CSE 847 (Spring 2021): Machine Learning— Homework 1

Instructor: Jiayu Zhou Due: Wednesday, Feb 10, 11:59PM Eastern Time. All submissions should be uploaded to the designated D2L dropbox.

1 Introduction

Questions in the textbook Pattern Recognition and Machine Learning:

- 1. (10 points) Page 58, Question 1.3
- 2. (10 points) Page 59, Question 1.6
- 3. (20 points) Page 59, Question 1.11

2 Linear Algebra I

1. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, E = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}.$$

If possible, compute the following:

- (a) $(2A)^T$
- (b) $(A B)^T$
- (c) $(3B^T 2A)^T$
- (d) $(-A)^T E$
- (e) $(C + D^T + E)^T$
- 2. (10 points) Which of the following are subspace of \mathbb{R}^2 ? Justify your answer.
 - (a) $\{(x,y) \in \mathbb{R}^2 | x^2 + y^2 = 0 \}$
 - (b) $\{(x,y) \in \mathbb{R}^2 | x^2 y^2 = 0 \}$
 - (c) $\{(x,y) \in \mathbb{R}^2 | x^2 y = 0 \}$
 - (d) $\{(x,y) \in \mathbb{R}^2 | x y = 0\}$
 - (e) $\{(x,y) \in \mathbb{R}^2 | x y = 1\}$
- 3. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

Is AB = BA? Justify your answer.

- 4. (a) (10 points) Let A be an $m \times n$ matrix with a rwo consisting entirely of zeros. Show that if B is an $n \times p$ matrix, then AB has a row of zeros.
 - (b) (10 points) Let A be an $m \times n$ matrix with a column consisting entirely of zeros, and let B be $p \times m$. Show that BA has a column of zeros.
- 5. (10 points) Let $A \in \mathbb{R}^{m \times r}$ and $B \in \mathbb{R}^{r \times n}$ be two matrices. Show that $\operatorname{rank}(AB) \leq \min\{\operatorname{rank}(A), \operatorname{rank}(B)\}$.