

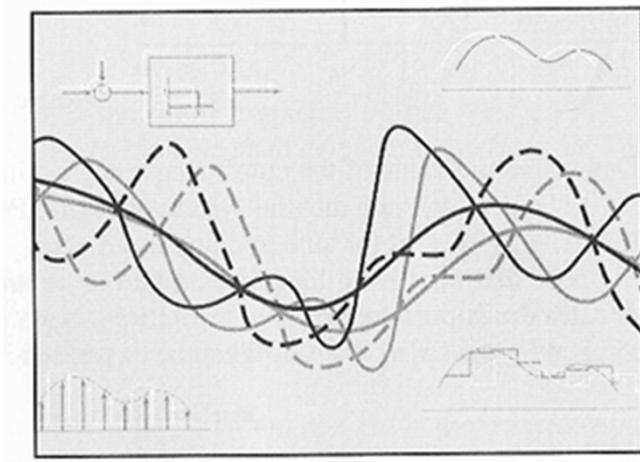
Spring 2013

信號與系統
Signals and Systems

Chapter SS-7
Sampling

Feng-Li Lian
NTU-EE
Feb13 – Jun13

Figures and images used in these lecture notes are adopted from
“Signals & Systems” by Alan V. Oppenheim and Alan S. Willsky, 1997



Introduction

[\(Chap 1\)](#)

LTI & Convolution

[\(Chap 2\)](#)

Bounded/Convergent

Periodic

FS

[\(Chap 3\)](#)

CT
DT

Aperiodic

FT

CT
DT

[\(Chap 4\)](#)
[\(Chap 5\)](#)

Unbounded/Non-convergent

LT

CT [\(Chap 9\)](#)

zT

DT [\(Chap 10\)](#)

Time-Frequency [\(Chap 6\)](#)

CT-DT

[\(Chap 7\)](#)

Communication [\(Chap 8\)](#)

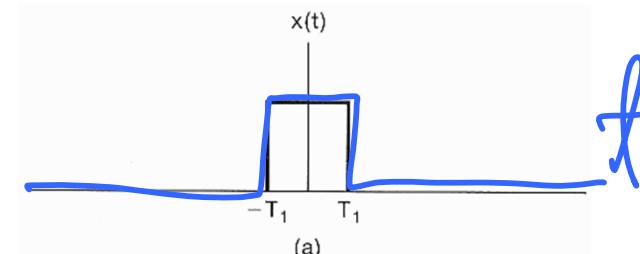
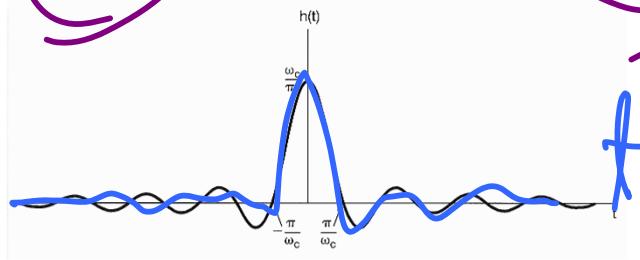
Control

[\(Chap 11\)](#)

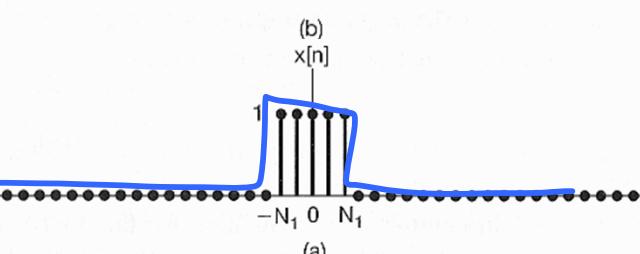
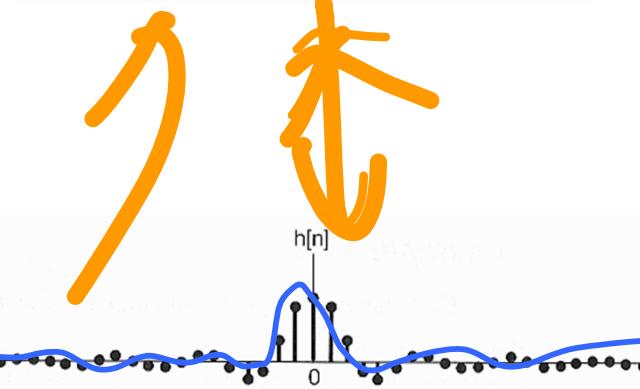
Digital
Signal
Processing
[\(dsp-8\)](#)

■ Time-Domain & Frequency-Domain Characterization:

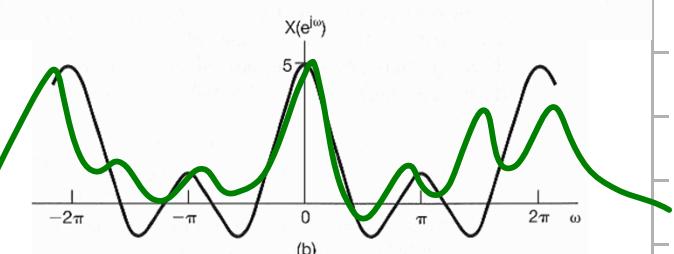
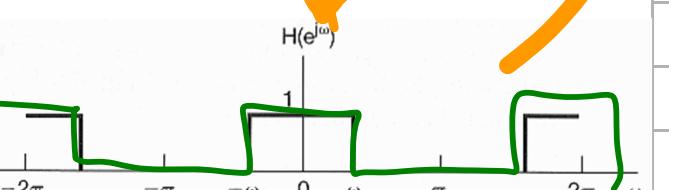
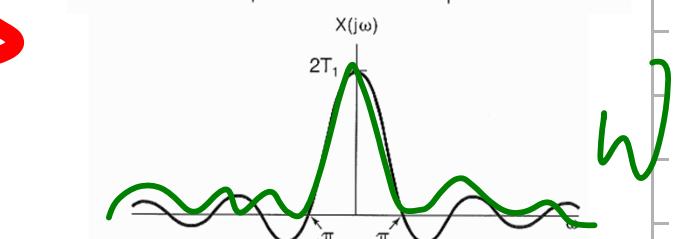
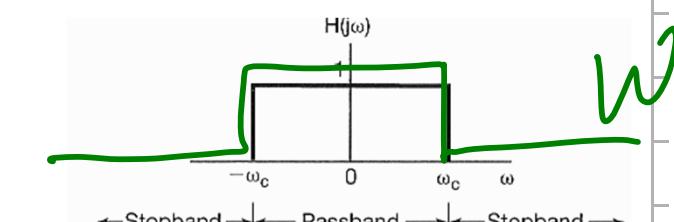
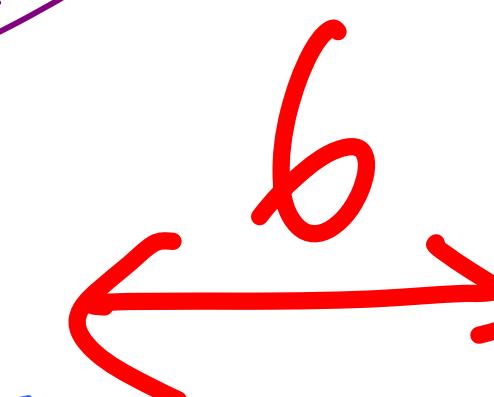
CT



$$h(t) \xleftrightarrow{\mathcal{F}} H(j\omega)$$

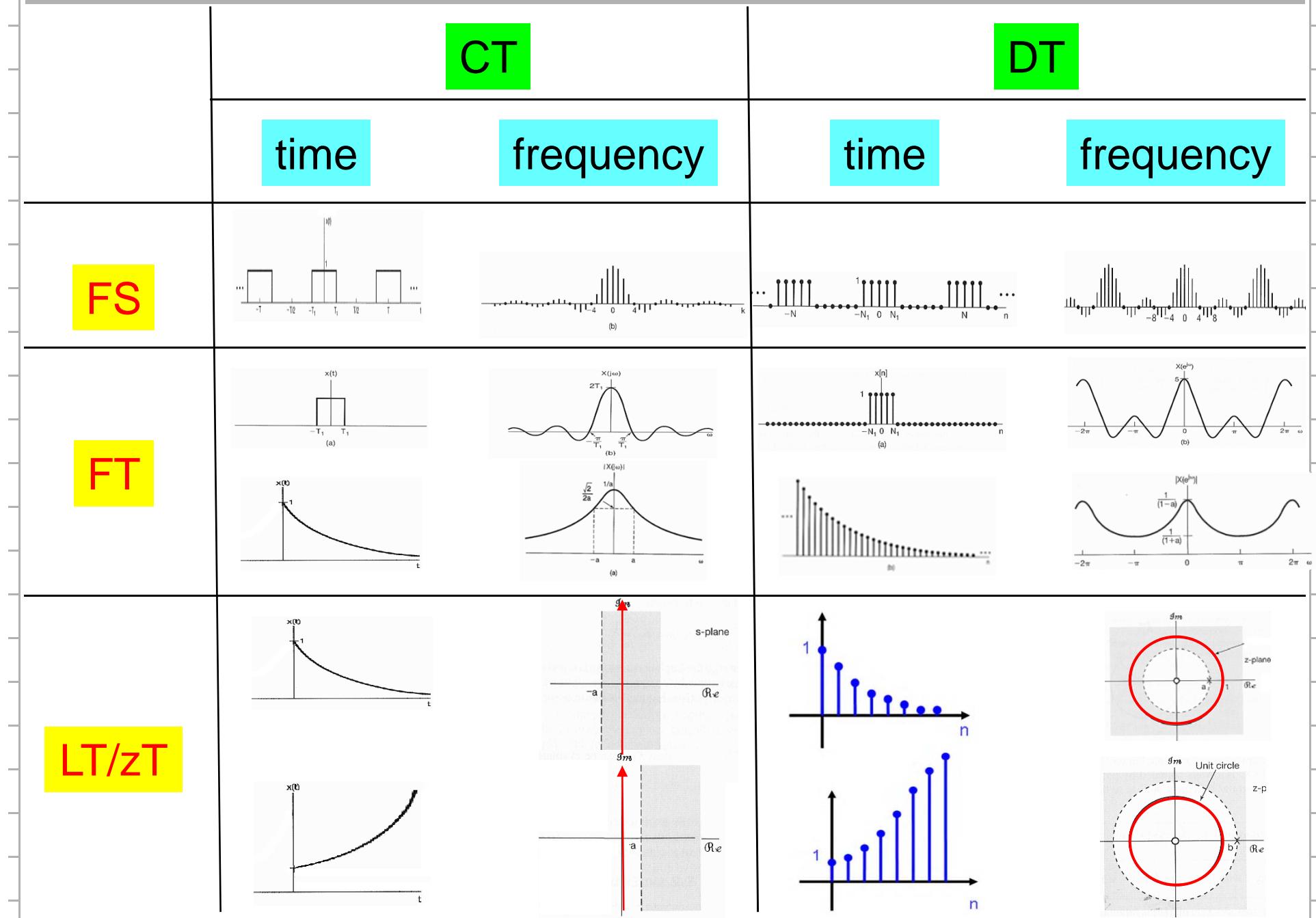


$$h[n] \xleftrightarrow{\mathcal{F}} H(e^{j\omega})$$



Fourier Series, Fourier Transform, Laplace Transform, z-Transform

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NTUEE-SS7-Sampling-3



- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

C.T.

y

■ Representation of CT Signals by its Samples

 $x_1(t)$

$t \in \mathbb{R}$

 $x_2(t)$

$\underline{\underline{t}}$

 $x_3(t)$

$\underline{\underline{t}}$

 $x_1(t)$

$t \in \mathbb{R}$

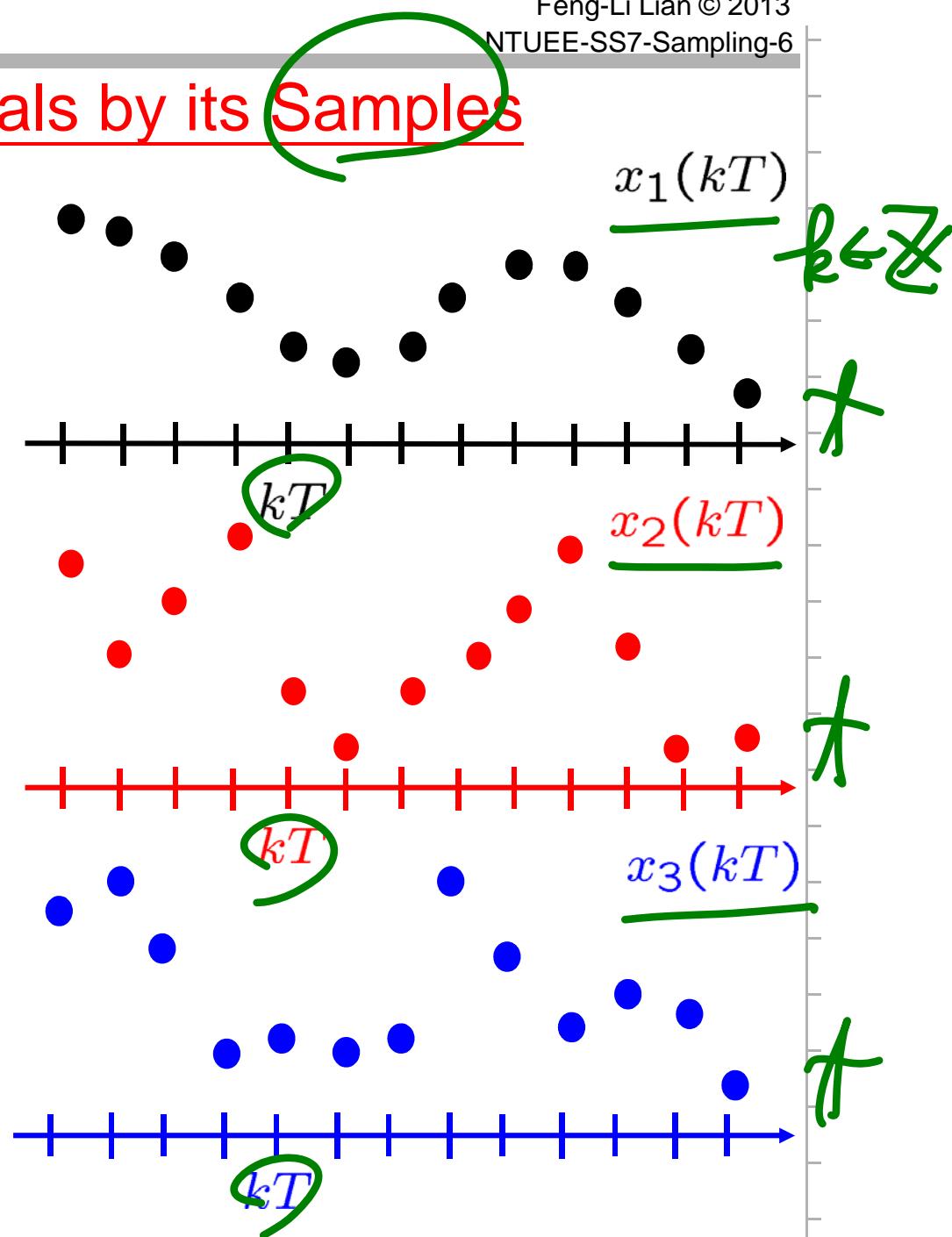
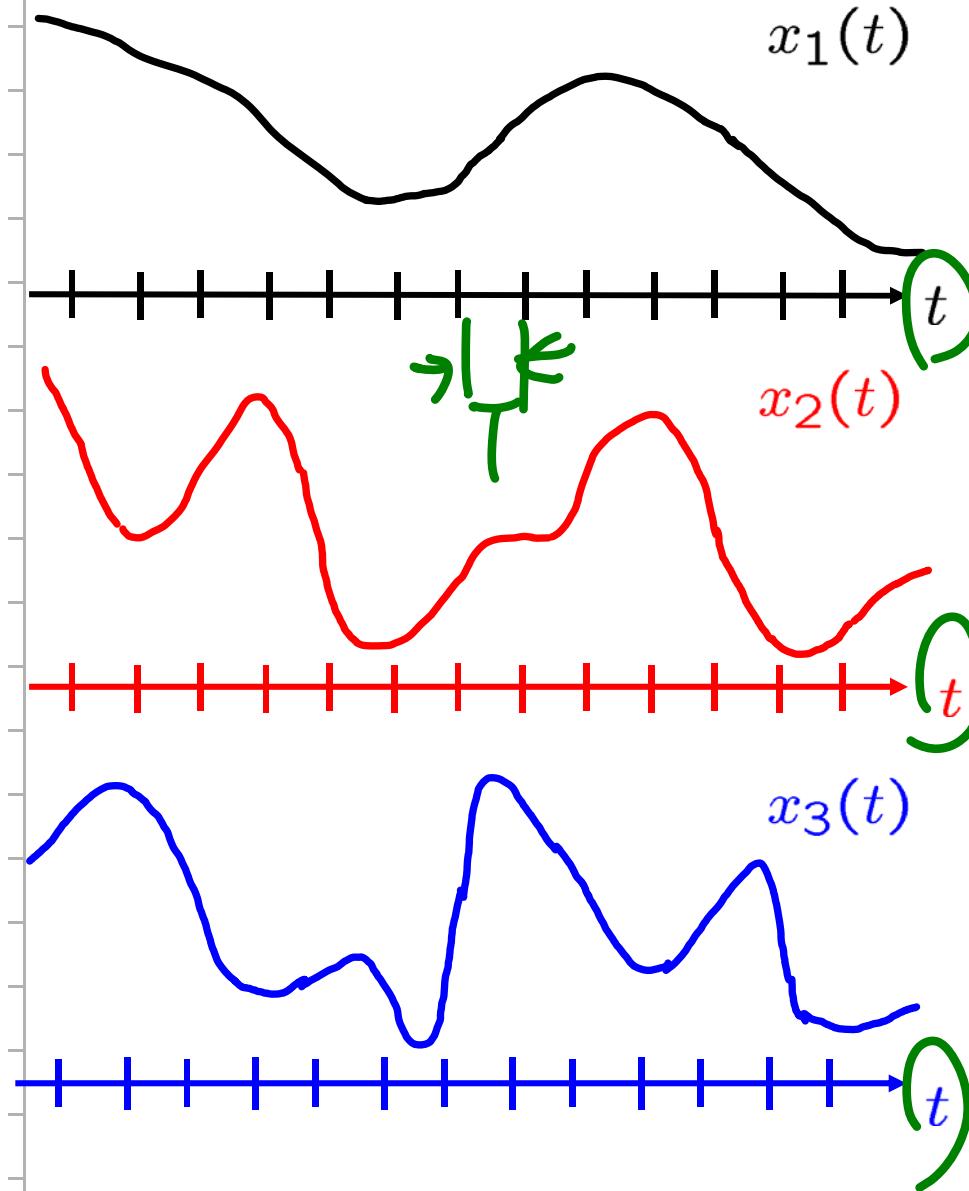
 $x_2(t)$

$\underline{\underline{t}}$

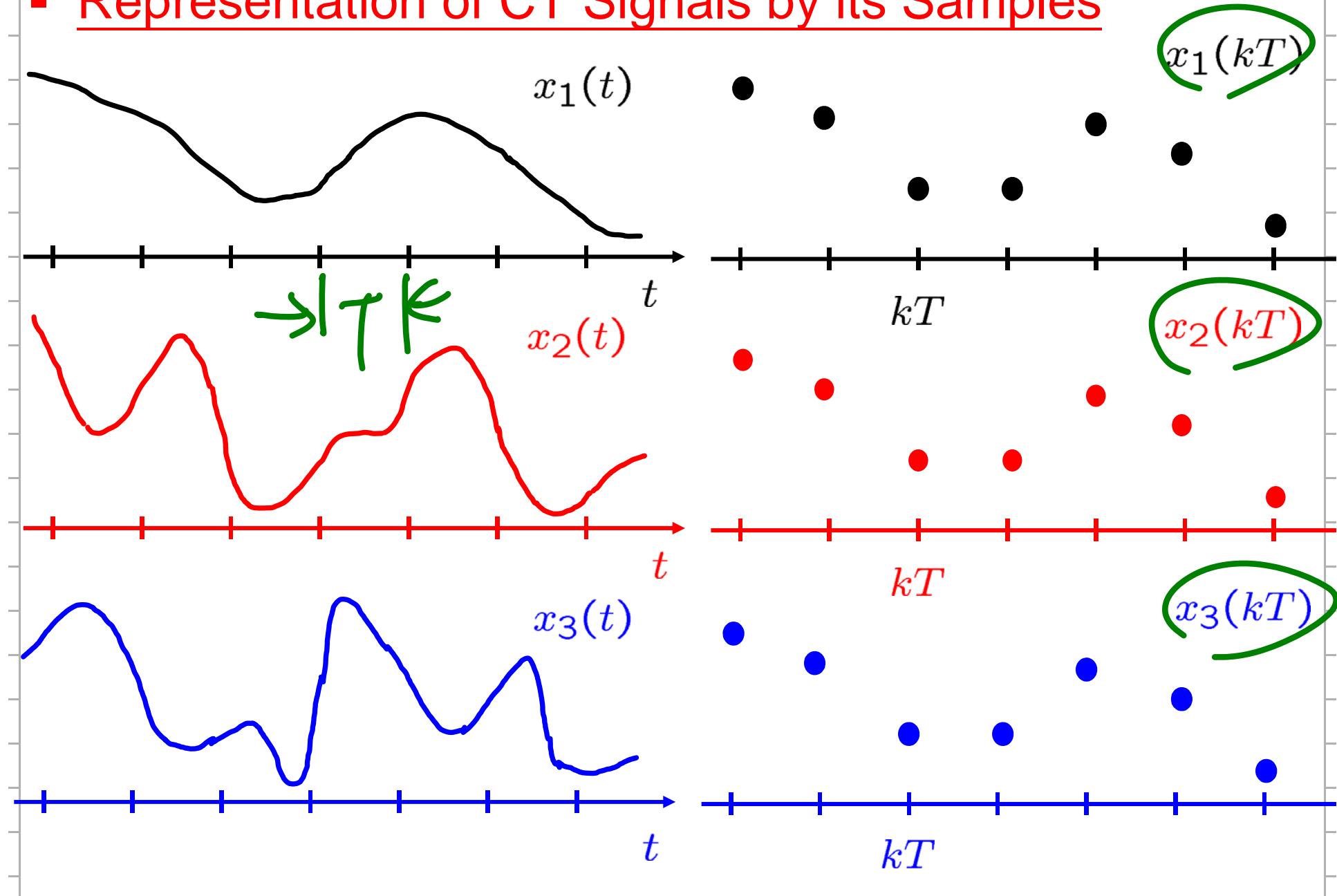
 $x_3(t)$

$\underline{\underline{t}}$

- Representation of CT Signals by its Samples

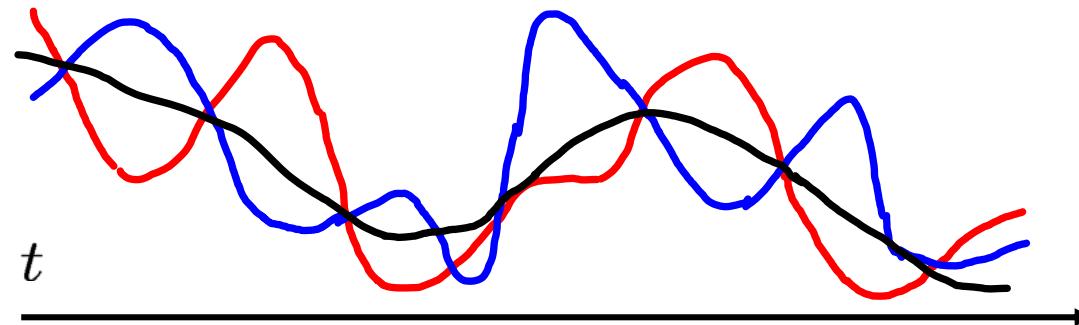


- Representation of CT Signals by its Samples

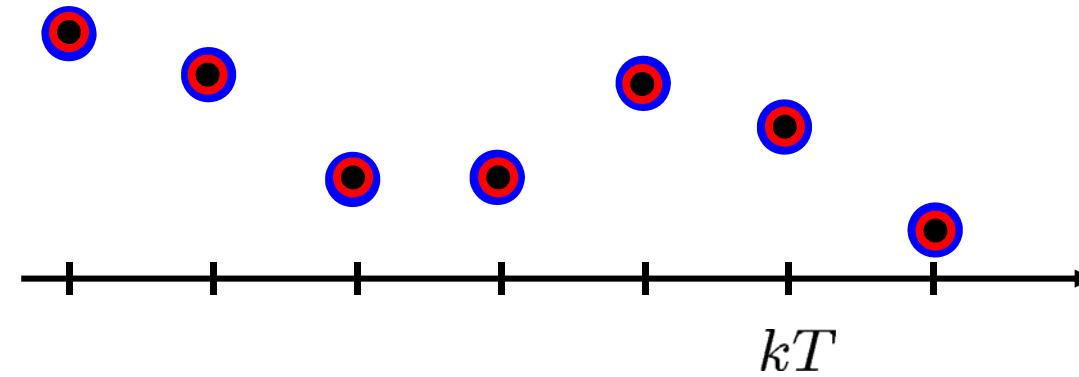


■ Representation of CT Signals by its Samples

$$x_1(t) \neq x_2(t) \neq x_3(t)$$



$$x_1(kT) = x_2(kT) = x_3(kT)$$



■ Impulse-Train Sampling:

$p(t)$: sampling function

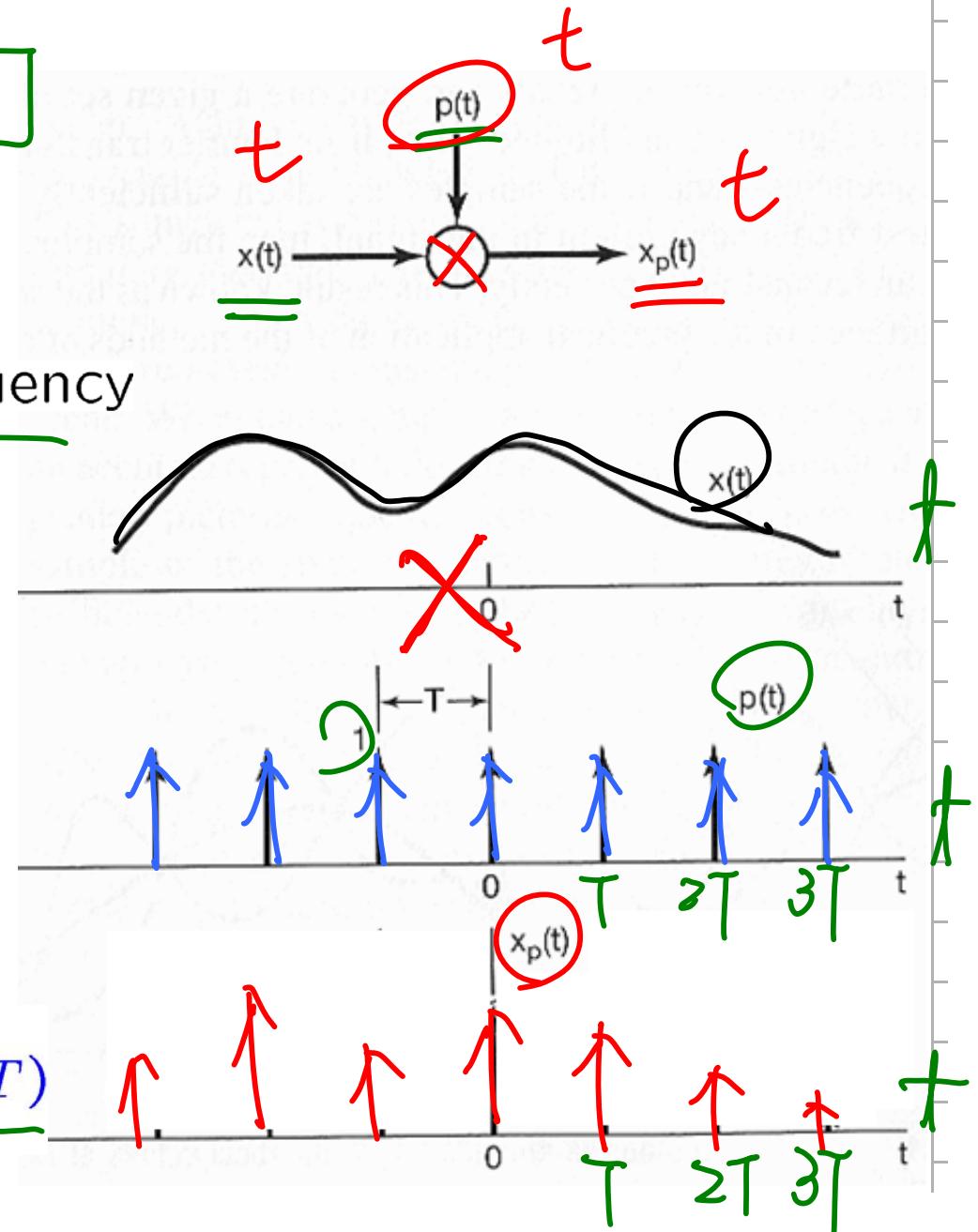
(T) : sampling period

$$w_s = \frac{2\pi}{T} : \text{sampling frequency}$$

$$\Rightarrow x_p(t) = x(t)p(t)$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t - nT)$$



■ Impulse-Train Sampling:

$$x_p(t) = x(t) p(t) \longleftrightarrow X_p(jw)$$

Eq 4.70, p. 322

$$\underline{x(t)} \longleftrightarrow \underline{X(jw)}$$

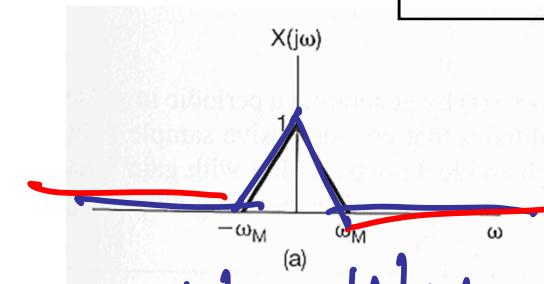
$$p(t) \longleftrightarrow P(jw) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(w - kw_s)$$

Ex 4.8, pp. 299-300

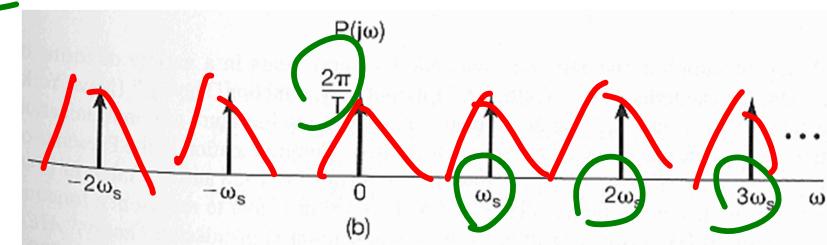
From multiplication property,

$$\begin{aligned} X_p(jw) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) P(j(w - \theta)) d\theta \\ &= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(w - kw_s)) \end{aligned}$$

Ex 4.21, p. 323



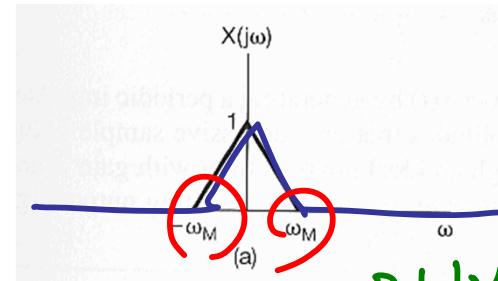
ω_M



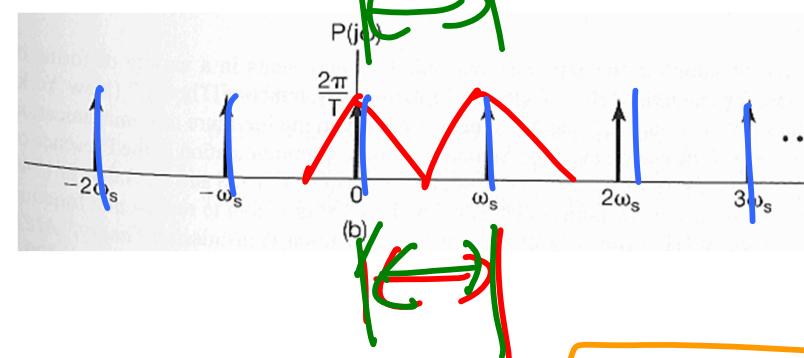
(b)

■ Impulse-Train Sampling:

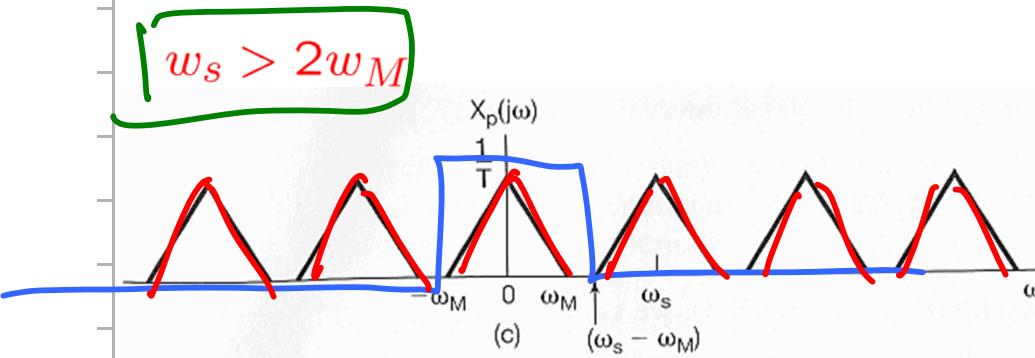
Ex 4.21, 4.22, pp. 323-4



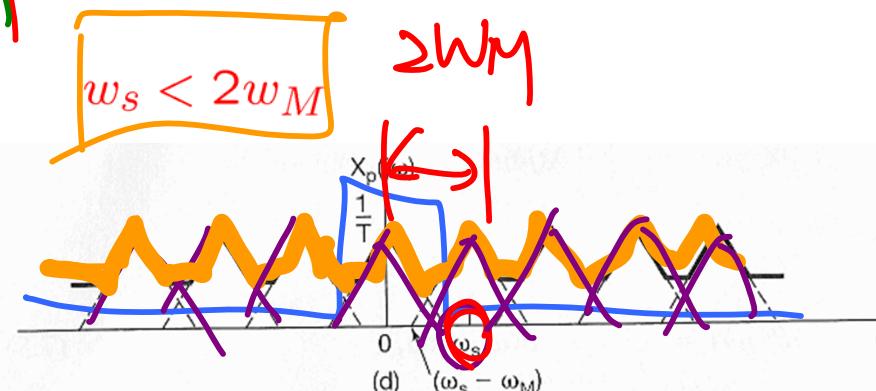
$2w_M$



ω_s



$\frac{1}{T}$



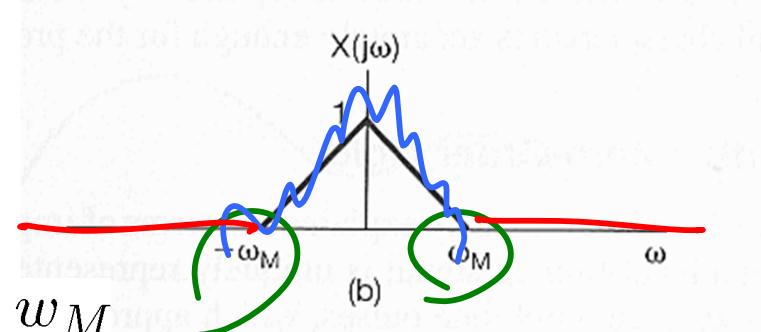
$w_s < 2w_M$

$w_s > 2w_M$

■ The Sampling Theorem:

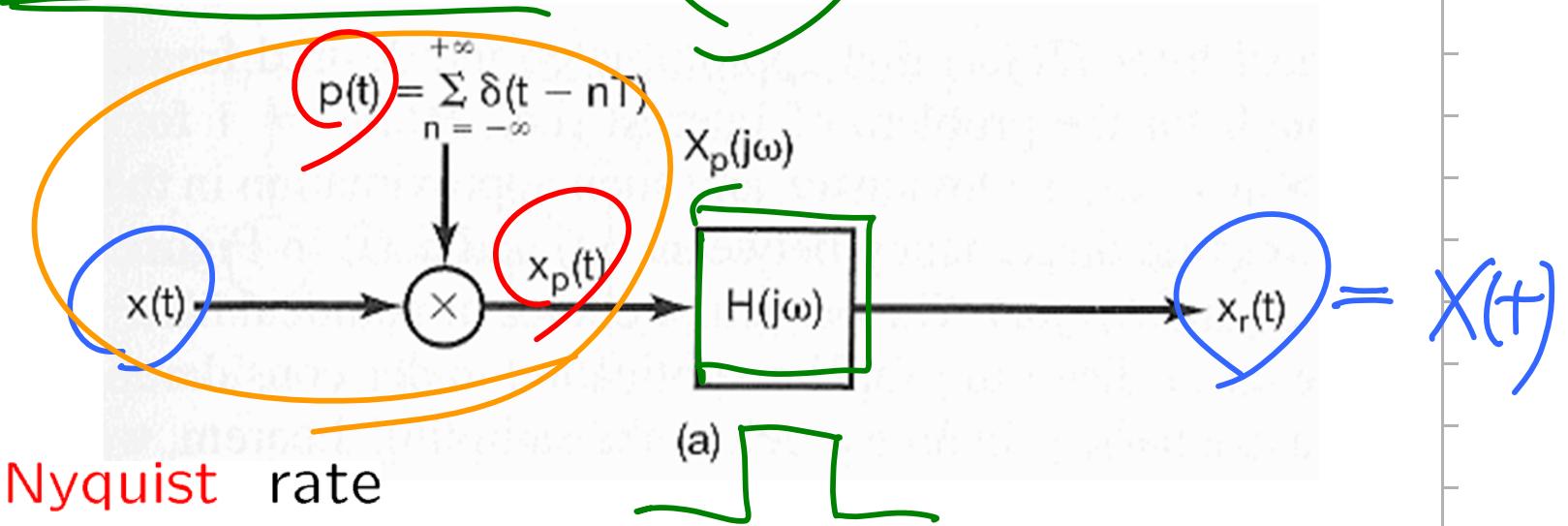
$x(t)$: a band-limited signal

\Leftrightarrow with $X(jw) = 0$ for $|w| > w_M$



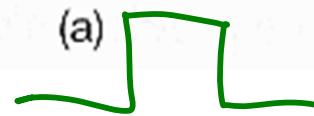
if $w_s > 2w_M$ where $w_s = \frac{2\pi}{T}$

$\Rightarrow x(t)$ is uniquely determined by $x(nT), n = 0, \pm 1, \pm 2, \dots$

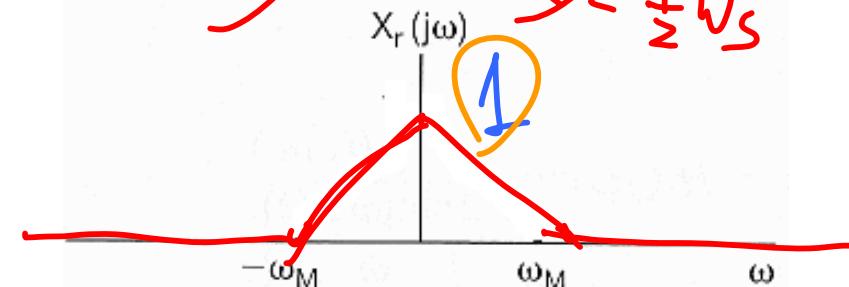
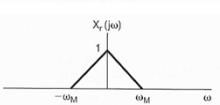
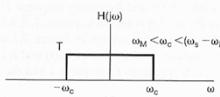
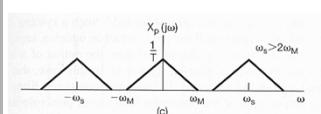
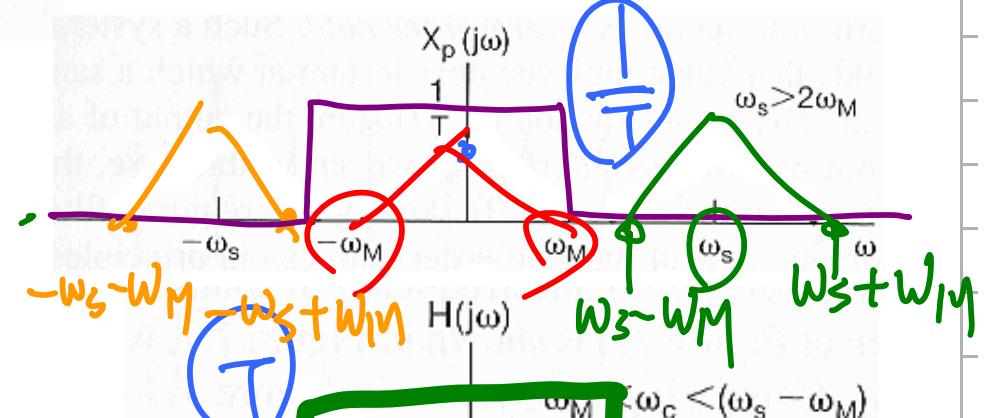
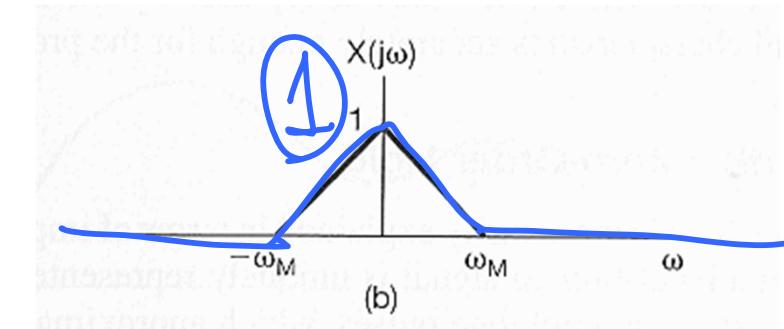
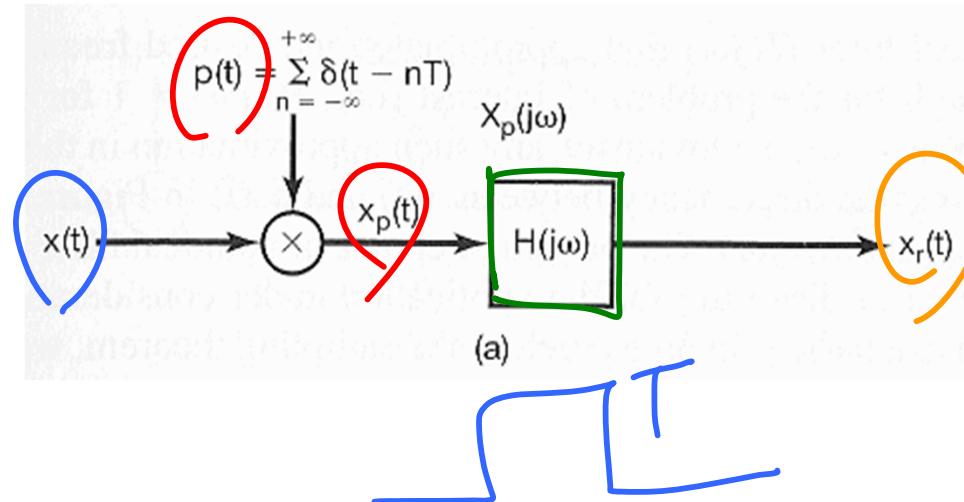


$\Rightarrow 2w_M$: Nyquist rate

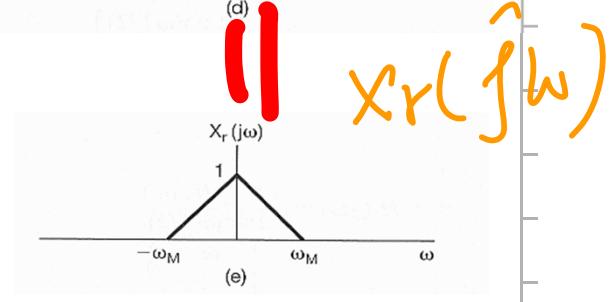
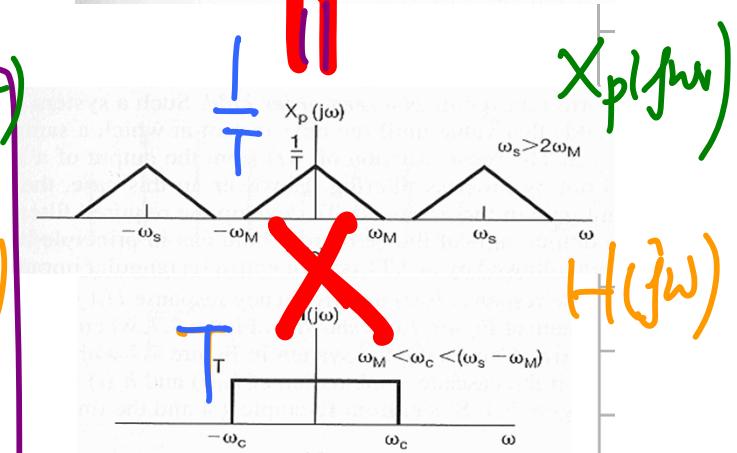
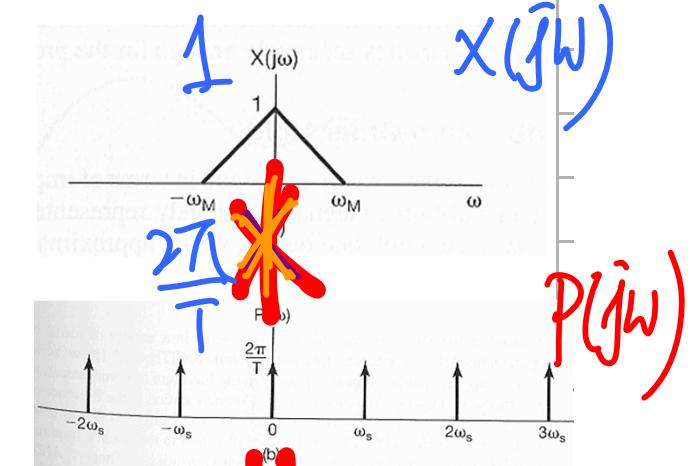
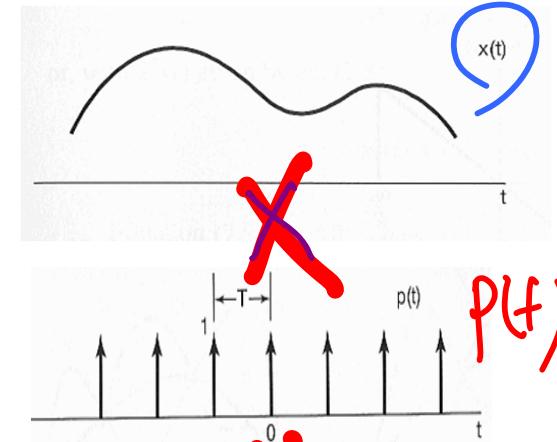
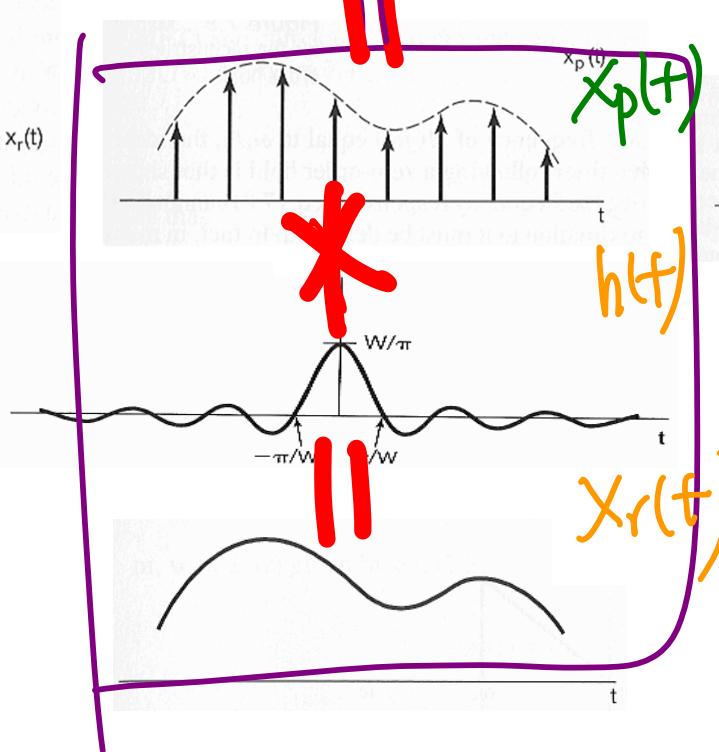
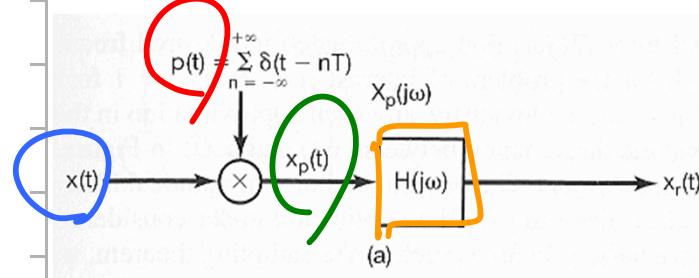
w_M : Nyquist frequency



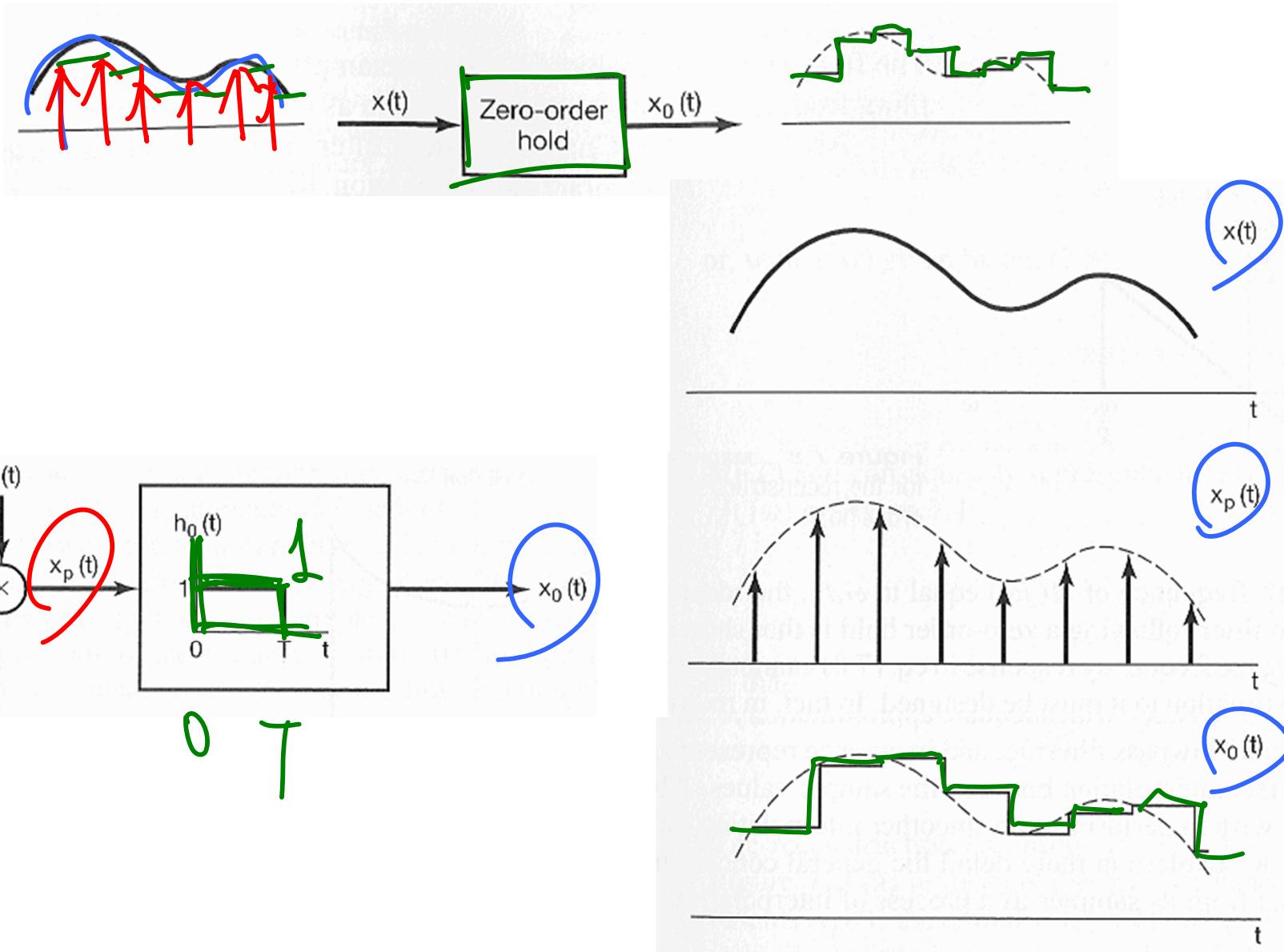
Exact Recovery by an Ideal Lowpass Filter:



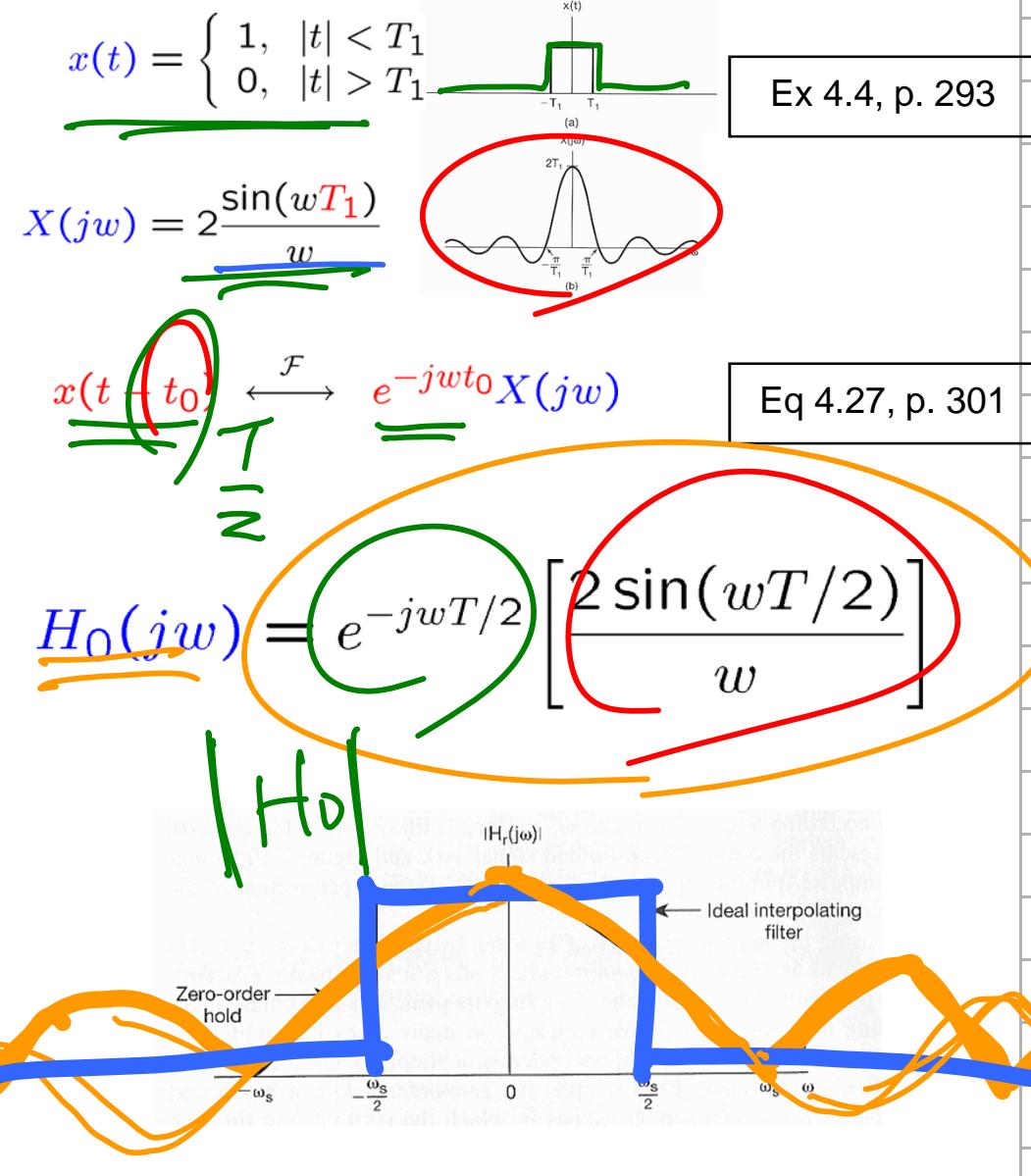
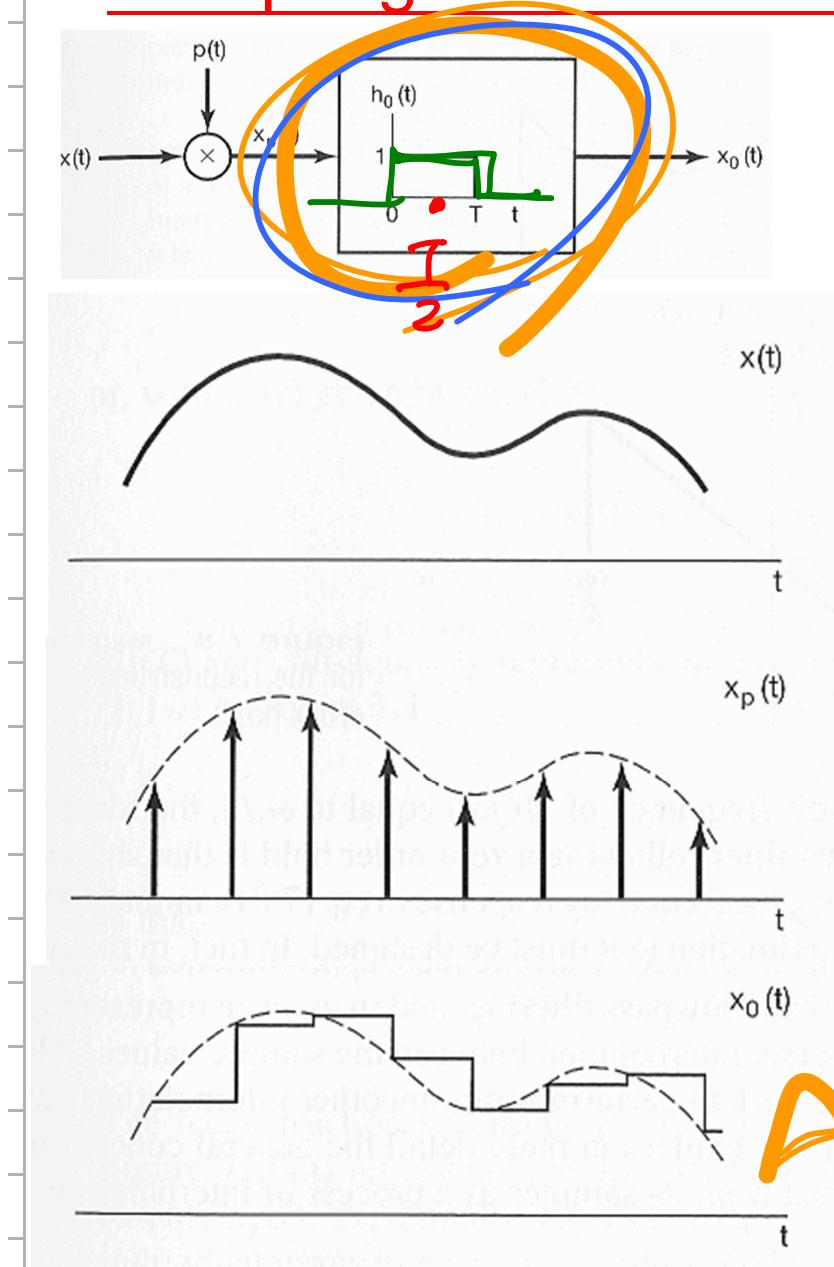
Exact Recovery by an Ideal Lowpass Filter:



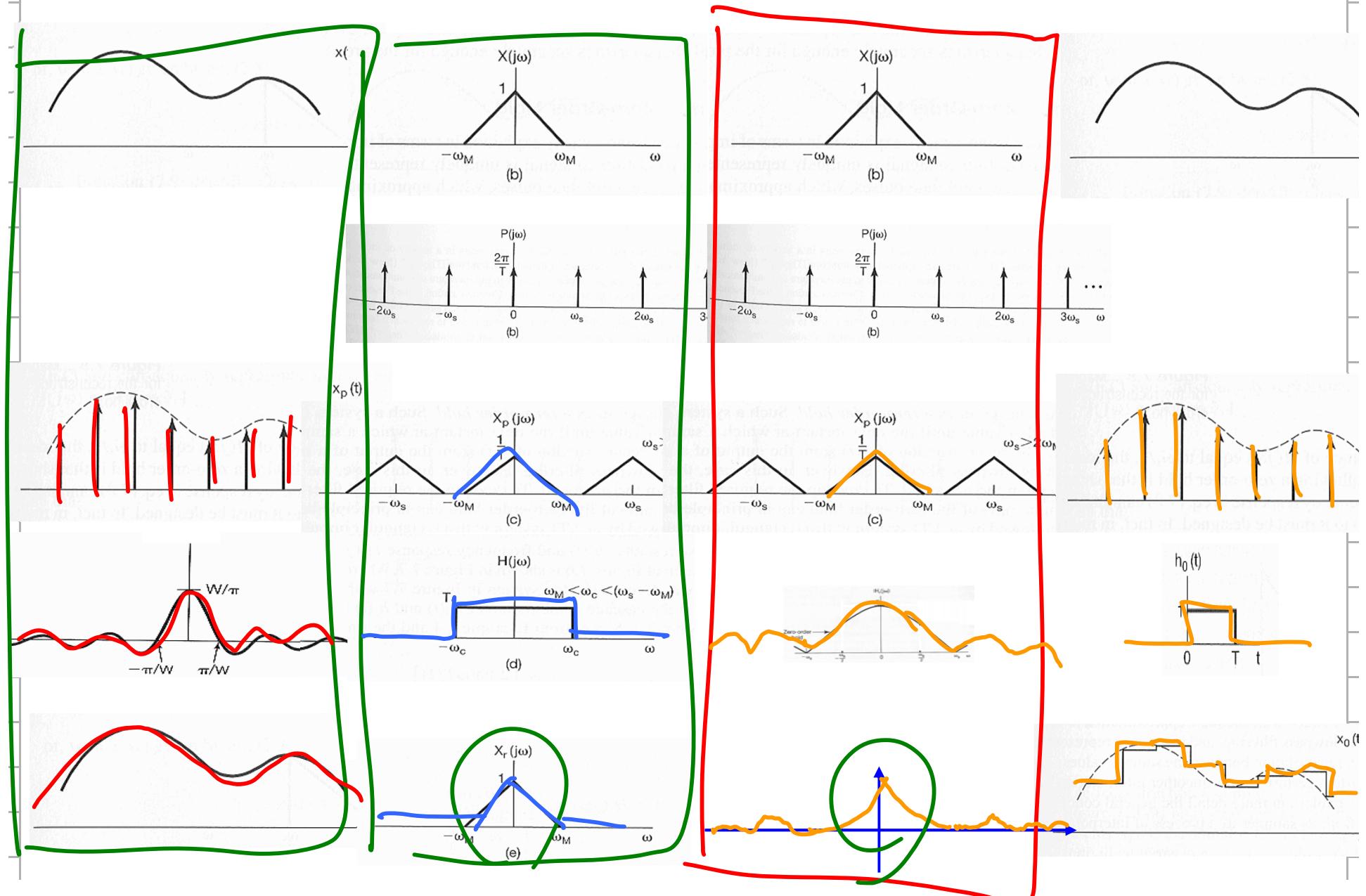
■ Sampling with Zero-Order Hold:



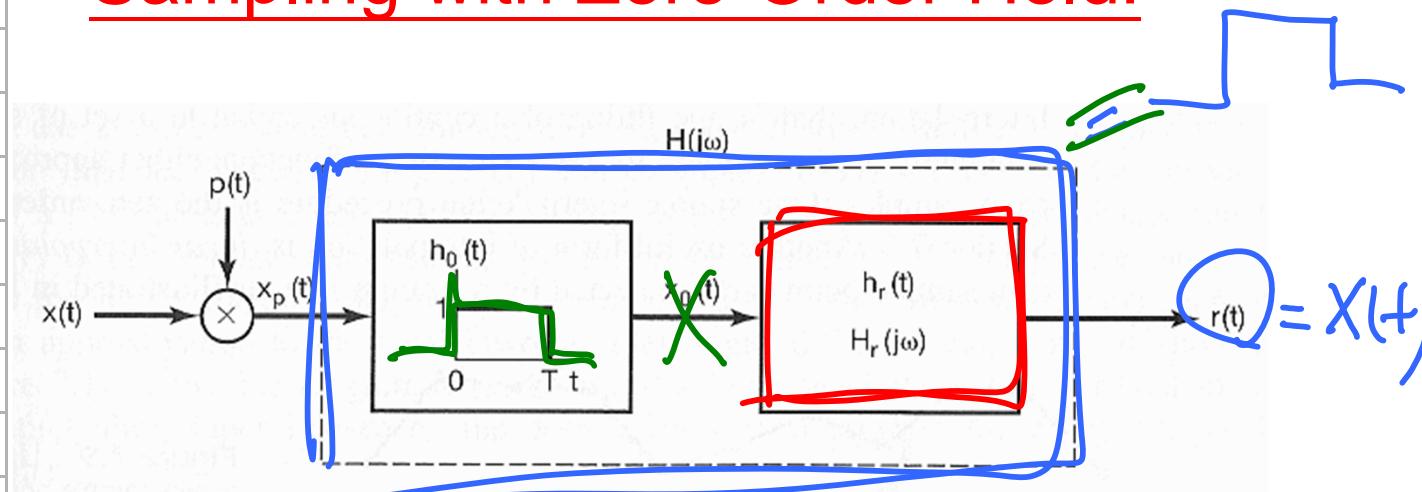
■ Sampling with Zero-Order Hold:



With Ideal Lowpass Filter & with Zero-Order Hold:



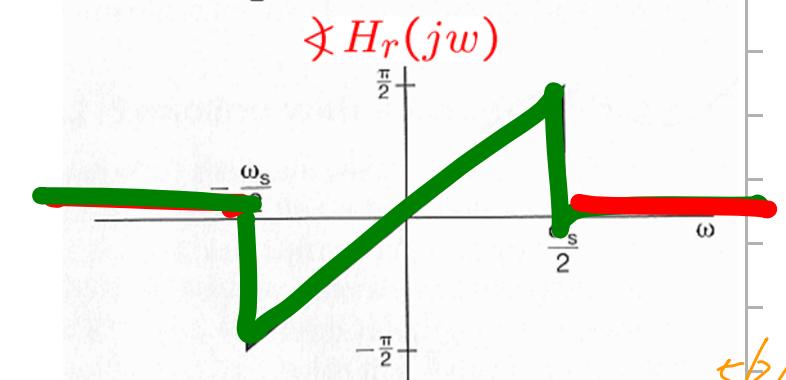
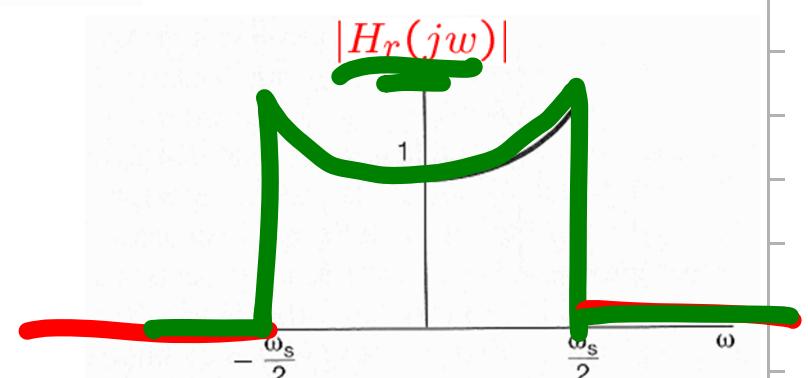
- Sampling with Zero-Order Hold:



$$H_0(jw) = e^{-jwT/2} \left[\frac{2 \sin(wT/2)}{w} \right]$$

$$H(jw) = H_0(jw)H_r(jw)$$

$$\Rightarrow H_r(jw) = \frac{e^{jwT/2} H(jw)}{\frac{2 \sin(wT/2)}{w}}$$



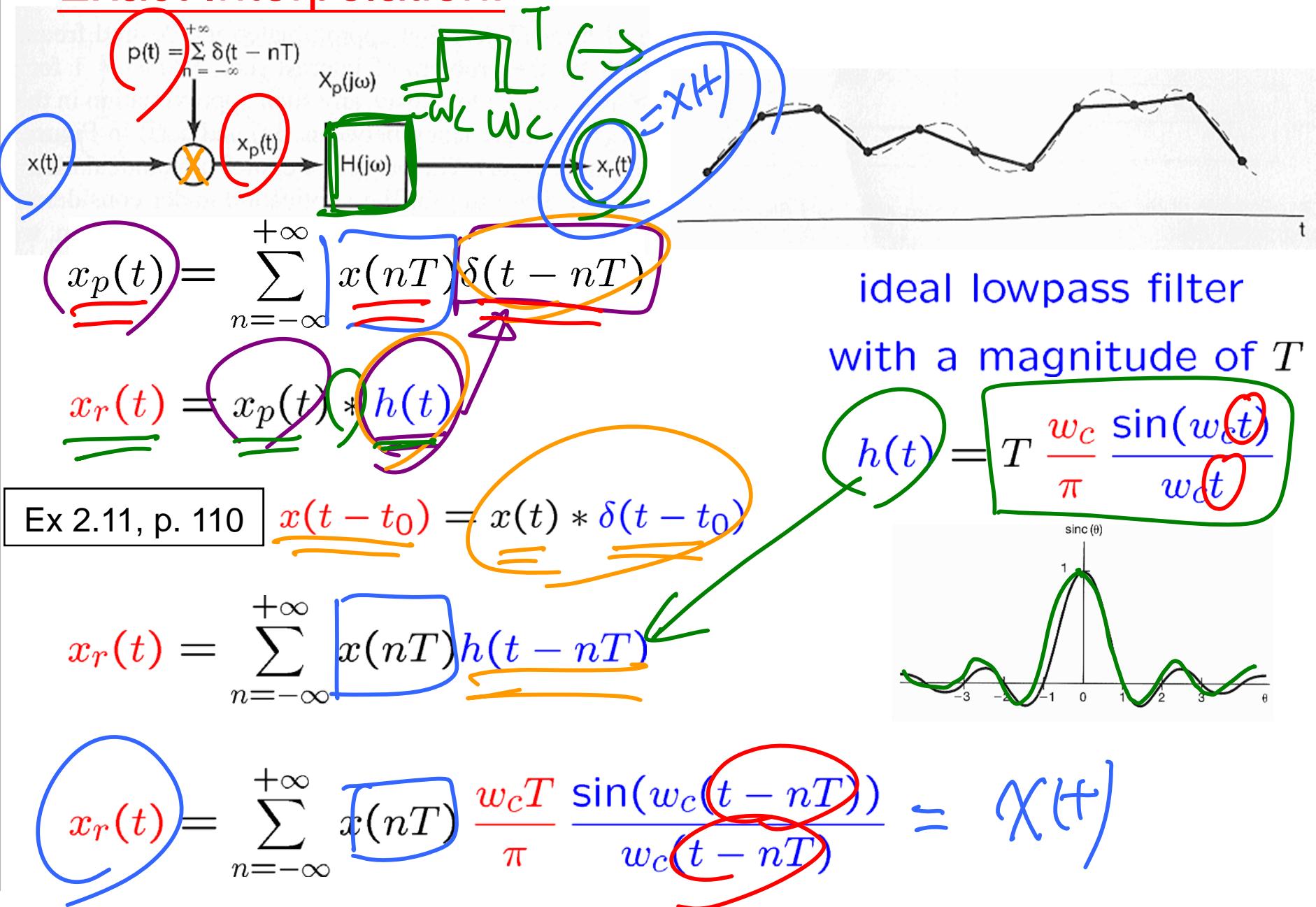
5/2/13
z=15pm

- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
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- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

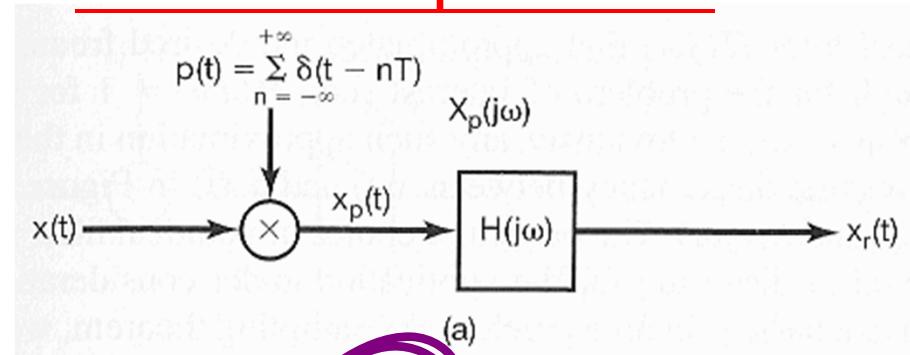
Reconstruction of a Signal from its Samples Using Interpolation

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Exact Interpolation:



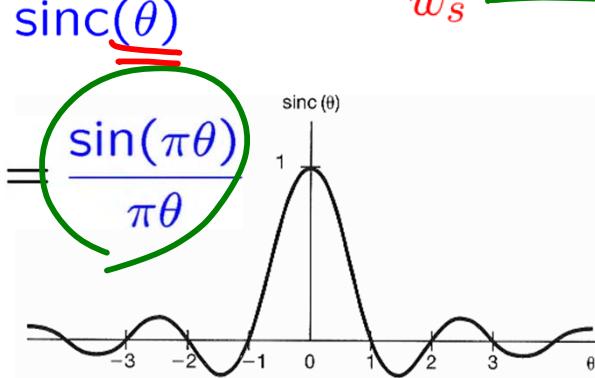
■ Exact Interpolation:



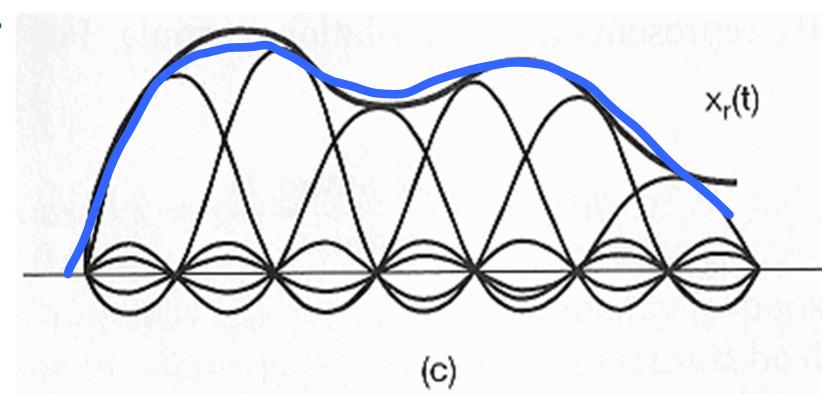
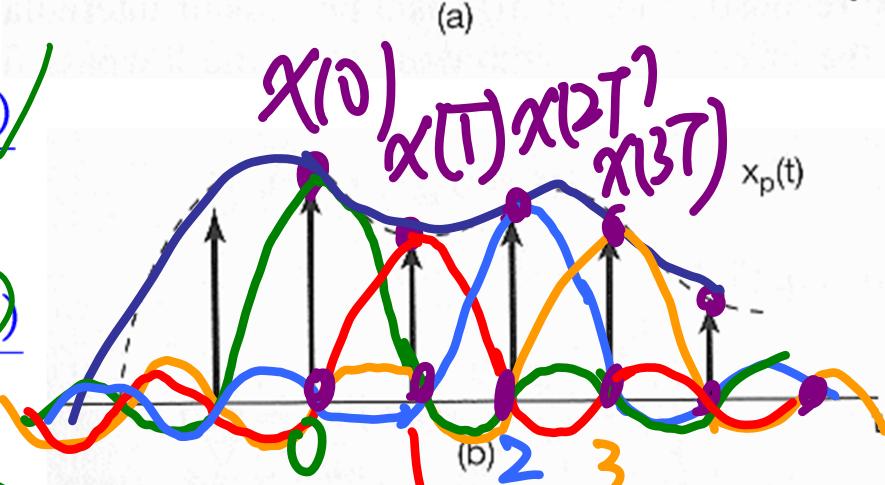
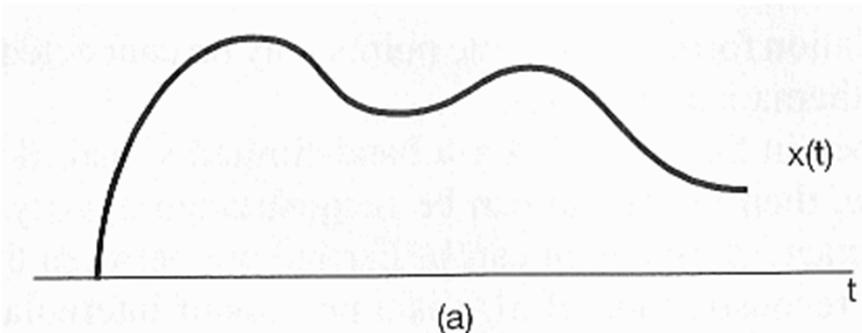
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{w_c T}{\pi} \frac{\sin(w_c(t - nT))}{w_c(t - nT)}$$

$$\frac{w_c}{\pi} \frac{2\pi}{w_s} \frac{\sin(\pi(w_c(t - nT)/\pi))}{\pi w_c(t - nT)/\pi}$$

$$\frac{2w_c}{w_s} \text{sinc}(\frac{w_c(t - nT)}{\pi})$$

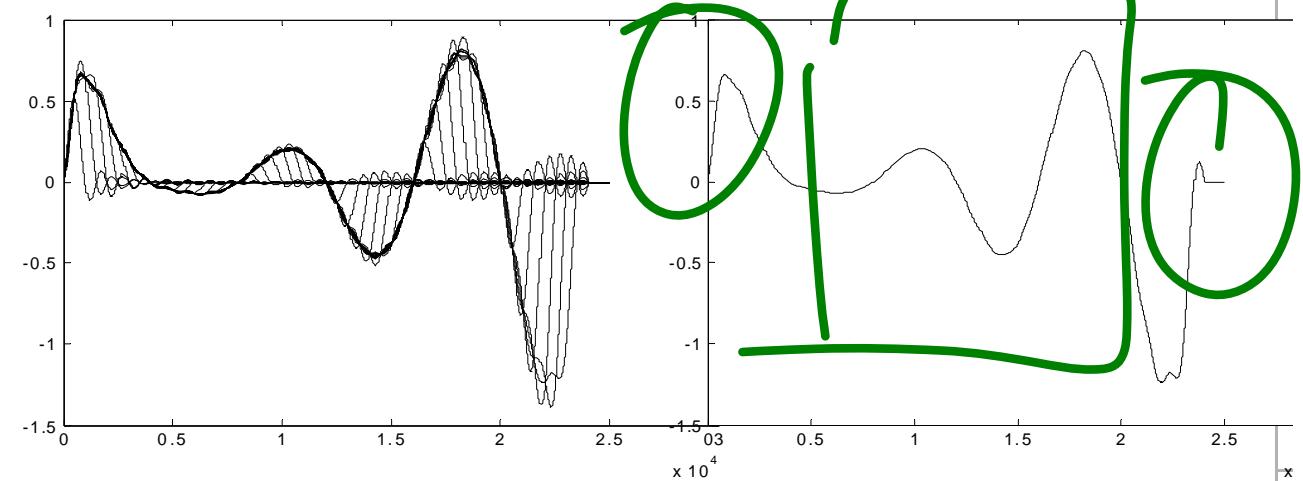
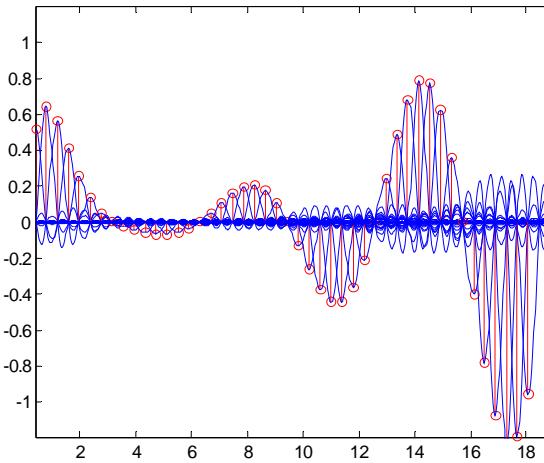
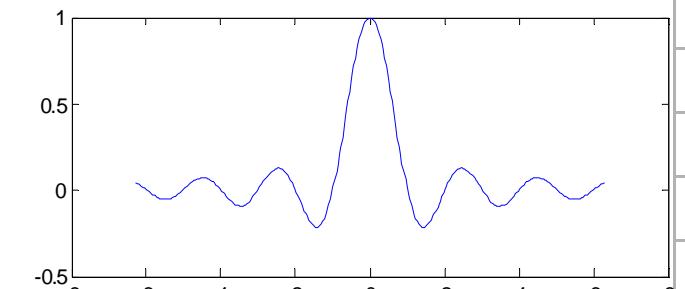
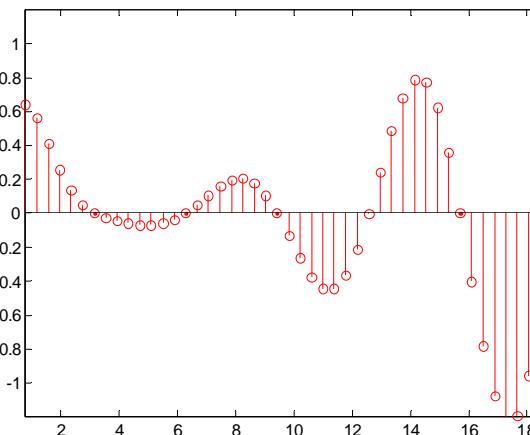
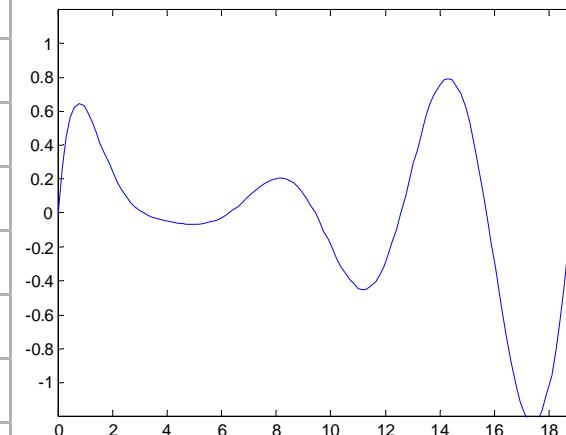


$n=0$
 $n=1$
 $n=2$
 $n=3$

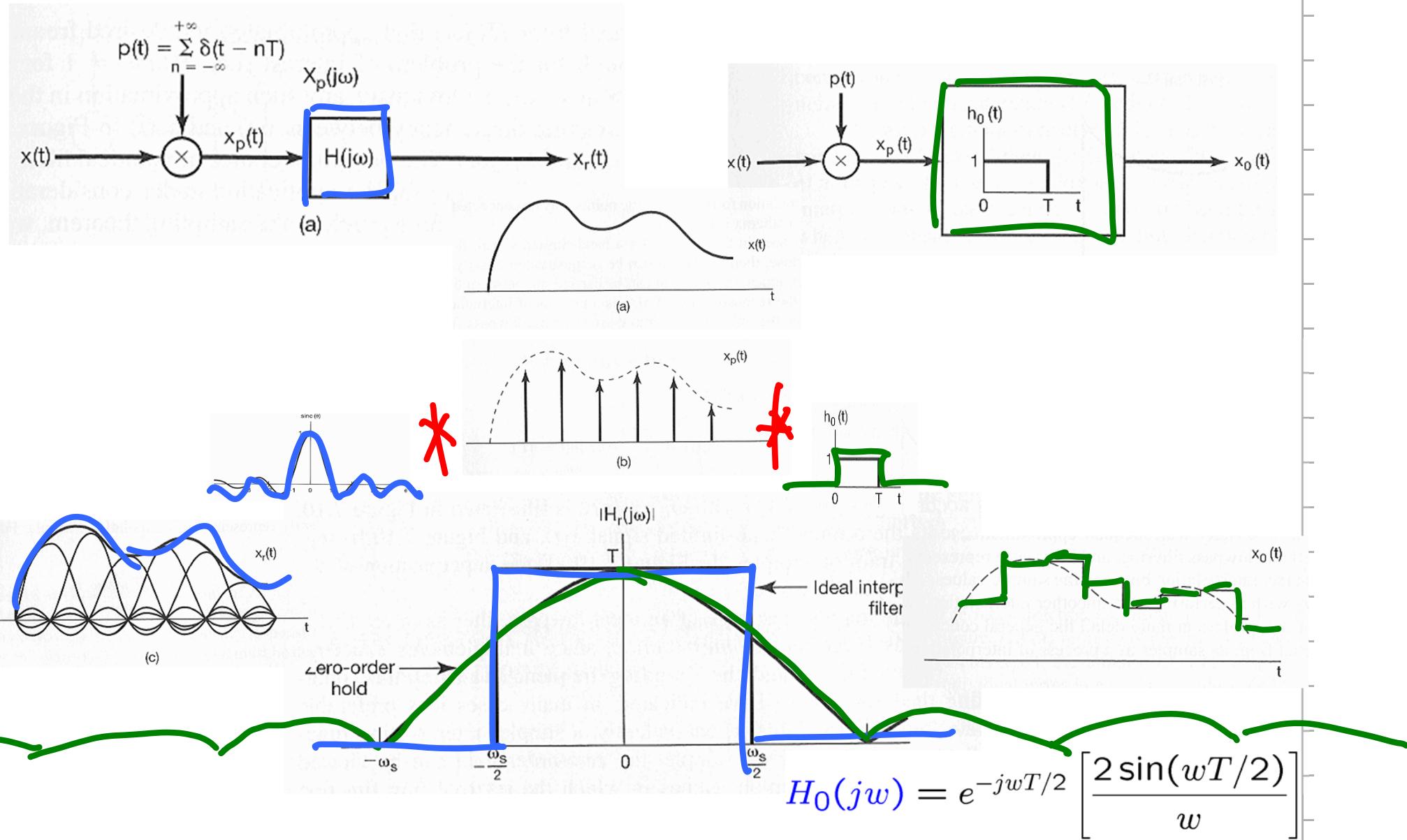


Reconstruction of a Signal from its Samples Using Interpolation

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■ Ideal Interpolating Filter & The Zero-Order Hold:



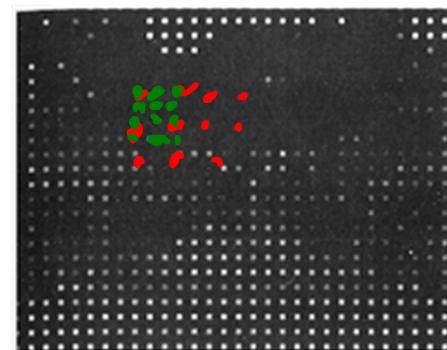
■ Sampling & Interpolation of Images:

original image

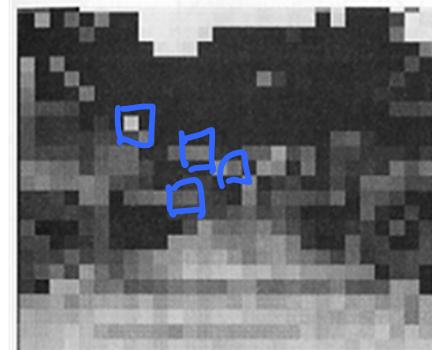


(a)

impulse sampling



zero-order hold

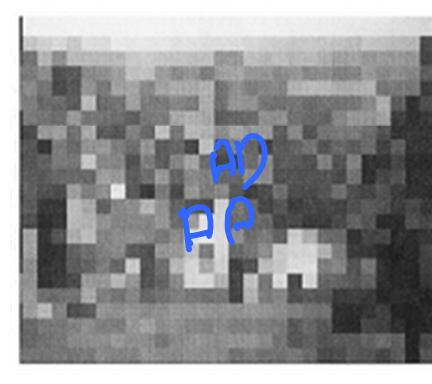
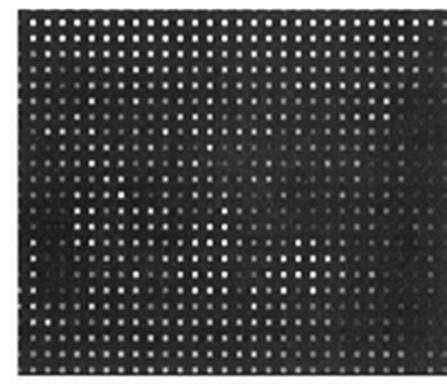


4 : 1

zero-order hold



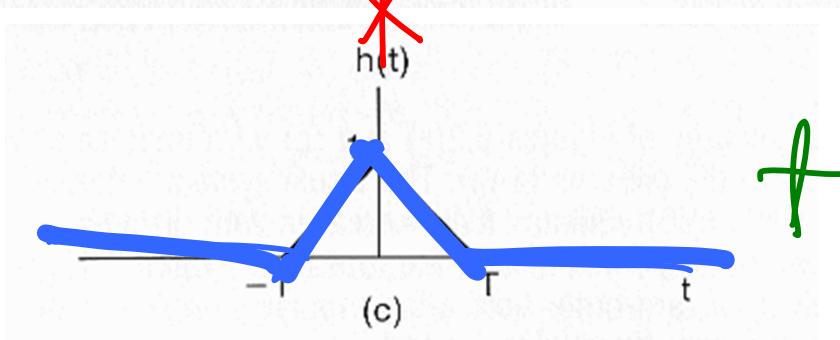
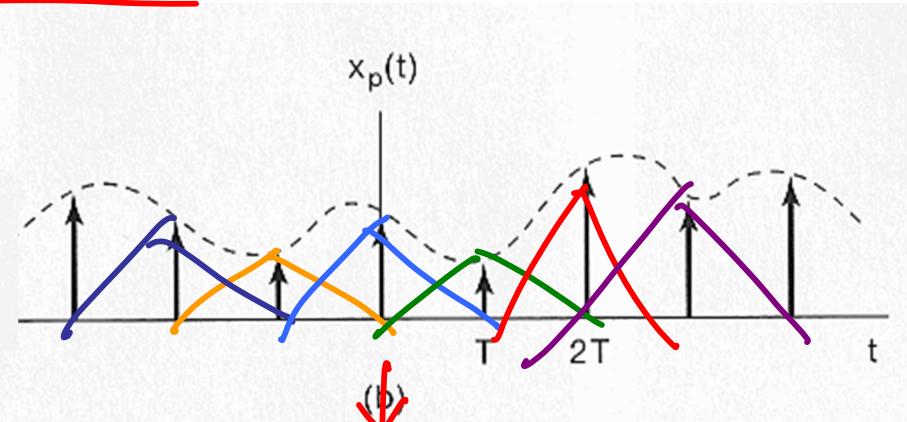
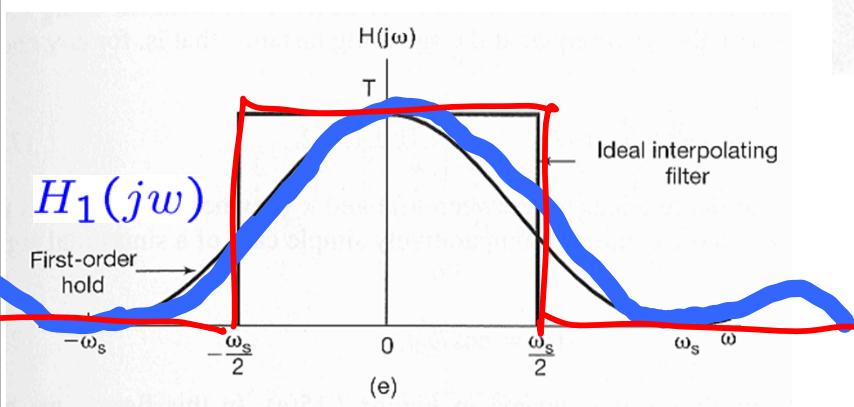
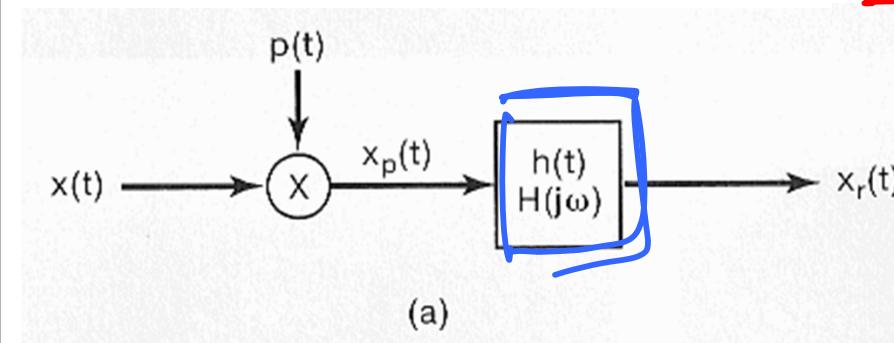
(g)



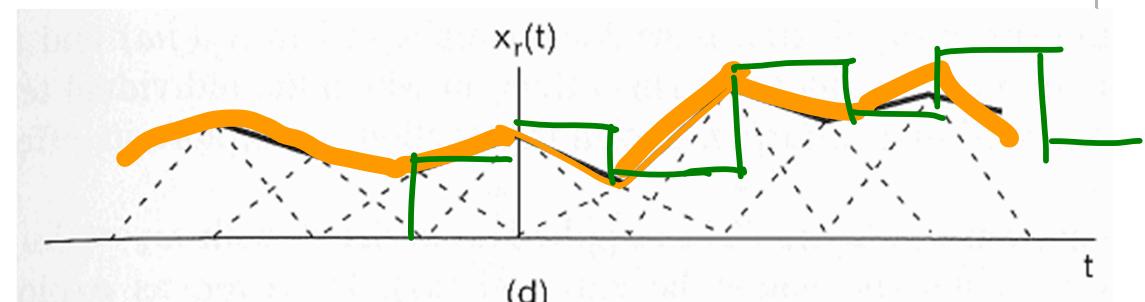
Reconstruction of a Signal from its Samples Using Interpolation

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■ Higher-Order Holds: 1st - order



$$H_1(j\omega) = \frac{1}{T} \left[\frac{\sin(\omega T/2)}{\omega/2} \right]^2$$

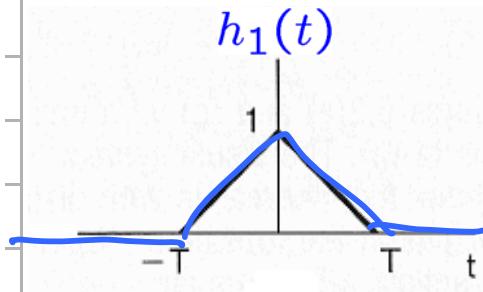
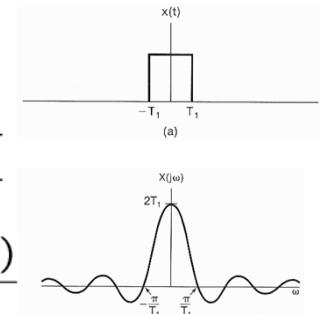
Reconstruction of a Signal from its Samples Using Interpolation

■ Higher-Order Holds:

Ex 4.4, p. 293

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$

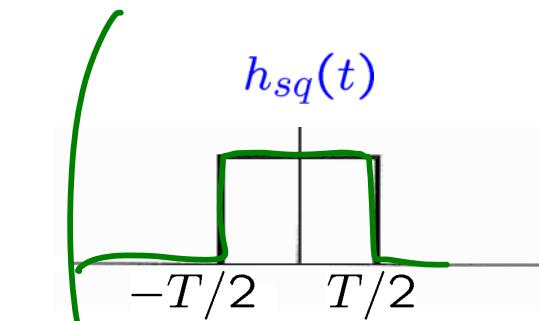
$$X(jw) = 2 \frac{\sin(wT_1)}{w}$$



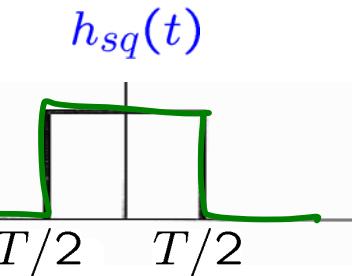
$$= \frac{1}{T}$$

$$= \frac{1}{T}$$

$$= \frac{1}{T} \left[\frac{\sin(wT/2)}{w/2} \right]^2$$

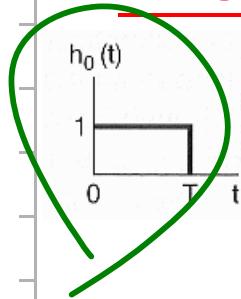


*



$$2 \frac{\sin(wT/2)}{w}$$

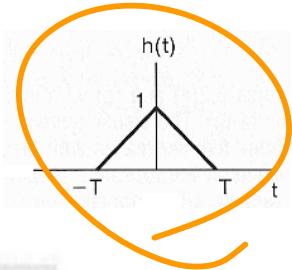
■ First-Order Hold on Image Processing:



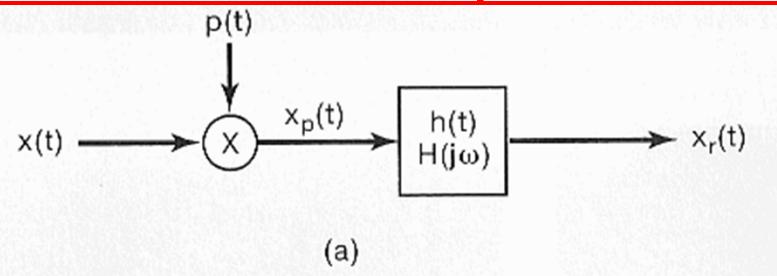
zero-order hold



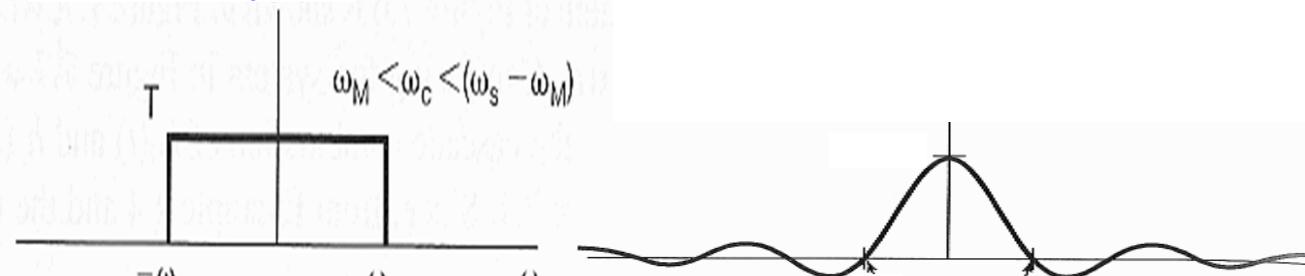
first-order hold



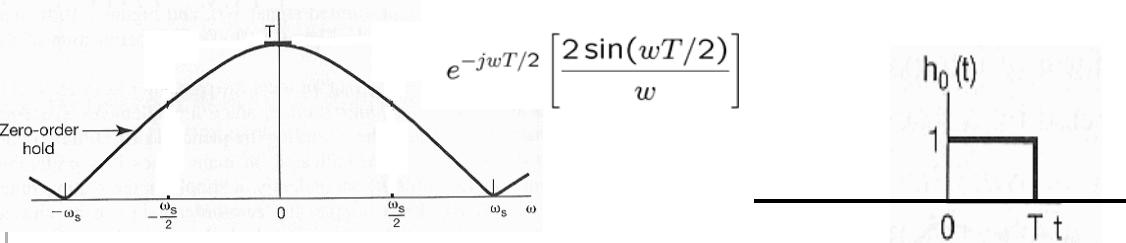
■ Three Filters: Ideal Lowpass, Zero-Order, First-Order



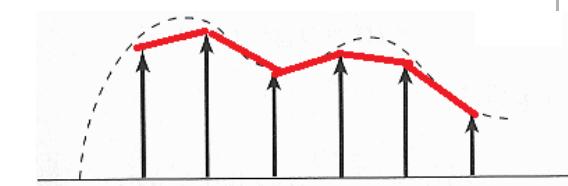
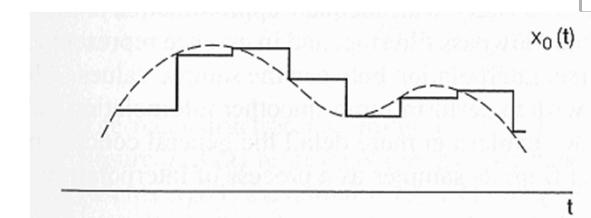
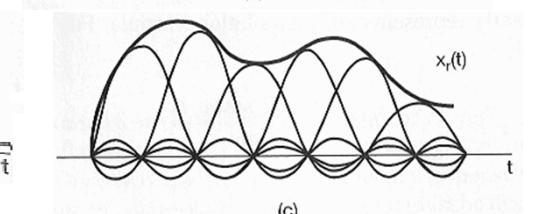
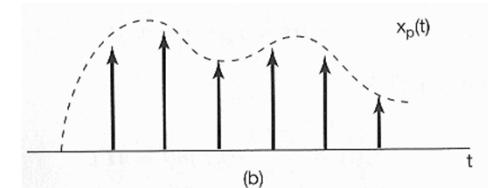
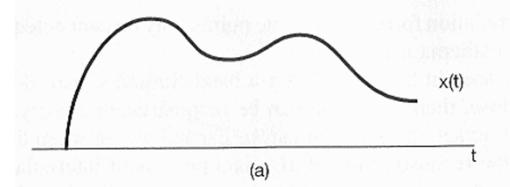
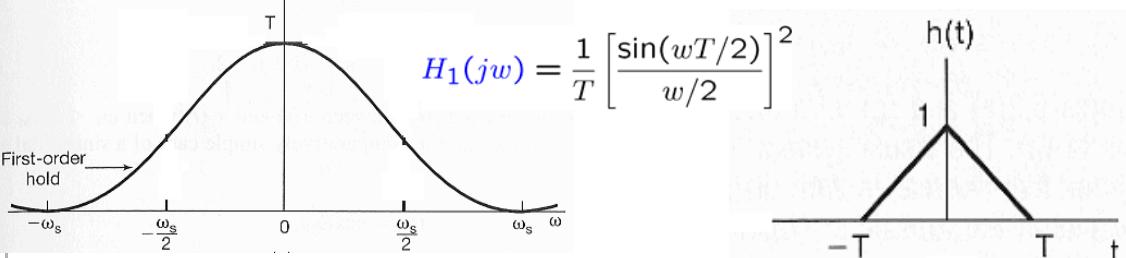
ideal lowpass



zero-order hold $H_0(jw) =$

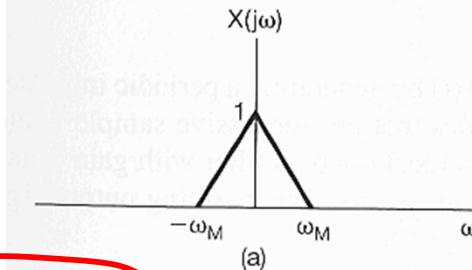


first-order hold

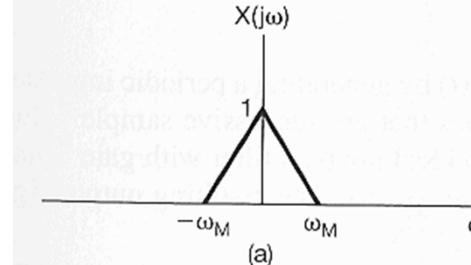
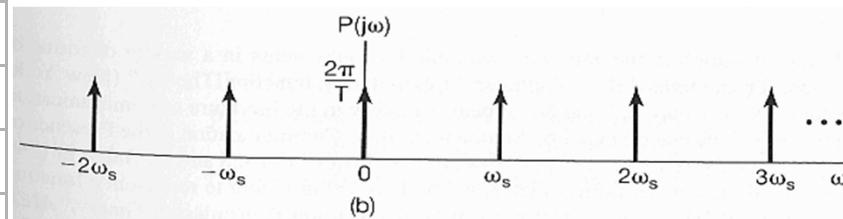


- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

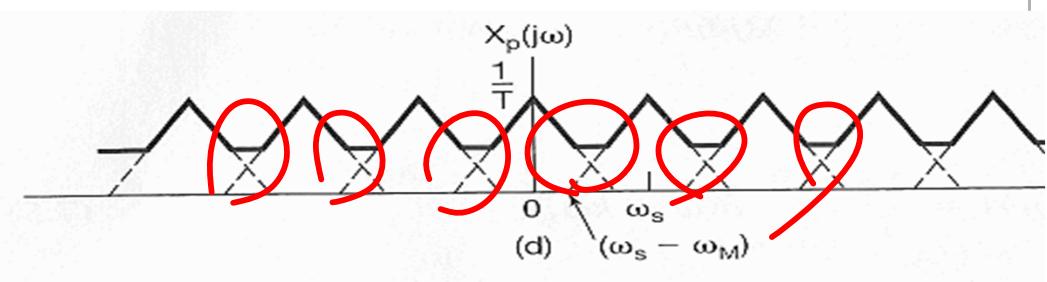
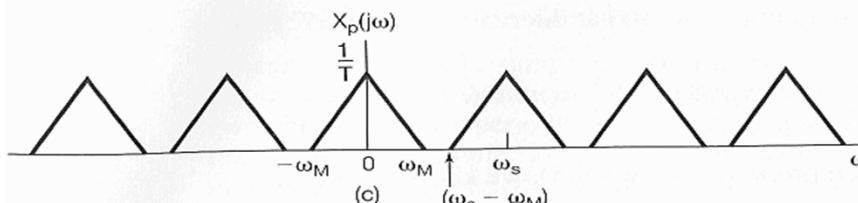
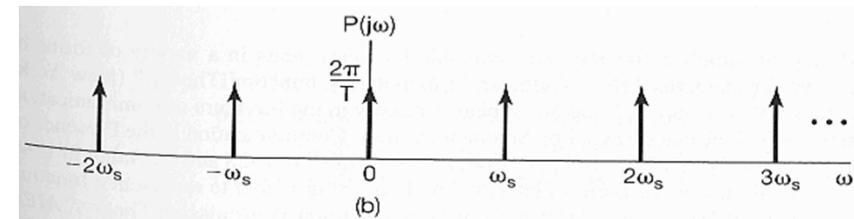
■ Overlapping in Frequency-Domain: Aliasing



$$w_s > 2w_M$$

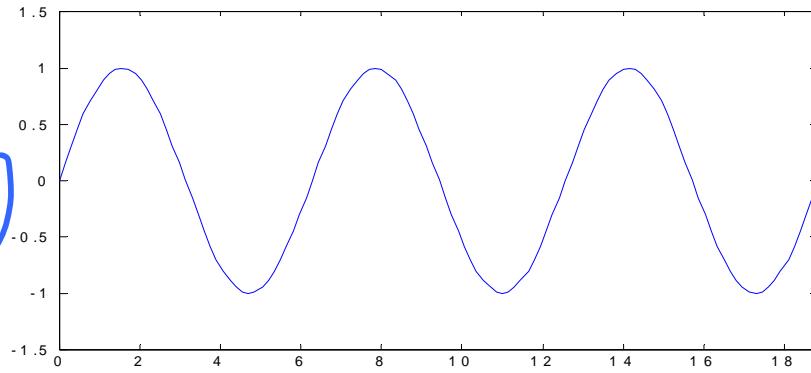


$$w_s < 2w_M$$

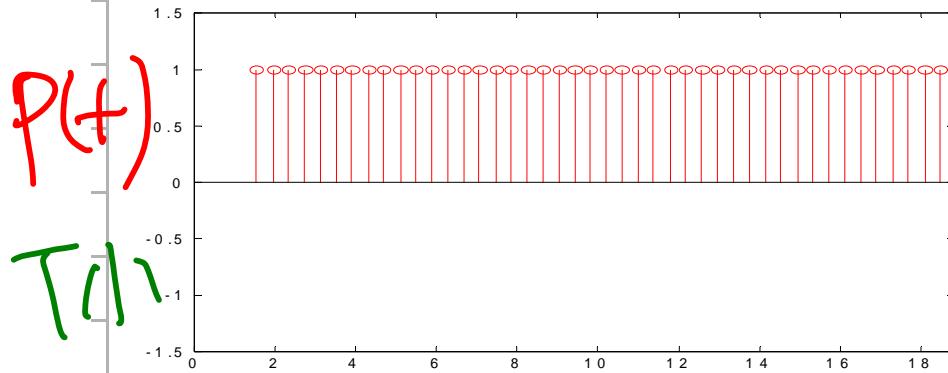


■ Overlapping in Frequency-Domain: Aliasing

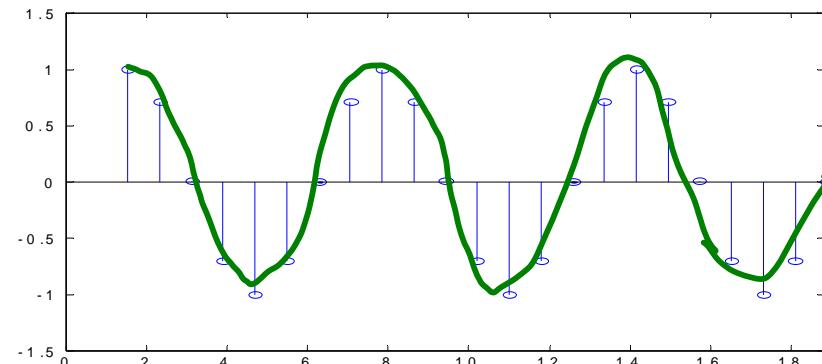
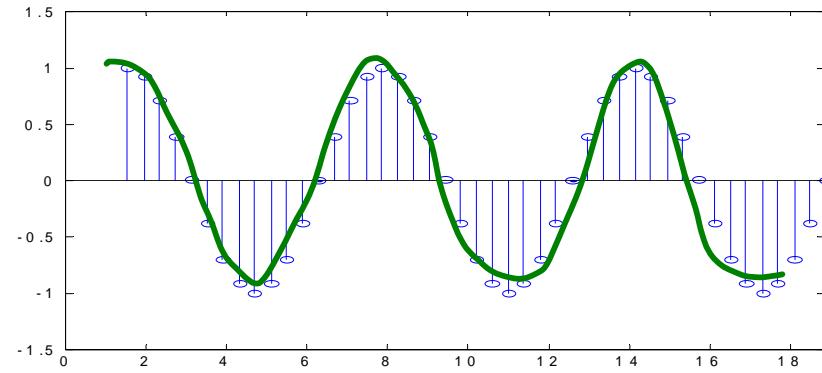
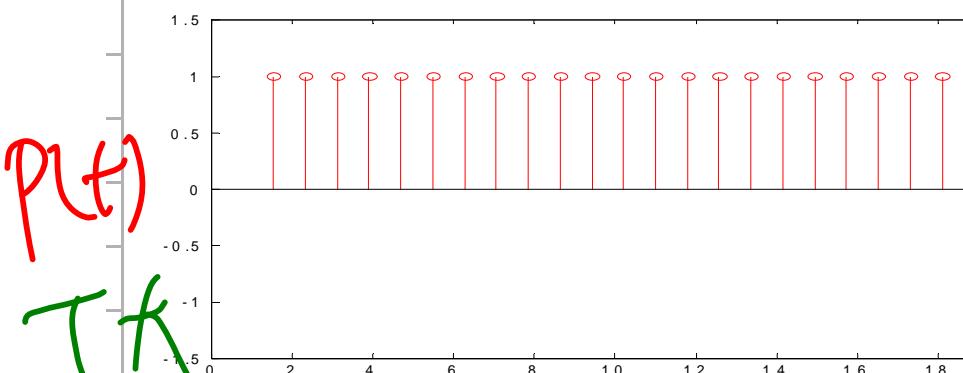
$X(t)$



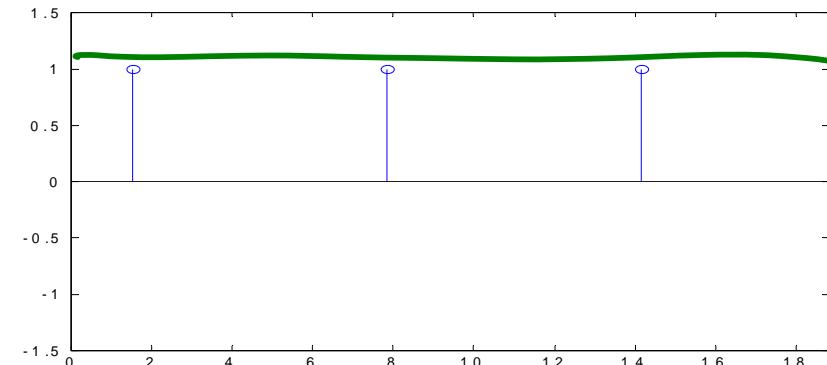
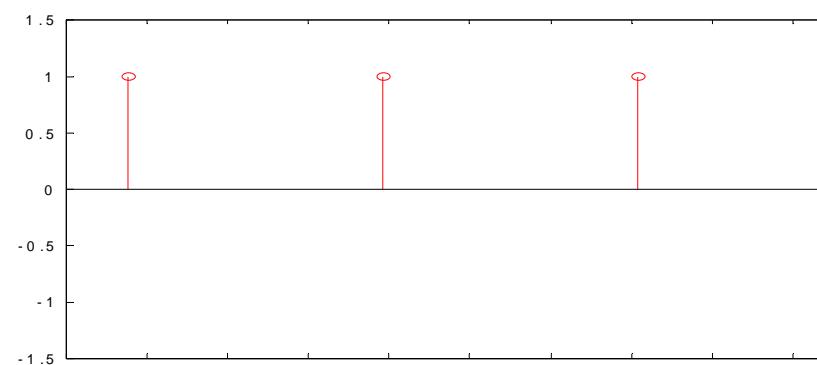
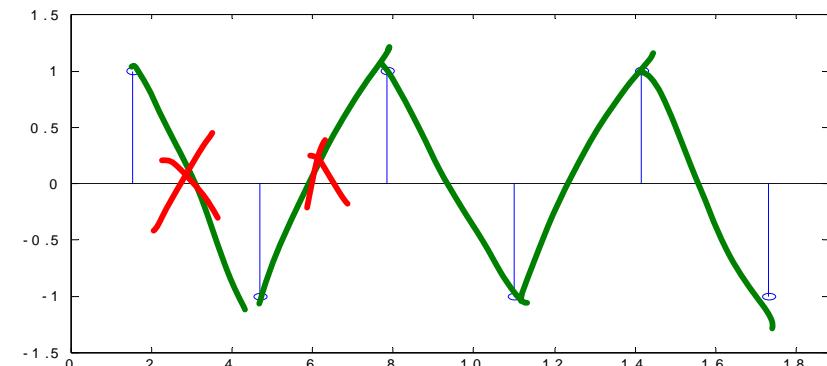
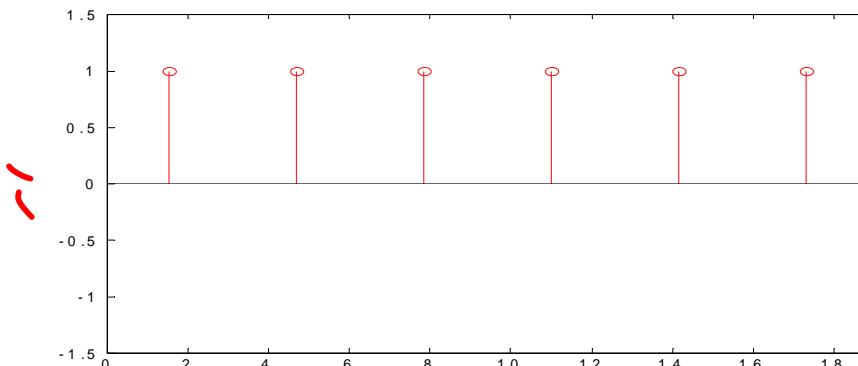
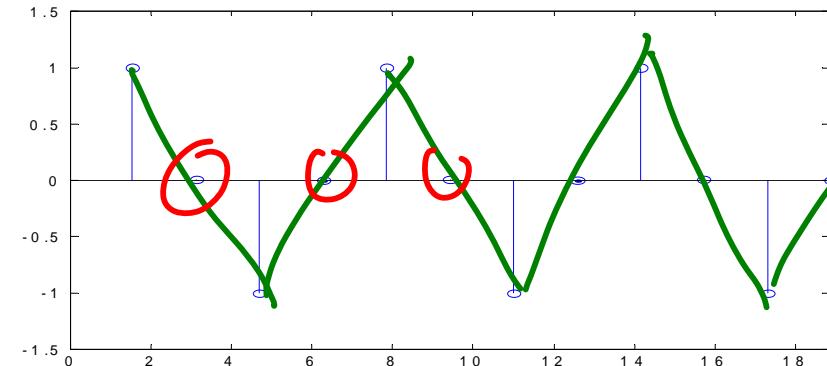
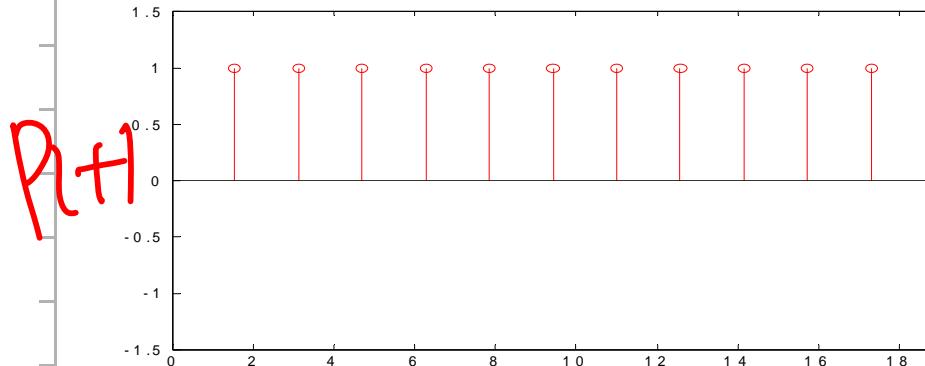
$P(t)$



$T(k)$



■ Overlapping in Frequency-Domain: Aliasing



▪ Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(\omega_0 t)$$

$-\omega_0$

ω_0

ω

$$w_s > 2\omega_0$$

$-w_s$

$-w_s/2$

$-\omega_0$

ω_0

$w_s/2$

$w_s - \omega_0$

w_s

$w_s + \omega_0$

$$w_s > 2\omega_0$$

$-w_s$

$-w_s/2$

$-\omega_0$

ω_0

$w_s/2$

w_s

$$w_s < 2\omega_0$$

$w_s/2$

ω_0

$w_s - \omega_0$

$w_s + \omega_0$

aliasing

$-w_s - \omega_0$

$w_s - \omega_0$

$w_s + \omega_0$

▪ Overlapping in Frequency-Domain: Aliasing

$$x(t) = \cos(\omega_0 t)$$

$$w_s > 2\omega_0$$

$$-w_s$$

$$-w_s/2$$

$$-\omega_0$$

$$\omega_0$$

$$w_s/2$$

$$w_s - \omega_0$$

$$w_s$$

$$w_s + \omega_0$$

$$w_s > 2\omega_0$$

$$-w_s$$

$$-w_0$$

$$\omega_0$$

$$w_s$$

$$w_s/2$$

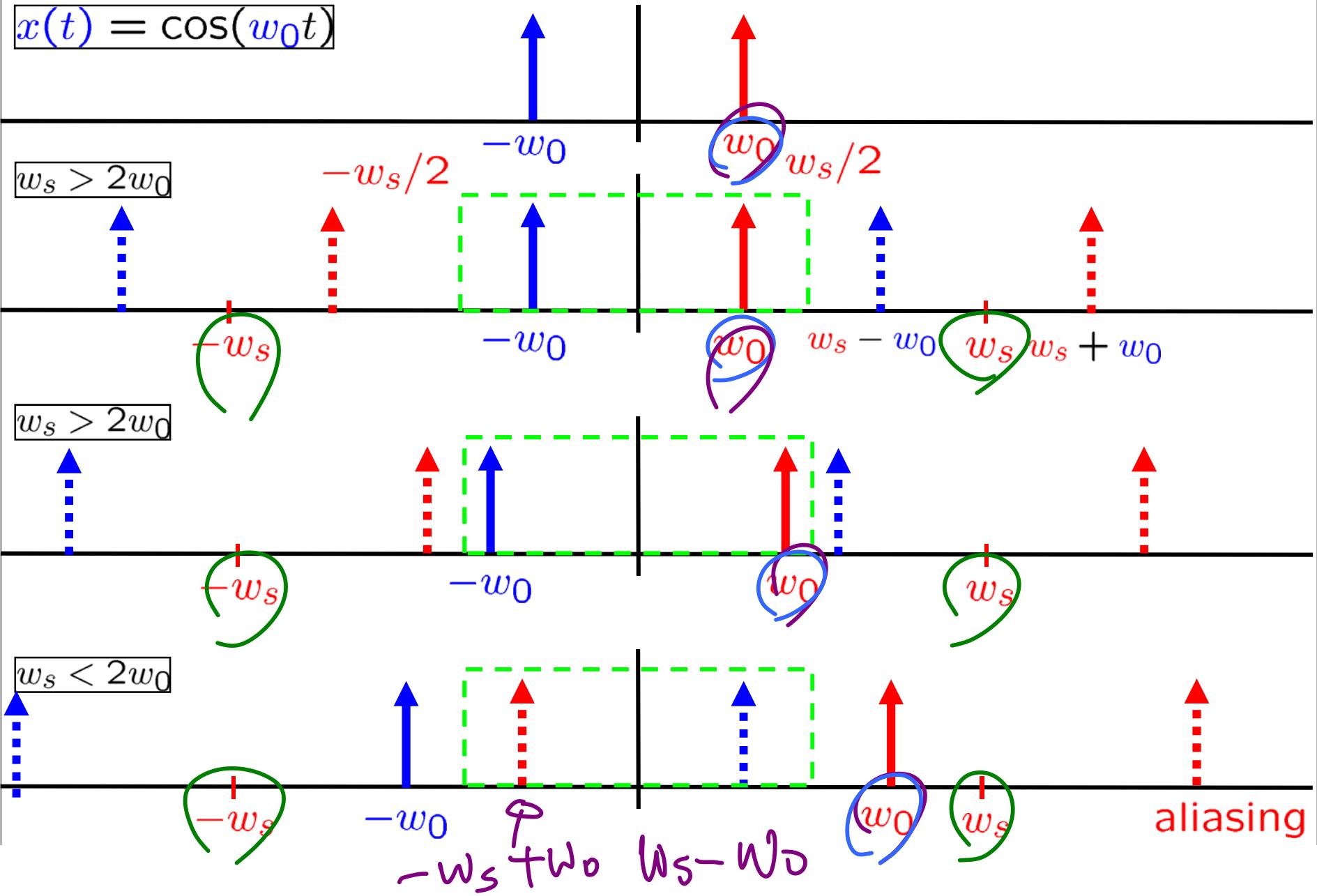
$$w_s < 2\omega_0$$

$$-w_s - w_0$$

$$w_s/2$$

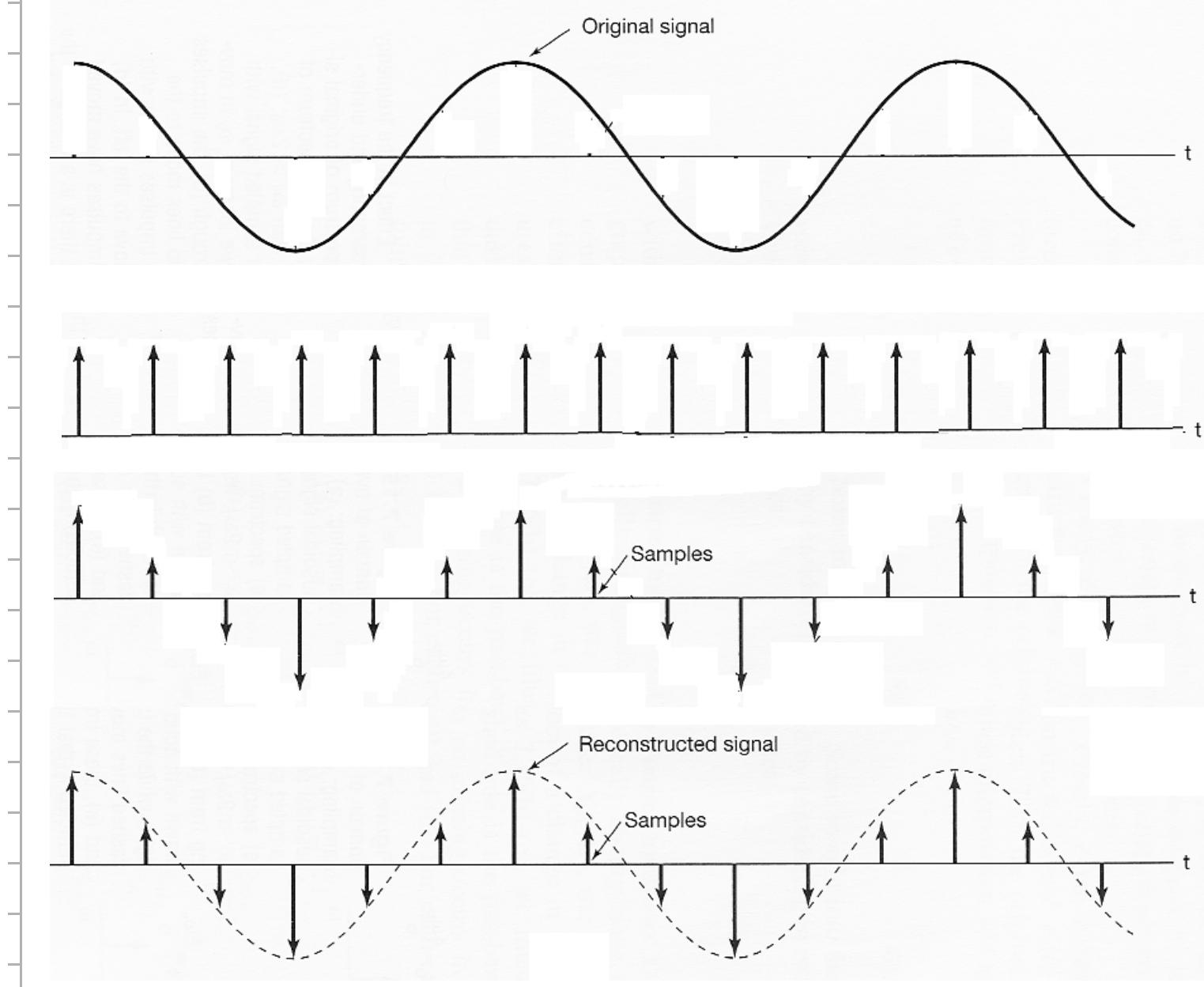
$$w_0 w_s$$

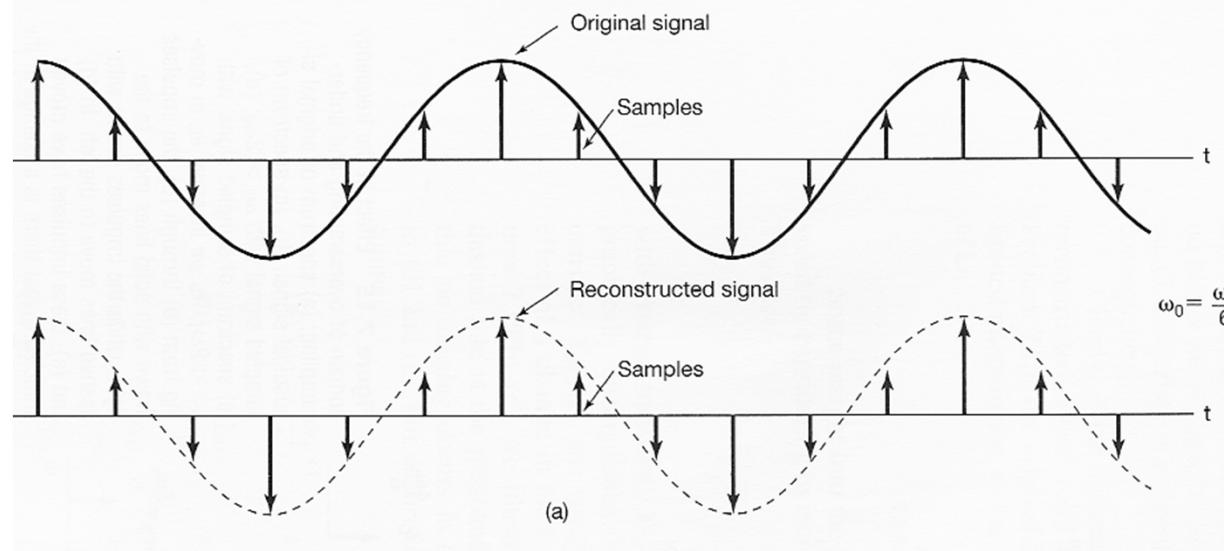
aliasing

▪ Overlapping in Frequency-Domain: Aliasing

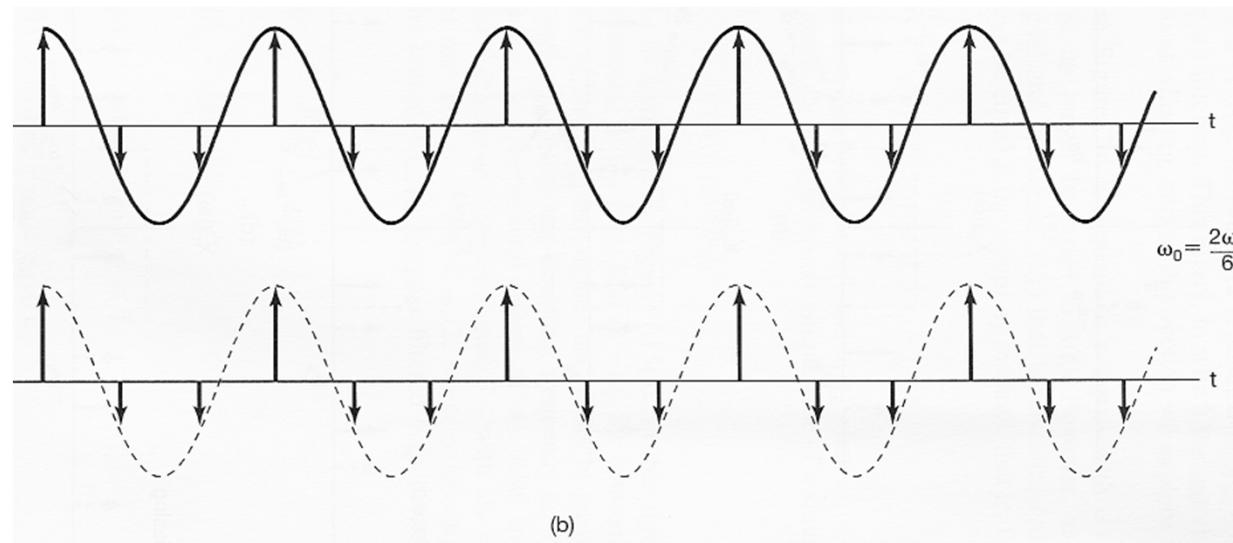
■ Overlapping in Frequency-Domain: Aliasing

$$w_0 = \frac{w_s}{6}$$



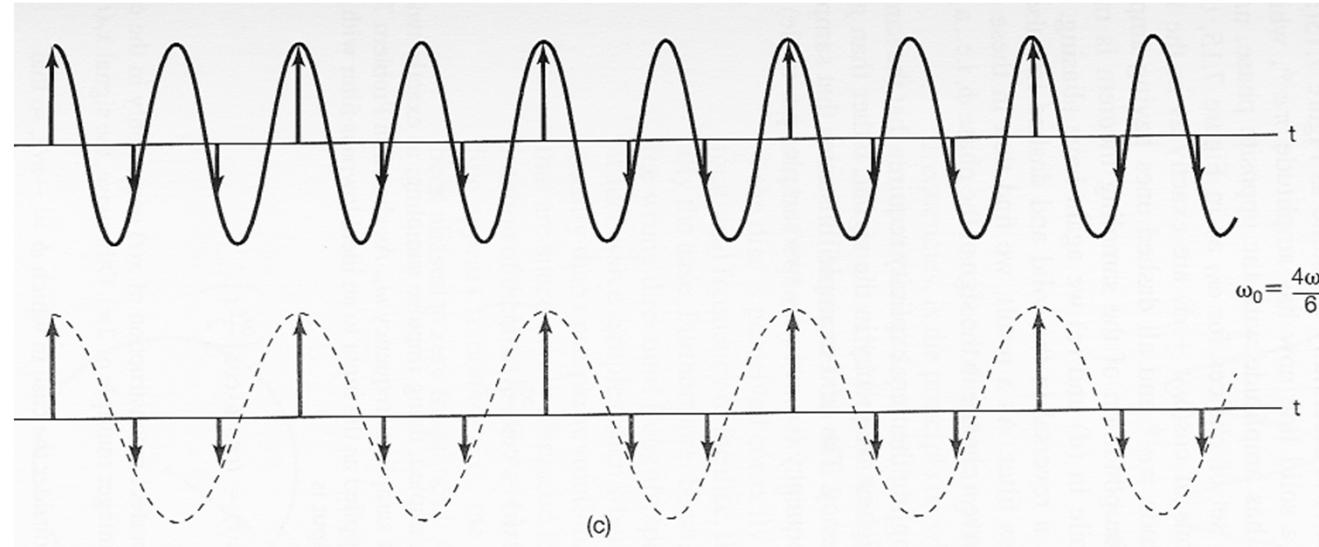
■ Overlapping in Frequency-Domain: Aliasing

$$\omega_0 = \frac{w_s}{6}$$



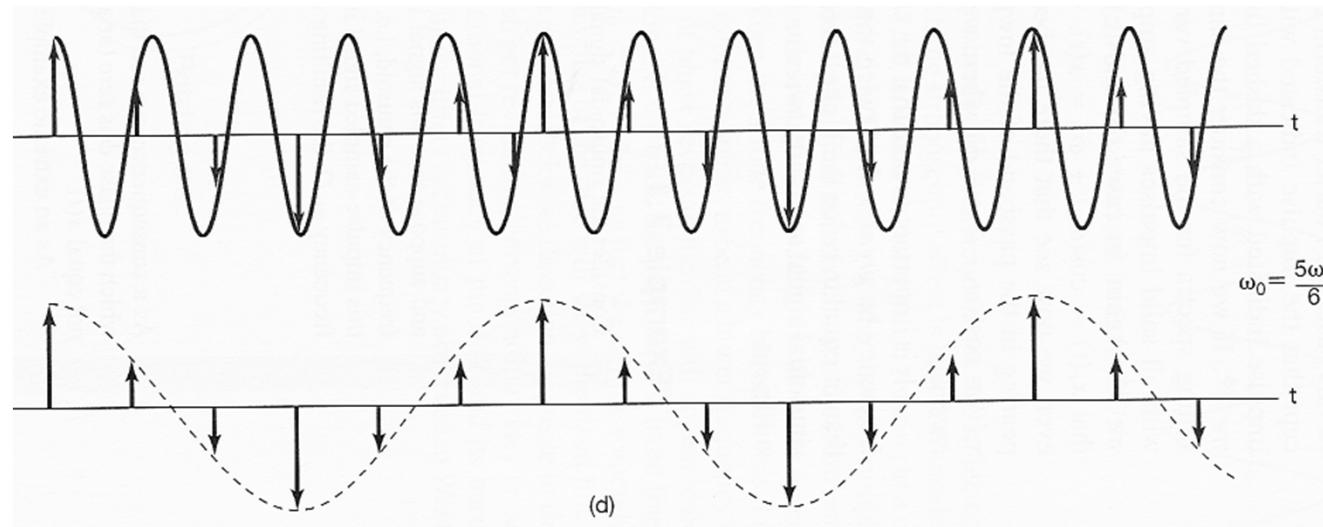
$$\omega_0 = \frac{2w_s}{6}$$

■ Overlapping in Frequency-Domain: Aliasing



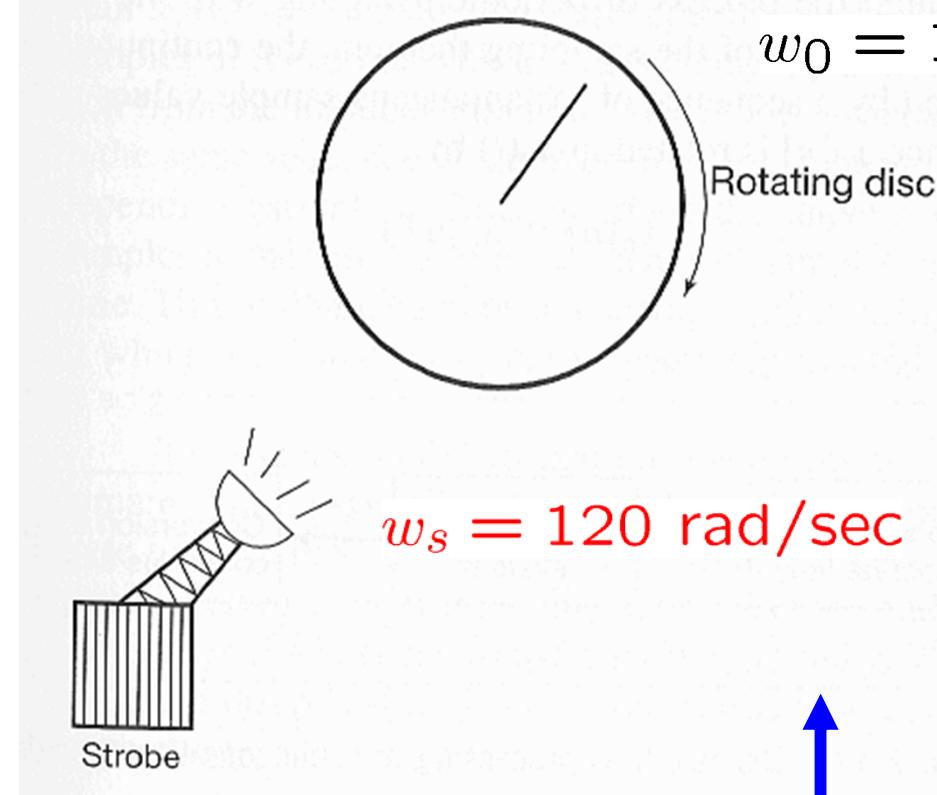
$$\omega_0 = \frac{4w_s}{6}$$

$$\begin{aligned} \omega_s - \omega_0 &= \omega_s - \frac{4w_s}{6} \\ &= \frac{2}{3} \omega_s \end{aligned}$$



$$\omega_0 = \frac{5w_s}{6}$$

$$\begin{aligned} \omega_s - \omega_0 &= \omega_s - \frac{5w_s}{6} \\ &= \frac{1}{6} \omega_s \end{aligned}$$

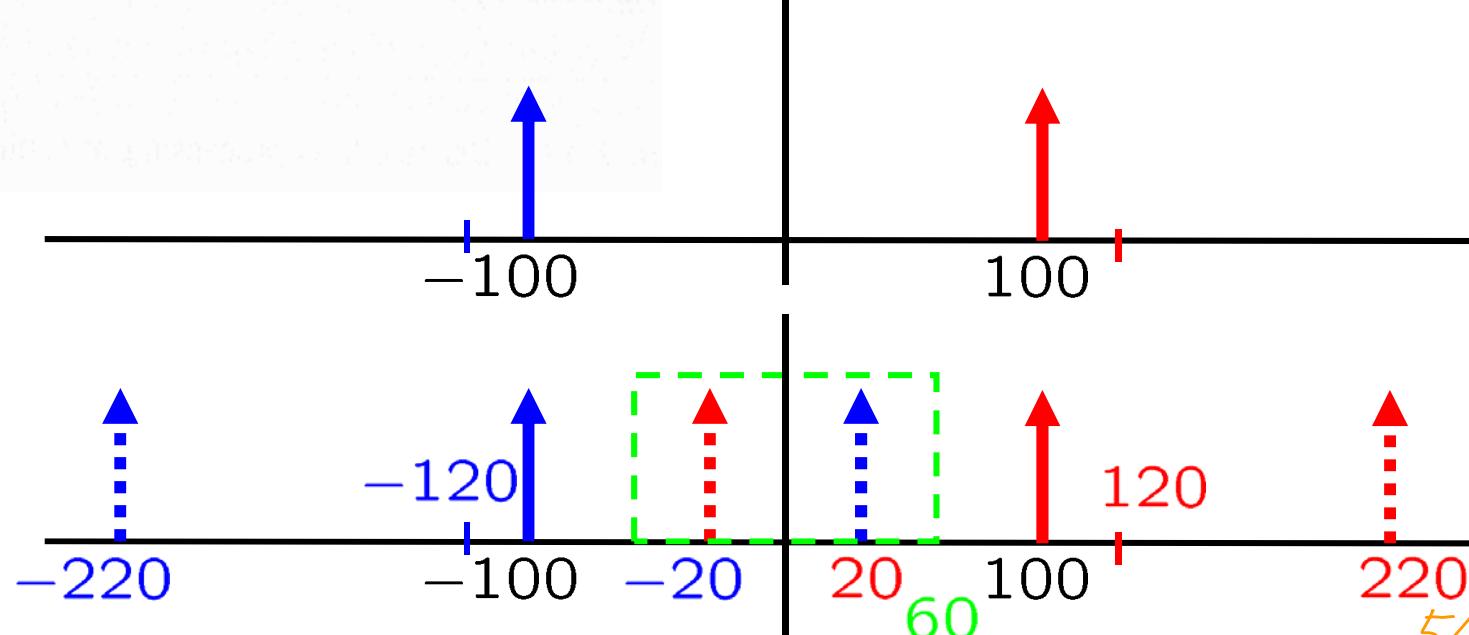
■ Strobe Effect:

$$w_0 = 100 \text{ rad/sec}$$

Rotating disc

$$\Rightarrow w = \pm w_s \pm w_0$$

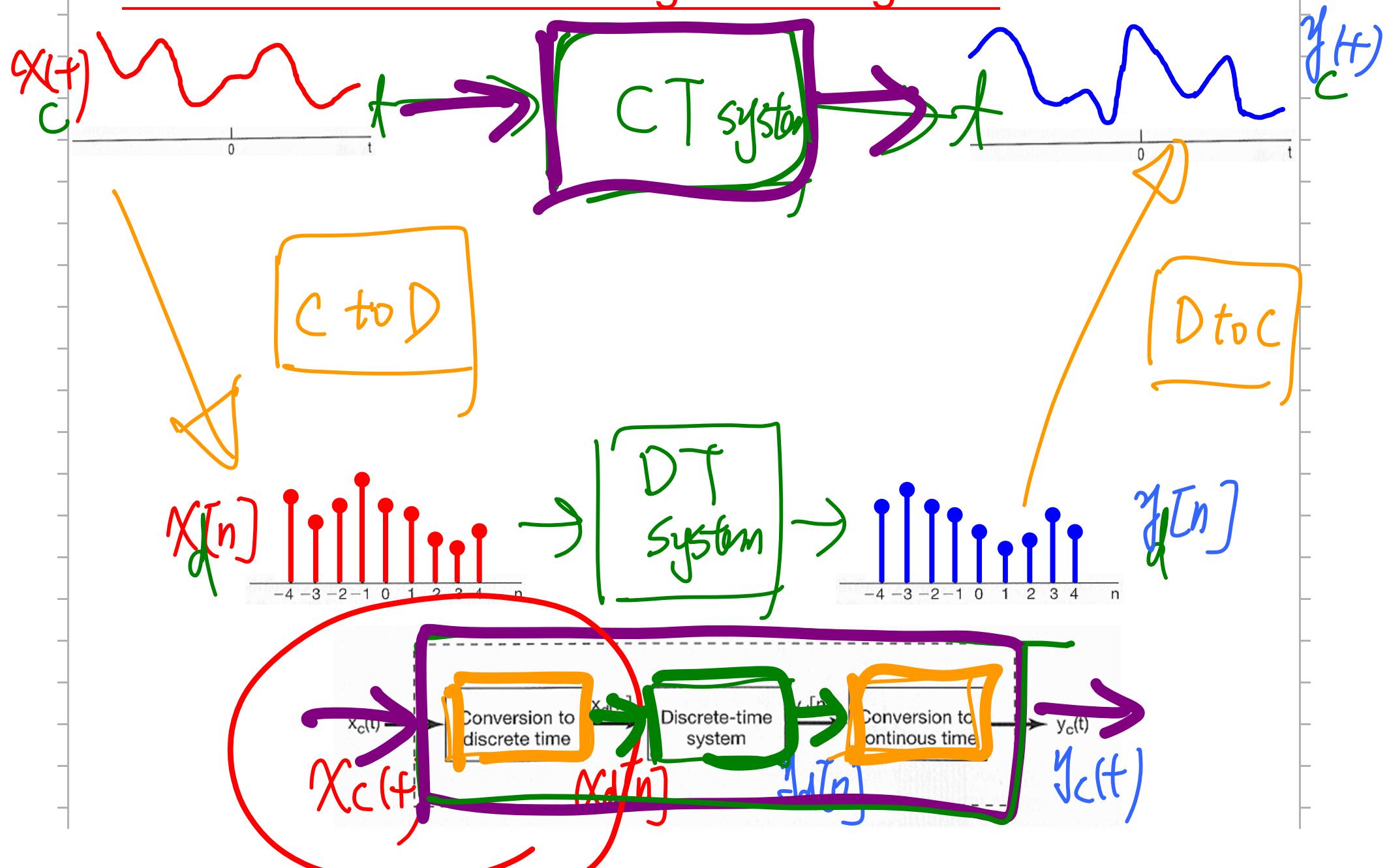
$$= +20, -20$$



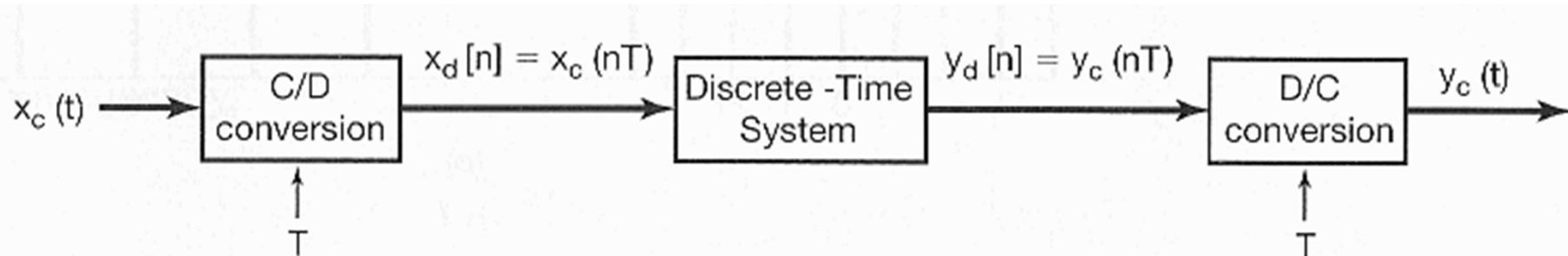
5/2/13
3=10pm

- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

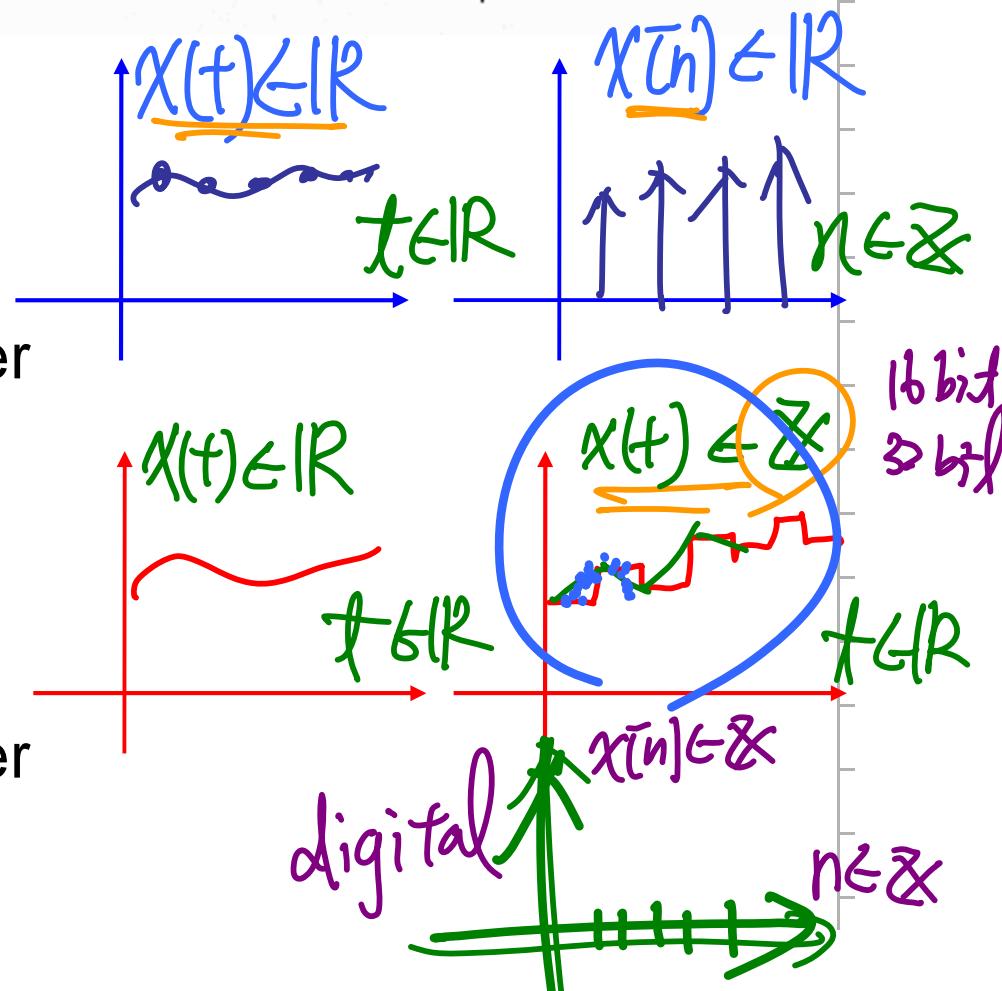
- Discrete-Time Processing of CT Signals:



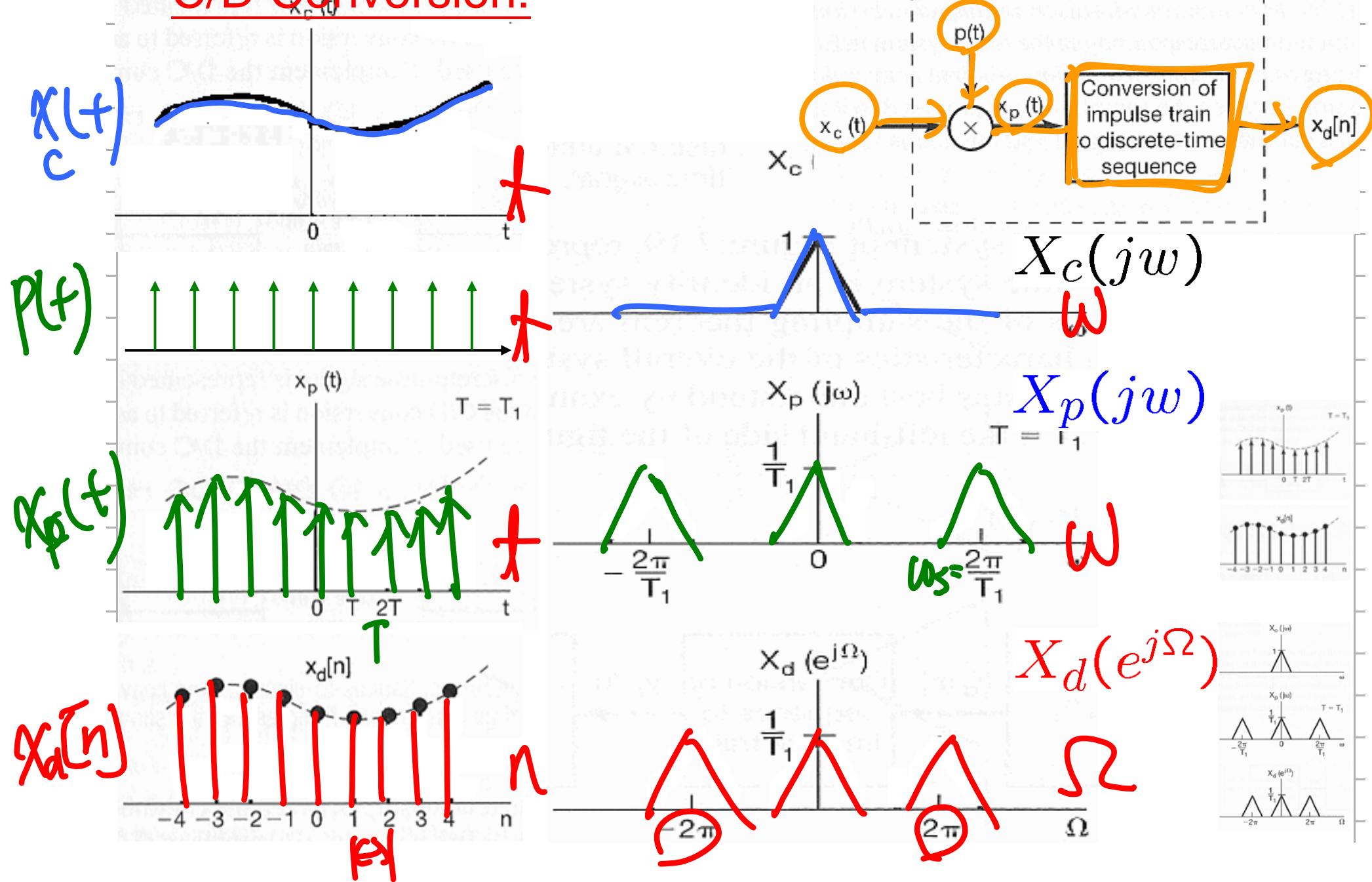
- C/D or A-to-D (ADC) and D/C or D-to-A (DAC):



- C/D: continuous-to-discrete-time conversion
- A-to-D: analog-to-digital converter
- D/C: discrete-to-continuous-time conversion
- D-to-A: digital-to-analog converter



C/D Conversion:

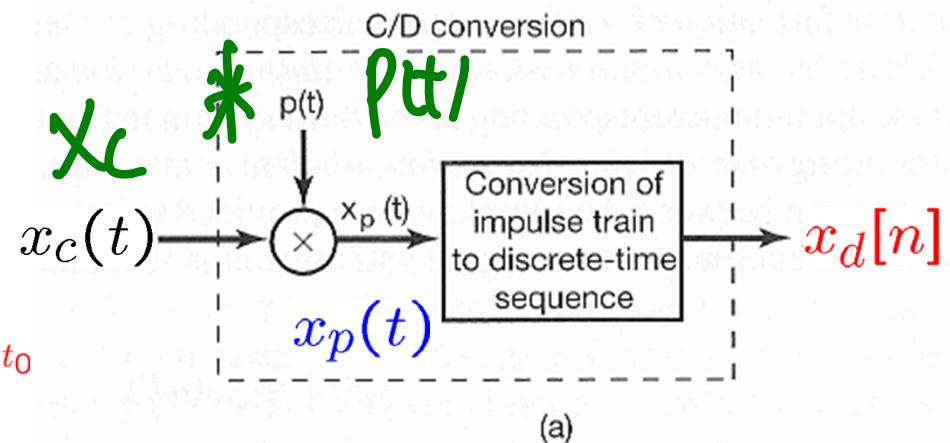


C/D Conversion:

$$\underline{x_p(t)} = \sum_{n=-\infty}^{+\infty} x_c(nT) \delta(t - nT)$$

Table 4.2, p. 329

$$\delta(t - t_0) \xleftrightarrow{\mathcal{F}} e^{-jw_0 t}$$



$$\underline{X_p(jw)} = \sum_{n=-\infty}^{+\infty} \underline{x_c(nT)} e^{-jwnT}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c(j(w - kw_s))$$

Eq 7.3, 7.6, p. 517

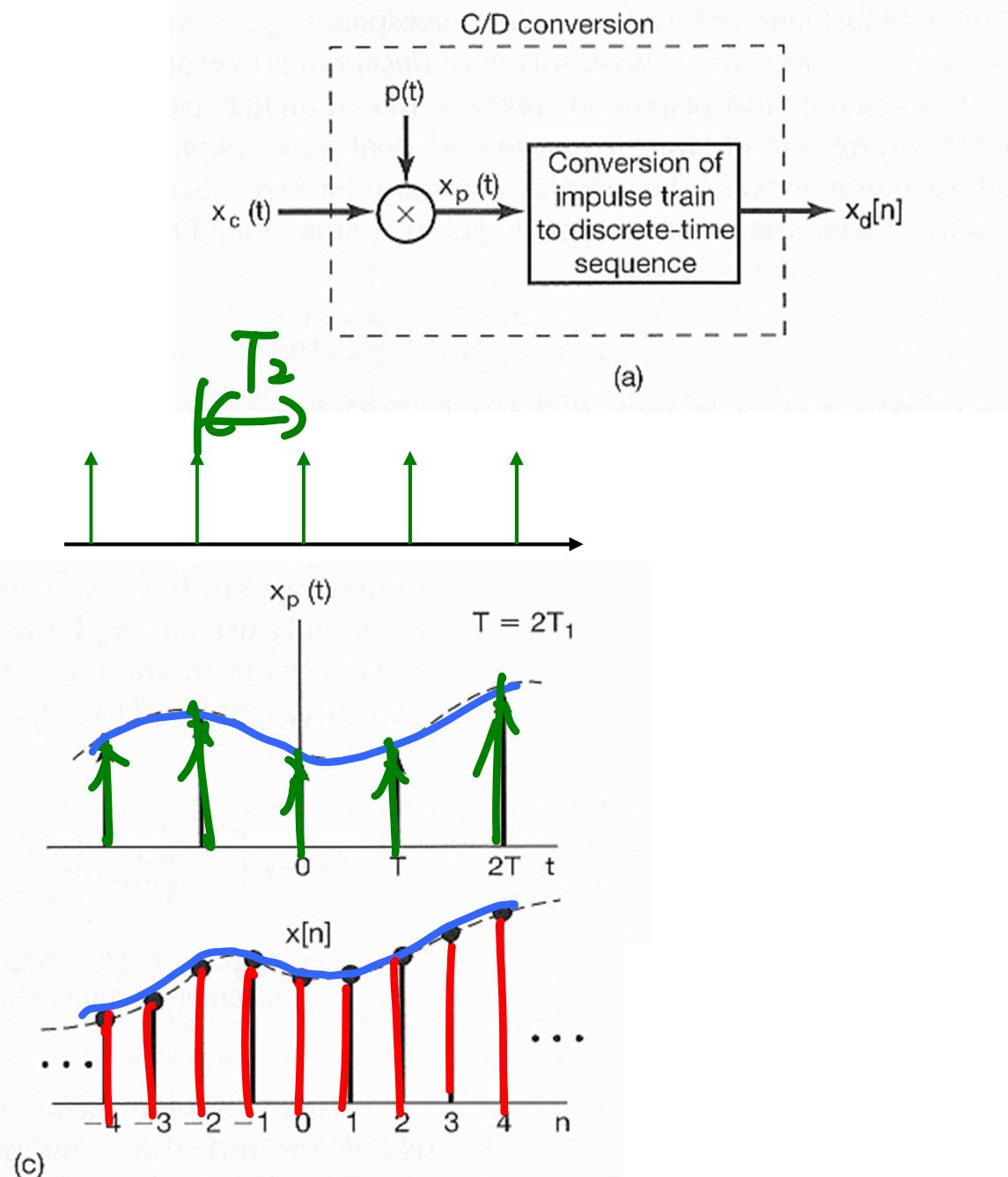
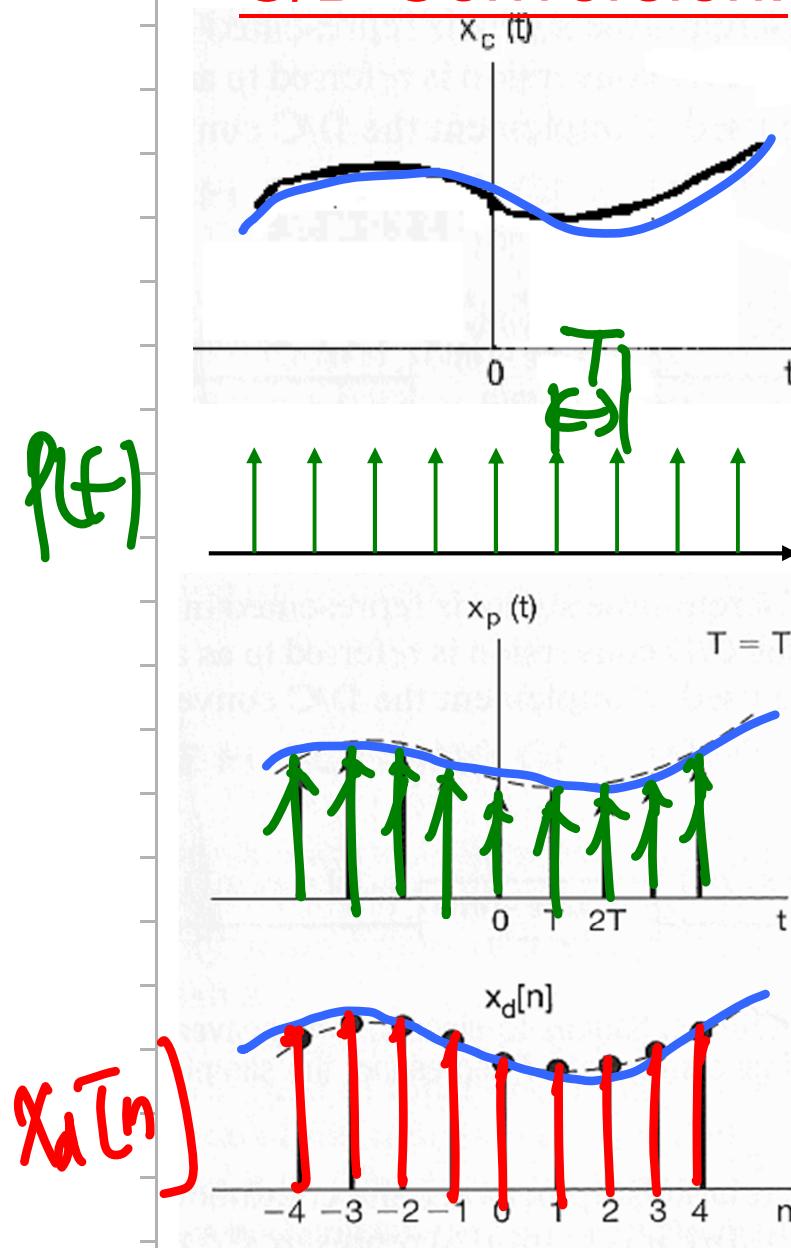
$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d[n] e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{+\infty} x_c(nT) e^{-j\Omega n}$$

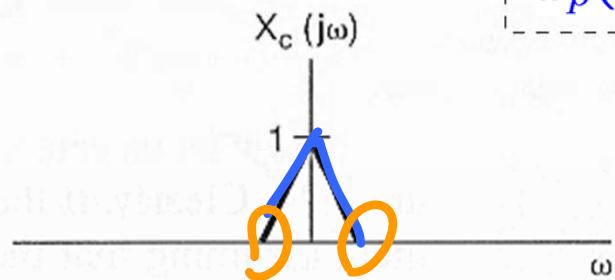
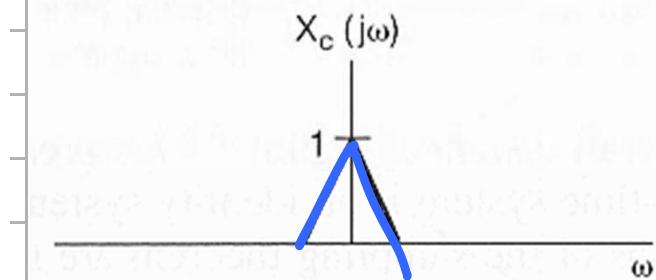
$$\rightarrow \boxed{X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right)}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(j\left(\frac{\Omega}{T} - k\frac{2\pi}{T}\right)\right)$$

C/D Conversion:

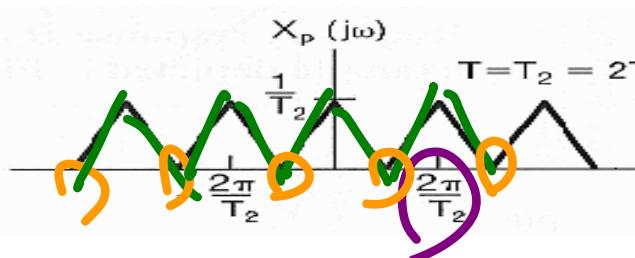
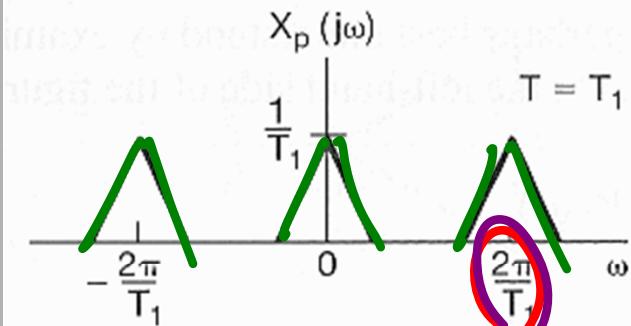


■ C/D Conversion:



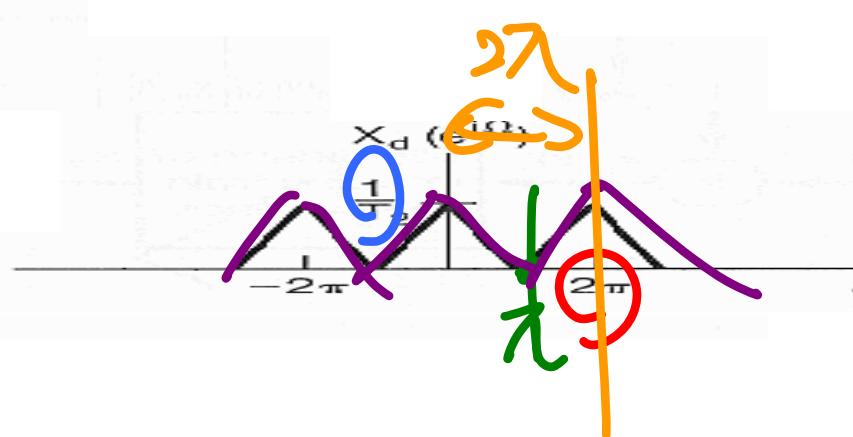
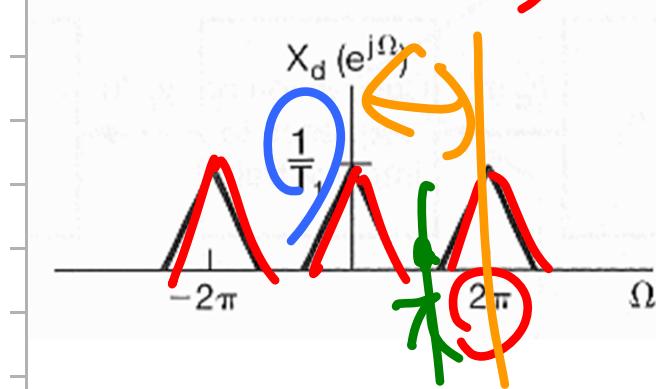
$x_d[n]$

(a)

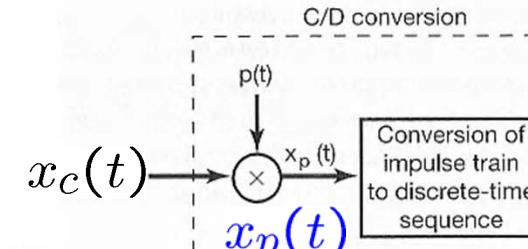


$X_c(jw)$

$X_p(jw)$



$X_d(e^{j\Omega})$



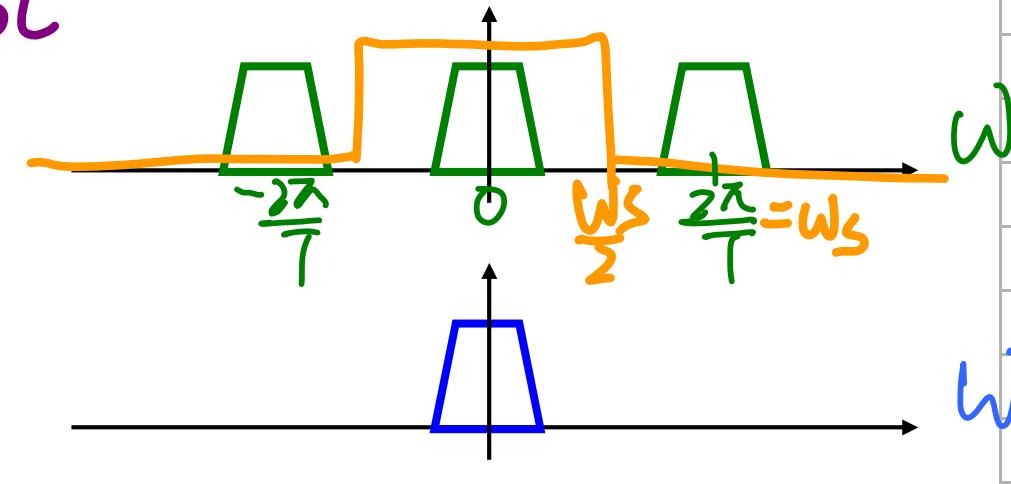
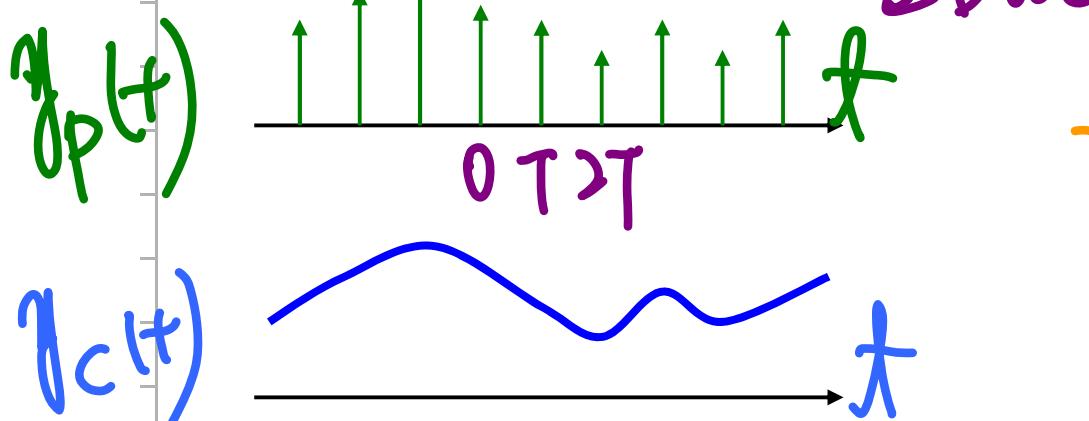
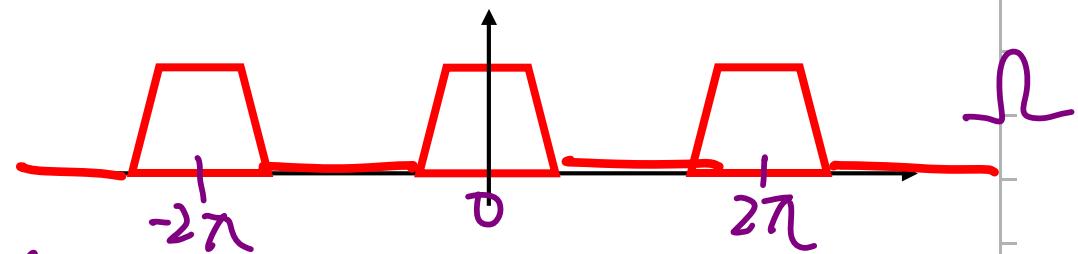
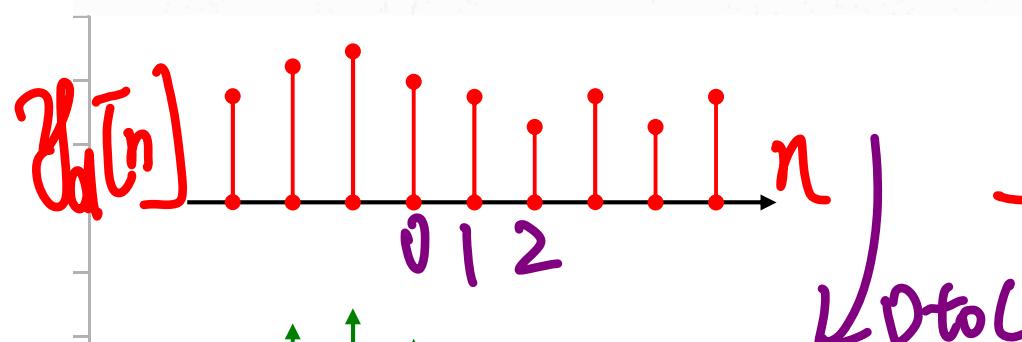
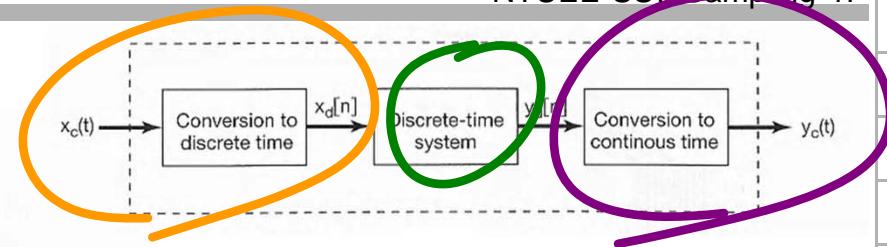
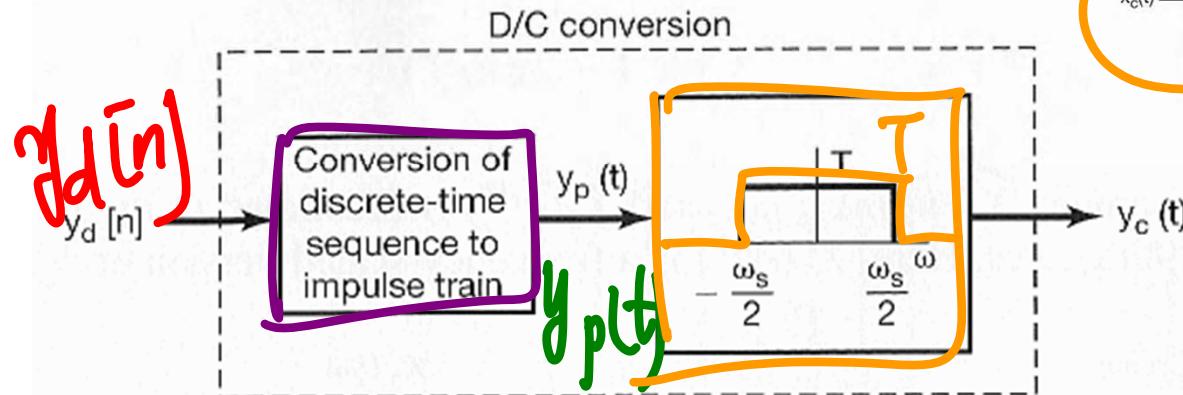
$x_p(t)$

$x_c(t)$

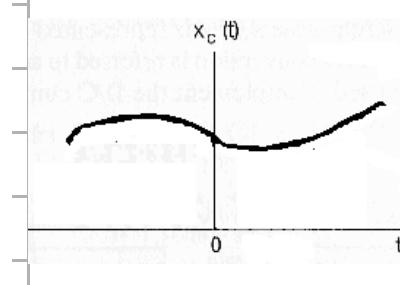
$x_p(t)$

Discrete-Time Processing of Continuous-Time Signals

D/C Conversion:

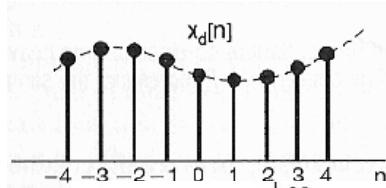


■ Overall System:



$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT) \delta(t - nT)$$

A graph of the impulse train $x_p(t)$ plotted against time t . The train consists of discrete impulses at regular intervals of T , with values $x_c(nT)$ at each impulse. Arrows point from the signal graph to the corresponding points on the impulse train.



$$\begin{aligned} X_p(jw) &= \sum_{n=-\infty}^{+\infty} x_c(nT) e^{-jwnT} \\ &= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c(j(w - kw_s)) \end{aligned}$$

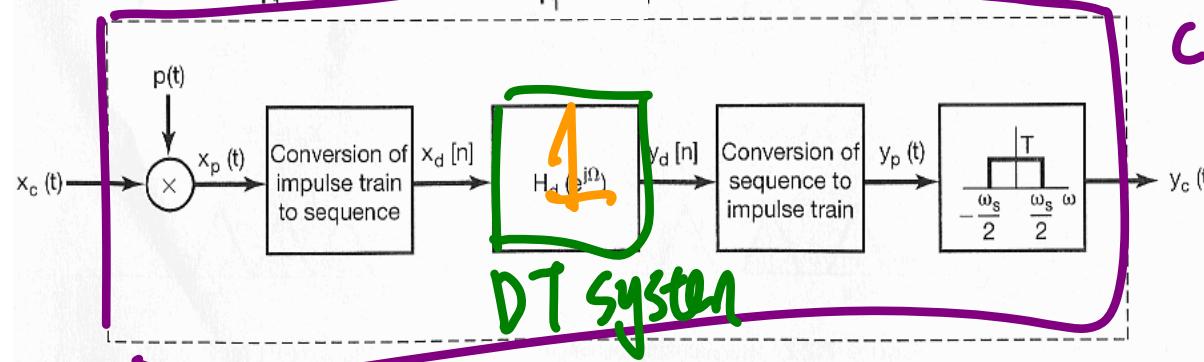
A graph of the frequency spectrum $X_p(jw)$ plotted against frequency ω . The spectrum is a periodic repetition of the continuous-time spectrum $X_c(j\omega)$ with period $2\pi/T$. The main lobe is centered at $\omega_s/2$.

$$\begin{aligned} X_d(e^{j\Omega}) &= \sum_{n=-\infty}^{+\infty} x_d[n] e^{-jn\Omega} = \sum_{n=-\infty}^{+\infty} x_c(nT) e^{-jn\Omega} \\ &= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(j\left(\frac{\Omega}{T} - k\frac{2\pi}{T}\right)\right) \end{aligned}$$

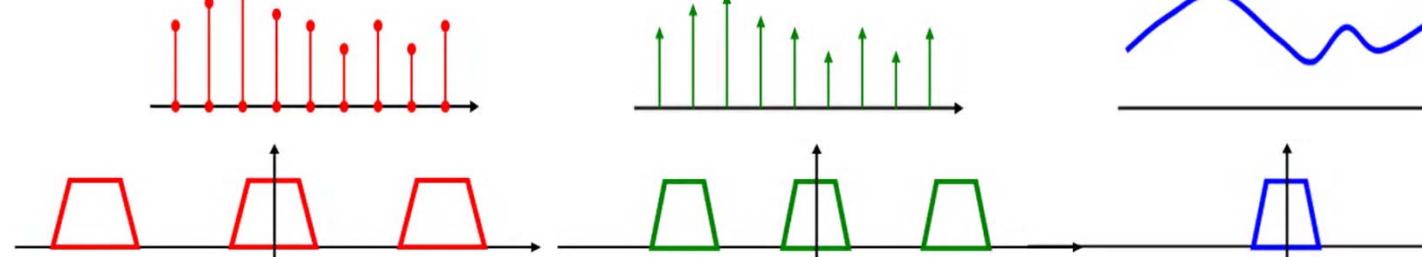
A graph of the frequency spectrum $X_d(e^{j\Omega})$ plotted against frequency Ω . The spectrum is a periodic repetition of the continuous-time spectrum $X_c(j\Omega)$ with period $2\pi/T$. The main lobe is centered at $\Omega_s/2$.

$$\Rightarrow X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right)$$

CT system



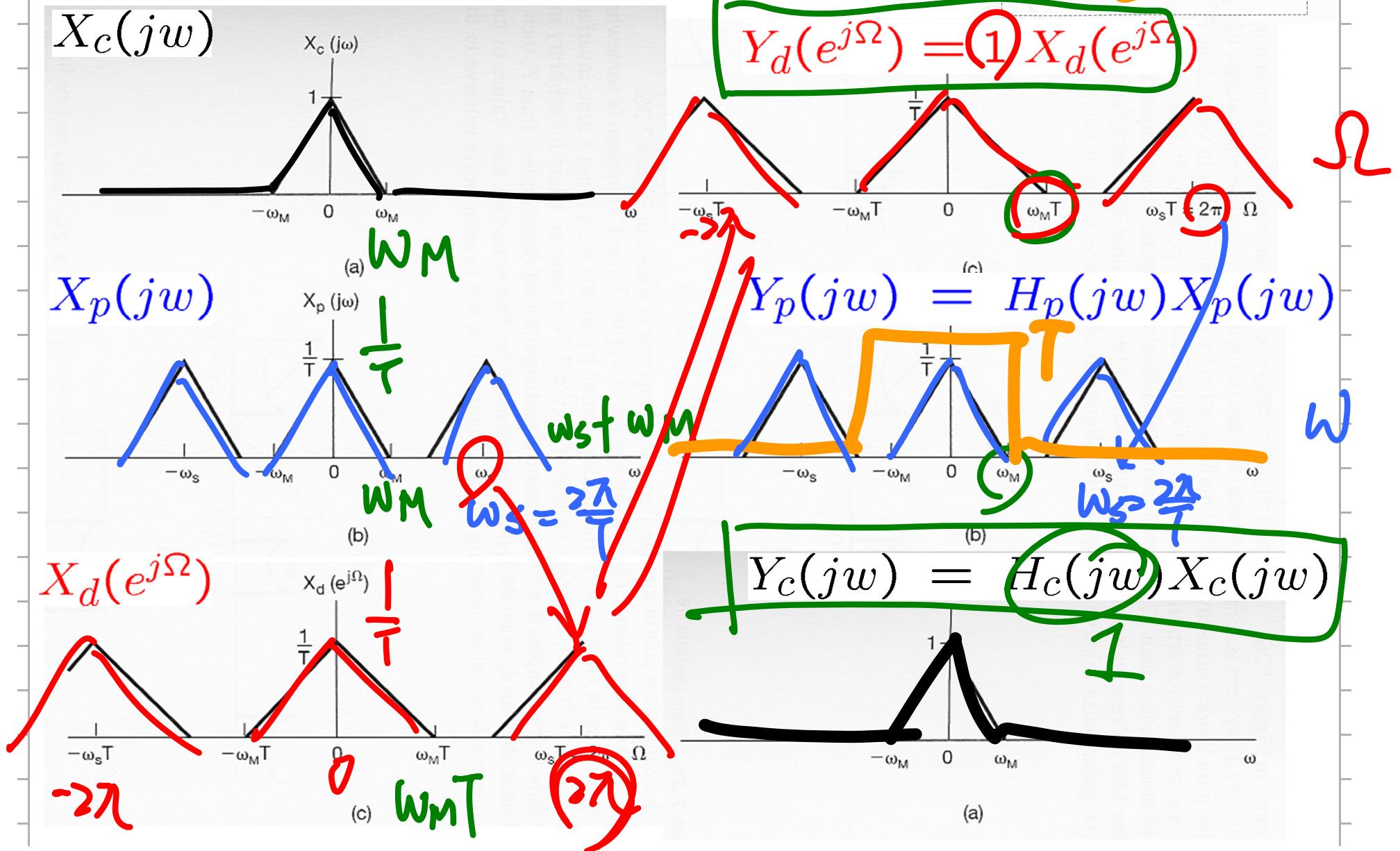
DT system



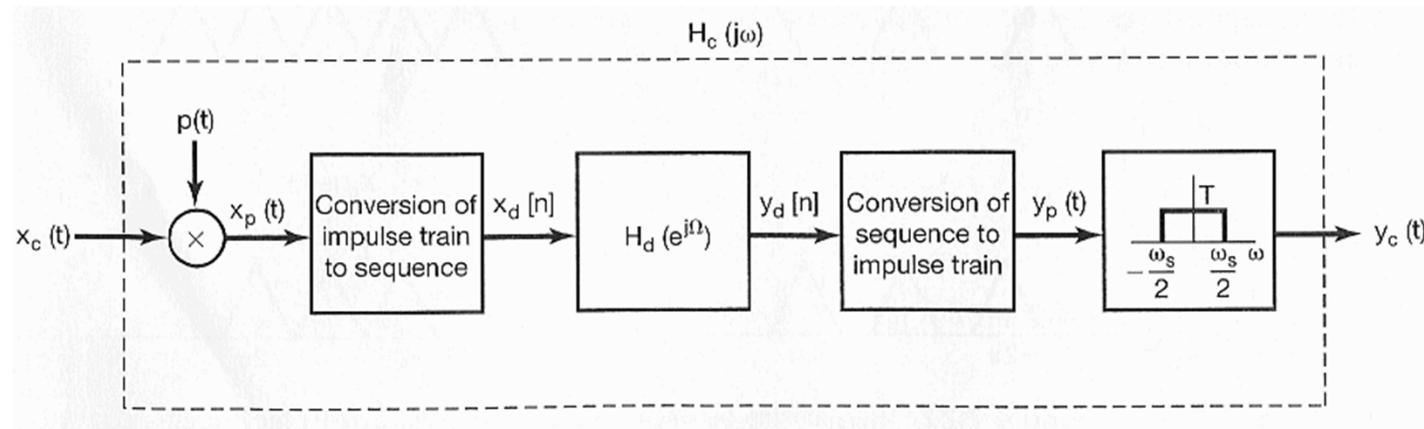
5/6/13

10:45 am

Frequency-Domain Illustration:

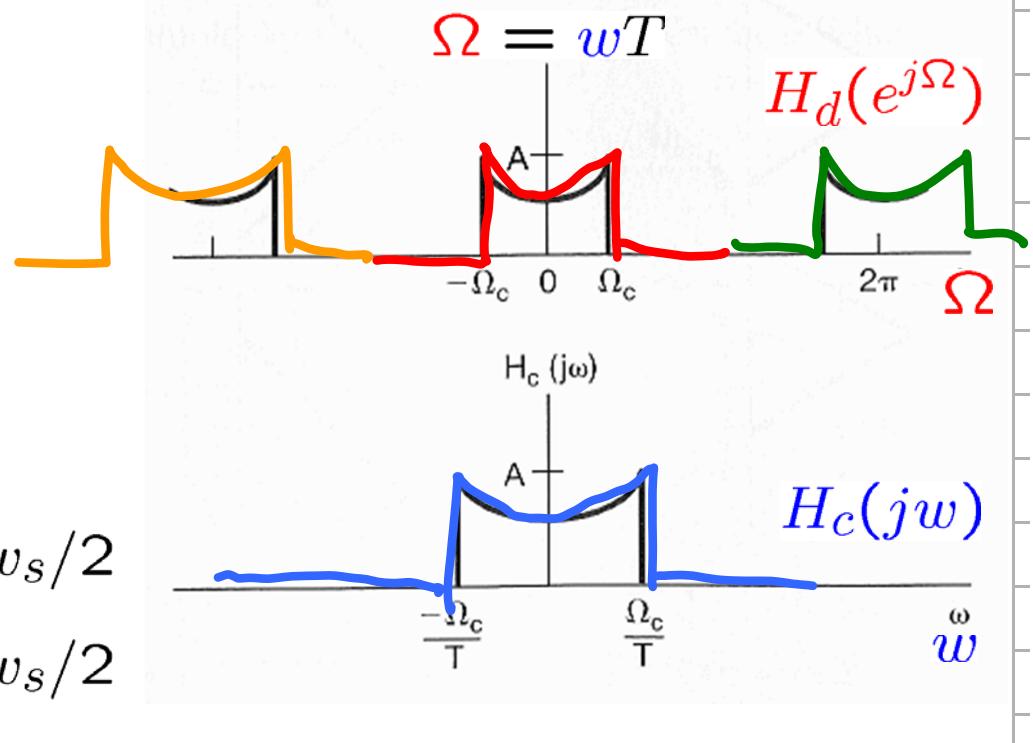


■ CT & DT Frequency Responses:

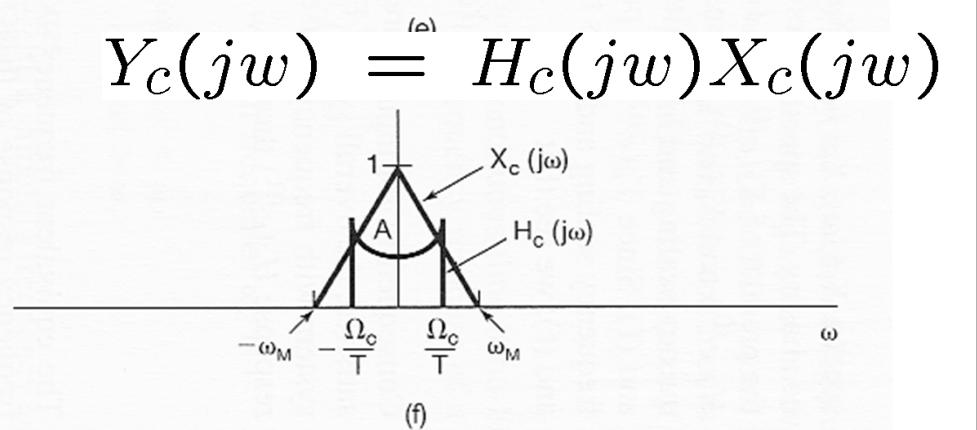
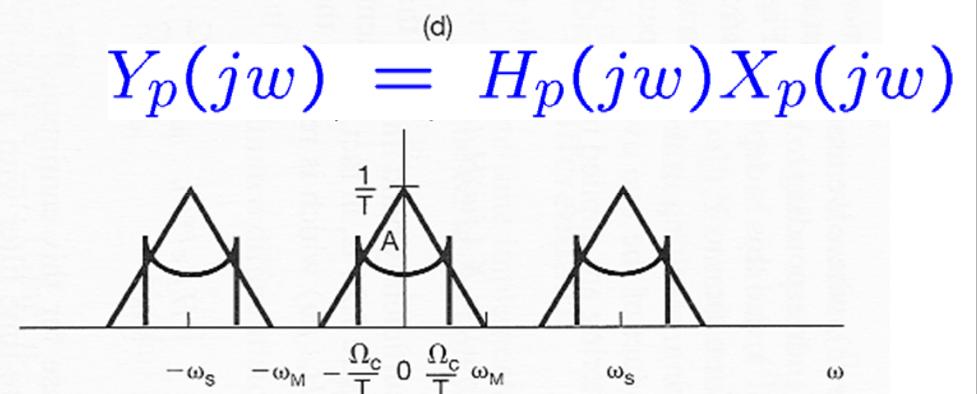
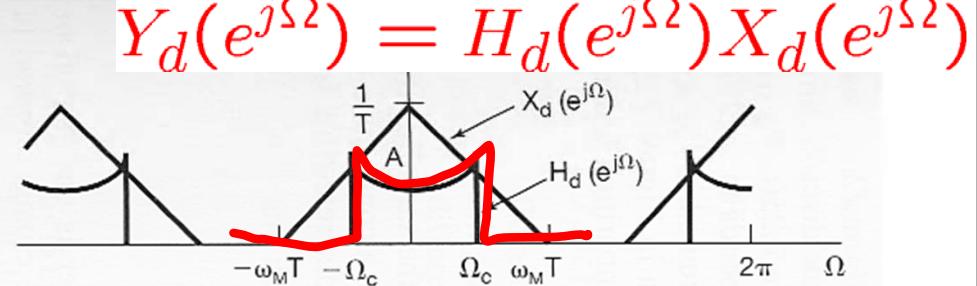
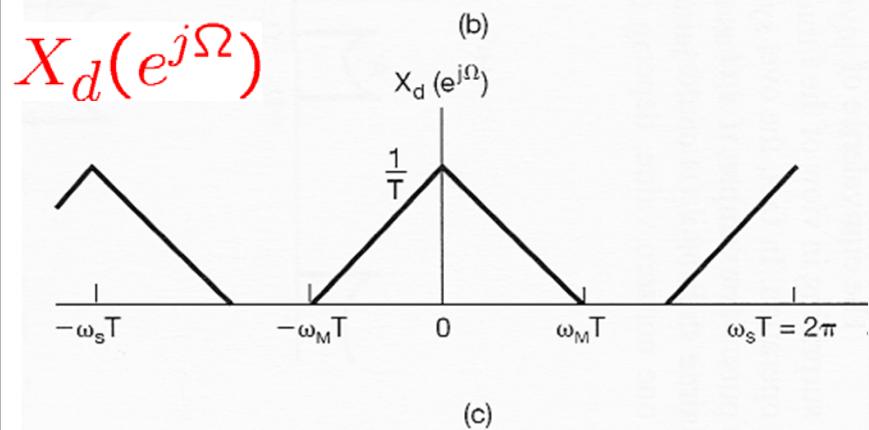
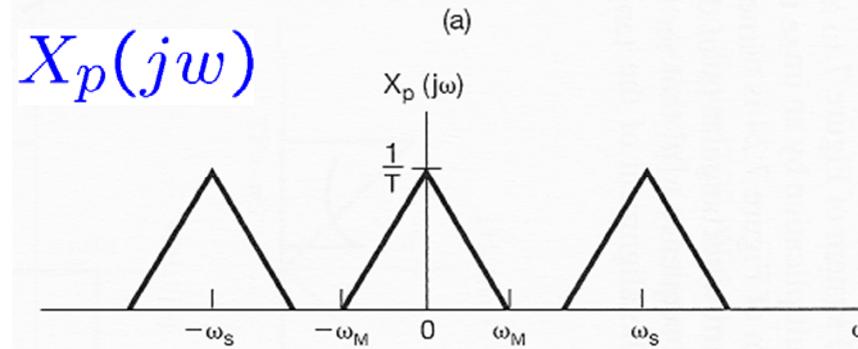
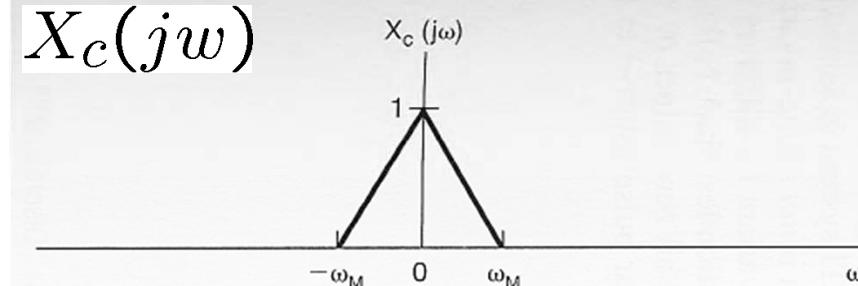


$$Y_c(jw) = X_c(jw) H_c(jw)$$

$$H_c(jw) = \begin{cases} H_d(e^{jwT}), & |w| < w_s/2 \\ 0, & |w| > w_s/2 \end{cases}$$

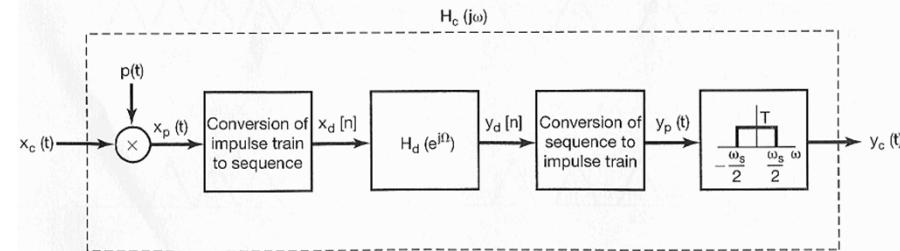


Frequency-Domain Illustration:



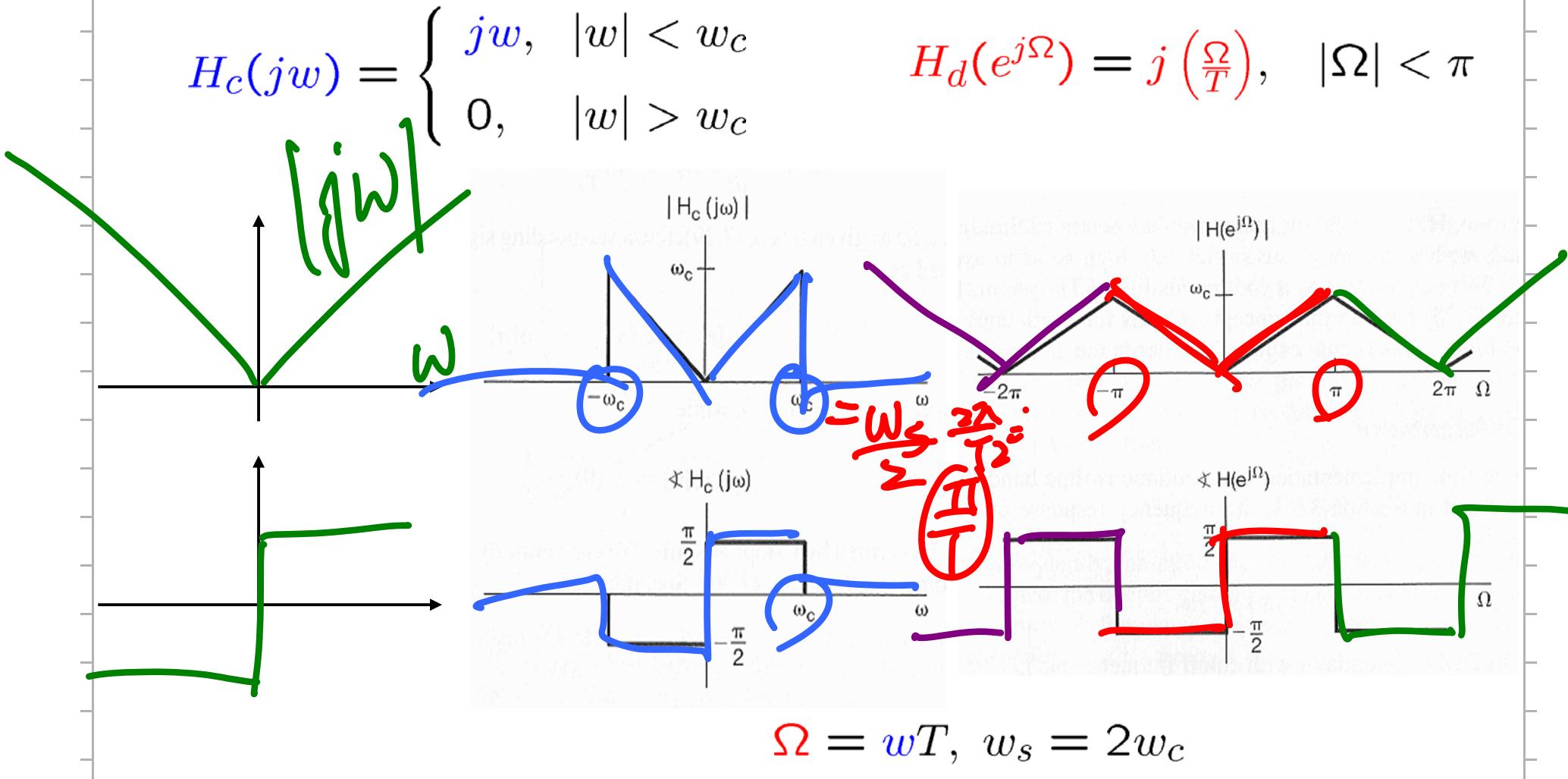
- Digital Differentiator:
(band-limited)

Ex 4.16, p. 317



$$H_c(jw) = \begin{cases} jw, & |w| < w_c \\ 0, & |w| > w_c \end{cases}$$

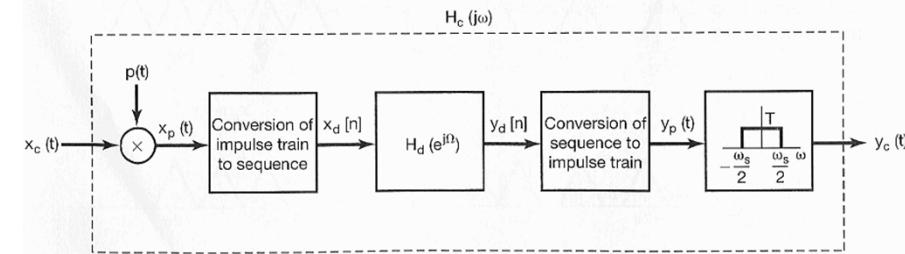
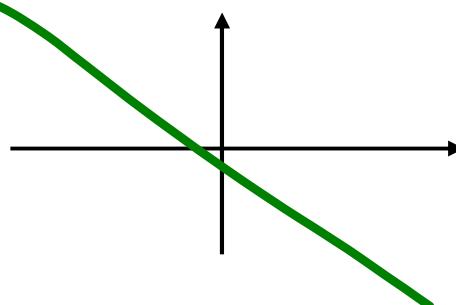
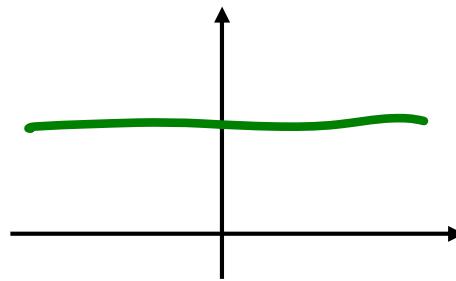
$$H_d(e^{j\Omega}) = j \left(\frac{\Omega}{T} \right), \quad |\Omega| < \pi$$



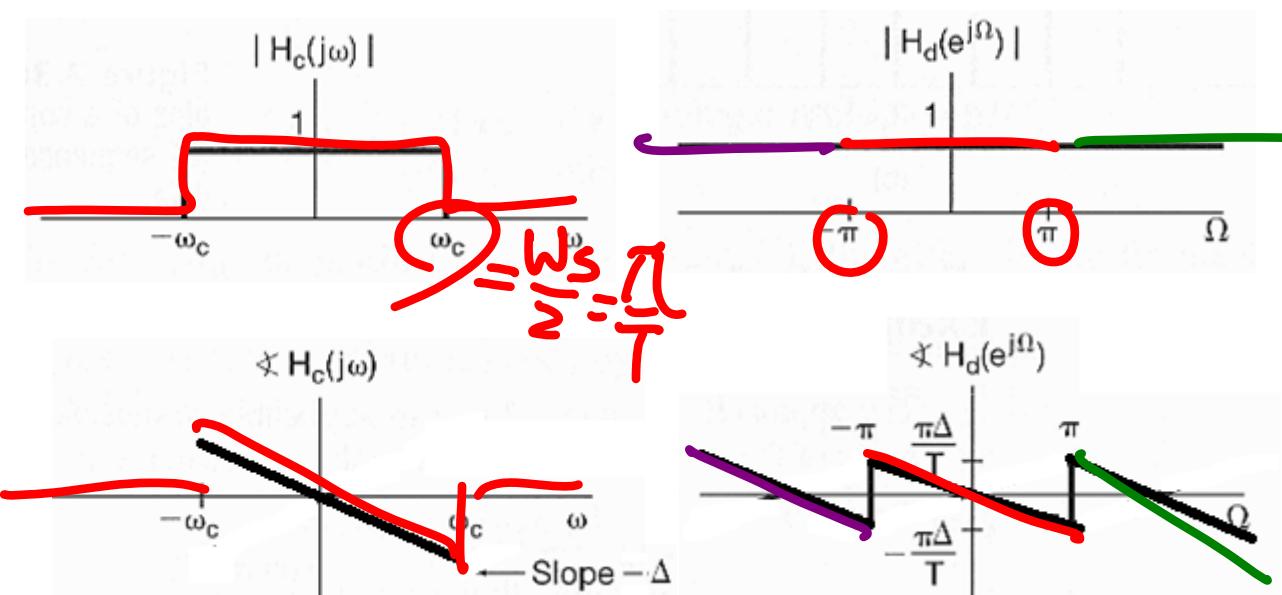
- Delay:
(band-limited)

Ex 4.15, p. 317

$$H_c(jw) = \begin{cases} e^{-jw\Delta}, & |w| < w_c \\ 0, & |w| > w_c \end{cases}$$



$$H_d(e^{j\Omega}) = e^{-j\Omega\Delta/T}, \quad |\Omega| < \pi$$

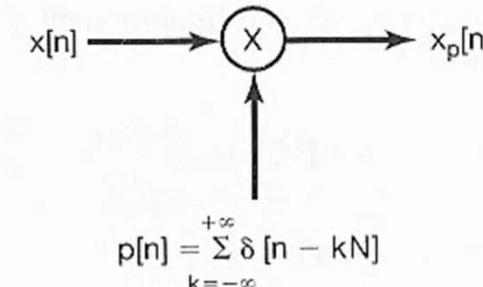


$$\Omega = wT, \quad w_s = 2w_c$$

- Representation of a Continuous-Time Signal by its Samples: The Sampling Theorem
- Reconstruction of a Signal from its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

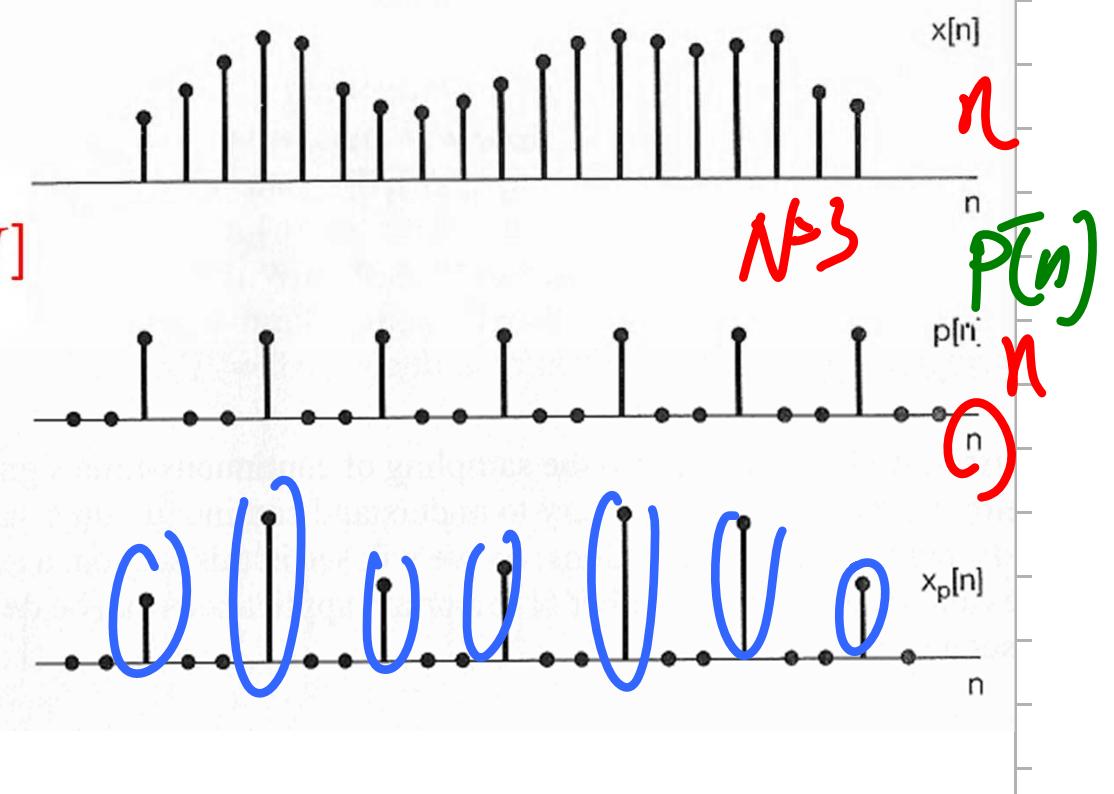
■ Impulse-Train Sampling:

$$x_p[n] = \begin{cases} x[n], & \text{if } n = kN \\ 0, & \text{otherwise} \end{cases}$$



$$x_p[n] = x[n] p[n]$$

$$= \sum_{k=-\infty}^{+\infty} x[kN] \delta[n - kN]$$

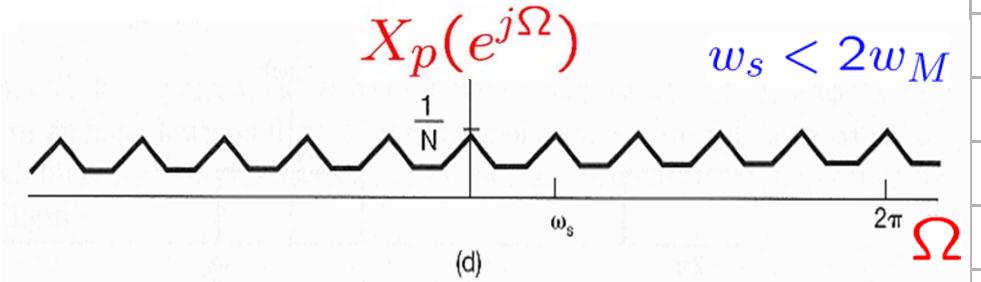
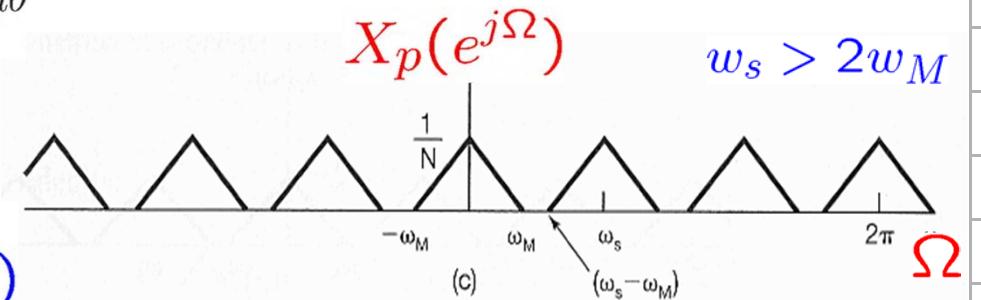
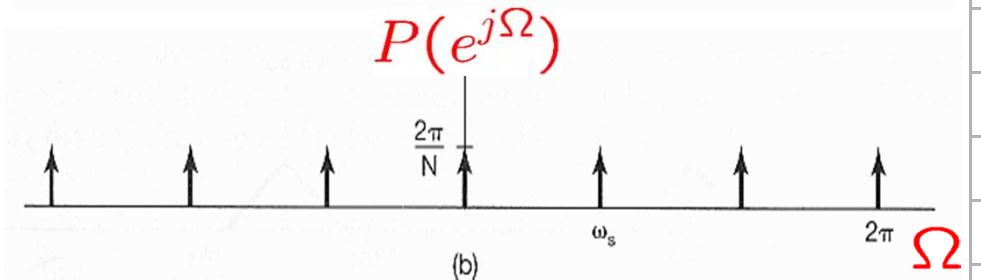
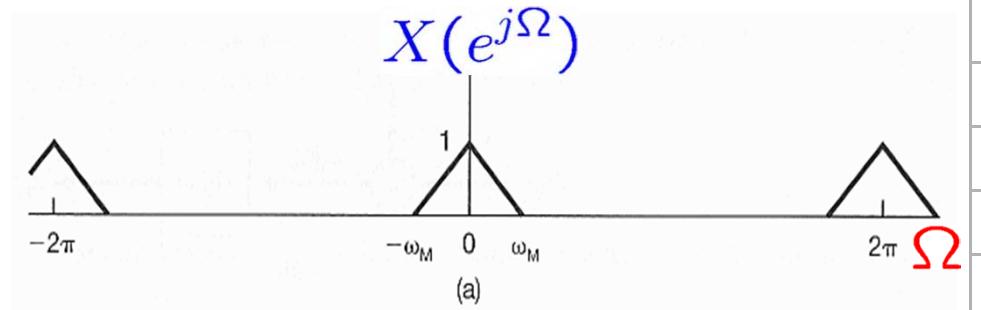


■ Impulse-Train Sampling:

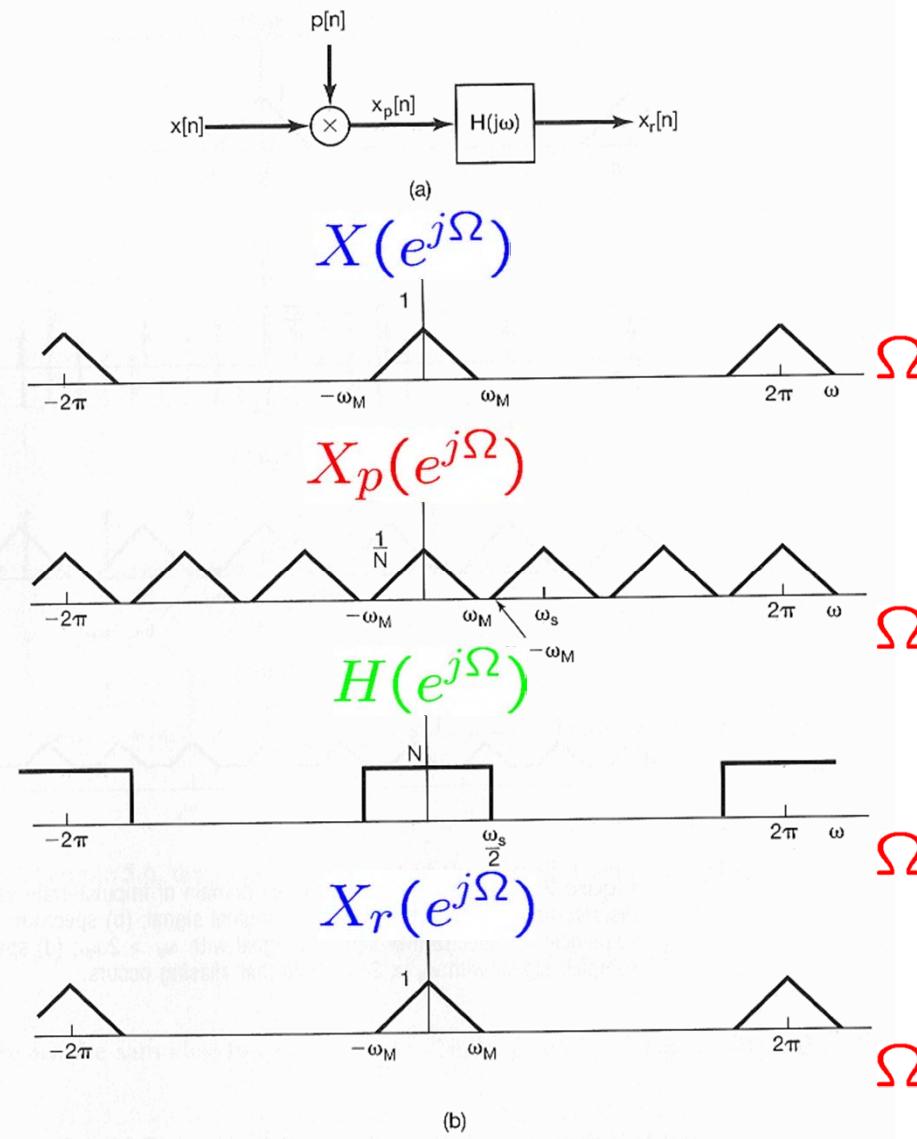
$$P(e^{j\Omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\Omega - kw_s)$$

$$X_p(e^{j\Omega}) = \frac{1}{2\pi} \int_{2\pi} P(e^{j\theta}) X(e^{j(\Omega-\theta)}) d\theta$$

$$\Rightarrow X_p(e^{j\Omega}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\Omega-kw_s)})$$

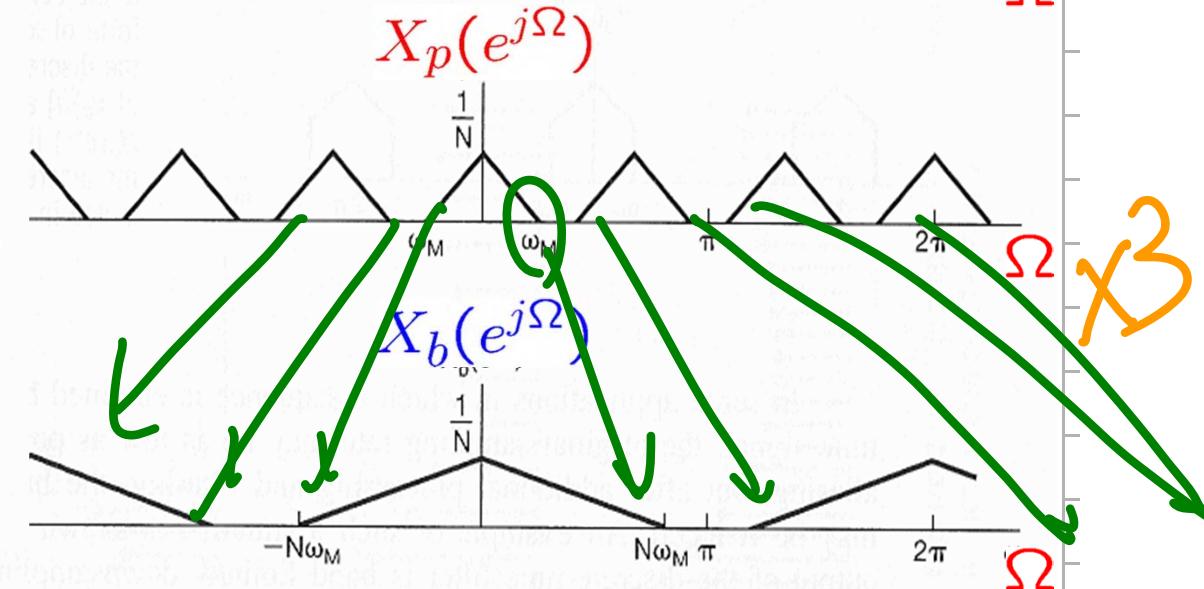
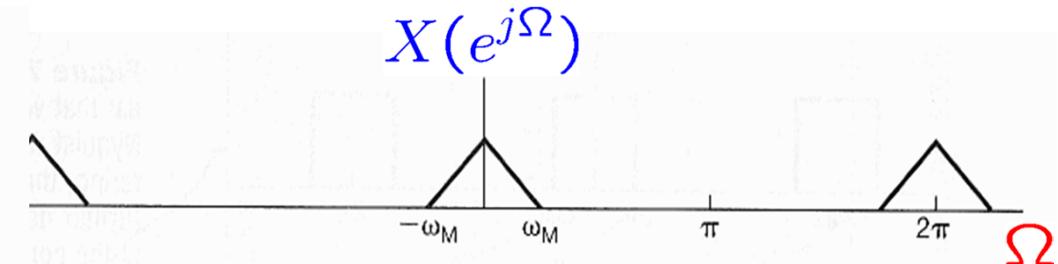
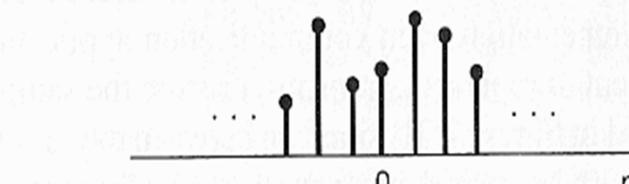
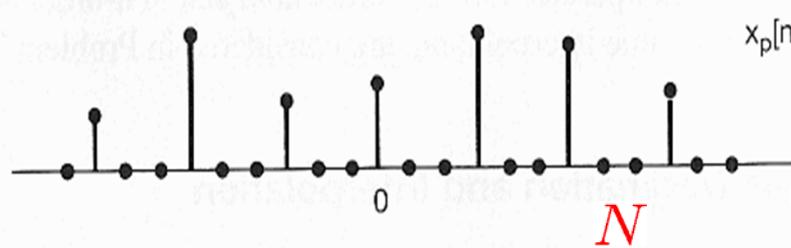
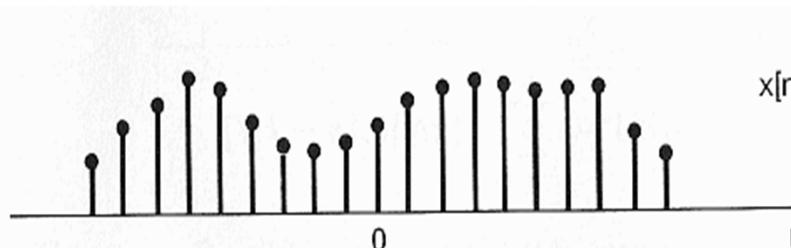


■ Exact Recovery Using Ideal Lowpass Filter:



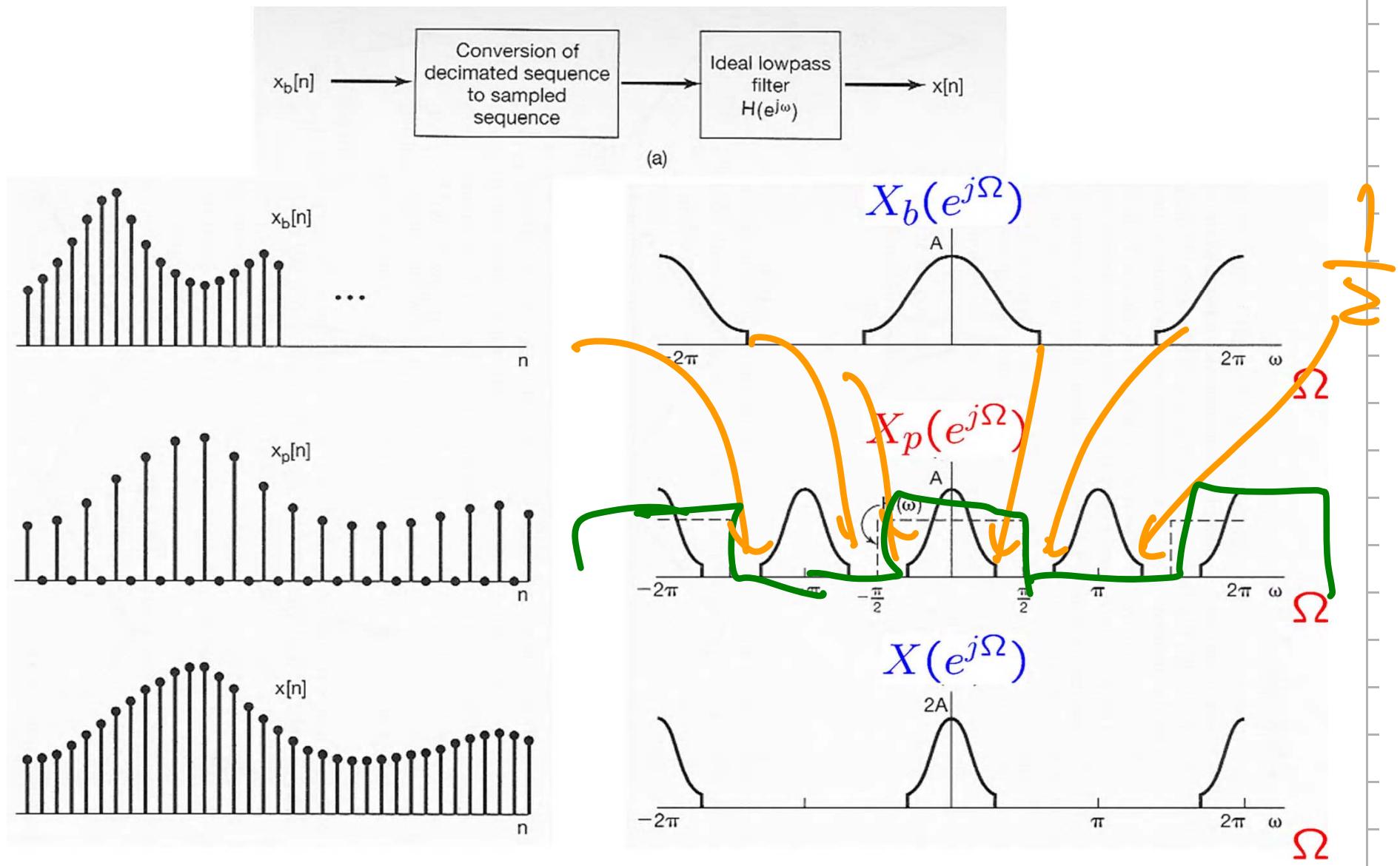
■ DT Decimation & Interpolation: Down-sampling

Eq 5.45, p. 378: Time expansion



$$X_b(e^{j\Omega}) = X_p(e^{j\Omega/N})$$

■ Higher Equivalent Sampling Rate: Up-sampling



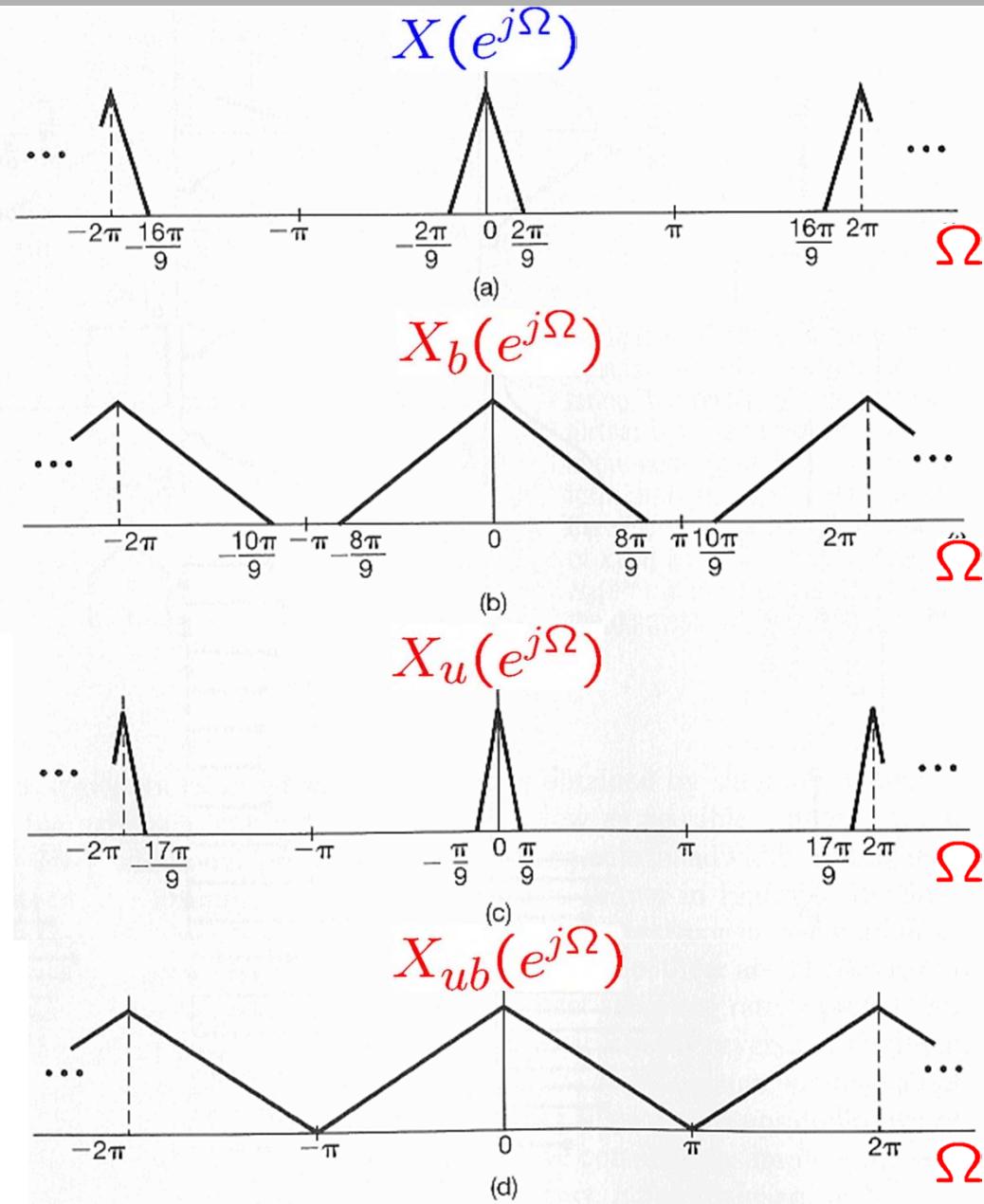
- Down-sampling
+ Up-sampling:

$$\frac{2\pi}{9} \times 4 < \pi$$

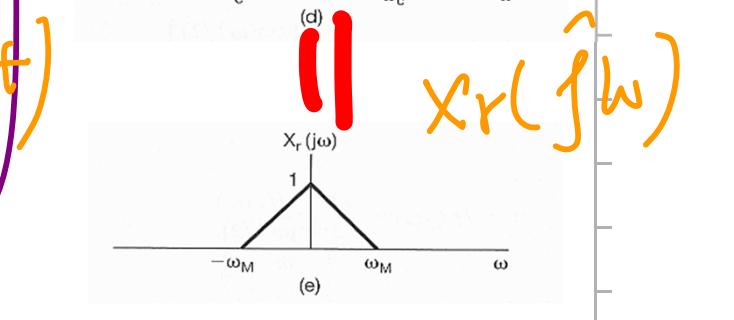
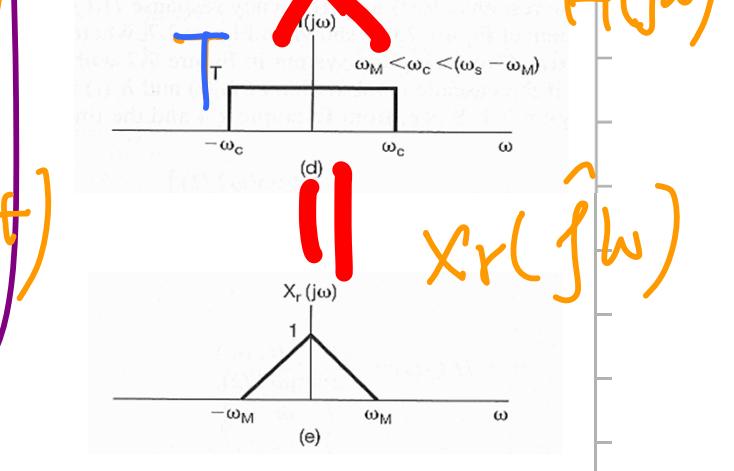
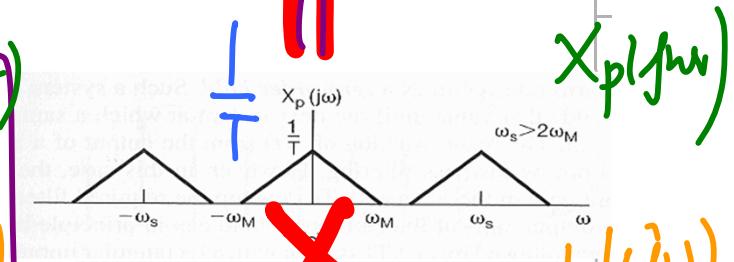
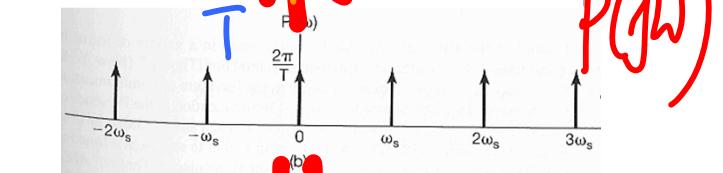
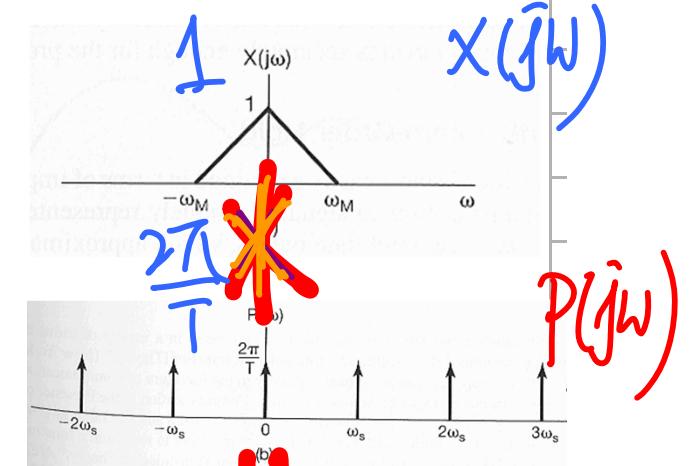
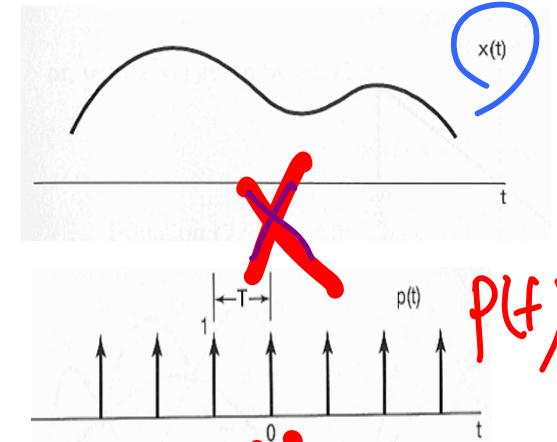
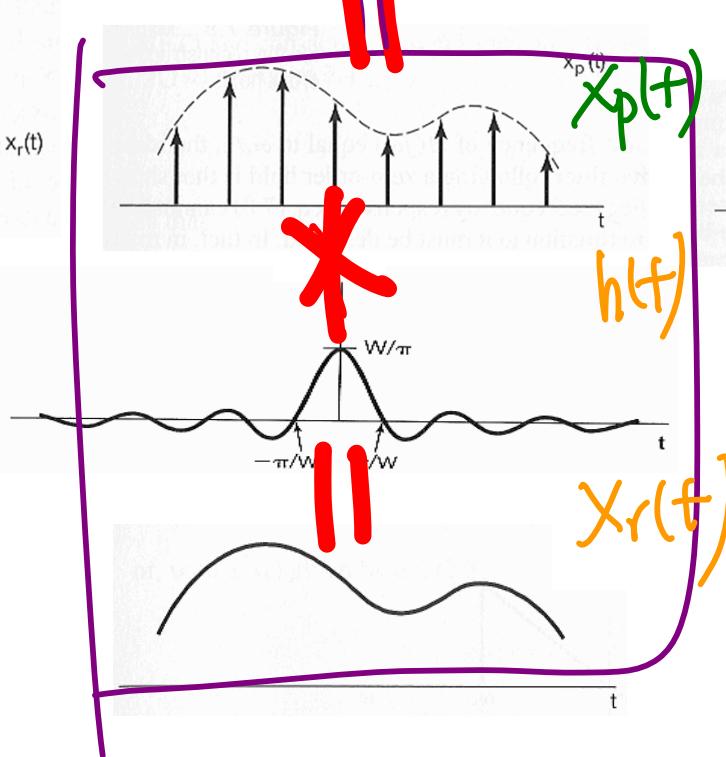
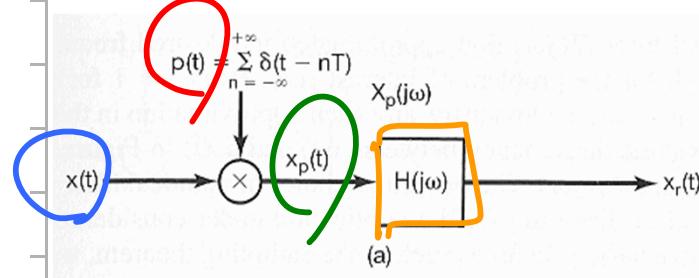
$$\frac{2\pi}{9} \times \frac{9}{2} = \pi$$

$$\frac{2\pi}{9} \times \frac{1}{2} = \frac{\pi}{9}$$

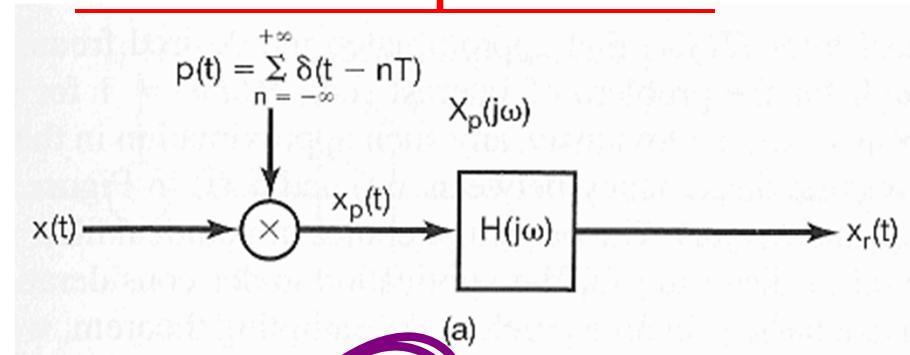
$$\frac{\pi}{9} \times 9 = \pi$$



Exact Recovery by an Ideal Lowpass Filter:



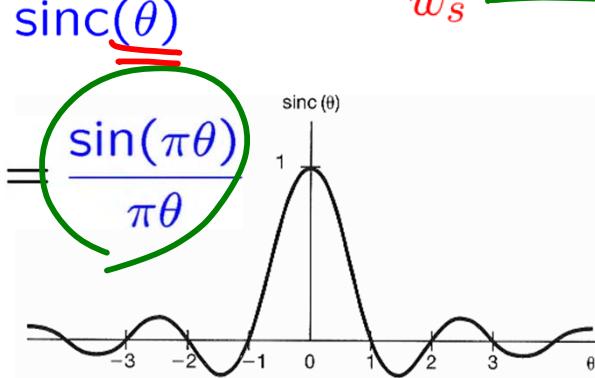
Exact Interpolation:



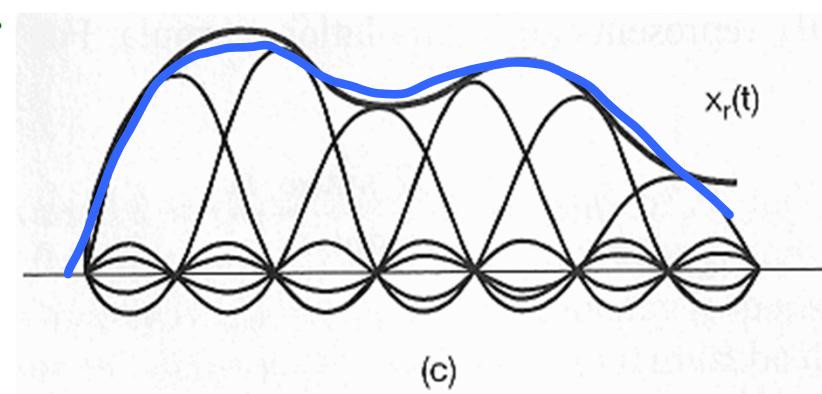
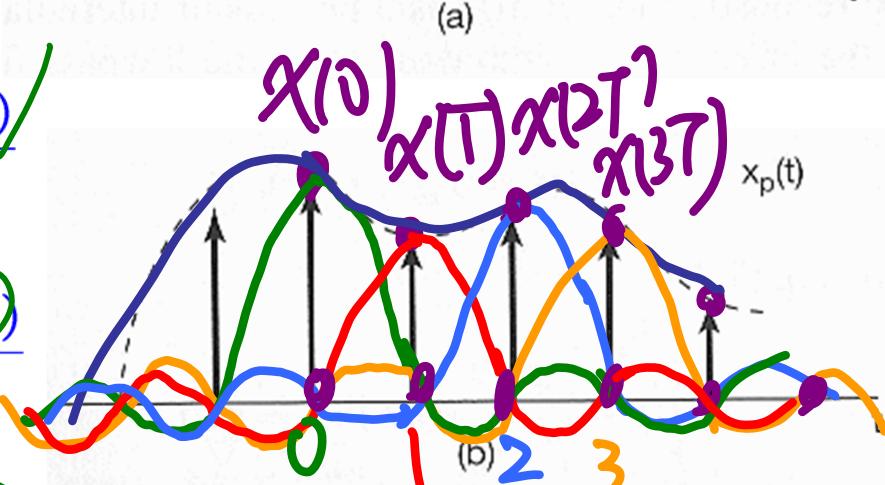
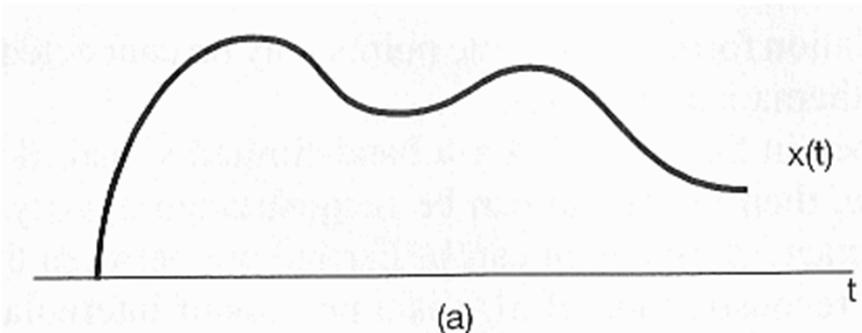
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT) \frac{w_c T}{\pi} \frac{\sin(w_c(t - nT))}{w_c(t - nT)}$$

$$\frac{w_c}{\pi} \frac{2\pi}{w_s} \frac{\sin(\pi(w_c(t - nT)/\pi))}{\pi w_c(t - nT)/\pi}$$

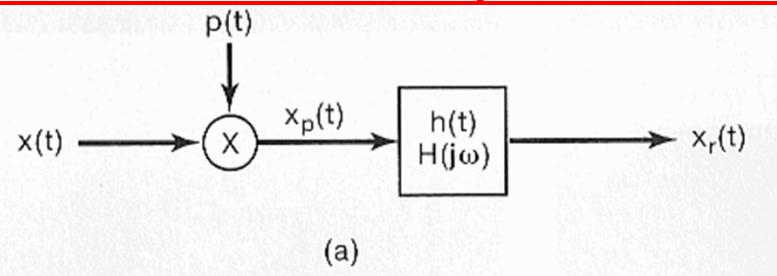
$$\frac{2w_c}{w_s} \text{sinc}(\frac{w_c(t - nT)}{\pi})$$



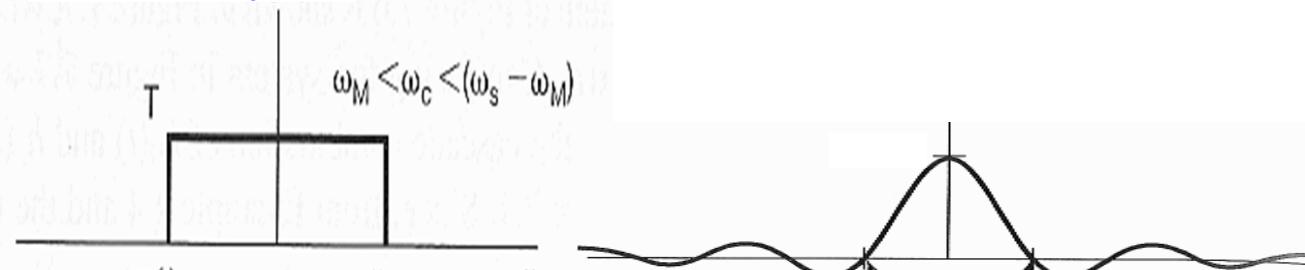
$n=0$
 $n=1$
 $n=2$
 $n=3$



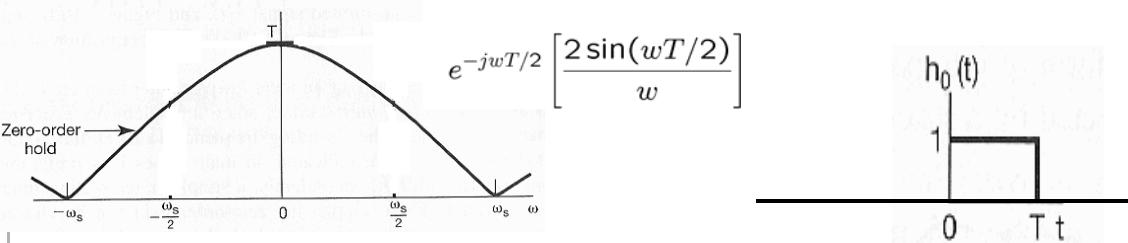
■ Three Filters: Ideal Lowpass, Zero-Order, First-Order



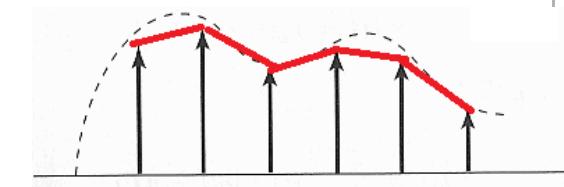
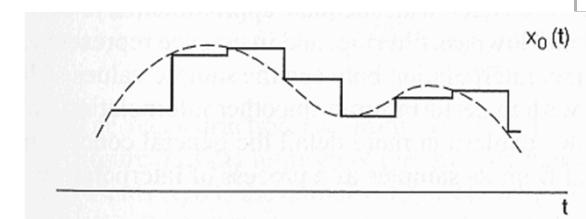
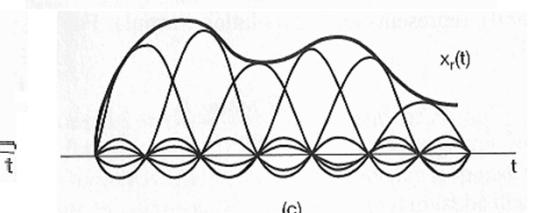
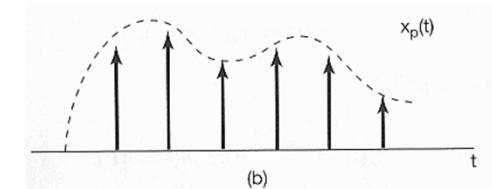
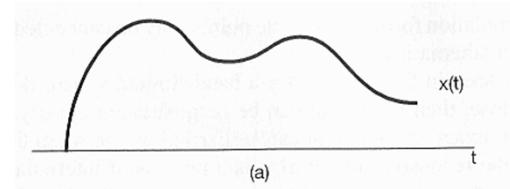
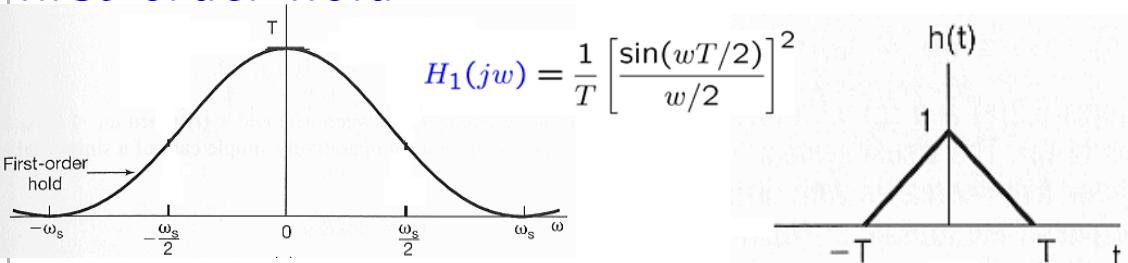
ideal lowpass



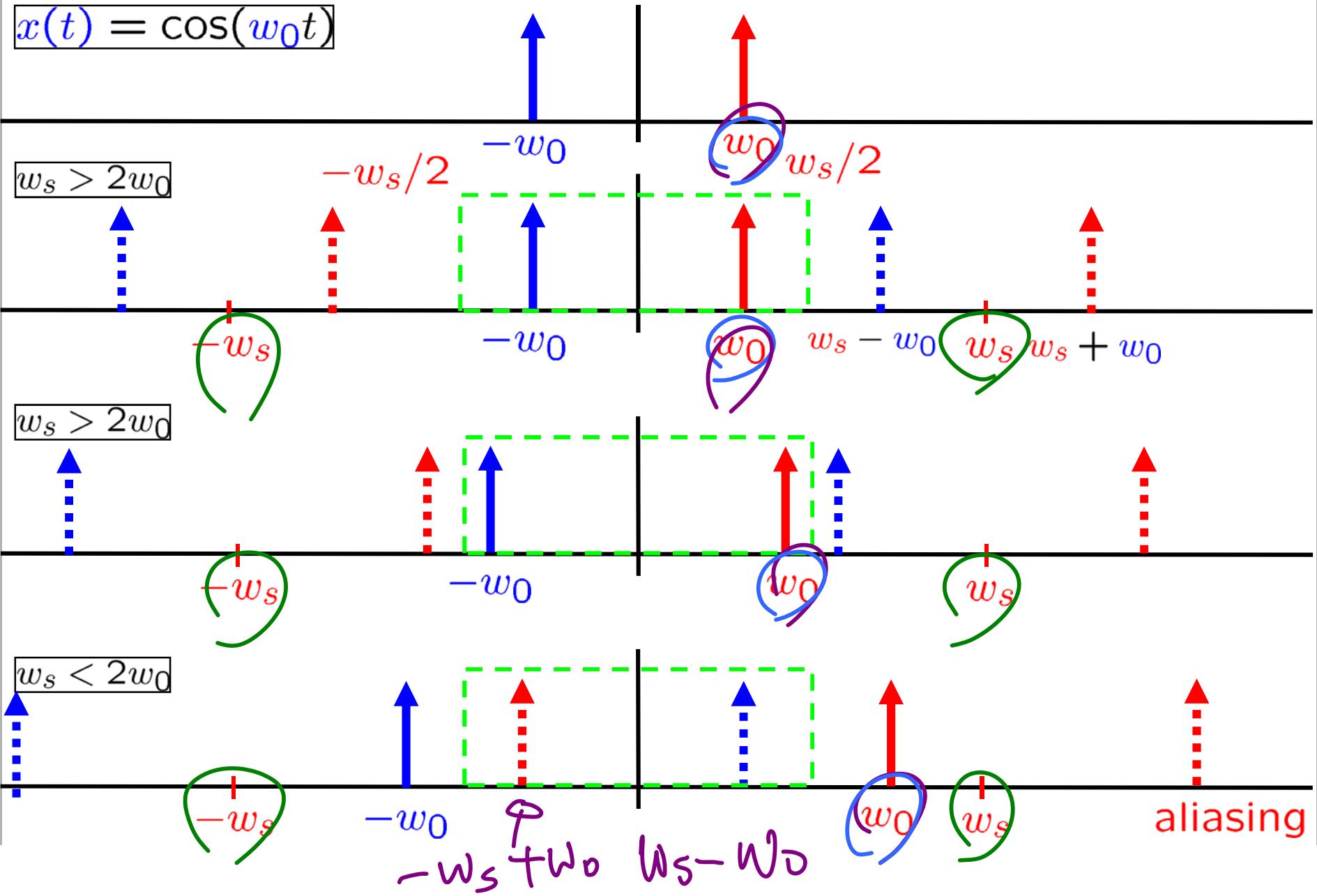
zero-order hold $H_0(jw) =$



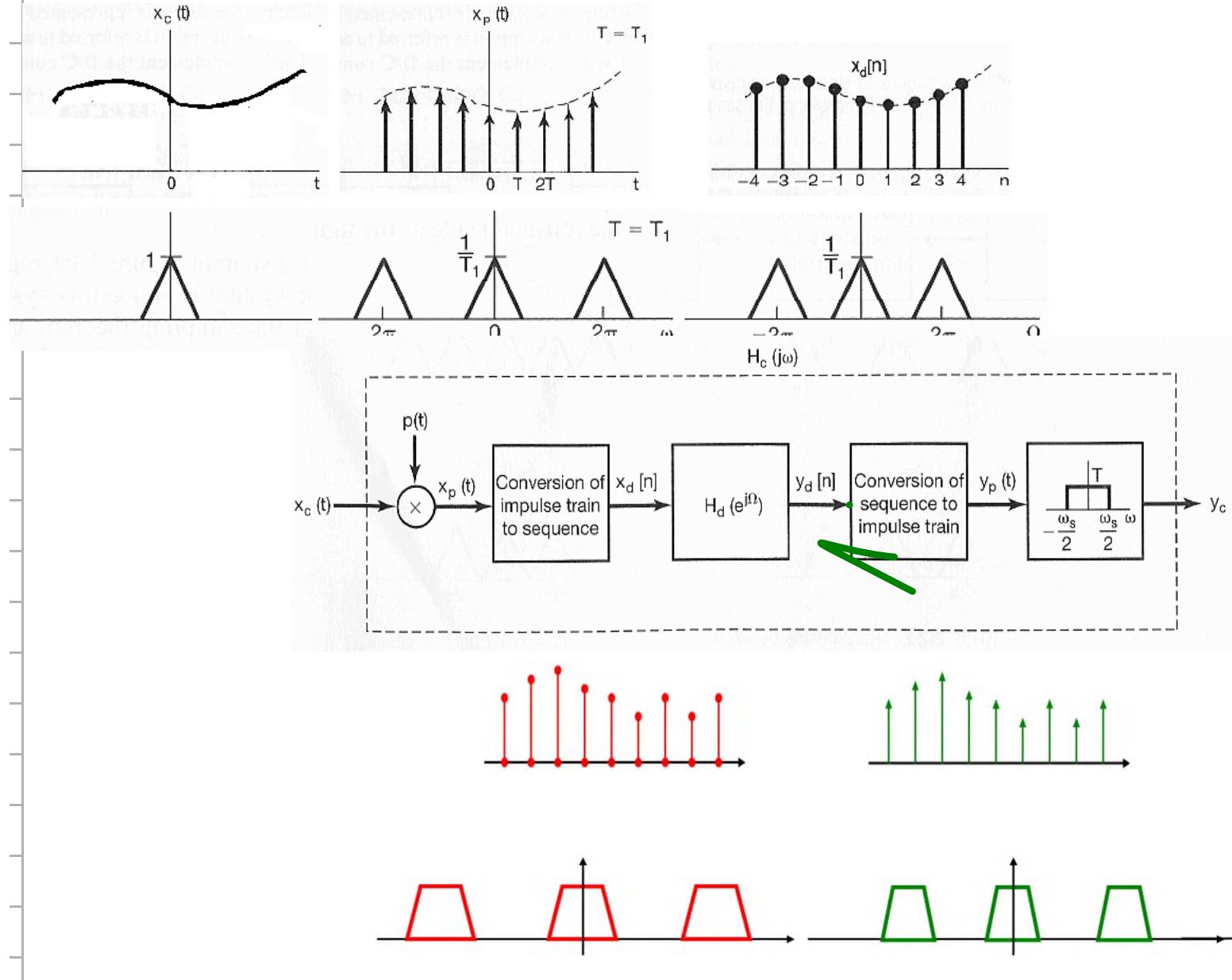
first-order hold



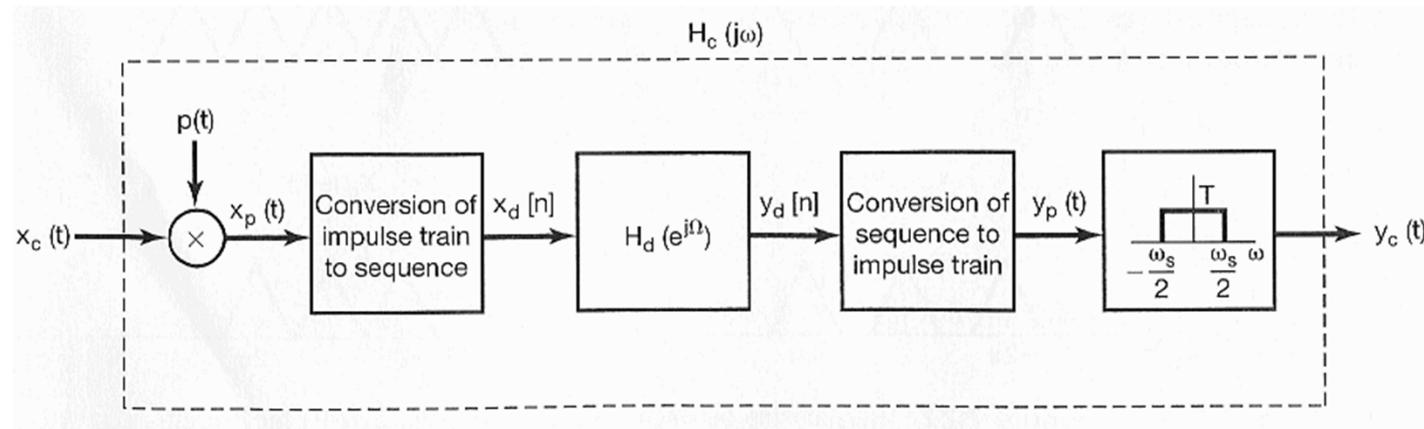
▪ Overlapping in Frequency-Domain: Aliasing



■ Discrete-Time Processing of CT Signals

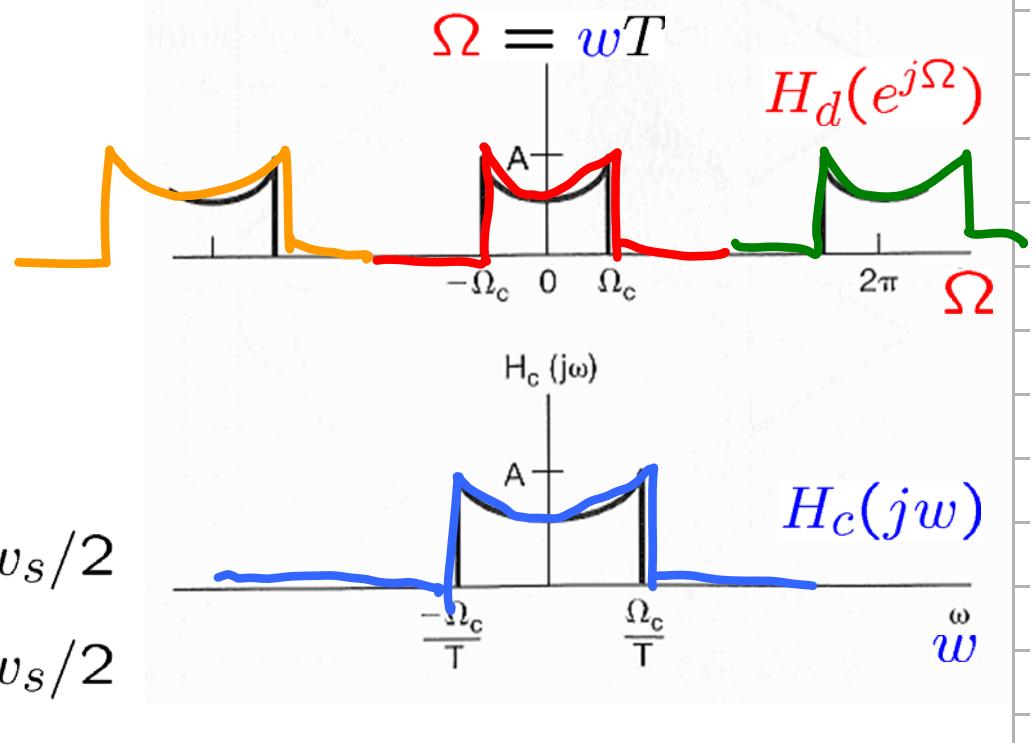


■ CT & DT Frequency Responses:

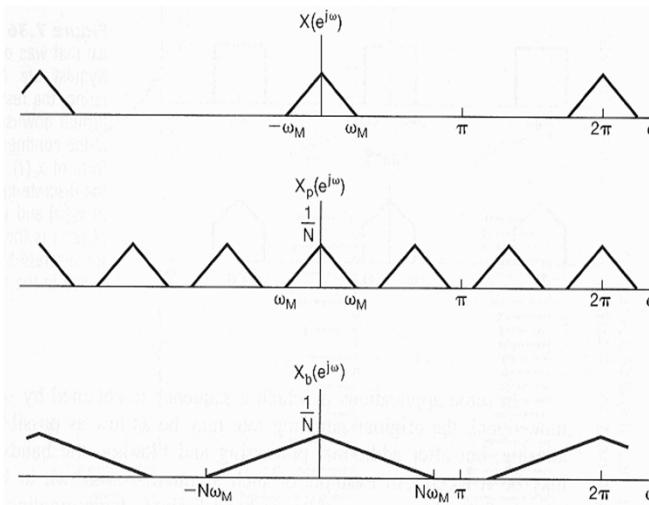
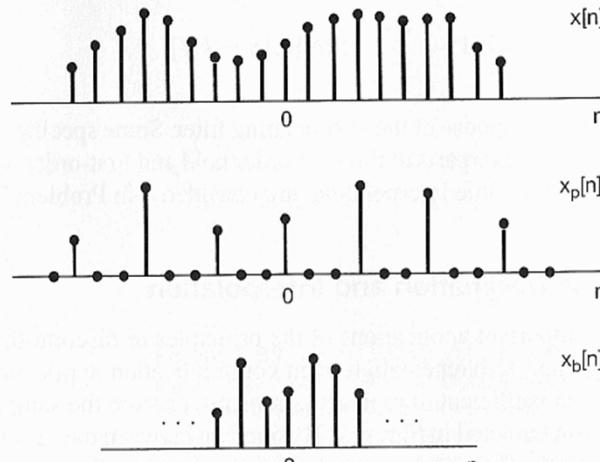


$$Y_c(jw) = X_c(jw) H_c(jw)$$

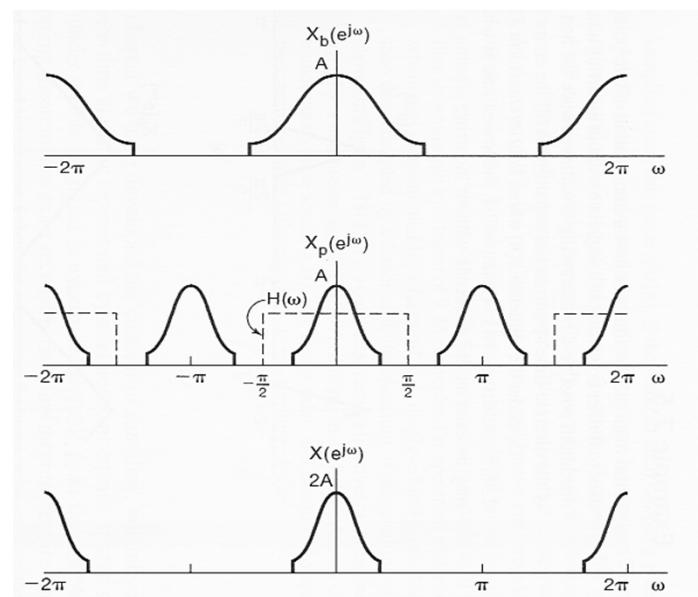
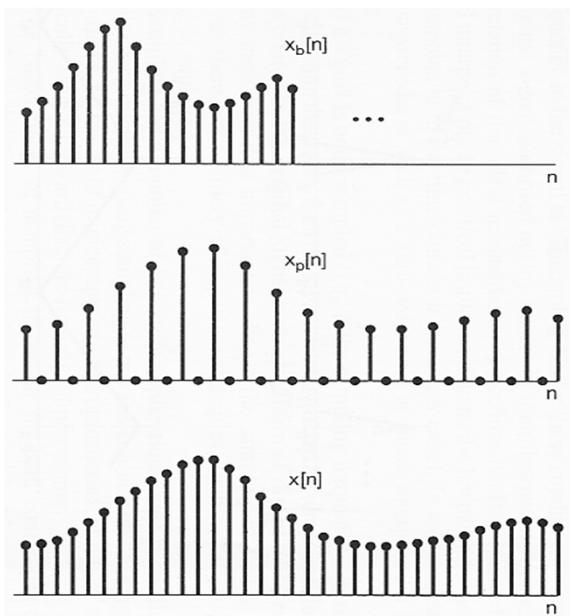
$$H_c(jw) = \begin{cases} H_d(e^{jwT}), & |w| < w_s/2 \\ 0, & |w| > w_s/2 \end{cases}$$

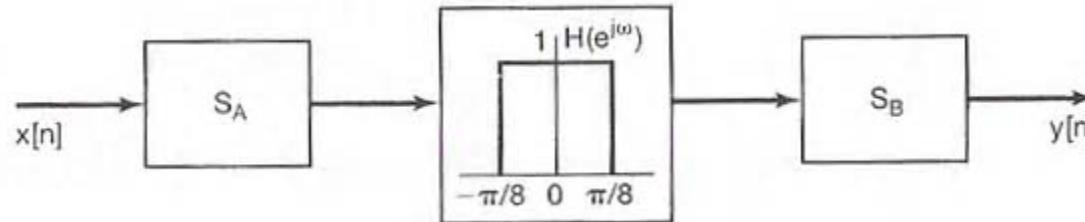
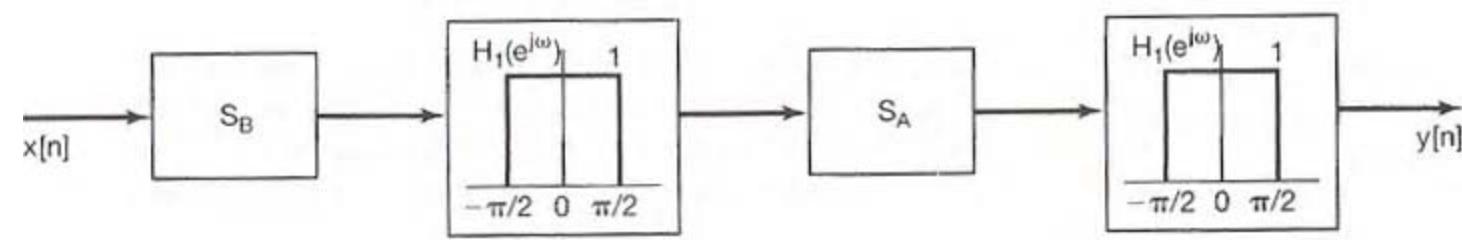


- Decimation (Down-sampling):**



- Interpolation (Up-sampling):**



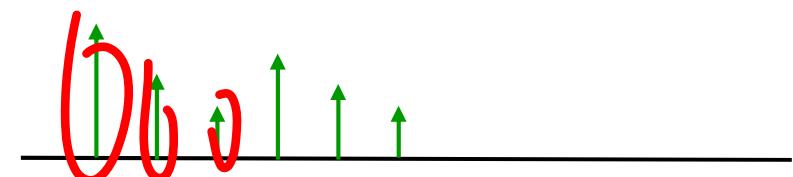
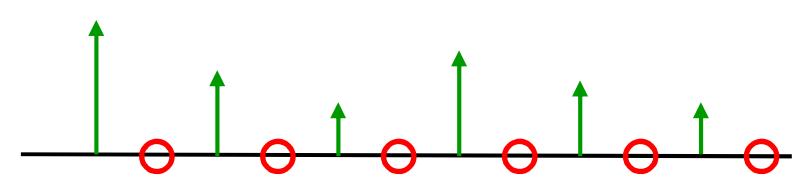
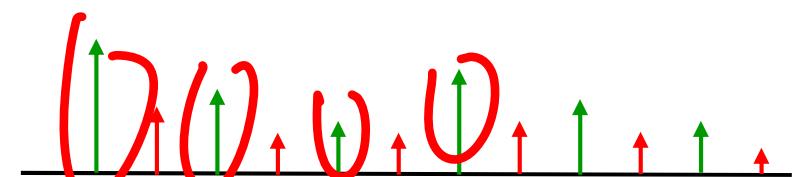
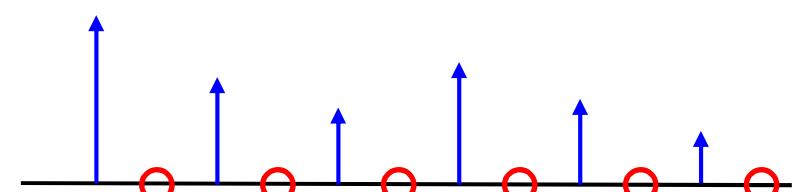
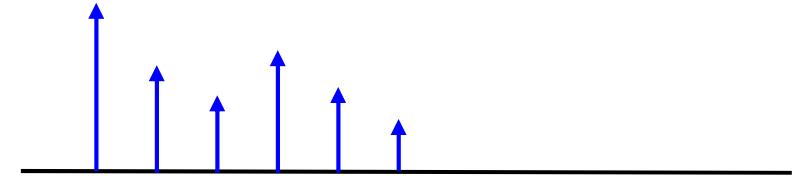
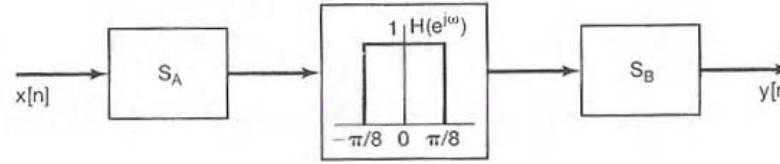
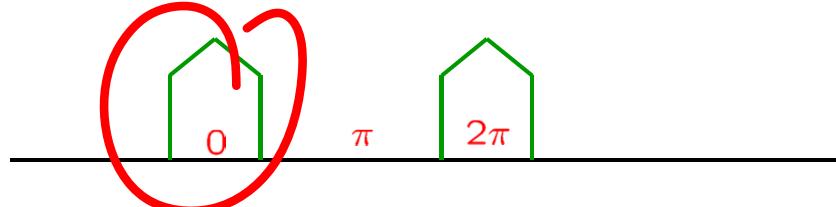
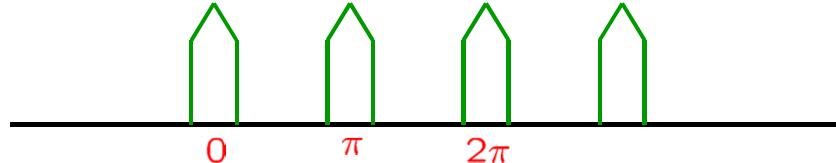
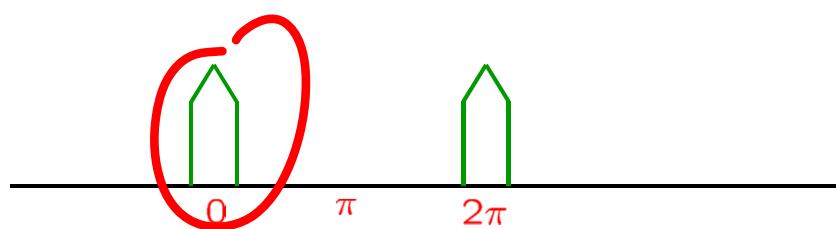
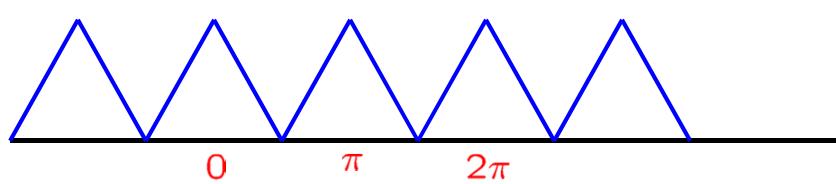
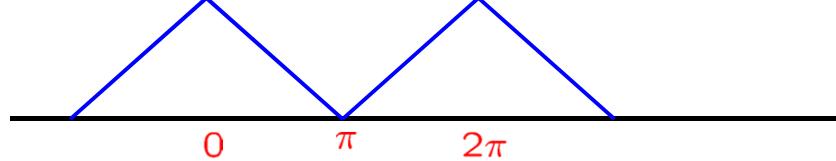
■ Problem 7.20 (p.560):**■ (a)****■ (b)**

- S_A : Inserting one zero after each sample
- S_B : Decimation 2:1, extracting every second sample
- Which corresponds to low-pass filtering with $\omega_c = \pi/4$?

Sampling of Discrete-Time Signals

- **Problem 7.20 (p.560):**

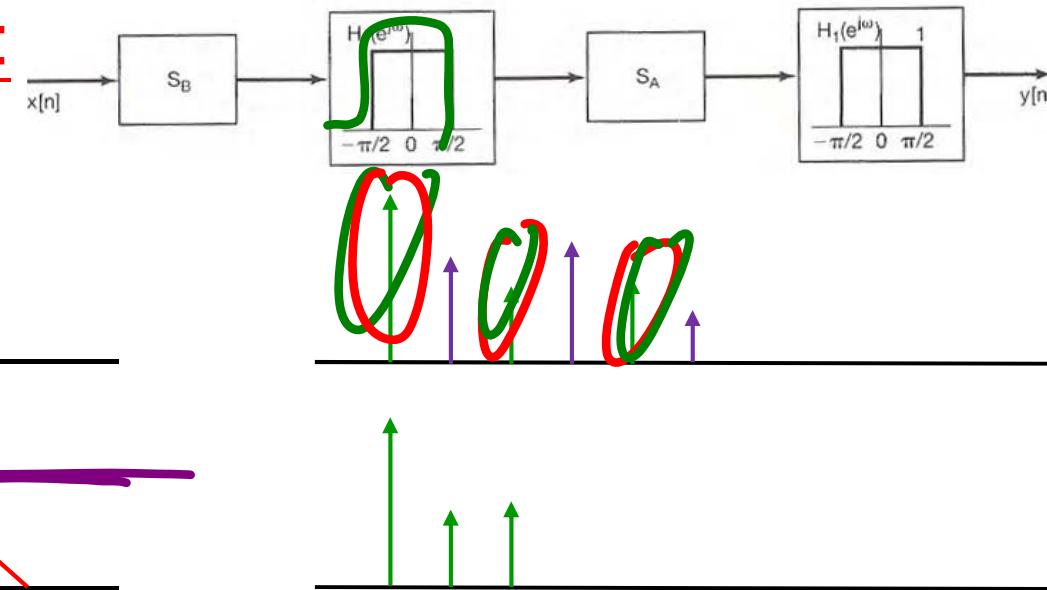
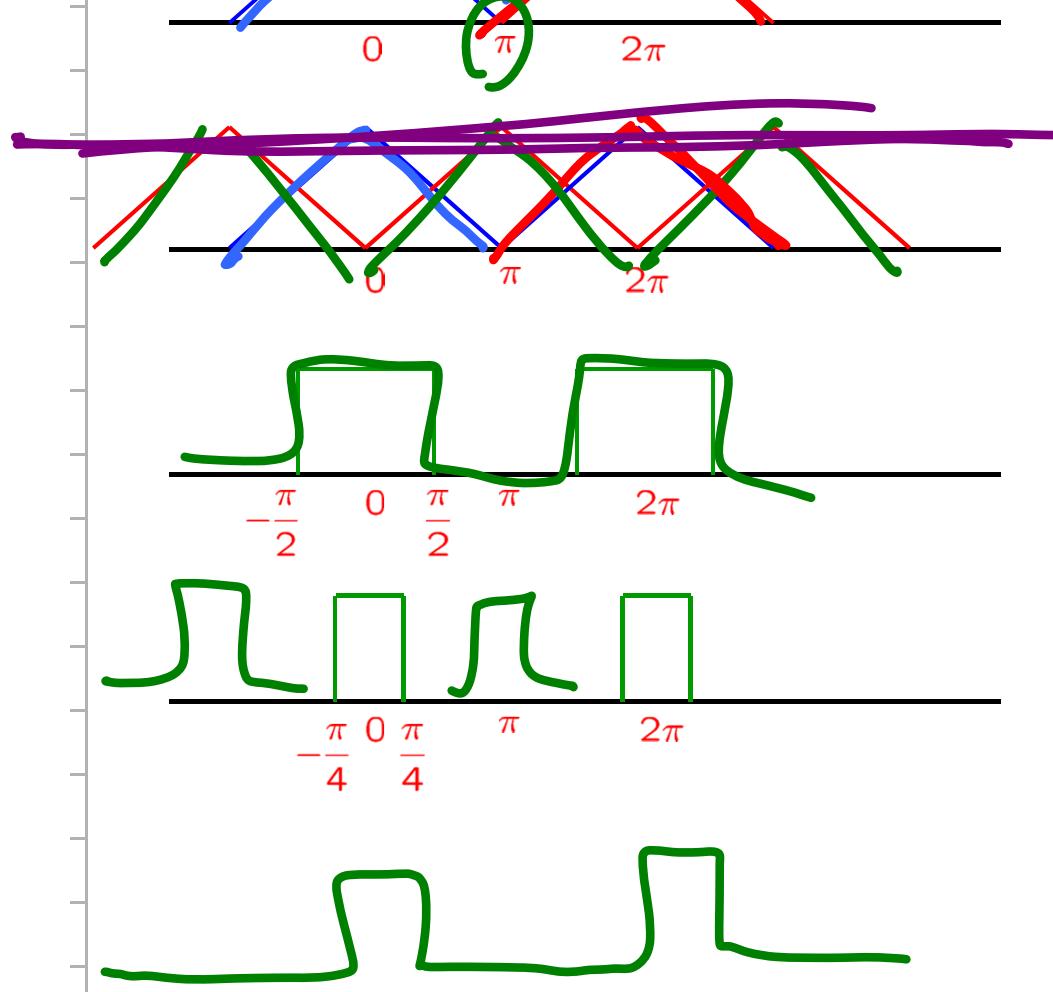
- (a)

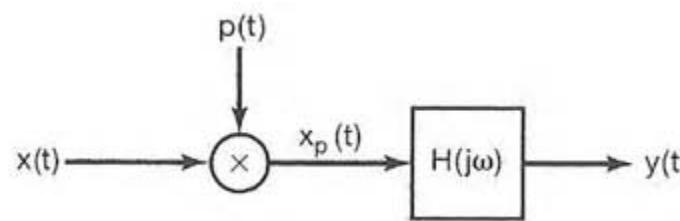
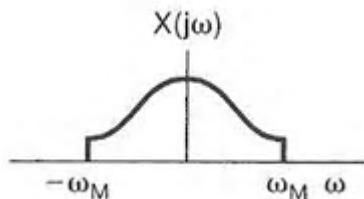
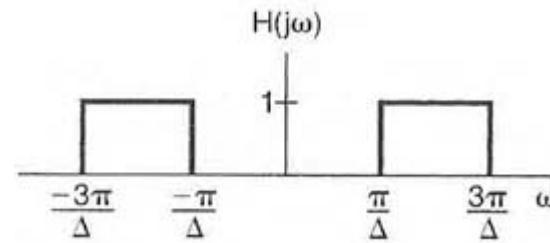
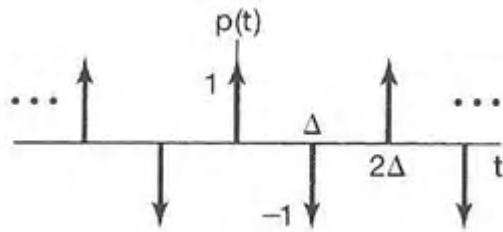


Sampling of Discrete-Time Signals

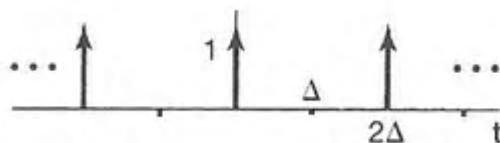
- Problem 7.20 (p.560):

- (b)



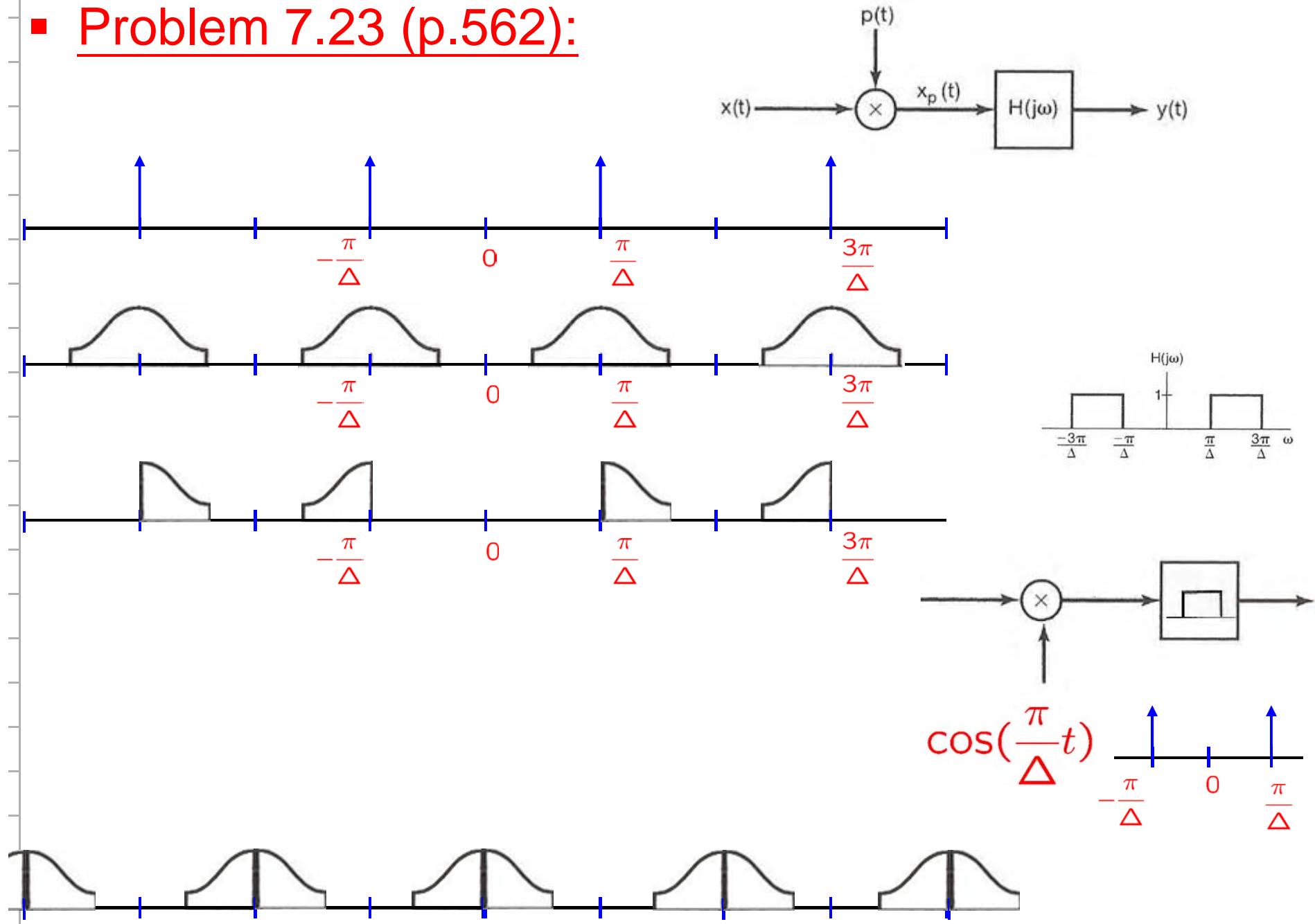
■ Problem 7.23 (p.562):

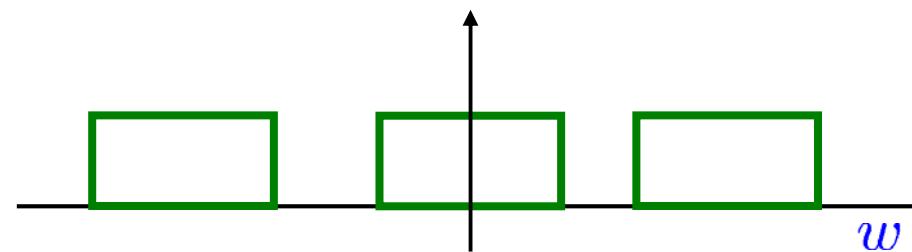
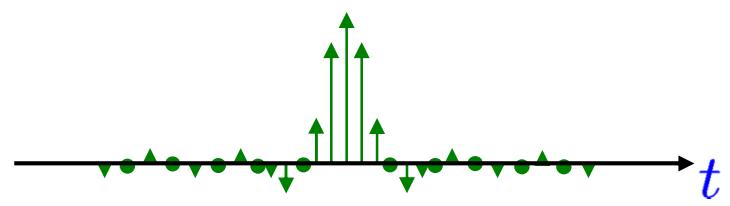
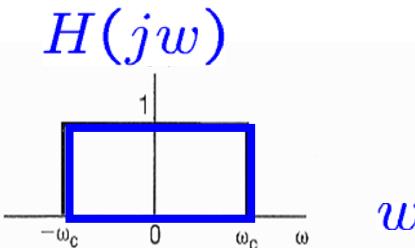
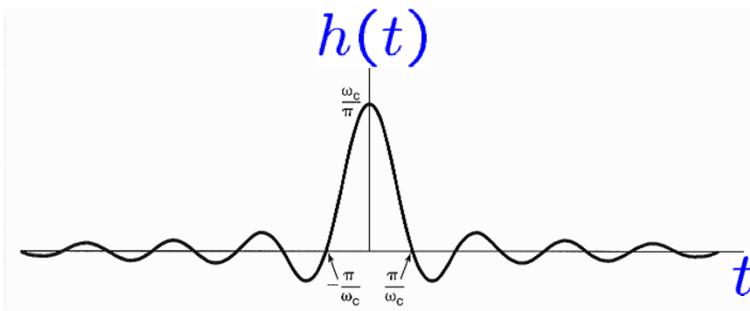
$$p_1(t)$$



$$p(t) = p_1(t) - p_1(t - \Delta)$$

■ Problem 7.23 (p.562):



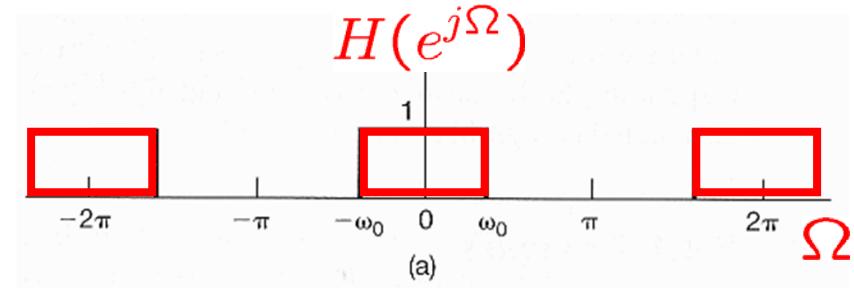
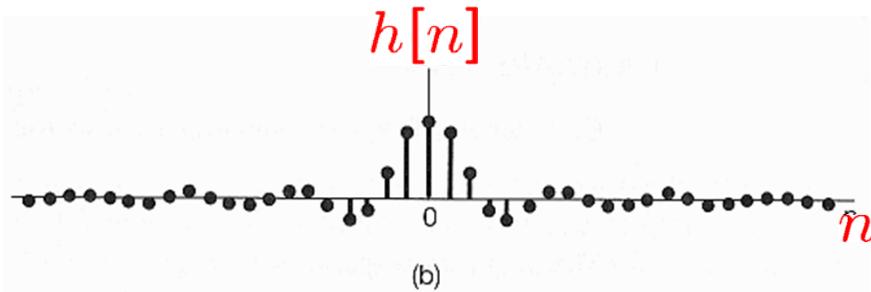


$$h(t) \xleftrightarrow{\mathcal{C.T.F.T.}} H(jw)$$

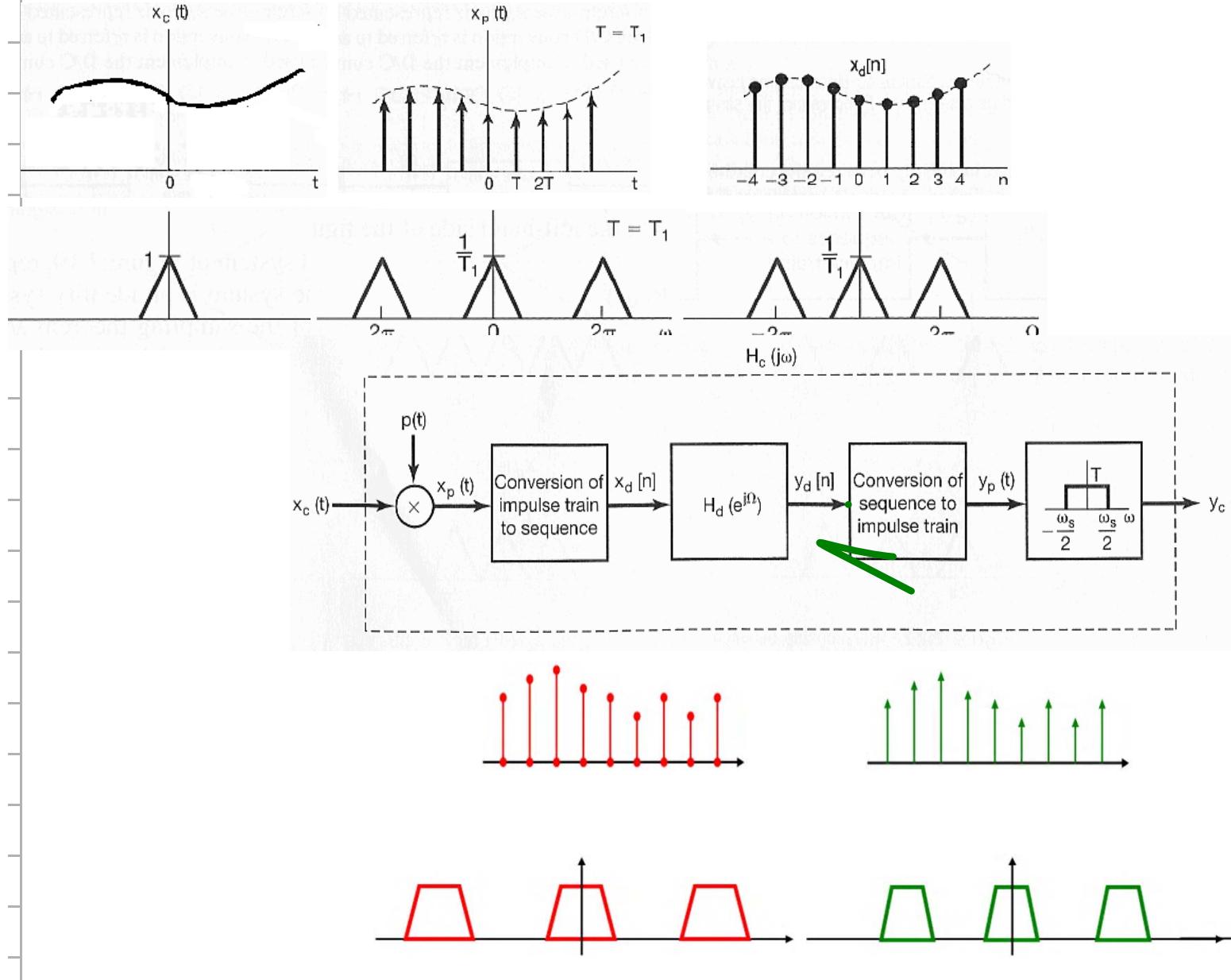
$$w_s = \frac{2\pi}{T}$$

$$\Omega = wT$$

$$h[n] \xleftrightarrow{\mathcal{D.T.F.T.}} H(e^{j\Omega})$$



■ Discrete-Time Processing of CT Signals



- Representation of of a CT Signal by Its Samples:
 - The Sampling Theorem
- Reconstruction of of a Signal from Its Samples
 - Using exact interpolation
 - Using zero-order hold
 - Using higher-order hold
- The Effect of Under-sampling
 - Overlapping in Frequency-Domain
 - Aliasing
- DT Processing of CT Signals
- Sampling of Discrete-Time Signals
 - Down-sampling
 - Up-sampling

Introduction

[\(Chap 1\)](#)

LTI & Convolution

[\(Chap 2\)](#)

Bounded/Convergent

Periodic

FS

[\(Chap 3\)](#)

CT
DT

Aperiodic

FT

CT
DT

[\(Chap 4\)](#)
[\(Chap 5\)](#)

Unbounded/Non-convergent

LT

CT [\(Chap 9\)](#)

zT

DT [\(Chap 10\)](#)

Time-Frequency [\(Chap 6\)](#)

CT-DT

[\(Chap 7\)](#)

Communication [\(Chap 8\)](#)

Control

[\(Chap 11\)](#)

Digital
Signal
Processing
[\(dsp-8\)](#)