Assignment 9
Papoullis Textbook Chapter 8 Ex 8.14

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Outline

Question

A coin is tossed once, and heads shows. Assuming that the probability p of heads is the value of the random variable $\mathbf{p}$ uniformly distributed in the interval (0.4,0.6), find the bayesian estimate.

Solution

Given that the probability is uniformly distributed in the interval (0.4,0.6). So, in this interval,

$$f(p)(0.6 - 0.4) = 1 \tag{1}$$

$$f(p) = 5 \tag{2}$$

$f(p)$

Therefore, $f(p)$ is defined in the following way,

$$f(p) = \begin{cases} 5, & 0.4 < p < 0.6 \\ 0, & \textit{otherwise} \end{cases}$$

$\hat{p}$

The estimate $\hat{p}$ is given by,

$$\hat{p} = \int_{0.4}^{0.6} pf(p)dp \tag{3}$$

$$= 5\int_{0.4}^{0.6} pdp \tag{4}$$

$$= 0.5 \tag{5}$$

Solving

The posterior density $f(p|M)$ in the interval (0.4,0.6), when M is 1 (because here it is only one head), is given by,

$$f(p|1) = \frac{pf(p)}{\int_{0.4}^{0.6} pf(p)dp} \tag{6}$$

$$= \frac{(p)(5)}{0.5} \tag{7}$$

$$= 10p \tag{8}$$

$f(p|1)$

Therefore, $f(p|1)$ is defined in the following way,

$$f(p|1) = \begin{cases} 10p, & 0.4 < p < 0.6 \\ 0, & otherwise \end{cases}$$

Updated $\hat{p}$

The updated estimate $\hat{p}$ of p is the conditional estimate of $\mathbf{p}$ assuming M, which is given by,

$$\hat{p} = \int_{0.4}^{0.6} p f(p|1) dp \tag{9}$$

$$= 10 \int_{0.4}^{0.6} p^2 dp \tag{10}$$

$$= 0.5067 \tag{11}$$