Matrix Theory Assignment 12

Ritesh Kumar
EE20RESCH11005

***Abstract*—This problem is all about to to introducing the concept of linear algebra over a filed.**

All the codes for this document can be found at

https://github.com/Ritesh622/Assignment_EE5609/tree/master/Assignment_12

1 Problem

If a and b are element of a filed $\mathbb{F}$ and $a \neq 0$, show that the ploynomial $1, ax + b, (ax+b)^2, (ax+b)^3, \dots$ form a basis of $\mathbb{F}[x]$.

2 solution

Let consider we have a set S such that,

$$S = \left\{1, ax+b, (ax+b)^2, (ax+b)^3, \dots\right\} \tag{2.0.1}$$

And let $\langle S \rangle$ be the subspace, that is spanned by S.

Since

$$1 \in S \tag{2.0.2}$$

and

$$ax + b \in S, \tag{2.0.3}$$

$$\implies b.1 + \frac{a}{a}(a + bx) \in \langle S \rangle \tag{2.0.4}$$

and hence, it follows

$$\implies x \in \langle S \rangle \tag{2.0.5}$$

Now to prove

$$x^2 \in \langle S \rangle \tag{2.0.6}$$

let consider another element form S which is

$$(ax+b)^2 \tag{2.0.7}$$

Subtracting $1.a^2 + 2.a.b.x$ from $(ax+b)^2$

$$\implies (ax+b)^2 - a^2 - 2.a.b.x = a^2.x^2 \tag{2.0.8}$$

$$\implies a^2.x^2 \in \langle S \rangle \tag{2.0.9}$$

$$\implies \frac{1}{a^2}.a^2.x^2 \in S. \tag{2.0.10}$$

$$\implies x^2 \in \langle S \rangle. \tag{2.0.11}$$

Now, Thus Hence using this concept with higher degree we can prove that,

$$x^n \in \langle S \rangle, \forall n \tag{2.0.12}$$

Consider,

$$S' = \left\{1, x, x^2, x^3, \dots\right\} \tag{2.0.13}$$

Hence we can say that, (2.0.13) span the space of all polynomials which form with the help of

$$(ax+b)^n \tag{2.0.14}$$

Hence we conclude that S spans the space of all polynomials. We can summarize our procedure step by step using table1

TABLE 1: Step for the solution

Sr. No.	Description	Mathematical representation
1.	Consider a set S	$S = \{1, ax+b, \dots\}$
2.	Provide a proof that subset S span the subspace $\langle S \rangle$	Given element are $\in S$
3.	Repeat step 2 for the higher degree of polynomial also lie in the subspace and the also lie in the subset S.	Given element are $\in S$
4.	After providing proof for all element $\in S$ find the basis .	$S' = \left\{1, x, x^2, x^3, \dots\right\}$
5.	show the element $\in S'$ are able to form all element S over $\mathbb{F}$.	Hence S form basis of $\mathbb{F}$