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Matrix Theory Assignment 10

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Abstract—This problem demonstrate a method of representation of transformations by Matrices.

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/ tree/master/Assignment 10

1 Problem

Let **T** be the linear operator on \mathbb{C} defined by

$$\mathbf{T}(x_1, x_2) = (x_1, 0). \tag{1.0.1}$$

Let β be the standard ordered basis for \mathbb{C} and let $\beta' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by

$$\alpha_1 = (1, i), \alpha_2 = (-i, 2).$$
 (1.0.2)

What is the matrix of **T** in the ordered basis β'

2 solution

Geometrically, **T** is a projection onto the x-axis.

We have

$$\alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix}$$
 (2.0.1)

For projection, let Consider Matrix A as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.2}$$

The matrix A is representation of the linear transformation T that is projection on x-axis. After applying linear operator T on it,

$$(\mathbf{T}(\alpha_1) \quad \mathbf{T}(\alpha_1)) = \mathbf{A} (\alpha_1 \quad \alpha_2)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} = \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix} (2.0.3)$$

Now, for finding the matrix of T in the ordered basis β' , we combine the 2.0.1 and 2.0.3 and use concept of row-reduction of the augmented matrix:

$$\begin{pmatrix} 1 & -i & 1 & -i \\ i & 2 & 0 & 0 \end{pmatrix} \xrightarrow{R_2 = R_2 - iR_1} \begin{pmatrix} 1 & -i & 1 & -i \\ 0 & 1 & -i & -1 \end{pmatrix} (2.0.4)$$

$$\xrightarrow{R_1 = R_1 + iR_2} \begin{pmatrix} 1 & 0 & 2 & -2i \\ 0 & 1 & -i & -1 \end{pmatrix} (2.0.5)$$

Hence, the matrix of T in the ordered basis β' is

$$\begin{pmatrix} 2 & 2i \\ -i & -1 \end{pmatrix} \tag{2.0.6}$$