

# Assignment 1

AI1110: Probability and Random Variables

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## 12.13.1.15: Question:

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Repeat this experiment till a coin is tossed. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

**Answer:**  $\frac{1}{2}$ .

## Solution:

Given that a die is thrown and if the outcome is a multiple of 3 i.e., 3 or 6 then another die is thrown, else a coin is tossed. The experiment is repeated till a coin is tossed.

Let  $k$  be the outcome of  $j^{\text{th}}$  die roll.

And  $X_j$  be a random variable such that

$$X_j = \begin{cases} 1 & k \in \{3, 6\} \\ 0 & k \in \{1, 2, 4, 5\} \end{cases} \quad (1)$$

$$\Pr(X_j = i) = \begin{cases} \frac{1}{3} & i = 1 \\ \frac{2}{3} & i = 0 \end{cases} \quad (2)$$

Let  $Y$  be a random variable for the coin toss then

$$Y = \begin{cases} 1 & \text{tail} \\ 0 & \text{head} \end{cases} \quad (3)$$

$$\Pr(Y = i) = \begin{cases} \frac{1}{2} & i = 1 \\ \frac{1}{2} & i = 0 \end{cases} \quad (4)$$

Let  $Z$  be a random variable which represents the number of times 3 has occurred in the die rolls.

Then  $Z \in \{0, 1, 2, \dots, \infty\}$

Need to Find, Conditional Probability of the event 'the coin shows a tail', given that 'at least one die shows a 3', i.e.,  $\Pr(Y = 1 | Z \geq 1)$

$$\Pr(Y = 1 | Z \geq 1) = \frac{\Pr(Y = 1, Z \geq 1)}{\Pr(Z \geq 1)} \quad (5)$$

The event of rolling the die is a Markov process because the outcome of the  $n^{\text{th}}$  die roll depends only on the outcome of the  $n - 1^{\text{th}}$  die roll i.e.,

$$\Pr(X_n = i_n | X_0 = i_0, \dots, X_{n-1} = i_{n-1}) = \Pr(X_n = i_n | X_{n-1} = i_{n-1}) \quad (6)$$

$$\Pr(X_n = 1 | X_{n-1} = 1) = \frac{1}{3} \quad (7)$$

$$\Pr(X_n = 0 | X_{n-1} = 1) = \frac{2}{3} \quad (8)$$

$$\Pr(X_n = 1 | X_{n-1} = 0) = 0 \quad (9)$$

$$\Pr(X_n = 0 | X_{n-1} = 0) = 0 \quad (10)$$

If we are tossing the coin after  $n$  die rolls then

$$\Pr(Y = 1 | X_n = 0) = \Pr(Y = 0 | X_n = 0) = \frac{1}{2} \quad (11)$$

$$\Pr(Y = 1 | X_n = 1) = \Pr(Y = 0 | X_n = 1) = 0 \quad (12)$$

The outcome of coin toss is depending only on the last die roll (since we are tossing the coin only when  $X_n = 0$ ) and this is independent of number of die rolls and also the number of times we get 3 before the last die roll.

Therefore,

$$\Pr(Y = 1, Z \geq 1) = \Pr(Y = 1) \cdot \Pr(Z \geq 1) \quad (13)$$

Substituting eq(13) in eq(5), we get

$$\Pr(Y = 1 | Z \geq 1) = \frac{\Pr(Y = 1) \cdot \Pr(Z \geq 1)}{\Pr(Z \geq 1)} \quad (14)$$

$$\Pr(Y = 1 | Z \geq 1) = \Pr(Y = 1) \quad (15)$$

$$\Pr(Y = 1 | Z \geq 1) = \frac{1}{2} \quad (16)$$

Hence,

Probability of the event 'the coin shows a tail', given that 'at least one die shows a 3' is  $\frac{1}{2}$ .

The below is the Markov Chain Diagram for this experiment.

States are labelled with numbers and they are defined as

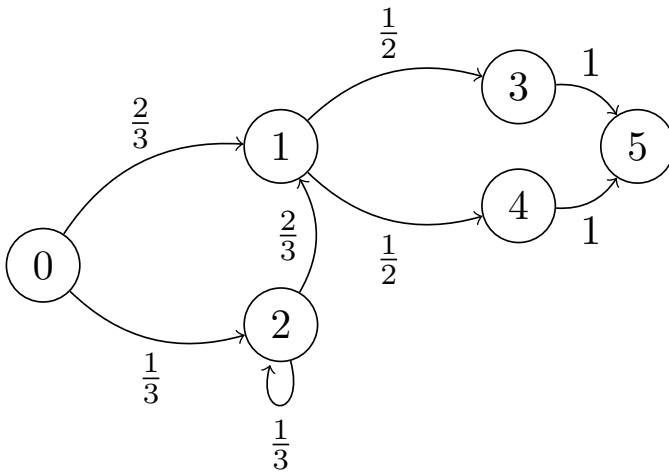


Fig. 0. Markov Chain Diagram

State 0 : Initial state , a die is to be rolled

State 1 : Dice is rolled and the outcome is not a multiple of 3 , a coin is to be tossed next.

State 2 : Dice is rolled and the outcome is multiple of 3, the die is to rolled again.

State 3 : Coin is tossed and outcome is Head.

State 4 : Coin is tossed and outcome is Tail.

State 5 : Terminal state and end of experiment.