Spring 2013

信號與系統
Signals and Systems

Chapter SS-6
Time & Frequency Characterization of Signals and Systems

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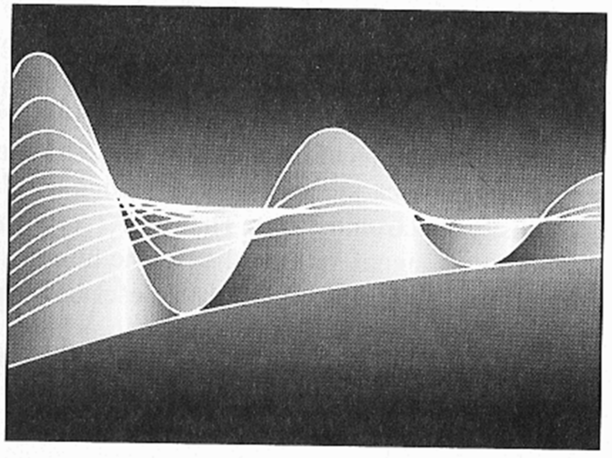
NTU-EE

Feb13 – Jun13



Figures and images used in these lecture notes are adopted from
"Signals & Systems" by Alan V. Oppenheim and Alan S. Willsky, 1997

Introduction (Chap 1)

LTI & Convolution (Chap 2)

Bounded/Convergent

Periodic

FS CT DT (Chap 3)

Aperiodic

FT CT (Chap 4) DT (Chap 5)

Unbounded/Non-convergent

LT CT (Chap 9)

zT DT (Chap 10)

Time-Frequency (Chap 6)

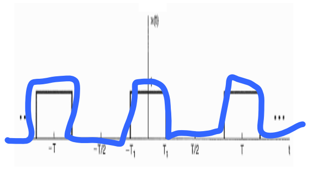
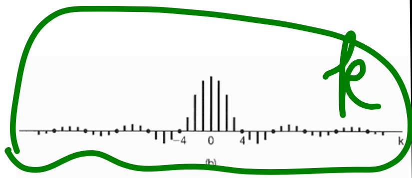
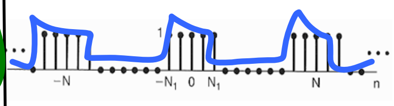
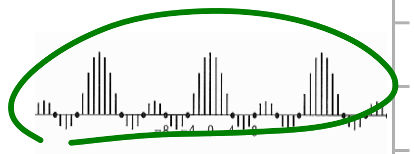
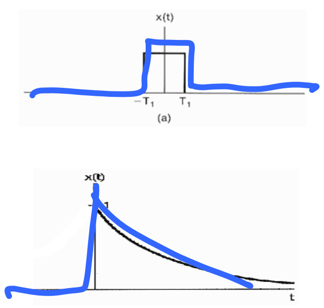
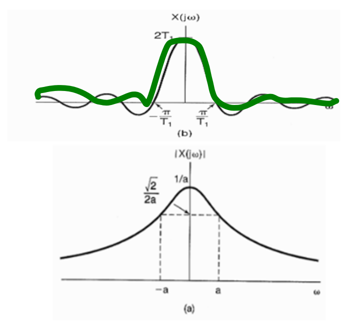
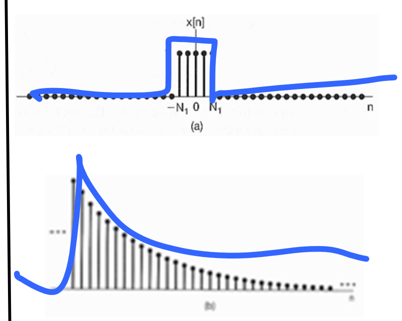
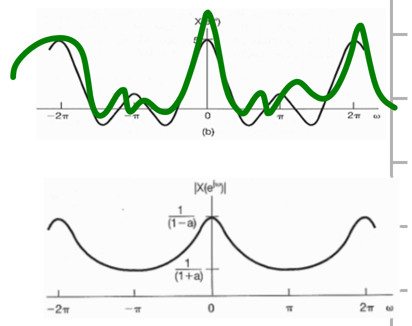
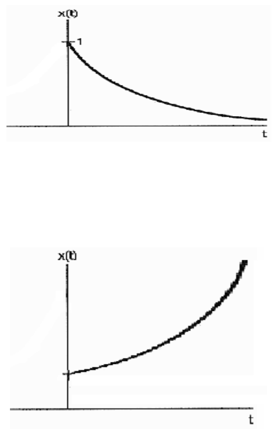
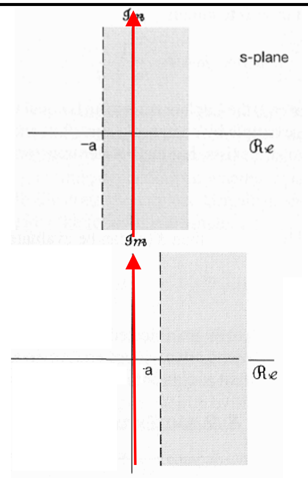
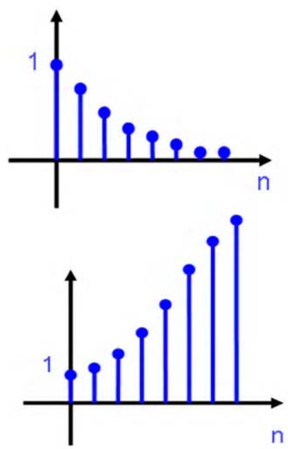
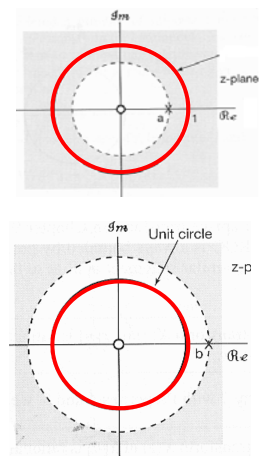
CT-DT (Chap 7)

Communication (Chap 8)

Control (Chap 11)

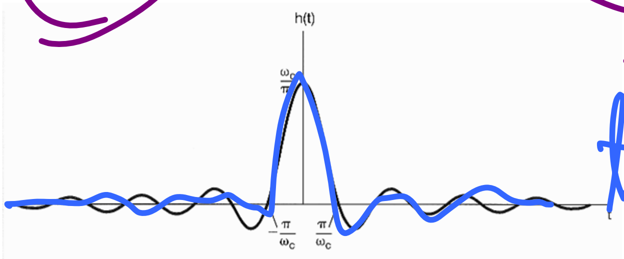
Digital Signal Processing (dsp-8)

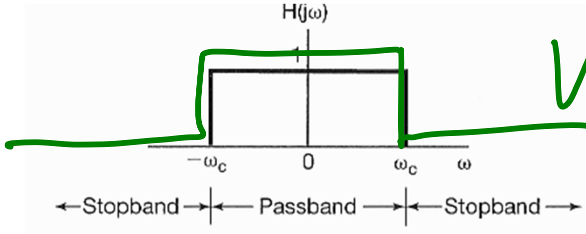
Fourier Series, Fourier Transform, Laplace Transform, z-Transform

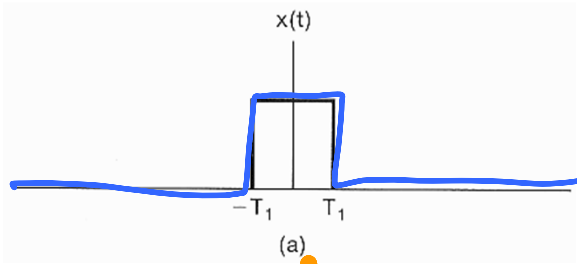
	CT		DT	
	time	frequency	time	frequency
FS				
FT				
LT/zT				

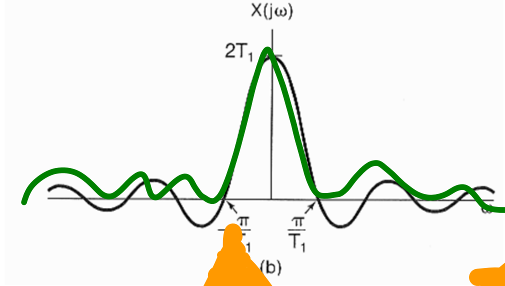
- Time-Domain & Frequency-Domain Characterization:

CT





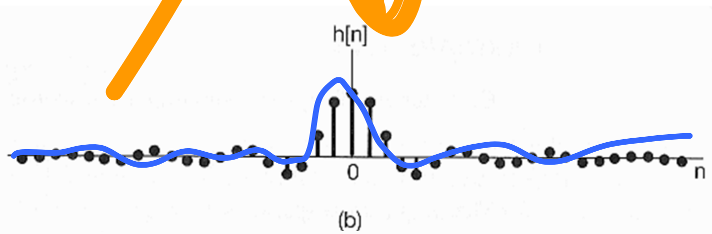


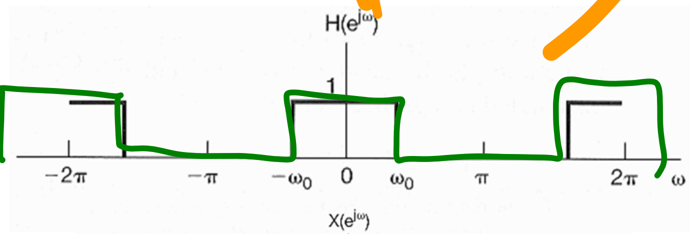


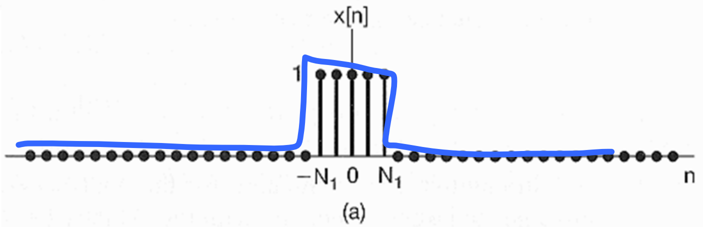
$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(jw)$$

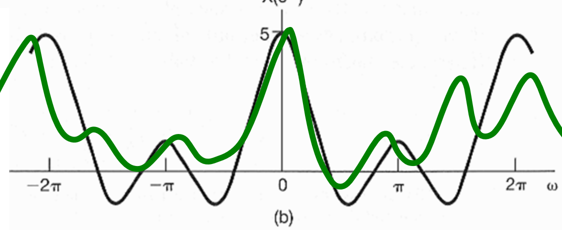
$$h[n] \overset{\mathcal{F}}{\longleftrightarrow} H(e^{jw})$$

DT









▪ Time-Domain & Frequency-Domain Characterization:

$$H(jw) = \int_{-\infty}^{\infty} h(t) e^{-jwt} dt \qquad H(e^{jw}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-jwn}$$

$\delta(t)/\delta[n]$		$h(t)/h[n]$
$x(t)/x[n]$	→ LTI System →	$y(t)/y[n]$
$X(jw)/X(e^{jw})$	$h(t)/h[n]$ $H(jw)/H(e^{jw})$	$Y(jw)/Y(e^{jw})$

$$x(t) \stackrel{\mathcal{F}}{\longleftrightarrow} X(jw) \qquad h(t) \stackrel{\mathcal{F}}{\longleftrightarrow} H(jw) \qquad y(t) \stackrel{\mathcal{F}}{\longleftrightarrow} Y(jw)$$

$$x[n] \stackrel{\mathcal{F}}{\longleftrightarrow} X(e^{jw}) \qquad h[n] \stackrel{\mathcal{F}}{\longleftrightarrow} H(e^{jw}) \qquad y[n] \stackrel{\mathcal{F}}{\longleftrightarrow} Y(e^{jw})$$

$$y(t) = x(t) * h(t) \stackrel{\mathcal{F}}{\longleftrightarrow} Y(jw) = X(jw)H(jw)$$

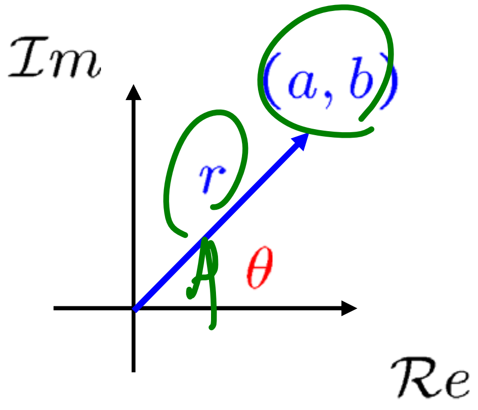
$$y[n] = x[n] * h[n] \stackrel{\mathcal{F}}{\longleftrightarrow} Y(e^{jw}) = X(e^{jw})H(e^{jw})$$

Introduction

Time-Domain	Convolution ⟷ Transformation	Frequency-Domain
Differential Eqns or Difference Eqns	System Model & Operations	Algebraic Equations
Convolution Multiplication	Techniques	Multiplication Convolution
Time-Domain Considerations	System Design	Frequency-Domain Considerations

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems (p. 427)
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order Continuous-Time Systems
- 1st-Order & 2nd-Order Discrete-Time Systems
- Time- & Frequency-Domain Analysis of Systems

Magnitude & Phase Representation:



$$a + jb \Rightarrow \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow a + jb = r e^{j\theta}$$

$$X(jw) = \mathcal{R}e\{X(jw)\} + j\,\mathcal{I}m\{X(jw)\} \qquad = \left|X(jw)\right| e^{j\sphericalangle X(jw)}$$

$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\,\mathcal{I}m\left\{X(e^{jw})\right\} \qquad = \left|X(e^{jw})\right| e^{j\sphericalangle X(e^{jw})}$$

$\left|X(jw)\right|$ or $\left|X(e^{jw})\right|$: magnitude

$\sphericalangle X(jw)$ or $\sphericalangle X(e^{jw})$: phase angle

▪ Magnitude Distortion & Phase Distortion: (p. 428)

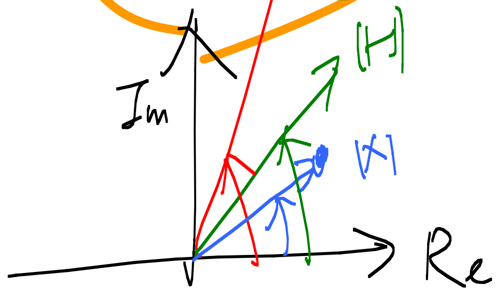
$X \rightarrow$ H LTI System $\rightarrow Y$

$$Y(jw) = X(jw)\, H(jw)$$
$$Y(e^{jw}) = X(e^{jw})\, H(e^{jw})$$

$$\Rightarrow |Y(jw)|e^{j\sphericalangle Y(jw)} = |X(jw)|e^{j\sphericalangle X(jw)}|H(jw)|e^{j\sphericalangle H(jw)}$$
$$= |X(jw)||H(jw)|e^{j(\sphericalangle X(jw)+\sphericalangle H(jw))}$$

$|Y|=|H||X|$

$$\Rightarrow \begin{cases} |Y(jw)| = |X(jw)|\,|H(jw)| & \text{magnitude distortion} \\ \sphericalangle Y(jw) = \sphericalangle X(jw) + \sphericalangle H(jw) & \text{phase distortion} \end{cases}$$



$$\Rightarrow \begin{cases} |Y(e^{jw})| = |X(e^{jw})|\,|H(e^{jw})| \\ \sphericalangle Y(e^{jw}) = \sphericalangle X(e^{jw}) + \sphericalangle H(e^{jw}) \end{cases}$$

$|H(jw)|$ or $|H(e^{jw})|$: gain of the system

$\sphericalangle H(jw)$ or $\sphericalangle H(e^{jw})$: phase shift of the system

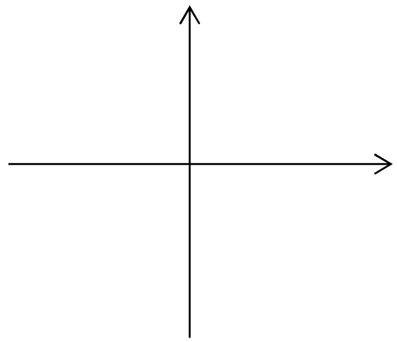
- **Log-Magnitude & Bode Plots: (p. 436)**

$$X \rightarrow \boxed{\begin{array}{c} H \\ \text{LTI System} \end{array}} \rightarrow Y$$

$$Y(jw) = X(jw)\, H(jw)$$

log log

$$\Rightarrow \begin{cases} |Y(jw)| & = |X(jw)|\, |H(jw)| \\ \sphericalangle Y(jw) & = \sphericalangle X(jw) + \sphericalangle H(jw) \end{cases}$$



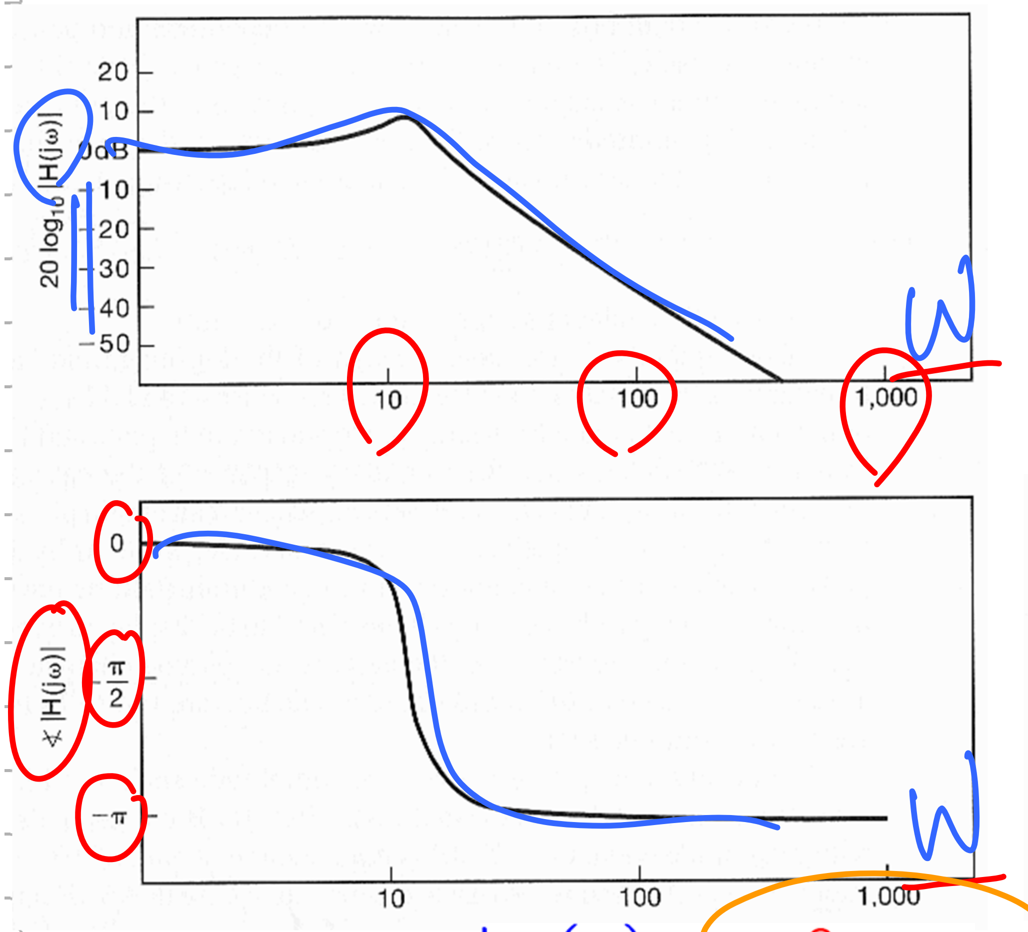
$$\Rightarrow \ \log|Y(jw)| = \log|X(jw)| + \log|H(jw)|$$

$$\Rightarrow 20 \log_{10}|Y(jw)| = 20 \log_{10}|X(jw)| + 20 \log_{10}|H(jw)|$$

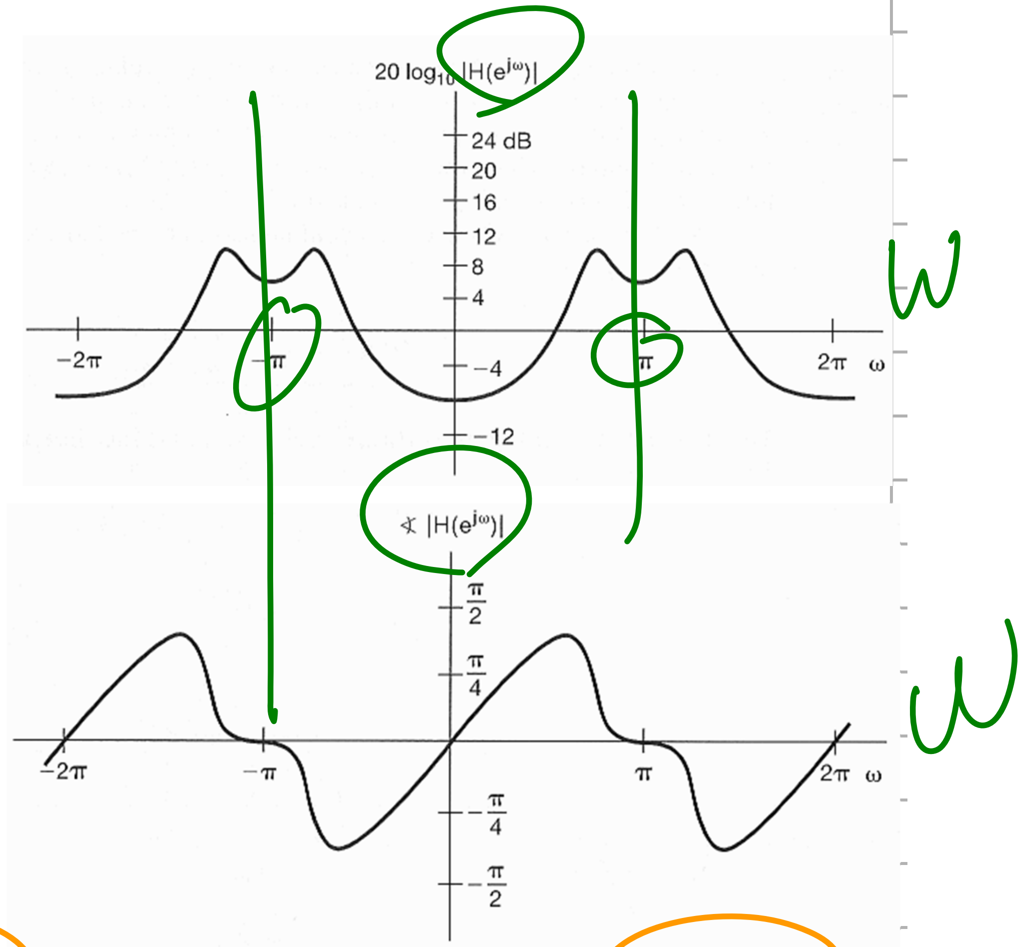
$$\Rightarrow \begin{cases} 20 \ \log_{10}(1) & = & 0 \ \ \text{dB} \\ 20 \ \log_{10}(10) & = & 20 \ \ \text{dB} \\ 20 \ \log_{10}(0.1) & = & -20 \ \ \text{dB} \end{cases}$$

▪ Log-Magnitude & Bode Plots: (p. 436)

Continuous-Time Bode plot



Discrete-Time Bode plot



$\log(w)$, $w: 0 \leftrightarrow \infty$

(w), $w: -\pi \leftrightarrow \pi$

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters (p. 448)
- 1st-Order & 2nd-Order Continuous-Time Systems
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- Time- & Frequency-Domain Analysis of Systems

▪ First-Order CT Systems: (p. 448)

$h(t)$

$x(t) \rightarrow$ LTI System $\rightarrow y(t)$

$$Y(jw) = X(jw)H(jw)$$

$$\mathcal{F}\left\{ \boxed{\tau \frac{d}{dt}y(t) + y(t) = x(t)} \right\}$$

$$\tau jw\, Y + 1\, Y = 1 X$$

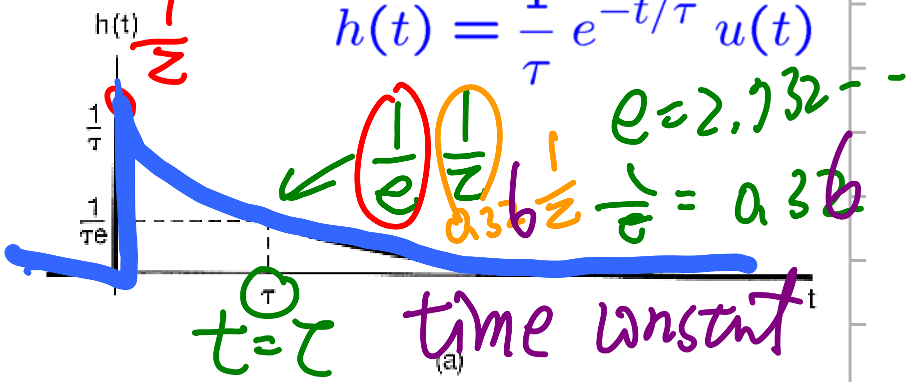
$$\Rightarrow \boxed{H(jw) = \frac{1}{jw\tau + 1}} = \frac{\frac{1}{\tau}}{jw + \frac{1}{\tau}}$$

$$\Rightarrow h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

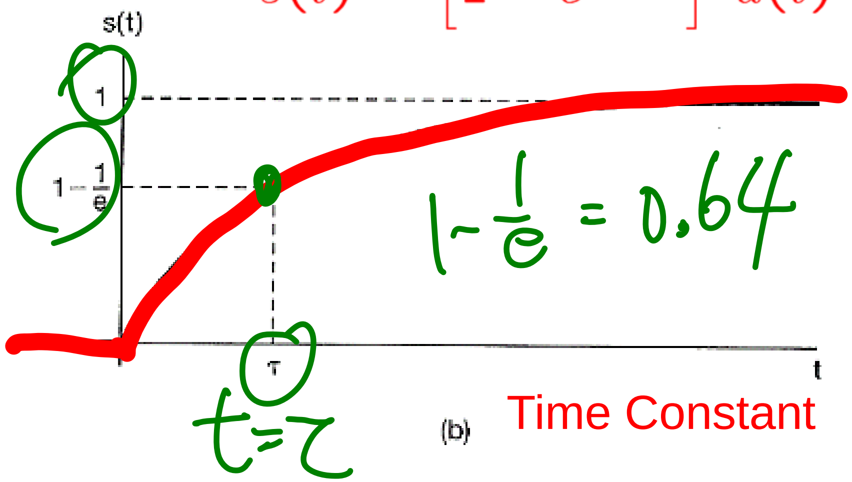
$$\Rightarrow s(t) = h(t) * u(t)$$

$$= \left[1 - e^{-t/\tau}\right] u(t)$$

$$h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$



$$s(t) = \left[1 - e^{-t/\tau}\right] u(t)$$



Time Constant

▪ First-Order CT Systems:

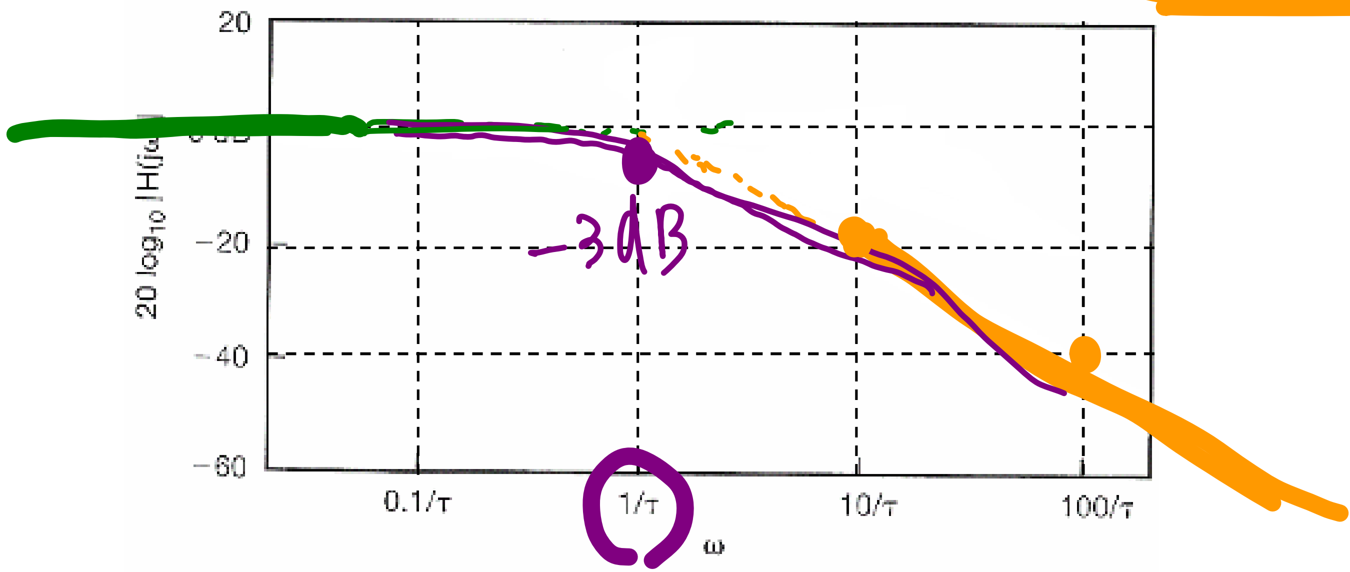
$$\boxed{H(jw) = \frac{1}{jw\tau + 1}}$$

$$|H(jw)| = \frac{1}{\sqrt{(w\tau)^2 + 1}}$$

$$20\log_{10}|H(jw)| = -10\log_{10}\left[(w\tau)^2 + \cancel{1}\right]$$

$$\approx \begin{cases} -10\log_{10}\left[(w\tau)^2+1\right] = 0 & w << \frac{1}{\tau} \\ -10\log_{10}\left[(w\tau)^2+1\right] = -10\log_{10}(2) \approx -3dB & w = \frac{1}{\tau} \\ -10\log_{10}\left[(w\tau)^2+1\right] = -20\log_{10}(w\tau) & w >> \frac{1}{\tau} \end{cases}$$

$$= -20\log_{10}(w) - 20\log_{10}(\tau)$$

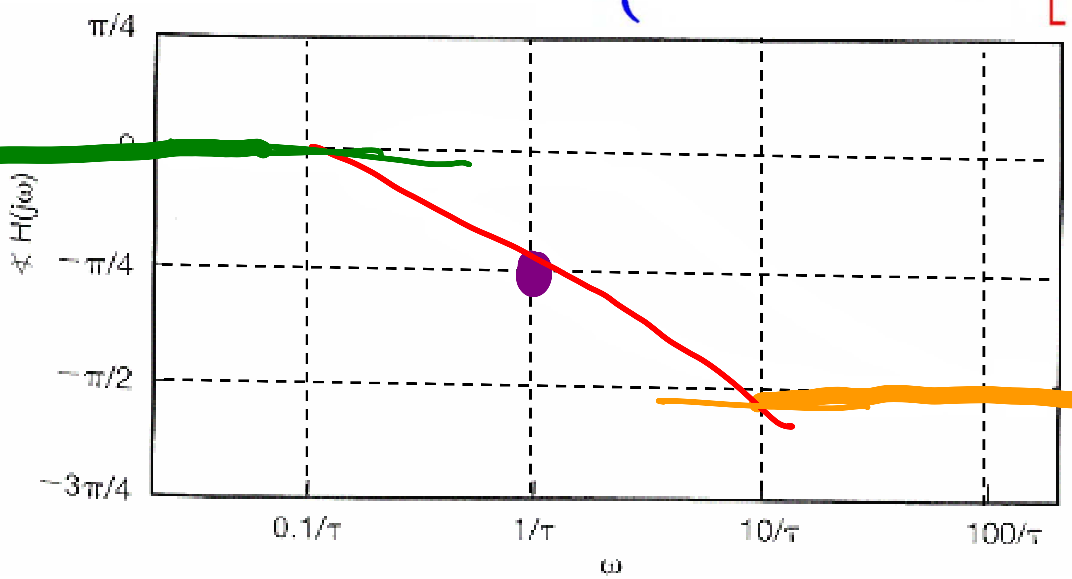


First-Order CT Systems:

$$H(jw) = \frac{1}{jw\tau + \cancel{1}}$$

$$\sphericalangle H(jw) = -\tan^{-1}(w\tau) \approx \begin{cases} 0 & w \le \frac{0.1}{\tau} \\ -\frac{\pi}{4} & w = \frac{1}{\tau} \\ -\frac{\pi}{2} & w \ge \frac{10}{\tau} \\ -(\frac{\pi}{4})[\log_{10}(w\tau) + 1] & \frac{0.1}{\tau} \le w \le \frac{10}{\tau} \end{cases}$$

$$= -(\tfrac{\pi}{4})\left[\log_{10}(w) + \log_{10}(\tau) + 1\right]$$



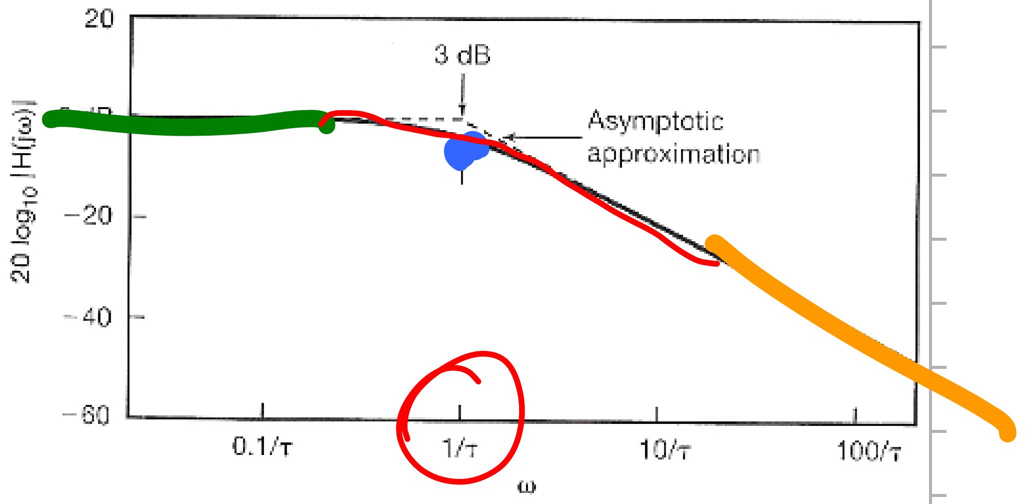
$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

▪ First-Order CT Systems:

$$H(jw) = \frac{1}{jw\tau + 1}$$

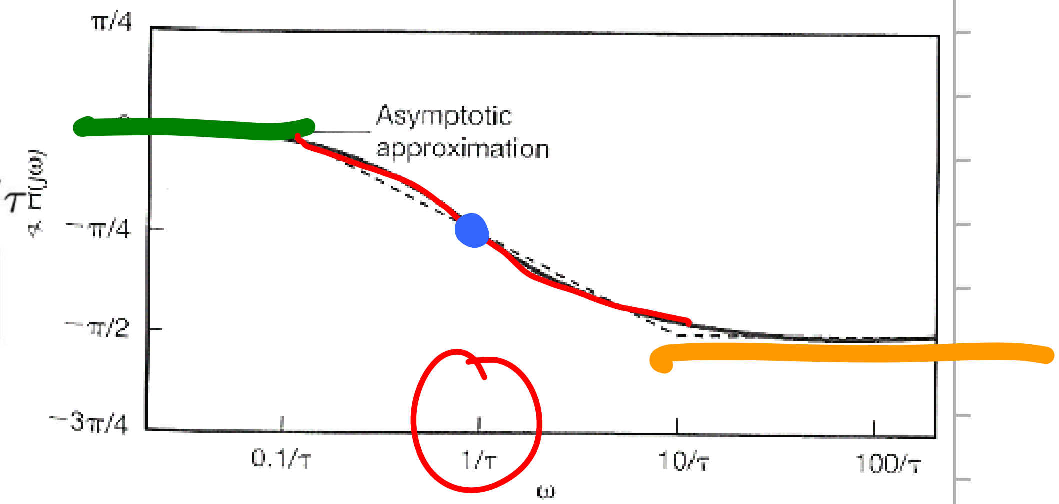
$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << 1/\tau \\ -10\log_{10}(2) \approx -3dB & w = 1/\tau \\ -20\log_{10}(w\tau) & w >> 1/\tau \\ \quad = -20\log_{10}(w) - 20\log_{10}(\tau) & \end{cases}$$



$$\sphericalangle H(jw) =$$

$$\begin{cases} 0 & w \le 0.1/\tau \\ -(\pi/4)[\log_{10}(w\tau) + 1] & 0.1/\tau \le w \le 10/\tau \\ \quad = -(\pi/4)\left[\log_{10}(w) + \log_{10}(\tau) + 1\right] & \\ -\pi/4 & w = 1/\tau \\ -\pi/2 & w \ge 10/\tau \end{cases}$$



Second-Order CT Systems: (p. 451)

zeta

$$\boxed{\frac{d^2}{dt^2}y(t)} + 2\zeta w_n \boxed{\frac{d}{dt}y(t)} + w_n^2 \boxed{y(t)} = w_n^2 \boxed{x(t)}$$

$$(jw)^2 Y \qquad jw\,Y \qquad Y \qquad X$$

$$\Rightarrow \quad H(jw) = \frac{w_n{}^2}{(jw)^2 + 2\zeta w_n (jw) + w_n{}^2}$$

$$= \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

$$\Rightarrow \quad H(jw) = \frac{w_n^2}{(jw - c_1)(jw - c_2)}$$

$$\begin{cases} c_1 = -\zeta w_n + w_n\sqrt{\zeta^2 - 1} \\ c_2 = -\zeta w_n - w_n\sqrt{\zeta^2 - 1} \end{cases}$$

Second-Order CT Systems:

$$\Rightarrow \quad H(jw) = \frac{{w_n}^2}{(jw)^2 + 2\cancel{\zeta} w_n (jw) + {w_n}^2}$$

$w = \pm w_n$

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

ζ : damping ratio

w_n : undamped natural frequency

$$\begin{cases} 0 < \zeta < 1 : & \text{underdamped} \\ \zeta = 1 : & \text{critically damped} \\ \zeta > 1 : & \text{overdamped} \end{cases}$$

Second-Order CT Systems:

$$H(jw) = \frac{w_n^2}{(jw - c_1)(jw - c_2)}$$

- For $\zeta = 1$, $\Rightarrow$ $c_1 = c_2 = -w_n$:

$$\Rightarrow H(jw) = \frac{w_n^2}{(jw + w_n)^2}$$

$$\Rightarrow h(t) = w_n^2 t e^{-w_n t} u(t)$$

- For $\zeta \neq 1$, $\Rightarrow$ c_1, c_2: unequal:

$$\Rightarrow H(jw) = \frac{M}{jw - c_1} - \frac{M}{jw - c_2}$$

$$M = \frac{w_n}{2\sqrt{\zeta^2 - 1}}$$

$$\Rightarrow h(t) = M\left[e^{c_1 t} - e^{c_2 t}\right] u(t)$$

Second-Order CT Systems:

$$M = \frac{w_n}{2\sqrt{\zeta^2 - 1}}$$

- For $0 < \zeta < 1$, c_1, c_2: complex:

$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1} = \frac{w_n^2}{(jw - c_1)(jw - c_2)}$$

$$\Rightarrow h(t) = M\left[e^{c_1 t} - e^{c_2 t}\right] u(t) \qquad \begin{cases} c_1 = -\zeta w_n + j w_n \sqrt{1-\zeta^2} \\ c_2 = -\zeta w_n - j w_n \sqrt{1-\zeta^2} \end{cases}$$

$$= \frac{w_n e^{-\zeta w_n t}}{2j\sqrt{1-\zeta^2}} \left\{ e^{j(w_n\sqrt{1-\zeta^2})t} - e^{-j(w_n\sqrt{1-\zeta^2})t} \right\} u(t)$$

$$= \frac{w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \left[\sin\left((w_n\sqrt{1-\zeta^2})t \right) \right] u(t)$$

$$\Rightarrow s(t) = h(t) * u(t) = \left\{ 1 + M\left[\frac{e^{c_1 t}}{c_1} - \frac{e^{c_2 t}}{c_2} \right] \right\} u(t) \qquad \zeta \neq 1$$

▪ Second-Order CT Systems:

ζ:

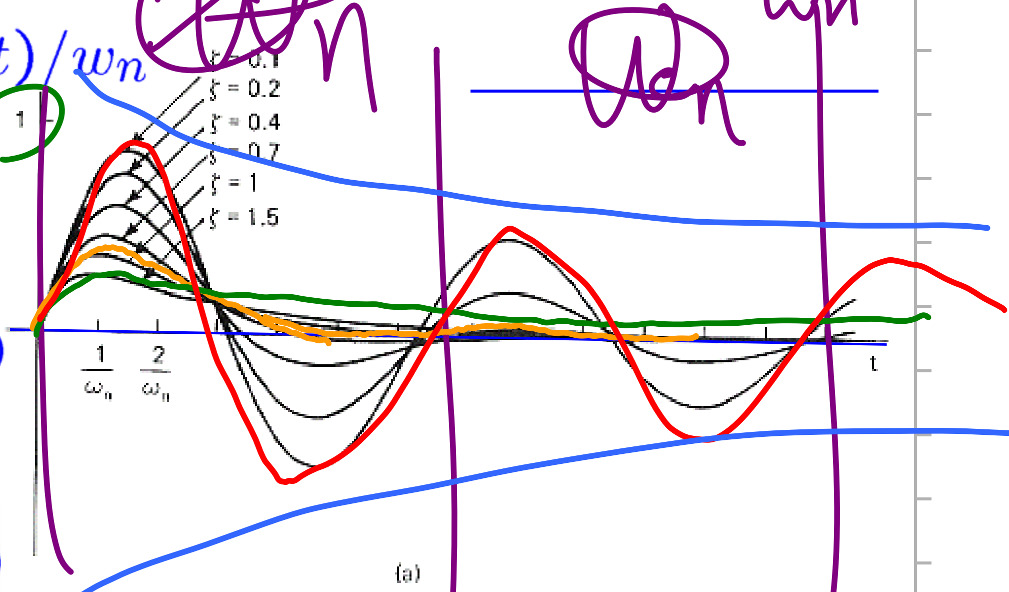
$$h(t) = \frac{w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \left[\sin\left((w_n \sqrt{1-\zeta^2}) t \right) \right] u(t)$$

$$\frac{h(t)}{w_n} = \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \left[\sin\left((w_n \sqrt{1-\zeta^2}) t \right) \right] u(t)$$

$$s(t) = \left\{ 1 + M \left[\frac{e^{c_1 t}}{c_1} - \frac{e^{c_2 t}}{c_2} \right] \right\} u(t)$$

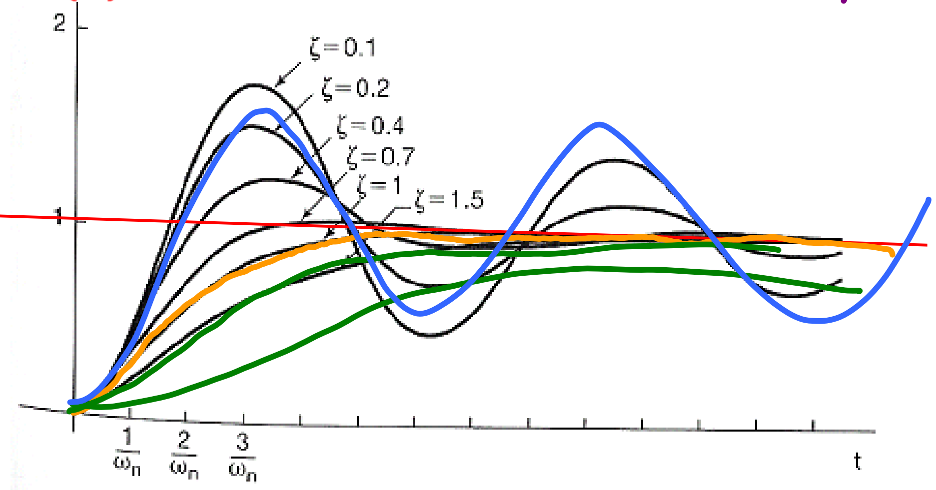
$h(t)/w_n$

$\cancel{1/w_n}$ $\quad$ $2\pi/w_n$ $\quad$ w_n



(a)

$s(t)$



(b)

- **Second-Order CT Systems:** $H(jw) = \dfrac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$

$$|H(jw)| = \frac{1}{\sqrt{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2}}$$

$$20\log_{10}|H(jw)| = -10\log_{10}\left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2\right\}$$

$$\left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2\right\} \qquad \left\{\left[1 - \left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2\right\}$$

$$Q = \frac{1}{2\zeta} \qquad \approx \begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$

$$|H(jw)| = \frac{1}{\sqrt{\left[1-\left(\frac{w}{w_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{w}{w_n}\right)^2}}$$

$$\Rightarrow \left(\frac{w}{w_n}\right)^4 - 2\left(\frac{w}{w_n}\right)^2 + 1 + 4\zeta^2\left(\frac{w}{w_n}\right)^2 = \left(\frac{w}{w_n}\right)^4 + (4\zeta^2-2)\left(\frac{w}{w_n}\right)^2 + 1$$

$$\Rightarrow \frac{d}{dw}\left\{\left(\frac{w}{w_n}\right)^4 + (4\zeta^2-2)\left(\frac{w}{w_n}\right)^2 + 1\right\} = 0$$

$$\Rightarrow \left(\frac{4w^3}{w_n^4}\right) + (4\zeta^2-2)\left(\frac{2w}{w_n^2}\right) = 0 \qquad \Rightarrow w\left(w^2 + (2\zeta^2-1)w_n^2\right) = 0$$

$$\Rightarrow w = 0, \pm\sqrt{1-2\zeta^2}\, w_n$$

- For $\zeta < \frac{\sqrt{2}}{2}$

$$\Rightarrow \max\{|H(jw)|\} \text{ at } w_{\max} = w_n\sqrt{1-2\zeta^2} \qquad |H(jw_{\max})| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- **Second-Order CT Systems:**

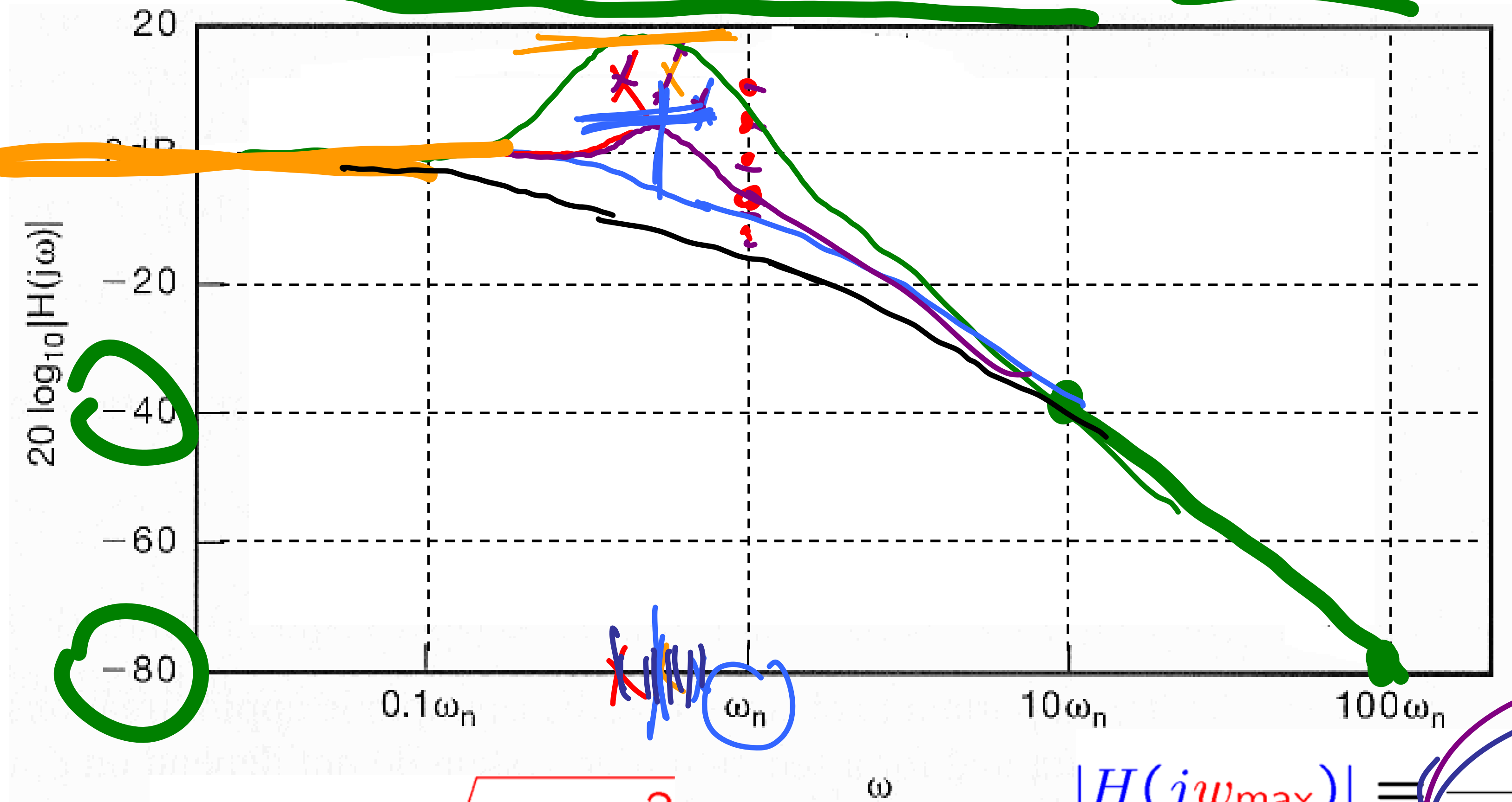
$$H(jw) = \frac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$$

$$\begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$

$$Q = \frac{1}{2\zeta}$$

$\zeta = 0.707$

$\frac{\sqrt{2}}{2}$



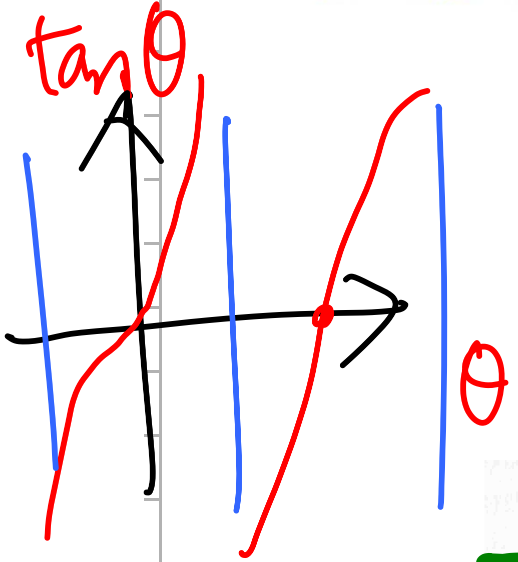
$$w_{\max} = w_n\sqrt{1-2\zeta^2}$$

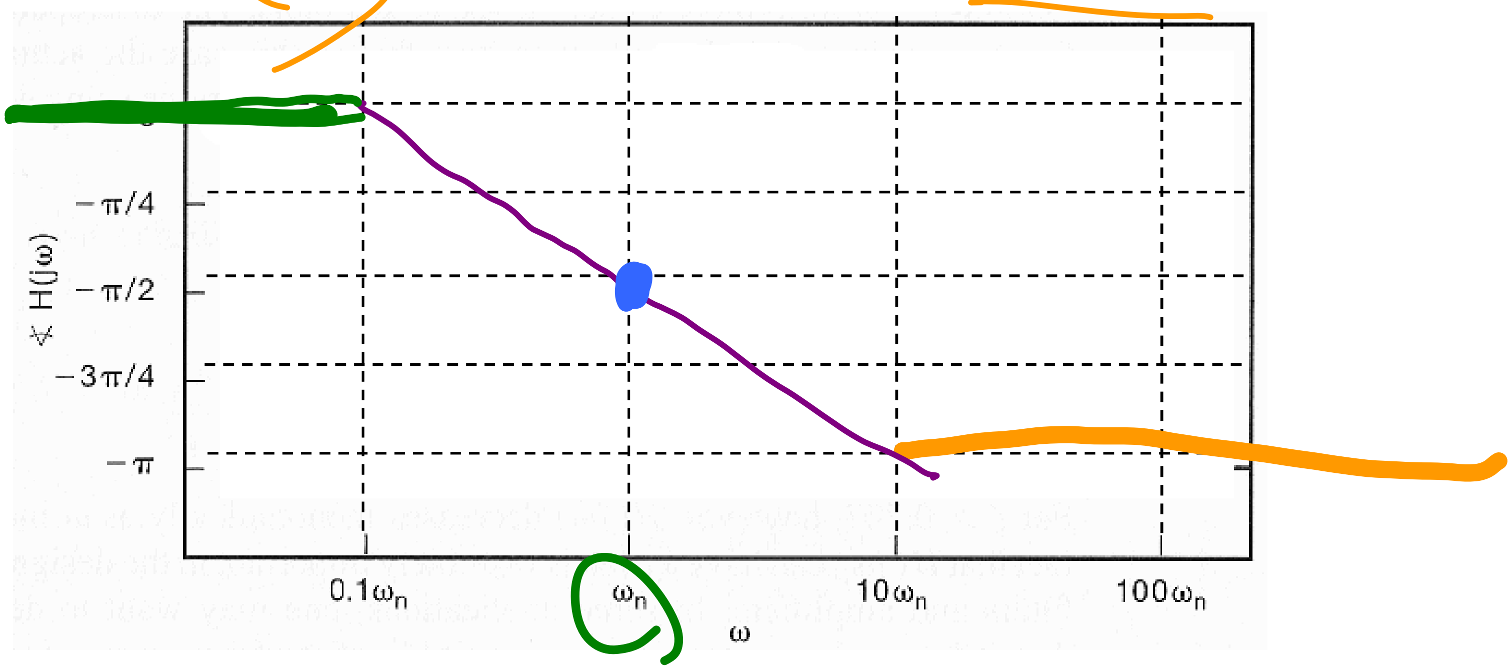
$$|H(jw_{\max})| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- **Second-Order CT Systems:** $H(jw) = \dfrac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$

$$\sphericalangle H(jw) = -\tan^{-1}\left(\frac{2\zeta(\frac{w}{w_n})}{1-(\frac{w}{w_n})^2}\right)$$

$$\approx \begin{cases} 0 & w \le 0.1 w_n \\ -\pi/2 & w = w_n \\ -(\pi/2)[\log_{10}(\frac{w}{w_n}) + 1] & 0.1 w_n \le w \le 10 w_n \\ -\pi & w \ge 10 w_n \end{cases}$$





■ Second-Order CT Systems: $H(jw) = \dfrac{1}{(j\frac{w}{w_n})^2 + 2\zeta(j\frac{w}{w_n}) + 1}$

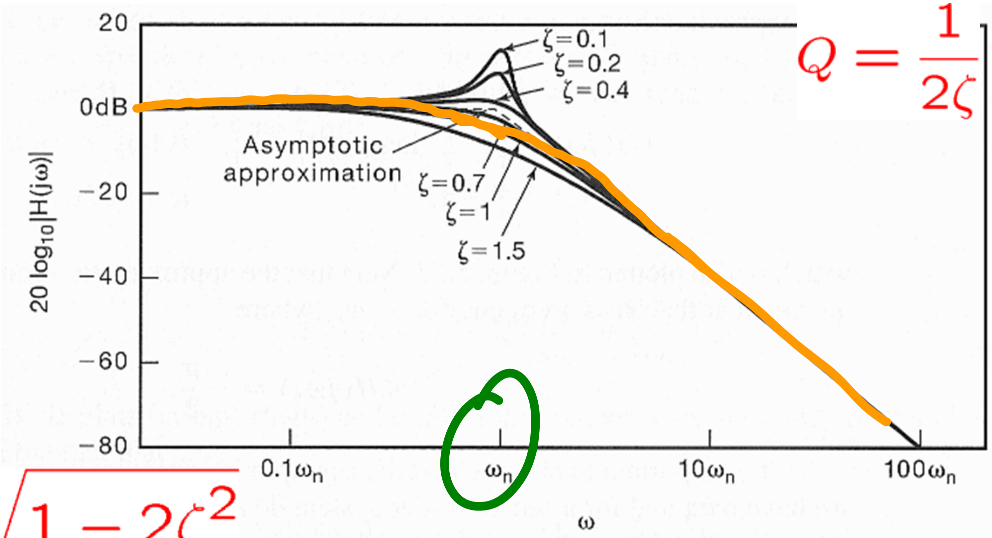
$$Q = \frac{1}{2\zeta}$$

ζ

w_n

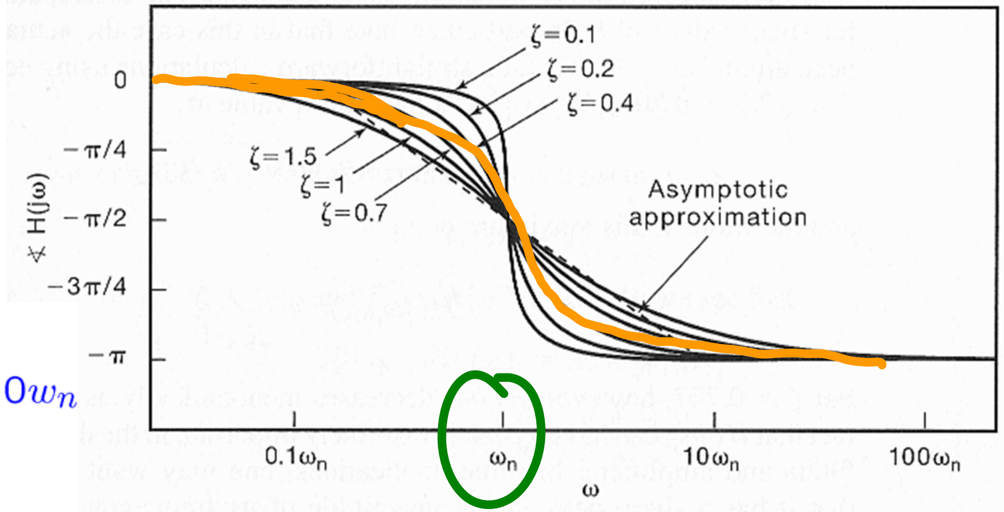
$$20\log_{10}|H(jw)| =$$

$$\begin{cases} 0 & w << w_n \\ -20\log_{10}(2\zeta) & w = w_n \\ -40\log_{10}(w) + 40\log_{10}(w_n) & w >> w_n \end{cases}$$



- For $\zeta < \frac{\sqrt{2}}{2}$ $\qquad w_{\max} = w_n\sqrt{1 - 2\zeta^2}$

$$\measuredangle H(jw) =$$



$$\begin{cases} 0 & w \le 0.1w_n \\ -(\pi/2)[\log_{10}(w/w_n) + 1] & 0.1w_n \le w \le 10w_n \\ -\pi/2 & w = w_n \\ -\pi & w \ge 10w_n \end{cases}$$

- **Example 6.4: (p.457)**

$$\hat{H}(jw) = \frac{w_n^2}{(jw)^2 + 2\zeta w_n (jw) + w_n^2}$$

$$H(jw) = \frac{2 \times 10^4}{(jw)^2 + \underbrace{100}_{2\zeta w_n}(jw) + \underbrace{10^4}_{w_n^2}}$$

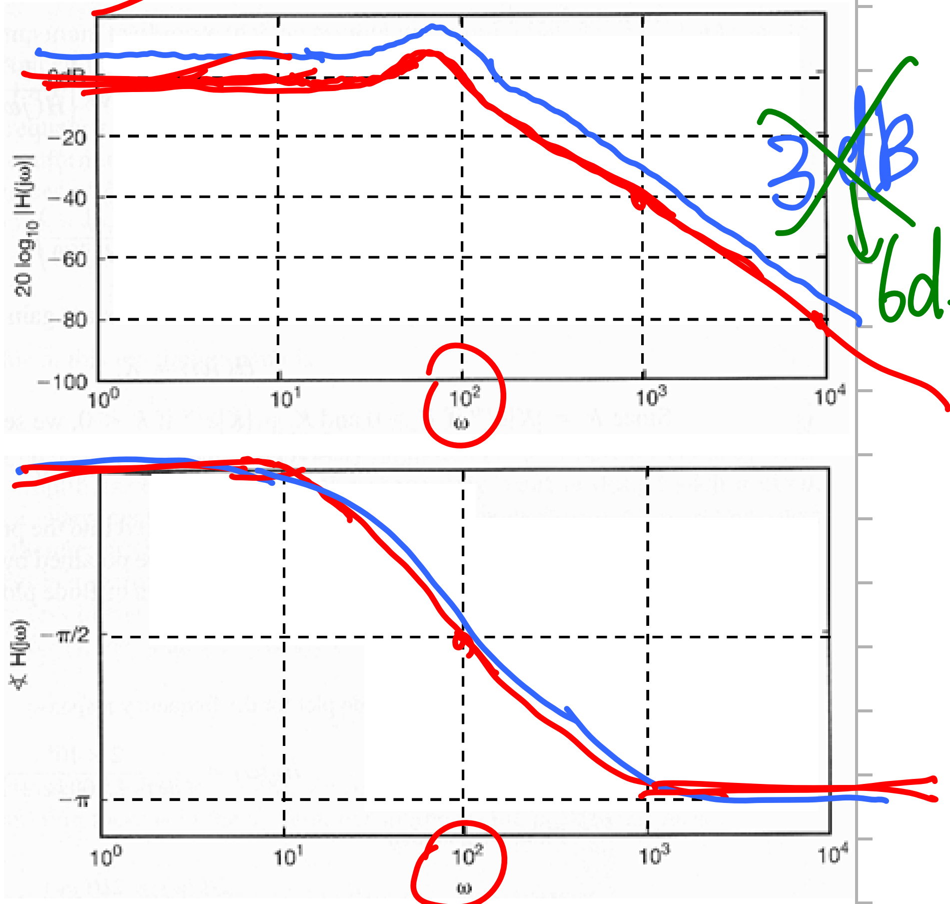
$$|H(jw)| = 2 \times |\hat{H}(jw)|$$

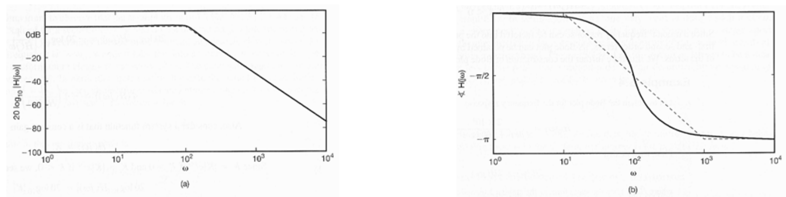
$$\sphericalangle H(jw) = \sphericalangle \hat{H}(jw)$$

$$\Rightarrow \begin{cases} w_n = 100 \\ \zeta = 1/2 \end{cases}$$

$$\Rightarrow 20\log_{10}|H(jw)| =$$

$$20\log_{10}(2) + 20\log_{10}|\hat{H}(jw)|$$

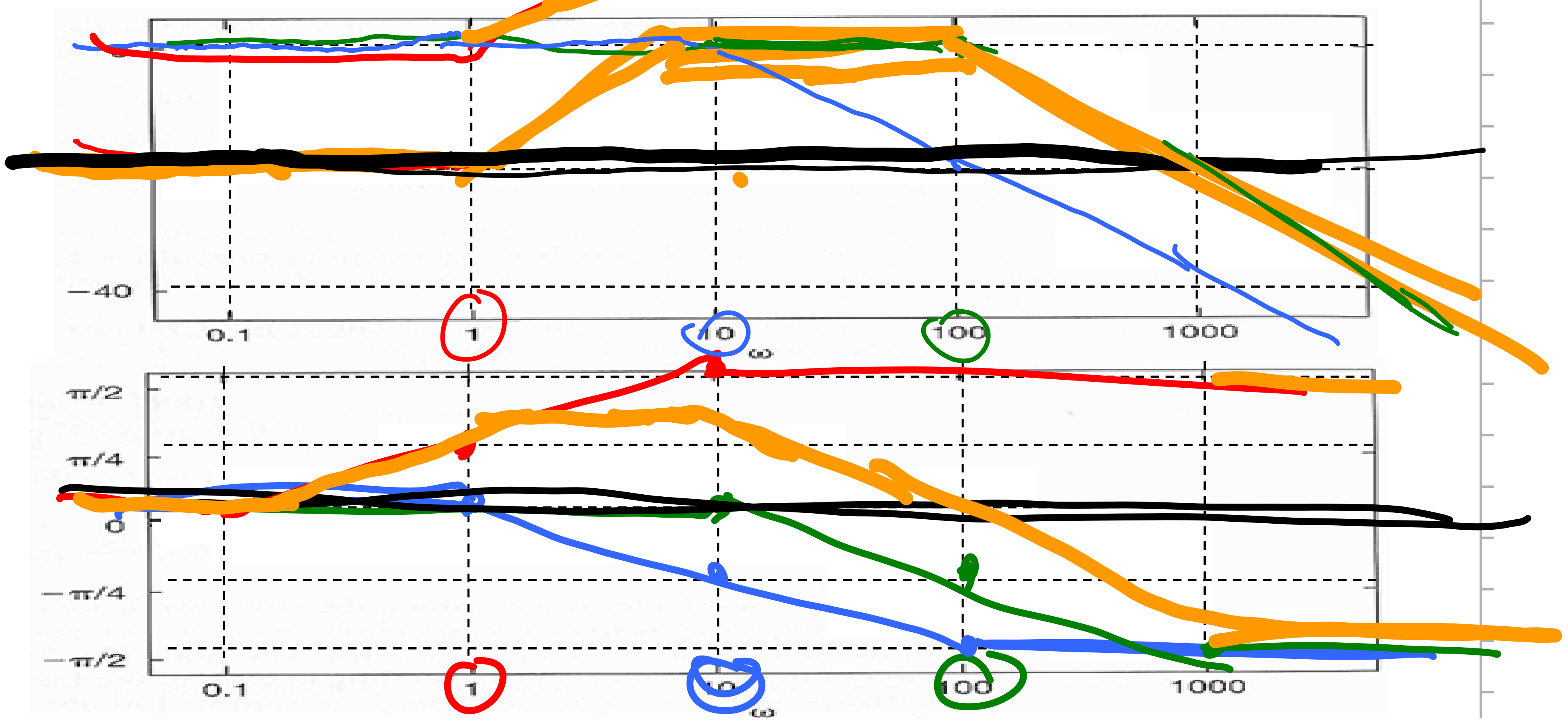




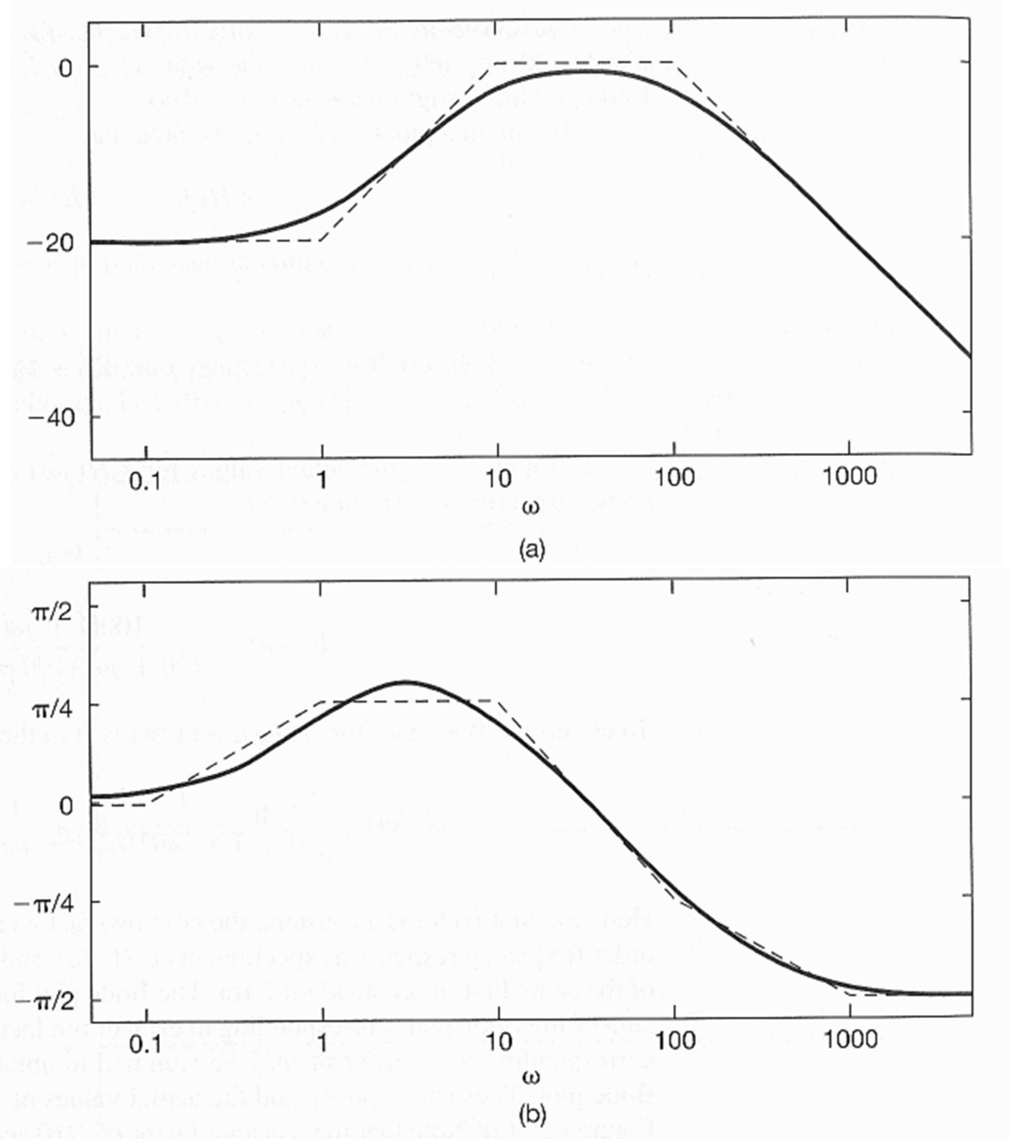
- Example 6.5:

$$H(jw) = \frac{100(1+jw)}{(10+jw)(100+jw)}$$

$$= \left(\frac{1}{10}\right)(1+jw)\left(\frac{1}{1+j\frac{w}{10}}\right)\left(\frac{1}{1+j\frac{w}{100}}\right)$$



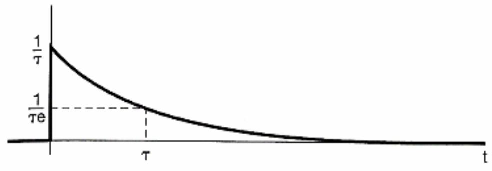
- Example 6.5:



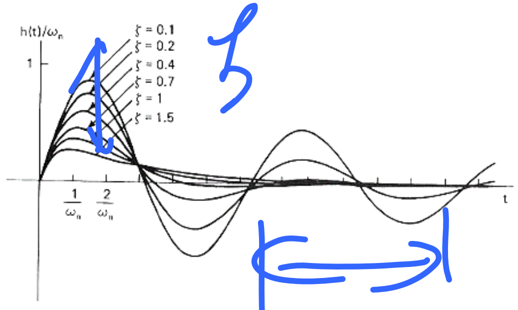
(a)
(b)

First-Order & Second-Order CT Systems

$h(t)$



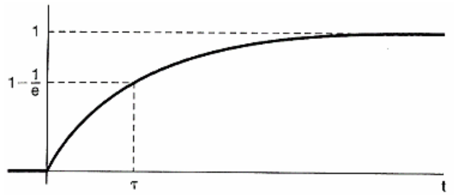
τ



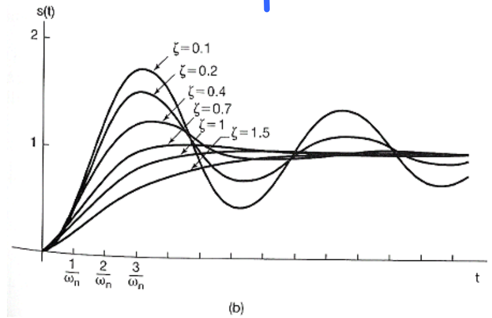
ζ

ω_n

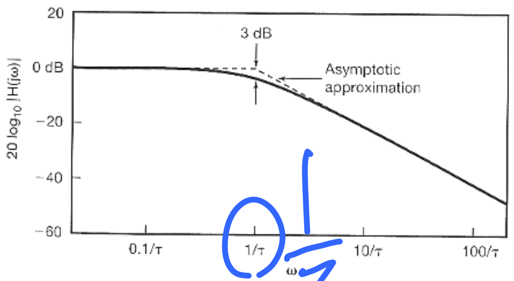
$s(t)$



τ

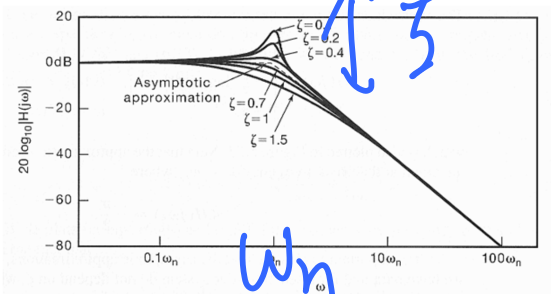


$20 \log_{10} |H(jw)|$



$\frac{1}{\tau}$

ω

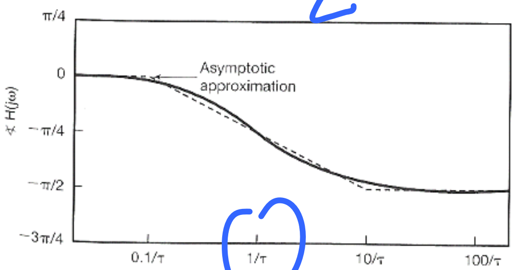


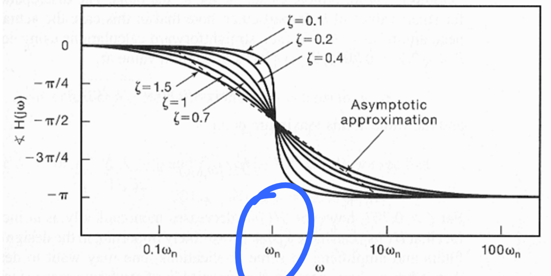
ζ

ω_n

ω

$\measuredangle H(jw)$





4/22/13

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order Continuous-Time Systems
- p.461 1st-Order & 2nd-Order Discrete-Time Systems
- Time- & Frequency-Domain Analysis of Systems

▪ First-Order DT Systems: (p.461)

$$Y(e^{jw}) = X(e^{jw})\ H(e^{jw})$$

$$x[n] \longrightarrow \boxed{\text{LTI System}} \longrightarrow y[n]$$

$$y[n] - a\ y[n-1] = x[n] \qquad |a| < 1$$

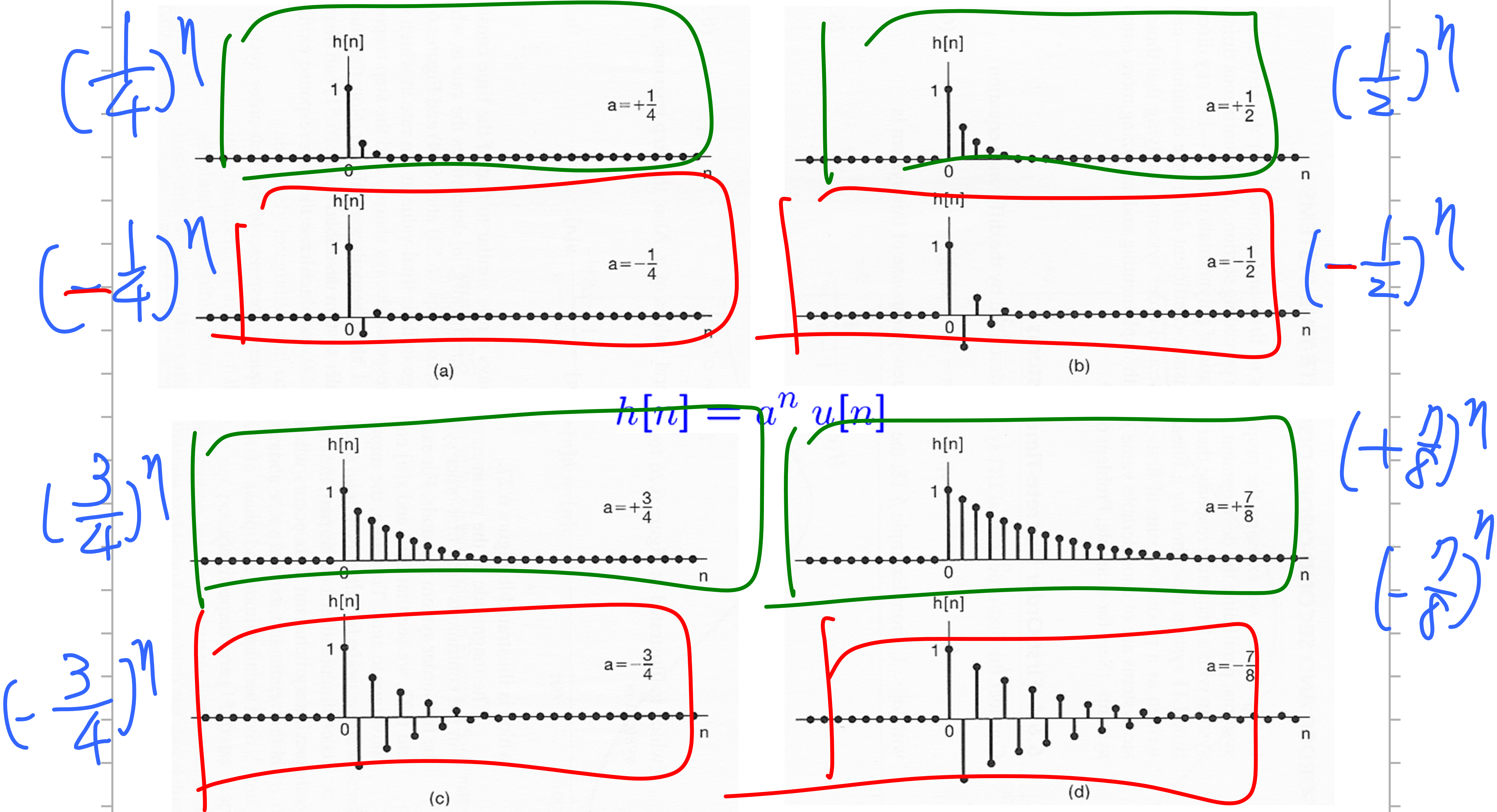
$$\Rightarrow\ H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

$|a| < 1$

$$\Rightarrow\ h[n] = a^n\, u[n]$$

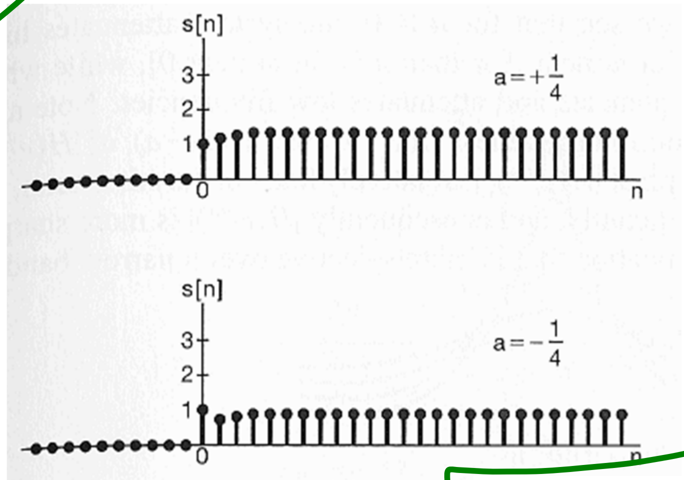
$$\Rightarrow\ s[n] = h[n] * u[n] = \frac{1 - a^{n+1}}{1 - a}\, u[n]$$

- **Impulse Response of First-Order DT Systems:**

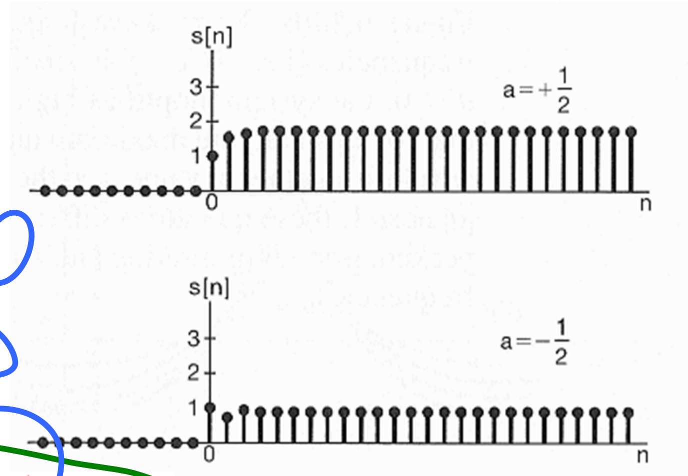


$h[n] = a^n \, u[n]$

■ Step Response of First-Order DT Systems:



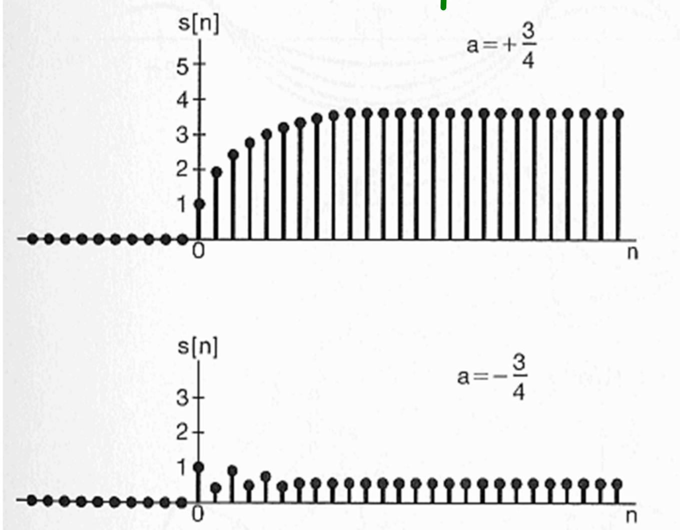
(a)



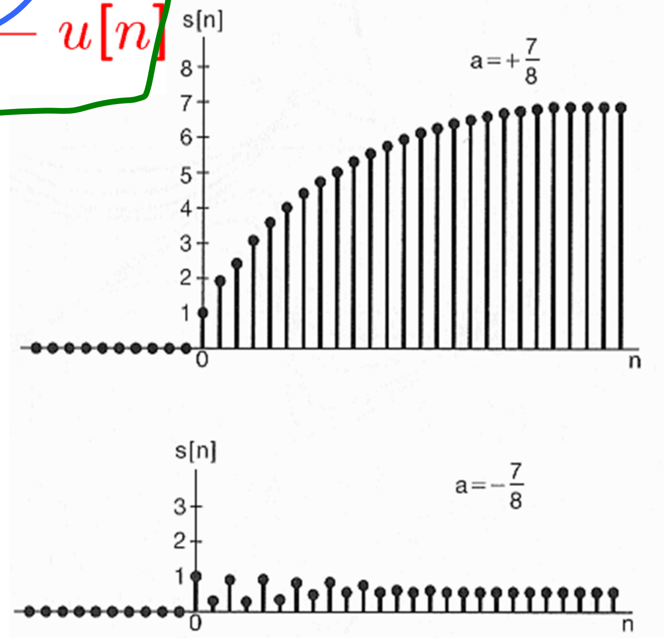
(b)

0

$$\boxed{s[n] = \frac{1 - a^{n+1}}{1 - a}\, u[n]}$$



(c)



(d)

▪ Magnitude & Phase of Frequency Response:

$$H(e^{jw}) = \frac{1}{1 - ae^{-jw}} = \frac{1}{1 - a\cos w + j a \sin w}$$

$$|H(e^{jw})| = \frac{1}{\sqrt{1 + a^2 - 2a\cos w}}$$

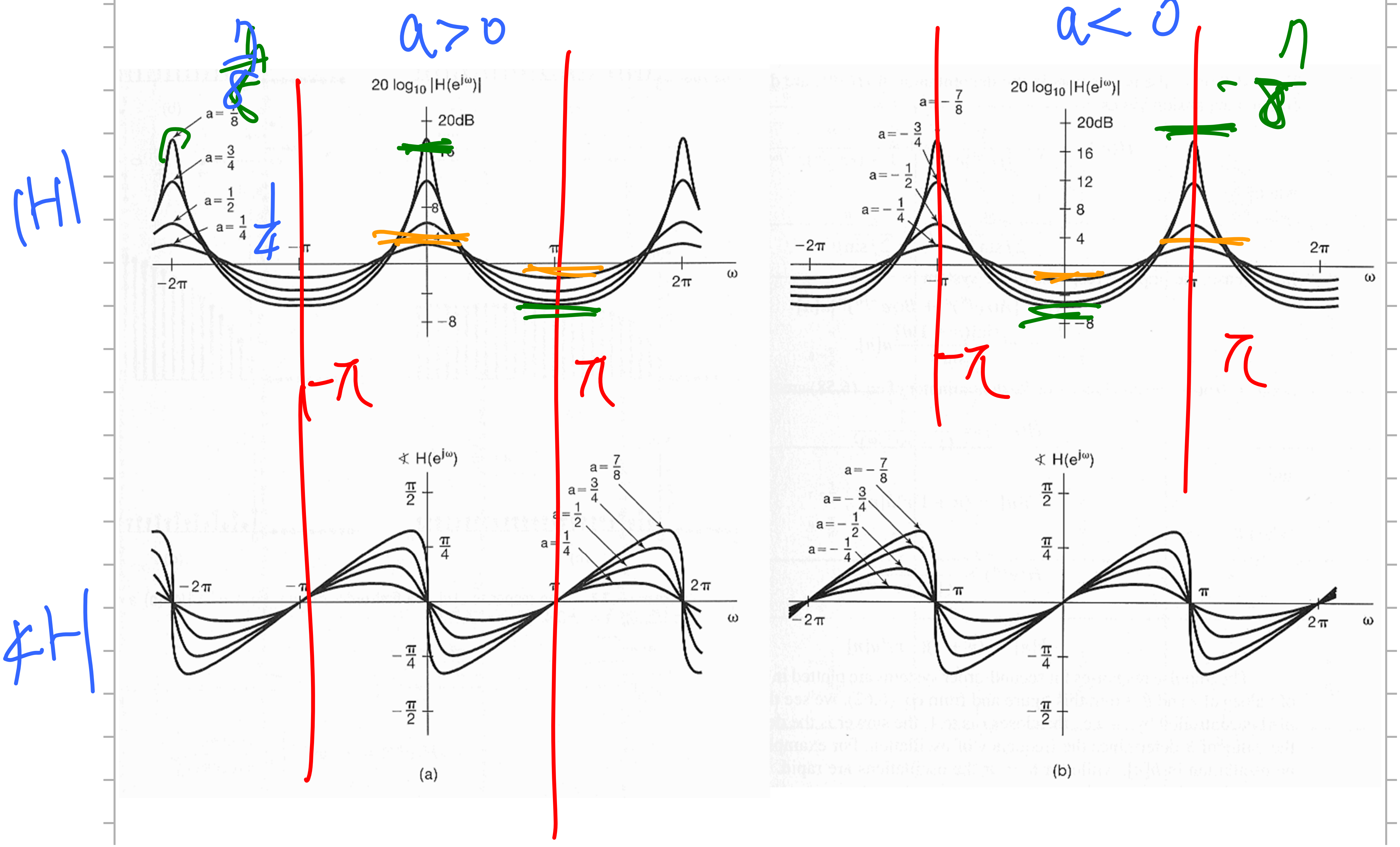
$w : 0 \to \infty$

$-\pi \quad \pi$

$$\measuredangle H(e^{jw}) = -\tan^{-1}\left[\frac{a\sin w}{1 - a\cos w}\right]$$

$(-\frac{7}{8})^n = (-1)^n (\frac{7}{8})^n$

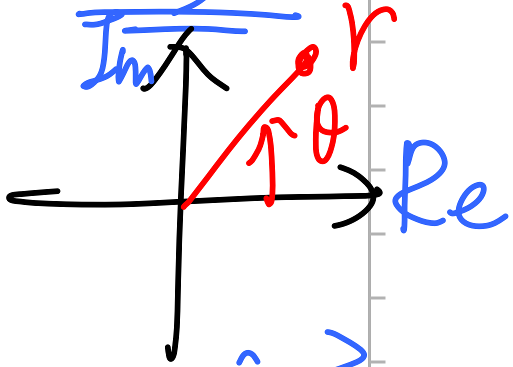
- Magnitude & Phase of Frequency Response:



■ Second-Order DT Systems: (p.465)

r, θ

$0 < r < 1$ and $0 \le \theta \le \pi$



$$y[n] - 2r\cos(\theta)\, y[n-1] + r^2 y[n-2] = x[n]$$

e^{-jw} $\quad$ e^{-j2w}

$$\Rightarrow H(e^{jw}) = \frac{1}{1 - 2r\cos(\theta)\, e^{-jw} + r^2 e^{-j2w}}$$

$(re^{-jw})^2$

$$\boxed{\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)}$$

$-r(e^{j\theta}e^{-jw}) - re^{-j\theta}e^{-jw}$

$$= \frac{1}{[1 - (re^{j\theta})e^{-jw}]\,[1 - (re^{-j\theta})e^{-jw}]}$$

Impulse Response of 2nd-Order DT Systems:

- For $\theta = 0$:

$$\Rightarrow \; H(e^{jw}) = \frac{1}{(1-r\ e^{-jw})^2} \quad \Rightarrow \; h[n] = (n+1)\ (r)^n\ u[n]$$

- For $\theta = \pi$:

$$\Rightarrow \; H(e^{jw}) = \frac{1}{(1+r\ e^{-jw})^2} \quad \Rightarrow \; h[n] = (n+1)\ (-r)^n\ u[n]$$

- For $\theta \neq 0$ or π:

$$\Rightarrow \; H(e^{jw}) = \frac{A}{1-(re^{j\theta})e^{-jw}} + \frac{B}{1-(re^{-j\theta})e^{-jw}}$$

$$A = \frac{e^{j\theta}}{2j\sin(\theta)}$$

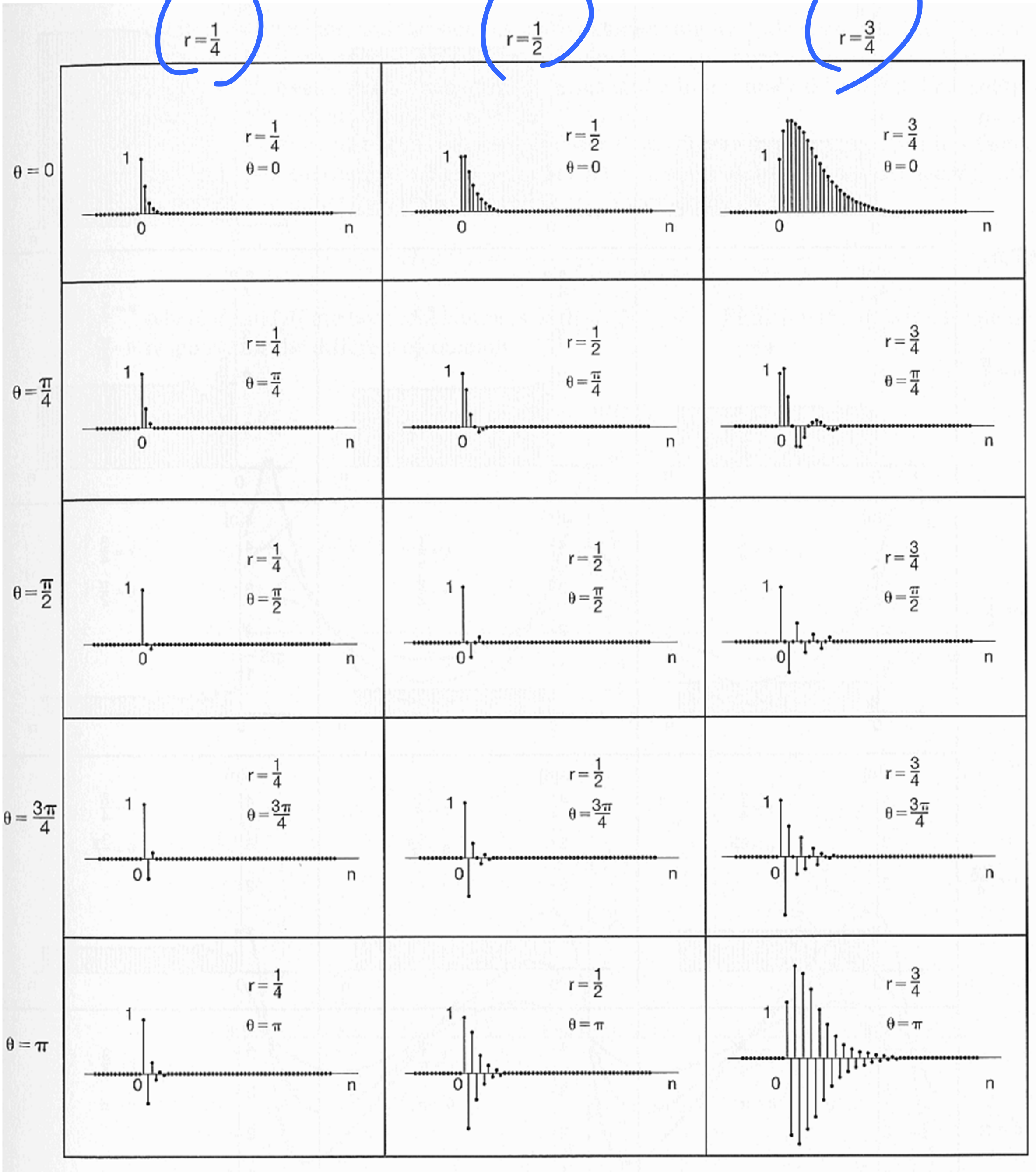
$$B = -\frac{e^{-j\theta}}{2j\sin(\theta)}$$

$$\Rightarrow \; h[n] = \left[A\boxed{(re^{j\theta})^n} + B\boxed{(re^{-j\theta})^n}\right]\ u[n] \quad = \boxed{r^n \frac{\sin[(n+1)\theta]}{\sin(\theta)}}u[n]$$

First-Order & Second-Order DT Systems

r

θ: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π



$$h[n] = (n+1)\,(r)^n\,u[n]$$

$$r^n \frac{\sin[(n+1)\theta]}{\sin(\theta)} u[n]$$

$$h[n] = (n+1)\,(-r)^n\,u[n]$$

■ Step Response of 2nd-Order DT Systems:

- For $\theta = 0$:

$$s[n] = \left[\frac{1}{(r-1)^2} - \frac{r}{(r-1)^2} r^n + \frac{r}{r-1} (n+1) r^n \right] u[n]$$

- For $\theta = \pi$:

$$s[n] = \left[\frac{1}{(r+1)^2} + \frac{r}{(r+1)^2} (-r)^n + \frac{r}{r+1} (n+1)(-r)^n \right] u[n]$$

- For $\theta \neq 0$ or π:

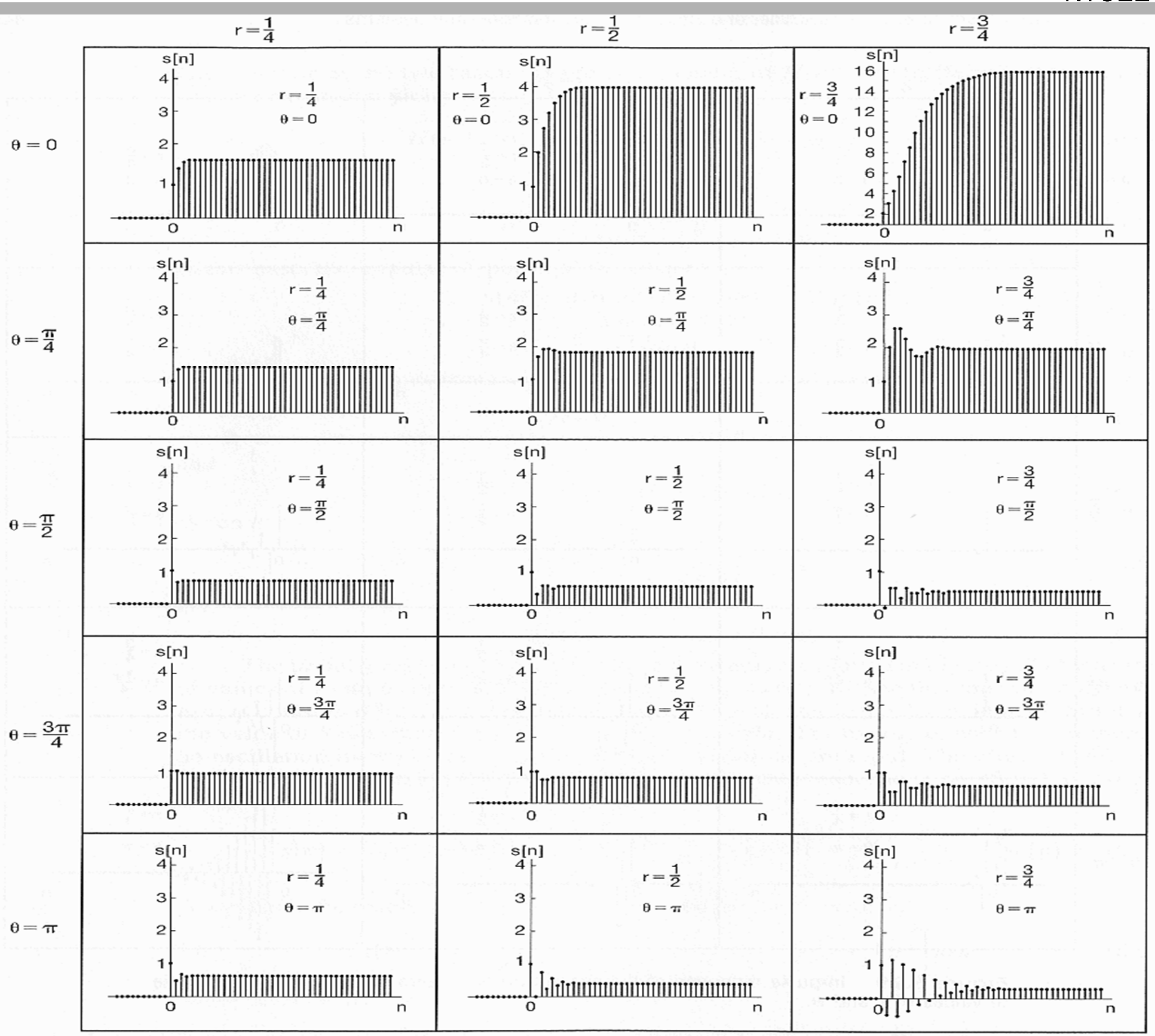
$$s[n] = h[n] * u[n]$$

$$A = \frac{e^{j\theta}}{2j \sin(\theta)}$$

$$B = -\frac{e^{-j\theta}}{2j \sin(\theta)}$$

$$= \left[A \left(\frac{1 - (re^{j\theta})^{n+1}}{1 - re^{j\theta}} \right) + B \left(\frac{1 - (re^{-j\theta})^{n+1}}{1 - re^{-j\theta}} \right) \right] u[n]$$

First-Order & Second-Order DT Systems



* Note: The plot for $r=\frac{3}{4}$, $\theta=0$ has a different scale from the others.

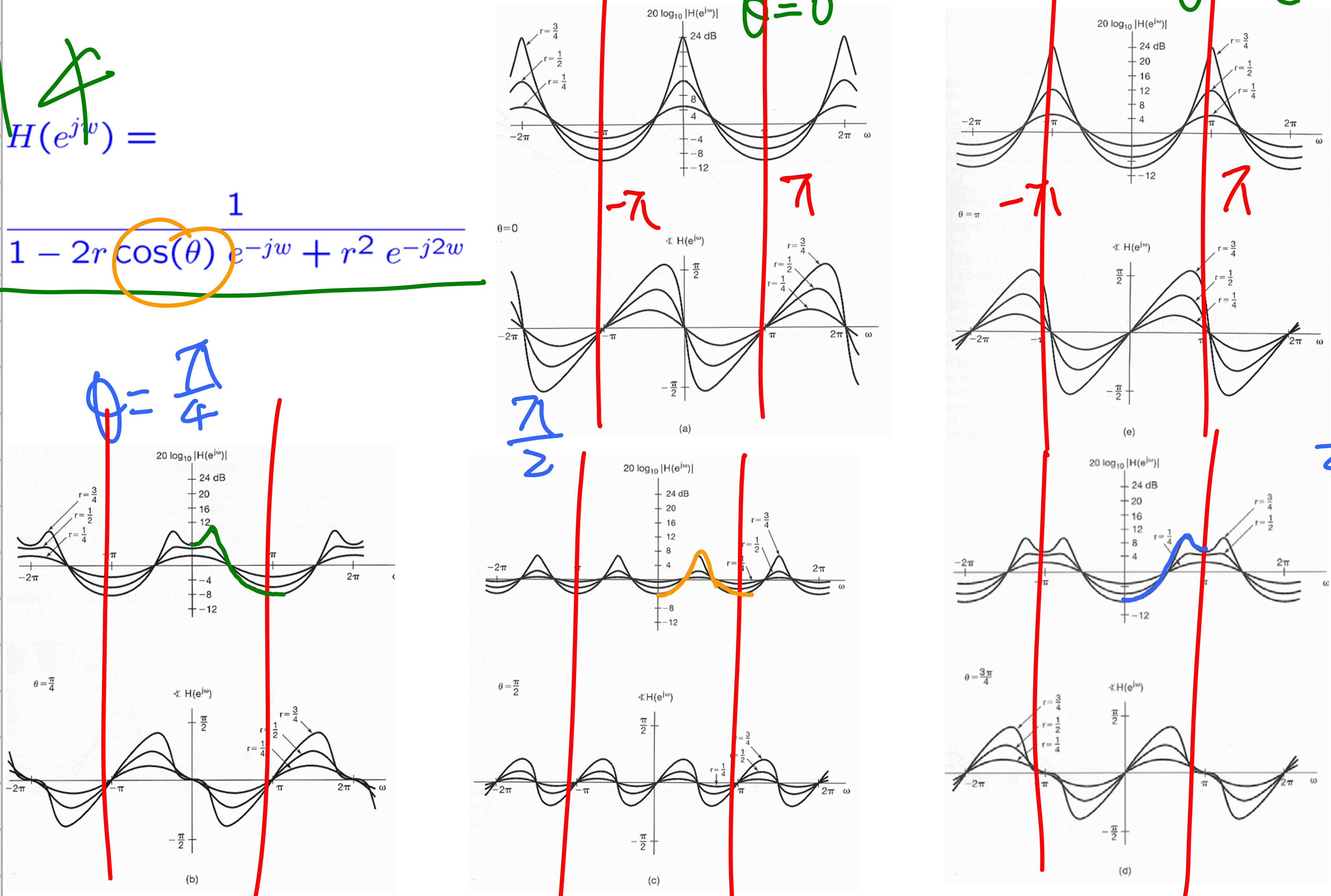
▪ Magnitude & Phase of Frequency Response:

$\theta = \pi \quad (-1)^n$

$|H| \; \measuredangle$

$$H(e^{jw}) = \frac{1}{1 - 2r\cos(\theta)\, e^{-jw} + r^2\, e^{-j2w}}$$

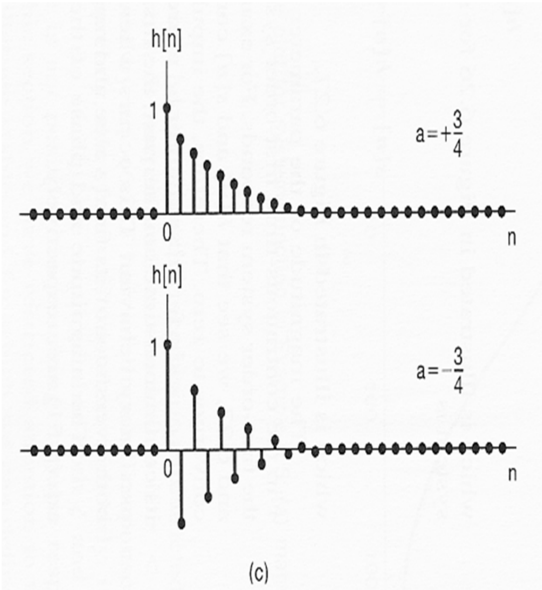
$\theta = \frac{\pi}{4}$ $\quad \theta = 0$ $\quad \frac{\pi}{2}$ $\quad \frac{3}{4}\pi$

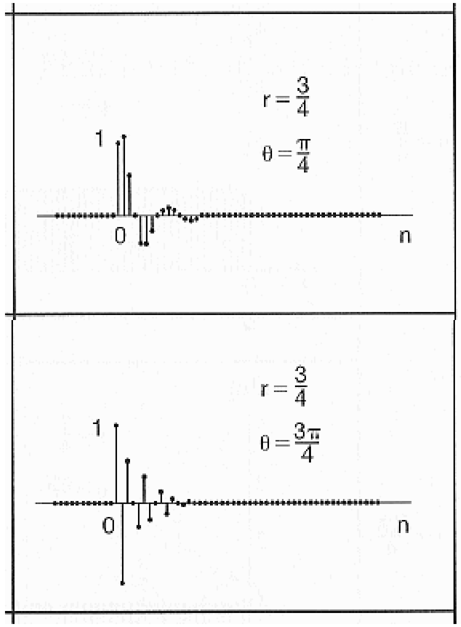


First-Order & Second-Order DT Systems

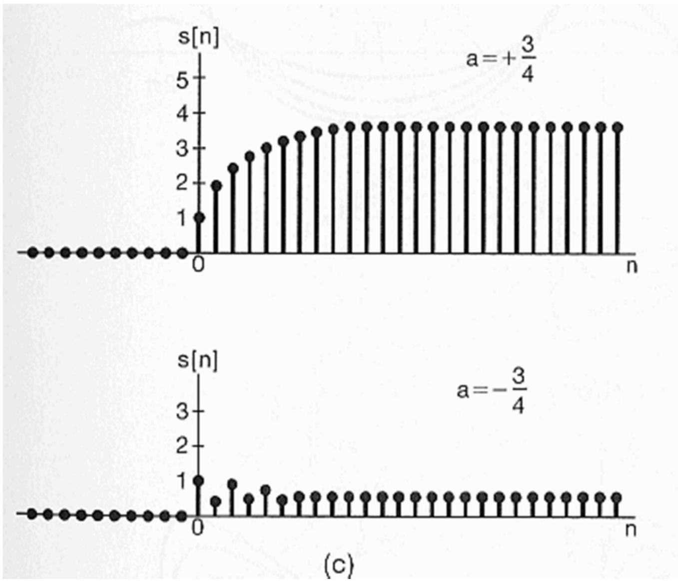
a r θ

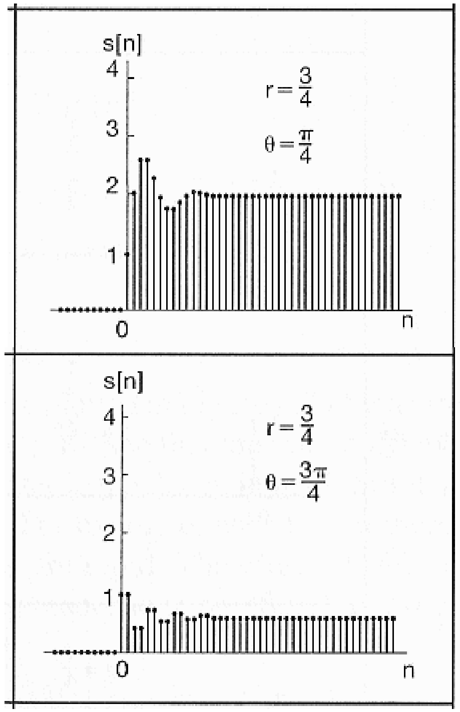
$h[n]$



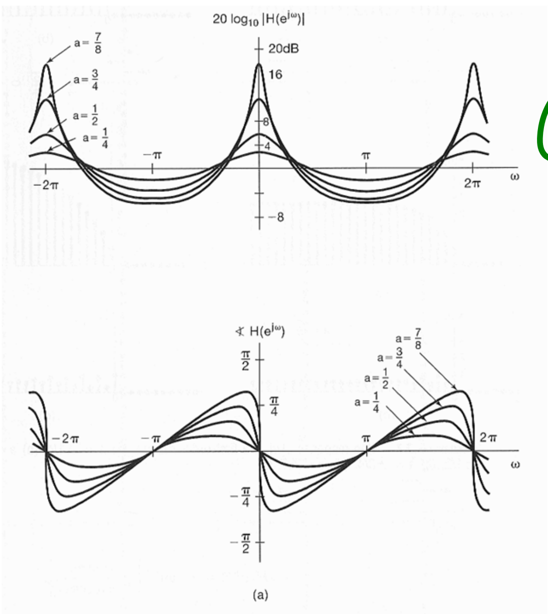


$s[n]$

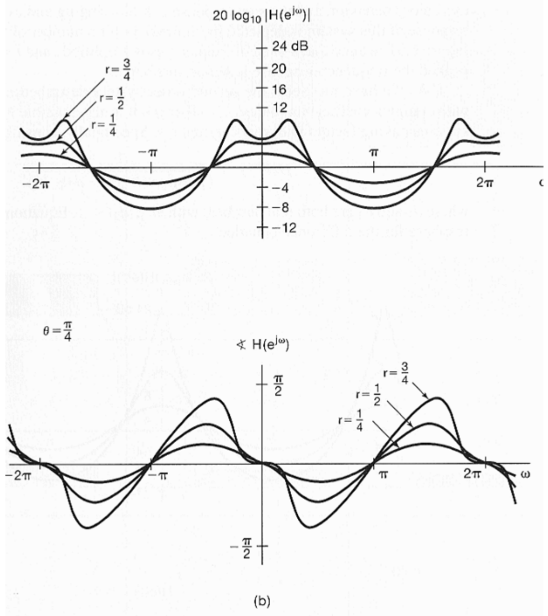




$20 \log_{10} |H(e^{jw})|$



a

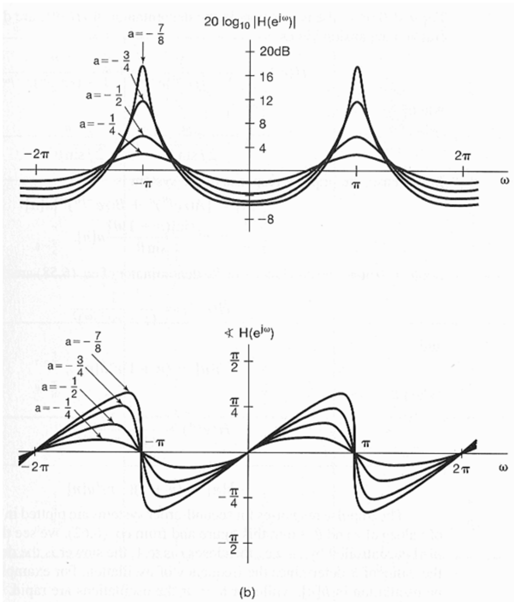


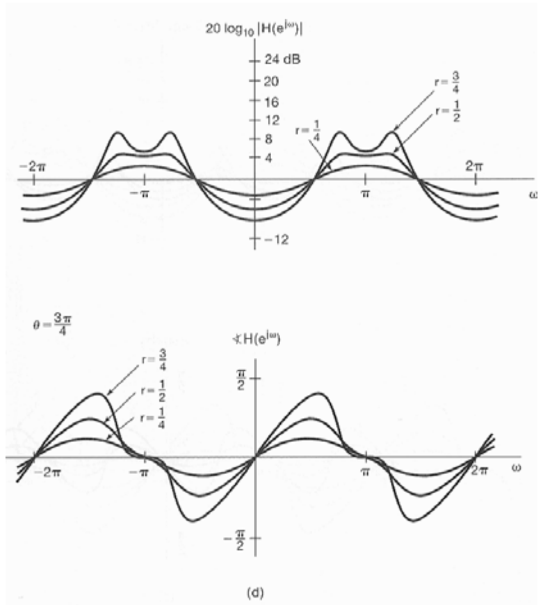
r

θ

$\measuredangle\, H(e^{jw})$

$20 \log_{10} |H(e^{jw})|$





$\measuredangle\, H(e^{jw})$

4/22/13
11:10am

- The Magnitude-Phase Representation of the Fourier Transform (p.423)
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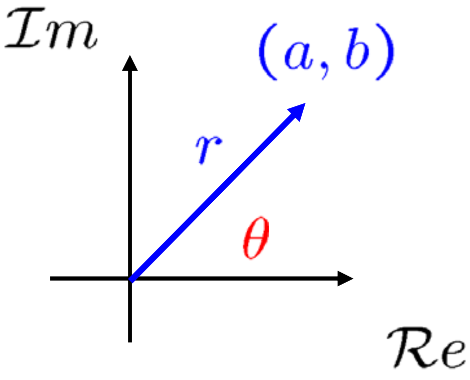
▪ Magnitude & Phase Representation:



$$a + jb \;\Rightarrow\; \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow\; a + jb = re^{j\theta}$$

$$X(jw) = \mathcal{R}e\{X(jw)\} + j\ \mathcal{I}m\{X(jw)\} \qquad = |X(jw)|\, e^{j\sphericalangle X(jw)}$$

$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\ \mathcal{I}m\left\{X(e^{jw})\right\} \qquad = |X(e^{jw})|\, e^{j\sphericalangle X(e^{jw})}$$

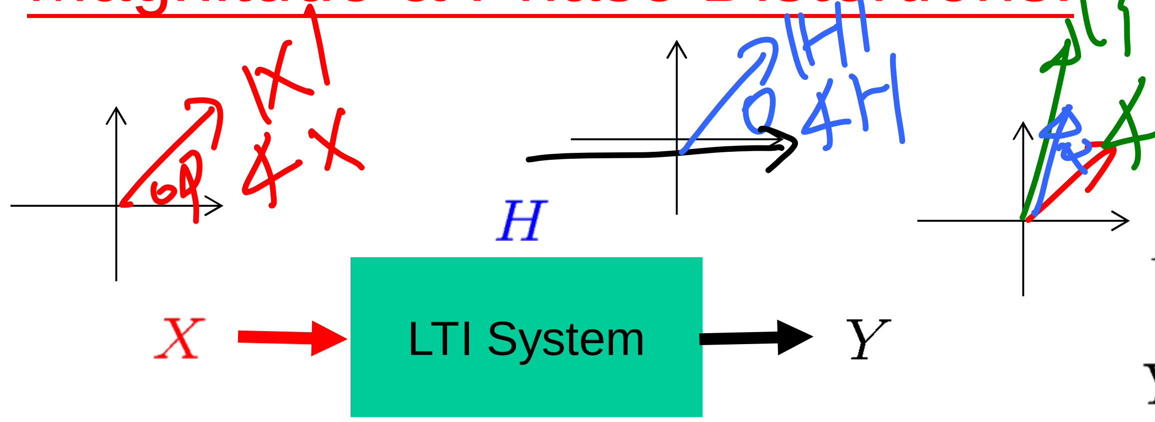
$|X(jw)|$ or $|X(e^{jw})|$: magnitude

$\sphericalangle X(jw)$ or $\sphericalangle X(e^{jw})$: phase angle

- Magnitude & Phase Distortions:

$|Y| = |X||H|$

$\sphericalangle Y = \sphericalangle X + \sphericalangle H$



$$Y(jw) = X(jw)\ H(jw)$$

$$Y(e^{jw}) = X(e^{jw})\ H(e^{jw})$$

$$\Rightarrow \quad |Y(jw)| = |X(jw)|\ |H(jw)|$$

$$|Y(e^{jw})| = |X(e^{jw})|\ |H(e^{jw})|$$

magnitude distortion

$$\Rightarrow \sphericalangle Y(jw) = \sphericalangle X(jw) + \sphericalangle H(jw)$$

$$\sphericalangle Y(e^{jw}) = \sphericalangle X(e^{jw}) + \sphericalangle H(e^{jw})$$

phase distortion

$|H(jw)|$ or $|H(e^{jw})|$: gain of the system

$\sphericalangle H(jw)$ or $\sphericalangle H(e^{jw})$: phase shift of the system

- **Magnitude & Phase Angle:** $A\cos(w_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{jw_0 t} + \frac{A}{2} e^{-j\phi} e^{-jw_0 t}$

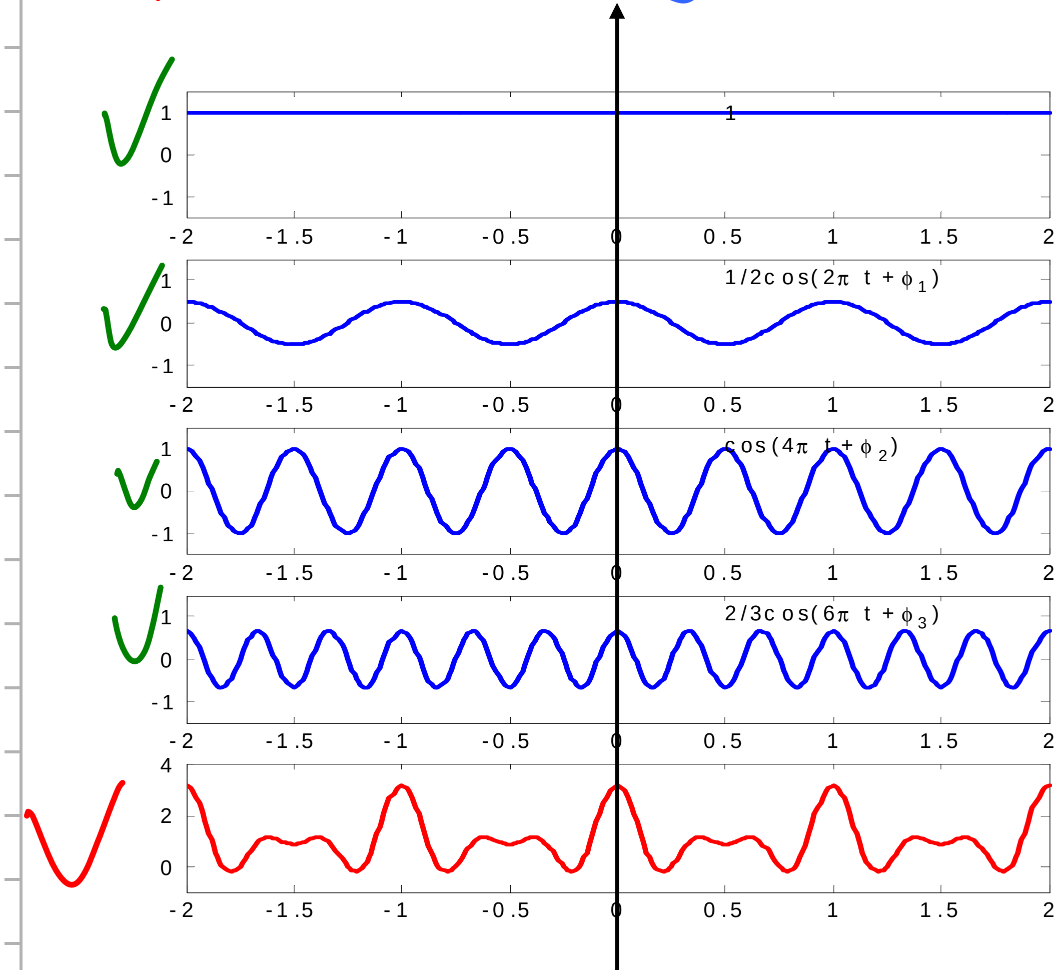
$$x(t) = 1\cos(0\pi t + \cancel{\phi_0}) + \frac{1}{2}\cos(2\pi t + \phi_1) + 1\cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$

0

ss3-17

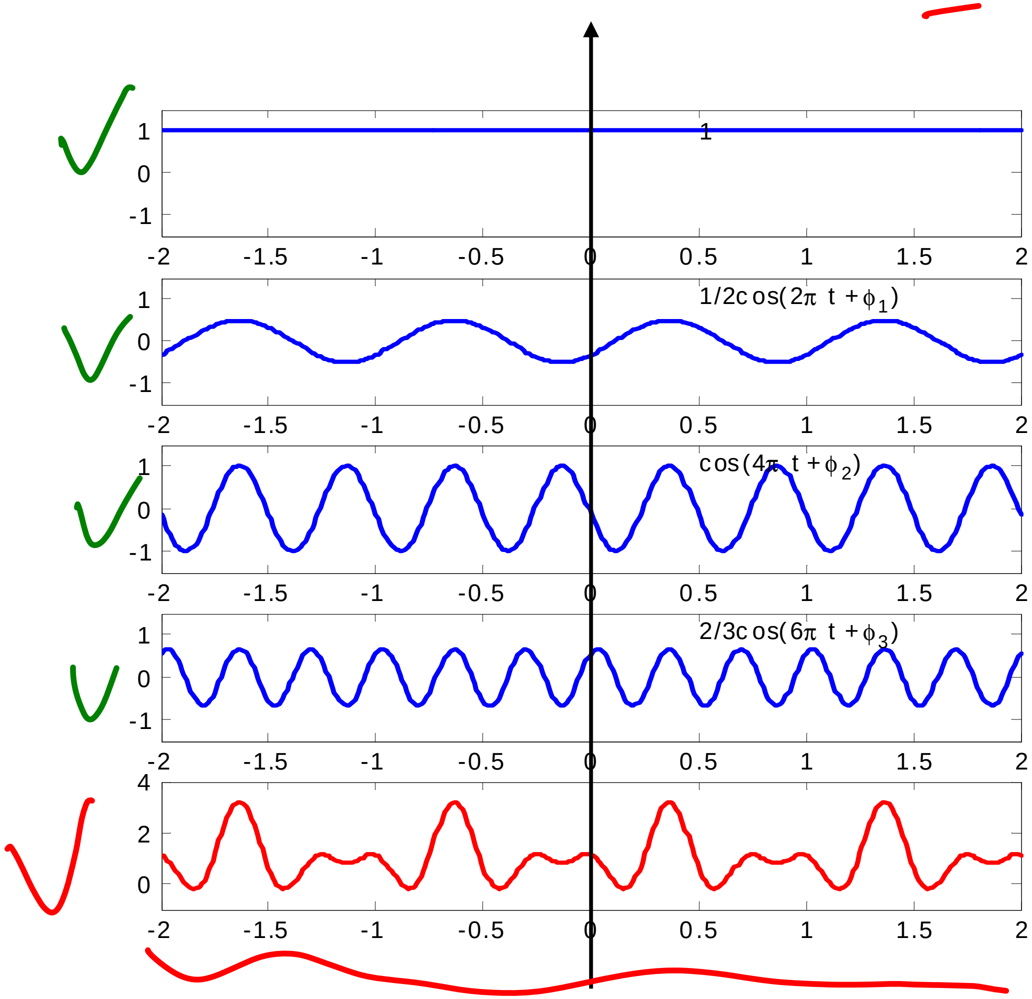
ss3-18

$$\left\{ \begin{array}{lcl} \phi_1 & = & 0 \ (rad) \\ \phi_2 & = & 0 \ (rad) \\ \phi_3 & = & 0 \ (rad) \end{array} \right.$$



- **Magnitude & Phase Angle:** $A\cos(w_0 t + \phi) = \frac{A}{2} e^{j\phi}\, e^{jw_0 t} + \frac{A}{2} e^{-j\phi}\, e^{-jw_0 t}$

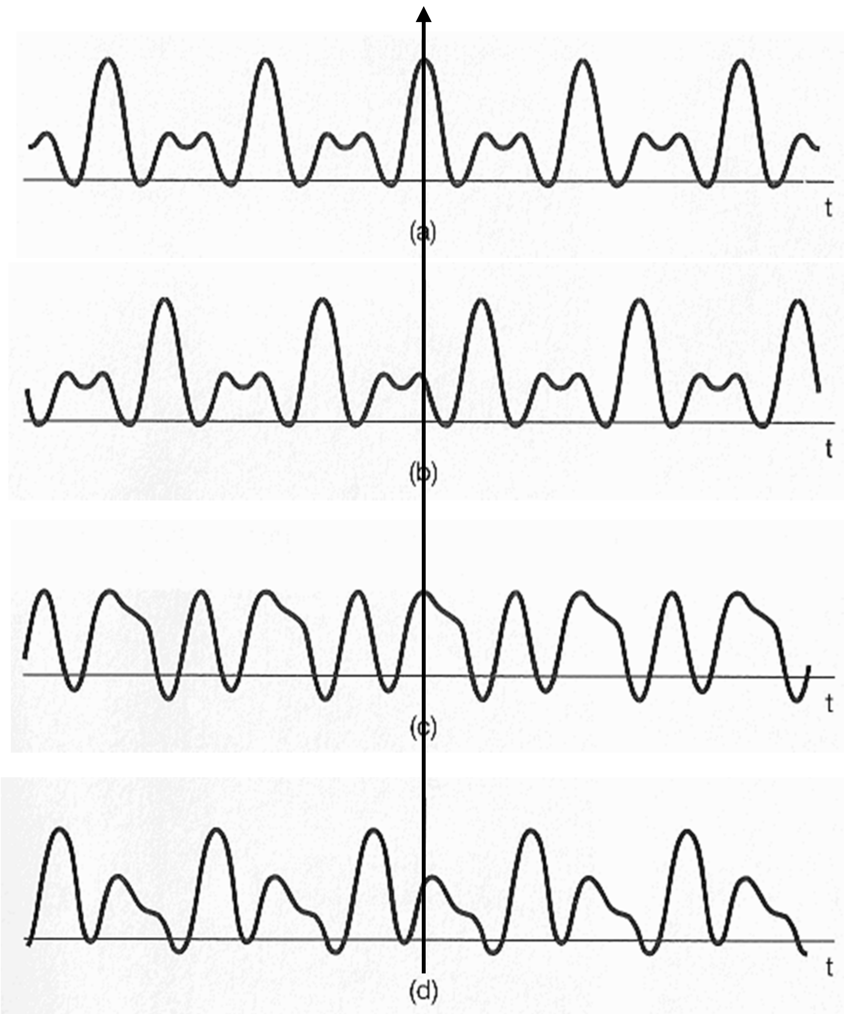
$$x(t) = 1\cos(0\pi t + \phi_0) + \frac{1}{2}\cos(2\pi t + \phi_1) + 1\cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$



$$\begin{cases} \phi_1 = 4 \ (rad) \\ \phi_2 = 8 \ (rad) \\ \phi_3 = 12 \ (rad) \end{cases}$$

- **Magnitude & Phase Angle:** $A\cos(w_0 t + \phi) = \frac{A}{2}\, e^{j\phi}\, e^{jw_0 t} + \frac{A}{2}\, e^{-j\phi}\, e^{-jw_0 t}$

$$x(t) = 1\cos(0\pi t + \phi_0) + \frac{1}{2}\cos(2\pi t + \phi_1) + 1\cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$



$$\begin{cases} \phi_1 = 0 \ (rad) \\ \phi_2 = 0 \ (rad) \\ \phi_3 = 0 \ (rad) \end{cases}$$

$$\begin{cases} \phi_1 = 4 \ (rad) \\ \phi_2 = 8 \ (rad) \\ \phi_3 = 12 \ (rad) \end{cases}$$

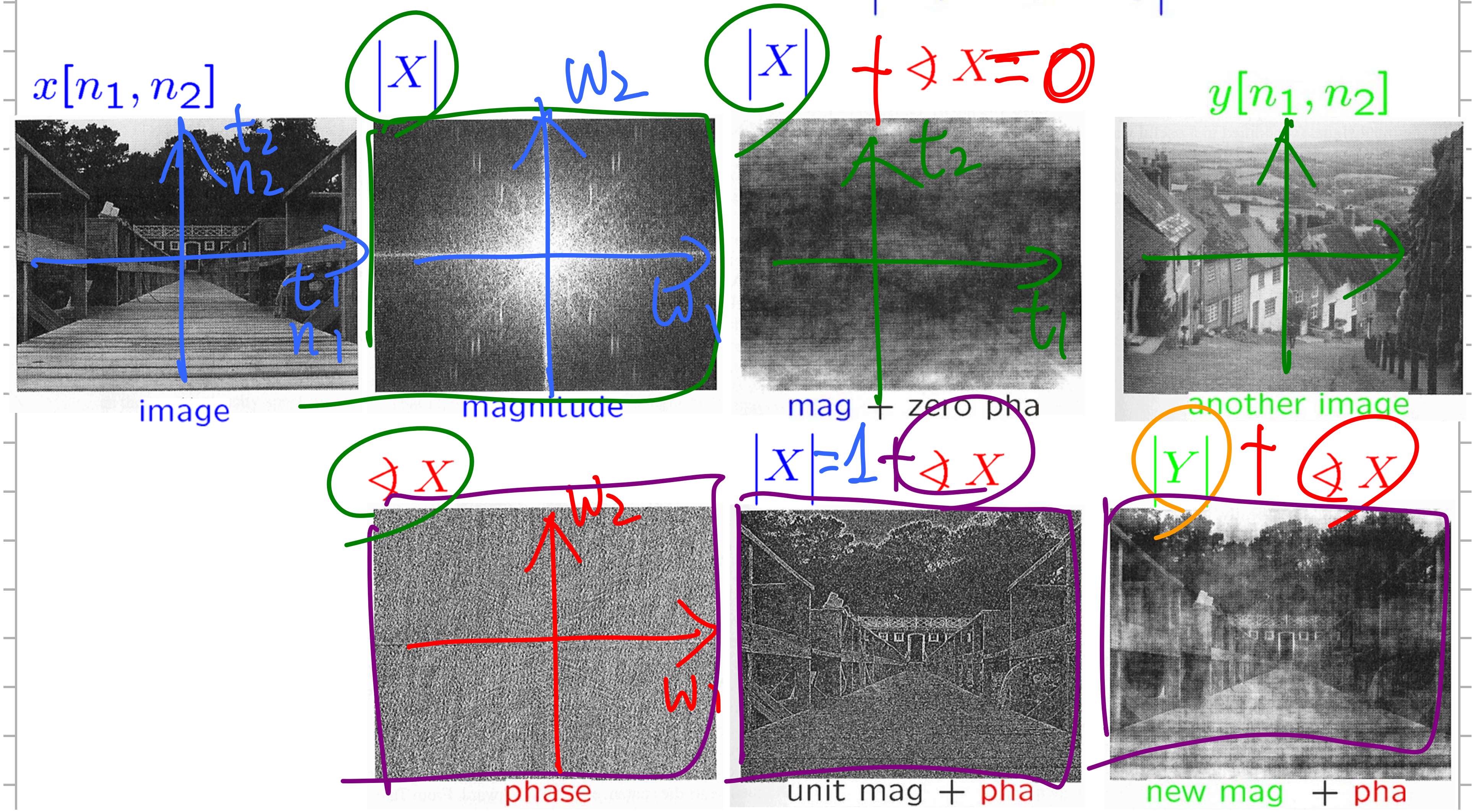
$$\begin{cases} \phi_1 = 6 \ (rad) \\ \phi_2 = -2.7 \ (rad) \\ \phi_3 = 0.93 \ (rad) \end{cases}$$

$$\begin{cases} \phi_1 = 1.2 \ (rad) \\ \phi_2 = 4.1 \ (rad) \\ \phi_3 = -7.02 \ (rad) \end{cases}$$

Magnitude & Phase Angle in Images:

$$x(t_1, t_2) \xleftrightarrow{\mathcal{F}} X(jw_1, jw_2) \qquad \left|X(jw_1, jw_2)\right| e^{j\sphericalangle X(jw_1, jw_2)}$$

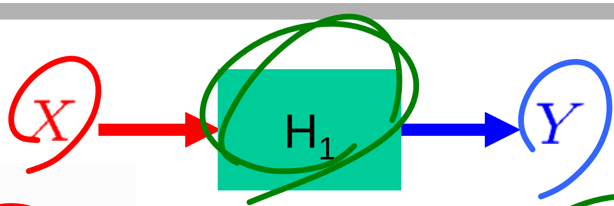
$$x[n_1, n_2] \xleftrightarrow{\mathcal{F}} X(e^{jw_1}, e^{jw_2}) \qquad \left|X(e^{jw_1}, e^{jw_2})\right| e^{j\sphericalangle X(e^{jw_1}, e^{jw_2})}$$

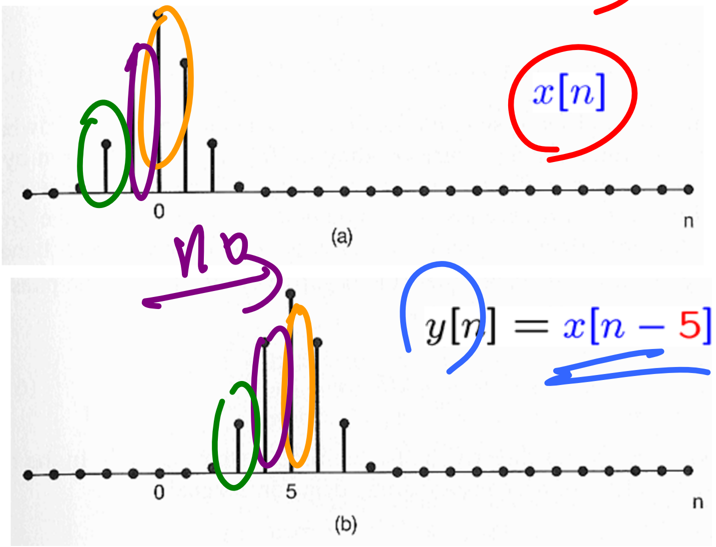


Outline

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems (p.427)
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order Continuous-Time Systems
- 1st-Order & 2nd-Order Discrete-Time Systems
- Time- & Frequency-Domain Analysis of Systems

- Linear Phase:





$n_0 = 5$

- $H_1(e^{jw}) = e^{-jwn_0}$

$$\Rightarrow \begin{cases} |H_1(e^{jw})| & = 1 \\ \sphericalangle\, |H_1(e^{jw})| & = -wn_0 \end{cases}$$

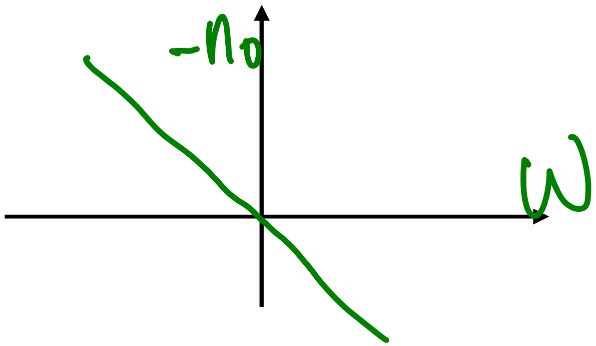
$$Y_1(e^{jw}) = H_1(e^{jw})\ X(e^{jw})$$

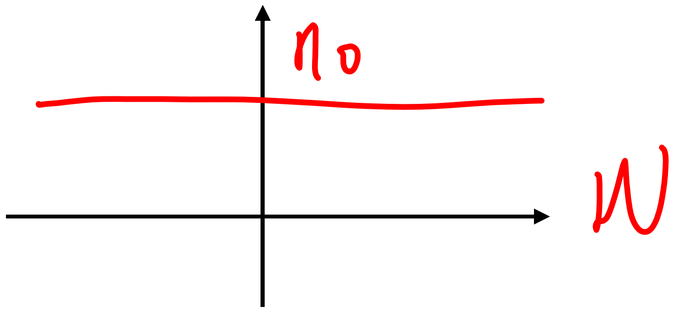
$$= \boxed{e^{-jwn_0}\quad X(e^{jw})}$$

$$\Rightarrow \quad y[n] = x[n - n_0]$$

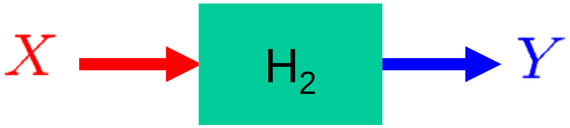
$$\sphericalangle\, H_1(e^{jw}) = -wn_0$$

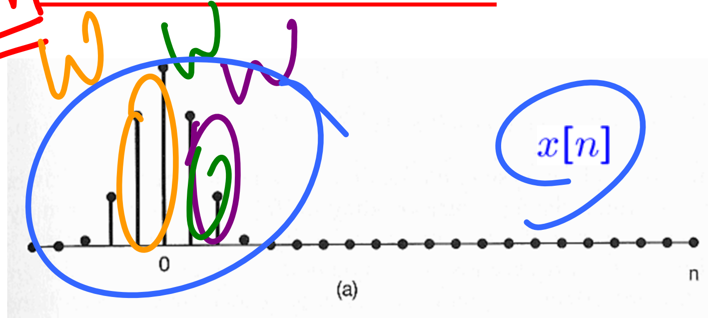
$$-\frac{d}{dw}\left\{\sphericalangle\, H_1(e^{jw})\right\} = n_0$$





Non ■ Linear Phase:



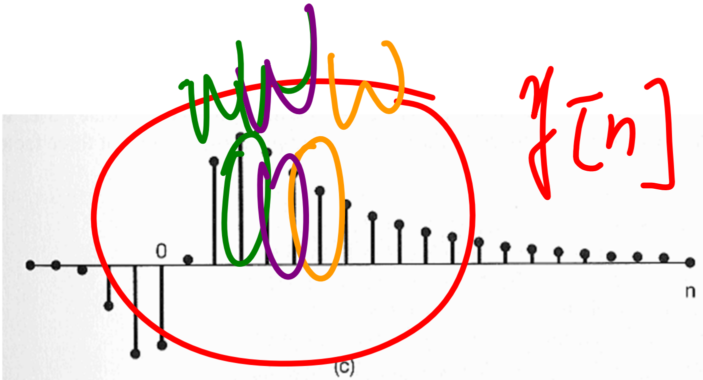


$$H_2(e^{jw}) = e^{jf(w)}$$

$f(w)$

- $Y_2(e^{jw}) = H_2(e^{jw})\,X(e^{jw})$

$$= e^{jf(w)}\,X(e^{jw})$$

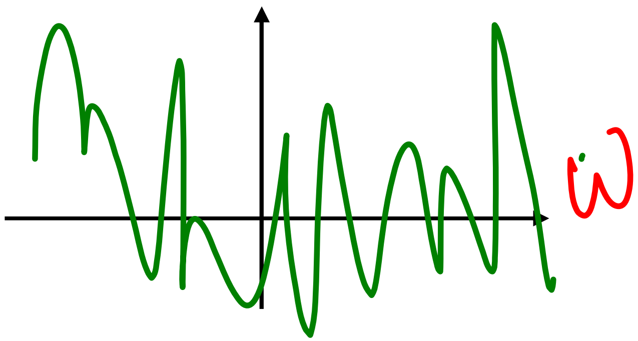


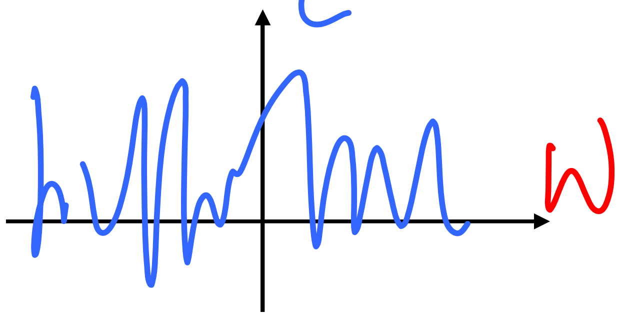
$$\sphericalangle H_2(e^{jw}) = f(w)$$

$$-\frac{d}{dw}\left\{\sphericalangle H_2(e^{jw})\right\} = -\frac{df}{dw}$$

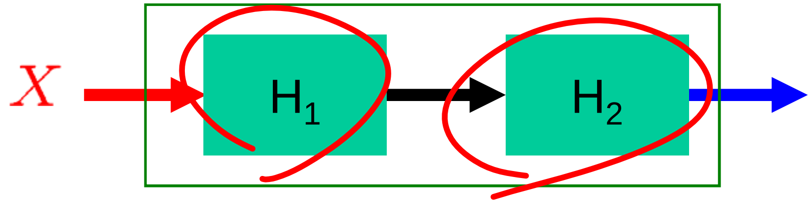
τ

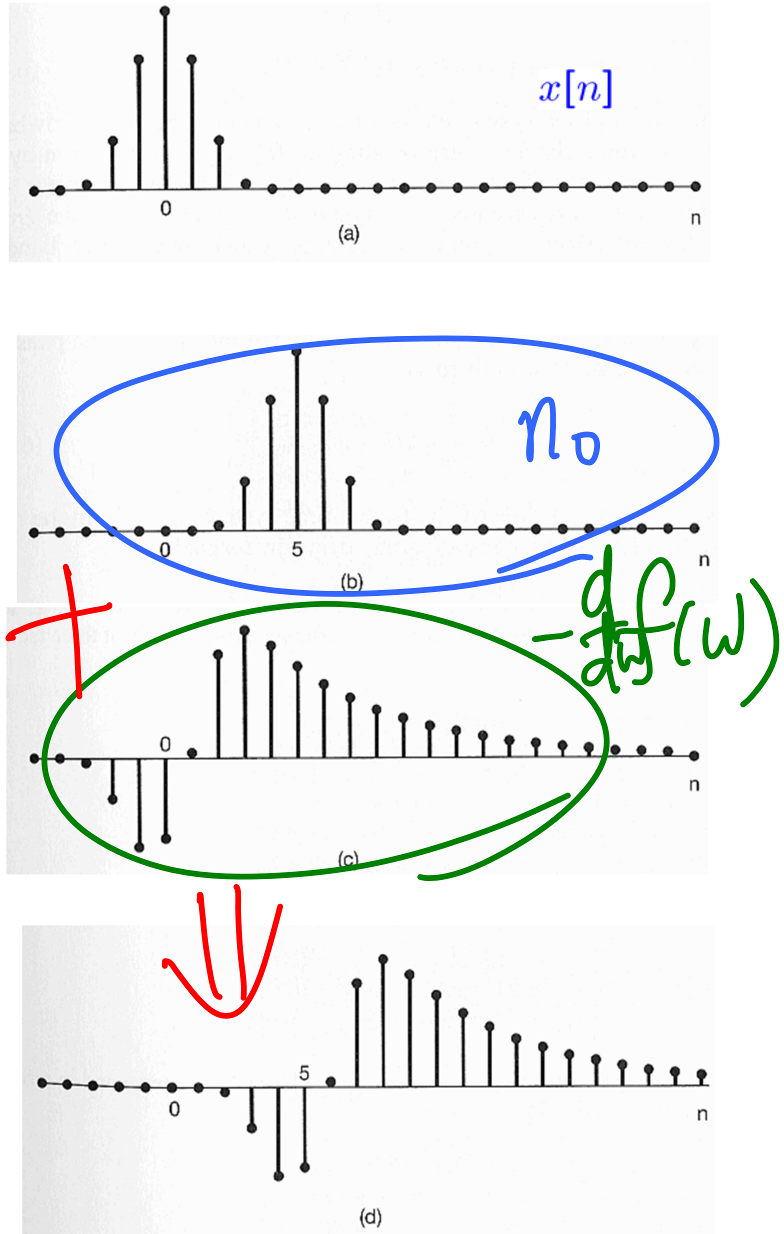
delay





- Linear Phase:





- $Y_3(e^{jw}) = H_2(e^{jw}) H_1(e^{jw}) \; X(e^{jw})$

$$H_3(e^{jw}) = H_2(e^{jw}) H_1(e^{jw})$$

$$= H_2(e^{jw}) e^{-jwn_0}$$

$$= e^{j(f(w) - wn_0)}$$

$e^{jf(w)}$

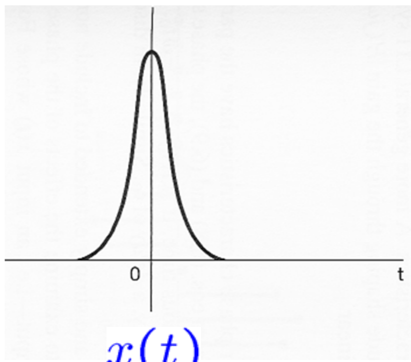
$$-\frac{d}{dw}\{\measuredangle H_3(e^{jw})\} = -\frac{df(w)}{dw} + n_0$$

- **Linear Phase:**

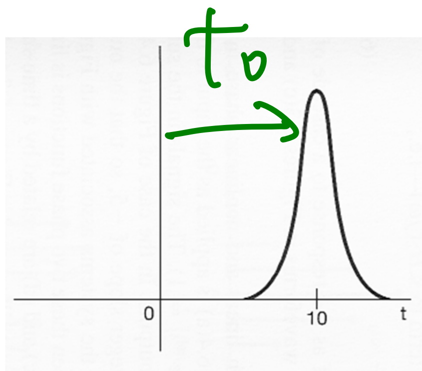
$$H_1(jw) = e^{-jwt_0}$$

$$\Rightarrow \begin{cases} |H_1(jw)| & = 1 \\ \sphericalangle |H_1(jw)| & = -wt_0 \end{cases}$$

$$\Rightarrow \ y(t) = x(t - t_0)$$



$x(t)$

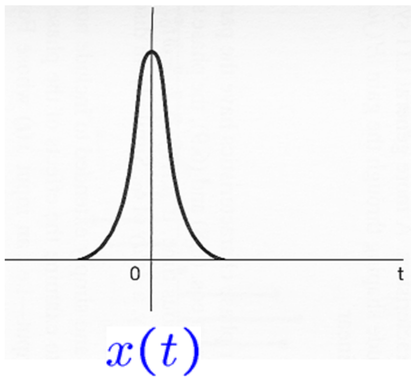


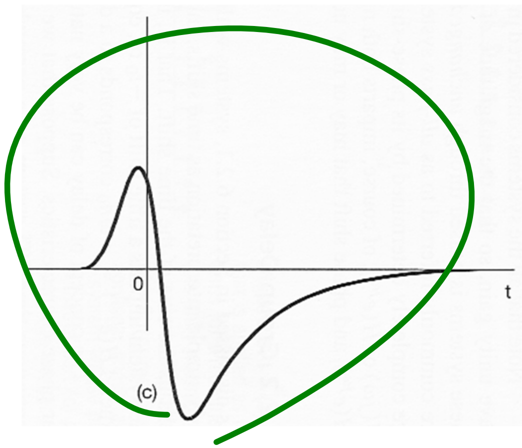
$y(t) = x(t - 10)$

$t_0 = 10$

▪ Linear Phase:

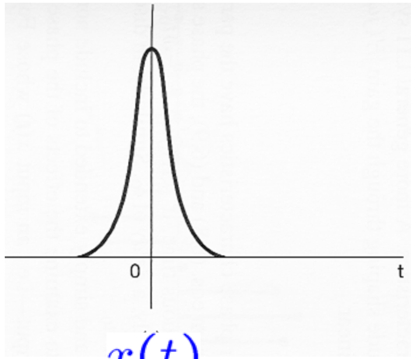
$$H_2(jw) = e^{jf(w)}$$



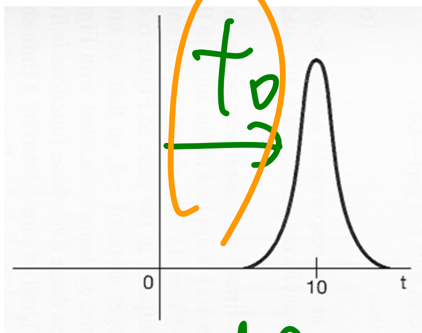


- Linear Phase:

$$H_3(jw) = H_2(jw)\, H_1(jw) = \boxed{H_2(jw)e^{-jwt_0}} = e^{j(f(w) - wt_0)}$$



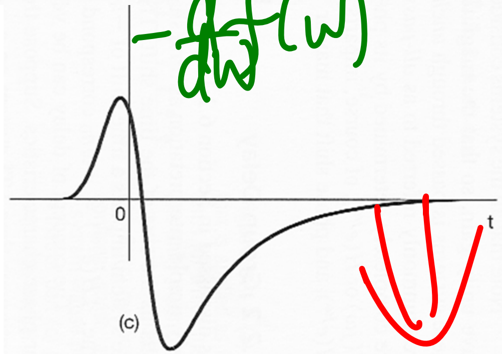
$x(t)$



(b)

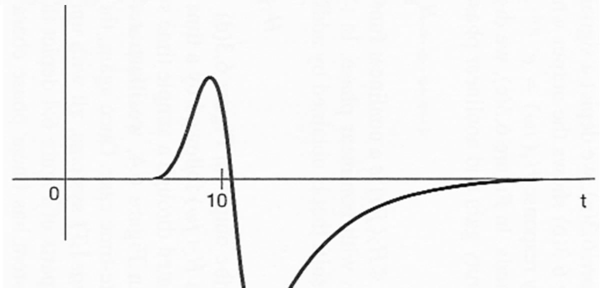
$+$

$-\frac{d}{dw} f(w)$



(c)

$-\frac{d f(w)}{dw} + t_0$



(d)

Group Delay & Phase:

- Linear Phase & Delay:

$$H_1(jw) = e^{-jwt_0} \quad \Rightarrow \quad y(t) = x(t - t_0) \quad \Rightarrow \quad \text{delay} = t_0$$

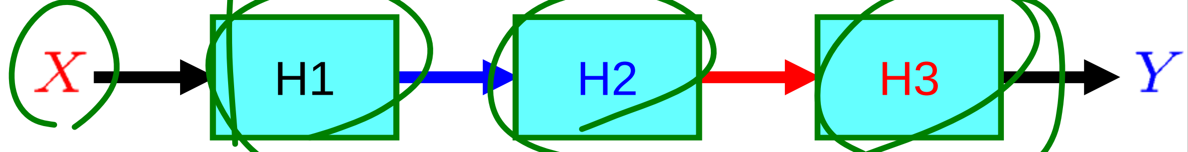
$$H_1(e^{jw}) = e^{-jwn_0} \quad \Rightarrow \quad y[n] = x[n - n_0] \quad \Rightarrow \quad \text{delay} = n_0$$

- Nonlinear Phase & Group Delay

$$H_2(jw) = e^{jf(w)} \quad \Rightarrow \quad \tau(w) = -\frac{d}{dw}\left\{\measuredangle H_2(jw)\right\}$$

$$= -\frac{d}{dw}\left\{f(w)\right\}$$

- **Example 6.1:**



$$H(jw) = \boxed{H_1(jw)\, H_2(jw)\, H_3(jw)}$$

$$H_i(jw) = \frac{(w_i)^2 + (jw)^2 - 2\zeta_i w_i (jw)}{(w_i)^2 + (jw)^2 + 2\zeta_i w_i (jw)} = \frac{1 + (\frac{jw}{w_i})^2 - 2j\zeta_i(\frac{w}{w_i})}{1 + (\frac{jw}{w_i})^2 + 2j\zeta_i(\frac{w}{w_i})}$$

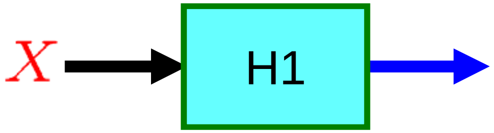
$$\Rightarrow \begin{cases} |H_i(jw)| & = 1 \\ \measuredangle H_i(jw) & = -2 \arctan\left[\frac{2\zeta_i(\frac{w}{w_i})}{1-(\frac{w}{w_i})^2}\right] \end{cases}$$

w_i

$$\Rightarrow \begin{cases} |H(jw)| & = 1 \\ \measuredangle H(jw) & = \measuredangle H_1(jw) + \measuredangle H_2(jw) + \measuredangle H_3(jw) \end{cases}$$

$$\Rightarrow \tau(w) = -\frac{d}{dw}\{\measuredangle H(jw)\}$$

Magnitude-Phase Representation of Freq Resp of LTI Systems



$$H_1(jw) = \frac{1 + (j\frac{w}{w_1})^2 - 2j\zeta_1(\frac{w}{w_1})}{1 + (j\frac{w}{w_1})^2 + 2j\zeta_1(\frac{w}{w_1})}$$

$$\begin{cases} w_1 & = & 315 & \text{rad/sec} \\ \zeta_1 & = & 0.066 & \\ f_1 & \approx & 50 & \text{Hz} \end{cases}$$

$$\Rightarrow \begin{cases} |H_1(jw)| & = & 1 \\ \measuredangle H_1(jw) & = & -2\ \arctan\left[\frac{2\zeta_1(\frac{w}{w_1})}{1-(\frac{w}{w_1})^2}\right] \end{cases} \longrightarrow$$

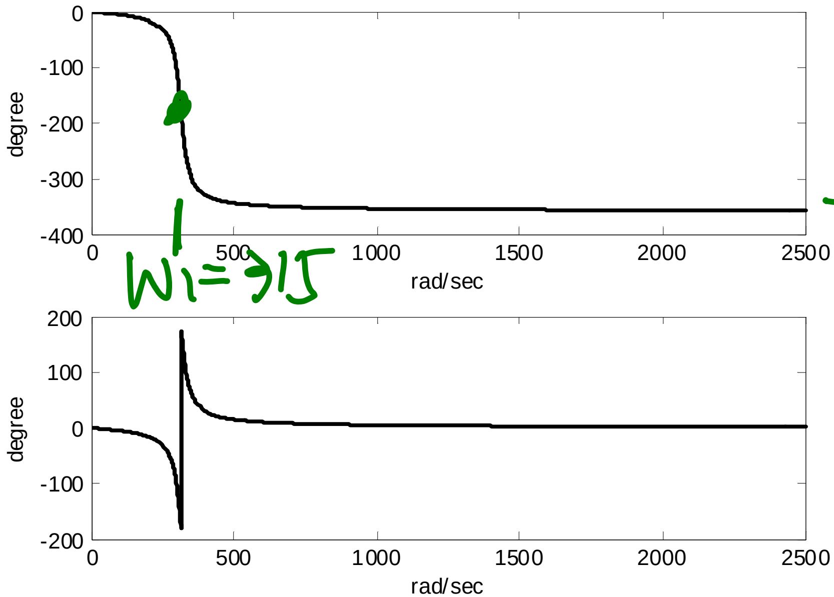
Matlab

$\arctan(\frac{0}{0})$

$-\pi \to \pi$

0

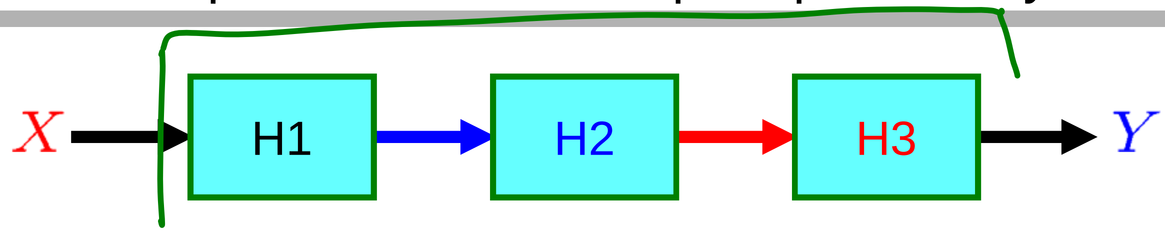
$\measuredangle H$



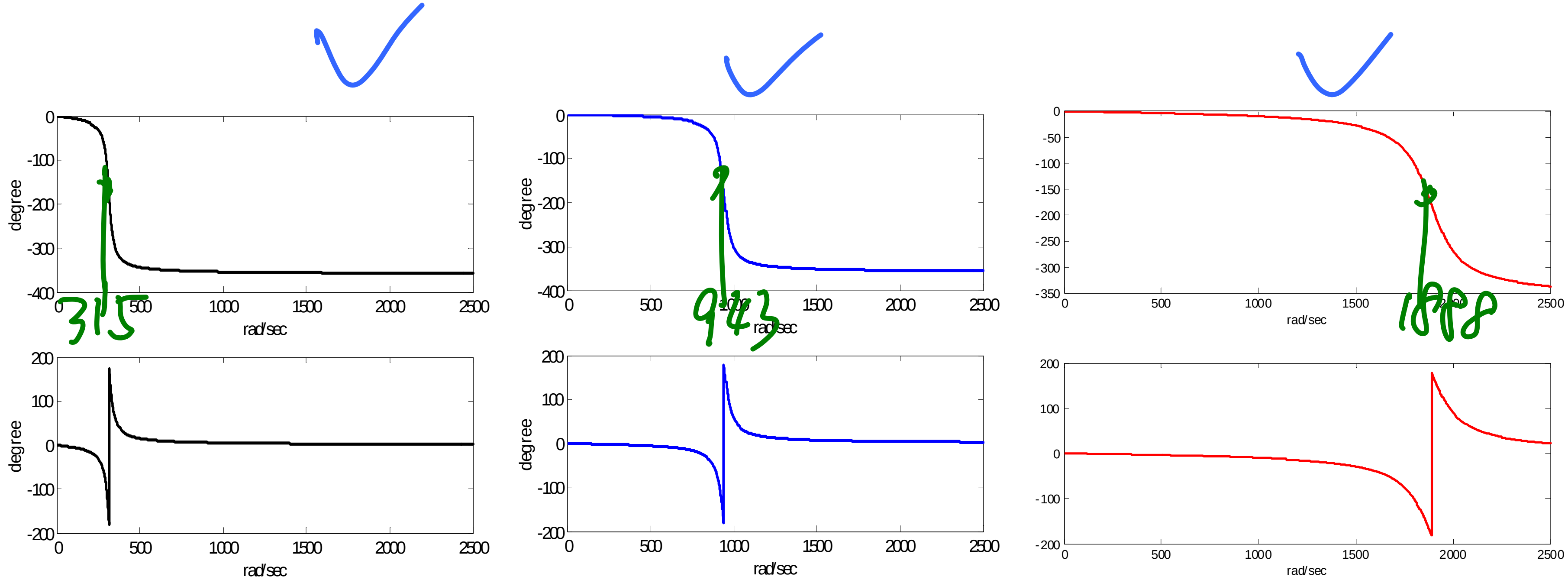
-2π

$w_1 = 315$

Magnitude-Phase Representation of Freq Resp of LTI Systems



$$\begin{cases} w_1 = 315 \ \text{rad/sec} \\ w_2 = 943 \ \text{rad/sec} \\ w_3 = 1888 \ \text{rad/sec} \end{cases} \qquad \begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$

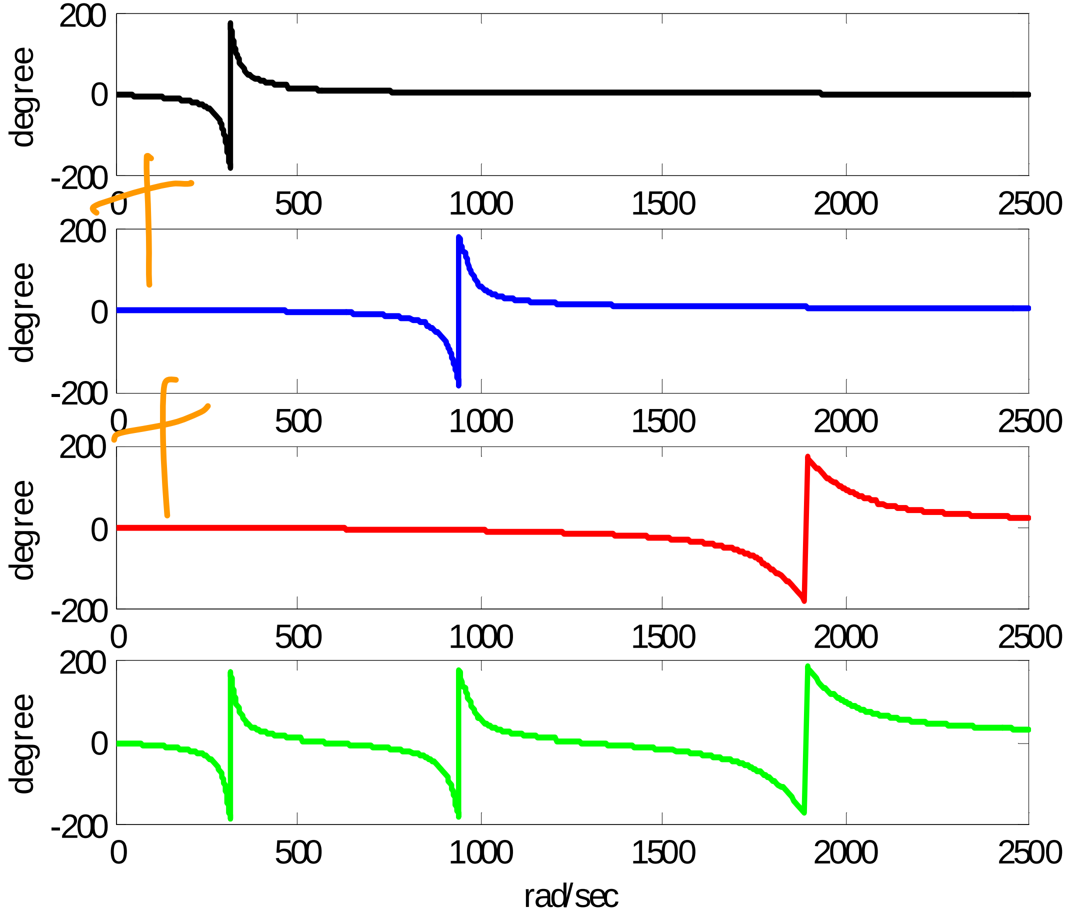


Magnitude-Phase Representation of Freq Resp of LTI Systems

$$\begin{cases} |H(jw)| &= 1 \\ \sphericalangle H(jw) &= \sphericalangle H_1(jw) + \sphericalangle H_2(jw) + \sphericalangle H_3(jw) \end{cases}$$

$$\begin{cases} w_1 &= 315 \quad \text{rad/sec} \\ w_2 &= 943 \quad \text{rad/sec} \\ w_3 &= 1888 \quad \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 &= 0.066 \\ \zeta_2 &= 0.033 \\ \zeta_3 &= 0.058 \end{cases}$$

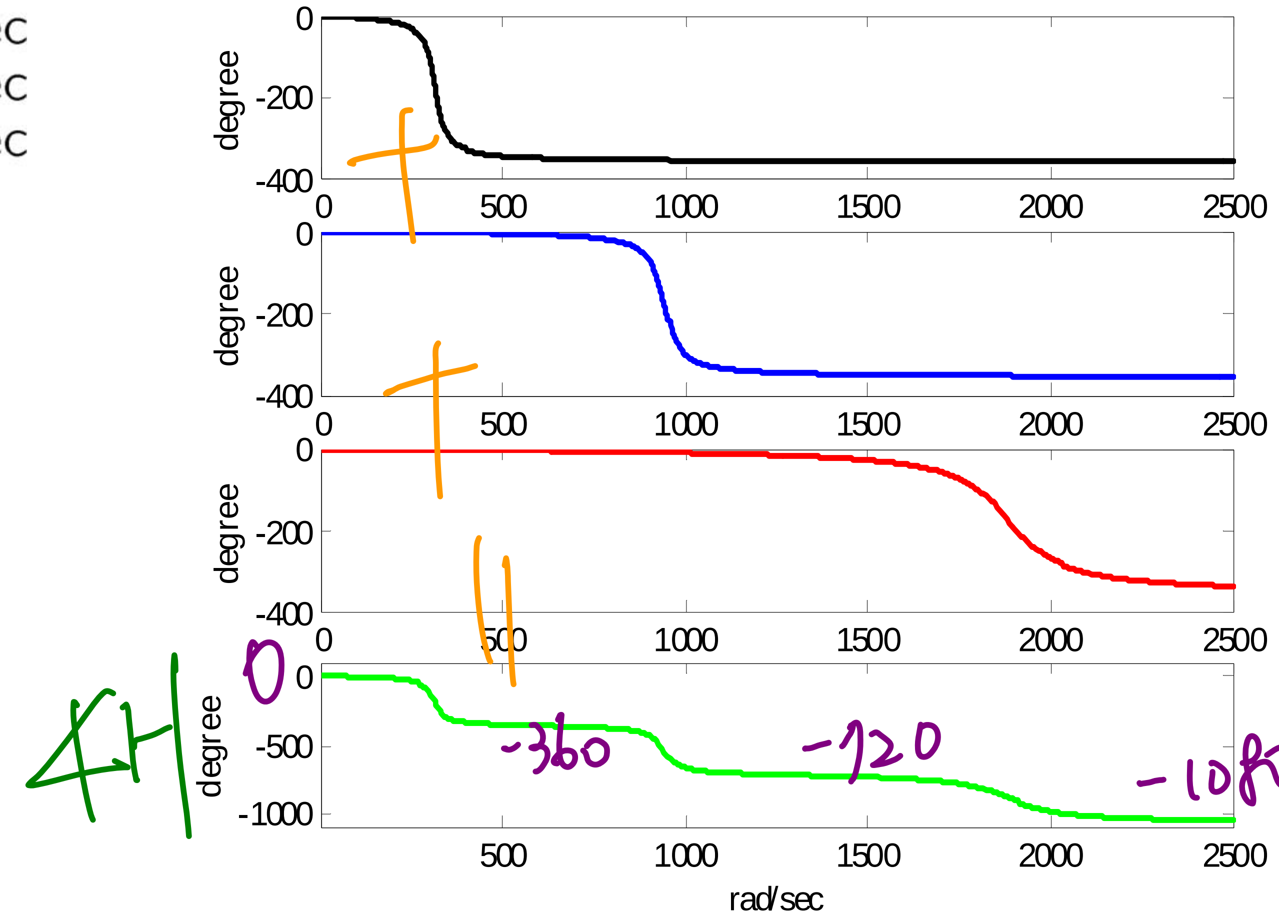


Magnitude-Phase Representation of Freq Resp of LTI Systems

$$\begin{cases} |H(jw)| &= 1 \\ \sphericalangle H(jw) &= \sphericalangle H_1(jw) + \sphericalangle H_2(jw) + \sphericalangle H_3(jw) \end{cases}$$

$$\begin{cases} w_1 = 315 & \text{rad/sec} \\ w_2 = 943 & \text{rad/sec} \\ w_3 = 1888 & \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$



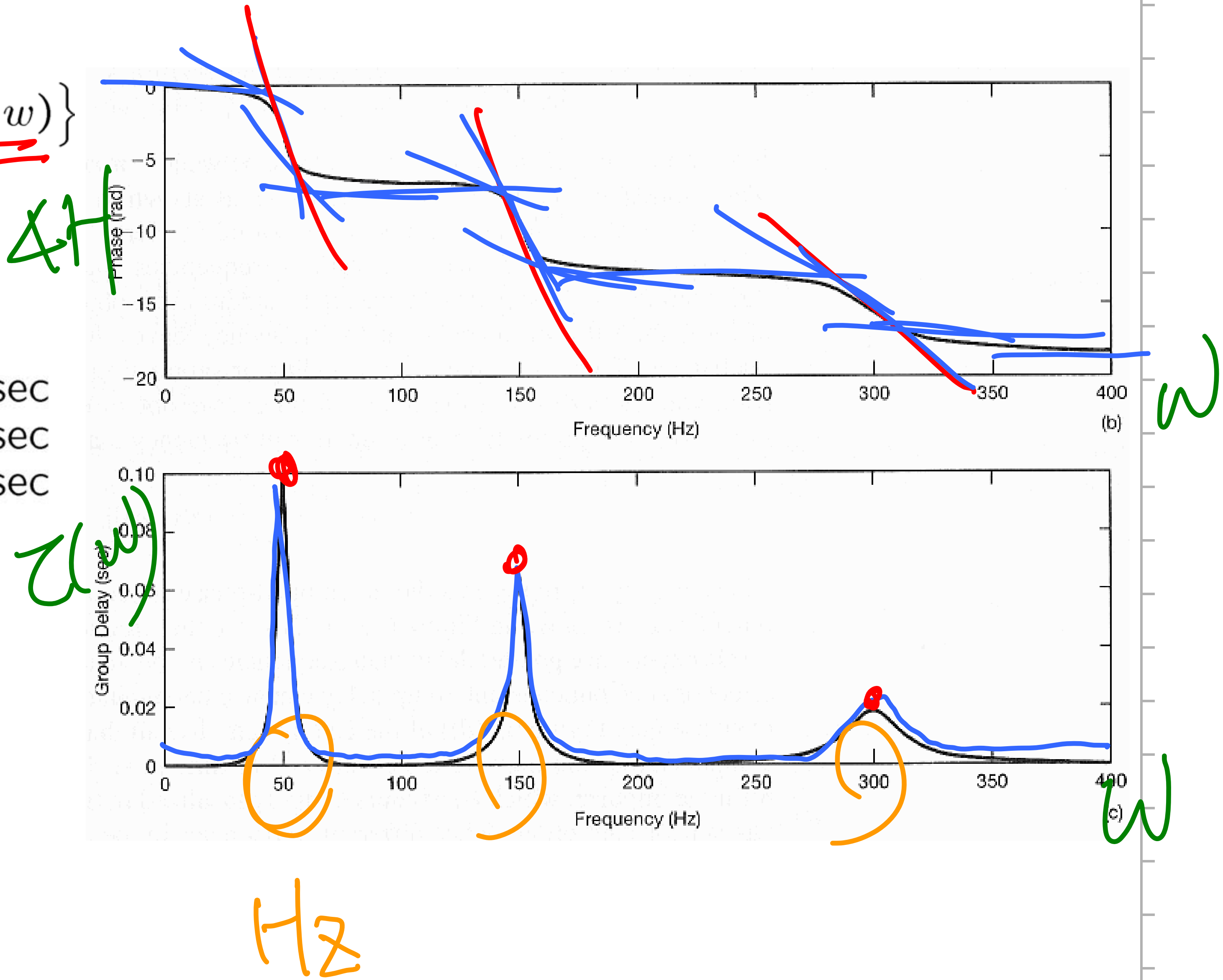
$$\tau(w) = -\frac{d}{dw}\{\sphericalangle H(jw)\}$$

$$\begin{cases} w_1 = 315 \text{ rad/sec} \\ w_2 = 943 \text{ rad/sec} \\ w_3 = 1888 \text{ rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$

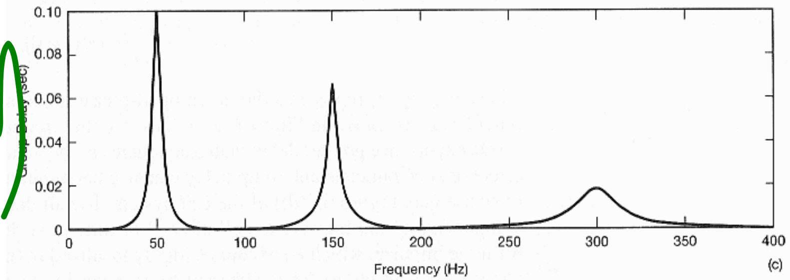
$$w_i = 2\pi f_i$$

$$\begin{cases} f_1 \approx 50 \text{ Hz} \\ f_2 \approx 150 \text{ Hz} \\ f_3 \approx 300 \text{ Hz} \end{cases}$$



Magnitude-Phase Representation of Freq Resp of LTI Systems

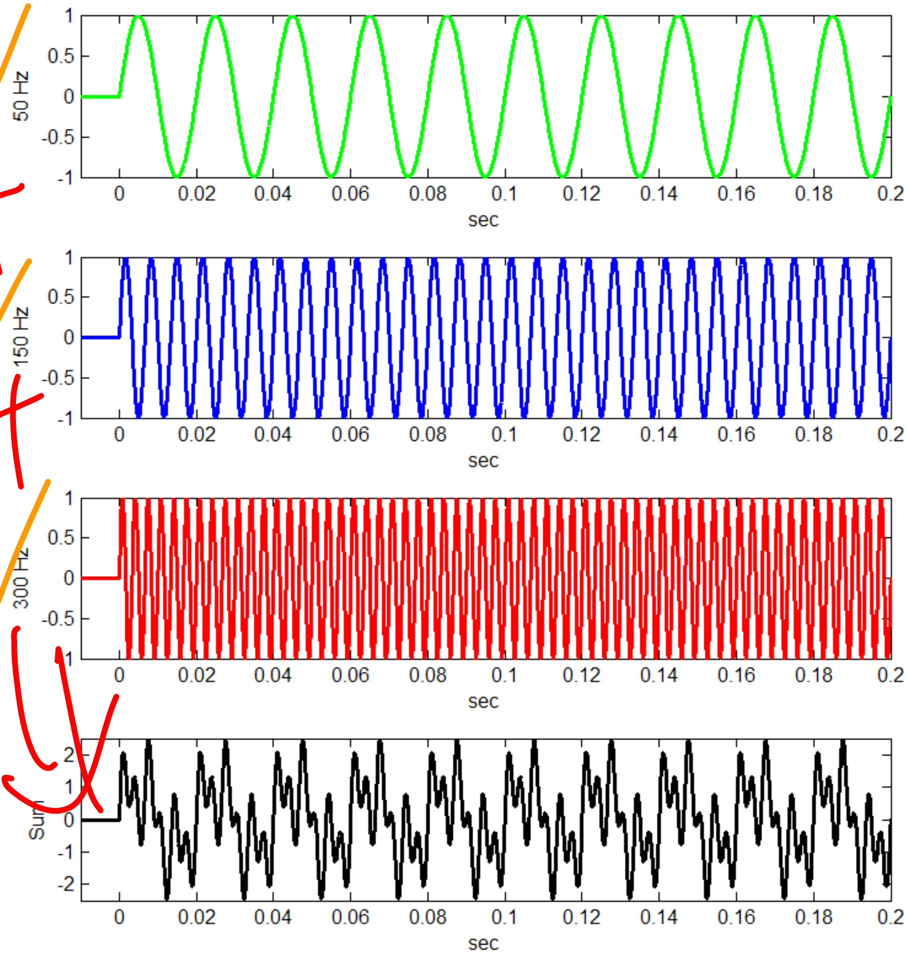
$\tau(\omega)$

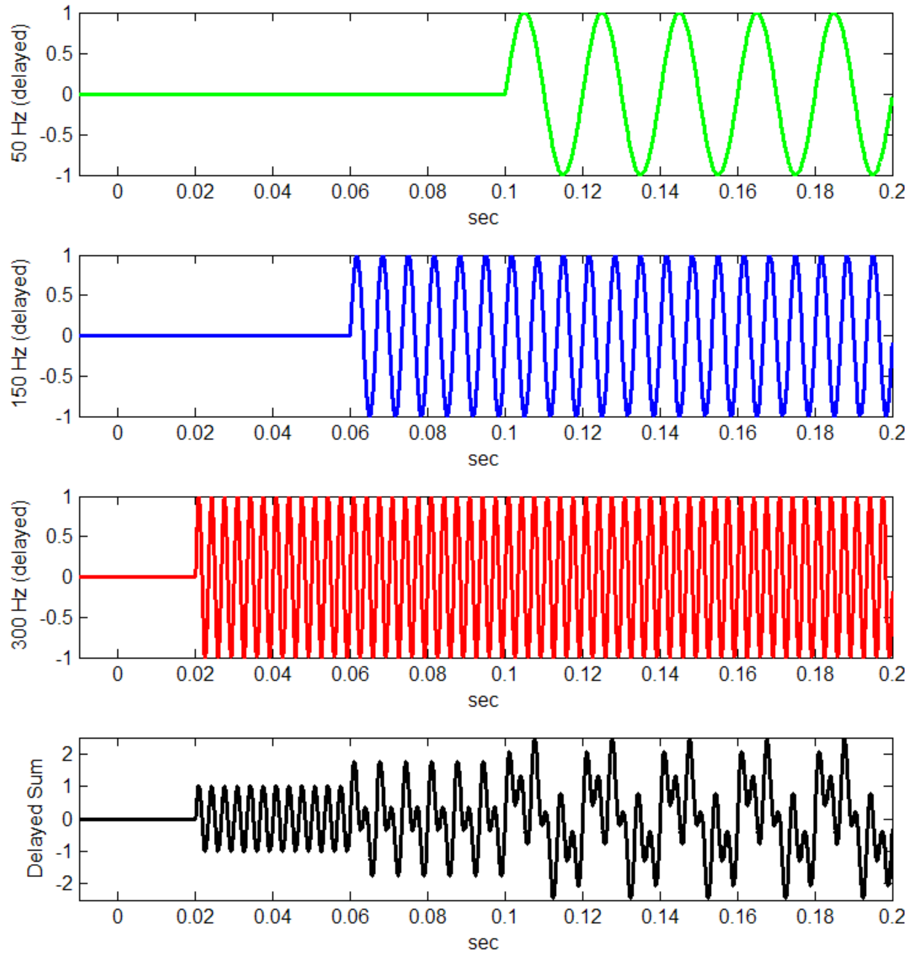


ω

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

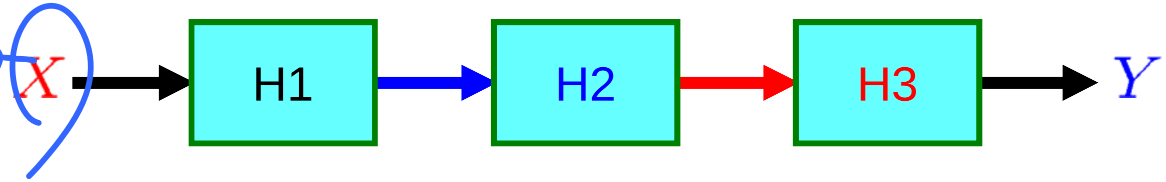
$$y(t) = y_1(t) + y_2(t) + y_3(t)$$



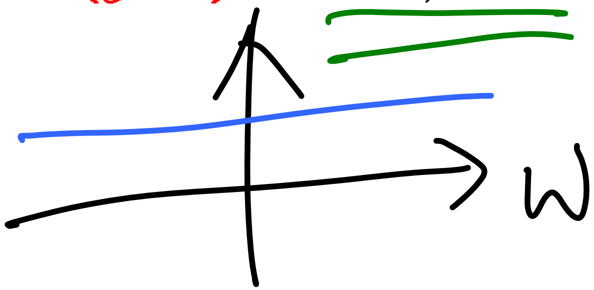


Magnitude-Phase Representation of Freq Resp of LTI Systems

$x(t) = \delta(t)$

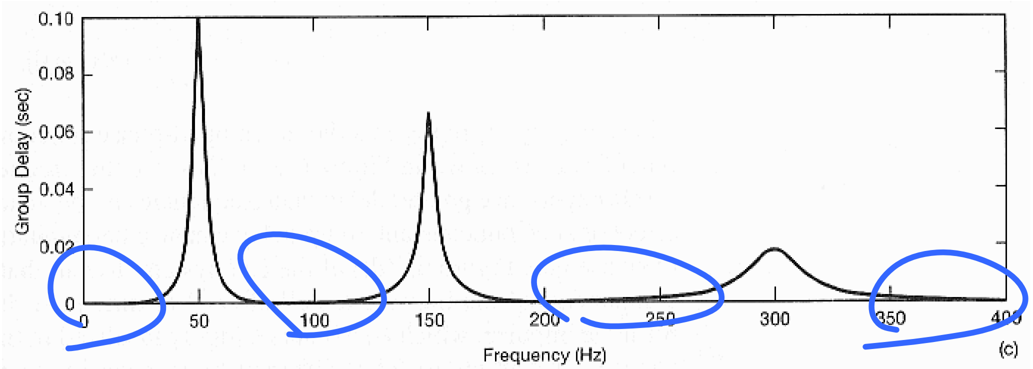


$X(jw) = 1, \ \forall w$



$$\begin{cases} |H(jw)| & = 1 \\ \sphericalangle H(jw) & = \sphericalangle H_1(jw) + \sphericalangle H_2(jw) + \sphericalangle H_3(jw) \end{cases}$$

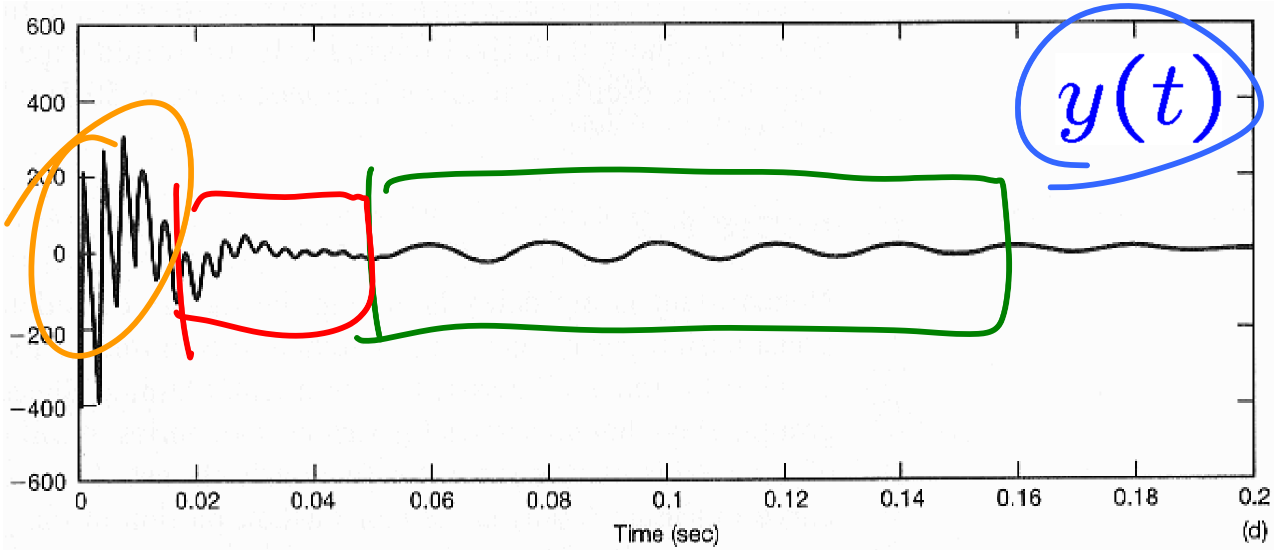
$$\begin{cases} w_1 = 315 & \text{rad/sec} \\ w_2 = 943 & \text{rad/sec} \\ w_3 = 1888 & \text{rad/sec} \end{cases}$$



$$\begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$

$$w_i = 2\pi f_i$$

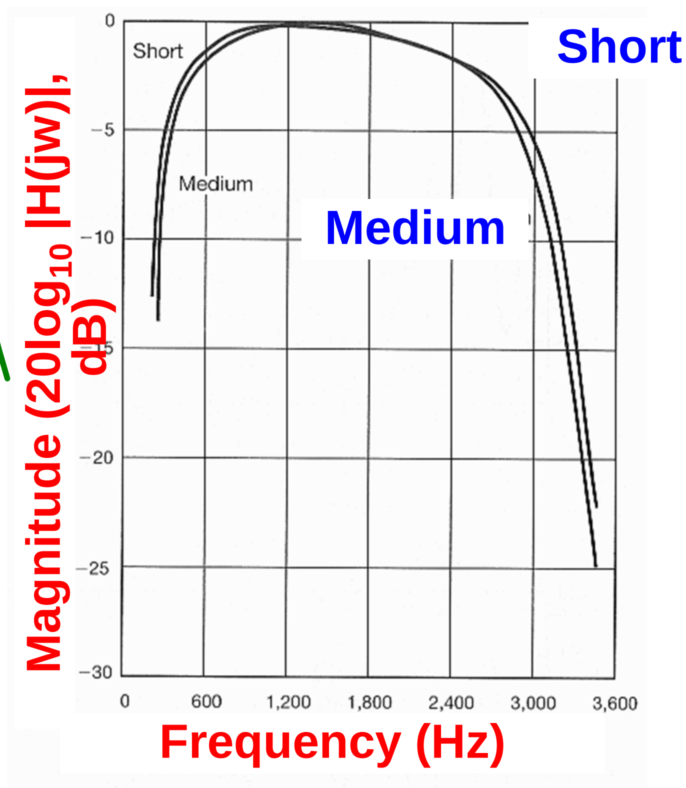
$$\begin{cases} f_1 \approx 50 & \text{Hz} \\ f_2 \approx 150 & \text{Hz} \\ f_3 \approx 300 & \text{Hz} \end{cases}$$



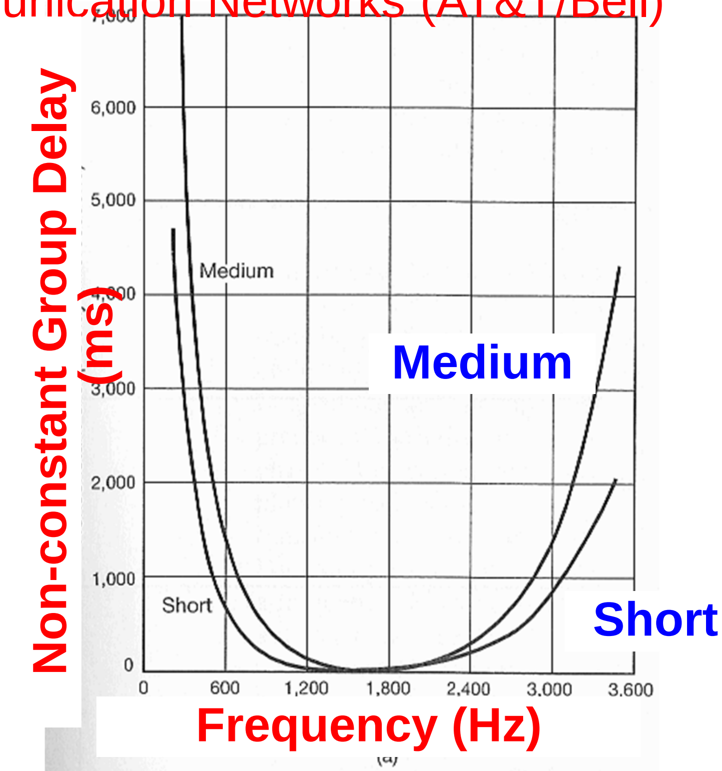
- Example 6.2:

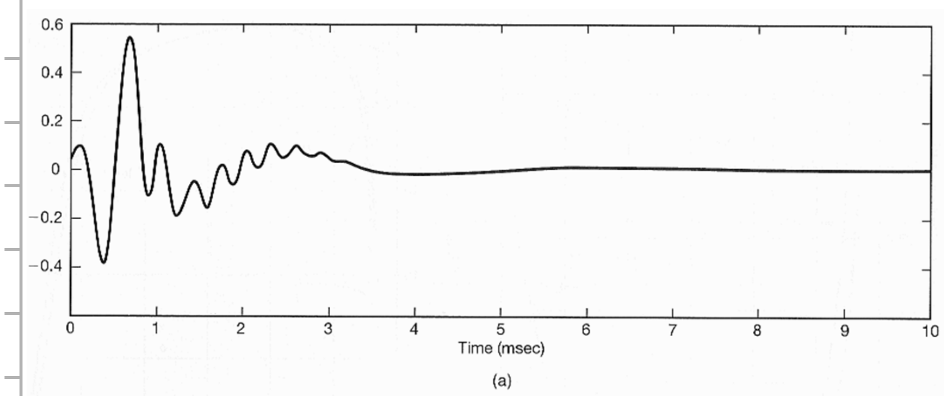
Analog Transmission Performance on the Switched Telecommunication Networks (AT&T/Bell)

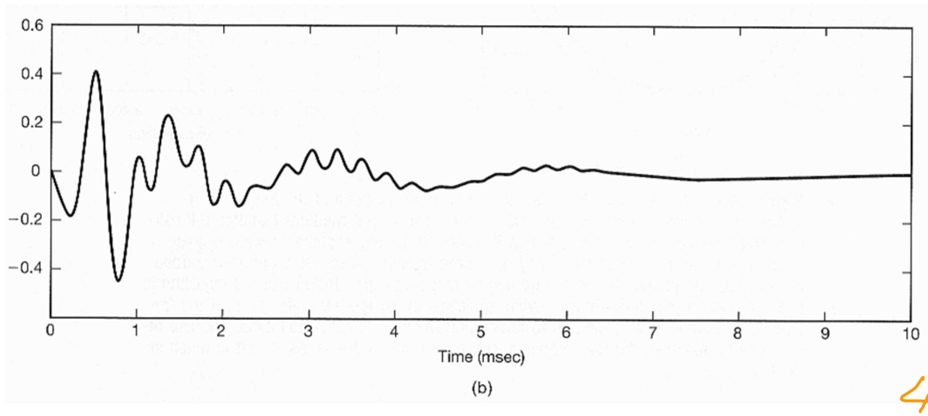
|H|



∠H
⇓
τ



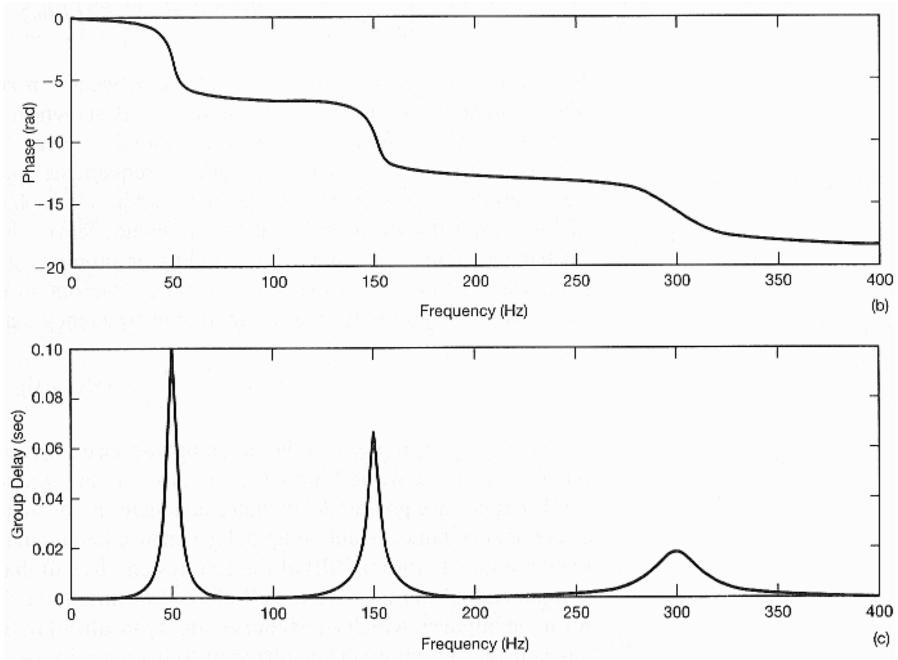




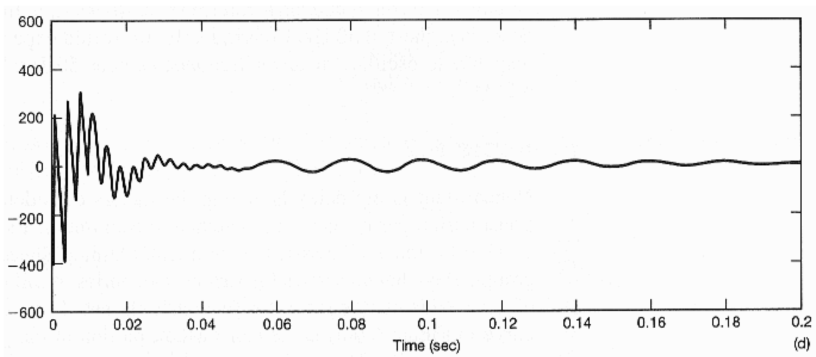
4/25/13
2:20pm

Phase Distortion and Group Delay

$$\tau(w) = -\frac{d}{dw}\left\{ \sphericalangle\, H(jw) \right\}$$



$$x(t) = \delta(t)$$



- The Magnitude-Phase Representation of the Fourier Transform
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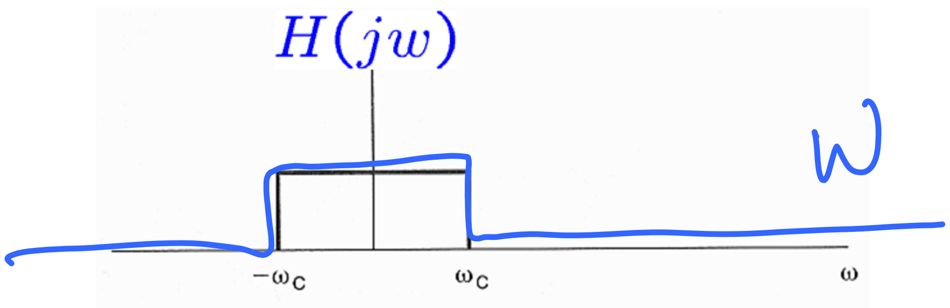
Ideal Lowpass Filters:

CT $$H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & |w| > w_c \end{cases}$$

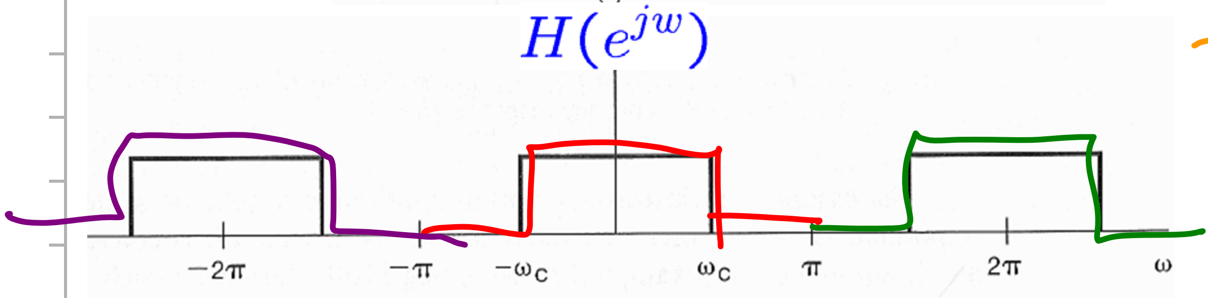
DT $$H(e^{jw}) = \begin{cases} 1, & |w| \le w_c \\ 0, & \boxed{w_c < |w| \le \pi} \end{cases}$$

–unit gain

–zero phase distortion

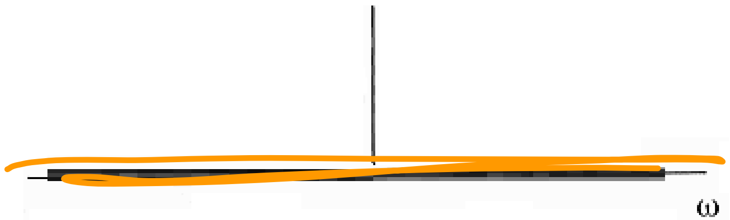


(a)



(b)

$\sphericalangle H(jw)$ $\quad$ $\sphericalangle H(e^{jw})$

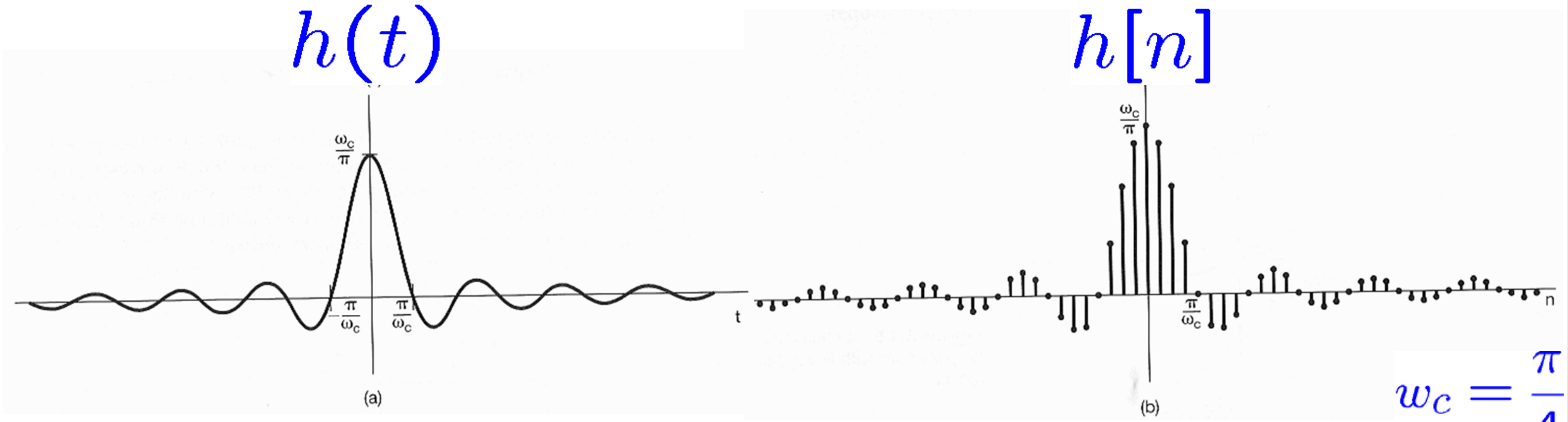


Ideal Lowpass Filters:

$$H(jw) = \begin{cases} 1, & |w| \leq w_C \\ 0, & |w| > w_C \end{cases}$$

$$H(e^{jw}) = \begin{cases} 1, & |w| \leq w_C \\ 0, & w_C < |w| \leq \pi \end{cases}$$

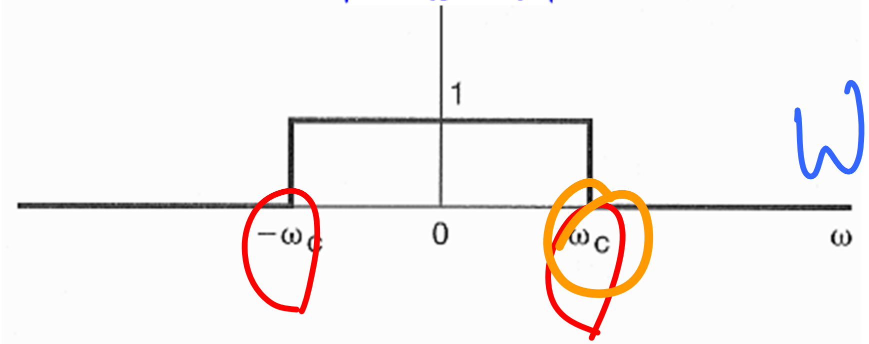
$$\Rightarrow \begin{cases} h(t) = \dfrac{\sin w_C t}{\pi t} \\ h[n] = \dfrac{\sin w_C n}{\pi n} \end{cases}$$



$w_C = \dfrac{\pi}{4}$

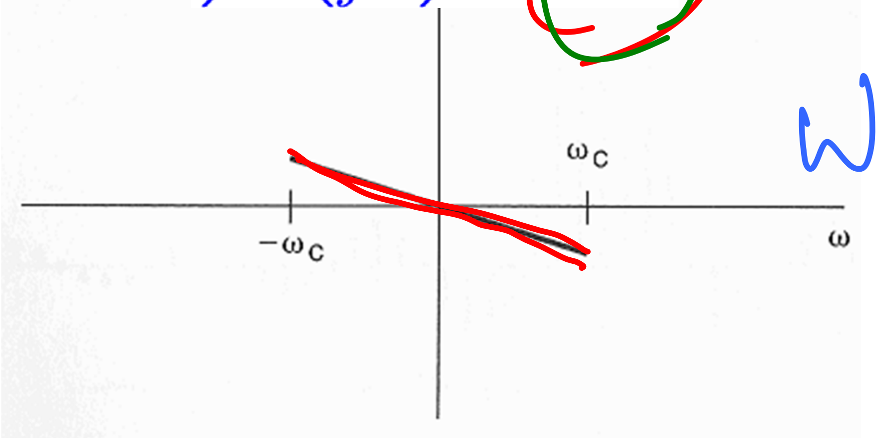
- Ideal Lowpass Filters with Linear Phase:

$|H(jw)|$



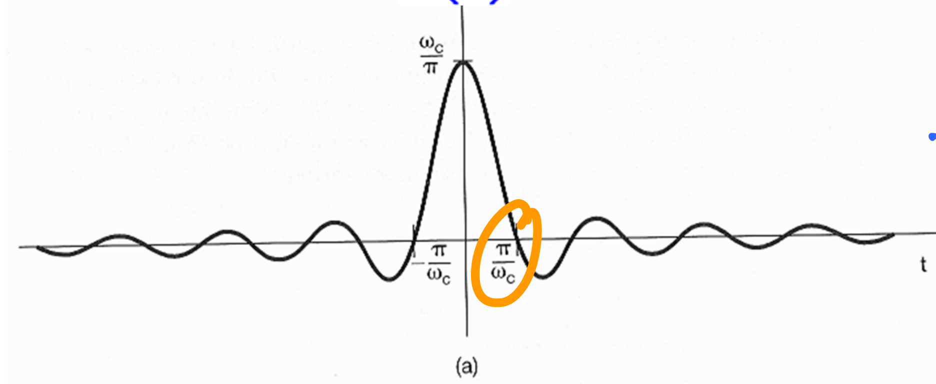
w

$\measuredangle H(jw) = -\alpha w$



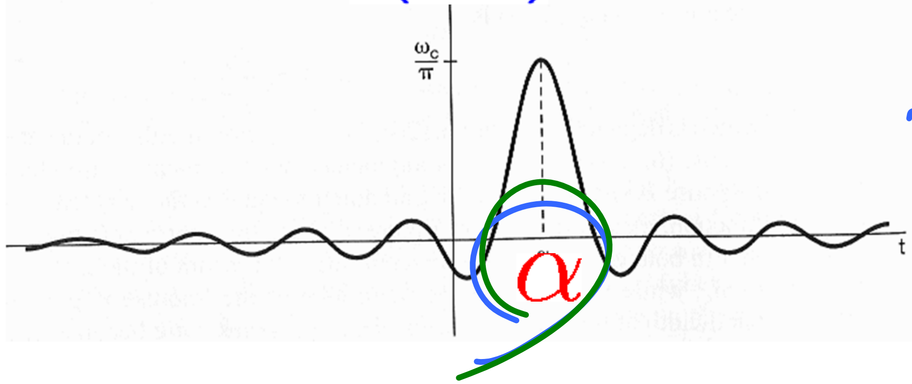
w

$h(t)$



t

$h(t-\alpha)$

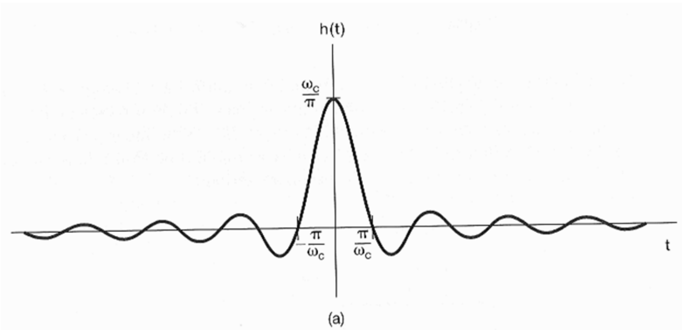


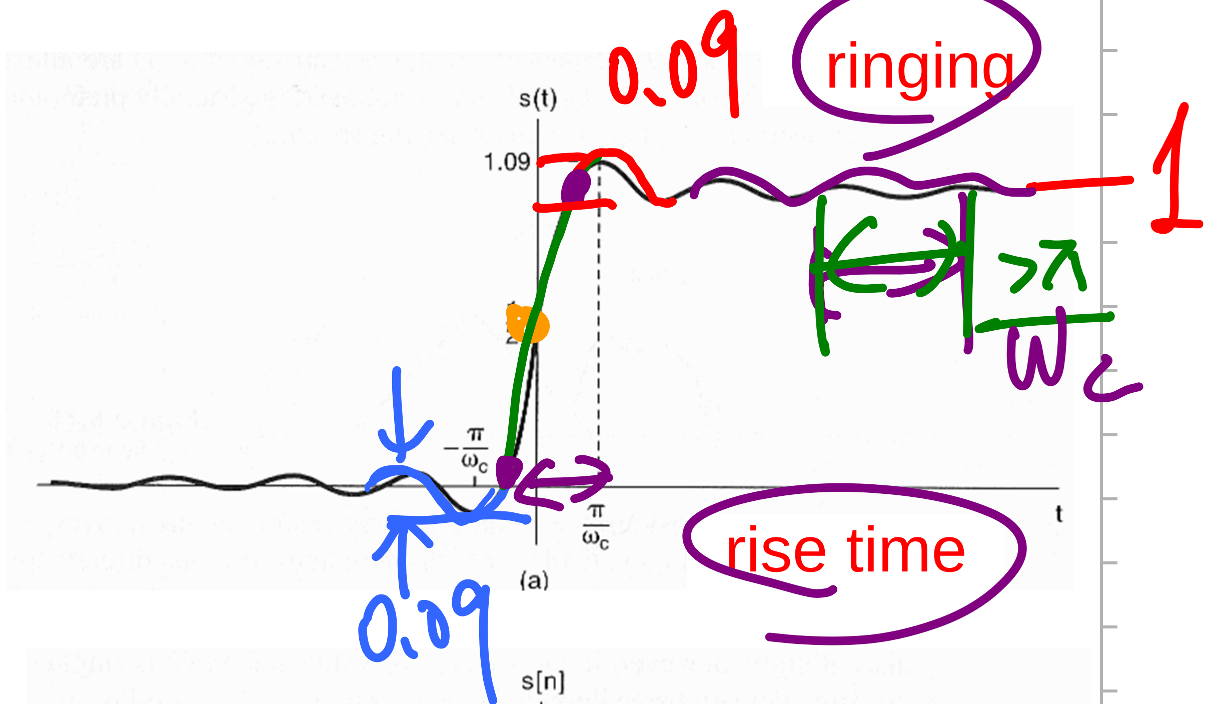
t

$\vec{\alpha}$

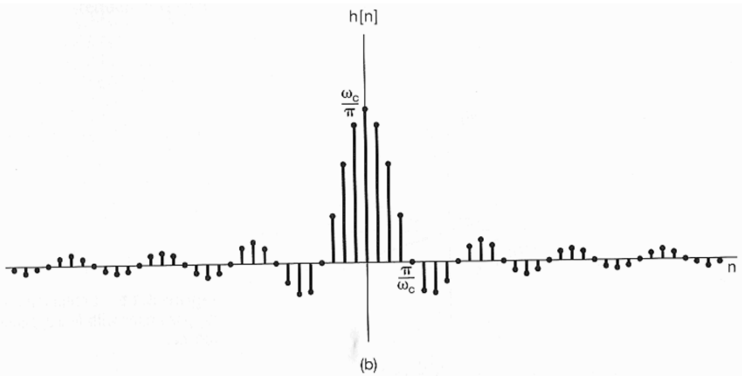
▪ Step Response of Ideal Lowpass Filters:

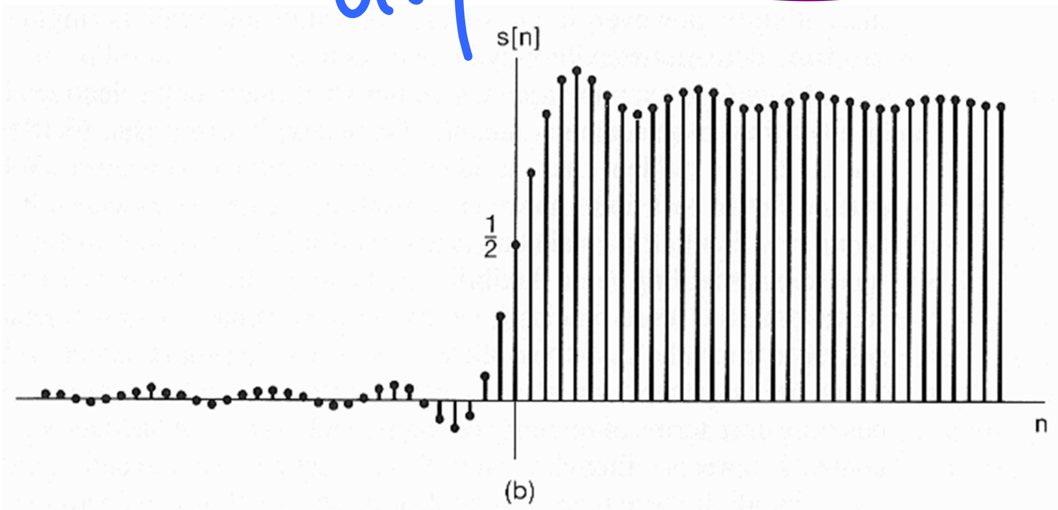
$$s(t) = \int_{-\infty}^{t} h(\tau) d\tau$$





$$s[n] = \sum_{m=-\infty}^{n} h[m]$$





Outline

- The Magnitude-Phase Representation of the Fourier Transform
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
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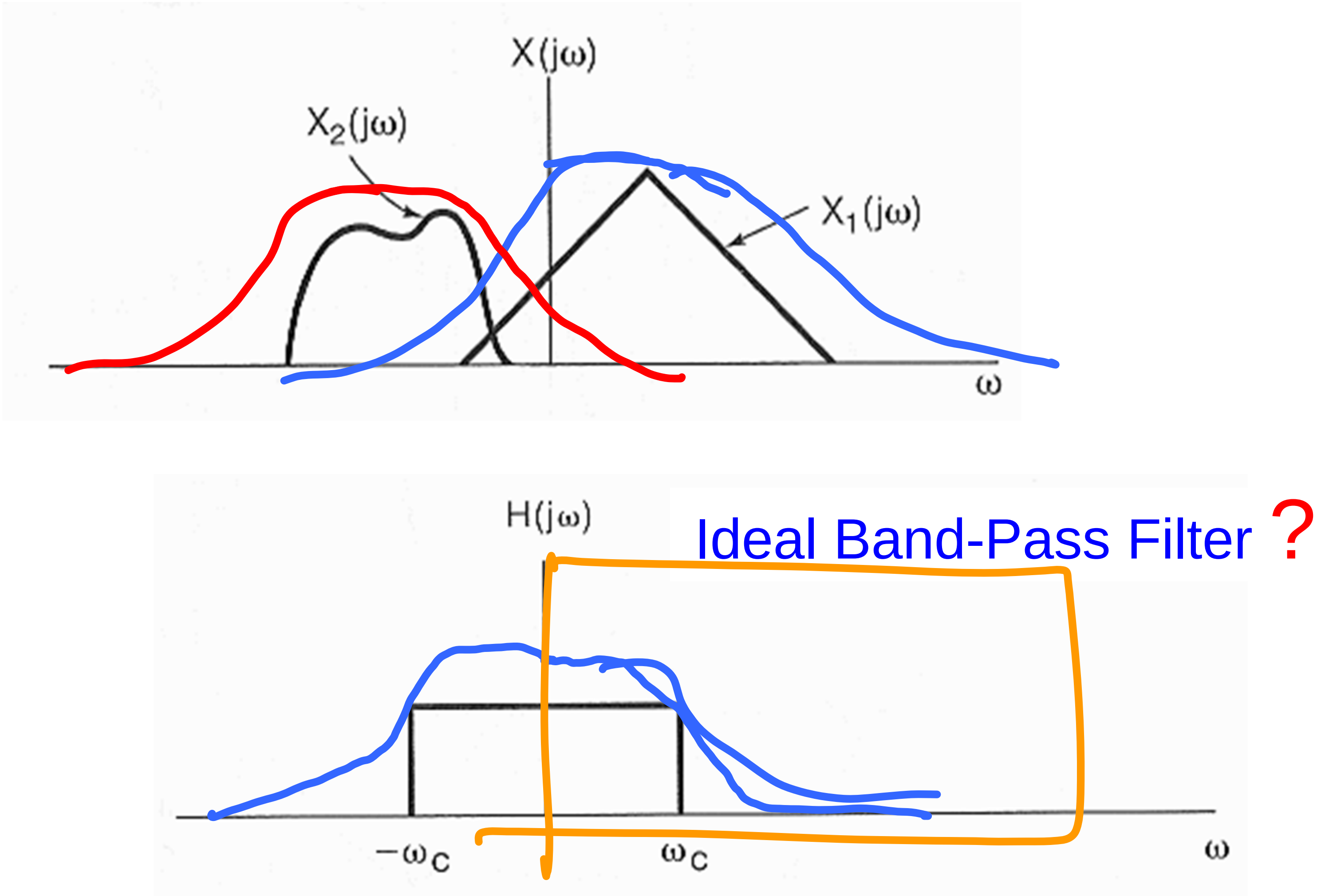
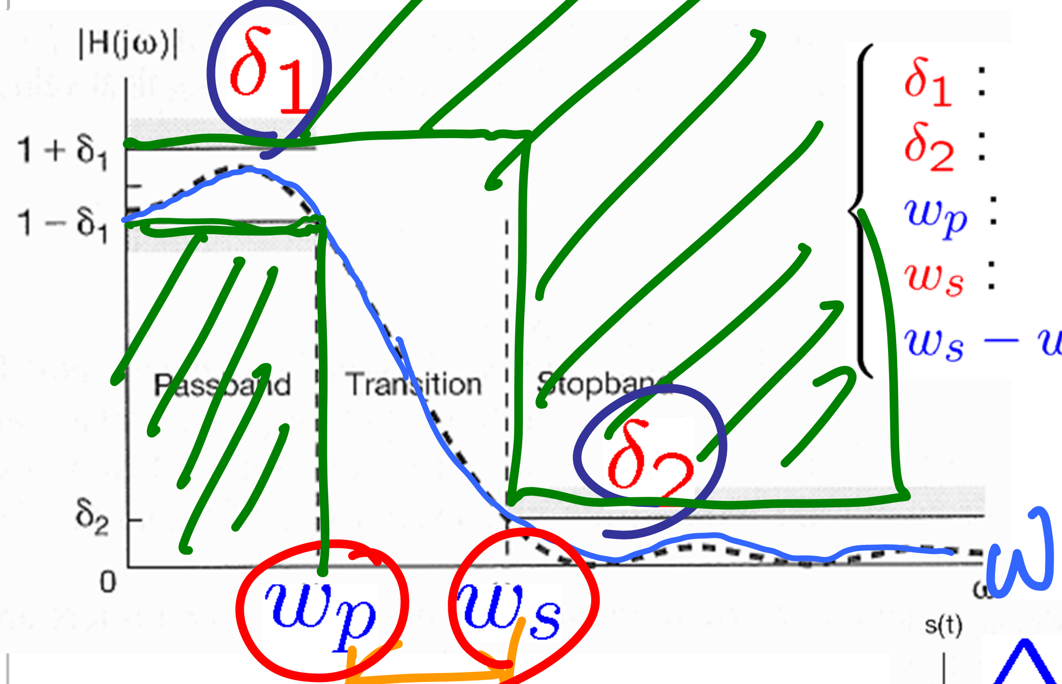
- Overlapping Spectra:



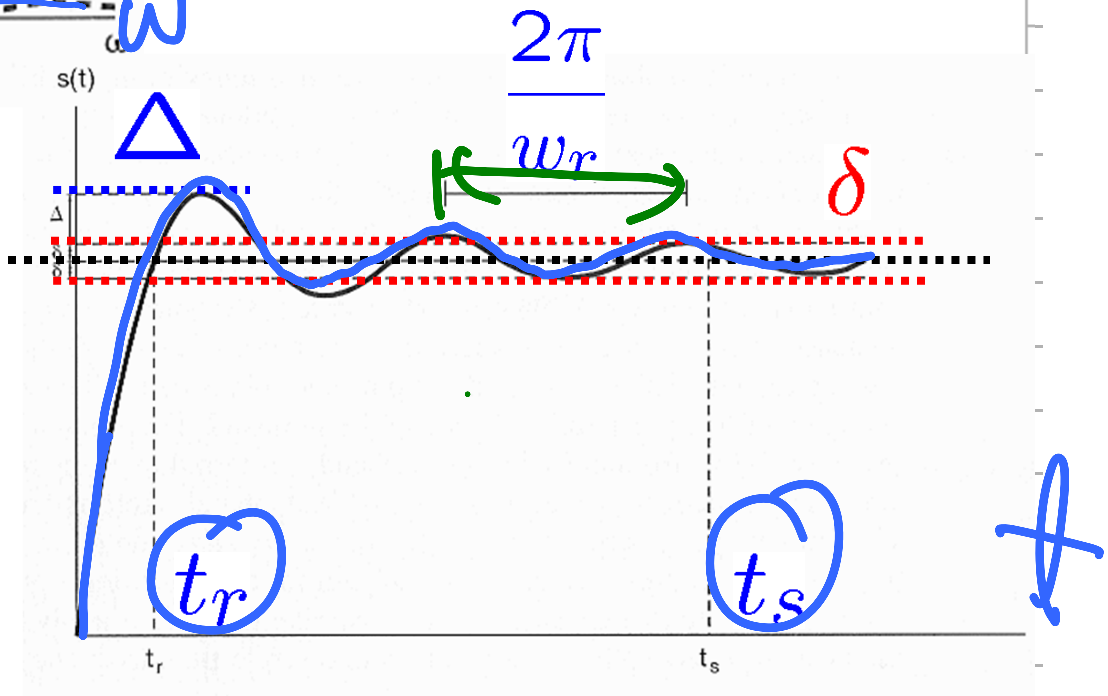
(a)

Desired Filter Characteristics:

$X \rightarrow \boxed{H} \rightarrow Y$



- δ_1 : allowable passband ripple
- δ_2 : allowable stopband ripple
- w_p : passband edge
- w_s : stopband edge
- $w_s - w_p$: transition band

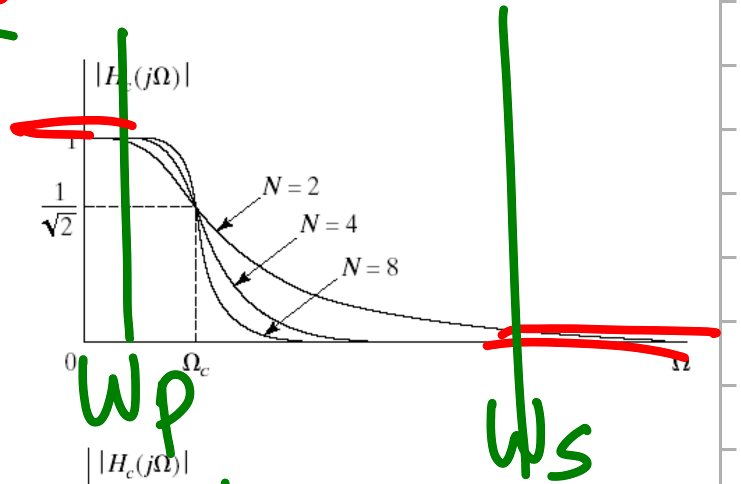


- Δ : overshoot
- δ : steady-state error
- w_r : ringing frequency
- t_r : rise time
- t_s : settling time

Three Frequently Used Filters:

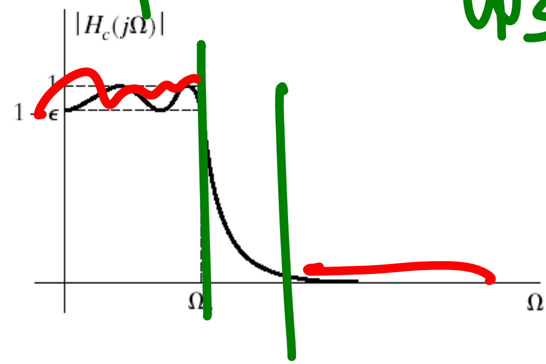
- Butterworth, Chebyshev, Elliptic filters

$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1 + (\frac{j\Omega}{j\Omega_c})^{2N}}$$



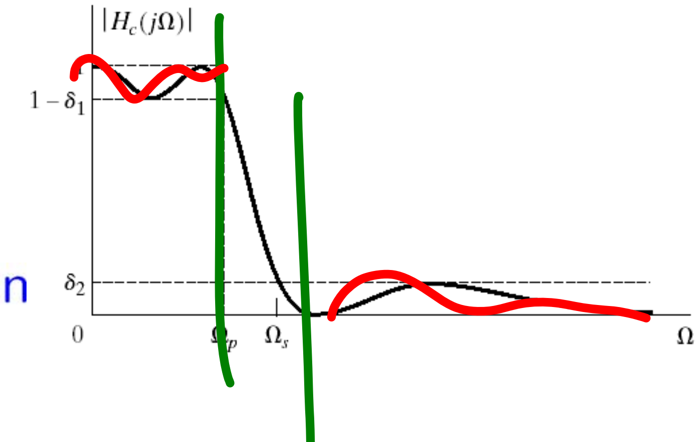
$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1 + \epsilon^2 V_N^2(\frac{\Omega}{\Omega_c})}$$

$$V_N(x) = \cos\left(N \cos^{-1} x\right)$$



$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1 + \epsilon^2 U_N^2(\Omega)}$$

$U_N(x)$: Jacabian elliptic function

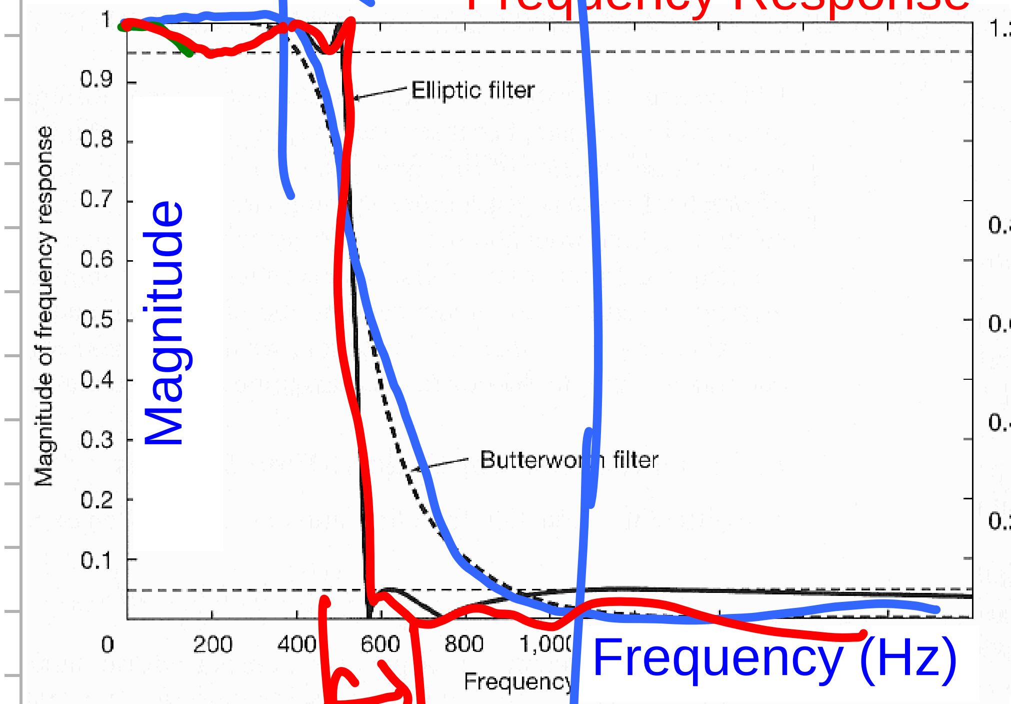


Example 6.3: Two Frequently Used Filters:

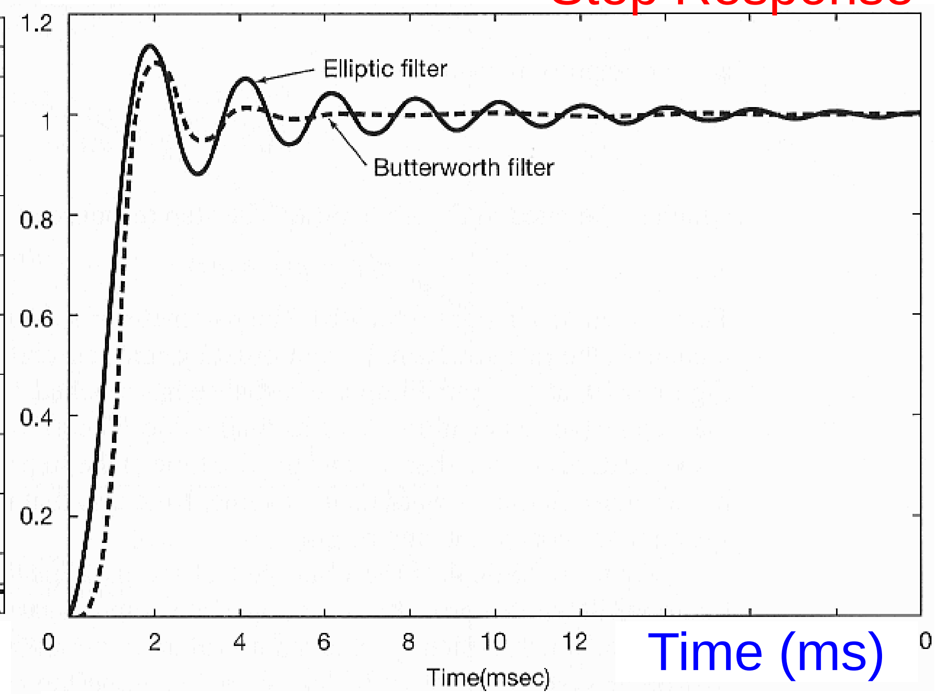
- Butterworth filter
- Elliptic filter

Fifth-order rational frequency response
Cutoff frequency = 500 Hz

Frequency Response



Step Response



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(p.472)

DT Non-recursive Filters:

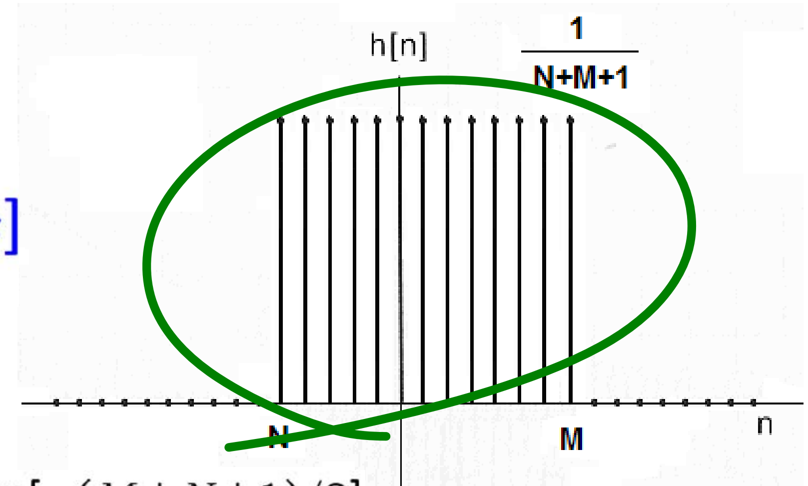
- Recursive or infinite impulse response (IIR) filters
  - > Impossible to design a causal, recursive filter with exactly linear phase (related to time delay)

$$y[n] - a\,y[n-1] = x[n] \qquad |a| < 1 \quad \Rightarrow \quad H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

- Non-recursive or finite impulse response (FIR) filters
  - > Can have exactly linear phase (related to time delay)

ss3-105

$$y[n] = \frac{1}{N+M+1} \sum_{k=-N}^{M} x[n-k]$$



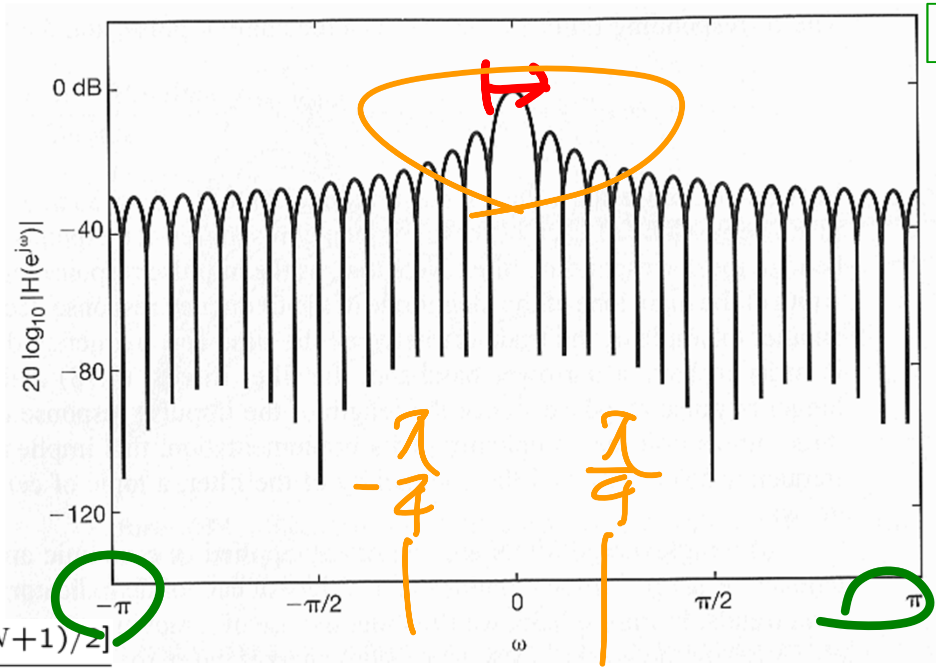
$$\Rightarrow \quad H(e^{jw}) = \frac{1}{N+M+1} e^{jw[(N-M)/2]} \frac{\sin[w(M+N+1)/2]}{\sin(w/2)}$$

ss3-106

Log-Magnitude Plots:

16 16

$N + M + 1 = 33$

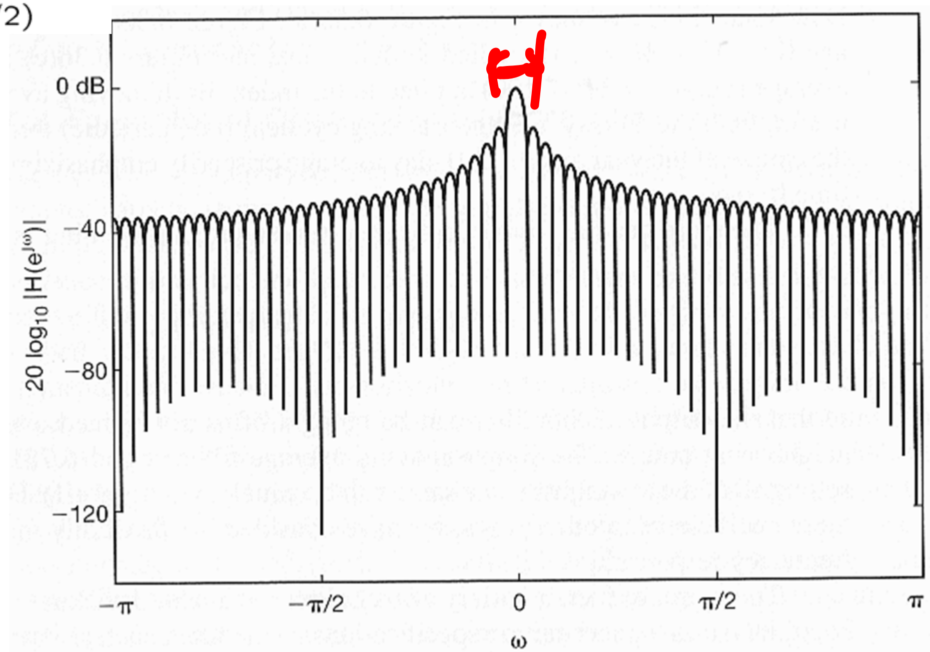


(a)

$$H(e^{jw}) = \frac{1}{N+M+1} e^{jw[(N-M)/2]} \frac{\sin[w(M+N+1)/2]}{\sin(w/2)}$$

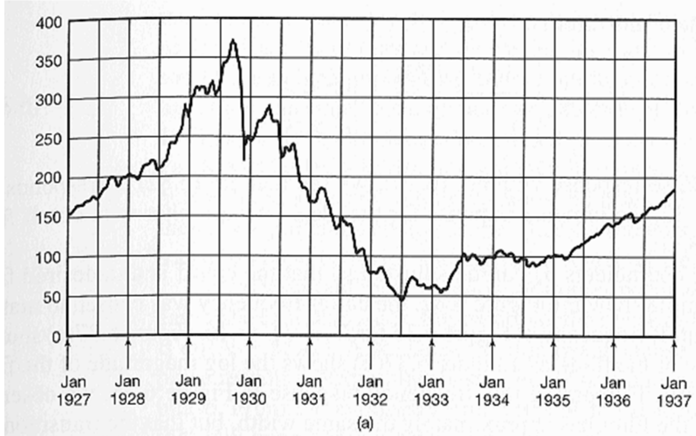
$N + M + 1 = 65$

32 32

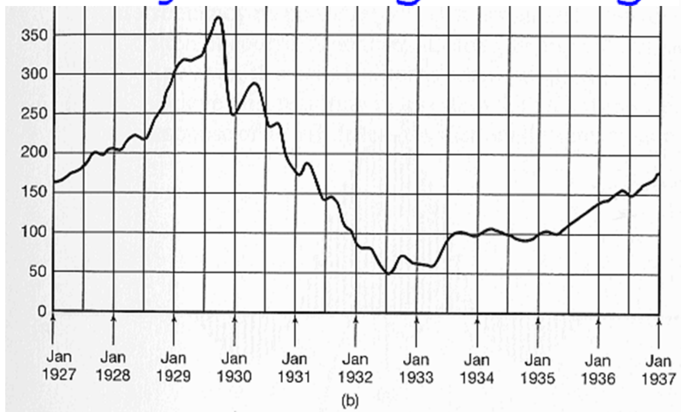


(b)

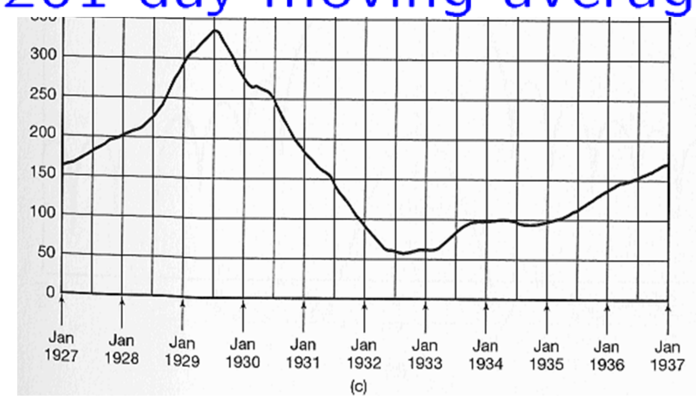
- Lowpass Filtering on Dow Jones Weekly Stock Market Index:



51-day moving average



201-day moving average



http://big5.jrj.com.cn/

ss3-104
ss3-108

General Form of DT Non-recursive Filters:

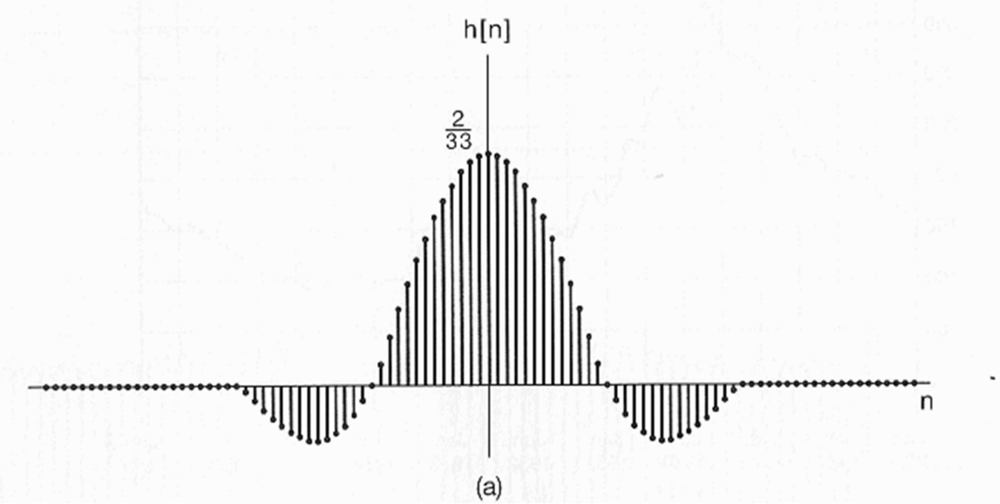
$$y[n] = \sum_{k=-N}^{M} b_k x[n-k] \qquad\qquad y[n] = \sum_{k=-N}^{M} \frac{1}{N+M+1} x[n-k]$$

b_k

- Let $N = M = 16$:

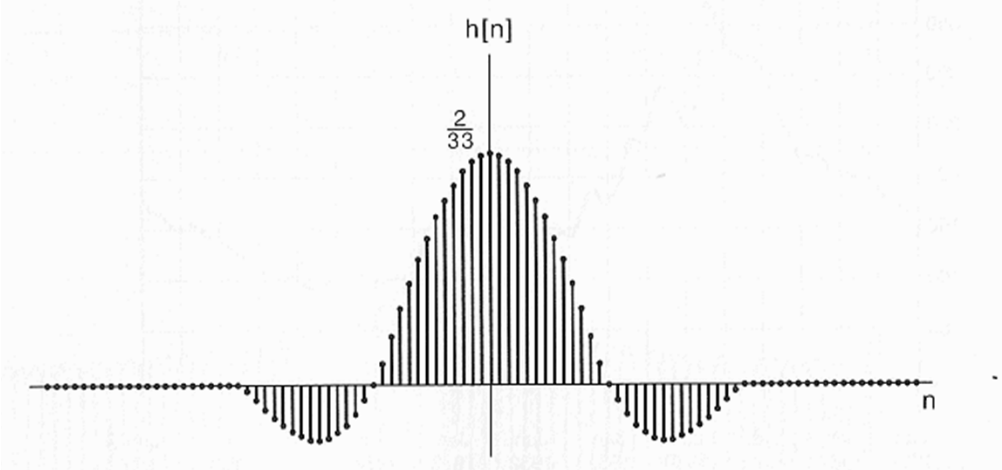
$$b_k = \begin{cases} \dfrac{\sin(2\pi k/33)}{\pi k}, & |k| \le 32 \\ 0, & |k| > 32 \end{cases}$$

$$h[n] = \begin{cases} \dfrac{\sin(2\pi n/33)}{\pi n}, & |n| \le 32 \\ 0, & |n| > 32 \end{cases}$$

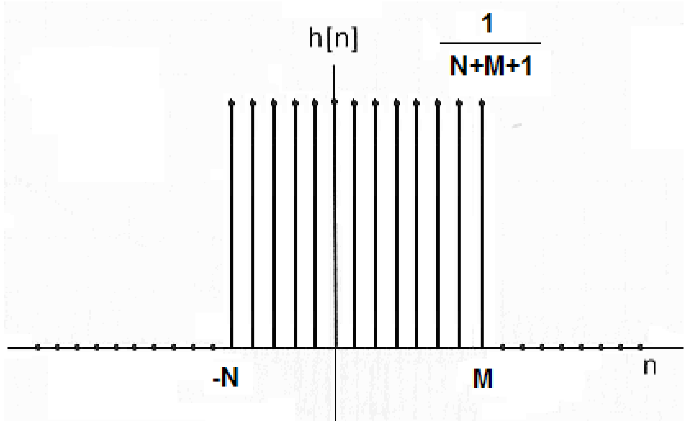


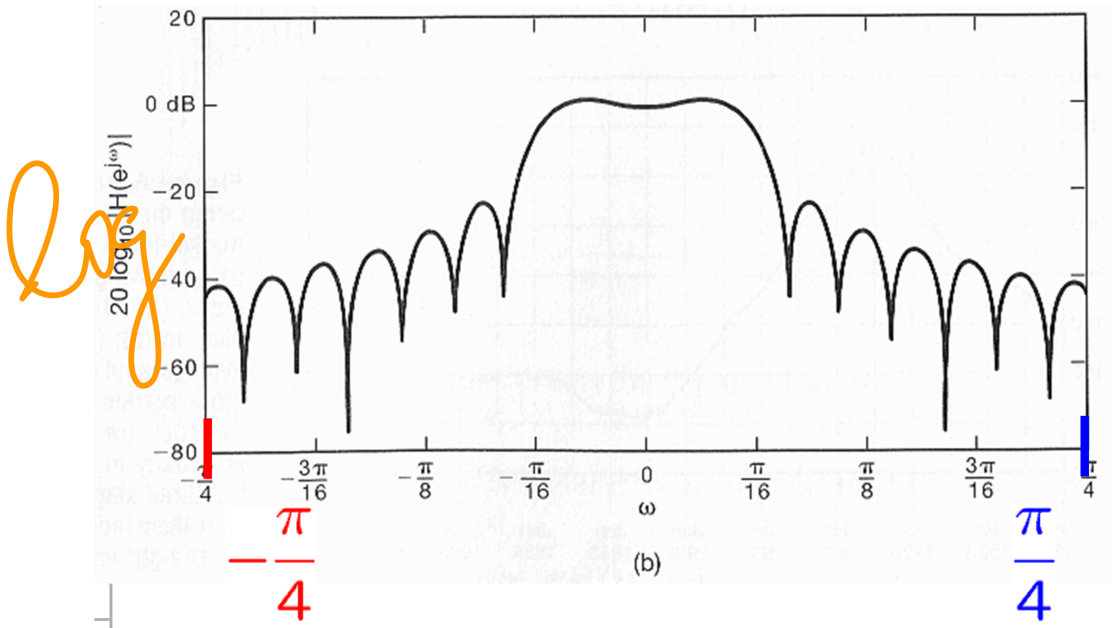
General Form of DT Non-recursive Filters:

ss6-82

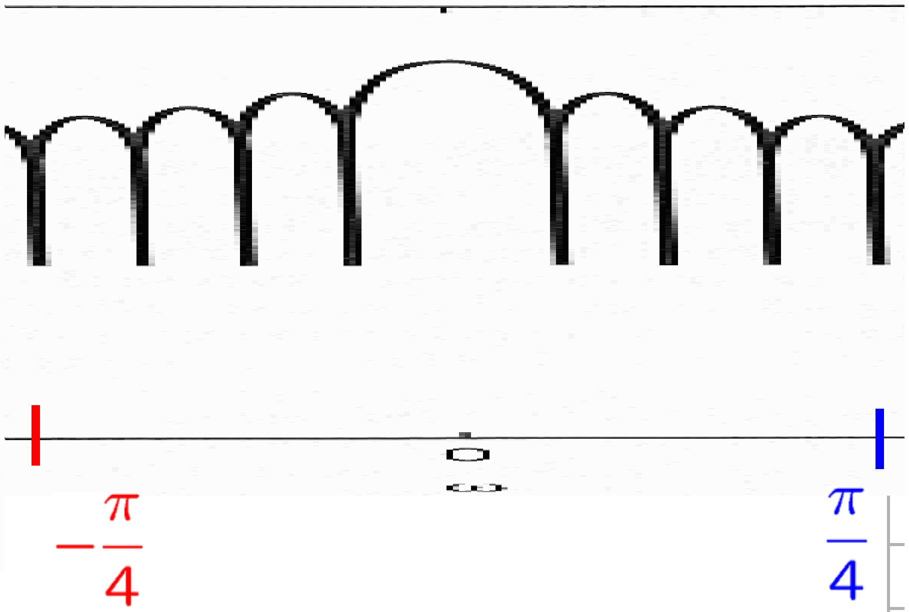


(a)

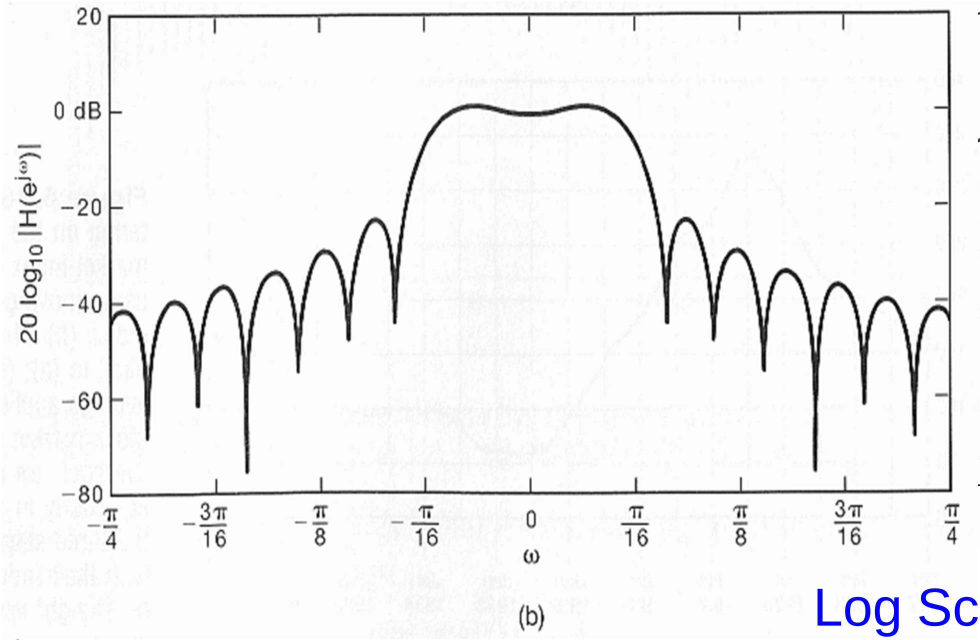


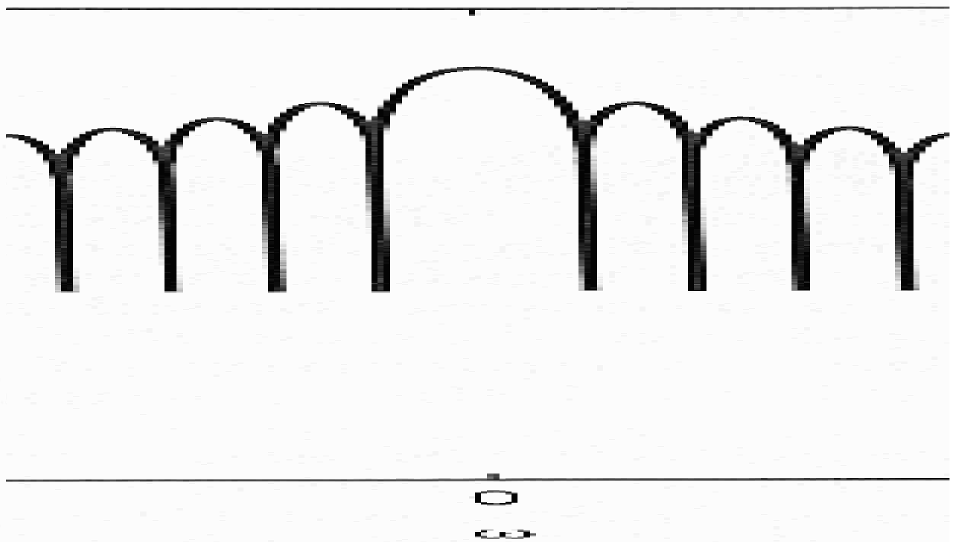


(b)



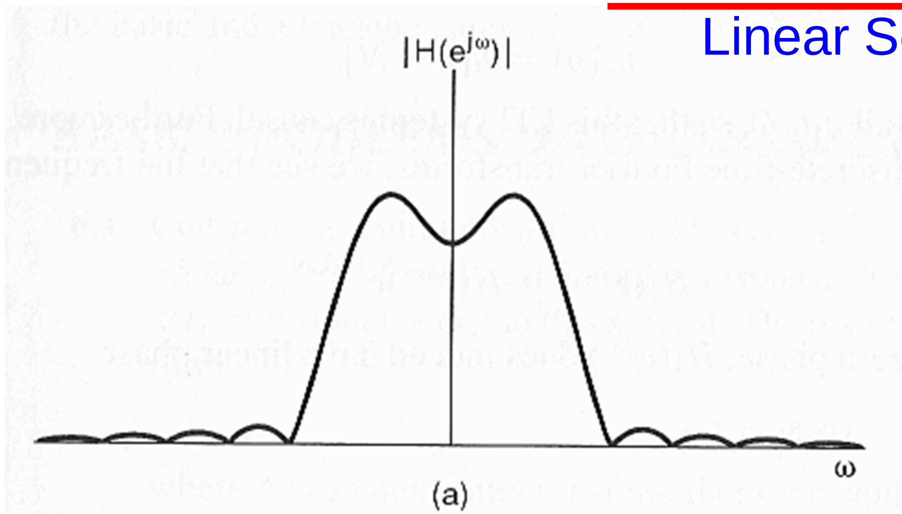
Comparison on a Linear Amplitude Scale:

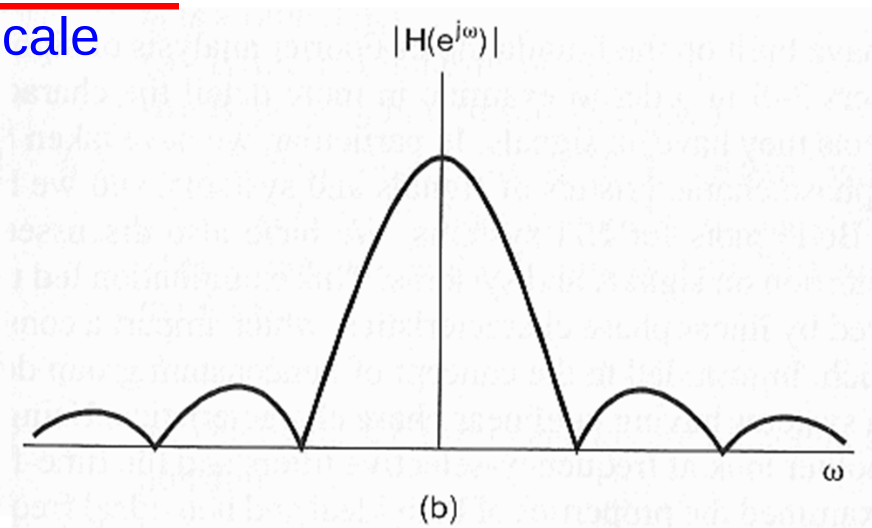




Log Scale

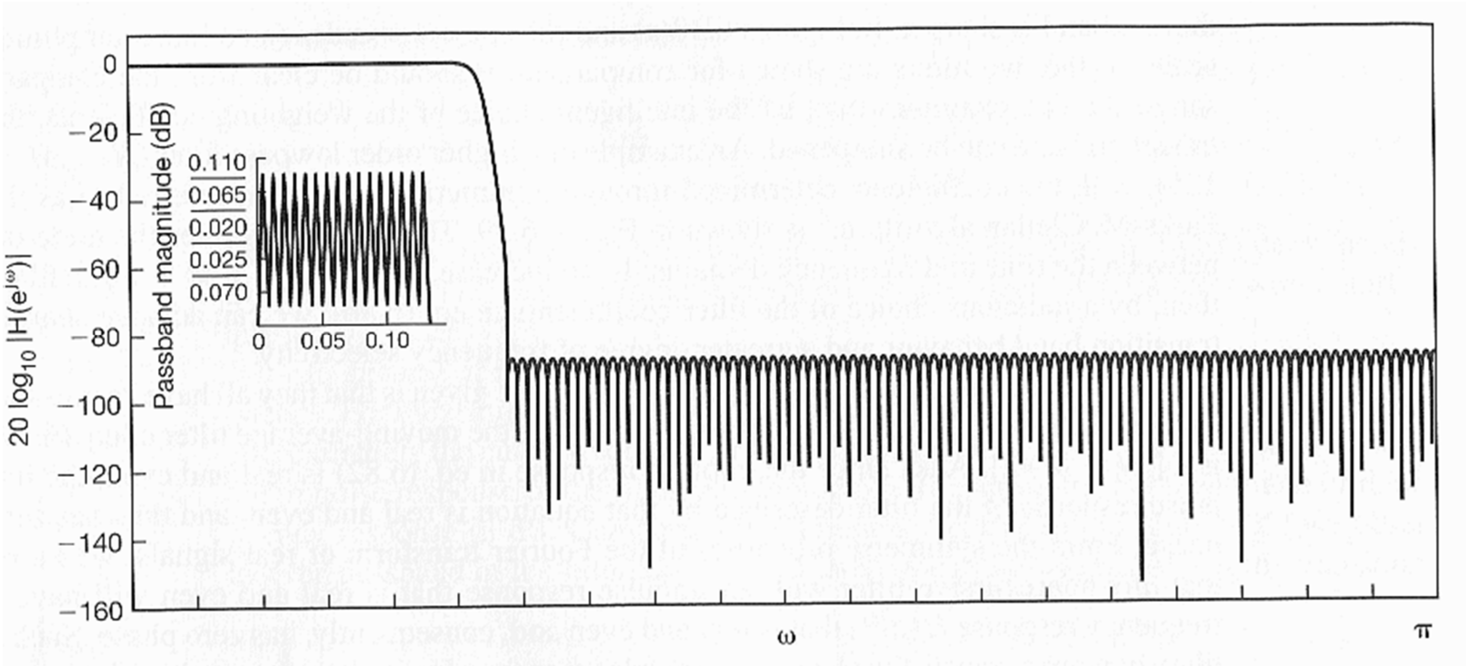
Linear Scale





- Lowpass Non-recursive Filter with 251 Coefficients:

$$y[n] = \sum_{k=-N}^{M} b_k \, x[n-k]$$



Coefficients determined by the Parks-McClellan algorithm

Summary: Chap 6 - 1

$$\frac{d}{dt}y(t) + ay(t) = ax(t)$$

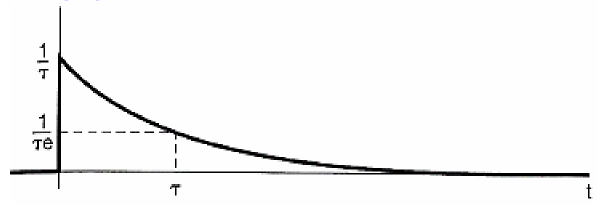
$$\Rightarrow \quad H(jw) = \frac{a}{jw + a}$$

$$\Rightarrow \quad H(s) = \frac{a}{s + a}$$

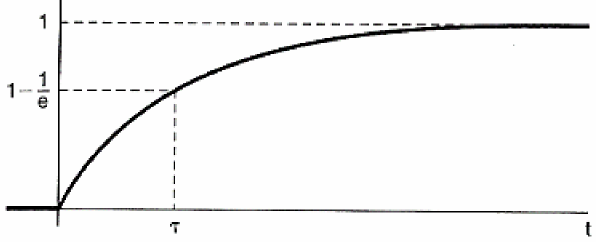
$$h(t) = a\, e^{-at}\, u(t)$$

$$s(t) = \left[1 - e^{-at}\right]\, u(t)$$

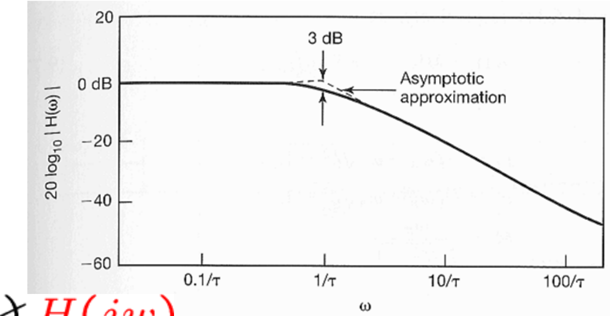
$h(t)$



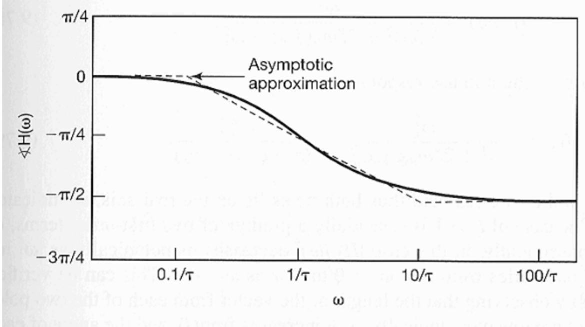
$s(t)$



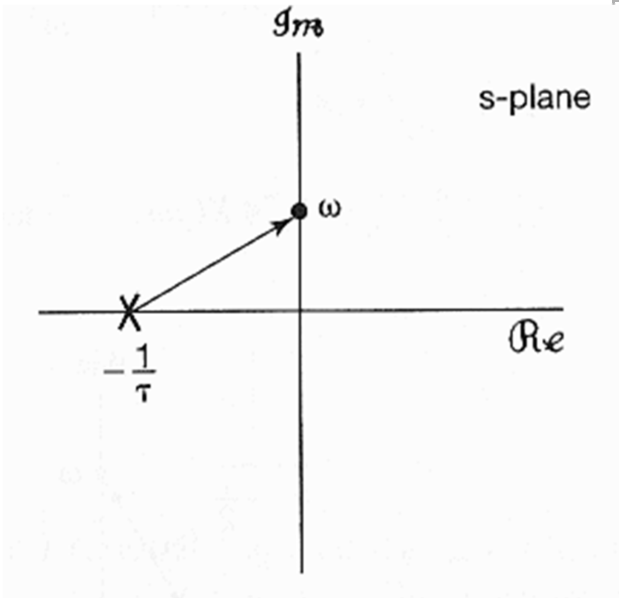
$20\log_{10}|H(jw)|$



$\measuredangle H(jw)$



$H(s)$



$$y[n] - a\,y[n-1] = x[n] \qquad |a| < 1$$

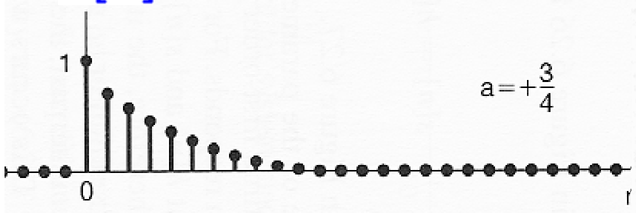
$$\Rightarrow H(e^{jw}) = \frac{1}{1 - ae^{-jw}}$$

$$\Rightarrow h[n] = a^n\,u[n]$$

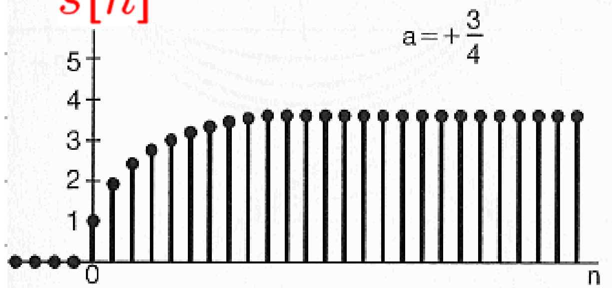
$$\Rightarrow s[n] = \frac{1 - a^{n+1}}{1 - a}\,u[n]$$

$$\Rightarrow H(z) = \frac{z}{z - a}, \quad |z| > |a|$$

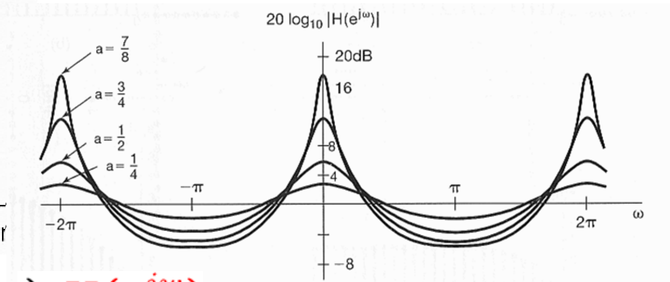
$h[n]$



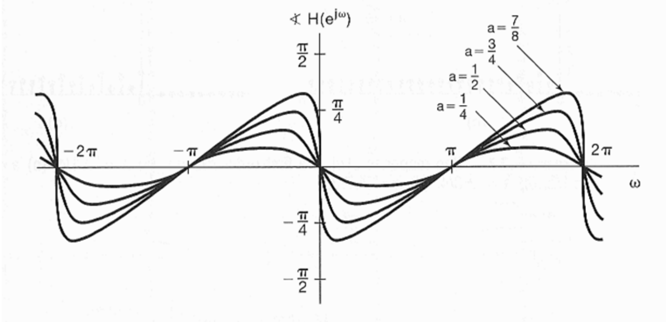
$s[n]$



$20\log_{10}|H(e^{jw})|$

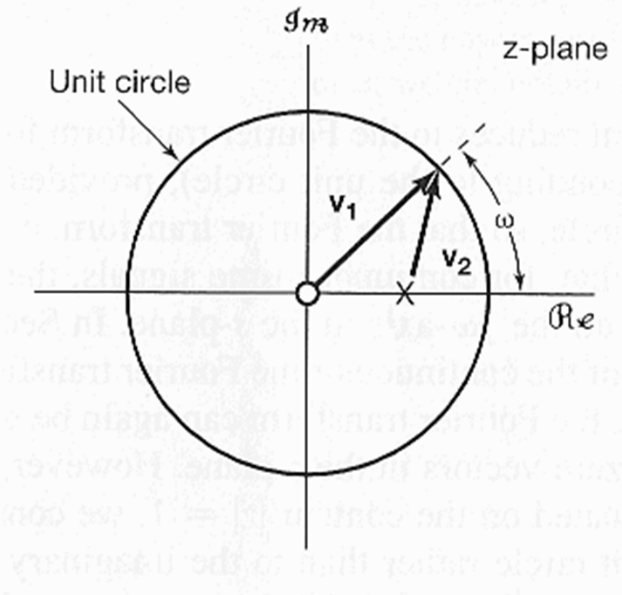


$\sphericalangle H(e^{jw})$



(a)

$H(z)$

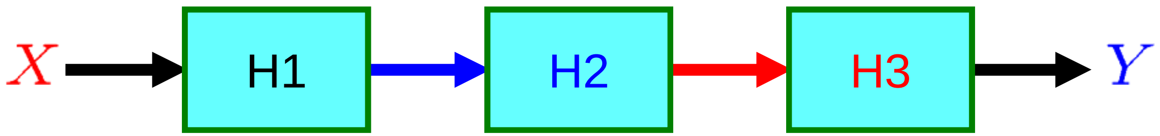


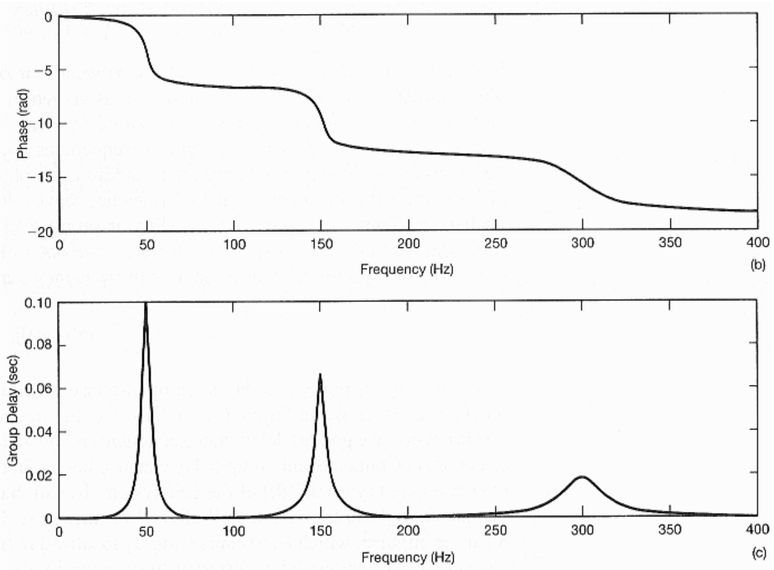
(a)

Summary: Chap 6 - 3

$$x(t) = \delta(t)$$

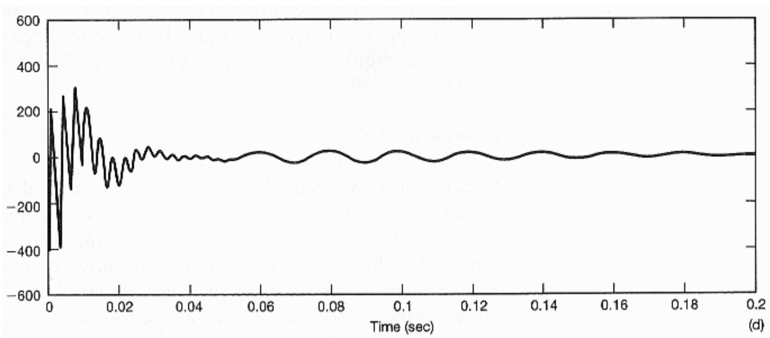
$$X(jw) = 1, \ \forall w$$

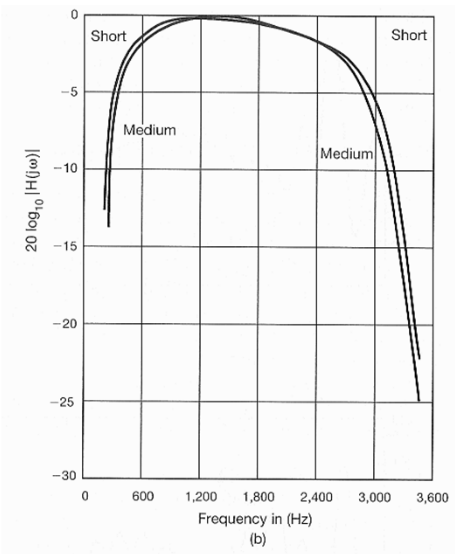


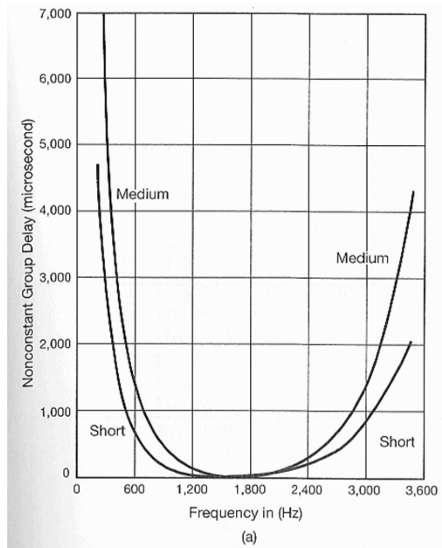


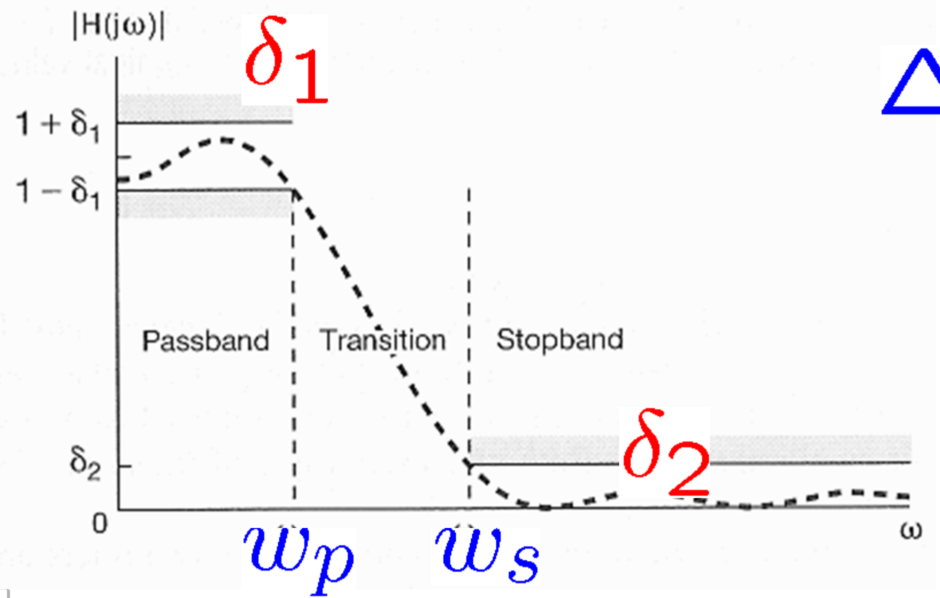
$$\tau(w) = -\frac{d}{dw}\left\{ \measuredangle H(jw) \right\}$$

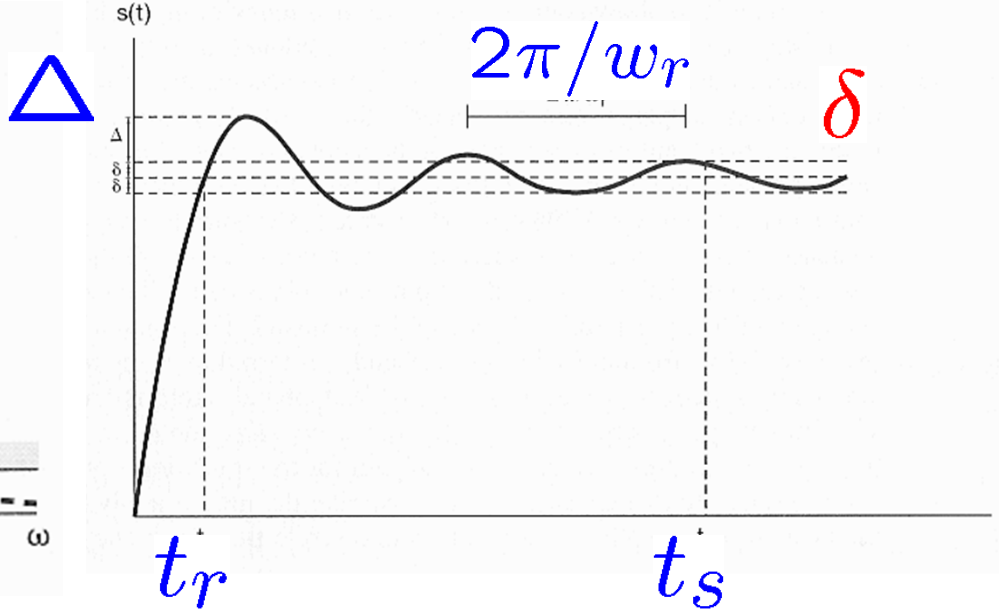
$$y(t)$$





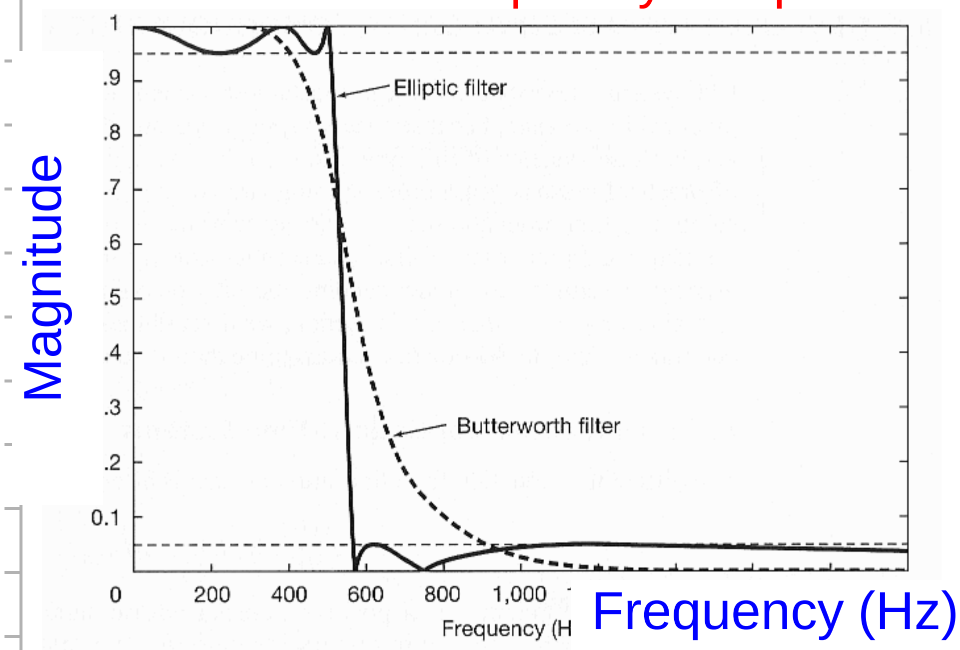


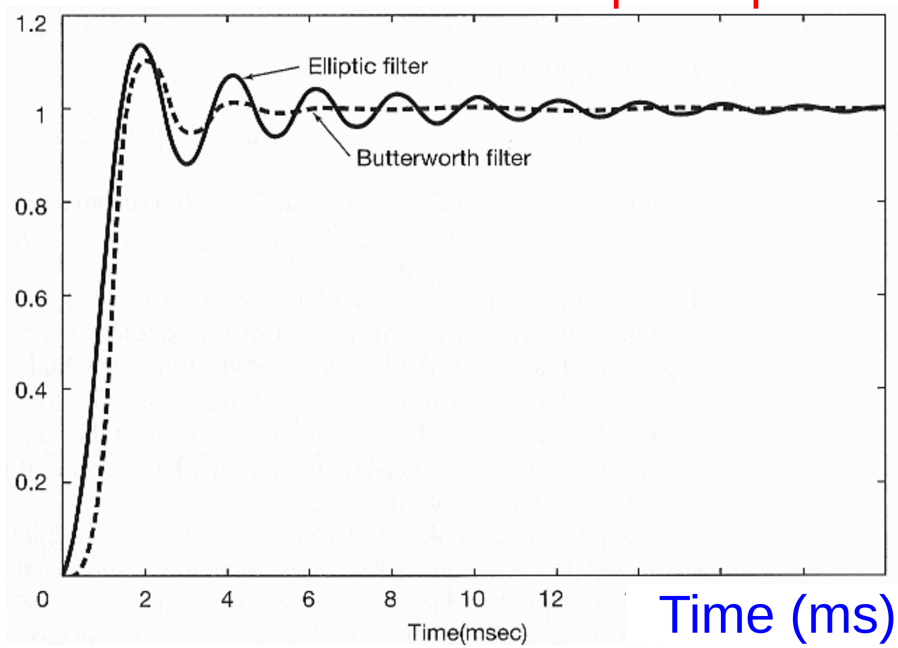




Frequency Response

Step Response





- The Magnitude-Phase Representation of the FT
- The Magnitude-Phase Representation of Frequency Response of LTI Systems
- Time-Domain Properties of Ideal Frequency-Selective Filters
- Time-Domain and Frequency-Domain Aspects of Non-ideal Filters
- 1st-Order & 2nd-Order CT Systems
- 1st-Order & 2nd-Order DT Systems
- Time- & Frequency-Domain Analysis of Systems