Part B

Each student is required to select a stock from the NIFTY 50 index and collect the daily stock prices of the selected stock for at least 100 days, say $S_0, S_1, \ldots$. *It must be ensured that no two student use the same set of data.* Consider a market consisting of two assets only, namely the selected stock and the risk-free cash Bond. The interest rate on the cash bond may be assumed to be constant, which needs to be identified by students based on the current market situation (justifying the choice).

Suppose that a European Call option is available based on the selected stock having maturity time $T = f(\text{roll number})$ days and strike price $K = 5T + S_0$, where f is a function defined in the R-code given below.

(a) Fit the selected data on share price by a suitable geometric Brownian motion. [Students do not need to check for the model fit]

(b) Compute the fair price of the Call option assuming the Black-Scholes model with the parameter values estimated in (a). Also, derive a Delta-hedged replicating portfolio for this derivative under the Black-Scholes model.

(c) Compute the fair price of the Call option assuming an Binomial lattice model with sub-interval length $\Delta t = 1$ days. The parameter values must be obtained by matching the means and variance of the underlying share with those obtained under the Black-Scholes model. Also, derive a Delta-hedged replicating portfolio for this derivative under the present binomial model.

(d) Repeat the exercise in part (c) with decreasing values of Δt to show that the option price as well as the replicating portfolio obtained under the binomial model converge to the same obtained under the Black-Scholes model in part (b) as $\Delta t \to 0$.

(e) Comments on the convergence rate based on suitable literature review and the results obtained for your choose stock price data.

[1+(2+2)+(3+3)+6+3]=20

```
# Function to calculate strike price and expiry based on roll number.
# Roll number must be entered in a valid format, namly MB2314 or MD1234.
f <- function(roll_number) {

        roll_number <- toupper(roll_number)
        last_two_digits <- as.numeric(substring(roll_number, nchar(roll_number) -
             1, nchar(roll_number)))

        value <- NA
        if (substring(roll_number, 2, 2) == "B") {
                value <- 2 * (last_two_digits %% 10) + last_two_digits
        } else if (substring(roll_number, 2, 2) == "D") {
                value <- last_two_digits
        } else {
                return("Make sure you have entered your actual roll number; there
                     seems to be an error.")
        }
        return(value)
}
```