

Matrix Theory Assignment 16

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All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_16

Similarly, eigen values of **B** is given as:

$$\begin{pmatrix} 2-\lambda & 10 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{pmatrix} \quad (2.1.4)$$

$$\Rightarrow -\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0 \quad (2.1.5)$$

$$\Rightarrow \lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 3. \quad (2.1.6)$$

1 PROBLEM

Consider a matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix} \quad (1.0.1)$$

and,

$$\mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (1.0.2)$$

Then which of following is true,

- 1) **A** and **B** is similar over the field of rational numbers.
- 2) **A** is diagonalizable over the field of rational numbers \mathbb{Q} .
- 3) **B** is the Jordan canonical form of **A**.
- 4) The minimal polynomial and the characteristic polynomial of **A** are the same.

2 SOLUTION

2.1 Part 1

Two matrix are said to be similar if their eigen values are same.

Eigen value of **A** is given as:

$$\begin{pmatrix} 2-\lambda & 2 & 1 \\ 0 & 2-\lambda & -1 \\ 0 & 0 & 3-\lambda \end{pmatrix} = 0 \quad (2.1.1)$$

$$\Rightarrow -\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0 \quad (2.1.2)$$

$$\Rightarrow \lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 3. \quad (2.1.3)$$

Hence, matrices **A** and **B** are similar.

2.2 Part B

Matrix **A** is diagonalizable if and only if there is a basis of \mathbb{R}^3 consisting of eigenvectors of **A**.

From (2.1.3), our eigenvalues for **A** are,

$$\lambda_1 = \lambda_2 = 2 \quad (2.2.1)$$

and,

$$\lambda_3 = 3. \quad (2.2.2)$$

Hence $\lambda_1 = \lambda_2$ is a repeated root with multiplicity two. Hence, We can get only two linearly independent eigenvectors for **A**, are given as :

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \quad (2.2.3)$$

But any basis for \mathbb{R}^3 consists of three vectors. Therefore there is no third eigenbasis for **A**, hence **A** is not diagonalizable.

2.3 part 3

From (2.1.3) we have eigenvalue $\lambda_1 = 2$ with geometric multiplicity 2. Hence the Jordan canonical form of **A** can be written as :

$$\mathbf{J}_A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad (2.3.1)$$

Hence **B** is the Jordan canonical form of **A**.

2.4 Part 4

From (2.1.3), the characteristic polynomial of this matrix is:

$$f(\lambda) = -\lambda^3 + 7\lambda^2 - 16\lambda + 12 = (\lambda - 2)^2(\lambda - 3) \quad (2.4.1)$$

Minimal polynomial for a matrix is a smallest polynomial for which

$$M_{\mathbf{A}}(x) = 0 \quad (2.4.2)$$

Using (2.4.2), we found minimal polynomial of \mathbf{A} is :

$$M_{\mathbf{A}}(x) = (x - 2)^2(x - 3) \quad (2.4.3)$$