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Dynamic trust game model between venture capitalists and entrepreneurs based on reinforcement learning theory

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Abstract
It has been extensively demonstrated that the trust between venture capitalists and entrepreneurs plays an important role in improving the success rate of entrepreneurship. However, little research has been performed concerning the dynamic nature of the trust between venture capitalists and entrepreneurs. Therefore, we attempt to reveal the motivations and process of trust between VCs and entrepreneurs from the perspective of dynamics. First, the theoretical model of dynamic trust was proposed. Second, reinforcement learning was introduced to design the multi-stage game model of dynamic trust, and the algorithm was illustrated. Reinforcement learning theory can reveal the reasons for dynamic changes in trust from a psychological perspective. Finally, the action strategies of venture capitalists and entrepreneurs were simulated. We observe that the dynamic characteristics of trust between venture capitalists and entrepreneurs are determined by entrepreneurs' efforts. Venture capitalists can prevent investment risks and maintain trust stability when fewer funds are invested in the early stage and when more funds are invested in the later stage. Since the dynamic process of trust is also the process of learning about the other, the dynamic process of trust is non-linear, and the relationship is adaptive.

Keywords Dynamic trust · Game model · Reinforcement learning · Venture capitalists

1 Introduction

The cooperation between venture capitalists (VCs) and entrepreneurs has been a heavily researched topic since Timmon and Bygrave suggested that the relationship between VCs and entrepreneurs was more important than money [1]. Numerous studies have confirmed that trust between VCs and entrepreneurs was important in the establishment and maintenance of partnerships and contributed to the transformation of partnerships from economic transactions to long-term relationships [2]. Venture capital was an important source of financing for start-ups, and trust played a key role in uncertain risks [3]. Investors who could provide the necessary capital face the same immeasurable uncertainty as the entrepreneur, and additionally, investors cannot rationally assess the skills, characteristics, and judgement of the entrepreneur. Many researchers have investigated trust as a critical factor in decision-making in various fields [4,5]. VCs who invested in start-ups faced high uncertainty and opportunism, since they usually receive limited information. Therefore, VCs are often interested in soft information, such as trust. Trust as a key informational institutional factor that affects information transmission and cooperation [6,7]. Investors and entrepreneurs meet, interact, and work together [8]. Therefore, interaction between them plays a key role in investments. Trust was expressed when an individual transferred his own resources to others and accepted the risks [9]. From the definition of trust, we can see that trust is often accompanied by risks and uncertainties. The cooperation between VCs and entrepreneurs faces greater risks and vulnerabilities due to technical and market uncertainties. Therefore, it is essential to evaluate the trust between VCs and entrepreneurs to reduce the risks of cooperation.

The research on the relationship between VCs and entrepreneurs is focused on the factors that influence the formation of trust and the impact of trust on entrepreneurial performance. The credibility of the trustee is assessed using three types of social information, including ability, goodwill, and integrity [10]. Initial trust between VCs and entrepreneurs is affected by cultural factors, such as nationality and location [11,12]. Cross-case analysis was adopted

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to analyse asymmetric trust between VCs and entrepreneurs, and the reasons for asymmetric trust were proposed [13,14]. The effect of trust on entrepreneurial performances has also attracted considerable attention. Trust between VCs and entrepreneurs has a positive impact on the performances. Trust can reduce transaction costs and promote relational investments, knowledge sharing and integration of complementary resources. Trust can reduce the search costs in social networks caused by the asymmetric information between VCs and entrepreneurs, which is beneficial for VCs making investment decisions and the establishment of cooperation. The game model is used by economics scholars to study the decisions of VCs.The study revealed that VCs adjust their decisions based on the evaluation of returns. The trust game model was used to discuss the financing mechanism of the start-ups, and the result showed that trust as social capital played a positive role in obtaining VCs' investments [15]. The game model combined with principal-agent theory was used to study the investment decisions of VCs [16,17]. The results revealed that VCs adjust their investment decisions according to the evaluation of returns. Wong et al. [18] and Sudek [19] find that trust is an important determinant of investment decisions in business angel financing.

From the above analysis, we can learn that there is little research regarding dynamic trust and game models that address the dynamic and uncertain trust between VCs and entrepreneurs. A simple trust game cannot explain how VCs and entrepreneurs adjust trust behaviours based on perceived trust. In addition, the simple trust game is a general model that does not consider the actual situation of entrepreneurship. What are the motivations for the dynamic change of trust between VCs and entrepreneurs in the context of entrepreneurial collaboration? What is the process of dynamic trust? Such questions are infrequently discussed by previous works, and the answers to them are of great significance for long-term cooperative relationships between VCs and entrepreneurs.

To solve the above questions, a theoretical model of dynamic trust between VCs and entrepreneurs was drawn from previous works. Then, reinforcement learning and game theory were combined with VC's staged investments to design a trust dynamic game. This paper both calculates the optimal action strategy and introduces the reinforcement learning theory to explain how VCs and entrepreneurs adjust their trust states and determine their strategies in the trust game. Therefore, we can have an in-depth comprehension of the dynamic trust between VCs and entrepreneurs.

We contribute to the research in the following ways. First, we develop a multi-stage trust game model between VCs and entrepreneurs. Although previous studies have examined the trust relationship between VCs and entrepreneurs, little research has investigated the dynamics of trust. Second, reinforcement learning theory is introduced to design the dynamic trust game between VCs and entrepreneurs based on the actual situation of entrepreneurial cooperation. To the best of our knowledge, there is little research that has combined learning theory with the dynamic trust game in the field of entrepreneurship. Reinforcement learning theory can reveal the reasons for dynamic changes in trust from a psychological level. The simulated results of the model are similar to cases in entrepreneurial cooperation, indicating that the model can explain the motivation and process of dynamic trust between VCs and entrepreneurs. Finally, we will provide the optimal action strategy for the maintenance and development of trust and explain how to make decisions in the trust game.

2 Related works

2.1 Reinforcement learning theory

Psychologists suggest that learning in social interactions relies on a basic mechanism of reinforcement learning. Rescorla and Wagner proposed a classic reinforcement learning model in 1972 that argued that the positive results of previous learning would lead to expectations of positive outcomes [20]. Reinforcement learning states that the agent continuously adjusts the action and updates the decision according to the rewarding signal issued by the interaction with the environment, and ultimately, the maximum values of the objective function are obtained [21]. Reinforcement learning theory illustrates how learning occurs. The gap between reality and expectations (an expected reward error) provides a learning signal [22,23].

In addition to the agents and the environment, the system of reinforcement learning also includes the following basic elements: the environmental model (non-essential), the reward function, the state function and the strategy [22]. The reward function refers to the positive or negative feedback from the environment when the agent interacts with it and determines whether the feedback is a reward or punishment. The state function is the accumulation of the reward functions. The state value is affected by both instantaneous returns and the experiences from prior interactions. The purpose of the state function is to obtain the global optimal solution and avoid falling into the local optimum. The state function can lay the foundation for future decision-making.

The reinforcement learning algorithm is a computational model that studies how future benefits are maximized by actions, including free parameters, such as learning rates or temperature parameters (describing the degree of individual inclinations to explore the potential strategies that may yield more benefits or the tendency to take advantage of the original strategies). The time difference algorithm uses the iterative function method in which the reward error provides the learning signal [24].

$$\delta_t = [reward_{t+1} + \gamma \times V(state_{t+1})] - V(state_t)$$

where δ_t is the expected deviation. $reward_{t+1}$ refers to the instantaneous gains at stage $t + 1$. γ is the discount factor. $V(state_{t+1})$ is the estimated value at stage $t + 1$. $V(state_t)$ is the estimated value at stage of t.

$$V(state_t) \quad + \partial \times \delta_t \rightarrow V(state_{t+1})$$

The agent updates the estimated values of the previous states according to the expected deviation. ∂ is the learning rate, which indicates the effect of the previous state on the current state. The instantaneous return and the expected future value are included in estimated values of a given state when the temporal difference algorithm is used.

The Q-learning algorithm proposed by Watkins is based on the temporal difference algorithm, which is a model-independent algorithm of reinforcement learning. The agent relies on experiences learned from past to make decisions [25,26]. $Q : S \times A \rightarrow R$, $Q(s, a)$ is the expected return when the agent performs the action a under state s . The agent selects the optimal strategy based on the maximum value of Q obtained from the current state, and the value of Q includes the instantaneous gain and expected future value.

2.2 Dynamic trust game

Trust involves interpersonal interaction strategies, dependent relationships, and game theory. Game theory studies the interactions of strategies and offers an alternative perspective to profoundly reveal the important role and production mechanism of trust. Therefore, research on trust based on game theory is quite common in sociology, economics, politics, and other disciplines. Berg first utilized the trust game to study trust behaviours among decision makers [27]. Evolutionary games can theoretically observe the dynamic process of trust. Repeated trust games (RTGs) allow us to empirically observe trust-based dynamics [28]. Chang argues that while the trustor can quickly assess the credibility of trusted parties, decision making in social relationships often requires a repetitive interaction in order to learn the individual's reputation [29]. Trust is essentially reciprocal, and the formation of trust between partners is a process of learning from each other. The entrepreneur makes a decision based on judgement but also the investor. The entrepreneur who is trusted has both good intentions and abilities, making it likely that the investment will generate a positive return. VCs' investment is a form of risk capital, and trust is an essential element of the VC-investment process [6]. Trust is also crucial to the investment decisions of VCs. Trust as a key informal institutional factor affects information transmission and cooperation. Trust is a way of forming expectations and a heuristic decision rule allowing investors to address the true uncertainty regarding the outcomes of funding start-ups [18]. Trust between subjects is a dynamic process that presents different characteristics at different stages of the relationship [30].The trustor determines the trustworthiness of the trustee in accordance with their interaction result with the trustee [31]. The information with which the trustor judges the trustworthiness of the trustee changes with the interactive process [32]. Economics scholars believe that investors, when cooperating with partners, constantly adjust their trust decisions based on the feedback from the cooperation. Therefore, trust is established during cooperation. Klabunde [27] explore the effects of trust between VCs and entrepreneurs on the success rate of entrepreneurship based on the evolutionary game model. These researchers conclude that trust behaviour of VCs on entrepreneurs expresses different features at different stages of enterprises. Trust is formed during direct interactions between VCs and entrepreneurs, and VCs adjust trust decisions based on entrepreneurs' efforts [33]. Berg et al. argued that both the expected share of the entrepreneur in the firm and the expected return of the VCs decreased over time [34]. The VCs provide the financial support to the start-up; therefore, the entrepreneurs have the obligation to make returns for the VCs. The VCs determined the credibility of the entrepreneurs according to their efforts, and entrepreneurs adjusted the trust status based on the VCs' investment at each stage [35,36]. The trust between VCs and entrepreneurs is formed by observing the strategies of the other's actions and the results of previous actions. VCs and entrepreneurs update their trust state and trust is established with steady gains. The cooperative relationship is gradually formed in the process of mutual learning [37]. Adjustments in the dynamic strategy and evolutionary stability are obtained through the repeated game. We propose a theoretical model of dynamic trust between VCs and entrepreneurs in Fig. 1 based on the previous works.

3 Model

3.1 Principles of the model

Reinforcement learning has been widely used to explain behavioural adaptation and the assessment of neural rewards. Reinforcement learning theory illustrates how learning occurs, showing that the deviation between actual results and expected results provide a learning signal [23,25]. Using mathematical estimation techniques, a set of parameter values is derived for each participant such that the model's performance best mimics the participant's actual values.

Investors can make profits from investments, but they will suffer losses if other participants do not follow the rules and do not keep their promises. Therefore, trust is risky and uncertain. Investors should expect such risks in economic

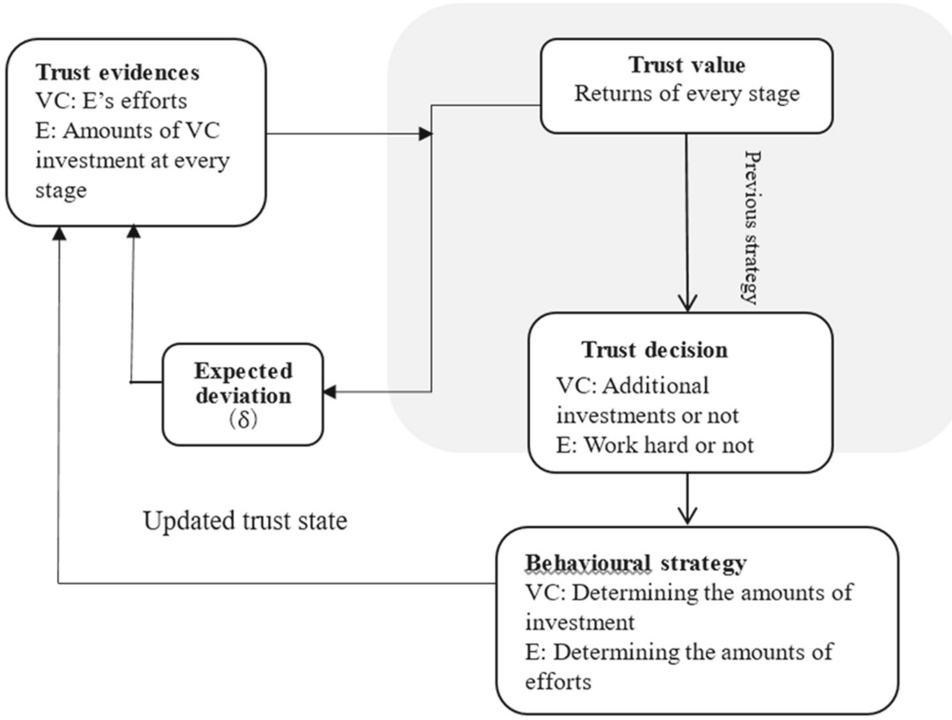

Fig. 1 Theoretical model of the dynamic trust between VCs and entrepreneurs

transactions and assess the other's credibility through different strategies. There is no requirement that participants are rational or that there is complete information on the conditions of the evolutionary game. The evolutionary game combines game theory with the analysis of the dynamic process.

The mathematics model of reinforcement learning explains how the agent makes the decision and the computational process of the agent in social decision-making (Chang [28]). The agent makes the optimal decision through repeated trial-and-error with the environment. Reinforcement learning theory emphasizes the active learning of the agent and is suitable for solving complex and dynamic decision-making problems. Individuals have limited but not complete rationality in the evolutionary game and need to learn through trial-and-error to achieve the game's equilibrium. The above process belongs to the dynamic problems of decision-making. Therefore, we adopt the reinforcement learning theory to explore the dynamic game of trust between VCs and entrepreneurs and reveal the evolutionary mechanism.

The agents, strategy sets, and spending functions are usually included in the dynamic game model. $\langle S, B_1, B_2, V_1, V_2, P\rangle$ is the strategy model for VCs and entrepreneurs in the dynamic trust game. S is the state space. VCs adjust the trust states according to the returns created by entrepreneurs' efforts. Likewise, entrepreneurs adjust their trust states according to additional investments.$\langle B_1, B_2\rangle$ is the action space. V_1 is the returns of VCs at every stage. V_2 is the returns of entrepreneurs at every stage. $S, B_1, B_2, \rightarrow V_1, V_2$ are the gains earned by VCs and entrepreneurs at every stage as determined by the state S and the actions of B_1, B_2. P is the probability of state transitions, which is used to measure the extent of the results of previous actions performed in the current state. The VCs determine the new action B_1', depending on whether entrepreneurs' efforts have led to an increase in V_1.The entrepreneurs determine the new action B_2' according to whether V_2 increases based on the VCs' investment. $\langle B_1', B_2'\rangle$ is the new set of actions, and the state S is transferred to S'.

3.2 Model design

We design the dynamic trust game model between VCs and entrepreneurs by combining the staged investment behaviour of VCs. Additionally, a learning model that matches the game model is given. VCs and entrepreneurs assess the credibility of each other based on the returns received at each stage. $EV(t)$ is the expected return of VCs at the stage $t.EW(t)$ is the expected return of entrepreneurs at stage $t.EW(t)$ and $EV(t)$are used to calculate the trustworthiness of VCs and entrepreneurs at stage t.

$$EV(t) = P_v(t)*V(t) \tag{1}$$

$$EW(t) = P_w(t)*W(t) \tag{2}$$

$$V(t) = \theta_t(e_t)(R_t - S_t + V_{t-1}) - [1 - \theta_t(e_t)F_t] \tag{3}$$

$$W(t) = \theta_t(e_t)(S_t + W_{t-1}) + [1 - \theta_t(e_t)F_t] - C(e_t) \tag{4}$$

where e_t is the effort of entrepreneurs and is difficult to observe. Therefore, the growth rate of the firm's total revenue ($\theta_t(e_t)$) as proposed by Klabunde, is used to measure the extent of entrepreneurs' efforts [27]. $\theta_t(e_t)$ is the function of the effort and is directly proportional to the extent of effort. $\theta_t(e_t)$ is defined as $\theta_t(e_t) = ae_t, a \in [0, 1], \theta_t(e_t) \in [0, 1]. V(t)$ is the total revenue of VCs in phase $t. S_t$ is the additional investments provided by VCs in the phase $t. V_{t-1}$ is the returns of VCs in the phase $t-1. W_{t-1}$ is the returns of entrepreneurs in the phase $t-1. F_t$ is a constant that denotes the returns stipulated by the initial contract for entrepreneurs when the growth rate of the firm's total revenues decreases in phase $t. C(e_t)$ is the costs of entrepreneurs' efforts. $P_v(t)$ and $P_w(t)$ are the probabilities of the state transition for $V(t)$ and $W(t)$.The maximum value of $V(t)$ is the objective of VCs' decisions.

$$\begin{aligned} Max\,V(t) &= \theta_t(e_t)(R_t - S_t + V_{t-1}) - [1 - \theta_t(e_t)Fe_t] \\ &s.t \quad R_t + V_{t-1} > 0, \quad t = 1, 2, ...t, S_t \geq 0 \end{aligned} \tag{5}$$

The maximum value of $W(t)$ is the objective of entrepreneurs' decisions.

$$\begin{aligned} Max\,W(t) &= \theta_t(e_t)({}_tS + W_{t-1}) + [1 - \theta_t(e_t)F_t] - C(e_t) \\ &s.t \quad t = 1, 2, ...t, e_t \geq 0 \end{aligned} \tag{6}$$

The following equations are obtained from (6).

$$\frac{dW(t)}{de_t} = \theta_t'(e_t)(S_t + W_{t-1}) - \theta_t'(e_t)F_t - C'(e_t) = 0$$

$$\theta_t'(e_t)(S_t + W_{t-1} - F_t) = C'(e_t) \tag{7}$$

$$e_t^* = S_t + W_{t-1} - F_t \tag{8}$$

The result from Eq. (7) is that the optimal investments S_t^* of VCs should meet the condition that the marginal expected returns equal the marginal costs of entrepreneurs' efforts. We conclude from Eq. (8) that entrepreneurs determine the optimal level of efforts based on the investments from the current phase and accumulated returns from previous actions.

3.3 Calculating probability of state transition

The probability of receiving rewards at each stage is uncertain, since the VCs' and entrepreneurs' action strategies in the current state are affected by the cumulative returns of previous actions. The transitional probability can be calculated utilizing the Rescorla–Wagner rule from reinforcement learning theory. The rule explores the learning process with a signal (δ) of anticipatory error. The anticipatory error is the difference between the result (R) and the expected result (V) of the previous stage. $\delta = R - V\delta = R - V$, $P_{t+1} = P_t + \partial_t * \delta_t$ .The agent updates their expected values of the following results with different learning rates depending on the error signal.

$$P_{v(t)} = \begin{cases} P_{v(t-1)} + \partial_v \left[1 - P_{v(t-1)}\right] & \text{entrepreneurs}' \\ & \text{efforts have increased} \\ P_{v(t-1)} - \partial_v \left[1 - P_{v(t-1)}\right] & \text{entrepreneurs}' \\ & \text{efforts have decreased} \end{cases}$$

$$P_{w(t)} = \begin{cases} P_{w(t-1)} + \partial_w \left[1 - P_{w(t-1)}\right] & \text{VCs have} \\ & \text{invested additional funds} \\ P_{w(t-1)} - \partial_w \left[1 - P_{w(t-1)}\right] & \text{VCs do not} \\ & \text{invest additional funds} \end{cases}$$

∂_v and ∂_w are the learning rates of VCs and entrepreneurs, respectively. VCs and entrepreneurs adjust their actions according to interaction results in the trust game; therefore, the learning rates vary over time. The probability distribution of Boltzmann is used to calculate the probabilities of state transition.

The VCs' probability of state transition in phase t is,

$$\mathrm{Pr}_{v(t)} = \frac{\boldsymbol{e}^{EV(t)}/\beta_v}{\boldsymbol{e}^{EV(t)}/\beta_v + \boldsymbol{e}^{1/\beta_v}} \tag{9}$$

The entrepreneurs' probability of state transition in phase t is,

$$\mathrm{Pr}_{w(t)} = \frac{\boldsymbol{e}^{EW(t)}/\beta_w}{\boldsymbol{e}^{EW(t)}/\beta_w + \boldsymbol{e}^{1/\beta_w}} \tag{10}$$

β_v and β_w are temperature parameters ranging from 0 to 1 that determine the tendency to repeat previously-rewarded interactions with a partner (exploit) versus trying alternate actions that might result in greater payoffs (explore). Higher values of β_v and β_w bias the system to explore and lower values bias it to exploit.

3.4 Procedure of algorithm

F_t is a constant. We set $F_t = 0, C(e_t) = e_t^2, V(0) = 0, W(0) = 0$. R_t is the total return of VCs in phase t. It is a constant and has no effect on the results of model. Setting $R_1 = R_2 = R_3 = ...R_t = 1$ is convenient for the analysis. Therefore, Eqs. (3), (4) and (8) are presented in the following form.

$$V(t) = ae_t(1 - S_t + V_{t-1}) \tag{11}$$

$$W(t) = ae_t(S_t + W_{t-1}) - e_t^2 \tag{12}$$

$$e_t^* = \frac{a}{2}(S_t + W_{t-1}) \tag{13}$$

The recursive equations of e_t^* are

$$e_1^* = \frac{a}{2}S_1$$
$$e_2^* = \frac{a}{2}S_2 + \frac{a^3}{8}S_1^2$$
$$e_3^* = \frac{a}{2}S_3 + \frac{a}{8}\left(aS_2 + \frac{a^3}{4}S_1^2\right)^2$$
$$e_4^* = \frac{a}{2}S_4 + \frac{a}{8}\left[aS_3 + \frac{a(aS_2 + \frac{a^3}{4}S_1^2)^2}{4}\right]^2$$
$$e_t^* = \frac{a}{2}S_t + \frac{a}{8}\left\{aS_{t-1} + \frac{a(aS_{t-2} + \frac{a^3}{4}S_{t-3}^2)^2}{4}]^2\right\}$$

The recursive equations of $V(t)$ are

$$\begin{aligned}
V_1 &= ae_1^*(1 - S_1)\\
V_2 &= ae_2^*(1 - S_2) + a^2e_2^*e_1^*(1 - S_1)\\
V_3 &= ae_3^*(1 - S_3) + a^2e_3^*e_2^*(1 - S_2) + a^3e_3^*e_2^*e_1^*(1 - S_1)\\
&\vdots\\
&\vdots\\
&.\\
&.\\
V_t &= ae_t^*(1 - S_t) + a^2e_t^*e_{t-1}^*(1 - S_{t-1})\\
&\quad + a^3e_t^*e_{t-1}^*e_{t-2}^*(1 - S_{t-2}) + \ldots\\
&\quad + a^te_t^*e_{t-1}^*\ldots e_3^*e_2^*e_1^*(1 - S_1)
\end{aligned}$$

The optimal investments of VCs at each stage are calculated. The optimal amounts $(S_1^*, S_2^*, S_3^*\ldots S_t^*)$ invested by VCs can be calculated when the recursive expressions of $(e_1^*, e_2^*, e_3^*\ldots e_t^*)$ are plugged into $V(t)$.

The optimal efforts of entrepreneurs at every stage are calculated. The extent of optimal effort can be calculated when the equations $(S_1^*, S_2^*, S_3^*\ldots S_t^*)$ are plugged into equations $(e_1^*, e_2^*, e_3^*\ldots e_t^*)$.

The returns $V(t)$ and $W(t)$ are calculated. The values of $V(t)$ and $W(t)$ can be calculated by plugging the mathematical expressions S_t^* and e_t^* jinto Eqs. (11) and (12).

Next, the state transition probabilities Pr $v(t)$ and Pr $w(t)$ are calculated. $V(t)$ and $W(t)$ are plugged into Eqs. (8) and (9). The logarithmic likelihood is used to estimate the parameters β_v and β_w of the learning model in Matlab, and the computational process is consistent with the action strategies (S_t and e_t) every stage.

Akaike information criterion (AIC) is used to evaluate the matching between model parameter settings and action strategies. AIC can measure whether the numbers of model parameters match the action strategies of the agents. The smaller value of AIC means that the matching between the parameters of the learning model and action strategies is better, and the decision-making behaviours of agents can be effectively explained by the parameters.

$$LLE = \sum_{t=1}^{k} \log\left(\Pr_{v,j(t)}\right) \tag{14}$$

$$LLE = \sum_{t=1}^{k} \log\left(\Pr_{w,j(t)}\right) \tag{15}$$

$$AIC = 2K - 2LLE \tag{16}$$

K is the numbers of parameters in the learning model. There are 4 parameters in this learning model (∂_v, ∂_w, β_v and β_w).Therefore,

$$AIC = 8 - 2LLE$$

4 Simulation of the model

A numerical simulation is carried out to reveal how the additional investments (S_t) and entrepreneurs' efforts (e_t) affect the dynamics of trust between VCs and entrepreneurs. We also estimate the values of the learning model parameters to explain the optimal strategies of VCs and entrepreneurs and reveal the evolutionary motivation and process of trust. The initial value of t is 1 and the simulated period is 1000 time units.

4.1 Estimation of the learning rates (∂_v and ∂_w) and temperature parameters (β_v and β_w)

The optimal action strategies (S_t^*, e_t^*) and returns of VCs and entrepreneurs at each stage can be calculated from the above game model. We assume to $a = 1$ estimate the parameters of the learning model to explain the strategies of VCs and entrepreneurs. The loglikelihood method is used to estimate the values of the learning model parameters in Matlab based on the above algorithm. The parameters (∂_v, ∂_w, β_v and β_w) are included in the general learning model.

The results of Table 1 and Fig. 2 show that the values of VCs' learning rate (∂_v) and temperature parameter (β_v) when entrepreneurs reduce their efforts are greater than

Table 1 Values of parameters (∂_v, ∂_w, β_v and β_w)

Parameters	∂_v	∂_w	β_v	β_w	Pr $v(t)$	Pr $w(t)$
$S_t^* > S_{t-1}^*$	0.15	0.36*	0.21	0.27	0.18	0.16
$S_t^* < S_{t-1}^*$	0.11	0.52*	0.19	0.46	0.12	0.42
$e_t^* > e_{t-1}^*$	0.37*	0.09	0.23	0.27	0.24	0.23
$e_t^* < e_{t-1}^*$	0.63*	0.12	0.45	0.15	0.57	0.28

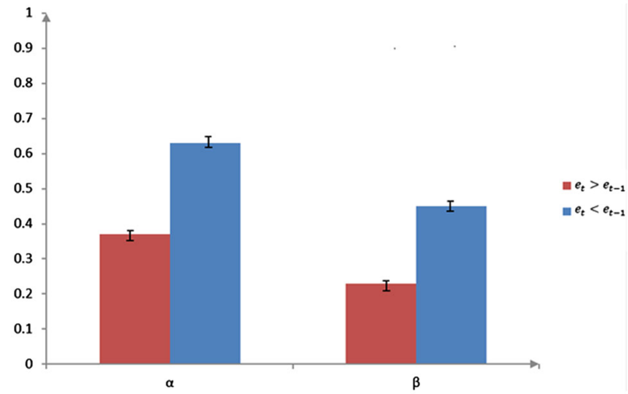

Fig. 2 Learning rate of VCs under the condition that entrepreneurs' efforts have increased or decreased

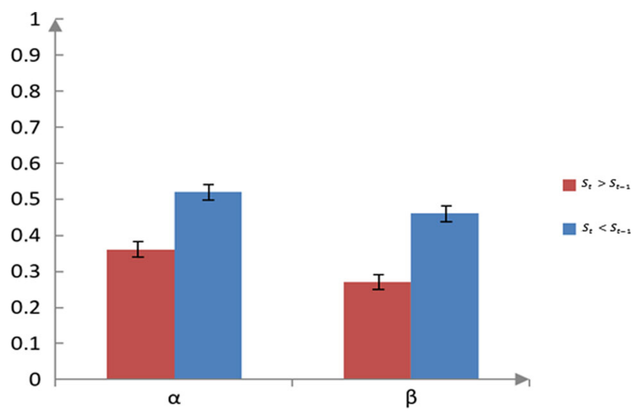

Fig. 3 Learning rate of entrepreneurs with an increase or decrease in investment

when entrepreneurs increase their efforts. The results demonstrate that VCs are more sensitive to the situation that the growth rate of earnings has declined caused by entrepreneurs' reduced efforts. In the situation, VCs tend to adopt new strategies and are less affected by the previous action strategies. Therefore, strategy selection is random.

The results of Table 1 and Fig. 3 show that the values of entrepreneurs' learning rate (∂_w) and temperature parameter (β_w) when VCs reduce their investments are greater than the values when VCs increase their investments. The results show that entrepreneurs are less sensitive to VCs' investment increases. Entrepreneurs are unlikely to update their original strategy when VCs' increase their investments in the early stage. Entrepreneurs' strategies are greatly influenced by the previous strategy results. Therefore, the original strategy is often adopted.

Table 2 shows that the AIC value of the learning model (∂_v, ∂_w, β_v and β_w are free parameters) is minimal. The AIC value of the model without the learning behaviour (∂_v, ∂_w, β_v and β_w are fixed) is maximal. The lower value of AIC shows that the number of model parameters is more reasonable. The results confirm that the action strategies of VCs

Table 2 Values of AIC in different learning models

Learning models	AIC
∂_v, ∂_w, β_v and β_w (all fixed)	59.37
∂_v, β_v (free), ∂_w, β_w (fixed)	46.18
∂_w, β_w (free), ∂_v, β_v (fixed)	45.24
∂_v, ∂_w, β_v and β_w (all free)	40.36

and entrepreneurs in the dynamic game can be effectively matched by the proposed learning model. It is also demonstrated that VCs and entrepreneurs updatetheir trust states according to the growth rate of returns at each stage and the dynamic process of trust is the process of learning from each other.

4.2 Effect of $\theta_t e(t)$ (the extent of entrepreneurs'efforts) on the dynamic change of trust

The optimal action strategy (S_t^*, e_t^*) at every stage determines the stability of the trust relationship between VCs and entrepreneurs. From the above analysis, we conclude that entrepreneurs' efforts are the basis of the strategic adjustment in the dynamic trust game. To further reveal the effect of entrepreneurs' effortson dynamic changes in trust, a numerical simulation is performed to reveal how VCs and entrepreneurs make trust decisions based on entrepreneurs'efforts. $\theta_t e(t) = a e_t, a \in [0, 1], \theta_t e(t) \in [0, 1]$, the initial value of t is 1 and the simulated period is 1000 time units. We calculate the values of the optimal solution when $a = 1$.

The result of Fig. 4 shows that the investments added by VCs at every stage increased at the beginning of t and entrepreneurs' efforts also increased when $a = 1$. After passing 23, the investments that VCs add to every stage are equal, and entrepreneurs devote equal efforts at every stage. Faced with the same action results, VCs and entrepreneurs tend to take the original strategy to ensure the benefits of each stage. VCs and entrepreneurs adjust their strategies according to previous experiences. The tendency to explore a new strategy is minimal; therefore, the trust relationship between VCs and entrepreneurs is stable.

As shown in Fig. 5, the fluctuation of entrepreneurs' efforts when a is randomly selected between [0, 1] leads to random fluctuations in the growth rate of returns. VCs cannot make investment decisions based on the results of previous actions due to stochastic changes in the growth rates of returns. The trust state is constantly adjusted as a result of the randomness caused by the action strategies of VCs and entrepreneurs. The trust state between VCs and entrepreneurs greatly fluctuate, leading to unstable trust. It is demonstrated that the

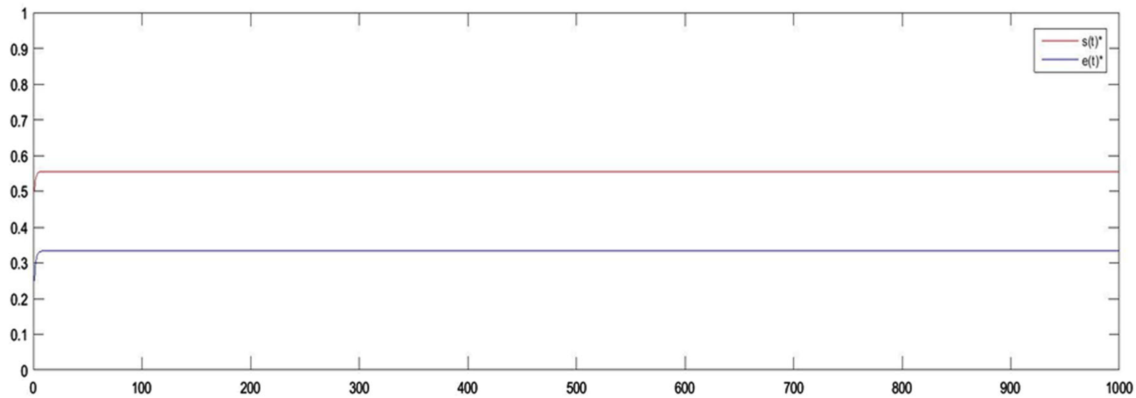

Fig. 4 Values of the optimal solution (S_t^*, e_t^*) when $a = 1$

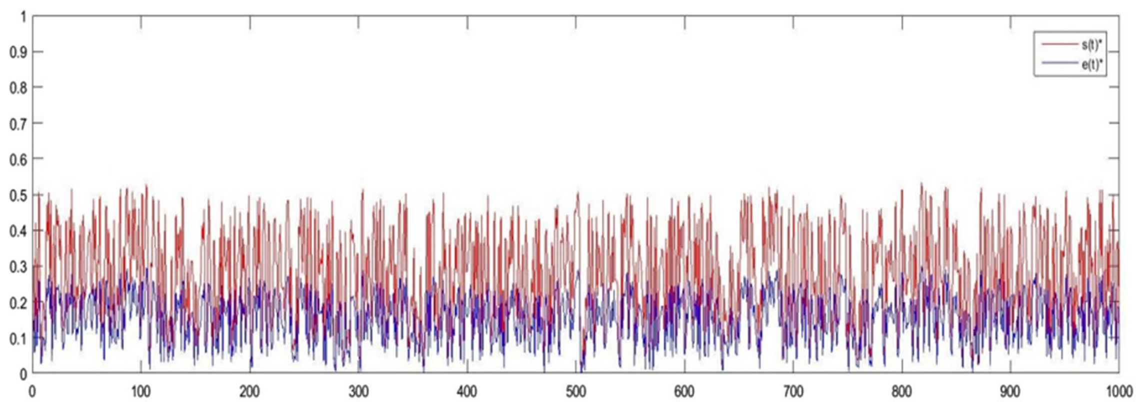

Fig. 5 Values of the optimal solution (S_t^*, e_t^*) when a is randomly selected between [0, 1].

degree of entrepreneurs' efforts is the key factor influencing dynamic changes in the trust relationship between VCs and entrepreneurs.

As shown in Fig. 6a–c, S_t^* and e_t^* continuously increase. VCs and entrepreneurs are reciprocal when entrepreneurs' efforts increase. The additional investments by VCs are to achieve capital appreciation. Entrepreneurs make great efforts to develop the business and manage the company to meet the requirements of VCs. The curve of e_t^* rises faster than S_t^* indicating that VCs are less inclined to explore new strategies when entrepreneurs' efforts have contributed to an increase in the growth rate of returns. The result is consistent with the results of the learning model in Table 1 and Fig. 2. We also reach the conclusion that it is conducive to the stability of trust between VCs and entrepreneurs when VCs invest small amounts of funds in the early stage and when more funds are invested in the later stage. This strategy observes entrepreneurs' efforts to prevent investment risks and is conducive to the stability of the trust relationship. The growth rate of returns brought by entrepreneurs' efforts has increased, and VCs made additional investments in start-ups. Entrepreneurs' trust on VCs is also rising when entrepreneurs can get financial support. The trust status between VCs and entrepreneurs is less adjusted. The trust relationship between VCs and entrepreneurs is ultimately stable and adaptive.

As illustrated in Fig. 7a–c, the optimal solution (S_t^*, e_t^*) rapidly decreases after a brief increase in the initial stage when the growth rate of returns continuously declines as entrepreneurs reduce efforts. When the level is reduced, the speed of change is faster than when the effort is increased. The varied rate of S_t^* when entrepreneurs' efforts are decreasing is higher than the rate when entrepreneurs increase their efforts. The results indicate that VCs are more sensitive when the level of entrepreneurs' efforts drops. The simulated results are consistent with the results of the learning model in Table 1 and Fig. 3. (S_t^*, e_t^*) increase temporarily in the initial stage. Returns decline in the early stage due to uncertainties in the market. However, the potential growth of start-ups cannot be determined by the decline in short-term earnings; therefore, additional investments are made by VCs in the early stage.

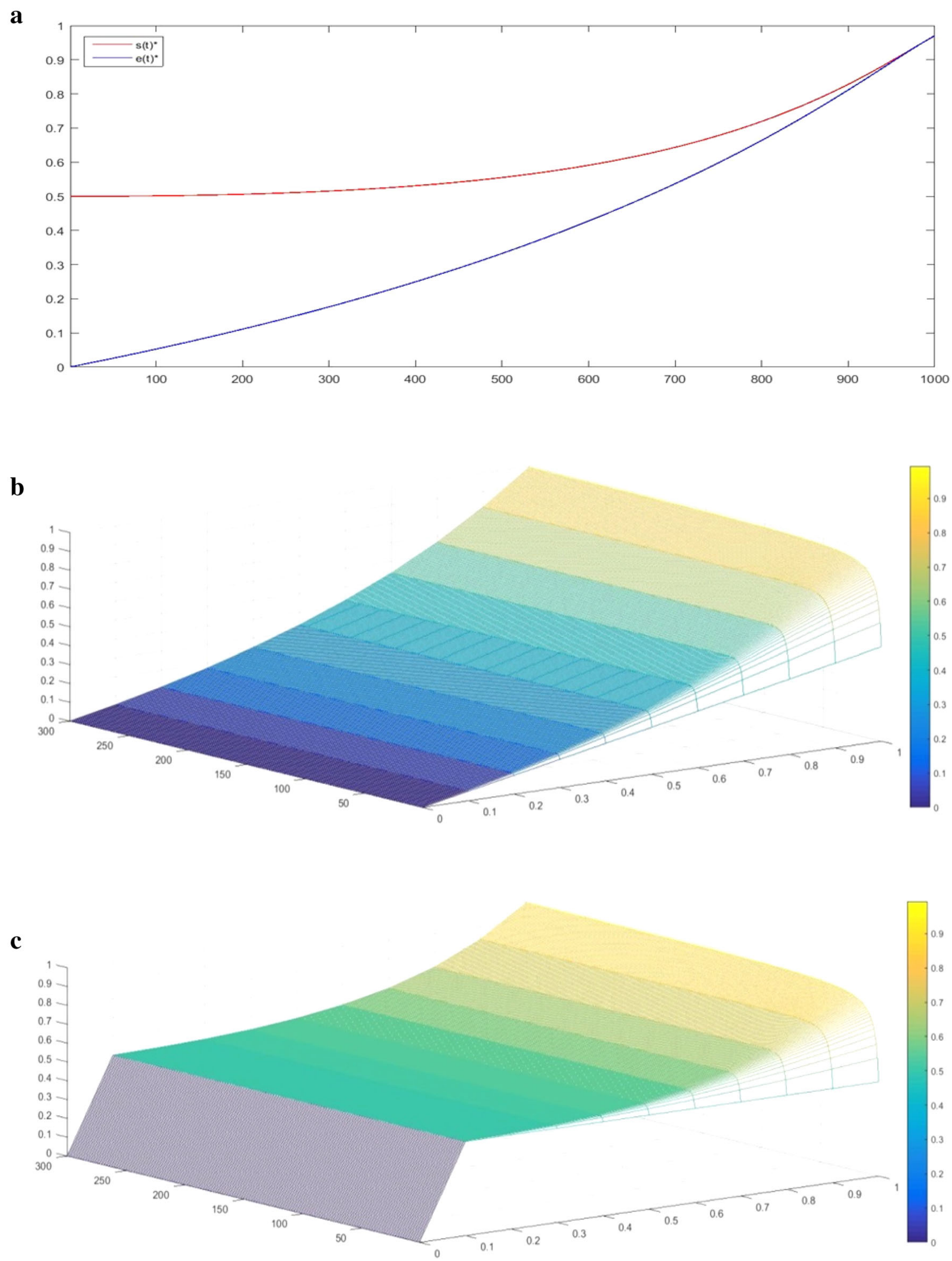

Fig. 6 **a** Values of the optimal solution (S_t^*, e_t^*) when a increases continuously between [0, 1], **b** Values of the optimal solution (e_t^*) when a increases continuously between [0, 1], **c** Values of the optimal solution (S_t^*) when a increases continuously between [0, 1]

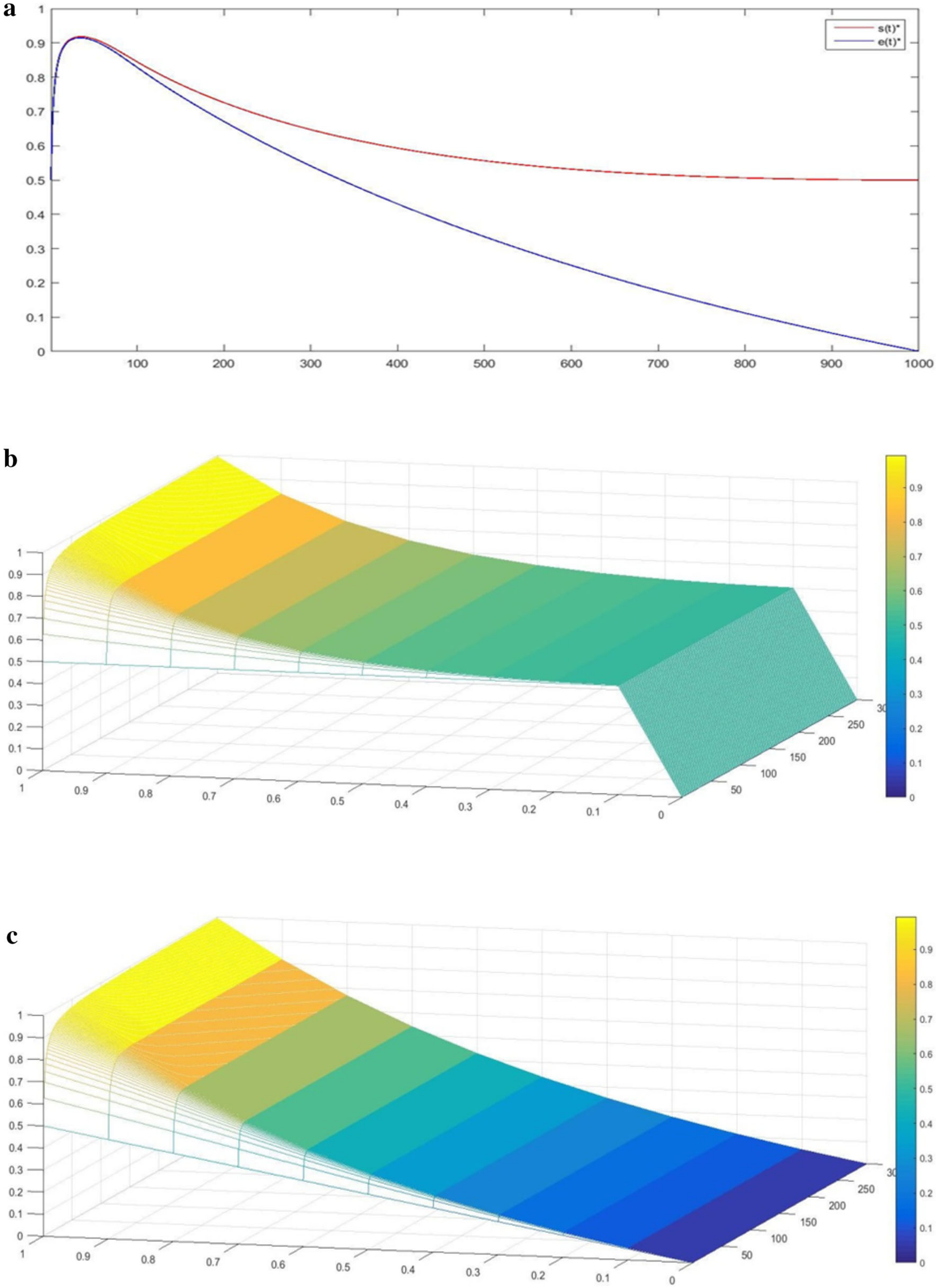

Fig. 7 **a** Values of the optimal solution (S_t^*, e_t^*) when a decreases continuously between [0, 1], **b** Values of the optimal solution (S_t^*) when a decreases continuously between [0, 1], **c** Values of the optimal solution (e_t^*) when a decreases continuously between [0, 1]

5 Conclusions

Combined with the staged investment behaviour of VCs, the dynamic game model of trust between VCs and entrepreneurs is developed. The motivation and process of dynamic trust are revealed. The numerical simulation of the model is conducted to analyse the learning rate and temperature parameters under different results to explain how VCs and entrepreneurs adjust their trust states in the game. The conclusions are as follows.

The dynamic process of trust between VCs and entrepreneurs is essentially the process of learning from each other. The dynamics of trust are determined by cooperative mutual learning. The degree of matching between the learning model and action strategy is superior to the non-learning model, confirming that VCs and entrepreneurs make trust decisions based on the interactive experience. Trust decisions are influenced by previous actions. The level of entrepreneurs' efforts is the motivation for dynamic changes of trust. VCs and entrepreneurs determine trust decisions based on returns at different stages, and strategies and trust states are updated accordingly. Therefore, the dynamic process of trust between VCs and entrepreneurs is essentially a process of mutual learning.

VCs and entrepreneurs are more concerned with negative results in the interaction process. The learning rate of entrepreneurs increases when the amount of investment provided by VCs is less than the previous stage. The learning rate of VCs rises when entrepreneurs cut back on their efforts, resulting in a decline in the growth rate of start-ups' returns. Entrepreneurs pay more attention to the situation when fewer funds are invested by VCs, and the temperature parameters are higher than when more funds are invested. Entrepreneurs are less affected by previous results. Entrepreneurs quickly adjust their strategy in the next stage and adopt a new strategy. VCs are more sensitive to the reduction of entrepreneurs' efforts, and the temperature parameters of VCs are higher in this situation. VCs quickly adjust their decisions and tend to explore a new strategy.

Entrepreneurs' efforts are the key factor that affects the stability of the trust relationship between VCs and entrepreneurs. It is beneficial to the stability of the trust between VCs and entrepreneurs when VCs invest fewer funds in the early stage and when more funds are invested in the later stage. VCs can monitor entrepreneurs' efforts by adding less investment in the early stage. Entrepreneurs can obtain financial support to grow, and VCs realize capital appreciation, if more funds are invested by VCs in the later stage. The trust relationship is stable, and mutual trust is formed. The evolutionary process of trust presents a non-linear feature and the trust relationship is ultimately stable and adaptive.

Implication

This paper confirmed that VCs' level of trust in entrepreneurs is determined by their evaluation of entrepreneurs' efforts in cooperation. VCs' trust in entrepreneurs is an important determinant of the success of venture enterprises that often face problems, such as insufficient funds and considerable market risks. Entrepreneurs should keep their promises and make strong efforts to manage the enterprises during their cooperation with VCs. VCs will quickly lose trust in entrepreneurs if they notice diminished efforts by entrepreneurs. Therefore, it is important for entrepreneurs to work hard after they receive funds from VCs, since the returns from the trust are the key determinant of VCs' investment decisions. Entrepreneurs should make great efforts to develop their business and realize growth in order to meet the requirements of VCs which, in the end, are valuable to strengthen their long-term business relationship with VCs.

VCs and entrepreneurs pay attention to the development of trust to develop long-term cooperative relationships between them. The speculation of entrepreneurs can be partly constrained by the contract, but the costs to establish and revise the contract are high. Therefore, trust is an important factor for the sustainable development of the cooperation in entrepreneurial situations with higher risks. Trust is an intangible resource that can reduce the opportunistic behaviour brought by the uncertainty in cooperation, simplify the mechanism of formal management and reduce transaction costs.

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