q

C

b_i

k

n

$$n = exp(O(k))$$

$q = 3$

$$n = subexp(k)$$

$$n \geq \Omega(k^2)$$

3

$$n \geq \tilde{\Omega}(k^3)$$

$$b \in 0,1^k \to x \in 0,1^n$$

x

$i \in [k]$

$\hat{b}_i \in 0, 1$

$$(q, \delta, \epsilon)$$

$$\triangle(x, Enc(b)) \leq \delta n$$

$$Dec^x(i) = b_i$$

$$\geq \frac{1}{2} + \epsilon$$

i

$$2^{\Omega(k)} \leq n$$

$n \leq 2^k$

$k^2 \leq n$

$$n \leq exp(k^{o(1)})$$

$O(1)$

$$k^{\frac{q}{q+1}} \leq n$$

$$k^{\frac{q+1}{q-1}} \leq n$$

$$k \leq n \leq 2^k$$

$$k^2 \leq n \leq exp(exp(\sqrt{\log k \log \log k}))$$

$k^3 \leq n$

$$(3, \delta, \epsilon)$$

$$n \geq \tilde{\Omega}_{\delta,\epsilon}(k^3)$$

$$k^3$$

$$H_1, H_2, \cdots H_k$$

H_i

$$H_i \subseteq \binom{[n]}{q}$$

$|H_i| \geq \delta n$

$$Dec(i)$$

$$C \leftarrow H_i$$

$$\sum_{j \in C} x_j$$

$$b \in 0,1^k \mapsto f = (\langle b, v \rangle)_{v \in 0,1}^k$$

$$b_i = f(e_i) = f(v) + f(v + e_i)$$

v

$$v + e_i$$

$$\approx \mathbb{Z}_m^h$$

ψ_b

$$val(\psi_b) = 1$$

$$b \in 0, 1^k$$

b

$$n \ll k^{\frac{q}{q-2}}$$

$$\psi \to A \to algval(\psi)$$

$$val(\psi) \leq algval(\psi)$$

$$algval(\psi) < 1$$

A

ψ

m

$\psi w/m$

b_c

$C \in H$

$$\sum_{j \in C} x_j = b_c$$

$$\psi_b \ is \ \sum_{j \in C}$$

$$xj = b_i, i \in [k], C \in H_i$$

$$\{H_1 \cdots H_k\}$$

$$b_i \in \{\pm 1\}$$

$$\prod_{j \in C} x_j = b_i$$

$C \in H_i$

$$n \ll k^{q/(q-2)}$$

$x \in \{\pm 1\}^n$

$$\frac{1}{2} + \frac{f(x)}{2}$$

$$f(x)$$

$$f(x) = \frac{1}{m} \sum_{i} b_i \sum_{C \in H_i} \prod_{j \in C} x_j$$

$$m = k \cdot \delta n$$

$$\max_{x \in \{\pm 1\}^n} f(x) < 1 \;\; when \; n \ll k^{\frac{q}{q-2}}$$

$$w.h.p. \quad \max_{x \in \{\pm 1\}^n} f(x) < 1 \ \ where \ f(x) = \frac{1}{m} \sum_i b_i \sum_{C \in H_i} \prod_{j \in C} x_j$$

$$when\ n \ll k^{\frac{q}{q-2}}$$

$$A \in \mathbb{R}^{N \times N}$$

$$f(x) \leq ||A||_{\infty \to 1} = \max_{z,w \in \{\pm 1\}^N} z^T A w$$

$$S \in \binom{[n]}{l}$$

$$x \mapsto y$$

$$y^T A y \propto f(x)$$

$$y_s := \prod_{j \in S} x_j$$

$$A(S,T)$$

$$y^T A y = \sum_{S,T} y_S y_T A(S,T) = \sum_{S,T} A(S,T) \prod_{j \in S \oplus T} x_j$$

$$S \oplus T = C \in h_i$$

$$\prod_{j \in S \oplus T} x_j = b_i$$

$$\implies A(S,T) = b_i$$

$$S \oplus T = C \in H_i$$

$$y^T A y = \sum_{i=1}^{k} b_i \sum_{C \in h_i} \sum_{S \oplus t = C} \prod_{j \in C} x_j = Dmf(x)$$

$D =$

S, T

$$S \oplus T = C$$

$$A_C(S, T) = 1$$

$$A_i = \sum_{C \in h_i} A_C$$

$$A = \sum_{i=1}^{k} b_i A_i$$

$$y_S := \prod_{j \in S} x_j$$

$$y^T A y = Dmf(x) \implies Dmf(x) \leq ||A||_{\infty \to 1}$$

D

$$|C| = q$$

$$D = \binom{q}{\frac{q}{2}} \binom{n-q}{l-\frac{q}{2}}$$

$$A_c \in \mathbb{R}^{N \times N}$$

$$N = \binom{n}{l}$$

$$||A||_{\infty\to 1} \geq Dm \max_x f(x) \geq Dm \geq D\delta nk$$

$$||A||_{\infty \to 1} \leq N ||A||_2$$

$$||A||_{\infty \to 1}$$