

AI1110: Assignment 9

Rishi Manoj - CS21BTECH11045

Abstract—This document contains the solution to Question of Chapter 8 in the Papoullis Textbook.

Chapter 8 Ex 8.14: A coin is tossed once, and heads shows. Assuming that the probability p of heads is the value of the random variable \mathbf{p} uniformly distributed in the interval $(0.4, 0.6)$, find the bayesian estimate.

Solution: Given that the probability is uniformly distributed in the interval $(0.4, 0.6)$. So, in this interval,

$$f(p)(0.6 - 0.4) = 1 \quad (1)$$

$$f(p) = 5 \quad (2)$$

Therefore, $f(p)$ is defined in the following way,

$$f(p) = \begin{cases} 5, & 0.4 < p < 0.6 \\ 0, & otherwise \end{cases}$$

The estimate \hat{p} is given by,

$$\hat{p} = \int_{0.4}^{0.6} pf(p)dp \quad (3)$$

$$= 5 \int_{0.4}^{0.6} p dp \quad (4)$$

$$= 0.5 \quad (5)$$

The posterior density $f(p|M)$ in the interval $(0.4, 0.6)$, when M is 1 (because here it is only one head), is given by,

$$f(p|1) = \frac{pf(p)}{\int_{0.4}^{0.6} pf(p)dp} \quad (6)$$

$$= \frac{(p)(5)}{0.5} \quad (7)$$

$$= 10p \quad (8)$$

Therefore, $f(p|1)$ is defined in the following way,

$$f(p|1) = \begin{cases} 10p, & 0.4 < p < 0.6 \\ 0, & otherwise \end{cases}$$

The updated estimate \hat{p} of p is the conditional

estimate of \mathbf{p} assuming M , which is given by,

$$\hat{p} = \int_{0.4}^{0.6} pf(p|1)dp \quad (9)$$

$$= 10 \int_{0.4}^{0.6} p^2 dp \quad (10)$$

$$= 0.5067 \quad (11)$$