Let $f_d(n, k)$ be the number of n-digit, base-d numbers where no digit occurs more than k times (note that leading zeroes are allowed in this definition)

Consider the mapping from a length-n string of base-10 digits to another such string, as follows:

1. Get the leftmost digit, and call it x.
2. Subtract x from all digits (including the leftmost one). If, after subtracting, the digit becomes less than zero, then add 10.

For example, 325990 maps to 092667, and 092667 maps to 092667.

From this, it is clear that:

1. Every length-n string maps to another length-n string that starts with a zero.
2. For every length-n string that starts with a zero, there are *exactly* 10 length-n strings that map to it. (one for every distinct value of x)

This means that the mapping described above is a perfect 10-to-1 mapping! It also means that among the $f_{10}(n, k)$ n-digit numbers where no digit occurs more than k times, $\frac{1}{10} f_{10}(n, k)$ begin with a zero, and so, the answer to the problem is $\frac{9}{10} f_{10}(n, k)$:)

Now how do we calculate $f_d(n, k)$? Simple:

$$f_d(n,k) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n > 0\text{, but } k = 0 \text{ or } d = 0 \\ \displaystyle\sum_{e=0}^{d} \binom{d}{e}\binom{n}{ek}\frac{(ek)!}{k!^e} f_{d-e}(n - ek, k - 1) & \text{if } n > 0\text{, } k > 0 \text{ and } d > 0 \end{cases}$$

To explain the last equality, let e be the number of digits that appear *exactly* k times. Then there are $\binom{d}{e}$ ways to choose those digits, $\binom{n}{ek}$ ways to choose the positions of those digits, $\frac{(ek)!}{k!^e}$ ways to order those digits in those positions, and $f_{d-e}(n-ek, k-1)$ ways to choose the rest of the digits. Summing for all possible values of e gives $f_d(n, k)$.

Author: kevinsogo