

# Matrix Theory : Assignment 3

Ritesh Kumar

Roll no. : EE20RESCH11005

**Abstract**—This problem is to demonstrate the way to prove the triangles are congruent and to prove a triangle as isosceles using matrix algebra.

## 1 PROBLEM

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal. Show that

$$1) \triangle ABC \cong \triangle ACF$$

## 2 SOLUTION

### 2.1 part 1

Let consider we have a triangle  $\triangle ABC$ . There are two altitudes BE and CF being drawn from the vertices B and C respectively.

In  $\triangle ABE$ , taking inner product of sides AE and EB we can write :

$$(\mathbf{A} - \mathbf{E})^T (\mathbf{E} - \mathbf{B}) = \|\mathbf{A} - \mathbf{E}\| \|\mathbf{E} - \mathbf{B}\| \cos \angle AEB \quad (2.1.1)$$

$$\Rightarrow \cos \angle AEB = \frac{(\mathbf{A} - \mathbf{E})^T (\mathbf{E} - \mathbf{B})}{\|\mathbf{A} - \mathbf{E}\| \|\mathbf{E} - \mathbf{B}\|} \quad (2.1.2)$$

In  $\triangle ACF$ , taking inner product of sides AF and FC :

$$(\mathbf{A} - \mathbf{F})^T (\mathbf{F} - \mathbf{C}) = \|\mathbf{A} - \mathbf{F}\| \|\mathbf{F} - \mathbf{C}\| \cos \angle AFC \quad (2.1.3)$$

$$\Rightarrow \cos \angle AFC = \frac{(\mathbf{A} - \mathbf{F})^T (\mathbf{F} - \mathbf{C})}{\|\mathbf{A} - \mathbf{F}\| \|\mathbf{F} - \mathbf{C}\|} \quad (2.1.4)$$

In triangle  $\triangle ABC$ ,

$$\cos \angle AFC = \cos \angle AEB \quad (CF \perp AB \text{ \& } BE \perp AC) \quad (2.1.5)$$

Given,

$$\|\mathbf{E} - \mathbf{B}\| = \|\mathbf{F} - \mathbf{C}\| \quad (2.1.6)$$

$$\angle FAC = \angle EAB \text{ (Common angle)} \quad (2.1.7)$$

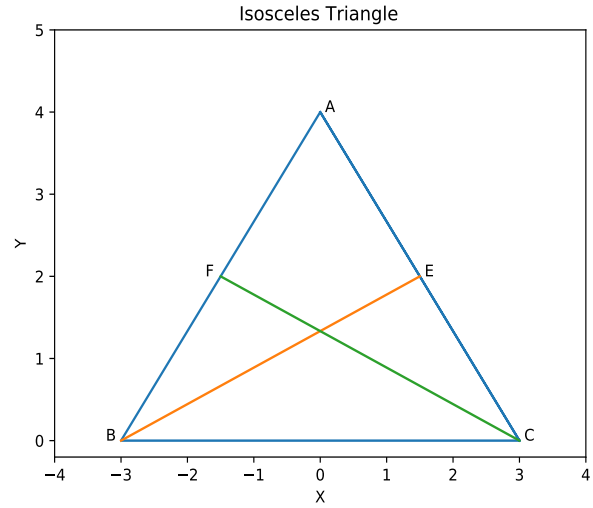


Fig. 1: Isosceles triangle

We know that if the two angles of triangles are equal then the third angle will also be equal. Hence,

$$\angle FCA = \angle EBA \quad (2.1.8)$$

Hence by ASA ( Angle - Side - Angle ) We can say that ,  $\triangle ABC \cong \triangle ACF$ .