

Geometrical Question

A circle passes through two points $(4,1)$ $(6,5)$ and also the center lies on the equation $4x+y=16$. Find the equation of the circle.

Geometrical question in terms of matrices

A circle passes through two points $\begin{bmatrix} 4 & 1 \end{bmatrix}$, $\begin{bmatrix} 6 & 5 \end{bmatrix}$ and also the center of the circle lies on the line $\begin{bmatrix} 4 & 1 \end{bmatrix} X=16$. Find the equation of the circle.

Solution

Let $A = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$. Let the center of the circle be O. The mid point of the chord AB is $C = \frac{A+B}{2} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Let the direction vector $AB = B - A$
which gives $AB = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

The line joining C and O is normal to the chord AB. The equation of OC is:

$$AB^T(x - C) = 0 \quad (1)$$

$$\text{which gives } \begin{bmatrix} 2 & 4 \end{bmatrix} x = 22$$

Solution

Given, the center of the circle also lies on line $\begin{bmatrix} 4 & 1 \end{bmatrix} X = 16$.
center O is the point of intersection of OC and the given line.

Let $S = \begin{bmatrix} 4 & 1 \\ 2 & 4 \end{bmatrix}$ The point of intersection is given by: $Sx = \begin{bmatrix} 16 \\ 22 \end{bmatrix}$. Then,

$$x = P^{-1}b \quad (2)$$

$$x = \frac{1}{14} \begin{bmatrix} 4 & -1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 16 \\ 22 \end{bmatrix}$$
$$x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

The obtained Solution is nothing but the center..

The radius can be obtained by computing the norm of (O-A) or (O-B)

$$\text{Radius} = ||(O - B)|| = ||[3 \ -1]|| = 3.16 \text{ units}$$

Let o be the center of the circle. Then

$$||x - c|| = r^2$$

$$(x - c)^T (x - c) = r^2$$

$$x^T x - 2c^T x = r^2 - c^T c$$

Therefore, the equation of the circle is given by,

$$x^T x - 2 \begin{bmatrix} 3 & 4 \end{bmatrix}^T x = 3.16 - \begin{bmatrix} 3 & 4 \end{bmatrix}^T \begin{bmatrix} 3 & 4 \end{bmatrix} \quad (3)$$

FIGURE

