Matrix Theory Assignment 9

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EE20RESCH11005

Abstract—**This problem demonstrate a method to find nature linear transformation.**

All the codes for the figure in this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_9

1 Problem

Let$\mathbf{T}$ and $\mathbf{U}$ be the linear operators on $\mathbb{R}^2$ defined by

$$\mathbf{T}(x_1, x_2) = (x_2, x_1) \tag{1.0.1}$$

and

$$\mathbf{U}(x_1, x_2) = (x_1, 0) \tag{1.0.2}$$

How would you describe T and U geometrically ?

2 solution

Geometrically, in the x-y plane, $\mathbf{T}$ is the reflection about the diagonal x = y and $\mathbf{U}$ is a projection onto the x-axis.

Let Consider Matrix A as

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.1}$$

The matrix $\mathbf{A}$ is representation of the linear transformation T across the line y=x with respect to the standard basis.

Let suppose

$$\mathbf{x_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2.0.2}$$

After applying linear operator $\mathbf{T}$ on it,

$$\mathbf{T}(x_1, x_2) = \mathbf{A}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.3}$$

$$\implies \mathbf{A}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{2.0.4}$$

Similarly

$$\mathbf{A}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{2.0.5}$$

Hence after applying Operator $\mathbf{T}$ on $\mathbf{x_1}$ and $\mathbf{x_2}$

$$\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{2.0.6}$$

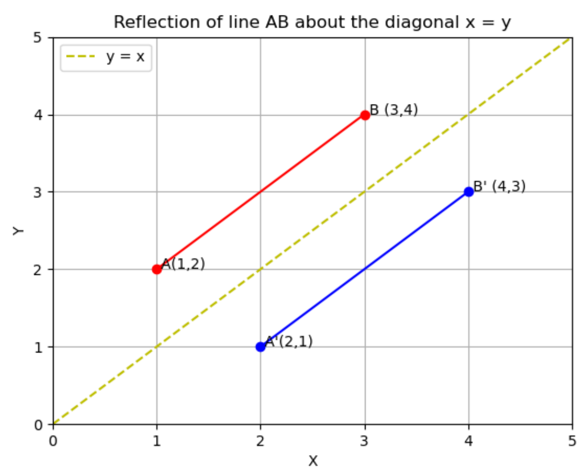

Fig. 1: Reflection of line AB about the x = y

For projection let Consider Matrix B as

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.7}$$

The matrix $\mathbf{B}$ is representation of the linear transformation $\mathbf{U}$ that is projection on x-axis.

Let suppose

$$\mathbf{x_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2.0.8}$$

After applying linear operator $\mathbf{U}$ on $\mathbf{x_1}$ and x_2,

$$\mathbf{T}(x_1, x_2) = \mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.9}$$

$$\implies \mathbf{B}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

Similarly

$$\mathbf{A}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.11}$$

Hence after applying Operator $\mathbf{U}$ on $\mathbf{x_1}$ and $\mathbf{x_2}$

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.12}$$

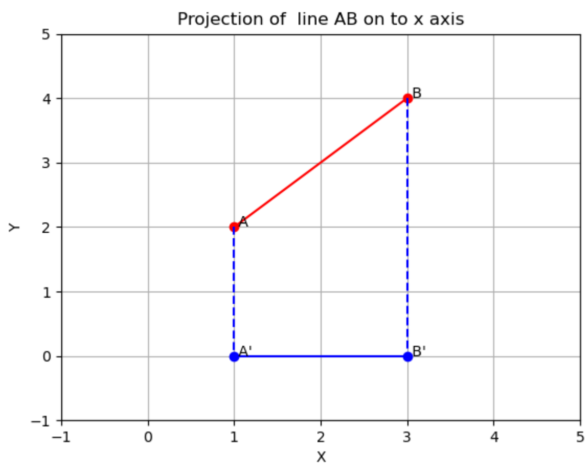

Fig. 2: Projection of AB onto x-axis