7-7.2-19

AI24BTECH11020 - RISHIKA KOTHA

Question:Equation of the circle with centre on the Y axis and passing through the origin and the point (2, 3) is

a)$3x^2 + 3y^2 - 13y = 0$

b)$3x^2 + 3y^2 + 13x + 3 = 0$

c)$6x^2 + 6y^2 - 13x = 0$

d)$x^2 + y^2 + 13x + 3 = 0$

Solution: from the given information,

$$x_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, x_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, n = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = 0 \tag{0.1}$$

$$\begin{pmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} -13 \\ 0 \\ 0 \end{pmatrix} \tag{0.2}$$

The augmented matrix is expressed as

$$\left(\begin{array}{ccc|c} 4 & 6 & 1 & -13 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array} \right) \tag{0.3}$$

performing sequences of row operations to transform into Echelon form

$$\xrightarrow{R_3 \to 4R_3 + R_1} \left(\begin{array}{ccc|c} 4 & 6 & 1 & -13 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 1 & -13 \end{array} \right) \xrightarrow{R_1 \to R_1 - R_3} \left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 1 & -13 \end{array} \right)$$

$$\xrightarrow{R_2 \to R_2 - R_3} \left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & -6 & 0 & 13 \\ 0 & 6 & 1 & -13 \end{array} \right) \xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & -6 & 0 & 13 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

$$\xrightarrow{R_1 \to R_1/4, R_2 \to R_2/-6} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -13/6 \\ 0 & 0 & 1 & 0 \end{array} \right) \tag{0.4}$$

$$u = \begin{pmatrix} 0 \\ -13/6 \end{pmatrix}, f = 0 \tag{0.5}$$

∴ the equation of the circle is $3x^2 + 3y^2 - 13y = 0$.

parameter	Description	value
C	Centre	$\begin{pmatrix} 0 \\ 13/6 \end{pmatrix}$
O	point1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
P	point2	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
r	radius	13/6