Challenge Problem

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***Abstract*—This document demonstrate if the vectors are orthogonal then they will linear independent.**

1 Problem

We have to prove that the vectors which are orthogonal are also linearly independent

2 Solution

Consider that we have the linear combinations of the vectors is :

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} + \cdots + c_n\mathbf{v_n} = 0 \tag{2.0.1}$$

if they are linearly independent then equation 2.0.1 holds good if and only if

$$c_1 + c_2 + a_3 + \cdots + c_n = 0 \tag{2.0.2}$$

Let $\mathbf{v_i}$ is a vector from the set :

$$S = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_n}\} \tag{2.0.3}$$

$$\tag{2.0.4}$$

We can write that,

$$\mathbf{v_i}.0 = 0 \tag{2.0.5}$$

$$\implies \mathbf{v_i}.\left(c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} + \cdots + c_n\mathbf{v_n}\right) = \mathbf{0} \tag{2.0.6}$$

$$\implies \mathbf{v_i}.c_1\mathbf{v_1} + \mathbf{v_i}.c_2\mathbf{v_2} + \mathbf{v_i}.c_3\mathbf{v_3} + \cdots + \mathbf{v_i}.c_n\mathbf{v_n} = \mathbf{0} \tag{2.0.7}$$

Since vectors are orthogonal,

$$\mathbf{v_i}.\mathbf{v_j} = 0, \text{for all i} \neq \text{j} \tag{2.0.8}$$

Hence,

$$\mathbf{c_i}\mathbf{v_i}.\mathbf{v_i} = \mathbf{c_i}\left\|\mathbf{v_i}\right\|^2 \tag{2.0.9}$$

For non zero vector $\left\|\mathbf{v_i}\right\|^2 \neq 0$, hence $c_i = 0$. Therefore we can say that vectors are linearly independent.