04-06-2023 shift-2

AI24BTECH11020 - Rishika

1) If the sum of the series $\frac{1}{1.(1+d)} + \frac{1}{(1+d)(1+2d)} + \cdots + \frac{1}{(1+9d)(1+10d)}$ is equal to 5,then 50d is equal to:
   a) 5
   b) 10
   c) 20
   d) 15
2) The solution of the differential equation $\left(x^2 + y^2\right)\mathrm{d}x\text{-}5xy\mathrm{d}y = 0,\ y(1) = 0$, is :
   a) $\left|x^2 - 2y^2\right|^6 = x$
   b) $\left|x^2 - 2y^2\right|^5 = x^2$
   c) $\left|x^2 - 4y^2\right|^6 = x$
   d) $\left|x^2 - 4y^2\right|^5 = x^2$
3) A variable line L passes through the point $(3, 5)$ and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is:
   a) 25
   b) 40
   c) 35
   d) 30
4) Let α,β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}\left(\alpha^6 + \beta^6\right)$, is :
   a) $x^2 - 180x + 9506 = 0$
   b) $x^2 - 195x + 9506 = 0$
   c) $x^2 - 195x + 9466 = 0$
   d) $x^2 - 190x + 9466 = 0$
5) Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that $f(1) = 40, f'(1) = 2andf''(1) = 4$. Then $a^2 + b^2 + c^2$ is equal to :
   a) 73
   b) 54
   c) 51
   d) 62
6) If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $\mathbb{R} - (\alpha,\beta)$. Then $12\alpha\beta$ is equal to:
   a) 36
   b) 32
   c) 24
   d) 40
7) Let three vectors $\overrightarrow{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}, \overrightarrow{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}, \overrightarrow{c} = x\hat{i} + y\hat{j} + z\hat{k}$ form a triangle such that $\overrightarrow{c} = \overrightarrow{a} - \overrightarrow{b}$ and the area of the triangle is $5\sqrt{6}$. If α is a positive real number, then $\left|\overrightarrow{c}\right|^2$ is equal to :
   a) 16
   b) 10
   c) 14
   d) 12

8) Let $|\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)| \leq \frac{1}{8}, \theta \in [0,2\pi]$. Find the sum of all $\theta \in [0,2\pi]$, where $\cos 3\theta$ attains its maximum value,is :
   a) 18π
   b) 9π
   c) 6π
   d) 15π
9) The coefficient of x^{70} in $x^2(1+x)^{98}+x^3(1+x)^{97}+x^4(1+x)^{96}+\cdots+x^{54}(1+x)^{46}$ is $\binom{99}{p}-\binom{46}{q}$. Then a possible value of $p+q$ is:
   a) 61
   b) 55
   c) 83
   d) 68
10) The shortest distance between the lines $\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$ and $\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$ is :
   a) $\frac{185}{\sqrt{563}}$
   b) $\frac{178}{\sqrt{563}}$
   c) $\frac{179}{\sqrt{563}}$
   d) $\frac{187}{\sqrt{563}}$
11) Let a circle passing through $(2,0)$ have its centre at the point (h,k). Let (x_c, y_c) be the point of intersection of the lines $3x+5y=1$ and $(2+c)x+5c^2y=1$. If $\lim\limits_{c\to1} x_c$ and $\lim\limits_{c\to1} y_c$, then the equation of the circle is :
   a) $25x^2+25y^2-2x+2y-60=0$
   b) $5x^2+5y^2-4x+2y-12=0$
   c) $25x^2+25y^2-20x+2y-60=0$
   d) $5x^2+5y^2-4x-2y-12=0$
12) Let $\lambda, \mu \in \mathbb{R}$. If the system of equations
   $3x+5y+\lambda z=3$
   $7x+11y-9z=2$
   $97x+155y+189z=\mu$
   has infinitely many solutions, then $\mu+2\lambda$ is equal to :
   a) 25
   b) 22
   c) 24
   d) 27
13) Let the line L intersect the lines $x-2=-y=z-1, 2(x+1)=2(y-1)=z+1$ and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L?
   a) $\left(\frac{-1}{3},-1,-1\right)$
   b) $\left(\frac{-1}{3},1,-1\right)$
   c) $\left(\frac{-1}{3},1,1\right)$
   d) $\left(\frac{-1}{3},-1,1\right)$
14) The frequency distribution of the age of students in a class of 40 students is given below. If the mean

Age	15	16	17	18	19	20
No of Students	5	8	5	12	x	y

   deviation about the median is 1.25, then $4x+5y$ is equal to:
   a) 47
   b) 43

c) 46
d) 44

15) The solution curve, of the differential equation $2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point $(0, 1)$ is a conic, whose vertex lies on the line :

a) $2x + 3y = 9$
b) $2x + 3y = 6$
c) $2x + 3y = -6$
d) $2x + 3y = -9$