Bonus Question

AI1110: Probability and Random Variables

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Question:
It is known that 10% of certain articles manufactured are defective.What is the probability that in a random sample space of 12 such articles, 9 are defective?

Solution:
Let X_i be a random variable corresponding to i^{th} article such that

$$X_i = \begin{cases} 1 & \text{if the article is defective} \\ 0 & \text{if the article is not defective} \end{cases} \tag{1}$$

$X_1, X_2, ..., X_{12}$ is a sequence of independent and identically distributed random variables.This sequence forms a Binomial Distribution with mean μ and variance σ^2
For this Binomial Distribution, n = 12 and p = 0.1.
The mean and standard deviation of the Binomial distribution are

$$\mu = np \tag{2}$$
$$\mu = 12 \times 0.1 \tag{3}$$
$$\mu = 1.2 \tag{4}$$
$$\sigma = \sqrt{np(1-p)} \tag{5}$$
$$\sigma = \sqrt{12 \times 0.1 \times 0.9} \tag{6}$$
$$\sigma = 1.04 \tag{7}$$

Let $S_n = \sum_{i=1}^{n} X_i$

$$\text{Standardized sample mean} = \frac{\frac{S_n}{n} - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{8}$$
$$= \frac{S_n - \mu n}{\sigma\sqrt{n}} \tag{9}$$

as $n \to \infty$,$\frac{S_n - \mu n}{\sigma\sqrt{n}} \to N(0,1)$
here N(0,1) denotes a standard normal distribution with mean 0 and variance 1.
Let, E be the event that exactly 9 articles are defective.
For the event E

$$S_n = 9 [\text{as exactly 9 articles are defective}]$$

Standardized sample mean for the event E is

$$z = \frac{S_n - np}{\sqrt{np(1-p)}} \tag{10}$$
$$z = \frac{9 - 1.2}{1.04} \tag{11}$$
$$z = 7.5 \tag{12}$$

From the Standard Normal Distribution table,
The probability of an event having standardized sample mean value greater than 7.5 is very low and is equal to $4.338751580235112 \times 10^{-13}$
Therefore, $\Pr(E) = 4.338751580235112 \times 10^{-13}$