Solving JEE 2008 GEOMETRY QUESTION USING MATRICES

Indian Institute of Technology Hyderabad

ee18btech11033@iith.ac.in
ee18btech11008@iith.ac.in

Geometrical Question

A circle passes through two points (4,1) (6,5) and also the center lies on the equation $4x+y=16$.Find the equation of the circle.

A circle passes through two points $\begin{bmatrix} 4 & 1 \end{bmatrix}$, $\begin{bmatrix} 6 & 5 \end{bmatrix}$ and also the center of the circle lies on the line $\begin{bmatrix} 4 & 1 \end{bmatrix} X = 16$. Find the equation of the circle.

Solution

Let A=$\begin{bmatrix}4\\1\end{bmatrix}$ and B= $\begin{bmatrix}6\\5\end{bmatrix}$.Let the center of the circle be O. The mid point of the chord AB is C $=\frac{A+B}{2}=\begin{bmatrix}5\\3\end{bmatrix}$

Let the direction vector AB=B-A

which gives AB=$\begin{bmatrix}2\\4\end{bmatrix}$

The line joining C and O is normal to the chord AB. The equation of OC is:

$$AB^T(x-C)=0 \tag{1}$$

which gives $\begin{bmatrix}2 & 4\end{bmatrix}$x=22

Solution

Given,the center of the circle also lies on line $\begin{bmatrix}4 & 1\end{bmatrix}X=16$.
center O is the point of intersection of OC and the given line.
Let $S = \begin{bmatrix}4 & 1\\2 & 4\end{bmatrix}$ The point of intersection is given by: $Sx=\begin{bmatrix}16\\22\end{bmatrix}$.Then,

$$x = P^{-1}b \tag{2}$$

$$x=\frac{1}{14}\begin{bmatrix}4 & -1\\-2 & 4\end{bmatrix}\begin{bmatrix}16\\22\end{bmatrix}$$

$$x=\begin{bmatrix}3\\4\end{bmatrix}$$

The obtained Solution is nothing but the center..

The radius can be obtained by computing the norm of (O-A) or (O-B)

Radius $= ||(O-B)|| = ||[3-1]|| = 3.16$ units

Let o be the center of the circle.Then

$$||x-c|| = \mathrm{r}^2$$

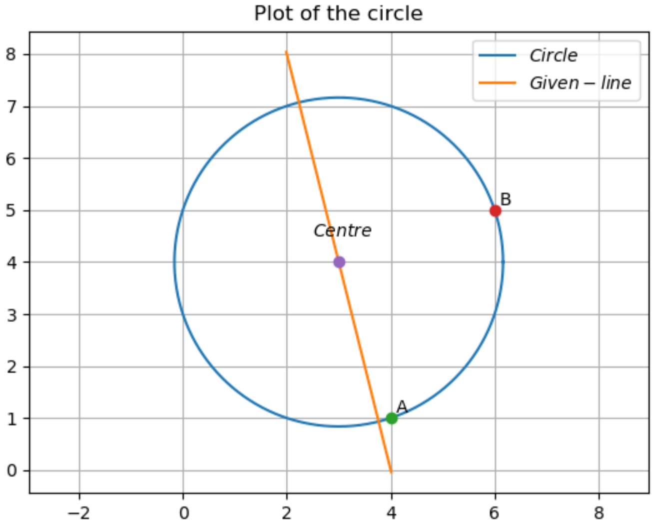
$$(x-c)^T(x-c) = r^2$$

$$\mathrm{x}^T x - 2c^T x = r^2 - c^T c$$

Therefore,the equation of the circle is given by,

$$\mathrm{x}^T x - 2\begin{bmatrix}3 & 4\end{bmatrix}^T x = 3.16 - \begin{bmatrix}3 & 4\end{bmatrix}^T \begin{bmatrix}3 & 4\end{bmatrix} (3)$$

FIGURE



Plot of the circle
Circle
Given – line
B
Centre
A
8
7
6
5
4
3
2
1
0
−2
0
2
4
6
8