Using the scribe structure pdf provided in the attached "Example Lecture Scribe" PDF, generate a lecture scribe for the provided Lecture 8 (mention lecture number) materials. the scribe must have all the topics included and it's content based on example pdf. This scribe is intended to serve as a reference material for a closed-notes exam preparation. You must prioritize correctness and maintain strict faithfulness to the provided context, including definitions, notation, theorem statements, and step-by-step proofs (if any), worked examples exactly as they appear in the slides. Identify every major topic in the lecture, For EACH identified topic, you must independently repeat the given structure in example scribe format. * Do not include any creative explanations, external intuition, or material not traceable to the attached files*. Organize the output to follow the logical flow and scope of the lecture to ensure it is suitable for reliable revision.

CSE400 - Fundamentals of Probability in Computing

Lecture 8: Gaussian, Uniform, Exponential, and Gamma Random Variables Scribe: Fagun Rathod - AU2440111

Topic 1: Continuous Random Variable (CRV) Moments

1. Definitions and Notation

- **nth order Central Moment**: $E[(X-\mu_X)^n] = \int_{-\infty}^{\infty}(x-\mu_X)^n f_X(x)dx$.
- **Mean (μ_X)**: $E[X] = \int_{-\infty}^{\infty} x f_X(x)dx$.
- **Variance (σ_X^2)**: The second central moment ($n = 2$), defined as $E[(X-\mu_X)^2]$.
- **Skewness (C_s)**: The third-order normalized central moment ($n = 3$), defined as $C_s = \frac{E[(X-\mu_X)^3]}{\sigma_X^3}$.
- **Kurtosis (C_k)**: The fourth central moment ($n = 4$), defined as $C_k = E[(X-\mu_X)^4]$.

2. Main Results / Theorems

- **Central Moment for n = 0**: $E[(X-\mu_X)^0] = E[1] = 1$.
- **Central Moment for n = 1**: $E[(X-\mu_X)^1] = E[X] - \mu_X = 0$.
- **Variance Identity**: $\sigma_X^2 = E[X^2] - \mu_X^2$.
- **Skewness Interpretation**:
  - Positive (+) value: PDF is Right Skewed.
  - Negative (-) value: PDF is Left Skewed.
- **Kurtosis Interpretation**: A large value indicates the RV X will have a large peak near the mean.
- **Linearity of Expectation**: For any constants a and b, $E[aX+b] = aE[X]+b$.
- **Expectation of Sums**: For any function $g(x) = \sum_{k=1}^{N} g_k(x)$, $E[\sum_{k=1}^{N} g_k(x)] = \sum_{k=1}^{N} E[g_k(x)]$.

3. Proofs / Derivations

Derivation of Variance Identity (σ_X^2):

- **Step 1**: Expand the quadratic term: $E[(X-\mu_X)^2] = E[X^2 - 2\mu_X X + \mu_X^2]$.
- **Step 2**: Apply linearity of expectation: $E[X^2] - 2\mu_X E[X] + \mu_X^2$.
- **Step 3**: Substitute $E[X] = \mu_X$: $E[X^2] - 2\mu_X^2 + \mu_X^2 = E[X^2] - \mu_X^2$.

Topic 2: Gaussian Random Variable

1. Definitions and Notation

- **Gaussian PDF**: A random variable X is Gaussian if its PDF is:

  $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$

- **Notation**: $X \sim N(m, \sigma^2)$ where m is the mean and σ^2 is the variance.
- **Standard Normal Distribution**: A Gaussian distribution where $m = 0$ and $\sigma^2 = 1$.

2. Standard Forms: Q-function and Error Functions

- **Error Function**: $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt$.
- **Complementary Error Function**: $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)dt$.
- **Φ-function (CDF of Standard Normal)**: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left(-\frac{t^2}{2}\right) dt$.
- **Q-function (Gaussian Tail Function)**: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt$.

3. Main Results / Theorems

- **Identity**: $Q(x) = 1 - \Phi(x)$.
- **Evaluating Gaussian CDF**: $F_X(x) = \Phi\left(\frac{x-m}{\sigma}\right)$.
- **Evaluating Tail Probabilities**: $\Pr(X > x) = Q\left(\frac{x-m}{\sigma}\right)$.
- **CDF in terms of Q-function**: $F_X(x) = 1 - Q\left(\frac{x-m}{\sigma}\right)$.
- **Symmetry**: $\Phi(-x) = 1 - \Phi(x)$.

4. Worked Examples

Example 1: Probability Calculation Given a PDF: $f_X(x) = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x+3)^2}{8}\right)$. Find probabilities in terms of Q-functions.

- **Step 1 (Identify Parameters)**: By comparison with the general form, $m = -3$ and $2\sigma^2 = 8 \Rightarrow \sigma^2 = 4 \Rightarrow \sigma = 2$.
- **Step 2 ($\Pr(X \le 0)$)**: $F_X(0) = \Phi\left(\frac{0-(-3)}{2}\right) = \Phi\left(\frac{3}{2}\right) = 1 - Q\left(\frac{3}{2}\right)$.
- **Step 3 ($\Pr(X > 4)$)**: $Q\left(\frac{4-(-3)}{2}\right) = Q\left(\frac{7}{2}\right)$.
- **Step 4 ($\Pr(|X+3| < 2)$)**: This implies $-2 < X + 3 < 2 \Rightarrow -5 < X < -1$. $F_X(-1) - F_X(-5) = \Phi(1) - \Phi(-1) = \Phi(1) - (1 - \Phi(1)) = 2\Phi(1) - 1 = 1 - 2Q(1)$.
- **Step 5 ($\Pr(|X-2| > 1)$)**: This implies $X < 1$ or $X > 3$. $F_X(1) + \Pr(X > 3) = 1 - Q\left(\frac{1-(-3)}{2}\right) + Q(\frac{3-$

$(-3)) = 1 - Q(2) + Q(3)$.

Topic 3: CDF Analysis Problem

1. Worked Examples

Example 2: Analyzing a non-Gaussian CDF Given $FX(x) = \{0, x2, 1, x < 0 0 \le x \le 1 x > 1$

- **Step 1 (Find PDF)**: $fX(x) = dxdFX(x) = 2x$ for $0 \le x \le 1$.
- **Step 2 (Mean)**: $\mu X = \int 01\ x(2x)dx = [32x3]01 = 32$.
- **Step 3 (Variance)**: $\sigma X2 = \int 01\ x2(2x)dx - (32)2 = 21 - 94 = 181$.
- **Step 4 (Skewness)**: $cs = \sigma 31 E[(X-\mu)3] = -52\ \ 2$ (Left-skewed).
- **Step 5 (Kurtosis)**: $ck = \sigma 41 E[(X-\mu)4] = 512$ (Platykurtic).

Topic 4: Applications in Computing

1. Definitions and Notation

- **Thermal Noise**: Random voltage in electronic circuits.
- **Jitter**: Packet delay variation in communication networks.
- **Generative Formula**: $X = \sigma Z + \mu$ (Transforming standard randomness Z into physical model X).
- **Sample Statistics**: Using μ^ and σ^ from raw noisy samples (histograms) to estimate a PDF.

2. Main Results / Theorems

- **Sensor Modeling**: Measured Value = True Value + Gaussian Noise.
- **Image Denoising**: Using Gaussian priors to distinguish real light from random thermal grain.
- **Tail Latency**: Modeling the "99th percentile" in distributed systems for SRE/DevOps.
- **Sensor Fusion (Kalman Filters)**: Guessing real locations between noisy GPS pings.

Would you like me to generate a set of practice problems based on the Q-function identities and the moments derivations covered in this lecture?