

# Matrix Theory Assignment 11

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**Abstract—This problem demonstrate a method of find the Transpose of linear transformations by Linear algebra.**

Using (1.0.2) in (2.0.4)

$$f(\mathbf{T}(x_1, x_2)) = f(-x_2, x_1) \quad (2.0.5)$$

All the codes for this document can be found at

From (1.0.1), we can write

[https://github.com/Ritesh622/Assignment\\_EE5609/tree/master/Assignment\\_11](https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_11)

$$f(-x_2, x_1) = -ax_2 + bx_1 \quad (2.0.6)$$

Hence,

$$\mathbf{T}^t f(x_1, x_2) = -ax_2 + bx_1 \quad (2.0.7)$$

## 1 PROBLEM

Let  $\mathbb{F}$  be a field and let  $f$  be the linear functional on  $\mathbb{F}^2$  defined by,

$$f(x_1, x_2) = ax_1 + bx_2 \quad (1.0.1)$$

For given linear operator  $\mathbf{T}$ , such that

$$\mathbf{T}(x_1, x_2) = (-x_2, x_1) \quad (1.0.2)$$

Let

$$g = \mathbf{T}^t f \quad (1.0.3)$$

Then find  $g(x_1, x_2)$

## 2 SOLUTION

We have,

$$g = \mathbf{T}^t f \quad (2.0.1)$$

$$\Rightarrow g(x_1, x_2) = \mathbf{T}^t f(x_1, x_2) \quad (2.0.2)$$

We know that, if  $\mathbf{V}$  and  $\mathbf{W}$  be vector spaces over the field  $\mathbb{F}$ . For each linear transformation  $\mathbf{T}$  from  $\mathbf{V}$  into  $\mathbf{W}$ , there is a unique linear transformation  $\mathbf{T}^t$  from  $\mathbf{W}^*$  into  $\mathbf{V}^*$  such that,

$$(\mathbf{T}^t g)(\alpha) = g(\mathbf{T}\alpha) \quad (2.0.3)$$

Where for every  $g$  in  $\mathbf{W}^*$  and  $\alpha$  in  $\mathbf{V}$ .

Now using (2.0.3) in (2.0.2) we can write,

$$\mathbf{T}^t f(x_1, x_2) = f(\mathbf{T}(x_1, x_2)) \quad (2.0.4)$$