Matrix Theory Assignment 10

Ritesh Kumar

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Abstract—This problem demonstrate a method of representation of transformations by Matrices.

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_10

1 PROBLEM

Let $\mathbf{T}$ be the linear operator on $\mathbb{C}$ defined by

$$\mathbf{T}(x_1, x_2) = (x_1, 0). \tag{1.0.1}$$

Let β be the standard ordered basis for $\mathbb{C}$ and let $\beta' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by

$$\alpha_1 = (1, i), \alpha_2 = (-i, 2). \tag{1.0.2}$$

What is the matrix of $\mathbf{T}$ in the ordered basis β'

2 SOLUTION

Let

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.1}$$

We have

$$\alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \tag{2.0.2}$$

So,

$$\beta' = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} \tag{2.0.3}$$

Geometrically, $\mathbf{T}$ is a projection onto the x-axis.

For projection, let Consider Matrix A as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.4}$$

The matrix $\mathbf{A}$ is representation of the linear transformation $\mathbf{T}$ that is projection on x-axis. After applying linear operator $\mathbf{T}$ on it,

$$\mathbf{T}(\beta') = \mathbf{A}\beta' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} = \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix} \tag{2.0.5}$$

Now, for finding the matrix of $\mathbf{T}$ in the ordered basis β', we combine the 2.0.2 and 2.0.5 and use concept of row-reduction of the augmented matrix:

$$\begin{pmatrix} 1 & -i & 1 & -i \\ i & 2 & 0 & 0 \end{pmatrix} \xleftarrow{R_2 = R_2 - iR_1} \begin{pmatrix} 1 & -i & 1 & -i \\ 0 & 1 & -i & -1 \end{pmatrix} \tag{2.0.6}$$

$$\xleftarrow{R_1 = R_1 + iR_2} \begin{pmatrix} 1 & 0 & 2 & -2i \\ 0 & 1 & -i & -1 \end{pmatrix} \tag{2.0.7}$$

Hence, the matrix of $\mathbf{T}$ in the ordered basis β' is

$$\begin{pmatrix} 2 & 2i \\ -i & -1 \end{pmatrix} \tag{2.0.8}$$

And this can also be represented using β and β'

$$\mathbf{T}(\beta) = \begin{pmatrix} 2 & 2i \\ -i & -1 \end{pmatrix}\beta' \tag{2.0.9}$$