**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans1: C

#

from scipy.stats import norm

nd= norm (45,8) #mean=45min sd=8min P[x>50]

#P(x>50)

p=1-nd.cdf (50)

p.round(4)

#

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans2A:

mean=38y, SD=6, sample=400,38<x<44, x>44

#mean=38y, sd=6, sample=400

from scipy.stats import norm

nd1= norm (38,6) #mean, sd

#p(38<x<44)

p1=nd1.cdf (44) -nd1.cdf (38)

p1. round (4) #0.3413(34.13%)

#p(x>44)

p2=1-nd1.cdf (44)

p2. round (4) #0.1587(15.87%)

#From above we can see that, ages between 38 to 44 years probability =34.13% which is more than that of, 15.87% =probability of age more than 44years.

So, the statement “More employees at the processing center are older than 44 than between 38 and 44” is False

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans2B:

#P(x<30)

p3=nd1.cdf (30)

p3

0.09121121972586788\*400

#36.484487890347154 employees

So, the statement “A training program for employees under the age of 30 at the center would be expected to attract about 36 employees” is True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans3: I.I.D =Independent Identically Distributed

#2\*X1 distribution remains same has before, only magnitude is changed

#(x1+X2) both X1 and X2 have identical distribution so merging of data would not change the distribution, only magnitude of data will change

So, difference of (2\**X*1 ) and (*X*1 + *X*2 ) magnitude of graph will change but distribution will remain same.

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans4: 48.5, 151.5

#mean=100, sd=20

import numpy as np

from scipy import stats

from scipy.stats import norm

np.round(stats.norm.interval(0.99, loc = 100, scale = 20),1)

#array ([ 48.5, 151.5])

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans5A: mean=5+7=12, sd=squrt(9+16)=5

#Profit1 ~ N (540,225)

Profit1= (stats.norm.interval(0.95, loc =540 , scale =225))

Profit1# (99.00810347848784, 980.9918965215122)

#Rs 99 million to 980.99 million that it contains 95% probability for the annual profit of the company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans5B:

→formula X= mean+ Z\*SD; wherein from z table, 5 percentile = -1.645

→ X = 540(-1.645) \*225

X = Rs 169.875 million

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans5C:

# Probability of Division 1 making a loss P(X<0)

Rs3=stats.norm.cdf(0,5,3)

Rs3#0.0477903522728147

# Probability of Division 2 making a loss P(X<0)

Rs4=stats.norm.cdf(0,7,4)

Rs4#0.040059156863817086

#Division 1 has 4.77% probability of making no profit which is higher than division 2=4% probability