Sinhgad College of Enginnering

Department of Information Technology

Second Year of Engineering

Discrete Mathematics (2019 pattern)

Unit II

Combinatorics and Discrete Probability

Discrete Probability: Discrete Probability, Conditional Probability, Bayes Theorem, Information and

Mutual Information, Applications of Combinatorics and Discrete Probability.

Consider the following experiments

- A coin is tossed
- A dice is thrown
- A card is drawn from pack of playing

In 1, either ‘Head’ or ‘Tail’

In 2, any one of the first 6 natural numbers

In 3, any one of the 52 playing cards may be drawn

Discrete Probability

Sample space: A set of all possible outcomes.

Denoted by ‘S’.

Each element of sample space is called sample point denoted by x.

For 1, S={H,T}

For 2, S={1,2,3,4,5,6}

For 3, sample space consists 52 playing cards may be drawn

Write the sample space for the following experiments

A coin is tossed twice or tossing pair of coins.

Sample space={TT,TH,HT,HH}

Sample points: TT,TH,HT,HH

A coin is tossed thrice or three coins are tossed

A dice is tossed twice or two coins are tossed

Sample space={TTT, TTH, THT, THH, HTT, HTH, HHT, HHH}

Sample space={ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Event

Event: A subset of sample space containing those sample points satisfying the given condition is called the event.

Example: Tossing pair of coins with resulting at least one head.

Sample Space= {TT, TH, HT, HH}

Event = {TH, HT, HH}

Let a dice be thrown .

Let E be the event that ‘Prime numbers occur on face’

$S=\{1,2,3,4,5,6\}$

$E=\{2,3,5\}$

TYPES OF EVENTS

Impossible Event:

The event occurs to the empty set is called an impossible event.

Let a dice be thrown

Let E be the event that 7 occurs on the face

S={1,2,3,4,5,6}, E=NULL

Certain Event:

Certain Event: The event covers the full space S is called the certain event. (ES)

Let a coin is tossed

Let E be the event that either head or tail may come up

Complementary Event:

If A is an event, complement of A is given by,

~A=S-A

Example: Tossing pair of coins with resulting at least one head.

Sample Space=S= {TT, TH, HT, HH}

Event =A= {TH, HT, HH}

Complement of A=~A={TT}

Compound Event

Compound event: If A and B are two events, the compound event is either A or B occurs.

It is denoted by, $A \cup B$

Example: Pair of die is rolled, sum on faces is 4 or less.

$A \cup B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$

example

Let a card be drawn from a pack of playing card.

Let A be the event that ‘Heart is drawn’

Let B be the event that ‘Queen is drawn’

Pack(52)

The compound of event is that queen of heart is drawn.

Product Event

If A and B are two events, the product event is both A and B occurs.

It is denoted by, $A \cap B$

Example: Pair of die is rolled, sum is even and perfect square.

$A \cap B = \{(1,3), (2,2),(3,1)\}$

Mutually Exclusive Events

If A and B are said to be mutually exclusive if occurrence or non occurrence A precludes B.

If A occurs B doesn’t occurs and vice versa.

$A \cap B = \Phi$

A: Head

B: Tail

Independent Events:

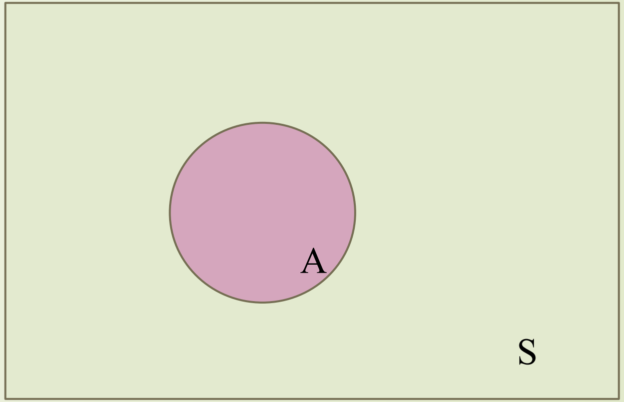
Occurrence or non occurrence of one doesn't affect occurrence or non occurrence of the other.

Example: Tossing coin independently.

Probability

Let S be the sample space containing ‘n’ sample points.

Let A be the event containing ‘m’ sample points



Probability

We define the probability of event A occurs is the number or ratio m/n

$$P(A) = m/n = n(A)/n(S)$$

Probability of sample point P(x):

$$0 <= P(x) <= 1,$$

$$\Sigma P(x_i) = 1$$

Probability of Impossible Event ??

Probability of Certain Event ??

Example: Tossing Coin experiment. $S=\{H,T\}$

$$P(x)=P(H)=P(T)=1/2$$

- Probability of an event (A):

$$A=(x_1,x_2,x_3,\dots.x_n)$$

$$P(A)=\Sigma\, P(x_i)$$

Example

Tossing Coin three times. Getting exactly one head.

S={TTT, TTH, THT, THH, HTT, HTH, HHT, HHH}

A= {HTT, THT, TTH}

P(A)= 1/8 + 1/8 + 1/8

=3/8

Example

An unbiased dice is thrown. Find the probability of events A and B

A: score is an even number

B: score is the number less than 5 and more than 2.

SOLUTION

We have,

S={1,2,3,4,5,6}

A={2,4,6} and B={3,4}

P(A)=n(A)/n(S)=3/6=1/2

P(B)=n(B)/n(S)=2/6=1/3

PROBLEM

Two dice are thrown. Find the probability of getting

A: the same score on first dice same as second dice

B: the score on 2^{nd} dice is greater than 1^{st} dice

C: sum of the numbers on upper surfaces is at most 5.

SOLUTION

A={(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

P(A)=n(A)/n(S)=6/36=1/6

B={(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6) }

P(B)=n(B)/n(S)=15/36=5/12

C={(1,1),(1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1) }

P(C)=n(C)/n(S)=10/36=5/18

Practice Example

Q.1 A coin is tossed two times. Find the probability that at least one head occurs

Q.2 A coin is tossed three times. Find the probability of getting

a) at most 1 head

b) two consecutive heads

SOLUTION

1. $\frac{3}{4}$

   a. $\frac{1}{2}$

   b. 3/8

EXAMPLE

Find the probability of drawing from a white ball from a box containing 6 white and 4 black balls.

solution

S={W1,W2,W3,W4,W5,W6,B1,B2,B3,B4}

A={W1,W2,W3,W4,W5,W6}

n(S)=10

n(A)=6

P(A)=6/10 =3/5

OTHER METHOD

1 ball out of 10 balls can be selected in C(10,1)= 10 different ways

So, sample space contains 10 sample points.

1 white out of 6 white of balls can be selected in C(6,1)=6 different ways

PROBLEM

A box contains 5 black, 6 white and 4 green balls. Two balls are drawn at random. Find the probability that

1. Both are black.
2. One is black and other is green.
3. Only one is black

SOLUTION

The box contains 15 balls.

So, the sample space contains 15 C2 = 15*14/1*2=105

Let A: event that both the drawn balls are black

5C2=10 sample points

P(A)=10/105=2/21

SOLUTION ctd..

Let B: event that one ball is black and other is green.

5C1 * 4C1=5*4= 20 sample points

P(B)=20/105=4/21

Let C: event that only one ball is black

5C1 * 10C1=5*10= 50 sample points

P(C)=50/105=10/21

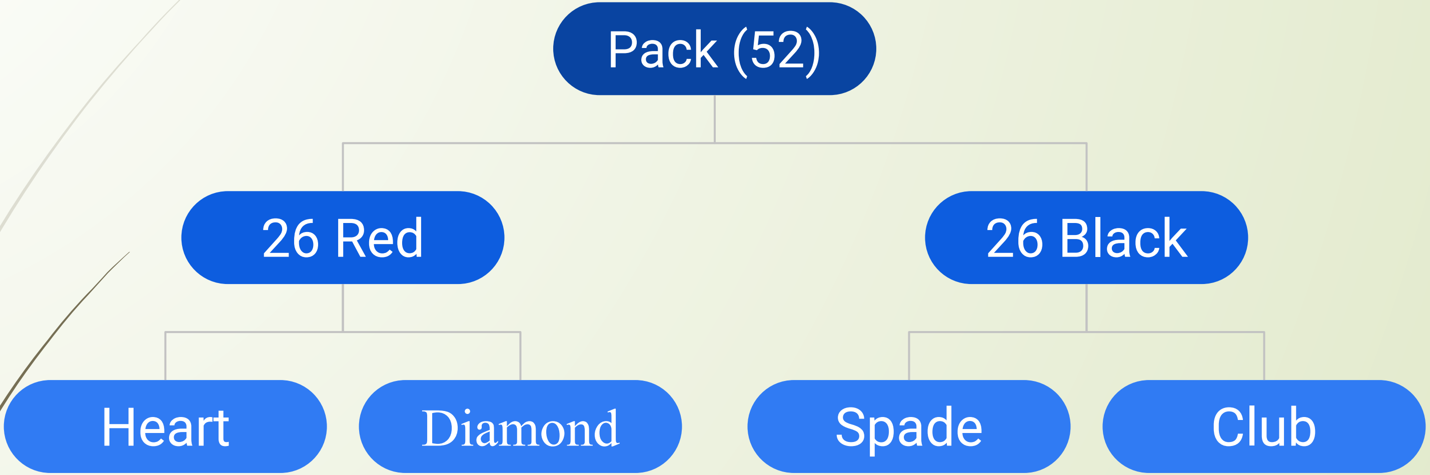
PROBLEM

A box contains 5 black, 6 white and 4 green balls. Two balls are drawn at random. Find the probability that

1. Both are black.

2. One is black and other is green.

3. Only one is black



FACE: KING, QUEEN, JACK (12 pictures)

Numbers: Ace, 2, 3, 4, ……….10

PROBLEM

For a well-shuffled pack of playing cards, 3 cards are drawn at random. Find the probability that

i) Three drawn card contains 2 kings and an ace

ii)at least two red cards

iii) at most two red cards

Solution

The sample space contains C(52,3)= sample points=52!/49!*3!
=26*17*50

Let A: event that contains 2 kings and an ace

A contains C(4,2)*C(4,1)=24

P(A)=6/5525

Solution ctd….

Let B: event that contains at least 2 red cards

B contains C(26,2)*C(26,1)+C(26,3)=26*25*17

P(B)=½

Let C: event that at most 2 red cards

(0 red and 3 black OR 1 red and 2 black OR

2 red and 1 black)

Solution ctd….

C(26,0)*C(26,3) +
C(26,1)*C(26,2) +
C(26,2)*C(26,1)
=19,500

P(C)=15/17

PROBLEM

A team of 5 is to be selected from 8 boys and 3 girls. Find the probability if it includes

i) exactly 2 girls

ii)2 particular girls

Solution

The sample space contains C(11,5)= sample points=11!/6!*5! =11*6*7

Let A: event that contains exactly two girls

A contains C(3,2)*C(8,3)=4*7*6

P(A)=4/11

Solution ctd….

Let B=event that particular 2 girls (i.e. 3 students out of 9 students)

Therefore, B contains C(9,3)=3*4*7

P(B)=2/11

PROBLEM

A team of 4 boys and 3 girls is to be selected from a group of 8 boys and 5 girls. Find the probability if committee includes a particular girl and a particular boy

SOLUTION

THERE ARE 13 STUDENTS.

The sample space contains C(8,4)*C(5,3)

= 700

Let A: committee includes a particular girl and a particular boy and girl

SOLUTION ctd...

A contains C(7,3)*C(4,2)=7*3*10

P(A)=3/10

PROBLEM

4 pairs of hand gloves are there in a closet. Two gloves are drawn at random. Find the probability that they are of the same hand.

SOLUTION

There are 8 gloves in closet. Sample space contains C(8,2)= 28

A: event that ‘they are of the same hand’

(Right hands OR Left hands)

=C(4,2)+C(4,2)=4*3

P(A)=3/7

PROBLEM

A bag contains 5 red, 4 blue and unknown number m of green balls. If probability of getting both the balls are green when two balls are selected at random is 1/7. Find the value of m.

SOLUTION

There are (9+m) balls in the bag. The sample space contains C(9+m,2)= (9+m)!/2!*(9+m-2)!

=(9+m)!/(7+m)!*2!

=(9+m)(8+m)/1*2

Let A: event that both drawn balls are green. A contains C(m,2)=m*(m-1)/2

SOLUTION

$P(A)=m(m-1)/(9+m)(8+m)$

But $P(A)=1/7$

$m^2 -4m-12=0$

$m=6$ or $m=-2$

Therefore, $m=6$

PROBLEM

If the letters of the word ‘RANDOM’ be arranged at random. What is the probability that two letters O and A will be at either extremes?

SOLUTION

There are 6 different letters in a given word. So, sample space contains P(6,6)=6!

For required event we have the following two types

A	P(4,4)	O

O	P(4,4)	A

SOLUTION

For each of the type we have

P(4,4)=4! Different arrangements

So, required event contains 2*4! Sample points.

Therefore, prob=2*4!/6! =1/15

PROBLEM

8 boys and 2 girls are sitting in a row for photograph. Find the probability that the girls

i) are seated together

ii)occupy end seats

SOLUTION

There are 10 students in all.

The sample space contains P(10,10)=10! Sample points.

When two girls are seated together, we can consider two girls as a single unit. So, we have following two types.

……. ……. ……. …….

For each of these 2 type, we have P(9,9)=9!

G1G2 G2G1

SOLUTION ctd….

Therefore, required event contains 2*9! sample points.

Required probability=2*9!/10!

=1/5

SOLUTION ctd….

For two girls occupy end seats, we have 2 types

For each type 8 boys can be adjusted in P(8,8)=8! different ways.

G1	**P(8,8)**	**G2**
G2	**P(8,8)**	**G1**

SOLUTION ctd….

So, this event contains 2*P(8,8) sample points.

Required Probability is

2*8!/10!=1/45

PROBLEM

The letters of the word 'EQUATION' are arranged in a row at random. Find the probability that

i) all the vowels are together

ii) the arrangements starts with a vowel and ends with a consonant

SOLUTION

There are 8 different letters in a given word of which 5 vowels and 3 consonants.

The sample space contains P(8,8)=8! Sample points.

When 5 vowels are kept together, we can consider 5 vowels as a single unit. So, we have 4 things in all.

……   ……

They can be permuted P(4,4)=4!

5 Vowels

SOLUTION ctd….

For each of these for 4! permutations we can arrange 5 vowels among themselves in $P(5,5)=5!$

So, this event contains $P(5,5)*P(4,4)$ sample points.

Required Probability is $5!*4!/8!=1/14$

SOLUTION ctd….

Vowels (5 ways)	P(6,6)	Consonants (3 ways)

So, this event contains 5*3*6! Sample points.

Hence, required probability = 5*3*6!/ 8!

= 15/56

PROBLEM

6 boys and 3 girls are to be seated in a row for a photograph. Find the probability that no two girls seat together

SOLUTION

There are 9 students.

The sample space contains P(9,9)=9! sample points.

For required event we arranged the boys

_ B1 _ B2 _ B3 _ B4 _ B5 _ B6 _

1 2 3 4 5 6 7 = *places*

6 boys among themselves can be permuted P(6,6)=6!

SOLUTION ctd….

For the girls there are 7 places available. 3 girls on these 7 places can be adjusted P(7,3)=7!/4!=7*6*5 different ways.

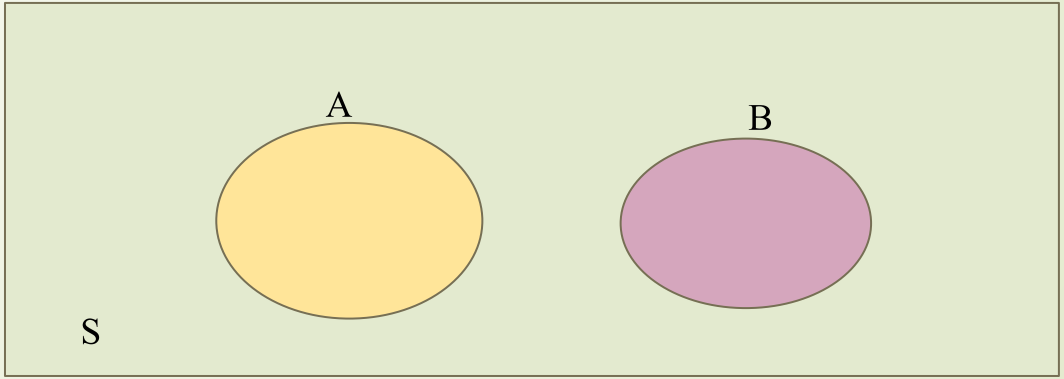
So, this event contains 6!*7*6*5 sample points.

Required Probability is 6!*7*6*5/9!=5/12

Addition Theorem

I) For exclusive events:

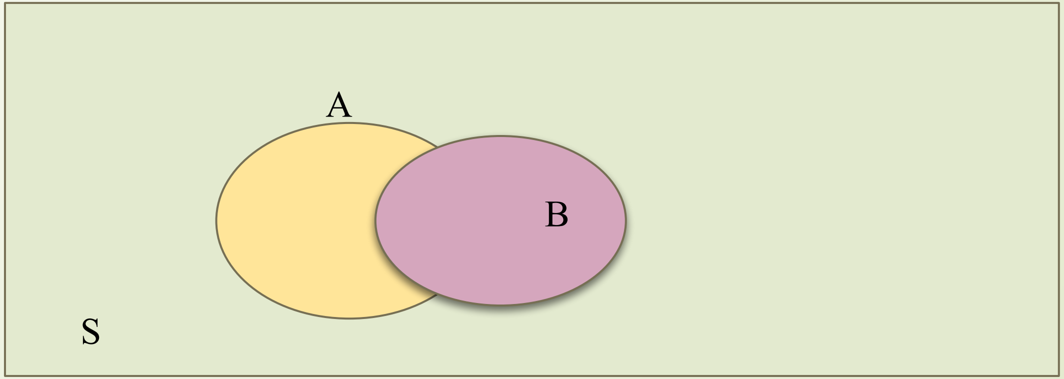
If A and B are exclusive events then $P(A \cup B) = P(A) + P(B)$



Addition Theorem

II) For non-exclusive events:

If A and B are non-exclusive events then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



PROBLEM

Find the probability that a card drawn from a pack of cards is either a king or a queen.

SOLUTION

The sample space contains 52 sample points.

Let A be the event that king is drawn.

Let B be the event that queen is drawn.

To find P(A U B),

A and B contains 4 points.

P(A U B)= P(A) + P(B)= 4/52 + 4/52=2/13

PROBLEM

Find the probability that a card drawn from a pack of cards is a red card or a picture card.

SOLUTION

The sample space contains 52 sample points.

Let A be the event that a red card is drawn.

Let B be the event that a picture card is drawn.

A contains 26 sample points and B contains 12 sample points.

SOLUTION ctd….

To find P(A U B),

A contains 26 sample points and B contains 12 sample points.

6 sample points are both red and picture card.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 26/52 + 12/52 - 6/52$
$= 32/52 = 8/13$

PROBLEM

Two dice green and red are thrown. Obtain the probability that number on green dice is odd or sum of the number on faces are 6.

SOLUTION

Sample Space contains (x,y) where x is number on Green dice and y is number on Red dice.

Sample Space contains 6*6= 36 sample points

SOLUTION ctd….

Let A: event that number on green dice is odd

={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),}

n(A)=18

SOLUTION ctd….

Let B: event that sum of the numbers on the faces is 6

$=\{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

$n(B)=5$

$P(A \cap B)=\{(1,5),(3,3),(5,1)\}$

SOLUTION ctd….

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 18/36 + 5/36 - 3/36$

$= 20/36$

$= 5/9$

PRACTICE EXAMPLE

Two dice are thrown. Obtain the probability that the total score obtained is perfect square or a multiple of 3

SOLUTION

n(S)=36

Let A: event that total score is a perfect square (i.e. sum =4 or 9)

Let B: event that total score is a multiple of 3 (i.e. sum =3,6,9,12)

SOLUTION ctd..

$n(A)=7$, $n(B)=12$, $n(A \cap B)=4$

Therefore,

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

$$= 7/36 + 12/36 - 4/36$$

$$= 15/36 \qquad = 5/12$$

PROBLEM

A box contains 7 red and 5 blue balls. Two balls are drawn at random. Find the probability that they are of same color.

SOLUTION

$n(S)=C(12,2)$

Let A: event that both drawn balls are red

Let B: event that both drawn balls are blue

$n(A)=C(7,2)=21$

$n(B)=C(5,2)=10$

$$P(A \cup B)= P(A) + P(B)$$
$$= 21/66 + 10/66$$
$$=31/66$$

CONDITIONAL PROBABILITY

Let a card is drawn from a pack of playing cards.

Let A be the event that red card is drawn.

Let B be the event that picture card is drawn.

$n(S)=52$, $n(A)=26$, $n(B)=12$

Therefore, $P(A \cap B)=P(AB)=6/52$

These probabilities are called as regular probabilities because sample space is full space.

Let a card be drawn from a pack of playing cards

And it is given that event A has occurred.

So, we can restrict the sample space to the set A only and the favorable sample points for event B to occur are only 6.

This probability of B under the assumption that event A as occurred is called conditional probability of B.

- Conditional Probability: Occurrence of one event depends on occurrence or non occurrence of the other.
- If event A already occurred, then conditional probability B with respect to A is,

$$P(B/A)=P(A\cap B)/P(A)$$

Formula

P(AB)=P(A).P(B/A)

OR P(B/A)=P(AB)/P(A)

LHS

=6/52

RHS

=26/52 * 6/26

=6/52

Exmaple

Let a dice be thrown

Let A: even number occurs on the face

Let B: number on face >=4

S={1,2,3,4,5,6} A={2,4,6} B={4,5,6}

A∩B={4,6} P(A∩B)=2/6

P(A∩B)=3/6*2/3 =2/6

Example: Two dies are rolled. What is the probability that the sum of the faces will not exceed 7? Given that at least one face shows 4.

Ans: $A = \{(a,b) \mid a+b <=7\}$

$B = \{ (a,4), (4,a)\}$

$A=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),$
$(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),$
$(4,3),(5,1),(5,2),(6,1)\}$

$B=\{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1),(4,2),$
$(4,3),(4,5),(4,6)\}$

$p(A \cap B)=p(AB) = 6/36$, $p(B)= 11/36$, $p(A)=21$
$p(A/B) = 6/11$

Example: What is the probability of a random bit string of length four contains at least two consecutive 0s, given that its first bit is a 0 ?

Ans:

A: "bit string contains at least two consecutive 0s"

B: "first bit of the string is a 0"

We know the formula $p(A|B) = p(A \cap B)/p(B)$.

$A \cap B = \{0000, 0001, 0010, 0011, 0100\}$

$p(A \cap B) = 5/16$

$p(B) = 8/16 = 1/2$

$p(A \mid B) = (5/16)/(1/2) = 10/16 = 5/8 = 0.625$

PROBLEM

What is the conditional probability that a family with two children has two boys, given they have at least one boy? Assume that each of the possibilities BB, BG, GB, and GG is equally likely, where B represents a boy and G represents a girl.

SOLUTION

Let E be the event that a family with two children has two boys, and let F be the event that a family with 2 children has at least one boy

Therefore, E={BB}, F={BB,GB,BG} and E∩F={BB}

$p(F)=\frac{3}{4}$ $p(E\cap F)=\frac{1}{4}$ $p(E/F)=(\frac{1}{4})/(\frac{3}{4})=1/3$

Independent Events

Event A is said to be independent of event B if the probability of A is unaffected by the happening or non-happening of the event B

i.e. event A is independent of event B

If $P(A) = P(A/B)$

Examples

Let a coin be tossed and dice be thrown.

A: head occurs on the coin.

B: 6 occurs on the dice

Let a mathematical problem is given to 2 students Alice and Bob

A: Alice solves the problem

B: Bob solves the problem

If A and B are independent events then

$$P(A \cap B) = P(AB) = P(A) * P(B)$$

Problem

A and B are two events such that $p(A)=\frac{1}{2}$ $p(B)=\frac{1}{3}$, $p(A \cup B)= \frac{2}{3}$. Check whether A and B are independent events.

Solution

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$\frac{2}{3} = \frac{1}{2} + \frac{1}{3} - p(AB)$

$p(AB) = \frac{1}{6}$

$P(A).p(B) = \frac{1}{2} * \frac{1}{3} = \frac{1}{6}$

$p(AB) = p(A).p(B)$

So, events A and B are independent events.

Problem

Find the probability that 'A can solve the problem is $\frac{1}{4}$ and that B can solve it is $\frac{3}{5}$' , the problem is solved by them independently.

Solution

Let A: A solves the problem and B: B solves the problem events A and B are independent events.

$p(A)=\frac{1}{4}$ and $p(B)=\frac{3}{5}$

$p(A).p(B)=\frac{1}{4} * \frac{3}{5} = 3/20$

Solution

The problem is solved if anyone can solve the problem i.e. A U B

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

$$= \tfrac{1}{4} + \tfrac{3}{5} - 3/20$$

$$= 7/10$$

Problem

A problem is given to 3 students A, B and C whose chances of solving them are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. Find the probability that the problem is not solved by them if they try independently.

Solution

Let A: A solves the problem, B: B solves the problem and C: C solves the problem

$p(A)=\frac{1}{4}$, $p(B)=\frac{3}{5}$ and $p(C)=\frac{1}{4}$

A': A cannot solve the problem $p(A')=1-\frac{1}{2}= \frac{1}{2}$

B': B cannot solve the problem $p(B')=1-\frac{1}{3} = \frac{2}{3}$

C': C cannot solve the problem $p(C')=1-\frac{1}{4} =\frac{3}{4}$

Problem is not solved if no one should solve the problem i.e. to find $p(A' \cap B' \cap C')$

$P(A').p(B').p(C') = \frac{1}{2} * \frac{2}{3} * \frac{3}{4} = \frac{1}{4}$

- Find the probability that problem is solved

Solution

Problem is solved and problem is not solved are complementary events of each other.

So, required probability $=1 - \frac{1}{4} = \frac{3}{4}$

PROBLEM

A box contains 3 white and 4 black balls. Another box contains 4 white and 5 black balls. If 1 ball is selected at random from each box then what is probability that

- They are of the same colour
- They are of the different colour

Solution

Let A: both drawn balls are white and B: both drawn balls are black

To find p(AUB)

A and B are exclusive events.

$$p(A \cup B) = p(A) + p(B)$$

$$= p(w1w2) + p(b1b2)$$

$$= p(w1)p(w2) + p(b1)p(b2)$$

$= 3/7 * 4/9 + 4/7 * 5/9$

$=12/63 + 20/63$

$=32/63$

Both are of he same colour and both are of the different colour are complementary events.

Therefore, required probability is $1 - 32/63 = 31/63$

Binomial Probability: The experiments is performed n times, probability of event occurs is p, if event occurs r times then its has r successes.

The probability that the trial has r successes is given as,

$$P(r)={}^{n}C_{r} * p^{r} * q^{n-r}$$

$q=1-p$

p=probability of success

Binomial Probability:

Example: An examination contains multiple choice questions is designed so that the probability of correct choice by guessing is 0.2. What is the probability that a student will not get more than two questions right out of 10 by guessing alone?

Ans: $n= 10, 0<= r <= 2, p=0.2, q= (1-0.2)= 0.8$

$$p(0<= r <= 2) = p(0) +p(1) + p(2)$$

$$= {}^{10}C_0 . (0.2)^0 . (0.8)^{10} + {}^{10}C_1 . (0.2)^1 . (0.8)^9 + {}^{10}C_2 . (0.2)^2 . (0.8)^8$$

$$=0.68$$

Information: Let A be an event with its probability p(A). Then ,

$$I(A) = -\log(p(A))$$

Example: A man is that when a pair of dice were rolled the result was 7. How much information is there in this message?

Ans: $A = \{(1,6), (2,5), (3,4), (4,3),(5,2),(6,1)\}$

$$p(A) = 6/36 = 1/6$$

$$I(A) = -\log(1/6) = \log_2 6 = 2.585 \text{ bits}$$

Mutual Information: Let A and B are two events. Mutual information between A and B is occurrence of A contained in statement of B.

$$I(A,B) = -\log(p(A)) + \log(p(A/B))$$

$$= -\log(p(A)) + \log(p(A \cap B)) - \log(p(B))$$

$$I(A,B) = I(B, A)$$

Mutual Information

$P(B|A)=P(A \cap B)/P(A)$

$I(B,A)=-\log P(B)+\log[P(A \cap B)]-\log P(A)$

$=-\log P(A)+\log(P(A \cap B)/P(B))$

$=-\log P(A)+\log P(A|B)$

$I(A,B)=I(B,A)$

Baye's Theorem

Let A1, A2....An are mutual events whose union is sample space S. Let B is any other event in S. Then,

$$p(Ai/B)=\frac{p(A_i).p(B/A_i)}{p(A_1).p(B/A_1)+p(A_2).p(B/A_2)+\ldots+p(A_n).p(B/A_n)}$$

Baye's Theorem

Example: In a bolt factory there are 4 machines A,B, C, D manufactures 20%, 25%, 10%, 45% of the total golds respectively. 2% of the bolts manufactured by A, 4% by B, 2% by C and 5% of D are found defective. A bolt is chosen at random and is found defective.What is the probabilty that it is manufactured by C?

Baye's Theorem

Ans:

$$p(C/E)=\frac{p(C).p(E/C)}{p(A).p(E/A)+p(B).p(E/B)+p(C).p(E/C)+p(D).p(E/D)}$$

$$p(C/E)=\frac{2/100*10/100}{2/100*20/100+4/100*25/100+2/100*10/100+5/100*45/100}$$

$$p(C/E)=20/385$$

$$p(C/E)=0.05194$$

Baye's Theorem

Example:

Consider a binary communication channel over which a 0 or a 1 is to be sent. Probability that 1 is sent is 0.6, and 0 is sent is 0.4. Due to noise in the channel, the probability that 1 is charged to 0 is 0.15 and that of 0 to 1 is 0.2. Suppose that 0 is received. What is the probability that 1 was sent?

Baye's Theorem

Ans: Let A den

ote the event that 1 was sent and B the event that 0 was received. Hence,

$P(A|B)=P(B|A).P(A)/P(B)$

$P(B|A)=0.15$

$P(A)=0.6$ $P(B)=(0.4)(0.8)+(0.6)(0.15)=0.41$

$P(A|B)=(0.15*0.16)/0.41=0.2195$


THANK
YOU