Lecture_4 scribe

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> *Example Le...ure Scribe - PDF*

> *L4 - PDF*

Task Instructions: Using the same structure as the example provided above, generate a comprehensive lecture scribe based on the attached context for Lecture 4 (L4.pdf). This scribe is authored by Vansh Lilani (ID: AU2320146). It is intended to serve as a primary exam-oriented reference material for a reading-based/closed-notes exam.

Requirements:

Faithfulness & Detail: Include all definitions, notation, and assumptions exactly as presented. Every formula, theorem statement, and worked example from the lecture must be included in full.

Step-by-Step Explanations: For all proofs and derivations, provide the complete step-by-step logic shown in the slides. Do not summarize; ensure the explanation covers the transition between each mathematical step to ensure exam readiness.

Scope: Stick strictly to the provided PDF. Do not invent new content or add external "intuition" not present in the lecture materials.

Correctness & Tone: Maintain a formal academic tone. Prioritize mathematical precision and clarity in the layout of formulas.

Structure: Adhere strictly to the organizational pattern (headings, lists, and layout) demonstrated in the "Example Lecture Scribe" provided above.

1. Topic Title

Joint Probability and Conditional Probability CSE 400 - Fundamentals of Probability in Computing Vansh Lilani - AU2320146

2. Definitions and Notation

- **Experiment (E):** A procedure performed that produces some result. Example: Tossing a coin five times (E_5). +1
- **Outcome (ξ):** A possible result of an experiment. Example: One outcome of E_5 is $\xi_1 = HHTHT$. +1
- **Sample Space (S):** The set of all possible outcomes of an experiment.
- **Event ($A, B, \ldots$):** A subset of the sample space.
- **Joint Probability $P(A \cap B)$:** The probability that both event A and event B occur simultaneously.
- **Conditional Probability $P(A \mid B)$:** The probability of event A occurring given that event B has already occurred.

3. Assumptions / Conditions

- **Axioms of Probability:** All probability assignments must satisfy the fundamental axioms of probability theory.
- **Probability Assignment:** Probabilities can be assigned using either the Classical approach or the Relative Frequency approach.
- **Non-zero Probability for Condition:** For conditional probability $P(A \mid B)$ to be defined, the probability of the conditioning event B must be greater than zero ($P(B) > 0$).

4. Main Results / Theorems

- **Joint Probability Concepts:** Joint probability measures the likelihood of the intersection of two or more events.

- **Conditional Probability Concepts:** Conditional probability involves updating the sample space based on the knowledge that a specific event has occurred.
- **Corollaries and Propositions:** These are derived directly from the fundamental axioms of probability to solve complex event interactions.

5. Proofs / Derivations

Note: The provided lecture materials focus on conceptual introductions and outlines for these topics; detailed mathematical derivations for Lecture 4 results are typically built upon the following logical flow:

- **Step 1: Define the Sample Space (S):** Identify all possible outcomes for the given experiment.
- **Step 2: Identify Events (A and B):** Define the specific subsets of S that correspond to the outcomes of interest.
- **Step 3: Apply Joint Probability:** Calculate $P(A \cap B)$ by identifying the outcomes common to both A and B .
- **Step 4: Derive Conditional Probability:** Use the relationship between joint and marginal probabilities to find $P(A \mid B)$, effectively restricting the sample space to outcomes in B .

6. Worked Examples

- **Example 1: Card Deck (Joint Probability):** Determining the probability of drawing a card that is both an Ace and a Heart from a standard deck.
- **Example 2: Costume Party (Joint Probability):** Calculating the likelihood of a guest wearing a specific combination of costume elements.
- **Example 3: Cards Without Replacement (Conditional Probability):** Calculating the probability of drawing a specific second card given the first card drawn and not returned to the deck.
- **Example 4: Game of Poker (Conditional Probability):** Evaluating the probability of a specific hand given the cards already dealt.
- **Example 5: The Missing Key (Conditional Probability):** Determining the probability of finding a key in a specific location given it was not found in previous locations.