A Rate-Splitting Approach to the Gaussian Multiple-Access Channel

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December 3, 2021

Introduction

1. This presentation is based on the paper.
B. Rimoldi and R. Urbanke, "A rate splitting approach to the Gaussian multiple access channel," IEEE Trans. Inf. Theory, vol. 42, pp. 364–375, Mar. 1996.

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2. Assumptions :
   - Channel is discrete-time and frame synchronous with power constraint
     $$P = (P_1, P_2 \ldots P_M) \tag{1}$$
     Where P_i is power constraint for user i.
   - Noise variance is σ^2.
   - "code" stands for the ideal random code. And decoding scheme use successive cancellation.

- Random codes have a decoding complexity of of the order of $2^{nR_{\text{sum}}}$.
- Rate touples component satsfying the condition

$$\sum_{i \in S} R_i \leq \frac{1}{2} \log_2 \left(1 + \frac{\sum_{i \in S} P_i}{\sigma^2} \right) \quad \forall S \subseteq \{1, \ldots, M\} \tag{2}$$

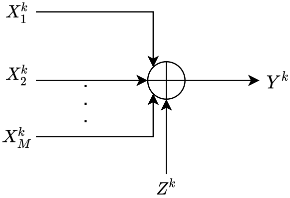

Figure 1: Gaussian multiple-access channel

RSMA

- RSMA is a multiple access technique in which we split the message of user in 2 parts.

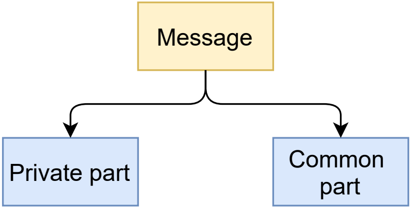

Figure 2: Message splitting

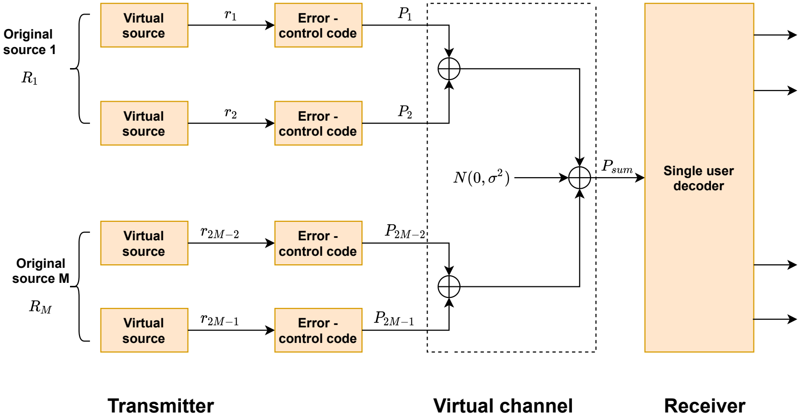

Figure 3: Multiple-access system based on rate-splitting multiple accessing

Single user decoding

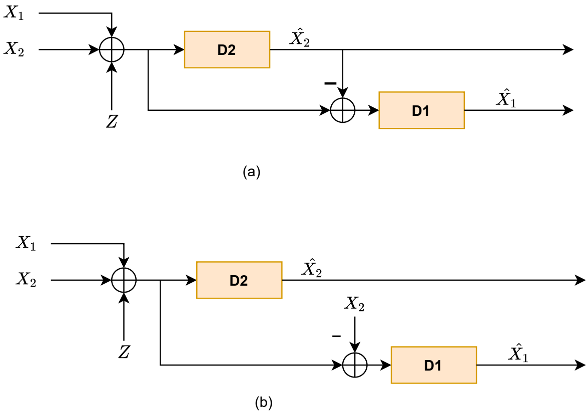

Figure 4: (a) Single-user decoder. (b) Genie-aided decoder

RSMA

- One user is split into two virtual users.
- Let define $C(P, \sigma^2)$ as,

$$C(P, \sigma^2) = \frac{1}{2} \log_2 \left(1 + \frac{P}{\sigma^2}\right) \tag{3}$$

- With chain rule,

$$C(P, \sigma^2) = C(P_1, \sigma^2) + C(P_2, \sigma^2 + P_1) \tag{4}$$

- For all nonnegative number P_1, P_2 and σ^2 with $P = P_1 + P_2$
- Rate R of a single user transmitting at capacity can be seen as a vertex

$$(R_1, R_2) = \big(C(P_1, \sigma^2), C(P_2, \sigma^2 + P_1)\big) \tag{5}$$

- Higher rate code can be obtained from two lower rate codes.

- Any rate tuple (R_1, R_2) in the dominant face of capacity region such that,

$$\begin{aligned} R_1 &< C(P_1, \sigma^2) \\ R_2 &< C(P_2, \sigma^2) \\ R_1 + R_2 &= C(P_1 + P_2, \sigma^2) \end{aligned}$$

- Let $\delta > 0$ be the unique number wich satisfies,

$$R_2 = C(P_2, \sigma^2 + \delta) \tag{6}$$

Now consider Gaussian MAC with noise power σ^2 and there is 3 virtual inputs.

- Power constraint is (p_1, p_2, p_3) such that,

$$\begin{aligned} p_1 &= \delta \\ p_2 &= P_2 \\ p_3 &= P_1 - \delta \end{aligned}$$

- Rate tuple (r_1, r_2, r_3) is ,

$$\begin{aligned} r_1 &= C(p_1, \sigma^2) \\ r_2 &= C(p_2, \sigma^2 + p_1) \\ r_3 &= C(p_2, \sigma^2 + p_1 + p_2) \end{aligned}$$

- Virtual user 2 has same rate and power constraint as the original user 2.
- We can also observe that,

$$\begin{aligned} r_1 + r_2 + r_3 &= C(p_1 + p_2 + p_3, \sigma^2) \\ &= C(P_1 + P_2, \sigma^2) = R_1 + R_2 \end{aligned}$$

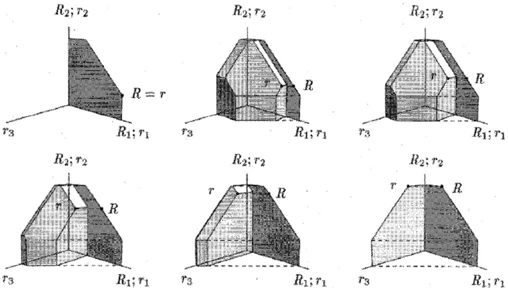

Fig. 4. Geometric relationship between $R = (R_1, R_2)$ and $r = (r_1, r_2, r_3)$ for fixed power constraint P and σ^2 and various points on the dominant face of the capacity region. Note that $r_1 + r_3 = R_1$.

Figure 5: Dominant face of capacity region

- The geometrical relationship between $R = (R_1, R_2)$ and $r = (r1, r2, r3)$ is that, they will lie on dominant face of capacity region.
- Define the quardruple $\left(M, P, R, \sigma^2\right)$ and a nonnegative δ_i, $i = 1, \ldots, M$ that satisfies

$$R_i = C(P_i, \delta_i + \sigma^2) \tag{7}$$

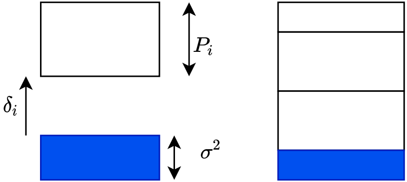

Figure 6: Block representation

Lemma 1

For a tight configuration $\left(M, P, R, \sigma^2\right)$ after a possible re-indexing, there exist at least one index i for which

$$\delta_i \leq \delta_{i+1} \leq \delta_i + P_i \tag{8}$$

where δ_i is a unique nonegative number that satisfies

$$R_i = C(P_i, \delta_i + \sigma^2). \tag{9}$$

proof:
We re-index so that $0 = \delta_0 \leq \delta_1 \leq \delta_2 \ldots, \leq \delta_M$ and assume that assume that claim is false i.e., that

$$\delta_{i+1} > \delta_i + P_i, i = 0, 1, \ldots, (M-1) \tag{10}$$

It follows that

$$\sum_{i=1}^{M} R_i = \sum_{i=1}^{M} C(P_i, \sigma^2 + \delta_i) < \sum_{i=1}^{M} C\left(P_i, \sigma^2 + \sum_{j<i} P_i\right) = C\left(\sum_{i=1}^{M} P_i, \sigma^2\right) \tag{11}$$

Lemma 2

Consider a quardruple $\left(M, P, R, \sigma^2\right)$ and assume that,

$$\delta_j \geq \delta_i + P_i \tag{12}$$

for some $i, j \in \{1, 2, 3 \ldots, M\}$. Let δ be the unique nonnegative numbe such that $R_i + R_j = C(P_i + P_j, \delta)$. Then

$$\delta_i \leq \delta \leq \delta_j - P_i \tag{13}$$

Proof :

$$\begin{aligned} C(P_i + P_j, \delta) &= R_i + R_j = C(P_i, \delta_i) + C(P_j, \delta_j) \\ &\leq C(P_i, \delta_i) + C(P_j, \delta_i + P_i) = C(P_i + P_j, \delta_i) \end{aligned}$$

From the inequality in (12) we can write that,

$$\begin{aligned} C(P_i + P_j, \delta) = R_i + R_j = C(P_i, \delta_i) + C(P_j, \delta_j) \\ \geq C(P_i, \delta_j - P_i) + C(P_j, \delta_j) = C(P_i + P_j, \delta_j - P_i) \end{aligned}$$

Quadruple for RSMA

The quardruple $\left(m, p, r, \sigma^2\right)$ is a spinoff of $\left(M, P, R, \sigma^2\right)$ if there exist a surjective mapping $\phi : \{1, \ldots, m\} \to \{1, \ldots, M\}$ such that for all $i \in \{1, \ldots, M\}$ we have,

$$P_i \geq \sum_{j \in \phi^{-1}(i)} p_j$$

and,

$$R_i \leq \sum_{j \in \phi^{-1}(i)} r_j$$

Theorem

For every M-user configuration (M, P, R, σ^2) *there exist a spinoff* (m, p, r, σ^2) *which is single-user-codable.Moreover, one can guarantee that one user is un-split and that no user split into more than two virtual users. Hence* $m \leq 2M - 1$

Proof :
Without loss of generality we may assume that (M, P, R, σ^2) is tight. By induction on M we can proof this.

- for $\mathrm{M} = 1$ the claim is trivially true.
- Consider 2 user case, i.e. M=2.
- Any rate tuple (R_1, R_2) in the dominant face of capacity region such that,

$$\begin{aligned} R_1 &< C(P_1, \sigma^2) \\ R_2 &< C(P_2, \sigma^2) \\ R_1 + R_2 &= C(P_1 + P_2, \sigma^2) \end{aligned}$$

let $\delta > 0$ be a unique number such that,

$$R_2 = C(P_2, \sigma^2 + \delta) \tag{14}$$

Now we assume that t it is true for an arbitrary positive M and consider the case of M+1 users.

- Using lemma 1 we may assume that, $\delta_M \leq \delta_{M+1}$ and $\delta_{M+1} \leq \delta_M + P_M$
- We reduce the original (M+1) user configuration $\left(M+1, P, R, \sigma^2\right)$ to an M-user configuration $\left(M, \hat{P}, \hat{R}, \sigma^2\right)$ where,

$$\begin{aligned} \hat{P}_i &= P_i, i = 1, 2, \ldots, M-1 \\ \hat{P_M} &= P_M + P_{M+1}, \\ \hat{R}_i &= R_i, i = 1, 2, \ldots, M-1 \\ \hat{R_M} &= R_M + R_{M+1} \end{aligned}$$

- By induction hypothesis, there exist a spinoff $\left(m, p, r, \sigma^2\right)$ of $\left(M, P, R, \sigma^2\right)$ with m $\leq$2M-1 and with property that one user is unsplit and no user is split into more than two users.

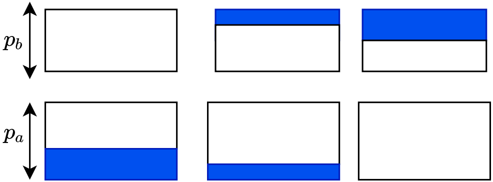

Figure 7: Block representation

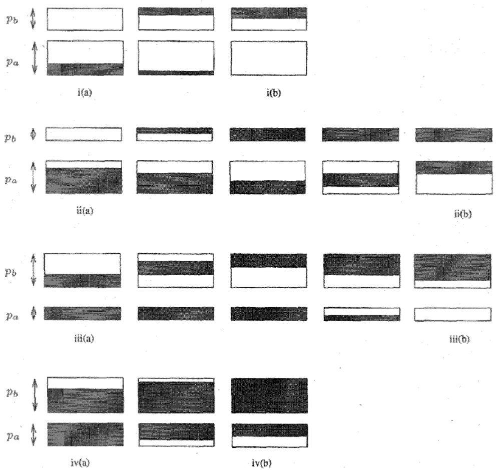

Figure 8: Block representation

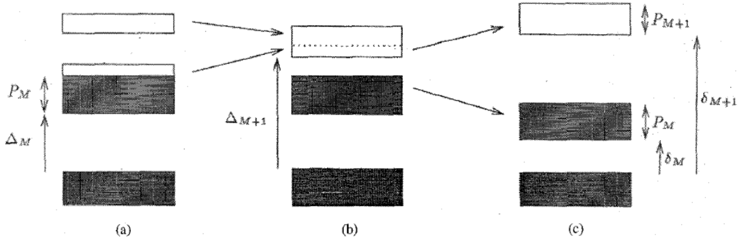

Fig. 8. Proof, by contradiction, that in Fig. 7(a) $\rho_M \geq R_M$.

Figure 9: Block representation

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$$2R_{\text{sum}} \leq \lim_{\text{M} \to \infty} M \log_2 \left(1 + \frac{P}{\sigma^2 + (M-1)P}\right) = 1/log_e 2[b/s/Hz] \quad (15)$$

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- There is no such upper bound for RSMA.

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- There is no near far problem in RSMA like SSMA.
- RSMA allows one to achieve any point in the capacity region of a time varying multipath channel.
- Its allows the user to use the his extra power for the benefits of another user.

further research

- How RSMA behaves with the practical codes.
- What is the effect of the imperfect cancellation at the receiver.
- Effect on the Rate with various coding scheme.

Thank You!