Matrix Theory Assignment 9

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Abstract—This problem demonstrate a method to find nature linear transformation.

All the codes for the figure in this document can be found at

1 PROBLEM

Let **T** and **U** be the linear operators on $\mathbb{R}^2$ defined by

$$\mathbf{T}(x_1, x_2) = (x_2, x_1) \qquad (1.0.1)$$

and

$$\mathbf{U}(x_1, x_2) = (x_1, 0) \qquad (1.0.2)$$

How would you describe T and U geometrically ?

2 SOLUTION

Geometrically, in the x-y plane, **T** is the reflection about the diagonal x = y and **U** is a projection onto the x-axis.

1) **Reflection**

Let Consider Matrix A as

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (2.0.1)$$

The matrix **A** is representation of the linear transformation T across the line y=x with respect to the standard basis.

Let suppose

$$\mathbf{x_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad (2.0.2)$$

After applying linear operator **T** on it,

$$\mathbf{T}(x_1, x_2) = \mathbf{A}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad (2.0.3)$$

$$\implies \mathbf{A}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad (2.0.4)$$

Similarly

$$\mathbf{A}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \qquad (2.0.5)$$

Hence after applying Operator **T** on $\mathbf{x_1}$ and $\mathbf{x_2}$

$$\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \qquad (2.0.6)$$
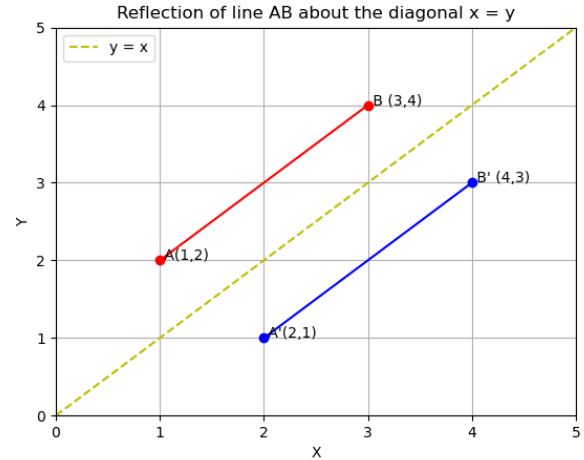

Fig. 1: Reflection of line AB about the x = y

2) **Projection**

For projection let Consider Matrix B as

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (2.0.7)$$

The matrix **B** is representation of the linear transformation **U** that is projection on x-axis.

Let suppose

$$\mathbf{x_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad (2.0.8)$$

After applying linear operator $\mathbf{U}$ on $\mathbf{x_1}$ and x_2,

$$\mathbf{T}(x_1, x_2) = \mathbf{U}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad (2.0.9)$$

$$\implies \mathbf{B}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2.0.10)$$

Similarly

$$\mathbf{A}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \qquad (2.0.11)$$
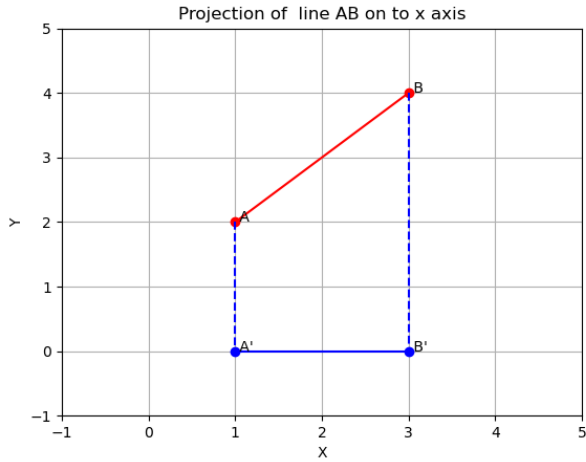

Fig. 2: Projection of AB onto x-axis

Hence after applying Operator $\mathbf{U}$ on $\mathbf{x_1}$ and $\mathbf{x_2}$

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \qquad (2.0.12)$$