Market Microstructure Modeling
In High-Frequency Cryptocurrency Markets

M.Stat Second Year Project by

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Abstract

An exchange is an marketplace where buyers and sellers trade financial assets, eg. company shares and contracts. Trades occur only when both parties agree on a common price. With multiple buyers and sellers placing limit orders and market orders at varying prices and quantities, the exchange employs a mechanism called limit order book (denoted by $\mathcal{L}(t)$ at time t, left of figure 1) to match them and execute trades, usually using price-time priority. For each asset's $\mathcal{L}(t)$, the exchange imposes a tick-size($\theta = 0.01$ USDT for figure 1) and lot-size($\sigma = 10^{-5}$ BTC for figure 1) parameter. To fully characterize $\mathcal{L}(t)$, we also need to specify the best bid($b(t)$) and best ask($a(t)$) prices highlighted in figure 1. $\mathcal{L}(t)$ is divided into the buy side $\mathcal{B}(t)$ and sell side $\mathcal{S}(t)$ which are shaded in green and pink in figure 1. With tick-size discretization, it is naturally interpreted as a set of queues (figure 2(a)), where each queue represents active buy or sell orders at a specific price, characterized by its distance from the $b(t)$ or $a(t)$. The i^{th} queue on $\mathcal{B}(t)$ (resp. $\mathcal{S}(t)$) is the set of active buy (resp. sell) limit orders with price $p_i^b = b(t) - i\theta$ (resp. $p_i^a = a(t) + i\theta$) like Buy 1-15 on the left of figure 1. The number of active buy (resp. sell) limit orders in the i^{th} queue on $\mathcal{B}(t)$ (resp. $\mathcal{S}(t)$) can be denoted by a random variable $X_{i,t}^b$ (resp. $X_{i,t}^a$). In $\mathcal{L}(t)$, the price evolution depends on the way orders are matched. Consider a buy (resp. sell) limit order x placed immediately after time t with price p and size ω. The arrival of this order can affect the state of $\mathcal{L}(t)$ in two ways: (1) If $p \le b(t)$ (resp. $p \ge a(t)$), x becomes active on arrival and does not change the price as shown in figure 2(b)-2(d). (2) If $b(t) < p < a(t)$ (resp. same for sell side), x becomes active on arrival and causes a price change $b(t^+) = p$ (resp. $a(t^+) = p$) as shown in figure 2(j)-2(k). If a market buy (resp. sell) order x is placed with the above specifications, then $p \ge a(t)$ (resp. $p \le b(t)$) is executed immediately upon arrival. Whenever such an order arrives, it is matched to the price of an active order, even if this is not the same as the price of the incoming order resulting in market trades (right of figure 1). Whether this order induces a price change depends on the size of the order: the best bid (resp. ask) price upon arrival of a sell (resp. buy) market order is $b(t^+) = \max(p_{i'}^b, p_{i^*}^b)$, where $i' = \max\{i \in \mathbb{W} : p_i^b < p, X_{i,t}^b \neq 0\}, i^* = \min\{i \in \mathbb{W} : \sum_{k=0}^{i} \mathbb{1}[X_{k,t}^b > \omega] = 1\}$ (similarly, we can get $a(t^+)$) shown in figure 2(e)-2(i). i' and i^* are rarely undefined which can happen when $\mathcal{B}(t) = \emptyset$ (resp. $\mathcal{S}(t) = \emptyset$). In this case, we define $b(t^+) = -\infty$ (resp. $a(t^+) = +\infty$).

This project focuses on developing and extending statistical models for order arrivals in $\mathcal{L}(t)$ in high-frequency cryptocurrency markets (figure 2(l)), using non-homogeneous Poisson process[1] with independent queue assumption and self/mutually exciting point process[2] to capture inter-dependence between queues, under the representation shown in figure 2(a). It also addresses estimation of order execution probabilities and forecasting of the asset's price using features derived from $\mathcal{L}(t)$ and recent trades like order imbalance ($\rho(t)$), bid-ask spread ($a(t) - b(t)$, shown in figure 2(o)). All these components are togetherly called **market microstructure**. Under this framework of developed market microstructure models, this project will explore designing efficient market-making strategies (illustrated in figure 2(n)) considering latency and capital constraints. One of the simpler utility functions which a market-making algorithm tries to maximize through its limit order placement at each time t is $u(x, s, q, t) = \mathbb{E}_t[-e^{-\gamma(X_T + q_T S_T)}]$ (the utility function has a shape similar the risk-averse curve in 2(m)), where X_T is the wealth, q_T is the inventory size, S_T is the price of the asset, each at terminal time T, and γ is the risk aversion parameter.

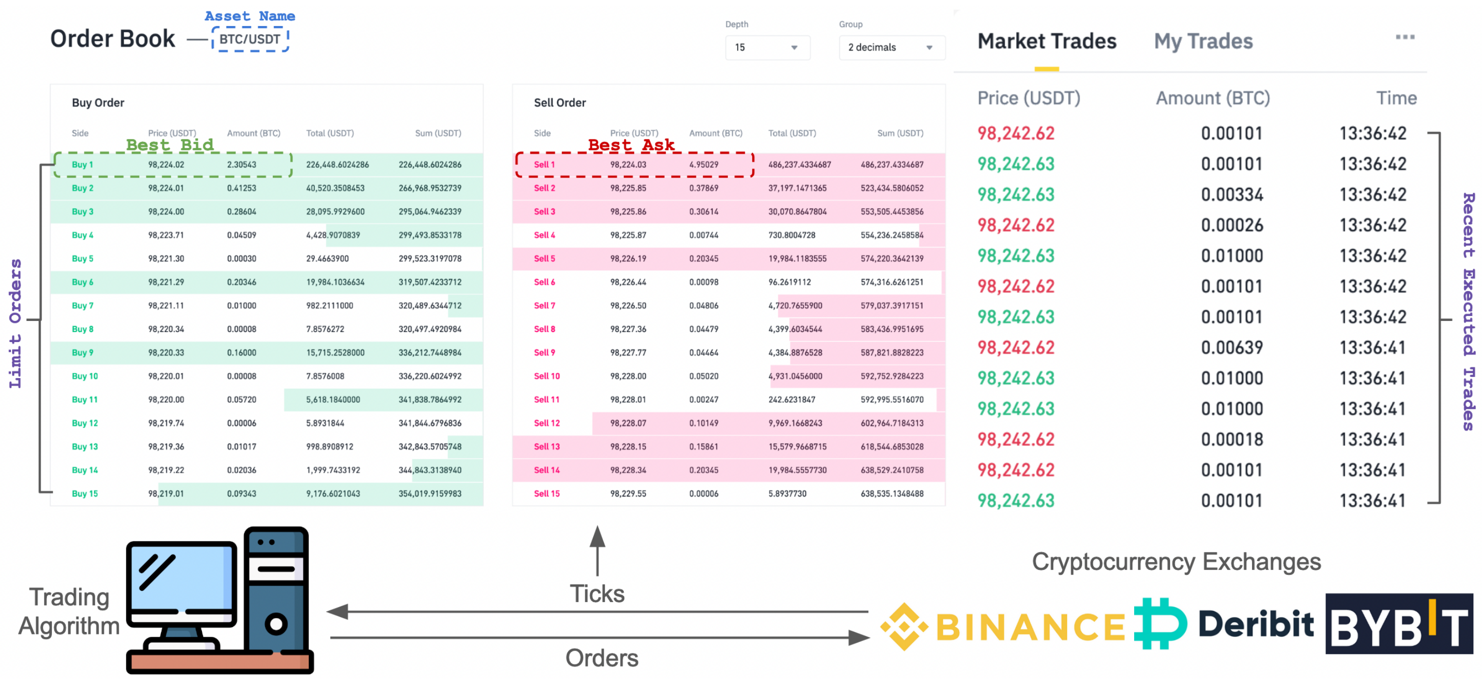

Figure 1: Limit orderbook and trades of BTCUSDT on Binance cryptocurrency exchange. The time taken between receiving market ticks and sending out processed order requests is usually in the submilliseconds range for High-Frequency Trading.

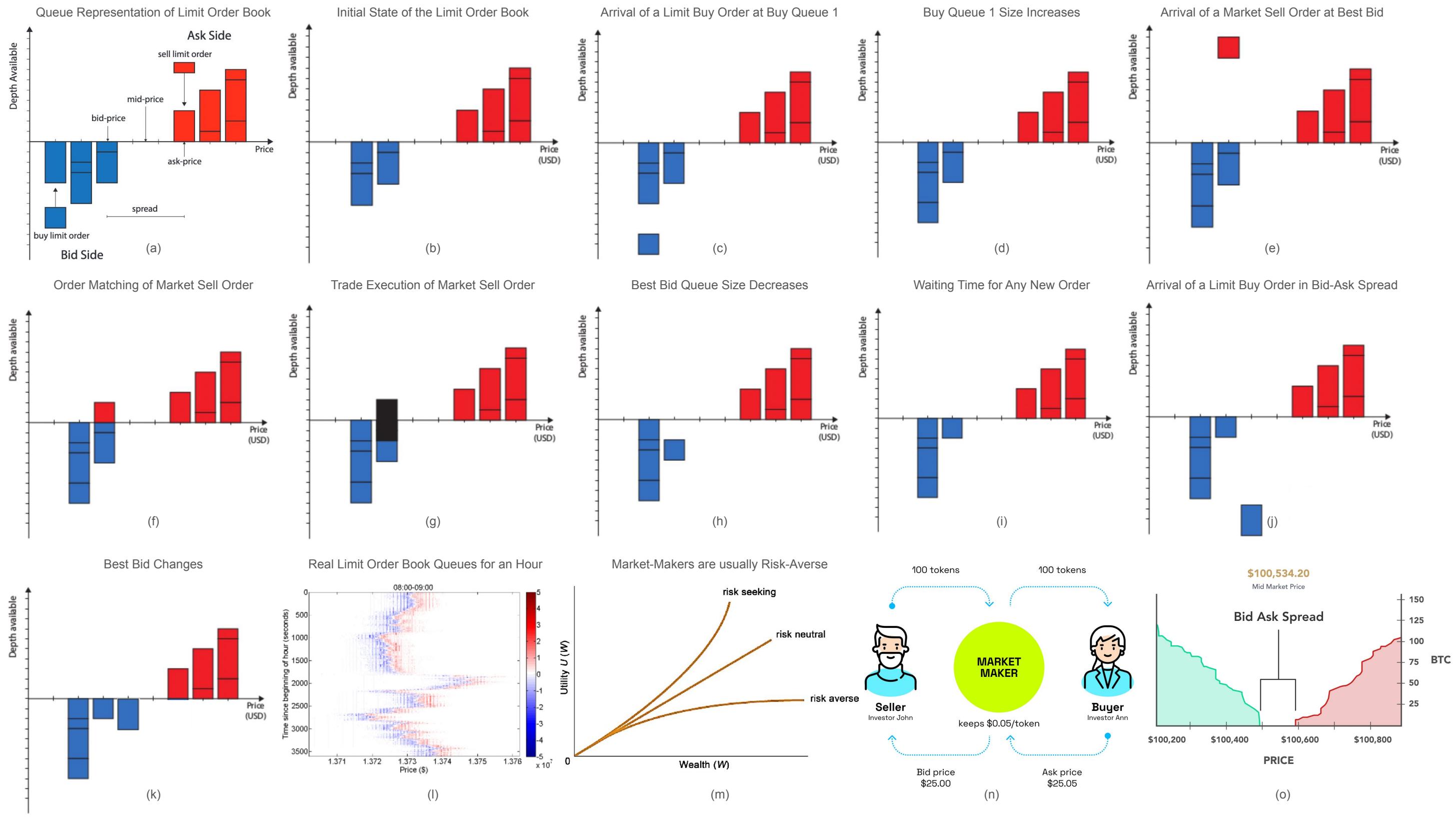

Figure 2: (a) shows the queue-based representation of $\mathcal{L}(t)$. (b) to (k) illustrates how $\mathcal{L}(t)$ evolves with arrival of new limit and market orders. (l) shows real-life $\mathcal{L}(t)$ queues and (m) illustrates the shape of a risk-averse market maker. (n) shows how market-making works and (o) the bid-ask spread.

Glossary

best ask The best ask price is the lowest price at which there is an active sell limit order at time t. It is highlighted in red on Sell 1 in figure 1.

best bid The best bid price is the highest price at which there is an active buy limit order at time t. It is highlighted in green on Buy 1 in figure 1.

bid-ask spread Bid-ask spread is the difference between the best ask and best bid i.e. $a(t) - b(t)$ at time t. It often provides idea about liquidity in the market i.e. selling or buying an asset using market orders. It is shown in figure 2(o).

buy side The set of all active buy limit orders.

cryptocurrency markets Similar to a stock market (eg. Bombay Stock Exchange) but for cryptocurrencies which are highly secure digital currency. Some of the popular cryptocurrency exchanges are Binance, Deribit and Bybit.

inventory size The amount or size of assets which a trading strategy holds. It is always associated with a risk that the price of the asset might move adversely incurring losses.

limit order book It is the collection of all active limit orders in a market at a particular time. Shown on the left of figure 1.

limit orders A buy (resp. sell) limit order submitted at time t with price p and size ω is a commitment to buy (resp. sell) up to ω units of an asset at a price no more (resp. no less) than p. This does not execute immediately and stay *active* in the limit order book till it is matched or *cancelled*.

lot-size It is the unit of quantity or size of an order for a limit order book. It is the smallest possible interval between consecutive sizes. From figure 1, we can see the lot-size is 10^{-5} BTC. It is rare to see fractional lot-sizes in stock markets, but it is common in cryptocurrency markets.

market ticks Any updates on the exchange like new buy/sell orders added/cancelled in $\mathcal{L}(t)$ and recently executed trades.

market-making Trading strategies which quote buy and sell limit orders on $\mathcal{L}(t)$ at all times aiming to profit from the bid-ask spread shown in figure 2(n) and 2(o) while managing the inventory risk viz. the risk associated with holding large quantity of an asset which results in losses if the price changes adversely.

market orders Orders that are immediately matched to a limit order resulting in a trade (right of figure 1).

order imbalance This is a measure of the buy versus sell pressure on an asset and it contains predictive power on both the arrival rates of market orders, and the direction and size of future price movements. It is measured as ratio of the quoted size imbalance to the total quoted size $\rho(t) = \frac{\sum_{k=0}^{\infty} X^b_{k,t} - \sum_{k=0}^{\infty} X^a_{k,t}}{\sum_{k=0}^{\infty} X^b_{k,t} + \sum_{k=0}^{\infty} X^a_{k,t}}$.

order execution probabilities The probability of a limit buy (resp. sell) order getting filled or executed when posting at a given price level δ tick-size away from the best bid (resp. best ask), conditional of the arrival of an market order. Naturally, it is decreasing function of δ. It can be seen from figure 1 that the limit orders at Buy 15 (resp. Sell 15) are far less likely to get executed than Buy 1 (resp. Sell 1) because of the way orders are matched.

price-time priority It is an algorithm for matching newly arrived market orders with active limit orders stated as follows,

- For active buy orders, priority is given to the active orders with the highest price.
- For active sell orders, priority is given to the active orders with the lowest price.
- Ties are broken by selecting the active order with the earliest submission time.

Hence, faster an order reaches the exchange, better are its chances of execution.

sell side The set of all active sell limit orders.

tick-size It is the unit of price of an order for a limit order book. It is the smallest possible interval between consecutive prices. From figure 1, we can see the tick-size is 0.01 USDT.

References

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[2] Konark Jain et al. *Limit Order Book Dynamics and Order Size Modelling Using Compound Hawkes Process.* 2024. arXiv: 2312.08927 [q-fin.TR]. URL: https://arxiv.org/abs/2312.08927.