Assignment 10
Papoullis Textbook Chapter 9 Ex 9.30

Rishi Manoj - CS21BTECH11045

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LaTeX

Outline

Question

The input of a linear system with $h(t) = Ae^{-\alpha t}U(t)$ is a process of $x(t)$ with $R_x(\tau) = N\delta(\tau)$ applied at $t = 0$ and disconnected at $t = T$. Find $E\left\{y^2(t)\right\}$.

Solution

Given, $h(t) = Ae^{-\alpha t}U(t)$, $R_x(\tau) = N\delta(\tau)$, applied at $t = 0$ and disconnected at $t = T$. Also $q(t) = N$ for $0 < t < T$ and 0 otherwise.

For $0 < t < T$

For $0 < t < T$, $E\left\{y^2(t)\right\}$ is given as,

$$E\left\{y^2(t)\right\} = N\int_0^t h^2(\tau)d\tau \tag{1}$$
$$= NA^2\int_0^t e^{-2\alpha\tau}d\tau \tag{2}$$
$$= \frac{NA^2}{2\alpha}(1 - e^{-2\alpha t}) \tag{3}$$

For $t \geq T$

For $t \geq T$,given $q(t) = 0$. So, $E\left\{y^2(t)\right\}$ is given as,

$$
\begin{aligned}
E\left\{y^2(t)\right\} &= q(t)\int_0^t h^2(\tau)d\tau && (4)\\
&= q(t)\int_0^T h^2(\tau)d\tau + q(t)\int_T^t h^2(\tau)d\tau && (5)\\
&= NA^2\int_0^T e^{-2\alpha\tau}d\tau + 0 && (6)\\
&= \frac{NA^2}{2\alpha}(1 - e^{-2\alpha T}) && (7)
\end{aligned}
$$

$E\left\{y^2(t)\right\}$

In the above cases $U(t)$ is taken as 1 as t is positive. Therefore,

$$E\left\{y^2(t)\right\} = \begin{cases} \frac{NA^2}{2\alpha}(1 - e^{-2\alpha t}), & 0 < t < T \\ \frac{NA^2}{2\alpha}(1 - e^{-2\alpha T}), & T \leq t \end{cases}$$