Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Bode Plot

1.1 Gain and Phase Margin

1.1.1. For a unity feedback system shown in Fig. 1.1.1, having transfer function given below in eq 1.1.1.1. Design the value of gain K for (*i*) a gain margin of 33 dB. (*ii*) Phase margin of 40°. (*iii*) to yield maximum peak overshoot of 20 percent for a step input.

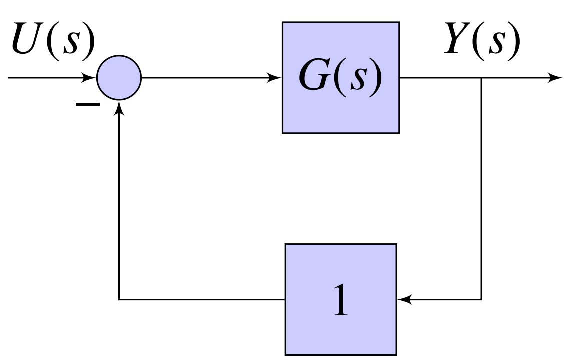

Fig. 1.1.1

$$G(s) = \frac{K}{(s+3)(s+9)(s+15)} \tag{1.1.1.1}$$

Solution:

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$$G(s)H(s) = \frac{K}{(s+3)(s+9)(s+15)} \tag{1.1.1.2}$$

For K=1 let:

$$B(s) = \frac{1}{(s+3)(s+9)(s+15)} \tag{1.1.1.3}$$

Gain of the given transfer function is:

$$= 20log(|G(s)H(s)|) \tag{1.1.1.4}$$
$$= 20log(K) + 20log|B(s)| \tag{1.1.1.5}$$

Phase of the given transfer function is:

$$= \angle G(s)H(s) \tag{1.1.1.6}$$
$$= \angle B(s) \tag{1.1.1.7}$$

Thus value of K has : a) no effect on phase. b) linear effect on gain.

1.1.2. (*i*) Given gain = 33dB

Solution: The following code generates Bode plot of $B(s)$ as shown in Fig 1.1.2

```
codes/ee18btech11033_1.py
```

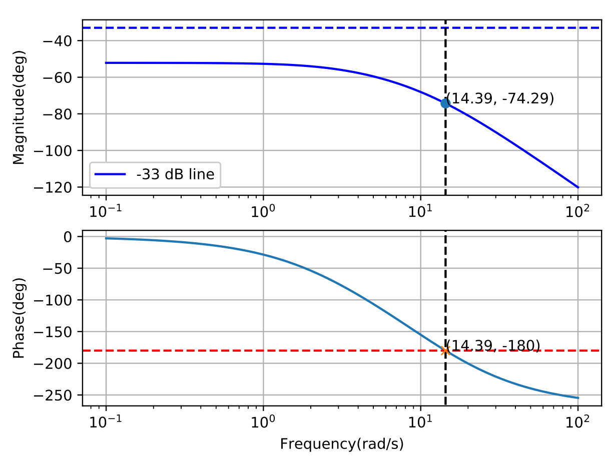

Fig. 1.1.2: Bode Plot of $B(s)$

Fig 1.1.2 shows how much the gain graph be shifted to get -33 dB gain at ω_{pc}. From the graph we can tell it should be shifted by a length $20log(K)$ which results in $K = 116.01$

1.1.3. Verify by substituting value of K obtained above.
Solution: The following code generates Fig 1.1.3.

```
codes/ee18btech11033_ver1.py
```

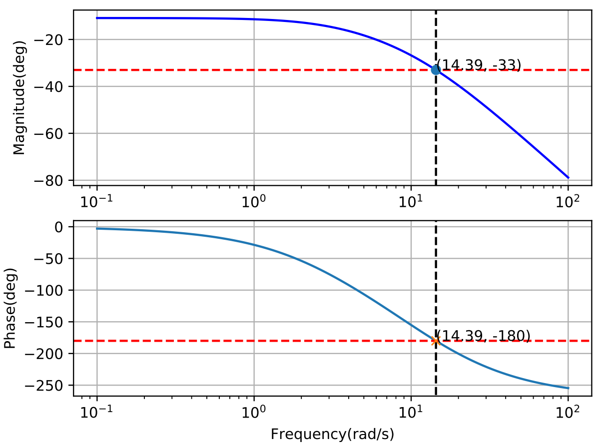

Fig. 1.1.3: Bode Plot of $G(s)$ with $K = 116.01$

1.1.4. (i) Given PM = 40°
Solution:

$$phase\ at\ \omega_{gc} = -180^\circ + PM \quad (1.1.4.1)$$
$$= -140^\circ \quad (1.1.4.2)$$

The following code generates Bode plot of $B(s)$ to obtain ω_{gc} as shown in Fig 1.1.4

```
codes/ee18btech11033_2.py
```

Fig 1.1.4 shows how much the gain graph be shifted to get 0 dB gain at ω_{gc}. From the graph we can tell it should be shifted by a length $20log(k)$ which results in $K = 1710.01$

1.1.5. Verify by substituting value of K obtained above.
Solution: The following code generates Fig 1.1.5.

```
codes/ee18btech11033_ver2.py
```

1.1.6. (iii) 20 percent peak overshoot in step response.

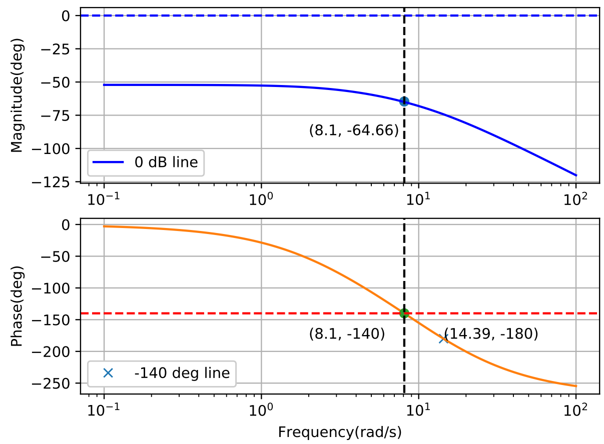

Fig. 1.1.4: Bode Plot of $B(s)$

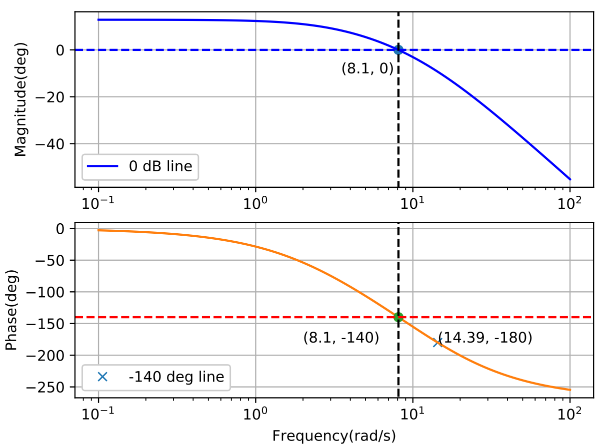

Fig. 1.1.5: Bode Plot of $G(s)$ with $K = 1710.01$

Solution:

$$\frac{G(s)}{1 + G(s)} \quad (1.1.6.1)$$
$$= \frac{K}{s^3 + 27s^2 + 207s + (405 + K)} = Y(s) \quad (1.1.6.2)$$

Step response-

$$\frac{K}{(s)\left[s^3 + 27s^2 + 207s + (405 + K)\right]} \quad (1.1.6.3)$$

By final value theorem, steady state value-

$$\lim_{s\to 0} sY(s) = \lim_{t\to\infty} y(t) = \frac{k}{405 + k} \quad (1.1.6.4)$$

So the value at peak should be $\frac{6k}{2025+5k}$. Now it is extremely difficult to find K from the given data. Since it a Third order system, there exist no explicit formula for peak time. Thus, trying a random value of K under the bound that satisfies routh hurwitz,and verifying the peak overshoot, is the only method that remains. Routh hurwitz criteria tells us that $k < 2389.5$. The following code generates Step input response of $y(t)$ to obtain k as shown in Fig 1.1.6

codes/ee18btech11033_3.py

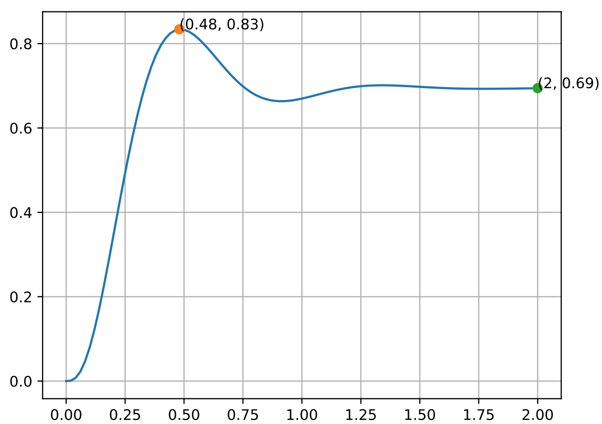

Fig. 1.1.6: Step Input Response $y(t)$

Fig 1.1.6 shows us that the peak value is 0.83 and the steady state value as 0.69 . We observe a 20 percent peak overshoot for $K = 920$.