Navigating Bias: Exploring Age and Gender in AI-Agent Decision-Making within Prisoner's Dilemma Framework *

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Abstract. The study examines the possible biases, such as sex and age, that may influence the decision-making of AI agents, with a specific focus on the ChatGPT 3.5 model. The study focuses on the Prisoner's Dilemma game, as it provides a distinctive framework to investigate these biases, in part because of its fundamental nature in comprehending cooperation and decision-making in various fields. Through thorough experimentation and analysis, our goal is to illuminate how age and gender dynamics impact the decisions made by AI agents, unveiling the intricacies embedded in algorithmic decision-making.

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1 Introduction

In the field of AI development, there is growing concern among policymakers and governments about the impact of artificial intelligence on society. To address this, the United States government has proposed a pioneering "Blueprint for an AI Bill of Rights," aimed at ensuring equitable AI design and deployment practices that protect individuals from algorithmic discrimination. Our study seeks to

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understand how age and gender dynamics influence AI decision-making, particularly within the context of widely used models like ChatGPT 3.5. This research seeks to inquire into how social characteristics may influence the decision made within the context of the game theory interactions, as well as understand the reasoning behind such decisions if the AI agent accompanies their answer with an explained strategy. This research is based on the research by Mei et al. attempting to look into behavioral aspects of AI chatbots. Our research reveals potential biases influenced by sex and age dynamics that shape the cognitive processes of AI agents. Departing from traditional analysis, we adopt the Prisoner's Dilemma framework, recognized for its value in illuminating cooperative dynamics and decision-making paradigms across diverse domains. The mission of this research is to unravel the complex interplay of age and gender influences on AI decision-making within the context of the Prisoner's Dilemma, paying the way for more transparent and socially responsible AI technologies. This research aspires not only to uncover the mysteries of AI bias but also to chart a path forward defined by integrity, inclusivity, and collective advancement.

2 Background

Re-imagining game theory in the age of AI and complex systems demands a holistic approach that transcends traditional boundaries. No longer confined to human players, this new landscape encompasses a diverse array of agents—from autonomous algorithms to sentient machines — each with their unique rationality and adaptability. To navigate this dynamic arena, game theory must evolve, integrating interdisciplinary insights to probe the complexities of human-AI interaction. It's not just about strategic decision-making; it's a reflection of the intricate dance between human cognition and artificial intelligence. This evolution is crucial as AI becomes increasingly intertwined with societal structures, raising profound questions of agency, responsibility, and fairness. In essence, reimagining game theory in this context is a call to arms — a call to chart a path towards a future where human and artificial intelligence can coexist harmoniously.

3 The Pioneers in the History of Game Theory

In the evolution of game theory, prominent theorists have continuously built upon the foundational work of their predecessors, paving the way for ground-breaking advancements and opening new avenues for exploration. From John von Neumann and Oskar Morgenstern's seminal "Theory of Games and Economic Behavior" to John Nash's formulation of the Nash Equilibrium, each contribution has expanded our understanding of strategic interactions. These successive developments not only deepen our comprehension of strategic decision-making but also inspire future scholars to explore novel territories within the discipline, perpetuating its evolution.

Traditional decision theory, the predecessor of game theory, delves into individual decision-making under uncertainty, prioritizing optimization of expected utility or minimization of loss. Game theory, as outlined in the seminal work "Theory of Games and Economic Behavior" by John von Neumann and Oskar Morgenstern in 1944, signifies a significant departure from traditional decision theory towards understanding strategic interactions [2]. Their foundational insights set the stage for subsequent advancements in the field. In the 1950s, John Nash's contributions, particularly his formulation of the Nash Equilibrium, established game theory as a separate discipline, linking individual decision-making with strategic interactions [3]. Initially focusing on pure strategies, Nash's equilibrium concept was later expanded in the 1960s by John Harsanyi and Reinhard Selten to incorporate mixed strategies, where players incorporate randomness into their actions [4]. This progression from pure to mixed strategies significantly deepened our understanding of strategic interactions.

While Nash primarily explored non-cooperative games, Lloyd S. Shapley introduced cooperative game theory, allowing players to form coalitions and make binding agreements, thereby expanding the scope of strategic interactions [5]. The transition from static to dynamic games was further advanced by von Neumann's minimax theorem and Nash's extension of static games into dynamic ones, introducing concepts such as subgame perfection [2] [3]. Finally, the shift from games with perfect information, emphasized by von Neumann and Morgenstern, to those with imperfect information, championed by John Harsanyi, enhanced the practical relevance of game theory in real-world situations [6]. These significant milestones in the evolution of game theory not only build upon the groundwork laid by earlier scholars but also open up new avenues for exploration, providing opportunities for future researchers in the field.

Imagining myself as one of the "founding fathers" of game theory, Lloyd Shapley [5], I embarked on a transformative journey aimed at addressing the limitations of non-cooperative game theory, particularly evident in the classic example of the "Prisoner's Dilemma" [7]. The specific challenge I aimed to overcome was the inability of non-cooperative frameworks to capture cooperative interactions and the potential benefits of forming alliances among players.

In the "Prisoner's Dilemma," two individuals are faced with the choice of cooperating or betraying one another to the authorities [7]. In the traditional non-cooperative approach, each player acts independently, often leading to a suboptimal outcome where both players betray each other to minimize personal risk. However, in reality, the players may communicate and agree to cooperate, leading to a mutually beneficial outcome where both refrain from betrayal.

Introducing cooperative game theory, my [Lloyd Shapley's] innovative framework enables players to form coalitions and negotiate binding agreements, thus fostering cooperation and leading to mutually beneficial outcomes. By allowing for the formation of coalitions and equitable distribution of gains among players, cooperative game theory outperforms previous non-cooperative approaches in achieving desired outcomes, fostering better coordination and collaboration among players, as illustrated in the context of the "Prisoner's Dilemma" game.

Illustration of Interaction with ChatGPT 4

In the exploration of our research topic, it was decided to ask ChatGPT to participate in the Prisoner's Dilemma game using the same text, however, across multiple chats and conversations - so that the previous decisions do not influence decisions with adjusted factors.

Here is the message template used to interact with ChatGPT 3.5:

"Let's play a Prisoner's Dilemma Game. You are player [one/two] and you need to make a direct choice of what would you do. Some important information about you is that you are [factor].

If Player 1 stays silent and Player 2 confesses, Player 1 gets the worst outcome (10 years) while Player 2 goes free (0 years).

If Player 2 stays silent and Player 1 confesses, Player 2 gets the worst outcome (10 years) while Player 1 goes free (0 years).

If they both confess, they both end up with a moderate outcome (5 years).

If they both stay silent, they both get a better outcome (1 year).

It was decided to explore the following factors:

- Sex (Male, Female)
- Age (Divided into 5 different age groups: Early Childhood (0-6 years), Childhood and Adolescence (6 to 18 years old), Young Adulthood (19 to 39 years old), Middle Adulthood (40 to 64 years old), Older Adulthood (65 years old and older))
- Neutral response (the prompt to give a choice is given without any characteristics)

	Player 2		
Player 1	Male	Female	Neutral
Male	"0,10"	"5,5"	"5,5"
Female	"1,1"	"10,0"	"10,0"
Neutral	"0,10"	"5.5"	"5,5"

Table 1. Observed Matrix for Gender

In the observed matrix 1, the rows represent the choices made by Player 1, while the columns represent the choices made by Player 2. The entries in the matrix represent the outcomes of the Prisoner's Dilemma game, where each entry is in the format "X,Y" denoting the years of imprisonment for Player 1 and Player 2 respectively.

Male vs. Male: Both players tend to receive moderate outcomes regardless of their choices.

Player 2 Player 1 Age С ECOA Ν YAMAEarly Childhood "1.1""1,1"1,1 "1,1""1,1"10,0 Childhood '1.1''1.1'''1.1''''1,1'''1,1'10.0 Young Adulthood 0,100,10'0.100.10'0.10 $^{'}5,5^{''}$ Middle Adulthood 0,10 0,100,100,10 0.10"5,5"Older Adulthood "1,1""1,1""1,1""1,1" "1,1" 10,0 Neutral 0,10" 0,10" 0,10" 0,10" "0,10["] "5,5"

Table 2. Observed Matrix for Age Groups

Male vs. Female: If a male player faces a female player, the male player tends to receive worse outcomes, while the female player receives better outcomes.

Male vs. Automatic: Similarly, if a male player faces an automatic player, the male player tends to receive worse outcomes.

Female (Figure 1) vs. Female (Figure 2): Both female players tend to receive moderate outcomes regardless of their choices.

Female vs. Automatic: When a female player faces an automatic player, the female player tends to receive worse outcomes.

Automatic vs. Automatic: Both automatic players tend to receive moderate outcomes regardless of their choices.

In the next observed matrix 2, the rows represent the choices made by Player 1, while the columns represent the choices made by Player 2. The entries in the matrix represent the outcomes of the Prisoner's Dilemma game, where each entry is in the format "X,Y" denoting the years of imprisonment for Player 1 and Player 2 respectively.

Early Childhood vs. Early Childhood: Both players tend to receive the worst outcomes regardless of their choices.

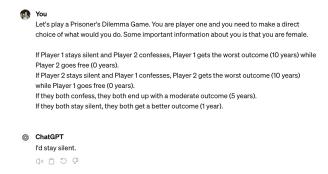
Early Childhood vs. Childhood: If a player in the early childhood age group faces a player in the childhood age group, the player in the early childhood age group tends to receive worse outcomes.

Young Adulthood vs. Young Adulthood and Middle Adulthood (Figure 3): Players in the young adulthood and middle adulthood age groups tend to receive the worst outcomes regardless of their opponent's age group.

Older Adulthood vs. Older Adulthood: Both players tend to receive the worst outcomes regardless of their choices. Automatic vs. Automatic: Both automatic players tend to receive moderate outcomes regardless of their choices.

In summary, in both observed matrices, certain matchups and age groups tend to lead to worse outcomes for specific players, highlighting the dynamics of the Prisoner's Dilemma game and the influence of gender and age on decisionmaking and outcomes.

6 CS/Econ 206 Computational Microeconomics, Duke Kunshan University



 ${\bf Fig.\,1.}$ An example of interaction with ChatGPT 3.5 - specifying first player as a female.

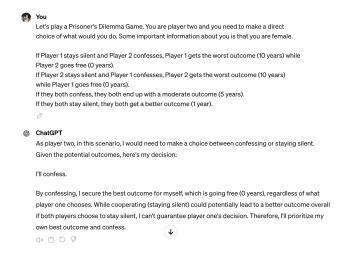


Fig. 2. An example of interaction with ChatGPT 3.5 - specifying second player as a female.

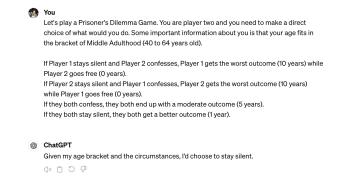


Fig. 3. An example of interaction with ChatGPT 3.5 - specifying second player as an person in Middle Adulthood.

5 Review Classic Games, Nash Equilibrium and the Analytical Tools

5.1 Exploring Inspirational Games in Strategic or Normal Form

Definition 1 (Strategic or Normal Form). a representation in game theory that outlines the strategies and associated payoffs for each player in a game, enabling analysis of strategic interactions independently of move sequences. A (finite, n-person) normal-form game is a tuple (N, A, u), where:

- -N is a finite set of n players, indexed by i;
- $-A = A_1 \times ... \times A_n$, where A_i denotes a finite set of actions available to player i. Each vector $a = (a_1, ..., a_n) \in A$ is called an action profile;
- $-u = (u_1, \ldots, u_n)$, where $u_i : A \to \mathbb{R}$ is a real-valued utility (or payoff) function for player i [8].

The strategic (normal) form stands as the most prevalent depiction of strategic interactions within game theory. In strategic game theory, players make decisions with the understanding that the outcome depends not only on their own actions but also on the actions of others. The Prisoner's Dilemma is one of the central examples of a strategic game.

The Prisoner's Dilemma is a game that has a significant impact on game theory. It is known for its simplicity and profound implications [7]. In this classic scenario, two people are arrested and interrogated separately for a crime that they supposedly committed together. Each prisoner faces a dilemma of whether to cooperate by remaining silent or to defect by betraying their partner and confessing. The outcome of the game depends on the decisions made by both prisoners (see Table 1).

- If both prisoners cooperate (remain silent), they each receive a moderate sentence, representing the optimal collective outcome.
- If both prisoners defect (confess), they both receive harsher sentences, although it's individually preferable to cooperating.
- If one prisoner defects while the other cooperates, the defector receives the most favorable outcome (freedom), while the cooperator faces the harshest punishment (longest sentence).

	Cooperate	Defect	
Player A	(3, 3)	(0, 5)	
	(5, 0)	(1, 1)	

Table 3. Prisoner's Dilemma Payoff Matrix

Nash Equilibrium

Definition 2 (Nash Equilibrium). a stable equilibrium in the dynamics of a strategic game, wherein each player possesses accurate expectations regarding the actions of others and makes rational decisions accordingly. [3] [9] In a strategic game, a strategy profile $s = (s_1, \ldots, s_n)$ constitutes a Nash equilibrium if, for every player i, the strategy s_i represents the optimal response to the strategies of all other players s_{-i} . [8]

Nash Equilibrium occurs when every player's strategy remains optimal considering the choices of others, leading to a stable outcome. In the Prisoner's Dilemma, despite mutual cooperation offering a superior outcome, each player's dominant strategy is defection, making reaching equilibrium challenging. Computational determination of Nash equilibrium in strategic games is facilitated through Python-based tools like Nashpy [10], enabling the definition and resolution of equilibria. Nashpy is a Python library tailored for game theory, specifically aimed at discovering Nash equilibria in two-player strategic games.

The following code (see Table 2) allows us to see that the Nash Equilibrium is achieved when both of the players either cooperate or defect:

```
import nashpy as nash
import numpy as np

# Create the game with the payoff matrix
g1 = np.array([[3, 3], [0, 5]])
# g1 is the row player
g2 = np.array([[5, 0], [1, 1]])
# g2 is the column player

# Form the game
game_1 = nash.Game(g1,g2)
game_1

# Find the Nash Equilibrium
equilibria = game_1.support_enumeration()
for eq in equilibria:
    print(eq)
```

Table 4. An example of a Nashpy code finding the Nash Equilibrium. Graciously provided by Professor Luyao Zhang for the students of CS/ECON 206 course.

The implications of the Prisoner's Dilemma extend beyond theoretical realms, permeating various disciplines including economics, politics, and psychology [9]. Serving as a powerful metaphor for real-world dynamics, this game illuminates the intricate interplay between individual incentives and collective outcomes.

Engaging with the Prisoner's Dilemma has enriched my comprehension of strategic interactions within complex systems. It underscores the pivotal roles of trust, cooperation, and communication in navigating scenarios where individual rationality intersects with broader societal interests. Through its nuanced exploration of human behavior and incentive structures, the Prisoner's Dilemma offers valuable insights applicable to diverse contexts and decision-making frameworks.

5.2 Delving into Extensive-Form Games

Definition 3 (Extensive-Form Games). a detailed description of the sequential structure of the decision problems encountered by the players in a strategic situation. There is perfect information in such a game if each player, when making any decision, is perfectly informed of all the events that have previously occurred. An extensive game with imperfect information differs in that the players may in addition be imperfectly informed about some (or all) of the choices that have already been made.

An extensive game has the following components:

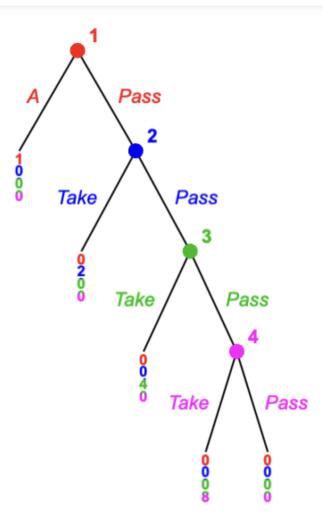
- A set N (the set of players).
- A set H of sequences (finite or infinite) that satisfies the following three properties:
 - 1. The empty sequence \emptyset is a member of H.
 - 2. If $(a_k)_{k=1,...,K} \in H$ (where K may be infinite) and L < K, $(a_k)_{k=1,...,L} \in H$.
 - 3. If an infinite sequence $(a_k)_{\infty k=1}$ satisfies $(a_k)_{k=1,\dots,L} \in H$ for every positive integer L then $(a_k)_{\infty k=1} \in H$.

Each member of H represents a history, where each component of a history is an action taken by a player. A history $(a_k)_{k=1,...,K} \in H$ is considered terminal if it is infinite or if there is no a_{K+1} such that $(a_k)_{k=1,...,K+1} \in H$. The set of terminal histories is denoted by Z. [9] [8]

The Centipede game, a classic example of an extensive-form game, captivates me due to its illustration of altruism/personal gain dynamics. In this sequential game, players must decide whether to continue passing or to seize a progressively increasing reward, balancing short-term gains against the risk of losing out on potential future payoffs (see Figure 4). Rosenthal original analysis underscores the strategic depth arising from anticipating opponents' moves and the complexities of timing and commitment. The game has regained popularity recently in the media through variations such as "take something, or double it and give it to the next person." This resurgence underscores its enduring relevance in illustrating strategic tension and psychological dynamics.

5.3 Critiquing Nash Equilibrium and Envisioning Innovations:

The scalability of Nash Equilibrium analysis to games with a higher number of players is a broader challenge within the field of game theory and strategic analysis. Nash Equilibrium itself struggles to provide actionable insights in games with a large number of players due to the exponential increase in complexity.



 ${\bf Fig.\,4.}$ Game Theory Explorer - Example of a Centipede Game.

This challenge extends to various analytical tools used in game theory, including Nashpy, as they rely on the foundational concepts of Nash Equilibrium for their analyses. Therefore, while the analytical tools may be useful for studying Nash Equilibria in simpler games or scenarios, the broader limitation of scalability to complex multi-player interactions is inherent to Nash Equilibrium theory itself, impacting all tools and methodologies based on it. The scalability challenge is particularly pronounced in the context of the prisoner's dilemma when multiple iterations involve more than two players. As more individuals are added to the game, the complexity increases exponentially, making it exceedingly difficult to compute Nash Equilibria analytically.

Another limitation of Nash Equilibrium is its assumption of complete information, which may not hold in many real-world scenarios. In practice, players often have imperfect or asymmetric information about their opponents' strategies, payoffs, or even the structure of the game itself. This asymmetry can lead to strategic uncertainty and deviations from equilibrium predictions as players make decisions based on their subjective beliefs or incomplete information. Consequently, traditional Nash Equilibrium analysis may fail to accurately capture the complexities of decision-making in situations of information asymmetry, highlighting the need for alternative modeling approaches that account for uncertainty and incomplete information. For instance, they might be uncertain about the likelihood of their partner defecting or cooperating. This asymmetry in information can lead to deviations from the Nash Equilibrium predictions, as players make decisions based on their subjective beliefs rather than objective knowledge.

To address these limitations, a new tool or conceptual framework could integrate advanced computational techniques with game theory principles. Reinforcement Learning within Game Theory offers a dynamic solution for modeling strategic interactions. Through allowing agents to learn and adapt their strategies over repeated interactions, we can grasp the evolving nature of decision-making in scenarios such as the prisoner's dilemma. Agents can employ strategies like tit-for-tat or forgiveness, leading to deeper analyses of long-term cooperation dynamics.

Moreover, Hierarchical Learning Architectures introduce an innovative framework for modeling multiplayer games. By organizing agents into hierarchical layers, we can capture strategic interactions at various levels of abstraction, accommodating the complexity of real-world interactions and enabling a more comprehensive understanding of strategic dynamics.

Additionally, Bayesian Game Theory provides a principled framework for addressing uncertainty and incomplete information. By modeling players' beliefs as probability distributions and updating them based on observed actions and outcomes, we can accurately capture strategic reasoning. Integrating Bayesian techniques into the analysis of the prisoner's dilemma allows for more realistic predictions of behavior and more informed decision-making.

These advancements not only improve scalability and adaptability but also capture the intricacies of real-world interactions, empowering decision-makers to

navigate strategic environments more effectively. I am excited about the potential of these approaches to push the boundaries of game theory and contribute to more effective decision-making in strategic contexts, as shown in Figure 5.



Fig. 5. Nash Equilibrium limitations and proposed solutions. Created with Markmap.

Bayesian (Subgame Perfect) Nash Equilibrium

Definition 4 (Subgame). In the context of a perfect-information extensive-form game, the subgame of G located at node h refers to the limited portion of G that includes only the descendants of h. The collection of subgames of G encompasses all such limited portions of the game rooted at various nodes within G. [8]

Definition 5 (Subgame Perfect Nash Equilibrium). According to Shoham and Leyton-Brown, Subgame-perfect equilibria (SPE) of a game G are defined as strategy profiles s such that, for any subgame G' of G, the strategy profile s, when applied within G', constitutes a Nash equilibrium of G'. According to Osborne and Rubinstein, a subgame-perfect equilibrium is characterized by a strategy profile s^* in Γ , where for any history h, the strategy profile s^* forms a Nash equilibrium of the subgame $\Gamma(h)$.

Theorem 1. Single-deviation Principle: In a multistage game with continuous play extending to infinity, a strategy profile qualifies as a subgame-perfect Nash equilibrium if, and only if, it successfully withstands the single-deviation test for each player at every stage. [12]

Proof. According to MIT OpenCourseWare, the single-deviation test assesses a strategy profile σ^* . At any stage, with fixed moves for other players as per σ^* , can a player improve their payoff by deviating from σ^* ? If so, σ^* fails the single-deviation test for that player at that stage. Failing this test for any player implies σ^* cannot be a subgame-perfect equilibrium, as it doesn't induce a Nash equilibrium in the subsequent subgame. Conversely, in a multistage game continuous at infinity, if σ^* passes the single-deviation test for every player at every stage, it qualifies as a subgame-perfect equilibrium.

Example 1. In the Centipede game, players take turns deciding whether to continue or stop, with each turn representing a stage. The single-deviation test assesses whether deviating from the equilibrium strategy at any stage would yield a higher payoff for a player. If a deviation leads to a higher payoff, the equilibrium strategy fails the single-deviation test, indicating it's not subgame-perfect.

Subgame-perfect Nash equilibrium is vital in understanding the limitations of Nash Equilibrium, as it offers a more stringent criterion that accounts for dynamic decision-making and strategic credibility over time. By requiring strategies to withstand deviations at each stage, subgame-perfect equilibrium reveals vulnerabilities in traditional Nash Equilibrium, particularly in sequential and repeated interactions. This distinction sheds light on the complexities of strategic behavior and equilibrium outcomes, offering insights into more realistic models and predictions in game theory.

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6 Game Theory Glossary Tables

Terminology	Definition	Author
	A type of game representation that lists	
Games	all players, their possible strategies, and	
	the resulting payoffs in a matrix or ta-	
	ble format. In this form, players make	
	decisions simultaneously without know-	
	ing the actions chosen by other players.	
Extensive-Form Games	A type of game representation that cap-	Selten
	tures the sequential nature of decision-	
	making by depicting the possible ac-	
	tions of players as nodes in a tree-like	
	structure. It includes information about	
	the order of play, possible choices at	
	each decision point, and payoffs asso-	
	ciated with different outcomes.	
Nash Equilibrium	A stable state in a game where each	Nash
Trash Equilisitani	player's strategy is optimal given the	Trabil
	strategies chosen by the other players,	
	and no player has an incentive to unilat-	
	erally deviate from their chosen strat-	
	egy.	
Sequential Games with	Games where players have complete in-	Rosenthal
Perfect Information	formation about previous actions and	TOSCITUITAT
Cricce imormation	outcomes, allowing for the analysis of	
	optimal strategies.	
Bayesian Games	Games in which players have private in-	Harcanyi
Dayesian Games	formation that affects their payoffs or	liaisanyi
	_ ~	
Utility Theory	beliefs about the game. A theory that models individual prefer-	Normann and Man
Ctility Theory		
	ences or choices in terms of utility func-	~
	tions, which assign numerical values to	
	outcomes or alternatives based on their	
7 0 0	desirability or satisfaction.	NT 1 NA
Zero-Sum Games	Games where the total utility or payoff	
	is constant, meaning one player's gain	genstern
	is exactly balanced by another player's	
) () ([D]	loss.	NT 1.3.5
Minimax Theorem	A fundamental result in game theory	
	that states that in zero-sum games with	genstern
	perfect information, there exists a value	
	known as the minimax value, and op-	
	timal strategies for both players that	
T 1 - T 2	achieve this value.	
Incomplete Information	A situation in game theory where play-	Harsanyi
	ers have imperfect or incomplete knowl-	
	edge about the game environment, in-	
	cluding the strategies, payoffs, or pref-	
	erences of other players.	

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