Matrix Theory : Assignment 2

Ritesh Kumar
Roll No: EE20RESCH11005

Abstract—**This is a problem to balance a chemical equation using system of linear equations.**

Download all codes from

1 PROBLEM

Balance the following chemical equation.

Zinc + Silver nitrate → Zinc nitrate + Silver
$$(1.0.1)$$
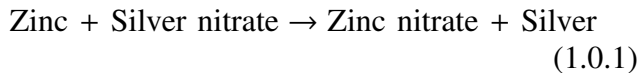
2 SOLUTION

Equation 1.0.1 can be written as :

$$Zn + AgNO_3 \rightarrow Ag + Zn(NO_3)_2 \qquad (2.0.1)$$

Suppose balance form of the equation is :

$$x_1 Zn + x_2 AgNo_3 \rightarrow x_3 Ag + x_4 Zn(NO_3)_2 \qquad (2.0.2)$$
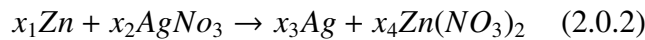
which results in the following equations:

$$(x_1 - 2x_4)Zn = 0 \qquad (2.0.3)$$
$$(x_2 - x_3)Ag = 0 \qquad (2.0.4)$$
$$(x_3 - 2x_4)N = 0 \qquad (2.0.5)$$
$$(3x_3 - 6x_4)O = 0 \qquad (2.0.6)$$

which can be expressed as

$$x_1 + 0.x_2 + 0.x_3 - x_4 = 0 \qquad (2.0.7)$$
$$0.x_1 + x_2 - x_3 + 0.x_4 = 0 \qquad (2.0.8)$$
$$0.x_1 + 0.x_2 + x_3 - 2.x_4 = 0 \qquad (2.0.9)$$
$$0.x_1 + 0.x_2 + 3x_3 - 6.x_4 = 0 \qquad (2.0.10)$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 3 & -6 \end{pmatrix} X = 0 \qquad (2.0.11)$$

where,

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad (2.0.12)$$

(2.0.11) can be reduced as follows:

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 3 & -6 \end{pmatrix} \xrightarrow{R_4 \leftarrow R_4 - 3R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
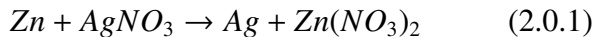$$(2.0.13)$$

Thus,

$$x_1 = x_4, \, x_2 = 2x_4, \, x_3 = 2x_4 \qquad (2.0.14)$$

$$\implies X = \begin{pmatrix} x_4 \\ 2x_4 \\ 2x_4 \\ x_4 \end{pmatrix} = x_4 \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \qquad (2.0.15)$$

by substituting $x_4 = 1$, we get :

$$\implies X = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \qquad (2.0.16)$$

Hence, (2.0.2) finally becomes

$$Zn + 2AgNo_3 \rightarrow 2Ag + Zn(NO_3)_2 \qquad (2.0.17)$$