AI1110 Assignment-1
ICSE Class-10 2017

Rishi Manoj
CS21BTECH11045

Q4(a): What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor?

Solution: Let $p(x) = 16x^3 - 8x^2 + 4x + 7$ and $d(x) = 2x + 1$.

Polynomial division of $p(x)$ with $d(x)$:

$$\begin{array}{r|l} & 8x^2 - 8x + 6 \\ \hline 2x+1 & 16x^3 - 8x^2 + 4x + 7 \\ & -16x^3 - 8x^2 \\ \hline & \quad -16x^2 + 4x \\ & \quad 16x^2 + 8x \\ \hline & \quad\quad 12x + 7 \\ & \quad\quad -12x - 6 \\ \hline & \quad\quad\quad 1 \end{array}$$

From the above division, it is clear that 1 has to be subtracted from the polynomial $p(x)$, so that $d(x)$ becomes factor of the resulting polynomial after subtraction.

Using remainder theorem:

Remainder theorem states that remainder of division of a polynomial $f(x)$ by a linear polynomial $x - r$ is equal to $f(r)$.So, remainder of $p(x)$ divided by $d(x)$ is $p\left(\frac{-1}{2}\right)$.

$$p\left(\frac{-1}{2}\right) = 16\left(\frac{-1}{2}\right)^3 - 8\left(\frac{-1}{2}\right)^2 + 4\left(\frac{-1}{2}\right) + 7 \tag{1}$$

$$= 1 \tag{2}$$

So, subtracting 1 from the given polynomial $p(x)$ gives a polynomial which has $d(x)$ as its factor.