

Bonus Question

AI1110: Probability and Random Variables

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Question:

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

Solution:

Let A be a random variable which represents the number of Heads obtained in 6 coin tosses.

And B be a random variable which represents the number of Tails obtained in 6 coin tosses.

Then,

$$A \in \{0, 1, 2, 3, 4, 5, 6\}$$

Similarly,

$$B \in \{0, 1, 2, 3, 4, 5, 6\}$$

$$\Pr(X = i) = \begin{cases} \Pr(A = 3) & i = 0 \\ \Pr(A = 2) + \Pr(A = 4) & i = 2 \\ \Pr(A = 1) + \Pr(A = 5) & i = 4 \\ \Pr(A = 0) + \Pr(A = 6) & i = 6 \end{cases} \quad (8)$$

The distribution of X is

$$\Pr(X = i) = \begin{cases} \binom{6}{3} \left(\frac{1}{2}\right)^6 & i = 0 \\ \binom{6}{2} \left(\frac{1}{2}\right)^6 + \binom{6}{4} \left(\frac{1}{2}\right)^6 & i = 2 \\ \binom{6}{1} \left(\frac{1}{2}\right)^6 + \binom{6}{5} \left(\frac{1}{2}\right)^6 & i = 4 \\ \binom{6}{0} \left(\frac{1}{2}\right)^6 + \binom{6}{6} \left(\frac{1}{2}\right)^6 & i = 6 \end{cases} \quad (9)$$

$$A + B = 6 \quad (1)$$

$$X = |A - B| \quad (2)$$

from eq(1)

$$X = |A - (6 - A)| \quad (3)$$

$$X = |2A - 6| \quad (4)$$

$$X = \begin{cases} 6 & A \in \{0, 6\} \\ 4 & A \in \{1, 5\} \\ 2 & A \in \{2, 4\} \\ 0 & A \in \{3\} \end{cases} \quad (5)$$

Hence, The possible values of X are $\{0, 2, 4, 6\}$

Here, A is a Bernoulli random variable with $n = 6$ and $p = \frac{1}{2}$ Therefore it can be written as

$$\Pr(A = k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & 0 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$\Pr(A = k) = \begin{cases} \binom{6}{k} \left(\frac{1}{2}\right)^6 & 0 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$