Least Squares for Channel Estimation

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***Abstract*—This paper focuses on least squares(LS) channel estimation for the following wireless communication networks-(a)Intelligent reflecting surface (IRS) enhanced orthogonal frequency division multiplexing (OFDM), where channel state information (CSI) is estimated using least squares which will be used for maximizing achievable rate. (b)Downlink multiple antenna wireless network**

***Index Terms*—Intelligent reflecting surface (IRS), OFDM, MIMO, least squares**

I. Introduction

(a)There is always a demand for higher data rates in wireless communications. For maintaining the power consumption at higher data rates, energy efficiency of wireless networks has to be increased. This can be achieved using massive MIMO and mmWave but they demand high implementation cost, energy consuming hardware and sophisticated signal processing.
Intelligent reflective surface (IRS) enhanced OFDM is able to achieve higher data rates while overcoming the above problems and in addition there is improvement in wireless link performance. In this paper we focus on LS channel estimation of IRS groups instead of estimating for each element of IRS to reduce training overhead and design complexity. Based on estimated channels at the base station(BS), the transmit power allocation and IRS group reflection coefficients are optimized at BS to maximize the achievable rate.
(b)Channel estimation of MIMO based on LS so as to achieve decrease in the complexity of computing channel coefficients and minimum error.

II. Linear Algebra Background

A. Vector spaces

1) Column space: Span of columns of $m \times n$ matrix A
$\mathcal{C}(A) = \{A \cdot x : x \in F^n\} \subseteq F^m$
2) Null space: The set of all solutions to a system Ax = 0
$\mathcal{N}(A) = \{x \in F^n : A \cdot x = 0\}$
3) Null space of A^T,
$\mathcal{N}(A^T) = \{x \in F^m : A^T \cdot x = 0\}$
4) $\mathcal{C}(A)$ and $\mathcal{N}(A^T)$ are orthogonal subspaces

B. Least squares

Consider system of linear equations, $A \cdot \bar{x} = \bar{b}, \quad \bar{x} \in \mathbb{R}^k, \quad \bar{b} \in \mathbb{R}^n$
There are cases where the above equation doesn't have solutions, meaning $\bar{b}$ is not in the column space of A. So find a solution which is close to $A \cdot \bar{x} = \bar{b}$, say $\hat{x}$ where $A \cdot \hat{x}$ is as close to $\bar{b}$ as possible.
In other words, minimise the distance between $A \cdot \hat{x}$ and $\bar{b}$.
minimize $\|A\hat{x} - \bar{b}\|$ which can also extend to
minimize $\|A\hat{x} - \bar{b}\|^2$

Another method to find least square estimate:
Closest vector to $\bar{b}$ in $C(A)$ is projection of $\bar{b}$ on to $\mathcal{C}(A)$. So to minimise the above equation, $A \cdot \hat{x} =$ Projection of $\bar{b}$ on to $C(A)$
Now subtract $\bar{b}$ from both sides,
$A \cdot \hat{x} - \bar{b} =$ Projection of $\bar{b}$ on to $C(A) - \bar{b}$,
where, (Projection of $\bar{b}$ on to $C(A) - \bar{b}$) is orthogonal to $C(A)$. Then $A \cdot \hat{x} - \bar{b}$ is orthogonal to $C(A)$.
As mentioned in section-A.4, $\mathcal{C}(A)$ and $\mathcal{N}(A^T)$ are orthogonal subspaces,hence $A\hat{x} - \bar{b} \in N(A^T)$
From A.3,

$$A^T(A\hat{x} - \bar{b}) = 0 \tag{1}$$
$$A^T A\hat{x} - A^T\bar{b} = 0 \tag{2}$$
$$A^T A\hat{x} = A^T\bar{b} \tag{3}$$
$$\hat{x} = (A^T A)^{-1} A^T\bar{b} \tag{4}$$

Hence, $\hat{x} = (A^T A)^{-1} A^T\bar{b}$ is the least square estimate

III. Application of least squares

(a) IRS enhanced OFDM wireless network [1]:
Grouping of IRS elements are done so as to decrease the complexity.
The combined composite BS-IRS and IRS-user channel matrix $\mathbf{V}' \in \mathbb{C}^{N\times K}$ comprises all the reflecting channels associated with the respective IRS elements groups, which superpose with each other in the resultant BS-IRS-user link as $h_r = \mathbf{V}'\bar{\phi}$ where, $h_r \in \mathbb{C}^{N\times 1}$ is BS-IRS-user reflected channel and $\bar{\phi} = [\bar{\phi}_1, \ldots, \bar{\phi}_K]^T \in \mathbb{C}^{K\times 1}$ represents the IRS group reflection coefficients.with $\bar{\phi}_k$ denoting the common reflection coefficient for the k-th group, $k \in \mathcal{K}$, with $\mathcal{K} = 1, ..., K$
To estimate the channel of each IRS group, on/off control approach is adapted, where all the elements of IRS will be off except for the elements of k-th group whose channel is being estimated i.e., $\bar{\phi} = e_k, \quad k \in \mathcal{K}$ and in total K+1 OFDM symbol durations are required.
Let $\mathbf{x}_p \in \mathbb{C}^{N\times 1}$ denote the pilot signal sent at every symbol duration in the first phase,and $X_p = \mathrm{diag}(\mathbf{x}_p)$. In the first symbol duration,BS-user direct channel, h_d is estimated so

all the IRS reflecting elements are switched off off, i.e., $\bar{\phi} = \mathbf{0}_{K\times 1}$.
The equivalent baseband received signal in the frequency domain after cyclic prefix removal is then given by,

$$s_0 = X_p F_N h_d + n_0 \tag{5}$$

where, F_N denotes $N \times N$ discrete fourier transform(DFT) and n_0 denotes the receiver noise vector with each element modelled by an independent CSCG random variable with mean zero and variance σ^2
Now, to estimate the channels corresponding to IRS groups, the equivalent baseband received signal in the frequency domain after CP removal is therefore given by,

$$s_k = X_p F_N (h_d + \nu'_k) + n_k, k \in \mathcal{K} \tag{6}$$

where, where n_k denotes the receiver noise vector at the (k + 1)-th pilot symbol duration, with $n_k \sim \mathcal{CN}(\mathbf{0}_{N\times 1}, \sigma^2 \mathbf{I}_N)$

Now using concepts from [2] :
The LS estimate for direct channel which minimizes $(s_0 - X_p F_N h_d)^H (s_0 - X_p F_N h_d)$ is given by,

$$(H_d)_{LS} = X_p^{-1} s_0 \tag{7}$$

Now by performing IDFT, F_N^H of above equation,

$$\hat{h}_d = [[\frac{1}{N} F_N^H X_p^{-1} s_0]_{1:L}^T, 0_{1\times(N-L)}]^T \tag{8}$$

$$\hat{h}_d = [[\frac{1}{N} F_N^H X_p^{-1} (X_p F_N h_d + n_0)]_{1:L}^T, 0_{1\times(N-L)}]^T \tag{9}$$

$$\hat{h}_d = h_d + \bar{n}_0 \tag{10}$$

Similarly, the LS estimate for IRS groups which minimizes $(s_k - X_p F_N (h_d + \nu'_k))^H (s_0 - X_p F_N (h_d + \nu'_k))$ is given by,

$$(H_d + U'_k)_{LS} = X_p^{-1} s_k \tag{11}$$

Now by performing IDFT, F_N^H of above equation,

$$\hat{\nu}'_k = [[\frac{1}{N} F_N^H X_p^{-1} s_k - \hat{h}_d]_{1:L_0}^T, 0_{1\times(N-L_0)}]^T \tag{12}$$

$$\hat{\nu}'_k = [[\frac{1}{N} F_N^H X_p^{-1} (X_p F_N (h_d + \nu'_k) + n_k) - \hat{h}_d]_{1:L_0}^T, 0]^T \tag{13}$$

$$\hat{\nu}'_k = \nu'_k + \bar{n}_k - \bar{n}_0, \quad k \in \mathcal{K} \tag{14}$$

where,

$$\bar{n}_0 = [[\frac{1}{N} F_N^H X_p^{-1} n_0]_{1:L}^T, 0_{1\times(N-L)}]^T \tag{15}$$

$$\bar{n}_k = [[\frac{1}{N} F_N^H X_p^{-1} n_k]_{1:L_0}^T, 0_{1\times(N-L_0)}]^T \text{for } k \in \mathcal{K} \tag{16}$$

By using the estimated channels, the transmit power allocation and IRS group reflection coefficients are optimized at the BS to maximize the achievable rate, which is given by, $r(p, \bar{\phi} | \hat{h}_d, \hat{V})$ [1]

(b) LS channel estimation for downlink MIMO network [3]:
Vector notation for the said network,(η_r and η_r are number of receiver and transmitter antennas respectively)

$$\begin{bmatrix} y_1(k) \\ \vdots \\ y_{\eta r}(k) \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1\eta t} \\ \vdots & \ddots & \vdots \\ h_{\eta r 1} & \cdots & h_{\eta r \eta t} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{\eta t}(k) \end{bmatrix} + \begin{bmatrix} \sigma_1(k) \\ \sigma_2(k) \\ \vdots \\ \sigma_{\eta r}(k) \end{bmatrix}$$

which can be written as,

$$\bar{y}(k) = \Theta \bar{x}(k) + \bar{\sigma}(k) \tag{17}$$

where $\bar{y}(k)$ is the $\eta_r \times 1$ receiver vector, Θ is the $\eta_r \times \eta_t$ MIMO channel matrix, $\bar{x}(k)$ is the $\eta_t \times 1$ transmitted symbol vector, and $\bar{\sigma}(k)$ is the $\eta_r \times 1$ noise vector, each measured at the k-th time instant.

$$Y = \Theta X + \Sigma \tag{18}$$

where, Here,X is the $\eta_t \times N$ pilot matrix, Y is the$\eta_r \times N$ output observation matrix, Σ is the $\eta_r \times N$ noise matrix comprising of unknown noise samples.
LS channel estimation,

$$\arg\min_{\Theta} \|\bar{y} - \Theta \bar{x}\|^2 \tag{19}$$

$$\hat{\Theta} = (X^T X)^{-1} X^T Y \tag{20}$$

IV. Conclusion

Even though there are many channel estimation methods which gives better performance, least squares channel estimation is widely used because this method enables computation of channel coefficients with lower complexity, minimum error, is easier and very simple to apply also .

References

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