Q1

Computing Gauss elimination on a 2-digit floating point computer.

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 4 & -1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ -9 \end{bmatrix}$$

In augmented form

$$\left[\begin{array}{ccc|c} 0.40\times10^1 & 0.10\times10^1 & 0.20\times10^1 & 0.90\times10^1 \\ 0.20\times10^1 & 0.40\times10^1 & -0.10\times10^1 & -0.50\times10^1 \\ 0.10\times10^1 & 0.10\times10^1 & -0.30\times10^1 & -0.90\times10^1 \end{array}\right]$$

R_i - ith row

$$R_1 \rightarrow \frac{R_1}{a_{11}}$$

$$\left[\begin{array}{ccc|c} 0.10\times10^1 & 0.25\times10^0 & 0.5\times10^0 & 0.22\times10^1 \\ 0.20\times10^1 & 0.40\times10^1 & -0.10\times10^1 & -0.50\times10^1 \\ 0.10\times10^1 & 0.10\times10^1 & -0.3\times10^1 & -0.90\times10^1 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1 * a_{21}$ & $R_3 \rightarrow R_3 - R_1 * a_{31}$

$$\left[\begin{array}{ccc|c} 0.10\times10^1 & 0.25\times10^0 & 0.5\times10^0 & 0.22\times10^1 \\ 0 & 0.35\times10^0 & -0.20\times10^1 & -0.94\times10^1 \\ 0 & 0.\cancel{0}8\times10^0 & -0.35\times10^1 & -0.11\times10^2 \end{array}\right]$$

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a_{22} is

$$R_2 \longrightarrow R_2 / a_{22}$$

$$\left[\begin{array}{ccc|c} 1.0 & 0.25\times10^{0} & 0.5\times10^{0} & 0.22\times10^{1} \\ 0 & 1.0 & -0.57\times10^{0} & -0.27\times10^{1} \\ 0 & 0.80\times10^{0} & -0.35\times10^{1} & -0.11\times10^{2} \end{array}\right]$$

$$R_3 \longrightarrow \underline{R_3} - R_2 * a_{32}.$$

$$\left[\begin{array}{ccc|c} 1.0 & 0.25\times10^{0} & 0.5\times10^{0} & 0.22\times10^{1} \\ 0 & 1.0 & -0.57\times10^{0} & -0.27\times10^{1} \\ 0 & 0 & -0.30\times10^{1} & -0.9\times10^{1} \end{array}\right]$$

now back substitution

$$x_3 = \frac{-0.9}{-0.3} = 3$$

$$x_2 + (-0.57)(x_3) = -2.7$$

$$x_2 = -2.7 + 0.57\times3$$
$$= -2.7 + 1.71 = -2.7 + 1.7$$
$$= -1$$

$$x_1 = 2.2 + (0.25 * 1) - (0.5 * 3)$$
$$= 2.2 + (0.25) - (1.5)$$
$$= 0.22\times10^{1} + 0.02\times10^{1} - 0.15\times10^{1} = 0.09\times10^{1}$$
$$= 0.9$$

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So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.9 \\ -1 \\ 3 \end{bmatrix}$

Question 3

Computational complexity of ~~of~~ gauss olimination of a tridiagonal matrix.

$$\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & \cdots \\ a_{21} & a_{22} & a_{23} & 0 & 0 & \cdots \\ 0 & a_{32} & a_{33} & a_{34} & & \\ & & \ddots & \ddots & \ddots & \\ & & & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Suppose the algorithm is at i^{th} step

then, operations are:

4 divisions at ith row to make $a_{ii} = 1$

then 4 multiplicat^n & 4 substraction to make

$a_{i-1,i+1}$ 0. (Including operation on b)

No further subtraction are required because of triadiagonal nature of the matrix.

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Total operations at i^{th} row = 12

except at 1 row, At first row there will be

3 multiplicat^n

3 division

& 3 subtract^n

$\therefore$ Total operation $= 9 + 12(n-2)$

$= 12n - 3$

Now Back substitution.

After elimination matrix will look like

$$\begin{bmatrix} a'_{11} & a'_{12} & & & \\ & a'_{22} & a'_{23} & & 0 \\ & & a'_{33} & \ddots & \\ & 0 & & \ddots & \ddots \\ & & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b'_1 \\ b'_2 \\ \vdots \\ \vdots \\ b'_n \end{bmatrix}$$

last row requires 1 division.

& other rows require 1 multiplicat^n, 1 subtract^n.

~~& 1 div~~ (division not required because we have 1's in diagonal except at ~~first~~ last row).

$\therefore$ Total operation in back substitut^n

$= 1 + n(2)$

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Total operations to obtain the solutⁿ of the system is

$$\text{Total Operat}^n = 12n-3 + 2n+1$$
$$= 14n-2$$
$$\approx O(n)$$

Hence, solutⁿ of $Ax=b$, with A being tridiagonal, can be obtain in $O(n)$ steps.

Question 5

~~Three~~ criteria for selecting library

1) I will check the timing measurements of the library to see which one is optimised. If they are using different algorithms then the package with less complexity will be selected. ~~without~~

2) How storage is managed in the ~~also~~ library and the storage requirements.

3) Usually libraries are free but if not, cost will also play a role. Also the library which is popularly used and has produced reliable results for others

will be preferred. All of this is ~~given~~ considered given that the library can solve the physics problem I am interested in solving.

Question 6

$$\frac{dy_1}{dx} = 32y_1 + 66y_2 + \frac{2x}{3} + \frac{2}{3} = f$$

$$\frac{dy_2}{dx} = -66y_1 - 133y_2 - \frac{1}{3}x - \frac{1}{3} = g$$

$$\left|\frac{\partial f}{\partial y_1}\right| = 32 \qquad \left|\frac{\partial g}{\partial y_1}\right| = |-66| = 66$$

$$\left|\frac{\partial f}{\partial y_2}\right| = 66 \qquad \left|\frac{\partial f}{\partial y_2}\right| = |-133| = 133$$

$\Rightarrow$ the derivates are finite and bounded
$\therefore$ they satisfy Lipschitz condition

$\Rightarrow$ This problem is well posed and solution by numerical method is correct.

Also, solution seem to overlap with analytic solution.

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Question 4

(e) Sample is a function randomly generated between 0 & 1 whose average is $\frac{1}{2}$ so DFT is a delta function and hence power spectrum is a ~~density~~ delta function at $K=0$.

(c) $K_q = 0, \frac{q}{n\Delta}$ with $q = 1 \ldots \frac{n}{2} - 1$

& $\frac{q}{n\Delta}$ with $q = -\frac{n}{2}, \ldots 1$.

$K_{max} = 0.499$

$K_{min} = -0.5$

and frequencies

$K'_{max} = 2\pi K_{max} = 3.135$

$K'_{min} = 2\pi K_{min} = 3.142$

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