AI1110: Assignment 9

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***Abstract*—This document contains the solution to Question of Chapter 8 in the Papoullis Textbook.**

Chapter 6 Ex 8.14: A coin is tossed once, and heads shows. Assuming that the probability p of heads is the value of the random variable **p** uniformly distributed in the interval (0.4,0.6), find the bayesian estimate.

Solution: Given that the probability is uniformly distributed in the interval (0.4,0.6). So, in this interval,

$$f(p)(0.6-0.4)=1 \tag{1}$$
$$f(p)=5 \tag{2}$$

Therefore, $f(p)$ is defined in the following way,

$$f(p)=\begin{cases}5, & 0.4<p<0.6\\ 0, & otherwise\end{cases}$$

The estimate $\hat{p}$ is given by,

$$\hat{p}=\int_{0.4}^{0.6} pf(p)dp \tag{3}$$
$$=5\int_{0.4}^{0.6} pdp \tag{4}$$
$$=0.5 \tag{5}$$

The posterior density $f(p|M)$ in the interval (0.4,0.6), when M is 1 (because here it is only one head), is given by,

$$f(p|1)=\frac{pf(p)}{\int_{0.4}^{0.6} pf(p)dp} \tag{6}$$
$$=\frac{(p)(5)}{0.5} \tag{7}$$
$$=10p \tag{8}$$

Therefore, $f(p|1)$ is defined in the following way,

$$f(p|1)=\begin{cases}10p, & 0.4<p<0.6\\ 0, & otherwise\end{cases}$$

The updated estimate $\hat{p}$ of p is the conditional estimate of **p** assuming M, which is given by,

$$\hat{p}=\int_{0.4}^{0.6} pf(p|1)dp \tag{9}$$
$$=10\int_{0.4}^{0.6} p^2dp \tag{10}$$
$$=0.5067 \tag{11}$$