LBC Complete Example

Linear Block Codes Complete example

- For a (6, 3) code, the generator matrix G is

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find
(a) All Corresponding Code Vectors
(b) Minimum Hamming dist.^{n} d_{min}
(c) Error detection & Error correction Capability.
(d) Parity check Matrix
(e) Find error if received Code is (100011)

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$\rightarrow G = [I_k : P]$

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\rightarrow [C] = [i][G]$

$= [m : P_c]$

$\rightarrow [P_c] = [i_m][P]$

$\rightarrow G = [I_k : P]$

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\rightarrow [C] = [i][G]$$

$$= [m : P_c]$$

$$\rightarrow [P_c] = [i_m][P]$$

$$[P_0 \; P_1 \; P_2] = [i_0 \; i_1 \; i_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$\rightarrow [C] = [i][G]$

$= [m : P_c]$

$\rightarrow [P_c] = [i_m][P]$

$$[P_0 \; P_1 \; P_2] = [i_0 \; i_1 \; i_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\rightarrow P_0 = (i_0 \oplus i_2)$

$P_1 = (i_1 \oplus i_2)$

$P_3 = (i_0 \oplus i_1)$

$$[P_0 \; P_1 \; P_2] = [i_0 \; i_1 \; i_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\rightarrow P_0 = (i_0 \oplus i_2)$

$P_1 = (i_1 \oplus i_2)$

$P_2 = (i_0 \oplus i_1)$

C	i_0	i_1	i_2	P_0	P_1	P_2
c_1	0	0	0	0	0	0
c_2	0	0	1	1	1	0
c_3	0	1	0	0	1	1
c_4	0	1	1	1	0	1
c_5	1	0	0	1	0	1
c_6	1	0	1	0	1	1
c_7	1	1	0	1	1	0
c_8	1	1	1	0	0	0

$$[P_0 \; P_1 \; P_2] = [i_0 \; i_1 \; i_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\Rightarrow P_0 = (i_0 \oplus i_2)$

$P_1 = (i_1 \oplus i_2)$

$P_2 = (i_0 \oplus i_1)$

C	i_0	i_1	i_2	P_0	P_1	P_2	W
C_1	0	0	0	0	0	0	-
C_2	0	0	1	1	1	0	3
C_3	0	1	0	0	1	1	3
C_4	0	1	1	1	0	1	4
C_5	1	0	0	1	0	1	3
C_6	1	0	1	0	1	1	4
C_7	1	1	0	1	1	0	4
C_8	1	1	1	0	0	0	3

$\boxed{d_{min} = 3}$

$$\begin{bmatrix} \cdots & & & \cdots & & \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

I_k P

Find (a) All Corresponding Code Vectors

(b) Minimum Humming dist.^{u} d_{min}

(c) Error detection & Error correction capability.

Parity check matrix

Find error if received code is (100011)

$[I_k : P]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$= [i][G]$

$= [m : P_c]$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$= [\,i\,][\,G\,]$

$= [\, m : P_c \,]$

$] = [\, i_m \,][\, P \,]$

$P_1\ P\ ] = [\, i_0 \ \ i_1 \ \ i_2 \,] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\oplus\, i_2)$

$(i_1 \oplus i_2)$

$(i_0 \oplus i_1)$

- $d_{min} = 3$
- Error detection
  - $\Rightarrow d_{min} \geq s+1$
  - $\Rightarrow 3 \geq s+1$
  - $\Rightarrow s \leq 2$
- It can detect 2bit error
- Error correction
  - $\Rightarrow d_{min} \geq 2t+1$
  - $\Rightarrow 3 \geq 2t+1$
  - $\Rightarrow t \leq 1$
  - It can correct 1 bit error

C	i_0	i_1	i_2	P_0	P_1	P_2	W
C_1	0	0	0	0	0	0	-
C_2	0	0	1	1	1	0	3

$$H = [\,P^T : I_{n-k}\,]$$

$\rightarrow$ $P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $\qquad I_{6-3} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\rightarrow$ $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

Find (a) All Corresponding Code Vectors

(b) Minimum Humming dist.^{n} d_{min}

(c) Error detection & Error correction capability.

(d) Parity check matrix

(e) Find error if received code is (100011)

: P]

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$[C] = [i][G]$

$= [m : P_c]$

$[P_c] = [i_m][P]$

→ $d_{min} = 3$

→ Error detection

$\Rightarrow d_{min} \geq s + 1$

$\Rightarrow 3 \geq s + 1$

$\Rightarrow s \leq 2$ error

2 bit

$\rightarrow$ $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

$\rightarrow$ $[S] = [Y][H^T]$

$\rightarrow$ $[Y] = [1\,0\,0\,0\,1\,1]$

$[H^T] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$S = YH^T$

$= [1\,0\,0\,0\,1\,1] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\rightarrow [S] = [Y][H^T]$

$\rightarrow [Y] = [1\ 0\ 0\ 0\ 1\ 1]$

$$[H^T] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = YH^T$$

$$= [1\ 0\ 0\ 0\ 1\ 1]\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 1\ 0]$$

$\rightarrow [S] = [Y][H^T]$

$\rightarrow [Y] = [1\ 0\ 0\ 0\ 1\ 1]$

$$[H^T] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = YH^T$

$$= [1\ 0\ 0\ 0\ 1\ 1] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & & 0 \\ 0 & 1 & \\ 0 & 0 & \end{bmatrix}$$

$= [1\ 1\ 0]$

$\rightarrow [Y] = [1\ 0\ 0\ 0\ 1\ 1]$

$$[H^T] = \begin{array}{c} 1 \\ 2 \\ 3 \\ \\ \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = YH^T$

$$= [1\ 0\ 0\ 0\ 1\ 1]\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ \boxed{1 \quad 1 \quad 0} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= [1\ 1\ 0]$

$\rightarrow$ Here error happens at 3^{rd} bit.

$e = [0\ 0\ 1\ 0\ 0\ 0]$

$Y = [1\ 0\ 0\ 0\ 1\ 1]$

$\rightarrow X = e + Y$

$= [1\ 0\ 1\ 0\ 1\ 1]$ (Tx)