Statistics and Data science

Theory: Difference in Difference (DiD)

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Introduction

- RDD measures a shock at an exogenous threshold.
- What if we are not interested only in a discontinuity but rather in a difference before/after (pre/post) ?
  - Do air quality policies work ?
  - $\Rightarrow$ Difference-in-Difference

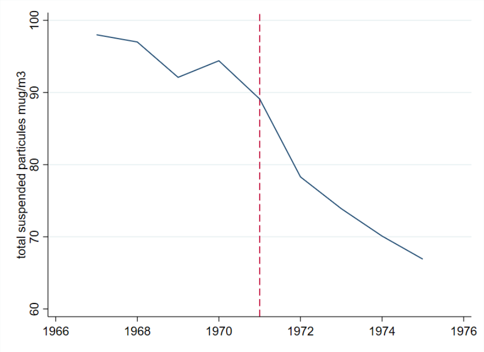
Overview

1. [week 1] The gold-standard : RCT, A/B testing
2. [week 2] Regression discontinuity design (RDD)
3. **[today] Difference-in-Difference (DiD)**
4. [week 4] Synthetic controls

Illustration : Clean Air Act

- Do air quality policies work ?
- The Clean Air Act, for example, was a policy dating back to the 60s aiming to put a cap on the total suspended particulates at the county level.
  - Pollution above threshold ⇒ county has to reduce pollution below the ceilling
  - Pollution below threshold ⇒ no action needed
- Is it ok to compare before and after for those who were above the ceiling ?

Illustration : Clean Air Act



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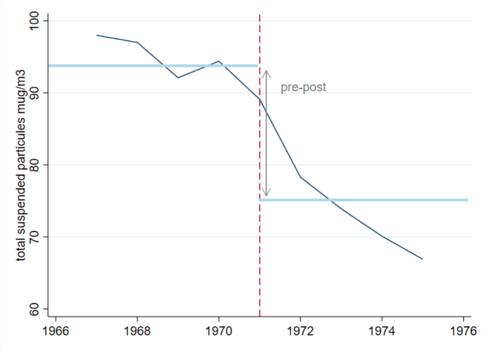
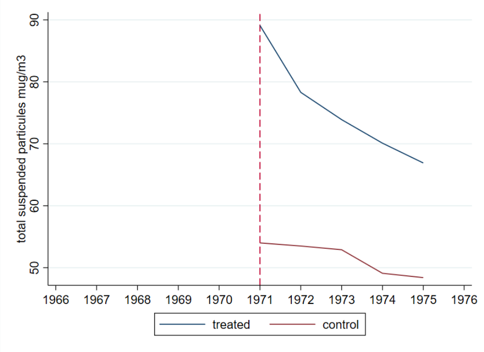


- What if there is an overall downward trend due to technology improvements for example ?
- What if on the contrary, there is an overall increase in pollution ?

Illustration : Clean Air Act

- Can we compare the level after the policy with the level for those without the policy ?



- What if there are other differences between the two (rural vs urban) ?

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  - Pros : Compares apples with apples (same place)
  - Cons : Time trend confounded

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- **Between** (control/treated) post comparison :
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  - Cons : Compares apples and oranges (other differences between the places confounded)

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- **Diff-in-Diff :** Look at the pre-post difference between the treated and control
  - Controls for the overall evolution through time
  - Compare Apples with Apples and Oranges with Oranges (within comparison)

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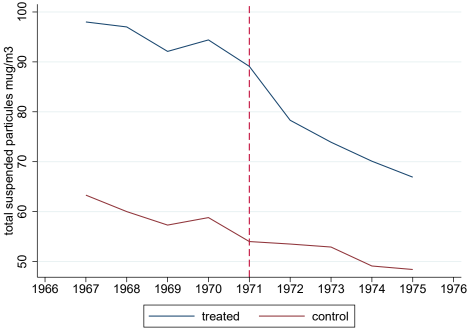
- Diff-in-Diff



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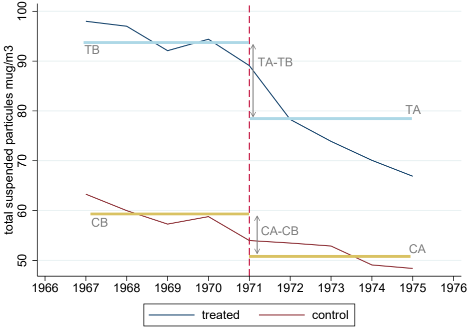

- **Diff-in-Diff= (Treated After- Treated Before) - (Control After - Control Before)**
- The first difference ($After - Before$) implies a within comparison (controlling for individual fixed characteristics).
- The second difference $-(ControlAfter - ControlBefore)$ allows to control for the evolution without treatment (counter-factual).

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- We are going to use a regression to compute this effect for two reasons :
  1. Easy to compute the confidence intervals
  2. We can include additional controls (to prevent an omitted variable bias)

Model Assumptions

- Diff-in-Diff exploits panel data

⇒ **Fundamental assumption** we can decompose into an additive model :

  - $E[Y_{0it}|i,t] = FE_i + FE_t$
  - FE_i is unit i fixed effect
    - Controls for individual characteristics fixed over time
    - **Assumption :** the difference is fixed over time $(FE_T - FE_C)$
  - FE_t is period t fixed effect
    - Controls for global (same for all the countries) time-varying shocks
    - **Assumption :** constant across units (e.g : technological improvement, COVID etc.)

Model Assumptions

- So the first difference controls for individual fixed effects.
  - $E(Y_{it}|i = Control; t = Post) - E(Y_{it}|i = Control; t = Pre)$
    $= (FE_C + FE_{Post}) - (FE_C + FE_{Pre})$
    $= FE_{Post} - FE_{Pre}$
  - $E(Y_{it}|i = Treated; t = Post) - E(Y_{it}|i = Treated; t = Pre)$
    $= (FE_T + FE_{Post} + \beta) - (FE_T + FE_{Pre})$
    $= \beta + (FE_{Post} - FE_{Pre})$
  - **with β the treatment effect**

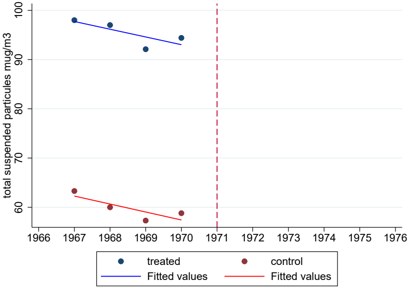
Model Assumptions

- The double difference allows to isolate the causal effect β :

$$[E(Y_{it}|i = Treated; t = Post) - E(Y_{it}|i = Treated; t = Pre)]$$
$$-$$
$$E(Y_{it}|i = Control; t = Post) - E(Y_{it}|i = Control; t = Pre)$$
$$= [\beta + (FE_{Post} - FE_{Pre})] - [FE_{Post} - FE_{Pre}]$$
$$= \beta$$

Identifying assumption

- **Parallel trends :** Without the treatment, the difference in outcome for the control and treatment groups would be identical in the "post" period.
  - Impossible to test : This is the fundamental problem of causal inference, we do not observe the outcome for the treated group for the post period without treatment.
- What can we do ?
  - We can test for parallel trends before treatment.



Identifying assumption

Clean Air Act

- As control counties and treated counties behave similarly before, they might behave similarly after (without treatment).

$\Rightarrow$ No other shocks

Regression Equation

Diff-in-Diff :
Diff-in-Diff exploits panel data

$$Y_{it} = \alpha + \gamma Treated_i + \lambda Post_t + \beta(Treated_i \cdot Post_t) + \epsilon_{it}$$

- $Treated_i$: dummy =1 if unit i treated (0 for the control group)
- $Post_t$: dummy =1 for the post period (0 for pre)

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Exemple (Clean Air Act) :

- $Treated_i$: county above pollution threshold before 1971
- $Post_t$: from 1971 onwards

Regression Equation

Coefficients interpretation

$$Y_{it} = \alpha + \gamma Treated_i + \lambda Post_t + \beta(Treated_i \cdot Post_t) + \epsilon_{it}$$

- $\alpha = E(Y_{it}|i = Control; t = Pre)$
  - Expected value of the outcome for the control group before the intervention.
    - *e.g. Pollution for the untreated counties before 1971.*
- $\gamma = E(Y_{it}|i = Treated; t = Pre) - E(Y_{it}|i = Control; t = Pre)$
  - Expected value of the difference between the outcome for the treated and control groups pre-treatment.
    - *e.g. Average pollution difference between treated and untreated counties before 1971.*
- $\lambda = E(Y_{it}|i = Control; t = Post) - E(Y_{it}|i = Control; t = Pre)$
  - Expected value of the difference Pre and Post for the control group.
    - *e.g Average pollution difference before 1971 vs after 1971 for the untreated counties.*

Regression Equation

Coefficients interpretation

$$Y_{it} = \alpha + \gamma Treated_i + \lambda Post_t + \beta(Treated_i \cdot Post_t) + \epsilon_{it}$$

- $\beta = E(Y_{it}|i = Treated; t = Post) - E(Y_{it}|i = Treated; t = Pre)$
  $-[E(Y_{it}|i = Control; t = Post) - E(Y_{it}|i = Control; t = Pre)]$
  - Expected value of the difference for the outcome between pre and post treatment for the treatment group compared to the same difference for the control group.
    - *e.g. Average difference in pollution pre-post for the treated counties compared to the pre-post difference for the untreated counties.*

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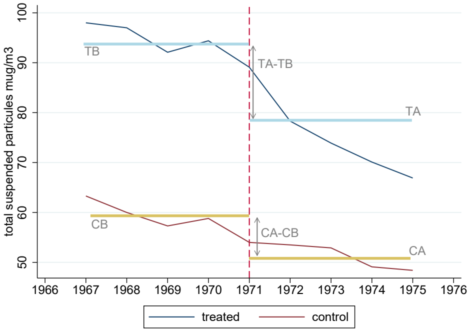

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Robustness and Limitations

- The main limitation comes from the impossibility to test the parallel trends assumption and finding a good control.
- Robustness tests allow us to see if the results are robust to different adjustments. It also allows for to challenge of the identification assumption.

1. Pick another control place,
2. Check between two control places the DiD effect (expect no effect),
3. Pick a placebo period,
4. Choose a placebo outcome (e.g. you study min. wages, and check the effects on the highest wages)