Matrix Theory Assignment 17

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EE20RESCH11005

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_17

1 Problem

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function. such that

$$\int_{\mathbb{R}^n} |f(x)dx| < \infty \tag{1.0.1}$$

Let A be a real $n \times n$ invertible matrix and for $x, y \in \mathbb{R}^n$. Let $\langle x, y\rangle$ denotes the standard inner product in $\mathbb{R}^n$ then, $\int_{\mathbb{R}^n} f(Ax)e^{i\langle y,x\rangle}dx =?$

1) $\int_{\mathbb{R}^n} f(x)e^{i\langle (A^{-1})^T y,x\rangle}\frac{dx}{|\det(A)|}$
2) $\int_{\mathbb{R}^n} f(x)e^{i\langle A^T y,x\rangle}\frac{dx}{|\det(A)|}$.
3) $\int_{\mathbb{R}^n} f(x)e^{i\langle (A^T)^{-1} y,x\rangle}dx$.
4) $\int_{\mathbb{R}^n} f(x)e^{i\langle A^{-1} y,x\rangle}\frac{dx}{|\det(A)|}$.

2 solution

Let consider,

$$Ax = t \tag{2.0.1}$$

$$\implies x = A^{-1}t \tag{2.0.2}$$

$$\implies dx = \frac{dx}{|\det(A)|} \tag{2.0.3}$$

Using (2.0.1) to (2.0.3), We can write:

$$\int_{\mathbb{R}^n} f(Ax)e^{i\langle y,x\rangle}dx = \int_{\mathbb{R}^n} f(t)e^{i\langle y,(A^{-1}t)\rangle}\frac{dt}{|\det(A)|} \tag{2.0.4}$$

We know that,

$$\langle x, y\rangle = x^T y = y^T x \tag{2.0.5}$$

$$\implies \langle y, (A^{-1}t)\rangle = (y^T A^{-1}t) \tag{2.0.6}$$

And

$$\implies \langle (A^{-1})^T y, t\rangle = ((A^{-1})^T y)^T t \tag{2.0.7}$$

$$\implies ((A^{-1})^T y)^T t = (y^T((A^{-1})^T)^T t) = (y^T A^{-1}t) \tag{2.0.8}$$

Hence, from (2.0.6) and (2.0.8)

$$\langle y, (A^{-1}t)\rangle = \langle (A^{-1})^T y, t\rangle \tag{2.0.9}$$

Using (2.0.9) in (2.0.4) We can write,

$$\implies \int_{\mathbb{R}^n} f(t)e^{i\langle (A^{-1})^T y,t\rangle}\frac{dt}{|\det(A)|} \tag{2.0.10}$$

replacing variable t with x.

$$\implies \int_{\mathbb{R}^n} f(t)e^{i\langle (A^{-1})^T y,x\rangle}\frac{dx}{|\det(A)|} \tag{2.0.11}$$

Hence option 1 is correct.