**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans = B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans -** We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question

a)Probabilty of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X d" 44).

Z = (X -µ )/ = (X - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 44) = Pr(Z d" (44 - 38)/6) = Pr(Z d" 1)=84.1345%

Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

b) Probabilty of employees less than age of 30 = Pr(X<30).

Z = (X - )/ = (30 - 38)/6

Thus the question can be answered by using the normal table to find

Pr(X d" 30) = Pr(Z d" (30 - 38)/6) = Pr(Z d" -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees).

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans-** As we know that if X <" N(µ1, Ã 1^2 ), and Y <" N(µ2, Ã 2^2 ) are two independent random variables then X + Y <" N(µ1 + µ2, Ã 1^2 + Ã 2^2 ) , and X " Y <" N(µ1 " µ2, Ã 1^2 + Ã 2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(aµ1 + bµ2, a^2Ã 1^2 + b^2Ã 2^2 ).

Therefore in the question

2X1~ N(2 u,4 Ã ^2) and

X1+X2 ~ N(µ + µ, Ã ^2 + Ã ^2 ) ~ N(2 u, 2Ã ^2 )

2X1-(X1+X2) = N( 4µ,6 Ã ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans- D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans=** Mean profit is RS 540 Million

Std deviation is RS 225 Millio Range is RS

State norms interval (0.95,540,225)

**Range is RS 99.0081034, 980991896**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans=** formula X= X=μ + Zσ; wherein from z table, 5 percentle = -1.645

X = 540(-1.645)\*225

**X = 169.875**

1. Which of the two divisions has a larger probability of making a loss in a given year?

Probability od division 1 making a loss p(X<0)

Stats. norm. cdf (0, 7, 3)

**0.0477903**