CSE/ECE 848
Introduction to Evolutionary Computation

Module 2, Lecture 8, Part 4a
More Theory--Implicit Parallelism

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The N^3 Argument (Implicit or Intrinsic Parallelism)

Assertion: A GA with pop size N can usefully process on the order of N^3 hyperplanes (schemata) in a generation

(WOW! If pop size N=100, N^3 = 1 million)

Derivation:

- Assume random population of size N
- Assume we need ϕ *instances of a schema* in population to claim we are "processing" it in a statistically significant way in one generation

The N^3 Argument (cont.)

Example: to have 8 samples (on average) of 2nd order schemata in a pop., (in which there are 4 distinct (CONFLICTING) schemata in each 2-position pair – for example, *0*0**, *0*1**, *1*0**, *1*1**), we'd need 4 bit patterns x 8 instances = 32 popsize.

In general, the highest ORDER of schema, θ, that is "processed" is log (N/ϕ); in our case, $\log(32/8) = \log(4) = 2$. (log means $\log_2$)

The N^3 Argument (cont.)

But the number of distinct schemata of order θ is $2^\theta\binom{L}{\theta}$, the number of ways to pick θ different positions and assign all possible binary values to each subset of the θ positions. (Our example, if L=6, yields 4*(6!/(4!*2!) = 4*30/2 = 60 distinct order-2 schemata)

So we are trying to argue that $2^\theta\binom{L}{\theta} \geq N^3$,

which implies that $2^\theta\binom{L}{\theta} \geq (2^\theta\varphi)^3$, since θ = log(N/ϕ).

The N^3 Argument (cont.)

Rather than proving anything general, Fitzpatrick & Grefenstette ('88) argued as follows: (remember that $\theta = \log(N/\phi)$).

To show:

$$2^\theta \binom{L}{\theta} \geq N^3$$

- Assume $L \geq 64$ and $2^6 \leq N \leq 2^{20}$
- Pick ϕ=8, which implies $3 \leq \theta \leq 17$
- By inspection (plug in N's, get θ's, etc.), the number of schemata processed is greater than N^3. So, as long as our population size is REASONABLE (64 to a million) and L is large enough (problem hard enough), the argument holds!
- (For N=64, L=64, ϕ=8, we get 8*(64*63*62/6) = 8*41,664=333,312 >> 64**3 = 262,144)
- But this deals with the initial population, and it does *not* necessarily hold for the latter stages of evolution. Still, it may help to explain why GA's can work so well…