Matrix Theory : Assignment 3

Ritesh Kumar
Roll no. : EE20RESCH11005

***Abstract*—This problem is to demonstrate the way to prove the triangles are congruent and to prove a triangle as isosceles using matrix algebra.**

1 Problem

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal. Show that

1) $\triangle$ABC $\cong$ $\triangle$ACF
2) AB = AC i.e, $\triangle$ABC is an isosceles triangle.

2 Solution

2.1 part 1

Let consider we have a triangle $\triangle$ABC. There are two altitudes BE and CF being drawn from the vertices B and C respectively.
In $\triangle$ABE, taking inner product of sides AE and EB we can write :

$$(\mathbf{A}-\mathbf{E})^T(\mathbf{E}-\mathbf{B}) = \|\mathbf{A}-\mathbf{E}\|\,\|\mathbf{E}-\mathbf{B}\|\cos\text{AEB} \tag{2.1.1}$$

$$\implies \cos\text{AEB} = \frac{(\mathbf{A}-\mathbf{E})^T(\mathbf{E}-\mathbf{B})}{\|\mathbf{A}-\mathbf{E}\|\,\|\mathbf{E}-\mathbf{B}\|} \tag{2.1.2}$$

In $\triangle$ACF, taking inner product of sides AF and FC :

$$(\mathbf{A}-\mathbf{F})^T(\mathbf{F}-\mathbf{C}) = \|\mathbf{A}-\mathbf{F}\|\,\|\mathbf{F}-\mathbf{C}\|\cos\text{AFC} \tag{2.1.3}$$

$$\implies \cos\text{AFC} = \frac{(\mathbf{A}-\mathbf{F})^T(\mathbf{F}-\mathbf{C})}{\|\mathbf{A}-\mathbf{F}\|\,\|\mathbf{F}-\mathbf{C}\|} \tag{2.1.4}$$

In triangle $\triangle$ABC,

$$\cos\text{AFC} = \cos\text{AEB}\,(\text{CF} \perp \text{AB}\ \&\ \text{BE} \perp \text{AC}) \tag{2.1.5}$$

Given,

$$\|\mathbf{E}-\mathbf{B}\| = \|\mathbf{F}-\mathbf{C}\| \tag{2.1.6}$$

$$\angle FAC = \angle EAB\,(\text{Common angle}) \tag{2.1.7}$$

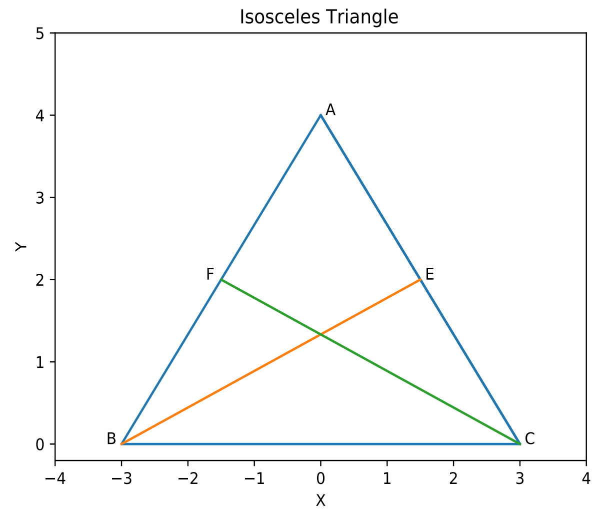

Fig. 2: Isosceles triangle

We know that if the two angles of triangles are equal then the third angle will also be equal.Hence,

$$\angle FCA = \angle EBA \tag{2.1.8}$$

Hence by ASA (Angle - Side - Angle) We can say that , $\triangle$ABC $\cong$ $\triangle$ACF.

2.2 part 2

we have given that ,

$$\|\mathbf{E}-\mathbf{B}\| = \|\mathbf{F}-\mathbf{C}\| \tag{2.2.1}$$

Hence we know that if the two sides of the triangle are equal then angles opposite to them are also equal. So we can have

$$\angle\text{ABC} = \angle\text{ACB} \tag{2.2.2}$$

In $\triangle$ABC we can have :

$$\implies \angle B = \angle C \tag{2.2.3}$$

Now in $\triangle$ ABC we have two equal angles, therefore corresponding opposite sides will be equal. That is,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.2.4}$$

Hence we can say that the $\triangle ABC$ is an isosceles triangle since its two sides and respective angles are equal.