Assignment 5

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1 Problem

Find QR decomposition of matrix

$$\mathbf{V} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Let $\mathbf{x}$ and $\mathbf{y}$ be the column vectors of the given matrix.

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{y} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.0.2}$$

The column vectors can be expressed as follows,

$$\mathbf{x} = k_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\mathbf{y} = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

$$k_1 = \|\mathbf{x}\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\mathbf{x}}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \mathbf{y}}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\mathbf{y} - r_1 \mathbf{u}_1}{\|\mathbf{y} - r_1 \mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T \mathbf{y} \tag{2.0.9}$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{1^2 + (-1)^2} = \sqrt{2} \tag{2.0.12}$$

$$\mathbf{u}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{2.0.13}$$

$$r_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -\sqrt{2} \tag{2.0.14}$$

$$\tag{2.0.15}$$

Similarly using equation (2.0.8), we can have

$$\mathbf{u}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 1 \tag{2.0.17}$$

Thus putting the values from (2.0.12) to (2.0.17) in (2.0.10) we obtain QR decomposition,

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & -\sqrt{2} \\ 0 & 1 \end{pmatrix} \tag{2.0.18}$$