

Matrix Theory : Assignment 3

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Abstract—This problem is to demonstrate the way to prove a triangle as isosceles using matrix algebra.

Download latex and python codes from

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_3

$$(C - A)^T (A - B) + \|A - B\|^2 + (B - F)^T (A - B) = 0 \quad (2.0.9)$$

1 PROBLEM

ABC is a triangle in which altitudes BE and CF to sides AC and AB, are equal. Show that

1) $AB = AC$ i.e, $\triangle ABC$ is an isosceles triangle.

2 SOLUTION

Let consider we have a triangle $\triangle ABC$. There are two altitudes BE and CF being drawn from the vertices B and C respectively. And we have ,

$$\|E - B\| = \|F - C\| \quad (2.0.1)$$

Let \mathbf{m}_{AB} and \mathbf{m}_{CF} are the direction vectors of AB and CF respectively. Since $AB \perp CF$ hence,

$$\mathbf{m}_{AB} \mathbf{m}_{CF} = 0 \quad (2.0.2)$$

$$(B - E)^T (A - C) = 0 \quad (2.0.3)$$

$$(B - A + A - C + C - E)^T (A - C) = 0 \quad (2.0.4)$$

$$(B - A)^T (A - C) + \|A - C\|^2 + (C - E)^T (A - C) = 0 \quad (2.0.5)$$

Similarly, $AC \perp BE$ hence,

$$\mathbf{m}_{AC} \mathbf{m}_{BE} = 0 \quad (2.0.6)$$

$$(C - F)^T (A - B) = 0 \quad (2.0.7)$$

$$(C - A + A - B + B - F)^T (A - B) = 0 \quad (2.0.8)$$

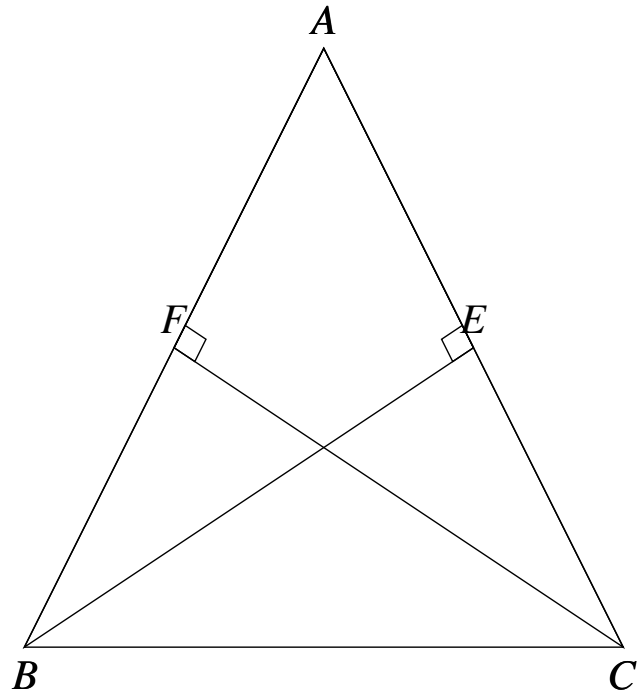


Fig. 1: Isosceles triangle ABC

In $\triangle ABC$, taking inner product of sides AB and AC we can write :

$$\Rightarrow \cos \angle BAC = \frac{(B - A)^T (A - C)}{\|B - A\| \|A - C\|} \quad (2.0.10)$$

and,

$$(C - A)^T (A - B) = \|C - A\| \|A - B\| \cos \angle ABC \quad (2.0.11)$$

From equation 2.0.10, and 2.0.11, we have ,

$$(B - A)^T (A - C) = (C - A)^T (A - B) \quad (2.0.12)$$

using equation 2.0.12 in 2.0.5 and 2.0.9 we can

write,

$$\begin{aligned} \|\mathbf{A} - \mathbf{C}\|^2 + (\mathbf{C} - \mathbf{E})^T (\mathbf{A} - \mathbf{C}) &= \|\mathbf{A} - \mathbf{B}\|^2 \\ &+ (\mathbf{B} - \mathbf{F})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.13) \end{aligned}$$

$$\begin{aligned} \|\mathbf{A} - \mathbf{C}\|^2 + (\mathbf{C} - \mathbf{E})^T (\mathbf{A} - \mathbf{C}) &= \|\mathbf{A} - \mathbf{B}\|^2 \\ &+ (\mathbf{B} - \mathbf{F})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.14) \end{aligned}$$

$$\begin{aligned} \|\mathbf{A} - \mathbf{C}\|^2 + (\mathbf{C} - \mathbf{A} + \mathbf{A} - \mathbf{B} + \mathbf{B} - \mathbf{E})^T (\mathbf{A} - \mathbf{C}) &= \\ \|\mathbf{A} - \mathbf{B}\|^2 + (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{C} + \mathbf{C} - \mathbf{F})^T (\mathbf{A} - \mathbf{B}) &= \\ (2.0.15) \end{aligned}$$

$$\begin{aligned} \|\mathbf{A} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2 + (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) + (\mathbf{B} - \mathbf{E})^T \\ (\mathbf{A} - \mathbf{C}) &= \|\mathbf{A} - \mathbf{B}\|^2 + \|\mathbf{A} - \mathbf{B}\|^2 + (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) \\ &+ (\mathbf{C} - \mathbf{F})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.16) \end{aligned}$$

since $\mathbf{BE} \perp \mathbf{AC}$ and $\mathbf{CF} \perp \mathbf{AB}$, hence :

$$(\mathbf{B} - \mathbf{E})^T (\mathbf{A} - \mathbf{C}) = \mathbf{0} \quad (2.0.17)$$

and,

$$(\mathbf{C} - \mathbf{F})^T (\mathbf{A} - \mathbf{B}) = \mathbf{0} \quad (2.0.18)$$

Now equation 2.0.16 become :

$$\begin{aligned} 2\|\mathbf{A} - \mathbf{C}\|^2 + (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) &= \\ 2\|\mathbf{A} - \mathbf{B}\|^2 + (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) &= \quad (2.0.19) \end{aligned}$$

Using equation 2.0.12 in equation 2.0.19,

$$\|\mathbf{A} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{B}\| \quad (2.0.20)$$