Hamming Code (All basics for 3.2)

Hamming Code Basics

→ It is given by RW Hamming.

→ It is used to detect and correct error.

→ In Hamming code, we send data along with parity bits or Redandent bits.

→ It is represented by (n, k) code.

total bits

message bits.

→ Parity bits $P = n - k$

→ It is represented by (n, k) code.

n → total bits

k → message bits.

→ Parity bits $p = n - k$

→ To Identify parity bits, It should satisfy given Cond.^{n}

$$\Rightarrow 2^p \geq p + k + 1$$

→ It is represented by (n, k) code.

n → total bits

k → message bits.

→ Parity bits $p = n - k$

→ To Identify parity bits, It should satisfy given Cond.^{n}

$$\Rightarrow 2^p \geq p + k + 1$$

→ So for $k = 4$ message bits.

$$\Rightarrow 2^p \geq p + 4 + 1$$

$$\Rightarrow 2^p \geq p + 5$$

For $p = 1$	For $p = 2$	For $p = 3$
$2^1 \geq 6$	$2^2 \geq 7$	$2^3 = 8$
×	×	✓

$\Rightarrow p = 3$ for $k = 4$ bits.

$\Rightarrow n = 3 + 4 = 7$ bits.

→ This is $(7, 4)$ code.

$\Rightarrow 2^p \geq p+4+1$

$\Rightarrow 2^p \geq p+5$

For $p=1$: $2^1 \geq 6$ ✗

For $p=2$: $2^2 \geq 7$ ✗

For $p=3$: $2^3 = 8$ ✓

$\Rightarrow p=3$ for $k=4$ bits.

$\Rightarrow n = 3+4 = 7$ bits.

→ This is (7,4) code.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

For p = 1

$2^1 \geq 6$

×

For p = 2

$2^2 \geq 7$

×

For p = 3

$2^3 = 8$

✓

⇒ $P = 3$ for $K = 4$ bits.

⇒ $n = 3 + 4 = 7$ bits.

→ This is (7, 4) code.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

→ P_1 →

For P1, Check 1 bit and skip 1 bit.

For $p = 1$	For $p = 2$	For $p = 3$
$2^1 \geq 6$	$2^2 \geq 7$	$2^3 = 8$
×	×	✓

$\Rightarrow p = 3$ for $k = 4$ bits.

$\Rightarrow n = 3 + 4 = 7$ bits.

→ This is (7,4) code.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$\rightarrow P_1 \rightarrow D_3\ D_5\ D_7$

$\rightarrow P_2 \rightarrow$

For P2, Check 2 bits and skip 2 bits.

For $p=1$: $2^1 \geq 6$ ×

For $p=2$: $2^2 \geq 7$ ×

For $p=3$: $2^3 = 8$ ✓

$\Rightarrow p = 3$ for $K = 4$ bits.

$\Rightarrow n = 3 + 4 = 7$ bits.

→ This is (7, 4) code.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

→ $P_1 \rightarrow D_3\ D_5\ D_7$

→ $P_2 \rightarrow D_3\ D_6\ D_7$

→ $P_4 \rightarrow$

For P4, Check 4 bits and skip 4 bits.

For $p = 1$: $2^1 \geq 6$ ✗

For $p = 2$: $2^2 \geq 7$ ✗

For $p = 3$: $2^3 = 8$ ✓

$\Rightarrow p = 3$ for $k = 4$ bits.

$\Rightarrow n = 3 + 4 = 7$ bits.

$\rightarrow$ This is (7, 4) code.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$\rightarrow P_1 \longrightarrow D_3\ D_5\ D_7$ (XOR)

$\rightarrow P_2 \longrightarrow D_3\ D_6\ D_7$ (XOR)

$\rightarrow P_4 \longrightarrow D_5\ D_6\ D_7$ (XOR).

0:00 - Digital Electronics Lecture Series
0:33 - Example to Generate Hamming code
1:03 - Number Parity bits identification in Hamming Code
4:12 - Representation of Hamming Code
4:43 - Position of Parity bits in Hamming Code
6:05 - Calculation of Parity bits in Hamming Code

Generation of Hamming Code

- 5 bit data 01101 is given. Represent given data in Hamming code.

  → $K = 5$ bits.

  → Identify parity bits.

  $$\Rightarrow 2^P \geq P + K + 1$$

  $$\Rightarrow 2^P \geq P + 6$$

For $P = 1$	For $P = 2$	For $P = 3$	For $P = 4$
$2^1 \geq 7$	$2^2 \geq 8$	$2^3 \geq 9$	$2^4 \geq 10$
×	×	×	✓

For $p=1$	For $p=2$	For $p=3$	For $\boxed{p=4}$
$2^1 \geq 7$	$2^2 \geq 8$	$2^3 \geq 9$	$2^4 \geq 10$
×	×	×	✓

→ So for $p=4$, $k=5$, $n=9$

→ This is $(9,5)$ hamming code.

9	8	7	6	5	4	3	2	1

→ So for $p = 4$, $k = 5$, $n = 9$

→ This is $(9, 5)$ hamming code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ Position of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ So for $p = 4$, $K = 5$, $n = 9$

→ This is (9, 5) hamming code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ Position of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3\ D_5\ D_7\ D_9$

For P1, Check 1 bit and skip 1 bit.

→ So for $p = 4$, $K = 5$, $n = 9$

→ This is $(9, 5)$ hamming code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ ~~Pattern~~ Pattern of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3\ D_5\ D_7\ D_9$

$P_2 \longrightarrow D_3\ D_6\ D_7$

For P2, Check 2 bits and skip 2 bits.

→ So for $p = 4$, $k = 5$, $n = 9$

→ This is (9, 5) hamming code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ Pos[illegible] of [illegible] bits.

→ Value of Parity bits.

$P_1 \longrightarrow D_3\ D_5\ D_7\ D_9$

$P_2 \longrightarrow D_3\ D_6\ D_7$

$P_4 \longrightarrow$

For P4, Check 4 bits and skip 4 bits.

→ So for $p = 4$, $K = 5$, $n = 9$

→ This is (9, 5) hamming code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ Pos... of ... bits.

P

→ Value of Parity bits.

$P_1 \rightarrow D_3\ D_5\ D_7\ D_9$

$P_2 \rightarrow D_3\ D_6\ D_7$

$P_4 \rightarrow D_5\ D_6\ D_7$

$P_8 \rightarrow$

For P8, Check 8 bits and skip 8 bits.

→ So for $p = 4$, $k = 5$, $n = 9$

→ This is $(9, 5)$ hamming code.

9	8	7	6	5	4	3	2	1
D_9	P_8	D_7	D_6	D_5	P_4	D_3	P_2	P_1

→ ~~Posttom~~ Position of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3\ D_5\ D_7\ D_9$

$P_2 \longrightarrow D_3\ D_6\ D_7$

$P_4 \longrightarrow D_5\ D_6\ D_7$

$P_8 \longrightarrow D_9$

For P8, Check 8 bits and skip 8 bits.

9	8	7	6	5	4	3	2	1
0 D_9	P_8	1 D_7	1 D_6	0 D_5	P_4	1 D_3	P_2	P_1

→ Pattern of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3\ D_5\ D_7\ D_9$

$P_2 \longrightarrow D_3\ D_6\ D_7$

$P_4 \longrightarrow D_5\ D_6\ D_7$

$P_8 \longrightarrow D_9$

→ $P_1 = 1 \oplus 0 \oplus 1 \oplus 0 = 0$

→ $P_2 = 1 \oplus 1 \oplus 1 = 1$

→ $P_4 = 0 \oplus 1 \oplus 1 = 0$

→ $P_8 = D_9 = 0$

9	8	7	6	5	4	3	2	1
0 D_9	P_8	1 D_7	1 D_6	0 D_5	P_4	1 D_3	P_2	P_1

→ ~~Pattern~~ Position of Parity bits.

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$P_8 = 2^3 = 8$

→ Value of Parity bits.

$P_1 \longrightarrow D_3 \; D_5 \; D_7 \; D_9$

$P_2 \longrightarrow D_3 \; D_6 \; D_7$

$P_4 \longrightarrow D_5 \; D_6 \; D_7$

$P_8 \longrightarrow D_9$

→ $P_1 = 1 \oplus 0 \oplus 1 \oplus 0 = 0$

→ $P_2 = 1 \oplus 1 \oplus 1 = 1$

→ $P_4 = 0 \oplus 1 \oplus 1 = 0$

→ $P_8 = D_9 = 0$

$\boxed{001100110}$

Hamming Code Error detection & Error Correction

* If Received Hamming Code is 1110101 with even Parity then detect and correct error.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

Hamming Code Error detection & Error Correction

If Received Hamming Code is 1110101 with even parity then detect and correct error.

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	1	1	0	1	0	1

$P_1 = 2^0 = 1$

$P_2 = 2^1 = 2$

$P_4 = 2^2 = 4$

$$\left.\begin{array}{l} P_1 = D_3 \oplus D_5 \oplus D_7 \\ 1 = 1 \oplus 1 \oplus 1 \end{array}\right] \text{—} \; P_1 = 0 \; ✓$$

$$\left.\begin{array}{l} P_2 = D_3 \oplus D_6 \oplus D_7 \\ 0 = 1 \oplus 1 \oplus 1 \end{array}\right] \text{—} \; P_2 = 1 \; \times$$

$$\left.\begin{array}{l} P_4 = D_7 \oplus D_6 \oplus D_5 \\ 0 = 1 \oplus 1 \oplus 1 \end{array}\right] \text{—} \; P_4 = 1 \; \times$$

$P_4 = D_7 \oplus D_6 \oplus D_5$
$0 = 1 \oplus 1 \oplus 1$ — $P_4 = 1$ ×

$P_4 P_2 P_1 = 110 = \underline{\underline{6}}^{th}$ bit error

E =	0	1	0	0	0	0	0
R =	1	1	1	0	1	0	1

→ Corrected data = $R \oplus E$

=	1	0	1	0	1	0	1

LBC Basics

Linear Codes basics & property with example

<u>Defination</u> - A Block code is said to be linear code if its codewords satisfy the condition that the sum of any two codewords gives another codeword.

i.e. $c_p = c_i + c_k$

<u>Property</u>

i) The all-zero wo... ... 0] is always a codeword

ii) Given any three ... such that

$c_p = c_i + c_k$

iii) Minimum distan...

Defination - A Block code is said to be linear code if its codewords satisfy the condition that the sum of any two codewords gives another codeword.

$$\text{i.e. } c_p = c_i + c_k$$

property

i) The all-zero words $[0, 0, 0, \ldots 0]$ is always a codeword.

ii) Given any three codewords c_i, c_j and c_k such that

$$c_p = c_i + c_k \text{ , then } d(c_i, c_j) = W(c_p)$$

iii) Minimum distance of the code

$$d_{min} = W_{min}$$

ii) Given any three codewords c_p, c_j and c_k such that

$$c_p = c_i + c_k \text{ , then } d(c_i, c_j) = W(c_p)$$

iii) Minimum distance of the code

$$d_{min} = W_{min}$$

- (7,4) Hamming Code

$$c_1 = 0\,0\,0\,1\,0\,1\,1 \quad \boxed{3}$$
$$c_{10} = 1\,0\,1\,0\,0\,1\,1 \quad \boxed{4} \qquad c_{11} = c_1 + c_{10}$$
$$c_{11} = 1\,0\,1\,1\,0\,0\,0$$

- $c_0 = [0\,0\,0\,0\,0\,0\,0]$

iii) Minimum distance of the code

$$d_{min} = W_{min}$$

- (7,4) Hamming Code

$$c_1 = 0001011 \quad \boxed{3}$$
$$c_{10} = 1010011 \quad \boxed{4} \qquad c_{11} = c_1 + c_{10}$$
$$c_{11} = 1011000 \quad \boxed{3}$$

- $c_0 = [0\ 0\ 0\ 0\ 0\ 0\ 0]$

$d(c_1, c_{10}) = 3$

$w(c_{11}) = 3$

$d(c_1, c_{10}) = 3 = w(c_{11})$

- (7, 4) Hamming Code

$$c_1 = 0\,0\,0\,1\,0\,1\,1 \quad 3$$
$$c_{10} = 1\,0\,1\,0\,0\,1\,1 \quad 4 \qquad c_{11} = c_1 + c_{10}$$
$$c_{11} = 1\,0\,1\,1\,0\,0\,0 \quad \boxed{3}$$

- $c_0 = [0\;0\;0\;0\;0\;0\;0]$

$$d(c_1, c_{10}) = 3 \qquad d(c_1, c_{10}) = 3 = W(c_{11})$$
$$W(c_{11}) = 3$$

- $c_{15} = [1\;1\;1\;1\;1\;1\;1]$, $\omega = 7$

other then c_{15} codes are having weight 3 & 4.

$$d_{min} = W_{min}$$

S.No.	m0	m1	m2	m3	p0	p1	p2
0	0	0	0	0	0	0	0
1	0	0	0	1	0	1	1
2	0	0	1	0	1	1	0
3	0	0	1	1	1	0	1
4	0	1	0	0	1	1	1
5	0	1	0	1	1	0	0
6	0	1	1	0	0	0	1
7	0	1	1	1	0	1	0
8	1	0	0	0	1	0	1
9	1	0	0	1	1	1	0
10	1	0	1	0	0	1	1
11	1	0	1	1	0	0	0
12	1	1	0	0	0	1	0
13	1	1	0	1	0	0	1
14	1	1	1	0	1	0	0
15	1	1	1	1	1	1	1

Show that (4, 3) Even-parity code is a linear and (4, 3) odd parity is not linear.

→ (4, 3) even parity code.

C	d_1	d_2	d_3	P
	0	0	0	
	0	0	1	
	0	1	0	

(4, 3) odd parity is not linear.

→ (4, 3) even parity code.

C	d_1	d_2	d_3	P
C_0	0	0	0	0
C_1	0	0	1	1
C_2	0	1	0	1
C_3	0	1	1	0
C_4	1	0	0	1
C_5	1	0	1	0
C_6	1	1	0	0
C_7	1	1	1	1

					W
0	0	1	1	C_1	2
0	1	0	1	C_2	2
0	1	1	0	$\boxed{C_3}$	2

$d_{min}(C_1, C_2) = 2$

$W(C_3) = 2$

– $\boxed{d_{min} = W}$

- Odd parity (4, 3) code

C	d_1	d_2	d_3	P
C_0	0	0	0	1
C_1	0	0	1	0
C_2	0	1	0	0
C_3	0	1	1	1
C_4	1	0	0	0
C_5	1	0	1	1
C_6	1	1	0	1
C_7	1	1	1	0

- Odd parity (4, 3) code

C	d_1	d_2	d_3	P
C_0	0	0	0	1
C_1	0	0	1	0
C_2	0	1	0	0
C_3	0	1	1	1
C_4	1	0	0	0
C_5	1	0	1	1
C_6	1	1	0	1
C_7	1	1	1	0

$$\begin{array}{cccc|c} 0 & 0 & 1 & 0 & C_1 \\ 0 & 1 & 0 & 0 & C_2 \\ \hline 0 & 1 & 1 & 0 & \end{array}$$

- $C_1 + C_2$ is not present in odd parity (4, 3) code
- It is not linear block code.

Generator Matrix to generate Codeword in LBC

Generator matrix in Linear code
to generate code words

- using a matrix to generate codewords is a better approach.

$$[c] = [i][G]$$

$[c]$ = Codeword
$[i]$ = Information words
$[G]$ = Generator matrix

- The Generator matrix of an (n, k) linear code has 'k' rows and 'n' columns.

Generator matrix for $(7, 4)$ code is given by

[i] = Information words

[G] = Generator matrix

- The Generator matrix of an (n, K) linear code has 'K' rows and 'n' colomns.

- Generator matrix for (7, 4) code is given by

$$[G] = [I : P]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$[G] = [I : P]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

Identity Matrix I_K Parity Matrix

$\rightarrow$ $[C] = [i][G]$

Example - Generate Codeword for $\underline{i = (1110)}$ with (7, 4) generator matrix code.

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$C = [i][G]$$

$$1.1 + 1.0 + 1.0 + 0.0$$

$$= [1\ 1\ 1\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [1$$

Determine the set of codewords for the (6, 3)

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$C = [i][G]$$

$$1.1 + 1.0 + 1.0 + 0.0$$

$$= [1\ 1\ 1\ 0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [1\ 1\ 1\ 0\ 1\ 0\ 0]$$

Determine the set of codewords for the (6, 3)

Example - Determine the set of codewords for the (6, 3) code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ $n = 6$

$K = 3$

→ Message bits = 3

$m_0 \; m_1 \; m_2$

code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ $n = 6$
$K = 3$
→ Message bits = 3

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$K = 3$

$\rightarrow$ Message bits $= 3$

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$\rightarrow C = [i][G]$

$\rightarrow C_0 = [0\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= [0\ 0\ 0\ 0\ 0\ 0]$

$\rightarrow C_1 = [0\ 0\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= [0\ 0\ 1\ 1\ 1\ 0]$

	m_0	m_1	m_2	p_0	p_1	p_2
c_0	0	0	0	0	0	0
c_1	0	0	1	1	1	0
c_2	0	1	0	1	0	1
c_3	0	1	1	0	1	1
c_4	1	0	0	0	1	1
c_5	1	0	1	1	0	1
c_6	1	1	0	1	1	0
c_7	1	1	1	0	0	0

$= [0\ 0\ 1\ 1\ 1\ 0]$

the set of codewords for the (6, 3) code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ $n = 6$

$K = 3$

→ Message bits = 3

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

→ $C = [i][G]$

$\boxed{C = [m, P]}$

→ $C_0 = [0\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= [0\ 0\ 0\ 0\ 0\ 0]$

→ $C_1 = [0\ 0\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= [0\ 0\ 1\ 1\ 1\ 0]$

Generator matrix in Linear code
to generate code words

$[G] = [I : P]$

- Using a matrix to generate codewords is a better approach.

$[C] = [i][G]$

$[C] = [m : P_c]$

$[P_c] = [m][P]$

$[C]$ = Codeword
$[i]$ ... ion words
... ator matrix

... trix of an (n, K) linear code
... 'n' columns.

... matrix for $(7, 4)$ code is given

$[G] = [I : P]$

Systematic Generator Matrix

Systematic Generator matrix in Linear block codes

- A Generator matrix $[G] = [I_k : P]$ is said to be in a Systematic form If It generates the Systematic codewords.

- Here

$$[I_k] \longrightarrow k \times k \text{ matrix}$$

$$[P] \longrightarrow k \times (n-k) \text{ matrix}$$

$$[G] \longrightarrow k \times n \text{ Matrix}$$

- In these matrix Information bits are placed together

- Codeword

Systematic Generator matrix in Linear block codes

- A Generator matrix $[G] = [I_k : P]$ is said to be in a Systematic form if it generates the systematic codewords.

$$[C] = [i][G] = [m, P]$$

- Here

$$[I_k] \longrightarrow k \times k \text{ matrix}$$

$$[P] \longrightarrow k \times (n-k) \text{ matrix}$$

$$[G] \longrightarrow k \times n \text{ matrix}$$

- In the ... atrix Inform... ... ether

- Cod...

- In these matrix Information bits are placed together
- Codeword

$$[C] = [i][G]$$

-

$$= [i_1, i_2, i_3, i_4] \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

- So when you get Codeword

$$C = (i_1, i_2, i_3, i_4, P_1, P_2, P_3)$$

- Ident[illegible] matrix Keeps Information together
- P[illegible]atrix generates Pa[illegible] bits

- Codeword

$$[c] = [i][G]$$

$$= [i_1, i_2, i_3, i_4]\left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

- So when you get Codeword

$$c = (i_1, i_2, i_3, i_4, p_1, p_2, p_3)$$

- Identity matrix keeps Information together
- Parity matrix generates Parity bits

$$= [i_1, i_2, i_3, i_4] \begin{array}{c} \begin{array}{cccccccc} i_1 & i_2 & i_3 & i_4 & & p_1 & p_2 & p_3 \end{array} \\ \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \end{array}$$

- So when you get Codeword

$$c = (i_1, i_2, i_3, i_4, p_1, p_2, p_3)$$

- Identity matrix keeps Information together
- Parity matrix generates Parity bits.

$$p_1 = i_1 + i_2 + i_3$$
$$p_2 = i_2 + i_3 + i_4$$
$$p_3 = i_1 + i_2 + i_4$$

The (5,3) linear code has the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [\, I_k : P \,]$$

- Determine systematic form of G.
- generate codeword for Information (0 1 1) with Systematic G & Non-systematic G.

The (5,3) linear code has the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [\, I_k : P \,]$$

- Determine systematic form of G.
- generate codeword for Information (0 1 1) with Systematic G & Non-systematic G.

$$[G] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[\text{}]{\text{Add } R_2 \text{ to } R_3} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$[G] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_2 \text{ to } R_3]{\text{Add}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Add R_3 to R_1

$$\begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} & \underbrace{\begin{matrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{matrix}}_{P} \end{bmatrix}$$

$$\boxed{G = [\, I_k : P \,]}$$

$$[G] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_2 \text{ to } R_3]{\text{Add}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Add R_3 to R_1

$$\begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} & \underbrace{\begin{matrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{matrix}}_{P} \end{bmatrix}$$

$$\boxed{G = [I_K : P]}$$

→ For Non Systematic $[G]$

$$c = [i][G]$$

$$= [0\ 1\ 1] \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1]$$

$$c = [i][G]$$

$$= [0\ 1\ 1]\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1]$$

$\downarrow$ R_3 to R_1

$$\left[\ \underbrace{\boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}}_{I}\ \underbrace{\boxed{\begin{matrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{matrix}}}_{P}\ \right]$$

$$\boxed{G = [I_k : P]}$$

$\rightarrow$ For Systametric $[G]$

$$c = [i][G]$$

$$= [0\ 1\ 1]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$= [\underbrace{0\ \ 1\ \ 1}_{\text{Information}}\ \ \underbrace{1\ \ 0}_{\text{Parity}}]$$

Parity check matrix

Parity Check Matrices in Linear block Codes with Examples.

- From Generator matrix $[G] = [I_k | P]$ we can identity parity matrix.
- By taking P^T we can make parity Check Matrix $[H]$

$$[H] = [P^T : I_{n-k}]$$

- Parity check matrix is used at R_x to decode data.

Example - Generate Parity check Matrix for (7,4) Code

$$[G] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Example - Generate Parity check Matrix for (7, 4) Code

$$[G] = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$\uparrow$ Identity matrix I

$\uparrow$ Parity Matrix P.

$$\left[\ \underbrace{0\ \ 0\ \ 0\ \ 1}_{\substack{\text{Identity} \\ \text{matrix} \\ I}}\ \ \underbrace{0\ \ 1\ \ 1}_{\substack{\text{Parity} \\ \text{Matrix} \\ P.}}\ \right]$$

$$\rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Identity Matrix I

Parity Matrix P.

$$\rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow I_{n-k} = I_{7-4} = I_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow I_{n-k} = I_{7-4} = I_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow [H] = [P^T : I_{n-k}]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Prove that GH^T and $CH^T = 0$.

Prove that GH^T and $CH^T = 0$.

$$\Rightarrow \quad GH^T = [\, I_k \mid P \,]\,[\, P^T \mid I_{n-k} \,]^T$$

$$= [\, I_k \mid P \,]\begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix}$$

$$= I_k P + P I_{n-k}$$

$$= P + P$$

$$P \begin{cases} \nearrow 0 \\ \searrow 1 \end{cases}$$

For Mod-2 add, $0 + 0 = 0$, $1 + 1 = 0$

$$\Rightarrow \boxed{GH^T = 0}$$

$$= [I_k | P] \begin{bmatrix} P \\ I_{n-k} \end{bmatrix}$$

$$= I_k P + P I_{n-k}$$

$$= P + P$$

$P \to 0$, $P \to 1$ For Mod-2 add, $0 + 0 = 0$, $1 + 1 = 0$

$$\Rightarrow \boxed{G H^T = 0}$$

$$\Rightarrow C H^T = [i][G][H^T]$$

$$= [i]\, 0$$

$$\boxed{C H^T = 0}$$