Q 1

Computing Gauss elimination on a 2-digit floating point computer.

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 4 & -1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ -9 \end{bmatrix}$$

In augmented form

$$\left[\begin{array}{ccc|c} 0.40\times10^{1} & 0.10\times10^{1} & 0.20\times10^{1} & 0.90\times10^{1} \\ 0.20\times10^{1} & 0.40\times10^{1} & -0.10\times10^{1} & -0.50\times10^{1} \\ 0.10\times10^{1} & 0.10\times10^{1} & -0.30\times10^{1} & -0.90\times10^{1} \end{array}\right]$$

R_i - ith row

$$R_1 \rightarrow \frac{R_1}{a_{11}}$$

$$\left[\begin{array}{ccc|c} 0.10\times10^{1} & 0.25\times10^{0} & 0.5\times10^{0} & 0.22\times10^{1} \\ 0.20\times10^{1} & 0.40\times10^{1} & -0.10\times10^{1} & -0.50\times10^{1} \\ 0.10\times10^{1} & 0.10\times10^{1} & -0.3\times10^{1} & -0.90\times10^{1} \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1 * a_{21}$ & $R_3 \rightarrow R_3 - R_1 * a_{31}$

$$\left[\begin{array}{ccc|c} 0.10\times10^{1} & 0.25\times10^{1} & 0.5\times10^{0} & 0.22\times10^{1} \\ 0 & 0.35\times10^{0} & -0.20\times10^{1} & -0.94\times10^{1} \\ 0 & 0.08\times10^{0} & -0.35\times10^{1} & -0.11\times10^{2} \end{array}\right]$$

a_{22} is

$R_2 \longrightarrow R_2 / a_{22}$

$$\left[\begin{array}{ccc|c} 1.0 & 0.25\times10^0 & 0.5\times10^0 & 0.22\times10^1 \\ 0 & 1.0 & -0.57\times10^0 & -0.27\times10^1 \\ 0 & 0.80\times10^0 & -0.35\times10^1 & -0.11\times10^2 \end{array}\right]$$

$R_3 \longrightarrow \underline{R_3} - R_2 * a_{32}$.

$$\left[\begin{array}{ccc|c} 1.0 & 0.25\times10^0 & 0.5\times10^0 & 0.22\times10^1 \\ 0 & 1.0 & -0.57\times10^0 & -0.27\times10^1 \\ 0 & 0 & -0.30\times10^1 & -0.9\times10^1 \end{array}\right]$$

now back substitution

$$x_3 = \frac{-0.9}{-0.3} = 3$$

$$x_2 + (-0.57)(x_3) = -2.7$$

$$x_2 = -2.7 + 0.57\times3$$

$$= -2.7 + 1.71 = -2.7 + 1.7$$

$$= -1$$

$$x_1 = 2.2 + (0.25 * 1) - (0.5 * 3)$$

$$= 2.2 + (0.25) - (1.5)$$

$$= 0.22\times10^1 + 0.02\times10^1 - 0.15\times10^1 = 0.09\times10^1$$

$$= 0.9$$

So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.9 \\ -1 \\ 3 \end{bmatrix}$

Question 3

Computational complexity of ~~g~~ gauss elimination of a tridiagonal matrix.

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & \cdots \\ a_{21} & a_{22} & a_{23} & 0 & 0 & \cdots \\ 0 & a_{32} & a_{33} & a_{34} & & \\ & & \ddots & \ddots & \ddots & \\ & & & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Suppose the algorithm is at i^{th} step

then, operations are:

4 divisions at ith row to make $a_{ii} = 1$

then 4 multiplicat^n & 4 substraction to make $a_{i-1, i+1}$ 0. (Including operation on b)

No further subtraction are required because of triadiagonal nature of the matrix.

Total operations at i^{th} row = 12

except at 1 row, At first row there will be

3 multiplicat^n

3 division

& 3 subtract^n

$\therefore$ Total operation $= 9 + 12(n-2)$

$= 12n - 3$

Now Back substitution.

After elimination matrix will look like

$$\begin{bmatrix} a'_{11} & a'_{12} & & 0 \\ & a'_{22} & a'_{23} & \\ & & a'_{33} & \ddots \\ 0 & & & \ddots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b'_1 \\ b'_2 \\ \vdots \\ b'_n \end{bmatrix}$$

last row requires 1 division.

& other rows require 1 multiplicat^n, 1 subtract^n

& 1 ~~div~~ (division not required because we have 1's in diagonal except at ~~first~~ last row).

$\therefore$ Total operation in back substitut^n

$= 1 + n(2)$

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Total operations to obtain the solutⁿ of the system is

$$\text{Total Operat}^n = 12n-3 + 2n + 1$$
$$= 14n - 2$$
$$\approx O(n)$$

Hence, solutⁿ of $Ax=b$, with A being tridiagonal, can be obtain in $O(n)$ steps.

Question 5

Three criteria for selecting library

1) I will check the timing measurements of the library to see which one is optimised. If they are using different algorithms then the package with less complexity will be selected.

2) How storage is managed in the library and the storage requirements.

3) Usually libraries are free but if not, cost will also play a role. Also the library which is popularly used and has produced reliable results for others

will be preferred. All of this is considered given that the library can solve the physics problem I am interested in solving.

Question 6

$$\frac{dy_1}{dx} = 32y_1 + 66y_2 + \frac{2x}{3} + \frac{2}{3} = f$$

$$\frac{dy_2}{dx} = -66y_1 - 133y_2 - \frac{1}{3}x - \frac{1}{3} = g$$

$$\left|\frac{\partial f}{\partial y_1}\right| = 32 \qquad \left|\frac{\partial g}{\partial y_1}\right| = |-66| = 66$$

$$\left|\frac{\partial f}{\partial y_2}\right| = 66 \qquad \left|\frac{\partial f}{\partial y_2}\right| = |-133| = 133.$$

$\Rightarrow$ the ~~pro~~ derivates are finite and bounded

$\therefore$ they satisfy Lipschitz condition

$\Rightarrow$ This problem is well posed and solution by numerical method is correct.

Also, solution seem to overlap with analytic solution.

Question 4

(e)
Sample is a function randomly generated between 0 & 1 whose average is $\frac{1}{2}$ so DFT is a delta function and hence power spectrum is a ~~density~~ delta function at $K=0$.

(c)

$$K_q = 0, \frac{q}{n\Delta} \quad \text{with } q = 1 \ldots \frac{n}{2} - 1$$

$$\& \; \frac{q}{n\Delta} \quad \text{with } q = -\frac{n}{2}, \ldots 1.$$

$$K_{max} = 0.499$$

$$K_{min} = -0.5$$

and frequencies

$$K'_{max} = 2\pi K_{max} = 3.135$$

$$K'_{min} = 2\pi K_{min} = 3.142$$