Lecture Scribe: Joint Probability and Conditional Probability
Course: CSE 400 — Fundamentals of Probability in Computing

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1 Topic Title

Joint Probability and Conditional Probability

2 Definitions and Notation

- **Experiment (E):** A procedure performed that produces a specific result.
  Example: Tossing a coin five times (E_5).

- **Outcome (ξ):** A possible result of an experiment.
  Example: One possible outcome of E_5 is $\xi_1 = HHTHT$.

- **Sample Space (S):** The set of all possible outcomes of an experiment.

- **Event:** A subset of the sample space.

- **Joint Probability:** The probability of two or more events occurring simultaneously.

- **Conditional Probability:** The probability of an event occurring given that another event has already occurred.

3 Assumptions / Conditions

Axioms of Probability: All probability assignments must satisfy the fundamental axioms:

1. **Non-negativity:** $P(A) \geq 0$ for any event A.

2. **Normalization:** $P(S) = 1$.

3. **Additivity:** For mutually exclusive events, the probability of their union is the sum of their individual probabilities.

Probability Assignment Approaches:

- **Classical Approach:** Assumes all outcomes in a finite sample space are equally likely.

- **Relative Frequency Approach:** Based on the limit of the frequency of an outcome over many trials.

4   Main Results / Theorems

- **Joint Probability Notation:** Represented as $P(A \cap B)$ or $P(A, B)$, indicating the probability that both Event A and Event B occur.

- **Conditional Probability Formula:** The probability of event A given event B is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) > 0$$

5   Proofs / Derivations

Note: Focus is on conceptual motivation in engineering contexts (e.g., Speech Recognition, Radar Systems).

Derivation of Multiplication Rule

1. **Step 1:** Start with the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

2. **Step 2:** Rearrange the formula to solve for the joint probability by multiplying both sides by $P(B)$.

3. **Step 3:** The resulting **Multiplication Rule** is:

$$P(A \cap B) = P(A|B)P(B)$$

6   Worked Examples

- **Example 1: Card Deck (Joint Probability):** Calculation of probabilities involving specific suits and ranks from a standard deck.

- **Example 2: Costume Party (Joint Probability):** Determining the likelihood of overlapping characteristics among party guests.

- **Example 3: Cards Without Replacement (Conditional Probability):** Calculating the probability of drawing a specific sequence of cards when the first card is not returned to the deck.

- **Example 4: Game of Poker:** Applying conditional probability to determine the strength of a hand as more cards are revealed.

- **Example 5: The Missing Key:** A logical probability problem determining the likelihood of finding a key in a specific location given it was not found in others.

Reference: Lecture 4 - Joint Probability and Conditional Probability by Dhaval Patel, PhD.