Matrix Theory Assignment 11

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Abstract—**This problem demonstrate a method of representation of transformations by Matrices.**

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_11

1 Problem

Let $\mathbb{F}$ be a filed and let f be the linear functional on $\mathbb{F}^2$ defined by,

$$f(x_1, x_2) = ax_1 + bx_2 \tag{1.0.1}$$

For given linear operator $\mathbf{T}$, such that

$$\mathbf{T}(x_1, x_2) = (-x_2, x_1) \tag{1.0.2}$$

Let

$$g = \mathbf{T}^t f \tag{1.0.3}$$

find $g(x_1, x_2)$

2 solution

We have,

$$g = \mathbf{T}^t f \tag{2.0.1}$$

$$\implies g(x_1, x_2) = \mathbf{T}^t f(x_1, x_2) \tag{2.0.2}$$

We know that, if $\mathbf{V}$ and $\mathbf{W}$ be vector spaces over the field $\mathbb{F}$. For each linear transformation $\mathbf{T}$ from $\mathbf{V}$ into $\mathbf{W}$, there is a unique linear transformation $\mathbf{T}^t$ from $\mathbf{W^*}$ into $\mathbf{V^*}$ such that,

$$(\mathbf{T}^t g)(\alpha) = g(\mathbf{T}\alpha) \tag{2.0.3}$$

Where for every g in $\mathbf{W^*}$ and α in $\mathbf{V}$.

Now using (2.0.3) in (2.0.2) we can write,

$$\mathbf{T}^t f(x_1, x_2) = f(\mathbf{T}(x_1, x_2)) \tag{2.0.4}$$

Using (1.0.2) in (2.0.4)

$$f(\mathbf{T}(x_1, x_2)) = f(-x_2, x_1) \tag{2.0.5}$$

From (1.0.1), we can write

$$f(-x_2, x_1) = -ax_2 + bx_1 \tag{2.0.6}$$

Hence,

$$\mathbf{T}^t f(x_1, x_2) = -ax_2 + bx_1 \tag{2.0.7}$$