + 9y \geq 360 \qquad (\text{third constraint})$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 75x + 50y$$

subject to constraints,

$$12x + 6y \leq 360$$

$$x \leq 20$$

$$6x + 9y \leq 360$$

$$x, y \geq 0$$

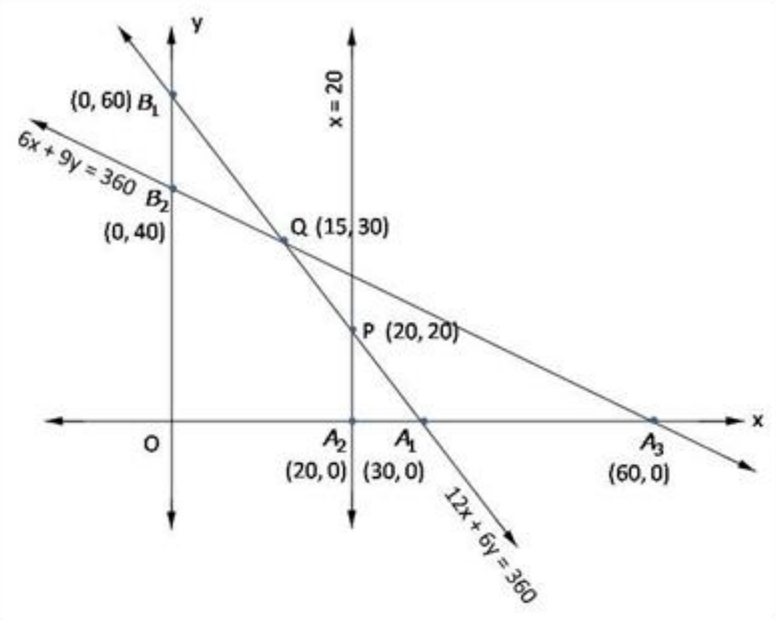
[Since production of tea cups can not be less than zero]

Region $12x + 6y \leq 360$: line $12x + 6y = 360$ meets axes at $A_1(30,0)$, $B_1(0,60)$ respectively. Region containing origin represents $12x + 6y \leq 360$ as $(0,0)$ satisfies $12x + 6y \geq 360$.

Region $x \leq 20$: line $x = 20$ is parallel to y – axes and meets x – axes at $A_2(20,0)$. Region containing origin represents $x \leq 20$ as $(0,0)$ satisfies $x \leq 20$.

Region $6x + 9y \leq 360$: line $8x + 9y = 360$ meets axes at $A_3(60,0)$, $B_2(0,40)$ respectively. Region containing origin represents $6x + 9y \leq 360$ as $(0,0)$ satisfies $6x + 9y \geq 360$.

Region $x, y \geq 0$: it represents first quadrant.



Shaded region OA_2PQB_2 is the feasible region. P is point obtained by solving $x = 20$ and $12x + 6y = 360$ and Q is point obtained by solving $12x + 6y = 360$ and $6x + 9y = 360$.

The value of $Z = 75x + 50y$ at

$O\ (0,0)$	$= 75(0) + 50(0) = 0$
$A_2\ (20,0)$	$= 75(20) + 50(0) = 1500$
$P\ (20,20)$	$= 75(20) + 50(20) = 2500$
$Q\ (15,30)$	$= 75(15) + 50(30) = 2624$
$B_2\ (0,40)$	$= 75(0) + 50(40) = 2000$

Hence, Z is maximum at $x = 15, Y = 30$

Therefore,

15 teacups of type A and 30 tea-cups f type B are needed to maximize profit

Question 8

A factory owner purchases two types of machines, A and B for his factory. The requirements and limitations for the machines are as follows:

	Area occupied by the machine	*Labour force for each machine*	*Daily output in units*
Machine *A*	1000 sq.m	12 men	60
Machine *B*	1200 sq.m	8 men	40

He has an area of 7600 sq.m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output?

Solution 8

Let required number of machine A and B are x and y respectively.

Since, production of each machine A and B are 60 and 40 units daily respectively,So, productions by x number of machine A and y number of machine B are $60x$ and $40y$ respectively, Let Z denote total output daily, so,

$$Z = 60x + 40y$$

Since, each machine of type A and B require 1000 sq.m and 1200 sq.m area so, x machine of type A and y machine of type B require $100x$ and $1200y$ sq.m area but, Total area available for machine is 7600 sq.m. so,

$$1000x + 1200y \le 7600$$

$$5x + 6y \le 38 \qquad \text{(first constraint)}$$

Since, each machine of type A and B require 12 men and 8 men to work respectively so, x machine of type A and y machine of type B require $12x$ and $8y$ men to work respectively but, Total 72 men available for work so,

$$12x + 8y \le 72$$

$$3x + 2y \le 18 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 60x + 40y$$

subject to constraints,

$$5x + 6y \le 38$$

$$3x + 2y \le 18$$

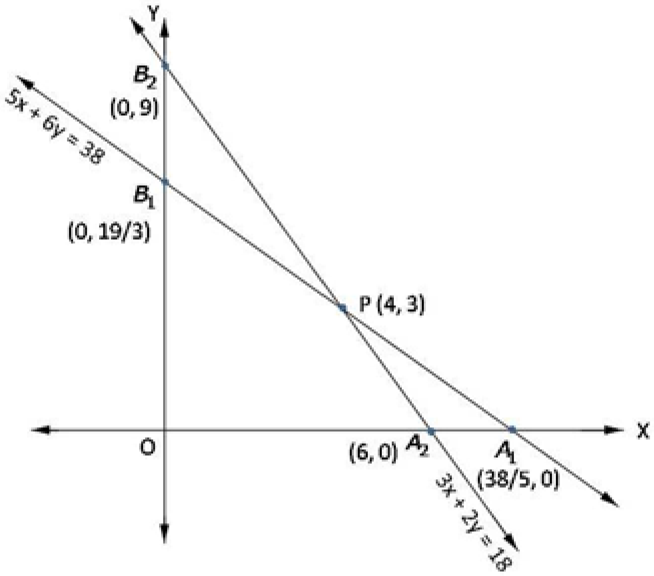
$$x, y \ge 0$$

$$\left[\text{Number of machines can not be less than zero}\right]$$

Region $5x + 6y \le 38$: line $5x + 6y = 38$ meets axes at $A_1\left(\frac{38}{5}, 0\right), B_1\left(0, \frac{19}{3}\right)$ respectively. Region containing origin represents $5x + 6y \le 38$ as origin satisfies $5x + 6y \ge 38$.

Region $3x + 2y \le 18$: line $3x + 2y = 18$ meets axes at $A_2(6,0)$, $B_2(0,9)$ respectively. Region containing origin represents $3x + 2y \le 18$ as $(0,0)$ satisfies $3x + 2y \le 18$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $O\,A_2 P B_1$ is the feasible region. $P(4,3)$ is obtained by solving $3x+2y=18$ and $5x+6y=38$

The value of $Z=60x+40y$ at

$$
\begin{aligned}
&O(0,0) && =60(0)+40(0)=0\\
&A_2(6,0) && =60(6)+40(0)=360\\
&P(4,3) && =60(4)+40(3)=360\\
&B_1\left(0,\frac{19}{3}\right) && =60(0)+40\left(\frac{19}{3}\right)=\frac{760}{3}
\end{aligned}
$$

Therefore maximum $Z=360$ at $x=4, Y=3$ or $x=6, y=0$

Output is maximum when 4 machines of type A and 3 machine of type B or 6 machines of type A and no machine of type B.

Question 9

A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 gm of silver and 1 gm of gold while that of type B requires 1 gm of silver and 2 gm of gold. The company can produce 9 gm of silver and 8 gm of gold. If each unit of type A birngs a profit of Rs 40 and that of type B Rs 50, find the number of units of each type that the company should produce to maximize the profit. What is the maximum profit?

Solution 9

Let number of goods A and B are x and y respectively.

Since, profits on each A and B are Rs 40 and Rs 50 respectively.So, profits on x of type A and y of type B are $40x$ and $50y$ respectively, Let Z be total profit on A and B, so,

$$Z = 40x + 50y$$

Since, each A and B require 3 gm and 1 gm of silver respectively. so, x of type A and y type B require $3x$ and y gm silver respectively but, Total silver available is 9 gm. so,

$$3x + y \le 9 \qquad \text{(first constraint)}$$

Since, each A and B require 1 gm and 2 gm of gold respectively. so, x of type A and y type B require x and $2y$ gm of gold respectively but, Total gold available is 8 gm, so,

$$x + 2y \le 8 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 40x + 50y$$

Subject to constraints,

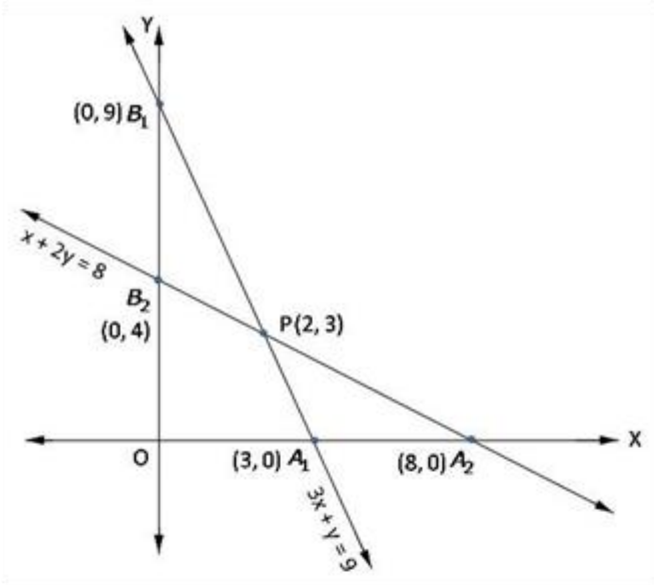
$$3x + y \le 9$$
$$x + 2y \le 8$$
$$x, y \ge 0$$

[Since production of A and B can not be less than zero]

Region $3x + y \le 9$: line $3x + y = 9$ meets axes at $A_1(3,0), B_1(0,9)$ respectively. Region containing origin represents $3x + y \le 9$ as $(0,0)$ satisfies $3x + y \ge 9$.

Region $x + 2y \le 8$: line $x + 2y = 8$ meets axes at $A_2(8,0)$, $B_2(0,4)$ respectively. Region containing origin represents $x + 2y \le 8$ as $(0,0)$ satisfies $x + 2y \le 8$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PB_2 is the feasible region.Point $P(2, 3)$ is obtained by solving $3x + y = 9$ and $x + 2y = 8$

The value of $Z = 40x + 50y$ at

$O(0,0) = 40(0) + 50(0) = 0$

$A_1(3,0) = 40(3) + 50(0) = 120$

$P(2,3) = 40(2) + 50(3) = 230$

$B_2(0,4) = 40(0) + 50(4) = 200$

Therefore maximum $Z = 230$ at $x = 2, Y = 3$

Hence,

Maximum profit = Rs 230 number of goods of type $A = 2$, type $B = 3$

Question 10

A manufacturer of Furniture makes two products: chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hrs on machine A and 6 hrs on machine B. A table requires 4 hrs on machine A and 2 hrs on machine B. There are 16 hrs of time per day available on machine A and 30 hrs on machine B. Profit gained by the manufacturer form a chair and a table is Rs 3 and Rs 5 respectively. Find with the help of graph what should be the daily production of each of the t products so as to maximize his profit.

Solution 10

Let daily production of chairs and tables be x and y respectively.

Since, profits on each chair and table are Rs 3 and Rs 5. So,
profits on x number of chairs and y number of tables are Rs $3x$ and Rs $5y$ respectively,
Let Z be total profit on table and chair, so,

$$Z = 3x + 5y$$

Since, each chair and table require 2 hrs and 4 hrs on machine A respectively. so,
x number of chair and y number of table require $2x$ and $4y$ hrs on machine A respectively but,
maximum time available on machine A be 16 hrs, so,

$$2x + 4y \le 16$$

$$x + 2y \le 8 \qquad \text{(first constraint)}$$

Since, each chair and table require 6 hrs and 2 hrs on machine B. so,
x number of chair and y number of table require $6x$ and $2y$ hrs on machine B respectively but,
maximum time available on machine B be 30 hrs, so,

$$6x + 2y \le 30$$

$$3x + y \le 15 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 3x + 5y$$

subject to constraints,

$$x + 2y \le 8$$

$$3x + y \le 15$$

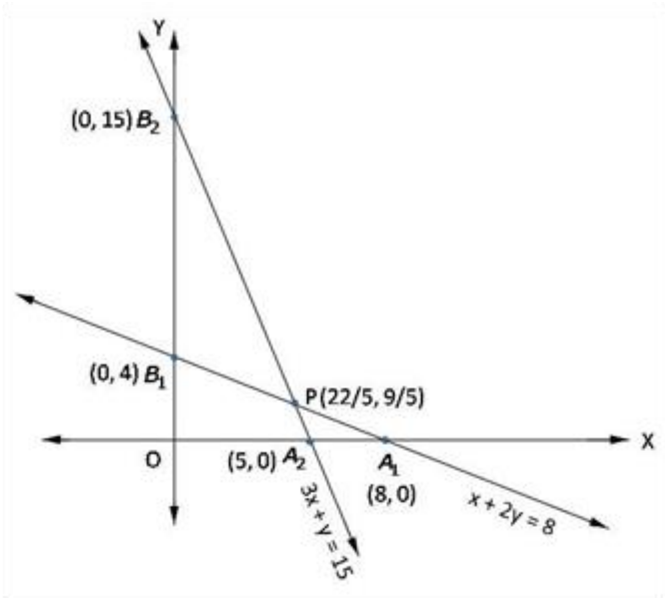
$$x, y \ge 0$$

[Since production of chair and table can not be less than zero]

Region $x + 2y \le 8$: line $x + 2y = 8$ meets axes at $A_1(8, 0), B_1(0, 4)$ respectively. Region containing origin represents $x + 2y \le 8$ as $(0,0)$ satisfies $x + 2y \le 8$.

Region $3x + y \le 15$: line $3x + y = 15$ meets axes at $A_2(5, 0), B_2(0,15)$ respectively. Region containing origin represents $3x + y \le 15$ as $(0,0)$ satisfies $3x + y \le 15$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $O\,A_2 P\,B_1$ representa feasible region.Point $P\left(\frac{22}{5},\frac{9}{5}\right)$ is obtained by solving $x + 2y = 8$ and $3x + y = 15$

The value of $Z = 3x + 5y$ at

$$O\,(0,0) \quad = 3(0) + 5(0) = 0$$
$$A_2\,(5,0) \quad = 3(5) + 5(0) = 15$$
$$P\left(\frac{22}{5},\frac{9}{5}\right) \quad = 3\left(\frac{22}{5}\right) + 5\left(\frac{9}{5}\right) = \frac{111}{5} = 22.2$$
$$B_1\,(0,4) \quad = 3(0) + 5(4) = 20$$

Maximum $Z = 22.2$ at $x = \frac{22}{5}, y = \frac{9}{5}$

Daily production of chair $= \frac{22}{5}$, table $= \frac{9}{5}$

maximum profit = Rs 22.2

Question 11

A furniture manufaturing company plans to make two products: chairs and tables.From its available resources which consists of 400 square feet to teak wood and 450 man-hours. It is known that to make a chair requires 5 square feet of wood and 10 man-hours and yeilds a profit of Rs 45, while each table uses 20 square feet of wood and 25 man-hours and yeilds a profit of Rs 80.

How many items of each product should be produced by the company so that the profit is maximum.

Solution 11

Let required production of chairs and tables be x and y.

Since, profits on each chair and table are Rs 45 and Rs 80, So,
profits on x number of chairs and y number of tables are Rs $45x$ and Rs $80y$,
Let Z be total profit on tables and chairs, so,

$$Z = 45x + 80y$$

Since, each chair and table require 5 sq.ft. and 20 sq.ft. of wood respectively. so,
x number of chair and y number of table require $5x$ and $20y$ sq.ft. of wood respectively but,
400 sq.ft. of wood is available, so,

$$5x + 20y \le 400$$

$$\Rightarrow \quad x + 4y \le 80 \qquad \text{(first constraint)}$$

Since, each chair and table require 10 and 25 men-hrs respectively. so,
x number of chairs and y number of tables require $10x$ and $25y$ men-hrs respectively but, only 450 men-hrs are available, so,

$$10x + 25y \le 450$$

$$\Rightarrow \quad 2x + 5y \le 90 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 45x + 80y$$

Subject to constraints,

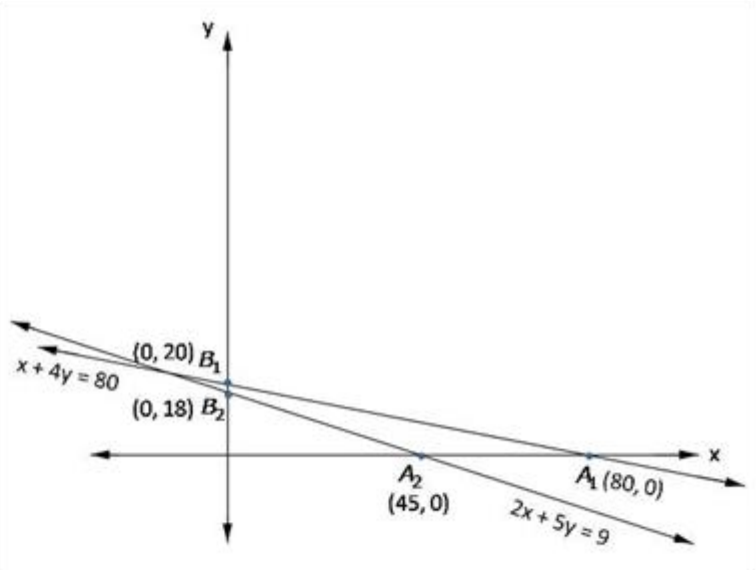
$$x + 4y \le 80$$
$$2x + 5y \le 90$$
$$x, y \ge 0$$

$$\left[\text{Since production of tabel and chair can not be less than zero}\right]$$

Region $x + 4y \le 80$: line $x + 4y = 80$ meets axes at $A_1(80,0)$, $B_1(0,20)$ respectively. Region containing origin represents $x + 4y \le 80$ as $(0,0)$ satisfies $x + 4y \le 80$.

Region $2x + 5y \le 90$: line $2x + 5y = 90$ meets axes at $A_2(45,0)$, $B_2(0,18)$ respectively. Region containing origin represents $2x + 5y \le 90$ as $(0,0)$ satisfies $2x + 5y \le 90$.

Region $x, y \geq 0$: it represents first quadrant.



Shaded region OA_2B_2 is the feasible region.

The value of $Z = 45x + 80y$ at

$O(0,0) \quad = 45(0) + 80(0) = 0$

$A_2(45,0) \quad = 45(45) + 80(0) = 2025$

$B_2(0,18) \quad = 45(0) + 80(18) = 1440$

Therefore,

Maximum $Z = 2025$ at $x = 45, y = 0$

Profit is maximum when number of chairs = 45, tables = 0

profit = Rs 2025

Question 12

A firm manufactures two products A and B. Each product is processed on two machine M_1 and M_2. Product A requires 4 minutes of processing time on M_1 and 8 min. on M_2; product B requires 4 minutes on M_1 and 4 min. on M_2. The machine M_1 is available for not more than 8 hrs 20 min. while machine M_2 is available for 10 hrs. during any working day. The products A and B are sold at a profit of Rs 3 and Rs 4 respectively.

Formulate the problem as a linear programming problem and find how many products of each type should be produced by the firm each day in order to get maximum profit.

Solution 12

Let required production of product A and B be x and y respectively.

Since, profit on each product A and B are Rs 3 and Rs 4 respectively, So,
profit on x product A and y product B are Rs $3x$ and Rs $4y$ respectively,
Let Z be the total profit on product, so,

$$Z = 3x + 4y$$

Since, each product A and B requires 4 minutes each on machine M_1. so,
x product A and y product B require $4x$ and $4y$ minutes on machine M_1 respectively
but maximum available time on machine M_1 is 8 hrs 20 min.=500 min. so,

$$4x + 4y \le 500$$

$$\Rightarrow \quad x + y \le 125 \qquad \text{(first constraint)}$$

Since, each product A and B requires 8 minutes and 4 min. on machine M_2 respectively. so,
x product A and y product B require $8x$ and $4y$ min. respectively on machine M_2
but, maximum available time on machine M_2 is 10 hrs =600 min. so,

$$8x + 4y \le 600$$

$$\Rightarrow \quad 2x + y \le 150 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 3x + 4y$$

subject to constraints,

$$x + y \le 125$$

$$2x + y \le 150$$

$$x, y \ge 0$$

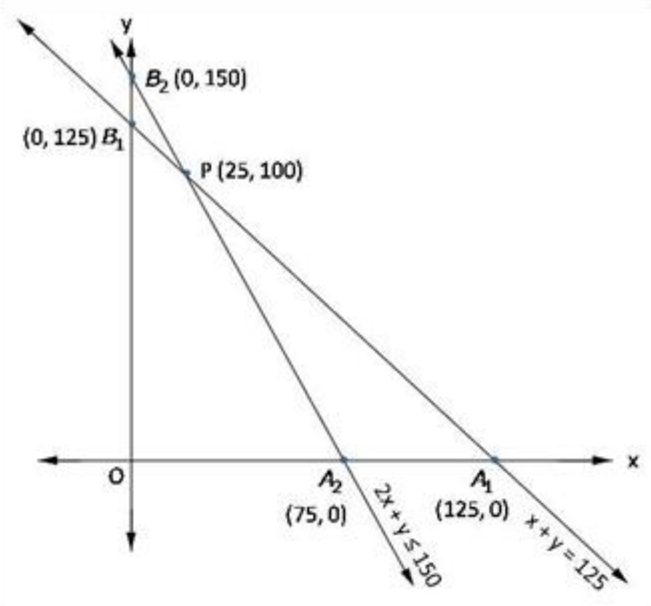
[Since number of product can not be less than zero]

Region $x + y \le 125$: line $x + y = 125$ meets axis at $A_1(125,0)$, $B_1(0,125)$ respectively. Region $x + y \le 125$ contains origin represents as $(0,0)$ satisfies $x + y \le 125$.

Region $2x + y \le 150$: line $2x + y = 150$ meets axis at $A_2(75,0)$, $B_2(0,150)$ respectively. Region containing origin represents $2x + y \le 150$ as $(0,0)$ satisfies $2x + y \le 150$

Region $x, y \ge 0$: it represents first quadrant.

Shaded region OA_2PB_1 is feasible region $P(25,100)$ is obtained by solving $x + y = 125$ and $2x + y = 150$



The value of $Z = 3x + 4y$ at

$O(0,0)$	$= 3(0) + 4(0) = 0$
$A_2(75,0)$	$= 3(75) + 4(0) = 225$
$P(25,100)$	$= 3(25) + 4(100) = 475$
$B_1(0,125)$	$= 3(0) + 4(125) = 500$

Maximum profit = Rs 500, product $A = 0$
product $B = 125$

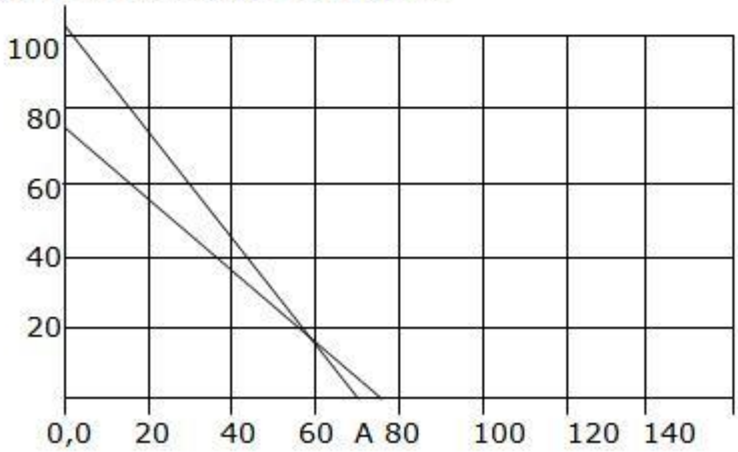
Question 13

A firm manufacturing two types of electric items, A and B, can make a profit of Rs 20 per unit of A and Rs 30 per unit of B. Each unit of A requires 3 motors and 4 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an export model requiring a voltage stabilizer which has a supply restricted to 65 units per month. Formulate the linear programing problem for maximum profit and solve it graphically.

Solution 13

	Item A	Item B	
	x	y	
Motors	3x	2y	≤ 210
Transformer	4x	4y	≤ 300
Profit Rs.	20x	30y	Maximize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that maximize the function $Z = 20x + 30y$, subject to the conditions
$3x + 2y \leq 210$; gives $x=0$, $y= 105$ & $y=0$, $x=70$
$4x + 4y \leq 300$; gives $x=0$, $y=75$ & $y=0$, $x=75$
$x, y \geq 0$. Plotting the constraints,



The feasible region is 80-B-A-0,0
Tabulating the value of Z at the corner points

Corner point	Value of $Z = 20x + 30y$
0, 0	0
0, 75	2250
70, 0	1400
60, 15	1650

The maximum occur with the production of 0 units of Item A and 75 units of Item B, with a value of Rs. 2250/-

Question 14

A factory uses three different resources for the manufacture of two different products, 20 units of the resources *A*, 12 units of *B* and 16 units of *C* being available. 1 unit of the first product requires 2, 2 and 4 units of the respective resources and 1 unit of the second product requires 4, 2 and 0 units of respective resources. It is known that the first product gives a profit of 2 monetary units per unit and the second 3. Formulate the linear programming problem. How many units of each product should be manufactured for maximizing the profit? Solve it graphically.

Solution 14

Let number of I product and II product produced are x and y respectively.

Since, profits on each unit of product I and product II are 2 and 3 monetary unit, So,
profits on x units of product I and y units of product II are $2x$ and $3y$ monetary units respectively,
Let Z be total profit, so,

$$Z = 2x + 3y$$

Since, each product I and II require 2 and 4 units of resources A, so,
x units of product I and y units of product II require $2x$ and $4y$ units of resource A respectively, but maximum available quantity of resource A is 20 units. so,

$$2x + 4y \le 20$$

$$\Rightarrow \quad x + 2y \le 10 \quad \text{(first constraint)}$$

Since, each product I and II require 2 and 4 units of resource B each, so,
x units of product I and y units of product II require $2x$ and $2y$ units of resource B respectively, but maximum available quantity of resource B is 12 units. so,

$$2x + 2y \le 12$$

$$\Rightarrow \quad x + y \le 6 \quad \text{(second constraint)}$$

Since, each units of product I require 4 units of resource C. It is not required by product II , so, x units of product I require $4x$ units of resource C, but maximum available quantity of resource C is 16 units. so,

$$4x \le 16$$

$$\Rightarrow \quad x \le 6 \quad \text{(Third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \le 10$$
$$x + y \le 6$$
$$x \le 4$$
$$x, y \ge 0$$

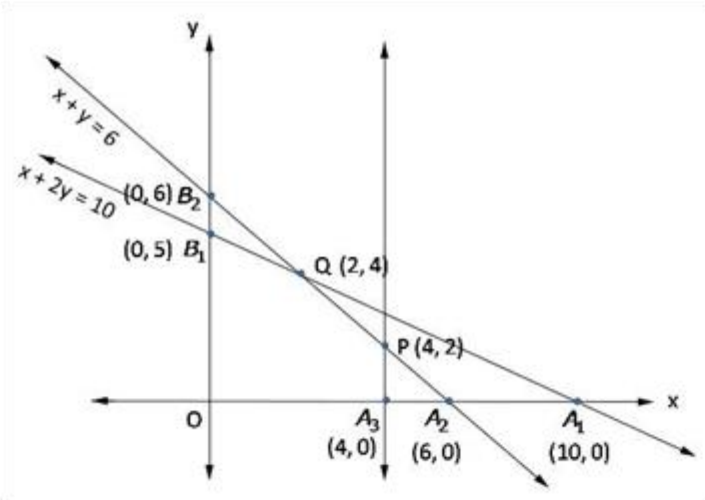
[Since production fo I and II can not be less than zero]

Region $x + 2y \le 10$: line $x + 2y = 10$ meets axes at $A_1(10, 0), B_1(0, 5)$ respectively. Region containing origin represents $x + 2y \le 10$ as $(0,0)$ satisfies $x + 2y \le 10$.

Region $x + y \le 6$: line $x + y = 6$ meets axes at $A_2(6,0)$, $B_2(0,6)$ respectively. Region containing origin represents $x + y \le 6$ as $(0,0)$ satisfies $x + y \le 6$.

Region $x \le 4$: line $x = 4$ is parallel to y - axis and meets y-axis at $A_3(4,0)$. Region containing origin represents $x \le 4$ as $(0,0)$ satisfies $x \le 4$

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_3PQB_1 represents feasible region $P(4,2)$ is obtained by solving $x = 4$ and $x + y = 6$, $Q(2,4)$ is obtained by solving $x + y = 6$ and $x + 2y = 10$.

The value of $Z = 2x + 3y$ at

$O(0,0)$	$= 2(0) + 3(0) = 0$
$A_3(4,0)$	$= 2(4) + 3(0) = 8$
$P(4,2)$	$= 2(4) + 3(2) = 14$
$Q(12,4)$	$= 2(12) + 3(4) = 16$
$B_1(0,5)$	$= 2(0) + 3(5) = 15$

Maximum $Z = 16$ at $x = 2, y = 4$
First product = 2 units, second product = 4 unit
Maximum profit = 16 monetary units

Question 15

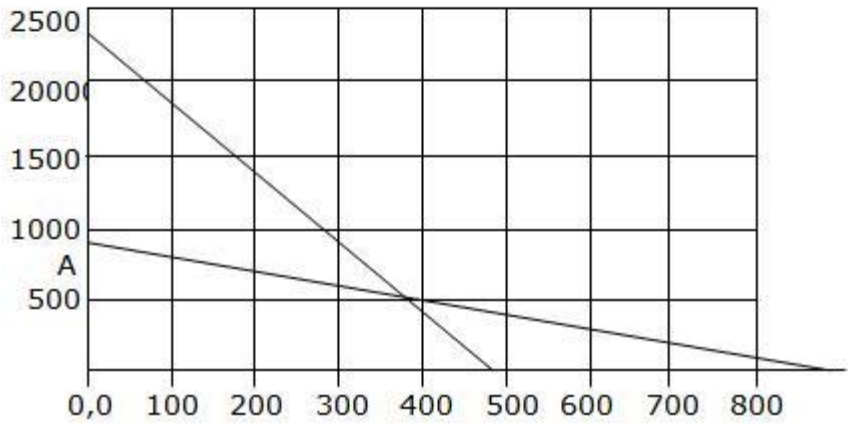
A publisher sells a hard cover edition of a text book for Rs 72.00 and a paperback edition of the same text for Rs 40.00. Costs to the publisher are Rs 56.00 and Rs 28.00 per book respectively in addition to weekly costs of Rs 9600.00. Both types require 5 minutes of printing time, although hardcover requires 10 minutes binding time and the paperback requires only 2 minutes. Both the printing and binding operations have 4,800 minutes available each week. How many of each type of book should be produced in order to maximize profit?

Solution 15

	Hardcover	Paperback	
	x	y	
Printing time	5x	5y	≤ 4800
Binding time	10x	2y	≤ 4800
Selling price Rs.	72x	40y	Maximize

The above LPP can be presented in a table above.

Aim is to find the values of x & y that maximize the function Z = 72x + 40y, subject to the conditions
5x + 5y ≤ 4800; gives x=0, y= 960 & y=0, x=960
10x + 2y ≤ 4800; gives x=0, y=2400 & y=0, x=480
x, y ≥ 0. Plotting the constraints,



The feasible region is A-B-480-0,0
Tabulating the value of Z at the corner points

Corner point	Value of Z = 72x + 40y
0, 0	0
0, 480	19200
360, 600	49920
480, 0	34560

The maximum occurs with the production of 360 units of Hardcover books and 600 units of Paperback books, with a value of Rs. 49920/-. This the selling price.

Cost price = fixed cost + variable cost

= 9600+56x360+28x600 = 46560

Profit = Selling price − cost price = 49920-46560

= Rs. 3360

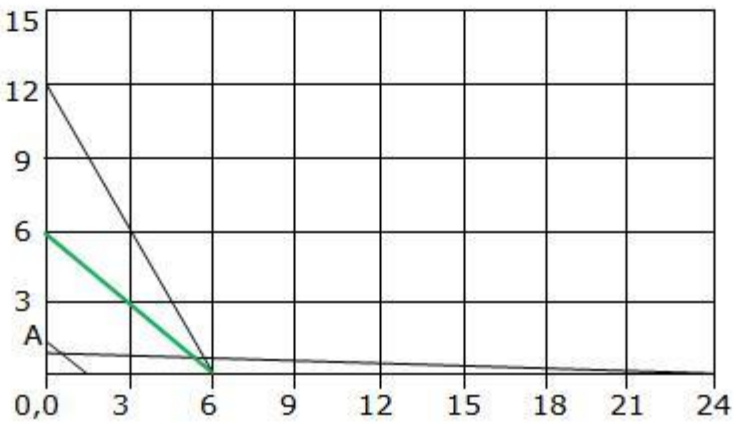
Question 16

A firm manufactures headache pills in two sizes *A* and *B*. Size *A* contains 2 grains of aspiring, 5 grains of bicarbonate and 1 grain of codeine; size *B* contain 1 grain of aspirin, 8 grains of bicarbonate and 66 grains of codeine.It has been found by users that it requires at least 12 grains of aspirin, 7.4 grians of bicarbonate and 24 grains of codeine for providing immediate effects. Determine graphically the least number of pills a patient should have to get immediate relief. Determine also the quantity of codeine consumed by patient.

Solution 16

	Pill size A	Pill size B	
	x	y	
Aspirin	2x	1.y	≥ 12
Bicarbonate	5x	8y	≥ 7.4
Codeine	1.x	66y	≥ 24
Relief	x	y	Minimize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that minimize the function Z = x + y, subject to the conditions
$2x + y \geq 12$; gives x=0, y= 12 & y=0, x=6
$5x + 8y \geq 7.4$; gives x=0, y=7.4/8 & y=0, x=7.4/5
$x + 66y \geq 24$; gives x=0, y=4/11 & y=0, x=24
$x, y \geq 0$. Plotting the constraints,



The feasible region is 12-C-24
Tabulating the value of Z at the corner points

Corner point	Value of Z = x + y
0, 12	12
24, 0	24
5.86, 0.27	6.13

The minimum occurs with 5.86 pills of size A and 0.27 pills of size B. since the feasible region is unbounded plot x+y < 6.13. the green line shows here are no common points with the unbounded feasible region so the obtained point is the point that gives minimum pills to be consumed.

Question 17

A chemical company produces two compounds, *A* and *B*. The following table gives the units of ingredients, *C* and *D* per kg of compounds *A* and *B* as well as minimum requirements of *C* and *D* and costs per kg of *A* and *B*. Find the quantities of *A* and *B* which would give a supply of *C* and *D* at a minimum cost.

	Compound		*Minimum requirement*
	A	*B*	
Ingredient *C*	1	2	80
Ingredient *D*	3	1	75
Cost (in Rs per kg)	4	6	

Solution 17

Let required quantity of compound A and B are x and y kg.

Since, cost of one kg of compound A and B are Rs 4 and Rs 6 per kg.So,
cost of x kg. of compound A and y kg. of compound B are Rs $4x$ and Rs $6y$ respectively, Let Z be the total cost of compounds, so,

$$Z = 4x + 6y$$

Since, compound A and B contain 1 and 2 units of ingredient C per kg. respectively, so, x kg. of compound A and y kg. of compound B contain x and $2y$ units of ingredient C respectively but minimum requirement of ingredient C is 80 units, so,

$$x + 2y \geq 80 \qquad \text{(first constraint)}$$

Since, compound A and B contain 3 and 1 unit of ingredient D per kg. respectively, so, x kg. of compound A and y kg. of compound B contain $3x$ and y units of ingredient D respectively but minimum requirement of ingredient D is 75 units, so,

$$3x + y \geq 75 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 4x + 6y$$

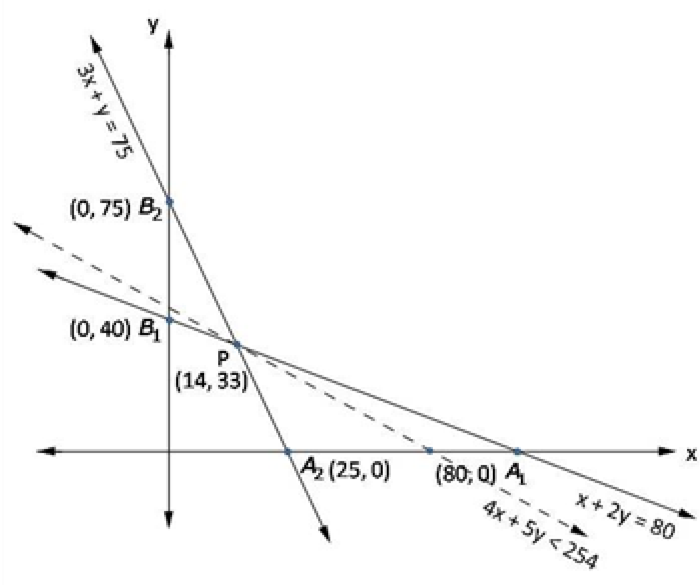
Subject to constraints,

$$x + 2y \geq 80$$
$$3x + y \geq 75$$
$$x, y \geq 0$$
$$\left[\text{Since production can not be less than zero}\right]$$

Region $x + 2y \geq 80$: line $x + 2y = 80$ meets axes at $A_1(80, 0)$, $B_1(0, 40)$ respectively. Region not containing origin represents $x + 2y \geq 80$ as $(0,0)$ does not satisfy $x + 2y \geq 80$.

Region $3x + y \geq 75$: line $3x + y = 75$ meets axes at $A_2(25, 0)$, $B_2(0, 75)$ respectively. Region not containing origin represents $3x + y \geq 75$ as $(0,0)$ does not satisfy $3x + y \geq 75$.

Region $x, y \geq 0$: it represents first quadrant.



Unbouded shaded region $A_1 P B_2$ represents feasible region. point P is obtained by solving $x + 2y = 80$ and $3x + y = 75$

The value of $Z = 4x + 6y$ at

$$A_1 (80, 0) \quad = 4(80) + 6(0) = 320$$
$$P (14, 33) \quad = 4(14) + 6(33) = 254$$
$$B_2 (0, 75) \quad = 4(0) + 6(75) = 450$$

Smallest value of $Z =$ 254 open half plane $4x + 6y < 254$ has no point in common with feasible region. so,

Smallest value is the minimum value.

Minimum cost=Rs 254
quantity of A = 14 kg
quantity of B = 33 kg

Question 18

A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours of assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

Solution 18

Let the company manufacture x souvenirs of type A and y souvenirs of type B. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Type A	**Type B**	**Availability**
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are

$5x + 8y \leq 200$

$10x + 8y \leq 240$ i.e., $5x + 4y \leq 120$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 6y$ ... (1)

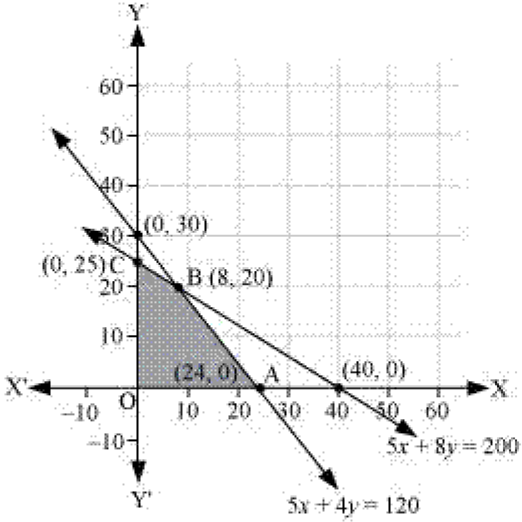
subject to the constraints,

$5x + 8y \leq 200$ ... (2)

$5x + 4y \leq 120$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

Corner point	$Z = 5x + 6y$	
A(24, 0)	120	
B(8, 20)	160	→ Maximum
C(0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

Question 19

A manufacturer makes two products A and B. Product A sells at Rs 200 each and takes $\frac{1}{2}$ hour to make. Product B sells at Rs 300 each and takes 1 hour to make. There is a permanent order for 14 of product A and 16 of product B. A working week consists of 40 hours of production and weekly turnover must not be less than Rs10000. If the profit on each of product A is Rs 20 and on product B is Rs 30, then how many of each should be produced so that the profit is maximum. Also, find the maximum profit.

Solution 19

Let required number of product A and B be x and y respectively.

Since, profit on each product A and B are Rs 20 and Rs 30 respectively.So, x number of product A and y number of product B gain profits of Rs $20x$ and Rs $30y$ respectively, Let Z be total profit then,

$$Z = 20x + 30y$$

Since, selling prices of each product A and B are Rs 200 and Rs 300 respectively, so, revenues earned by selling x units of product A and y units of product B are $200x$ and $300y$ respectively but weekly turnover must not be less than Rs 10000, so,

$$200x + 300y \geq 10000$$

$$2x + 3y \geq 100 \qquad \text{(first constraint)}$$

Since, each product A and B require $\frac{1}{2}$ and 1 hr.to make so, x units of product A and y units of product B are $\frac{1}{2}x$ and y hrs. to make respectively but working time available is 40 hrs maximum, so,

$$\frac{1}{2}x + y \leq 40$$

$$x + 2y \leq 80 \qquad \text{(second constraint)}$$

There is a permanent order of 14 and 16 of product A and B respectively, so,

$$x \geq 14$$

$$y \geq 16 \qquad \text{(third and fourth constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 20x + 30y$$

Subject to constraints,

$$2x + 3y \geq 100$$

$$x + 2y \leq 80$$

$$x \geq 40$$

$$y \geq 14$$

$$x, y \geq 0$$

$$\left[\text{Since production can not be less than zero}\right]$$

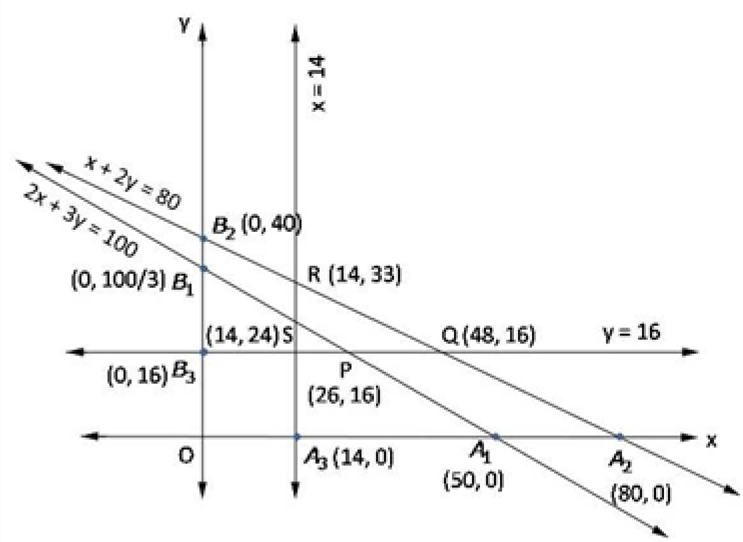
Region $2x + 3y \geq 100$: line $2x + 3y = 100$ meets axes at $A_1(50,0), B_1\left(0, \frac{100}{3}\right)$ respectively. Region not containing origin represents $2x + 3y \geq 100$ as $(0,0)$ does not satisfy $2x + 3y \geq 100$.

Region $x+2y \le 80$: line $x+2y=80$ meets axes at $A_2(80,0), B_2(0,40)$ respectively. Region not containing origin represents $x+2y \le 80$ as $(0,0)$ satisfies $x+2y \le 80$.

Region $x \ge 14$: line $x = 14$ is parallel to y-axis and meets x-axis at $A_3(14,0)$. Region not containing origin represents $x \ge 14$ as $(0,0)$ does not satisfy $x \ge 14$.

Region $y \ge 16$: line $y = 16$ is parallel to x-axis and meets y-axis at $B_3(0,16)$. Region not containing origin represents $y \ge 16$ as $(0,0)$ does not satisfy $y \ge 16$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $PQRS$ represents feasible region. Point $P(26,16)$ is obtained by solving $y = 16$ and $2x+3y = 100$, $Q(48,16)$ is obtained by solving $y = 16$ and $x+2y = 80$, $R(14,33)$ is obtained by solving $x = 14$ and $x+2y = 80$, $S(14,24)$ is obtained by solving $x = 14$ and $2x+3y = 100$

The value of $Z = 20x + 30y$ at

$P(26,16)$	$= 20(26) + 30(16) = 1000$
$Q(48,16)$	$= 20(48) + 3(16) = 1440$
$R(14,33)$	$= 20(14) + 3(33) = 1270$
$S(14,24)$	$= 20(14) + 3(24) = 1000$

maximum $Z = 1440$ at $x = 48, y = 16$

Number product $A = 48$, product $B = 16$

maximum profit = Rs 1440

Question 20

A manufacturer produces two types of steel trunks. He has two machines *A* and *B*. For completing, the first type of the trunk requires 3 hours on machine *A* and 3 hours on machine *B*, whereas the second type of the trunk requires 3 hours on machine *A* and 2 hours on machine *B*. Machines *A* and *B* can work at most for 18 hours and 15 hours per day respectively. How many trunks of each type must the make each day to make maximum profit?

Solution 20

Let required number of trunk I and trunk II be x and y respectively.

Since, profit on each trunk I and trunk II are Rs 30 and Rs 25 respectively.So, profit on x trunk of type I and y trunk of type II are Rs $30x$ and Rs $25y$ respectively, Let total profit on trunks be Z, so,

$$Z = 30x + 25y$$

Since, each trunk I and trunk II is reequired to work 3 hrs each on machine A, so, x trunk I and y trunk II is required $3x$ and $3y$ hrs respectively to work on machine A but machine A can work for at most 18 hrs, so,

$$3x + 3y \le 18$$

$$\Rightarrow \quad x + y \le 6 \qquad \text{(first constraint)}$$

Since, each trunk I and II is reequired to work 3 hrs and 2 hrs on machine B, so, x trunk I and y trunk II is required $3x$ and $2y$ hrs to work respectively on machine B but machine B can work for at most 15 hrs, so,

$$3x + 2y \le 15 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 30x + 25y$$

Subject to constraints,

$$x + y \le 6$$

$$3x + 2y \le 15$$

$$x, y \ge 0$$

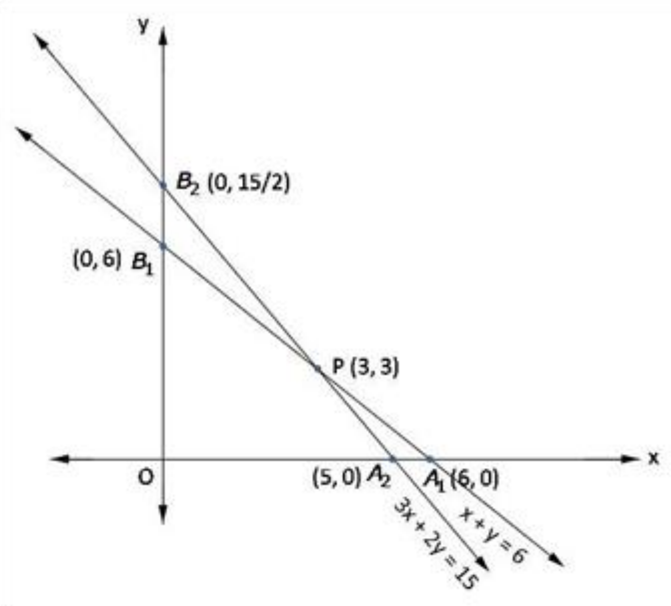
[Since production of trunk can not be less than zero]

Region $x + y \le 6$: line $x + y = 6$ meets axes at $A_1(6,0)$, $B_1(0,6)$ respectively. Region containing origin represents $x + y \le 6$ as $(0,0)$ satisfies $x + y \le 6$.

Region $3x + 2y \le 15$: line $3x + 2y = 15$ meets axes at $A_2(5,0)$, $B_2\left(0, \frac{15}{2}\right)$ respectively. Region containing origin represents $3x + 2y \le 15$ as $(0,0)$ satisfies $3x + 2y \le 15$.

Region $x, y \ge 0$: it represents first quadrant.

Shaded region A_2PB_1 represents feasible region. Point $P(3,3)$ is obtained by solving $x + y = 6$ and $3x + 2y = 15$,



The value of $Z = 30x + 25y$ at

$A_2(5,0)$ $= 30(5) + 25(0) = 150$

$P(3,3)$ $= 30(3) + 25(3) = 165$

$B_1(0,6)$ $= 30(0) + 25(6) = 150$

$O(0,0)$ $= 30(0) + 25(0) = 0$

maximum $Z = 165$ at $x = 3, y = 3$

Trunk of type $A = 3$, type $B = 3$

maximum profit = Rs 165

Question 21

A manufacturer of patent medicines is preparing a production plan on medicines, A and B, There are sufficient raw materials available to make 20000 bottles of A and 40000 bottles of B, but there are only 45000 bottles into which either of the medicines can be put. Further, it takes 3 hours to prepare enough material to fiil 1000 bottles of A, it takes 1 hour prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is Rs 8 per bottle for A and Rs 7 per bottle for B. How should the manufacturer schedule his producation in order to maximize his profit?

Solution 21

Let production of each bottle of A and B are x and y respectively.

Since, profits on each bottle of A and B are Rs 8 and Rs 7 per bottle respectively.So, profit on x bottles of A and y bottles of B are $8x$ and $7y$ respectively, Let Z be total profit on bottles so,

$$Z = 8x + 7y$$

Since, it takes 3 hrs and 1 hr to prepare enough material to fill 1000 bottlesof type A and B respectively, so, x bottles of A and y bottles of B are preparing is $\frac{3x}{1000}$hrs and $\frac{y}{100}$ hrs respectively but total 66 hrs are available, so,

$$\frac{3x}{1000} + \frac{y}{1000} \le 66$$

$$\Rightarrow \quad 3x + y \le 66000 \qquad \text{(first constraint)}$$

Since, row material available to make 2000 bottles of A and 4000 bottles of B but there are 45000 bottles into which either of medicines can be put so,

$$\Rightarrow \quad x \le 20000 \qquad \text{(second constraint)}$$

$$y \le 40000 \qquad \text{(third constraint)}$$

$$x + y \le 45000 \qquad \text{(fourth constraint)}$$

$$x, y \ge 0$$

[Since production of bottles can not be less than zero]

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 8x + 7y$

Subject to constriants,

$$3x + y \le 66000$$

$$x \le 20000$$

$$y \le 40000$$

$$x + y \le 45000$$

$$x, y \ge 0$$

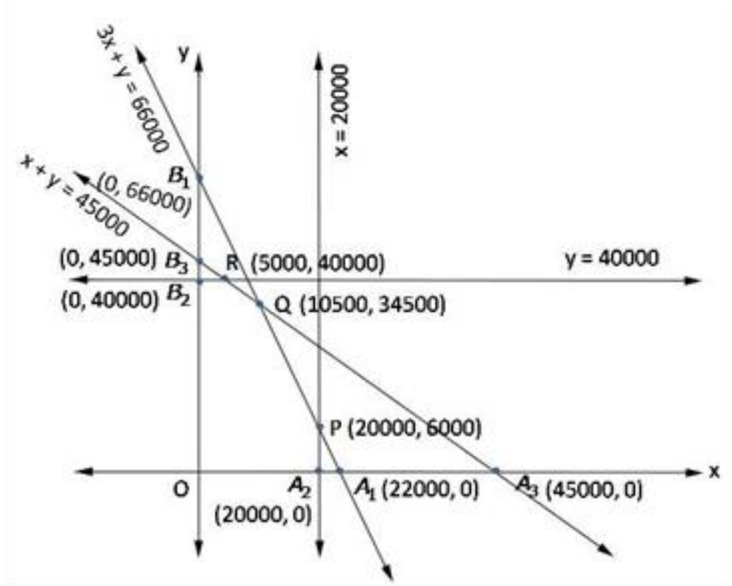
Region $3x + y \le 66000$: line $3x + y = 66000$ meets axes at $A_1(22000, 0)$, $B_1(0, 66000)$ respectively. Region containing origin represents $3x + y \le 66000$ as $(0,0)$ satisfies $3x + y \le 66000$.

Region $x \le 20000$: line $x = 20000$ is parallel to y-axis and meets x-axis at $A_2(20000, 0)$. Reg
containing origin represents $x \le 20000$ as $(0,0)$ satisfies $x \le 20000$.

Region $y \le 40000$: line $y = 40000$ is parallel to x-axis and meets y-axis at $B_2(0, 40000)$. Reg
containing origin represents $y \le 40000$ as $(0,0)$ satisfies $y \le 40000$.

Region $x + y \le 45000$: line $x + y = 45000$ meets axes at $A_3(45000,0)$, $B_3(0,45000)$ respectiv
containing origin represents $x + y \le 45000$ as $(0,0)$ satisfies $x + y \le 45000$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PRB_2 represents feasible region. Point $P(20000,6000)$ is obtained by solving $x = 20000$ and $3x + y = 66000$, $Q(10500, 34500)$ is obtained by solving $x + y = 45000$ and $3x + y = 66000$, $R(15000,40000)$ is obtained by solving $x + y = 45000$, $y = 40000$

The value of $Z = 8x + 7y$ at

$O(0, 0)$	$= 8(0) + 7(0) = 0$
$A_2(20000, 0)$	$= 8(20000) + 7(0) = 160000$
$P(20000, 6000)$	$= 8(20000) + 7(6000) = 202000$
$Q(10500, 34500)$	$= 8(10500) + 7(34500) = 325500$
$R(5000, 4000)$	$= 8(5000) + 7(40000) = 32000$
$B_2(0, 40000)$	$= 8(0) + 7(40000) = 250000$

maximum $Z = 325500$ at $x = 10500, y = 34500$

Number bottles A type = 10500, B type = 34500

maximum profit = Rs 325500

Question 22

A aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats of first class. However, at least 4 times as many passengers prefer to travel by economy class to the first class. Determine how many each type of tickets must be sold in order to maximize the profit for airline. What is the maximum profit.

Solution 22

Let required number of first class and economy class tickets be x and y respectively.

Each ticket of first class and economy class make profit of Rs 400 and Rs 600 respectively.So, x ticket of first class and y tickets of economy class make profits of Rs $400x$ and Rs $600y$ respectively, Let total profit be Z, so,

$$Z = 400x + 600y$$

Given, aeroplane can carry a miximum of 200 passengers,so,

$$\Rightarrow \quad x + y \le 200 \qquad \text{(first constraint)}$$

Given, airline reservesa at least 20 seats for first class,so,

$$\Rightarrow \quad x \ge 20 \qquad \text{(second constraint)}$$

Given, at least 4 times as mnay passengers prefer totravel by economy class to the first class ,so,

$$y \ge 4x$$

$$\Rightarrow \quad 4x - y \le 0 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 400x + 600y$

Subject to constriants,

$$x + y \le 200$$

$$x \ge 20$$

$$4x - y \le 0$$

$$x, y \ge 0$$

[Since seats of both the classes can not be less than zero]

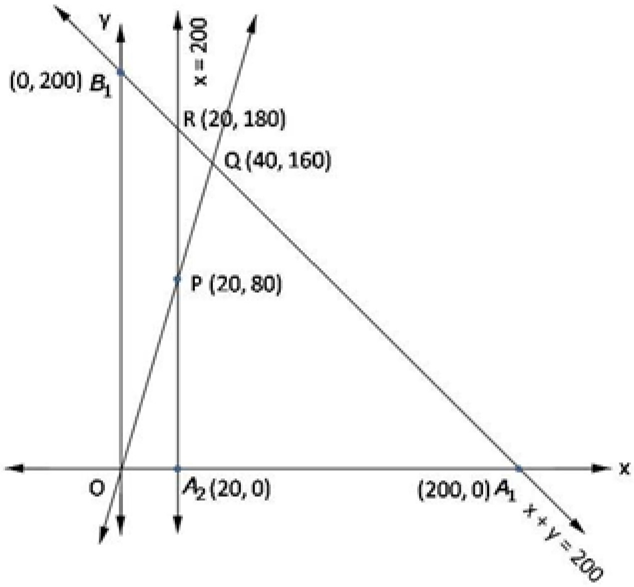
Region $x + y \le 200$: line $x + y = 200$ meets axes at $A_1(200,0)$, $B_1(0,200)$ respectively.

Region containing origin represents $x + y \le 200$ as $(0,0)$ satisfies $x + y \le 200$.

Region $x \geq 20$: line $x = 200$ is parallel to y-axis and meets x-axis at $A_2(20, 0)$. Region not containing origin represents $x \geq 20$ as $(0,0)$ does not satisfy $x \geq 20$.

Region $4x - y \leq 0$: line $4x - y = 0$ passes through origin and $P(20,80)$.region containing $B(0, 200)$ represents $4x - y \leq 0$ as $(0,200)$ satisfies $4x - y \leq 0$.

Region $x, y \geq 0$: it represents first quadrant.



Shaded region PQR represents feasible region. $Q(40,160)$ is obtained by solving $x + y = 200$ and $4x - y = 0$, $R(20,180)$ is obtained by solving $x = 20$ and $x + y = 200$

The value of $Z = 400x + 600y$ at

$P(20,80) \quad = 400(20) + 600(80) = 56000$

$Q(40,160) \quad = 400(40) + 600(160) = 112000$

$R(20,180) \quad = 400(20) + 600(180) = 116000$

so,

maximum Z = Rs 116000 at $x = 20, y = 180$

Number of first class ticket = 20,
Number of economy class ticket= 180
maximum profit = Rs 116000

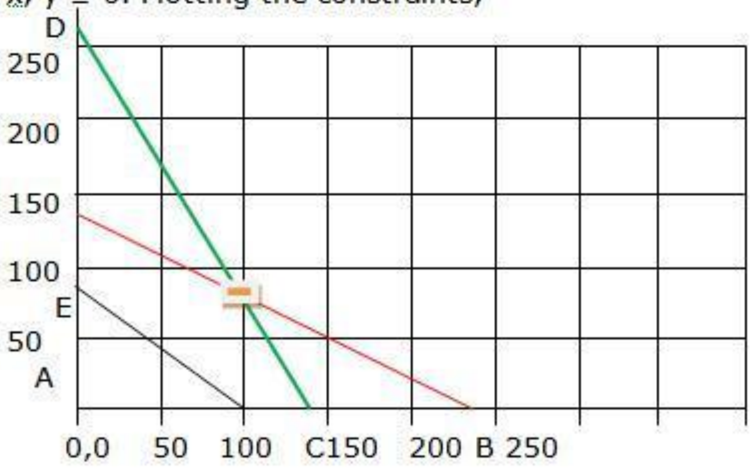
Question 23

A gardener has supply of fertilizer of type I which consists of 10% nitrogen and 6% phosphoric acid and type II fertilizer which consists of 5% nitrogen and 10% phosphoric acid . After testing the soil condition,he finds that he needs at least 14 kg of nitrogen and 14 kg. phosporic acid for his crop. If the type I fertilizer costs 60 paise per kg and type II fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Solution 23

	Type I	Type II	
	x	y	
Nitrogen	0.1x	0.05y	≥ 14
Bicarbonate	0.06x	0.1y	≥ 14
Cost	0.6x	0.4y	Minimize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that minimize the function Z = 0.6x + 0.4y, subject to the conditions
$0.1x + 0.05y \geq 14$; gives x=0, y= 280 & y=0, x=140
$0.06x + 0.1y \geq 14$; gives x=0, y=140 & y=0, x=233.33
x, y ≥ 0. Plotting the constraints,



The feasible region is the unbounded region D-C-B

Corner point	Value of Z = 0.6x + 0.4y
0, 280	112
233.33, 0	140
100, 80	92

The minimum occurs at x=100, y=80 with a value of 92
Since the region is unbounded plot $0.6x + 0.4y \leq 92$
Plotting the points, we get line E-100.
There are no common points so x=100, y=80 with a value of 92 is the optimal minimum.

Question 24

Anil wants to invest at most Rs 12000 in Saving Certificates and National Saving Bonds. According to rules, he has to invest at least Rs 2000 in Saving Certificates and at least Rs 4000 in National Saving Bonds. If the rate of interest on saving certificate is 8% per annum and the rate of interest on National Saving Bond is 10% per annum, how much money should he invest to earn maximum yearly income? Find also his maximum yearly income.

Solution 24

Let he invests Rs x and Rs y in saving certificate (sc) and National saving bond (NSB) respectively.

Since, rate of interest on SC is 8% annual and on NSB is 10% annual,So, interest on Rs x of SC is $\frac{8x}{100}$ and Rs y of NSB is $\frac{10x}{100}$ per annum.

Let Z be total interest earned so,

$$Z = \frac{8x}{100} + \frac{10y}{100}$$

Given he wants to invest Rs 12000 is total

$$x + y \le 12000 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = \frac{8x}{100} + \frac{10y}{100}$$

Subject to constriants,

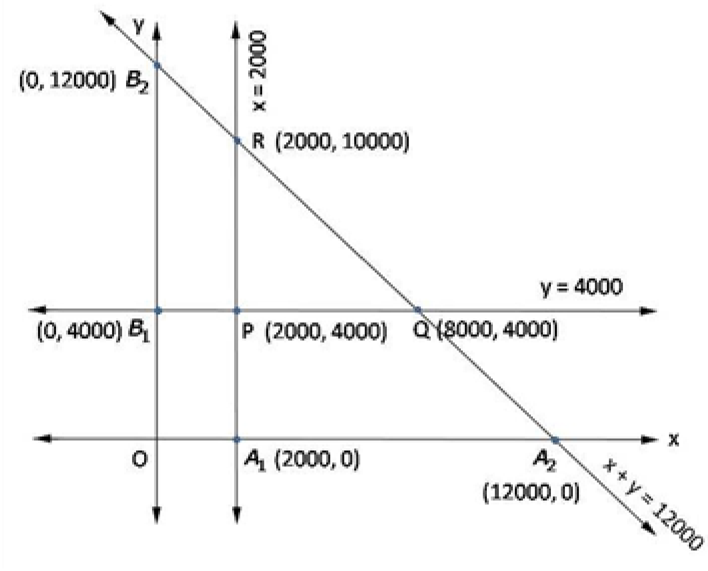
$$x \ge 2000$$
$$y \ge 4000$$
$$x + y \le 12000$$
$$x, y \ge 0 \qquad [\text{Since investment can not be less than zero}]$$

Region $x \ge 2000$: line $x = 2000$ is parallel to y-axis and meets x-axis at $A_1(2000, 0)$. Region not containing origin represents $x \ge 2000$ as $(0,0)$ does not satisfy $x \ge 2000$

Region $y \ge 4000$: line $y = 4000$ is parallel to x-axis and meets y-axis at $B_1(0, 4000)$. Region not containing origin represents $y \ge 4000$ as $(0,0)$ does not satisfy $y \ge 4000$.

Region $x + y \le 12000$: line $x + y = 12000$ meets axes at $A_2(12000, 0)$, $b_2(0, 1200)$ respectively. Region containing represents $x + y \le 12000$ as $(0,0)$ satisfies $x + y \le 12000$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region PQR represents feasible region. $P(2000, 4000)$ is obtained by solving $x = 2000$ and $y = 4000$, $Q(8000, 4000)$ is obtained by solving $x + y = 12000$ and $y = 4000$ $R(2000, 10000)$ is obtained by solving $x = 2000$ and $y + x = 1200$

The value of $Z = \frac{8x}{100} + \frac{10y}{100}$ at

$$P(2000, 4000) \qquad = \frac{8}{100}(2000) + \frac{10}{100}(4000) = 560$$

$$Q(8000, 4000) \qquad = \frac{8}{100}(8000) + \frac{10}{100}(4000) = 1040$$

$$R(2000, 10000) \qquad = \frac{8}{1000}(2000) + \frac{10}{100}(10000) = 1160$$

so,

maximum $Z =$ Rs 1160 at $x = 2000, y = 10000$

He should invest Rs 2000 in Saving
Certificates and 1000 in National
Saving scheme, maximum Interest = Rs 1160

Question 25

A man owns a field of area 1000 sq.m. He wants to plant fruit trees in it. He has a sum of Rs 1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 sq. m of ground per tree and costs Rs 20 per tree and type B requires 20 sq. m of ground per tree and costs Rs 25 per tree. When fully grown, type A produces an average of 20 kg of fruit which can be sold at a profit of Rs 2.00 per kg and type B produces an average of 40 kg of fruit which can be sold at a profit of Rs 1.50 per kg. How many of each type should be planeted to achieve maximum profit when the trees are fully grown? What is the maximum profit?

Solution 25

Let required number of trees of type A and B be Rs x and Rs y respectively.

Since, selling price of 1 kg of type A is Rs 2 and growth is 20 kg per tree, so, revenue from type A is Rs 40x , selling price of 1 kg of type B is Rs 1.5 and growth 40 kg per tree, so, revenue from type B is Rs $60y$. Total revenue is $(40x + 60y)$. Costs of each tree of type A and B are Rs 20 and Rs 25, so,costs of x trees of type A and y trees of type B are Rs $20x$ and $25y$ respectively. Total cost is Rs $(20x + 25y)$

Let Z be total profit so,

$$Z = (40x - 60y) - (20x + 25y)$$
$$Z = 20x + 35y$$

Since he has Rs 1400 to invest so,

$$\text{cost} \le 1400$$

$$\Rightarrow \quad 20x + 35y \le 1400$$
$$\Rightarrow \quad 4x + 5y \le 280 \qquad (\text{ first constraint})$$

Since each tree of type A and B needs 10 sq. m and 20 sq. m of ground respectively so, x trees of type A and y trees of type B need $10x$ sq. m and $20y$ sq. m of ground respectively. but total ground available is 1000 sq. m so,

$$10x + 20y \le 1000$$

$$\Rightarrow \quad x + 2y \le 100 \qquad (\text{second constraint})$$
$$x, y \ge 0$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 20x + 35y$$

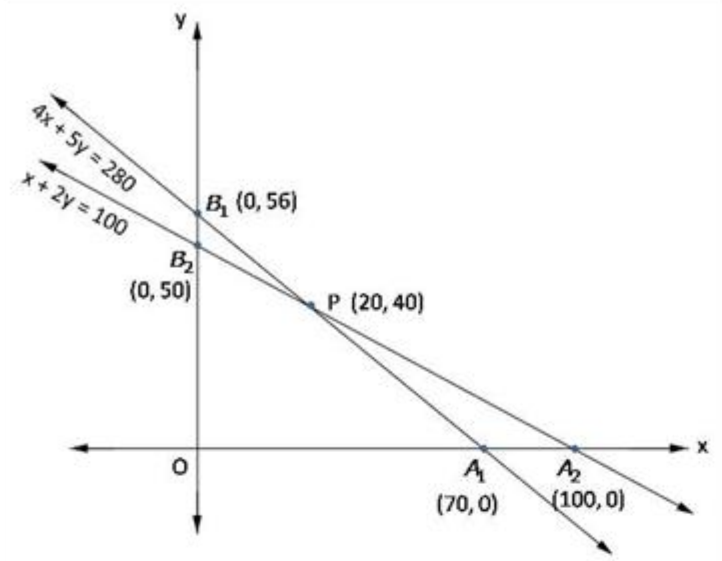
Subject to constriants,

$$4x + 5y \le 280$$
$$\Rightarrow \quad x + 2y \le 100$$
$$x, y \ge 0 \qquad [\text{Since number of trees can not be less than zero}]$$

Region $4x + 5y \le 280$: line $4x + 5y = 280$ meets axes at $A_1\left(70,0\right)$, $B_1\left(0,56\right)$ respectively.
Region containing origin represents $4x + 5y \le 280$ as $\left(0,0\right)$ satisfies $4x + 5y \le 280$.

Region $x + 2y \le 100$: line $x + 2y = 100$ meets axes at $A_2\left(100,0\right)$, $B_2\left(0,50\right)$ respectively.
Region containing origin represents $x + 2y \le 100$ as $\left(0,0\right)$ satisfies $x + 2y \le 100$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_1PB_2 the feasible region. $P\left(20,40\right)$ is obtained by solving $x + 2y = 100$ and $4x + 5y = 280$,

The value of $Z = 20x + 35y$ at

$O\left(0,0\right)$	$= 20\left(0\right) + 35\left(0\right) = 0$
$A_1\left(70,0\right)$	$= 20\left(0\right) + 35\left(0\right) = 1400$
$P\left(20,40\right)$	$= 20\left(20\right) + 35\left(40\right) = 1800$
$B_2\left(0,50\right)$	$= 20\left(0\right) + 35\left(50\right) = 1750$

maximum $Z = 1800$ at $x = 20, y = 40$

20 trees of type A , 40 trees of type B, profit = Rs 1800

Question 26

A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?

Solution 26

Let the cottage industry manufacture x pedestal lamps and y wooden shades. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, Z = $5x + 3y$

The mathematical formulation of the given problem is

Maximize Z = $5x + 3y$ ... (1)

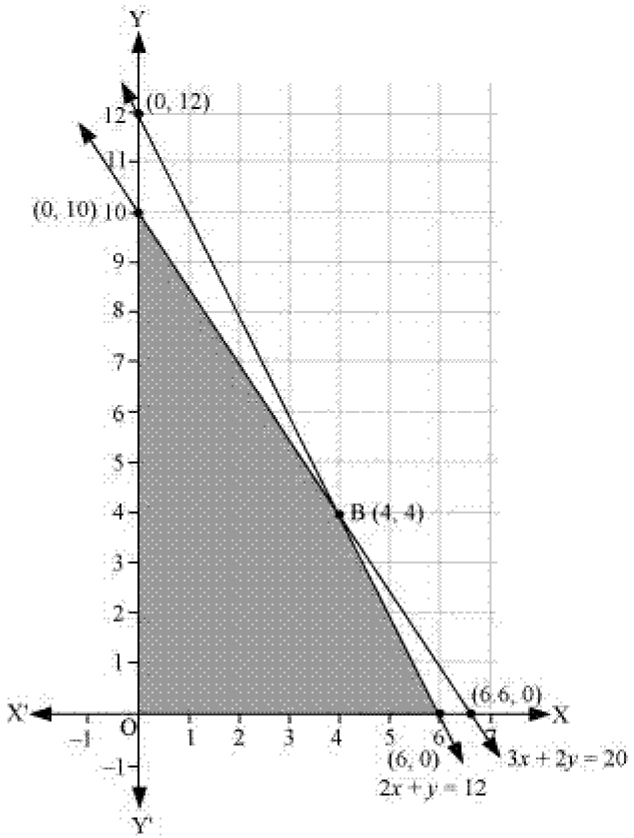
subject to the constraints,

$2x + y \leq 12$ ... (2)

$3x + 2y \leq 20$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	**Z = 5*x* + 3*y***	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

Question 27

A producer has 30 and 17 units of labour and capital respectively which he can use to produce two type of goods x and y. To produce one unit of x, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of y. If x and y are priced at Rs 100 and Rs 120 per unit respectively, how should be producer use his resources to maximize the total revenue? Solve the problem graphicallly.

Solution 27

Let required number of goods of type x and y be x_1 and x_2 respectively.

Since, selling prices of each goods of type x and y are Rs 100 and Rs 120 respectively, so, selling price of x_1 units of goods of type x and x_2 units of goods of type y are Rs $100x$ and Rs $120y$ respective respectively
Let Z be total revenue, so

$$Z = 100x_1 + 120x_2.$$

Since each unit of goods x and y require 2 and 3 units of labour, so, x_1 unit of x and x_2 unit of y require $2x_1$ and $3x_2$ units of labour units but maximum labour units available is 30 units, so,

$$2x_1 + 3x_2 \le 30 \qquad \text{(first constraint)}$$

Since each unit of goods x and y require 3 and 1 unit of capital so, x_1 unit of x and x_2 unit of y require $3x_1$ and x_2 units of capital respectively but maximum units available for capital is 17 , so,

$$3x_1 + x_2 \le 17 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 20x + 35y$$

Subject to constriants,

$$2x_1 + 3x_2 \le 30$$

$$\Rightarrow \quad 3x_1 + x_2 \le 17$$

$$x_1, x_2 \ge 0 \qquad \left[\text{Since production of goods can not be less than zero}\right]$$

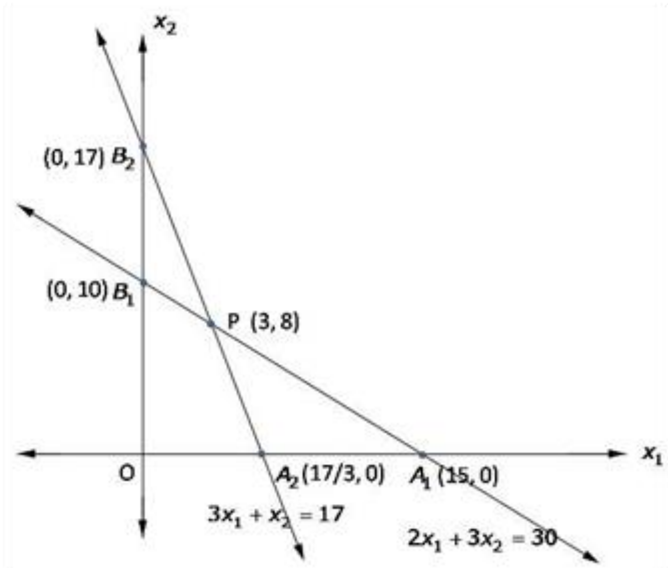
Region $2x_1 + 3x_2 \le 30$: line $2x + 3y = 30$ meets axes at $A_1(15,0)$, $B_1(0,10)$ respectively.
Region containing origin represents $2x_1 + 3x_2 \le 30$ as $(0,0)$ satisfies $2x_1 + 3x_2 = 30$.

Region $3x_1 + x_2 \le 17$: line $3x_1 + x_2 \le 17$ meets axes at $A_2\left(\frac{17}{3},0\right)$, $B_2(0,17)$ respectively.

Region containing origin represents $3x_1 + x_2 \le 17$ as $(0,0)$ satisfies $3x_1 + x_2 \le 17$.

Region $x_1, x_2 \ge 0$: it represent first quandrant shaded region OA_2PB_1 represents feasible region. Point $P(3,8)$ is obtained by solving

$2x_1 + 3x_2 = 30$ and $3x_1 + x_2 = 17$



The value of $Z = 100x_1 + 120x_2$ at

$O(0,0) \quad = 100(0) + 120(0) = 0$

$A_2\left(\frac{17}{3},0\right) \quad = 100\left(\frac{17}{3}\right) + 120(0) = \frac{1700}{3} = 566\frac{2}{3}$

$P(3,8) \quad = 100(3) + 120(8) = 1260$

$B_1(0,10) \quad = 100(0) + 120(10) = 1200$

maximum $Z = 1260$ at $x = 3, y = 8$

goods of type $x = 3$, type $y = 8$
maximum profit = Rs 12160

Question 28

A firm manufactures two types of products A and B and sells them at a profit of Rs 5 per unit of type A and Rs 3 per unit of type B. Each product is processed on two machines M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 , whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_1 are respectivley available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximinze the profit. Solve the problem grphically.

Solution 28

Let required number of product A and B be x and y respectively.

Since,profit on each product A and B are Rs 5 and Rs 3 respectively, so,
profits on x product A and y product B are Rs $5x$ and Rs $3y$ respectively
Let Z be total profit so

$$Z = 5x + 3y$$

Since each unit of product A and B require one min. each on machine M_1, so,
x unit of product A and y units of product B require x and y min. respectivley
on machine M_1 but M_1 can work at most $5 \times 60 = 300$ min.,so

$$x + y \leq 300 \qquad \text{(first constraint)}$$

Since each unit of product A and B require 2 and one min.respectively on machine
M_2, so, x unit of product A and y units of product B require $2x$ and y min. respectivley
on machine M_2 but M_2 can work at most $6 \times 60 = 360$ min.,so

$$2x + y \leq 360 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 5x + 3y$

Subject to constriants,

$$x + y \leq 300$$

$$\Rightarrow \quad 2x + y \leq 360$$

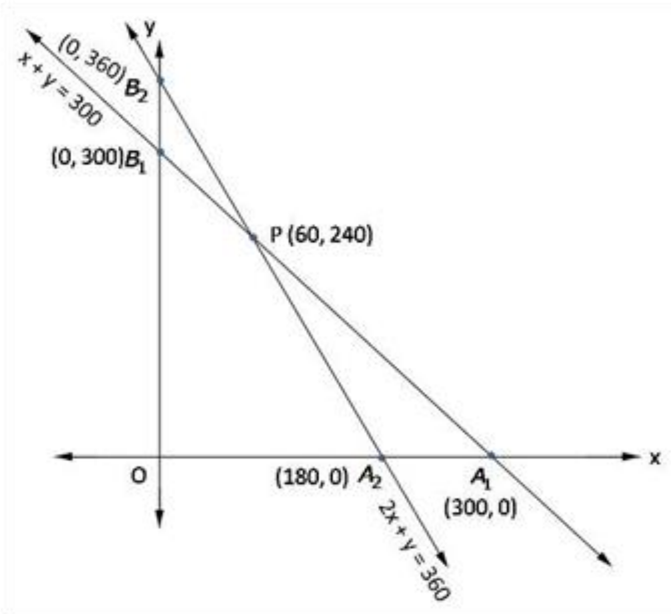
$$x, y \geq 0 \qquad \left[\text{Since production can not be less than zero}\right]$$

Region $x + y \le 300$: line $x + y = 300$ meets axes at $A_1(300,0)$, $B_1(0,300)$ respectively.
Region containing origin represents $x + y \le 300$ as $(0,0)$ satisfies $x + y = 300$.

Region $2x + y \le 360$: line $2x + y = 360$ meets axes at $A_2(180,0)$, $B_2(0,360)$ respectively.
Region containing origin represents $2x + y \le 360$ as $(0,0)$ satisfies $2x + y \le 360$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P(60,240)$ is obtained by solving
$x + y = 300$ and $2x + y = 360$



The value of $Z = 5x + 3y$ at

$O(0,0) \quad = 5(0) + 3(0) = 0$

$A_2(180,0) \quad = 5(180) + 3(0) = 900$

$P(60,240) \quad = 5(60) + 3(240) = 1020$

$B_1(0,300) \quad = 5(0) + 3(300) = 900$

maximum $Z = 1020$ at $x = 60, y = 240$

Number of product $A = 60$, product $B = 240$
maximum profit = Rs 1020

Question 29

A small firm manufaturers items A and B. The total number of item A and B that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hour. If the profit on one unit of item A be Rs 300 and one unit of item B be Rs 160, how many of each type of item be produced to maximize the profit? Solve the problem graphically.

Solution 29

Let required quantity of item A and B produced be x and y respectively.

Since,profits on each item A and B are Rs 300 and Rs 160 respectively, so, profits on x unit of item A and y units of item B are Rs $300x$ and Rs $160y$ respectively
Let Z be total profit so

$$Z = 300x + 160y$$

Since one unit of item A and B require one and $\frac{1}{2}$ hr respectively, so, x units of item A and y units of item B require x and $\frac{1}{2}y$ hr. respectivley but maximum time available is 16 hours.,so

$$x + \frac{1}{2}y \le 16$$

$$\Rightarrow \quad 2x + y \le 32 \qquad \text{(first constraint)}$$

Given, manufacturer can produce at most 24 items, so,

$$\Rightarrow \quad x + y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 300x + 160y$

Subject to constriants,

$$2x + y \le 32$$

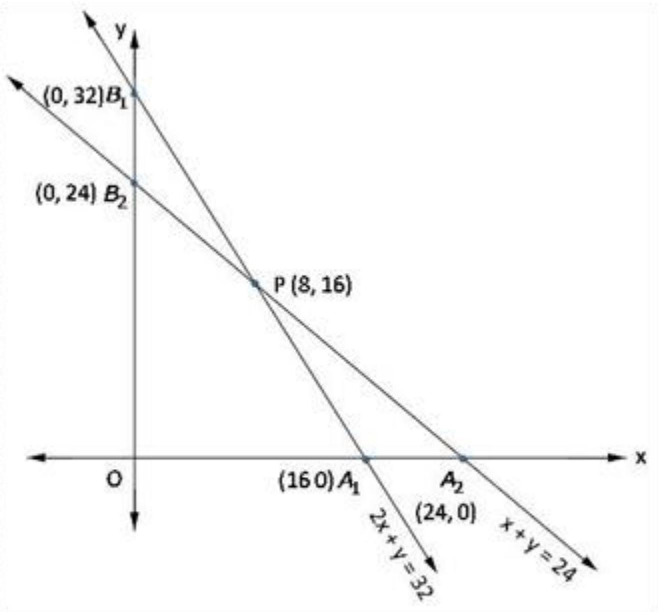
$$x + y \le 24$$

$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Region $2x + y \le 32$: line $2x + y = 32$ meets axes at $A_1(16,0)$, $B_1(0,32)$ respectively.
Region containing origin represents $2x + y \le 32$ as $(0,0)$ satisfies $2x + y \le 32$.

Region $x + y \le 24$: line $x + y = 24$ meets axes at $A_2(24,0)$, $B_2(0,24)$ respectively.
Region containing origin represents $x + y \le 24$ as $(0,0)$ satisfies $x + y \le 24$.

Region $x, y \ge 0$: it represent first quandrant



Shaded region OA_1PB_2 represents feasible region.
Point P is obtained by solving

$x + y = 24$ and $2x + y = 32$

The value of $Z = 300x + 160y$ at

$O(0,0) \quad = 300(0) + 160(0) = 0$
$A_1(16,0) \quad = 300(180) + 160(0) = 4800$
$P(8,16) \quad = 300(8) + 160(16) = 4960$
$B_2(0,24) \quad = 300(0) + 160(24) = 3640$

maximum $Z = 4960$

Number of item $A = 8$, item $B = 16$
maximum profit = Rs 4960

Question 30

A company manufactures two type of toys A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling.Type B requires 8 minutes each for cutting and 8 minutes each for assembling.There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is Rs 50 each on type A and Rs 60 each on type B. How many toys of each type should the company manufacture in a day to maximize the profit?

Solution 30

Let number of toys of type A and B produced are x and y respectively.

Since,profits on each unit of toys A and B are Rs 50 and Rs 60 respectively, so, profits on x units of toys A and y units of toy B are Rs $50x$ and Rs $60y$ respectively
Let Z be total profit so

$$Z = 50x + 60y$$

Since each unit of toy A and toy B require 5 min. and 8 min. on cutting, so, x units of toy A and y units of toy B require $5x$ and $8y$ min. respectivley but maximum time available for cutting $3 \times 60 = 180$ min.,so

$$5x + 8y \le 180 \qquad \text{(first constraint)}$$

Since each unit of toy A and toy B require 10 min. and 8 min. for assembling, so, x units of toy A and y units of toy B require $10x$ and $8y$ min. for assembling respectivley but maximum time available for assembling is $4 \times 60 = 240$ min.,so

$$10x + 8y \le 240$$

$$\Rightarrow \quad 5x + 4y \le 120 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 50x + 60y$$

Subject to constriants,

$$5x + 8y \le 180$$

$$5x + 4y \le 120$$

$$x, y \ge 0 \qquad [\text{Since production can not be less than zero}]$$

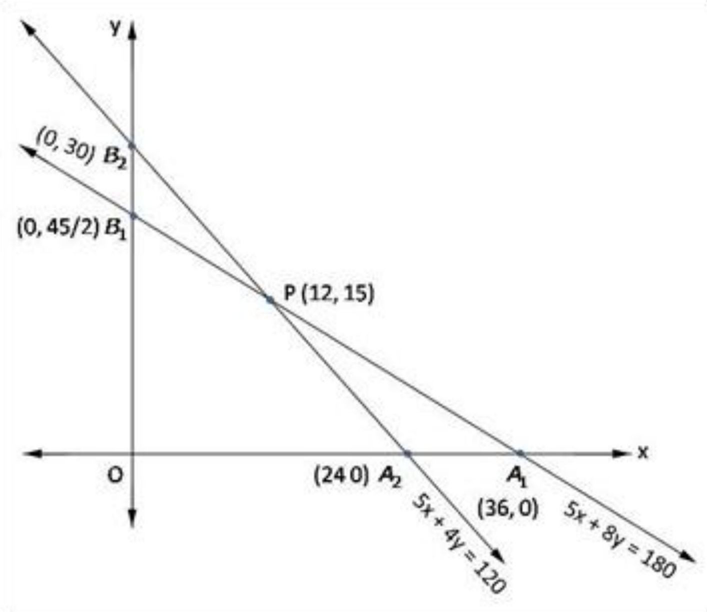
Region $5x + 8y \le 180$: line $5x + 8y = 180$ meets axes at $A_1(36,0)$, $B_1\left(0,\frac{45}{2}\right)$ respectively.
Region containing origin represents $5x + 8y \le 180$ as $(0,0)$ satisfies $5x + 8y \le 180$.

Region $5x + 4y \le 120$: line $5x + 4y = 120$ meets axes at $A_2(24,0)$, $B_2(0,30)$ respectively.
Region containing origin represents $5x + 4y \le 120$ as $(0,0)$ satisfies $5x + 4y \le 120$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P(12,15)$ is obtained by solving

$5x + 8y = 180$ and $5x + 4y = 120$



The value of $Z = 50x + 60y$ at

$O(0,0)$	$= 50(0) + 60(0) = 0$
$A_2(24,0)$	$= 50(24) + 60(0) = 1200$
$P(12,15)$	$= 50(12) + 60(15) = 1500$
$B_1\left(0,\frac{45}{2}\right)$	$= 50(0) + 60\left(\frac{45}{2}\right) = 1350$

Maximum $Z = 1500$ at $x = 12$, $y = 15$

Number of toys $A = 12$, toys $B = 15$
maximum profit = Rs 1500

Question 31

A company manufactures two articles A and B. There are two deparments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the first deparment is 60 hours a week and that of other deparment is 48 hours per week. The product of each unit of article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing . If the profit is Rs 6 for each unit of A and Rs 8 for each unit of B, find the number of unit of A and B to be produced per week in order to have maximum profit.

Solution 31

Let required number of product A and B are x and y respectively.

Since,profits on each unit of product A and product B are Rs 6 and Rs 8 respectively, so, profits on x units of product A and y units of product B are Rs $6x$ and Rs $8y$ respectively
Let Z be total profit so

$$Z = 6x + 8y$$

Since each unit of product A and B require 4 and 2 hrs for assembling respectively, so, x units of product A and y units of product B require $4x$ and $2y$ hrs for assembling respectivley but maximum time available for assembling is 60 hrs.,so

$$4x + 2y \le 60$$

$$2x + y \le 30 \qquad \text{(first constraint)}$$

Since each unit of product A and B require 2 and 4 hrs for finishing, so, x units of product A and y units of product B require $2x$ and $4y$ hrs for finishing respectivley but maximum time available for finishing is 48 hrs.,so

$$2x + 4y \le 48$$

$$x + 2y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 6x + 8y$

Subject to constriants,

$$2x + y \le 30$$

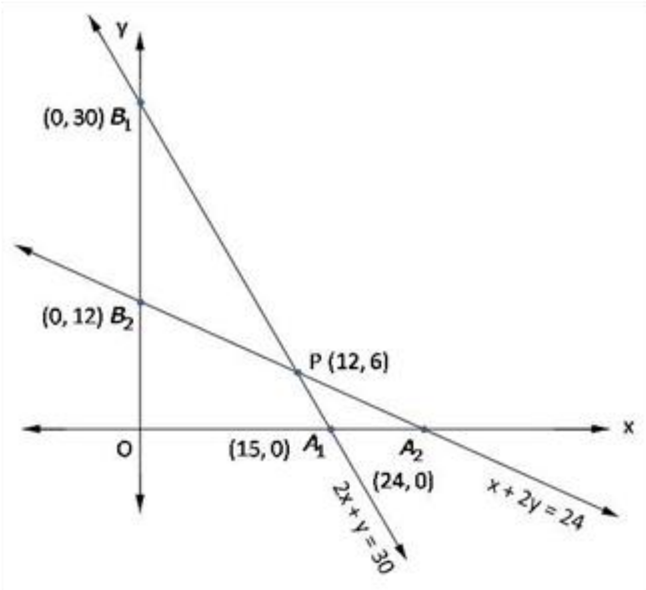
$$x + 2y \le 24$$

$$x, y \ge 0 \qquad \left[\text{Since production of both can not be less than zero}\right]$$

Region $2x + y \le 30$: line $2x + y = 24$ meets axes at $A_1(15,0)$, $B_1(0,30)$ respectively.
Region containing origin represents $2x + y \le 30$ as $(0,0)$ satisfies $2x + y \le 30$.

Region $x + 2y \le 24$: line $x + 2y = 24$ meets axes at $A_2(24,0)$, $B_2(0,12)$ respectively.
Region containing origin represents $x + 2y \le 24$ as $(0,0)$ satisfies $x + 2y \le 24$.

Region $x, y \ge 0$: it represent first quandrant



Shaded region OA_1PB_2 represents feasible region.
Point $P(12, 6)$ is obtained by solving
$x + 2y = 24$ and $x + 2y = 30$

The value of $Z = 6x + 8y$ at

$O(0,0)$	$= 6(0) + 8(0) = 0$
$A_1(15, 0)$	$= 6(15) + 8(0) = 90$
$P(12, 6)$	$= 6(12) + 8(6) = 120$
$B_2(0,12)$	$= 6(0) + 8(12) = 96$

maximum $Z = 120$ at $x = 12$, $y = 6$

Number of product $A = 12$, product $B = 6$
maximum profit = Rs 120

Question 32

A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and half an hour to make an item B. The maximum time available per day is 16 hours. The profit on an item of A is Rs 300 and on one item of B is Rs 160. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

Solution 32

Let x & y be the No. of items of A & B respectively.

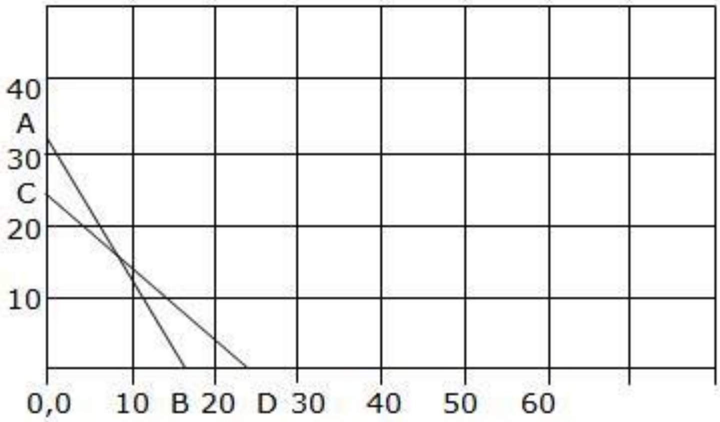
$x + y = 24$ (total No. of items constraint)

$x + 0.5y \leq 16$ (time constraint)

$x, y \geq 0$

$Z = 300x + 160y$ (profit function to be maximized)

Plotting the inequalities gives,



The feasible region is 0,0-C-F-B

Corner point	Value of Z = 300x + 160y
0, 0	0
0, 24	3840
16, 0	4800
8 ,16	4960

The firm must produce 8 items of A and 16 items of B to maximize the profit at Rs. 4960/-

Question 33

A company sells two different products, *A* and *B*. The two products are produced in a common production process, which has a total capacity of 500 man-hours. It takes 5 hours to produce a unit of *A* and 3 hours to produce a unit of *B*. The market has been surveyed and company officials feel that the maximum number of units of *A* that can be sold is 70 and that for *B* is 125. If the profit is Rs 20 per unit for the product *A* and Rs 15 per unit for the product *B*, how many units of each product should be sold to maximize profit?

Solution 33

Let required number of product A and B are x and y respectively.

Since,profits on each unit of product A and product B are Rs 20 and Rs 15 respectively, so, x units of product A and y units of product B give profit of Rs $20x$ and Rs $15y$ respectively
Let Z be total profit so

$$Z = 20x + 15y$$

Since each unit of product A and B require 5 and 3 man-hrs respectively, so, x units of product A and y units of product B require $5x$ and $3y$ man-hrs respectivley but maximum time available for is 500 man-hrs., so

$$5x + 3y \le 500 \qquad \text{(first constraint)}$$

Since maximum number that product A and B can be sold is 70 and 125 respectively, so,

$$x \le 70 \qquad \text{(second constraint)}$$

$$y \le 125 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 15y$

Subject to constriants,

$$5x + 3y \le 500$$

$$x \le 70$$

$$y \le 70$$

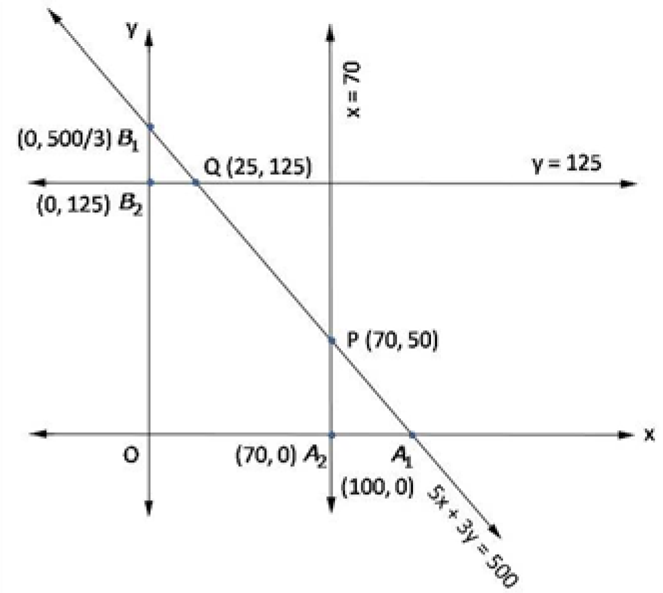
$$x, y \ge 0 \qquad \text{[Since production of both can not be less than zero]}$$

Region $5x+3y \le 500$: line $5x+3y=500$ meets axes at $A_1(100,0)$, $B_1\left(0,\frac{500}{3}\right)$ respectively. Region containing origin represents $5x+3y \le 500$ as $(0,0)$ satisfies $5x+3y \le 500$.

Region $x \le 70$: line $x=70$ is parallel to y - axis meets x-axes at $A_2(70,0)$. Region containing origin represents $x \le 70$ as $(0,0)$ satisfies $x \le 70$.

Region $y \le 125$: line $y=125$ is parallel to x - axis meets y-axes at $B_2(0,125)$. with y – axis. Region containing origin represents $y \le 125$ as $(0,0)$ satisfies $y \le 125$.

Region $x, y \ge 0$: it represent first quandrant.



Shaded region OA_2PQB_2 represents feasible region.

Point $P(70,50)$ is obtained by solving $x = 70$

Point $Q(25,125)$ is obtained by solving $y = 125$ and $5x + 3y = 500$.

The value of $Z = 20x + 15y$ at

$O(0,0) \quad = 20(0) + 15(0) = 0$

$A_2(70,0) \quad = 20(70) + 15(0) = 1400$

$P(70,50) \quad = 20(70) + 15(50) = 2150$

$Q(25,125) \quad = 20(25) + 15(125) = 2375$

$B_2(0,125) \quad = 20(0) + 15(125) = 1875$

maximum $Z = 2375$ at $x = 25$, $y = 125$

Number of product $A = 25$, product $B = 125$

maximum profit = Rs 2375

Question 34

A box manfacturer makes large and small boxes from a large piece of cardboard.. The large boxes require 4 sq. metre per box while the small boxes require 3 sq. metre per box. The manufacturer is required to make at leas three large boxes and at least twice as many small boxes as large boxes. If 60 sq. metre of carvoard is in stock, and if the profits on the large and small boxes are Rs 3 and Rs 2 per box, how many of each should be made in order to maximize the total profit?

Solution 34

Let required quantity of large and small boxes are x and y respectively.

Since,profits on each unit of large and small boxes are Rs 3 and Rs 2 respectively, so, profit on x units of large and y units of small boxes are Rs $3x$ and Rs $2y$ respectively
Let Z be total profit so

$$Z = 3x + 2y$$

Since each large and small box require 4 sq. m. and 3 sq. m. cardboard respectively, so, x units of large and y units of small boxes require $4x$ and $3y$ sq.m. cardboard respectivley but only 60 sq. m. of cardboard is available,so

$$4x + 3y \le 60 \qquad \text{(first constraint)}$$

Since manufacturer is required to make at least three large boxes,so,

$$x \ge 3 \qquad \text{(second constraint)}$$

Since manufacturer is required to make at least twice as many small boxes as large boxes ,so,

$$y \ge 2x \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 15y$

Subject to constriants,

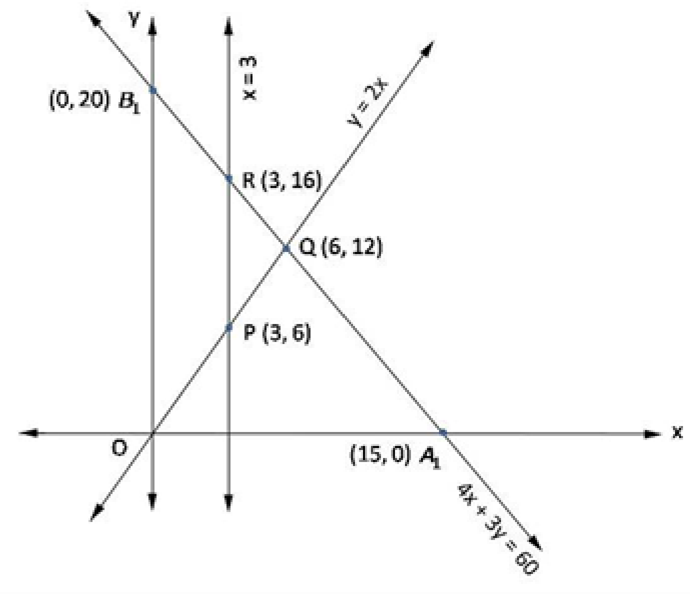
$$4x + 3y \le 60$$
$$x \ge 3$$
$$y \ge 2x$$
$$x, y \ge 0 \qquad [\text{Since production can not be less than zero}]$$

Region $4x + 3y \le 60$: line $4x + 3y = 60$ meets axes at $A_1(15,0)$, $B_1(0,20)$ respectively. Region containing origin represents $4x + 3y \le 60$ as $(0,0)$ satisfies $4x + 3y \le 60$.

Region $x \ge 3$: line $x = 3$ is parallel to y - axis meets x-axes at $A_2(3,0)$. Region containing origin represents $x \ge 70$ as $(0,0)$ satisfies $x \ge 3$.

Region $y \ge 2x$: line $y = 2x$ is passes throgh origin and $P(3,6)$. Region containging $B_1(0,20)$ represents $y \ge 2x$ as $(0,20)$ satisfies $y \ge 2x$.

Region $x, y \ge 0$: it represent first quandrant.



Shaded region PQR represents feasible region.

Point $Q(6,12)$ is obtained by solving $y = 2x$ and $4x + 3y = 60$

Point $R(3,16)$ is obtained by solving $x = 3$ and $4x + 3y = 60$.

The value of $Z = 3x + 2y$ at

$P(3,6) = 3(3) + 2(6) = 21$

$Q(6,12) = 3(6) + 2(12) = 42$

$R(3,16) = 3(3) + 2(16) = 41$

maximum $Z = 42$ at $x = 6$, $y = 12$

Number of large box = 6, small box =12

maximum profit = Rs 42

Question 35

A manufacturer makes two products, A and B. Product A sells at Rs. 200 each and takes ½ hour to make. Product B sells at Rs.300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consist of 40 hours of production and the weekly turn over must not be less than Rs. 1000. If the profit on each of product A is Rs. 20 and an product B is Rs. 30, then how many of each should be produced so that the profit is maximum? Also find the maximum profit.

Solution 35

The given data can be written in the tabular form as follows:

Product	A	B	Working week	Turn over
Time	0.5	1	40	
Prise	200	300		10000
Profit	20	30		
Permanent order	14	16		

Let x be the number of units of A and y be the number of units of B produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 20x + 30y$

Subject to $0.5x + y \le 40$,

$200x + 300y \ge 10000$

and $x \ge 14$, $y \ge 16$

To solve the LPP we draw the lines,

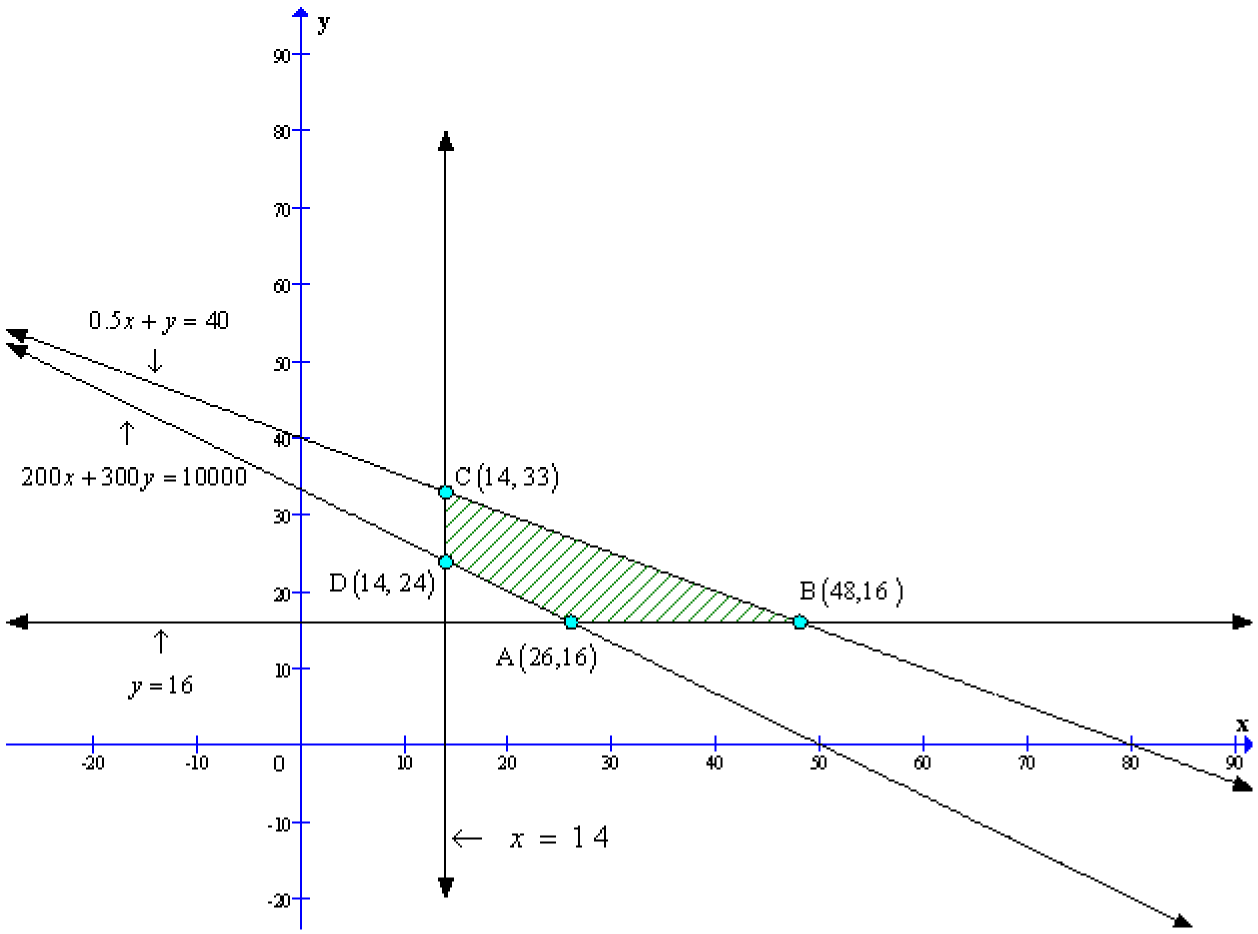
$0.5x + y = 40$,

$200x + 300y = 10000$

$x = 14$

$y = 16$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(26, 16),B(48,16), C(14, 33) and D(14, 24).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 20x + 30y
A(26, 16)	Z = 1000
B(48, 16)	Z = 1440
C(14, 33)	Z = 1270
D(14,24)	Z = 600

48 units of product A and 16 units of product B should be produced to earn the maximum profit of Rs. 1440.

Question 36

If a young man drives his vehicle at 25 km/hr, he has to spend Rs. 2 per km on petrol. If he drives is at a faster speed of 40 km/hr, the petrol cost increases to Rs. 5/ per km. He has Rs. 100 to spend on petrol and travel within one hour. Express this as an LPP solve the same.

Solution 36

Let the distance covered with the speed of 25 km/hr be x.

Let the distance covered with the speed of 40 km/hr be y.

Then the mathematical model of the LPP is as follows:

Maximize Z = x + y

Subject to $2x + 5y \le 100$,

$$\frac{x}{25} + \frac{y}{40} \le 1$$

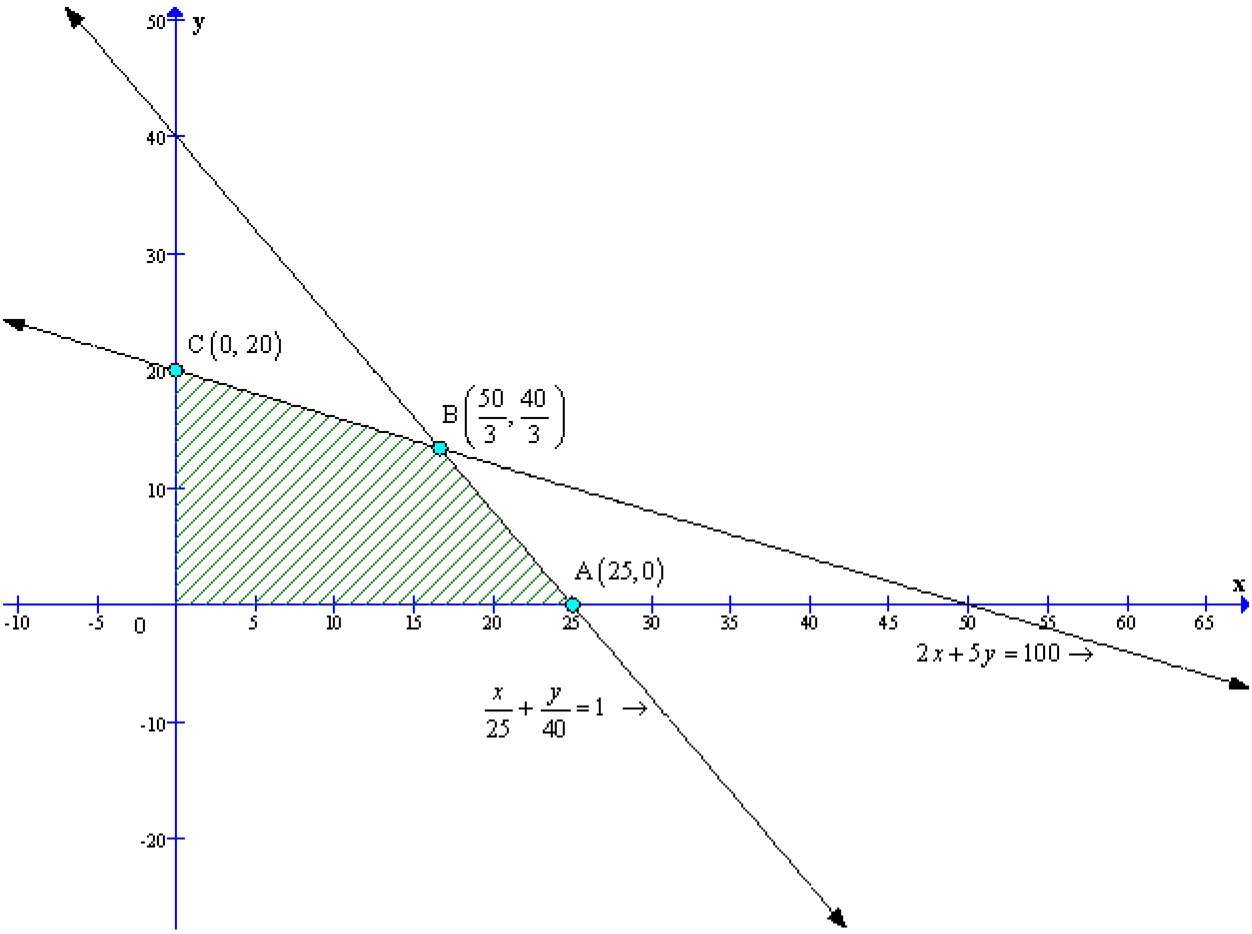
and $x \ge 0,\ y \ge 0$

To solve the LPP we draw the lines,

$2x + 5y = 100$,

$$\frac{x}{25} + \frac{y}{40} = 1$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A(25, 0), B\left(\frac{50}{3}, \frac{40}{3}\right)$ and $C(0, 20)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = x + y$
$A(25, 0)$	$Z = 25$
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 30$
$C(0, 20)$	$Z = 20$

The distance covered at the speed of 25km/hr is $\frac{50}{3}$km and

The distance covered at the speed of 40km/hr is $\frac{40}{3}$km.

Maximum distance travelled is 30 km.

Question 37

An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distance (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km)		
From/To	**A**	**B**
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

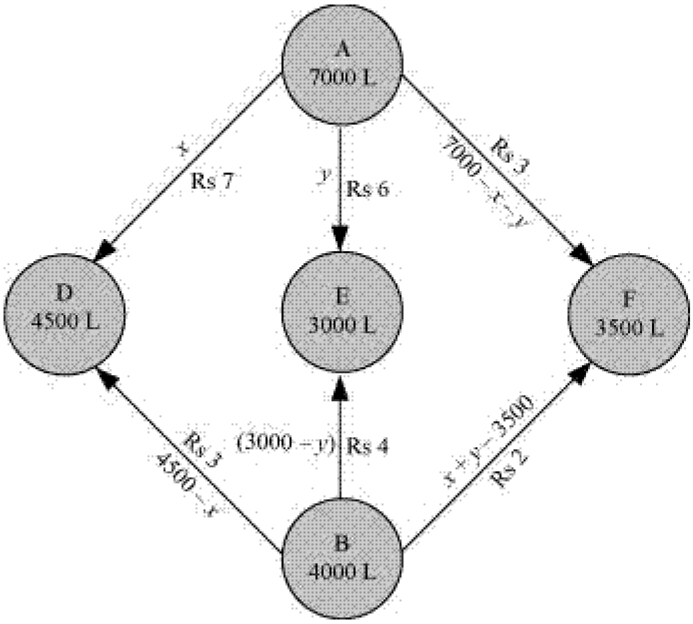
Solution 37

Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, $(7000 - x - y)$ will be supplied from A to petrol pump F.

The requirement at petrol pump D is 4500 L. Since x L are transported from depot A, the remaining $(4500 - x)$ L will be transported from petrol pump B.

Similarly, $(3000 - y)$ L and $3500 - (7000 - x - y) = (x + y - 3500)$ L will be transported from depot B to petrol pump E and F respectively.

The given problem can be represented diagrammatically as follows.



$$x \geq 0,\ y \geq 0,\ \text{and}\ (7000 - x - y) \geq 0$$

$$\Rightarrow x \geq 0,\ y \geq 0,\ \text{and}\ x + y \leq 7000$$

$$4500 - x \geq 0,\ 3000 - y \geq 0,\ \text{and } x + y - 3500 \geq 0$$
$$\Rightarrow x \leq 4500,\ y \leq 3000,\ \text{and } x + y \geq 3500$$

Cost of transporting 10 L of petrol = Re 1

Cost of transporting 1 L of petrol = $\text{Rs } \frac{1}{10}$

Therefore, total transportation cost is given by,

$$z = \frac{7}{10} \times x + \frac{6}{10} y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$
$$= 0.3x + 0.1y + 3950$$

The problem can be formulated as follows.

Minimize $z = 0.3x + 0.1y + 3950$ ... (1)

subject to the constraints,

$x + y \leq 7000$ ...(2)

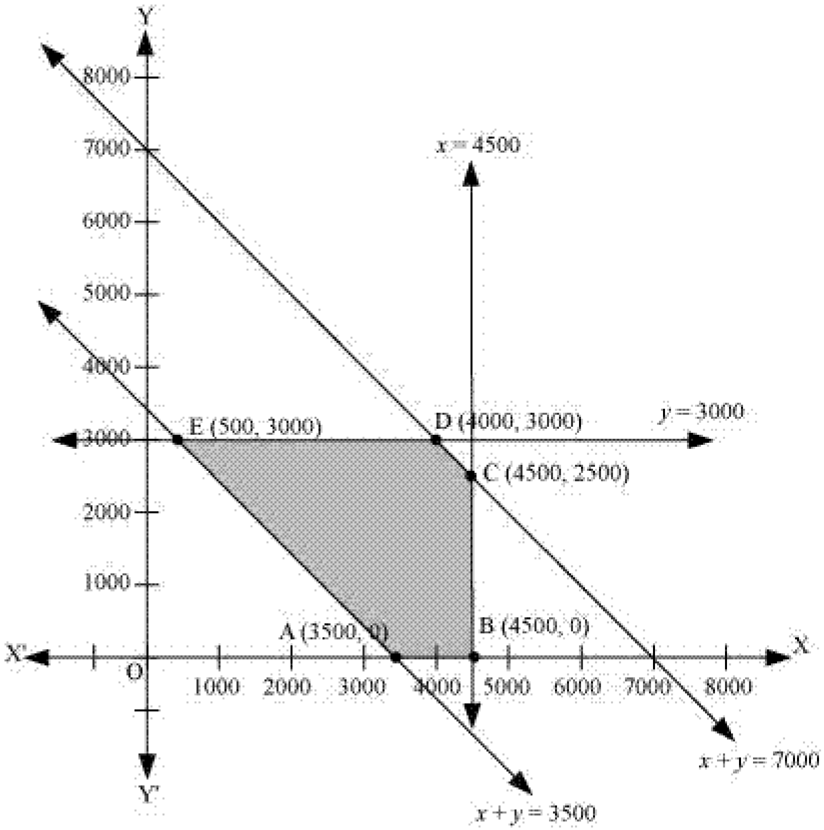
$x \leq 4500$ ...(3)

$y \leq 3000$ ...(4)

$x + y \geq 3500$ ...(5)

$x,\ y \geq 0$ ...(6)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000), and E (500, 3000).

The values of z at these corner points are as follows.

Corner point	$z = 0.3x + 0.1y + 3950$	
A (3500, 0)	5000	
B (4500, 0)	5300	
C (4500, 2500)	5550	
D (4000, 3000)	5450	
E (500, 3000)	4400	→ Minimum

The minimum value of z is 4400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E, and F respectively.

The minimum transportation cost is Rs 4400.

Question 38

A small firm manfactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring Rs 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

Solution 38

Let required number of gold rings and chains are x and y respectively.

Since,profits on each ring and chains are Rs 300 and Rs 190 respectively, so,
profit on x units of ring and y units of chains are Rs $300x$ and Rs $190y$ respectively
Let Z be total profit so

$$Z = 300x + 190y$$

Since each unit of ring and chain require 1 hr and 30 min. to make respectively, so,
x units of rings and y units of rings require $60x$ and $30y$ min. to make respectivley,
but total time available to make is $16 \times 60 = 960$, so

$$60x + 30y \le 960$$

$$\Rightarrow \quad 2x + y \le 32 \qquad \text{(first constraint)}$$

Given, total number of rings and chains manufactured is at most 24, so,

$$x + y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 300x + 160y$$

Subject to constriants,

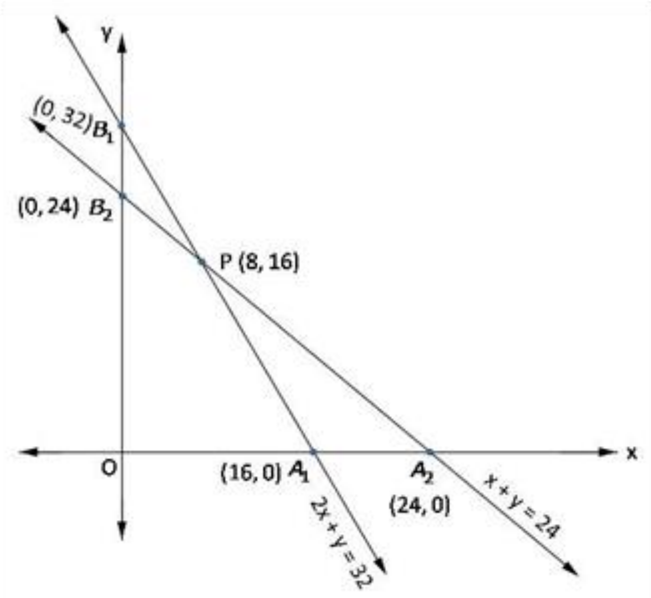
$$2x + y \le 32$$
$$x + y \le 24$$
$$x, y \ge 0 \qquad [\text{Since production can not be less than zero}]$$

Region $2x + y \le 32$: line $2x + y = 32$ meets axes at $A_1(16,0)$, $B_1(0,32)$ respectively.
Region containing origin represents $2x + y \le 32$ as $(0,0)$ satisfies $2x + y \le 32$.

Region $x + y \le 24$: line $x + y = 24$ meets axes at $A_2(24,0)$, $B_2(0,24)$ respectively.
Region containing origin represents $x + y \le 24$ as $(0,0)$ satisfies $x + y \le 24$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(8,16)$ is obtained by solving $2x + y = 32$ and $x + y = 24$.



The value of $Z = 300x + 160y$ at

$O(0,0)$	$= 300(0) + 160(0) = 0$
$A_1(16,0)$	$= 300(16) + 160(0) = 4800$
$P(8,16)$	$= 300(8) + 160(16) = 4960$
$B_2(0,24)$	$= 300(0) + 160(24) = 3840$

maximum $Z = 4960$ at $x = 8$, $y = 16$

Number of rings = 8, chains = 16
maximum profit = Rs 4960

Question 39

A library has to accomodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weight 1 kg and $1\frac{1}{2}$ kg each respectively. The shelf is 96 cm long and atmost can support a weight of 21 kg .How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically.

Solution 39

Let required number of books of type I and II be x and y respectively.

Let Z be total number of books in the shelf ,so,

$$Z = x + y$$

Since 1 book of type I and II 6 cm and 4 cm. thick respectively, so, x books of type I and y books of type II has thickness of $6x$ and $4y$ cm. respectivley, but shelf is 96 cm. long ,so

$$6x + 4y \le 96$$

$$\Rightarrow \quad 3x + 2y \le 48 \qquad \text{(first constraint)}$$

Since 1 book of type I and II weight 1 kg and $1\frac{1}{2}$ kg respectively, so, x books of type I and y books of type II weight x kg and $\frac{3}{2}y$ kg respectivley, but shelf can support at most 21 kg,so

$$x + \frac{3}{2}y \le 21$$

$$\Rightarrow \quad 2x + 3y \le 42 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$3x + 2y \le 48$$

$$2x + 3y \le 42$$

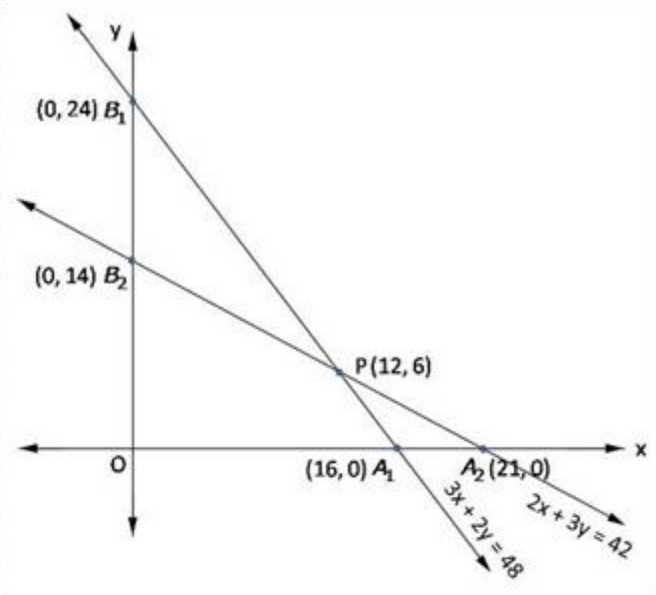
$$x, y \ge 0 \qquad \text{[Since number of books can not be less than zero]}$$

Region $3x + 2y \le 48$: line $3x + 2y = 48$ meets axes at $A_1(16,0)$, $B_1(0,24)$ respectively.
Region containing origin represents $3x + 2y \le 48$ as $(0,0)$ satisfies $3x + 2y \le 48$.

Region $2x + 3y \le 42$: line $2x + 3y = 42$ meets axes at $A_2(21,0)$, $B_2(0,14)$ respectively.
Region containing origin represents $2x + 3y \le 42$ as $(0,0)$ satisfies $2x + 3y \le 42$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(12,6)$ is obtained by solving $2x + 3y = 42$ and $3x + 2y = 48$



The value of $Z = x + y$ at

$O(0,0) = 0 + 0 = 0$

$A_1(16,0) = 16 + 0 = 16$

$P(12,6) = 12 + 6 = 18$

$B_2(0,14) = 0 + 14 = 14$

maximum $Z = 18$ at $x = 12$, $y = 6$

Number of books of type I = 12, type II = 6

Question 40

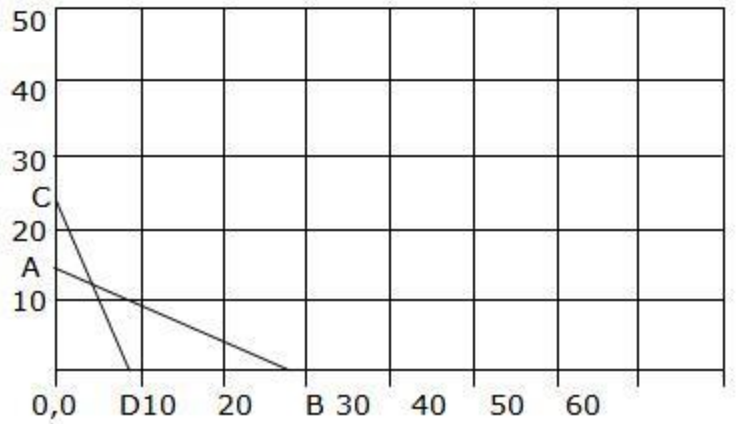
A factory makes tennis rackets and cricket bats A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hour of machine time and 24 hours of craftman's time. If the profit on racket and on a bat is Rs 20 and Rs 10 respectively, find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an LPP and solve it graphically.

Solution 40

Let x & y be the No. of tennis rackets and cricket bats produced.

$1.5x + 3y \le 42$ (constraint on machine time)
$3x + y \le 24$ (constraint on craftsman's time)
$Z = 20x + 10y$ (Maximize profit)
$x, y \ge 0$
plotting the inequalities we have,

when x=0, y= 14 and when y=0, x=28 and
when x=0, y= 24 and when y=0, x=8



The feasible region is given by 0,0-A-F-D
Tabulating Z and corner points we have

Corner point	Value of Z = 20x + 10y
0, 0	0
0, 14	140
4, 12	200
8, 0	160

The factory must manufacture 4 tennis rackets and 12 cricket bats to earn the maximum profit of Rs. 200/-

Question 41

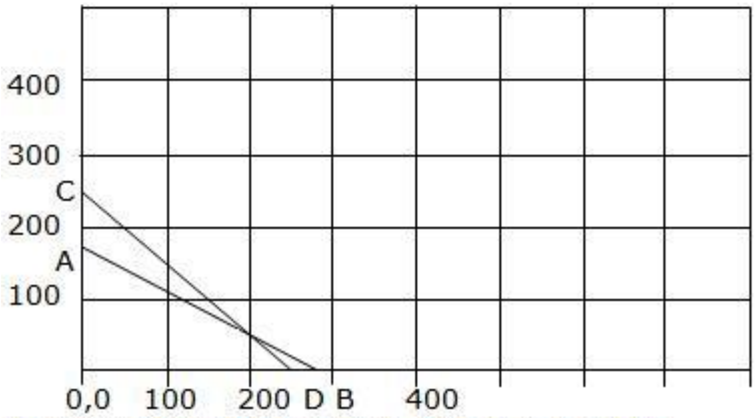
A merchant plans to sell two types of personal computers-a desktop model and a portable model that will cost Rs 25,000 and Rs 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and his profit on the desktop model is Rs 4500 and on the portable model is RS 5000. Make an LPP and solve it graphically.

Solution 41

Let x & y be the No. of desktop model and portable model of personal computers stocked.

$x + y \le 250$ (constraint on total demand of computers)
$25000x + 40000y \le 70,00,000$ (constraint on cost)
$Z = 4500x + 5000y$ (Maximize profit)
$x, y \ge 0$

plotting the inequalities we have,
when x=0, y= 250 and when y=0, x=250 and line CD
when x=0, y= 175 and when y=0, x=280



The feasible region is given by 0,0-A-E-D-0,0

Tabulating Z and corner points we have

Corner point	Value of Z = 4500x + 5000y
0, 0	0
0, 175	8,75,000
250, 0	11,25,000
200, 50	11,50,000

The merchant must stock 200 desktop models and 50 portable models to earn a maximum profit of Rs. 11,50,000/-

Question 42

A cooperative society of formers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs. 10,500 and Rs. 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so at to maximise the total profit of the society?

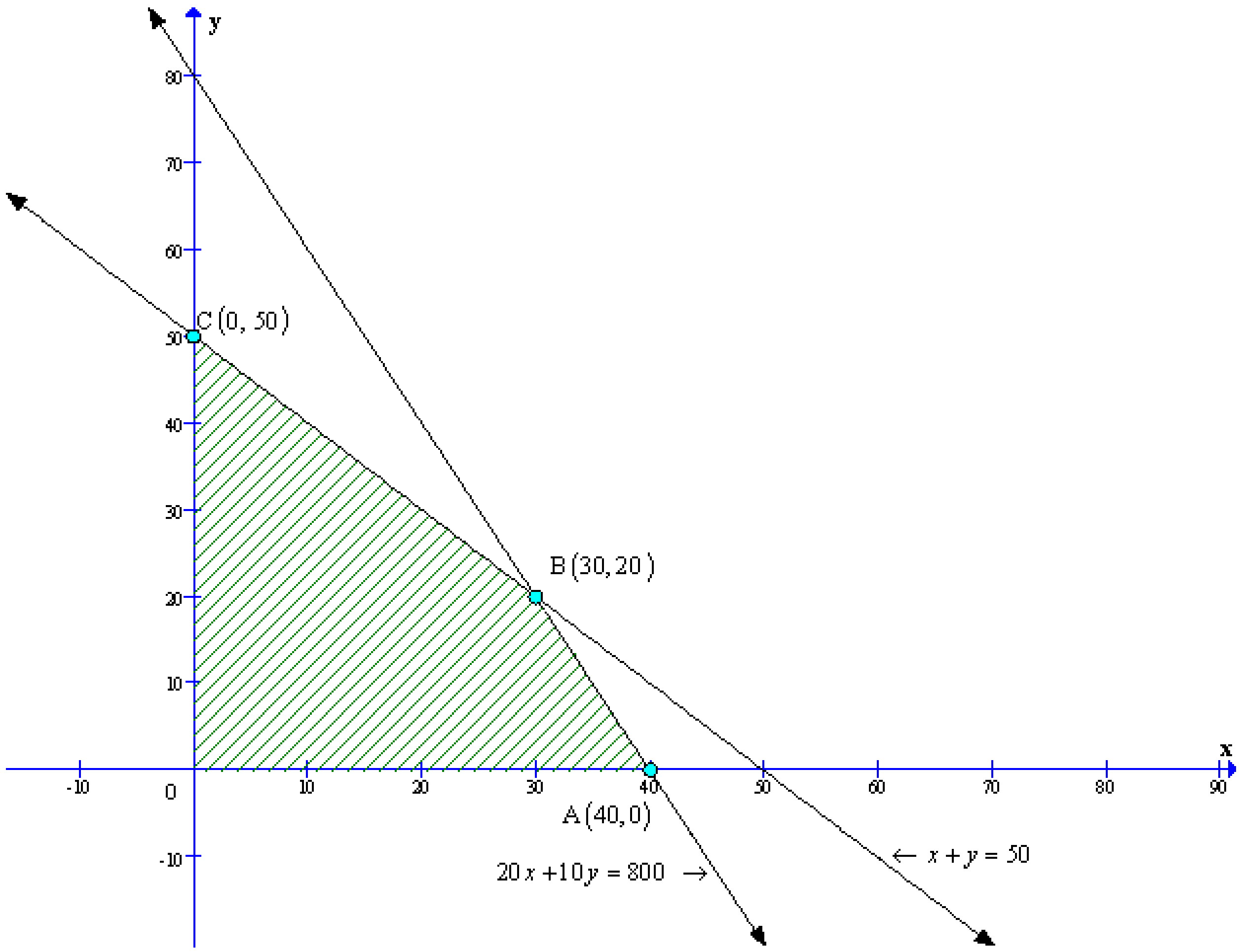
Solution 42

Let x hectores of land grows crop X.
Let y hectores of land grows crop Y.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 10{,}500x + 9{,}000y$
Subject to $x + y \leq 50$,
$20x + 10y \leq 800$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$x + y = 50$,
$20x + 10y = 800$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 0), B(30, 20) and C(0, 50).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 10,500x + 9,000y
A(40, 0)	Z = 4,20,000
B(30, 20)	Z = 4,95,000
C(0, 50)	Z = 4,50,000

30 hectors of land should be allocated to crop X and
20 hectors of land should be allocated to crop Y to maximize the profit.
The maximum profit that can be eared is Rs. 4,95,000.

Question 43
A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 1200 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

Solution 43
The given data can be written in the tabular form as follows:

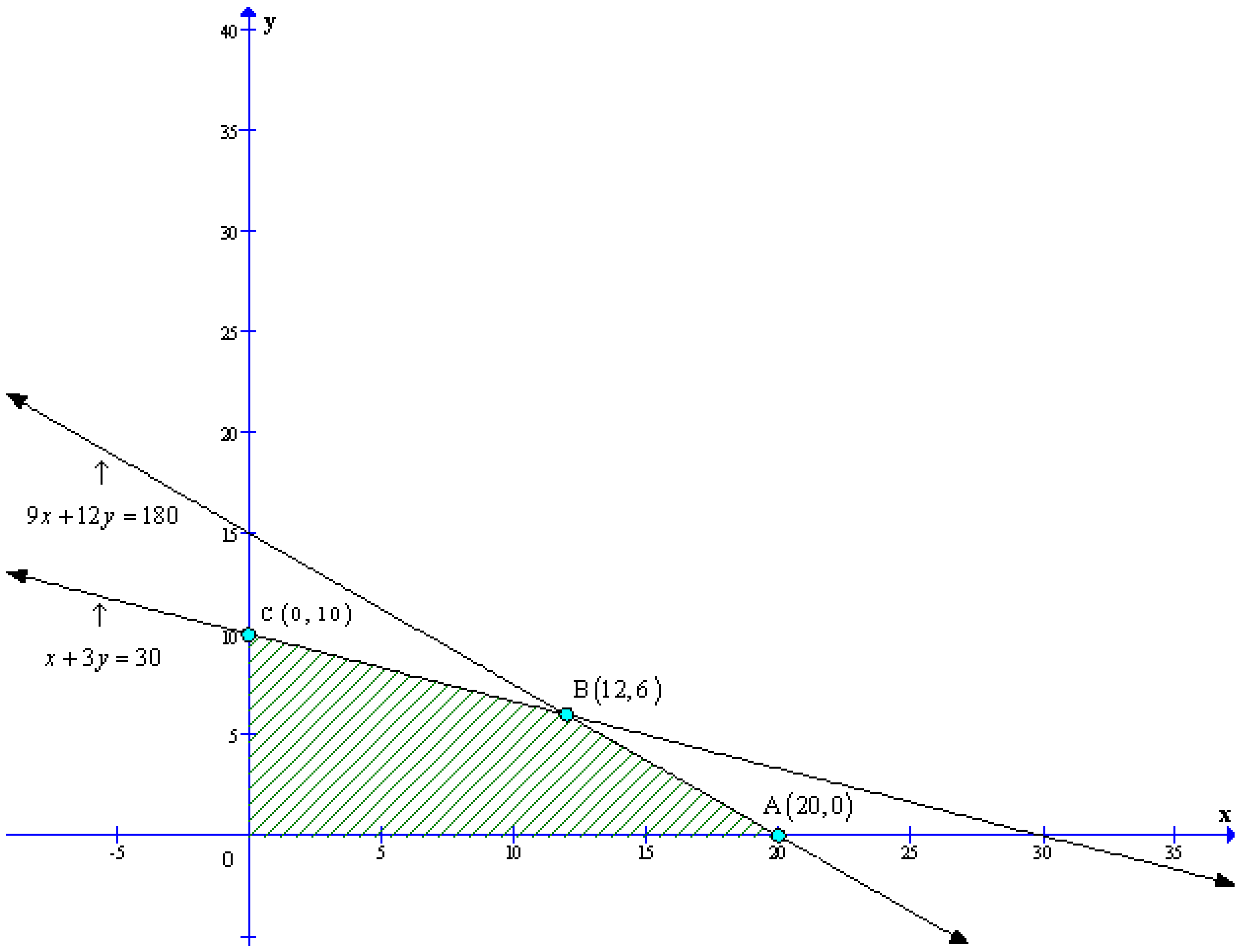
Model	A	B	Maximum hours
Fabricating	9	12	180
Finishing	1	3	30
Profit	8000	12000	

Let x be the number of pieces of A and y be the number of pieces of B manufactured to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 8000x + 12000y$
Subject to $9x + 12y \le 180$,
$x + 3y \le 30$
and $x \ge 0,\ y \ge 0$

To solve the LPP we draw the lines,
$9x + 12y = 180$,
$x + 3y = 30$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 0), B(12, 6) and C(0, 10).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 8{,}000x + 12{,}000y$
A(20, 0)	Z = 1,60,000
B(12, 6)	Z = 1,68,000
C(0, 10)	Z = 1,20,000

12 pieces of Model A and 6 pieces of Model B should be eaned maximize the profit.
The maximum profit that can be eared is Rs. 1,68,000.

Question 44

A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftrnan's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

i. What number of rackets and bats must be made if the factory is to Work at full capacity?
ii. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the maximum profit of the factory when it works at full capacity.

Solution 44

The given data can be written in the tabular form as follows:

Product	Racket	Bat	Maximum hours
Machine	1.5	3	42
Craftman	3	1	24
Profit	20	10	

Let x be the number of rackets and y be the number of bats made to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 20x + 10y$

Subject to $1.5x + 3y \leq 42$,

$3x + y \leq 24$

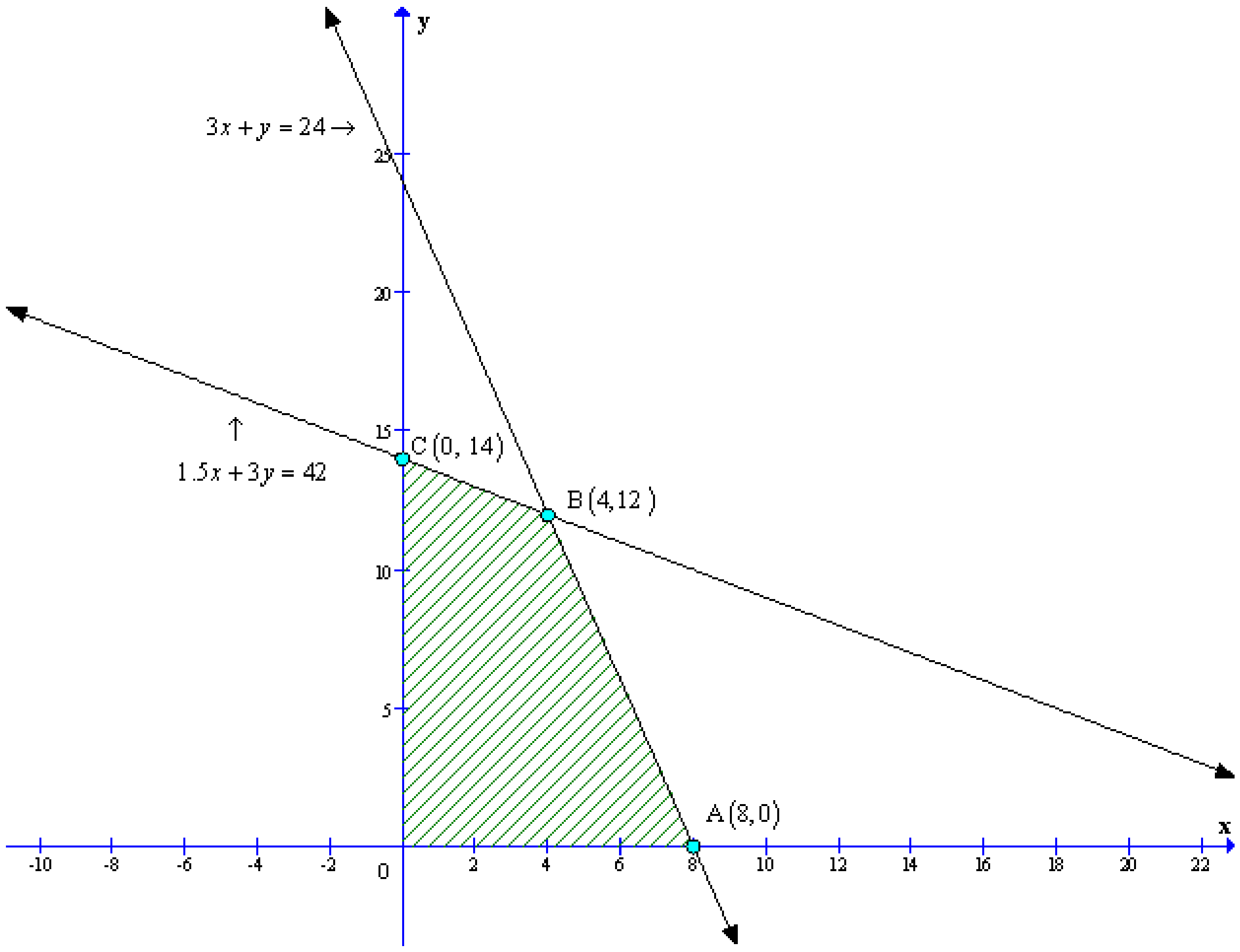
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,

$1.5x + 3y = 42$,

$3x + y = 24$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(8, 0),B(4,12) and C(0, 14).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 20x + 10y
A(8, 0)	Z = 160
B(4, 12)	Z = 200
C(0, 14)	Z = 140

4 rackets and 12 bats must be made if the factory is to work at full capacity.
The maximum profit that can be eared is Rs. 200.

Question 45

A merchant plans to sell two types of personal computers a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

Solution 45

Let x be the number of desktop computers and y be the number of portable computers which merchant should stock to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 4500x + 5000y$

Subject to $25000x + 40000y \leq 70,00,000$

$x + y \leq 250$

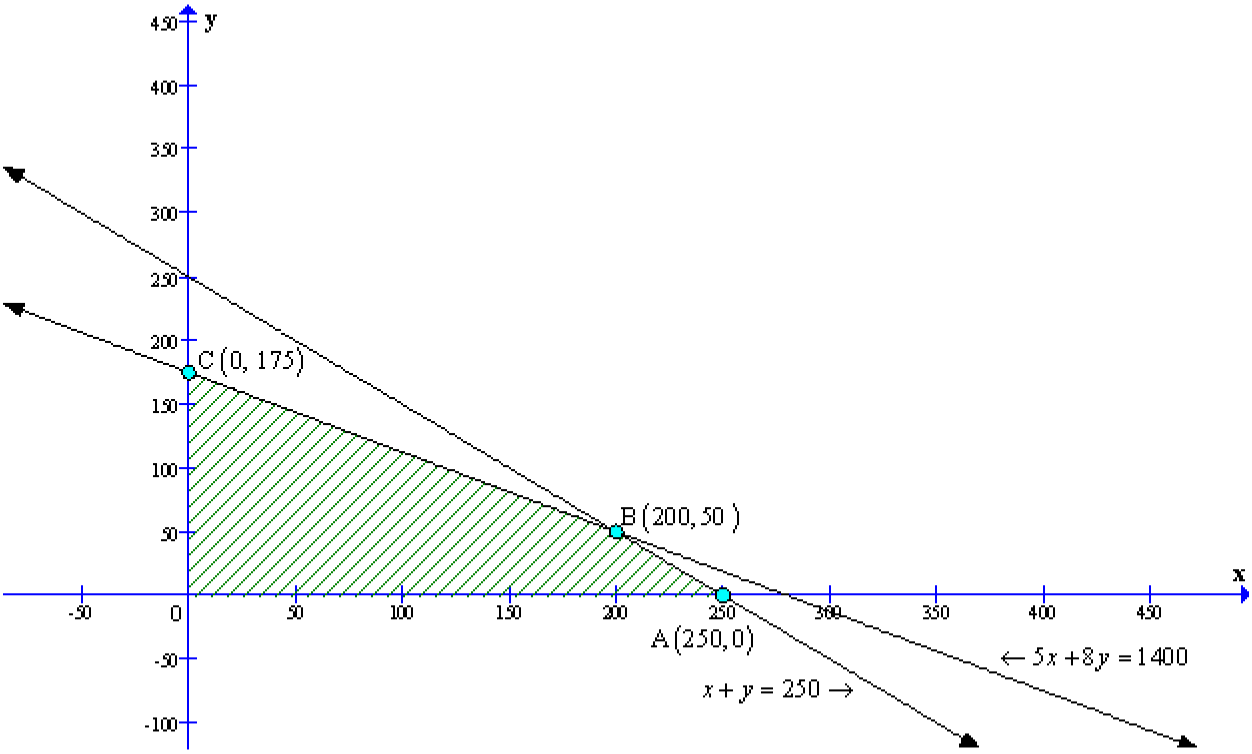
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,

$5x + 8y = 1,400$

$x + y = 250$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(250, 0), B(200,50) and C(0, 175).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 4500x + 5000y$
A(250, 0)	Z = 11,25,000
B(200, 50)	Z = 11,50,000
C(0, 175)	Z = 8,75,000

The merchant should stock 200 personal computer and 50 portable computers to earn maxim
The maximum profit that can be eared is Rs. 11,50,000.

Question 46

A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs. 12 and Rs. 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

Solution 46

Let x be the number of dolls of type A and y be the number of dolls of type B should be produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 12x + 16y$

Subject to $x + y \leq 1200$

$\frac{1}{2}x - y \geq 0$

$x - 3y \leq 600$

and $x \geq 0$, $y \geq 0$

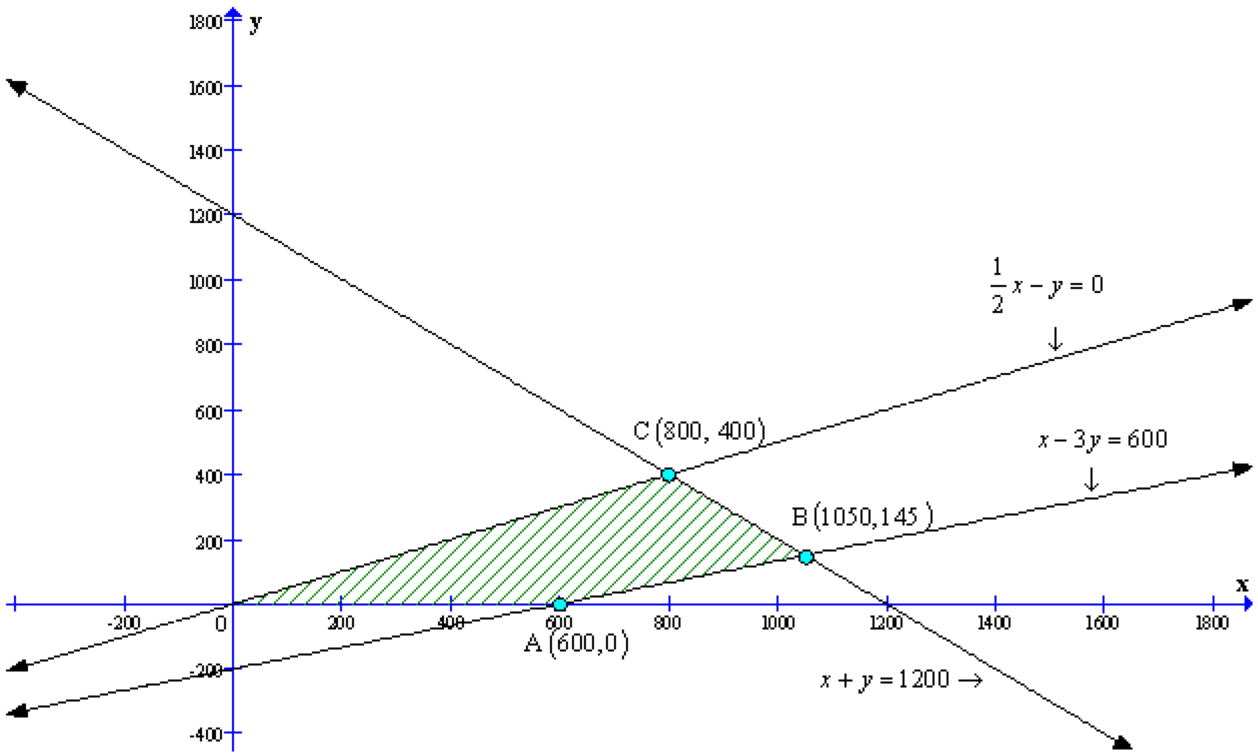
To solve the LPP we draw the lines,

$x + y = 1200$

$\frac{1}{2}x - y = 0$

$x - 3y = 600$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(600, 0), B(1050, 145) and C(800, 400).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 12x + 16y$
A(600, 0)	Z = 7200
B(1050, 145)	Z = 14920
C(800, 400)	Z = 16000

The toy company should manufacture 800 dolls of type A and 400 dolls of type B to earn max
The maximum profit that can be eared is Rs. 16,000.

Question 47

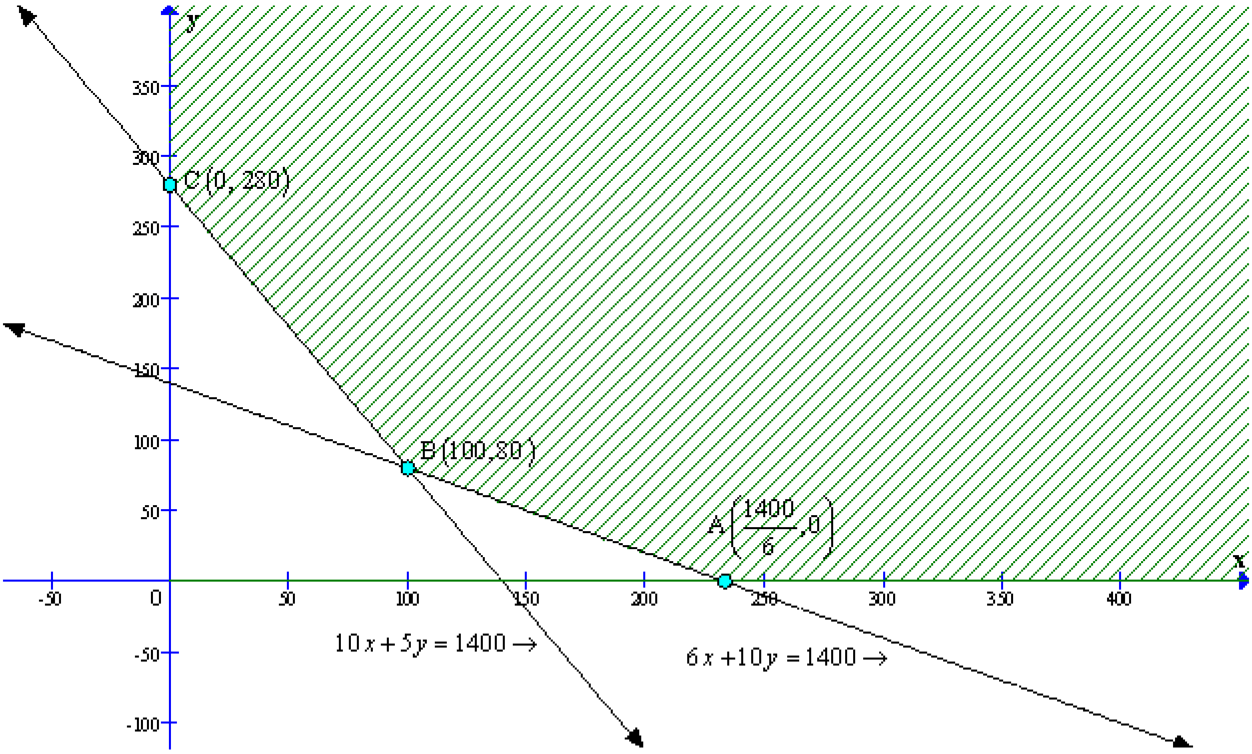
There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs. 6/kg and F_2 costs Rs. 5 /kg, determine how much of each type of fertiliser should be used so that nutrient requirement are met at a minimum cost. What is the minimum cost?

Solution 47

Let x kg of fertiliser F_1 and y kg of fertiliser F_2
should be used to minimise the cost.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 6x + 5y$
Subject to $10x + 5y \geq 1400$
$6x + 10y \geq 1400$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$10x + 5y = 1400$
$6x + 10y = 1400$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{1400}{6},\ 0\right)$, $B(100, 80)$ and $C(0, 280)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 6x + 5y$
$A\left(\frac{1400}{6},\ 0\right)$	$Z = 1400$
$B(100, 80)$	$Z = 1000$
$C(0, 280)$	$Z = 1400$

100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 to earn minimise the cost.
The maximum cost Rs. 1,000.

Question 48

A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines.
The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs. 600 and Rs. 400 on items M and N respectively. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Solution 48

Let x units of item M and y units of item N
should be produced to maximise the cost.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 600x + 400y$

Subject to $x + 2y \leq 12$

$2x + y \leq 12$

$x + 1.25y \geq 5$

and $x \geq 0$, $y \geq 0$

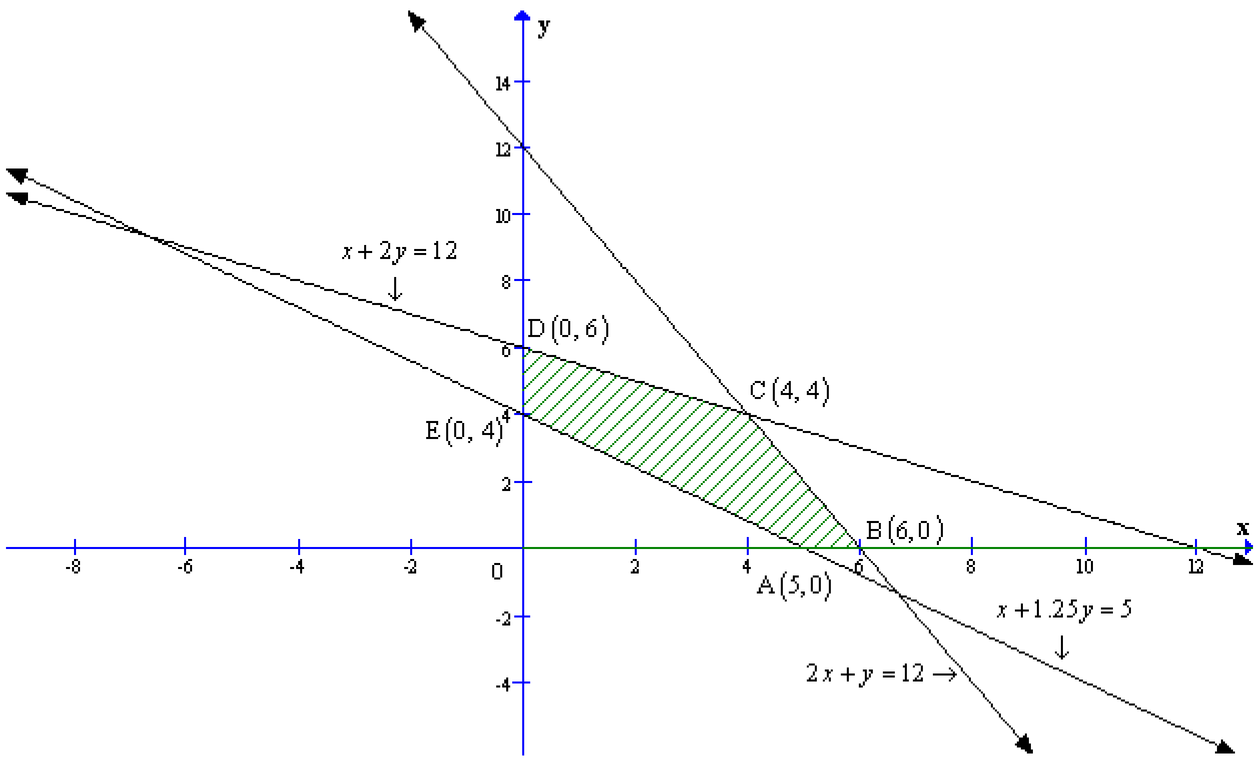
To solve the LPP we draw the lines,

$x + 2y = 12$

$2x + y = 12$

$x + 1.25y = 5$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDE are A(5, 0), B(6, 0), C(4, 4), D(0, 6) and E(0, 4).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 600x + 400y
A(5, 0)	Z = 3000
B(6, 0)	Z = 3600
C(4, 4)	Z = 4000
D(0, 6)	Z = 2400
E(0, 4)	Z = 1600

4 units of item M and 4 units of item N should be produced to maximise the profit.
The maximum profit is Rs. 4,000.

Question 49

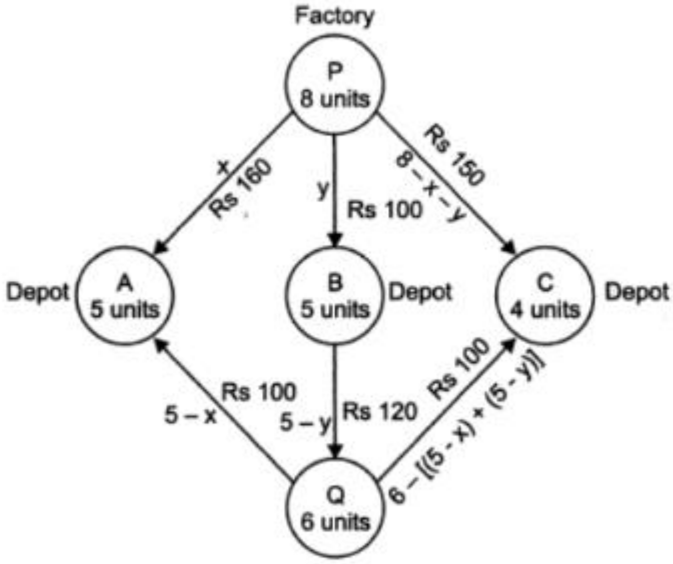
There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

To/from	Cost (in Rs.)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost?

Solution 49

Let x and y units of commodity be transported from factory P to the depots at A and B respectively.
Then (8 - x - y) units will be transported to depot at C.
The flow is shown below.

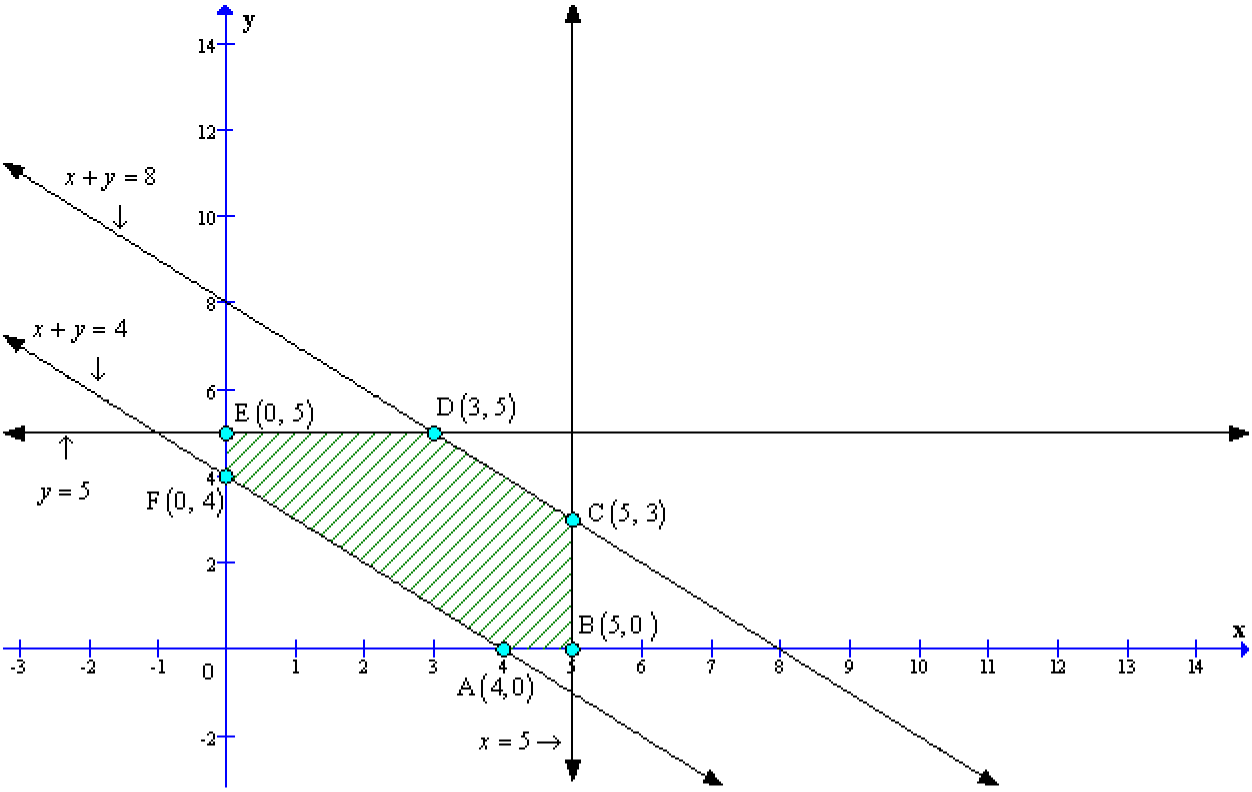

Factory
P
8 units
x
Rs 160
y
Rs 100
Rs 150
8 – x – y
Depot
A
5 units
B
5 units
Depot
C
4 units
Depot
Rs 100
5 – x
5 – y
Rs 120
Rs 100
6 – [(5 - x) + (5 - y)]
Q
6 units

Hence we have, $x \geq 0$, $y \geq 0$ and $8 - x - y \geq 0$
i.e. $x \geq 0$, $y \geq 0$ and $x + y \geq 8$
Now, the weekly requirement of the depot at A is 5 units of the commodity.
Since x units are transported from the factory at P, remaimning $(5 - x)$ units need to be transported from the factory at Q.
$\therefore 5 - x \geq 0 \Rightarrow x \leq 5$
Simillarly, $(5 - y)$ and $6 - (5 - x + 5 - y) = x + y - 4$ units are to be transported from the factory at Q to the depots at B and C respectively.
$\therefore 5 - y \geq 0$ and $x + y - 4 \geq 0$
$\Rightarrow y \leq 5$ and $x + y \geq 4$

Total transportation cost Z is given by
$$Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100(x + y - 4) + 150(8 - x - y)$$
$$Z = 10(x - 7y + 190)$$

So the mathematical model of given LPP is as follows.
Minimize $Z = 10(x - 7y + 190)$
Subject to $x + y \leq 8$
$x \leq 5, y \leq 5$
$x + y \geq 4$
$x \geq 0, y \geq 0$

To solve the LPP we draw the lines,
$x + y = 8$
$x = 5, y = 5$
$x + y = 4$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDEF are A(4, 0),B(5,0), C(5, 3), D(3, 5), E(0, 5) and F(0, 4).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 10(x - 7y + 190)$
A(4, 0)	Z = 1940
B(5, 0)	Z = 1950
C(5, 3)	Z = 1740
D(3,5)	Z = 1580
E(0,5)	Z = 1550
F(0,4)	Z = 1620

Deliver 0, 5, 3 units from factory at P and 5, 0, 1 from the factory at Q to the depots at A, B a
The minimum transportation cost is Rs. 1550.

Question 50

A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Types of Toys	Machines		
	I	II	III
A	12	18	6

B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs. 7.50 and that on each toy of type B is Rs.5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Solution 50

Let the mixture contains x toys of type A and y toys of type B.

Type of toys	No. of toys	Machine I (in min)	Machine II (in min)	Machine III (in min)	Profit Rs.
A	x	12x	18x	6x	7.5x
B	y	6y	0	9y	5y
Total	x+y	12x + 6y	18x	6x + 9y	7.5x + 5y
Requirement		360	360	360	

Then the mathematical model of the LPP is as follows:

Maximize $Z = 7.5x + 5y$

Subject to $12x + 6y \leq 360 \Rightarrow 2x + y \leq 60$

$18x \leq 360 \Rightarrow x \leq 20$

$6x + 9y \leq 360 \Rightarrow 2x + 3y \leq 120$

and $x \geq 0$, $y \geq 0$

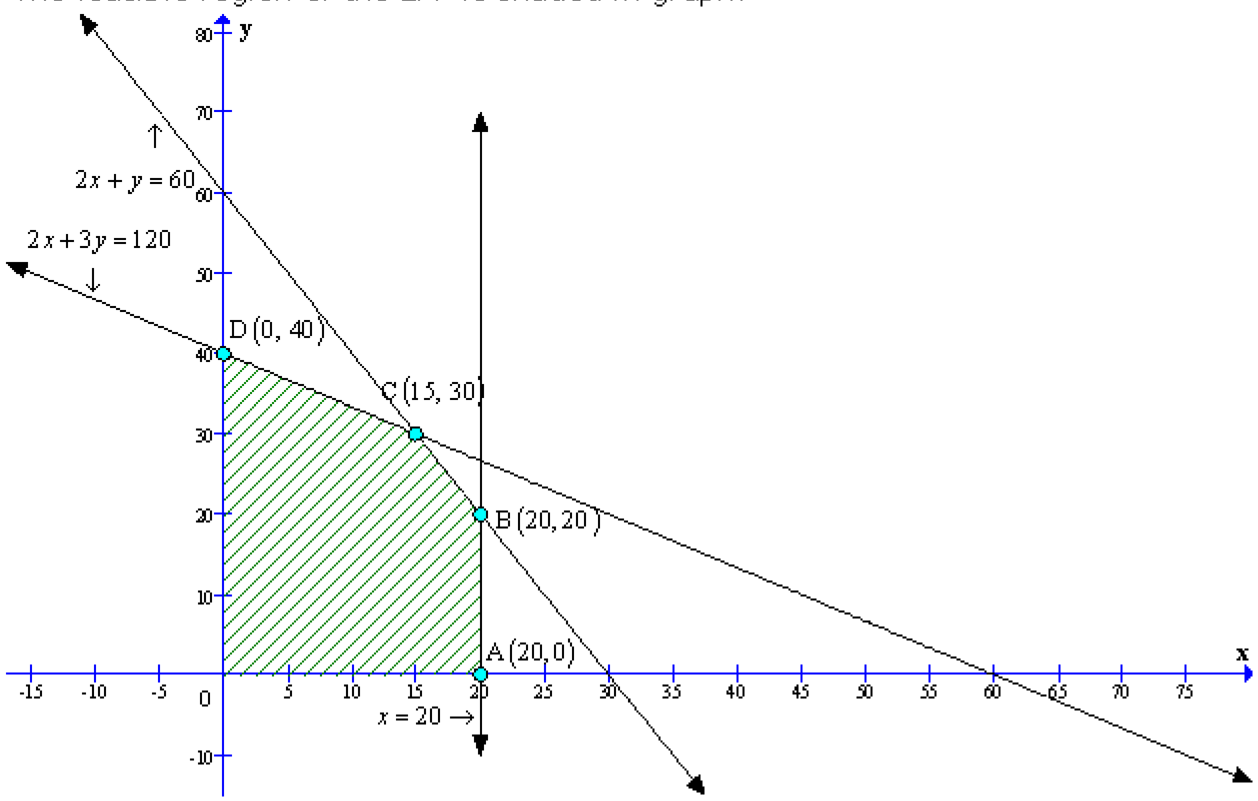
To solve the LPP we draw the lines,

$2x + y = 60$

$x = 20$

$2x + 3y = 120$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(20, 0), B(20,20), C(15, 30) and D(0, 40).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 7.5x + 5y$
A(20, 0)	Z = 150
B(20, 20)	Z = 250
C(15, 30)	Z = 262.5
D(0, 40)	Z = 200

Manufacturer should make 15 toys of type A and 30 toys of type B to maximize the profit.
The maximum profit that can be earned is Rs. 262.5

Question 51

An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Solution 51

Let x be the number of executive class tickets and y be the number of economic class tickets.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 1000x + 600y$

Subject to $x + y \leq 200$

$x \geq 20$

$y \geq 4x \Rightarrow -4x + y \geq 0$

and $x \geq 0,\ y \geq 0$

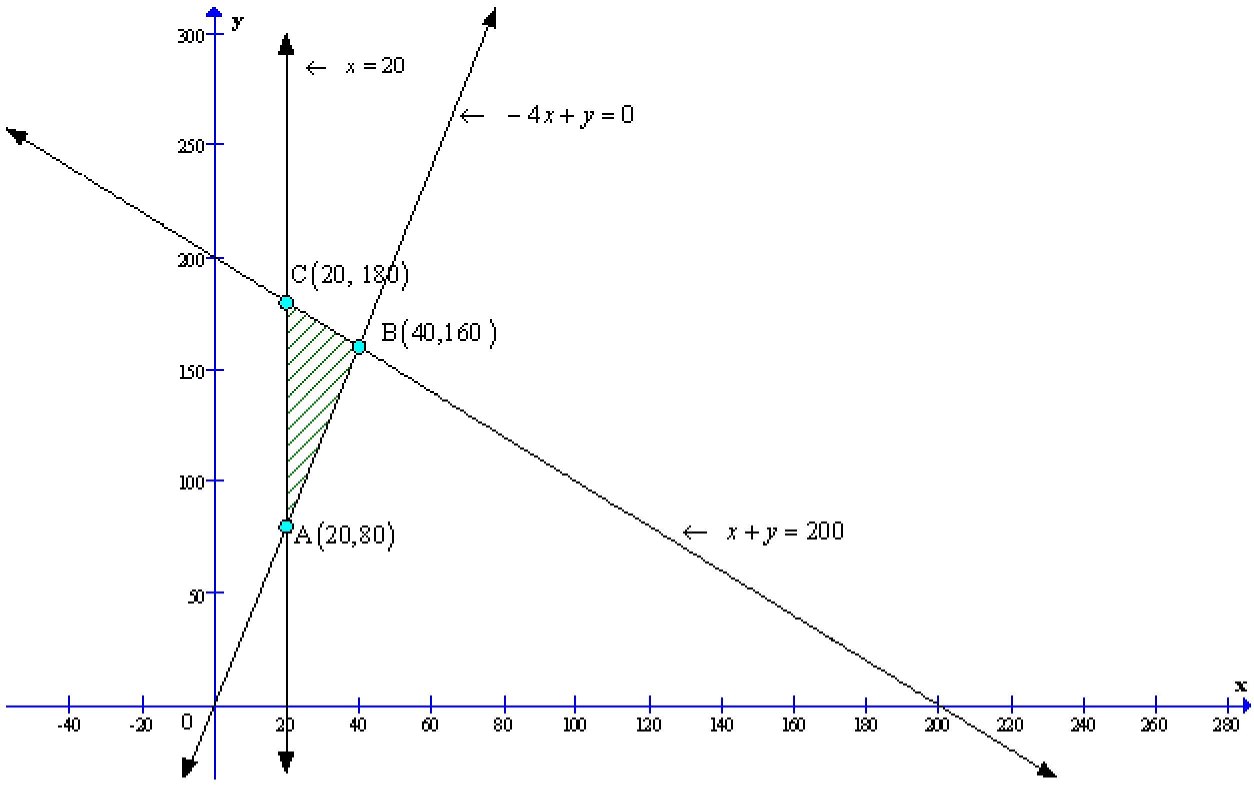
To solve the LPP we draw the lines,

$x + y = 200$

$x = 20$

$-4x + y = 0$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 80),B(40,160) and C(20, 180).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 1000x + 600y$
A(20, 80)	Z = 68,000
B(40, 160)	Z = 1,36,000
C(20, 180)	Z = 1,28,000

40 tickets of executive class and 160 tickets of economic class must be sold to maximize the profit.
The maximum profit that can be earned is Rs. 1,36,000.

Question 52

A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at Rs. 100 and Rs. 120 per unit respectively, how should he use his resources to maximize the total revenue? Form the above as an LPP and solve graphically. Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?

Solution 52

Let x units of workers and y units of capital are required to maximize the total revenue.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 100x + 120y$

Subject to $2x + 3y \le 30$

$3x + y \le 17$

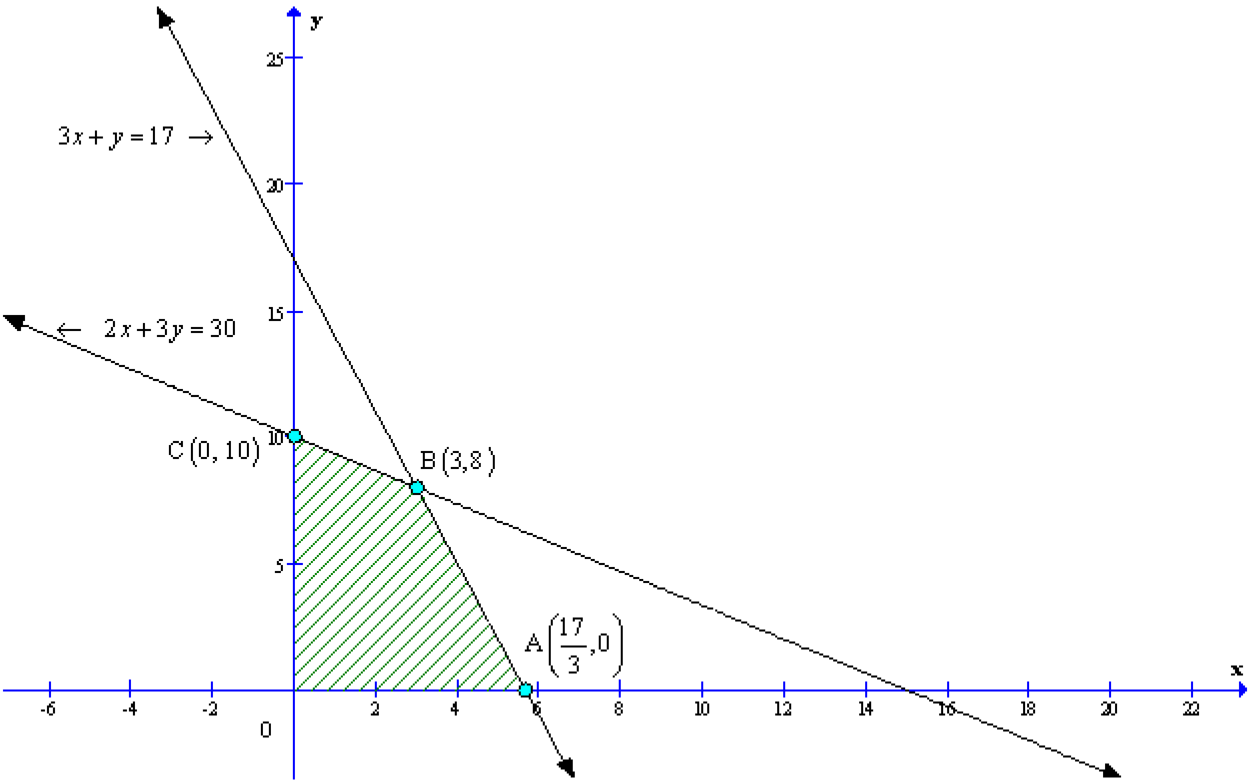
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,

$2x + 3y = 30$

$3x + y = 17$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{17}{3}, 0\right), B(3, 8)$ and $C(0, 10)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 100x + 120y$
$A\left(\frac{17}{3}, 0\right)$	$Z = 566.67$
$B(3, 8)$	$Z = 1260$
$C(0, 10)$	$Z = 1200$

3 units of workers and 8 units of capital must be used to maximize the profit.
The maximum profit that can be earned is Rs. 1260.

Yes, because efficiency of a person does not depend on sex (male or female).

Chapter 30 - Linear programming Exercise Ex. 30.5

Question 1

Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

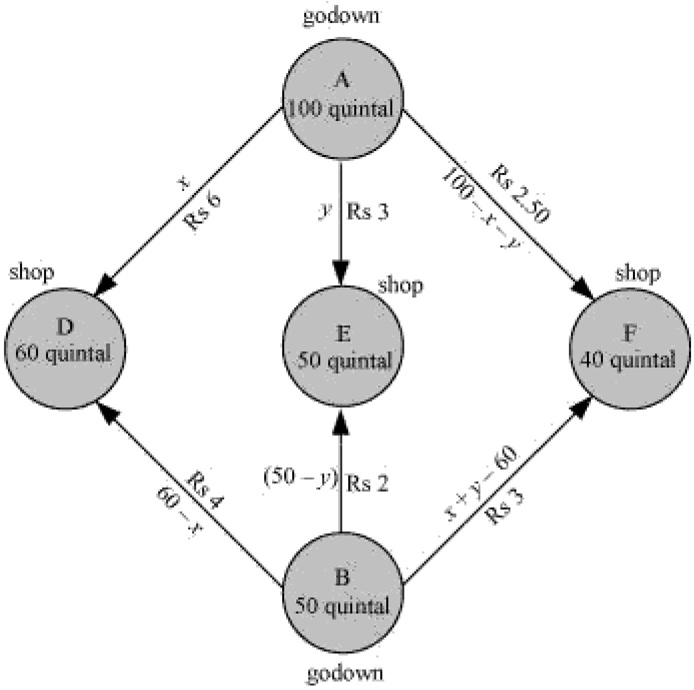
Solution 1

Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, $(100 - x - y)$ will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining $(60 - x)$ quintals will be transported from godown B.

Similarly, $(50 - y)$ quintals and $40 - (100 - x - y) = (x + y - 60)$ quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows.



$$x \geq 0,\ y \geq 0,\ \text{and } 100 - x - y \geq 0$$
$$\Rightarrow x \geq 0,\ y \geq 0,\ \text{and } x + y \leq 100$$

$$60 - x \geq 0,\ 50 - y \geq 0,\ \text{and } x + y - 60 \geq 0$$
$$\Rightarrow x \leq 60,\ y \leq 50,\ \text{and } x + y \geq 60$$

Total transportation cost z is given by,

$$\begin{aligned} z &= 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60) \\ &= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180 \\ &= 2.5x + 1.5y + 410 \end{aligned}$$

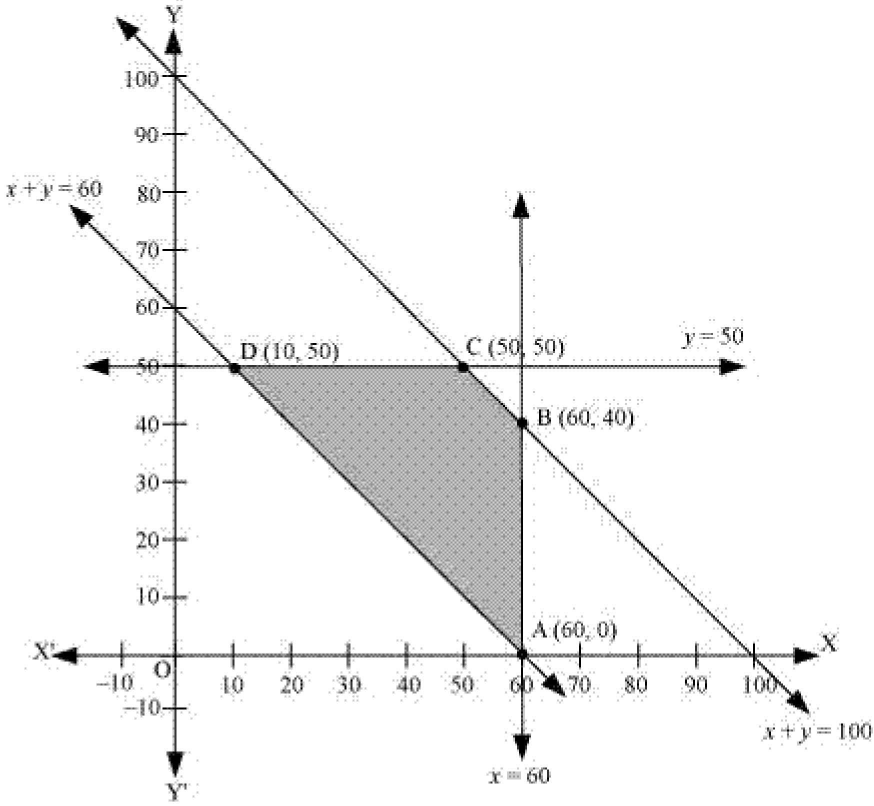
The given problem can be formulated as

Minimize $z = 2.5x + 1.5y + 410$ ... (1)

subject to the constraints,

$$x + y \leq 100 \qquad ...(2)$$
$$x \leq 60 \qquad ...(3)$$
$$y \leq 50 \qquad ...(4)$$
$$x + y \geq 60 \qquad ...(5)$$
$$x,\ y \geq 0 \qquad ...(6)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (60, 0), B (60, 40), C (50, 50), and D (10, 50).

The values of z at these corner points are as follows.

Corner point	$z = 2.5x + 1.5y + 410$	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	→ Minimum

The minimum value of z is 510 at (10, 50).

Thus, the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals, and 40 quintals respectively and from B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively.

The minimum cost is Rs 510.

Question 2

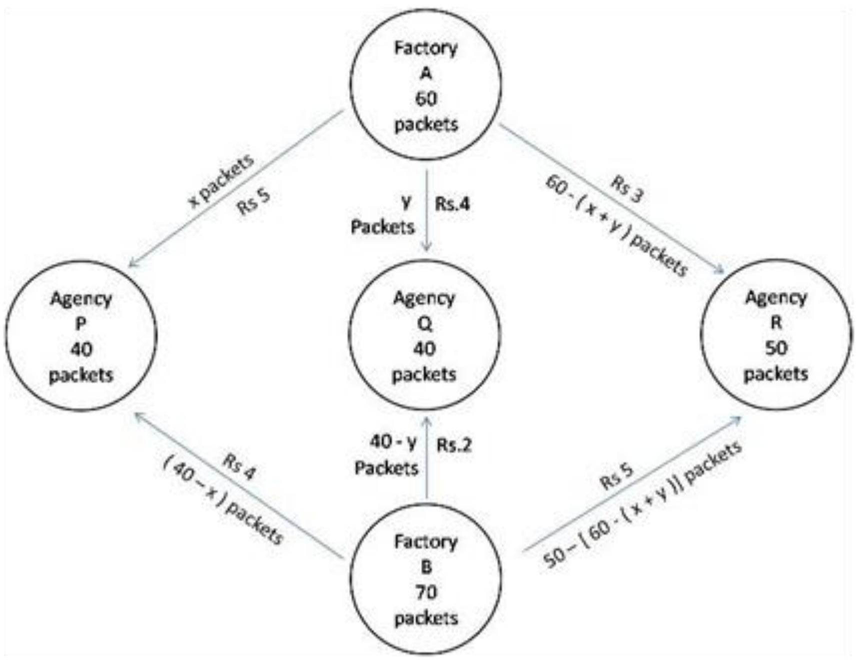
A medical company has factories at two places, A and B. From these places, supply is made to each of its three agencies situated at P, Q and R. The monthly requirements of the agencies are respectively 40,40 and 50 packets of the medicines, while the production capacity of the factories, A and B, are 60 and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given below:

Transportation Cost per packet(in Rs)		
From / To	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost?

Solution 2

The given information can be exhibited diagramatically as below:



Let factory A transports x packets to agency P and y packet to agency Q. Since factory A has capacity of 60 packets so, rest $\left[60-(x+y)\right]$ packets transported to agency R.

Since requirements are always non negative so,

$\Rightarrow \quad x, y \geq 0 \quad$ (first constraint)

and $\quad 60-(x+y) \geq 0$

$(x+y) \leq 60 \quad$ (second constraint)

Since requirement of agency P is 40 packet but it has recieved x packet, so $(40-x)$ packets are transported from factory B, requirement of agency Q is 40 packets but it has recieved y packets, so $(40-y)$ packets are transported from factory B. Requirement of agency R is 50 packets but it has recieved $(60-x-y)$ packets from factory A, so $50-\left[60-x-y\right]=(x+y-10)$ is transported from factory B, As the requirements of agencies P, Q, R are always non negative, so,

$40-x \geq 0$

$\Rightarrow \quad x \leq 40 \quad$ (third constraint)

$40-y \geq 0$

$\Rightarrow \quad y \leq 40 \quad$ (fourth constraint)

$x+y-10 \geq 0$

$\Rightarrow \quad x+y \geq 10 \quad$ (fifth constraint)

Costs of transportation of each packet from factory A to agency P, Q, R are Rs 5,4,3 respectively and costs of transportation of each packet from factory B to agency P, Q, R are Rs 4,2,5 respectively,

Let Z be total cost of transportation so,

$$\begin{aligned} Z &= 5x+4y+3\left[60-x-y\right]+4(40-x)+2(40-y)+5(x+y-10) \\ &= 5x+4y+180-3x-3y+160-4x+80-2y+5x+5y-50 \\ &= 3x+4y+370 \end{aligned}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z=3x+4y+370$

subject to constraints,

$x, y \ge 0$

$x + y \le 60$

$x \le 40$

$y \le 0$

$x + y \ge 10$

Region $x, y \ge 0$: It is represents first quandrant.

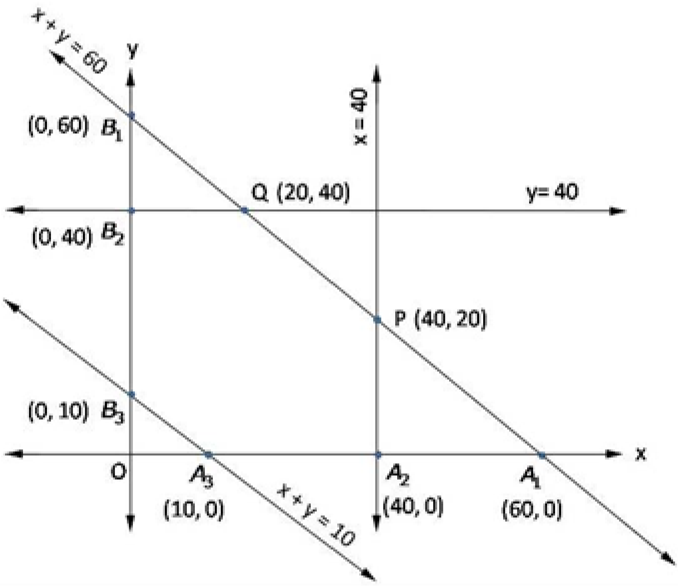
Region $x + y \le 60$: line $x + y = 60$ meets axes at $A_1(60,0)$, $B_1(0,60)$ respectively.
Region containing origin represents $x + y \le 60$ as $(0,0)$ satisfies $x + y \le 60$.

Region $x \le 40$: line $x = 40$ is parallel to y-axis and meets x-axis at $A_2(40,0)$.
Region containing origin represents $x \le 40$ as $(0,0)$ satisfies $x \le 40$.

Region $y \le 40$: line $y = 40$ is parallel to x-axis and meets y-axis at $B_2(0,40)$.
Region containing origin represents $y \le 40$ as $(0,0)$ satisfies $y \le 40$.

Region $x + y \ge 10$: line $x + y = 10$ meets axes at $A_2(10,0)$, $B_3(0,10)$ respectively.
Region containing origin represents $x + y \ge 10$ as $(0,0)$ does not satisfy $x + y \ge 10$.

Shaded region $A_3A_2PQB_2B_3$ represents feasible region.
Point $P(40, 20)$ is obtained by solving $x = 40$ and $x + y = 60$
Point $Q(20, 40)$ is obtained by solving $y = 40$ and $x + y = 60$



The value of $Z = 3x + 4y + 370$ at

$$\begin{aligned}
&A_3\left(10,0\right) && = 3\left(10\right) + 4\left(0\right) + 370 = 400 \\
&A_2\left(40,0\right) && = 3\left(40\right) + 4\left(0\right) + 370 = 490 \\
&P\left(40,20\right) && = 3\left(40\right) + 4\left(20\right) + 370 = 570 \\
&Q\left(20,40\right) && = 3\left(20\right) + 4\left(40\right) + 370 = 590 \\
&B_2\left(0,40\right) && = 3\left(0\right) + 4\left(40\right) + 370 = 530 \\
&B_3\left(0,10\right) && = 3\left(0\right) + 4\left(10\right) + 370 = 410
\end{aligned}$$

minimum $Z = 400$ at $x = 10$, $y = 0$

From $A \rightarrow P =$ 10 packets

From $A \rightarrow Q =$ 0 packets

From $A \rightarrow R =$ 50 packets

From $B \rightarrow P =$ 30 packets

From $B \rightarrow Q =$ 40 packets

From $B \rightarrow R =$ 0 packets

minimum cost = Rs 400

Chapter 30 - Linear programming Exercise Ex. 30RE

Question 1(i)

Solve the following liner programming problem graphically:

Maximise $Z = 4x + y$ ...(1)

Subject to the constraints:

$x + y \leq 50$ ...(2)

$3x + y \leq 90$ ...(3)

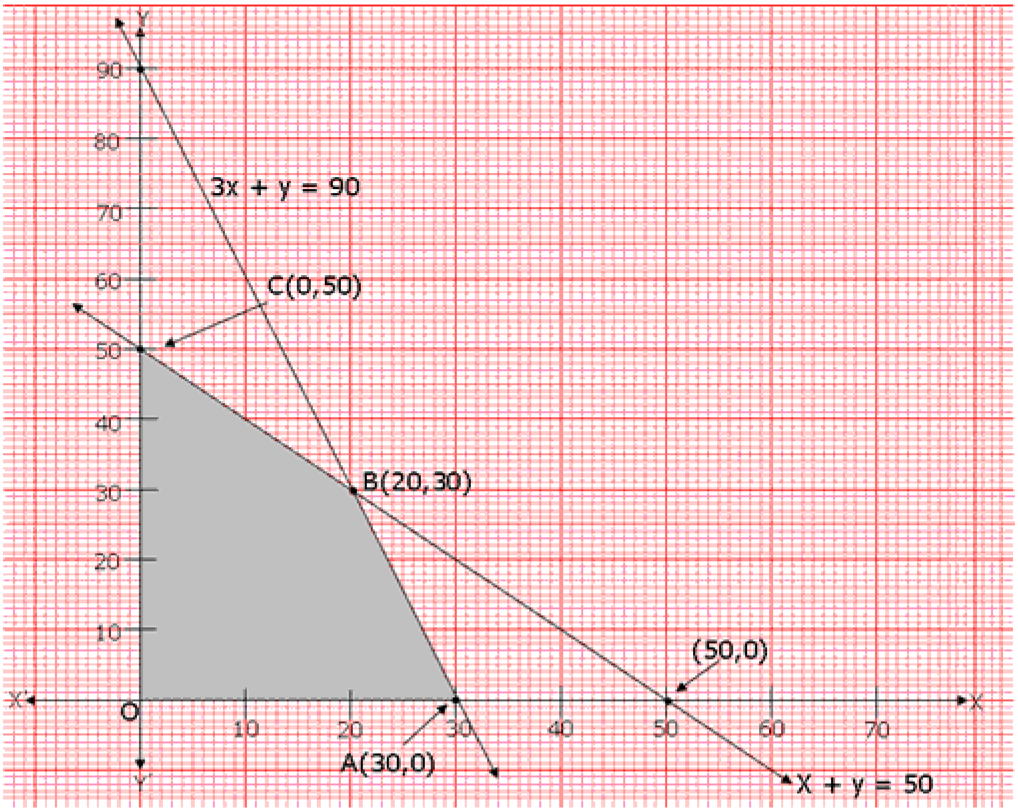
$x \geq 0, y \geq 0$ ...(4)

Solution 1(i)

The shaded region in fig., is the feasible region determined by the system of constraints (2) to (4). We observe that the feasible region OABC is bounded. So, not use Corner Point Method to determine the maximum value of Z.

The coordinates of the corner points O,A,B and C are (0,0), (30,0), (20,30) and (0,50) respectively. Now we evaluate Z at each corner point.

Corner Point	Corresponding value of Z	
(0,0)	0	
(30,0)	**120** ⟵	Maximum
(20,0)	110	
(0,50)	50	



Hence, maximum value of Z is 120 at the point (30.0)

Question 1(ii)

Solve the following linear programming problem graphically:

Minimise Z = 200x + 500y ...(1)

Subject to the constraints:

$x + 2y \geq 10$ ...(2)

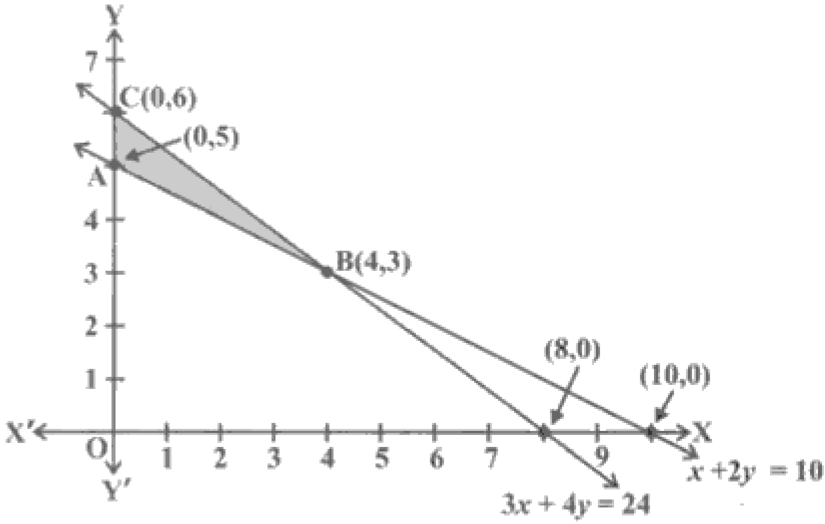
$3x + 4y \leq 24$ ...(3)

$x \geq 0,\ y \geq 0$ ...(4)

Solution 1(ii)

The shaded region in fig., is the feasible region ABC determined by the system of constraints (2) to (4), which is bounded. The coordinates of corner points

Corner Point	Corresponding value of Z	
(0,5)	2500	
(4,3)	**2300** $\longleftarrow$	Minimum
(0,6)	3000	



A,B and C are (0,5), (4,3) and (0,6) respectively. Now we evaluate $Z = 200x + 500y$ at these points.

Hence, minimum value of Z is 2300 attained at the point (4,3)

Question 1(iii)

Solve the following problem graphically:

Minimise and Maximise $Z = 3x + 9y$ ...(1)

Subject to the constraints: $x + 3y \leq 60$ ...(2)

$x + y \geq 10$ ...(3)

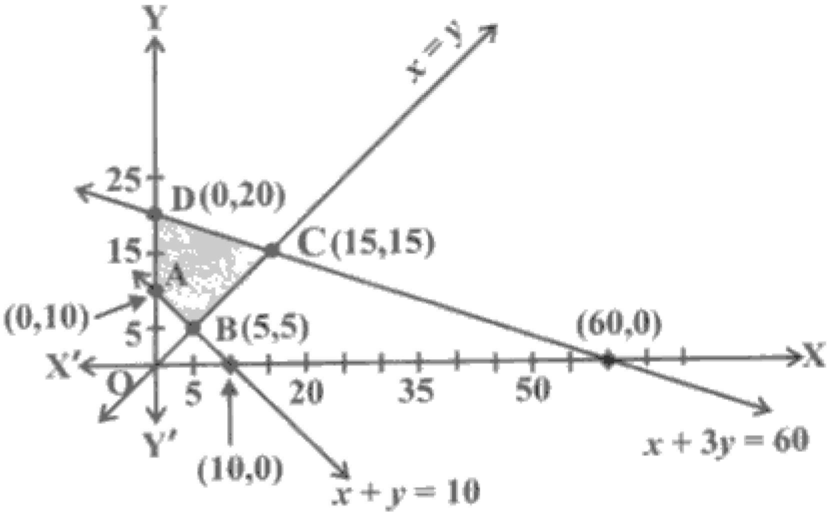
$x \leq y$ ...(4)

$x \geq 0,\ y \geq 0$ ...(5)

Solution 1(iii)

First of all, let us graph the feasible region of the system of linear inequalities (2) to (5). The feasible region ABCD is shown in the fig.,. Note that the region is bounded. The coordinates of the corner points A,B,C and D are (0,10), (5,5), (15,15) and (0,20) respectively.

Corner Point	Corresponding value of Z = 3x + 9y	
A (0,10)	90	
B (5,5)	**60** ⟵	Minimum
C (15,15)	**180**	Maximum (Multiple optimal solutions)
D (0,20)	**180**	



We now find the minimum and maximum value of Z. From the table, we find that the minimum value of Z is 60 at the point B(5,5) of the feasible region.
The maximum value of Z on the feasible region occurs at the two corner points C(15,15) and D(0,20) and it is 180 in each case.

Question 1(iv)

Minimise Z = 3x + 2y

Subject to the constraints:

$X + y \geq 8$ ...(1)

$3x + 5y \leq 15$ ...(2)

$X \geq 0,\ y \geq 0$ ...(3)

Solution 1(iv)

Let us graph the inequalities (1) to (3) fig.,. Is there any feasible region? Why is so?

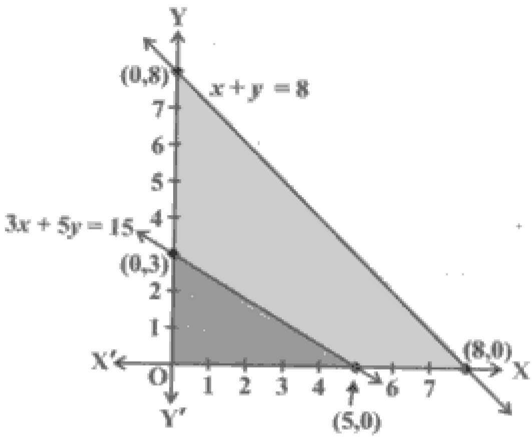
From fig., you can see that there is no point satisfying all the constraints simultaneously. Thus, the problem is having no feasible region and hence no feasible solution.

Remarks From the examples which we have discussed so far, we notice some general features of linear programming problems:

(i) The feasible region is always a convex region.

(ii) The maximum (or minimum)

Solution of the objective function occurs at the vertex (Corner) of the feasible region. If two corner points produce the same maximum (or minimum) value of the objective function, then every point on the line segment joining these points will also give the same maximum (or minimum) value.



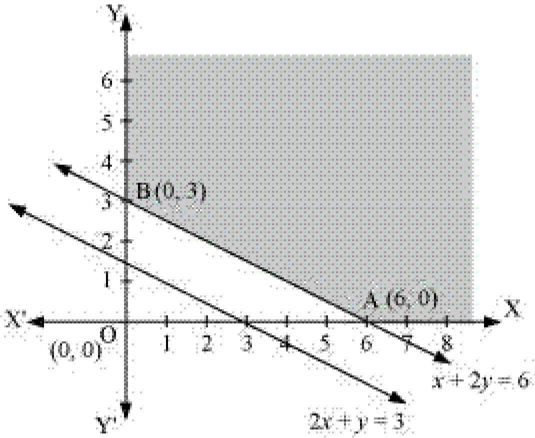
Question 1(v)

Minimise Z = $x + 2y$

subject to $2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0$.

Solution 1(v)

The feasible region determined by the constraints, $2x + y \geq 3$, $x + 2y \geq 6$, $x \geq 0$, and $y \geq 0$, is as follows.



The corner points of the feasible region are A (6, 0) and B (0, 3).

The values of Z at these corner points are as follows.

Corner point	$Z = x + 2y$
A(6, 0)	6
B(0, 3)	6

It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line $x + 2y = 6$, then Z = 6

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line, $x + 2y = 6$

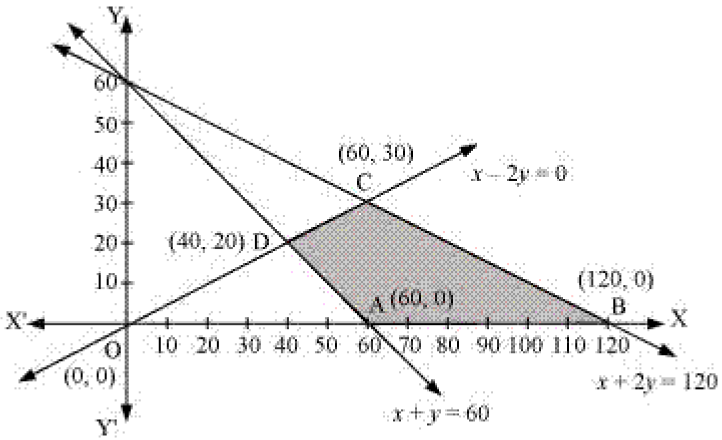
Question 1(vi)

Minimise and Maximise $Z = 5x + 10y$

subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$.

Solution 1(vi)

The feasible region determined by the constraints, $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$, is as follows.



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20).

The values of Z at these corner points are as follows.

Corner point	$Z = 5x + 10y$	
A(60, 0)	300	→ Minimum
B(120, 0)	600	→ Maximum
C(60, 30)	600	→ Maximum
D(40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

Question 2

A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,5000 and Rs 9,000 respectivley. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more thatn 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainages from this land. How much land should be allocated to each crop so as to miximise the total profit of the society?

Solution 2

Let x hectare of land be allocated to crop X and y hectare to crop Y. Obviously, $x \geq 0$, $y \geq 0$.

Profit per hectare on crop X = ₹ 10500

Profit per hectare on crop Y = ₹ 9000

The mathematical formulation of the problem is as follows:

Maximise Z = 10500 x + 9000 y

Subject to the constraints:

$x + y \leq 50$ (constraint related to land) ...(1)

$20x + 10y \leq 8000$ (constraint related to use of herbicide)

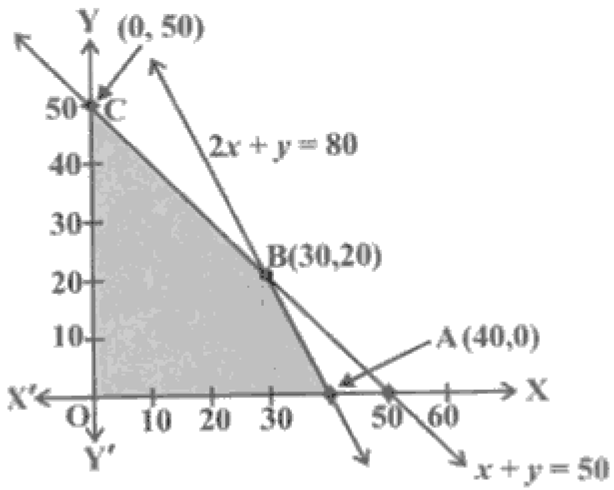
$2x + y \leq 80$ ...(2)

$x \geq 0, y \geq 0$ (non negative constraint) ...(3)

Let us draw the graph of the system of inequalities (1) to (3). The feasible region OABC is shown (shaded) in the fig., Observe that the feasible region is bounded.

The coordinates of the corner points O,A,B and C are (0,0),(40,0),(30,20) and (0,50) respectively. Let us evaluate the objective function Z = 10500 x + 9000y at these vertices to find which one gives the maximum profit.

Corner Point	Z = 10500 x + 9200y
O (0,0)	0
A(40,0)	420000
B(30,20)	495000 ⟵ Maximum
C(0,50)	450000



Question 3

A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

Solution 3

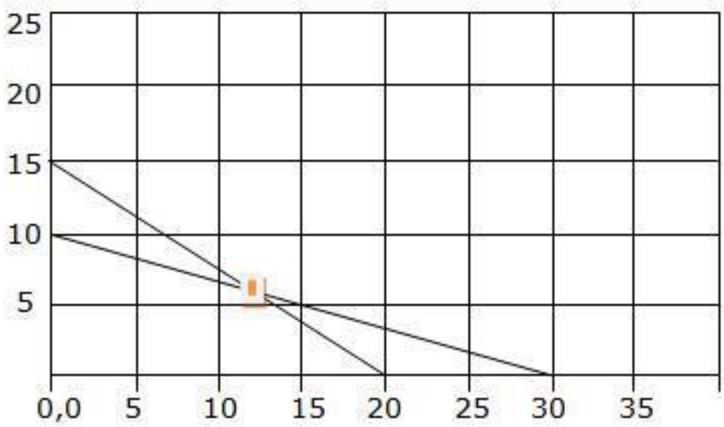
Let x & y be the No. of model A and model B of a product.

$9x + 12y \le 180$ (constraint on labour hr for fabricating)

$1x + 3y \le 30$ (constraint on labour hr for finishing)
$Z = 8000x + 12000y$ (Maximize profit)

$x, y \ge 0$; plotting the inequalities we have,

when x=0, y= 15 and when y=0, x=20 and
when x=0, y= 10 and when y=0, x=30



The feasible region is given by 0,0-10-E-20-0,0

Tabulating Z and corner points we have

Corner point	Value of Z = 8000x + 12000y
0, 0	0
0, 10	1,20,000
12, 6	1,68,000
20, 0	1,60,000

12 pieces of model A and 6 pieces of model B should be manufactured per week to realize a maximum profit of Rs.1, 68,000 /- per week.

Question 4

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?

Solution 4

Let the mixture contain x kg of food P and y kg of food Q. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	**Vitamin B (units/kg)**	**Cost (Rs/kg)**
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B. Therefore, the constraints are

$3x + 4y \geq 8$

$5x + 2y \geq 11$

Total cost, Z, of purchasing food is, $Z = 60x + 80y$

The mathematical formulation of the given problem is

Minimise $Z = 60x + 80y$ ... (1)

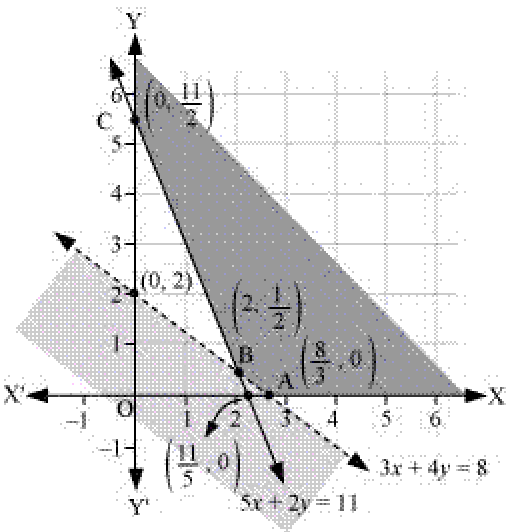
subject to the constraints,

$3x + 4y \geq 8$ ... (2)

$5x + 2y \geq 11$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$, and $C\left(0, \frac{11}{2}\right)$.

The values of Z at these corner points are as follows.

Corner point	Z = 60*x* + 80*y*	
$A\left(\frac{8}{3}, 0\right)$	160	$\left.\begin{matrix} \\ \end{matrix}\right\} \rightarrow$ Minimum
$B\left(2, \frac{1}{2}\right)$	160	
$C\left(0, \frac{11}{2}\right)$	440	

As the feasible region is unbounded, therefore, 160 may or may not be the minimum value of Z.

For this, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 4y < 8$

Therefore, the minimum cost of the mixture will be Rs 160 at the line segment joining the points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$.

Question 5

One kind of cake requires 200g flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?

Solution 5

Let there be x cakes of first kind and y cakes of second kind. Therefore,
$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Flour (g)	Fat (g)
Cakes of first kind, x	200	25
Cakes of second kind, y	100	50
Availability	5000	1000

$$\therefore 200x + 100y \leq 5000$$
$$\Rightarrow 2x + y \leq 50$$
$$25x + 50y \leq 1000$$
$$\Rightarrow x + 2y \leq 40$$

Total numbers of cakes, Z, that can be made are, Z = x + y

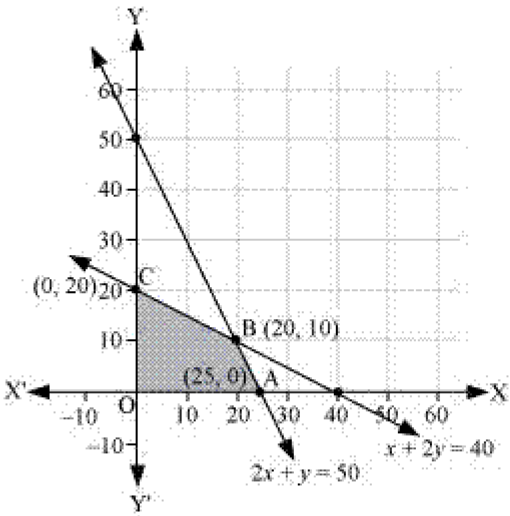
The mathematical formulation of the given problem is

Maximize Z = x + y ... (1)

subject to the constraints,

$$2x + y \leq 50 \qquad ...(2)$$
$$x + 2y \leq 40 \qquad ...(3)$$
$$x, y \geq 0 \qquad ...(4)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

Corner point	$Z = x + y$	
A(25, 0)	25	
B(20, 10)	30	→ Maximum
C(0, 20)	20	
O(0, 0)	0	

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

Question 6

A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

Solution 6

(i) Let the number of rackets and the number of bats to be made be x and y respectively.

The machine time is not available for more than 42 hours.

$$\therefore 1.5x + 3y \leq 42 \qquad \text{...(1)}$$

The craftsman's time is not available for more than 24 hours.

$$\therefore 3x + y \leq 24 \qquad \text{...(2)}$$

The factory is to work at full capacity. Therefore,

$1.5x + 3y = 42$

$3x + y = 24$

On solving these equations, we obtain

$x = 4$ and $y = 12$

Thus, 4 rackets and 12 bats must be made.

(ii) The given information can be complied in a table as follows.

	Tennis Racket	Cricket Bat	Availability
Machine Time (h)	1.5	3	42
Craftsman's Time (h)	3	1	24

$1.5x + 3y \leq 42$

$3x + y \leq 24$

$x, y \geq 0$

The profit on a racket is Rs 20 and on a bat is Rs 10.

$\therefore Z = 20x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 20x + 10y$ ... (1)

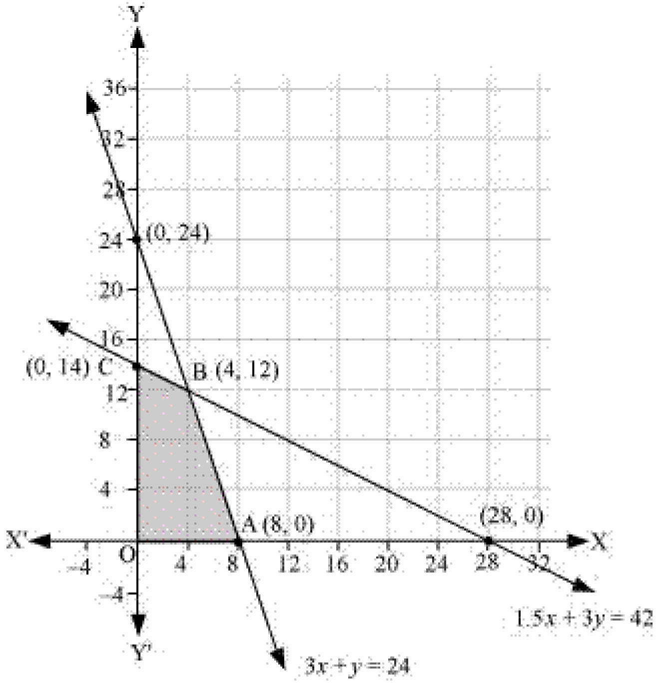
subject to the constraints,

$1.5x + 3y \leq 42$ ... (2)

$3x + y \leq 24$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (8, 0), B (4, 12), C (0, 14), and O (0, 0).

The values of Z at these corner points are as follows.

Corner point	**$Z = 20x + 10y$**	
A(8, 0)	160	
B(4, 12)	200	→ Maximum
C(0, 14)	140	
O(0, 0)	0	

Thus, the maximum profit of the factory when it works to its full capacity is Rs 200.

Question 7

A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods *A* and *B*. To produce one unit of *A*, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of *B*. If *A* and *B* are priced at Rs. 100 and Rs. 120 per unit respectively, how should he use his resources to maximize the total revenue? Form the above as an LPP and solve graphically. Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate?

Solution 7

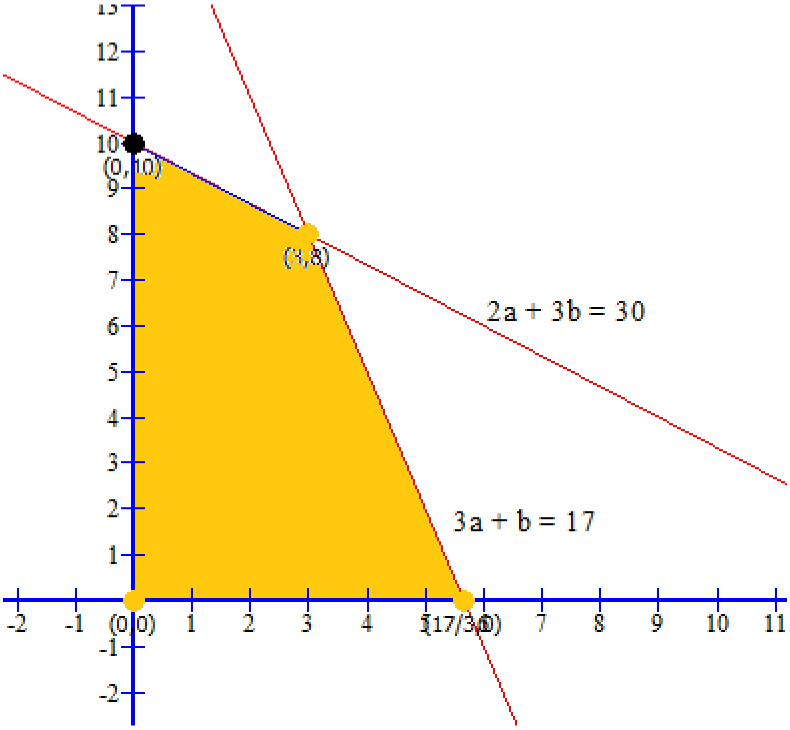
Let 'a' be the number of units of A and 'b' be the number of units of B.

	A	B	Total number of workers
Workers	2	3	30
Capital	3	1	17
Cost	100	120	

$\therefore\ 2a + 3b \le 30$

Also, $3a + b \le 17$

Maximum Profit $= 100a + 120b$



Corner points formed for:

Corner Points	Profit	Remarks
(0, 0)	Z = 100 × 0 + 120 × 0 = 0	
(0, 10)	Z = 100 × 0 + 120 × 10 = 1200	
(3, 8)	Z = 100 × 3 + 120 × 8 = 1260	Maximum
(17/3, 0)	Z = 100 × 17/3 + 120 × 0 = 1700/3	

Revenue is maximum when x = 3, y = 8.
Maximum Profit = Rs. 1260

Question 8

A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit?

Solution 8

Let x and y be the number of dolls of type A and B respectively that are produced per week.

The given problem can be formulated as follows.

Maximize $z = 12x + 16y$ ... (1)

subject to the constraints,

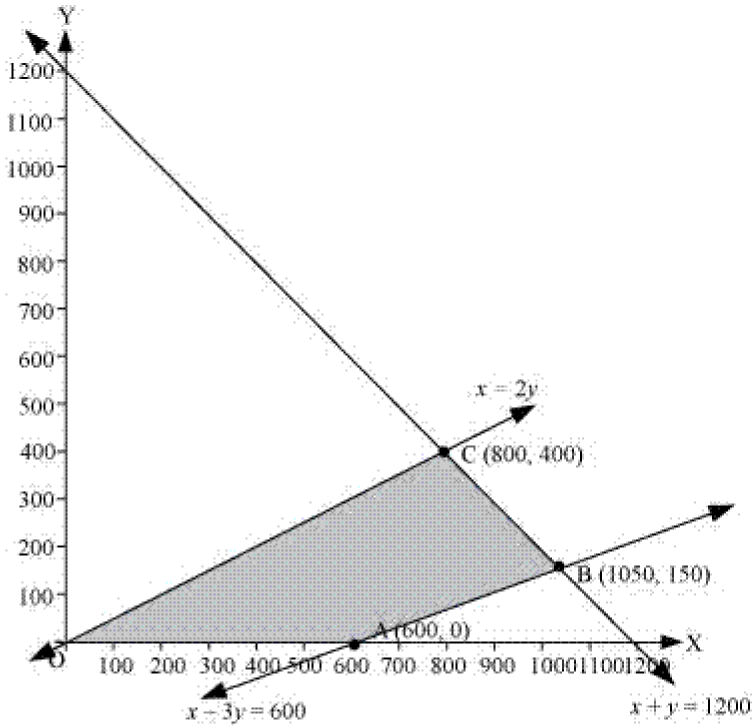
$$x + y \leq 1200 \qquad ...(2)$$

$$y \leq \frac{x}{2} \Rightarrow x \geq 2y \qquad ...(3)$$

$$x - 3y \leq 600 \qquad ...(4)$$

$$x,\ y \geq 0 \qquad ...(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (600, 0), B (1050, 150), and C (800, 400).

The values of z at these corner points are as follows.

Corner point	$z = 12x + 16y$	
A (600, 0)	7200	
B (1050, 150)	15000	
C (800, 400)	16000	→ Maximum

The maximum value of z is 16000 at (800, 400).

Thus, 800 and 400 dolls of type A and type B should be produced respectively to get the maximum profit of Rs 16000.

Question 9

There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 cost Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Solution 9

Let the farmer buy x kg of fertilizer F_1 and y kg of fertilizer F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Nitrogen (%)	**Phosphoric Acid (%)**	**Cost (Rs/kg)**
F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

$\Rightarrow$ 10% of x + 5% of $y \geq 14$

$$\frac{x}{10} + \frac{y}{20} \geq 14$$
$$2x + y \geq 280$$

F_1 consists of 6% phosphoric acid and F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

$\Rightarrow$ 6% of x + 10% of $y \geq 14$

$$\frac{6x}{100} + \frac{10y}{100} \geq 14$$
$$3x + 56y \geq 700$$

Total cost of fertilizers, $Z = 6x + 5y$

The mathematical formulation of the given problem is

Minimize $Z = 6x + 5y$ ... (1)

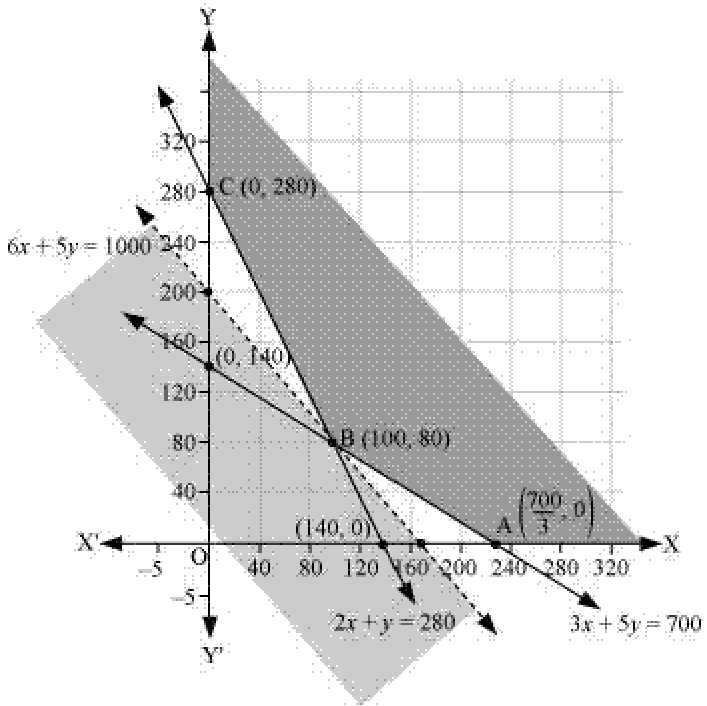
subject to the constraints,

$2x + y \geq 280$ ... (2)

$3x + 5y \geq 700$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3},0\right), B(100,80)$, and $C(0,280)$.

The values of Z at these points are as follows.

Corner point	$Z = 6x + 5y$	
$A\left(\frac{700}{3},0\right)$	1400	
B(100, 80)	1000	→ Minimum
C(0, 280)	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with

$6x + 5y < 1000$

Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost. The minimum cost is Rs 1000.

Question 10

A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium. 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

Solution 10

Let x and y be the number of packets of food P and Q respectively. Obviously $x \geq 0$, $y \geq 0$. Mathematical formulation of the given problem is as follows:

Subject to the constraints

$12x + 3y \geq 240$ (constraint on calcium) i.e. $4x + y \geq 80$ ...(1)

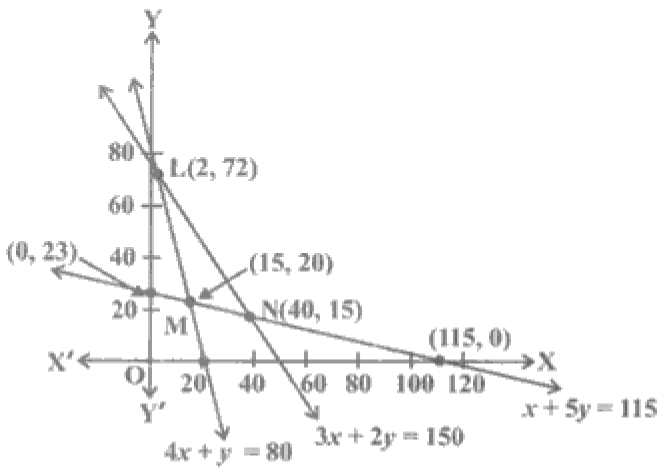
$4x + 20y \geq 460$ (constraint on iron) i.e. $x + 5y \geq 115$ ...(2)

$6x + 4y \leq 300$ (constraint on cholesterol), i.e. $3x + 2y \leq 150$ ...(3)

$X \geq 0$, $y \geq 0$ ...(4)

Let us graph the inequalities (1) to (4).

The feasible region (shaded) determined by the constraints (1) to (4) is shown in fig., and note that it is bounded.



The coordinates of the corner points L,M and N are (2,72), (15,20) and (40,15) respectively. Let us evaluate Z at these points:

Corner Point	$Z = 6x + 3y$	
(2,72)	228	
(15,20)	**150** ⟵	Maximum
(40,15)	285	

From the table, we find that Z is minimum at the point (15,20). Hence, the amount of vitamin A under the constraints given in the problem will be minimum, if 15 packets of food P and 20 packets of food Q are used in the special diet. The minimum amount of vitamin A will be 150 units.

Question 11

A manufacturer has three machines 'I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 units of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? Why will be the maximum profit?

Solution 11

Let x and y be the number of items M and N respectively.

Total profit on the production = Rs (600x + 400y)

Mathematical formulation of the given problem is as follows:

Maximise Z = 600x + 400y

Subject to the constraints:

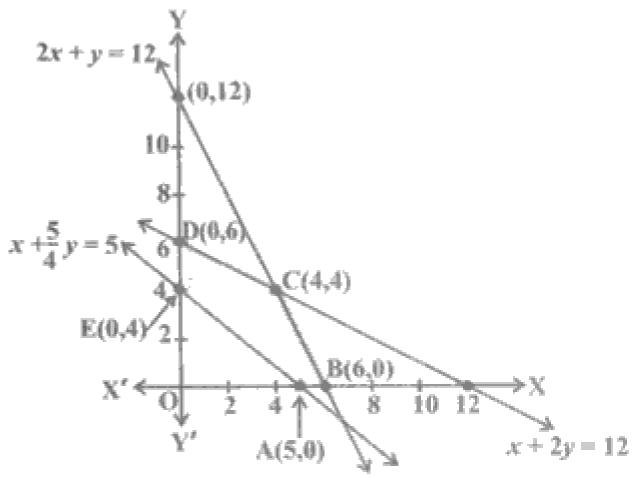
$X + 2y \leq 12$ (constraint of Machine I) ...(1)

$2x + y \leq 12$ (constraint on Machine II) ...(2)

$X + \frac{5}{4} y \geq 5$ (constraint on Machine III) ...(3)

$X \geq 0, y \geq 0$ ...(4)

Let us draw the graph of constraints (1) to (4). ABCDE is the feasible region (shaded) as shown in fig., determined by the constraints (1) to (4). Observe that the feasible region is bounded. Coordinates of the corner points A.B.C.D and E are (5,0) (6,0), (4,4), (0,6) and (0,4) respectively.



Let us evaluate Z = 600x + 400y at these corner points.

Corner Point	Z = 600x + 400y
(5,0)	3000
(6,0)	3600
(4,4)	**4000** ⟵ Maximum
(0,6)	2400
(0,4)	1600

We see that the point (4,4) is giving the maximum value of Z. Hence, the manufacturer has to produce 4 units of each item to get the maximum profit of Rs. 4000.

Question 12

There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A,B and C. The weekly requirements of the depots are respectively 5,5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

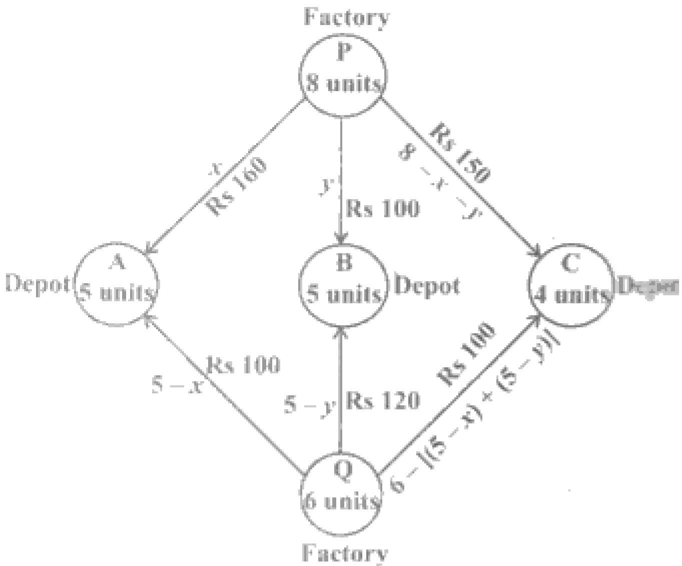
From/To	Cost (in Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost?

Solution 12

The problem can be explained diagrammatically as follows fig.,.

Let x units and y units of the commodity be transported from the factory at P to the depots at A and B respectively. Then (8 − x − y) units will be transported to depot at C (Why?)



Hence, we have $x \geq 0$, $y \geq 0$ and $8 - x - y \geq 0$

i.e., $x \geq 0$, $y \geq 0$ a and $x + y \leq 8$

Now, the weekly requirement of the depot at A is 5 units of the commodity. Since x units are transported from the factory at P, the remaining (5 − x) units need to be transported from the factory at Q. Obviously. $5 - x \geq 0$, i.e. $x \leq 5$.

Similarly, (5 − y) and 6 − (5 − x + 5 − y) = x + y − 4 units are to be transported from the factory at Q to the depots at B and C respectively.

Thus, $5 - y \geq 0$, $x + y - 4 \geq 0$,

i.e. $y \leq 5$, $x + y \geq 4$

Total transportation cost Z is given by

$Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100(x + y - 4) + 150(8 - x - y)$

$= 10(x - 7y + 190)$

Therefore, the problem reduces to

Minimise $Z = 10(x - 7y + 190)$

Subject to the constraints:

$$\begin{aligned} x \geq 0,\ y \geq 0 &\quad \ldots(1)\\ x + y \leq 8 &\quad \ldots(2)\\ x \leq 5 &\quad \ldots(3)\\ y \leq 5 &\quad \ldots(4)\\ x + y \geq 4 &\quad \ldots(5) \end{aligned}$$

Question 13

A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional elements A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Solution 13

Let the farmer mix x bags of brand P and y bags of brand Q.

The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	**Vitamin B (units/kg)**	**Cost (Rs/kg)**
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The given problem can be formulated as follows.

Minimize $z = 250x + 200y$ ... (1)

subject to the constraints,

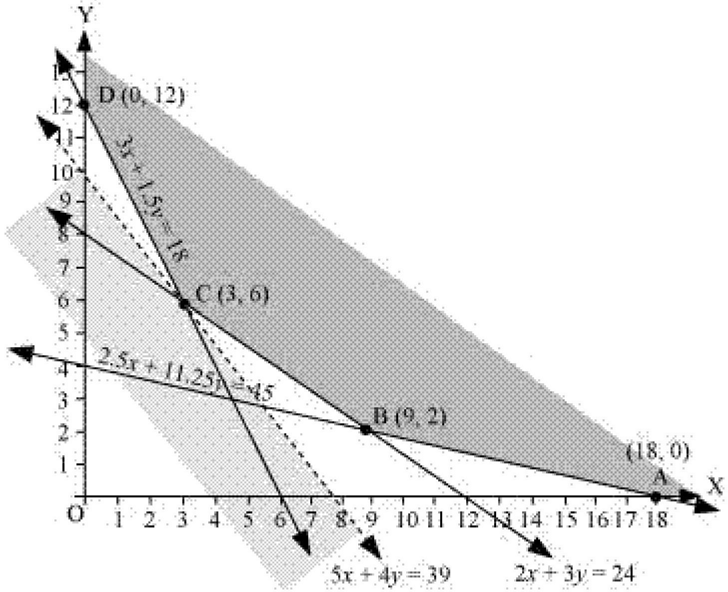
$3x + 1.5y \geq 18$ ...(2)

$2.5x + 11.25y \geq 45$ ...(3)

$2x + 3y \geq 24$ ...(4)

$x,\ y \geq 0$ ...(5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6), and D (0, 12).

The values of z at these corner points are as follows.

Corner point	z = 250*x* + 200*y*	
A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	→ Minimum
D (0, 12)	2400	

As the feasible region is unbounded, therefore, 1950 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 4y < 39$

Therefore, the minimum value of z is 2000 at (3, 6).

Thus, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimize the cost to Rs 1950.

Question 14

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg food is given below:

Food	**Vitamin A**	**Vitamin B**	**Vitamin C**
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

Solution 14

Let the mixture contain x kg of food X and y kg of food Y.

The mathematical formulation of the given problem is as follows.

Minimize $z = 16x + 20y$ ... (1)

subject to the constraints,

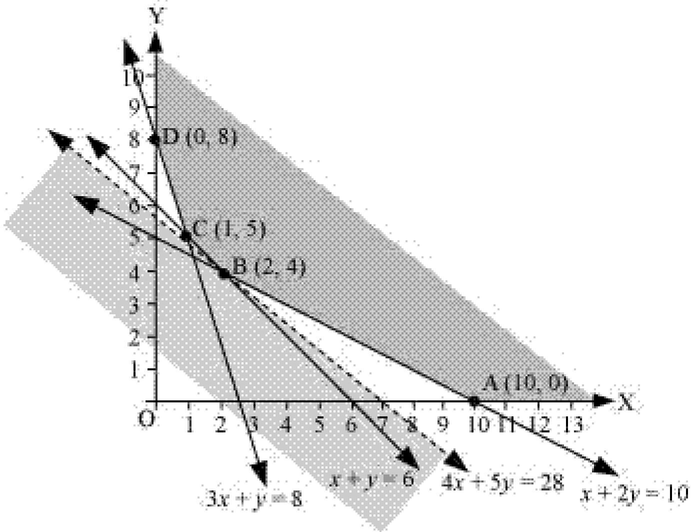
$$x + 2y \geq 10 \quad \ldots(2)$$

$$x + y \geq 6 \quad \ldots(3)$$

$$3x + y \geq 8 \quad \ldots(4)$$

$$x,\ y \geq 0 \quad \ldots(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5), and D (0, 8).

The values of z at these corner points are as follows.

Corner point	**$z = 16x + 20y$**	
A (10, 0)	160	
B (2, 4)	112	→ Minimum
C (1, 5)	116	
D (0, 8)	160	

As the feasible region is unbounded, therefore, 112 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, $16x + 20y < 112$ or $4x + 5y < 28$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $4x + 5y < 28$

Therefore, the minimum value of z is 112 at (2, 4).

Thus, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is Rs 112.

Question 15

A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Type of toys	**Machines**		
	I	**II**	**III**
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Solution 15

Let x and y toys of type A and type B respectively be manufactured in a day.

The given problem can be formulated as follows.

Maximize $z = 7.5x + 5y$ ... (1)

subject to the constraints,

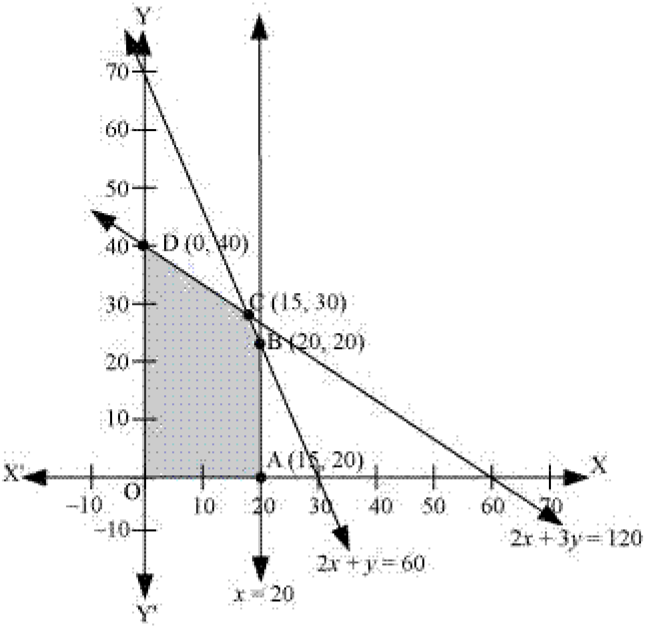
$$2x + y \le 60 \quad ...(2)$$

$$x \le 20 \quad ...(3)$$

$$2x + 3y \le 120 \quad ...(4)$$

$$x, y \ge 0 \quad ...(5)$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 0), B (20, 20), C (15, 30), and D (0, 40).

The values of z at these corner points are as follows.

Corner point	**Z = 7.5x + 5y**	
A(20, 0)	150	
B(20, 20)	250	
C(15, 30)	262.5	→ Maximum
O(0, 40)	200	

The maximum value of z is 262.5 at (15, 30).

Thus, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximize the profit.

Question 16

An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Solution 16

Let the airline sell x tickets of executive class and y tickets of economy class.

The mathematical formulation of the given problem is as follows.

Maximize $z = 1000x + 600y$ ... (1)

subject to the constraints,

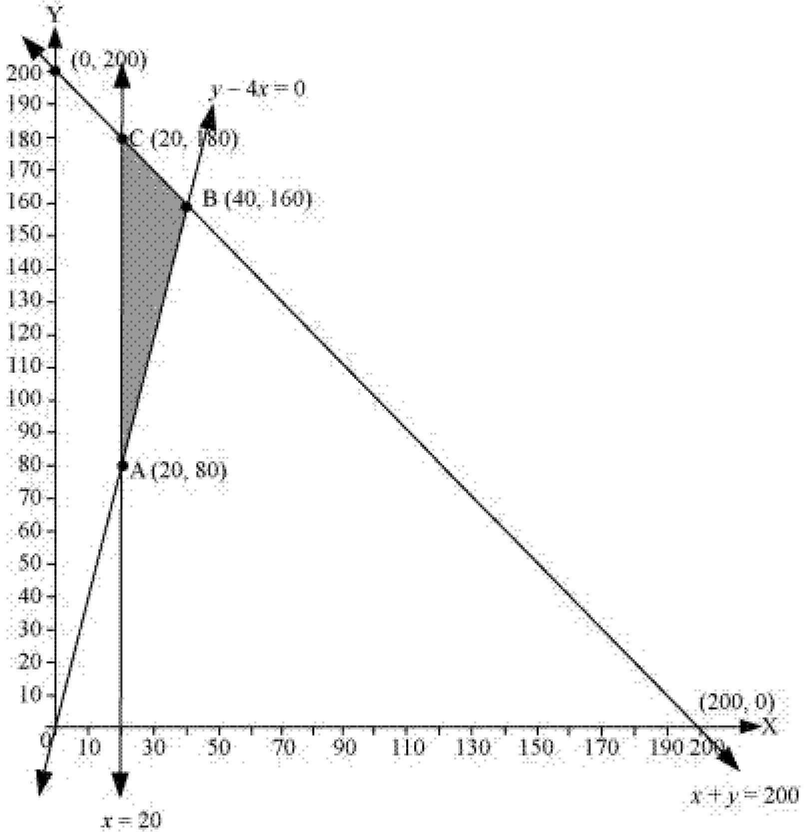
$$x + y \le 200 \quad ...(2)$$

$$x \ge 20 \quad ...(3)$$

$$y - 4x \ge 0 \quad ...(4)$$

$$x, y \ge 0 \quad ...(5)$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of z at these corner points are as follows.

Corner point	$z = 1000x + 600y$	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of z is 136000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

Question 17

A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Solution 17

Let the fruit grower use *x* bags of brand P and *y* bags of brand Q.

The problem can be formulated as follows.

Minimize $z = 3x + 3.5y$ ... (1)

subject to the constraints,

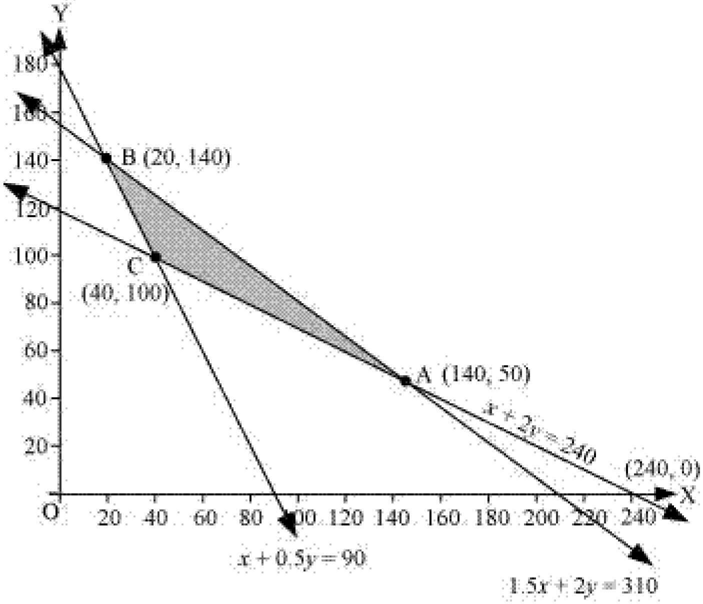
$$x + 2y \geq 240 \qquad ...(2)$$

$$x + 0.5y \geq 90 \qquad ...(3)$$

$$1.5x + 2y \leq 310 \qquad ...(4)$$

$$x, y \geq 0 \qquad ...(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (240, 0), B (140, 50), and C (20, 140).

The values of *z* at these corner points are as follows.

Corner point	$z = 3x + 3.5y$	
A (140, 50)	595	
B (20, 140)	550	
C (40, 100)	470	→ Minimum

The maximum value of *z* is 470 at (40, 100).

Thus, 40 bags of brand P and 100 bags of brand Q should be added to the garden to minimize the amount of nitrogen.

The minimum amount of nitrogen added to the garden is 470 kg.

Question 18

A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5,760 to invest and has space for at most 20 items for storage. An electronic sewing machine cost him Rs. 360 and a manually operated sewing machine Rs. 240. He can sell an electronic sewing machine at profit of Rs. 22 and a manually operated sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a LPP and solve it graphically.

Solution 18

Let 'x' be the number of electronic sewing machines
and 'y' be the number of manual machines.

	Electronic sewing machines	Manually operated sewing machines	Total
Number of machines	x	y	20
Cost of the machine	360	240	5760
Profit per machine	22	18	

Let $Z=22x+18y$

We need to maximize the function Z

Constraints are:

$x+y \leq 20$

$360x+240y \leq 5760$

Thus, the given linear programming problem is:

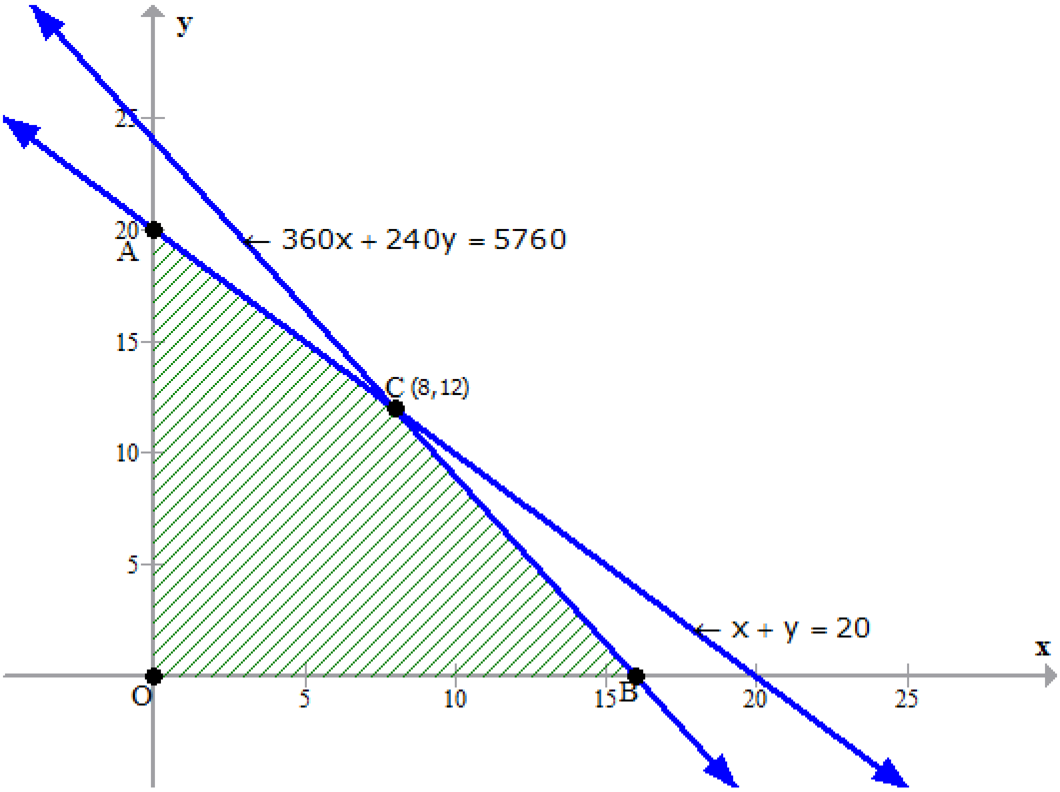
Max. Profit$=22x+18y$

such that,

$x+y \leq 20$

$360x+240y \leq 5760$

Let us plot the constraints and find the feasible region through graph.



Let us list all the corner points in the table

Corner points	Profit $Z=22x+18y$	Result
$O(0,0)$	$Z=22\times0+18\times0$	0
$A(0,20)$	$Z=22\times0+18\times20$	360
$B(16,0)$	$Z=22\times16+18\times0$	352
$C(8,12)$	$Z=22\times8+18\times12$	392

Thus the profit is maximum when the dealer buys 8 electronic sewing machines and 12 manual operated sewing machines.

Question 19

A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of Rs. 80 on each piece of type A and Rs. 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

Solution 19

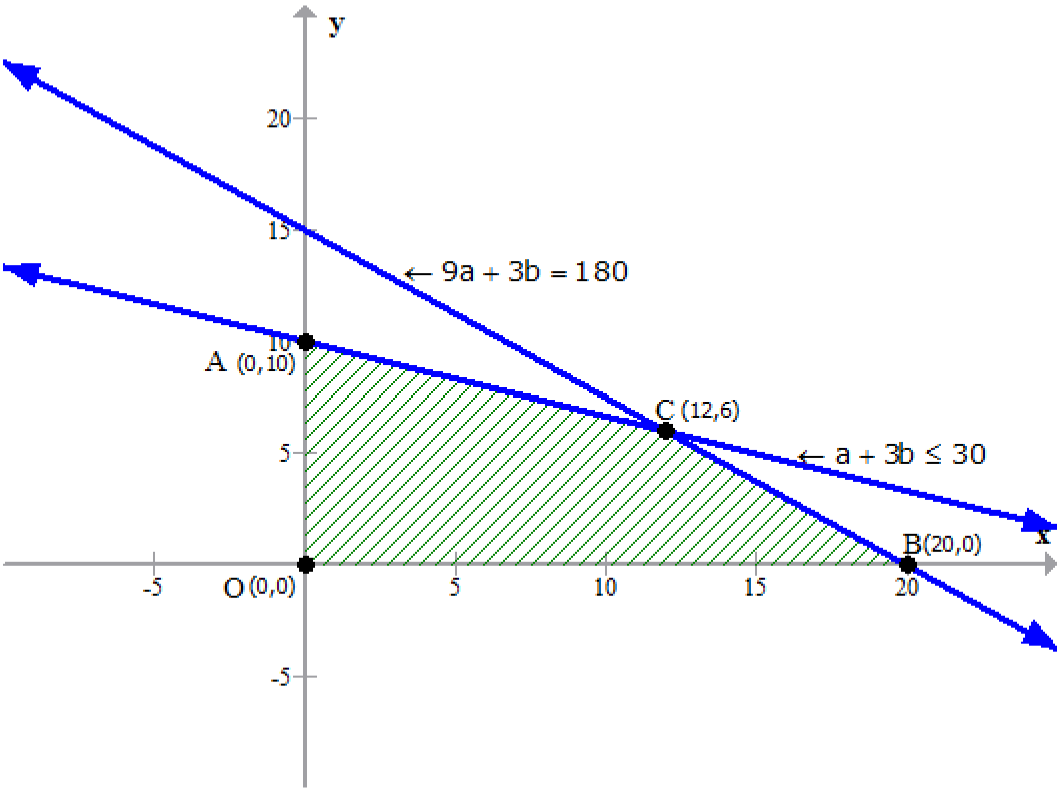
Let 'a' be the number of units of A and 'b' be the number of units of B.

	A	B	Total number of hours
Hours for Fabricating	9	12	180
Hours for Finishing	1	3	30
Profit	80	120	

$\therefore\ 9a + 12b \leq 180$

Also, $a + 3b \leq 30$

Maximum Profit $= 80a + 120b$



Let us list all the corner points in the table

Corner points	Profit $Z=80a+120b$	Result
O(0,0)	$Z=80\times0+120\times0$	0
A(0,10)	$Z=80\times0+120\times10$	1200
B(16,0)	$Z=80\times16+120\times0$	1280
C(12,6)	$Z=80\times12+120\times6$	1680

Thus profit is maximum and is equal to Rs.1680

The company should manufacture 12 Type A machines and 6 Type B machines to maximize their profit.

Chapter 30 - Linear programming Exercise MCQ

Question 1

The solution set of the inequation $2x + y > 5$ is

a. half plane that contains the origin
b. open half plane not contains the origin
c. whole xy-plane except the points lying on the line
2x + y =5
d. none of these

Solution 1

Correct option: (b)

Given inequation is $2x + y > 5$.

Consider, $2x + y = 5$

$$\Rightarrow \frac{2x}{5} + \frac{y}{5} = 1$$

$$\Rightarrow \frac{x}{\frac{5}{2}} + \frac{y}{5} = 1$$

Here, x – intercept and y – intercept are 2.5 and 5.

As $2x + y > 5$ hence open half plane not containing origin.

Question 2

Objective function of a LPP is

a. a constraint
b. a function to be optimized
c. a relation between the variable
d. none of these

Solution 2

Correct option: (b)

Objective function of a LPP is always maximized or minimized. Hence, it is optimized.

Question 3

Which of these following sets are convex?

a. $\{(x, y): x^2 + y^2 \geq 1\}$
b. $\{(x, y): y^2 \geq x\}$
c. $\{(x, y): 3x^2 + 4y^2 \geq 5\}$
d. $\{(x, y): y \geq 2, y \leq 4\}$

Solution 3

Correct option: (d)

Set of points between two parallel lines. Hence, set is connected. Set is convex.

Question 4

Let X_1 and X_2 are optimal solutions of a LPP, then

a. $X = \lambda X_1 + (1- \lambda) X_2$, $\lambda \in R$ is also an optimal solution
b. $X = \lambda X_1 + (1- \lambda) X_2$, $0 \leq \lambda \leq 1$ gives an optimal solution
c. $X = \lambda X_1 + (1+ \lambda) X_2$, $0 \leq \lambda \leq 1$ gives an optimal solution
d. $X = \lambda X_1 + (1+ \lambda) X_2$, $\lambda \in R$ given an optional solution

Solution 4

Correct option: (b)

If $x_1, x_2 \in A$ and $\lambda \in [0,1] \Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A$

then set is convex.

Also if x_1, x_2 are optimal solutions then convex combination will have an optimal solution.

Question 5

The maximum value of Z = 4x + 2y subjected to the constraints 2x + 3y ≤ 18, x + y ≥ 10; x, y ≥ 0 is

a. 36
b. 40
c. 20
d. None of these

Solution 5

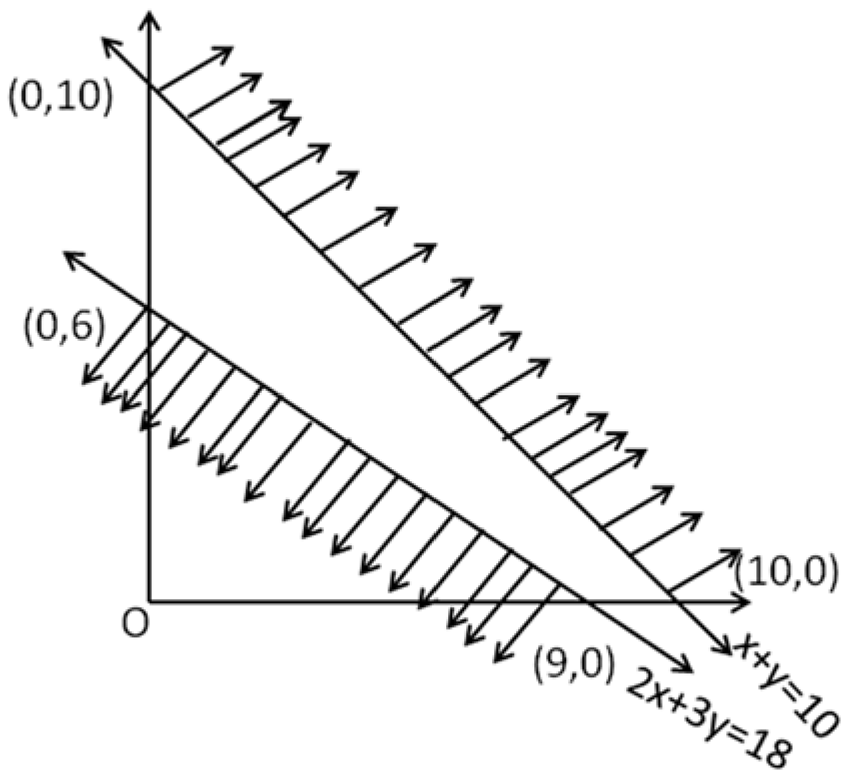
Correct option: (d)

Consider, $2x + 3y = 18$

x	y	(x, y)
0	6	(0, 6)
9	0	(9, 0)

Consider, $x + y = 10$

x	y	(x, y)
0	10	(0, 10)
10	0	(10, 0)



From the graph we conclude that no feasible region exist.

Question 6

The maximum value of the objective function is attained at the points

a. given by intersection of inequations with the axes only
b. given by intersection of inequations with x-axis only
c. given by corner points of the feasible region
d. none of these

Solution 6

Correct option: (c)

The maximum value of the objective function is attained at the points given by corner points of the feasible region.

Question 7

The maximum value of $Z = 4x + 3y$ subjected to the constraints $3x + 2y \geq 160$, $5x + 2y \geq 200$, $x + 2y \geq 80$; $x, y \geq 0$ is

a. 320
b. 300
c. 230
d. none of these

Solution 7

Correct option: (d)

If we put x=0 and y=0 in all the equations then we get contradiction. Hence, region is on open half plane not containing origin. The region is unbounded we can not find the maximum value of the feasible region.

Question 8

Consider a LPP given by

Minimum $Z = 6x + 10y$

Subjected to $x \geq 6$; $y \geq 2$; $2x + y \geq 10$; $x, y \geq 0$

Redundant constraints in this LPP are

a. $x \geq 0, y \geq 0$
b. $x \geq 6, 2x + y \geq 10$
c. $2x + y \geq 10$
d. none of these

Solution 8

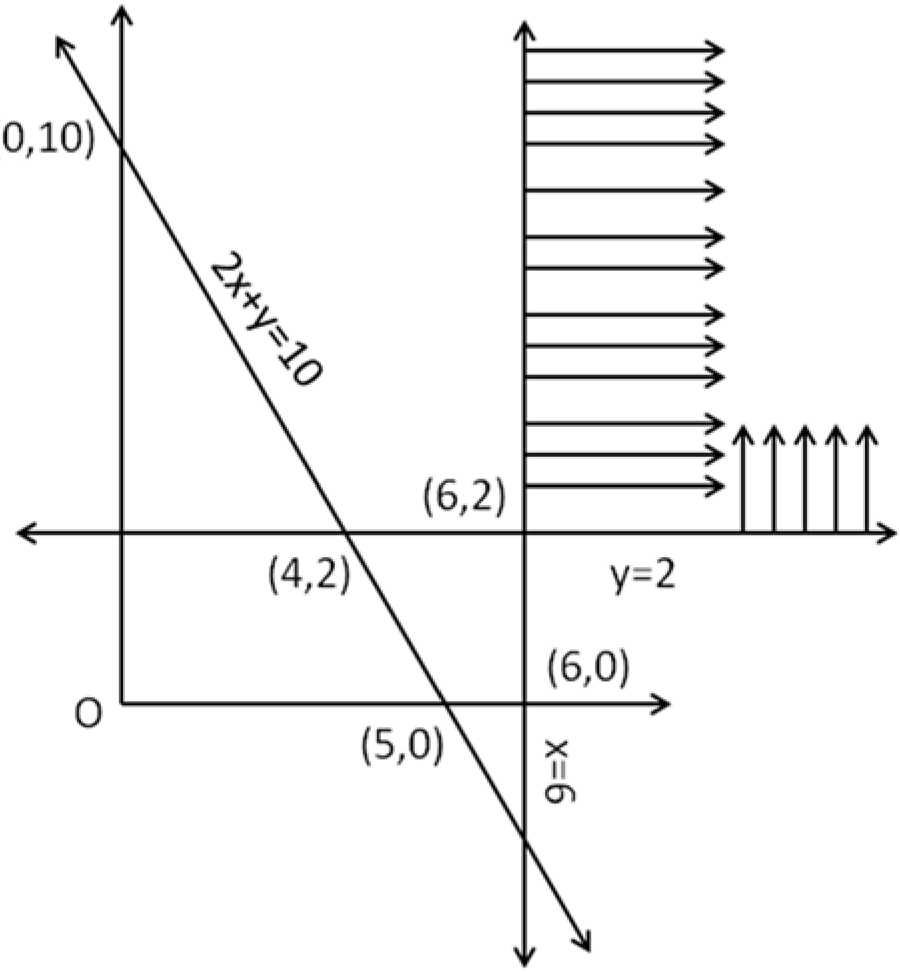
Correct option: (c)

Consider, $x = 6$

and $y = 2$

Now $2x + y = 10$

x	y	(x, y)
0	10	(0, 10)
5	0	(5, 0)



Minimum Z will be at $2x + y \geq 10$.

Question 9

The objective function $Z = 4x + 3y$ can be maximized subjected to the constraints $3x + 4y \leq 24$, $8x + 6y \leq 48$, $x \leq 5$, $y \leq 6$; $x, y \geq 0$

a. At only one point
b. At two points only
c. At an infinite number of points
d. None of these

Solution 9

Correct option: (c)

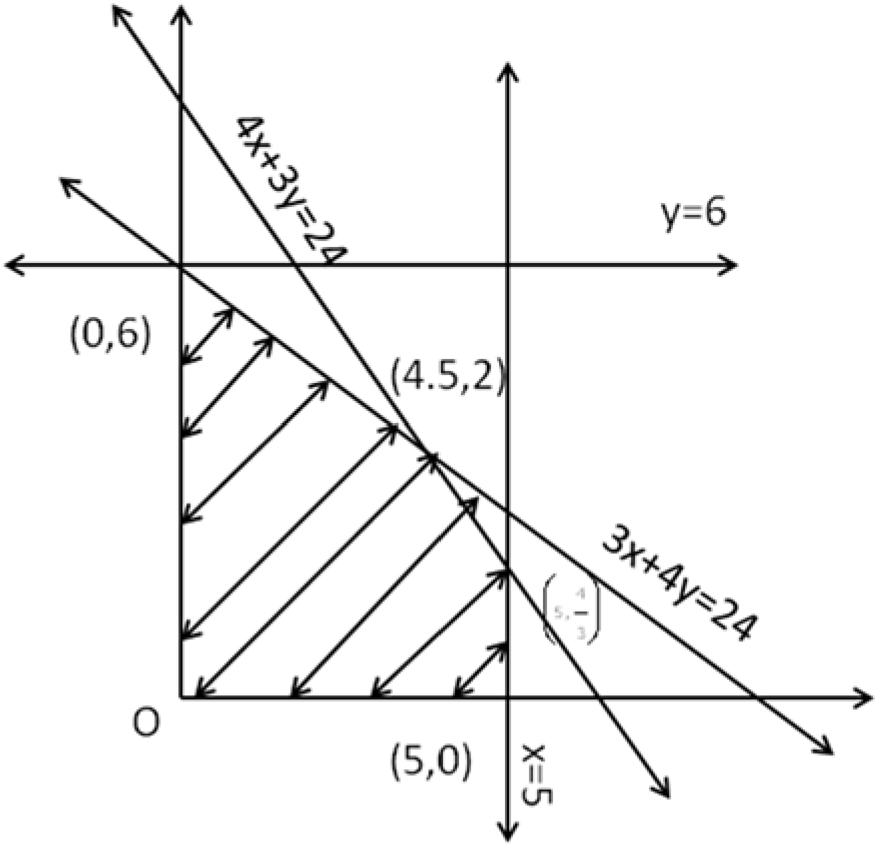
Consider, $3x + 4y = 24$

x	y	(x, y)
0	6	(0, 6)
8	0	(8, 0)

Also $8x + 6y = 48 \Rightarrow 4x + 3y = 24$

x	y	(x, y)
0	8	(0, 8)
6	0	(6, 0)

and $x = 5,\ y = 6$



$Z = 4x + 3y$

At $(0, 6) \Rightarrow Z = 18$

At $(5, 0) \Rightarrow Z = 20$

At $(4.5, 2) \Rightarrow Z = 24$

At $\left(5, \frac{4}{3}\right) \Rightarrow Z = 24$

Hence, maximum Z is at two points.

$\Rightarrow$ Maximum Z is at an infinite number of points.

Question 10

If the constraints in a linear programming problem are changed

a. the problem is to be re-evaluated
b. solution is not defined
c. the objective function has to be modified
d. the change in constraints is ignored

Solution 10

Correct option: (a)

Optimization of objective function is depend on constraints. Hence, if the constraints in a linear programming problem are changed the problem is to be re-evaluated.

Question 11

Which of the following statement is correct?

a. Every LPP admits an optimal solution
b. A LPP admits unique optimal solution
c. If a LPP admits two optimal solution it has an infinite number of optimal solution
d. The set of all feasible solutions of a LPP is not a converse set

Solution 11

Correct option: (c)

Optimal solution of LPP has three types.

1. Unique
2. Infinite
3. Does not exist.

Hence, it has infinite solution if it admits two optimal solution.

Question 12

Which of the following is not a convex set?

a. $\{(x, y): 2x + 5y < 7\}$
b. $\{(x, y): x^2 + y^2 \leq 4\}$
c. $\{x: |x| = 5\}$
d. $\{(x, y): 3x^2 + 2y^2 \leq 6\}$

Solution 12

Correct option: (c)

As $|x|=5$ will only on x-axis. Hence, set is not connected to any two points between the set.

Hence, it is not convex.

Question 13

By graphical method, the solution of linear programming problem

Maximize $Z = 3x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1 \geq 0$, $x_2 \geq 0$, is

a. $x_1 = 2$, $x_2 = 0$, $Z = 6$
b. $x_1 = 4$, $x_2 = 6$, $Z = 36$
c. $x_1 = 4$, $x_2 = 3$, $Z = 27$
d. $x_1 = 4$, $x_2 = 6$, $Z = 42$

Solution 13

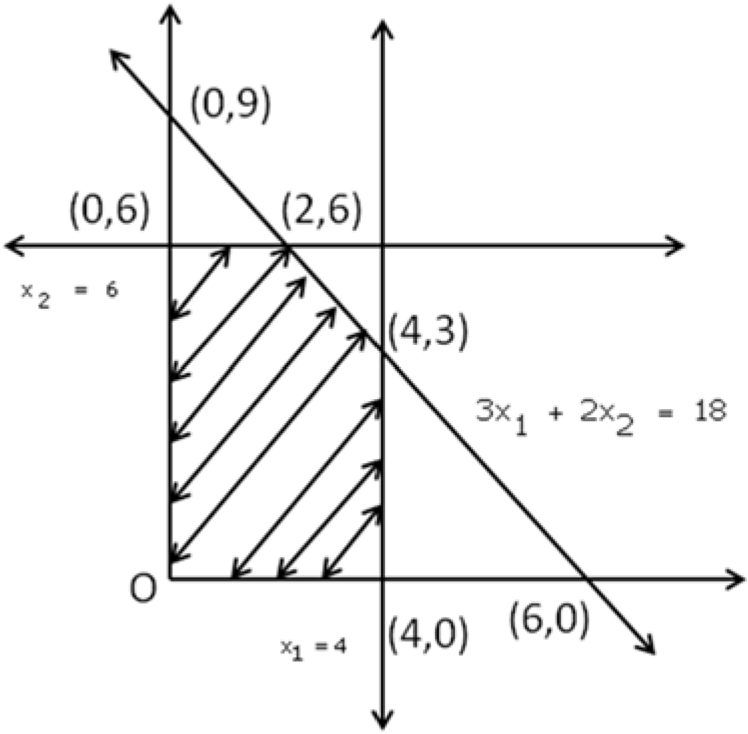
Correct option: (b)

Consider, $x_2 = 6$

and $x_1 = 4$

Now $3x_1 + 2x_2 = 18$

x_1	x_2	(x_1, x_2)
0	9	(0,9)
6	0	(6,0)



$Z = 3x_1 + 5x_2$

At $(0,6) \Rightarrow Z = 30$

At $(4,0) \Rightarrow Z = 12$

At $(4,3) \Rightarrow Z = 27$

At $(2,6) \Rightarrow Z = 36$

MAximum $Z = 36$, $x_1 = 2$, $x_2 = 6$

Question 14

The region represented by the inequation system x, y ≥ 0, y ≤ 6, x + y ≤ 3 is

a. unbounded in first quadrant
b. unbounded in first and second quadrants
c. bounded in first quadrant
d. none of these

Solution 14

Correct option: (c)

As region is on origin side it is always bounded. Also, given that x,y ≥ 0 it is bounded in the first quadrant.

NOTE: Answer not matching with back answer.

Question 15

The point at which the maximum value of x + y, subject to the constraints $x + 2y \leq 70$, $2x + y \leq 95$, x, $y \geq 0$ is obtained is

a. (30, 25)
b. (20, 35)
c. (35, 20)
d. (40, 15)

Solution 15

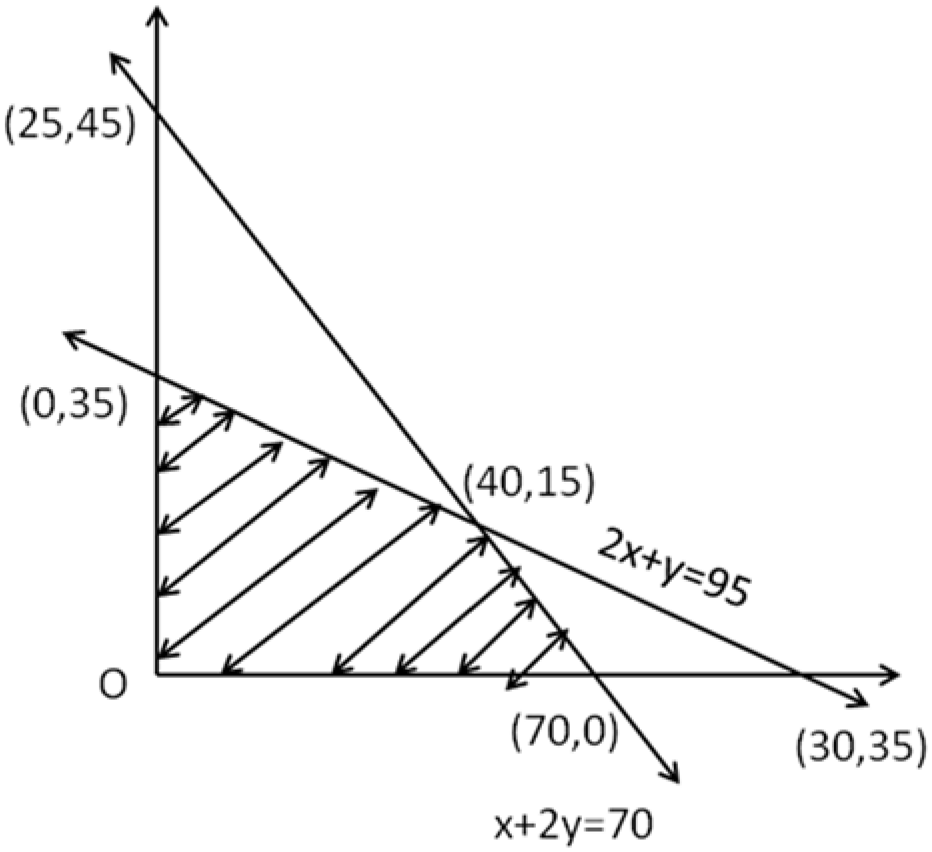
Correct option: (d)

Consider, $x + 2y = 70$

x	y	(x, y)
0	35	(0, 35)
70	0	(70, 0)

Consider, $2x + y = 95$

x	y	(x, y)
25	45	(25, 45)
30	35	(30, 35)



$Z = x + y$

At $(0, 35) \Rightarrow Z = 35$

At $(70, 0) \Rightarrow Z = 70$

At $(40, 15) \Rightarrow Z = 55$

Maximum value is at $(70, 0)$.

But there is no option hence, maximum value is at $(40, 15)$.

Question 16

The value of objective function is maximum under linear constraints

a. at the centre of feasible region
b. at (0, 0)
c. at any vertex of feasible region
d. the vertex which is maximum distance from (0, 0)

Solution 16

Correct option: (c)

To find maximum or minimum value of the region we use the coordinates of the vertices of feasible region. Hence, the value of objective function is maximum under linear constraints at any vertex of the feasible region.

Note: Answer not matching with back answer.

Question 17

The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0,5). Let $Z = p x + q y$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is

a. $p = q$
b. $p = 2q$
c. $p = 3q$
d. $q = 3p$

Solution 17

Correct option: (d)

Given that $Z=px + qy$

Maximum value at (3, 4) = maximum value at (0, 5)

$3p+4q=5q$

$q=3p$