STAT 241 - Business Statistics
Case Study: Quality Control

Case Problem:
Quality Associates, Inc., a consulting firm, advises its clients about sampling and statistical procedures that can be used to control their manufacturing processes.

In one particular application, a client gave Quality Associates a vast sample of 10,000 observations taken during a time in which the client's process was operating satisfactorily. The standard deviation of this sample was 0.21; hence with so much data, the **population standard deviation** was assumed to be 0.21.

Quality Associates then suggested that random samples of size 30 be taken hourly to monitor the process on an ongoing basis. By analyzing the new samples, the client could quickly learn whether the process was operating satisfactorily. When the process was not operating satisfactorily, corrective action could be taken to eliminate the problem.

Risto Rushford
STAT 241-A01

We are testing to determine if the manufacturing processes are operating satisfactorily as per design specifications. The summarized data given is:

$\sigma = 0.21$ $\quad$ $n_{1,2,3,4}$=30

And our hypothesis statement for each sample tested is:

$H_0 : \mu = 12$

$H_1 : \mu \neq 12$ $\qquad$ Let $\alpha = .01$

1) Conduct a hypothesis test for each sample at the $\alpha = .01$ significance level. Determine if H_0 should be rejected. Determine what action, if any, should be taken. Conclusions below are made from referencing the data tables in Appendix A[i]			
8:00 AM Sample 1	9:00 AM Sample 2	10:00 AM Sample 3	11:00 AM Sample 4
Conclusion: H_0 should not be rejected, there is insufficient evidence to conclude $\mu \neq 12$	Conclusion: H_0 should not be rejected, there is insufficient evidence to conclude $\mu \neq 12$	Conclusion: H_0 should be rejected, there is sufficient evidence to conclude $\mu \neq 12$	Conclusion: H_0 should not be rejected, there is insufficient evidence to conclude $\mu \neq 12$
Decision: While three of the four samples taken had no statistically significant difference between the sample and population means, the 10AM sample was significantly different given our testing. Thus, I must conclude that the manufacturing processes must be reevaluated to determine the source of the variation.			
The p-value for this test is **0.2621**	The p-value for this test is **0.3572**	The p-value for this test is **0.0041**	The p-value for this test is **0.0325**

2) Check if the assumption that $\sigma = 0.21$ is valid: $H_0 : \sigma = 0.21 \quad verses \quad H_1 : \sigma \neq 0.21$ Let $\alpha = .01$ Expressed in terms of variance (for software calculations): σ^2 = 0.0441 Conclusions below are made from referencing the data tables in Appendix B[ii]			
8:00 AM Sample 1	9:00 AM Sample 2	10:00 AM Sample 3	11:00 AM Sample 4
Conclusion: $\sigma = 0.21$ Fail to reject, there is insufficient evidence to conclude $\sigma \neq 0.21$	Conclusion: $\sigma = 0.21$ Fail to reject, there is insufficient evidence to conclude $\sigma \neq 0.21$	Conclusion: $\sigma = 0.21$ Fail to reject, there is insufficient evidence to conclude $\sigma \neq 0.21$	Conclusion: $\sigma = 0.21$ Fail to reject, there is insufficient evidence to conclude $\sigma \neq 0.21$
Decision: As per the data of the analysis for σ, no further tests are needed to verify the appropriate value of the standard deviation.			
The p-value for this test is **0.3211**	The p-value for this test is **0.4181**	The p-value for this test is **0.4959**	The p-value for this test is **0.4858**

3) Compute limits for the sample mean, $\bar{x}$, around $\mu = 12$ such that if a new sample mean is within these limits, the process will be operating satisfactorily.[iii] Our z-scores will be ±2.576, so: LCL = $12 - 2.576\frac{\sigma}{\sqrt{n}}$ = 11.901 UCL = $12 + 2.576\frac{\sigma}{\sqrt{n}}$ = 12.099 Conclusions below are made from referencing the data table in Appendix C			
8:00 AM Sample 1	9:00 AM Sample 2	10:00 AM Sample 3	11:00 AM Sample 4
Conclusion: $\bar{x}_1$ = 11.9570 This value is within the critical limits.	Conclusion: $\bar{x}_2$ = 12.0353 This value is within the critical limits.	Conclusion: $\bar{x}_3$ = 11.8900 This value falls below the lower critical limit.	Conclusion: $\bar{x}_4$ = 12.0820 This value is within the critical limits.
Decision: As with the first test, we find that the 8, 9 and 11 AM samples fall within the expectations of the control procedure being used. However, the 10 AM sample falls outside of the lower critical limit, thus lending weight to my conclusion that the manufacturing processes should be reevaluated to determine the source of the error.			

4) Throughout this analysis we have used the α = .01 significance level. This is considered to be small, and to some extent tolerances are acceptable. If we were to adjust the significance level for this analysis to α = .05 or .10 then we would have to change our conclusions about the 11 AM sample as well because the sample mean would r outside of the critical limits of the population mean. In order to determine if a higher significance level would be ideal, we need to examine, based on the product being made, how much error is acceptable in a finished product, and how expensive would it be to either fix those errors or allow a certain proportion of them through?

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[i] **Appendix A:** Data for Part 1:

QA Data Descriptive Summary

	8:00 AM	9:00 AM	10:00 AM	11:00 AM
Mean	11.9570	12.0353	11.8900	12.0820
Median	11.9450	12.0250	11.9350	12.0800
Mode	11.9300	12.0000	11.9500	12.0200
Minimum	11.5200	11.5900	11.3600	11.6400
Maximum	12.3200	12.3900	12.2200	12.4700
Range	0.8	0.8	0.86	0.83
Variance	0.0486	0.0455	0.0430	0.0427
Standard Deviation	0.2205	0.2133	0.2073	0.2066
Coeff. of Variation	1.84%	1.77%	1.74%	1.71%
Skewness	-0.2108	-0.2428	-0.5362	-0.3907
Kurtosis	-0.6370	-0.4257	0.1133	-0.1355
Count	30	30	30	30
Standard Error	0.0403	0.0389	0.0379	0.0377

Appendix A (continued) **Z Tests For each sample:**

8:00 AM Sample:

Z Test of Hypothesis for the Mean

Data	
Null Hypothesis $\mu=$	**12**
Level of Significance	**0.01**
Population Standard Deviation	**0.21**
Sample Size	**30**
Sample Mean	**11.957**
Intermediate Calculations	
Standard Error of the Mean	0.0383
***Z* Test Statistic**	**-1.1215**
Two-Tail Test	
Lower Critical Value	**-2.5758**
Upper Critical Value	**2.5758**
***p*-Value**	**0.2621**
Do not reject the null hypothesis	

9:00 AM Sample:

Z Test of Hypothesis for the Mean

Data	
Null Hypothesis $\mu=$	**12**
Level of Significance	**0.01**
Population Standard Deviation	**0.21**
Sample Size	**30**
Sample Mean	**12.0353**
Intermediate Calculations	
Standard Error of the Mean	0.0383
***Z* Test Statistic**	**0.9207**
Two-Tail Test	
Lower Critical Value	**-2.5758**
Upper Critical Value	**2.5758**
***p*-Value**	**0.3572**
Do not reject the null hypothesis	

10:00 AM Sample:

Z Test of Hypothesis for the Mean

Data	
Null Hypothesis $\mu=$	**12**
Level of Significance	**0.01**
Population Standard Deviation	**0.21**
Sample Size	**30**
Sample Mean	**11.89**
Intermediate Calculations	
Standard Error of the Mean	0.0383
***Z* Test Statistic**	**-2.8690**
Two-Tail Test	
Lower Critical Value	**-2.5758**
Upper Critical Value	**2.5758**
***p*-Value**	**0.0041**
Reject the null hypothesis	

11:00 AM Sample:

Z Test of Hypothesis for the Mean

Data	
Null Hypothesis $\mu=$	**12**
Level of Significance	**0.01**
Population Standard Deviation	**0.21**
Sample Size	**30**
Sample Mean	**12.082**
Intermediate Calculations	
Standard Error of the Mean	0.0383
***Z* Test Statistic**	**2.1387**
Two-Tail Test	
Lower Critical Value	**-2.5758**
Upper Critical Value	**2.5758**
***p*-Value**	**0.0325**
Do not reject the null hypothesis	

ii **Appendix B:** Data for Part 2

8:00 AM Sample:

Chi-Square Test of Variance

Data		
Null Hypothesis	**σ^2=**	**0.0441**
Level of Significance		**0.01**
Sample Size		**30**
Sample Standard Deviation		**0.2205**
Intermediate Calculations		
Degrees of Freedom		29
Half Area		0.005
Chi-Square Statistic		**31.9725**
Two-Tail Test		
Lower Critical Value		13.1211
Upper Critical Value		52.3356
***p*-Value**		**0.3211**
Do not reject the null hypothesis		

9:00 AM Sample:

Chi-Square Test of Variance

Data		
Null Hypothesis	**σ^2=**	**0.0441**
Level of Significance		**0.01**
Sample Size		**30**
Sample Standard Deviation		**0.2133**
Intermediate Calculations		
Degrees of Freedom		29
Half Area		0.005
Chi-Square Statistic		**29.9186**
Two-Tail Test		
Lower Critical Value		13.1211
Upper Critical Value		52.3356
***p*-Value**		**0.4181**
Do not reject the null hypothesis		

10:00 AM Sample:

Chi-Square Test of Variance

Data		
Null Hypothesis	**σ^2=**	**0.0441**
Level of Significance		**0.01**
Sample Size		**30**
Sample Standard Deviation		**0.2073**
Intermediate Calculations		
Degrees of Freedom		29
Half Area		0.005
Chi-Square Statistic		**28.2591**
Two-Tail Test		
Lower Critical Value		13.1211
Upper Critical Value		52.3356
***p*-Value**		**0.4959**
Do not reject the null hypothesis		

11:00 AM Sample:

Chi-Square Test of Variance

Data		
Null Hypothesis	**σ^2=**	**0.0441**
Level of Significance		**0.01**
Sample Size		**30**
Sample Standard Deviation		**0.2066**
Intermediate Calculations		
Degrees of Freedom		29
Half Area		0.005
Chi-Square Statistic		**28.0686**
Two-Tail Test		
Lower Critical Value		13.1211
Upper Critical Value		52.3356
***p*-Value**		**0.4858**
Do not reject the null hypothesis		

[iii] **Appendix C:** Data for Part 3

Confidence Interval Estimate for the Mean

Data	
Population Standard Deviation	**0.21**
Sample Mean	**12**
Sample Size	**30**
Confidence Level	**99%**

Intermediate Calculations	
Standard Error of the Mean	0.0383
Z Value	-2.5758
Interval Half Width	0.0988

Confidence Interval	
Interval Lower Limit	**11.90**
Interval Upper Limit	**12.10**