LARGE SAMPLE TEST-1

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TEST OF SIGNIFICANCE FOR LARGE SAMPLES

- If the sample size $n > 30$, the sample is taken as large sample. For such sample we apply normal test.
- Under large sample test, the following are the important tests to test the significance.

1. Testing of significance for single mean

2. Testing of significance for difference of mean

TESTING THE HYPOTHESIS THAT THE POPULATION MEAN $= \mu$

- To test whether the difference between sample mean and population mean is significant or not
- Under the null hypothesis that there is no difference between the sample mean and population mean.

- The test statistic is $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ where σ is the standard deviation of the population
- If σ is not known, we use the test statistic $z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$, where s is the standard deviation of the Sample

CONFIDENCE LIMITS

- If the level of significance is α and z_α is the critical value $-z_\alpha < z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < z_\alpha$

- The limit of the population mean μ are given by $\bar{x} - z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right)$

CONFIDENCE LIMITS

- At 5% of level of significance, 95% confidence limits are

$$\bar{x} - 1.96\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$$

- At 1% level of significance, 99% confidence limits are

$$\bar{x} - 2.58\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + 2.58\left(\frac{\sigma}{\sqrt{n}}\right)$$

EXAMPLE-1

Measurements of the weights of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of 0.824 newton and a standard deviation of 0.042 newton. Find 95% confidence limits for the mean weight of all ball bearings.

- **Step 1:** For 95% confidence level the critical value $z_\alpha = 1.96$
- **Step 2:** Since the sample size is large and population standard deviation is not know
- The confidence interval is $\bar{x} \pm 1.96\frac{s}{\sqrt{n}}$
- $0.824 \pm 1.96\frac{0.042}{\sqrt{200}} = (0.8182, 0.8298)$

EXAMPLE-2

Cardiac patients were implanted pacemakers to control heartbeats. A plastic connector module mounts on top of the pacemaker. A random sample of 75 modules has an average of 0.31 inches. Assuming standard deviation of 0.0015 inches and normal distribution, find 95% confidence interval for the mean size of the connector module.

Step 1: For 95% confidence level the critical value $z_\alpha = 1.96$

Step 2: Since the sample is large and population S.D. is known,

The confidence interval is $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

The confidence interval is $(0.3097\,, 0.3103)$

EXAMPLE-3

A simple random sample of size 65 was drawn in the process of estimating the mean annual income of 950 families of a certain township. The mean and the standard deviation of the sample were found to be Rs. 4730 and Rs. 765 respectively. Find a 99% confidence interval for the population mean.

SOLUTION

Step 1: For 99% confidence level the critical value $z_\alpha = 2.58$

Step 2: since the sample is drawn from a finite population and

since $n/N = 65/950 = 0.068$ is greater than 0.05,

we use finite population multiplier

$$\sqrt{(N-n)/(N-1)}$$

The confidence interval is $\bar{x} \pm 2.58\left(\frac{s}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}\right)$

$\textit{The confidence interval is}\,(4493.59\,, 4966.40)$

EXAMPLE-4

A machine is set to produce metal plates of thickness 1.5 cms with standard deviation 0.2 cm. A sample of 100 plates produced by the machine gave an average thickness of 1.52 cms. Is the machine fulfilling the purpose?

- SOLUTION:
- Null hypothesis $H_0: \mu = 1.5$
- Alternative Hypothesis $H_a: \mu \neq 1.5$

Test statistic: Since the population standard deviation is given

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = 1$$

Level of significance: $\alpha = 0.05$

Critical value: The value of z_{α} at 5% level of significance from the table $= 1.96$

Decision: Since the computed value of $|z| = 1$ is less than the critical value $z_{\alpha} = 1.96$, the null hypothesis is accepted

The machine is fulfilling its purpose.

EXAMPLE-5

A random sample of 50 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 10% level of significance?

SOLUTION

Null hypothesis $H_0: \mu = 5.4$

Alternative Hypothesis $H_a: \mu \neq 5.4$

Test statistic: Since sample S.D. s is known and since sample is large

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Level of significance: $\alpha = 0.1$

Critical value: The value of z_α at 10% level of significance from the table $= 1.64$

Decision: Since the computed value of $|z|$ is greater than the critical value $z_\alpha = 1.64$, the null hypothesis is rejected.

$\therefore$ The sample is not drawn from the population with mean 5.4

EXAMPLE-6

Can it be concluded that the average life-span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation of 8.9 years?

SOLUTION

- Null hypothesis $H_0: \mu = 70$ years
- Alternative Hypothesis $H_a: \mu > 70$ years

- Test statistic: $z = \frac{\bar{x}-\mu}{s/\sqrt{n}}$
- Since we are given standard deviation of the sample, we put $\bar{x} = 71.8, \mu = 70, s = 8.9,\ n = 100$
- $\therefore\ z = \frac{71.8-70}{8.9/\sqrt{100}} = 2.02$

Level of significance: $\alpha = 0.05$

Critical value: The value of z_α at 5% level of significance is 1.64

Decision: Since the computed value of $z = 2.02$ is greater than the critical value $z_\alpha = 1.64$, the null hypothesis is rejected

It can be concluded that the average life-span of an Indian is more than 70 years

TESTING THE HYPOTHESIS USING P-VALUE

A university claims that the average height of male students is **170 cm**. A researcher believes the average height is **different** from this claim. To test this, a random sample of **100 students** is taken. The mean of the sample is 172 and standard deviation is 6 cm.

Solution:

Null Hypothesis: $H_0: \mu = 172$ cm

Alternative Hypothesis $H_a: \mu \neq 172$ cm

Significance Level

We set α = 0.05 (5% level of significance).

the test-Statistic

$$z = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$\therefore z = \frac{172-170}{6/\sqrt{100}} = 3.33$$

TESTING THE HYPOTHESIS USING P-VALUE

Find the p-value

For a **two-tailed test**, we find the probability of $Z > 3.33$ using a standard normal table:

$$P(Z > 3.33) \approx 0.00043$$

Since it is a two-tailed test:

$$p = 2 \times 0.00043 = 0.00086$$

Compare p-value with α

p-value = 0.00086

$\boldsymbol{\alpha = 0.05}$

Since **p-value < 0.05**, we **reject H0**

Conclusion

There is strong statistical evidence to conclude that the **average height of male students is significantly different from 170 cm.**