

Bonus Question

AI1110: Probability and Random Variables

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Question:

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample space of 12 such articles, 9 are defective?

Solution:

Let X_i be a random variable corresponding to i^{th} article such that

$$X_i = \begin{cases} 1 & \text{if the article is defective} \\ 0 & \text{if the article is not defective} \end{cases} \quad (1)$$

X_1, X_2, \dots, X_{12} is a sequence of independent and identically distributed random variables. This sequence forms a Binomial Distribution with mean μ and variance σ^2

For this Binomial Distribution, $n = 12$ and $p = 0.1$. The mean and standard deviation of the Binomial distribution are

$$\mu = np \quad (2)$$

$$\mu = 12 \times 0.1 \quad (3)$$

$$\mu = 1.2 \quad (4)$$

$$\sigma = \sqrt{np(1-p)} \quad (5)$$

$$\sigma = \sqrt{12 \times 0.1 \times 0.9} \quad (6)$$

$$\sigma = 1.04 \quad (7)$$

$$\text{Let } S_n = \sum_{i=1}^n X_i$$

$$\text{Standardized sample mean} = \frac{\frac{S_n - \mu}{n}}{\frac{\sigma}{\sqrt{n}}} \quad (8)$$

$$= \frac{S_n - \mu n}{\sigma \sqrt{n}} \quad (9)$$

$$\text{as } n \rightarrow \infty, \frac{S_n - \mu n}{\sigma \sqrt{n}} \rightarrow N(0, 1)$$

here $N(0,1)$ denotes a standard normal distribution with mean 0 and variance 1.

Let, E be the event that exactly 9 articles are

defective.

For the event E

$$S_n = 9 [\text{as exactly 9 articles are defective}]$$

Standardized sample mean for the event E is

$$z = \frac{S_n - np}{\sqrt{np(1-p)}} \quad (10)$$

$$z = \frac{9 - 1.2}{1.04} \quad (11)$$

$$z = 7.5 \quad (12)$$

From the Standard Normal Distribution table,

The probability of an event having standardized sample mean value than 7.5 is very low and is equal to $4.338751580235112 \times 10^{-13}$

$$\text{Therefore, } \Pr(E) = 4.338751580235112 \times 10^{-13}$$

