ECON 206 Problem Set 2

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Abstract. This article presents some of the author's reflections to topics in game theory discussed in the course ECON 206.
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1 Introduction

Previous frameworks in game theory only consider rational/partially rational agents who assume their counterparts to be modellable in a normal sense as well. With the appearance of AGIs which will have their own ways of thinking, however, the decision of conventional agents (humans or organizational entities) might also be based on their perception of whether the opponent is silicon-based. For instance, in a hawk-dove game which has a partially ethical framing, I predict that AGIs might be more inclined to the dove strategy from alignment with values such as altruism during the training process, which means that their opponent agents should be more inclined towards a hawk strategy. Hence, I propose to additionally study the inclinations of AGIs and see how to detect if one is present in the game (as shown in Figure 1), and subsequently find how to maximize utility in those games. This is equivalent to adding a model-based additional step in decision trees in games, similar to guessing games, which incorporates prior information.

2 Background

I envision that as AI becomes more powerful and our understanding of human behavior increases, we can use add a third component to game theory, which I call Doppelgänger Agents. Doppelgänger Agents are based on AGI systems that are conditioned upon the game environment and predicted human actions, and are updated during each iteration based on ground-truth observations of

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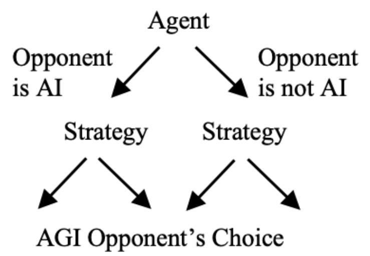

Fig. 1. Illustration of split decision tree based on perception of opponent.

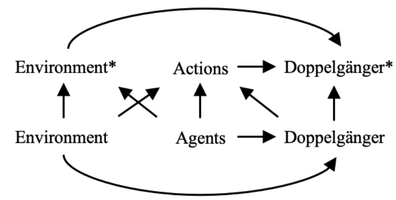

Fig. 2. Illustration for the concept of Doppelgänger agents.

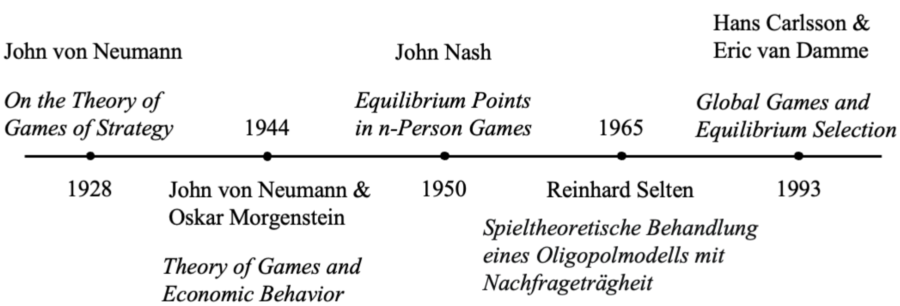

Fig. 3. Timeline for game theory history.

the human agents. AI agents can similarly have their own Doppelgänger Agents based on simulations that indicate the agent's tendencies. Predictions of future reactions are then forwarded back to regular Agents, which then affects their decisions, and the iteration continues, as shown in Figure 2. This will enable the effective incorporation of information into the game theoretic system and thus encourage the formation of more realistic modelling and more rational decisions.

A The Pioneers in the History of Game Theory

A timeline for pioneers in the history of game theory is shown in Figure 3. The referenced papers are cited as [1, 2, 3, 4, 5].

B Review Classic Games, Nash Equilibrium and the Analytical Tools

Definition of subgames Subgames are smaller parts of a game, represented as subtrees of decision trees in extensive-form games. Some Nash equilibria require threats, and when a threat does not reach a Nash equilibrium for a subgame, the threat is referred to as incredible. Formally, subgames are defined as follows, as described by Shoham and Leyton-Brown [6]:

Definition 1. *Given a perfect-information extensive-form game G, the subgame of G rooted at node h is the restriction of G to the descendants of h. The set of subgames of G consists of all of subgames of G rooted at some node in G.*

Subgame-perfect equilibria and their existence Given Definition 1, we can proceed to define the concept of a subgame-perfect equilibrium as follows [6]:

Definition 2. *The subgame-perfect equilibria of a game G are all strategy profiles s such that for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G.*

By Kuhn's theorem (proposed in Kuhn [7]), presented as follows, we can prove the existence of subgame-perfect equilibria [8]:

Theorem 1. *Every finite extensive game with perfect information has a subgame perfect equilibrium.*

Proof. By the one-deviation property one can inductively define a strategy profile by going back to each previous endpoint in the history and deducing future subgame strategies.

B.1 Exploring Inspirational Games in Strategic or Normal Form

The Colab link[1] is an implementation of Chicken (as illustrated in Figure 4) based on Nashpy. As I have described in class, Chicken is a very classic game, and is equivalent to the dove-hawk game, where the best strategy is the opposite of the opponent's strategy. The players can either go or swerve, and if both go they die in a car crash. If both swerve, nothing happens. If one swerves, he will be teased as a coward by the other. This game is significant because it is a very classical competitive game, and also because it does not have a fixed-strategy Nash equilibrium. It has very widespread implications for ecology [9], political science [10], behavioral science [11], fuzzy mathematics [12], *etc.*

B.2 Delving into Extensive-Form Games

The Colab link[2] is an implementation of the stripped-down poker game (as illustrated in Figure 5) from Reiley, Urbancic, and Walker [13] using PyGambit. Alice gets dealt a card that has a 50-50 probability of being a King or a Queen. Alice knows what the card is but Bob doesn't. First Alice, then Bob, can choose to bet or fold. If a player folds the other immediately wins, but if both bet,

[1] *https://colab.research.google.com/drive/1oCoQZipFgvCJ1yTYAWV8p4tegxJqT2Cf?usp=sharing*

[2] *https://colab.research.google.com/drive/11JLpW1Ab5MKqNW6rnB25RSiAq3R-5XYD?usp=sharing*

Fig. 4. Decision tree for the Game of Chicken.

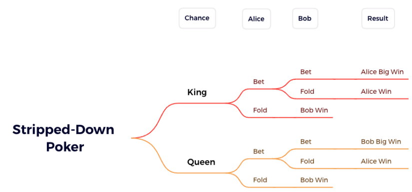

Fig. 5. Decision tree for the Game of Stripped-Down Poker.

Alice wins big (twice the pot) with a King and Bob wins big with a Queen. Hence Alice should obviously always bet when she gets a King, but can choose to "bluff" (bet) when she has a Queen to fool Bob into folding.

PyGambit correctly gives the Nash Equilibrium strategy to always bet with a King and bluff with a 1/3 probability with a Queen. This game is extremely simple but insightful because it shows how signaling (Alice's choice) can matter in a game with hidden information.

B.3 Critiquing Nash Equilibrium and Envisioning Innovations:

The Nash equilibrium only considers a fixed strategy independent of previous choices. Furthermore, the tools used for computing it do not contain tools for storing the "state" of a game. A game (interestingly an extensive-form game) that illustrates both insufficiencies is the Gunboat (also called No-Press) variant of the board game Diplomacy, which is a chess-like turn-based correspondence game that can be modeled as an extensive-form game (as in Figure 6). Since each turn's moves are simultaneously determined, Nash equilibria often occur.

For instance, very commonly occurring scenarios known as "guessing games" in Diplomacy jargon can be seen as the matching pennies game, where if the defending troop and the attacking troop target the same region then the attack fails, and if they target different regions the attack succeeds. Similarly, cooperation between countries in a military operation often can be modeled as a battle of the sexes, where if both countries choose "move" or "support" the operation fails, but if one chooses "move" and the other chooses "support" the operation is successful, and the one which chose "move" gets to occupy the targeted region.

However, while constructing trees of possible moves is plausible, it would be much more efficient to save the board's state and use that to find equilibria (as in Figure 7). That does not apply to most current game theoretical tools. Hence I wish to develop a game theoretical tool that could save a representation of the current state. There would be a programmable board representation, which can be used as a factor for determining the policy. Gunboat Diplomacy, for instance, can be modeled as a connected graph, where each vertex is a region, adjacent regions are connected by an edge, and each military force is characterized as a set of attributes on the vertex. My experience with coding and knowledge of game theory would help with constructing such a system.

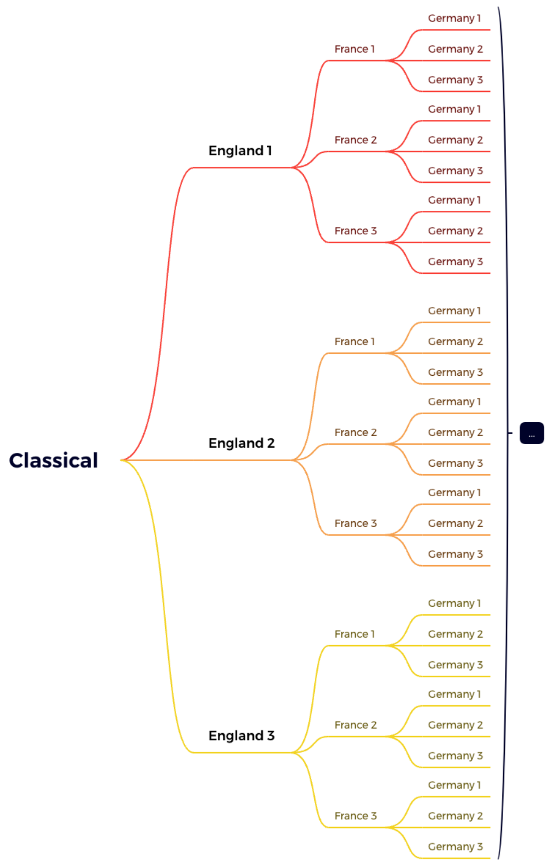

Fig. 6. Tree for Gunboat Diplomacy in classical fashion.

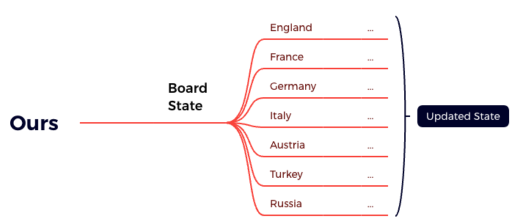

Fig. 7. Tree for Gunboat Diplomacy with states added

C Game Theory Glossary Tables

Table 1. Glossary

Term	**Definition**
Trembling Hand Perfect Equilibrium	An equilibrium that assumes the players have a small but nonzero probability of making suboptimal moves inconsistent with their policy. [14]
Congestion Games	Games where players choose between different sets of limited resources, and where choosing one set would have negative impacts on the utility of players making the same choice. [15]
Strong Nash Equilibrium	An equilibrium in cooperative games where no coalition can be formed to increase the utility of all its members simultaneously. [16]
Oddness of Nash Equilibria	The phenomenon that any game, given minor perturbation of the utilities, has an odd number of Nash equilibria. [17]
Non-credible Threat	A threat in a policy that is not subgame-perfect. [18]

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