

# AI1110: Assignment 7

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**Abstract**—This document contains the solution to Question of Chapter 6 in the Papoullis Textbook.

**Chapter 6 Ex 13:** Let  $z = x^2 + y^2$ . Determine  $f_z(z)$ .

**Solution:**  $F_z(z)$  can be defined in the following way,

$$F_z(z) = P \{x^2 + y^2 \leq z\} \quad (1)$$

$$= \int \int_{x^2 + y^2 \leq z} f_{xy}(x, y) dx dy \quad (2)$$

Here,  $x^2 + y^2 \leq z$  represents the area of a circle and the radius of the circle is  $\sqrt{z}$ . Therefore,

$$F_z(z) = \int_{y=-\sqrt{z}}^{\sqrt{z}} \int_{x=-\sqrt{z-y^2}}^{\sqrt{z-y^2}} f_{xy}(x, y) dx dy \quad (3)$$

From this we get,

$$f_z(z) = \int_{-\sqrt{z}}^{\sqrt{z}} \frac{1}{2\sqrt{z-y^2}} \left\{ f_{xy}(\sqrt{z-y^2}, y) + f_{xy}(-\sqrt{z-y^2}, y) \right\} dy \quad (4)$$