Assignment 7
Papoullis Textbook Chapter 6 Example 13

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L^AT_EX

Outline

Question

Let $z = x^2 + y^2$. Determine $f_z(z)$.

Solution

$F_z(z)$ can be defined in the following way,

$$F_z(z) = P\left\{x^2 + y^2 \leq z\right\} \tag{1}$$

$$= \int\int_{x^2+y^2 \leq z} f_{xy}(x, y)dxdy \tag{2}$$

Solving

Here, $x^2 + y^2 \leq z$ represents the area of a circle and the radius of the circle is $\sqrt{z}$. Therefore,

$$F_z(z) = \int_{y=-\sqrt{z}}^{\sqrt{z}} \int_{x=-\sqrt{z-y^2}}^{\sqrt{z-y^2}} f_{xy}(x,y)dxdy \tag{3}$$

$f_z(z)$

From the above equation we get,

$$f_z(z) = \int_{-\sqrt{z}}^{\sqrt{z}} \frac{1}{2\sqrt{z-y^2}} \left\{ f_{xy}(\sqrt{z-y^2}, y) + f_{xy}(-\sqrt{z-y^2}, y) \right\} dy \quad (4)$$