AI1110: Assignment 8

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Abstract—This document contains the solution to Question of Chapter 6 in the Papoullis Textbook.

Chapter 6 Ex 6.70: The random variables x and y are $N(3, 4, 1, 4, 0.5)$. Find $f(y|x)$ and $f(x|y)$.

Solution: Given $N(3, 4, 1, 4, 0.5)$.

$$\mu_1 = 3 \tag{1}$$

$$\mu_2 = 4 \tag{2}$$

$$\sigma_1^2 = 1 \tag{3}$$

$$\sigma_2^2 = 1 \tag{4}$$

$$r = 0.5 \tag{5}$$

$f(y|x)$ is given by,

$$f(y|x) = \frac{1}{\sigma_2\sqrt{2\pi(1-r^2)}} exp\left(\frac{-((y-\eta_2)-r\sigma_2(x-\eta_1)/\sigma_1)^2}{2\sigma_2^2(1-r^2)}\right) \tag{6}$$

Substituiting the corresponding values and simplifying gives,

$$f(y|x) = \frac{1}{2\sqrt{2\pi(1-(0.5)^2)}} exp\left(\frac{-((y-4)-(0.5)(2)(x-3)/1)^2}{2(4)(1-(0.5)^2)}\right) \tag{7}$$

$$f(y|x) = \frac{1}{\sqrt{6\pi}} exp\left(\frac{-(y-x-1)^2}{6}\right) \tag{8}$$

$f(x|y)$ is given by,

$$f(x|y) = \frac{1}{\sigma_1\sqrt{2\pi(1-r^2)}} exp\left(\frac{-((x-\eta_1)-r\sigma_1(y-\eta_2)/\sigma_2)^2}{2\sigma_1^2(1-r^2)}\right) \tag{9}$$

Substituiting the corresponding values and simplifying gives,

$$f(y|x) = \frac{1}{1\sqrt{2\pi(1-(0.5)^2)}} exp\left(\frac{-((x-3)-(0.5)(1)(y-4)/1)^2}{2(1)(1-(0.5)^2)}\right) \tag{10}$$

$$f(y|x) = \frac{\sqrt{2}}{\sqrt{3\pi}} exp\left(\frac{-(4y-x-13)^2}{24}\right) \tag{11}$$