AI1110 Assignment-1
ICSE Class-10 2017

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Q4(a): What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x+1$ as a factor?
Solution: Let $p(x) = 16x^3 - 8x^2 + 4x + 7$ and $d(x) = 2x+1$.
Polynomial division of $p(x)$ with $d(x)$:

$$\begin{array}{r|rrrr}
 & & 8x^2 & -8x & +6 \\
\hline
2x+1 & 16x^3 & -8x^2 & +4x & +7 \\
 & -16x^3 & -8x^2 & & \\
\hline
 & & -16x^2 & +4x & \\
 & & 16x^2 & +8x & \\
\hline
 & & & 12x & +7 \\
 & & & -12x & -6 \\
\hline
 & & & & 1
\end{array}$$

From the above division, it is clear that 1 has to be subtracted from the polynomial $p(x)$, so that $d(x)$ becomes factor of the resulting polynomial after subtraction.

Using remainder theorem:
Remainder theorem states that remainder of division of a polynomial $f(x)$ by a linear polynomial $x-r$ is equal to $f(r)$.So, remainder of $p(x)$ divided by $d(x)$ is $p\left(\frac{-1}{2}\right)$.

$$p(x) \mod (2x+1) = p\left(\frac{-1}{2}\right) \tag{1}$$

$$p\left(\frac{-1}{2}\right) = 16\left(\frac{-1}{2}\right)^3 - 8\left(\frac{-1}{2}\right)^2 + 4\left(\frac{-1}{2}\right) + 7 \tag{2}$$

$$= 1 \tag{3}$$

Therefor, the remainder is:

$$p(x) \mod (2x+1) = 1 \tag{4}$$

So, subtracting 1 from the given polynomial $p(x)$ gives a polynomial which has $2x+1$ as its factor.