Matrix Theory : Assignment 3

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***Abstract*—This problem is to demonstrate the way to prove the triangles are congruent and to prove a triangle as isosceles using matrix algebra.**

1 Problem

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal. Show that

1) $\triangle ABC \cong \triangle ACF$

2 Solution

2.1 part 1

Let consider we have a triangle $\triangle ABC$. There are two altitudes BE and CF being drawn from the vertices B and C respectively.
In $\triangle ABE$, taking inner product of sides AE and EB we can write :

$$\left(\mathbf{A}-\mathbf{E}\right)^T\left(\mathbf{E}-\mathbf{B}\right) = \left\|\mathbf{A}-\mathbf{E}\right\|\left\|\mathbf{E}-\mathbf{B}\right\| \cos AEB \tag{2.1.1}$$

$$\implies \cos AEB = \frac{\left(\mathbf{A}-\mathbf{E}\right)^T\left(\mathbf{E}-\mathbf{B}\right)}{\left\|\mathbf{A}-\mathbf{E}\right\|\left\|\mathbf{E}-\mathbf{B}\right\|} \tag{2.1.2}$$

In $\triangle ACF$, taking inner product of sides AF and FC :

$$\left(\mathbf{A}-\mathbf{F}\right)^T\left(\mathbf{F}-\mathbf{C}\right) = \left\|\mathbf{A}-\mathbf{F}\right\|\left\|\mathbf{F}-\mathbf{C}\right\| \cos AFC \tag{2.1.3}$$

$$\implies \cos AFC = \frac{\left(\mathbf{A}-\mathbf{F}\right)^T\left(\mathbf{F}-\mathbf{C}\right)}{\left\|\mathbf{A}-\mathbf{F}\right\|\left\|\mathbf{F}-\mathbf{C}\right\|} \tag{2.1.4}$$

In triangle $\triangle ABC$,

$$\cos AFC = \cos AEB \, (CF \perp AB \,\&\, BE \perp AC) \tag{2.1.5}$$

Given,

$$\left\|\mathbf{E}-\mathbf{B}\right\| = \left\|\mathbf{F}-\mathbf{C}\right\| \tag{2.1.6}$$

$$\angle FAC = \angle EAB \, (\text{Common angle}) \tag{2.1.7}$$

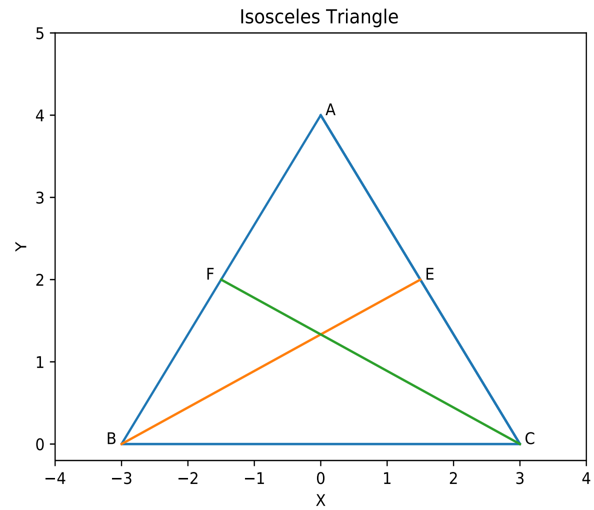

Fig. 1: Isosceles triangle

We know that if the two angles of triangles are equal then the third angle will also be equal.Hence,

$$\angle FCA = \angle EBA \tag{2.1.8}$$

Hence by ASA (Angle - Side - Angle) We can say that , $\triangle ABC \cong \triangle ACF$.

2.2 part 2

we have given that ,

$$\left\|\mathbf{E}-\mathbf{B}\right\| = \left\|\mathbf{F}-\mathbf{C}\right\| \tag{2.2.1}$$

Hence we know that if the two sides of the triangle are equal then angles opposite to them are also equal. So we can have

$$\angle ABC = \angle ACB \tag{2.2.2}$$

In $\triangle ABC$ we can have :

$$\implies \angle B = \angle C \tag{2.2.3}$$

Now in $\triangle$ ABC we have two equal angles, therefore corresponding opposite sides will be equal. That is,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.2.4}$$

Hence we can say that the $\triangle ABC$ is an isosceles triangle since its two sides and respective angles are equal.