Engineering Applications and Probability Theory Fundamentals

Study Summary

2026

Topic: Engineering Applications

Speech Recognition System

- **Mechanism:** Uses vocabulary sets (e.g., Hello, Yes, No, Bye) and templates to match signals.
- **Processing:** Input signal $x(t)$ is processed into word representations $x(w)$.
- **Variations:** Templates must account for different speakers (male, female, child) and noise/interference.

Radar System

Operates on **Hypothesis Testing**:

- H_0: No target present ($Y_i = W_i'$).
- H_1: Target present ($Y_i = S_i + W_i$).
- **Outcomes:** Classified as False Alarm or Miss Detect (P_M).
- **Key Relationship:** $P_D + P_M = 1$ (where P_D is Probability of Detection).

Communication Network

- **Standards:** Wi-Fi 802.11 a/b/g/n/ac/ax.
- **Bands:** 2.4 GHz, 5 GHz, 6 GHz.
- **QoS Metrics:** Delay and Latency.

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Topic: Introduction to Probability Theory

Experiment (E): A procedure that produces a result. Example: E_5 (tossing a coin five times).

Outcome (ξ): A possible result. Example: $\xi_1 = HHTHT$.

Event: A set of outcomes. Example: $C = \{\text{outcomes with even number of heads}\}$.

Sample Space (S): The set of all distinct outcomes. Must be:

1. **Mutually Exclusive:** Only one outcome can occur at a time.

2. **Collectively Exhaustive:** No other outcomes are possible.

S can be **Discrete**, **Countably Infinite**, or **Continuous** (e.g., random number in $[0, 1)$).
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Topic: Axioms of Probability

Definition: Probability is a measure of the likelihood of events, mapping an event to a numerical value.

The Three Axioms

1. **Axiom 1:** For any event A, $0 \leq Pr(A) \leq 1$.
2. **Axiom 2:** $Pr(S) = 1$.
3. **Axiom 3:** If $A \cap B = \emptyset$ (mutually exclusive), then $Pr(A \cup B) = Pr(A) + Pr(B)$.

   For an infinite sequence of mutually exclusive events A_i:

$$Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} Pr(A_i)$$

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Topic: Corollaries and Propositions

- **Corollary 2.1:** For M finite mutually exclusive sets: $Pr\left(\bigcup_{i=1}^{M} A_i\right) = \sum_{i=1}^{M} Pr(A_i)$.
- **Proposition 2.1:** $Pr(A^c) = 1 - Pr(A)$.
- **Proposition 2.2:** If $A \subset B$, then $Pr(A) \leq Pr(B)$.
- **Proposition 2.3:** $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$.
- **Proposition 2.4 (Inclusion-Exclusion):**

$$Pr\left(\bigcup_{i=1}^{M} A_i\right) = \sum Pr(A_i) - \sum Pr(A_{i_1} A_{i_2}) + \cdots + (-1)^{M+1} Pr(A_1 \ldots A_M)$$

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Topic: Joint and Conditional Probability

Joint Probability

Probability of the intersection of events: $Pr(A, B)$ or $Pr(A \cap B)$.

- **Classical:** Identify common atomic outcomes.
- **Relative Frequency:** $\lim_{n \to \infty} \frac{n_{A,B}}{n}$.

Conditional Probability

Probability of A given B has occurred:

$$Pr(A|B) = \frac{Pr(A,B)}{Pr(B)}, \quad Pr(B) > 0$$

Product Rule: $Pr(A,B) = Pr(A|B)Pr(B) = Pr(B|A)Pr(A)$.
Chain Rule (M events):

$$Pr(A_1, \ldots, A_M) = Pr(A_M|A_1 \ldots A_{M-1}) \ldots Pr(A_2|A_1)Pr(A_1)$$

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Example: The Missing Key

Scenario: K: Key in jacket ($Pr(K) = 0.8$). L: Key in Left pocket ($Pr(L) = 0.4$). R: Key in Right pocket ($Pr(R) = 0.4$).

Problem: Find $Pr(R|L^c)$ (Probability key is in right pocket given it wasn't in the left).
Solution:

$$Pr(R|L^c) = \frac{Pr(R \cap L^c)}{Pr(L^c)}$$

Since the key cannot be in both pockets simultaneously ($R \subset L^c$), $Pr(R \cap L^c) = Pr(R)$.

$$Pr(R|L^c) = \frac{Pr(R)}{1 - Pr(L)} = \frac{0.4}{1 - 0.4} = \frac{0.4}{0.6} = \frac{2}{3}$$