

Group 21 Homework 1 Report

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October 5, 2025

Note: The code that produced the results for this report can be found in our code.

1 Mean-Variance Optimization

1.1 Summary Statistics

Table 1 presents the annualized mean excess returns, volatilities, and Sharpe ratios for all ten assets. Statistics are annualized by multiplying monthly means by 12 and monthly standard deviations by $\sqrt{12}$.

Table 1: Annualized Summary Statistics of Excess Returns

Asset	Mean	Volatility	Sharpe Ratio
BWX	-0.0077	0.0828	-0.0932
DBC	-0.0053	0.1666	-0.0318
EEM	0.0293	0.1762	0.1665
EFA	0.0618	0.1509	0.4094
HYG	0.0414	0.0759	0.5449
IEF	0.0164	0.0634	0.2586
IYR	0.0749	0.1687	0.4441
PSP	0.0926	0.2134	0.4338
SPY	0.1281	0.1428	0.8971
TIP	0.0205	0.0511	0.4011

Best Sharpe Ratio: SPY has the highest Sharpe ratio of 0.8971, delivering 12.81% annualized excess return with 14.28% volatility.

Worst Sharpe Ratio: BWX has the worst Sharpe ratio of -0.0932 , with negative excess returns of -0.77% and 8.28% volatility.

1.2 Descriptive Analysis

Table 2 shows the correlation matrix of asset returns.

Highest correlation: (EFA, PSP) ≈ 0.90 , reflecting strong co-movement between developed international equities and private equity proxies.

Lowest correlation: (DBC, IEF) ≈ -0.30 , indicating commodities and intermediate Treasuries often move in opposite directions, providing diversification benefits.

TIPS Performance: TIPS delivered an annualized excess return of 2.05% with volatility of 5.11% and Sharpe ratio of 0.4011. TIPS outperformed domestic nominal Treasuries (IEF: 1.64% return, 0.26 Sharpe ratio) and substantially outperformed foreign bonds (BWX: -0.77% return, -0.09 Sharpe ratio), demonstrating superior risk-adjusted returns relative to both.

1.3 The MV Frontier

Table 3 presents the tangency portfolio weights and performance metrics.

Table 2: Correlation Matrix of Asset Returns

	BWX	DBC	EEM	EFA	HYG	IEF	IYR	PSP	SPY	TIP
BWX	1.00	0.19	0.62	0.60	0.60	0.58	0.55	0.53	0.44	0.68
DBC	0.19	1.00	0.51	0.50	0.46	-0.30	0.28	0.45	0.43	0.11
EEM	0.62	0.51	1.00	0.82	0.69	0.03	0.58	0.75	0.69	0.38
EFA	0.60	0.50	0.82	1.00	0.79	0.04	0.70	0.90	0.85	0.39
HYG	0.60	0.46	0.69	0.79	1.00	0.19	0.74	0.81	0.79	0.54
IEF	0.58	-0.30	0.03	0.04	0.19	1.00	0.32	0.02	0.00	0.75
IYR	0.55	0.28	0.58	0.70	0.74	0.32	1.00	0.75	0.75	0.60
PSP	0.53	0.45	0.75	0.90	0.81	0.02	0.75	1.00	0.89	0.41
SPY	0.44	0.43	0.69	0.85	0.79	0.00	0.75	0.89	1.00	0.38
TIP	0.68	0.11	0.38	0.39	0.54	0.75	0.60	0.41	0.38	1.00

Table 3: Tangency Portfolio Weights and Performance

Asset	Weight	Asset	Weight
BWX	-0.8506	DBC	-0.0716
EEM	0.0264	EFA	0.0687
HYG	0.2906	IEF	0.8812
IYR	-0.2466	PSP	-0.3330
SPY	1.0596	TIP	0.1753
<i>Portfolio Performance</i>			
Mean Return		0.1285	
Volatility		0.0875	
Sharpe Ratio		1.4692	

The tangency portfolio takes large long positions in SPY (105.96%) and IEF (88.12%), with substantial short positions in BWX (-85.06%). The portfolio is leveraged, with gross exposure $\sum_i |w_i| = 368.37\%$.

Alignment with Sharpe Ratios: The ranking of weights does not perfectly align with individual Sharpe ratios. While SPY (highest Sharpe) receives the largest weight, IEF (Sharpe 0.26) receives the second-largest weight due to its diversification benefits (near-zero correlation with equities). PSP is shorted despite a respectable Sharpe (0.43) because its returns are highly redundant with SPY (correlation 0.89). This demonstrates that optimal portfolios depend on both expected returns and the correlation structure (i.e., marginal contributions to risk).

1.4 TIPS

Tables 4 and 5 present the tangency portfolio under two counterfactual scenarios.

Table 4: Tangency Portfolio without TIPS

Asset	Weight	Asset	Weight
BWX	-0.8793	DBC	-0.0637
EEM	0.0325	EFA	0.0602
HYG	0.3163	IEF	1.0200
IYR	-0.2474	PSP	-0.3394
SPY	1.1010		
<i>Portfolio Performance</i>			
Mean Return		0.1327	
Volatility		0.0905	
Sharpe Ratio		1.4675	

Table 5: Tangency Portfolio with TIPS Return Adjusted (+0.12%)

Asset	Weight	Asset	Weight
BWX	-0.6697	DBC	-0.1214
EEM	-0.0118	EFA	0.1226
HYG	0.1284	IEF	0.0047
IYR	-0.2411	PSP	-0.2926
SPY	0.7985	TIP	1.2825
<i>Portfolio Performance</i>			
Mean Return			0.1204
Volatility			0.0746
Sharpe Ratio			1.6124

Impact of Excluding TIPS: Excluding TIPS results in minimal degradation—the Sharpe ratio declines from 1.4692 to 1.4675 (a 0.12% decrease). The portfolio reallocates TIPS weight primarily to IEF and SPY, suggesting TIPS are largely redundant given IEF’s presence (correlation 0.75).

Impact of Adjusting TIPS Returns: With a modest 0.12% increase in expected return, TIPS becomes dominant (128.25% weight) while IEF is nearly eliminated (0.47%). The Sharpe ratio improves to 1.6124 (a 9.75% enhancement), and volatility drops to 7.46%.

Conclusion: Based on historical data, TIPS do not significantly expand the investment opportunity set. Excluding them barely affects portfolio efficiency, and their high correlation with IEF indicates substantial redundancy. However, TIPS are highly sensitive to expected return assumptions. Harvard should consider TIPS as a tactical allocation if forward-looking inflation expectations exceed historical levels or if the university has inflation-sensitive liabilities requiring hedging. For pure mean–variance optimization based on historical performance, TIPS do not provide substantial standalone value beyond existing nominal Treasury holdings.

Interpretation and robustness: The small impact from excluding TIPS is consistent with their high correlation with IEF and comparable risk per unit of return in the historical sample. The large reallocation to TIPS under a +12 bps expected-return shock reflects how mean–variance solutions amplify small changes when assets are highly substitutable (TIP–IEF correlation ≈ 0.75) yet offer slightly better Sharpe or lower covariance with equities. In practice, to avoid extreme leverage/shorts and estimation error sensitivity, Harvard should evaluate TIPS under constraints (e.g., weight bounds, turnover controls) and with robust inputs (e.g., shrinkage means/covariances or Bayesian priors tied to real yield curves and inflation expectations). Under such robust, constrained optimization, TIPS are still unlikely to be a separate strategic sleeve on historical data alone, but they remain valuable as a tactical and liability-hedging instrument when real yields or inflation risk premia are favorable.

2 Allocations

2.1 Comparison of EW, RP and MV Portfolios

In this section, we will compare the performance of the equally-weighted (EW), risk-parity (RP) and mean-variance (MV) portfolio under the targeted mean excess return of 1%, i.e. $\bar{\mu}^{\text{port}} = 0.01$. First, we compare the weights of each portfolio after rescaling to the target mean:

Ticker	EW	RP	MV
BWX	0.2655	0.5406	-0.7942
DBC	0.2655	0.1336	-0.0669
EEM	0.2655	0.1194	0.0247
EFA	0.2655	0.1627	0.0641
HYG	0.2655	0.6427	0.2713
IEF	0.2655	0.9206	0.8228
IYR	0.2655	0.1302	-0.2302
PSP	0.2655	0.0814	-0.3109
SPY	0.2655	0.1816	0.9894
TIP	0.2655	1.4181	0.1637

Table 6: Comparison of Portfolio Weights

Notice that only the mean-variance position has short positions, whilst the risk-parity portfolio has a lot of weight in TIP and IEF, which makes sense, as they generally have low volatility. Next, we will look at the performance of each portfolio: Here we can see that the mean-variance optimized

	EW	RP	MV
Annualized Return	0.1200	0.1200	0.1200
Annualized Volatility	0.2692	0.2639	0.0817
Sharpe Ratio	0.4457	0.4547	1.4692

Table 7: Comparison of Performance

portfolio provides the lowest volatility by a large margin and therefore the best sharpe ratio. Whilst the risk-parity portfolio has slightly lower volatility than the equally-weighted portfolio and therefore also only offers slight improvement in terms of sharpe ratio over the EW portfolio.