Parameter fitting of M3-Chandrayan data using Hapke Model

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Abstract:*his project focuses on refining surface parameter estimation utilizing the Hapke model, a pivotal tool in remote sensing and planetary science. By integrating observational data with computational modeling techniques, we aim to optimize key parameters such as surface roughness and scattering properties. Employing optimization algorithms and statistical methods, we seek to achieve the best fit between model predictions and observed data. The refined parameter estimates enhance our understanding of surface composition, geological processes, and environmental conditions on various celestial bodies. This endeavor underscores the significance of the Hapke model in advancing planetary exploration and scientific interpretations of remote sensing data, offering valuable insights for future missions and exploration.*

I. THEORY

A. Introduction

The Hapke model, developed by Bruce Hapke in the 1980s, is a cornerstone in remote sensing and planetary science for characterizing the reflectance properties of particulate surfaces. It provides a theoretical framework to understand how light interacts with rough, porous surfaces commonly found on celestial bodies like the Moon, Mars, and asteroids.
At its core, the Hapke model incorporates parameters describing surface roughness, particle size distribution, and composition to simulate the bidirectional reflectance distribution function (BRDF) of a surface. It considers both single and multiple scattering processes, accounting for variations in illumination and viewing geometry.

B. Bidirectional Reflectance Distribution Function (BRDF)

The Bidirectional Reflectance Distribution Function (BRDF) is a fundamental concept in remote sensing and computer graphics used to describe how light is scattered or reflected from a surface in different directions. Essentially, the BRDF characterizes the surface's reflectance properties as a function of the incident light direction and the viewing direction.

Here's a breakdown of its components:

1. **Bidirectional:** The BRDF considers both the direction of incoming light (illumination direction) and the direction of outgoing light (viewing direction). This bidirectional aspect is crucial because the reflectance of a surface can vary significantly depending on the angles of illumination and observation.

2. **Reflectance:** The BRDF quantifies the ratio of reflected radiant flux (light) to incident radiant flux at a particular wavelength and direction. It describes how efficiently the surface reflects light in a given direction compared to the total light incident upon it.

3. **Distribution Function:** The BRDF describes the distribution of reflected light across different outgoing directions relative to the surface normal. This distribution function encapsulates the surface's microstructure and optical properties, including factors such as surface roughness, texture, and composition.

In practical terms, the BRDF provides a mathematical model or function that predicts how much light will be reflected in a specific direction based on the surface's characteristics and the incoming and outgoing light angles. This model is essential for interpreting remote sensing data, generating synthetic images in computer graphics, and understanding how light interacts with various surfaces in the real world.

The Hapke model is a widely used theoretical framework in remote sensing and planetary science for characterizing the reflectance properties of particulate surfaces, such as those found on the Moon, Mars, and asteroids. It was developed by Bruce Hapke in the 1980s and has since become a cornerstone in the analysis of planetary surfaces.

The model incorporates several key components:

1. **Surface Roughness:** The Hapke model considers both macroscopic and microscopic surface roughness features, which influence the scattering and reflection of light. Surface roughness parameters are crucial for accurately simulating the reflectance properties of particulate surfaces.

2. **Particle Properties:** It accounts for the optical properties of particles present on the surface, including their size distribution, shape, and composition. These particle properties affect how light interacts with the surface and contribute to the overall reflectance behavior.

3. **Scattering Mechanisms:** The model incorporates various scattering mechanisms, including single scattering, multiple scattering, and shadowing effects. By considering these mechanisms, the Hapke model can simulate the complex reflectance behavior observed in planetary surfaces.

4. **Phase Function:** It includes a phase function to describe the angular distribution of scattered light. This phase function accounts for the directionality of scattering and is essential for accurately modeling the reflectance properties of particulate surfaces.

In summary, the Hapke model provides a comprehensive framework for simulating the reflectance properties of particulate surfaces in remote sensing applications. By incorporating surface roughness, particle properties, scattering mechanisms, and phase functions, the model enables scientists to analyze and interpret remote sensing data from planetary surfaces with high accuracy.

In this model, the radiance factor I/F is written as:

$$\begin{aligned}\frac{I}{F} =& LS(\theta_i, \theta_e) K_w \\ & \times \left[p(g)(1 + B_{S0} B_S(g)) + M(\theta_i, \theta_e)\right] \\ & \times \left[1 + B_{C0} B_C(g)\right] S(\theta_i, \theta_e, \psi) \qquad (1)\end{aligned}$$

where:

- The Lommel-Seeliger function (LS) is given by:

$$LS(\theta_i, \theta_e) = \frac{\cos(\theta_i)}{\cos(\theta_i) + \cos(\theta_e)}$$

- For the phase function $p(g)$, we employed the Henyey-Greenstein double-lobed single particle phase function, given by:

$$p(g) = \frac{1 - b^2}{(1 - b^2)(1 + c)} \frac{1 - c}{(1 + c)^2} + \frac{2(1 - 2b\cos(g) + b^2)^{3/2}}{2(1 + 2b\cos(g) + b^2)^{3/2}}$$

where $b(0 \leq b \leq 1)$ is the shape-controlling parameter and $c(-1 \leq c \leq 1)$ is the relative strength of backward and forward lobes.

- The SHOE function $B_S(g)$ is given by:

$$BS(g) = \frac{1}{1 + \frac{\tan(g/2)}{h_S}}$$

where h_S is the angular width parameter of SHOE.

- The IMSA function $M(\theta_i, \theta_e)$ is given by:

$$M(\theta_i, \theta_e) = \frac{H(\cos(\theta_e)/\cos(\theta_i), w)}{K}$$

where $H(x, w)$ is the Ambartsumian-Chandrasekhar H function, approximated by:

$$H(x, w) \approx \begin{cases} \frac{1 - 2r_0 x(1+x)^{-1}}{\ln(1 - wxr_0) + 2x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

and r_0 is the diffusive reflectance, given by:

$$r_0 = \frac{1 - \sqrt{1 - w}}{1 + \sqrt{1 - w}}$$

- The CBOE function $B_C(g)$ is given by:

$$BC(g) = \frac{1 + \frac{1}{2}\left(1 - \exp\left[-\frac{\tan(g/2)}{h_C}\right]\right)}{\frac{\tan(g/2)}{h_C}\left[2\left(1 + \frac{\tan(g/2)}{h_C}\right)^2\right]}$$

where h_C is the angular width parameter of CBOE.

- The Shadow function $S(\theta_i, \theta_e, \psi)$ is taken as 1, as no significant shadowing effect is considered.

- The porosity factor K is taken as 1.

- The amplitude of Coherent Backscatter Opposition Effect BC_0 is taken as 0, as very little coherent backscattering happens on the surface.

So now the simplified Hapke model we have is,

$$\frac{I}{F} = \frac{\omega}{4}\frac{\mu}{\mu + \mu_0}\left[p(g)(1 + B_{S0}B_S) + M(\mu_0, \mu)\right]$$

where $\mu = \cos(e)$ and $\mu_0 = \cos(i)$.

C. Hapke Parameters

The Hapke parameters include values such as the single scattering albedo, the scattering phase function, the opposition effect width parameter, the surface roughness, and other parameters that characterize the properties of the surface material. These parameters are typically derived from laboratory measurements of the material's optical properties and are used to constrain the model when analyzing remote sensing data.

- **Single Scattering Albedo (w)**: The single scattering albedo represents the ratio of scattering to extinction for a single scattering event. It quantifies the fraction of incident light that is scattered rather than absorbed or transmitted by a medium. Mathematically, it is defined as the scattering coefficient divided by the sum of the scattering and absorption coefficients.

- **Henyey-Greenstein Double-Lobed Single Particle Phase Function Parameter (b)**: The Henyey-Greenstein phase function is used to describe the angular distribution of scattered light by a single particle. The parameter b determines the asymmetry of the phase function. It controls the forward-backward asymmetry of the scattering pattern. Positive values of b indicate forward scattering dominance, while negative values indicate backward scattering dominance.

- **Henyey-Greenstein Double-Lobed Single Particle Phase Function Parameter (c)**: The parameter c is related to parameter b through the formula , $c = 3.29\exp(-17.4b^2) - 0.908$. It is used to describe the shape of the phase function. The relationship between b and c accounts for the double-lobed nature of the scattering pattern.

- **Amplitude of Shadow Hiding Opposition Effect (SHOE) (B_{so})**: SHOE refers to the phenomenon where shadows on the surface of a particulate medium can hide or enhance the opposition surge, which is an increase in brightness observed when the phase angle (the angle between the illumination and observation directions) approaches zero. B_{so} represents the amplitude of this effect.

- **Angular Width of SHOE (h_s)**: h_s represents the angular width of the Shadow Hiding Opposition Effect. It characterizes the width of the phase angle range over which the effect occurs.

II. METHODOLOGY

We retrieve 21 sets of lunar surface image datacube of highland region with thermally corrected radiance values.Now we have to binned the data to get the optimized fitted parameters of hapke model.

A. Extracting the image data and radiance diance from the datacubes

retrieved datacubes from the Moon Mineralogy Mapper (M3) instrument aboard the Chandrayaan-1 spacecraft. These datacubes likely contain spectral information collected over the surface of the Moon. Here's a breakdown of the dimensions of your datacubes:

- **First Dimension (Samples)**: This dimension represents the spatial extent along one direction, likely horizontally across the surface of the Moon. Each sample corresponds to a specific point or pixel in the image.

- **Second Dimension (Lines)**: This dimension represents the spatial extent along another direction, likely vertically across the surface of the Moon. Each line corresponds to a row of pixels in the image.

- **Third Dimension (Bands)**: This dimension corresponds to the spectral bands at which the images were taken by the M3 instrument. Each band captures the intensity of light reflected or emitted at a specific wavelength range. These bands provide spectral information about the surface composition and properties of the Moon.

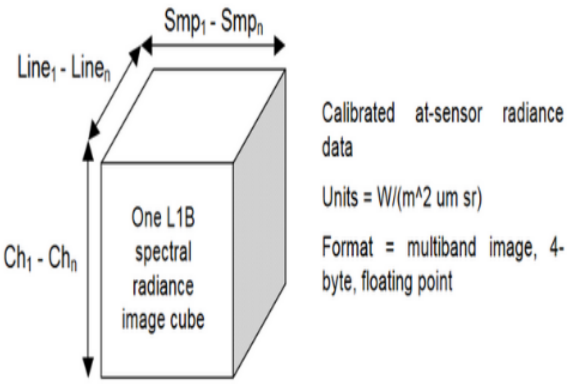


FIG. 1: Datacube Structure

The image datacube is loaded using the spectral library, and radiance datacube is simply in the .npy format so I have used the numpy library to load the data.

Here is the schematic how the angles are calculated from the satellite.

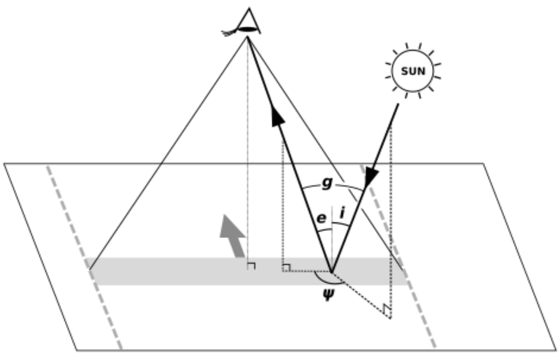


FIG. 2: Schematic diagram of incidence (i), emission (e), phase (g), and azimuthal (ψ) angles under the geometric configuration of the WAC. The narrow solid lines indicate the WAC field of view. The foot print of one WAC frame (gray zone), advancing direction (gray arrow), and the widths of single WAC image (gray dashed bold line) along the orbit track are illustrated.

To-Sun Azimuth (deg)	To-Sun Zenith (deg)	To-M3 Azimuth (deg)	To-M3 Zenith (deg)	Phase (deg)	To-Sun Path Length (au-0.983752357517)	To-M3 Path Length (m)	Facet Slope (deg)	Facet Aspect (deg)	Facet Cos(i) (unitless)

FIG. 3: Metadata from image datacube

Here from this we can understand the structure of the datacube of the image.From this information we extract the incidence, emission and phase angles values where the second column(To-Sun Zenith (deg)) is the incidence angle, the fourth column(To-M3 Zenith (deg)) contains the emission angle and the fifth column(Phase (deg)) contains the phase angle.
But in the radiance datacube it contains radiance values of different lines with different samples at 85 bands, where bands referred to as wavelengths. In the radiance datacube the band ranges from visible to infrared range, which can be analysed from this histogram given,

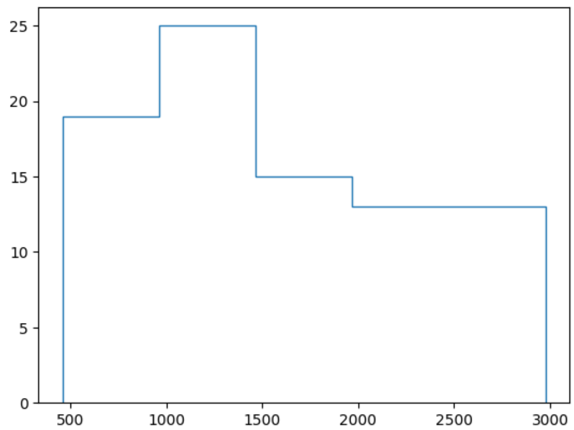


FIG. 4: Distribution of the wavelength

We can see that the maximaum number of wavelength lies between 1000-1500 nm range which is mostly in IF region. After this teps we correct the radiance value using the solar flux to get the reflectance values.

$$\frac{I}{F} = \frac{\text{radiance} \times \pi}{\text{solar flux}} \tag{2}$$

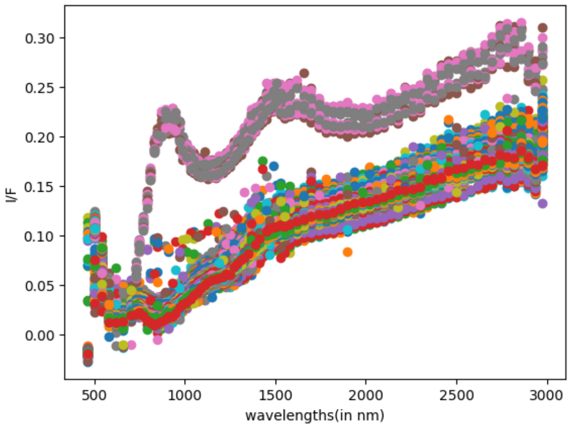


FIG. 5: Radiance vs wavelength of an arbitrary sample from the given images

We plotted the wavelength vs reflectance value to check whether the values are coming correct or not, we verified with the previous literature and got correct values. Here is a visualization of lunar surface image from a sample data we loaded.

FIG. 6: Lunar Surface

B. Binning of the data

In this section we are mentioning how the binning is done, binning is a process to reduce the number of datapoints without losing much information about the initial dataset.In our initial datasets we have huge number of datapoints so we have binned the data to reduce the number. First we concatenate all the datapoints of 21 images we extracted and binned the data as per this given algorithm below.

Algorithm: Binning Process

1. **Initialize Binned Matrix:**
   - Create a multi-dimensional array to store binned data.
   - Dimensions of the array depend on the number of bins desired along each angle (phase, incidence, and emission).
2. **Round Off Angles:**
   - Round off the incidence angles, emission angles, and phase angles to the nearest integer.
   - Ensure that the rounded angles fall within the range $[0, \text{num\_bins} - 1]$.
3. **Iterate Through Data Points:**
   - For each data point:
     - Determine the rounded phase angle, incidence angle, and emission angle of the data point.
     - Update the corresponding bin in the binned matrix:
       * Add the radiance value of the data point to the sum of radiance in the bin.
       * Increment the count of data points in the bin.
4. **Extract Non-Zero Bins:**
   - Identify the bins where either the sum of radiance or the count of data points is non-zero.
5. **Return Binned Matrix:**
   - Return the binned matrix containing only the non-zero elements.

In this way we reduce the number of datapoints in our array and store the phase,incidence,emission angle and radiance values in a single array, thus changing the whole datacube from 3-dimensional shape to 1-dimesional. This process is only done for a single band, as the reflectance spectra(Reflectance vs phase angle) or phase curve are different for different bands or wavelengths.

C. Distribution of phase angle and variation of phase function with phase angle

After getting the binned data we try to check the ditributions of some varaibles in the datasets. The mean phase angle of 40-60 degrees in M3 Chandrayaan data offers significant insights into the satellite observations of the lunar surface. This range suggests that the majority of observations were taken under specific lighting and viewing conditions. Such phase angles often correspond to intermediate illumination scenarios, where shadows cast by surface features are moderately pronounced, allowing for detailed surface characterization.

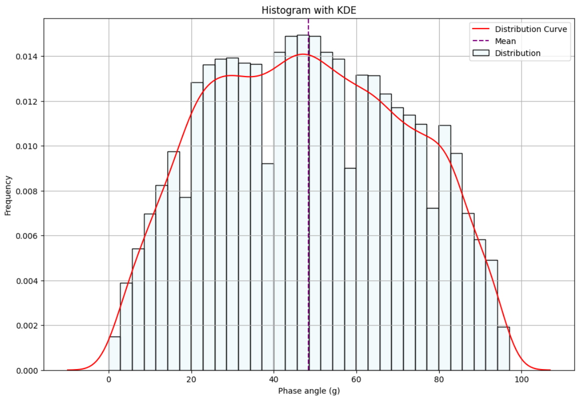


FIG. 7: Distribution of phase angles in the datasets

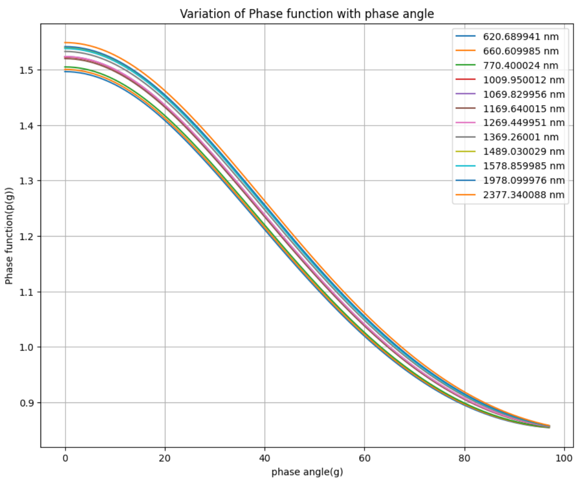


FIG. 8: Phase function vs phase angle variation at different wavelengths

The maximum phase function value of 1.5 at near 0 degrees phase angle suggests strong backscattering behavior, where light is predominantly reflected backward from the surface. This peak indicates that the surface exhibits higher reflectivity when illuminated directly facing the observer. As the phase angle increases, the phase function gradually decreases, reaching near 0 phase function value at around 100 degrees phase angle. This decreasing trend implies a shift towards forward scattering behavior, where light is scattered more towards the direction of the incident light.

The observed phase function curve provides insights into the surface properties and light scattering behavior of the studied planetary surface. The peak at low phase angles indicates a relatively smooth or less rough surface, with backscattering dominating the scattering process. As the phase angle increases, the decrease in phase function value suggests increased surface roughness or porosity, leading to more diffuse scattering and reduced reflectivity at larger angles. This information can be valuable for interpreting remote sensing data, understanding surface.

III. REFLECTANCE SPECTRA

Reflectance spectra of the lunar surface refer to measurements or observations of the amount of light reflected or emitted by the Moon's surface across different wavelengths of the electromagnetic spectrum. These spectra provide valuable insights into the composition, mineralogy, and physical properties of lunar surface materials.

he reflectance spectra of the lunar surface provide crucial insights into its composition, mineralogy, and surface processes. Here are key features and insights derived from lunar reflectance spectra:

1. **Mineral Identification**: Reflectance spectra exhibit characteristic absorption bands corresponding to minerals present on the lunar surface. These bands arise due to molecular vibrations and electronic transitions within minerals, enabling identification of specific mineral species such as pyroxenes, olivines, plagioclase feldspars, and ilmenite.

2. **Maturity and Weathering**: The shape and depth of absorption features in reflectance spectra are indicative of surface ma-

turity and degree of space weathering. Mature lunar soils typically exhibit broader and shallower absorption bands compared to immature or freshly exposed surfaces, reflecting the effects of prolonged exposure to micrometeorite impacts and solar wind irradiation.

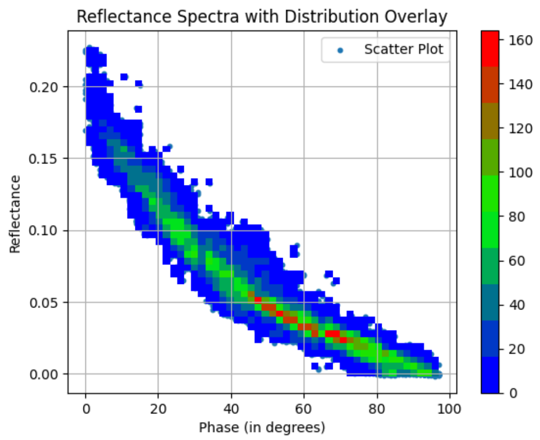
3. **Iron Content**: The depth of the 1 and 2 micron absorption bands in the near-infrared region is sensitive to the abundance of iron-bearing minerals such as pyroxenes and olivines. Higher iron content results in deeper absorption bands, providing quantitative information about iron abundance in lunar surface materials.

4. **Regolith Grain Size**: Variation in reflectance spectra with phase angle can provide insights into the regolith grain size distribution. Coarser-grained surfaces tend to exhibit stronger opposition effects, resulting in steeper spectral slopes at low phase angles.

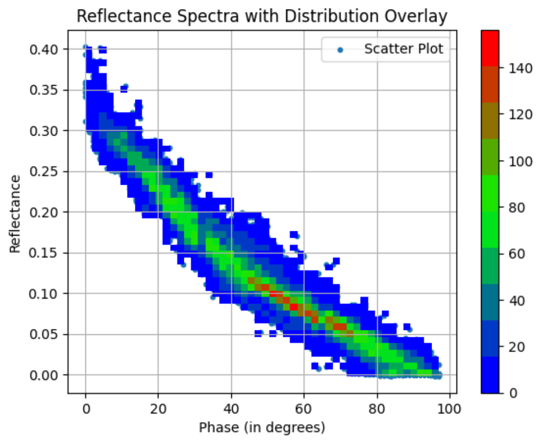
5. **Anorthosite Composition**: The continuum slope of reflectance spectra in the visible and near-infrared regions is indicative of the abundance of anorthosite, a primary component of the lunar highlands. Steeper slopes suggest higher anorthosite content, while flatter slopes indicate greater basaltic component.

6. **Spacecraft Observations**: Reflectance spectra acquired by spacecraft instruments (e.g., Clementine, Lunar Reconnaissance Orbiter) offer comprehensive coverage of the lunar surface, enabling detailed mapping of mineralogical and compositional variations across different lunar terrains.

Here are two plots at specific band we got,,



(a) Phase curve at 620.689941 nm



(b) Phase curve at 1578.859985 nm

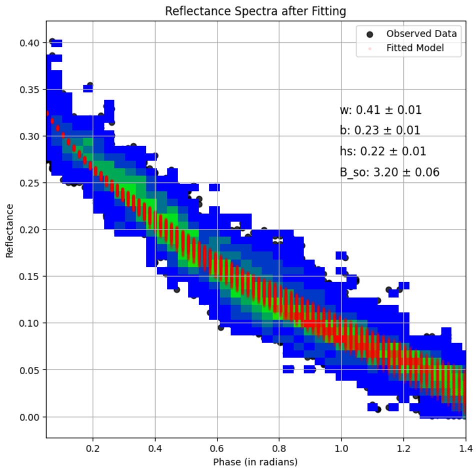
FIG. 9: Plot (a) is in visible range and plot (b) is in near infra-red range

Here from the plot we conclude that the denser regions are between 40-60 degrees phase angle, that was previously checked by the histogram given above where most of the datapoints recieved by the satellite are from 40-60 degrees.

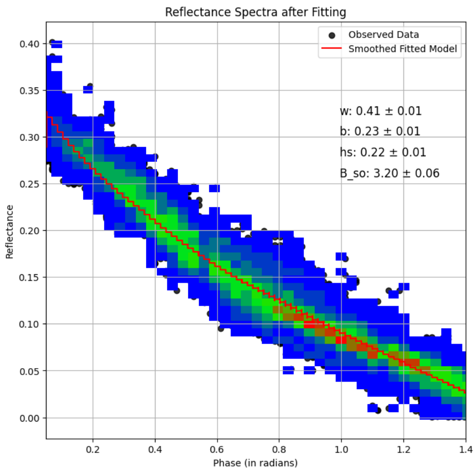
In this project we have choosed specific bands from 620-2400 nm to get the reflectance spectra from the binned dataset, after that fitting the spectra using hapke model to get the parameters.Here we are covering a whole range of EM spectrum from visible to IF region.

IV. FITTING OF REFLECTANCE SPECTRA

Using the simplified equation(1) of hapke model we fit the reflectance spectra to get the hapke parameters. Using scipy we fit the parameters as the data is large enough after the binning process we used inbuilt module to fit the data.



(a) Fitting of the spectra at 1578.859985 nm



(b) Smoothing spectra at 1578.859985 nm

FIG. 10: Plot (a) is fitted spectra and plot (b) is smoothed fitted spectra

Due to our binning process the fitted spectra is much spreaded so I smoothed the fitted line according to this algorithm.

Algorithm: Smooth Fitted Line

Input:

- Phase angles: Input data representing some independent variable.
- Parameters for a fitted model, obtained from a curve fitting procedure.

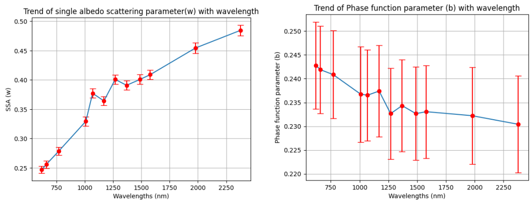
Steps:

1. **Generate Fitted Values**:
   - Calculate the fitted values using a function I/F with input phase angles and the parameters. This function models the relationship between phase angles and the fitted values.
2. **Smoothing Process**:
   - Define a window size for the moving average smoothing. This window size determines how many adjacent data points are considered in the smoothing process. Larger window sizes result in smoother curves but may blur out fine details.
   - Perform the smoothing operation:
     - For each data point, calculate the average value of itself and its neighboring points within the window.
     - Assign this average value as the smoothed value for the data point.
     - Move the window along the data, repeating the process for each point.
3. **Output**:
   - The function returns the smoothed values, representing the fitted curve after smoothing.

The smooth fitted line function takes fitted values, likely obtained from a curve fitting procedure, and smooths them using a moving average window of a specified size. This process reduces noise in the data and helps to emphasize underlying trends, making the fitted curve more interpretable and useful for analysis. Adjusting the window size allows for fine-tuning the level of smoothing applied to the curve.

V. VARIATION OF THE FITTED PARAMETERS WITH WAVELENGTHS

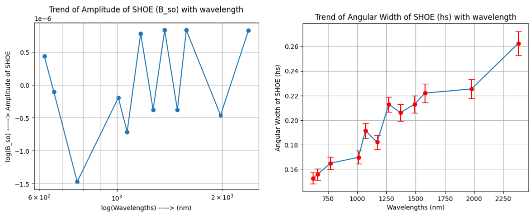
Here are the plot of the variation of fitted parameters with wavelengths.



(a) Variation in Single albedo Scaterring

(b) Variation in Asymetry factor

FIG. 11: Variation of w and b with wavelength



(a) Variation in SHOE amplitude

(b) Variation in SHOE angular width

FIG. 12: Variation of B_{S0} and h_s with wavelength

I have verified with previous literature the values are nearly correct here in the B_{S0} part I got negligible variation at the specified wavelengths, so I have plotted the difference between their mean with the fitted datapoints in the array. Also the trends are nearly same as off the previous papers I have studied.

VI. CONCLUSION

The single scattering albedo parameter (w) increasing with increasing wavelengths suggests that the surface becomes more reflective and less absorptive at longer wavelengths. This phenomenon could occur due to material properties, such as reduced absorption or increased scattering efficiency at longer wavelengths, resulting in a higher fraction of light being scattered rather than absorbed. Additionally, changes in chemical composition or surface roughness may contribute to the observed trend.

The decrease in the single particle phase function parameter (b) with increasing wavelengths in the Hapke model indicates that the surface becomes more dominated towards forward scattering as the wavelength increases. This could be due to absorption effects, changes in particle size distribution, or variations in surface roughness, all of which influence the scattering behavior of the surface. Furthermore, variations in particle shape or composition may also play a role in shaping the scattering properties across different wavelengths.

The amplitude (B_{so}) and angular width (h_s) of the shadow hiding and enhancement (SHOE) effect increase with wavelength due to wavelength-dependent surface roughness and particle size effects. Longer wavelengths interact with larger surface features, leading to broader and more pronounced shadow hiding and enhancement phenomena. This behavior is often associated with multiple scattering effects and can provide valuable insights into surface microstructure and composition.

In summary, the observed trends in the Hapke model parameters across different wavelengths reflect complex interactions between surface properties, material composition, and light-matter interactions. Understanding these relationships is essential for interpreting remote sensing data and gaining insights into the physical properties and processes governing planetary surfaces.

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