Matrix Theory Assignment 8

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***Abstract*—This problem demonstrate a method to find weather given transformation is linear or not.**

All the codes for the figure in this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_8

1 Problem

Find weather given functions $\mathbf{T}$ from $\mathbb{R}^2$ into $\mathbb{R}^2$ are linear transformations or not

$$\mathbf{T}(x_1, x_2) = ({x_1}^2, x_2) \tag{1.0.1}$$

2 solution

Let

$$\mathbf{A} = \begin{pmatrix} x_1, x_2 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.1}$$

and

$$\mathbf{Q} = \begin{pmatrix} x_1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.2}$$

We know that if we have $\mathbf{P}$ as a fixed matrix with entries in the field $\mathbb{F}$ and $\mathbf{Q}$ be a fixed matrix over $\mathbb{F}$. then we can define a function $\mathbf{T}$ from the space $\mathbb{F}^{m\times n}$ into itself by $\mathbf{T}(\mathbf{A}) = \mathbf{PAQ}$. Then $\mathbf{T}$ is a linear transformation from $\mathbb{F}$ into $\mathbb{F}$.

Now given transformation can be written as :

$$\mathbf{T}(x_1, x_2) = \mathbf{PAQ} \tag{2.0.3}$$

$$\implies \mathbf{T}(x_1, x_2) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} x_1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.4}$$

We can observe that matrix $\mathbf{Q}$ is not a fixed matrix. Hence we can conclude that given transformation is not linear.

Let

$$\mathbf{T}(1, 0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.5}$$

$$\implies \mathbf{T}(1, 0) = (1, 0) \tag{2.0.6}$$

$$\mathbf{T}(-1, 0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{T}(-1, 0) = (1, 0) \tag{2.0.8}$$

If $\mathbf{T}$ were a linear transformation then we would have

$$\mathbf{T}((1, 0)) = (1, 0) \tag{2.0.9}$$

$$\implies T(-1(1, 0)) = -1.\mathbf{T}(1, 0) \tag{2.0.10}$$

$$\implies -1.(1, 0) = (-1, 0) \tag{2.0.11}$$

which is a contradiction, since

$$(1, 0) \neq (-1, 0). \tag{2.0.12}$$

Hence non-linear transformation.