AI1110: Assignment 10

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***Abstract*—This document contains the solution to Question of Chapter 9 in the Papoullis Textbook.**

Chapter 9 Ex 9.30: The input of a linear system with $h(t) = Ae^{-\alpha t}U(t)$ is a process of $x(t)$ with $R_x(\tau) = N\delta(\tau)$ applied at $t = 0$ and disconnected at $t = T$. Find $E\left\{y^2(t)\right\}$.

Solution: Given, $h(t) = Ae^{-\alpha t}U(t)$, $R_x(\tau) = N\delta(\tau)$, applied at $t = 0$ and disconnected at $t = T$. Also $q(t) = N$ for $0 < t < T$ and 0 otherwise. For $0 < t < T$, $E\left\{y^2(t)\right\}$ is given as,

$$E\left\{y^2(t)\right\} = N\int_0^t h^2(\tau)d\tau \tag{1}$$

$$= NA^2\int_0^t e^{-2\alpha\tau}d\tau \tag{2}$$

$$= \frac{NA^2}{2\alpha}(1 - e^{-2\alpha t}) \tag{3}$$

For $t \geq T$,given $q(t) = 0$. So, $E\left\{y^2(t)\right\}$ is given as,

$$E\left\{y^2(t)\right\} = q(t)\int_0^t h^2(\tau)d\tau \tag{4}$$

$$= q(t)\int_0^T h^2(\tau)d\tau + q(t)\int_T^t h^2(\tau)d\tau \tag{5}$$

$$= NA^2\int_0^T e^{-2\alpha\tau}d\tau + 0 \tag{6}$$

$$= \frac{NA^2}{2\alpha}(1 - e^{-2\alpha T}) \tag{7}$$

In the above cases $U(t)$ is taken as 1 as t is positive. Therefore,

$$E\left\{y^2(t)\right\} = \begin{cases} \frac{NA^2}{2\alpha}(1 - e^{-2\alpha t}), & 0 < t < T \\ \frac{NA^2}{2\alpha}(1 - e^{-2\alpha T}), & T \leq t \end{cases}$$