Matrix Theory Assignment 17

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EE20RESCH11005

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_17

1 Problem

Let $f:\mathbb{R}^n \to \mathbb{R}^n$ be a continuous function. such that

$$\int_{\mathbb{R}^n} |f(x)dx| < \infty \tag{1.0.1}$$

Let A be a real $n\times n$ invertible matrix and for $x,y \in \mathbb{R}^n$. Let $<x,y>$ denotes the standard inner product in $\mathbb{R}^n$ then, $\int_{\mathbb{R}^n} f(Ax)e^{i<y,x>}dx = ?$

1) $\int_{\mathbb{R}^n} f(x)e^{i<(A^{-1})^T y,x>}\frac{dx}{|\det(A)|}$
2) $\int_{\mathbb{R}^n} f(x)e^{i<A^T y,x>}\frac{dx}{|\det(A)|}$.
3) $\int_{\mathbb{R}^n} f(x)e^{i<(A^T)^{-1} y,x>}dx$.
4) $\int_{\mathbb{R}^n} f(x)e^{i<A^{-1} y,x>}\frac{dx}{|\det(A)|}$.

2 solution

Let consider,

$$Ax = t \tag{2.0.1}$$

$$\implies x = A^{-1}t \tag{2.0.2}$$

$$\implies dx = \frac{dx}{|\det(A)|} \tag{2.0.3}$$

Using (2.0.1) to (2.0.3), We can write:

$$\int_{\mathbb{R}^n} f(Ax)e^{i<y,x>}dx = \int_{\mathbb{R}^n} f(t)e^{i<y,(A^{-1}t)>}\frac{dt}{|\det(A)|} \tag{2.0.4}$$

We know that,

$$<x,y> = x^T y = y^T x \tag{2.0.5}$$

$$\implies <y,(A^{-1}t)> = (y^T A^{-1}t) \tag{2.0.6}$$

And

$$\implies <(A^{-1})^T y, t> = ((A^{-1})^T y)^T t \tag{2.0.7}$$

$$\implies ((A^{-1})^T y)^T t = (y^T((A^{-1})^T)^T t) = (y^T A^{-1} t) \tag{2.0.8}$$

Hence, from (2.0.6) and (2.0.8)

$$<y,(A^{-1}t)> = <(A^{-1})^T y, t> \tag{2.0.9}$$

Using (2.0.9) in (2.0.4) We can write,

$$\implies \int_{\mathbb{R}^n} f(t)e^{i<(A^{-1})^T y,t>}\frac{dt}{|\det(A)|} \tag{2.0.10}$$

Hence option 1 is correct.