Matrix Theory Assignment 11

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***Abstract*—This problem demonstrate a method of find the Transpose of linear transformations by Linear algebra.**

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_11

1 Problem

Let $\mathbb{F}$ be a filed and let f be the linear functional on $\mathbb{F}^2$ defined by,

$$f(x_1, x_2) = ax_1 + bx_2 \tag{1.0.1}$$

For given linear operator $\mathbf{T}$, such that

$$\mathbf{T}(x_1, x_2) = (-x_2, x_1) \tag{1.0.2}$$

Let

$$g = \mathbf{T}^t f \tag{1.0.3}$$

Then find $g(x_1, x_2)$

2 solution

The linear operator $\mathbf{T}$ can be represented as a matrix $\mathbf{A}$ as follows

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2.0.1}$$

$$\tag{2.0.2}$$

Let suppose,

$$\mathbf{X_1} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \mathbf{X_2} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \tag{2.0.3}$$

And,

$$\mathbf{U} = \begin{pmatrix} a & b \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{T}(x_1, x_2) = \mathbf{AX_1} \tag{2.0.5}$$

$$\implies \mathbf{AX_1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{X_1} = \mathbf{X_2} \tag{2.0.6}$$

And (1.0.1) can be written as

$$f(x_1, x_2) = \mathbf{UX_1} \tag{2.0.7}$$

Now, we have given,

$$g = \mathbf{T}^t f \tag{2.0.8}$$

$$\implies g(x_1, x_2) = \mathbf{T}^t f(x_1, x_2) \tag{2.0.9}$$

We know that, if $\mathbf{V}$ and $\mathbf{W}$ be vector spaces over the field $\mathbb{F}$. For each linear transformation $\mathbf{T}$ from $\mathbf{V}$ into $\mathbf{W}$, there is a unique linear transformation $\mathbf{T}^t$ from $\mathbf{W^*}$ into $\mathbf{V^*}$ such that,

$$(\mathbf{T}^t g)(\alpha) = g(\mathbf{T}\alpha) \tag{2.0.10}$$

Where for every g in $\mathbf{W^*}$ and α in $\mathbf{V}$.

Now using (2.0.10) in (2.0.9) we can write,

$$\mathbf{T}^t f(x_1, x_2) = f(\mathbf{T}(x_1, x_2)) \tag{2.0.11}$$

$$\implies f(\mathbf{T}(x_1, x_2)) = \mathbf{UAX_1} \tag{2.0.12}$$

$$f(\mathbf{T}(x_1, x_2)) = \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \mathbf{UX_2} \tag{2.0.13}$$

$$\implies f(\mathbf{T}(x_1, x_2)) = -ax_2 + bx_1 \tag{2.0.14}$$

Hence,

$$f(\mathbf{T}(x_1, x_2)) = -ax_2 + bx_1 = g(x_1, x_2)$$

$$= \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} = \mathbf{UX_2} \tag{2.0.15}$$