Matrix Theory Assignment 10

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Abstract—**This problem demonstrate a method of representation of transformations by Matrices.**

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_10

1 Problem

Let $\mathbf{T}$ be the linear operator on $\mathbb{C}$ defined by

$$\mathbf{T}(x_1, x_2) = (x_1, 0). \tag{1.0.1}$$

Let β be the standard ordered basis for $\mathbb{C}$ and let $\beta' = \{\alpha_{\mathbf{1}}, \alpha_{\mathbf{2}}\}$ be the ordered basis defined by

$$\alpha_{\mathbf{1}} = (1, i), \alpha_{\mathbf{2}} = (-i, 2). \tag{1.0.2}$$

What is the matrix of $\mathbf{T}$ in the ordered basis β'

2 solution

Let

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.1}$$

We have

$$\alpha_{\mathbf{1}} = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_{\mathbf{2}} = \begin{pmatrix} -i \\ 2 \end{pmatrix} \tag{2.0.2}$$

So,

$$\beta' = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} \tag{2.0.3}$$

Geometrically, $\mathbf{T}$ is a projection onto the x-axis.

For projection, let Consider Matrix A as

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.4}$$

The matrix $\mathbf{A}$ is representation of the linear transformation $\mathbf{T}$ that is projection on x-axis. After applying linear operator $\mathbf{T}$ on it,

$$\mathbf{T}(\beta') = \mathbf{A}\beta' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} = \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix} \tag{2.0.5}$$

Now, for finding the matrix of $\mathbf{T}$ in the ordered basis β',we combine the 2.0.2 and 2.0.5 and use concept of row-reduction of the augmented matrix:

$$\begin{pmatrix} 1 & -i & 1 & -i \\ i & 2 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2=R_2-iR_1} \begin{pmatrix} 1 & -i & 1 & -i \\ 0 & 1 & -i & -1 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_1=R_1+iR_2} \begin{pmatrix} 1 & 0 & 2 & -2i \\ 0 & 1 & -i & -1 \end{pmatrix} \tag{2.0.7}$$

Hence, the matrix of $\mathbf{T}$ in the ordered basis β' is

$$\mathbf{B} = \begin{pmatrix} 2 & 2i \\ -i & -1 \end{pmatrix} \tag{2.0.8}$$

And this can also be represented using β and β', which shows the relation between $\mathbf{T}$, β and β' .

$$\mathbf{T}(\beta) = \mathbf{A}\beta = \mathbf{B}\beta^{'} \tag{2.0.9}$$