A Smart Contract Formula for LTV ratio

The Risk DAO

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1 Introduction

Determining the correct Loan-To-Value (LTV) of a collateral asset is crucial to balance the loans default risk and the usability of the platform. This is done by running excessive daily simulations and integrating a domain specific knowledge.

In this work, we try to find a simple "uniswap-like" formula for the LTV ratio. We present a simple formula that can be calculated by a smart contract (e.g., when coupled with B.Protocol's planned on-chain data.

2 The Formula

Aiming for the simplest possible formula, we abstract smart contract security considerations, and assumptions on user behavior. However, the formula will have an adjustable confidence parameter that could compensate for these aspects.

At the core level, the goal of setting an LTV ratio that is smaller than 1, is to compensate for the potential price decrease during the liquidation process (which need not be in a single tx, and could take time). For this purpose, we aim to answer the following question:

> How much the price will decrease before a liquidation is fully executed?

This is a very open ended question, as even the size of the liquidation is unknown. For this purpose, we make the following three assumptions:

1. *Price behavior.* There is a discrete basic time unit, and the price follows a log-normal random walk distribution. Formally, $\ln(\frac{P_{t+1}}{P_t}) \sim \mathcal{N}(0, \sigma)$. We note that in particular, there is an implicit assumption that a liquidation will not change the price of the asset, however, a different assumption could potentially be embedded at the confidence level factor.

2. *DEX liquidity.* We define liquidity as the total quantity that is available for sale with a given price slippage. We assume that every time the liquidity is depleted, it will be fully restored after a constant time interval T. This

can be justified, e.g., by people arbitraging the existing DEX liquidity with Binance orderbook, and/or for additional liquidity to come from another chain (if the lending market is on an L2). As usual, the confidence parameter will also embed the confidence in this assumption.

3. *Liquidation sizes.* The size of liquidation that a lending market should be prepared to, is proportional to the total debt in the system. We note that in practice, the current user account composition also have great affect on it, however, as a design choice, we chose to abstract away the current user composition.

Given the above assumptions, our proposed formula is:

$$LTV = e^{-c \cdot \frac{\sigma}{\sqrt{\ell/d}}} - \beta$$

where:

- σ is the price volatility between the collateral and debt asset (normalized to the base asset price).
- β is the liquidation bonus.
- ℓ is the available liquidity with slippage of β.
- d is the borrow cap of the debt asset.
- c is a confidence level factor. The higher c is, the odds for insolvency are decreasing.

In the reminder of the section we prove the correctness of the formula (subject to the simplifying assumptions).

Consider an arbitrary liquidation of size $r \cdot d$ (for an arbitrary constant $0 < r < 1$). Suppose it takes T minutes for the DEX liquidity to get restored. In this case, a liquidation will be composed of $n = \frac{r \cdot d}{\ell}$ iterations, where each iteration will take T minutes, and will execute $\frac{1}{n}$-th of the liquidation. And in total $N = n \cdot T$ time units will elapse before the liquidation is fully executed. Denote the price change in time unit i by Δ_i. Recall that if Δ_i is small, then $\ln(1 + \Delta_i) \approx \Delta_i$. Hence, we get that p_N, the price at time N adheres to $\frac{p_N}{p_0} = e^{\sum_{t=1}^{N} \Delta_t}$. As $\Delta_i \sim \mathcal{N}(0, \sigma)$, we get that $\sum_{t=1}^{N} \Delta_t \sim \mathcal{N}(0, \sigma\sqrt{N})$.

Recall that for a lognormal distribution with std σ and $\mu = 0$, the formula to represent the smallest value in 95% of the cases is $e^{-z_{95} \cdot \sigma}$. Similarly, for every x there is a constant z_x such that the smallest value if x faction of the cases is $e^{-z_x \cdot \sigma}$.

Hence, in our case the smallest value of $\frac{p_N}{p_0}$ is x% of the cases is

$$e^{-z_x \cdot \sigma \cdot \sqrt{N}}$$

by the definition of N we get that this is equal to

$$e^{-z_x \cdot \sigma \cdot \sqrt{T \cdot \frac{r \cdot d}{\ell}}}$$

$=$

$$e^{-z_x \cdot \sqrt{T \cdot r} \cdot \sigma \cdot \sqrt{\frac{d}{\ell}}}$$

With $c = z_x \cdot \sqrt{T \cdot r}$, we get that for $x\%$ of the cases, the price will not go down below $e^{-\frac{c \cdot \sigma}{\sqrt{\ell / d}}}$. Thus, an initial LTV of $e^{-\frac{c \cdot \sigma}{\sqrt{\ell / d}}} - \beta$ will prevent insolvency in $x\%$ of the cases.

The proof is completed.

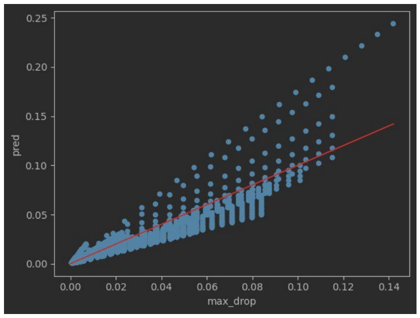
3 Experimental results

99.9% of all DeFi liquidations are executed in a single transaction. And thus we take real liquidation data from Binance Futures ETH-USD pair. In order to test different volatility levels, we amplify the price changes and multiply them by a stretching constant that ranges between 0 and 2. The most synthetic part in our experiments is the behavior of DEX liquidity. We follow the formula assumption that there is a fixed DEX liquidity size, which is restored every 30 minutes.

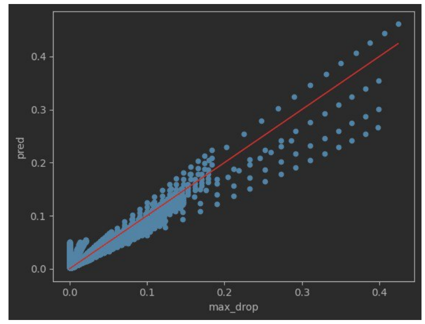
We take the 3 worst months (in terms of liquidations) in the history of Binance Futures, namely, January 2020, March 2020 and February 2021. Having price trajectory and liquidation time and sizes in hand, we simulate the DEX liquidity, and the behavior of the liquidators. And keep track of the maximum price drop of an open liquidation.

We ran the simulation for various volatility, liquidity, and liquidation size normalization factors. We then find the c factor that best approximates the results of each month according to the R^2 metric.

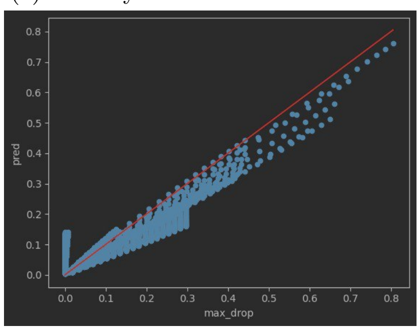
We plot the results in Figure 1.



(a) January 2020. 1.166. $R^2 = 0.795$.



(b) February 2021. $c = 2.583$. $R^2 = 0.833$.

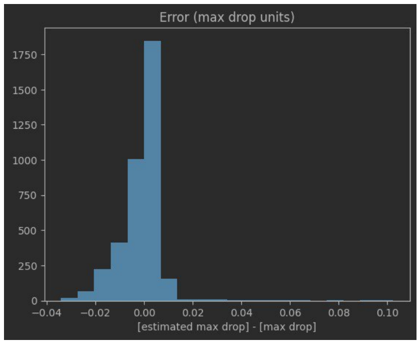


(c) March 2020. $c = 5.988$. $R^2 = 0.706$.

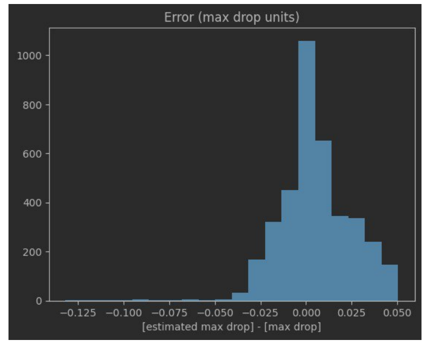
Figure 1: Comparison of simulation results and the results of the formula. For x=0.2, we take all the input parameters for which the simulated max price decrease (max drop) is 0.2, and then plot, in the y axis, the different results and formula give. The red line draws the ideal outcome, i.e., an y=x curve, where the formula returns exactly the same results as the simulation. The R^2 score is calculated w.r.t the red line.

As expected, March 2020 (the peak of the COVID fud) is best approximated with a higher c value, while for the other months, lower c was suffice.

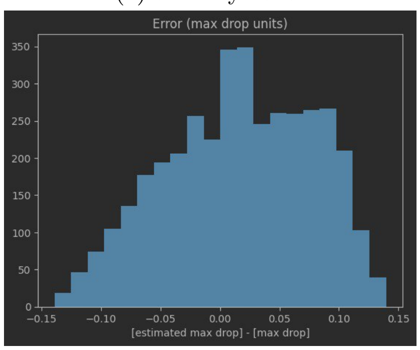
Typically in DeFi, LTV ratios are round numbers, e.g., multiples of 5% (0.05). Figure 2 depicts the absolute deviation between the formula and the simulation results.



(a) January 2020.



(b) February 2021.



(c) March 2020.

Figure 2: Deviation of the formula w.r.t simulated results.

We see that the vast majority of the results are within a radius of 0.05. Further, March 2020 is the month with the biggest absolute deviation. However, it is also unlikely for any model to accurately simulate 80% price decrease for high volatility assets. And thus, the deviation towards the high max drops is also due to inaccuracy in the simulation model we used.

4 Applications

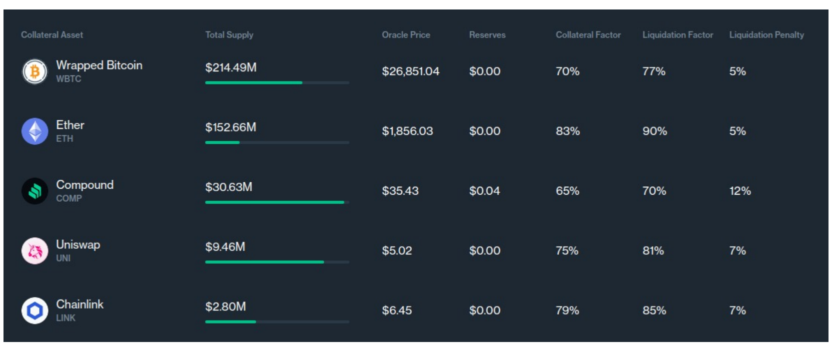
With the formula, one has to decide on a value of c instead of a value to the LTV. This process is more efficient as once c is selected, the LTV ratio could change automatically based on the market conditions (volatility and liquidity). Moreover, once such data is available on-chain (e.g., via B.Protocol oracle), it will be possible for a smart contract to automatically decide on the LTV ratio of assets, and change it dynamically over time.

Hence, the formula can reduce a lot of future decision making overhead, and give rise to more automated and permissionless lending market.

The formula can also be used as a ranking formula, to compare the different risk between different configurations and platforms. For example, a new platform can estimate the average c value of an existing project, and decides if they want to take similar or different risk with their assets. An existing platform can give more transparency to their community about which asset configurations are currently less or more risky.

We demonstrate both of these approaches with the help of Compound III USDC market. This mark is relatively easy to analyze as it contains only one debt asset. The market is depicted in Figure 3. The calculated c values are displayed in Table 1.

One can use the calculated c value as a ranking formula, and we observe that despite having the lowest LTV ratio, COMP token has the lowest c value, and thus has the most aggressive LTV configuration. One can also use the calculated c values in order to set the configuration to his own lending market. The value of c in Compound III ranges between 1 to 6, where most of the assets are in the 4-6 zone. Hence, a possible approach is to apply a c value of 5 to all of a lending market asset, and automatically (possibly with a smart contract) configure the LTVs according to that c value. However, the reader should be aware that more research is needed in order to calculate the c value of a platform. LTV requires a governance vote to be configured, and thus, the current values need not reflect the original intent of the Compound DAO. In a future research, we plan to average to liquidity results over time and present a more robust scheme on how to reason proper c values.



Collateral Asset	Total Supply	Oracle Price	Reserves	Collateral Factor	Liquidation Factor	Liquidation Penalty
Wrapped Bitcoin WBTC	$214.49M	$26,851.04	$0.00	70%	77%	5%
Ether ETH	$152.66M	$1,856.03	$0.00	83%	90%	5%
Compound COMP	$30.63M	$35.43	$0.04	65%	70%	12%
Uniswap UNI	$9.46M	$5.02	$0.00	75%	81%	7%
Chainlink LINK	$2.80M	$6.45	$0.00	79%	85%	7%

Figure 3: Compound III market. Data was fetched on 31/5/2023.

	LTV	Liq. bonus	Borrow cap	DEX liquidity	STD ratio	c
WBTC	77%	5%	323	50	1.18	6.61
ETH	90%	5%	651	90	1	1.90
COMP	70%	12%	32	0.16	1.339	1.04
UNI	81%	7%	11.6	1.6	1.154	4.11
LINK	85%	7%	5.28	2.7	0.88	6.77

Table 1: Compound III USDC mainnet data and calculated c values. Fetched on the 31/5/2023. DEX liquidity was fetched from 1inch aggregator. STD was taken from uniswap and sushiswap trading history, for a 3 month period. LTV are Compound's liquidation thresholds. Borrow cap is the collateral supply cap. Borrow caps and DEX liquidity are in millions (USD). STD ratio of Ethereum vs USD was set to 1, and the rest are relative to it. The presented value of c was multiplied by 100 for the sake of easier presentation.