Assignment 1

AI1110: Probability and Random Variables

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12.13.1.15: Question:
Consider the experiment of throwing a die,if a multiple of 3 comes up,throw the die again and if any other number comes,toss a coin.Repeat this experiment till a coin is tossed.Find the conditional probability of the event 'the coin shows a tail',given that 'at least one die shows a 3'.
Answer:$\frac{1}{2}$**.**
Solution:
The given experiment is a Markov Process and the following is the Markov Chain Diagram for this experiment.

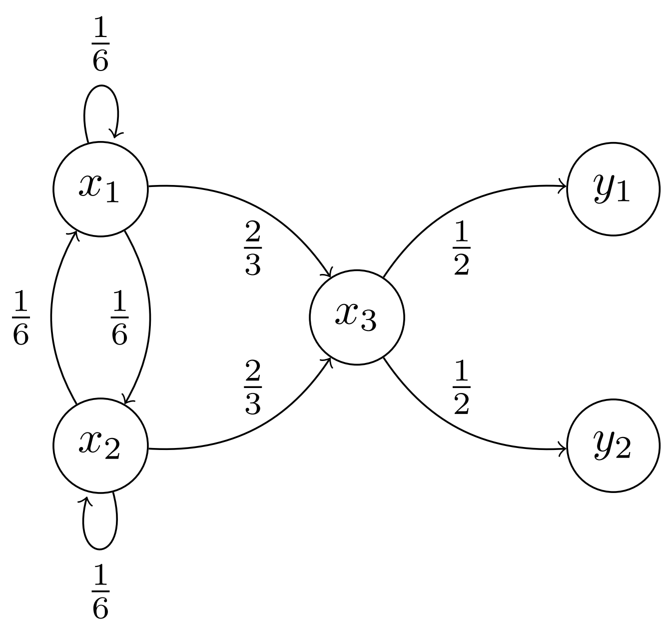

Fig. 0. Markov Chain Diagram

The states in the Diagram are defined as follows,
State x_1 : Die roll results in 3 and the die is to be rolled again.
State x_2 : Die roll results in 6 and the die is to be rolled again.
State x_3 : Die roll results in not a multiple of 3 and a coin is to be tossed now.
State y_1 : Coin toss results in Tail
State y_2 : Coin toss results in Head
Need to Find, Conditional Probability of the event 'the coin shows a tail',given that 'at least one die shows a 3', i.e., $\Pr(y_1|x_1, x_3)$
$\Pr(y_1|x_1, x_3)$ refers to Conditional Probability of being in state y_1 given that the Markov chain has been in state x_1 atleast once and has also visited state x_3.

$$\Pr(y_1|x_1, x_3) = \frac{\Pr(y_1, x_3|x_1)}{\Pr(x_3|x_1)} \tag{1}$$

$\Pr(y_1, x_3|x_1)$ refers to the Probability that Markov Chain transitions from state x_1 to state x_3 and then to state y_1.
This Markov Chain is time homogenous . So,

$$\Pr(y_1, x_3|x_1) = \Pr(y_1|x_3)\Pr(x_3|x_1) \tag{2}$$

$$\Pr(y_1|x_1, x_3) = \frac{\Pr(y_1|x_3)\Pr(x_3|x_1)}{\Pr(x_3|x_1)} \tag{3}$$

$$\Pr(y_1|x_1, x_3) = \Pr(y_1|x_3) \tag{4}$$

$$\Pr(y_1|x_1, x_3) = \frac{1}{2} \tag{5}$$

Hence,
Probability of the event 'the coin shows a tail',given that 'at least one die shows a 3' is $\frac{1}{2}$.