

# Computer Vision

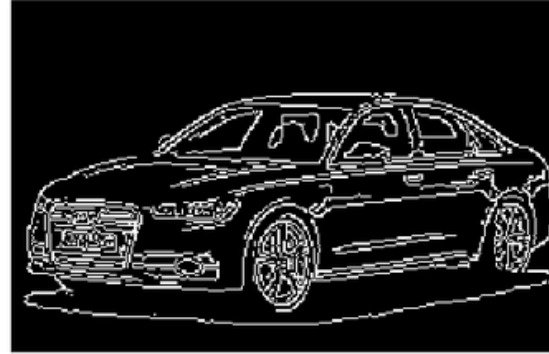
## Feature Extraction

(Corner and interest point detection, Local variant feature detectors and descriptors)

# Contents

- Corner and interest point detection
- Local variant feature detectors and descriptors

# Why Feature Descriptor?

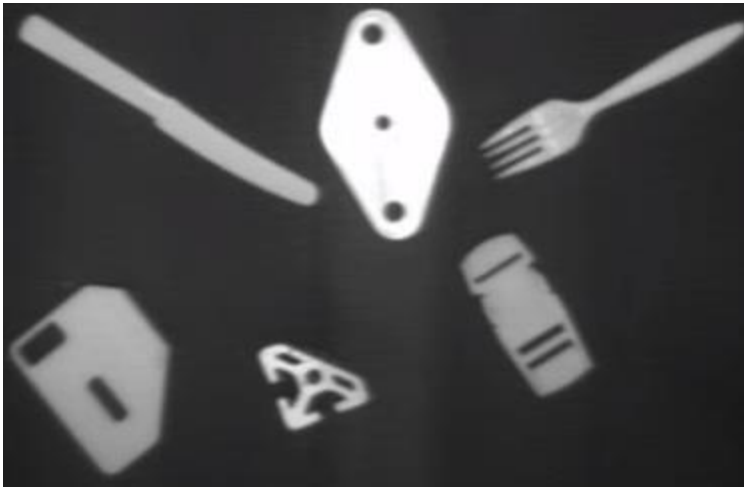


- Edges can be used to differentiate cat and dog
- Easy to classify cat and dog

- Shape and edges are known
- Can differentiate the two images
- Can not be used to match two objects
- Feature descriptor is a representation of the image
- It contains important information about the image

# Object Detection

- Detects and match features which are descriptive and unique

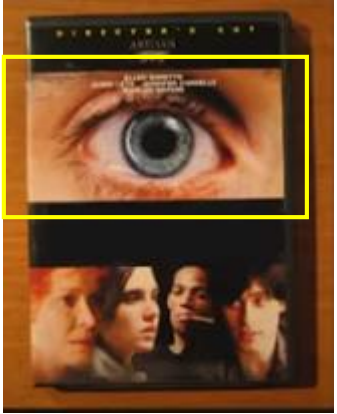


- Apply thresholding to convert it to binary
- Compare geometric properties and recognize the objects
- Also recognize position and orientation



- More complex
- Still, can apply thresholding
- And recognize letter and identify license plates

# Object Detection



Template



Image

Find key point (interest points) and match them to recognize objects

- Find template in image
- Template may not exactly be same as the object to be detected in image
- Objects in image may be
  - Rotated
  - scaled
  - occluded
- To detect such object create several templates with different rotations and scales
- Technique would be computationally expensive
- Use key points to reduce complexity

# Feature descriptors

- Interest points can be particular elements like unique points, edges, or corners
- Define local patches surrounding interest points
- Extract feature descriptors for each key point
- Match descriptors in images to find correspondence for image matching and other applications

# Local and Global Descriptors



Global feature representation



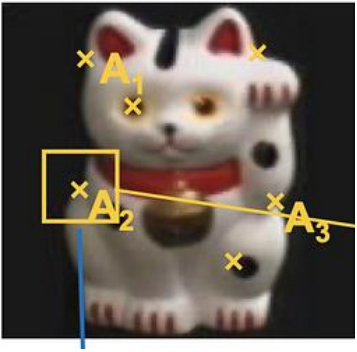
Local feature representation

# Local and Global Descriptors

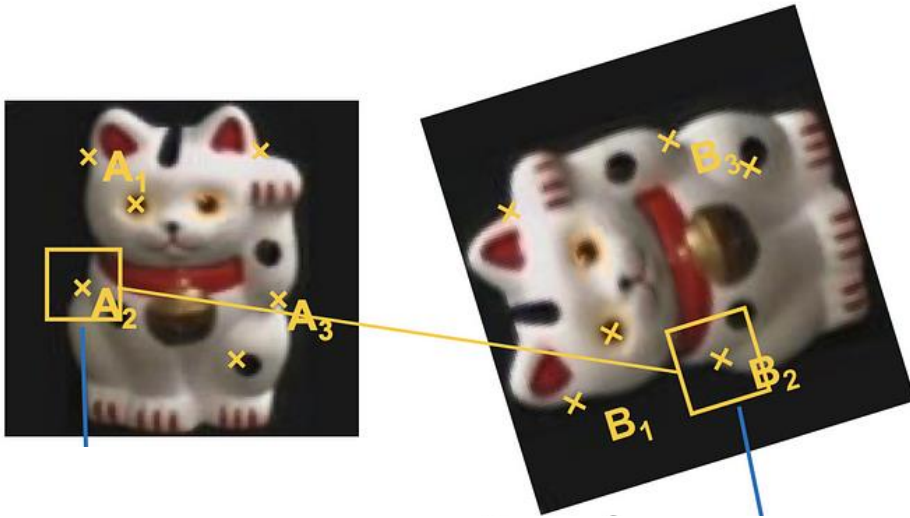
- Global Descriptors
  - Describe the image as a whole
  - Contour representations, Shape descriptors, Texture features
  - DoG, HOG, histograms of optical flow (HOF) etc are few examples
  - Limitations: difficult to detect objects with occlusions, profile variations
- Local Descriptors
  - Describes a patch within an image
  - Use multiple local descriptors to match an object
  - More accurate than global
  - Provides more robustness against occlusions and profile variations
  - SIFT, SURF, LBP, BRISK, MSER, and FREAK are examples



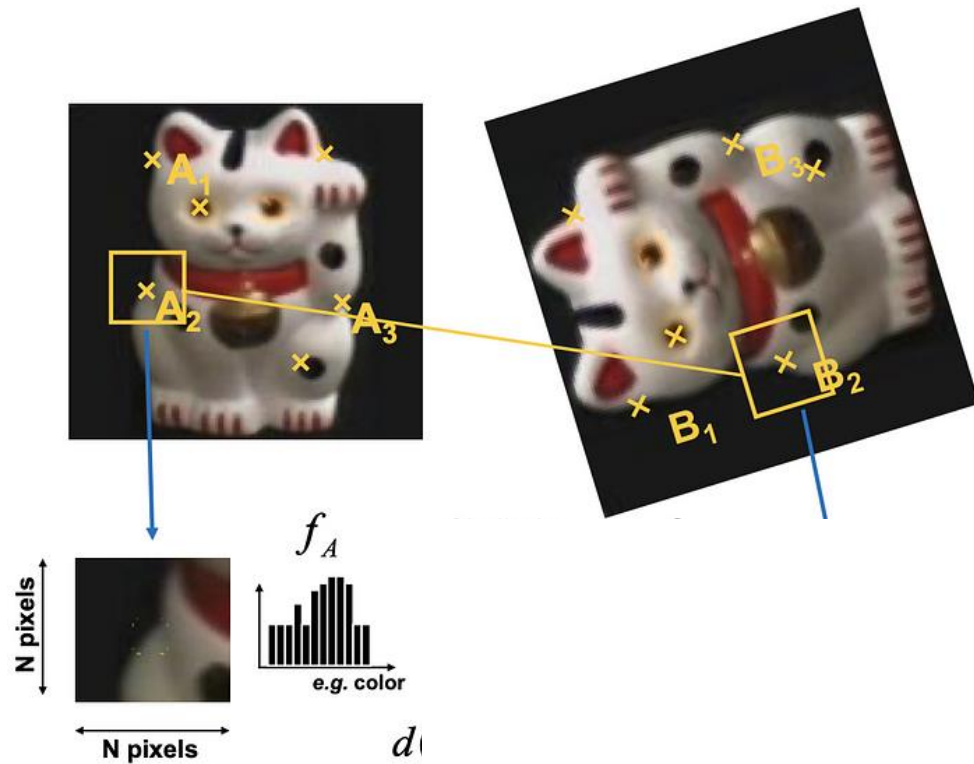
# Overview of Local Features Matching



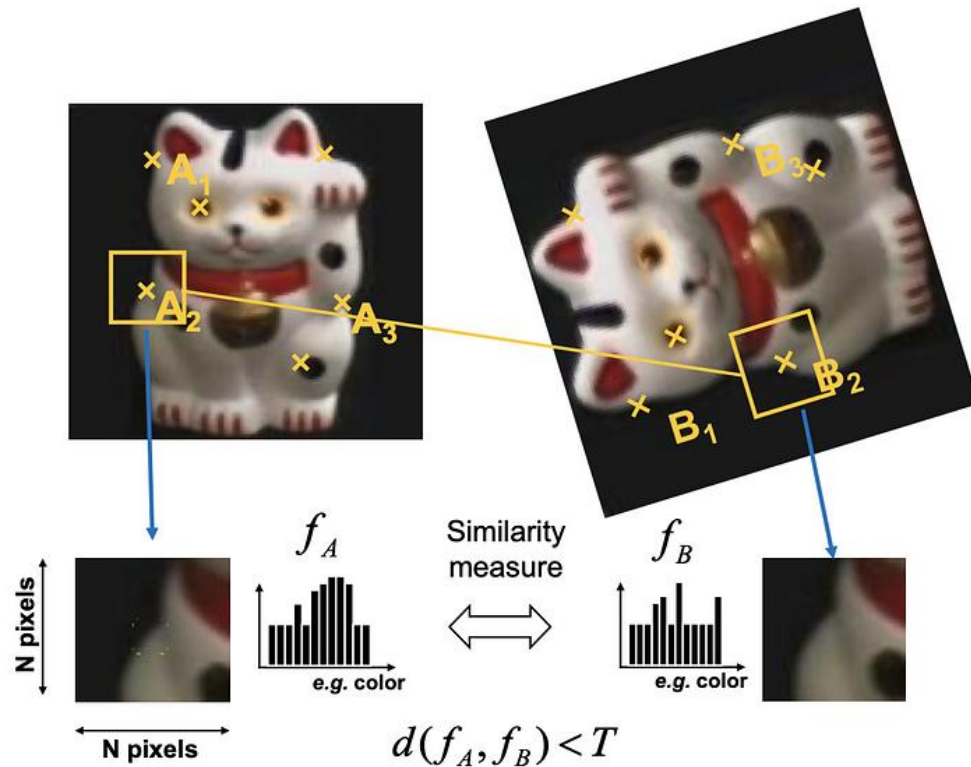
# Overview of Local Features Matching



# Overview of Local Features Matching



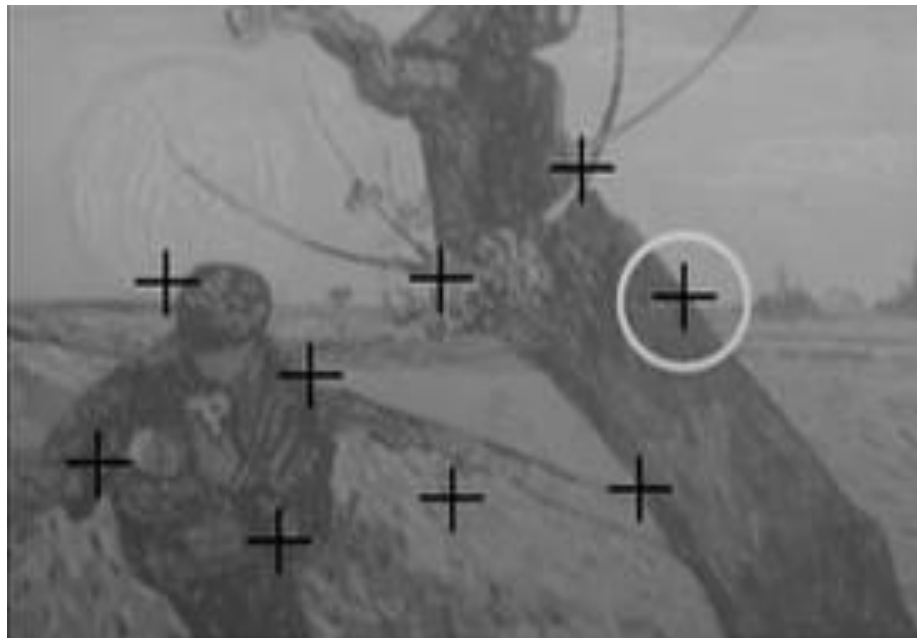
# Steps for Local Features Matching



- Determine distance between local descriptors of two interest points
- Match the local descriptors between images
- If matched, determine correspondence between two patches
- Apply same correspondence to other interest points

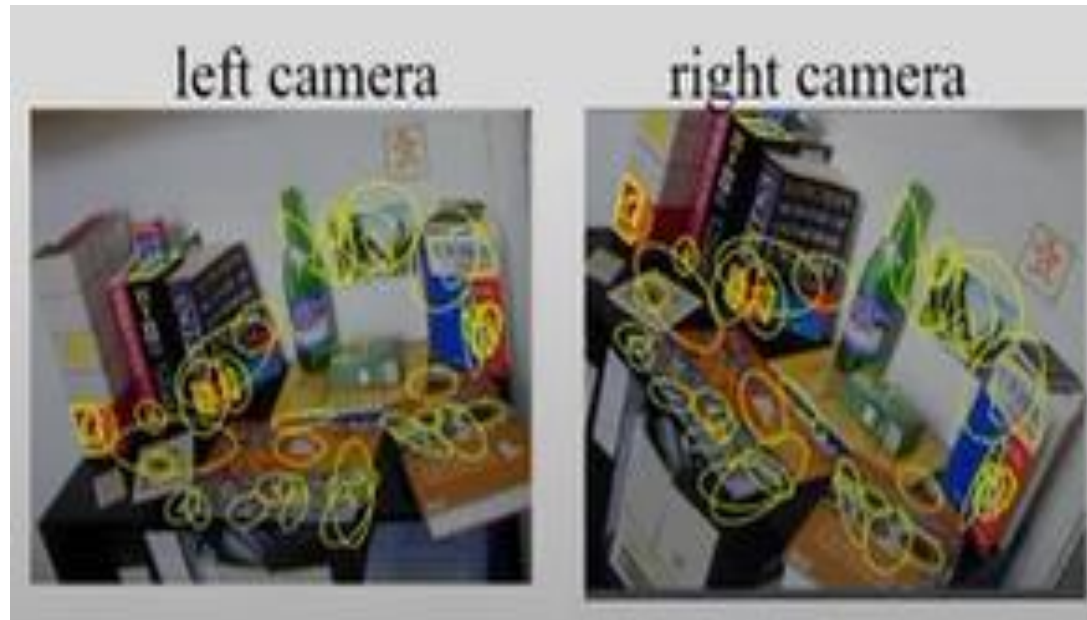
# How to define interest point?

- There may be a change of one or more image properties (color, texture, intensity etc)
- Interest points should be robust to these changes
- Corner point is an example of interest point



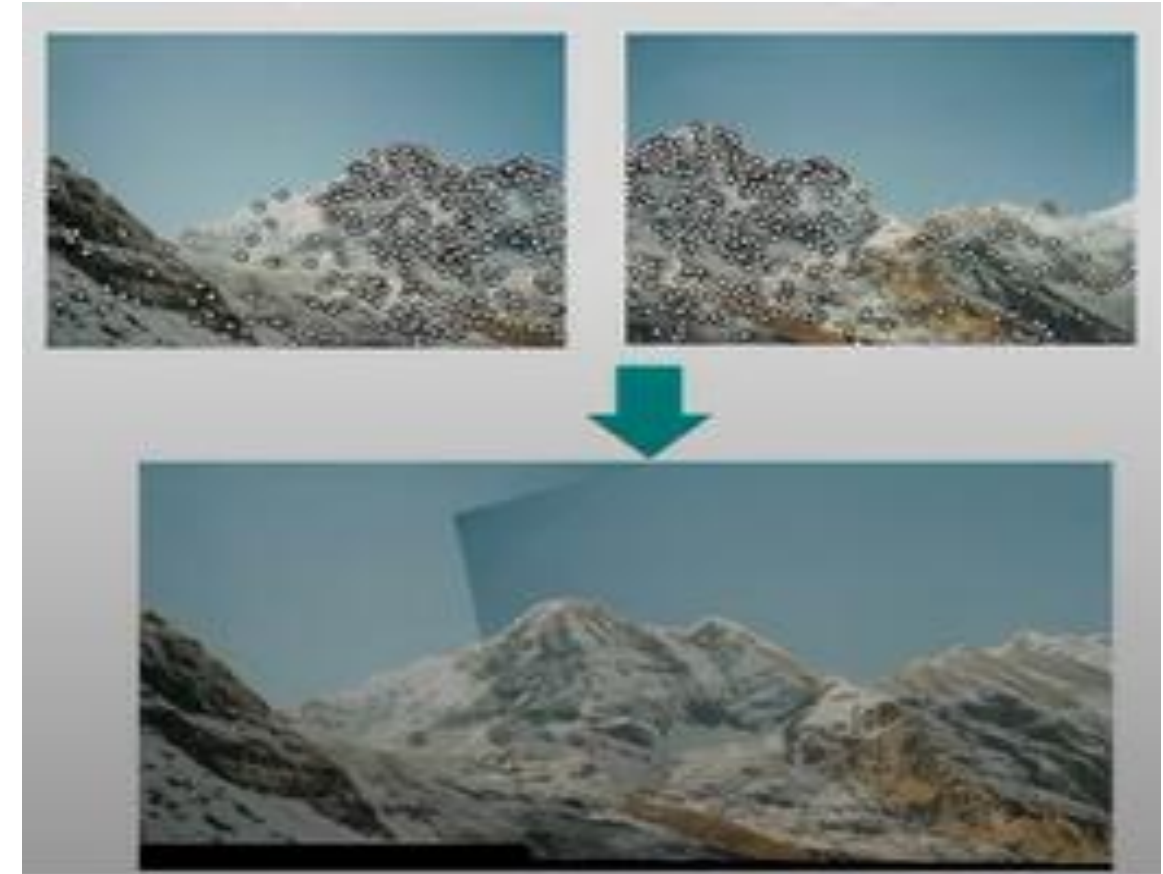
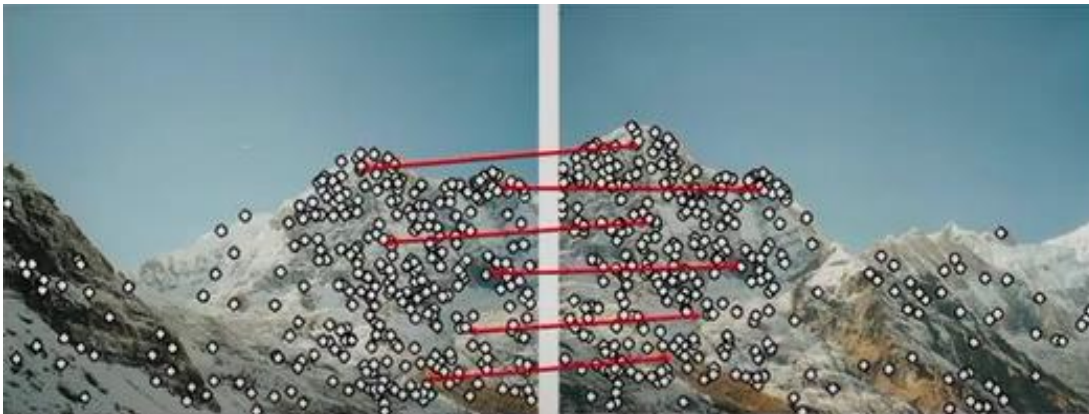
# Interest point Detection and Applications

- Stereo image matching
  - Choose corner interest points on both images
  - Apply image matching between left and right image for stereo correspondence



# Interest point Detection and Applications

- Join two images to get a bigger image
  - Choose interest point (corner point)
  - Use correspondence between interest points for image alignment

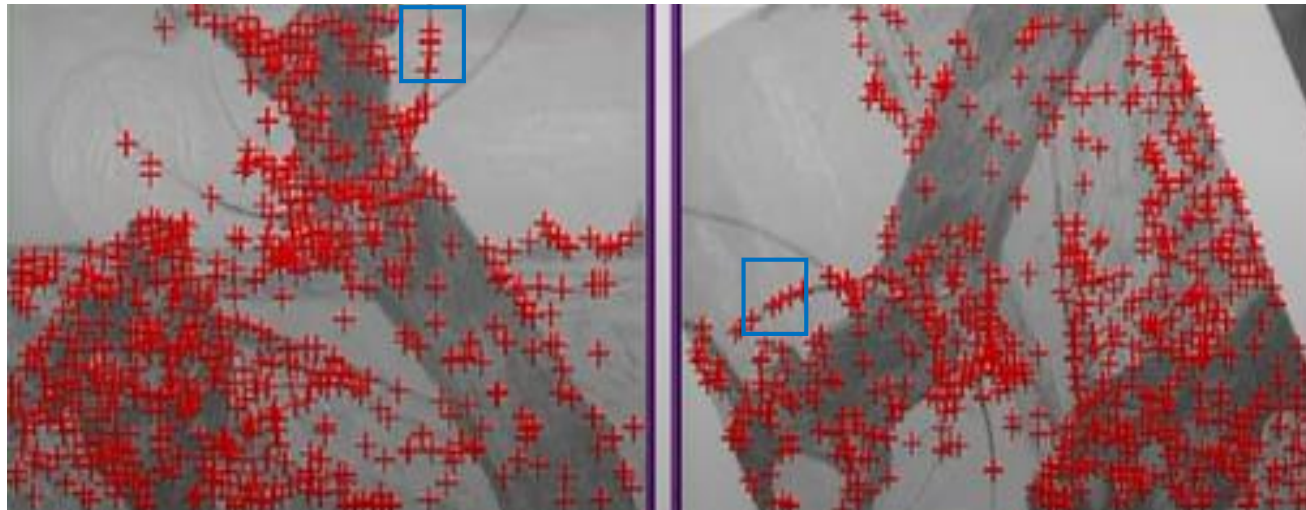


Panorama Stitching



# Characteristics of good features of interest points

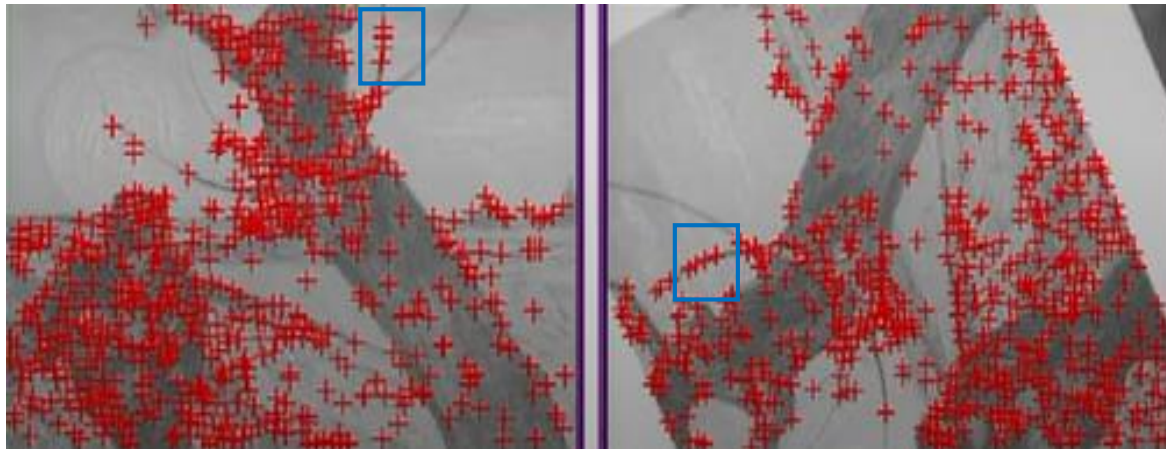
- Repeatability
  - Find same feature in multiple images irrespective of geometric and photometric transformations
- Saliency
  - Each feature is distinct/unique
- Compactness
  - fewer features than number of image pixels which represent entire image





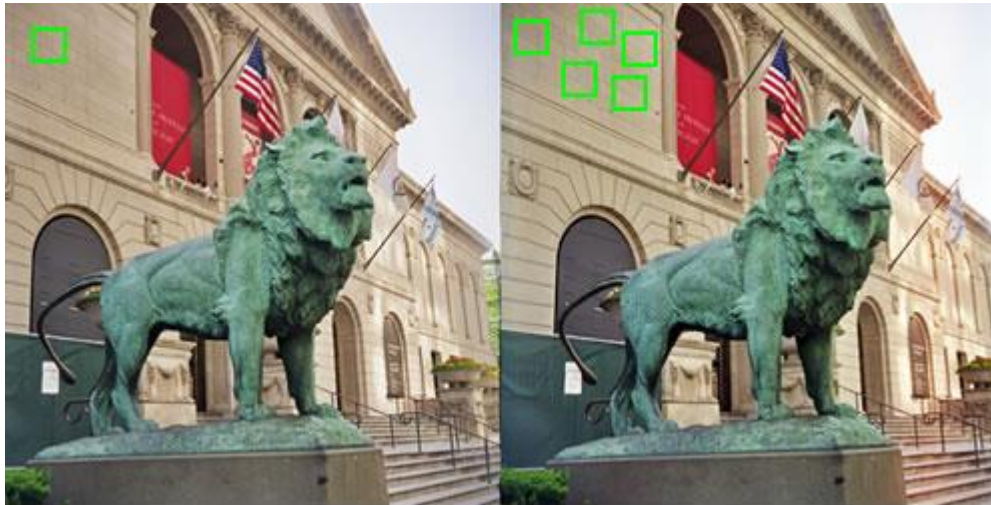
# Characteristics of good features

- Efficiency
  - Computationally faster for real time applications
- Locality
  - Occupy small area of image
  - Robust to clutter and occlusion
- Covariant
  - Should be detected in corresponding locations despite geometric and photometric variations



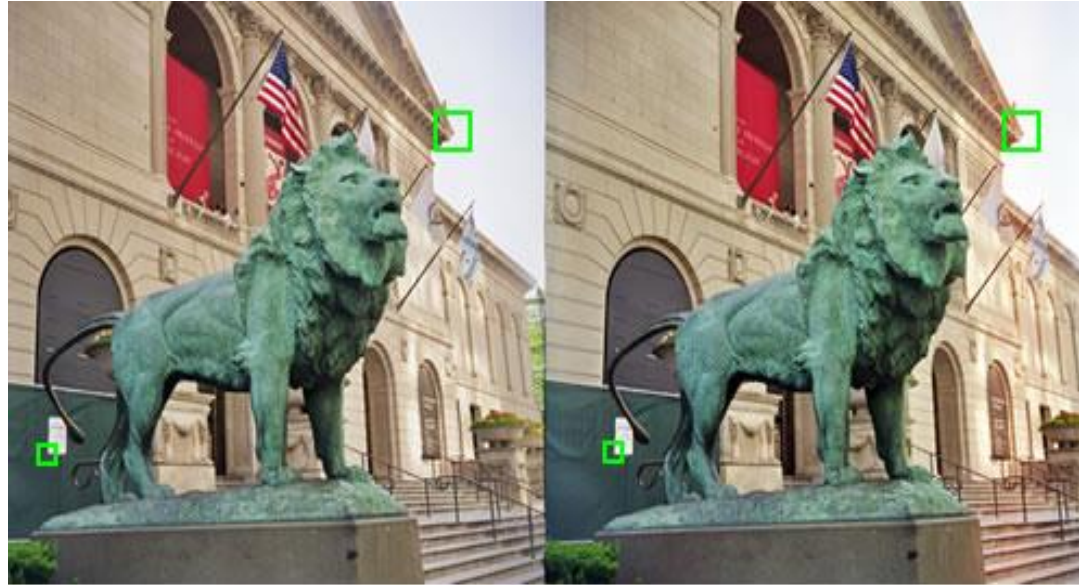
# Patches of images for matching

- Select a patch in one image
- Match it with a patch in the other image



- Patch should have unique shape in the image

# Patch Matching



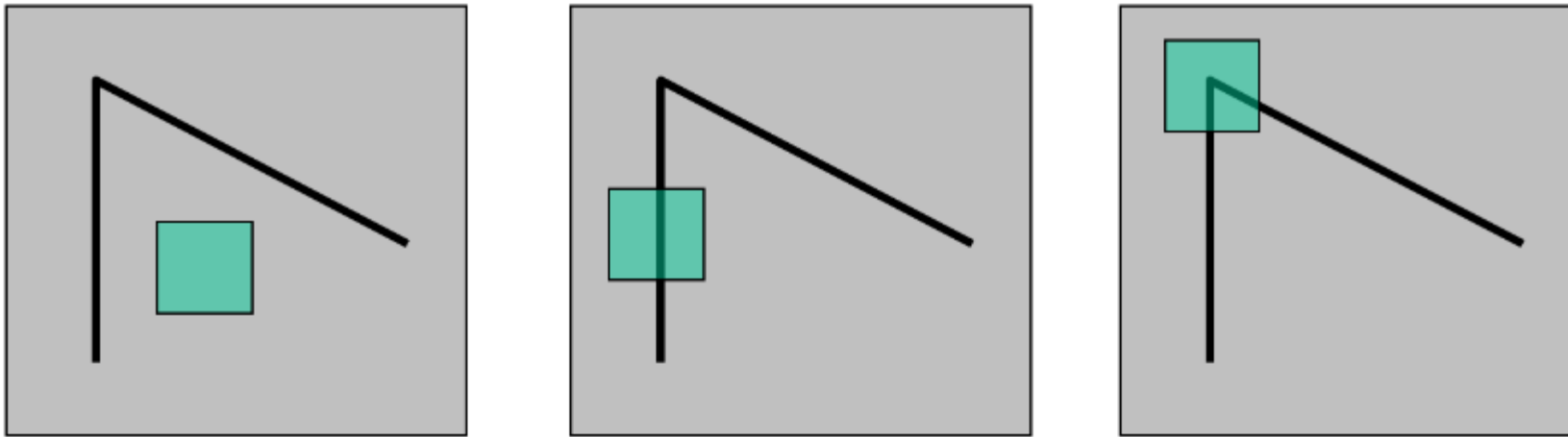
- Corners are unique points in images
- Locate patch around corners and map it to image feature

# Corner Point

- Corner points are special case of interest points
- Corner –
  - Is the intersection of two or more edge segments
  - Well localized or have a well-defined location in the image space
  - Maintain their stability against scaling and rotation
  - Can compute the interest points accurately
  - Offers effective detection

# Corner Point

- Window with a fixed size is chosen to detect the corner
- If window is shifted,



Flat portion: No change in pixel intensities in all directions

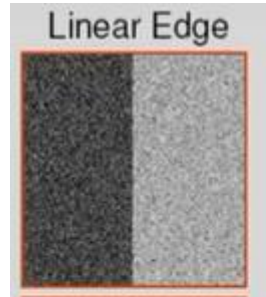
Edge: No change in pixel intensities along edge direction

Corner: Significant change in pixel intensities in all directions

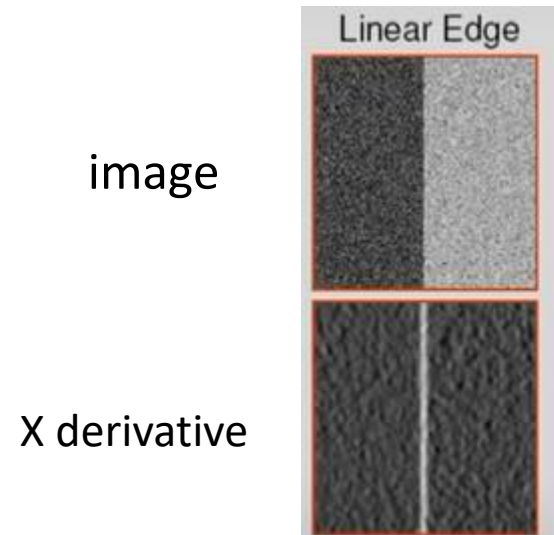
Difference in pixel intensities in window can be used to identify corners

# Examples of Derivatives

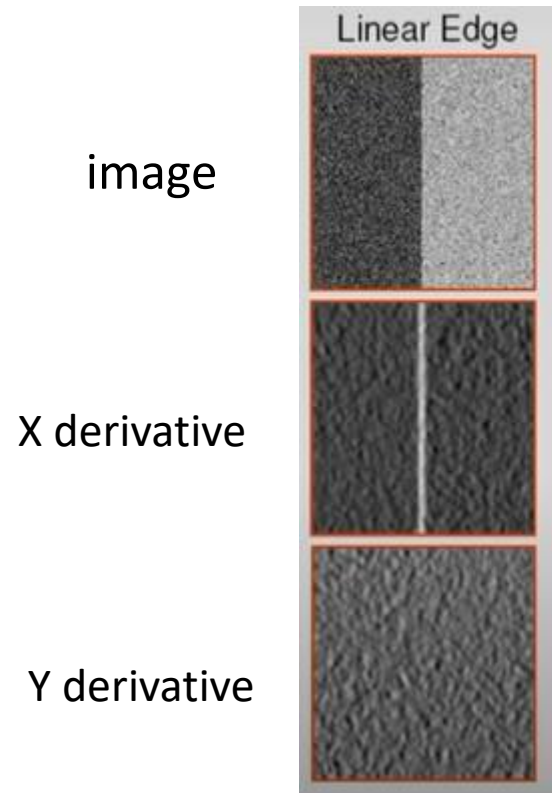
image



# Examples of Derivatives

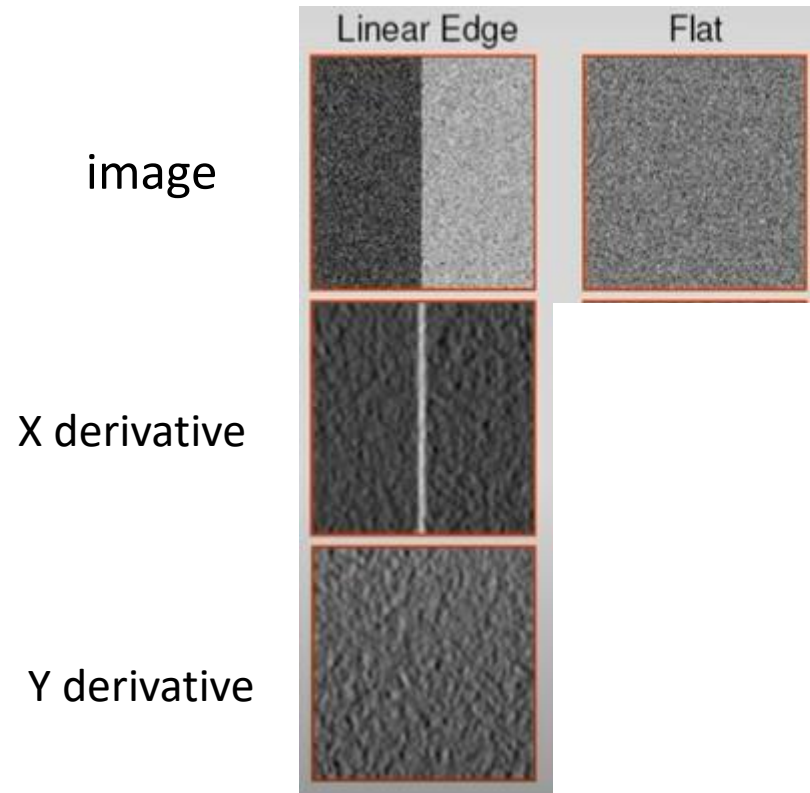


# Examples of Derivatives

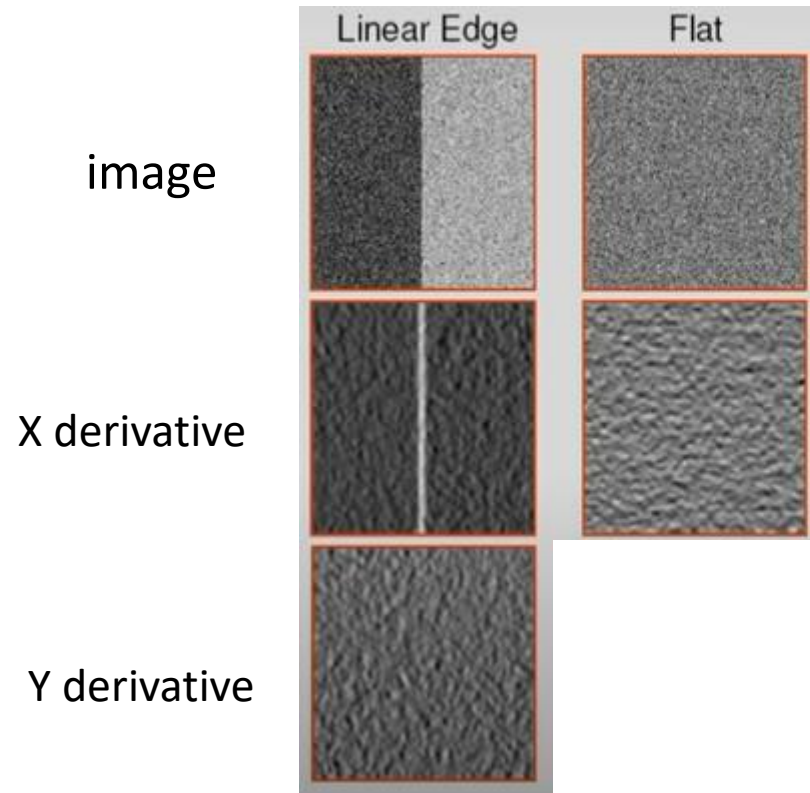




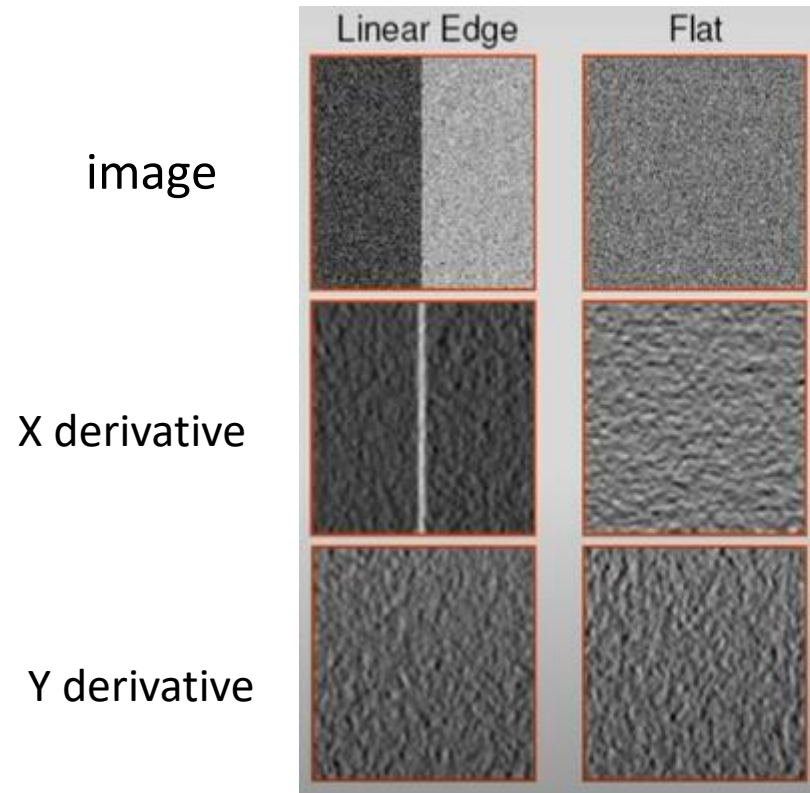
# Examples of Derivatives



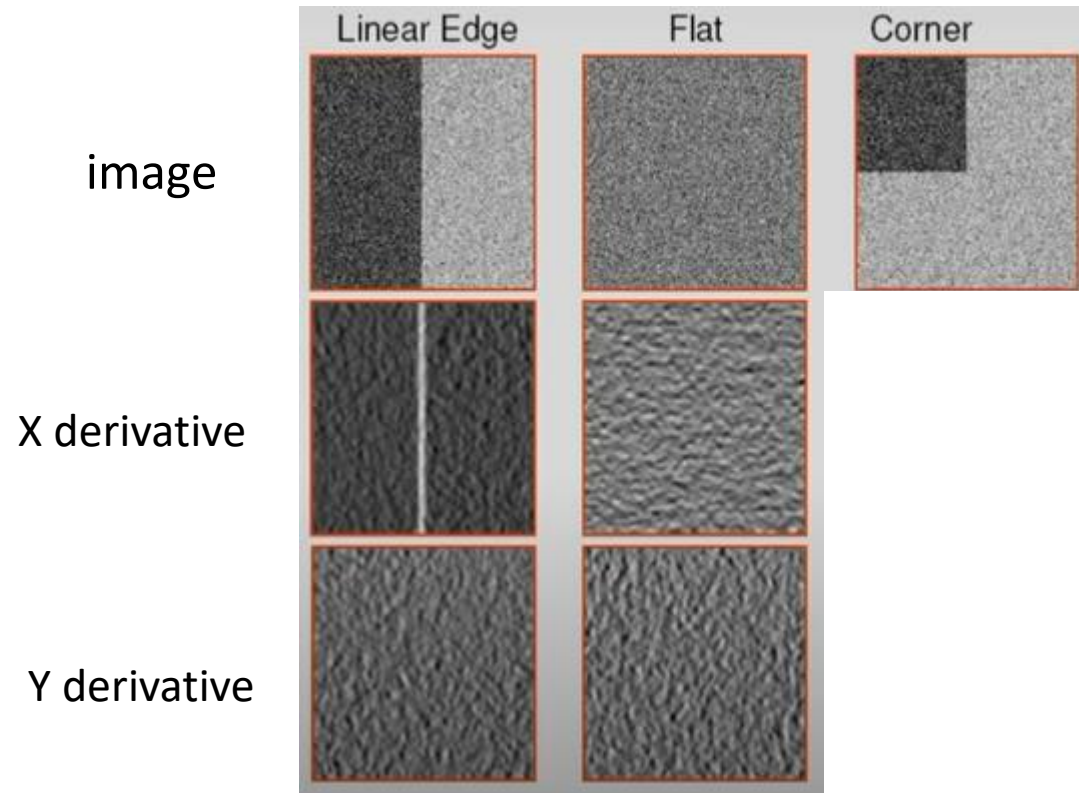
# Examples of Derivatives



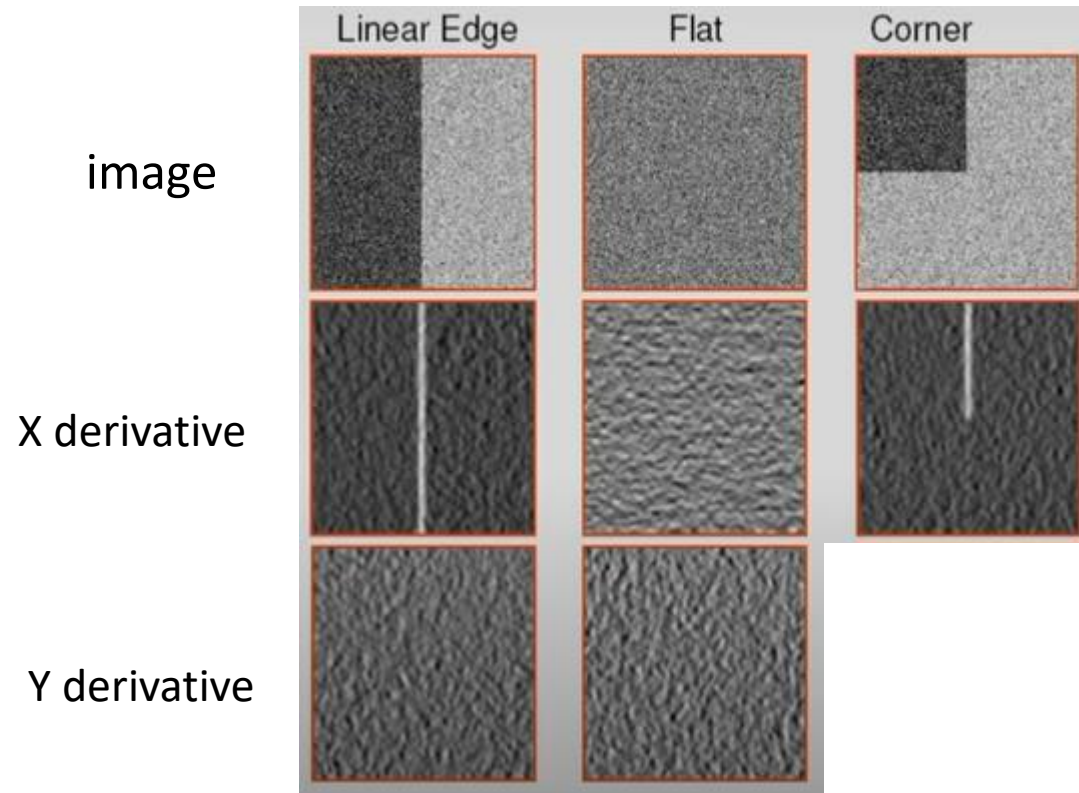
# Examples of Derivatives



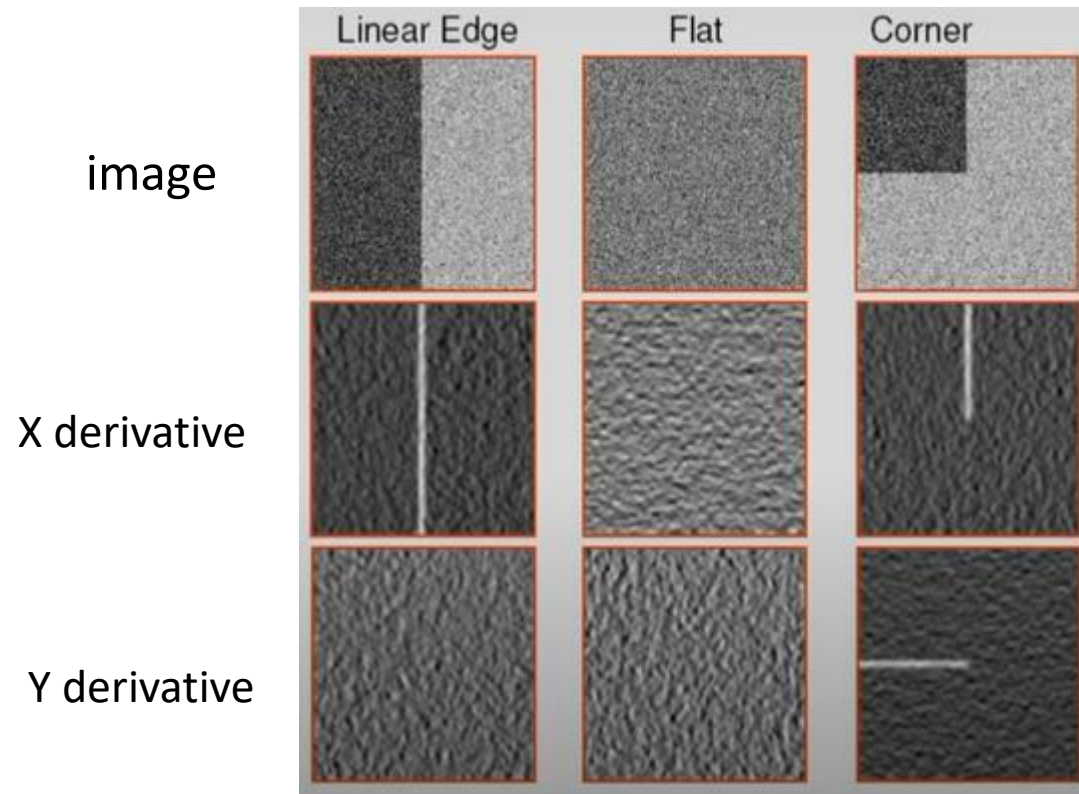
# Examples of Derivatives



# Examples of Derivatives



# Examples of Derivatives



# Corner detector

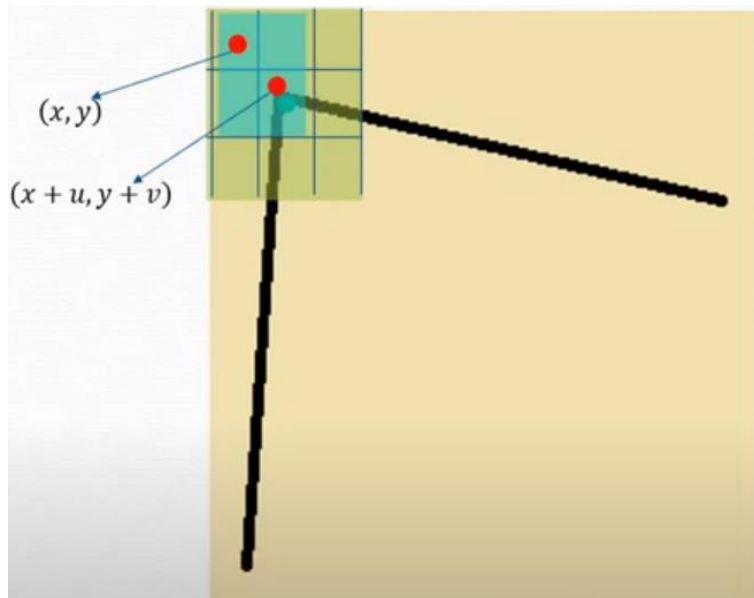
- Locate interest points where the surrounding neighborhood shows edges in more than one direction
- These points are image corners
- Change in intensity is tested for the shift (u,v) of a window

$$E(u, v) = \sum_{x,y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

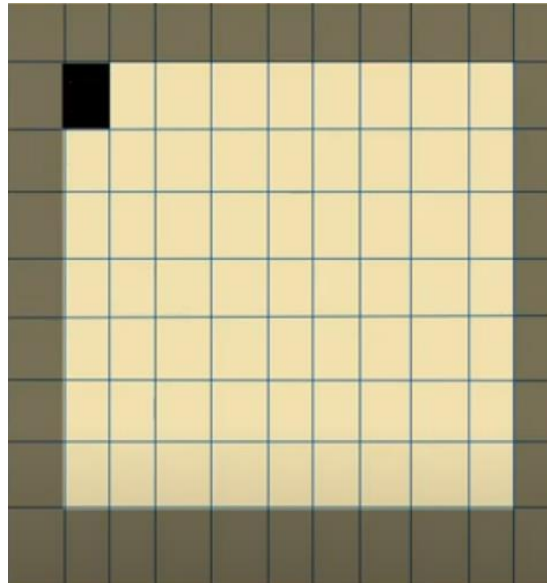
- For nearly constant patches, change of intensity,  $E(u,v)$  is almost 0
- For different patches,  $E(u,v)$  is large
  - Then it is corner point

# Corner detector

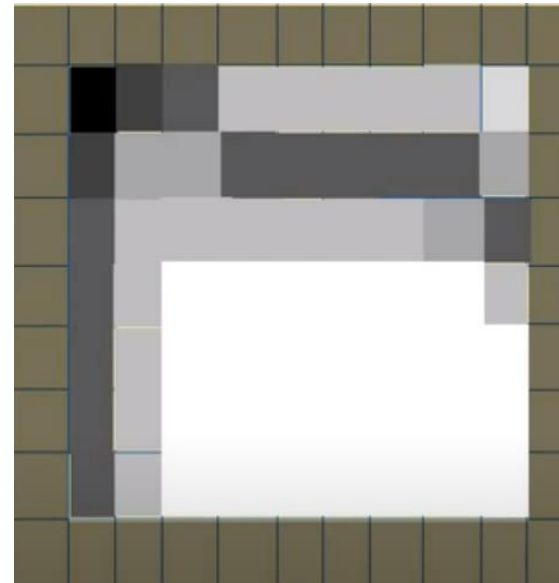
$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$



$E(u, v)$  for window at the corner point



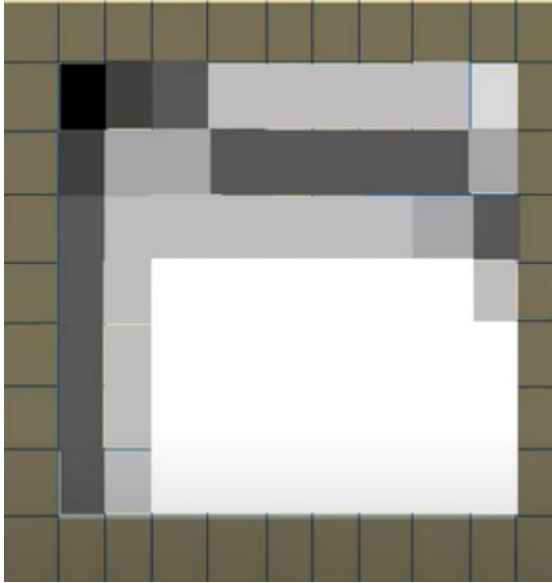
$E(u, v)$  for window at each point of image



Maximum value: black, Minimum value: white



# Corner detector



Maximum value: black,  
Minimum value: white

- $E(u,v)$  for
  - Corner has maximum value
  - Edges has relatively low value
  - Patches have minimum value
- Entire process of calculation is computationally expensive, especially for large images
- Harris corner detector is computationally less expensive

# Harris Corner Detector

- For small shifts,  $(u,v)$ ,  $E(u,v)$  can be approximated as

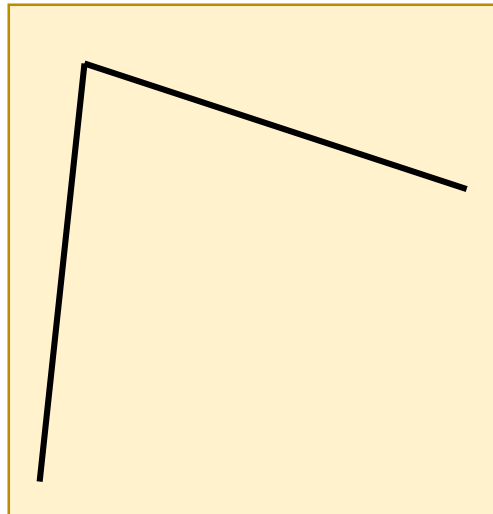
$$E(u, v) = [u, v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

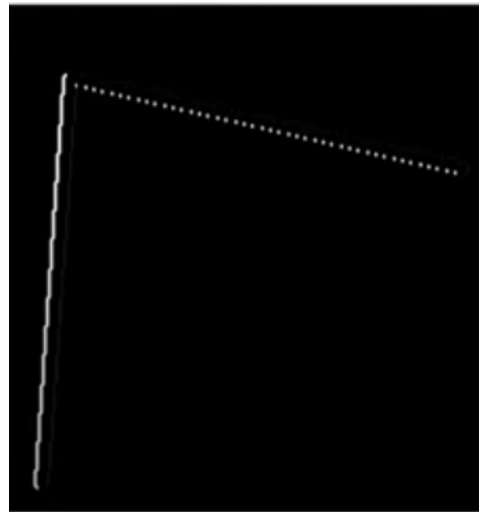
- $I_x$  and  $I_y$  are derivatives in x and y directions in window (patch)

# Harris Corner Detector

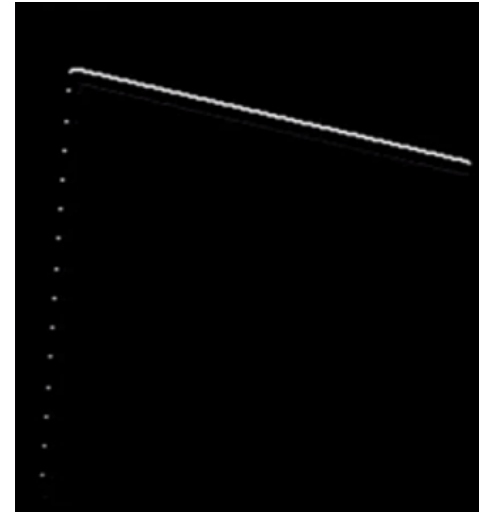
- Apply prewitt or sobel operator to determine gradients,  $I_x$  and  $I_y$
- For perfect vertical edge,  $I_y = 0$  and for perfect horizontal edge,  $I_x = 0$
- For flat regions,  $I_x$  and  $I_y$  are zero



Image



$I_x$



$I_y$

Prewitt operator

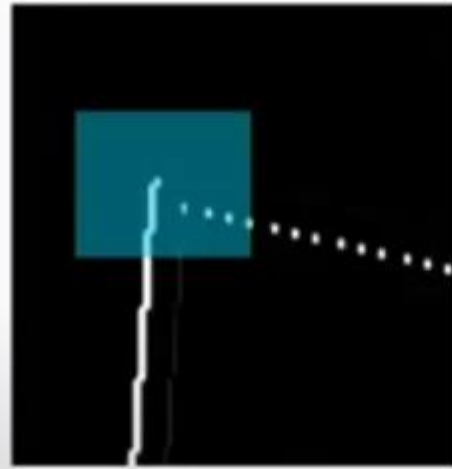
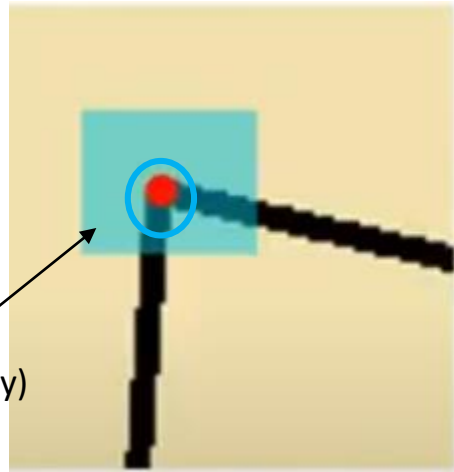
1	0	-1
1	0	-1
1	0	-1

To generate  $I_x$

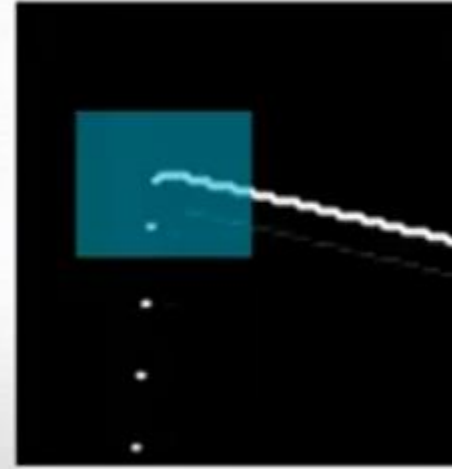
1	1	1
0	0	0
-1	-1	-1

To generate  $I_y$

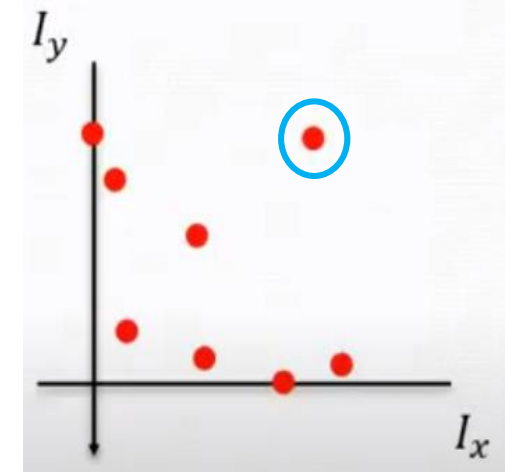
# Harris Corner Detector



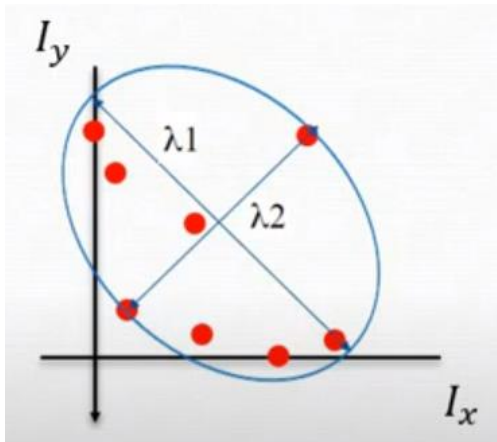
$I_x$



$I_y$



Gradients of points in window



$\lambda_1$  and  $\lambda_2$  are large

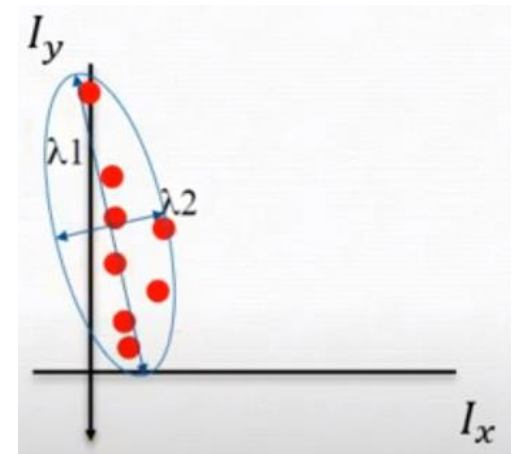
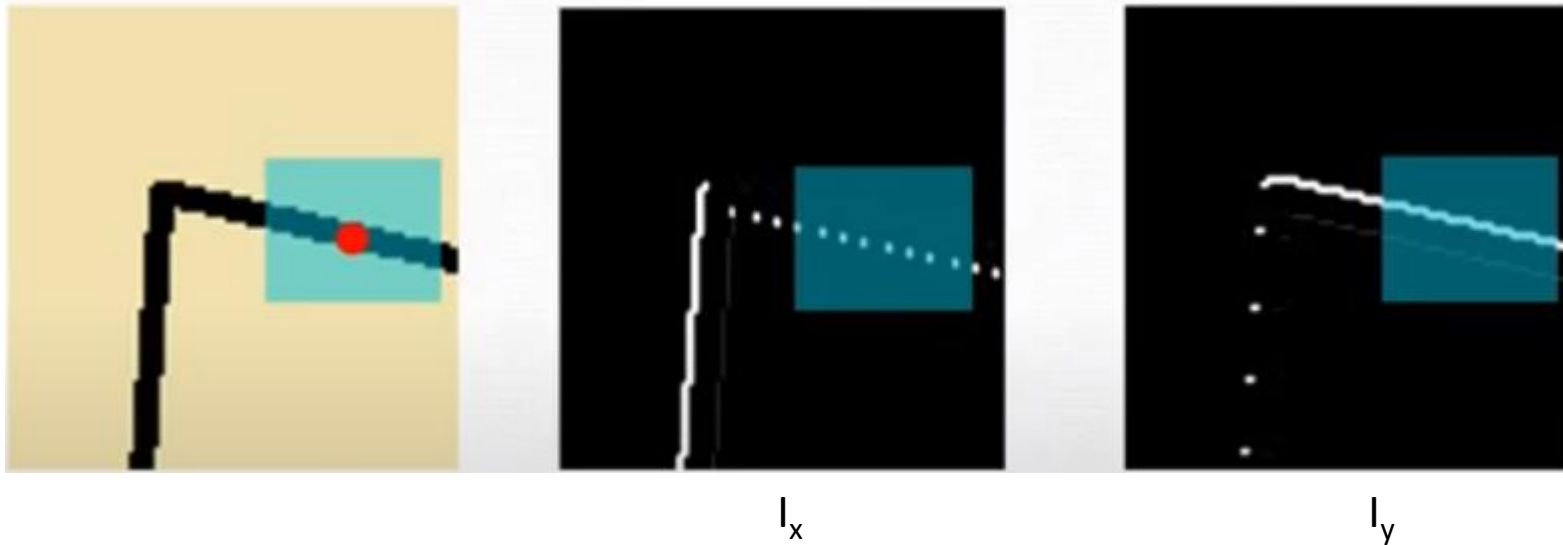
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- $\lambda_1$  and  $\lambda_2$  are Eigen values of  $M$
- $\lambda_1$  corresponds to major axis and  $\lambda_2$  corresponds to minor axis

If  $\lambda_1$  and  $\lambda_2$  are large, point at the center of window is a corner

# Harris Corner Detector

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

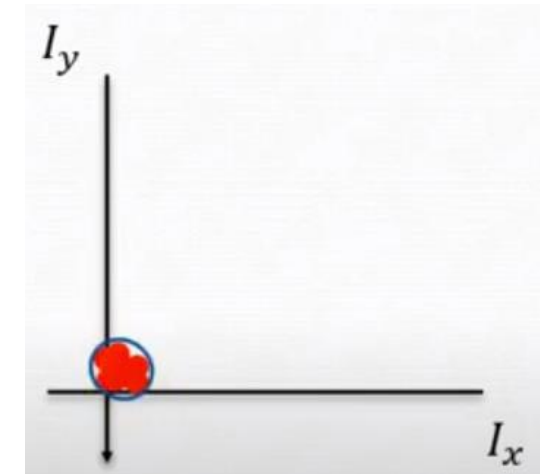
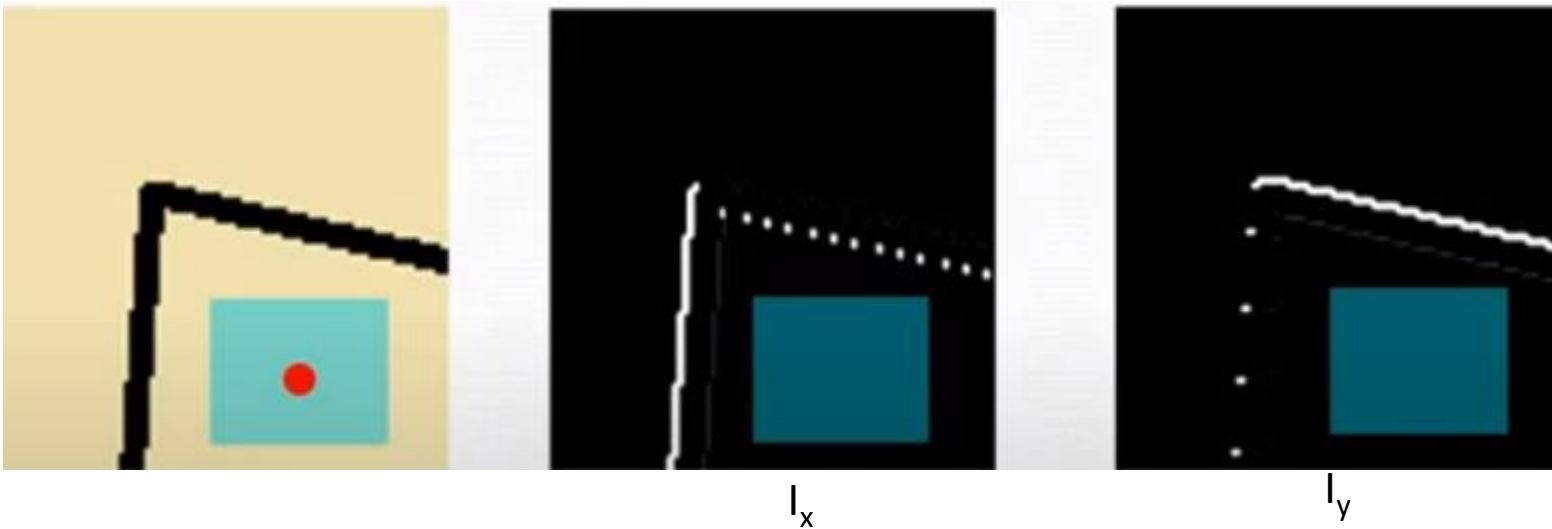


Gradients of points in window

Point at the center of window is on the edge

# Harris Corner Detector

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

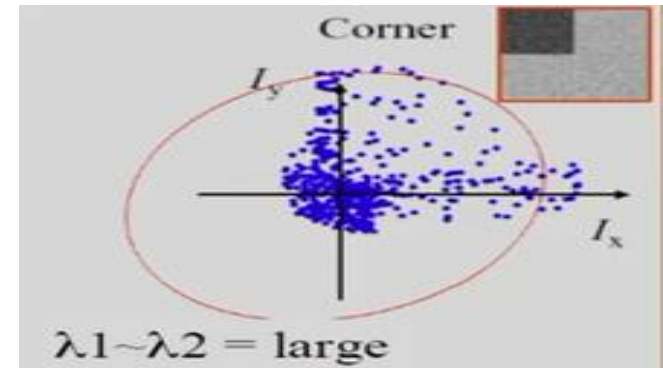
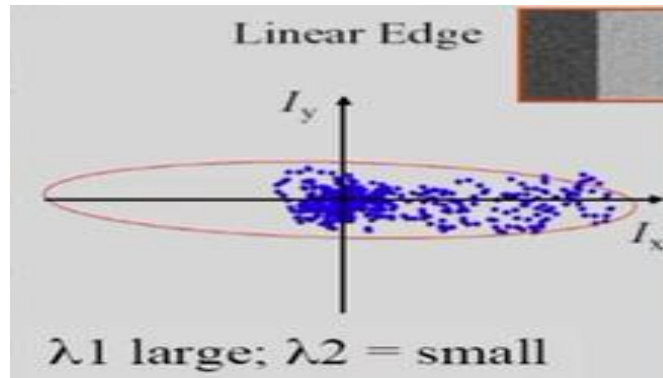
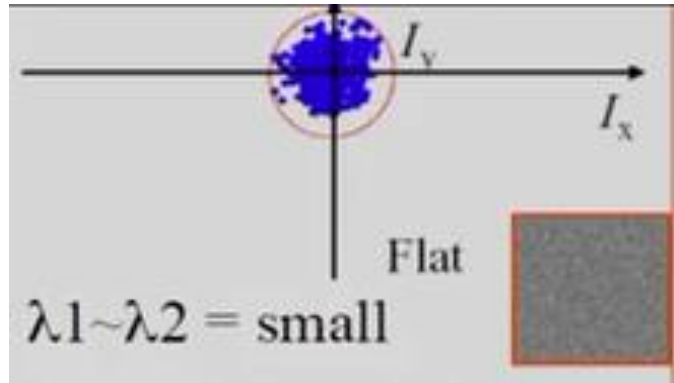


Gradients of points in window

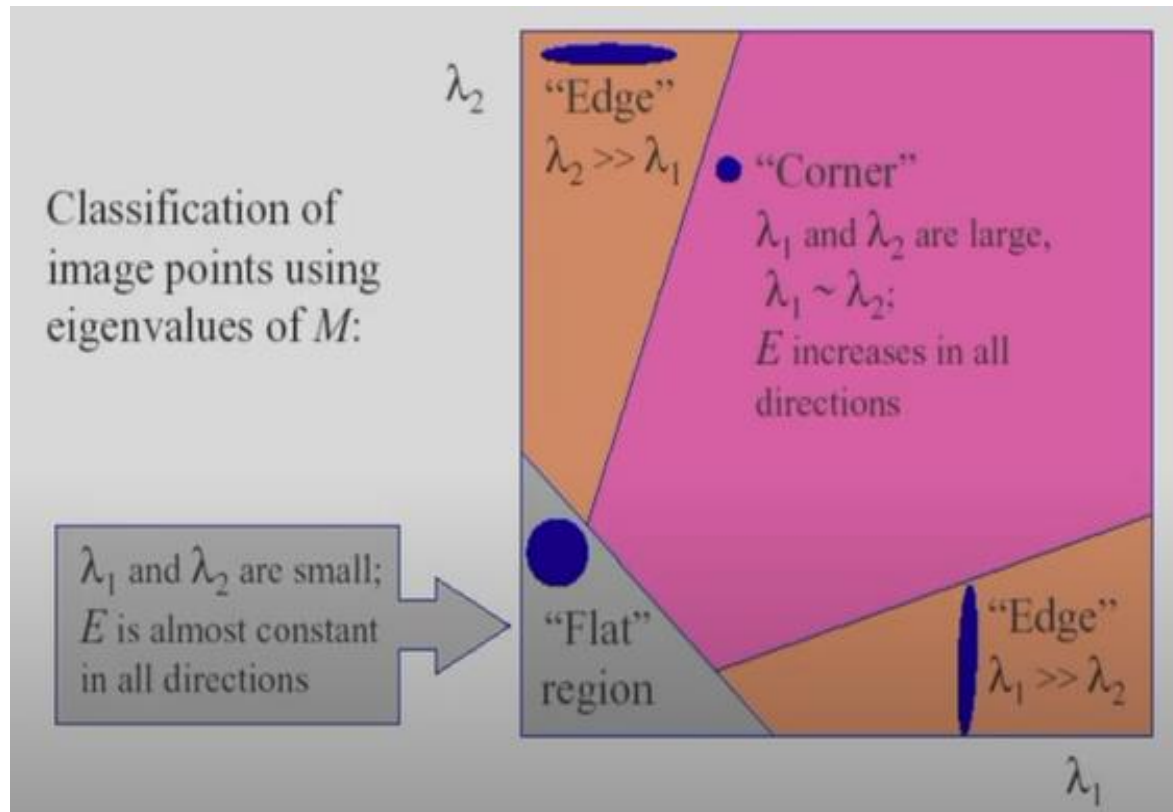
Point at the center of window is in flat portion of image

Eigen values of  $M$  can identify corner, edge and flat patch

# Eigen Values for Corner Point Detection



# Eigen Values for Corner Point Detection



- One eigenvalue is significantly high
  - derivative with respect to one direction is much stronger than the other
  - pixel lies on an edge
- Both eigenvalues are small
  - pixel intensities do not change in any direction
  - pixel lies on a flat region
- Both eigenvalues are large
  - pixel intensities largely change in both x and y direction
  - pixel lies at the corner



# Harris Corner Detector

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Computation of eigen value is costly
- Instead, compute corner response,
  - $R = \det\{M\} - k (\text{trace}\{M\})^2$

Where  $\text{Det}\{M\} = \lambda_1 \times \lambda_2$  and  $\text{Trace}\{M\} = \lambda_1 + \lambda_2$

- Or,

$$\text{Det}\{M\} = I_x^2 I_y^2 - I_{xy}^2$$

$$\text{Trace}\{M\} = I_x^2 + I_y^2$$

- Empirically determined value of  $k$  is  $0.04 - 0.06$
- $R$  is +ve and large for corner points
- $R$  is -ve and large magnitude for edge
- $|R|$  is small for flat region

# Ex: Harris Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

window

Image

1	0	-1
2	0	-2
1	0	-1

Sobel mask for  $I_x$

1	2	1
0	0	0
-1	-2	-1

Sobel mask for  $I_y$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

# Ex: Harris Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

1	0	-1
2	0	-2
1	0	-1

Sobel mask for  $I_x$

1	2	1
0	0	0
-1	-2	-1

Sobel mask for  $I_y$

0	0	0	0	0
0	-20	10	0	0
0	-10	30	0	0
0	0	40	0	0
0	0	0	0	0

$I_x$

0	0	0	0	0
0	-20	-10	0	0
0	10	30	40	0
0	0	0	0	0
0	0	0	0	0

$I_y$

# Ex: Harris Corner Detector

0	0	0	0	0
0	-20	10	0	0
0	-10	30	0	0
0	0	40	0	0
0	0	0	0	0

$I_x$

0	0	0	0	0
0	-20	-10	0	0
0	10	30	40	0
0	0	0	0	0
0	0	0	0	0

$I_y$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

0	0	0	0	0
0	400	100	0	0
0	100	900	0	0
0	0	1600	0	0
0	0	0	0	0

$I_x^2$  (square of each element of  $I_x$ )

0	0	0	0	0
0	400	100	0	0
0	100	900	1600	0
0	0	0	0	0
0	0	0	0	0

$I_y^2$  (square of each element of  $I_y$ )

0	0	0	0	0
0	400	-100	0	0
0	-100	900	0	0
0	0	0	0	0
0	0	0	0	0

$I_x I_y$  (multiplication of each element of  $I_x$  and  $I_y$ )

# Ex: Harris Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0	0	0
0	400	100	0	0
0	100	900	0	0
0	0	1600	0	0
0	0	0	0	0

$I_x^2$

0	0	0	0	0
0	400	100	0	0
0	100	900	1600	0
0	0	0	0	0
0	0	0	0	0

$I_y^2$

0	0	0	0	0
0	400	-100	0	0
0	-100	900	0	0
0	0	0	0	0
0	0	0	0	0

$I_x I_y$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} 3100 & 1100 \\ 1100 & 3100 \end{bmatrix}$$

$$\sum I_x^2 = 3100 \quad \sum I_y^2 = 3100$$

$$\sum I_x I_y = 1100$$

$$\begin{aligned} \text{Determinant of } M &= 3100 \times 3100 - 1100 \times 1100 \\ &= 9610000 - 12100 \\ &= 8400000 \end{aligned}$$

$$\begin{aligned} \text{Trace of } M &= 3100 + 3100 \\ &= 6200 \end{aligned}$$

# Ex: Harris Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

$$M = \begin{bmatrix} 3100 & 1100 \\ 1100 & 3100 \end{bmatrix}$$

Determinant of M = 8400000

Trace of M = 6200

0	0	0	0	0
0				0
0		R = 6862400		0
0				0
0	0	0	0	0

$$R = \det\{M\} - k (\text{trace}\{M\})^2, k = 0.04$$

$$R = 8400000 - 0.04 (6200)^2 \\ = 6862400$$

- For threshold = 3000000
- $R > \text{threshold}$
- Therefore, point at the center of window is a corner point

# Ex: Harris Corner Detector

0	0	0	0	0
0	0	0	0	0
10	10	10	10	10
0	0	0	0	0
0	0	0	0	0

Image

1	0	-1
2	0	-2
1	0	-1

Sobel mask for  $I_x$

1	2	1
0	0	0
-1	-2	-1

Sobel mask for  $I_y$

# Ex: Harris Corner Detector

0	0	0	0	0
0	0	0	0	0
10	10	10	10	10
0	0	0	0	0
0	0	0	0	0

Image

1	0	-1
2	0	-2
1	0	-1

Sobel mask for  $I_x$

1	2	1
0	0	0
-1	-2	-1

Sobel mask for  $I_y$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$I_x$

0	0	0	0	0
0	-40	-40	-40	0
0	0	0	0	0
0	40	40	40	0
0	0	0	0	0

$I_y$



# Ex: Harris Corner Detector

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$I_x$

0	0	0	0	0
0	-40	-40	-40	0
0	0	0	0	0
0	40	40	40	0
0	0	0	0	0

$I_y$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$I_x^2$

0	0	0	0	0
0	1600	1600	1600	0
0	0	0	0	0
0	1600	1600	1600	0
0	0	0	0	0

$I_y^2$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$I_x I_y$

# Ex: Harris Corner Detector

0	0	0	0	0
0	0	0	0	0
10	10	10	10	10
0	0	0	0	0
0	0	0	0	0

Image

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$I_x^2$

0	0	0	0	0
0	1600	1600	1600	0
0	0	0	0	0
0	1600	1600	1600	0
0	0	0	0	0

$I_y^2$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

$I_x I_y$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & 9600 \end{bmatrix}$$

$$\sum I_x^2 = 0 \quad \sum I_y^2 = 9600$$

$$\sum I_x I_y = 0$$

$$\begin{aligned} \text{Determinant of } M \\ &= 0 \times (9600) - 0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{Trace of } M \\ &= 9600 + 0 \\ &= 9600 \end{aligned}$$

# Ex: Harris Corner Detector

0	0	0	0	0
0	0	0	0	0
10	10	10	10	10
0	0	0	0	0
0	0	0	0	0

Image

$$M = \begin{bmatrix} 0 & 0 \\ 0 & 9600 \end{bmatrix}$$

Determinant of  $M = 0$

Trace of  $M = 9600$

0	0	0	0	0
0				0
0		-3686400		0
0				0
0	0	0	0	0

Corner Response at the center of window,  $R$

$$R = \det\{M\} - k (\text{trace } \{M\})^2$$

Where  $k$  is  $0.04 - 0.06$

$$\begin{aligned} R &= 0 - 0.04 (9600)^2 \\ &= -3686400 \end{aligned}$$

- For threshold = -3000000
- $R$  is negative and  $<$  negative threshold
- Therefore, point at the center of window is edge point

# Harris Corner Detector



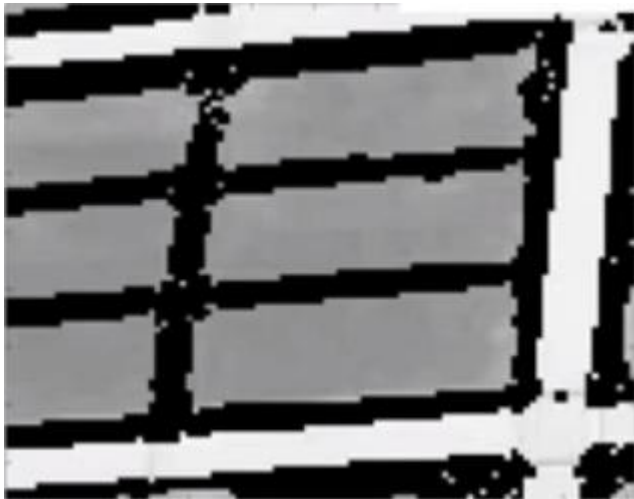
Image



$R < -10000$  for edges



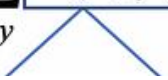
$R > 10000$  for corners



For  $-10,000 < R < 10,000$   
Neither edges nor corners

# Choice of Window Function

- By default, uniform window is used
- If uniform window is chosen as the window function, Harris detector is not rotation invariant

$$M = \sum_{x,y} \boxed{w(x,y)} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$


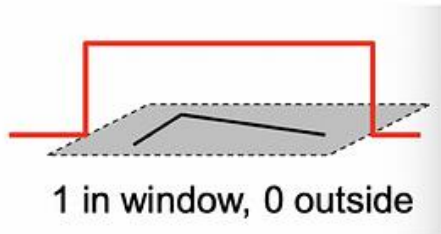
# Choice of Window Function

- By default, uniform window is used
- If uniform window is chosen as the window function, Harris detector is not rotation invariant

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Problem: not rotation invariant

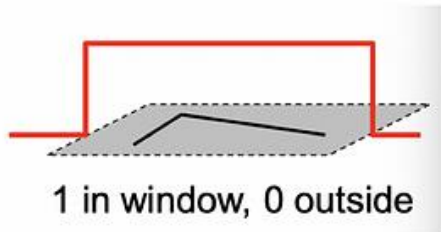
# Choice of Window Function

- By default, uniform window is used
- If uniform window is chosen as the window function, Harris detector is not rotation invariant

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Problem: not rotation invariant

Option 2: Smooth with Gaussian

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

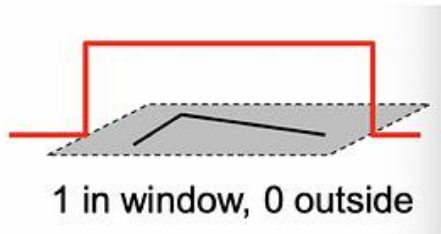
# Choice of Window Function

- By default, uniform window is used
- If uniform window is chosen as the window function, Harris detector is not rotation invariant

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

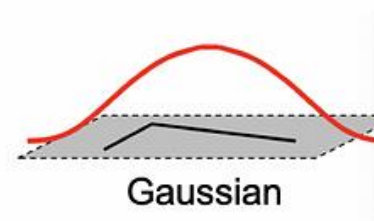
$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Problem: not rotation invariant

Option 2: Smooth with Gaussian

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$





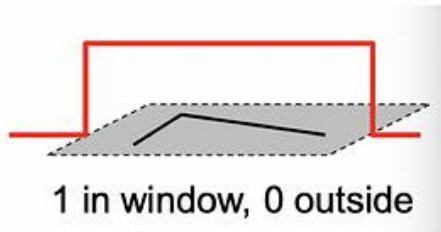
# Choice of Window Function

- By default, uniform window is used
- If uniform window is chosen as the window function, Harris detector is not rotation invariant

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Option 1: uniform window

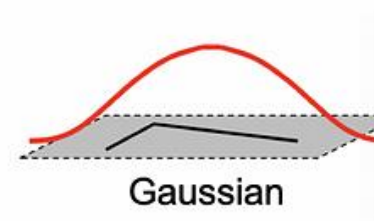
$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Problem: not rotation invariant

Option 2: Smooth with Gaussian

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Gaussian provides weighted sum  
Result is rotation invariant

Multiply  $I_x^2$ ,  $I_y^2$   
and  $I_x I_y$  by  
Gaussian  
window

# Ex: Harris Corner Detector (with Gaussian Window)

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0	0	0
0	400	100	0	0
0	100	900	0	0
0	0	1600	0	0
0	0	0	0	0

$I_x^2$

0	0	0	0	0
0	400	100	0	0
0	100	900	1600	0
0	0	0	0	0
0	0	0	0	0

$I_y^2$

0	0	0	0	0
0	400	-100	0	0
0	-100	900	0	0
0	0	0	0	0
0	0	0	0	0

$I_x I_y$

0	0	0	0	0
0	1	2	1	0
0	2	4	2	0
0	1	2	1	0
0	0	0	0	0

Gaussian Window, G

0	0	0	0	0
0	400	200	0	0
0	200	3600	0	0
0	0	3200	0	0
0	0	0	0	0

$G I_x^2$

0	0	0	0	0
0	400	200	0	0
0	200	3600	3200	0
0	0	0	0	0
0	0	0	0	0

$G I_y^2$

0	0	0	0	0
0	400	-200	0	0
0	-200	3600	0	0
0	0	0	0	0
0	0	0	0	0

$G I_x I_y$

# Ex: Harris Corner Detector (with Gaussian Window)

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0	0	0
0	400	200	0	0
0	200	3600	0	0
0	0	3200	0	0
0	0	0	0	0

$GI_x^2$

0	0	0	0	0
0	400	200	0	0
0	200	3600	3200	0
0	0	0	0	0
0	0	0	0	0

$GI_y^2$

0	0	0	0	0
0	400	-200	0	0
0	-200	3600	0	0
0	0	0	0	0
0	0	0	0	0

$GI_x I_y$

$$M = \sum_{x,y} G(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$\sum GI_x^2 = 400 + 200 + 200 + 3600 + 3200$$

$$\sum GI_y^2 = 400 + 200 + 200 + 3600 + 3200$$

$$\sum GI_x I_y = 400 - 200 - 200 + 3600$$

# Ex: Harris Corner Detector (with Gaussian Window)

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0	0	0
0	400	200	0	0
0	200	3600	0	0
0	0	3200	0	0
0	0	0	0	0

$GI_x^2$

0	0	0	0	0
0	400	200	0	0
0	200	3600	3200	0
0	0	0	0	0
0	0	0	0	0

$GI_y^2$

0	0	0	0	0
0	400	-200	0	0
0	-200	3600	0	0
0	0	0	0	0
0	0	0	0	0

$GI_x I_y$

$$\sum GI_x^2 = 400 + 200 + 200 + 3600 + 3200$$

$$\sum GI_y^2 = 400 + 200 + 200 + 3600 + 3200$$

$$\sum GI_x I_y = 400 - 200 - 200 + 3600$$

$$M = \begin{bmatrix} GI_x^2 & GI_x I_y \\ GI_x I_y & GI_y^2 \end{bmatrix}$$

$$R = \det\{M\} - k (\text{trace}\{M\})^2$$

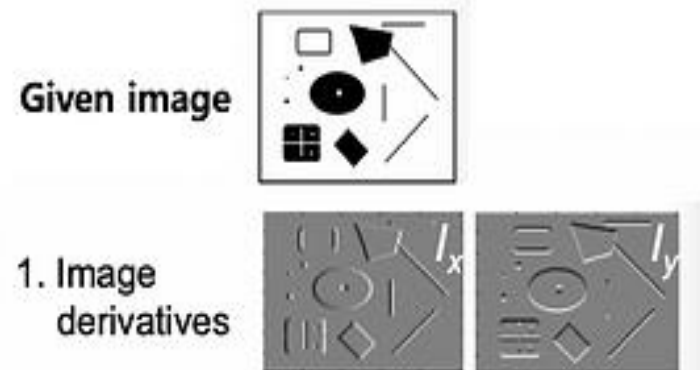
$$R =$$

# Steps of Harris Detector

Given image

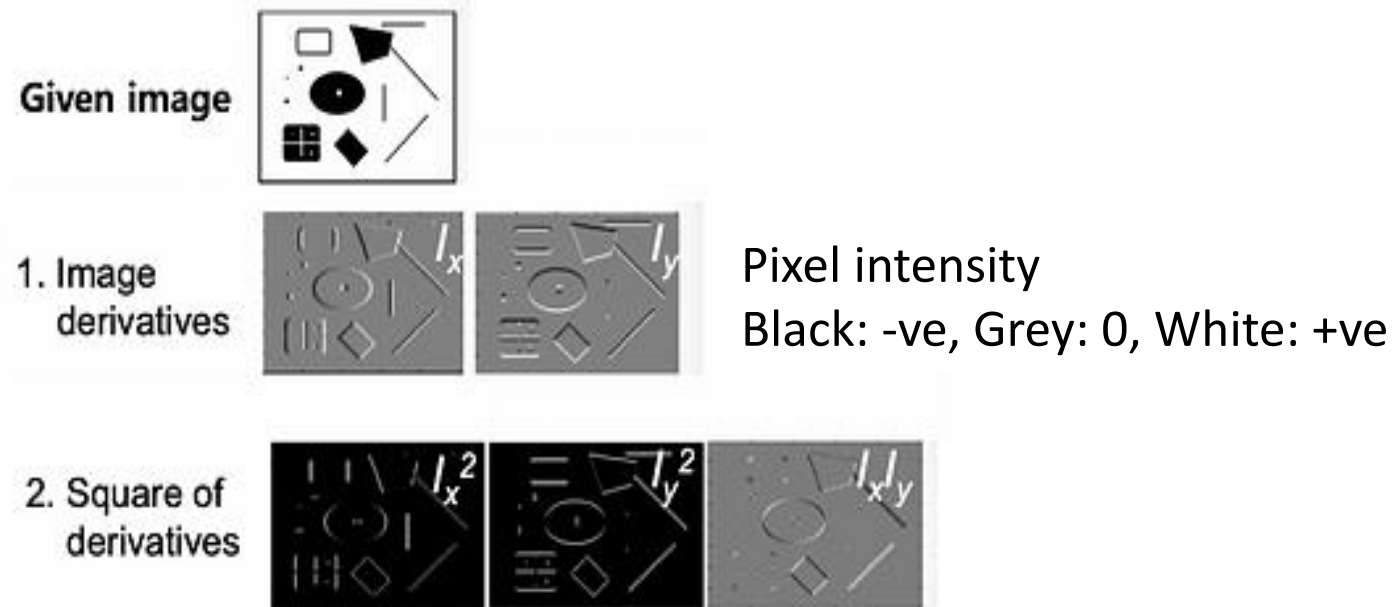


# Steps of Harris Detector

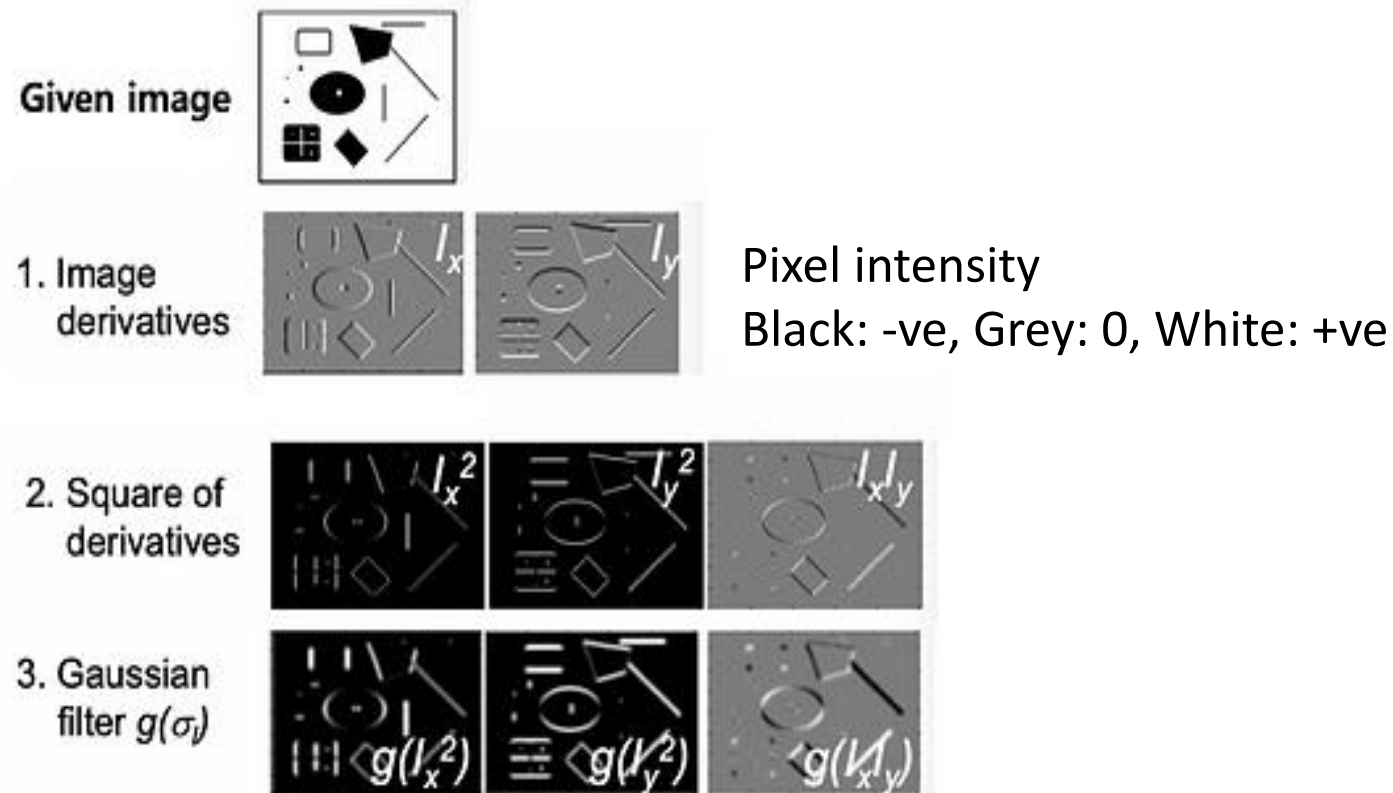


Pixel intensity  
Black: -ve, Grey: 0, White: +ve

# Steps of Harris Detector

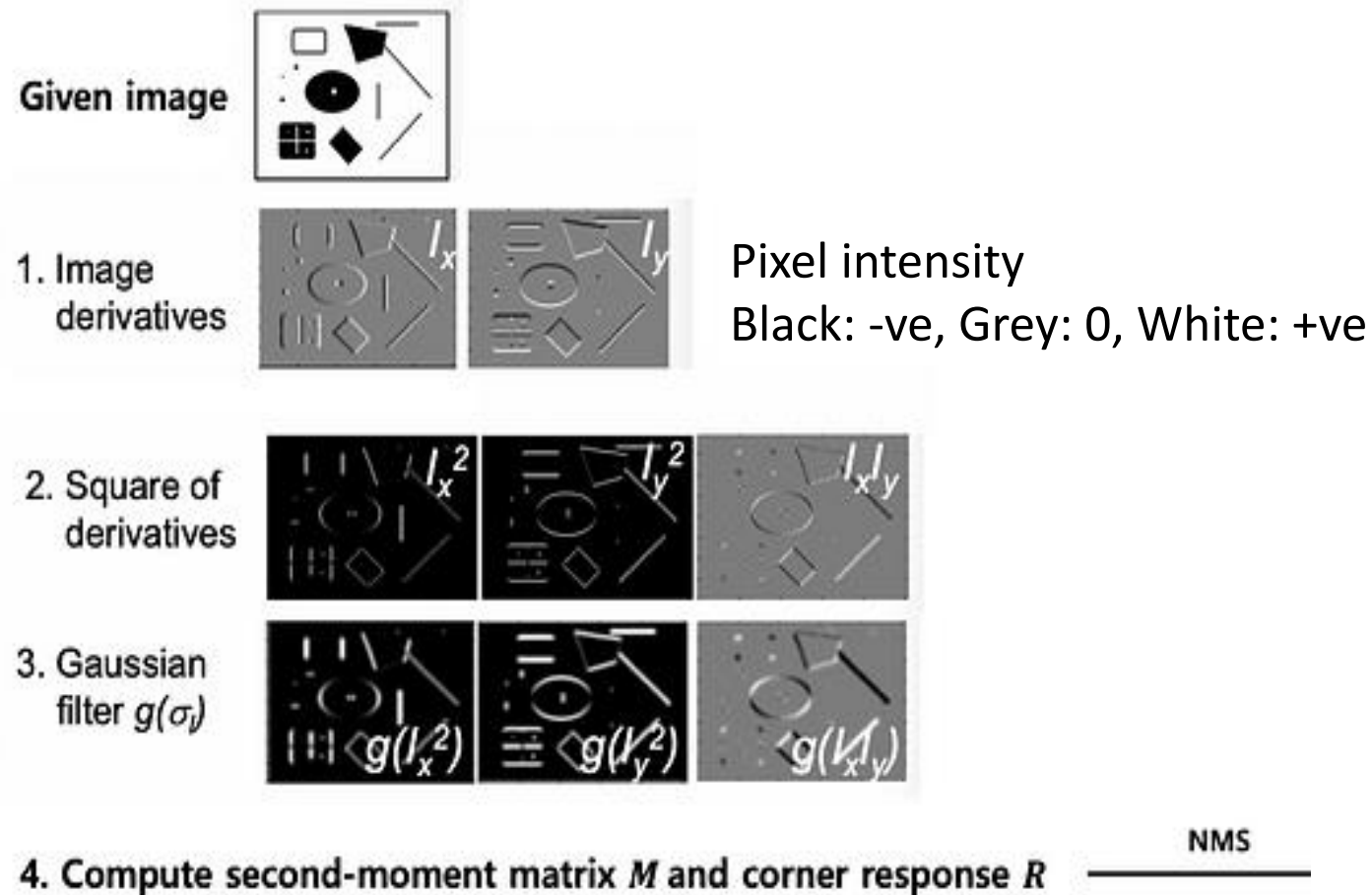


# Steps of Harris Detector

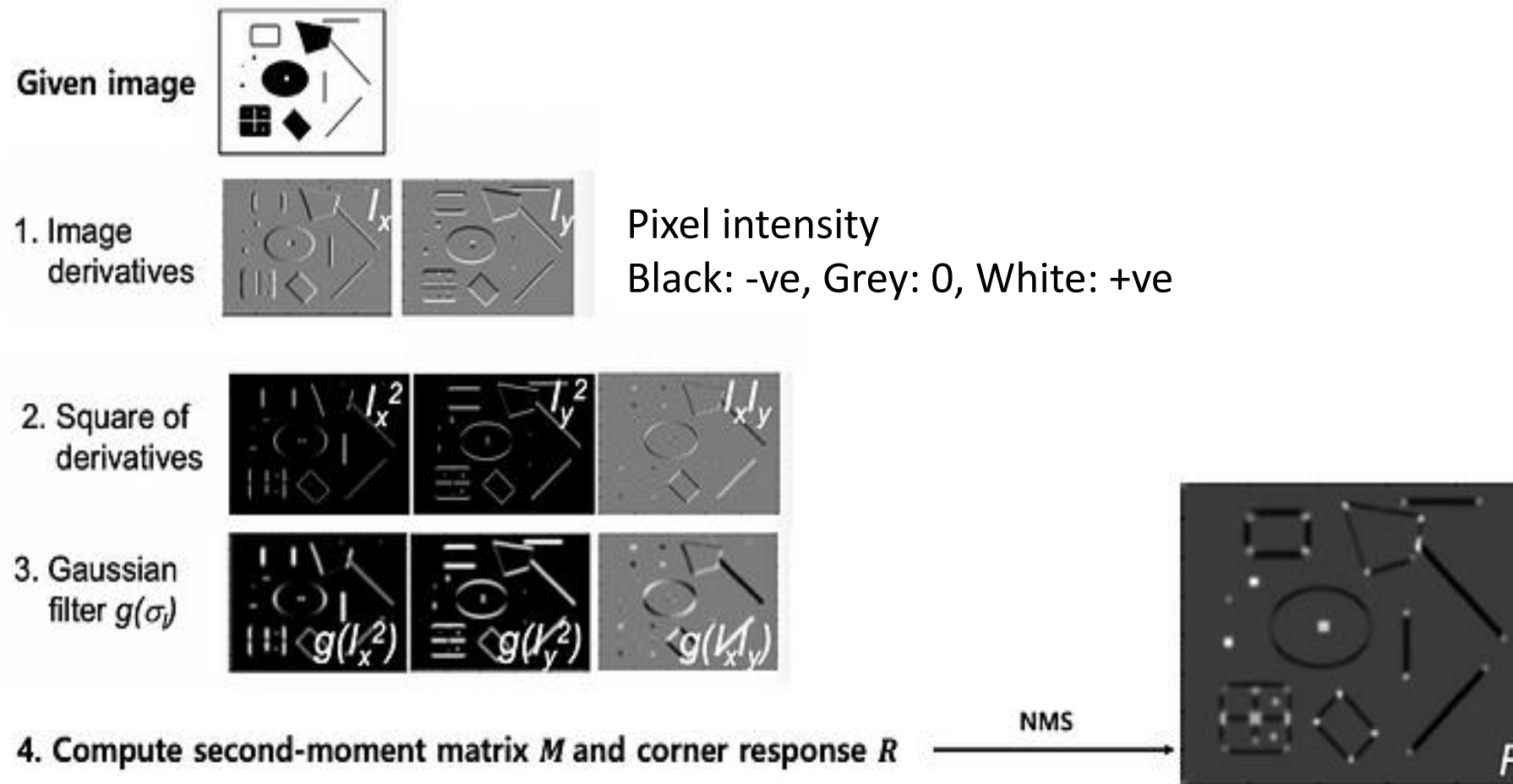




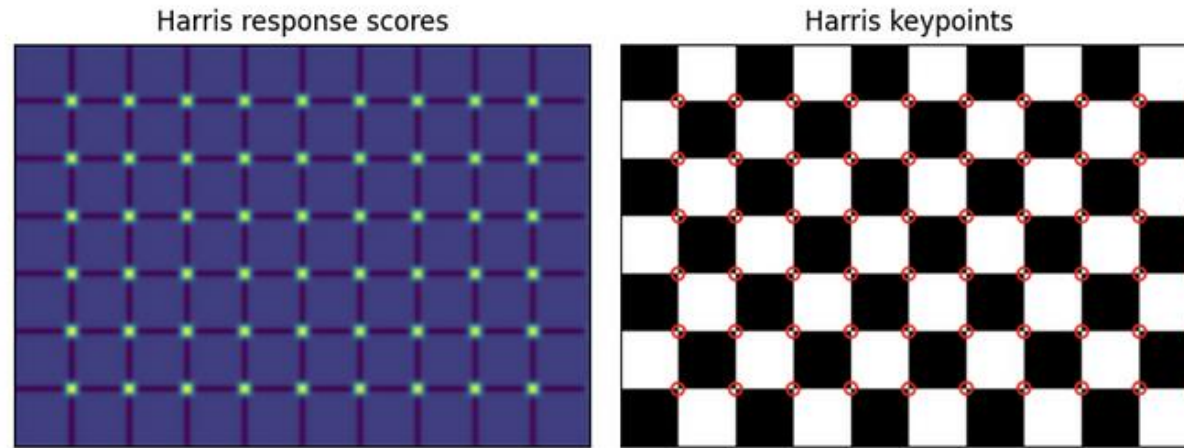
# Steps of Harris Detector



# Steps of Harris Detector

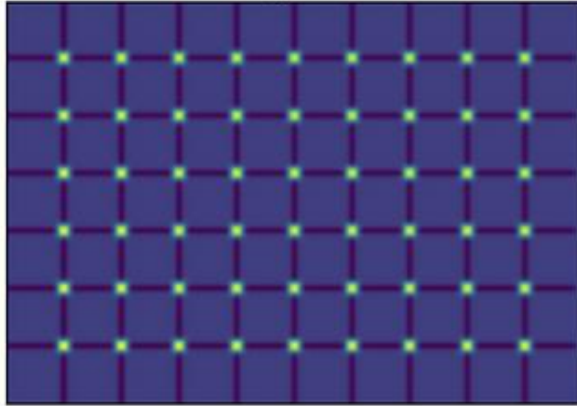


# Harris Corner Detector

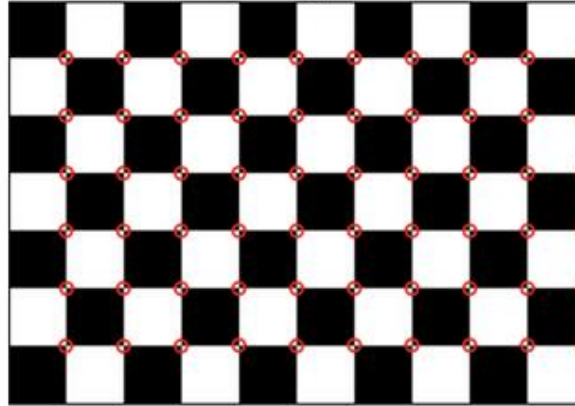


# Harris Corner Detector

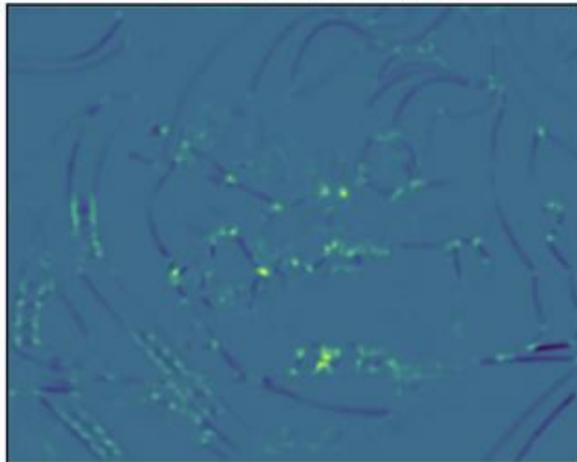
Harris response scores



Harris keypoints



Harris response scores



Harris keypoints



# Harris Corner Detector



Image 1

# Harris Corner Detector

Image after rotation and  
variation in illumination



Image 1

Image 2

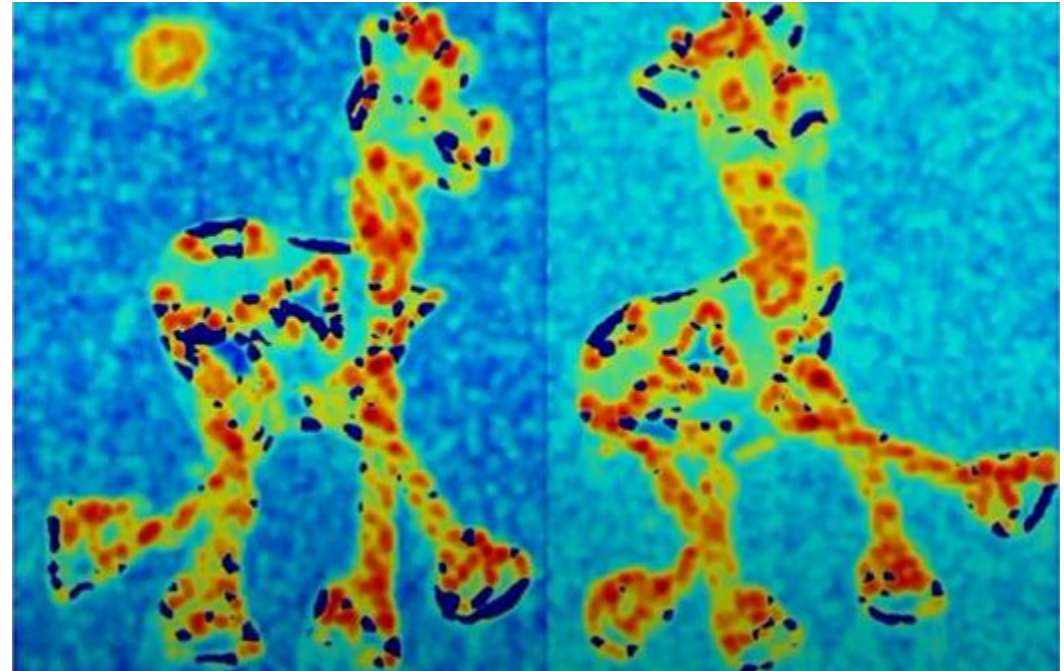


# Harris Corner Detector



Image

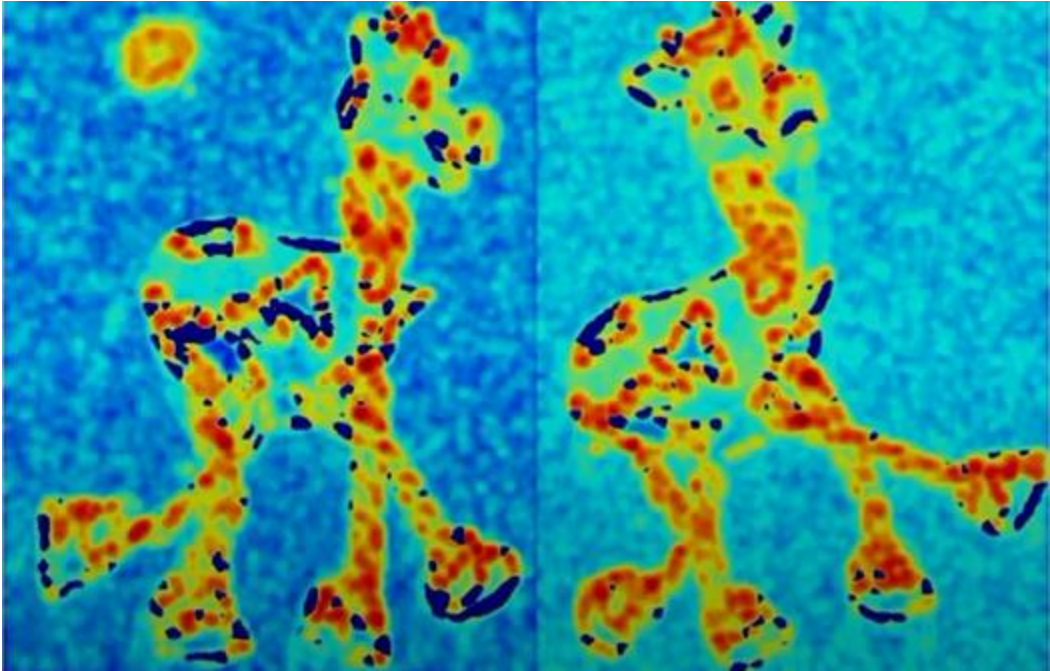
Image after rotation  
and variation in  
illumination



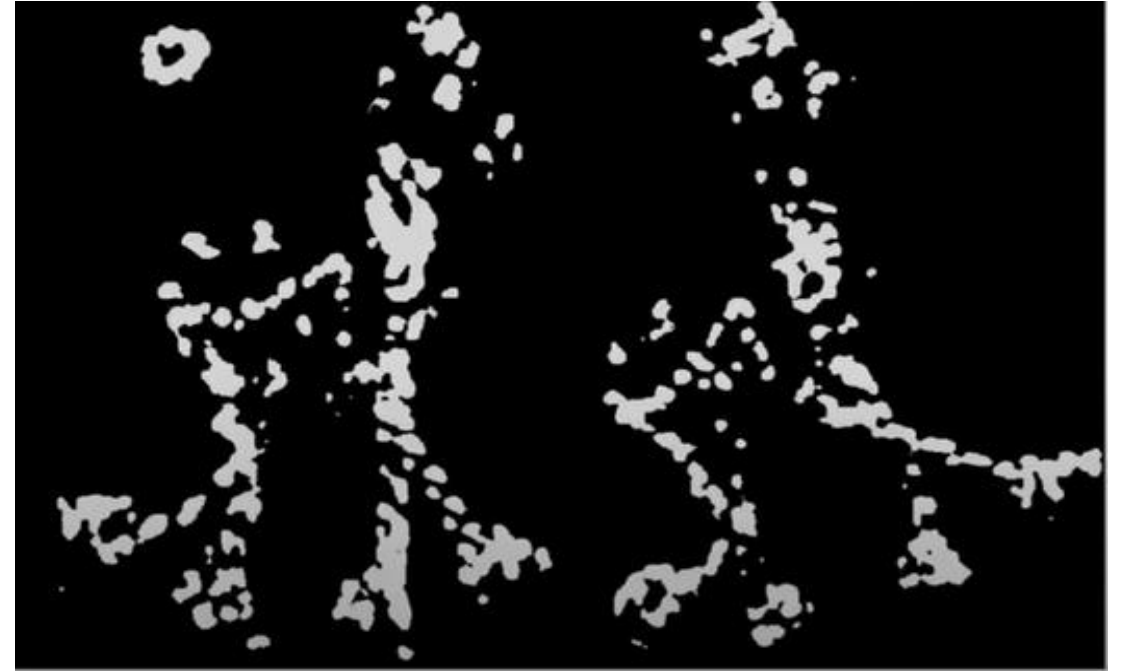
Corner response,  $R$

Blue for low or negative  $R$   
Red for positive  $R$

# Harris Corner Detector



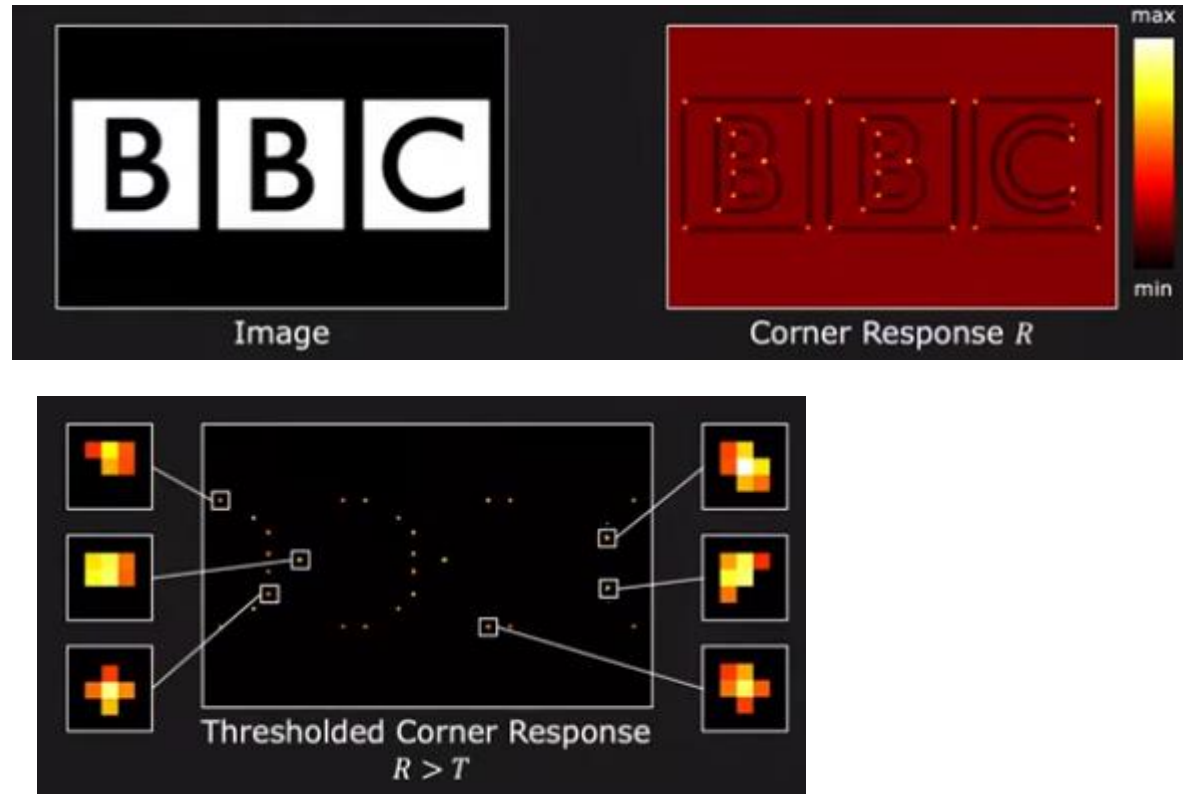
Corner response,  $R$



Points with large value of  $R$  ( $R > \text{threshold}$ )



# Harris Corner Detector



- After thresholding on  $R$ , we get clusters of corner points not a single point

# Non Maximal Suppression

- Slide a window of size  $k$  over the cluster
- At each position, if pixel at the center of window is maximum, label it as a corner and retain it
- Else label it as not corner and suppress it



White pixel represents maximum value of  $R$

# Non Maximal Suppression

- Slide a window of size  $k$  over the cluster
- At each position, if pixel at the center of window is maximum, label it as a corner and retain it
- Else label it as not corner and suppress it



White pixel represents maximum value of  $R$

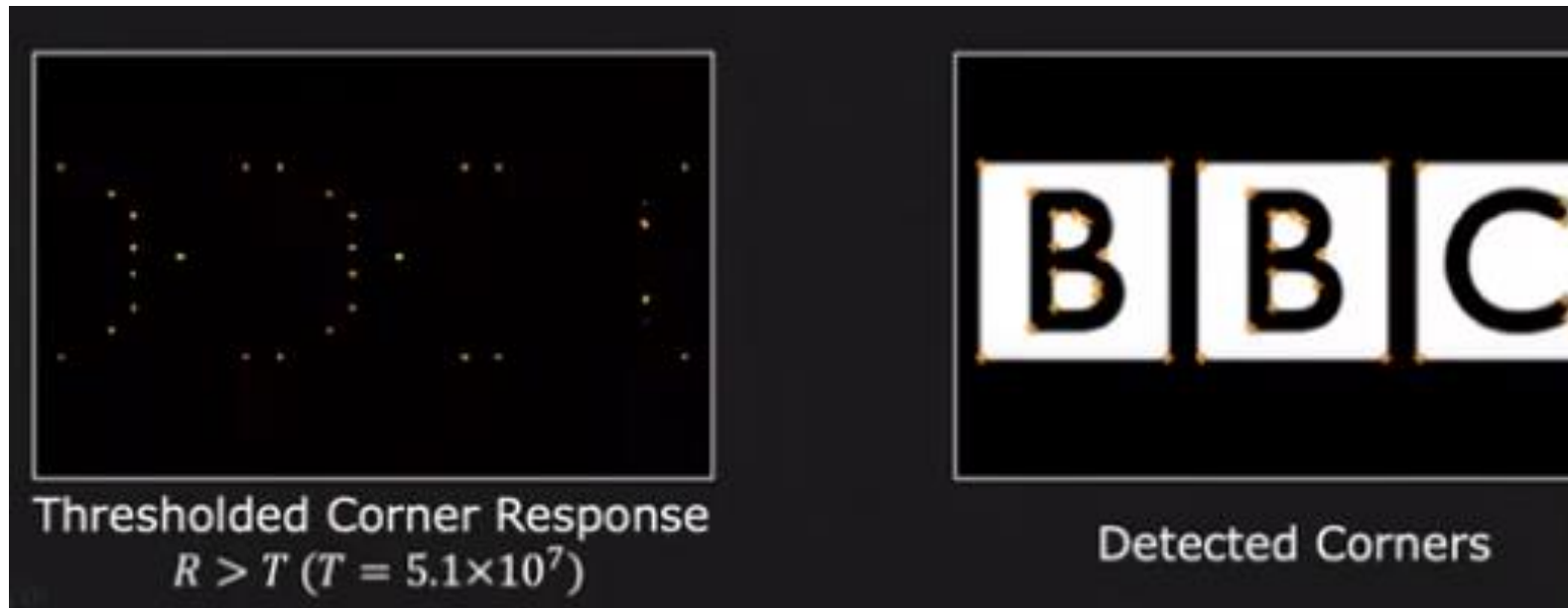
# Non Maximal Suppression

- Slide a window of size  $k$  over the cluster
- At each position, if pixel at the center of window is maximum, label it as a corner and retain it
- Else label it as not corner and suppress it



White pixel represents maximum value of  $R$

# Harris Corner Detector



# Hessian Corner Detector

- Harris and Hessian detectors are similar
- Harris detector uses first moment and corner response function  $R$  for corner detection
- Hessian detector uses Second moment and determinant to detect corners
- Hessian detector provides good performance in terms of computation time and accuracy
- Given a pixel, the Hessian of pixel is

$$H = \sum W(x, y) \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

- $I_{xx}$  and  $I_{yy}$  are second order derivative in x and y direction respectively
- $I_{xy}$  is first order derivative in x direction ( $I_x$ ) and then derivative of  $I_x$  in y direction

$$\text{Det}(H) = I_{xx}I_{yy} - (I_{xy})^2$$

If  $\text{det}(H)$  is high then it is a corner point

# Ex: Hessian Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0
1	-2	1
0	0	0

Mask for Second  
order derivative,  
 $I_{xx}$  in x direction

0	1	0
0	-2	0
0	1	0

Mask for Second  
order derivative,  
 $I_{yy}$  in y direction

1	0	-1
2	0	-2
1	0	-1

Mask for first  
order derivative,  $I_x$   
in x direction

1	2	1
0	0	0
-1	-2	-1

Mask for first  
order derivative of  
 $I_x$  in y direction  
that is  $I_{xy}$

# Ex: Hessian Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0
1	-2	1
0	0	0

Mask for Second  
order derivative,  
 $I_{xx}$  in x direction

0	1	0
0	-2	0
0	1	0

Mask for Second  
order derivative,  
 $I_{yy}$  in y direction

1	0	-1
2	0	-2
1	0	-1

Mask for first  
order derivative,  $I_x$   
in x direction

1	2	1
0	0	0
-1	-2	-1

Mask for first  
order derivative of  
 $I_x$  in y direction  
that is  $I_{xy}$

0	0	0	0	0
0	-10	0	0	10
0	-20	10	0	0
0	-20	10	0	0
0	10	0	0	0

$I_{xx}$

0	0	0	0	0
0	-10	-20	-20	10
0	0	10	10	0
0	0	0	0	0
0	10	0	0	0

$I_{yy}$

0	0	0	0	0
0	-20	10	0	0
0	-10	30	0	0
0	0	40	0	0
0	0	0	0	0

$I_x$

0	0	0	0	0
0	-50	-50	-30	10
0	-70	-80	-30	0
0	10	50	30	0
0	10	0	0	0

$I_{xy}$



# Ex: Hessian Corner Detector

0	0	0	0	0
0	10	10	10	10
0	10	0	0	0
0	10	0	0	0
0	10	0	0	0

Image

0	0	0	0	0
0	-10	0	0	0
0	-20	10	0	0
0	-20	10	0	0
0	0	0	0	0

$I_{xx}$

0	0	0	0	0
0	-10	-20	-20	0
0	0	10	10	0
0	0	0	0	0
0	0	0	0	0

$I_{yy}$

0	0	0	0	0
0	-50	-50	-30	10
0	-70	-80	-30	0
0	10	50	30	0
0	10	0	0	0

$I_{xy}$

$$\text{Hessian, } H = \sum \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

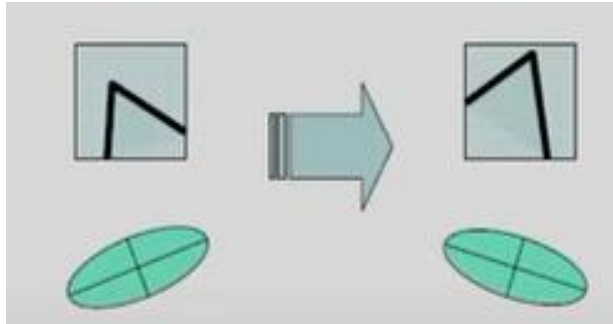
$$\text{Hessian, } H = \begin{bmatrix} -30 & -220 \\ -220 & -30 \end{bmatrix}$$

$$|\text{Det}(H)| = |900 - 48400| = 47500$$

- For threshold = 15000
- It is a corner point

# Harris and Hessian Corner Detectors

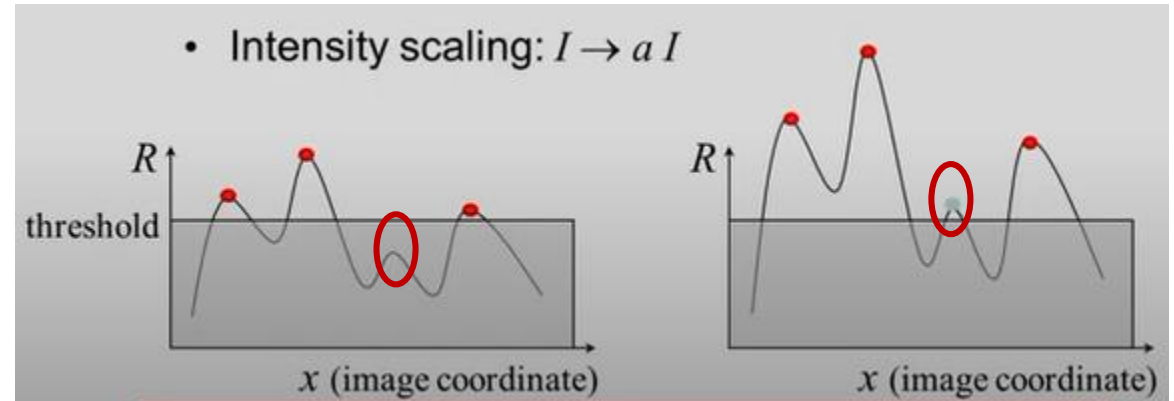
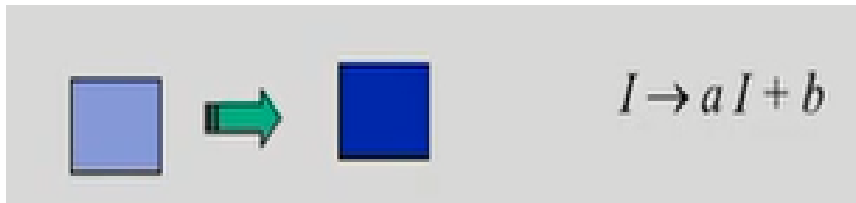
- Corner points should be detected at corresponding locations in other image even if image is rotated
- Harris and Hessian detectors are rotation invariant



- Ellipse rotates but shape (Eigen values) remains the same
- Also, corner points should be invariant to photo metric transformation and geometric transformation

# Harris and Hessian Corner Detectors

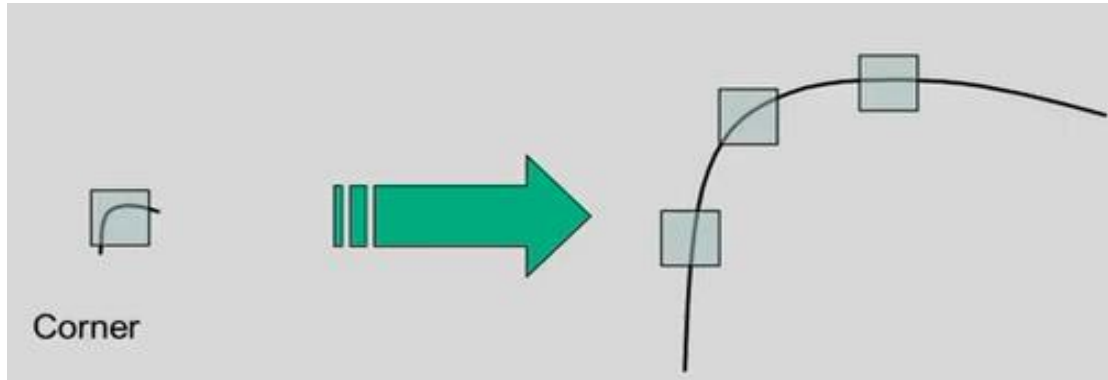
- Photometric/ intensity transformation



- For Hessian Detector,
  - Intensity shift ( $I \rightarrow I + b$ ) does not change derivative
  - Therefore  $R$  is invariant to shift in intensity
  - If  $I$  is scaled ( $I \rightarrow a \times I$ )
  - Then false point appears after scaling
- Harris detector
  - partially invariant to affine intensity change if intensity change is minor

# Harris and Hessian Corner Detectors

- Image scaling in size



- Corner gets magnified and becomes bigger than the size of the window by zooming
- After scaling, corner points are classified as edges
- Harris and Hessian can not detect the corner if image is up scaled
- They are not invariant to scaling

# Applications of Interest points

- Image alignment
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval

# References

- <https://nptel.ac.in/courses/108103174>
- [https://sbme-tutorials.github.io/2018/cv/notes/6\\_week6.html](https://sbme-tutorials.github.io/2018/cv/notes/6_week6.html)
- <https://medium.com/data-breach/introduction-to-harris-corner-detector-32a88850b3f6>
- <https://www.baeldung.com/cs/harris-corner-detection>
- <https://www.codingninjas.com/codestudio/library/harris-corner-detection>
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