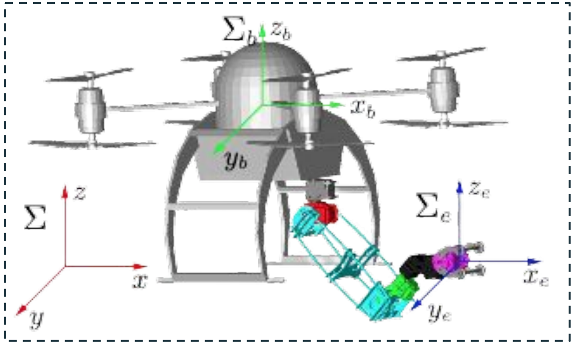
PROJECT-1 ENPM-667
Control of Robotics Systems

Adaptive Control for UAVs Equipped with a Robotic Arm



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INTRODUCTION

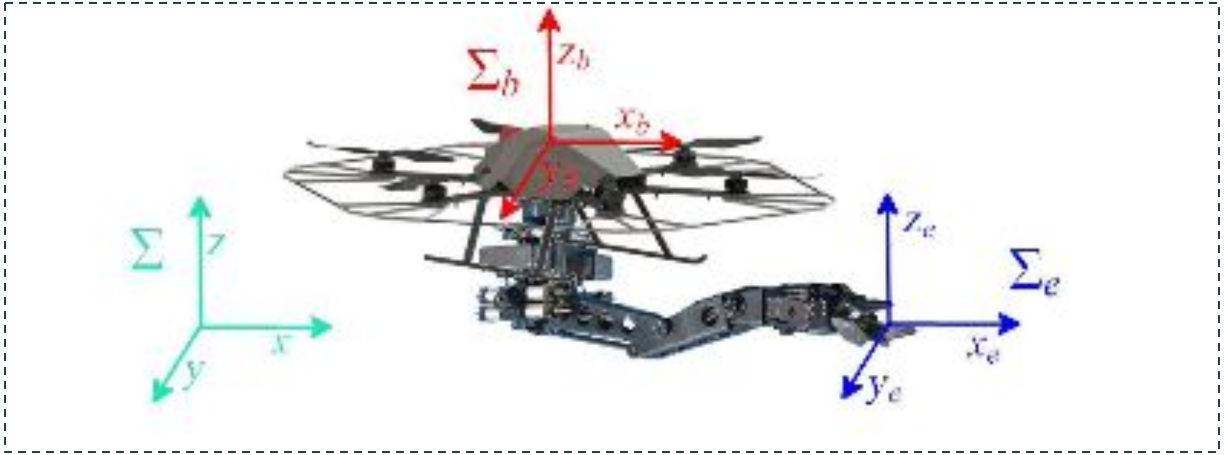
1. **The problem to be addressed:** Motion control of the end-effector of a robot manipulator mounted on a quadrotor helicopter

2. **Approach to solve the problem:** A hierarchical control architecture. (Top layer and bottom layer)

3. **Elaborate explanation:** In the top layer, an inverse kinematics algorithm computes the motion references for the actuated variables.

   An adaptive model-based motion control algorithm is in charge of tracking the motion references in the bottom layer: a vehicle position controller, an attitude controller, while the manipulator controller computes the joint torques.

MODELING



Quadrotor and robotic arm system with the corresponding frames

MODELING

Kinematics:

Aim: To find the end-effector velocities both linear and angular.

Approach:

- The position and rotation matrix are calculated with respect to the world frame.
- The obtained equations are then differentiated to obtain the velocities.
- Generalized end-effector velocities are obtained using Jacobian and transformation matrices.
- Different DOF constraints are then incorporated to simplify the obtained equation according to the available control inputs.

The obtained equation is:

$$\boldsymbol{v}_e = \boldsymbol{J}_\eta(\boldsymbol{q},\phi_b)\frac{\boldsymbol{d\eta}_b}{\boldsymbol{dt}} + \boldsymbol{J}_\sigma(\boldsymbol{q},\phi_b)\frac{\boldsymbol{d\sigma}_b}{\boldsymbol{dt}} + \boldsymbol{J}_{eb}(\boldsymbol{q},\boldsymbol{R}_b)\frac{\boldsymbol{dq}}{\boldsymbol{dt}}$$
$$= \boldsymbol{J}_\zeta(\boldsymbol{\sigma}_b,\zeta)\frac{\boldsymbol{d\zeta}}{\boldsymbol{dt}} + \boldsymbol{J}_\sigma(\boldsymbol{\sigma}_b,\zeta)\frac{\boldsymbol{d\sigma}_b}{\boldsymbol{dt}}$$

where $\boldsymbol{J}_\eta$ is is composed by the first 4 columns of $\boldsymbol{J}_b\boldsymbol{T}_A(\phi_b)$, $\boldsymbol{J}_\sigma$ by the last 2 columns of $\boldsymbol{J}_b\boldsymbol{T}_A(\phi_b)$ and $\boldsymbol{J}_\zeta = [\boldsymbol{J}_\eta \quad \boldsymbol{J}_{eb}]$.

MODELING

Dynamics:

Aim: To obtain the equations of dynamic model.

Approach:

- To get the desired output, Euler-Lagrange formulation is used.
- For the application of the formula the kinetic energy and potential energy of the system are computed and their values are substituted in the formula to get the dynamic model formula.
- Kinetic energy of the system are the sum of UAV kinetic energy and arm kinetic energy, similarly the potential energy of the system is also obtained.

Equation of dynamic model:

$$\boldsymbol{M}(\xi)\ddot{\xi} + \boldsymbol{C}(\xi,\dot{\xi})\dot{\xi} + \boldsymbol{g}(\xi) + \boldsymbol{d}(\xi,\dot{\xi}) = \boldsymbol{u},$$

where $\xi = \begin{bmatrix} \boldsymbol{x}_b^T & \boldsymbol{q}^T \end{bmatrix}^T \in \mathbb{R}^{(6+n\times 1)}$,

$\boldsymbol{M}$: Symmetric and positive definite inertia matrix of the system,

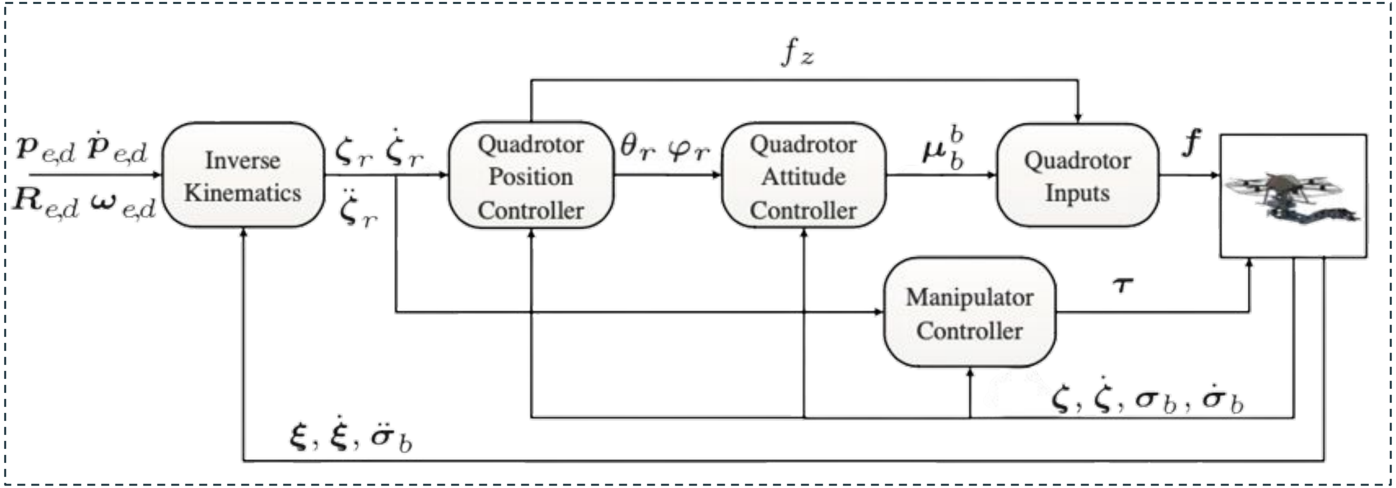
$\boldsymbol{C}$: Matrix of Coriolis and centrifugal terms,

$\boldsymbol{g}$: Vector of gravity forces,

$\boldsymbol{d}$: Disturbances, such as aerodynamic effects, and modeling uncertainties,

$\boldsymbol{u}$: Vector of inputs,

KINEMATIC CONTROL SCHEME



Block scheme of the control architecture

KINEMATIC CONTROL SCHEME

Inverse Kinematics:

Aim:

Computing the trajectory references for the motion control loops at the bottom layer.

Approach:

- The time derivative of the differential kinematics (velocity equation derived in the kinematics section) is computed.
- The error in the desired values is taken into consideration that gives the trajectory references for the motion control loops at the bottom layer.

$$\ddot{\zeta}_r = \boldsymbol{J}_{\zeta}^{\dagger}(\boldsymbol{\sigma}_b, \boldsymbol{\zeta})(\dot{\boldsymbol{v}}_{e,d} + \boldsymbol{K}_v(\boldsymbol{v}_{e,d} - \boldsymbol{v}_e) + \boldsymbol{K}_p \boldsymbol{e})$$

$$-\boldsymbol{J}_{\zeta}^{\dagger}(\boldsymbol{\sigma}_b, \boldsymbol{\zeta})(\dot{\boldsymbol{J}}_{\zeta}(\boldsymbol{\sigma}_b, \dot{\boldsymbol{\zeta}}) + \boldsymbol{J}_{\sigma}(\boldsymbol{\sigma}_b, \boldsymbol{\zeta})\ddot{\boldsymbol{\sigma}}_b + \dot{\boldsymbol{J}}_{\sigma}(\boldsymbol{\sigma}_b, \boldsymbol{\zeta})\dot{\boldsymbol{\sigma}}_b)$$

$$-\boldsymbol{N}(\boldsymbol{J}_{\zeta})(k_N \boldsymbol{I}_{N+4} + \dot{\boldsymbol{N}})\dot{\boldsymbol{\zeta}},$$

where $\boldsymbol{N}(\boldsymbol{J}_{\zeta})$ is a projector onto the null space of the Jacobian $\boldsymbol{J}_{\zeta}$; is a possible choice is $\boldsymbol{N}(\boldsymbol{J}_{\zeta}) = \boldsymbol{I}_{n+4} - \boldsymbol{J}_{\zeta}^{\dagger}\boldsymbol{J}_{\zeta}$.

where $\boldsymbol{J}_{\zeta}^{\dagger} = \boldsymbol{J}_{\zeta}^{T}(\boldsymbol{J}_{\zeta}\boldsymbol{J}_{\zeta}^{T})^{-1}$ is a right pseudoinverse of $\boldsymbol{J}_{\zeta}$, $\boldsymbol{K}_v$ and $\boldsymbol{K}_p$ are symmetric positive definite gain matrices, given by:

$$\boldsymbol{K}_p = \begin{bmatrix} \boldsymbol{K}_{(p,P)} \\ \boldsymbol{K}_{(p,O)} \end{bmatrix}, \boldsymbol{K}_v = \begin{bmatrix} \boldsymbol{K}_{(v,P)} \\ \boldsymbol{K}_{(v,O)} \end{bmatrix} = \boldsymbol{K}_p = \begin{bmatrix} \boldsymbol{K}_{(v,P)} \\ \boldsymbol{K}_{(v,O)}\boldsymbol{I}_3, \end{bmatrix}$$

KINEMATIC CONTROL SCHEME

Motion Control:

Aim: Achieve the desired motion

Approach:

- Dynamic system model is modified by making few terms in the matrix zero.
- The new adaptive control law is considered.
- The values of auxiliary input are selected and the disturbances are estimated.

$$\overline{\boldsymbol{M}}(\boldsymbol{\xi})\ddot{\boldsymbol{\xi}} + \boldsymbol{C}(\boldsymbol{\xi},\dot{\boldsymbol{\xi}})\dot{\boldsymbol{\xi}} + \boldsymbol{g}(\boldsymbol{\xi}) + \boldsymbol{d}(\boldsymbol{\xi},\dot{\boldsymbol{\xi}}) + \Delta\boldsymbol{M}(\boldsymbol{\xi})\ddot{\boldsymbol{\xi}} = \boldsymbol{u}, \quad (18)$$

where the matrix $\overline{\boldsymbol{M}}$ is obtained by setting to zero the second and third columns of $\boldsymbol{M}_{12}$

$$\overline{\boldsymbol{M}}(\xi) = \begin{bmatrix} \boldsymbol{M}_{11} & \overline{\boldsymbol{M}}_{12} & \boldsymbol{M}_{13} \\ \boldsymbol{M}_{12}^T & \boldsymbol{M}_{22} & \boldsymbol{M}_{23} \\ \boldsymbol{M}_{13}^T & \boldsymbol{M}_{23}^T & \boldsymbol{M}_{33} \end{bmatrix},$$

with $\overline{\boldsymbol{M}}_{12} = [\boldsymbol{m}_{12} \quad \boldsymbol{0}_3 \quad \boldsymbol{0}_3]$, where $\boldsymbol{m}_{12}$ denotes the first coloumn $\boldsymbol{M}_{12}$, and $\boldsymbol{0}_3$ is the (3×1) null vector. In order to globally linearize the closed-loop dynamics, the following adaptive control law can be considered:

$$\boldsymbol{u} = \boldsymbol{M}(\boldsymbol{\xi})\boldsymbol{\alpha} + \boldsymbol{C}(\boldsymbol{\xi},\dot{\boldsymbol{\xi}})\dot{\boldsymbol{\xi}} + \boldsymbol{g}(\boldsymbol{\xi}) + \widehat{\boldsymbol{d}}(\boldsymbol{\xi},\dot{\boldsymbol{\xi}}), \quad (19)$$

where the auxiliary input $\boldsymbol{\alpha}$ can be partitioned according to (13) as $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_1^T \ \boldsymbol{\alpha}_2^T \ \boldsymbol{\alpha}_3^T]^T$ with $\boldsymbol{\alpha}_\phi = [\alpha_\psi \ \alpha_\phi \ \alpha_\varphi]^T$. The term $\widehat{\boldsymbol{d}} = [\widehat{\boldsymbol{d}}_1^T \ \widehat{\boldsymbol{d}}_2^T \ \widehat{\boldsymbol{d}}_3^T]$ is an estimate of the disturbance $\boldsymbol{d}$ in (18). The auxiliary controls $\boldsymbol{\alpha}_3$ and $\boldsymbol{\alpha}_1$ and α_ψ can be chosen as

$$\boldsymbol{\alpha}_3 = \ddot{\boldsymbol{q}}_r + \boldsymbol{K}_{q,V}(\dot{\boldsymbol{q}}_r - \dot{\boldsymbol{q}}) + \boldsymbol{K}_{q,P}(\boldsymbol{q}_r - \boldsymbol{q}),$$

$$\boldsymbol{\alpha}_1 = \ddot{\boldsymbol{p}}_r + \boldsymbol{K}_{p,V}(\dot{\boldsymbol{p}}_r - \dot{\boldsymbol{p}}) + \boldsymbol{K}_{p,P}(\boldsymbol{p}_r - \boldsymbol{p}),$$

$$\alpha_\psi = \ddot{\psi} + k_{\psi,V}(\dot{\psi}_r - \dot{\psi}) + K_{\psi,P}(\psi_r - \psi),$$

where $\boldsymbol{K}_{*,V}, \boldsymbol{K}_{*,P} (* = \{q, p\})$ are symmetric positive definite matrices and $k_{\psi,V}, k_{\psi,P}$ are positive scalar gains.

KINEMATIC CONTROL SCHEME

Motion Control- Quadrotor position controller:

Aim: To compute reference trajectories for the roll and pitch angles to be fed to the inner loop.

Approach:

- The force vector equation is computed using the equations in the previous slides.
- Then the values of the roll and pitch angles are computed to be fed in the inner loop by analysing the input vector matrix previously derived in the dynamics section of the exercise.

On the basis of (19), the following expression of $\boldsymbol{u}_f$

$$\boldsymbol{u}_f = \boldsymbol{M}_{11}\boldsymbol{\alpha}_1 + \boldsymbol{m}_{12}\alpha_\psi + \boldsymbol{M}_{13}\boldsymbol{\alpha}_3 + \boldsymbol{C}_1\dot{\boldsymbol{\xi}} + \boldsymbol{g}_1 + \widehat{\boldsymbol{d}}_1 (20)$$

where the function dependencies have been dropped for notation compactness It can be noted that, since the manipulator links are much lighter than the vehicle body, the elements of matrix $\boldsymbol{M}_{12}$ are often negligible with respect to those of $\boldsymbol{M}_{11}$ (Arleo et al., 2013). Therefore, in practice, $\boldsymbol{u}_f$ in (20) is very close to the ideal control input. In view of (11), u_f depends on the attitude of the quadrotor via the relation

$$\boldsymbol{u}_f = \boldsymbol{h}(\boldsymbol{f}_z, \boldsymbol{\sigma}_b) \Longrightarrow \begin{bmatrix} u_{f,x} \\ u_{f,y} \\ u_{f,z} \end{bmatrix} = \begin{bmatrix} (c_\psi s_\theta c_\varphi + s_\psi s_\varphi) f_z \\ (s_\psi s_\theta c_\varphi - c_\psi s_\varphi) f_z \\ c_\theta c_\varphi f_z \end{bmatrix} \quad (21)$$

Therefore, the total thrust, f_z, and reference trajectories for the roll and pitch angles to be fed to the inner loop can be computed as

$$f_z = ||\boldsymbol{u}_f||, (22)$$

$$\theta_r = arctan\left(\frac{u_{f,x}c_\psi + u_{f,y}s_\psi}{u_{f,z}}\right), (23)$$

$$\phi_r = arcsin\left(\frac{u_{f,x}s_\psi - u_{f,y}c_\psi}{||u_f||}\right), (24)$$

KINEMATIC CONTROL SCHEME

Motion Control- Quadrotor attitude controller:

Aim: To compute the control inputs.

Approach:

- The control input equations have already been deduced in the previous section, however, to get the values of the control inputs derivative of the roll and pitch angles are required.
- Once they are computed, the noise that will affect these values has to be taken into account and modifications are made using filters and new control input equations are derived by using simple PD control laws.
- Vehicle torque is then computed. Further derivation of Quadrotor inputs and manipulator control becomes straightforward.

KINEMATIC CONTROL SCHEME

Motion Control- Quadrotor attitude controller(continued...):

$$\alpha_\theta = \ddot{\theta}_r + K_{\theta,V}(\dot{\theta}_r - \dot{\theta}) + K_{\theta,P}(\theta_r - \theta), (25)$$

$$\alpha_\varphi = \ddot{\varphi}_r + K_{\varphi,V}(\dot{\varphi}_r - \dot{\varphi}) + K_{\varphi,P}(\varphi_r - \varphi) (26)$$

where $k_{\theta,V}, k_{\theta,P}, k_{\varphi,V}, k_{\varphi,P}$ positive scalar gains. It is worth noticing that (25) and (26) require the knowledge of the time derivative of θ_r and φ_r, that can not be directly obtained by (21), but only via numerical differentiation. Since in a practical scenario θ_r and φ_r are likely to be affected by noise, it can be realistic compute the reference velocities ($\dot{\theta}_r$ and $\dot{\varphi}_r$) by using suitable filters but their derivative can be very noisy, thus it is possible to modify (25) and (26) by adopting simple PD control laws as

$$\overline{\alpha}_\theta = k_{\theta,V}(\dot{\theta}_r - \dot{\theta}) + k_{\theta,P}(\theta_r - \theta),$$

$$\overline{\alpha}_\varphi = k_{\varphi,V}(\dot{\varphi}_r - \dot{\varphi}) + k_{\varphi,P}(\varphi_r - \varphi).$$

Finally, $\boldsymbol{u}_\mu$ can be computed as

$$\boldsymbol{u}_\mu = \boldsymbol{M}_{12}^T \boldsymbol{\alpha}_1 + \boldsymbol{M}_{22}\boldsymbol{\alpha}_2 + \boldsymbol{M}_{23}\boldsymbol{\alpha}_3 + \boldsymbol{C}_2\dot{\boldsymbol{\xi}} + \boldsymbol{g}_2 + \widehat{\boldsymbol{d}}_2,$$

and, from (13), the vehicle torques as

$$\boldsymbol{\mu}_b^b = \boldsymbol{R}_b^T(\phi_b)\boldsymbol{T}^{-T}(\phi_b)\boldsymbol{u}_\mu.$$

Computation of quadrotor inputs. Once f_z and $\boldsymbol{\mu}_b^b$ have been computed, the four actuation forces of the vehicle rotors can be easily obtained by inverting the (14) ,i.e.,

$$\boldsymbol{f} = \boldsymbol{\Gamma}^{-1}\begin{bmatrix} \boldsymbol{f}_z \\ \boldsymbol{\mu}_b^b \end{bmatrix}$$

Manipulator control . Finally, the torques acting on the manipulator joints can be computed as

$$\boldsymbol{u}_\mu = \boldsymbol{M}_{12}^T\boldsymbol{\alpha}_1 + \boldsymbol{M}_{23}^T\boldsymbol{\alpha}_2 + \boldsymbol{M}_{33}\boldsymbol{\alpha}_3 + \boldsymbol{C}_3\dot{\boldsymbol{\xi}} + \boldsymbol{g}_3 + \widehat{\boldsymbol{d}}_3$$

KINEMATIC CONTROL SCHEME

Uncertainties Estimation:

Aim:

To find the compensation term by taking into consideration the modeling uncertainties and also the perturbation given the implementation of the control law.

$$\boxed{\ddot{\boldsymbol{\xi}} = \boldsymbol{\alpha} - \Delta\boldsymbol{\alpha} - \boldsymbol{M}(\boldsymbol{\xi})^{-1}(\Delta\boldsymbol{M}\overline{\boldsymbol{\alpha}} + \boldsymbol{d} - \widehat{\boldsymbol{d}})}$$

$$\ddot{\boldsymbol{\xi}} = \boldsymbol{\alpha} - \boldsymbol{\delta} + \widehat{\boldsymbol{\delta}} = \boldsymbol{\alpha} - \widetilde{\boldsymbol{\delta}}, (28),$$

where

$$\boldsymbol{\delta} = \Delta\boldsymbol{\alpha} + \boldsymbol{M}(\boldsymbol{\xi})^{-1}(\Delta\boldsymbol{M}\overline{\boldsymbol{\alpha}} + \boldsymbol{d}), \ \widehat{\boldsymbol{\delta}} = \boldsymbol{M}(\boldsymbol{\xi})^{-1}\widehat{\boldsymbol{d}}$$

Approach:

- By taking into consideration the control laws and inputs the closed law dynamic equation is computed.
- However, this equation is then modified by taking into account the perturbation terms and modeling uncertainties.

KINEMATIC CONTROL SCHEME

Uncertainties Estimation (Continued...):

- A good approximation of the term δ can be made resorting to the parametric model.
- The elements of the regressor matrix can be chosen as Radial Basis Functions (RBFs).
- By applying the Universal Interpolation theorem, Interpolation error can be reasonably assumed and bounded.
- Interpolation error can be reasonably assumed norm bounded,
- δ can be obtained by integrating the following update law for the unknown parameters χ.

A good approximations of the term $\boldsymbol{\delta}$ can be obtained by resorting to a parametric model, i.e.,

$$\boldsymbol{\delta} = \boldsymbol{\Lambda}(\boldsymbol{\xi})\boldsymbol{\chi} + \boldsymbol{\varsigma}$$

where $\boldsymbol{\Lambda}$ is a $(n+6 \times p)$ regressor matrix, $\boldsymbol{\chi}$ is a vector of constant parameters and $\boldsymbol{\varsigma}$ is the interpolation error. Of course, not all uncertainties can be rigorously

wide class of functions (Caccavale et al., 2013). The elements of the regressor matrix can be chosen as Radial Basis Functions (RBFs)

$$\lambda_{i,h}(\boldsymbol{\xi}) = exp\left(-\frac{||\boldsymbol{\xi} - \boldsymbol{c}_{i,h}||}{2\sigma^2}\right)$$

where $c_{i,h}$ and σ are the centroids and the width of the function, respectively. According to the Universal Interpolation Theorem any continuous function can be approximated (in the $\mathscr{L}_p - norm$ sense, $p \in [1,\infty)$)

curacy. Thus, the interpolation error $\boldsymbol{\varsigma}$ in can be reasonably assumed norm bounded,

$$||\boldsymbol{\varsigma}(t) \le \bar{\varsigma}||, \ \forall t \ge 0$$

An estimate,

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\Lambda}(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})\hat{\boldsymbol{\chi}}, (29)$$

, of $\boldsymbol{\delta}$ can be obtained by integrating the following update law for the unknown parameters $\boldsymbol{\chi}$

$$\dot{\hat{\boldsymbol{\chi}}} = \frac{1}{\beta}\boldsymbol{\Lambda}^T\boldsymbol{B}^T\boldsymbol{Q}\begin{bmatrix}\tilde{\boldsymbol{\xi}} \\ \dot{\tilde{\boldsymbol{\xi}}}\end{bmatrix}, (30)$$

where β is a positive scalar gain, $\boldsymbol{Q}$ is a symmetric and positive definite matrix, $\boldsymbol{B} = [\boldsymbol{O}_{6+n} \ \boldsymbol{I}_{6+n}]$ and $\tilde{\boldsymbol{\xi}} = \boldsymbol{\xi}_r - \boldsymbol{\xi}$ is the inner loop error. Then, $\hat{\boldsymbol{d}}$ can be easily obtained as $\hat{\boldsymbol{d}} = \boldsymbol{M}(\boldsymbol{\xi})\hat{\boldsymbol{\delta}}$.

STABILITY ANALYSIS

Inner loop:

Aim: To prove inner loop stability.

Approach:

- Inner loop error is derived from the closed loop dynamics.
- Interpolation error is considered 0 and then the state space form is rearranged.
- Theorem 1 is stated and proved to finally proved the inner loop stability.
- Proof is given by using the Lyapunov candidate function.

By considering (27), the following dynamics for the inner loop error $\tilde{\xi}$ can be derived

$$\ddot{\tilde{\xi}} = -\boldsymbol{\Omega}_V \dot{\tilde{\xi}} - \boldsymbol{\Omega}_P \tilde{\xi} + \tilde{\boldsymbol{\delta}}, (31)$$

where ($* =$ V, P)

$$\boldsymbol{\Omega}_* = \begin{bmatrix} -\boldsymbol{K}_{1,*} & O_3 & O_3 \\ O_3 & -\boldsymbol{K}_{2,*} & O_3 \\ O_3 & O_3 & -\boldsymbol{K}_{3,*} \end{bmatrix}, (32)$$

with $\boldsymbol{K}_{2,*} = diag\{K_{\psi,*}, K_{\theta,*}, K_{\varphi,*}\}$. Let us rearrange (31) in the state space form, by considering $\tilde{\boldsymbol{z}} = [\tilde{z}_1^T \ \tilde{z}_2^T]^T = [\tilde{\xi}^T \ \dot{\tilde{\xi}}^T]^T$ and assuming $\boldsymbol{\varsigma} = \boldsymbol{0}$,(30) as

$$\dot{\tilde{\boldsymbol{z}}} = \boldsymbol{\Omega}\tilde{\boldsymbol{z}} + \boldsymbol{B}\tilde{\boldsymbol{\delta}} = \boldsymbol{\Omega}\tilde{\boldsymbol{z}} + \boldsymbol{B}\boldsymbol{\Lambda}\tilde{\boldsymbol{\chi}}, (33)$$

where $\tilde{\chi} = \chi - \hat{\chi}$ and $\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{O}_{6+n} & \boldsymbol{I}_{6+n} \\ -\boldsymbol{\Omega}_V & -\boldsymbol{\Omega}_P \end{bmatrix}$, The following theorem can be stated for the inner loop error convergence.

STABILITY ANALYSIS

Inner loop(Continued...):

Theorem 1. Given the system and the update law (30), for any set of positive definite matrix gains $\boldsymbol{K}_{1,*}, \boldsymbol{K}_{2,*}$ and $\boldsymbol{K}_{3,*} (* = V, P)$, the equilibrium $\tilde{\boldsymbol{z}} = \boldsymbol{0}$ is globally asymptotically stable, while the parameters error χ.

Proof In order to analyze the stability of the system (33), the following Lyapunov candidate function could be considered

$$\boldsymbol{V}_i = \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{z}^T \boldsymbol{Q}\boldsymbol{z} + \frac{\beta}{2}\tilde{\chi}^T\tilde{\chi}.$$

The time derivative of V_i yields

$$V_i = -\boldsymbol{z}^T \boldsymbol{P}_z \boldsymbol{z} + \boldsymbol{z}^T \boldsymbol{Q}\boldsymbol{B}\boldsymbol{\Lambda}\tilde{\chi} + \beta\dot{\tilde{\chi}}^T\tilde{\chi}.$$

where $\boldsymbol{P}_z = -(\boldsymbol{Q}\boldsymbol{\Omega} + \boldsymbol{\Omega}^T\boldsymbol{Q})$ is the symmetric and positive definite solution of the Lyapunov equation that always exists since $\boldsymbol{\Omega}$ is Hurwitz. By assuming the parameter χ constant or, at least, slowly-varying, and by considering the update law (30), V_i can be rewritten as

$$\dot{\boldsymbol{V}}_i = -\boldsymbol{z}^T \boldsymbol{P}_z \boldsymbol{z} + \boldsymbol{z}^T \boldsymbol{Q}\boldsymbol{B}\boldsymbol{\Lambda}\tilde{\chi} + \beta\dot{\tilde{\chi}}^T\tilde{\chi} = -\boldsymbol{z}^T \boldsymbol{P}_z \boldsymbol{z}$$

Since $\boldsymbol{P}_z$ is positive definite, $\dot{\boldsymbol{V}}_i$ is negative semi-definite; this guarantees the boundedness of $\boldsymbol{z}$ and $\tilde{\chi}$. By invoking the Barbalat's lemma (Khalil, 1996), it can be recognized that $\dot{V}_i \to 0$, which implies the global asymptotic convergence to $\boldsymbol{0}$ of $\boldsymbol{z}$ as t $\to\infty$, while, as usual in direct adaptive control (Astrom and Wittenmark, 1995), $\tilde{\chi}$ is only guaranteed to be uniformly bounded, i.e., $||\tilde{\chi}|| \leq \bar{\chi}$.

STABILITY ANALYSIS

Kinematic control outer loop:

Aim: To prove the outer loop stability.

Approach:

- Considering the and perfect acceleration tracking, inverse kinematics error.
- Theorem 2 is used to prove the asymptotic stability using Lyapunov function.
- The proof of the theorem concludes asymptotically convergent to zero.

Under the assumption of perfect acceleration tracking (i.e.$\ddot{\zeta} = \ddot{\zeta}_r$), by substituting (14) in (13), the following holds

$$\dot{\boldsymbol{v}}_{e,d} - \dot{\boldsymbol{v}}_e = -\boldsymbol{K}_v(v_{e,d} - \boldsymbol{v}_e) - \boldsymbol{K}_p\boldsymbol{e}, (34)$$

Let us consider, the following inverse kinematics error

$$\epsilon = \begin{bmatrix} \epsilon_P \\ \epsilon_O \end{bmatrix}, \epsilon_P = \begin{bmatrix} e_P \\ e_{\dot{P}} \end{bmatrix}, \epsilon_O = \begin{bmatrix} e_O \\ \widetilde{\omega}_e \end{bmatrix},$$

where e_P and e_O are defined in (15), while $\widetilde{\omega}_e = \omega_{e,d} - \omega_e$.

STABILITY ANALYSIS

Kinematic control outer loop(Continued...):

Theorem 2 Given the system (34), there exists a set of positive definite matrix gains $\boldsymbol{K}_p$ and $\boldsymbol{K}_v$ chosen as in the kinematic control scheme with $k_{v,O} > \frac{1}{2}$, such that the equilibrium $\epsilon = 0$ is exponentially stable.

Proof

To prove the symptotic stability of the equilibrium point $\epsilon = 0$ let us consider the following candidate Lyapunov function (Chiaverini and Siciliano, 1999)

$$V_o = (\eta_d - \eta)^2 + (\epsilon_d - \epsilon)^T(\epsilon_d - \epsilon) + \tilde{\omega}_e^T \tilde{\omega}_e + \epsilon_P^T Q_P \epsilon_P$$

where $\eta(\eta_d)$ and $\epsilon(\epsilon_d)$ are the scalar part and the vector part of the unit quaternion representing the end-effector (desired) orientation and Q_P is a symmetric and positive matrix. The time derivative of V_o is given by

$$\dot{V}_o = -\boldsymbol{e}_O^T \boldsymbol{K}_{p,O} e_O - 2\tilde{\omega}_e^T K_{v,O} \tilde{\omega} - 2\tilde{\omega}_e^T K_{p,O} e_O$$

$$+\epsilon_P^T (Q_P A_P + A_P^T Q_P)\epsilon_P$$

, where $K_{*,O} (* = v, p)$ is the matrix including the last three row of matrix K_*, and

$$A_P = \begin{bmatrix} O_3 & I_3 \\ -K_{p,P} & -K_{v,P} \end{bmatrix}.$$

with $K_{*,P} (* = v, p)$ the matrix including the first three row of matrix K_*. Since A_P is Hurwitz, always exists a matrix P_P , symmetric and positive definite, solution of the Lyapunov equation in (46), hence

$$\dot{V}_o \leq -\lambda_m(K_{p,O})||e_O||^2 - 2\lambda_m(K_{v,O})||\tilde{\omega}_e||^2$$

$$-2\lambda_M(K_{p,O})||\tilde{\omega}_e||||e_O|| - \lambda_m(P_p,)||e_p||^2$$

$$\leq -\begin{bmatrix} ||e_O|| \\ ||\tilde{\omega}_e|| \end{bmatrix}^T \Xi \begin{bmatrix} ||e_O|| \\ ||\tilde{\omega}_e|| \end{bmatrix} - \lambda_m(P_p,)||e_p||^2$$

with

$$\Xi = \begin{bmatrix} \lambda_m(K_{p,O}) & -\lambda_M(K_{p,O}) \\ -\lambda_M(K_{p,O}) & 2\lambda_m(K_{v,O}) \end{bmatrix},$$

and $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ representing the minimum and maximum eigenvalue of a matrix. If the following holds

$$\lambda_m(K_{v,o}) > \frac{\lambda_M^2(K_{p,o})}{2\lambda_m(K_{p,o})},$$

matrix Ξ is positive definite, therefore $\dot{V}_o$ can be upper bounded as

$$\dot{V}_o \leq -\lambda_m(\Xi)||\epsilon_O||^2 - \lambda_m(\Xi)||\epsilon_P||^2 \leq -\lambda_\epsilon||\epsilon||^2$$

where $\lambda_\epsilon = min\{\lambda_m(\Xi), \lambda_m(\Xi)\}$.

Thus, since $\dot{V}_o$ is negative definite the error ϵ is asymptotically convergent to zero.

STABILITY ANALYSIS

Two-loops dynamics:

Aim: To prove that all unknown parameters and errors are converging to zero in the absence of interpolation error.

Approach:

- The Lyapunov candidate function is stated and it is straightforward ε and z are globally asymptotically convergent to zero.
- Theorem 3 along with the Corollary 1 and Theorem 2 is used to prove the the above stated aim.
- It is stated that manipulator joints are revolute joints and are bounded and the perturbation term is thus Vanishing.

For the two-loops dynamics, by considering the following Lyapunov candidate function

$$V = V_o(\epsilon) + V_i(z),$$

it is straightforward derived from (42) and (47) that $\dot{V}$ is negative definite and both ϵ and z are globally asymptotically convergent to zero. Moreover the con-

Theorem 3 Under the assumption of perfect acceleration tracking, in the presence of the PE condition and in the absence of the interpolation error($\varsigma = 0$) , on the basis of Corollary 1 and Theorem 2,ϵ,$\hat{\zeta}$and $\hat{\chi}$ are exponentially convergent to zero.

STABILITY ANALYSIS

Two-loops dynamics(Continued...):

- Perturbation term is upper bounded when the PE condition is satisfied.
- However, it is non-vanishing but it is bounded and when this condition is not satisfied.
- The errors are bounded but are not converging to 0 when PE is not satisfied.

In practice, the inner loop cannot guarantee instantaneous perfect tracking of the of the desired acceleration, therefore by considering the error $\hat{\ddot{\xi}} = \ddot{\xi}_r - \ddot{\xi}$, the outer loop error dynamics becomes

$$\dot{\boldsymbol{v}}_{e,d} - \dot{\boldsymbol{v}}_e = -\boldsymbol{K}_v(\boldsymbol{v}_{e,d} - \boldsymbol{v}_e) - \boldsymbol{K}_p\boldsymbol{e} + \boldsymbol{J}_\xi\ddot{\xi}, (34)$$

The perturbation term $J_\zeta\tilde{\ddot{\zeta}}$ can be upper bounded as The norm $||\boldsymbol{J}_\xi||$ is bounded since all the manipulator joints are revolute joints, therefore the first term can be viewed as a vanishing perturbation.

$$||J_\zeta\tilde{\ddot{\zeta}}|| \leq ||J_\zeta||||\tilde{\ddot{\zeta}}|| \leq ||J_\zeta||||\dot{z}_2|| \leq ||J_\zeta||(||\Omega||||z|| + ||\tilde{\delta}||)$$
$$\leq ||J_\zeta||||\omega||||z|| + ||J_\zeta||||\Lambda||||\chi||.$$

In the case $\boldsymbol{\varsigma} = \boldsymbol{0}$ the second term in the above equation is, at least, bounded, since, from Theorem 1,χ is at least bounded, while, as already specified, $\boldsymbol{\Lambda}$ is a norm bounded. Therefore, the convergence properties of the

SIMULATION RESULTS

Aim: To develop and verify the model in the an environment.

Approach:

- Python is used to visualize the outputs.
- DH parameters table is made for the 5-DOF manipulator and gain values and other noise parameters are decided upon in such a way that the conditions mimic the practical scenarios as closely as possible.
- Trajectory of the drone is plotted compared with the desired trajectory, changes in them are plotted with respect to time.

Joint	d[mm]	a[mm]	alpha[rad]	theta[rad]
1	0	0	$-\pi/2$	q1
2	0	150	$\pi/2$	q2
3	0	8	0	q3
4	0	0	$-\pi/2$	q4
5	18	0	0	q5

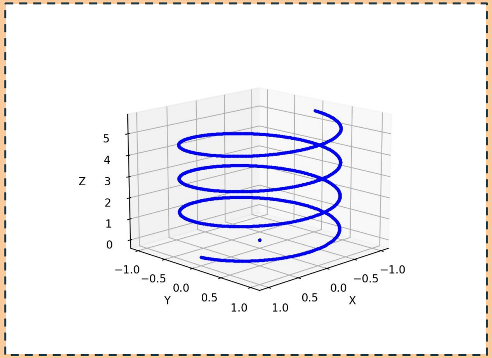
SIMULATION RESULTS

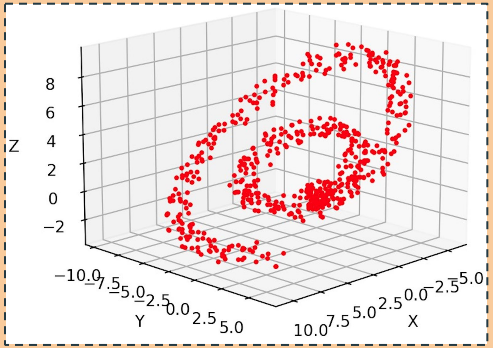
- Noises generated are: Joint position noise, Robot position noise, Robot orientation noise, Attitude rate noise.
- All the measured data have been considered available at a frequency rate of 250 Hz.
- Moreover, a rotation of pi/5 rad along roll, pitch and yaw axes is required as well.
- The following are also selected: Drag coefficient=7.5x10^-7, thrust coefficient = 3x10^-5.
- Compensation of the Centrifugal and Coriolis terms, C, has been neglected.
- Links masses selected are as follows: Link1- 80g, Link2- 10g, Link3- 6g, Link- 2g, Link5- 1.5g.

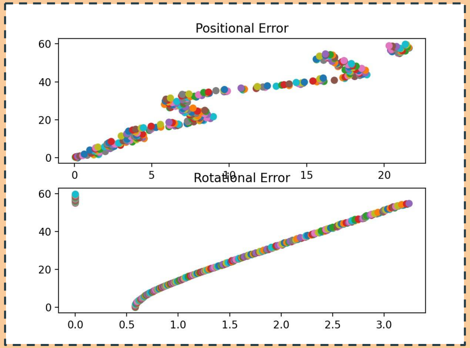
Gains	Values
$K_{p,P}$	$12I_3$
$K_{P,V}$	$5I_3$
$K_{\psi,P}$	8
$K_{\psi,V}$	3
$K_{\phi,P}$	2
$K_{\phi,V}$	1
β	4
$K_{Q,P}$	$140I_5$
$K_{Q,V}$	$20I_5$
$K_{\theta,P}$	2
$K_{\theta,V}$	1
K_P	$7.5I_6$
K_V	$0.6I_6$

SIMULATION RESULTS

Simulation results are as follows:







Desired Trajectory

Obtained End-effector trajectory

Rotation and position error from the desired and obtained trajectory