

Report on A Rate-Splitting Approach to the Gaussian Multiple-Access Channel

Ritesh Kumar
(EE20RESCH11005)

Under the supervision of

Dr. Shashank Vatedka
Assistant Professor
Department of Electrical Engineering
IIT Hyderabad

1 Introduction

The main idea described in this paper is to put some light on the structure of the capacity region of the Gaussian multiple-access channel (MAC) and suggest a new and promising strategy for the implementation of a practical system.

In this paper [1], the authors proposed a new scheme called rate splitting multiple-accessing (RSMA). This scheme allows user to split their data and signal into two parts. In this paper, authors have considered,

- Channel is discrete-time and frame synchronous with power constraint $P = (P_1, P_2, \dots)$, where P_i is power constraint for the user i and noise variance σ^2
- Its capacity region is the subset of \mathbf{R}^M containing rate tuples (R_1, \dots, R_M) .
- Rate tuples component satisfying the condition

$$\sum_{i \in S} R_i \leq \frac{1}{2} \log_2 \left(1 + \frac{\sum_{i \in S} P_i}{\sigma^2} \right) \quad \forall S \subseteq \{1, \dots, M\} \quad (1)$$

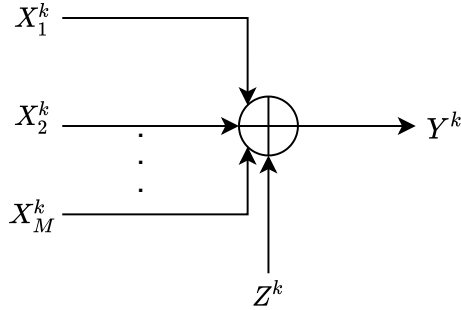


Figure 1: Gaussian multiple-access channel

- Where $Y^k = \sum_{i=1}^M X_i^k + Z^k$

RSMA is a code-division multiple access scheme, for the M-user Gaussian multiple-access channel, for which the effort of finding the codes for M users of encoding and decoding is that of at most $2M-1$ independent point-to-point Gaussian channels.

- In this scheme decoder of user M can decode the data by considering the codewords of other users as Gaussian noise. Hence, the decoder of user M can decode considering the codewords of user $1, \dots, M-1$ as noise. This kind of decoding procedure is known variously as onion peeling, stripping, and successive cancellation.

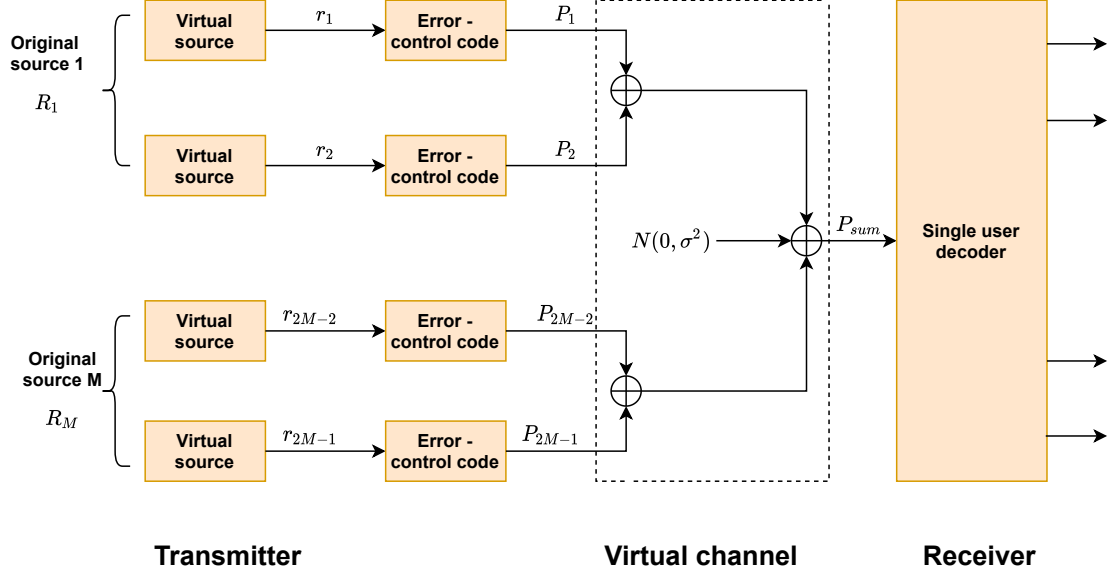


Figure 2: A multiple-access system based on rate-splitting multiple accessing

2 Vertex achievability by single user coding

- Vertices (corner) of the capacity region are indeed achievable via single user coding.
- In figure 3(a) we do single-user decoding for a two-user multiple-access system

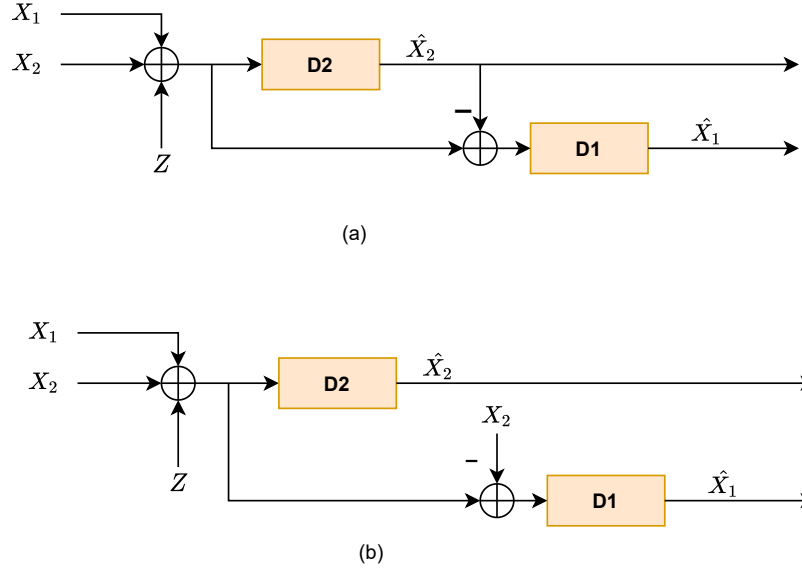


Figure 3: (a) Single-user decoder (b) Genie-aided decoder

In figure 3(b) we consider the case where X_2 is always removed from the input to $D1$. The rest of the procedure is the same in both. In first part of the figure 3 we decode X_2 first

then we decode X_1 by subtracting estimated value of X_2 from the received codeword. But in the second case which is "Genie-aided decoder", which follows the Wozencraft and Jacobs Postulate [2], D_1 always knows the value of X_2 and then decode the value of X_1 .

3 Rate-splitting multiple accessing achieves any point in the capacity region

Consider $C(P, \sigma^2)$ as the capacity of a white Gaussian noise channel with power P and noise σ^2 , hence,

$$C = \frac{1}{2} \log_2 \left(1 + \frac{P}{\sigma^2} \right) \quad (2)$$

With the help of chain rule we can verify that,

$$C(P, \sigma^2) = C(P_1, \sigma^2) + C(P_2, \sigma^2 + P_1) \quad (3)$$

equation (3) is valid for all nonnegative number P_1, P_2 , and σ^2 such that $P = P_1 + P_2$. The chain rule says that the rate R of a single user transmitting at capacity can be seen as vertex

$$(R_1, R_2) = (C(P_1, \sigma^2), C(P_2, \sigma^2 + P_1)) \quad (4)$$

[3] equation (4) shows the vertex point of a two user virtual channel described by $(P - 1, P_2)$ and σ^2 , where $P_1 + P_2 = P$ and $R_1 + R_2 = R$.

The advantage of splitting the user of the point-to-point channel into two virtual users may be seen as, we can achieve a higher rate code from the two lower rate codes.

Let consider a two user case for the more illustrative of the idea behind rate splitting. Let the noise variance is σ^2 and power constraint (P_1, P_2) be arbitrary, and consider any rate tuple (R_1, R_2) in the dominant face of capacity region [4] such that,

$$\begin{aligned} R_1 &< C(P_1, \sigma^2) \\ R_2 &< C(P_2, \sigma^2) \\ R_1 + R_2 &= C(P_1 + P_2, \sigma^2) \end{aligned}$$

Let $\delta > 0$ be the unique number which satisfies,

$$R_2 = C(P_2, \sigma^2 + \delta) \quad (5)$$

Now consider Gaussian MAC with noise power σ^2 and there is 3 virtual inputs. Power constraint is (p_1, p_2, p_3) such that,

$$\begin{aligned} p_1 &= \delta \\ p_2 &= P_2 \\ p_3 &= P_1 - \delta \end{aligned}$$

Rate tuple (r_1, r_2, r_3) is ,

$$\begin{aligned} r_1 &= C(p_1, \sigma^2) \\ r_2 &= C(p_2, \sigma^2 + p_1) \\ r_3 &= C(p_3, \sigma^2 + p_1 + p_2) \end{aligned}$$

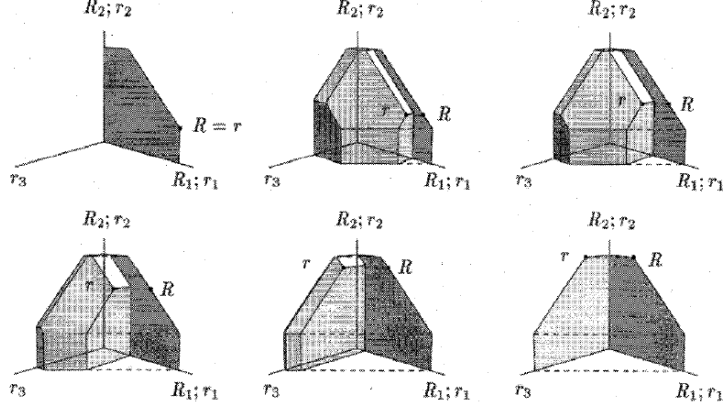


Fig. 4. Geometric relationship between $R = (R_1, R_2)$ and $r = (r_1, r_2, r_3)$ for fixed power constraint P and σ^2 and various points on the dominant face of the capacity region. Note that $r_1 + r_3 = R_1$.

Figure 4: Dominant face of capacity region

Virtual user 2 has the same rate and power constraint as the original user 2. We can also observe that,

$$\begin{aligned} r_1 + r_2 + r_3 &= C(p_1 + p_2 + p_3, \sigma^2) \\ &= C(P_1 + P_2, \sigma^2) = R_1 + R_2 \end{aligned}$$

Hence we can observe here user does need not to know at what rate user two is operating as long as his rate is known since there is no constraint on the rate of user one w.r.t user 2. And this helps a particular user to operate in the dominant face of the capacity region (more precisely on the capacity boundary without considering the operating region of another user).

The geometrical relationship between $R = (R_1, R_2)$ and $r = (r_1, r_2, r_3)$ is that, they will lie on dominant face of capacity region. Define the quadruple (M, P, R, σ^2) and a nonnegative δ_i , $i = 1, \dots, M$ that satisfies

$$R_i = C(P_i, \delta_i + \sigma^2) \quad (6)$$

The bottom box (in blue) in figure 5 represent the noise and, its height is propositional to the

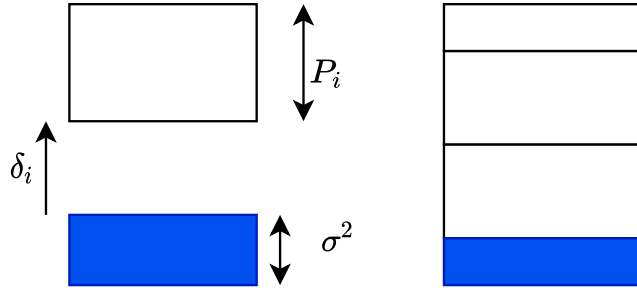


Figure 5: Block representation

noise and power σ^2 . The user is represented by the white rectangle whose height is proportional to

power constraint P and to δ respectively. Width represents nothing. it's just for convenience. For generalization for M-user author proposed two lemmas as, lemma 1 and lemma 2.

Lemma 1. *For a tight configuration (M, P, R, σ^2) after a possible re-indexing, there exists at least one index i for which*

$$\delta_i \leq \delta_{i+1} \leq \delta_i + P_i \quad (7)$$

Here, tight configuration means δ_i is very small.

Proof. We re-index so that $0 = \delta_0 \leq \delta_1 \leq \delta_2 \dots \leq \delta_M$ and assume that claim is false i.e., that

$$\delta_{i+1} > \delta_i + P_i, i = 0, 1, \dots, (M-1) \quad (8)$$

It follows that

$$\sum_{i=1}^M R_i = \sum_{i=1}^M C(P_i, \sigma^2 + \delta_i) < \sum_{i=1}^M C\left(P_i, \sigma^2 + \delta_i + \sum_{j < i} P_j\right) = C\left(\sum_{i=1}^M P_i, \sigma^2\right) \quad (9)$$

This is not possible since according to the assumption we have to consider optimal rate i.e we are operating on the boundary of the capacity region. So inequality given in (11) is not possible. Hence there must exist a δ_i that satisfies (7).

Lemma 2. *Consider a quadruple (M, P, R, σ^2) and assume that*

$$\delta_j \geq \delta_i + P_i \quad (10)$$

for some $i, j \in \{1, 2, \dots, M\}$. Let δ be the unique nonnegative number such that $R_i + R_j = C(P_i + P_j, \delta)$. Then

$$\delta_i \leq \delta \leq \delta_j - P_i \quad (11)$$

That is if two "nonoverlapping" users are combined by putting their power together, to keep their sum-rate constant, the bottom user has to move down.

Proof. The inequality (11) follows from the relation from,

$$C(P_i + P_j, \delta) = R_i + R_j = C(P_i, \delta_i) + C(P_j, \delta_j) \quad (12)$$

$$\leq C(P_i, \delta_i) + C(P_j, \delta_j + P_i) \quad (13)$$

$$= C(P_i + P_j, \delta_i) \quad (14)$$

strict inequality given in (10) follows from,

$$C(P_i + P_j, \delta) = R_i + R_j = C(P_i, \delta_i) + C(P_j, \delta_j) \quad (15)$$

$$\geq C(P_i, \delta_i - P_i) + C(P_j, \delta_j) \quad (16)$$

$$= C(P_i + P_j, \delta_i - P_i) \quad (17)$$

□

The notion of the splitting users into virtual users and channel inputs in a meaningful ways is formalized by the following definition.

Definition 1. *The quadruple (m, p, r, σ^2) is a spinoff of (M, PR, σ^2) if there exist a surjective mapping $\phi: \{1, \dots, m\} \rightarrow \{1, \dots, M\}$ such that for all $i \in \{1, \dots, M\}$ we have*

$$P_i \geq \sum_{j \in \phi^{-1}(i)} p_j \quad (18)$$

and

$$R_i \leq \sum_{j \in \phi^{-1}(i)} r_j \quad (19)$$

where $\phi^{-1}(i)$ is the set of all $j \in \{1, \dots, m\}$ that map into i by mean of ϕ

Theorem 1. For every M -user configuration (M, P, R, σ^2) there exist a spinoff (m, p, r, σ^2) which is single-user-codable. Moreover, one can guarantee that one user is un-split and that no user split into more than two virtual users. Hence $m \leq 2M - 1$

Proof :

Without loss of generality we may assume that (M, P, R, σ^2) is tight. By induction on M , we can prove this. for $M = 1$ the claim is trivially true. Consider 2 user cases, i.e. $M=2$. Any rate tuple (R_1, R_2) in the dominant face of the capacity region such that,

$$\begin{aligned} R_1 &< C(P_1, \sigma^2) \\ R_2 &< C(P_2, \sigma^2) \\ R_1 + R_2 &= C(P_1 + P_2, \sigma^2) \end{aligned}$$

let $\delta > 0$ be a unique number such that,

$$R_2 = C(P_2, \sigma^2 + \delta) \quad (20)$$

Now we assume that, it is true for an arbitrary positive M and consider the case of $M+1$ users. Using lemma 1 we may assume that, $\delta_M \leq \delta_{M+1}$ and $\delta_{M+1} \leq \delta_M + P_M$. We reduce the original $(M+1)$ user configuration $(M+1, P, R, \sigma^2)$ to an M -user configuration $(M, \hat{P}, \hat{R}, \sigma^2)$ where,

$$\begin{aligned} \hat{P}_i &= P_i, i = 1, 2, \dots, M-1 \\ \hat{P}_M &= P_M + P_{M+1}, \\ \hat{R}_i &= R_i, i = 1, 2, \dots, M-1 \\ \hat{R}_M &= R_M + R_{M+1} \end{aligned}$$

By the induction hypothesis, there exist a spinoff (m, p, r, σ^2) of $(M, \hat{P}, \hat{R}, \sigma^2)$ with $m \leq 2M-1$ and with the property that one user is unsplit and no user is split into more than two users.

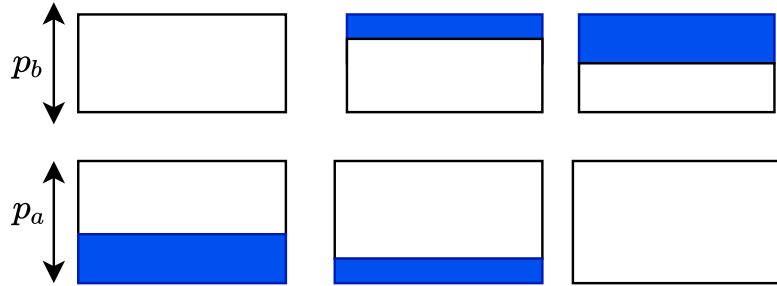


Figure 6: Block representation

The author claimed that for better decoding and encoding procedure, we have to perform splitting, at most into two virtual users and at least one user must be unsplit. That is for low complex coding and irrespective of the information of another user particularly power constraints. If we

split the user into more than two virtual sources, then we are not able to achieve the optimal rate in the capacity region. This is represented in figure 6. Where $\hat{P}_i = p_a + p_b$ and corresponding rate is \hat{R}_i \square

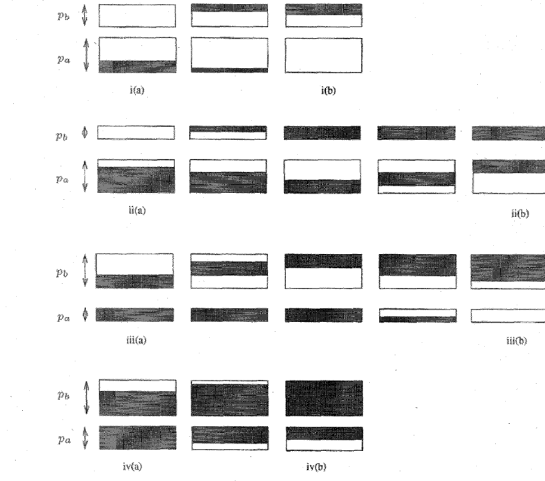


Figure 7: Block representation

4 Advantages of RSMA

Any point in the capacity region of a discrete-time synchronous Gaussian MAC is achievable via RSMA at relatively low coding complexity. There is no need for synchronization among users. Spread-spectrum multiple access (SSMA) has an upper bound on the spectral efficiency as, there is no such upper bound for RSMA. For discrete-time Gaussian MAC, one can achieve rate tuples via the "time-sharing" approach. For continuous-time channel one can also do "frequency-sharing between vertices" Interference reduction resulting from intermittent transmitters. Cellular reuse factor of 1 for RSMA. Channel estimation is better than a narrowband system. There is no near-far problem in RSMA like SSMA. RSMA allows one to achieve any point in the capacity region of a time-varying multipath channel. It allows the user to use his extra power for the benefit of another user. That is if user 1 is willing to increase his power to the point that he can achieve his rate viewing the user as noise, then user 2 can transmit the same rate as if the channel were entirely dedicated to him..

5 Further research

How RSMA behaves with the practical codes is not addressed by the author. What is the effect of the imperfect cancellation at the receiver is also an area to discover and need further research. Effect on the Rate with various coding schemes like turbo coding is further research needed to find an entire family of such codes with rates that vary over a broad range.

References

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