

MATGEO: 7-7.2-19

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Question

Equation of the circle with centre on the Y axis and passing through the origin and the point (2, 3) is

① $3x^2 + 3y^2 - 13y = 0$

② $3x^2 + 3y^2 + 13x + 3 = 0$

③ $6x^2 + 6y^2 - 13x = 0$

④ $x^2 + y^2 + 13x + 3 = 0$

(MATGEO 7-7.2-19)

Solution: Theory

parameter	Description	value
C	Centre	$\begin{pmatrix} 0 \\ 13/6 \end{pmatrix}$
O	point1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
P	point2	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
r	radius	13/6

Given Data

From the given information,

$$x_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, x_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, n = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = 0 \quad (1)$$

$$\begin{pmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ f \end{pmatrix} = \begin{pmatrix} -13 \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

The augmented matrix is expressed as

$$\left(\begin{array}{ccc|c} 4 & 6 & 1 & -13 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right) \quad (3)$$

Row Operations

performing sequences of row operations to transform into Echelon form

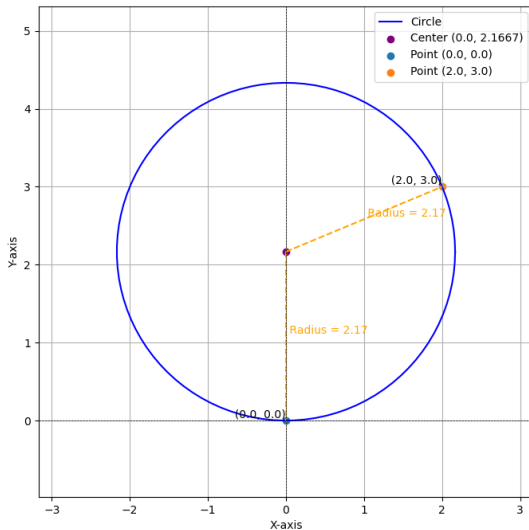
$$\xleftrightarrow{R_3 \rightarrow R_1 - 4R_3} \left(\begin{array}{ccc|c} 4 & 6 & 1 & -13 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 1 & -13 \end{array} \right) \xleftrightarrow{R_1 \rightarrow R_1 - R_3} \left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 6 & 1 & -13 \end{array} \right) \quad (4)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - R_3} \left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & -6 & 0 & 13 \\ 0 & 6 & 1 & -13 \end{array} \right) \xleftrightarrow{R_3 \rightarrow R_3 + R_2} \left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & -6 & 0 & 13 \\ 0 & 0 & 1 & 0 \end{array} \right) \quad (5)$$

$$\xleftrightarrow{R_1 \rightarrow R_1/4, R_2 \rightarrow R_2/-6} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -13/6 \\ 0 & 0 & 1 & 0 \end{array} \right) \quad (6)$$

$$u = \begin{pmatrix} 0 \\ -13/6 \end{pmatrix}, f = 0 \quad (7)$$

Graph



```
1 #include <stdio.h>
2 void find_u_and_f(float matrix[2][2], float rhs[2], float u[2], float *f) {
3     float factor;
4     if (matrix[0][0] != 0) {
5         factor = matrix[0][0];
6         for (int i = 0; i < 2; i++) {
7             matrix[0][i] /= factor;
8         }
9         rhs[0] /= factor;
10    }
11    factor = matrix[1][0];
12    for (int i = 0; i < 2; i++) {
13        matrix[1][i] -= factor * matrix[0][i];
14    }
15    rhs[1] -= factor * rhs[0];
16    if (matrix[1][1] != 0) {
17        factor = matrix[1][1];
18        matrix[1][1] /= factor;
19        rhs[1] /= factor;
20    }
21    u[1] = rhs[1];
22    u[0] = rhs[0] - matrix[0][1] * u[1];
23    *f = 0;
24    int main() {
25        float centerX = 0.0f, centerY = 2.1667f;
26        float radius = 2.1667f;
27        float points[2][2] = { {0.0f, 0.0f}, {2.0f, 3.0f} };
28        float *radiusPtr = &radius;
29        float matrix[2][2] = {{4, 6}, {0, 1}};
```

```
30 float rhs[2] = {-13, 0};
31 float u[2];
32 float f;
33 find_u_and_f(matrix, rhs, u, &f);
34 printf("u = [%.2f, %.2f]\n", u[0], u[1]);
35 printf("f = %.2f\n", f);
36 printf("Equation of the circle: 3x^2 + 3y^2 - 13y = 0\n");
37
38 FILE *file = fopen("data.txt", "w");
39 if (file == NULL) {
40     perror("Error opening file");
41     return 1;
42 }
43 fprintf(file, "Center: %.4f, %.4f\n", centerX, centerY);
44 fprintf(file, "Radius: %.4f\n", *radiusPtr);
45 fprintf(file, "Points: (%.1f, %.1f), (%.1f, %.1f)\n",
46         points[0][0], points[0][1], points[1][0], points[1][1]);
47 fclose(file);
48
49 printf("Data written to data.txt successfully.\n");
50 return 0;
51 }
```


Python Code

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 def read_data(file_name):
4     with open(file_name, 'r') as file:
5         lines = file.readlines()
6         center = tuple(map(float, lines[0].strip().split(':')[1].strip().split(',')))
7         radius = float(lines[1].strip().split(':')[1].strip())
8         points_line = lines[2].strip().split(':')[1].strip()
9         points = [tuple(map(float, p.strip().strip('{}').split(','))) for p in points_line.split(',')
10                    ('')]
11     return center, radius, points
12 center, radius, points = read_data('data.txt')
13 theta = np.linspace(0, 2 * np.pi, 100)
14 x_circle = radius * np.cos(theta) + center[0]
15 y_circle = radius * np.sin(theta) + center[1]
16 plt.figure(figsize=(8, 8))
17 plt.plot(x_circle, y_circle, label='Circle', color='blue')
18 plt.scatter(center[0], center[1], color='purple', label=f'Center {center}')
19 for point in points:
20     plt.scatter(*point, label=f'Point {point}')
21 for point in points:
22     plt.plot([center[0], point[0]], [center[1], point[1]], color='orange', linestyle='--')
23     mid_x = (center[0] + point[0]) / 2
24     mid_y = (center[1] + point[1]) / 2
25     plt.text(mid_x, mid_y, f'Radius = {radius:.2f}', fontsize=10, color='orange',
26              verticalalignment='bottom')
27 for point in points:
28     plt.text(point[0], point[1], f' {point}', fontsize=10, verticalalignment='bottom',
29              horizontalalignment='right')
30 plt.axis('equal')
31 plt.xlim(center[0] - radius - 1, center[0] + radius + 1)
32 plt.ylim(center[1] - radius - 1, center[1] + radius + 1)
33 plt.axhline(0, color='black', linewidth=0.5, ls='--')
34 plt.axvline(0, color='black', linewidth=0.5, ls='--')
35 plt.xlabel('X-axis')
36 plt.ylabel('Y-axis')
37 plt.legend()
38 plt.grid()
39 plt.show()
```