ITC : Mod 1 (Tutorial 1)

→ Self - Information

$I_c(p) = -\log_2(P)$

$\log_2 a = \frac{\log_{10} a}{\log_{10} 2}$ ↳ 0.301

$I = \sum_{j=1}^{M} -\log_2(P_j)$

→ Entropy

$H(s) = -\sum_{i=1}^{M} p_i \cdot \log_2 p_i$ bits/symbol

→ Information Rate

$R = r \cdot H$ bits/sec

H : Amt of info generated per sec

r : rate of msg

$T = \frac{1}{r}$

T : time taken for 1 msg

(Tutorial 2)

→(2)

Mutual Information

$$I(x,y) = I(x) + I(y)$$

Joint Entropy

$$H(x,y) = -\sum_{x=1}^{m}\sum_{y=1}^{n} P(x,y) \log_2 P(x,y)$$

Conditional Probability

$$H(X/Y) = \sum\sum P(xy) \log_2 \left[\frac{1}{P(x/y)}\right] = \sum_{i}^{n} P(Y=i)\, H(X/Y=i)$$

Relation of conditional & Joint

$$H(X,Y) = H(X/Y) + H(Y)$$

Marginal Entropy

$$H(x) = -\sum_{i=1}^{m} P(x_i) \log_2 P(x_i)$$

Shannon's

$$C = B \log_2 (1 + SNR)$$

(1)&(3)

Channel Matrix

$$P = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix} \begin{matrix} \longrightarrow \text{Probabilities when 0 is sent} \\ \longrightarrow \text{Probabilities when 1 is sent} \end{matrix}$$

CM for Binary Symmetric channel (BSC) {10% chance of error}

$$P = \begin{bmatrix} P(0/0) & P(1/0) \\ P(0/1) & P(1/1) \end{bmatrix}$$

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

Channel Point Probability Matrix

$P(x,y) = P(x) \cdot P(y/x)$

JP that both x & y happen tog = (Prob of x) . (Conditional Prob that y happens given x has already happened)

$P(0,0) = P(0) \cdot P(0|0)$

$P(0,1) = P(0) \cdot P(1|0)$

$P(1,0) = P(1) \cdot P(0|1)$

$P(1,1) = P(1) \cdot P(1|1)$

Marginal Entropy

$$H(X) = -\sum_{x \in X} P(x) \log_2 P(x)$$

→ Special Channels

* Binary Symmetric Channel (BSC)

P of being received incorrectly : error

1-P of being received correctly : सब सही

Capacity $C = 1 - H(P)$

↳ binary entropy

* Binary Erasure Channel (BEC)

Either received correctly or lost.

Prob of erasure = E

Prob of correct reception = 1 - E

Capacity $C = 1 - E$

* Z Channel

0 → safe

1 → risky → might flip to 0

If P = 0.3 (30% error rate but only for 1)

Channel Capacity

$$C = \max_{P(x)} I(X:Y)$$

↑ input prob distribution

↑ mutual information b/w X & Y

For BSC : $C = 1 - H(P)$

$$H(P) = -(P \log_2 P + (1-P)\log_2(1-P))$$

For BEC : $C = 1 - E$

Z channel is asymmetric

0 - correct

1 - flip to 0

Joint, Conditional, Marginal Entropy numericals.

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① Matrix of $P(x,y)$ given & $P(x)$, $P(Y)$ given

ⓐ $P(x/y) = \frac{P(x,y)}{P(Y)}$

Just divide row wise → similarly for $P(Y/x)$

ⓑ $H(X) = -\sum_{j=1}^{m} P(x_j) \log_2 P(x_j)$

Similarly for $H(Y)$

ⓒ $H(x,y) = H(x/Y) + H(Y)$

② $H(X) = -\sum_{i=1}^{n} p(x_i) \log P(x_i)$

Given $P(Y/X)$ matrix

$H(x,y) = -\sum_{j=1}^{m} \sum_{i=1}^{n} P(x_i,y_i) \log P(x_i,y_i)$ ← Basically just find by adding entropies of whole matrix

③ $I(x,y) = H(Y) - H(Y/x)$

$H(Y/X) = -\sum \sum P(x_i y_i) \log_2 P(x_i y_i)$

$P(x,y) = [P(x)]_d \, P(Y/X)$

Given $P(Y/X)$ matrix

④ Just multiply $P(x)$ values to $P(Y/X)$ matrix to get $P(x,y)$

$H(x) = -\sum p(x_i) \log P(x_i)$

$H(x,y) = I(x,y) = H(X) + H(Y/X)$

$I(x,y) =$ ____