

Dynamic Arrays

Issue of static arrays? Fixed size

Dynamic Arrays: We need not worry about size

T.C of accessing i^{th} element : $O(1)$

Java :

```
ArrayList<Integer> arr = new ArrayList<>();  
arr.add(50);  
arr.add(100);  
arr.add(1000);
```

```
for (i=0; i<arr.size(); i++) {  
    print( arr.get(i));  
}
```

y

C++

```
vector<int> a; //vector is created
```

```
a.push_back(60);
```

```
//a = {10, 20, 30, 40, 50, 60} after insertion at end
```

```
a.clear(); // a={}
```

```
for(int i=0; i<a.size(); i++) {
```

```
    cout<<a[i];
```

```
} //iterating the vector
```

Python

```
thislist = [] //created list
```

```
thislist.append("orange") //added orange at end
```

```
thislist.clear() //cleared the list
```

```
for i in range(len(thislist)):
```

```
    print(thislist[i]) //iterating on the list
```

Stock Portfolio Performance Tracking

Problem Statement :

Given an array representing the daily profit or loss from a particular stock over a period of days, write a function that calculates the total profit or loss over a given range of days. The function should efficiently handle multiple queries for different ranges without recalculating the sum for each query.

Example:

Stock_Prices[] = [-5, 10, 20, 40, 50, -10, 80, -90, -20, -10]
0 1 2 3 4 5 6 7 8 9

Start Day

1

5

End

3

8

Net Profit / Loss

$10 + 20 + 40 = 70$

$-10 + 80 - 90 - 20 = -40$

Question: Range sum queries

Given an array of size N , and Q queries of the format s (start index) and e (end index), return the sum of elements from s to e

$$N \leq 10^6$$

$$Q \leq 10^6$$

$A =$

-3	6	2	4	5	2	8	-9	3	1
0	1	2	3	4	5	6	7	8	9

Q queries

<u>s</u>	<u>e</u>	<u>Sum</u>
1	3	$6 + 2 + 4 = 12$
2	7	$2 + 4 + 5 + 2 + 8 - 9 = 12$
4	8	$5 + 2 + 8 - 9 + 3 = 9$
0	2	$-3 + 6 + 2 = 5$
7	7	-9

queries :

[[1,3],
[2,7]
[4,8]
[0,2]
]

Brute Force :

For each query, iterate from s to e and find the sum

Pseudocode

```
Function querySum(Queries[[]], Array[], querySize, size){  
    for(i -> 0 to Queries.length-1){  
        L = Queries[i][0]  
        R = Queries[i][1]  
  
        sum = 0  
        for( j -> L to R){  
            sum += Array[j]  
        }  
        print(sum)  
    }  
}
```

$$T.C: O(P \cdot N)$$

$$S.C: O(1)$$

$$N = 10^6$$

$$P = 10^6$$

$$10^6 \times 10^6 = 10^{12} \Rightarrow TLE$$

Efficient Approach

250-3, 45 overs

Ex.

Scores:	2	8	14	29	31	49	65	79	88	97
	1	2	3	4	5	6	7	8	9	10

$$\text{runs in } 7^{\text{th}} \text{ over} = \text{scores}[7] - \text{scores}[6]$$

$$= 65 - 49 = 16$$

Quiz: Runs from 6^{th} to 10^{th} over

$$\text{runs} = \text{scores}[10] - \text{scores}[5]$$

$$= 97 - 31 = 66$$

Quiz: 10^{th} over

$$\text{runs} = \text{scores}[10] - \text{scores}[9] = 97 - 88 = 9$$

$$S \dots \dots e = \text{scores}[e] - \text{scores}[S-1]$$

Ques: 3rd to 6th

$$\text{store}[6] - \text{store}[2] = 49 - 8 = 41$$

Given any over range, we are able to find the runs stored just by subtracting 2 values)

We are able to do this because, after every over we know the cumulative sum from 1st over until current over

Prefix Sum Array

$PS[i] =$ sum of all ely from index 0 i
 $= a[0] + a[1] + \dots + a[i-1] + a[i]$

A = 2 5 -1 7 1
 0 1 2 3 4

PS = 2 7 6 13
 0 1 2 3 4

$$PS[0] = A[0]$$

$$PS[1] = A[0] + A[1]$$

$$PS[2] = PS[1] + A[2] = 7 + (-1) = 6$$

$$PS[3] = PS[2] + A[3] = 6 + 7 = 13$$

$$PS[4] : PS[3] + A[4] = 13 + 1 = 14$$

$$PS[i] = PS[i-1] + A[i]$$

(0...i) (0...i-1)

// Construct

int PS[N];

// Declared an array of size N

PS[0] = A[0];

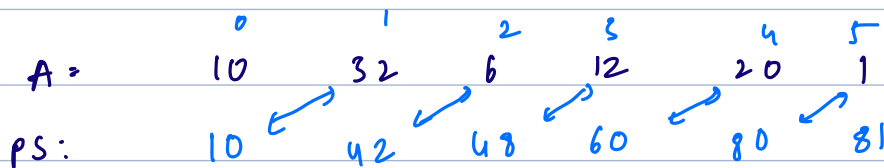
for(i=1; i<N; i++) {

 PS[i] = PS[i-1] + A[i];

}

T.C: $O(N)$

Ex:



A =

A: -3 6 2 4 5 2 8 -9 3 1
 0 1 2 3 4 5 6 7 8 9

PS: -3 3 5 9 14 16 24 15 18 19
 0 1 2 3 4 5 6 7 8 9

Q queries

s

e

sum

1

3

$$PS[3] - PS[0] = 9 - (-3) = 12$$

2

7

$$PS[7] - PS[1] = 15 - 3 = 12$$

4

8

$$PS[8] - PS[3] = 18 - 9 = 9$$

0

2

$$PS[2] = 5$$

7

7

$$PS[7] - PS[7-1] = 15 - 24 = -9$$

$$\text{sum}(s, \dots, e) = PS[e] - PS[s-1] \quad \checkmark \quad (\text{if } s=0)$$

```
Function querySum(Queries[][], Array[], querySize, size){
```

```
    //calculate pf array
```

```
    pf[N]
```

```
    pf[0] = A[0];
```

```
    for(i -> 1 to N-1){
```

```
        pf[i] = pf[i-1] + A[i];
```

```
    }
```

} O(N)

```
    //answer queries
```

```
    for( i -> 0 to Queries.length-1){
```

```
        L = Queries[i][0];
```

```
        R = Queries[i][1];
```

```
        if(L == 0) {
```

```
            sum = pf[R]
```

```
        }
```

```
        else {
```

```
            sum = pf[R] - pf[L - 1];
```

```
        }
```

```
        print(sum);
```

```
    }
```

```
}
```

T.C: (N + Q)

S.C: O(N)

↓
p.s array

Sum of a Range : Prefix Sum Technique
(Product)
(XOR)

8:32

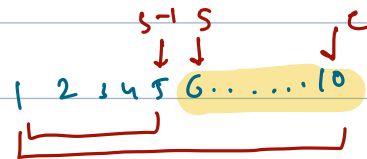
Question:

Given an array of size N and Q queries with start (s) and end (e) index. For every query, return the sum of all **even indexed elements** from s to e .

Example

$A[] = \{ 2, 3, 1, 6, 4, 5 \}$

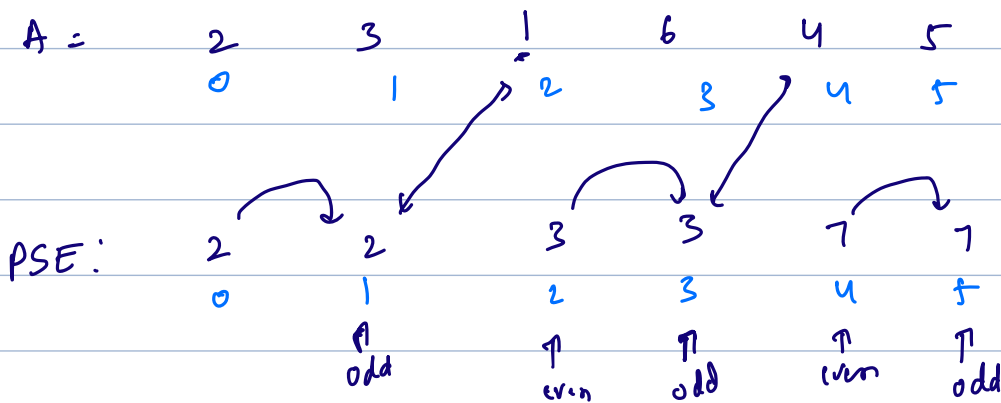
Query :
1 3 Ans
2 5 $A[2] = 1$
0 4 $A[2] + A[4] = 1 + 4 = 5$
3 3 $A[0] + A[2] + A[4] = 2 + 1 + 4 = 7$
3 3 0



Efficient Approach

Construct PS array for even indexed elements

$PSE[i] =$ Sum of even indexed elements from index 0 to index i



```
int PSE[N];
```

```
PSE[0] = A[0];
```

```
for (i = 1; i < N; i++) {
```

```
    if (i % 2 == 0) {
```

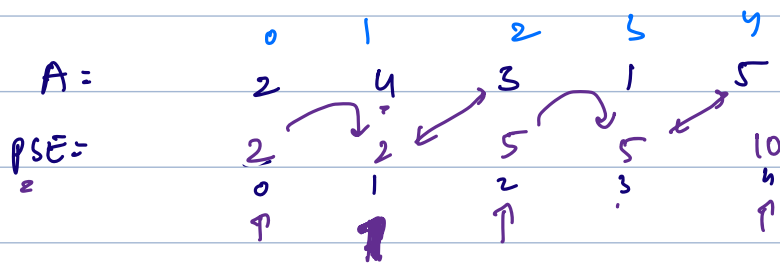
```
        PSE[i] = PSE[i-1] + A[i];
```

```
    }
    else {
```

```
        PSE[i] = PSE[i-1];
```

```
}
```

```
}
```



S

C

Sum

1

3

$PSE[3] - PSE[1-1] = 3$

0

4

$PSE[4] = 10$

3

3

$PSE[3] - PSE[2] = 5 - 5 = 0$

```
Function sumOfEvenIndexed(Array[], Queries[][], N){
    // prefix sum for even indexed elements
    PSe[N];

    PSe[0] = Array[0]

    for(i -> 1 to N-1){
        if(i % 2 == 0){
            PSe[i] = PSe[i-1] + Array[i];
        }
        else {
            PSe[i] = PSe[i-1];
        }
    }

    for(i -> 0 to Queries.length-1) {
        s = Queries[i][0]
        e = Queries[i][1]
        if(s == 0){
            print(PSe[e])
        }
        else {
            print(PSe[e]-PSe[s-1])
        }
    }
}
```

Construct PSE array
OCN)

T.C: $O(N+Q)$

S.C: $O(N)$
↳ pse array

Question: Sum of odd indexed elements

Steps

i) construct PSO array

psor[i] = sum of odd indexed els from 0 to i;

for (

if $(i \frac{1}{2} = 1)$ {

$$p_{sol}^i = p_{sol}^{i-1} + A \varepsilon^i;$$

3

2500

$$p_{3091} = p_{3091-1} ;$$

y

3

Question:

Given an array of size N, count the number of special index in the array.

Note: **Special Indices** are those after removing which, sum of all **EVEN** indexed elements is equal to sum of all **ODD** indexed elements.

A =

4	3	2	7	6	-2
0	1	2	3	4	5

Remove

i = 0

3	2	7	6	-2
0	1	2	3	4

→ $S_o = 8$ $S_e = 8$ ✓

i = 1

4	2	7	6	-2
0	1	2	3	4

→ $S_o = 9$ $S_e = 9$ ✗

i = 2

4	3	7	6	-2
0	1	2	3	4

→ $S_o = 9$ $S_e = 9$ ✓

Quiz: Sum of odd indexed

⇒ A =

4	1	3	7	10
0	1	2	3	4

Sum of even index

A =

4	1	7	10
0	1	2	3

sum of odd index

Ans = $S_{odd}[0, 1] + S_{even}[3, 4]$

Ques: Sum of odd indexed els

A =

2	3	1	4	0	-1	2	-2	10	8
0	1	2	3	4	5	6	7	8	9

Ans: $S_{\text{odd}}[0, 2] + S_{\text{even}}[4, 9]$

~~3~~
3

~~15~~

3 + (0 + 2 + 10) = 15

Ques: Sum of even indexed

A =

2	3	1	4	0	-1	2	-2	10	8
0	1	2	3	4	5	6	7	8	9

A' =

2	3	1	0	-1	2	-2	10	8
0	1	2	3	4	5	6	7	8

Sum = $S_{\text{even}}[0, 2] + S_{\text{odd}}[4, 9]$

A =

A_0	A_1	A_2	...	A_{i-1}	A_i	A_{i+1}	...	A_{N-1}
-------	-------	-------	-----	-----------	-------	-----------	-----	-----------

A'

After removing index i ,

$$S_{\text{odd}} = S_{\text{odd}}[0, i-1] + S_{\text{even}}[i+1, N-1]$$

$$\quad \quad \quad \underbrace{P_{\text{So}}[i-1]} + \underbrace{P_{\text{Se}}[N-1] - P_{\text{Se}}[i]}$$

$$S_{\text{even}} = S_{\text{even}}[0, i-1] + S_{\text{odd}}[i+1, N-1]$$

$$\quad \quad \quad \underbrace{P_{\text{Se}}[i-1]} + \underbrace{P_{\text{So}}[N-1] - P_{\text{So}}[i]}$$

$$\left\{ \begin{array}{l} S_{\text{odd}}[0, i-1] = P_{\text{So}}[i-1] \quad \checkmark \\ S_{\text{even}}[0, i-1] = P_{\text{Se}}[i-1] \end{array} \right. \Rightarrow \begin{array}{l} i = 0 \\ i \neq 0 \end{array}$$

$$\left\{ \begin{array}{l} S_{\text{even}}[i+1, N-1] = P_{\text{Se}}[N-1] - P_{\text{Se}}[i] \\ S_{\text{odd}}[i+1, N-1] = P_{\text{So}}[N-1] - P_{\text{So}}[i] \end{array} \right.$$

Pseudocode

```
Function count_special_index(arr[], n)
{
    // prefix sum for even indexed elements
    PSe[n];
    // prefix sum for odd indexed elements
    PSo[n];

    // Say we have already calculated PSe and PSo

    // Code to find Special Indices
    count = 0;
    for (i -> 0 to n-1) {
        if (i == 0) {
            Se = PSo[n-1] - PSo[i]; // sum from [i+1 n-1]
            So = PSe[n-1] - PSe[i]; // sum from [i+1 n-1]
        }
        else {
            Se = PSe[i-1] + PSo[n-1] - PSo[i]; // sum even from [0 to i-1] and odd from [i+1 n-1]
            So = PSo[i-1] + PSe[n-1] - PSe[i]; // sum odd from [0 to i] and even from [i+1 n-1]
        }

        if (Se == So) {
            count++;
        }
    }

    return count;
}
```

$$T.C: O(N) + O(N) + O(1) = O(N)$$

$$S.C: \begin{array}{cc} O(N) & O(N) \\ P_{\text{Se}} & P_{\text{So}} \end{array} = O(N)$$

Sold :