Lecture_8

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L8_S1_A - PDF

Example Le...ure Scribe - PDF

Task Instructions: Using the same structure as the example provided above, generate a comprehensive lecture scribe based on the attached context for Lecture 4 (L4.pdf). This scribe is authored by Vansh Lilani (ID: AU2320146). It is intended to serve as a primary exam-oriented reference material for a reading-based/closed-notes exam.

Requirements:

Faithfulness & Detail: Include all definitions, notation, and assumptions exactly as presented. Every formula, theorem statement, and worked example from the lecture must be included in full.

Step-by-Step Explanations: For all proofs and derivations, provide the complete step-by-step logic shown in the slides. Do not summarize; ensure the explanation covers the transition between each mathematical step to ensure exam readiness.

Scope: Stick strictly to the provided PDF. Do not invent new content or add external "intuition" not present in the lecture materials.

Correctness & Tone: Maintain a formal academic tone. Prioritize mathematical precision and clarity in the layout of formulas.

Structure: Adhere strictly to the organizational pattern (headings, lists, and layout) demonstrated in the "Example Lecture Scribe" provided above.

1. Topic Title

Lecture 8: Gaussian, Uniform, Exponential, and Gamma Random Variables CSE 400: Fundamentals of Probability in Computing Vansh Lilani - AU2320146

+2

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2. Definitions and Notation

- **Continuous Random Variable (CRV)**: Defined by its Probability Density Function (PDF) and Cumulative Distribution Function (CDF).
- **Gaussian Random Variable**: A random variable X is Gaussian if its PDF is given by: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$ +2
- **Notation**: $X \sim N\left(m, \sigma^2\right)$, where m is the mean (μ_X) and σ^2 is the variance. +1
- **Standard Normal Distribution**: A Gaussian distribution where $m = 0$ and $\sigma^2 = 1$.
- ** n^{th} order Central Moment**: Defined as $E\left[(X - \mu_X)^n\right]$. +1
- **Skewness (C_s)**: A measure of the symmetry of the PDF, defined as $C_s = \frac{E\left[(X-\mu_X)^3\right]}{\sigma_{X3}}$.
- **Kurtosis (C_k)**: A measure of the "peakiness" of the PDF, defined as $C_k = E\left[(X - \mu_X)^4\right]$. +1
- **Error Function ($erf(x)$)**: Defined as $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right) dt$.
- **Complementary Error Function ($erfc(x)$)**: Defined as $erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp\left(-t^2\right) dt$.

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3. Assumptions / Conditions

- **PDF Normalization**: For any valid PDF, the total area under the curve must equal 1: $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$.
- **Constants in Expectation**: When calculating expectations, a and b are treated as constants.
- **Linearity of Expectation**: Assumes that the expectation of a sum of functions is equal to the sum of their individual expectations.

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4. Main Results / Theorems

- **Theorem on Linear Transformation of Expectation**: For any constant a and b , $E[ax+b] = aE[X] + b$. +1
- **Theorem on Sum of Functions**: For any function $g(x)$ that is a sum of several other functions $g_k(x)$, the expectation is: $E\left[\sum_{k=1}^{N} g_k(x)\right] = \sum_{k=1}^{N} E[g_k(x)]$.
- **Symmetry and Skewness**:
  - If $C_s > 0$, the PDF is right-skewed.
  - If $C_s < 0$, the PDF is left-skewed.
- **Kurtosis Significance**: A large value of Kurtosis indicates the random variable X will have a large peak near the mean.

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5. Proofs / Derivations

Derivation of Central Moments ($n = 0, 1, 2$)

- **For $n = 0$ **: $E\left[(X - \mu_X)^0\right] = E[1] = 1$
- **For $n = 1$ **: Step 1: Expand the expectation: $E[X - \mu_X] = E[X] - \mu_X$. Step 2: Substitute μ_X for $E[X]$: $\mu_X - \mu_X = 0$. *Result*: The first central moment is always 0. +2

- **For $n = 2$ (Variance σ_{X2})**: Step 1: Start with the definition: $\sigma_{X2} = E\left[(X - \mu_X)^2\right]$. Step 2: Expand the square: $E\left[X^2 - 2\mu_X X + \mu_{X2}\right]$. Step 3: Apply linearity of expectation: $E\left[X^2\right] - 2\mu_X E\left[X\right] + \mu_{X2}$. Step 4: Substitute $E\left[X\right] = \mu_X$: $E\left[X^2\right] - 2\mu_{X2} + \mu_{X2}$. Step 5: Simplify: $\sigma_{X2} = E\left[X^2\right] - \mu_{X2}$. +2

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6. Worked Examples

Example 1: Gaussian PDF and CDF Parameters

Consider a Gaussian Random Variable with mean $m = 3$ and standard deviation $\sigma = 2$.

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- **PDF**: $f_X\left(x\right) = \frac{1}{\sqrt{2\pi(2^2)}} \exp\left(-\frac{(x-3)^2}{2(2^2)}\right) = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x-3)^2}{8}\right)$. +3
- **CDF**: $F_X\left(x\right) = Pr\left(X \leq x\right)$, represented as an S-shaped curve centered at $x = 3$. +2

Example 2: Linearity of Expectation with Constants

If $Y = aX + b$, find $E\left[Y\right]$.

- Step 1: $E\left[Y\right] = E\left[aX + b\right]$.
- Step 2: Using the theorem $E\left[ax + b\right] = aE\left[X\right] + b$, the result is directly obtained as $a\mu_X + b$.