Topics: Jackknife, bootstrap | Due on February 23, 2024

Name of student:

Roll number:

1. Comparison of confidence intervals [5]

Consider data $X_1, \ldots, X_n$ generated from a population distribution F with density f. We are interested in constructing confidence intervals for each of the following parameters: (i) the mean $\mu = \int x \, dF(x)$, (ii) the median $m = F^{-1}(1/2)$, (iii) the inter-quartile range $\psi = F^{-1}(3/4) - F^{-1}(1/4)$ and (iv) the differential entropy $\theta = - \int f(x) \log f(x) \, dx$. Via Monte Carlo simulations, compare the expected lengths and coverage probabilities of 95% confidence intervals based on (whichever is applicable)

(a) "exact resampling" (i.e. you are allowed to generate multiple independent samples of size n from F);

(b) asymptotic theory;

(b) jackknife;

(c) non-parametric bootstrap;

(d) parametric bootstrap;

(e) smoothed bootstrap.

Perform these comparisons for each of the following family of distributions (as F): (i) $\mathcal{N}(0,1)$, (ii) $\mathcal{N}(\mu, 1)$, (iii) $\mathcal{N}(\mu, \sigma^2)$, (iv) $\mathsf{Cauchy}(m, \gamma)$ and (v) $\frac{1}{2}\mathcal{N}(\mu, 1) + \frac{1}{2}\mathsf{Cauchy}(2\mu, 1)$. Explain your findings. (**Note:** You must submit your code along with your solution.)