

# Matrix Theory Assignment 7

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**Abstract**—This problem demonstrate a method to find the solution of the given system of equation using linear algebra.

All the codes for the figure in this document can be found at

[https://github.com/Ritesh622/Assignment\\_EE5609/tree/master/Assignment\\_7](https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_7)

## 1 PROBLEM

Consider the system of the equations

$$x_1 - x_2 + 2x_3 = 1 \quad (1.0.1)$$

$$x_1 - 0x_2 + 2x_3 = 1 \quad (1.0.2)$$

$$x_1 - 3x_2 + 4x_3 = 2 \quad (1.0.3)$$

Does this system have a solution ? If so describe explicitly all solutions.

## 2 SOLUTION

From equation (1.0.1) to (1.0.3) we can write,

$$\begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & 2 \\ 1 & -3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.1)$$

$$\Rightarrow \mathbf{Ax} = \mathbf{b} \quad (2.0.2)$$

Where,

$$(2.0.3)$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & 2 \\ 1 & -3 & 4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (2.0.4)$$

Solving the matrix  $\mathbf{A}$  for rank we get,

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & 2 \\ 1 & -3 & 4 \end{pmatrix} \xleftrightarrow{R_2=R_1-2R_1} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -2 \\ 1 & -3 & 4 \end{pmatrix} \quad (2.0.5)$$

$$\xleftrightarrow{R_3=R_3-R_1} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -2 \\ 0 & -2 & 2 \end{pmatrix} \quad (2.0.6)$$

$$\xleftrightarrow{R_3=R_3+R_2} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.7)$$

Hence, rank  $(\mathbf{A}) = 2$ . Now solving the augmented matrix of (2.0.2) we get,

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 1 & -3 & 4 & 2 \end{pmatrix} \xleftrightarrow{R_2=R_1-2R_1} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & -2 & -1 \\ 1 & -3 & 4 & 2 \end{pmatrix} \quad (2.0.8)$$

$$\xleftrightarrow{R_3=R_3-R_1} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & -2 & -1 \\ 0 & -2 & 2 & 1 \end{pmatrix} \quad (2.0.9)$$

$$\xleftrightarrow{R_3=R_3+R_2} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.10)$$

We have rank  $(\mathbf{A}) = \text{rank}(\mathbf{A} : \mathbf{b}) = 2 < n$ , where  $n = 3$ . Hence we have infinite no of solutions for given system of equations.

Using Gauss - Jordan elimination method to getting the solution,

$$\begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 1 & -3 & 4 & 2 \end{pmatrix} \xleftrightarrow{R_2=R_1-2R_1} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & -2 & -1 \\ 1 & -3 & 4 & 2 \end{pmatrix} \quad (2.0.11)$$

$$\xleftrightarrow{R_3=R_3-R_1} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & -2 & -1 \\ 0 & -2 & 2 & 1 \end{pmatrix} \quad (2.0.12)$$

$$\xleftrightarrow{R_2=\frac{R_2}{2}} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -\frac{1}{2} \\ 0 & -2 & 2 & 1 \end{pmatrix} \quad (2.0.13)$$

$$\xleftrightarrow{R_3=R_3+2R_2} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.14)$$

$$\xleftrightarrow{R_1=R_1+R_2} \begin{pmatrix} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & -1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.15)$$

$$\implies x_1 + x_3 = \frac{1}{2}, x_2 - x_3 = -\frac{1}{2} \quad (2.0.16)$$

$$\implies x_2 = -\frac{1}{2} + x_3, x_1 = \frac{1}{2} - x_3 \quad (2.0.17)$$

From equation (2.0.16) and (2.0.17)

$$\mathbf{x} = \begin{pmatrix} \frac{1}{2} - x_3 \\ -\frac{1}{2} + x_3 \\ x_3 \end{pmatrix} \quad (2.0.18)$$