Bayesian_Inference_MNIST

November 9, 2025

Bayesian Inference on MNIST dataset

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
from scipy.stats import multivariate_normal as mvn
```

Loading train and test files

[2]:
```
#load train dataset
df=pd.read_csv('C:/Users/rishd/OneDrive/Desktop/Data Cleaning DS/MNIST/
 ↪MNIST_train.csv')
df
```

[2]:

	Unnamed: 0	index	labels	0	1	2	3	4	5	6	…	774	775	776	\
0	0	0	5	0	0	0	0	0	0	0	…	0	0	0	
1	1	1	0	0	0	0	0	0	0	0	…	0	0	0	
2	2	2	4	0	0	0	0	0	0	0	…	0	0	0	
3	3	3	1	0	0	0	0	0	0	0	…	0	0	0	
4	4	4	9	0	0	0	0	0	0	0	…	0	0	0	
…	…	…	…	..	..	..	..	..	..	..	…	…	…	…	
59995	59995	59995	8	0	0	0	0	0	0	0	…	0	0	0	
59996	59996	59996	3	0	0	0	0	0	0	0	…	0	0	0	
59997	59997	59997	5	0	0	0	0	0	0	0	…	0	0	0	
59998	59998	59998	6	0	0	0	0	0	0	0	…	0	0	0	
59999	59999	59999	8	0	0	0	0	0	0	0	…	0	0	0	

	777	778	779	780	781	782	783
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
…	…	…	…	…	…	…	…
59995	0	0	0	0	0	0	0
59996	0	0	0	0	0	0	0

```
59997    0    0    0    0    0    0    0
59998    0    0    0    0    0    0    0
59999    0    0    0    0    0    0    0

[60000 rows x 787 columns]
```

[3]:
```
#load test dataset
df1=pd.read_csv('C:/Users/rishd/OneDrive/Desktop/Data Cleaning DS/MNIST/
 ↪MNIST_test.csv')
df1
```

[3]:
```
      Unnamed: 0  index  labels  0  1  2  3  4  5  6  …  774  775  776  777  \
0              0      0       7  0  0  0  0  0  0  0  …    0    0    0    0
1              1      1       2  0  0  0  0  0  0  0  …    0    0    0    0
2              2      2       1  0  0  0  0  0  0  0  …    0    0    0    0
3              3      3       0  0  0  0  0  0  0  0  …    0    0    0    0
4              4      4       4  0  0  0  0  0  0  0  …    0    0    0    0
…            …    …     …  .. .. .. .. .. .. ..  …  …  …  …  …
9995        9995   9995       2  0  0  0  0  0  0  0  …    0    0    0    0
9996        9996   9996       3  0  0  0  0  0  0  0  …    0    0    0    0
9997        9997   9997       4  0  0  0  0  0  0  0  …    0    0    0    0
9998        9998   9998       5  0  0  0  0  0  0  0  …    0    0    0    0
9999        9999   9999       6  0  0  0  0  0  0  0  …    0    0    0    0

      778  779  780  781  782  783
0       0    0    0    0    0    0
1       0    0    0    0    0    0
2       0    0    0    0    0    0
3       0    0    0    0    0    0
4       0    0    0    0    0    0
…   …  …  …  …  …  …  …
9995    0    0    0    0    0    0
9996    0    0    0    0    0    0
9997    0    0    0    0    0    0
9998    0    0    0    0    0    0
9999    0    0    0    0    0    0

[10000 rows x 787 columns]
```

[4]:
```
df.shape
```

[4]:
```
(60000, 787)
```

[5]:
```
df1.shape
```

[5]:
```
(10000, 787)
```

Dropping the first two index columns of train and test datasets

[6]:
```
#dropping first two columns as they are index columns and will not be needed in␣
 ↪the analysis
df = df.iloc[:,2:]
df
```

[6]:

	labels	0	1	2	3	4	5	6	7	8	…	774	775	776	777	778	779	\
0	5	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
1	0	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
2	4	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
3	1	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
4	9	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
…	…	..	..	..	..	..	..	..	..	..	…	…	…	…	…	…	…	
59995	8	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
59996	3	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
59997	5	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
59998	6	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
59999	8	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	

	780	781	782	783
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
…	…	…	…	…
59995	0	0	0	0
59996	0	0	0	0
59997	0	0	0	0
59998	0	0	0	0
59999	0	0	0	0

[60000 rows x 785 columns]

[7]:
```
#dropping first two columns of the test data
df1 = df1.iloc[:,2:]
df1
```

[7]:

	labels	0	1	2	3	4	5	6	7	8	…	774	775	776	777	778	779	\
0	7	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
1	2	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
2	1	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
4	4	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
…	…	..	..	..	..	..	..	..	..	..	…	…	…	…	…	…	…	
9995	2	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
9996	3	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	
9997	4	0	0	0	0	0	0	0	0	0	…	0	0	0	0	0	0	

```
9998       5  0  0  0  0  0  0  0  0  0  …    0    0    0    0    0    0
9999       6  0  0  0  0  0  0  0  0  0  …    0    0    0    0    0    0

      780  781  782  783
0       0    0    0    0
1       0    0    0    0
2       0    0    0    0
3       0    0    0    0
4       0    0    0    0
…   …   …  …   …
9995    0    0    0    0
9996    0    0    0    0
9997    0    0    0    0
9998    0    0    0    0
9999    0    0    0    0

[10000 rows x 785 columns]
```

Converting dataframe to a 2-D array for train and test datasets

[8]:
```
#converting dataframe to an array for train data
X_train=df.to_numpy()
X_train
```

[8]:
```
array([[5, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       [4, 0, 0, …, 0, 0, 0],
       …,
       [5, 0, 0, …, 0, 0, 0],
       [6, 0, 0, …, 0, 0, 0],
       [8, 0, 0, …, 0, 0, 0]])
```

[9]:
```
#converting dataframe to an array for test data
X_test=df1.to_numpy()
X_test
```

[9]:
```
array([[7, 0, 0, …, 0, 0, 0],
       [2, 0, 0, …, 0, 0, 0],
       [1, 0, 0, …, 0, 0, 0],
       …,
       [4, 0, 0, …, 0, 0, 0],
       [5, 0, 0, …, 0, 0, 0],
       [6, 0, 0, …, 0, 0, 0]])
```

[10]:
```
#Y is the target variable that is the feature label in this dataset
y_train=X_train[:,0]
y_train
```

[10]:
```
array([5, 0, 4, …, 5, 6, 8])
```

[11]:
```
y_train.shape
```

[11]:
```
(60000,)
```

[12]:
```
#take all features excluding the first one for train dataset
X_train = X_train[:,1:]
X_train
```

[12]:
```
array([[0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       …,
       [0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0]])
```

[13]:
```
X_train.shape
```

[13]:
```
(60000, 784)
```

[14]:
```
#y for test data
y_test = X_test[:,0]
y_test
```

[14]:
```
array([7, 2, 1, …, 4, 5, 6])
```

[15]:
```
y_test.shape
```

[15]:
```
(10000,)
```

[16]:
```
#take all features excluding the first one for test dataset
X_test = X_test[:,1:]
X_test
```

[16]:
```
array([[0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       …,
       [0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0],
       [0, 0, 0, …, 0, 0, 0]])
```

[17]:
```
X_test.shape
```

[17]:
```
(10000, 784)
```

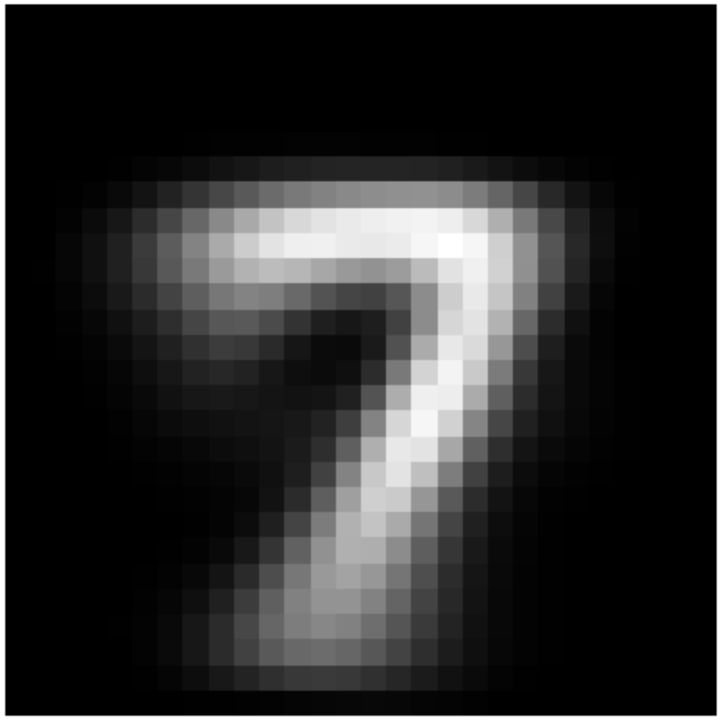
Scaling train and test data

[18]:
```
#scale the training and test dataset to improve accuracy
#we divide each value by 255 as there are 255 pixels
X_train_scaled = X_train.astype(np.float32) / 255.0
X_test_scaled = X_test.astype(np.float32) / 255.0
```

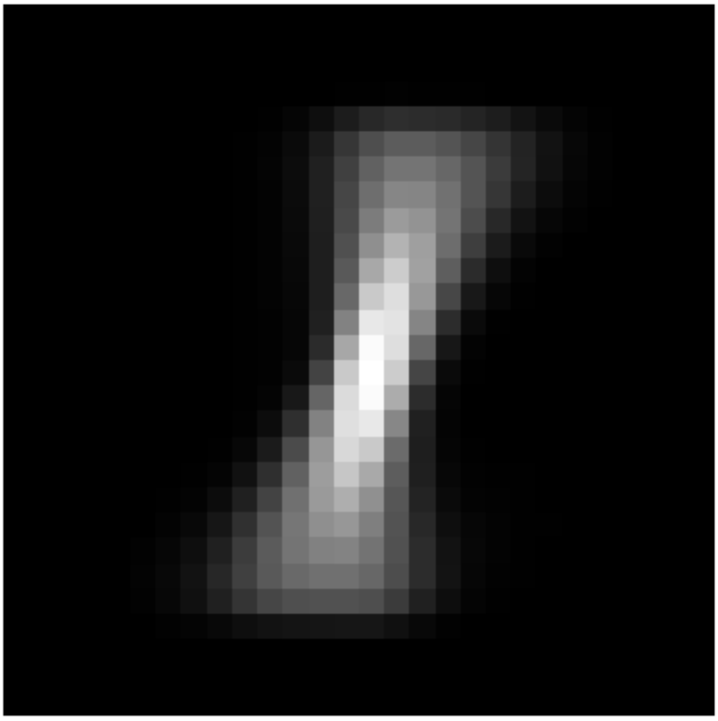
Visualizing MNIST Dataset

[19]:
```
#img = X_train_scaled[0]            # takes first row
#label = y_train[0]        #takes first digit

#img = img.reshape(28, 28)          #converts single 784 vector to 28*28 matrix

#plt.imshow(img, cmap='gray')
#plt.title(f"Label: {label}")
#plt.axis('off')
#plt.show()


def show_me(X):
    plt.imshow(X.reshape(28,28),cmap='gray')
    plt.axis('off')
    plt.show()

show_me(X_train_scaled[135])
```

[20]:
```
#average of all sevens in the dataset

def show_me_all_mean(X,y,k):
    show_me(sum(X[y==k,:]/len(X[y==k,:])))

show_me_all_mean(X_train_scaled,y_train,7)
```



[21]:
```
#average of all ones in the dataset

def show_me_all_mean(X,y,k):
    show_me(sum(X[y==k,:]/len(X[y==k,:])))

show_me_all_mean(X_train_scaled,y_train,1)
```



Naives Bayes Classifier

[22]:
```
class GaussNB():      #contains methods and variables
    def fit(self,X,y,epsilon=1e-1):                    #hyper paramter tuned␣
↪to 1e-1, added to variance to prevent division by zero
        self.likelihoods=dict()          #stores mean and var
        self.priors=dict()               #stores prior probbilities

        self.K = set(y.astype(int))         #stores labels as unique digits␣
↪from 0-9

        for k in self.K:
            X_k = X[y==k]
            self.likelihoods[k] = {"mean":X_k.mean(axis=0),"cov":X_k.
↪var(axis=0)+epsilon}        #P(X|y)
            self.priors[k] = len(X_k)/len(X)                  #P(y)



    def predict(self,X):
        N,D = X.shape
        p_hat = np.zeros((N,len(self.K)))           #needs to be a tuple
```

```
        for k,l in self.likelihoods.items():
            p_hat[:,k] = mvn.logpdf(X,l["mean"],l["cov"]) + np.log(self.
↪priors[k])             #log (X|y)*P(y) =  log P(X|y)+log P(y)

        return p_hat.argmax(axis=1)          #returns index of highest␣
↪posterior probability
```

[23]:
```
def accuracy(y,y_hat):
    return np.mean(y==y_hat)
```

[24]:
```
gnb = GaussNB()
```

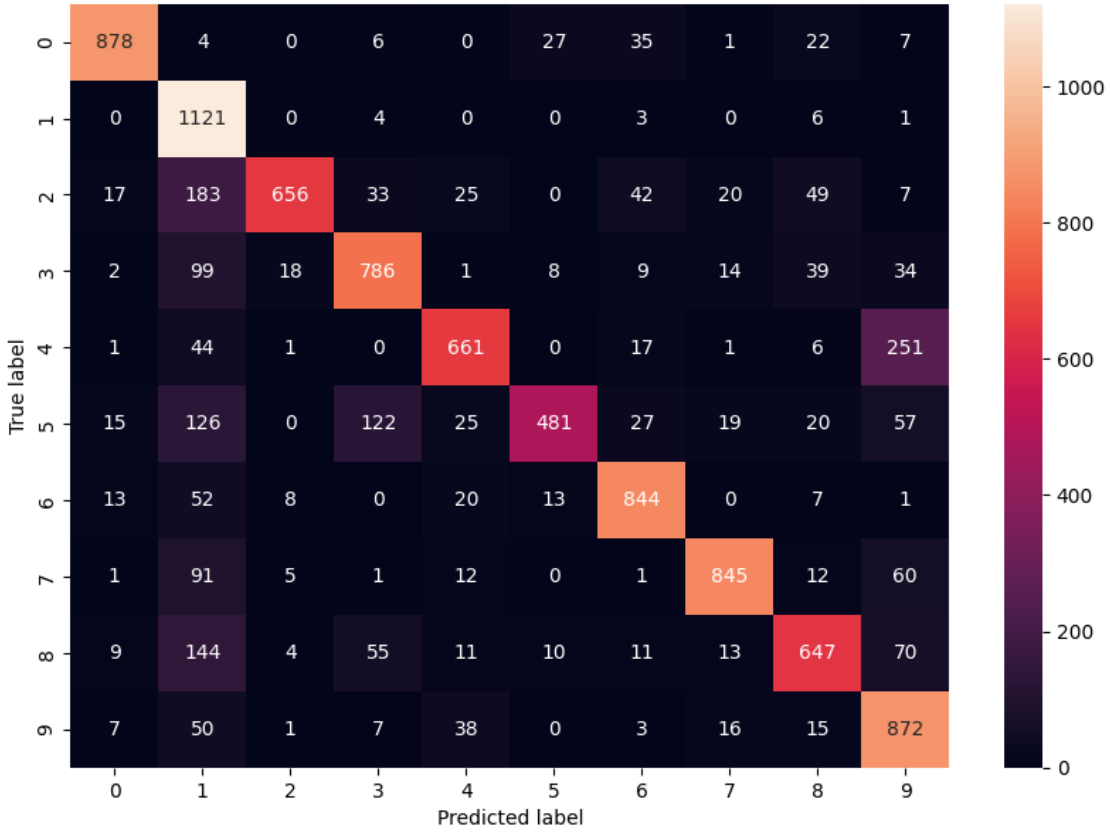
[25]:
```
#fit the model on train data
gnb.fit(X_train_scaled,y_train)
```

[26]:
```
#make predictions on the test data
y_hat = gnb.predict(X_test_scaled)
```

[27]:
```
#compare accuracies of predicted and actual test data labels
accuracy(y_test,y_hat)
```

[27]:
```
np.float64(0.7791)
```

[28]:
```
#confusion matrix for model evaluation
def ConfusionMatrix(y_actual,y_predicted):
  plt.figure(figsize=(10,7))
  y_actu = pd.Series(y_actual, name='Actual')
  y_pred = pd.Series(y_predicted, name='Predicted')
  cm = pd.crosstab(y_actu, y_pred)                          #creates a pivot table␣
↪that tells us how y_yest and y_hat relate with each other
  ax = sns.heatmap(cm, annot=True, fmt="d")
  plt.ylabel('True label')
  plt.xlabel('Predicted label')


ConfusionMatrix(y_test,y_hat)
```



Non-Naive Bayes Classifier

[29]:
```
class GaussBayes():

  def fit(self, X, y, epsilon=1e-3):                    #fit method

    self.likelihoods =dict()                           #likelihood probability
    self.priors = dict()                                #prior probability

    self.K = set(y.astype(int))

    for k in self.K:
      X_k = X[y==k,:]
      N_k, D = X_k.shape
      mu_k = X_k.mean(axis=0)

      self.likelihoods[k] = {"mean": mu_k,
                             "cov":(1/(N_k-1)) * np.matmul((X_k-mu_k).T,␣
↪X_k-mu_k)+epsilon*np.identity(D)}          #compute covariance
      self.priors[k] = len(X_k)/len(X)
```

```
  def predict(self, X):                                    #predict method
    N, D = X.shape
    P_hat =np.zeros((N, len(self.K)))

    for k,l in self.likelihoods.items():
      P_hat[:,k] = mvn.logpdf(X, l["mean"],l["cov"])+np.log(self.priors[k])
                          #posterior probability

    return P_hat.argmax(axis=1)
```

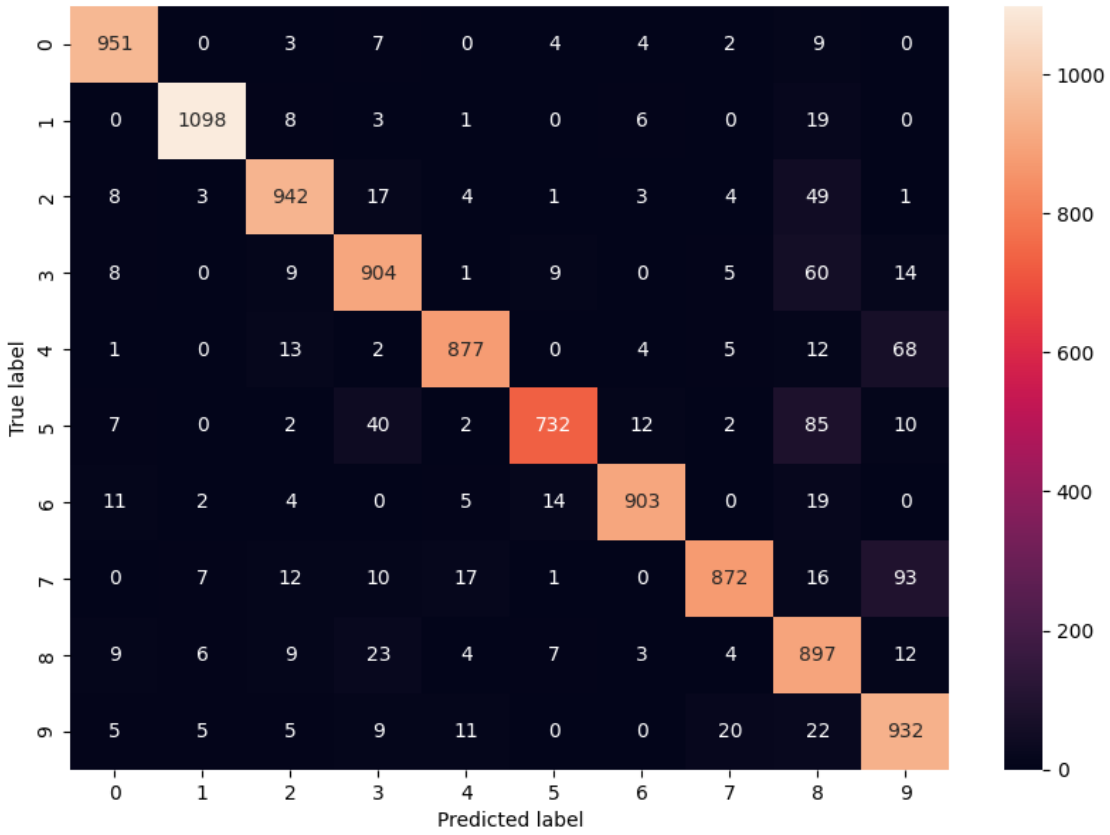
[30]:
```
#fit the model on train dataset
gbays = GaussBayes()
gbays.fit(X_train_scaled,y_train)
```

[31]:
```
#predict the model on the test dataset
y_hat_bayes = gbays.predict(X_test_scaled)
```

[32]:
```
accuracy(y_test,y_hat_bayes)
```

[32]:
```
np.float64(0.9108)
```

[33]:
```
#confusion matrix for model evaluation
def ConfusionMatrix(y_actual,y_predicted):
  plt.figure(figsize=(10,7))
  y_actu = pd.Series(y_actual, name='Actual')
  y_pred = pd.Series(y_predicted, name='Predicted')
  cm = pd.crosstab(y_actu, y_pred)
  ax = sns.heatmap(cm, annot=True, fmt="d")
  plt.ylabel('True label')
  plt.xlabel('Predicted label')


ConfusionMatrix(y_test,y_hat_bayes)
```



KNN Classifier

[ ]:
```
#not included in the ppt as non-naive bayes model performed well on MNIST␣
↪dataset
```

[34]:
```
class KNNClassifier():

    def fit(self,X,y):
        self.X = X
        self.y = y

    def predict(self,X,K,epsilon=1e-3):
        N = len(X)
        y_hat = np.zeros(N)

        for i in range(N):
            dist2 = np.sum((self.X-X[i])**2,axis=1)      #not taking sqrt cz it␣
↪is a slow process
            idxt = np.argsort(dist2)[:K]                 #returns sorted indices of␣
↪nearest neighbors
```

```
            gamma_k = 1/(np.sqrt(dist2[idxt]+epsilon))

            y_hat[i] = np.bincount(self.y[idxt],weights=gamma_k).argmax()

        return y_hat
```

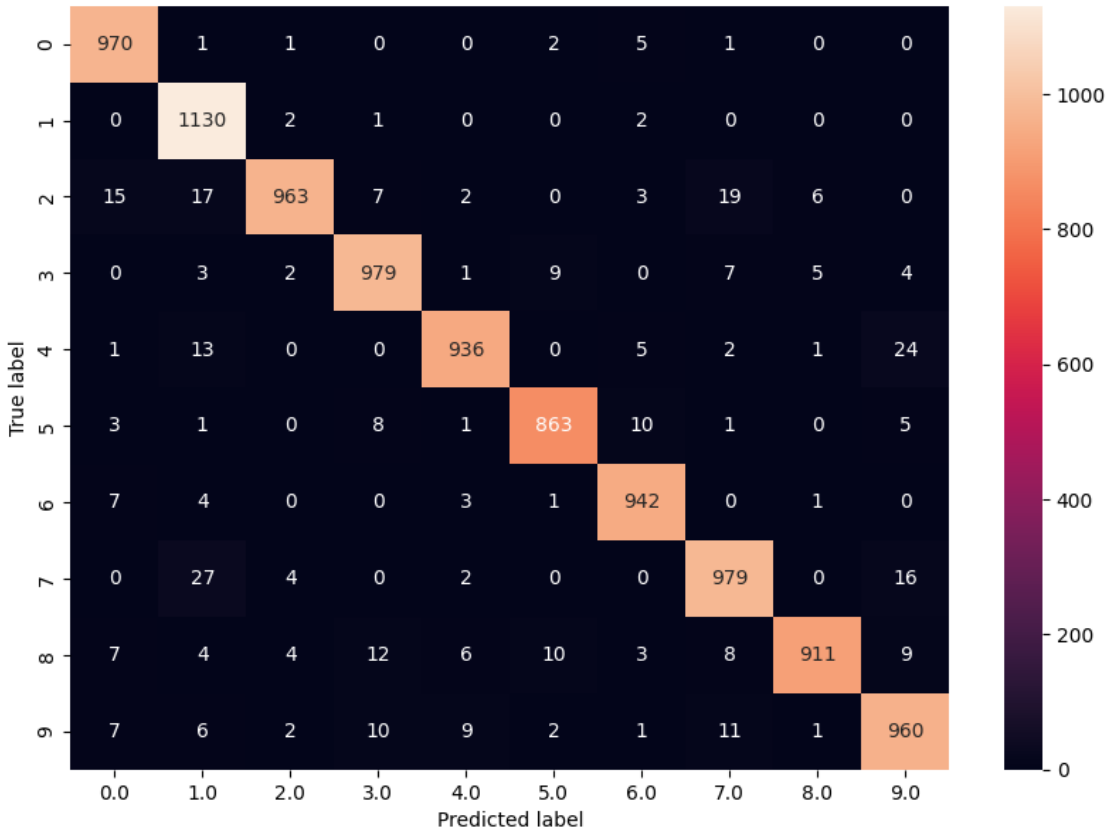
[35]:
```
knn_instance =  KNNClassifier()
knn_instance.fit(X_train_scaled,y_train)
```

[36]:
```
y_hat_knn = knn_instance.predict(X_test_scaled,K=20)
```

[37]:
```
accuracy(y_test,y_hat_knn)
```

[37]:
```
np.float64(0.9633)
```

[38]:
```
#confusion matrix
def ConfusionMatrix(y_actual,y_predicted):
  plt.figure(figsize=(10,7))
  y_actu = pd.Series(y_actual, name='Actual')
  y_pred = pd.Series(y_predicted, name='Predicted')
  cm = pd.crosstab(y_actu, y_pred)
  ax = sns.heatmap(cm, annot=True, fmt="d")
  plt.ylabel('True label')
  plt.xlabel('Predicted label')


ConfusionMatrix(y_test,y_hat_knn)
```


True label
Predicted label
0 970 1 1 0 0 2 5 1 0 0
1 0 1130 2 1 0 0 2 0 0 0
2 15 17 963 7 2 0 3 19 6 0
3 0 3 2 979 1 9 0 7 5 4
4 1 13 0 0 936 0 5 2 1 24
5 3 1 0 8 1 863 10 1 0 5
6 7 4 0 0 3 1 942 0 1 0
7 0 27 4 0 2 0 0 979 0 16
8 7 4 4 12 6 10 3 8 911 9
9 7 6 2 10 9 2 1 11 1 960
0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0
1000
800
600
400
200
0