Exercise 2

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Contents

1 Rocket Acceleration, Specific Impulse

1.1 Strategy and Expectations

For this exercise I went further into the rocket concept and used several parameters taken from PSAS rocket designs to build the thrust curve of the LV4 (Launch Vehicle 4) rocket that is currently being planned. I simplified the model by placing specific impulse (I_{sp}) directly as a parameter instead of as a calculated equation. Although I could model it as a stock of its own. I may look at that for the third exercise.
Unlike with my previous model, this begins with the rocket at 0 meters and shows its position for the duration of 600 seconds. This allows one to play with the dash board and view the different thrust curves of different mass profiles (airframe and propellant mass can be changed using sliders), the specific impulse, and the propellant burn time.
This model demonstrated the complex relationship between rocket mass, fuel properties and propellant burn time. When one adjusts the sliders, they will notice that the effects are exponential as the rocket acceleration quickly changes the flight profile (see section **Parameters** below).

1.2 Model Development

To build this model I needed to make some simplifying assumptions:

- Time is measured in seconds, with time-steps of 1 second for a total of 600 seconds.
- Specific impulse can be used as its own parameter (instead of derived from fuel information)
- Propellant mass and burn time can be adjusted independently of each other

1.3 Parameters

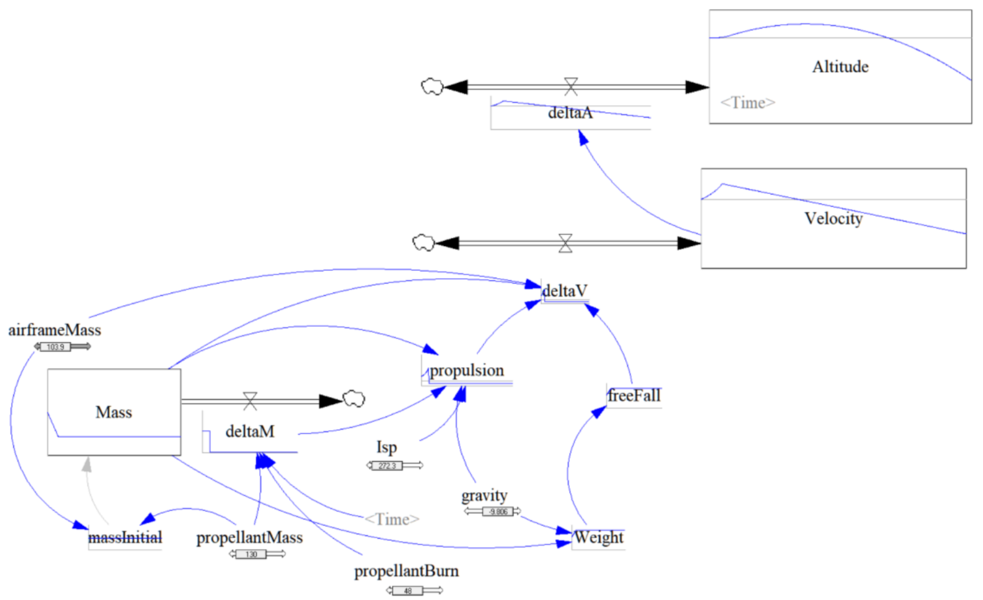

Figure 1: the parameters used in the rocket acceleration curve model

Again I used Open Rocket as a means of validating at least what my graph results should look like. The equations are fairly simple for this model after the simplifying assumptions were made (which are significant compared to real rocket equations).

1.3.1 Propulsion

$$a_P = -I_{sp} \cdot g \cdot m_\Delta + g \tag{1}$$
$$I_{sp} = 272 \tag{2}$$
$$g = -9.801 \tag{3}$$

With A_P = acceleration from propulsion, I_{sp} = specific impulse, g = gravity m_Δ = mass flow (explained later).

Propulsion was the most important parameter of the model as it set the stage for the other parameters to have their effect on vehicle acceleration. Other important elements to the propulsion component of the model are mass and propellant burn time.

1.3.2 Mass

$$M = m_{af} + m_p - m_\Delta \tag{4}$$
(5)

With M - total mass (a stock), m_{af}= airframe mass, m_p is propellant mass, and m_Δ = mass flow.
Mass is the most limiting part of the Rocket Equation, as a certain mass of fuel is needed to propel the mass of the rocket, and then additional fuel to propel the initial fuel in the tanks. This is why rockets that send payloads into space are so large. Mass flow rate is the rate at which the propellant (usually a fuel and an oxidizer) is consumed and blasted out of the rocket nozzle to create thrust. This, combined with I_{sp} are what makes a rocket fly.
Specific impulse is the effective exhaust velocity of the propellant, and is related to the mass flow rate and the chemical properties of the propellant.

1.3.3 Free Fall

$$w = M \cdot g \tag{6}$$
(7)

With w = weight, M = mass, g = gravity.

This will be the most familiar component of the model to the casual reader as this is the constant force that we all feel. However I made the free fall parameter to be able to factor in a parachute. I was not able to make a parachute work in this model (for similar reasons as last time), but I at least know why: velocity grows quickly in this model with constant acceleration, and the equations used to model how parachutes affect freefall velocity use velocity squared. Every time the model would break when the air friction reached a value of -1×10^{17}, and even with RK4 auto, it would break, usually at 40-55 seconds in depending on the parameters. When I saw that this problem was still occurring after building the model from scratch, I moved on to focus on other model aspects for the time being.

1.4 Model Performance and adjusting Parameters

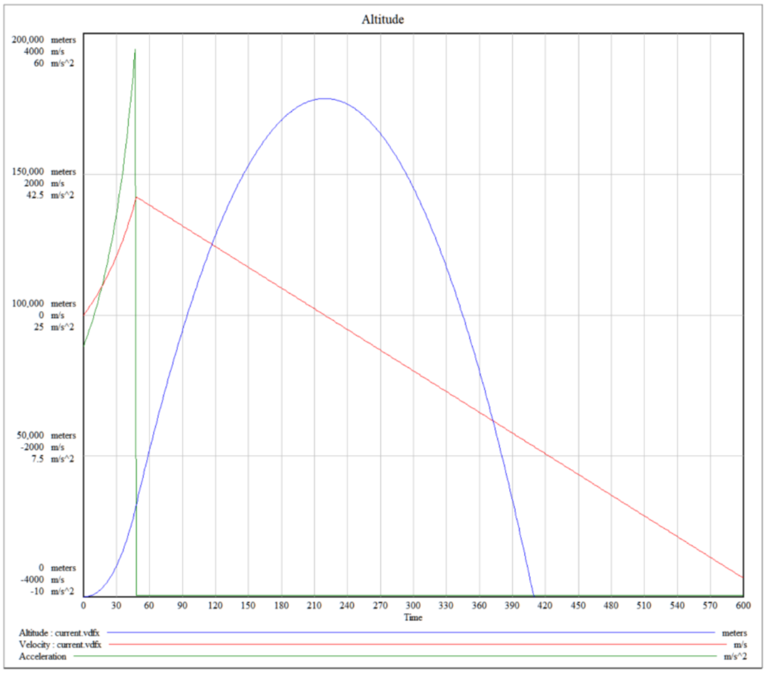

Figure 2: An initial run of the model plotting altitude, velocity and acceleration.

Notice in the graph that the rocket begins with an esponentially increasing thrust curve. Then right at the inflection point you can see that acceleration (due to thrust) drops to zero, and velocity decreases linearly. This is the same as what I would see in the Open Rocket simulations, which is a validating point at least as to the behavior of the model. This thrust curve is obtained using initial values for the following parameters:

$$m_{af} = 103.9 \tag{8}$$
$$m_p = 130 \tag{9}$$
$$t_{burn} = 48 \tag{10}$$
$$I_{sp} = 272.3 \tag{11}$$

Most of these variables are explained in prior section, except t_{burn} = propellant burn time.

1.4.1 Propellant Burn Time

This parameter has a balancing effect on the total altitude that a rocket can achieve. With all other things held equal, increasing the burn time can decrease the apogee (highest altitude) or eliminate the rocket from ever leaving the ground. Or by decreasing the burn time, the mass flow rate is so much quicker that the rocket will shoot off into the sky.

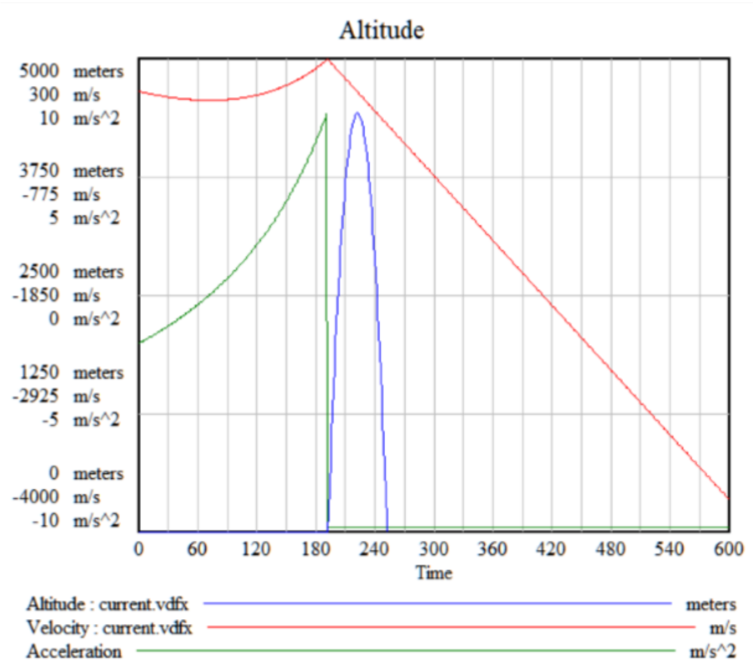

Figure 3: Flight profile with $t_{burn} = 190$ seconds

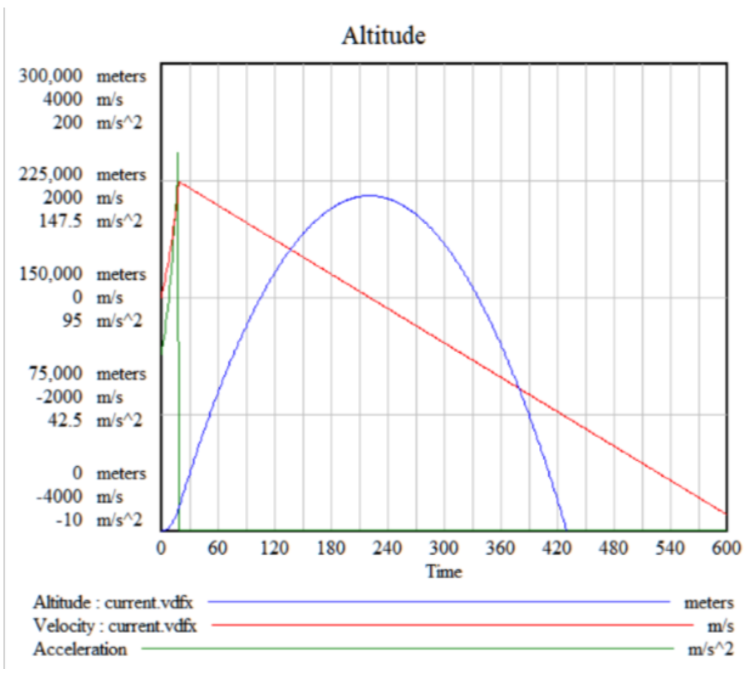

Figure 4: Flight profile with $t_{burn} = 20$ seconds

Changing other constants have similar effects, for example adjusting the I_{sp} upwards can increase the initial acceleration of the rocket, adding even more altitude.

1.5 Conclusions

This was an interesting way to build off of the model that I used in Exercise 1 for this class, and it required me to look back at a rocketry text book which has inspired me for what I will work on for Exercise 3: a rocket test stand. I found a section in the book that describes how various parameters of a rocket engine relate to each other in detail, presumably which would allow me to build a higher fidelity model.

1.6 What I learned

I learned more about causal tracing in this exercise, and while I didn't fix the problem that was plaguing me in the first one, I have a good sense of why it was happening and how to avoid it going forward. This modeling exercise also helped clarify and build my own understanding of mass flow rate and how it is so crucial to the Rocket Equation and propulsion. I'd like to continue to build on this understanding and insight that using Vensim has given me (as opposed to simply solving equations).

1.7 Potential Improvements

I had suggested in the previous exercise that I could find ways to add in more control elements. That is still true here. I would also be interested to tie I_{sp} directly to the mass flow rate.