

Matrix Theory Assignment 10

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Abstract—This problem demonstrate a method of representation of transformations by Matrices.

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_10

1 PROBLEM

Let \mathbf{T} be the linear operator on \mathbb{C} defined by

$$\mathbf{T}(x_1, x_2) = (x_1, 0). \quad (1.0.1)$$

Let β be the standard ordered basis for \mathbb{C} and let $\beta' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by

$$\alpha_1 = (1, i), \alpha_2 = (-i, 2). \quad (1.0.2)$$

What is the matrix of \mathbf{T} in the ordered basis β'

2 SOLUTION

Geometrically, \mathbf{T} is a projection onto the x-axis.

We have

$$\alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \quad (2.0.1)$$

For projection, let Consider Matrix \mathbf{A} as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.0.2)$$

The matrix \mathbf{A} is representation of the linear transformation \mathbf{T} that is projection on x-axis. After applying linear operator \mathbf{T} on it,

$$\mathbf{T}\alpha_1 = \mathbf{A}\alpha_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{T}\alpha_2 = \mathbf{A}\alpha_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -i \\ 2 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} \quad (2.0.4)$$

Now, for finding the matrix of \mathbf{T} in the ordered basis β' , we combine the 2.0.1, 2.0.3 and 2.0.4 and use concept of row-reduction of the augmented matrix:

$$\begin{pmatrix} 1 & -i & 1 & -i \\ i & 2 & 0 & 0 \end{pmatrix} \xrightarrow{R_2=R_2-iR_1} \begin{pmatrix} 1 & -i & 1 & -i \\ 0 & 1 & -i & -1 \end{pmatrix} \quad (2.0.5)$$

$$\xrightarrow{R_1=R_1+iR_2} \begin{pmatrix} 1 & 0 & 2 & -2i \\ 0 & 1 & -i & -1 \end{pmatrix} \quad (2.0.6)$$

Hence, the matrix of \mathbf{T} in the ordered basis β' is

$$\begin{pmatrix} 2 & 2i \\ -i & -1 \end{pmatrix} \quad (2.0.7)$$