

CSE/ECE 848

Introduction to

Evolutionary Computation

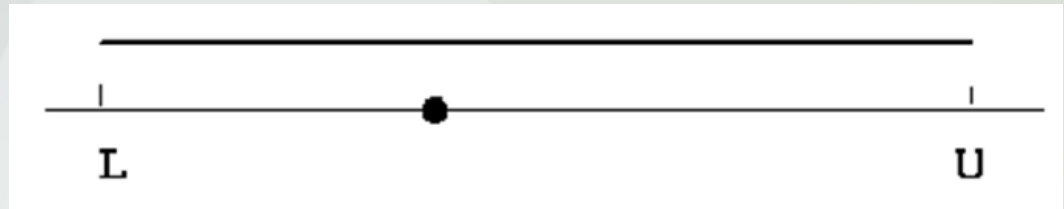
Module 2, Lecture 9, Part 3

Variants of BGAs and RGAs

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Real-parameter Mutation Operators

- Variable-wise mutation operators
- Every variable is mutated with prob. p_m ($\approx 1/n$)
- x_i is mutated to x'_i (perturbed in neighborhood)
- Uniform Mutation: (Not recommended)



- Gaussian Mutation
- Polynomial Mutation
- Mutation clock (effective for bit-wise mutation)
 - Estimate the next bit (variable) to be mutated

Reference: Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*. Chichester, London: Wiley

Gaussian Mutation Operator

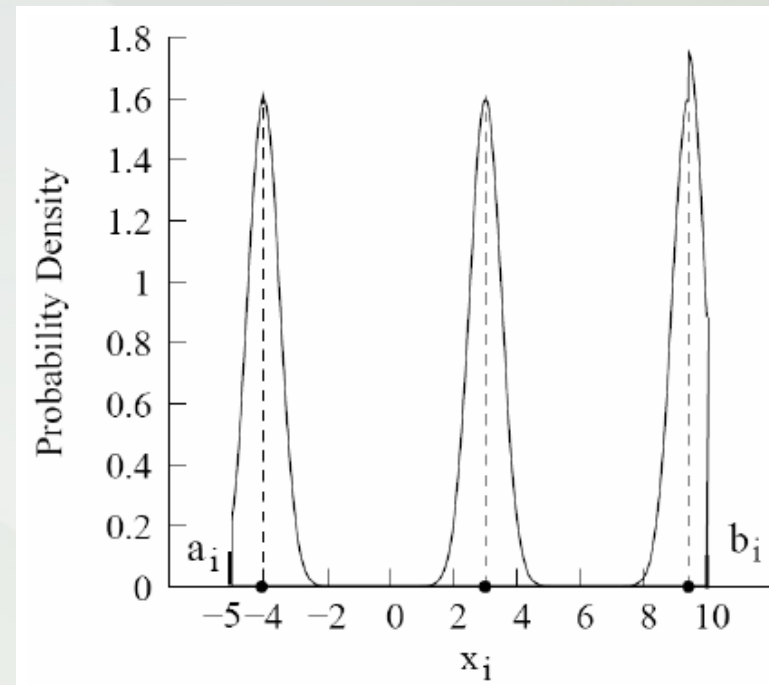
- Use a truncated Gaussian distribution with σ :
- Create a random number $u_i \in [0,1]$ for every var.

$$x'_i = x_i + \sqrt{2}\sigma(b_i - a_i)\text{erf}^{-1}(u'_i), \text{ erf}() \text{ is the Error function}$$

$$u'_i = \begin{cases} 2u_L(1-2u_i), & \text{if } u_i \leq 0.5, \\ 2u_R(2u_i-1), & \text{otherwise,} \end{cases}$$

$$u_L = 0.5 \left(\text{erf} \left(\frac{a_i - x_i}{\sqrt{2}(b_i - a_i)\sigma} \right) + 1 \right)$$

$$u_R = 0.5 \left(\text{erf} \left(\frac{b_i - x_i}{\sqrt{2}(b_i - a_i)\sigma} \right) + 1 \right)$$



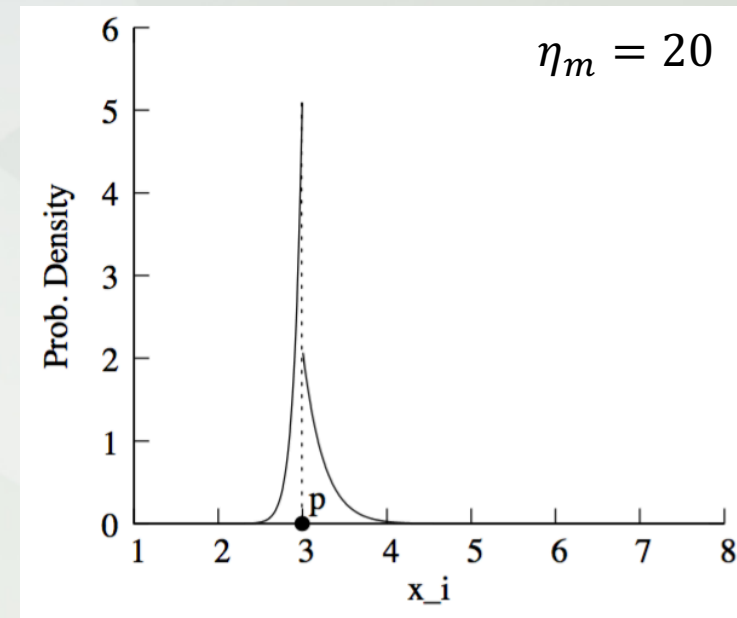
- Vector-wise Gaussian mutation possible

Boundary handling included

Polynomial Mutation Operator

- For the chosen variable, parent is $p \in [a, b]$
- Choose a random number $u \in [0, 1]$
- If $u \leq 1/2$, calculate $\delta_L = (2u)^{\frac{1}{1+\eta_m}} - 1$; $\eta_m \in [20, 100]$
 else calculate $\delta_R = 1 - (2(1 - u))^{\frac{1}{1+\eta_m}}$
- Calculate mutated child:
 - $p' = p + \delta_L(p - a)$, if $u \leq 1/2$;
 - $p' = p + \delta_R(b - p)$, otherwise
- Illustration: $p = 3.0 \in [1, 8]$

Reference: [Deb, K. and Deb, D. \(2014\).
 Analysing mutation schemes for real-parameter
 genetic algorithms. Int. J. Artificial Intelligence
 and Soft Computing, 4\(1\), Inderscience
 Enterprises Ltd., 1–28.](#)



Boundary Handling in Real-parameter EAs

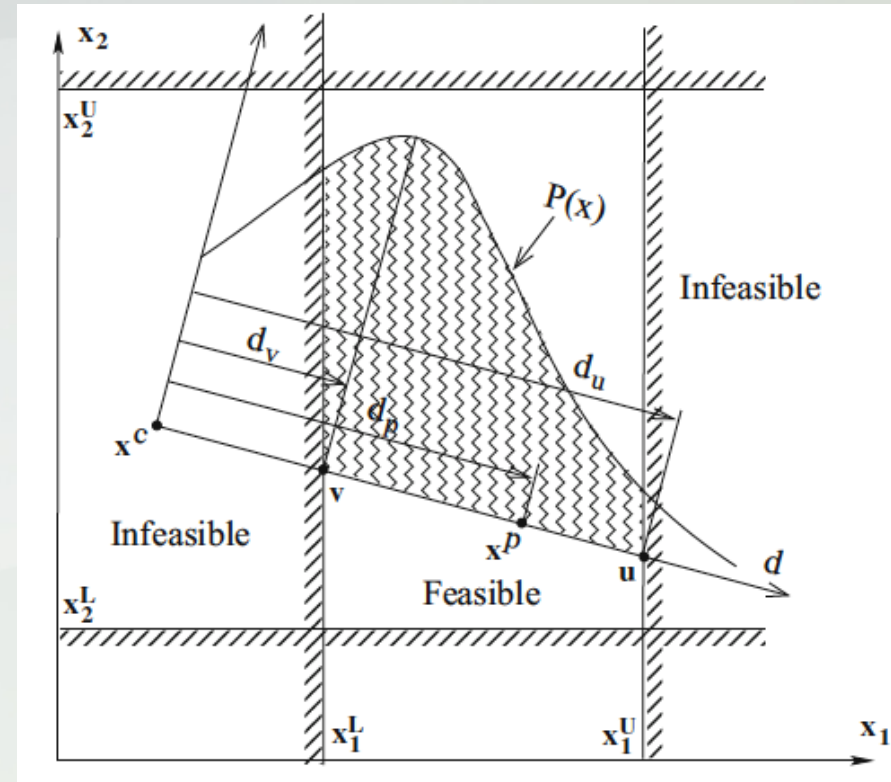
- When a boundary-less search creates a child outside variable bounds, bring it back, but how?
- Inverse Parabolic penalty:
Repair child x^c to create \vec{y} :

$$d' = d_v + \alpha d_v \tan \left(r \tan^{-1} \frac{a - d_v}{\alpha d_v} \right)$$

$$\vec{y} = \vec{x}^c + d'(\vec{x}^p - \vec{x}^c)$$

$r \in [0,1]$
 $\alpha = 1.2$
 $a = d_u$

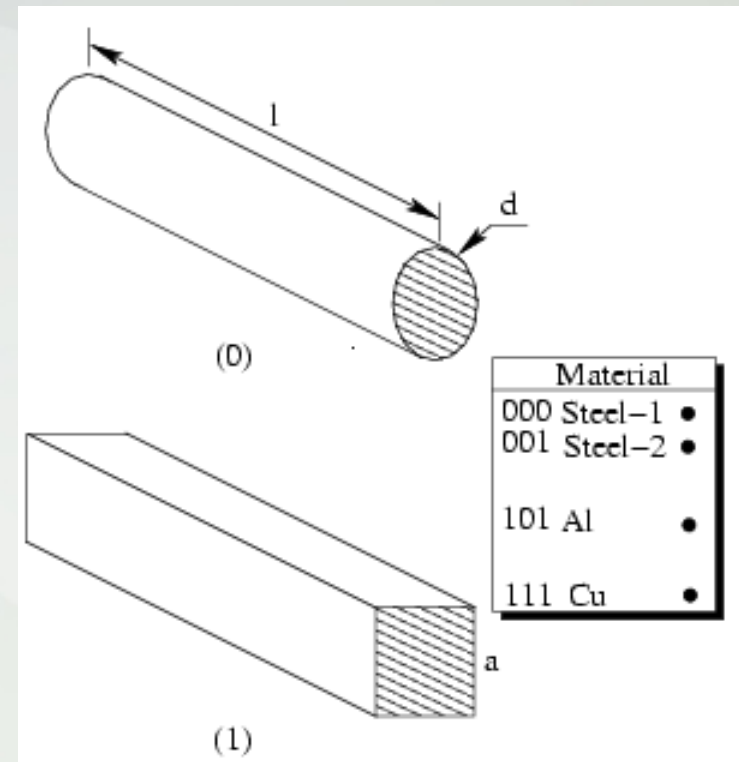
Reference: Padhye, N., Mittal, P., and Deb, K. (2015). Feasibility preserving constraint handling strategies for real-parameter evolutionary optimization. *Computational Optimization and Applications*, 62(3), 851–890.



Mixed Variable Handling

- ▶ EAs are excellent for handling mixed variables
- ▶ A mixed representation: **[(1) 14 23.457 (101)]**
 - ▶ (1): circular or square cross-section
 - ▶ 14 mm: diameter/side
 - ▶ 23.457 mm: length
 - ▶ (101): material
- ▶ Otherwise, many opt. runs
- ▶ Recomb & mut. on similar var.
- ▶ Permutation + real + discrete

Reference: [Deb, K., and Goyal. M. \(1996\). A combined genetic adaptive search \(GeneAS\) for engi-neering design. Computer Science and Informatics, 26\(4\), 30–45.](#)



End of Module 2, Lecture 9, Part 3

- Real-parameter mutation creates a perturbed point
 - Gaussian mutation
 - Polynomial mutation
- Boundary handling is required (with RGAs, DE, PSO, etc.)
- Handling mixed variables relatively easy with EAs