Modeling Quadruped Gait Bifurcations

Weslee Nguyen

Departments of Neuroscience and Mathematics at the University of Arizona

INTRODUCTION

Why model quadruped gaits?

- **Biology**: Neural gait mechanisms unknown, so model refines hypotheses
- **Diagnostics**: Movement issues found when data shifts away from model
- **Biomimicry**: Robotic movement is natural, flexible, and adaptable

How do we model quadruped gaits?

- **Central Pattern Generators**: Neural circuits that produce gaits
  - Modeled via cells (a system of ODEs) which gait transition occurs via a **Bifurcation**: Dynamical system behavior changes by varying parameter
    - **Supercritical Hopf Bifurcation**: As a parameter varies, solutions attract from a fixed point (standing) to a stable oscillation (some gait)

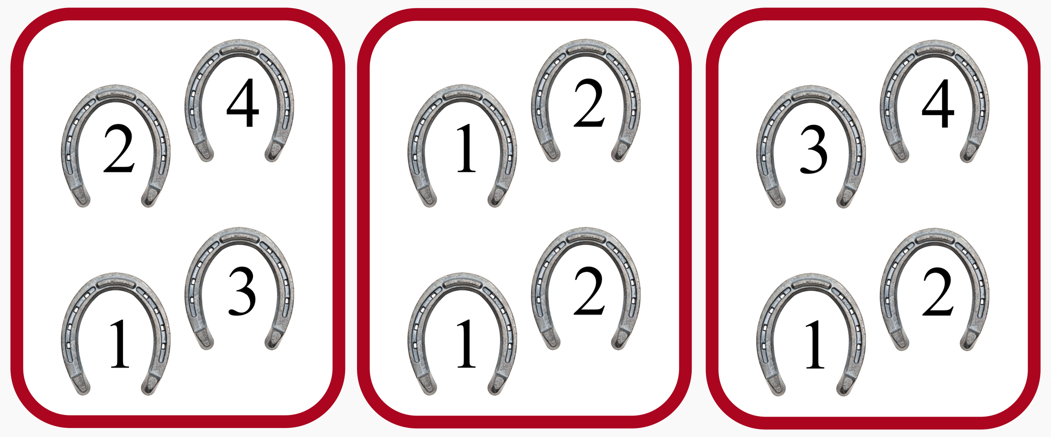
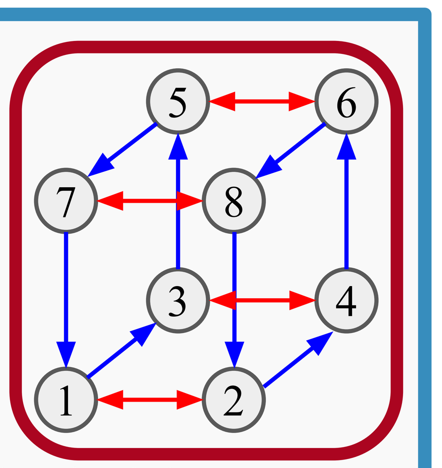


Figure 1: Spatiotemporal patterns of relevant gaits. Numbers denote order of movement, starting with the left hind. (**L.**) Walk. (**Mid.**) Pace. (**R.**) Transverse Gallop.

METHODS



- **Idea**: Symmetries of gait ↔ symmetries of model
  - Ex: Pace has ipsilateral legs "identical", so model still makes pace when swapping ipsilateral cells

Figure 2: The bottom layer corresponds to legs. 1 is left hind, 2 is right hind, 3 is left front, and 4 is right front. 4 - 8 can be viewed as higher-order neurons. The CPG model is all 8 cells with the gait being x_1 to x_4.

- Simplify cell as a neuron and model firing by the FitzHugh Nagumo Eqns
  - f_1: membrane potential, f_2: recovery variable; set (a, b, c) = (0.02, 0.2, 0.44) reproduce the results of Golubitsky et al. (1993)

$$f_1(x,y) := c\left(x + y - \frac{x^3}{3}\right) \qquad f_2(x,y) := -\frac{1}{c}(x - a + by)$$

- Then the ith cell has the 2D system for i = 1, …, 8 and indices modulo 8

$$\begin{aligned} \dot{x}_i &:= f_1(x_i, y_i) + \boxed{\alpha}(x_{i-2} - x_i) + \boxed{\gamma}(x_{i+\epsilon} - x_i) \\ \dot{y}_i &:= f_2(x_i, y_i) + \boxed{\beta}(x_{i-2} - x_i) + \boxed{\delta}(x_{i+\epsilon} - x_i) \end{aligned} \qquad \epsilon = \begin{cases} +1 & i \text{ odd} \\ -1 & i \text{ even} \end{cases}$$

Ipsilateral and **Contralateral** Coupling Strengths

- We also performed sensitivity and bifurcation analyses

RESULTS

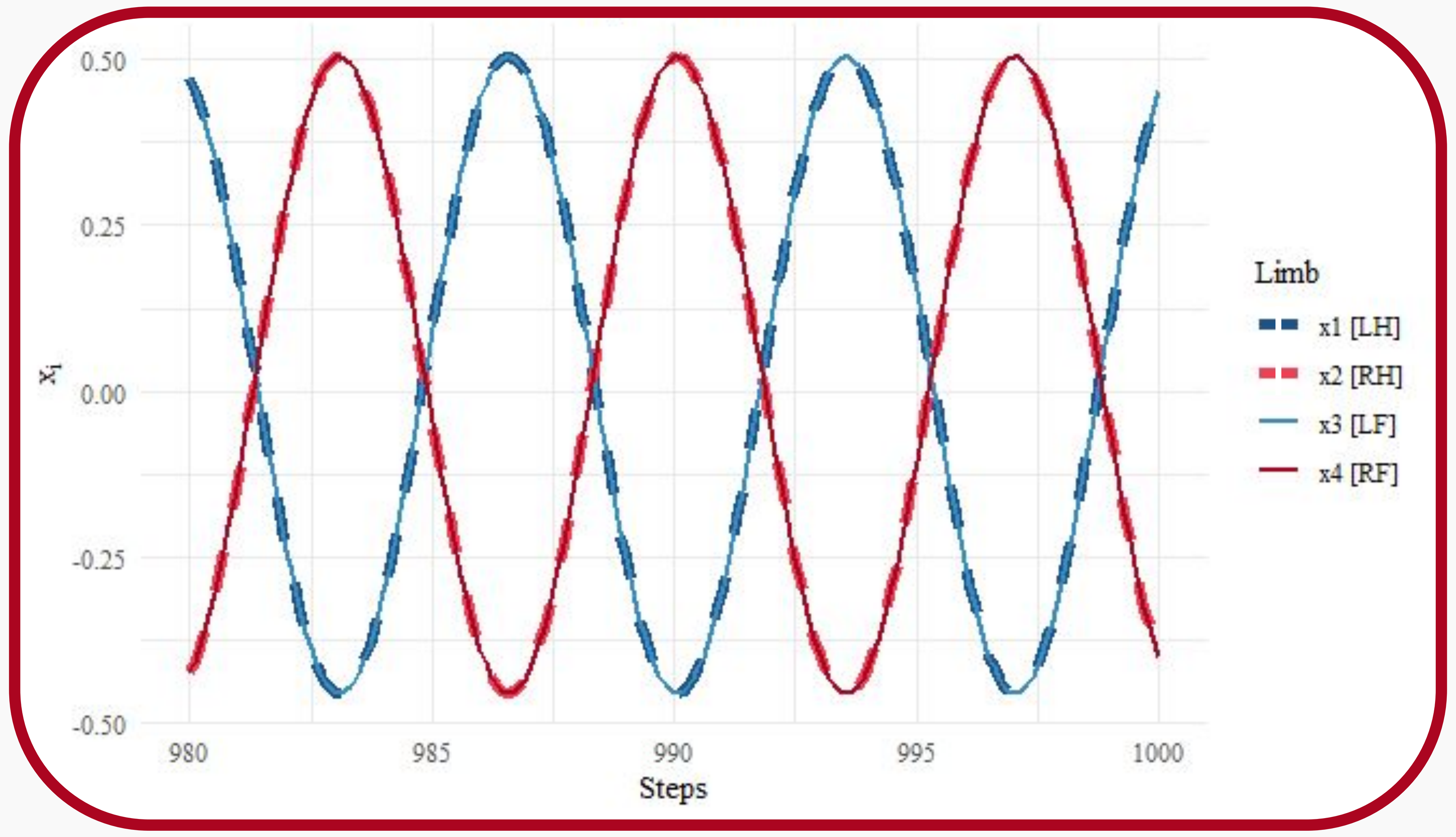


Figure 3: Example gait. Gait is defined spatiotemporally: right order and timing. Pace has ipsilateral legs moving at the same time (order) in a 2 beat cycle (timing). This model had strong ipsilateral and weak contralateral couplings (α, β, γ, δ) = (0.1, 0.1, -0.1, -0.1), producing pace as expected.

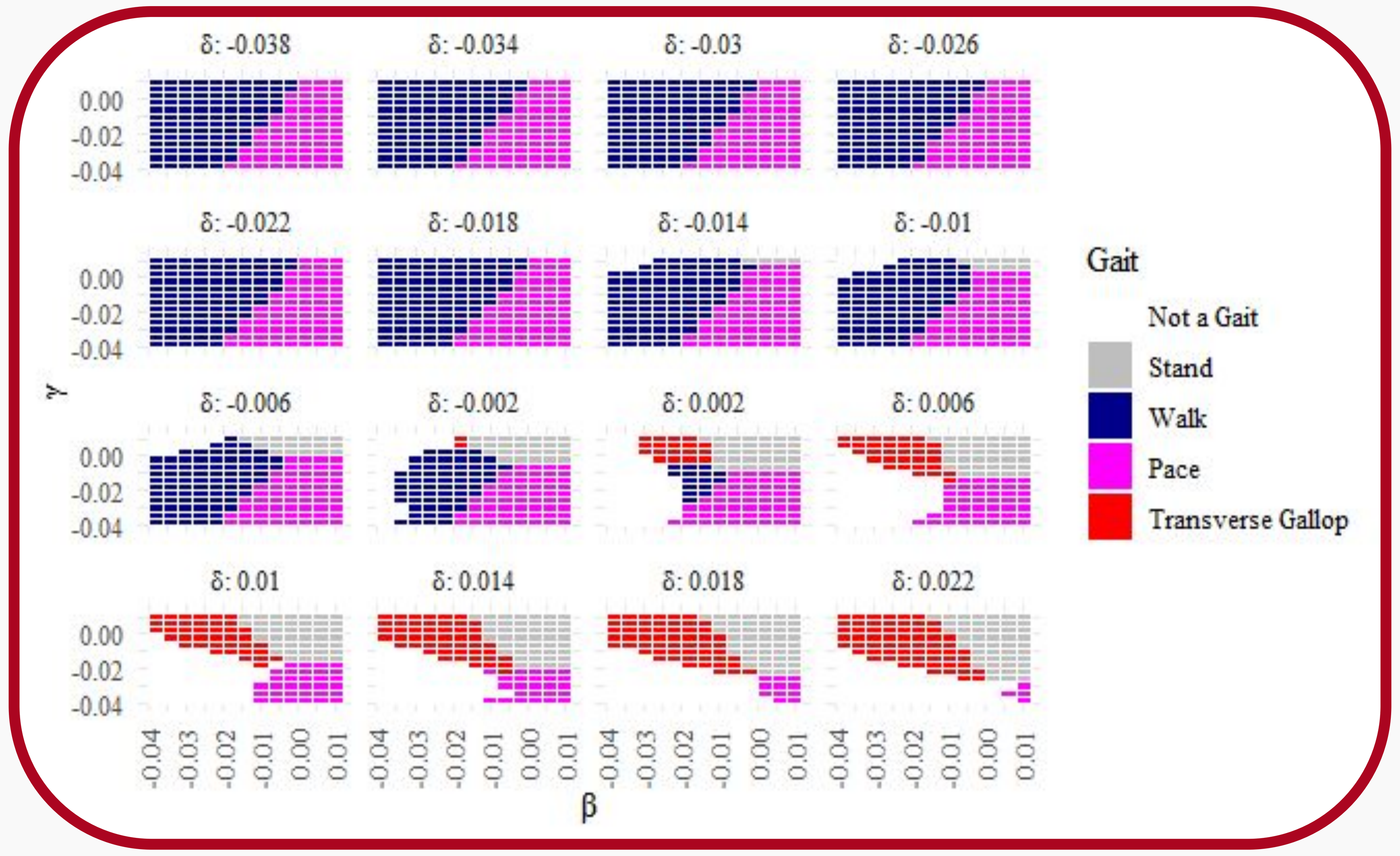


Figure 4: Strengthening / weakening coupling strengths matches expected gait. We fix α = 0.1 and vary β, γ, and δ like a cube, which then we cut βγ-planes. With α > 0, pace requires the other ipsilateral coupler β > 0. As walk has the left legs move before the right and transverse gallop has the back legs move before the front, then walk prioritizes ipsilateral coupling and transverse gallop contralateral. As δ increases, walk is swapped for transverse gallop. All couplers positive makes stand.

CONTACT: *nguyenw@arizona.edu*

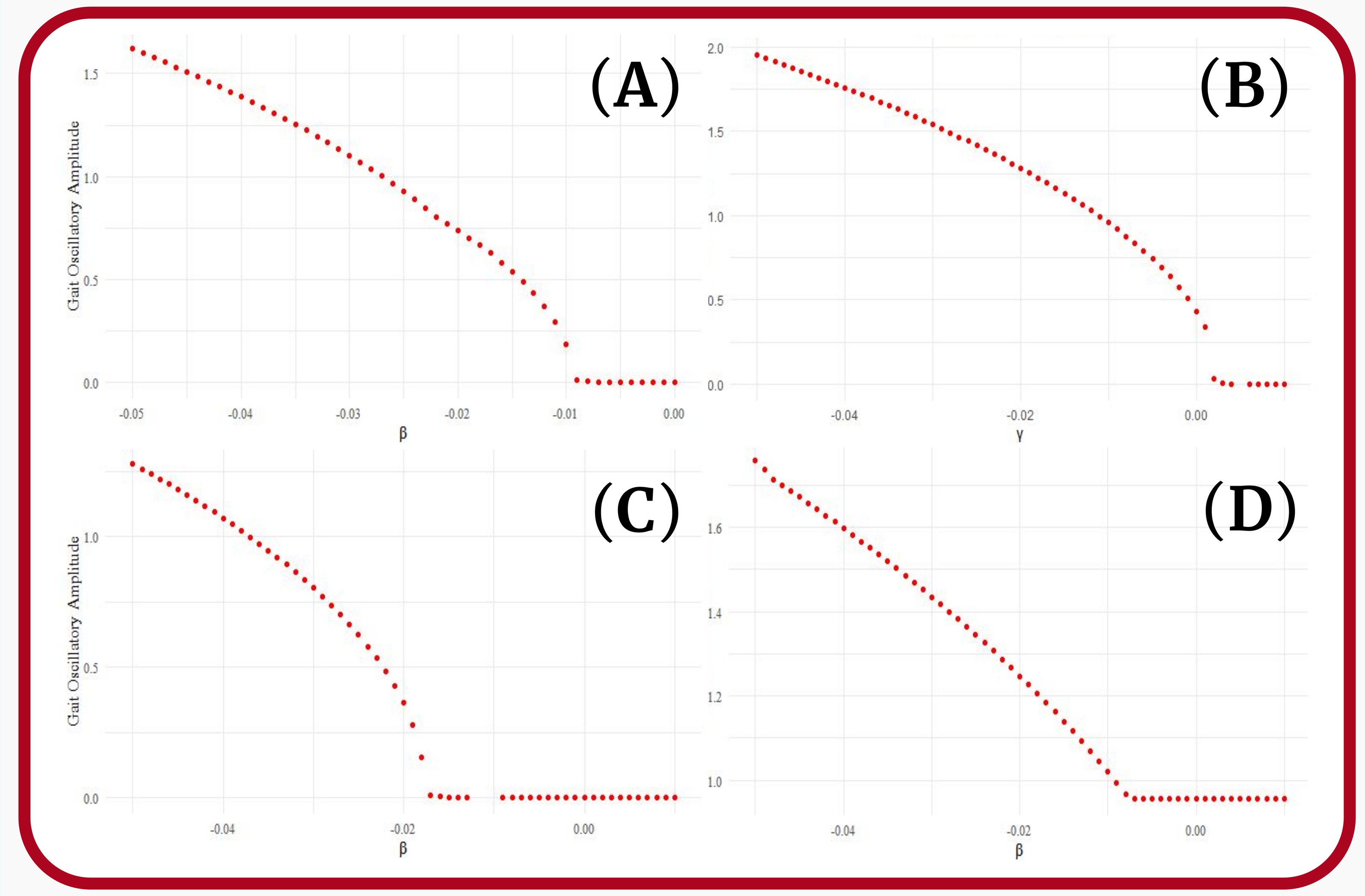


Figure 5: Transition of (**A**) walk, (**B**) pace, (**C**) transverse gallop (*left side of plots*) to stand (*right*) are smooth. When fixing parameters and varying only one, each resembles a Supercritical Hopf Bifurcation. The y-axis is the max - min of x_1 to x_4 where the gait converges (namely: gait oscillatory amplitude error < 0.002). (**D**) Transition of walk (*left*) to pace (*right*) is similar.

DISCUSSION

- Model is good as it is
  - **Interpretable**: Relative coupling strengths successfully predicted gaits
  - **Stable**: Perturbing initial conditions didn't change 3 tested gaits (60 trials)
    - Walk also stabilizes without chaotic transitions during the bifurcation
- Model can be improved as it is
  - **Simplistic**: one cell is likely many, not one, neurons
  - … and **complicated**: there's 7 parameters with many couplings possible
  - **Abstract**: a real life gait has no obvious pairing to specific parameters
  - **Lacking Feedback**: Would (**1**) mark "higher" v. "lower" cells, (**2**) adapt to movement needs, and (**3**) model top-down and bottom-up processes
    - Proprioceptive for lower and vestibular for higher to balance

ACKNOWLEDGEMENTS

- Work supported by the National Science Foundation through DMS-1937229
- Also supported by the Data Driven Discovery RTG at the University of Arizona
- Guided by Dr. Kevin Lin and Dr. Shay Gilpin

REFERENCES

Collins, J. J. & Stewart, N. (1992). Coupled Nonlinear Oscillators and the Symmetries of Animal Gaits. *Nonlinear Science*, *3*(1), 349-392. https://doi.org/10.1007/BF02429870

Golubitsky, M., Stewart, I., Buono, P., & Collins, J. J. (1998). A Modular Network for Legged Locomotion. *Physica D*, *115*(1-2), 56-72. https://doi.org/10.1016/S0167-2789(97)00222-4