Lecture Scribe Generation Instructions

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Purpose is to generate exam-oriented lecture notes (lecture scribe) strictly from the provided lecture PPT and my notes. The scribe must serve as a precise reference for exams.

Scope is to use only the content explicitly present in the provided material. Do not add external explanations, background knowledge, interpretations or examples. If a definition, assumption, example, derivation, or proof is not present in the material, do not include it.

Structural constraint :

Follow the exact structure and order of the lecture PPT. Preserve topic sequence and hierarchy.

Example Lecture Scribe (structure only)
Topic:
Sample Topic Title

Definition:
A formal definition as stated in the lecture.

Assumptions:
List of assumptions used.

Key Results:
• Statement of a theorem or result.

Proof:

Step 1: First step as presented.
Step 2: Second step as presented.

Conclusion

Example:
A short worked example exactly as shown in the lecture.

Instruction

Using the same structure as the example above, generate the lecture scribe for the provided lecture PPT and notes.
The scribe must:
• Be structured exactly like the example
• Include definitions, assumptions, results, proofs, derivations, and worked examples only if they appear in the provided material
• Use simple, clear language while preserving correctness
• Remain fully aligned with the lecture content and ordering

What a lecture scribe is?
A lecture scribe is a faithful, exam-ready reconstruction of what was taught in
class, written so that a student can later rely on it as reference material while studying.
A correct lecture scribe should allow a student to:
• Recall definitions and notation
• Reconstruct proofs and derivations
• Follow example solutions step by step
• Understand logical dependencies between concepts
A lecture scribe should answer the question:
"If I study only this document, can I reliably revise what was taught?"

What it's not.
A lecture scribe is not:

• A creative explanation
• A simplified tutorial
• A textbook rewrite
• A solution manual for unseen problems
• A place to add intuition not discussed in class
Everything included must be traceable to the provided context

Topic: Engineering Applications

Key Results:

- **Speech Recognition System:**
  - The system uses vocabulary sets (e.g., Hello, Yes, No, Bye) and templates to match signals .
  - The input signal $x(t)$ is processed into word representations $x(w)$.
  - Templates must account for variations such as different speakers (male, female, child) and noise or interference .
- **Radar System:**
  - The system operates on hypothesis testing:
    - H_0: No target present ($Y_i = W_i'$).
    - H_1: Target present ($Y_i = S_i + W_i$).
  - Outcomes are classified as:
    - False Alarm.
    - Miss Detect (P_M).
  - Relationship: $P_D + P_M = 1$.
- **Communication Network:**
  - Networks (e.g., Wi-Fi 802.11 a/b/g/n/ac/ax) operate across different frequency bands (2.4 GHz, 5 GHz, 6 GHz) .
  - Quality of Service (QoS) metrics include delay and latency.

Topic: Introduction to Probability Theory

Definition:

- **Experiment (E):** A procedure performed that produces some result.
  - *Example:* Tossing a coin five times (E_5).
- **Outcome (ξ):** A possible result of an experiment.
  - *Example:* One outcome of E_5 is $\xi_1 = HHTHT$.
- **Event:** A certain set of outcomes of an experiment.
  - *Example:* Event $C =$ {all outcomes consisting of an even number of heads} .
- **Sample Space (S):** The collection or set of "all possible" distinct outcomes. These outcomes must be:
  - **Mutually Exclusive:** You can get heads or tails, but not both .
  - **Collectively Exhaustive:** You cannot get anything other than heads or tails.
  - S is the universal set of outcomes and can be Discrete, Countably infinite, or Continuous .

Examples:

- Flipping a fair coin once.
- Rolling two dice.
- Random number generator with interval $[0, 1)$ (Continuous sample space) .

Topic: Axioms of Probability

Definition:

- **Probability:** A measure of the likelihood of various events, or a function of an event that produces a numerical quantity measuring the likelihood of that event.

Key Results:

- **Axiom 1:** For any event A, $0 \leq Pr(A) \leq 1$.
- **Axiom 2:** If S is the sample space for a given experiment, $Pr(S) = 1$.

- **Axiom 3:** If $A \cap B = \emptyset$ (mutually exclusive), then $Pr(A \cup B) = Pr(A) + Pr(B)$.
  - For an infinite number of mutually exclusive sets A_i where $A_i \cap A_j = \emptyset$ for all $i \neq j$:
    $Pr(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} Pr(A_i)$.

Topic: Corollaries and Propositions from Probability Axioms

Key Results:

- **Corollary 2.1:** For M finite number of mutually exclusive sets A_i, where $A_i \cap A_j = \emptyset$:
  $Pr(\bigcup_{i=1}^{M} A_i) = \sum_{i=1}^{M} Pr(A_i)$.
  - *Note:* Axiom 3 implies Corollary 2.1 for finite sample spaces, but Axiom 3 provides necessary generality for infinite sample spaces .
- **Proposition 2.1:** $Pr(A^c) = 1 - Pr(A)$.
- **Proposition 2.2:** If $A \subset B$ then $Pr(A) \leq Pr(B)$.
- **Proposition 2.3:** For any sets (not necessarily mutually exclusive), $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$.
- **Proposition 2.4 (Inclusion-Exclusion for M sets):**
  $Pr(A_1 \cup A_2 \cup \cdots \cup A_M) = \sum_{i=1}^{M} Pr(A_i) - \sum_{i_1 < i_2} Pr(A_{i_1} A_{i_2}) + ... + (-1)^{r+1} \sum_{i_1 < i_2 < \cdots < i_r} Pr(A_{i_1} A_{i_2} ... A_{i_r}) + \cdots + (-1)^{M+1} Pr(A_1 A_2 ... A_M)$.

Topic: Assigning Probabilities

Definition:

- **Classical Approach:** Used when outcomes can be counted.
  - *Example 2.6:* Coin flipping, compute $Pr(H)$ and $Pr(T)$.
  - *Example 2.7:* Dice rolling, compute $Pr(\text{even number})$.
  - *Example 2.8:* Pair of dice, compute $Pr(A)$ where A is sum equal to five.
- **Relative Frequency Approach:** Let $n_{A,B}$ be the number of times events occur in n trials.

- $Pr(A, B) = \lim_{n \to \infty} \frac{n_{A,B}}{n}$.

Key Results:

- **Inference:** To get an exact probability measure using relative frequency, the event must be repeated an infinite number of times.
- **Drawback:** Many random phenomena are not repeatable.

Topic: Joint Probability

Motivation: All events are not mutually exclusive. We are interested in the probability of the intersection of two events, AB.

Notation: Denoted as $Pr(A, B)$ or $Pr(A \cap B)$.

Calculation:

- **Classical:** Express A and B in terms of atomic outcomes, identify outcomes common to both, and calculate probabilities .
- **Relative Frequency:** $Pr(A, B) = \lim_{n \to \infty} \frac{n_{A,B}}{n}$.

Example 1: Card Deck Example

- **Problem:** Consider a set of 52 playing cards.
  - $A =$ {Red card selected}
  - $B =$ {Number card selected} (Ace is a number card)
  - $C =$ {Heart card selected}
  - Find: $Pr(A), Pr(B), Pr(C), Pr(A, B), Pr(A, C), Pr(B, C)$.
  - **Solution:**
  - $Pr(A) = 26/52 = 1/2$ (26 red cards).
  - $Pr(B) = 40/52 = 5/26$ (10 number cards in each of 4 suits).
  - $Pr(C) = 13/52 = 1/4$ (13 hearts).
  - $Pr(A, B) = 20/52 = 5/26$ (10 red number cards in hearts + 10 in diamonds).
  - $Pr(A, C) = 13/52 = 1/4$ (all 13 hearts are red).

- $Pr(B, C) = 10/52$ (10 number cards that are hearts).

Example 2: Costume Party Example

- **Problem:** Alex selects one top and one bottom randomly.
  - **Clothing:** 3 white t-shirts, 1 green cape, 2 pajama pants, 4 polka-dot boxers .
  - **Find:** Probability outfit is the green cape paired with polka-dot boxers.
- **Solution:**
  - **Step 1 (Tops):** Total Tops $= 3 + 1 = 4$. $Pr(\text{Cape}) = 1/4$.
  - **Step 2 (Bottoms):** Total Bottoms $= 2 + 4 = 6$. $Pr(\text{Boxers}) = 4/6 = 2/3$.
  - **Step 3 (Joint):** Total Probability $= Pr(\text{Cape}) \times Pr(\text{Boxers}) = (1/4) \times (4/6) = 1/6$.
  - **Result:** Option C.

Topic: Conditional Probability

Motivation: Occurrence of one event may be dependent upon another.

Definition: The probability of A conditioned on knowing B occurred is $Pr(A|B) = \frac{Pr(A,B)}{Pr(B)}$ where $Pr(B) > 0$.

Key Results:

- **Product Rule:** $Pr(A, B) = Pr(A|B)Pr(B) = Pr(B|A)Pr(A)$.
- **Three Events:** $Pr(A, B, C) = Pr(C|A, B)Pr(A, B) = Pr(C|A, B)Pr(B|A)Pr(A)$.
- **Chain Rule (M events):**

  $Pr(A_1, A_2, ...A_M) = Pr(A_M|A_1, A_2, ...A_{M-1}) \times Pr(A_{M-1}|A_1, A_2, ...A_{M-2}) \cdots Pr(A_2|A_1) \cdot Pr(A_1)$.

Example 3: Cards Without Replacement

- **Problem:** Select two cards. The first card is not returned to the deck. Find $Pr(B|A)$ where A is "First card was a Spade" and B is "Second card was a Spade" .

- **Solution:**
  - **Initial State:** 52 cards, 13 Spades.
  - **Event A Occurs:** One Spade removed.
  - **Remaining State:** 51 cards remaining, 12 Spades remaining .
  - **Calculation:** $Pr(B|A) = \frac{\text{Remaining Spades}}{\text{Remaining Total}} = \frac{12}{51}$.

Example 4: Game of Poker

- **Problem:** Probability of being dealt a flush (all five cards of the same suit) .
- **Solution:**
  - **Part 1 (Flush in Spades):**
    - $Pr(\text{1st is Spade}) = 13/52$
    - $Pr(\text{2nd is Spade}) = 12/51$
    - $Pr(\text{Spade Flush}) = \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}$.
  - **Part 2 (Any Flush):**
    - There are 4 mutually exclusive suits.
    - $Pr(\text{Any Flush}) = 4 \times Pr(\text{Spade Flush})$.

Example 5: The Missing Key

- **Scenario:**
  - K: Key is in jacket ($Pr(K) = 0.8$)
  - L: Key is in Left Pocket ($Pr(L) = 0.4$)
  - R: Key is in Right Pocket ($Pr(R) = 0.4$) .
- **Problem:** If a search of the left-hand pocket does not find the key, what is the conditional probability it is in the other pocket? Goal: Find $Pr(R|L^c)$.
- **Solution:**
  - Calculation: $Pr(R|L^c) = \frac{Pr(R \cap L^c)}{Pr(L^c)}$.
  - Since R implies L^c (key cannot be in both), $Pr(R \cap L^c) = Pr(R)$.
  - $Pr(R|L^c) = \frac{Pr(R)}{1 - Pr(L)} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = \frac{2}{3}$.
  - **Result:** Option C.