Worksheet on "Logistic Regression and Kernel Methods"

CS5691 PRML Jul–Nov 2025

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1. Consider a binary classification problem whose features are in $\mathbb{R}^2$. Suppose the predictor learned by logistic regression is $(\theta_0 + \theta_1 x1 + \theta_2 x2)$, where $\theta_0 = 4, \theta_1 = 1, \theta_2 = 0$. Find and plot the curve along which P(class 1) = 1/2 and the curve along which P(class 1) = 0.95.

2. Consider the logistic regression model

$$z = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

with parameters

$$\theta_0 = -1, \qquad \theta_1 = 2, \qquad \theta_2 = -0.5.$$

(a) For the point $x = (1, 2)$, compute

$$z \quad \text{and} \quad P(\text{class} = 1) = \sigma(z) = \frac{1}{1 + e^{-z}}.$$

(b) Find the decision boundary, i.e. the set of (x_1, x_2) for which $P(\text{class} = 1) = 0.5$. Express the boundary in slope–intercept form $x_2 = ax_1 + b$.Consider the logistic regression model

$$z = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

with parameters

$$\theta_0 = -1, \qquad \theta_1 = 2, \qquad \theta_2 = -0.5.$$

3. A potential cancer therapy drug "Y" was found to bind to certain cancer proteins in an experimental setup. Its binding efficiency was found to be based on its length (length), presence of β-sheets (structure; 1 = β-sheet is present, 0 = α-helix), and presence of hydrophobic side chains (HSC; 1 = side chains are hydrophobic, 0 otherwise). Using protein data from TCGA (a public database), logistic regression was performed to understand the drug behavior. The results are summarized below in a table.

	Coefficient	Standard error	Z	p-value
length	0.9	0.20	3.16	0.002
structure	0.3	0.77	2.73	0.006
HSC	2.5	0.26	1.99	0.047
constant	-25	3.28	-2.48	0.013

Determine the log odds of binding of drug "Y" to a Protein that has 20 amino acids, α-helix structure, and hydrophobic side chains. Will you predict this protein to bind or not bind to the drug? (Recall that odds is the ratio of the probability of binding to that of not binding.)

4. Check if the following kernels are valid.

(a) $k(x, y) = x - y$ in $\mathbb{R}^1$

(b) $k(x,y) = x^2$ in $\mathbb{R}^1$

(c) $k(x,y) = (x^\top y + c)^2$ in $\mathbb{R}^d,\ c \geq 0$

(d) $k(x,y) = \exp\left(-\frac{\|x-y\|^2}{2\sigma^2}\right)$, with $\sigma > 0$