Mod 4 - ITC

Cyclic Codes

→ Basics

① Linearity Property
② Cyclic shifting

→ Non - Systematic codes

$C(x) = m(x) \cdot g(x)$ $\quad$ $C = [msg, parity]$ ✗

① Get $m(x)$ using msg
② Get $g(x)$
③ Get $C(x)$
④ Convert to bits

→ Systematic codes

$C(x) = x^{n-k} m(x) + P(x)$ $\quad$ $C = [msg, parity]$ ✓

where,

$$P(x) = \text{remainder}\left[\frac{x^{n-k} m(x)}{g(x)}\right]$$

→ Generator Matrix

$[G] = [I : P]$

(n, k) $\quad$ $n \rightarrow$ col

$k \rightarrow$ rows

1st row : $\text{Rem}\left[\frac{x^{n-1}}{g(x)}\right]$

2nd row : $\text{Rem}\left[\frac{x^{n-2}}{g(x)}\right]$

⋮

→ Encoder designing & Syndrome calculation

Create diagram based on $g(x)$

Then make table → For 7,4 atleast

m	P_1	P_2	P_3
	$m \oplus P_3$	$P_3 \oplus P_1$	P_2
	0	0	0
,			
,			
,			

Extra:

For Syndrome

$S = [P_3 \ P_2 \ P_1]$

→ CRC

① Data & Divisor given

② Append Divisor length−1 '0' to Data

③ XOR continuously

④ Last 3 bits are CRC

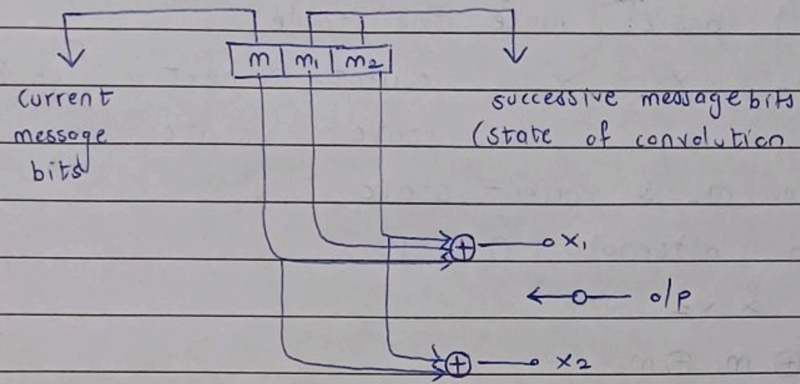
⑤ T_x = Data : CRC

For error,

just check if CRC ≠ 0

Convolution codes

→ **Basics**



$x_1 = m_0 \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

State Table

m_1	m_2	state
0	0	a
0	1	b
1	0	c
1	1	d

k = no. of msg bits = 1

n = no. of encoded o/p bits = 2

K = constraint length = 3

r = code rate = $\frac{k}{n}$ = $\frac{1}{2}$

(n,k) = ~~cost~~ code dimensions = $(2,1)$

→ **Code tree**

① One i/p will be given

② Draw shift register diagram at each step, inputting bits one by one

③ Find $x_1\ x_2$ at each register

④ Determine state - a, b, c, d at each shift register

⑤ Add one extra register with m = – & shifted m_1, m_2, also find its state.

⑥ Draw code tree

a) Start with horizontal line
- down step : i/p 1
- up step : i/p 0

b) Write state at each step up/down

c) Write $x_1\ x_2$ at each line

→ Code trellis & State diagram

① Based on all basics, make the table

m	m_1	m_2	x_1	x_2	current state	next state

[of 8 rows]

② First fill m_1, m_2 & current state

③ Then fill m, alternate 0 & 1

④ Calculate x_1 & x_2

$$x_1 = m \oplus m_1 \oplus m_2$$

$$x_2 = m \oplus m_2$$

⑤ Calculate next state by considering $m \rightarrow m_1 \rightarrow m_2 \rightarrow$

Code trellis

ⓐ ⓐ

ⓑ ⓑ

ⓒ ⓒ

ⓓ ⓓ

—— i/p 0

- - - - i/p 1

State diagram

ⓑ

ⓐ ⓓ

ⓒ

⟶ i/p 0

- - -> i/p 1

⑥ Write $x_1 x_2$ value on each line

→ BCH codes
(t bit error correction possibility)

Formulas :

Block length $n = 2^m - 1$

No. of msg bits $k \geqslant n - mt$

Minimum distance $d_{min} \geqslant 2t+1$

Degree (r) of n, k $= n - k$

$K = n - r$

In Questions :

n

GF( )

~~p(x)~~

t

} Given (any two of these)

k

g(x)

} To find

Remember

If $n = 7$

$x^7+1 = (x+1)(x^3+x+1)(x^3+x^2+1)$

Min Poly	Elements of GF(8)
$(x+1)$	α^7
(x^3+x+1)	$\alpha^1, \alpha^2, \alpha^4$
(x^3+x^2+1)	$\alpha^3, \alpha^6, \alpha^5$

If $n = 15$

$$x^{15} - 1 = (x+1)(x^4+x+1)(x^4+x^3+x^2+x+1)(x^2+x+1)(x^4+x^3+1)$$

Min Poly	Elem of GF(16)
$x+1$	α^{15}
x^4+x+1	$\alpha^1, \alpha^2, \alpha^4, \alpha^8$
$x^4+x^3+x^2+x+1$	$\alpha^3, \alpha^6, \alpha^{12}, \alpha^9$
x^2+x+1	α^5, α^{10}
x^4+x^3+1	$\alpha^7, \alpha^{14}, \alpha^{13}, \alpha^{11}$

→ How to solve?

① Have GF & $p(x)$

$GF(8)$, $p(x) = x^3+x+1$

$GF(16)$, $p(x) = x^4+x+1$

② Find $\alpha_1, \alpha_2 \ldots$ by mod $p(x)$

③ Use the remember part & the below eqn

$g(x) = LCM\,[f_1(x), f_2(x), f_3(x) \ldots f_{2t}(x)]$

$f_1 \rightarrow \alpha^1$; $f_2 \rightarrow \alpha^2$: Simply Multiply their Min Poly's

④ Use degree of $g(x)$ to find k

$n - k =$ degree of $g(x)$