Control Systems

G V V Sharma*

CONTENTS

Abstract—**The objective of this manual is to introduce control system design at an elementary level.**

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1 BODE PLOT

1.1 Gain and Phase Margin

1.1.1. For a unity feedback system shown in Fig. 1.1.1, having transfer function given below in eq 1.1.1.1. Design the value of gain K for (*i*) a gain margin of 33 dB. (*ii*) Phase margin of 40°. (*iii*) to yield maximum peak overshoot of 20 percent for a step input.
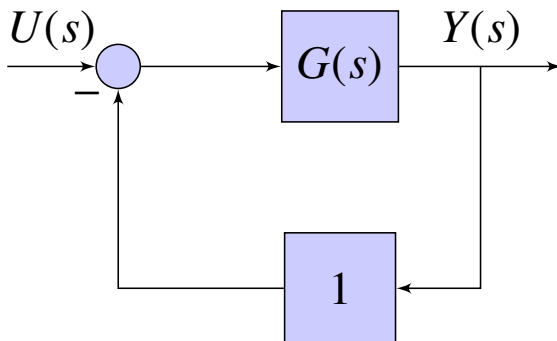

Fig. 1.1.1

$$G(s) = \frac{K}{(s+3)(s+9)(s+15)} \quad (1.1.1.1)$$

Solution:

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

$$G(s)H(s) = \frac{K}{(s+3)(s+9)(s+15)} \quad (1.1.1.2)$$

For K=1 let:

$$B(s) = \frac{1}{(s+3)(s+9)(s+15)} \quad (1.1.1.3)$$

Gain of the given transfer function is:

$$= 20log(|G(s)H(s)|) \quad (1.1.1.4)$$
$$= 20log(K) + 20log|B(s)| \quad (1.1.1.5)$$

Phase of the given transfer function is:

$$= \angle G(s)H(s) \quad (1.1.1.6)$$
$$= \angle B(s) \quad (1.1.1.7)$$

Thus value of K has : a) no effect on phase. b) linear effect on gain.

1.1.2. (*i*) Given gain = 33dB
Solution: The following code generates Bode plot of $B(s)$ as shown in Fig 1.1.2

codes/ee18btech11033_1.py

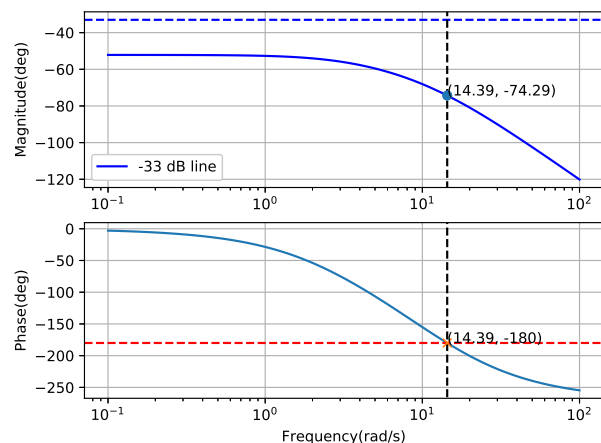

Fig. 1.1.2: Bode Plot of $B(s)$

Fig 1.1.2 shows how much the gain graph be shifted to get -33 dB gain at ω_{pc}. From the

graph we can tell it should be shifted by a length $20log(K)$ which results in $K = 116.01$

1.1.3. Verify by substituting value of K obtained above.

Solution: The following code generates Fig 1.1.3.
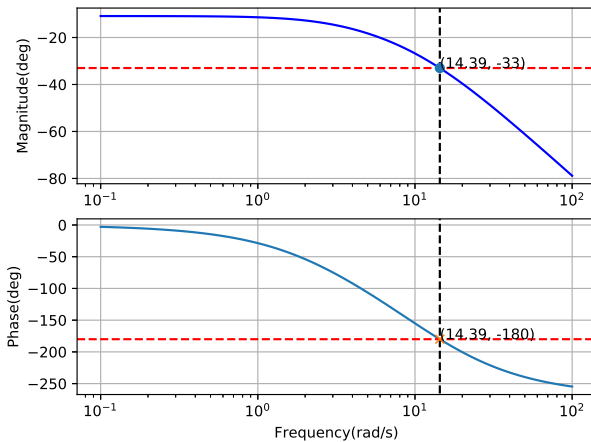
```
codes/ee18btech11033_ver1.py
```


Fig. 1.1.3: Bode Plot of $G(s)$ with $K = 116.01$

1.1.4. (*i*) Given PM = 40°

Solution:

$$phase\ at\ \omega_{gc} = -180° + PM \quad (1.1.4.1)$$
$$= -140° \quad (1.1.4.2)$$

The following code generates Bode plot of $B(s)$ to obtain ω_{gc} as shown in Fig 1.1.4

```
codes/ee18btech11033_2.py
```

Fig 1.1.4 shows how much the gain graph be slided to get 0 dB gain at ω_{gc}. From the graph we can tell it should be shifted by a length $20log(k)$ which results in $K = 1710.01$

1.1.5. Verify by substituting value of K obtained above.

Solution: The following code generates Fig 1.1.5.

```
codes/ee18btech11033_ver2.py
```

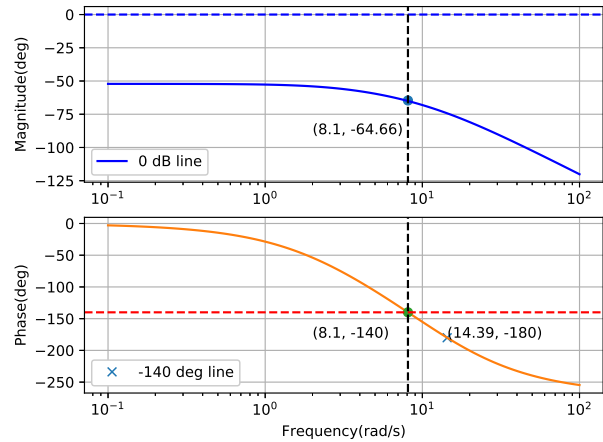1.1.6. (*iii*) 20 percent peak overshoot in step response.
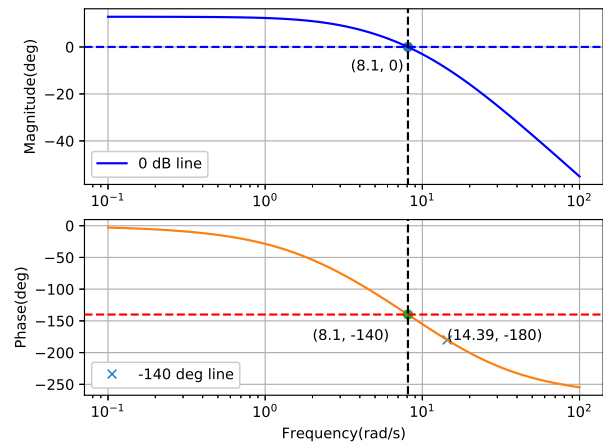

Fig. 1.1.4: Bode Plot of $B(s)$

Fig. 1.1.5: Bode Plot of $G(s)$ with $K = 1710.01$

Solution:

$$\frac{G(s)}{1 + G(s)} \quad (1.1.6.1)$$

$$= \frac{K}{s^3 + 27s^2 + 207s + (405 + K)} = Y(s) \quad (1.1.6.2)$$

Step response-

$$\frac{K}{(s)\,[s^3 + 27s^2 + 207s + (405 + K)]} \quad (1.1.6.3)$$

By final value theorem, steady state value-

$$\lim_{s \to 0} sY(s) = \lim_{t \to \infty} y(t) = \frac{k}{405} \quad (1.1.6.4)$$

So the value at peak should be $\frac{6k}{2025}$. Now

it is extremely difficult to find K from the given data. Since it a Third order system, there exist no explicit formula for peak time. Thus, trying a random value of K under the bound the satisfies routh hurwitz, then taking inverse Laplace and differentiating to get peak time and thus overshoot, is the only method that remains.