Matrix Theory Assignment 12

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***Abstract*—This problem is all about to to introducing the concept of linear algebra over a filed.**

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_12

1 Problem

If a and b are element of a filed $\mathbb{F}$ and $a \neq 0$, show that the ploynomial $1, ax + b, (ax+b)^2, (ax+b)^3, \dots$ form a basis of $\mathbb{F}[x]$.

2 solution

Let consider we have a set S such that,

$$S = \left\{1, ax+b, (ax+b)^2, (ax+b)^3, \dots\right\} \quad (2.0.1)$$

And let $\langle S \rangle$ be the subspace, that is spanned by S. We know that if any set which is forms with combination of two elements, and if no two elements of S have the same degree in S, then all elements in S will be independent set in $\mathbb{F}[x]$.
Since $1 \in S, ax + b \in S$, and hence $b.1 + \frac{a}{a}(a + bx) \in \langle S \rangle$, it follows $x \in \langle S \rangle$.

Now to prove $x^2 \in \langle S \rangle$ let consider another element form S which is $(ax+b)^2$.
Subtracting , $1.a^2 + 2.a.b.x$ from $(ax+b)^2$

$$(ax+b)^2 - a^2 - 2.a.b.x = a^2.x^2 \quad (2.0.2)$$

from 2.0.2 we conclude that $a^2.x^2$ will also $\in \langle S \rangle$. Now, $\frac{1}{a^2}.a^2.x^2 \in S$. Thus $x^2 \in \langle S \rangle$. Hence using this concept with higher degree we can prove that, $x^n \in \langle S \rangle \ \forall\ n$.
Consider,

$$S' = \left\{1, x, x^2, x^3, \dots\right\} \quad (2.0.3)$$

Hence we can say that, (2.0.3) span the space of all polynomials which form with $(ax+b)^n$. Hence we conclude that S spans the space of all polynomials.