Lecture - 1

The History and Weirdness of Quantum Mechanics

→ Before 1900's classical theory and Maxwell's theory of electromagnetism explained the world very but there were small gaps in the knowledge.

→ Questions that couldn't be answered with existing theories.

→ That began the study of small stuff

Some theories that changed the Course :-

① Planck's Radiation Law :-

$$\boxed{E(f) = \frac{8\pi}{c^3} \frac{hf^3}{e^{hf/k_BT} - 1}}$$

$h = 6.626 \times 10^{-34}$ kg m^2/s

② Einstein's Photoelectric effect :-

↳ Discrete nature of light,

↳ When light hits a metal e^-'s are emitted

↳ Higher the frequency more the emitted e^-'s energy

↳ Higher the Intensity more no. of e^-'s are emitted

→ This made einstein take plancks constant seriously.

↳ Watch Genius TV Series (Albert einstein Bio – Nat Geo)
(It has this scene.)

→ So energy is Coming in chunks where $E = hf$

- Energy $\propto f$ → so higher the freq, higher the E
- Energy of emitted electron is $hf - W$, where W is the work done to remove an e^-.
- The Intensity of light gives total no. of quanta.

→ Einstein won Noble prize for this but he never was the one to advocate Quantum Mechanics.

- "God doesn't play dice" — Albert Einstein

③ The atomic Collapse Puzzle :-

- The e^-'s should lose energy and fall into the nucleus and die (annihilate the atom), but that doesn't happen.
- You being still alive to read this is the proof.
- Niehls Bohr came up with discretization concept.
- He said e^-'s in an atom take only a specific values of energies which are multiples of h.
  - $\frac{1}{2}h, \frac{3}{2}h, \ldots\ldots$
  - Every atom also have a lowest possible energy called ground state energy.

→ All these energy levels are a particular energy value (eigenvalue) with a corresponding eigenvector

→ Hamiltonian (matrix) describes the energy change in a system which means the e^{-}'s hopping b/w different energy level by releasing or absorbing energy.

→ Schrodinger equation can be used to model these changes in time.

→ To model anything that is changing we need a differential equation.

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi}$$

→ Schrödinger's Equation

→ A lot of great Scientists were thinking debating b/w the wave nature and the particle nature

↳ Many theories and experiments revealed many insights about both particle and wave nature.

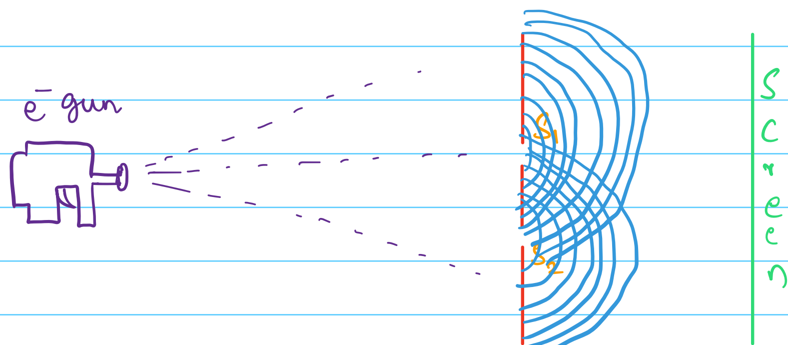
↳ But finally Louis De Broglie came up with his theory of matter waves in his Ph.D thesis

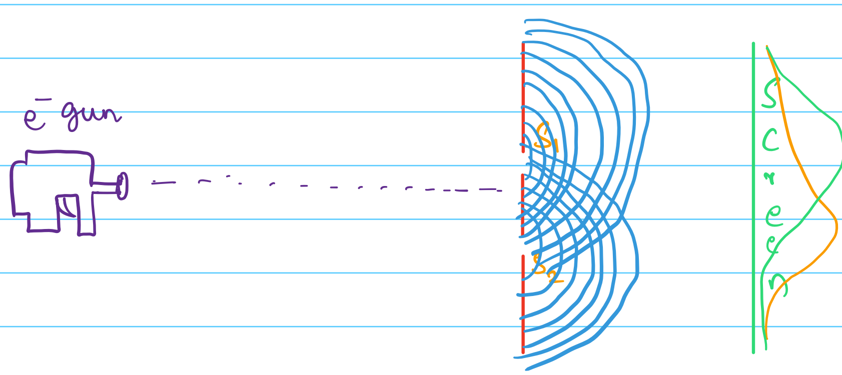
(4) Matter waves and Interference

↳ Young's Double slit exp :- Shooting e^- beams at two slits in a screen produced Interference pattern just like waves (ripples in water)

↳ But the weird thing is this Interference pattern formed even when single e^-'s are shot.

↳ It seemed like the e^-'s are Interacting with themselves.





→ This was the beginning of the Quantum mechanics.

↳ The Sherödinger equation (S.E) that i have written above is not yet produced at this time

→ In 1920's & 30's work of Shrödinger, Heisenberg, Dirac, pauli, Von Neumann (pronounced fon Noiman) and others led to the formation of a revolution Called Quantum Mechanics.

There are many weird properties of Quantum mechanics that made a lot of scientist (even Einstein) to not support the theory.

→ lets talk about some of those properties before going into formalising Quantum mechanics.

Weird properties of Quantum Mechanics

① Indeterminism :- In classical mechanics every thing is deterministic which means, If you are given all the variables in a situation, you can tell what happens at each point in space and time

Example:- If a car racing towards a wall, If you know the current position, velocity, accelaration, position of the wall, then you can say where the car will be at what time and when it will hit the wall with certainity.

→ But Quantum Mechanics is Indeterministic.

↳ In classical mechanics probabilities are discussed when there is incomplete Information about the system but in Quantum. everything is described using probabilities.

↳ If you know the position, then you know the speed of the election & vice versa in QM.

② Interference :-

↳ In quantum mechanics different States have their probability amplitudes just like a wave has an amplitude.

↳ Square of those probality amplitudes is the probability of being in that State

↳ In the double slit experiment Prob Amplitudes associated to both the slits were P_A & P_B

↳ Combined $P_A + P_B \rightarrow$ probability $= |P_A + P_B|^2$

$$|P_A + P_B|^2 = |P_A|^2 + |P_B|^2 + \underline{2 P_A P_B}$$

③ Uncertainity :- You don't know anything about the quantum System. But Suddenly you shined a light to See its position: Now the wave nature Collapsed and you get particle with defined position.

↳ But If you try to measure the probability distribution of different states of the wave, the you have its momentum, Its spread, then you don't have its position.

↳ So position & momentum both Canot be measured simultaneously

$$\boxed{(\Delta x)^2 (\Delta p)^2 \geq \frac{\hbar^2}{4}} \quad \boxed{\hbar = \frac{h}{2\pi}}$$

→ You can get $\psi(\tilde{p})$ by fourier transforming $\psi(x)$

↳ But there is no joint function $\psi(x, p)$

→ This uncertainity is not just for position & momentum but also holds for a lot of other variables.

(4) Superposition :— An electron can not just be in any one energy state but can be in a probabilistic (linear) combination of different energy states.

→ If $|0\rangle$ is ground state & $|1\rangle$ is first excited state then

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$ where

α, β are probability amplitudes associated to state $|0\rangle$ & $|1\rangle$

→ So when you measure (observe) the system n times state $|0\rangle$ is observed with prob $|\alpha|^2$ & state $|1\rangle$ with prob $|\beta|^2$

→ The set of all states forms a complex vector space (called Hilbert space).

→ The time evolution of these states can be described using S.E (Schrödinger eq.)

→ The various measurable physical quantities like energy are represented using matrices on the Hilbert space

$\alpha|0\rangle + \beta|1\rangle$ Can be described using Schrödinger Cat.

$$\alpha \left| \substack{Cat \\ Alive} \right\rangle + \beta \left| \substack{Cat \\ Dead} \right\rangle$$

→ Untill you open the box you don't know.

⑤ **Entanglement** :- One of the most complex Quantum mechanical property.

→ A quantum system with Individual Subsystems that Cannot be written as products of Individual States are said to be entangled

↳ You Cannot describe one System without the other.

↳ The joint System have a well defined State but Individual sub Systems doesn't.

$$\psi(x, y) \neq \psi(x) \cdot \psi(y)$$

↳ Measuring one System disturbs the other because of their very Strong Correlation.

→ So now how Can we do Computation using all these beautifully weird properties.

↳ Why does this Quantum Computing Idea Started.

Quantum Computing :-

→ Current transistor size : 2nm
→ Atomic size : 0.2 – 0.5 nm

→ If we try to reduce transistor size more quantum properties (like quantum tunneling), kicks in.

↳ So there is a limit with classical computers described moore's law.

↳ So scientists thought why can't we leverage the quantum phenomenon to build a new type of computer.

Advents :-

→ In a classical computer to get output of a function for n values takes n iterations

↳ But a quantum computer can solve the function for all n values in one shot.

↳ David Duetsch & Richard Josza came up with this algorithm in 1985 marking the power of quantum parallelism.

↳ Peter Shor while working at Bell labs came up with Shor's algorithm that can factor large Integers into prime factors. which is considered to be a very complex problem in classical computing

↳ This started the next revolution of Quantum Computing. and the field flourished from there.

↳ Shor's Algorithm can break RSA encryption used by most of the financial Banks & Companies. RSA is powered by prime factorization.

→ Charles Bennett, Artur Ekert, Gilles Brassard, and others also contributed a lot which will be discussed in next lecture notes.

→ This is some history and weirdness of Quantum.

To Be Continued ......
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