Prove that H(X,Y) = H(X/Y) + H(Y)H(X,Y) = H(Y/X) + H(X)

UNITZ

Prove that

$$H(XY) = H(X/Y) + H(Y)$$

we know that $P(x_i, y_j) = P(x_i/y_j) P(y_j)$
 $EP(x_i, y_j) = P(y_j)$
 $H(X,Y) = \sum_{i=1}^{N} \sum_{j=1}^{N} P(x_i, y_j) \log \frac{1}{P(x_i/y_j)} P(y_j)$
 $= \sum_{i=1}^{N} \sum_{j=1}^{N} P(x_i, y_j) \log \frac{1}{P(x_i/y_j)} P(y_j)$
 $= \sum_{i=1}^{N} \sum_{j=1}^{N} P(x_i, y_j) \log \frac{1}{P(x_i/y_j)} + \sum_{i=1}^{N} \sum_{j=1}^{N} P(x_i/y_j) \log \frac{1}{P(x_i/y_j)}$
 $= \sum_{i=1}^{N} \sum_{j=1}^{N} P(x_i/y_j) \log \frac{1}{P(x_i/y_j)} + \sum_{i=1}^{N} \sum_{j=1}^{N} P(x_i/y_j) \log \frac{1}{P(x_i/y_j)}$





$$H(x,y) = \sum_{i=1}^{j} \sum_{j=1}^{j} p(x_{i}, y_{j}) \log \frac{1}{p(x_{i}, y_{j})}$$

$$= \sum_{i=1}^{j} \sum_{j=1}^{j} p(x_{i}, y_{j}) \log \frac{1}{p(x_{i}/y_{j})} \cdot P(y_{j})$$

$$= \sum_{i=1}^{j} \sum_{j=1}^{j} p(x_{i}, y_{j}) \log \frac{1}{p(x_{i}/y_{j})} + \sum_{i=1}^{j} \sum_{j=1}^{j} p(x_{i}, y_{j}) \log \frac{1}{p(y_{i})}$$

$$= H(x/y) + \sum_{j=1}^{j} \sum_{i=1}^{j} P(y_{i}) \log \frac{1}{p(y_{j})}$$

$$= H(x/y) + H(y)$$

$$= H(x/y) + H(y)$$



$$H(XY) = H(Y/X') + H(X)$$

$$P(x_{i}, y_{i}) = P(y_{i}/x_{i}) \cdot P(x_{i})$$

$$\sum_{j=1}^{n} P(x_{i}, y_{j}) = P(x_{i})$$

$$H(XY) = \sum_{i=1}^{n} \sum_{j=1}^{n} P(x_{i}, y_{i}) \cdot \log \frac{1}{P(y_{i}/x_{i})} P(x_{i})$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} P(x_{i}, y_{j}) \cdot \log \frac{1}{P(y_{i}/x_{i})} P(x_{i})$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} P(x_{i}, y_{j}) \cdot \log \frac{1}{P(y_{i}/x_{i})} + \sum_{i=1}^{n} \left\{ \sum_{j=1}^{n} P(x_{i}, y_{j}) \cdot \log \frac{1}{P(x_{i})} \right\}$$

$$= H(Y/X) + \sum_{i=1}^{n} P(x_{i}) \cdot \log \frac{1}{P(x_{i})}$$

$$= H(Y/X) + H(X)$$



Proofs of mutual information properties

EEC Classes GGSIPU, UPTU, Mumbai Univ., Pune Univ., GTU, Anna Univ., PTU and Others EEC Classes

Information Theory and Coding - Video Lecture Series (For B.Tech, MCA, M.Tech) Proofs of Mutual Information Properties:

10 1(x; Y) = I(Y; X)

From Probability theory.

J.P [P(xi yi) = P(xi/yi) P(yi) - (i)

P(xi yi) = P(yi/xi) P(xi) - (ii)

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Information Theory and Coding - Video Lecture Series (For B.Tech, MCA, M.Tech) Proofs of Mutual Information Properties:

From Probability - theory.

From Probability - theory.

$$P(xi yi) = P(xi/yi) P(yi) - (i)$$
 $P(xi yi) = P(yi/xi) P(xi) - (ii)$

$$\frac{P(xi|yi)}{P(xi)} = \frac{P(yi|xi)}{P(xi)} - (iii)$$

$$I(X;Y) = \underbrace{\stackrel{n}{\leq}}_{i=1} \underbrace{\stackrel{m}{\leq}}_{j=1} P(xi,yj) \log_2 \frac{P(xi)yj}{P(xi)} - (iv)$$

$$\frac{P(xi|y_j)}{P(xi)} = \frac{P(y_j|x_i)}{P(y_j)} - (iii)$$

$$I(x; Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i|y_j)}{P(x_i)} - (iv)$$

$$I(Y; X) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(y_j|x_i)}{P(y_j)} - (v)$$





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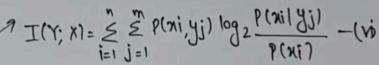
Proofs of Mutual Information Properties:

Prop. 1:- (i) I(X; Y) = I(Y; X)

$$\frac{P(xi|y_j)}{P(xi)} = \frac{P(y_j|x_i)}{P(y_j)} = \frac{P(y_j|x_i)}{P(y_j)}$$

$$I(X;Y) = \begin{cases} \sum_{i=1}^{n} P(x_i,y_i) \log_2 \frac{P(x_i|y_i)}{P(x_i)} \end{cases}$$
 (iv)

$$\frac{P(x_{i}|y_{j})}{P(x_{i})} = \frac{P(y_{j}|x_{i})}{P(y_{j})} = \frac{P(x_{i}|y_{j})}{P(x_{i},y_{j})} = \frac{P(x_{i}|y_{j})}{P(x_{i})} = \frac{P(x_{i}|y_{j})}{P(x_{i})} = \frac{P(x_{i}|y_{j})}{P(x_{i})} = \frac{P(y_{j}|x_{i})}{P(y_{j})} = \frac{P(y_{j}|x_{i})}{P(y_{j})}$$



From (iv) and (vi)
$$I(X;Y) = I(Y;X) Proved.$$

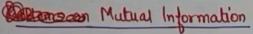






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Ques) Show that the the state .

$$I(X;Y) = H(X) - H(X|Y)$$
$$= H(Y) - H(Y|X)$$

$$H(X|Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(ni, yj) \log_2 \left(\frac{1}{P(ni|yj)} \right)$$

$$H(X|Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \left(\frac{1}{P(x_i|y_j)} \right)$$

$$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i|y_j)}{P(x_i)}$$





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Mutual Information

Ques) Show that decapt the trans.

$$I(x;Y) = H(x) - H(x|Y)$$
$$= H(Y) - H(Y|X)$$

$$H(X|Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(ni, y_j) \log_2 \left(\frac{1}{P(ni|y_j)} \right)$$

$$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(xi,yj) \log_{2} \frac{P(xi|yj)}{P(xi)}$$

$$T(x;Y) = \underbrace{\frac{n}{2}}_{i=1}^{m} \underbrace{\frac{n}{j=1}}_{j=1}^{m} P(ni,yi) \underbrace{\frac{p(ni|yi)}{p(ni)}}_{p(ni)} - \underbrace{\frac{n}{2}}_{i=1}^{m} \underbrace{\frac{n}{j=1}}_{j=1}^{m} P(ni,yi) \underbrace{\frac{p(ni|yi)}{p(ni)}}_{i=1} - \underbrace{\frac{n}{j=1}}_{j=1}^{m} P(ni,yi) \underbrace{\frac{1}{p(ni|yi)}}_{p(ni)}$$

$$I(X;Y) = \sum_{i=1}^{\infty} \sum_{j=1}^{m} P(xi,yj) \log_{2} \left(\frac{1}{P(xi)}\right) - H(X|Y)$$

$$I(X;Y) = \begin{cases} P(x_i,y_j) = P(x_i) \\ P(x_i) \log_2 \left(\frac{1}{P(x_i)}\right) + H(X|Y) \end{cases}$$





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Mutual Information

Prove: - Mutual Info" is always tre.

I(X:Y)>0

$$I(x;Y) = \begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{m} P(xi,yj) \log_{2} \frac{P(xi|yj)}{P(xi)} \end{cases}$$

coe Know that, P(xilyj) = P(xi, yj)

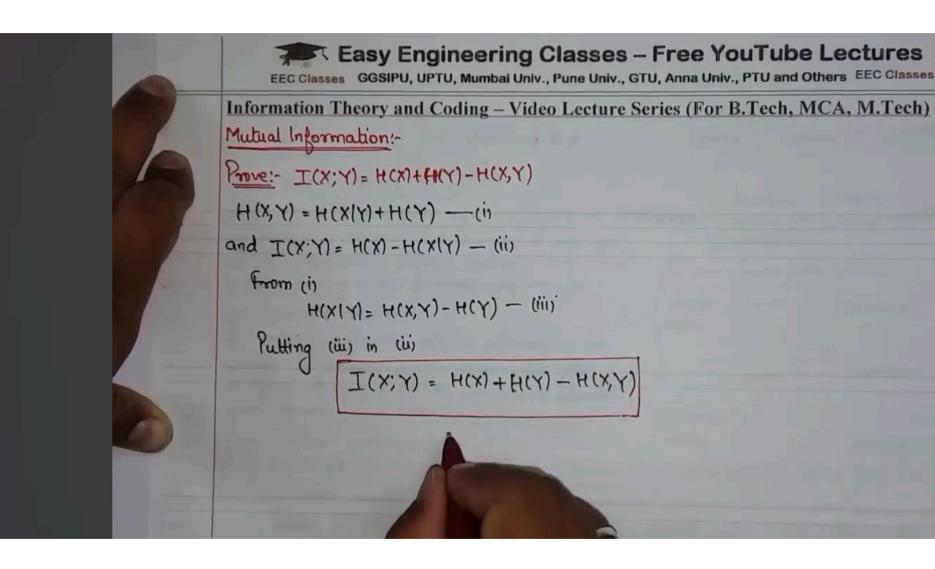
P(yj)

$$I(x; Y) = \begin{cases} \begin{cases} x \\ y \\ y \end{cases} \\ = 1 \end{cases} \begin{cases} y \\ y \\ y \end{cases} = \begin{cases} y \\ y \\ y \end{cases} \end{cases}$$

$$I(x; Y) = -\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P(xi, y_j) \log_2 \frac{P(xi)P(y_j)}{P(xi, y_j)}$$

$$-I(x;Y) = \begin{cases} x \\ z \\ i = 1 \end{cases} p(xi,yi) \log_{2} \frac{P(xi)P(yi)}{P(xi,yi)}$$

$$\begin{cases} \sum_{k=1}^{m} P_{k} \log_{2}\left(\frac{q_{k}}{P_{k}}\right) \leq 0 \end{cases}$$



Entropy proof

M Pi Ii Ii = -log Pi Σ P: (-log (Pi)) - \(\sum_{i=1} \) Pi \(\log \((Pi \) \)



← 1)ITC_Module... < ☆ :

8:00

b.	Entropy (H)
(.	Entropy is average information per message when many messages are transferred through a channel.

	Entropy Derivation
2.	consider M number of different and independent
	messages m, m2, m3 my having probablities
	PI, P2, P3 PM. The source generates sequence-
	of L independent messages over a long period
	of time and L>>>> M.
	The number of messages m, with L messages is
	P. L. Similarly for mm it is pm. L
	The amount of information in each m, is
	$I_1 = -\log_b P_1$
	The total information in all m, messages is
	m, II total = - P, L log P,
1899	Therefore for all mm messages
	T = -P I loa P.
	TM total = - PM L log PM
186	Total amount of messages in L message is
	THE PERSON NAMED IN COLUMN TWO IS NOT THE PERSON NAMED IN COLUMN TO SERVICE AND ADDRESS OF THE PERSON NAMED IN COLUMN TO SERVICE AND ADDRE
1310	I total = I total + I2 total + + Im total
(3)	Ttotal = -P, L log P, - P2 L log P2 PML log PM
	2.1120
18	Average = Itotal = -P, Llogp, PMLlogp
00	
	$M = - \sum_{i=1}^{M} P_i \log P_i$
	information $ \frac{M}{Entropy}(H) = \frac{1}{1+otal} = \underbrace{E \text{ Pilog } 1}_{i=1} = -\underbrace{E \text{ Pilog Pi}}_{i=1} $

Comparison of two sources



 $T(x,y) = \log 1 + \log 1$ p(x) p(y)

 $\log 1 = I(x)$

p(x)

log | = I(y)

p(y)

T(x,y) = T(x) + T(y)

Mutual Information measures dependence blw 2 random variables If x & y are independent, I(x,y) = 0