CSE/ECE 848 Introduction to Evolutionary Computation

Module 3 - Lecture 14 - Part 4

Comparison of EC Methods: Test Problems

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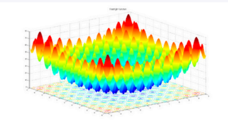
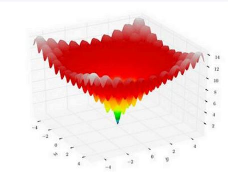
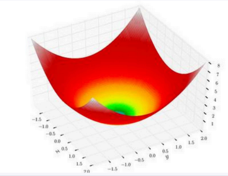
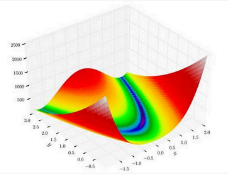
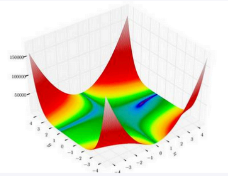
Test Problems

- Real world data sets
  - Often difficult to obtain, but see Kaggle
- Random variants of real world data sets
  - Synthetic data sets, constructed from real ones for purposes of public release
- Standard Benchmark test problems
  - Published, or online libraries
- Randomly generated instances
  - Self-generated or standardized and in the literature

Function Optimization Benchmarks

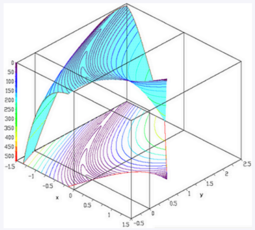
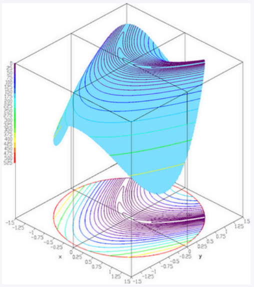
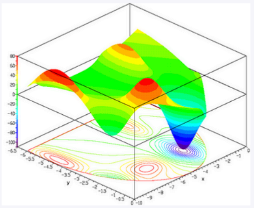
- https://en.wikipedia.org/wiki/Test_functions_for_optimization

Test functions for single-objective optimization [edit]

Name	Plot	Formula	Global minimum	Search domain
Rastrigin function		$f(\mathbf{x}) = An + \sum_{i=1}^{n} \left[x_i^2 - A\cos(2\pi x_i)\right]$ where: $A = 10$	$f(0,\dots,0) = 0$	$-5.12 \le x_i \le 5.12$
Ackley function		$f(x,y) = -20\exp\left[-0.2\sqrt{0.5\left(x^2+y^2\right)}\right] - \exp\left[0.5\left(\cos 2\pi x + \cos 2\pi y\right)\right] + e + 20$	$f(0,0) = 0$	$-5 \le x, y \le 5$
Sphere function		$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2$	$f(x_1,\dots,x_n) = f(0,\dots,0) = 0$	$-\infty \le x_i \le \infty$, $1 \le i \le n$
Rosenbrock function		$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2\right]$	$\text{Min} = \begin{cases} n=2 & \rightarrow \quad f(1,1) = 0, \\ n=3 & \rightarrow \quad f(1,1,1) = 0, \\ n>3 & \rightarrow \quad f(\underbrace{1,\dots,1}_{n \text{ times}}) = 0 \end{cases}$	$-\infty \le x_i \le \infty$, $1 \le i \le n$
Beale function		$f(x,y) = \left(1.5 - x + xy\right)^2 + \left(2.25 - x + xy^2\right)^2 + \left(2.625 - x + xy^3\right)^2$	$f(3, 0.5) = 0$	$-4.5 \le x, y \le 4.5$

Function Optimization Benchmarks II

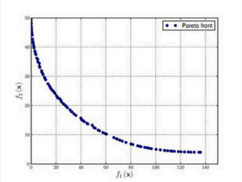
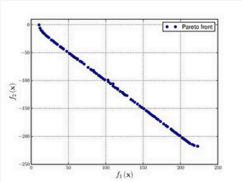
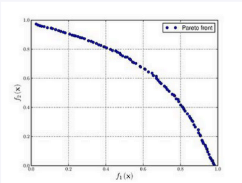
Test functions for constrained optimization [edit]

Name	Plot	Formula	Global minimum	Search domain
Rosenbrock function constrained with a cubic and a line[10]		$f(x,y) = (1-x)^2 + 100(y-x^2)^2$, subjected to: $(x-1)^3 - y + 1 \leq 0$ and $x + y - 2 \leq 0$	$f(1.0, 1.0) = 0$	$-1.5 \leq x \leq 1.5$, $-0.5 \leq y \leq 2.5$
Rosenbrock function constrained to a disk[11]		$f(x,y) = (1-x)^2 + 100(y-x^2)^2$, subjected to: $x^2 + y^2 \leq 2$	$f(1.0, 1.0) = 0$	$-1.5 \leq x \leq 1.5$, $-1.5 \leq y \leq 1.5$
Mishra's Bird function - constrained[12][13]		$f(x,y) = \sin(y) e^{\left[(1-\cos x)^2\right]} + \cos(x) e^{\left[(1-\sin y)^2\right]} + (x-y)^2$, subjected to: $(x+5)^2 + (y+5)^2 < 25$	$f(-3.1302468, -1.5821422) = -106.7645367$	$-10 \leq x \leq 0$, $-6.5 \leq y \leq 0$

Function Optimization Benchmarks III

Test functions for multi-objective optimization [edit]

[*further explanation needed*]

Name	Plot	Functions	Constraints	Search domain
Binh and Korn function:[5]		$\text{Minimize} = \begin{cases} f_1(x,y) = 4x^2 + 4y^2 \\ f_2(x,y) = (x-5)^2 + (y-5)^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = (x-5)^2 + y^2 \leq 25 \\ g_2(x,y) = (x-8)^2 + (y+3)^2 \geq 7.7 \end{cases}$	$0 \leq x \leq 5,$ $0 \leq y \leq 3$
Chankong and Haimes function:[16]		$\text{Minimize} = \begin{cases} f_1(x,y) = 2 + (x-2)^2 + (y-1)^2 \\ f_2(x,y) = 9x - (y-1)^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) = x^2 + y^2 \leq 225 \\ g_2(x,y) = x - 3y + 10 \leq 0 \end{cases}$	$-20 \leq x,y \leq 20$
Fonseca–Fleming function:[17]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$		$-4 \leq x_i \leq 4,$ $1 \leq i \leq n$
				

EC: F1 - F25

Unimodal Functions (5):

- F_1: Shifted Sphere Function
- F_2: Shifted Schwefel's Problem 1.2
- F_3: Shifted Rotated High Conditioned Elliptic Function
- F_4: Shifted Schwefel's Problem 1.2 with Noise in Fitness
- F_5: Schwefel's Problem 2.6 with Global Optimum on Bounds

Multimodal Functions (20):

- **Basic Functions** (7):
  - F_6: Shifted Rosenbrock's Function
  - F_7: Shifted Rotated Griewank's Function without Bounds
  - F_8: Shifted Rotated Ackley's Function with Global Optimum on Bounds
  - F_9: Shifted Rastrigin's Function
  - F_{10}: Shifted Rotated Rastrigin's Function
  - F_{11}: Shifted Rotated Weierstrass Function
  - F_{12}: Schwefel's Problem 2.13
- **Expanded Functions** (2):

2

  - F_{13}: Expanded Extended Griewank's plus Rosenbrock's Function (F8F2)
  - F_{14}: Shifted Rotated Expanded Scaffer's F6

Hybrid Composition Functions (11):

- F_{15}: Hybrid Composition Function
- F_{16}: Rotated Hybrid Composition Function
- F_{17}: Rotated Hybrid Composition Function with Noise in Fitness
- F_{18}: Rotated Hybrid Composition Function
- F_{19}: Rotated Hybrid Composition Function with a Narrow Basin for the Global Optimum
- F_{20}: Rotated Hybrid Composition Function with the Global Optimum on the Bounds
- F_{21}: Rotated Hybrid Composition Function
- F_{22}: Rotated Hybrid Composition Function with High Condition Number Matrix
- F_{23}: Non-Continuous Rotated Hybrid Composition Function
- F_{24}: Rotated Hybrid Composition Function
- F_{25}: Rotated Hybrid Composition Function without Bounds

Combinatorial Optimization Benchmarks - General Criteria

- Test suites should contain problems resistant to hill-climbers
- Test problems should be non-linear, non-separable, non-symmetric
- Test suites should contain scalable functions (eg dimensionality)
- Problems with scalable evaluation costs (eg, dimensional scaling in real world problems)
- Problems should have a canonical form

From: Whitley et al. Evaluating Evolutionary Algorithms
Artificial Intelligence, 85(1-2):245–276, 1996

Libraries

- TSP lib   http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/
  Discrete and Combinatorial Optimization Problem Library
- CSP lib   https://www.csplib.org/ Constraint Problems
- MIP lib   http://miplib.zib.de/   Mixed Integer Problems

Shir et al. Compiling a Benchmarking Test-Suite for CombinatorialBlack-Box Optimization: A Position Paper, GECCO 2018 Companion

Genetic Programming Benchmarks - Earlier Practice

- Symbolic Regression
- Classification
- Binary Functions
- Predictive Modelling
- Path Finding and Planning
- Others

McDermott et al. Genetic Programming Needs Better Benchmarks, GECCO 2012

Genetic Programming Benchmarks - Criteria

- Tunably difficult
- Varied
- Relevant
- Fast
- Accommodating to Implementers
- Easy to interpret and compare
- Representation-independent
- Precisely-defined
- Current

McDermott et al. Genetic Programming Needs Better Benchmarks, GECCO 2012

GP Benchmarks

- Symbolic Regression

Name	Vars	Objective Function	Training Set	Testing Set	Funct
Koza-1, Nguyen-2 [32, 45]	1	$x^4 + x^3 + x^2 + x$	U[-1, 1, 20]	None	Koza
Koza-2 [33]	1	$x^5 - 2x^3 + x$	U[-1, 1, 20]	None	Koza
Koza-3 [33]	1	$x^6 - 2x^4 + x^2$	U[-1, 1, 20]	None	Koza
Nguyen-1 [45]	1	$x^3 + x^2 + x$	U[-1, 1, 20]	None	Koza
Nguyen-3 [45]	1	$x^5 + x^4 + x^3 + x^2 + x$	U[-1, 1, 20]	None	Koza
Nguyen-4 [45]	1	$x^6 + x^5 + x^4 + x^3 + x^2 + x$	U[-1, 1, 20]	None	Koza
Nguyen-5 [45]	1	$\sin(x^2)\cos(x) - 1$	U[-1, 1, 20]	None	Koza
Nguyen-6 [45]	1	$\sin(x) + \sin(x + x^2)$	U[-1, 1, 20]	None	Koza
Nguyen-7 [45]	1	$\ln(x+1) + \ln(x^2+1)$	U[0, 2, 20]	None	Koza
Nguyen-8 [45]	1	$\sqrt{x}$	U[0, 4, 20]	None	Koza
Nguyen-9 [45]	2	$\sin(x) + \sin(y^2)$	U[0, 1, 20]	None	Koza
Nguyen-10 [45]	2	$2\sin(x)\cos(y)$	U[0, 1, 20]	None	Koza
Nguyen-11 [45] (Omit)	2	x^y	U[0, 1, 20]	None	Koza
Nguyen-12 [45] (Omit)	2	$x^4 - x^3 + \frac{y^2}{2} - y$	U[0, 1, 20]	None	Koza
Pagie-1 [50]	2	$\frac{1}{1+x^{-4}} + \frac{1}{1+y^{-4}}$	E[-5, 5, 0.4]	None	Koza
Korns-1 [30]	5	$1.57 + (24.3\,v)$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-2 [30]	5	$0.23 + 14.2\,\frac{v+y}{3\,w}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-3 [30]	5	$-5.41 + 4.9\,\frac{v-x+\frac{y}{w}}{3\,w}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-4 [30]	5	$-2.3 + 0.13\,\sin(z)$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-5 [30]	5	$3 + 2.13\,\ln(w)$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-6 [30]	5	$1.3 + 0.13\,\sqrt{x}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-7 [30]	5	$213.80940889(1 - e^{-0.54723748542\,x})$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-8 [30]	5	$6.87 + 11\,\sqrt{7.23\,x\,v\,w}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-9 [30]	5	$\frac{\sqrt{x}}{\ln(y)}\,e^{\frac{z}{v^2}} \longrightarrow \frac{\sqrt{x}}{\ln y}\,\frac{e^z}{v^2}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-10 [30]	5	$0.81 + 24.3\,\frac{2\,y+3\,z^2}{4\,(v)^3+5\,(w)^4}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-11 [30]	5	$6.87 + 11\cos(7.23\,x^3)$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-12 [30]	5	$2 - 2.1\cos(9.8\,x)\,\sin(1.3\,w)$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-13 [30]	5	$32 - 3\,\frac{\tan(x)}{\tan(y)}\,\frac{\tan(z)}{\tan(v)}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-14 [30]	5	$22 - 4.2\,(\cos(x) - \tan(y))\,\frac{\tanh(z)}{\sin(v)}$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Korns-15 [30]	5	$12 - 6\,\frac{\tan(x)}{e^y}(\ln(z) - \tan(v))$	U[-50, 50, 10000]	U[-50, 50, 10000]	Korns
Keijzer-1 [28]	1	$0.3\,x\sin(2\pi x)$	E[-1, 1, 0.1]	E[-1, 1, 0.001]	Keijzer
Keijzer-2 [28]	1	$0.3\,x\sin(2\pi x)$	E[-2, 2, 0.1]	E[-2, 2, 0.001]	Keijzer
Keijzer-3 [28]	2	$0.3\,x\sin(2\pi x)$	E[-3, 3, 0.1]	E[-3, 3, 0.001]	Keijzer
Keijzer-4 [28]	1	$x^3 e^{-x}\cos(x)\sin(x)(\sin^2(x)\cos(x) - 1)$	E[0, 10, 0.05]	E[0.05, 10.05, 0.05]	Keijzer
Keijzer-5 [28]	3	$\frac{30xz}{(x-10)y^2}$	x, $y \longrightarrow z$: U[-1, 1, 1000] $z \longrightarrow y$: U[1, 2, 1000]	x, $y \longrightarrow z$: U[-1, 1, 10000] $z \longrightarrow y$: U[1, 2, 10000]	Keijzer
Keijzer-6 [28]	1	$\sum_i^x \frac{1}{i}$	E[1, 50, 1]	E[1, 120, 1]	Keijzer
Keijzer-7 [28]	1	$\ln x$	E[1, 100, 1]	E[1, 100, 0.1]	Keijzer
Keijzer-8 [28]	1	$\sqrt{x}$	E[0, 100, 1]	E[0, 100, 0.1]	Keijzer
Keijzer-9 [28]	1	$\mathrm{arcsinh}(x)$ *i.e.*, $\ln(x + \sqrt{x^2+1})$	E[0, 100, 1]	E[0, 100, 0.1]	Keijzer
Keijzer-10 [28]	2	x^y	U[0, 1, 100]	E[0, 1, 0.01]	Keijzer
Keijzer-11 [28]	2	$xy + \sin((x-1)(y-1))$	U[-3, 3, 20]	E[-3, 3, 0.01]	Keijzer
Keijzer-12 [28]	2	$x^4 - x^3 + \frac{y^2}{2} - y$	U[-3, 3, 20]	E[-3, 3, 0.01]	Keijzer
Keijzer-13 [28]	2	$6\sin(x)\cos(y)$	U[-3, 3, 20]	E[-3, 3, 0.01]	Keijzer
Keijzer-14 [28]	2	$\frac{8}{2+x^2+y^2}$	U[-3, 3, 20]	E[-3, 3, 0.01]	Keijzer
Keijzer-15 [28]	2	$\frac{x^3}{5} + \frac{y^3}{2} - y - x$	U[-3, 3, 20]	E[-3, 3, 0.01]	Keijzer
Vladislavleva-1 [64]	2	$\frac{e^{-(x-1)^2}}{1.2+(y-2.5)^2}$	U[0.3, 4, 100]	E[-0.2, 4.2, 0.01]	Vladisl
Vladislavleva-2 [64]	1	$e^{-x}x^3(\cos x\sin x)(\cos x\sin^2 x - 1)$	E[0.5, 10, 0.1]$\longrightarrow$E[0.05, 10, 0.1]	E[-0.5, 10.5, 0.05]	Vladisl
Vladislavleva-3 [64]	2	$e^{-x}x^3(\cos x\sin x)(\cos x\sin^2 x - 1)(y-5)$	x: E[0.05, 10, 0.1] y: E[0.05, 10.05, 2]	x: E[-0.5, 10.5, 0.05] y: E[-0.5, 10.5, 0.5]	Vladisl
Vladislavleva-4 [64]	5	$\frac{10}{5+(x-3)^2+(y-3)^2+(z-3)^2+(v-3)^2+(w-3)^2}$	U[0.05, 6.05, 1024]	U[-0.25, 6.35, 5000]	Vladisl
Vladislavleva-5 [64]	3	$30\frac{(x-1)(z-1)}{y^2(x-10)}$	x: U[0.05, 2, 300] y: U[1, 2, 300] z: U[0.05, 2, 300]	x: E[-0.05, 2.1, 0.15] y: E[0.95, 2.05, 0.1] z: E[-0.05, 2.1, 0.15]	Vladisl
Vladislavleva-6 [64]	2	$6\sin(x)\cos(y)$	U[0.1, 5.9, 30]	E[-0.05, 6.05, 0.02]	Vladisl
Vladislavleva-7 [64]	2	$(x-3)(y-3) + 2\sin((x-4)(y-4))$	U[0.05, 6.05, 300]	U[-0.25, 6.35, 1000]	Vladisl
Vladislavleva-8 [64]	2	$\frac{(x-3)^4+(y-3)^3-(y-3)}{(y-2)^4+10}$	U[0.05, 6.05, 50]	E[-0.25, 6.35, 0.2]	Vladisl

Table 3: Symbolic Regression Benchmark Candidates. Variable names are, in order, x, y, z, v, w. Some benchmarks intentionally omit variables from the function. U[a,b,c] is c uniform random samples drawn from a to b, inclusive, for the variable. E[a,b,c] is a grid of points evenly spaced (for this variable) with an interval of c, from a to b inclusive. Testing and training sets are independent. See Table 2 for function sets.

McDermott et al. Genetic Programming Needs Better Benchmarks, GECCO 2012

GP Benchmarks III

- Symbolic Regression
- Feynman equations, from Feynman's Lectures on Physics

Udrescu and Tegmark, AI Feynman: A physics-inspired method for symbolic regression, Science Advances, 6, 2020

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Table 4. Tested Feynman equations, part 1. Abbreviations in the "Methods used" column: da, dimensional analysis; bf, brute force; pf, polyfit; ev, set two variables equal; sym, symmetry; sep, separability. Suffixes denote the type of symmetry or separability (sym–, translational symmetry; sep*, multiplicative separability; etc.) or the preprocessing before brute force (e.g., bf-inverse means inverting the mystery function before bf).

Feynman Eq.	Equation	Solution Time (s)	Methods Used	Data Needed	Solved By Eureqa	Solved W/o da	Noise Tolerance
I.6.20a	$f = e^{-\theta^2/2}/\sqrt{2\pi}$	16	bf	10	No	Yes	10^{-2}
I.6.20	$f = e^{-\frac{\theta^2}{2\sigma^2}}/\sqrt{2\pi\sigma^2}$	2992	ev, bf-log	10^2	No	Yes	10^{-4}
I.6.20b	$f = e^{-\frac{(\theta-\theta_1)^2}{2\sigma^2}}/\sqrt{2\pi\sigma^2}$	4792	sym–, ev, bf-log	10^3	No	Yes	10^{-4}
I.8.14	$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	544	da, pf-squared	10^2	No	Yes	10^{-4}
I.9.18	$F = \frac{Gm_1m_2}{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$	5975	da, sym–, sym–, sep∗, pf-inv	10^6	No	Yes	10^{-5}
I.10.7	$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$	14	da, bf	10	No	Yes	10^{-4}
I.11.19	$A = x_1y_1 + x_2y_2 + x_3y_3$	184	da, pf	10^2	Yes	Yes	10^{-3}
I.12.1	$F = \mu N_n$	12	da, bf	10	Yes	Yes	10^{-3}
I.12.2	$F = \frac{q_1q_2}{4\pi\epsilon r^2}$	17	da, bf	10	Yes	Yes	10^{-2}
I.12.4	$E_f = \frac{q_1}{4\pi\epsilon r^2}$	12	da	10	Yes	Yes	10^{-2}
I.12.5	$F = q_2E_f$	8	da	10	Yes	Yes	10^{-2}
I.12.11	$F = q(E_f + Bv\sin\theta)$	19	da, bf	10	Yes	Yes	10^{-3}
I.13.4	$K = \frac{1}{2}m(v^2+u^2+w^2)$	22	da, bf	10	Yes	Yes	10^{-4}
I.13.12	$U = Gm_1m_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$	20	da, bf	10	Yes	Yes	10^{-4}
I.14.3	$U = mgz$	12	da	10	Yes	Yes	10^{-2}
I.14.4	$U = \frac{k_{spring}x^2}{2}$	9	da	10	Yes	Yes	10^{-2}
I.15.3x	$x_1 = \frac{x-ut}{\sqrt{1-u^2/c^2}}$	22	da, bf	10	No	No	10^{-3}
I.15.3t	$t_1 = \frac{t-ux/c^2}{\sqrt{1-u^2/c^2}}$	20	da, bf	10^2	No	No	10^{-4}
I.15.10	$p = \frac{m_0v}{\sqrt{1-v^2/c^2}}$	13	da, bf	10	No	Yes	10^{-4}
I.16.6	$v_1 = \frac{u+v}{1+uv/c^2}$	18	da, bf	10	No	Yes	10^{-3}
I.18.4	$r = \frac{m_1r_1+m_2r_2}{m_1+m_2}$	17	da, bf	10	Yes	Yes	10^{-2}
I.18.12	$\tau = rF\sin\theta$	15	da, bf	10	Yes	Yes	10^{-3}
I.18.16	$L = mrv\sin\theta$	17	da, bf	10	Yes	Yes	10^{-3}
I.24.6	$E = \frac{1}{4}m(\omega^2+\omega_0^2)x^2$	22	da, bf	10	Yes	Yes	10^{-4}
I.25.13	$V_e = \frac{q}{C}$	10	da	10	Yes	Yes	10^{-2}

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