Bonus Question

AI1110: Probability and Random Variables

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Question:
Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times.What are possible values of X?

Solution:
Let A be a random variable which represents the number of Heads obtained in 6 coin tosses.
And B be a random variable which represents the number of Tails obtained in 6 coin tosses.
Then,
$A \in \{0,1,2,3,4,5,6\}$
Similarly,
$B \in \{0,1,2,3,4,5,6\}$

$$A + B = 6 \tag{1}$$
$$X = |A - B| \tag{2}$$

from eq(1)

$$X = |A - (6 - A)| \tag{3}$$
$$X = |2A - 6| \tag{4}$$

$$X = \begin{cases} 6 & A \in \{0,6\} \\ 4 & A \in \{1,5\} \\ 2 & A \in \{2,4\} \\ 0 & A \in \{3\} \end{cases} \tag{5}$$

Hence, The possible values of X are {0,2,4,6}
Here, A is a Bernoulli random variable with n = 6 and $p = \frac{1}{2}$ Therefore it can be written as

$$\Pr(A = k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & 0 \le k \le 6 \\ 0 & otherwise \end{cases} \tag{6}$$

$$\Pr(A = k) = \begin{cases} \binom{6}{k} (\frac{1}{2})^6 & 0 \le k \le 6 \\ 0 & otherwise \end{cases} \tag{7}$$

$$\Pr(X = i) = \begin{cases} \Pr(A = 3) & i = 0 \\ \Pr(A = 2) + \Pr(A = 4) & i = 2 \\ \Pr(A = 1) + \Pr(A = 5) & i = 4 \\ \Pr(A = 0) + \Pr(A = 6) & i = 6 \end{cases} \tag{8}$$

The distribution of X is

$$\Pr(X = i) = \begin{cases} \binom{6}{3}(\frac{1}{2})^6 & i = 0 \\ \binom{6}{2}(\frac{1}{2})^6 + \binom{6}{4}(\frac{1}{2})^6 & i = 2 \\ \binom{6}{1}(\frac{1}{2})^6 + \binom{6}{5}(\frac{1}{2})^6 & i = 4 \\ \binom{6}{0}(\frac{1}{2})^6 + \binom{6}{6}(\frac{1}{2})^6 & i = 6 \end{cases} \tag{9}$$

The CDF of Random Variable X can be written as

$$F_X(k) = \Pr(X \le k) = \sum_{i=0}^{k} \Pr(X = i) \tag{10}$$

Therefore,

$$F_X(0) = \Pr(X = 0) = \binom{6}{3}(\frac{1}{2})^6 \tag{11}$$

$$F_X(2) = \sum_{i=0}^{2} \Pr(X = i) = \binom{6}{3}(\frac{1}{2})^6 + \binom{6}{2}(\frac{1}{2})^6 + \binom{6}{4}(\frac{1}{2})^6 \tag{12}$$

$$F_X(4) = \sum_{i=0}^{4} \Pr(X = i) = \binom{6}{3}(\frac{1}{2})^6 + \binom{6}{2}(\frac{1}{2})^6 + \binom{6}{4}(\frac{1}{2})^6 + \binom{6}{1}(\frac{1}{2})^6 + \binom{6}{5}(\frac{1}{2})^6 \tag{13}$$

$$F_X(6) = \sum_{i=0}^{6} \Pr(X = i) = \binom{6}{3}(\frac{1}{2})^6 + \binom{6}{2}(\frac{1}{2})^6 + \binom{6}{4}(\frac{1}{2})^6 + \binom{6}{1}(\frac{1}{2})^6 + \binom{6}{5}(\frac{1}{2})^6 + \binom{6}{0}(\frac{1}{2})^6 + \binom{6}{6}(\frac{1}{2})^6 \tag{14}$$