Matrix Theory : Assignment 2

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***Abstract*—This is a problem to balance a chemical equation using system of linear equations.**

Download all codes from

https://github.com/Debolena/EE5609/blob/master/Assignment_2/assignment_2.py

1 Problem

Balance the following chemical equation.

$$\text{Zinc} + \text{Silver nitrate} \rightarrow \text{Zinc nitrate} + \text{Silver} \tag{1.0.1}$$

2 Solution

Equation 1.0.1 can be written as :

$$Zn + AgNO_3 \rightarrow Ag + Zn(NO_3)_2 \tag{2.0.1}$$

Suppose balance form of the equation is :

$$x_1 Zn + x_2 AgNo_3 \rightarrow x_3 Ag + x_4 Zn(NO_3)_2 \tag{2.0.2}$$

which results in the following equations:

$$(x_1 - 2x_4)Zn = 0 \tag{2.0.3}$$
$$(x_2 - x_3)Ag = 0 \tag{2.0.4}$$
$$(x_3 - 2x_4)N = 0 \tag{2.0.5}$$
$$(3x_3 - 6x_4)O = 0 \tag{2.0.6}$$

which can be expressed as

$$x_1 + 0.x_2 + 0.x_3 - x_4 = 0 \tag{2.0.7}$$
$$0.x_1 + x_2 - x_3 + 0.x_4 = 0 \tag{2.0.8}$$
$$0.x_1 + 0.x_2 + x_3 - 2x_4 = 0 \tag{2.0.9}$$
$$0.x_1 + 0.x_2 + 3x_3 - 6x_4 = 0 \tag{2.0.10}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 3 & -6 \end{pmatrix} \mathbf{X} = \mathbf{0} \tag{2.0.11}$$

where,

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{2.0.12}$$

(2.0.11) can be reduced as follows:

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 3 & -6 \end{pmatrix} \xleftrightarrow{R_4 \leftarrow R_4 - 3R_3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.13}$$

Thus,

$$x_1 = x_4, x_2 = 2x_4, x_3 = 2x_4 \tag{2.0.14}$$

$$\implies \quad \mathbf{X} = \begin{pmatrix} x_4 \\ 2x_4 \\ 2x_4 \\ x_4 \end{pmatrix} = x_4 \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \tag{2.0.15}$$

by substituting $x_4 = 1$, we get :

$$\implies \quad \mathbf{X} = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 1 \end{pmatrix} \tag{2.0.16}$$

Hence, (2.0.2) finally becomes

$$Zn + 2AgNo_3 \rightarrow 2Ag + Zn(NO_3)_2 \tag{2.0.17}$$