EE3025 ASSIGNMENT- 1

RISHITHA - EE18BTECH11033

Download all python codes from

```
https://github.com/Rishithapawar/EE3025/tree/
    master/Assignment−1/codes
```

and latex-tikz codes from

```
https://github.com/Rishithapawar/EE3025/tree/
    master/Assignment−1
```

1 Problem

The command

```
output_signal = signal.lfilter(b,a,
    output_signal)
```

in Problem 2.3 is executed through following difference equation

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{1.0.1}$$

where input signal is $x(n)$ and output signal is $y(n)$ with intial values all 0. Replace **signal.filtfilt** with your own routine and verify

2 Solution

The function **signal.filtfilt(b,a,x)** filters the input using Butter worth filter .This function applies a digital filter twice, once forward and once backward. The combined filter has zero phase shift. Analytically, let $v(n)$ denote the output of the first filtering operation(forward) and let $h(n)$ be the impulse response of the filter. Then:

$$v(n) = (h * x)(n) \tag{2.0.1}$$

For the second filtering operation (backward), we "flip" $v(n)$ to obtain $v(-n)$ and apply the filter again:

$$w(n) = (h * flip(v))(n) \tag{2.0.2}$$

The final output is then the above result flipped:

$$y = flip(w) = flip(h * flip(v)) = flip(h) * v \tag{2.0.3}$$

$$y(n) = flip(h * (h * x)) = (flip(h) * (h * x))(n) \tag{2.0.4}$$

Using the properties of z-transform

$$\mathcal{Z}\{flip(x)\} = X(z^{-k}) \tag{2.0.5}$$

Applying Z-transform to (2.0.4) gives:

$$Y(z) = H(x^{-1}(H(z)X(z)) \tag{2.0.6}$$

For real filters h, this reduces to

$$Y(exp(jwT)) = |H(exp(jwT))|^2 \tag{2.0.7}$$

$$X(exp(jwT)) \tag{2.0.8}$$

Using the properties of z-transform

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{2.0.9}$$

$$\mathcal{Z}\{y(n-m)\} = z^{-m}Y(z) \tag{2.0.10}$$

where $X(z)$ and $Y(z)$ are the respective z-transforms of $x(n)$ and $y(n)$ respectively.

Converting the difference equation into its z-transform equation

$$Y(z) \sum_{m=0}^{M} a(m)\, z^{-m} = X(z) \sum_{k=0}^{N} b(k)\, z^{-k} \tag{2.0.11}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b(k)\, z^{-k}}{\sum_{m=0}^{M} a(m)\, z^{-m}} \tag{2.0.12}$$

From the coefficients b,a and from (2.0.12) evaluating $H(K)$.
Using built in fft command evaluating $X(K)$ from $x(n)$
From

$$Y(K) = H(K)\, X(K) \tag{2.0.13}$$

Using built in ifft command evaluating $y(n)$ from $Y(K)$.

Below is the following python code for the above question

codes/ee18btech11033.py

Below is the soundfile constructed from output signal y using own routine filter

codes/Sound_With_ReducedNoise_7.1.wav

3 Verification

Plotting the time domain output signal evaluated from both own routine filter and signal.filtfilt command and Plotting the frequency domain response evaluated from both own routine and signal.filtfilt.

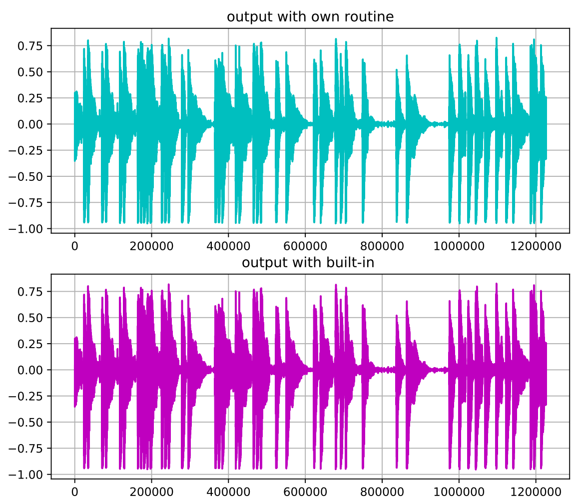

Fig. 0: Time domain response

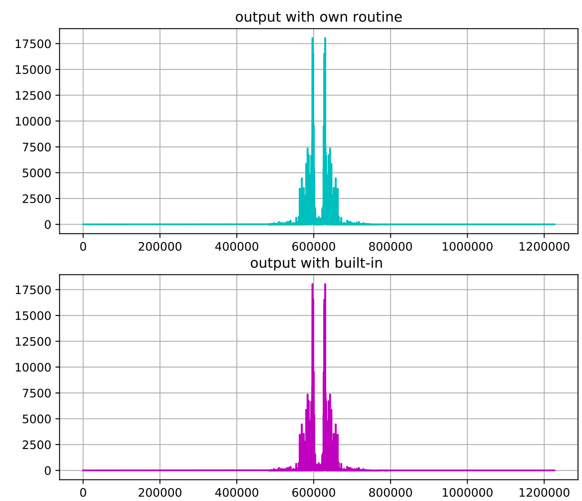

Fig. 0: Frequency domain response