

Assignment 1

AI1110: Probability and Random Variables

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12.13.1.15: Question:

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Repeat this experiment till a coin is tossed. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Answer: $\frac{1}{2}$.

Solution:

Given that a die is thrown and if the outcome is a multiple of 3 i.e., 3 or 6 then another die is thrown, else a coin is tossed. The experiment is repeated till a coin is tossed.

Let k be the outcome of the die roll.

And X be a random variable such that

$$X = \begin{cases} 1 & k \in \{3, 6\} \\ 0 & k \in \{1, 2, 4, 5\} \end{cases} \quad (1)$$

$$\Pr(X = i) = \begin{cases} \frac{1}{3} & i = 1 \\ \frac{2}{3} & i = 0 \end{cases} \quad (2)$$

Let Y be a random variable for the coin toss then

$$Y = \begin{cases} 1 & \text{tail} \\ 0 & \text{head} \end{cases} \quad (3)$$

$$\Pr(Y = i) = \begin{cases} \frac{1}{2} & i = 1 \\ \frac{1}{2} & i = 0 \end{cases} \quad (4)$$

Let Z be a random variable which represents the number of times 3 has occurred in the die rolls.

Then $Z \in \{0, 1, 2, \dots, \infty\}$

Need to Find, Conditional Probability of the event 'the coin shows a tail', given that 'at least one die shows a 3', i.e., $\Pr(Y = 1|Z \geq 1)$

$$\Pr(Y = 1|Z \geq 1) = \frac{\Pr(Y = 1, Z \geq 1)}{\Pr(Z \geq 1)} \quad (5)$$

The outcome of coin toss(i.e., head or tail) is independent of the number of die rolls or the outcome of any die roll.

Therefore,

$$\Pr(Y = 1, Z \geq 1) = \Pr(Y = 1) \cdot \Pr(Z \geq 1) \quad (6)$$

Substituting eq(6) in eq(5), we get

$$\Pr(Y = 1|Z \geq 1) = \frac{\Pr(Y = 1) \cdot \Pr(Z \geq 1)}{\Pr(Z \geq 1)} \quad (7)$$

$$\Pr(Y = 1|Z \geq 1) = \Pr(Y = 1) \quad (8)$$

$$\Pr(Y = 1|Z \geq 1) = \frac{1}{2} \quad (9)$$

Hence,

Probability of the event 'the coin shows a tail', given that 'at least one die shows a 3' is $\frac{1}{2}$.