Assignment 8
Papoullis Textbook Chapter 6 Ex 6.70

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LaTeX

Outline

Question

The random variables x and y are $N(3, 4, 1, 4, 0.5)$. Find $f(y|x)$ and $f(x|y)$.

Solution

Given $N(3, 4, 1, 4, 0.5)$.

$$\mu_1 = 3 \tag{1}$$
$$\mu_2 = 4 \tag{2}$$
$$\sigma_1^2 = 1 \tag{3}$$
$$\sigma_2^2 = 1 \tag{4}$$
$$r = 0.5 \tag{5}$$

$f(y|x)$

$f(y|x)$ is given by,

$$f(y|x) = \frac{1}{\sigma_2\sqrt{2\pi(1-r^2)}} exp\left(\frac{-((y-\eta_2)-r\sigma_2(x-\eta_1)/\sigma_1)^2}{2\sigma_2^2(1-r^2)}\right) \quad (6)$$

Solving for $f(y|x)$

Substituiting the corresponding values and simplifying gives,

$$f(y|x) = \frac{1}{2\sqrt{2\pi(1 - (0.5)^2)}} exp \left(\frac{-((y - 4) - (0.5)(2)(x - 3)/1)^2}{2(4)(1 - (0.5)^2)} \right) \quad (7)$$

$$f(y|x) = \frac{1}{\sqrt{6\pi}} exp \left(\frac{-(y - x - 1)^2}{6} \right) \quad (8)$$

$f(x|y)$

$f(x|y)$ is given by,

$$f(x|y) = \frac{1}{\sigma_1\sqrt{2\pi(1-r^2)}} exp\left(\frac{-((x-\eta_1) - r\sigma_1(y-\eta_2)/\sigma_2)^2}{2\sigma_1^2(1-r^2)}\right) \quad (9)$$

Solving for $f(x|y)$

Substituiting the corresponding values and simplifying gives,

$$f(y|x) = \frac{1}{1\sqrt{2\pi(1-(0.5)^2)}} exp\left(\frac{-((x-3)-(0.5)(1)(y-4)/1)^2}{2(1)(1-(0.5)^2)}\right) \tag{10}$$

$$f(y|x) = \frac{\sqrt{2}}{\sqrt{3\pi}} exp\left(\frac{-(4y-x-13)^2}{24}\right) \tag{11}$$