# 04-06-2023 shift-2

AI24BTECH11020 - Rishika

1) If gcd $(m,n)=1$ and $1^2-2^2+3^2-4^2+\ldots+(2021)^2-(2022)^2+(2023)^2=1012m^2n$ then $m^2-n^2$ is equal to:
   a) 180
   b) 220
   c) 200
   d) 240
2) The area bounded by the curves $y=|x-1|+|x-2|$ and $y=3$ is equal to:
   a) 5
   b) 4
   c) 6
   d) 3
3) For the system of equations
   $x+y+z=6$
   $x+2y+\alpha z=10$
   $x+3y+5z=\beta$, which one of the following is NOT true :
   a) System has a unique solution for $\alpha=3,\beta\neq 14$.
   b) System has a unique solution for $\alpha=-3,\beta=14$.
   c) System has no solution for $\alpha=3,\beta=24$.
   d) System has infinitely many solutions for $\alpha=3,\beta=14$.
4) Among the statements :
   $(S1)$: $(p\Rightarrow q)\vee((\sim p)\wedge q)$ is a tautology
   $(S2)$: $(q\Rightarrow p)\Rightarrow((\sim p)\wedge q)$ is a contradiction
   a) only $(S2)$ is True
   b) only $(S1)$ is True
   c) neither $(S1)$ and $(S2)$ is True
   d) both $(S1)$ and $(S2)$ are True
5) $\lim_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\cdots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$ is equal to:
   a) $\frac{1}{\sqrt{2}}$
   b) $\sqrt{2}$
   c) 1
   d) 0
6) Let P be a square matrix such that $P^2=I-P$.For $\alpha,\beta,\gamma,\delta\in\mathbb{N}$, if $P^\alpha+P^\beta=\gamma I-29P$ and $P^\alpha-P^\beta=\delta I-13P$, then $\alpha+\beta+\gamma-\delta$ is equal to:
   a) 40
   b) 22
   c) 24
   d) 18
7) A plane $P$ contains the line of intersection of the plane $\vec{r}(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}(2\hat{i}+3\hat{j}+4\hat{k})=-5$. If $P$ passes through the point $(0,2,-2)$, then the square of distance of the point $(12,12,18)$ from the plane $P$ is :

a) 620
b) 1240
c) 310
d) 155

8) Let $f(x)$ be a function satisfying $f(x) + f(\pi - x) = \pi^2, \forall x \in \mathbb{R}$. Then $\int_0^\pi f(x)\sin x dx$ is equal to :
a) $\frac{\pi^2}{2}$
b) $\pi^2$
c) $2\pi^2$
d) $\frac{\pi^2}{4}$

9) If the coefficients of $x^7$ in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ and $x^{-7}$ in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ are equal, then :
a) $64ab = 243$
b) $32ab = 729$
c) $729ab = 32$
d) $243ab = 64$

10) If the tangents at the points **P** and **Q** are the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $\mathbf{R}\left(\frac{9}{4}, 2\right)$, then the area of the triangle $PQR$ is :
a) $\frac{5}{4}$
b) $\frac{13}{4}$
c) $\frac{5}{8}$
d) $\frac{13}{8}$

11) Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$ ,where $p$ and $q$ are co-prime, then $q - p$ is equal to
a) 1
b) 2
c) 4
d) 3

12) In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is $\alpha$ and the number of persons who speak only Hindi is $\beta$ then the eccentricity of the ellipse $25\left(\beta^2 x^2 + \alpha^2 y^2\right) = \alpha^2\beta^2$ is :
a) $\frac{\sqrt{129}}{12}$
b) $\frac{\sqrt{117}}{12}$
c) $\frac{\sqrt{119}}{12}$
d) $\frac{3\sqrt{15}}{12}$

13) If the solution curve $f(x, y) = 0$ of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y, x > 0$, passes through the points $(1, 0)$ and $(\alpha, 2)$, then $\alpha^\alpha$ is equal to :
a) $e^{\sqrt{2}e^2}$
b) $e^{e^2}$
c) $e^{2e^{\sqrt{2}}}$
d) $e^{2e^2}$

14) Let the sets $A$ and $B$ denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{\lceil x \rceil - x}}$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to $x$. Then among the statements :
(S1) : $A \cap B = (1, \infty) - \mathbb{N}$ and
(S2) : $A \cup B = (1, \infty)$
a) only $(S1)$ is true
b) neither $(S1)$ nor $(S2)$ is true
c) only $(S2)$ is true

d) both $(S1)$ and $(S2)$ are true

15) Let $a \neq b$ be two-zero real numbers. Then the numbers of elements in the set $X = \{z \in \mathbb{C} : Re(az^2 + bz) = a \text{ and } Re(bz^2 + az) = b\}$ is equal to:

a) 0
b) 2
c) 1
d) 3