Fitting Different Factor Models for Market Risk Management

Rishi Dey Chowdhury (MB2320), Indian Statistical Institute Kolkata

April 18, 2025

Abstract

This report explores the application of different factor models — namely the Capital Asset Pricing Model (CAPM) and the Fama-French Three-Factor Model (FF3) — to asset returns data for market risk management purposes. We detail the theoretical background, methodology, results, and risk decomposition analysis based on factor exposures.

Contents

1 Introduction

Managing market risk is fundamental in finance. Factor models offer a structured way to understand and decompose the sources of asset returns and associated risks. In this project, we fit two widely-used factor models (CAPM and FF3) to a set of asset returns, evaluate their effectiveness, and use them to assess portfolio risk.

2 Theoretical Background

2.1 Capital Asset Pricing Model (CAPM)

The CAPM expresses the expected excess return of an asset as proportional to the market excess return:

$$R_i - R_f = \alpha_i + \beta_{iM}(R_M - R_f) + \varepsilon_i \tag{1}$$

where R_i is the return of asset i, R_f is the risk-free rate, R_M is the return of the market portfolio, β_{iM} measures the sensitivity to the market return, and ε_i is the idiosyncratic error term.

Economic Intuition CAPM postulates that in equilibrium, investors are only compensated for bearing systematic risk, measured by beta (β), because idiosyncratic risk can be diversified away.

Key Assumptions

- Investors are rational and risk-averse.
- Markets are frictionless (no taxes, no transaction costs).
- All investors have homogeneous expectations.
- There exists a risk-free asset.
- Investors can borrow and lend at the risk-free rate.
- Investors optimize portfolios solely based on mean and variance (mean-variance optimization).
- All assets are infinitely divisible and perfectly liquid.

Interpretation of Beta The beta coefficient measures an asset's sensitivity to movements in the overall market. A beta greater than 1 implies higher volatility than the market; less than 1 implies lower volatility.

2.2 Fama-French Three-Factor Model (FF3)

The FF3 model extends CAPM by including two additional factors:

$$R_i - R_f = \alpha_i + \beta_{iM}(R_M - R_f) + \beta_{iS}SMB + \beta_{iH}HML + \varepsilon_i \tag{2}$$

where SMB (Small Minus Big) captures the size premium and HML (High Minus Low) captures the value premium.

Economic Motivation Empirical evidence showed that CAPM alone could not explain certain anomalies such as the size effect (small-cap stocks tend to outperform large-cap stocks) and the value effect (high book-to-market stocks outperform low book-to-market stocks). FF3 aims to capture these effects by introducing the SMB and HML factors.

Interpretation of Factors

- **SMB (Size Factor)**: Captures the excess return of small-cap stocks over large-cap stocks.
- **HML (Value Factor)**: Captures the excess return of high book-to-market stocks over low book-to-market stocks.

Key Assumptions

- Markets are competitive and frictionless.
- Risk factors (Market, Size, Value) are priced by investors.
- Systematic differences in stock returns are driven by exposures to these risk factors.
- The factors are constructed to be orthogonal or nearly orthogonal.

2.3 Risk Decomposition

Total return variance can be decomposed into:

$$\mathrm{Var}(R_p) = \text{Factor Risk} + \text{Idiosyncratic Risk} \tag{3}$$

where:

- Factor Risk $= \boldsymbol{\beta}_p^T \Sigma_F \boldsymbol{\beta}_p$
- Idiosyncratic Risk $= \sum_i w_i^2 \sigma_{\varepsilon,i}^2$

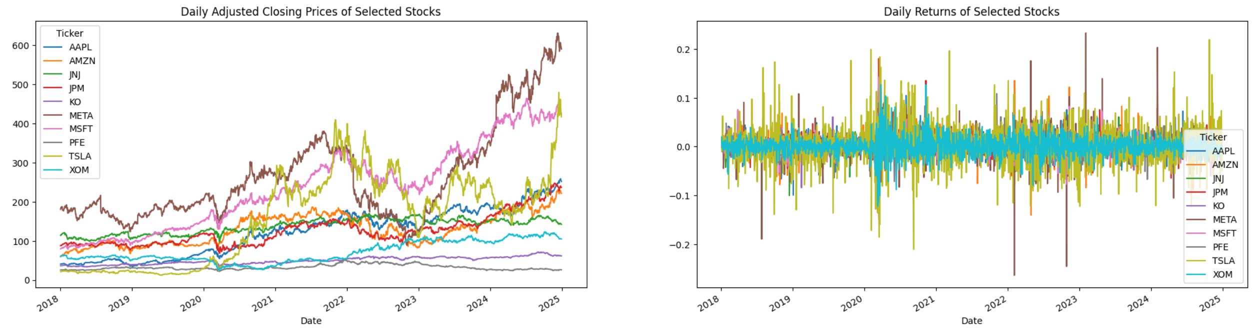


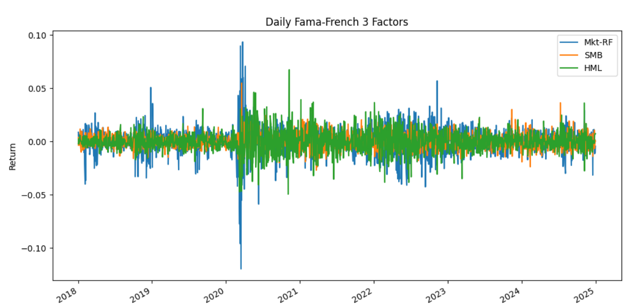
Figure 1: The daily returns and adjusted closing price of the selected stocks

3 Methodology and Results

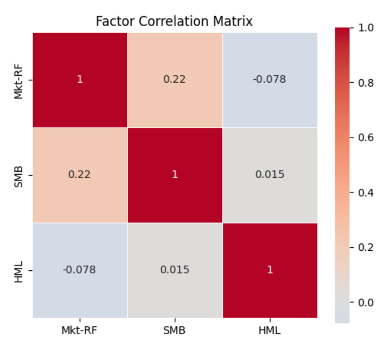
3.1 Data

- **Asset returns:** Data for 10 major stocks of the US market which includes Apple Inc., Microsoft Corporation, Amazon.com, Inc., JPMorgan Chase & Co., Johnson & Johnson, Exxon Mobil Corporation, Pfizer Inc., Tesla, Inc., Meta Platforms, Inc., The Coca-Cola Company is collected from Yahoo Finance from 3^{rd} January, 2018 to 30^{th} December, 2024. The daily returns were computed from the adjusted daily closing price of these stocks. The prices and returns are shown in figure 1.

- **Factor returns:** Fama-French 3-Factor daily data is obtained from Kenneth French's data library which provides us with 3 major factors, as shown in figure 2a, used for modelling the excess stock returns and performing risk decomposition as described in subsection 2.3. The covariance between these factors are low as shown in figure 2b suggesting independence between them which is an assumption made by the factor models.



(a) FF3 factor values



(b) Correlation matrix between the FF3 factors

Figure 2: The factors used in the FF3 factor models are completely uncorrelated and their values explain to great extent the returns of each asset.

3.2 Model Fitting and Performance Comparison

To model the returns of the selected assets and identify the two risk components factor risk and idiosyncratic risk, we do the following,

1. Calculate daily excess returns for each asset by subtracting the daily returns of the US treasury bill from the asset's returns.

2. Fit CAPM and FF3 linear regressions on each asset and obtain the Ordinary Least Squares coefficients. For CAPM, we only use one factor i.e. market excess returns to explain the stock returns whereas in FF3 we use all the 3 factors market excess returns, SMB and HML.

3. Evaluate and compare the regression performance R^2, AIC and BIC.

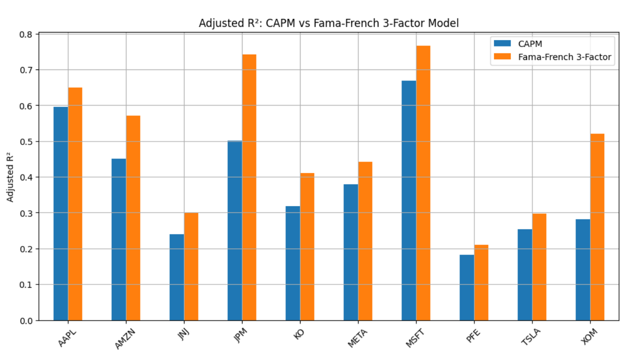
The estimated coefficients of the CAPM and FF3 model are shown in table 1 and 2 respectively. The comparison between their performances are shown in subfigure 3a and 3b of figure 3. It shows that across all metrics FF3 explains the market risks and returns of the assets better compared to CAPM. The AIC and BIC improvements are also substantial.

	AAPL	AMZN	JNJ	JPM	KO	META	MSFT	PFE	TSLA	XOM
Constant (α_i)	0.000569	0.000338	-0.000117	0.000146	0.000023	0.000318	0.000461	-0.000213	0.001632	0.000017
Market Beta Factor (β_{iM})	1.166061	1.142014	0.473484	1.035446	0.546297	1.276622	1.167358	0.537224	1.586761	0.814368

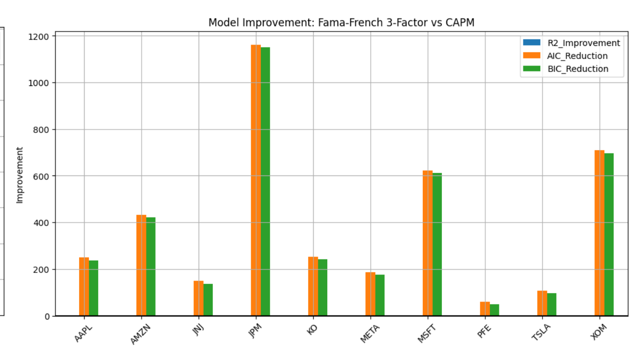
Table 1: CAPM regression coefficients for each asset.

	AAPL	AMZN	JNJ	JPM	KO	META	MSFT	PFE	TSLA	XOM
Constant (α_i)	0.000491	0.000249	-0.000161	0.000207	-0.000006	0.000235	0.000356	-0.000250	0.001658	0.000103
Market Beta Factor (β_{iM})	1.179768	1.114778	0.531383	1.112056	0.609986	1.256849	1.192741	0.587875	1.460075	0.874829
SMB Beta Factor (β_{iS})	-0.305689	-0.148886	-0.407725	-0.169943	-0.386084	-0.164144	-0.444452	-0.350803	0.670193	-0.016910
HML Beta Factor (β_{iH})	-0.380873	-0.731681	0.130028	0.899367	0.265866	-0.641504	-0.465945	0.125330	-0.721295	0.940476

Table 2: FF3 regression coefficients for each asset.



(a) The Adjusted R^2 for CAPM and FF3 model fit on each asset's returns.



(b) The AIC and BIC improvement of FF3 compared to the CAPM for each asset.

Figure 3: CAPM and FF3 factor model comparison for each of the selected assets. FF3 significantly outperforms the CAPM in all evaluation metrics.

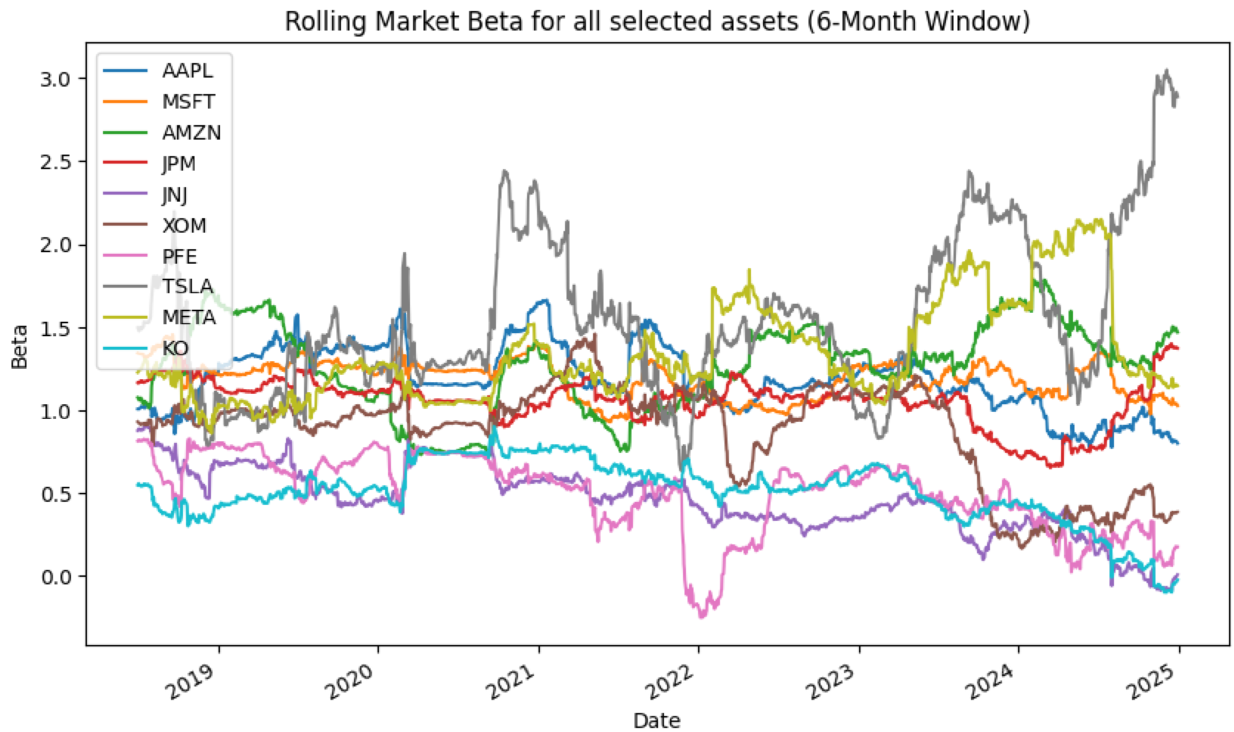


Figure 4: The rolling market beta estimates for each asset based on a 6-month look back window.

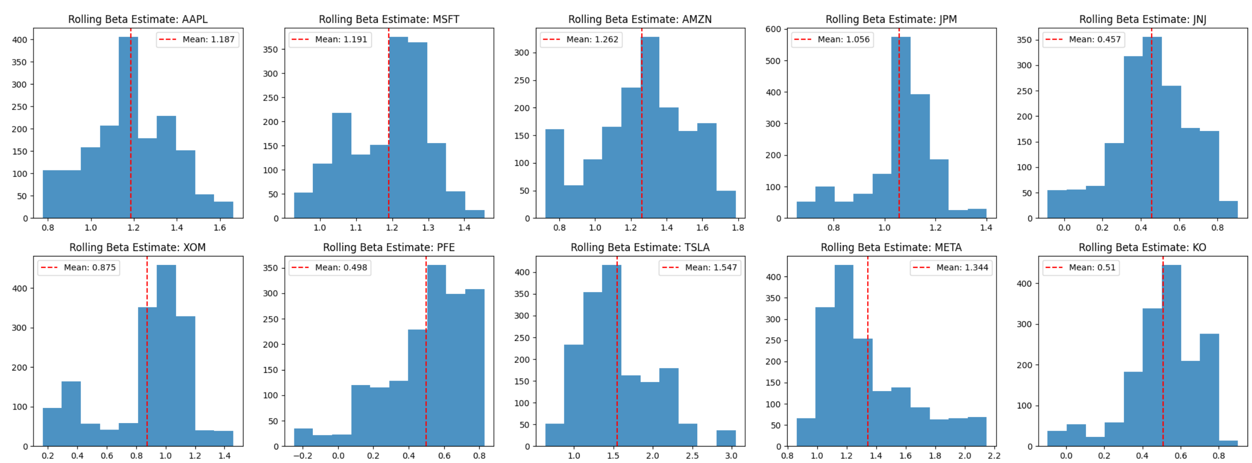


Figure 5: Rolling market beta distribution for each asset based on a 6-month look back window.

In the above FF3 model, we obtained a single estimate for all the market betas β_{iM} based on the data we have. But usually in practice, due to varying market conditions rolling estimates are obtained based on a lookback window. Here, I take the lookback window of 6-months to compute rolling market betas for each asset shown in figure 4. Performing t-test for the significance of α_i in FF3 and CAPM, we fail to reject the null hypothesis for each asset signifying it is 0.

3.3 Portfolio Construction and Risk Estimation

To understand how these factor models help decompose the risk of a portfolio and capture the risk exposure. We look at the following,

- Construct an equal-weighted portfolio on each asset.
- Compute portfolio-level factor exposures
- Decompose portfolio risk into factor and idiosyncratic components

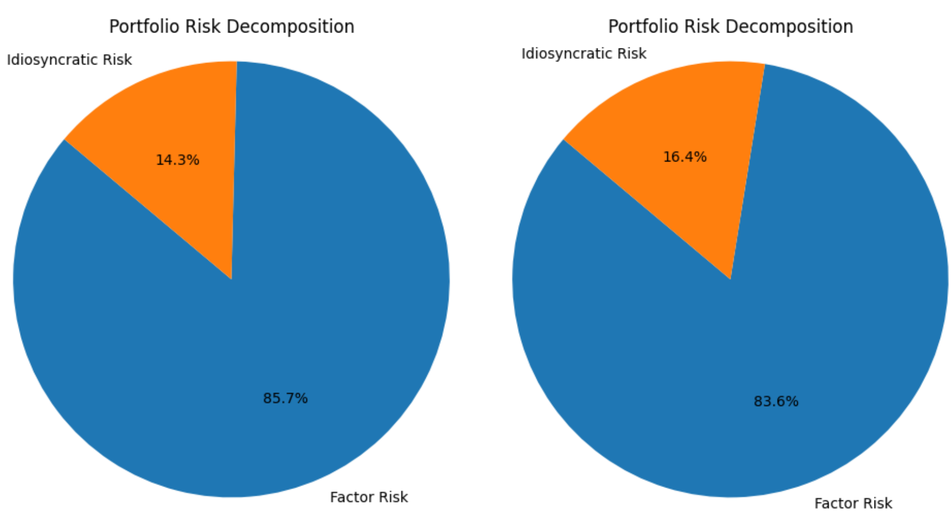
The residual variances are important in computing the market risks associated with our portfolio and they are stated in table 3 and 4. The decomposition of the risk of our portfolio into the idiosyncratic and the factor risk, shown in figure 6, reveals better explainability of the factor risk by FF3 compared to CAPM. This might be helpful for portfolio managers to separate the asset specific portfolio risk from the general market risk i.e. the factor risk.

	AAPL	AMZN	JNJ	JPM	KO	META	MSFT	PFE	TSLA	XOM
Residual Variance ($\sigma^2_{\epsilon,i}$)	0.000130	0.000202	0.000106	0.000090	0.000089	0.000389	0.000077	0.000203	0.001131	0.000183

Table 3: FF3 residual variance.

	AAPL	AMZN	JNJ	JPM	KO	META	MSFT	PFE	TSLA	XOM
Residual Variance ($\sigma^2_{\epsilon,i}$)	0.000150	0.000259	0.000116	0.000174	0.000103	0.000433	0.000110	0.000210	0.001203	0.000274

Table 4: CAPM residual variance.



(a) FF3 portfolio risk decomposition. (b) CAPM portfolio risk decomposition.

Figure 6: Comparison of the portfolio risk decomposition based on FF3 and CAPM.

Above, I found these risk based on the entire time data. But again, due to practical importance I used the rolling estimates to find the factor risk and idiosyncratic risk across time for the portfolio with equal weights. This is similar to what we did in 4. The result is shown in figure 7.

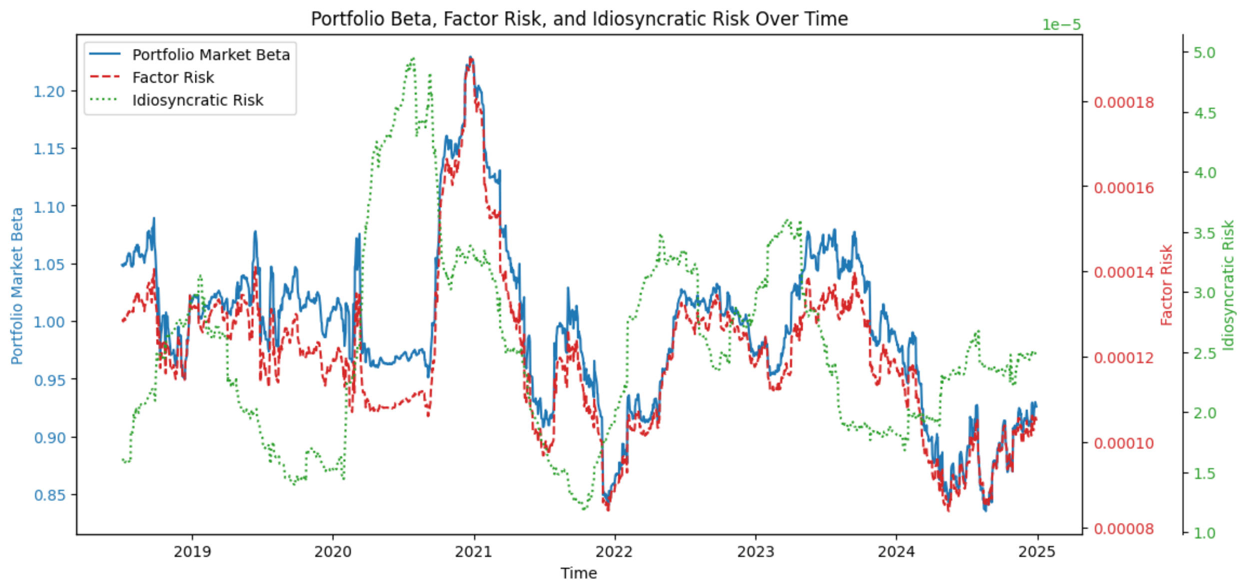


Figure 7: Rolling portfolio market beta and the two types of risks associated with it for the equal-weighted portfolio.

We observe how the factor risk closely follows the market and increases and decreases as the market beta varies i.e. this risk is closely associated with the current overall market scenario. The idiosyncratic risk is however specific to the portfolio due to the asset specific risk apart from the common market driven risk component.

4 Conclusion

The Fama-French Three-Factor Model demonstrated superior explanatory power over CAPM by capturing additional size and value effects. Most portfolio risk was attributable to systematic factors rather than idiosyncratic noise. These models are useful for portfolio managers to assess their portfolio specific risk and the risk associated with the portfolio as a whole because of the overall market movements. This help distinguish and minimize both of the risks separately and in a way desired by the portfolio manager or investor. Future work could include:

- Expanding to Fama-French Five-Factor Model
- Testing with different weighting schemes (e.g., value-weighted portfolio)

5 References

- Fama, Eugene F., and Kenneth R. French. *Common risk factors in the returns on stocks and bonds*. Journal of Financial Economics (1993).
- Kenneth French Data Library: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`