CS 70

Discrete Mathematics and Probability Theory

Spring 2025

Rao

DIS 0A

Propositional Logic Intro

Proposition: A statement with a truth value; it is either true or false.

Propositions can be combined to form more complicated expressions, using the following operations:

Operators		(Quantifiers	Implication	Implication operations	
\wedge	and	\forall	for all	Implication	$P \Longrightarrow Q$	
\vee	or	Ξ	there exists	Inverse	$\neg P \Longrightarrow \neg Q$	
\neg	not			Converse	$Q \Longrightarrow P$	
\Longrightarrow	implies			Contrapositive	$\neg Q \Longrightarrow \neg P$	
\equiv	equivalent to					

Further, for an implication $P \Longrightarrow Q$ where P is the *hypothesis* and Q is the *conclusion*, it is useful to know that $P \Longrightarrow Q \equiv \neg P \lor Q$. Additionally, observe that any implication is logically equivalent to its contrapositive.

DeMorgan's Laws: The following identities can be helpful when simplifying expressions and distributing negations.

- $\neg (P \land Q) \equiv \neg P \lor \neg Q$
- $\neg (P \lor Q) \equiv \neg P \land \neg Q$
- $\neg(\forall x P(x)) \equiv \exists x (\neg P(x))$
- $\neg(\exists x P(x)) \equiv \forall x(\neg P(x))$

1 Propositional Practice

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Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

Recall that \mathbb{R} is the set of reals, \mathbb{Q} is the set of rationals, \mathbb{Z} is the set of integers, and \mathbb{N} is the set of natural numbers. The notation " $a \mid b$ ", read as "a divides b", means that a is a divisor of b.

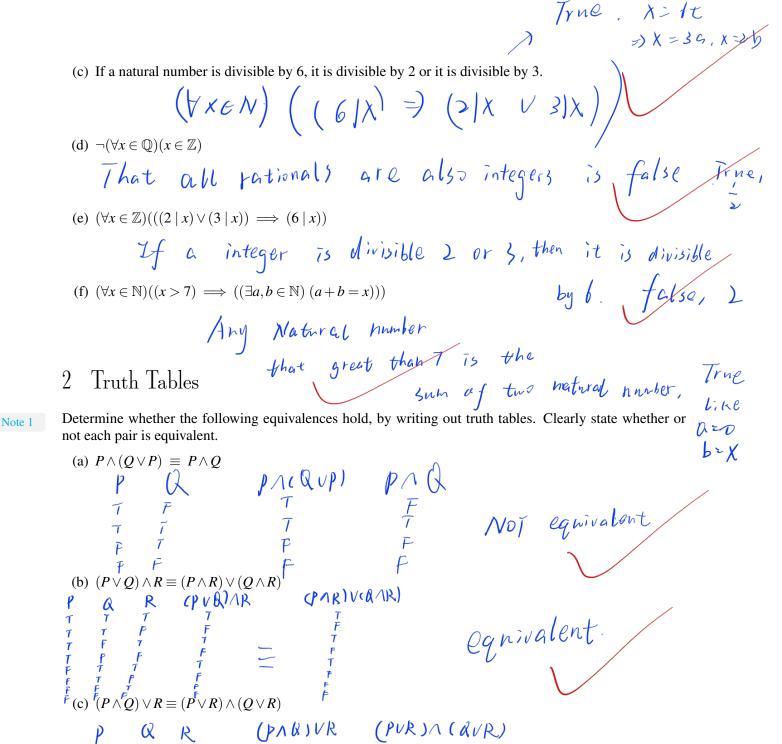
(a) There is a real number which is not rational.

Fre (X & Q) True, like T

(b) All integers are natural numbers or are negative, but not both.

Note 1

Note 1



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2

Egnivalent

3 Implication

Note 0 Note 1 Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement P(x, y) that would make the implication false).

- (a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

 True

 The Figure 1.

True

choose a for X in

antecendent

then x could be a

in consequence

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