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* given, circles passes through point $(\frac{1}{2})$ & both are touching axes.

find eqn of circles.

Solⁿ: Since both passes through $(\frac{1}{2})$, it will lie on both circles.

& let centre = $(\pm r, \pm r)$
Since both touches the axes
radius will be r .

touching point $(\frac{1}{2})$ is +ve hence $r > 0$.

Now eqn of circle

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$(x - r)^2 + (y - r)^2 = r^2$$

$$(1 - r)^2 + (2 - r)^2 = r^2$$

$$r^2 - 6r + 5 = 0 \Rightarrow r = 1, 5$$

Hence eqn of circles are

$$(x - 1)^2 + (y - 1)^2 = 1$$

$$(x - 5)^2 + (y - 5)^2 = 25$$

or

$$\|X - 0\|^2 = 1 \quad - (i)$$

$$\|X - 0\|^2 = 25 \quad - (ii)$$

Ansmethod

(ii)

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$B = \begin{pmatrix} r \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 \\ r \end{pmatrix}$$

Circle will pass through all these points Hence

$$\left\| \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 0 \right\|^2 = r^2$$

$$\left\| \begin{pmatrix} r \\ 0 \end{pmatrix} - 0 \right\|^2 = r^2$$

$$\left\| \begin{pmatrix} 0 \\ r \end{pmatrix} - 0 \right\|^2 = r^2$$

again not able to find the soln from here onwards.

3rd method

circles will pass through
 $\begin{pmatrix} r \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ r \end{pmatrix}$ & $\frac{1}{2}$ hence

we can write

using

$$2(r-0)c - f = r^2 \quad \left\{ \begin{array}{l} X^T X - 2C^T X + f = 0 \end{array} \right.$$

$$2(0-r)c - f = r^2$$

$$2(12)c - f = 5$$

$$\begin{pmatrix} 2 & 4 & -1 & 5 \\ 2r & 0 & -1 & r^2 \\ 0 & 2r & -1 & r^2 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 4 & -1 & 5 \\ 0 & 2r-1 & -1 & r^2 \\ 2r & 0 & -1 & r^2 \end{pmatrix} \begin{pmatrix} c \\ f \end{pmatrix} = \begin{pmatrix} 5 \\ r^2 \\ r^2 \end{pmatrix}$$

\Downarrow

$$\begin{pmatrix} 1 & 0 & 0 & \frac{r^2-5}{2(r-3)} \\ 0 & 1 & 0 & \frac{r^2-5}{2(r-3)} \\ 0 & 0 & 1 & \frac{r(3r-5)}{r-3} \end{pmatrix}$$

Hence $C = \begin{pmatrix} \frac{r^2-5}{2(r-3)} \\ \frac{r^2-5}{2(r-3)} \end{pmatrix}$

$$f = \frac{r(3r-5)}{r-3}$$

if you putting these in

$$x^T x - 2C^T x + f = 0$$

to get solution for r .? then how