Matrix Theory Assignment 13

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***Abstract*—This problem is all about to introducing the concept of characteristic polynomial over a filed.**

All the codes for this document can be found at

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https://github.com/Ritesh622/Assignment_EE5609/
    tree/master/Assignment_13
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1 Problem

Let A be the an $n \times n$ diagonal matrix with characteristic polynomial

$$(x-c_1)^{d_1}(x-c_2)^{d_2}\dots(x-c_k)^{d_k} \tag{1.0.1}$$

Where $c_1, c_2, \dots c_k$ are distinct. Let $\mathbf{V}$ the space of $n \times n$ matrices B such that

$$AB = BA \tag{1.0.2}$$

Prove that the dimension of $\mathbf{V}$ is,

$$d_1{}^2 + d_2{}^2 \dots + d_k{}^2 \tag{1.0.3}$$

2 solution

Let consider we have a matrix A which is a diagonal matrix, which is given as

$$A = \begin{bmatrix} c_1I & 0 & 0 & \dots & 0 & 0 \\ 0 & c_2I & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & . & \dots & . & c_kI \end{bmatrix} \tag{2.0.1}$$

Consider B as :

$$B = \begin{bmatrix} B_{11} & B_{12} & . & \dots & . & B_{1k} \\ B_{21} & B_{22} & . & \dots & . & B_{2k} \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ B_{k1} & B_{k2} & . & \dots & . & B_{kk} \end{bmatrix} \tag{2.0.2}$$

Where B_{ij} has dimension $d_i \times d_j$. Since we have given ,

$$AB = BA \tag{2.0.3}$$

$$\implies \begin{bmatrix} c_1B_{11} & c_1B_{12} & . & \dots & . & c_1B_{1k} \\ c_2B_{21} & c_2B_{22} & . & \dots & . & c_2B_{2k} \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ c_kB_{k1} & c_kB_{k2} & . & \dots & . & c_kB_{kk} \end{bmatrix} =$$

$$\begin{bmatrix} c_1B_{11} & B_{12} & . & \dots & . & c_1B_{1k} \\ c_2B_{21} & B_{22} & . & \dots & . & c_2B_{2k} \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ c_kB_{k1} & c_kB_{k2} & . & \dots & . & c_kB_{kk} \end{bmatrix} \tag{2.0.4}$$

Hence, from above equation 2.0.4 we can conclude,

$$c_i \neq c_j, \forall i \neq j \tag{2.0.5}$$

$$\implies B_{ij} = 0, \forall i \neq j \tag{2.0.6}$$

We can have $B_{11}, B_{22} \dots$ any arbitrary matrices. From (2.0.4) we can have

$$D(B_{ij}) = d_i^2 \tag{2.0.7}$$

Where D represents dimension of matrix. Therefore the dimension of the space of all such B_{ij}'s matrices is given as :

$$d_1{}^2 + d_2{}^2 \dots + d_k{}^2 \tag{2.0.8}$$