

Example Lecture Scribe (structure only):

Lecture 0: Introduction to Logic

## 1. Definitions

Proposition: A declarative statement that is either true or false, but not both.

Notation: Denoted by lowercase letters  $p, q, r$ .

Compound Proposition: A proposition formed from existing propositions using logical operators.

## 2. Logical Operators

Negation ( $\neg p$ ): The statement "it is not the case that  $p$ ".

Truth Table: If  $p$  is True,  $\neg p$  is False.

## 3. Theorems and Proofs

Theorem 1 (De Morgan's Law):  $\neg(p \land q) \equiv \neg p \lor \neg q$

Proof:

Construct a truth table for both sides.

Observe that for all truth value assignments of  $p$  and  $q$ , the column for  $\neg(p \land q)$  is identical to  $\neg p \lor \neg q$ . Thus, the statements are logically equivalent.

## 4. Worked Example

Problem: Show that  $p \lor \neg p$  is a tautology.

Solution:

Step 1: Assign logical values.

Step 2: If  $p=T$ , then  $\neg p=F$ , so  $T \lor F = T$ .

Step 3: If  $p=F$ , then  $\neg p=T$ , so  $F \lor T = T$ .

Conclusion: In all cases, the result is True.

Instructions:

Using the same structure as the example above, generate a lecture scribe for the provided context (lecture slides/textbook).

Requirements:

Purpose: The scribe must serve as exam reference material.

Content: Include definitions, notation, assumptions, theorem statements, proofs, derivations, and worked examples exactly as presented in the lecture materials.

Faithfulness: Do not introduce new examples, creative explanations, or intuition not discussed in the context. Everything must be traceable to the provided files.

Format: Use clear Markdown headers, bullet points for definitions, and LaTeX formatting for all mathematical notation.

directly give me the latex code



latex

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\documentclass{article}
\usepackage{amsmath}
\usepackage{amssymb}
\usepackage{geometry}
\geometry{a4paper, margin=1in}

\title{CSE400: Fundamentals of Probability in Computing \& Lecture 8: Gaussian, Uniform, Exponential, and Gamma Random Variables}
\author{Lecture Scribe}
\date{January 29, 2026}

\begin{document}

\maketitle

\section{Continuous Random Variables: Moments}

\subsection{Central Moments}
The  $n$ -th order central moment is defined as the expectation of the random variable centered around its mean.
\begin{equation}
[cite_start]E[(x - \mu_X)^n] = \int_{-\infty}^{\infty} (x - \mu_X)^n f_X(x) dx \quad

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\text{[cite: 20, 51]}
\end{equation}

\textbf{Specific Moments:}
\begin{itemize}
    [cite_start]\item \textbf{Zeroth Moment ($n=0$):}  $E[(x - \mu_X)^0] = E[1] = 1$ [cite: 55].
    [cite_start]\item \textbf{First Moment ($n=1$):}  $E[(x - \mu_X)] = E[x] - \mu_x = 0$ [cite: 56, 58].
        \item \textbf{Second Moment ($n=2$):} Defined as the Variance ( $\sigma_X^2$ ).
            \begin{equation}
                [cite_start]\sigma_X^2 = E[(x - \mu_X)^2] = E[x^2] - \mu_X^2 \quad \text{[cite: 59, 61]}
            \end{equation}
        \end{itemize}
    \end{equation}
\end{itemize}

\subsection{Higher Order Moments (Shape)}
\begin{itemize}
    \item \textbf{Skewness ($n=3$):} A measure of the symmetry of the Probability Density Function (PDF).
        \begin{equation}
            [cite_start]C_s = \frac{E[(x - \mu_X)^3]}{\sigma_X^3} \quad \text{[cite: 81]}
        \end{equation}
        \begin{itemize}
            [cite_start]\item If  $C_s$  is positive (+), the PDF is Right Skewed[cite: 82].
            [cite_start]\item If  $C_s$  is negative (-), the PDF is Left Skewed[cite: 83].
        \end{itemize}
    \end{itemize}
    \item \textbf{Kurtosis ($n=4$):} A measure of the "tailedness" or peak of the distribution.
        \begin{equation}
            [cite_start]C_k = \frac{E[(x - \mu_X)^4]}{\sigma^4} \quad \text{[cite: 86, 341]}
        \end{equation}
        [cite_start]A large value indicates the random variable will have a large peak near the mean[cite: 88, 89].
    \end{itemize}
\end{itemize}

\section{Theorems on Expectation}

\subsection{Linearity of Expectation}
For any constants  $a$  and  $b$ :
\begin{equation}
    [cite_start]E[ax + b] = a E[x] + b \quad \text{[cite: 94]}
\end{equation}

\subsection{Sum of Functions}
For a sum of several functions  $g(x) = g_1(x) + g_2(x) + \dots + g_N(x)$ :
\begin{equation}
    [cite_start]E[g(x)] = E[g_1(x) + g_2(x) + \dots + g_N(x)] = E[g_1(x)] + E[g_2(x)] + \dots + E[g_N(x)]
\end{equation}

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[cite_start]E\left[\sum_{k=1}^N g_k(x)\right] = \sum_{k=1}^N E[g_k(x)] \quad
\text{[cite: 97]}
\end{equation}

\section{Gaussian Random Variable}

\subsection{Definition}
A Gaussian random variable  $X \sim \mathcal{N}(\mu, \sigma^2)$  is defined by the PDF:
\begin{equation}
[cite_start]f_X(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{[cite: 119]}
\end{equation}
Where  $\mu$  is the mean and  $\sigma^2$  is the variance. [cite_start]If  $\mu=0$  and  $\sigma^2=1$ , it is a Standard Normal Distribution[cite: 121].

\subsection{Standard Functions}
Several standard integrals are used to evaluate Gaussian probabilities:
\begin{itemize}
\item \textbf{Error Function:}

$$[cite_start]\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad \text{[cite: 166].}$$

\item \textbf{Complementary Error Function:}

$$[cite_start]\text{erfc}(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt \quad \text{[cite: 167].}$$

\item \textbf{\$Phi\$-function (CDF of Standard Normal):} Represents the area under the left tail.

$$[cite_start]\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt \quad \text{[cite: 175, 250]}$$


\item \textbf{Q-function (Gaussian Tail Function):} Represents the area under the right tail.

$$[cite_start]Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \quad \text{[cite: 176, 254]}$$


\end{itemize}

\subsection{Key Identities and Relations}
\begin{itemize}
\item  $Q(x) = 1 - \Phi(x)$  [cite: 214].
 $\Phi(-x) = 1 - \Phi(x)$  [cite: 305].
\item The CDF of a general Gaussian variable can be expressed as:

$$[cite_start]F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) = 1 - Q\left(\frac{x-\mu}{\sigma}\right) \quad \text{[cite: 211, 214]}$$


\item Tail probabilities for a general Gaussian variable:

$$[cite_start]P(X > x) = Q\left(\frac{x-\mu}{\sigma}\right)$$




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[cite_start]Pr(X > x) = Q\left(\frac{x - m}{\sigma}\right) \quad \text{[cite: 205]}
\end{equation}
\end{itemize}

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## \section{Worked Example 1: Gaussian Probabilities}

\textbf{Problem:}

A random variable  $X$  has a PDF given by:

$$f_X(x) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x+3)^2}{8}}$$

[cite\_start]Find probabilities in terms of Q-functions[cite: 262].

\textbf{Solution:}

\begin{enumerate}

\item \textbf{Identify parameters:}

Comparing to the standard form  $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$ , we get:

[cite\_start]\$\$ m = -3, \quad \sigma^2 = 4 \Rightarrow \sigma = 2 \quad \text{[cite: 287]}

\$\$

\item \textbf{Calculate  $\Pr(X \leq 0)$ :

[cite\_start]\$\$ \Pr(X \leq 0) = F\_X(0) = \Phi\left(\frac{0 - (-3)}{2}\right) = \Phi(1.5) = 1 - Q(1.5) \quad \text{[cite: 297]}

\item \textbf{Calculate  $\Pr(X > 4)$ :

[cite\_start]\$\$ \Pr(X > 4) = Q\left(\frac{4 - (-3)}{2}\right) = Q(3.5) \quad \text{[cite: 298]}

\item \textbf{Calculate  $\Pr(|X+3| < 2)$ :

\$\$ |X+3| \quad \text{[cite\_start]} < 2 \Rightarrow -2 < X+3 < 2 \Rightarrow -5 < X < -1 \quad \text{[cite: 303]}

\$\$

Pr(-5 < X < -1) = F\_X(-1) - F\_X(-5) = \Phi(1) - \Phi(-1) \$\$

Using  $\Phi(-1) = 1 - \Phi(1)$ :

[cite\_start]\$\$ = \Phi(1) - (1 - \Phi(1)) = 2\Phi(1) - 1 = 1 - 2Q(1) \quad \text{[cite: 307]}

\item \textbf{Calculate  $\Pr(|X-2| > 1)$ :

\$\$ |X-2| > 1 \Rightarrow X < 1 \text{ or } X > 3 \quad \text{[cite: 308]}

\$\$ Pr = F\_X(1) + Pr(X > 3) \$\$

\$\$ F\_X(1) = \Phi\left(\frac{1 - (-3)}{2}\right) = \Phi(2) = 1 - Q(2) \$\$

\$\$ Pr(X > 3) = Q\left(\frac{3 - (-3)}{2}\right) = Q(3) \$\$

[cite\_start]\$\$ \text{Total} = 1 - Q(2) + Q(3) \quad \text{[cite: 309]}

\end{enumerate}

## \section{Worked Example 2: CDF Analysis}

\textbf{Problem:}

The CDF of a random variable is given by:

$$F_X(x) = \begin{cases} 0, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

[cite\_start]Find the Mean, Variance, Skewness, and Kurtosis[cite: 322, 323].

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\textbf{Solution:}
\begin{enumerate}
    \item \textbf{Find PDF:}
        [cite_start]$$ f_X(x) = \frac{d}{dx}F_X(x) = 2x, \quad 0 \leq x \leq 1 \quad \text{[cite: 333]} $$

    \item \textbf{Calculate Mean ( $\mu_X$ ):}
        [cite_start]$$ \mu_X = \int_0^1 x(2x) dx = \int_0^1 2x^2 dx = \left[\frac{2x^3}{3}\right]_0^1 = \frac{2}{3} \quad \text{[cite: 333]} $$

    \item \textbf{Calculate Variance ( $\sigma_X^2$ ):}
        $$ \sigma_X^2 = \int_0^1 x^2(2x) dx - \mu_X^2 = \int_0^1 2x^3 dx - \left(\frac{2}{3}\right)^2 $$
        [cite_start]$$ = \left[\frac{2x^4}{4}\right]_0^1 - \frac{4}{9} = \frac{1}{2} - \frac{4}{9} = \frac{9-8}{18} = \frac{1}{18} \quad \text{[cite: 340]} $$

    \item \textbf{Skewness and Kurtosis:}
        [cite_start]$$ C_s = \frac{\text{E}[(X-\mu)^3]}{\sigma^3} = -\frac{2\sqrt{2}}{5} \quad \text{[Distribution is left-skewed] [cite: 341, 343]} $$
        [cite_start]$$ C_k = \frac{\text{E}[(X-\mu)^4]}{\sigma^4} = \frac{12}{5} \quad \text{[Platykurtic] [cite: 341, 344]} $$
\end{enumerate}

\section{Applications}
Gaussian Random Variables are used to model:
\begin{itemize}
    - [cite_start]\item \textbf{Thermal Noise:} Noise voltage in electronic circuits[cite: 314].
- [cite_start]\item \textbf{Measurement Error:} Sensor readings where Measured Value = True Value + Gaussian noise[cite: 315].
- [cite_start]\item \textbf{Jitter:} Packet delay variation in communication networks[cite: 316].
- [cite_start]\item \textbf{Density Estimation:} Using sample statistics (histogram) to estimate the distribution parameters  $\hat{\mu}$  and  $\hat{\sigma}$ [cite: 397, 404].

\end{itemize}
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