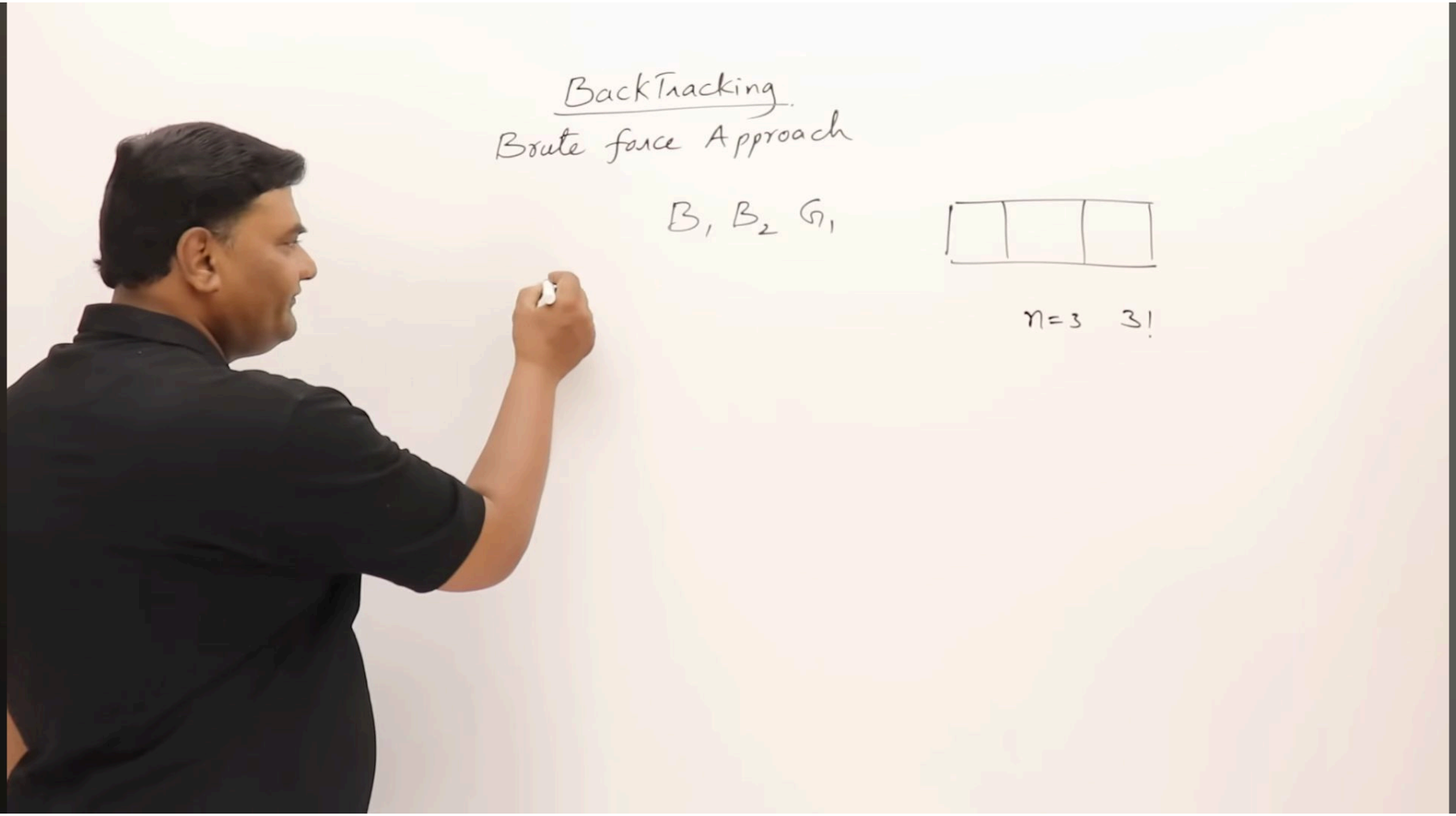
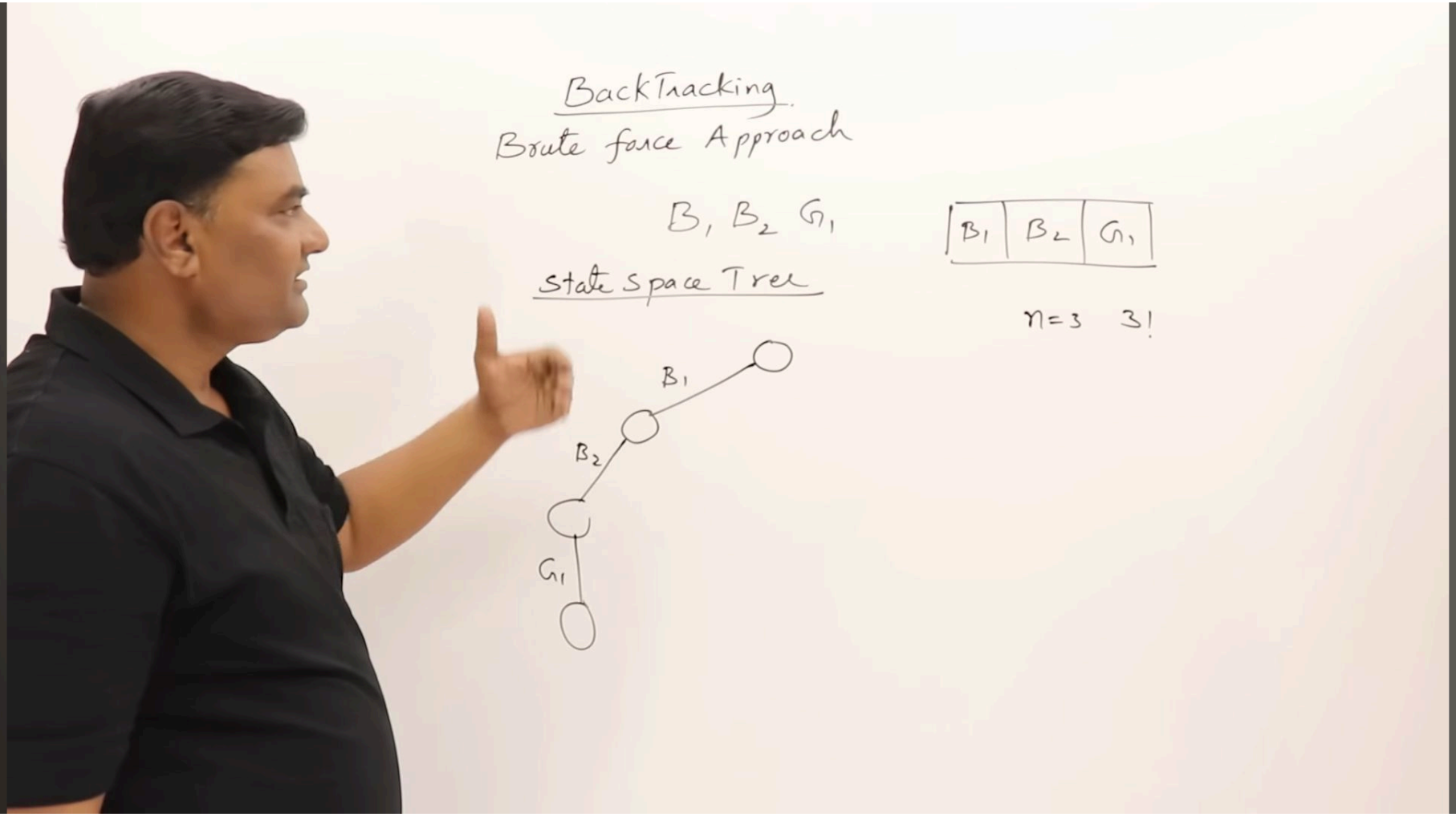
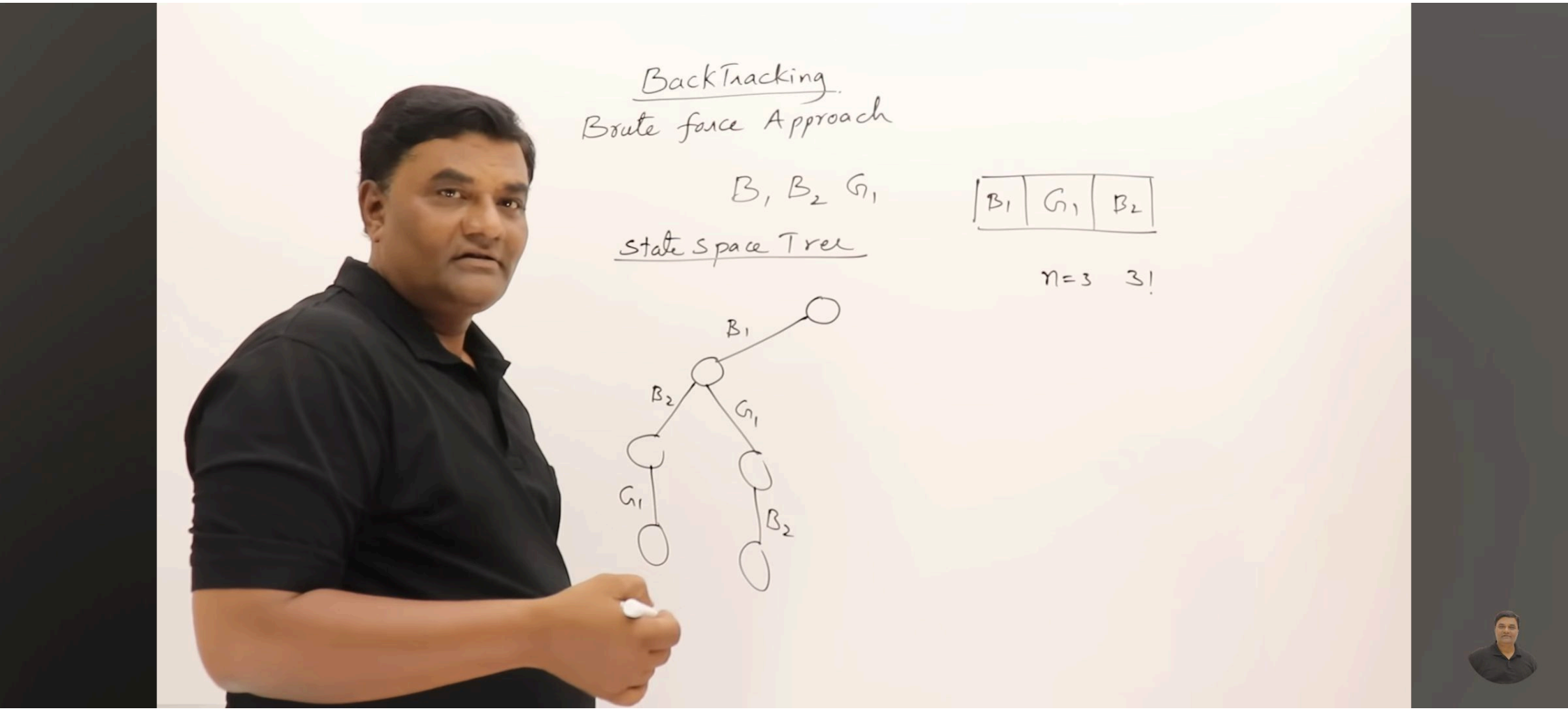
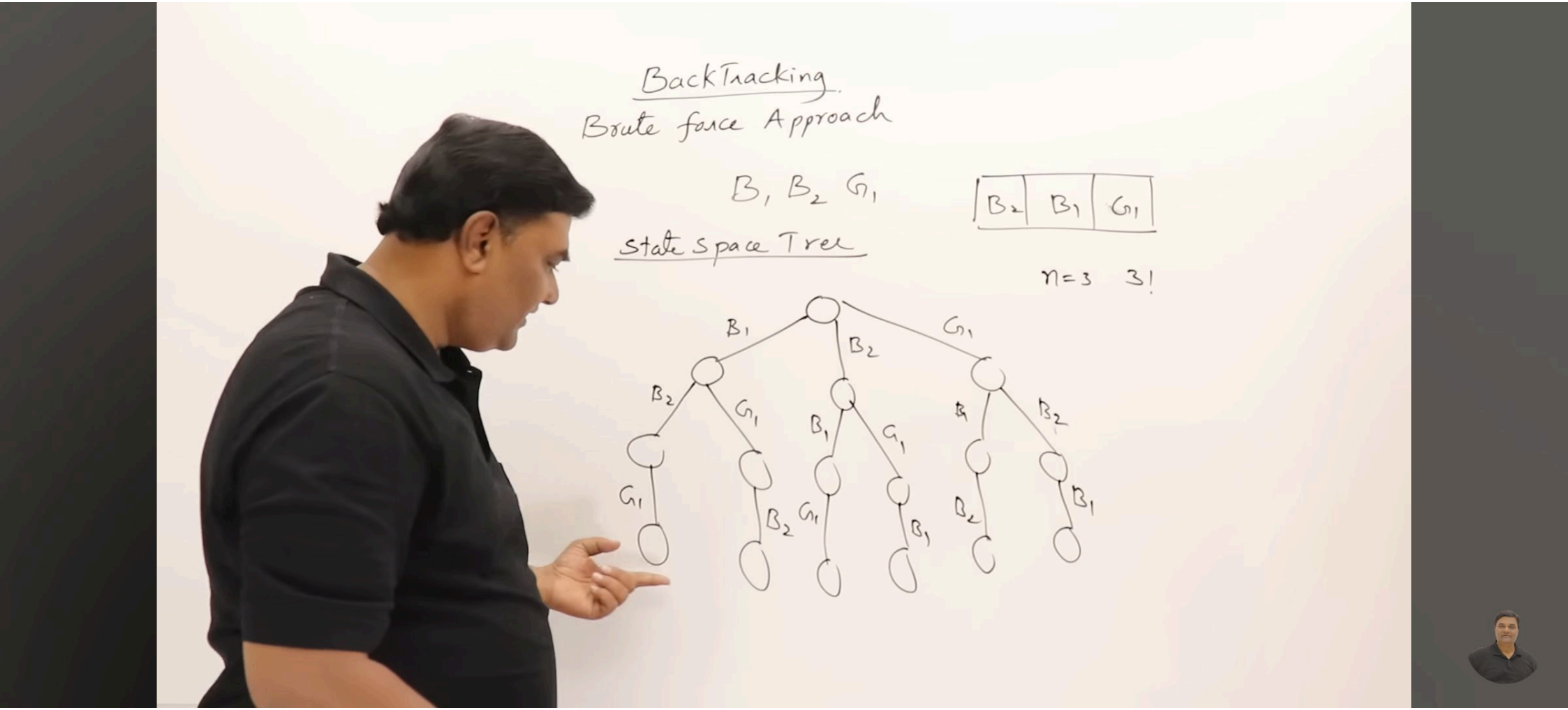
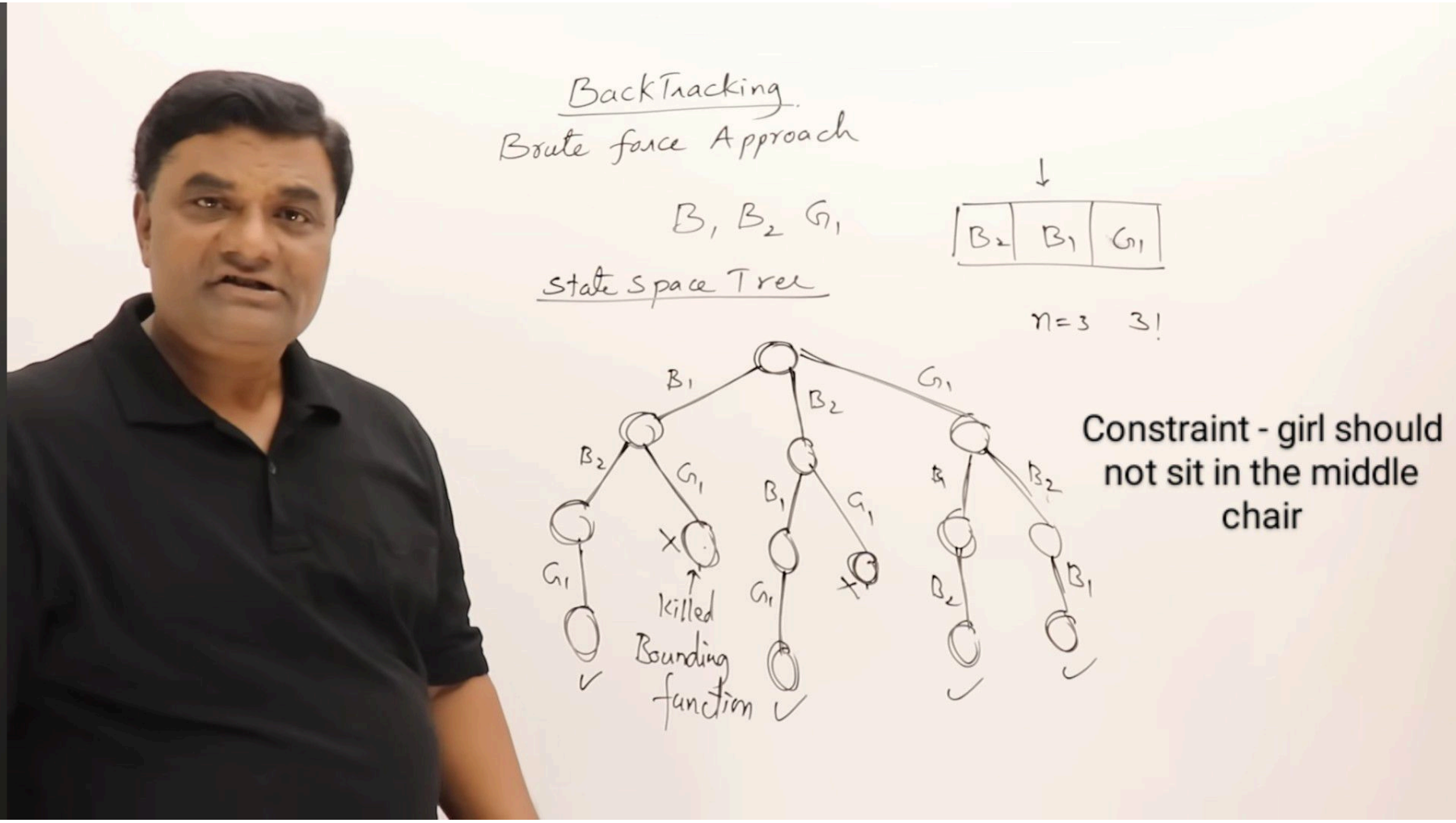
Introduction to Backtracking

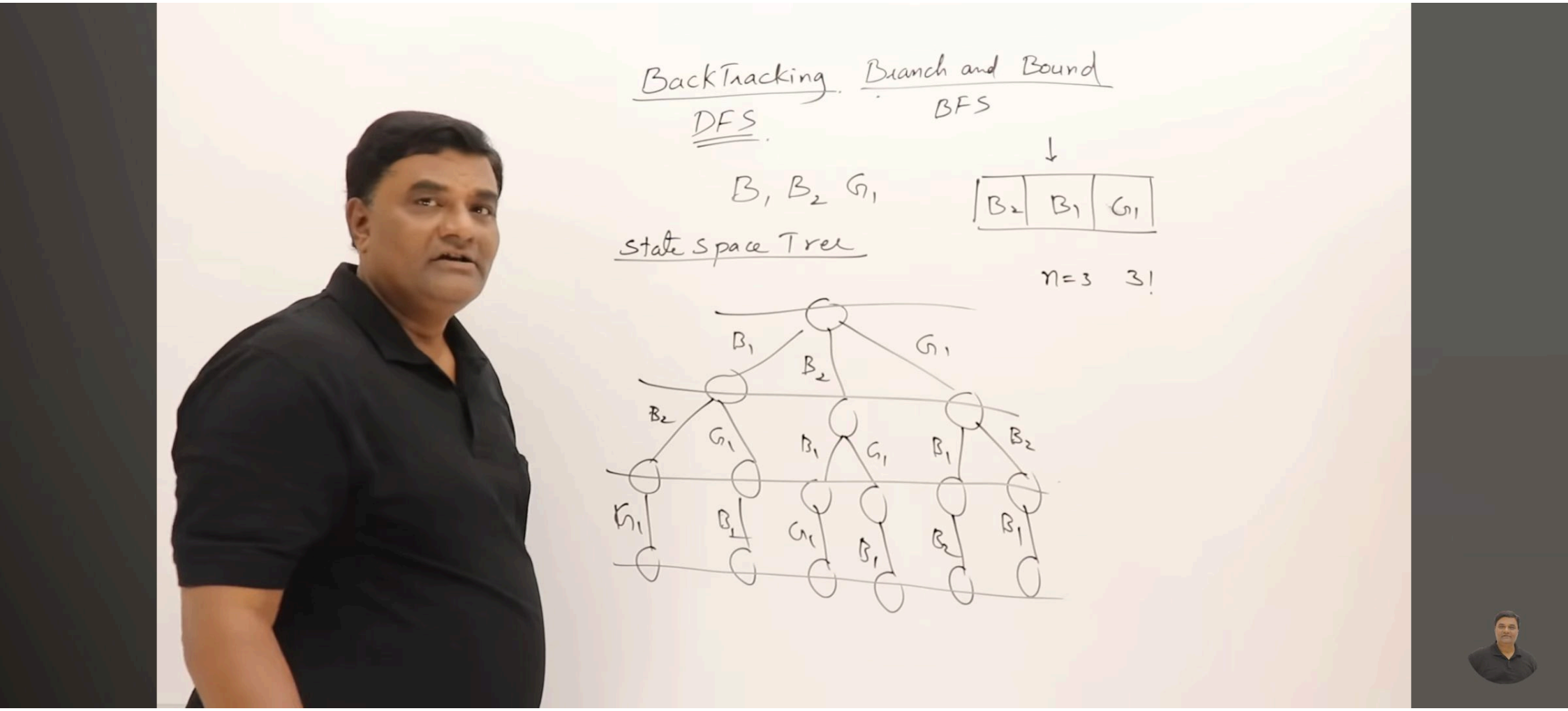

BackTracking
Brute force Approach
B1 B2 G1
n=3 3!


BackTracking
Brute force Approach
B1 B2 G1
B1 B2 G1
State Space Tree
B1
B2
G1
n=3 3!

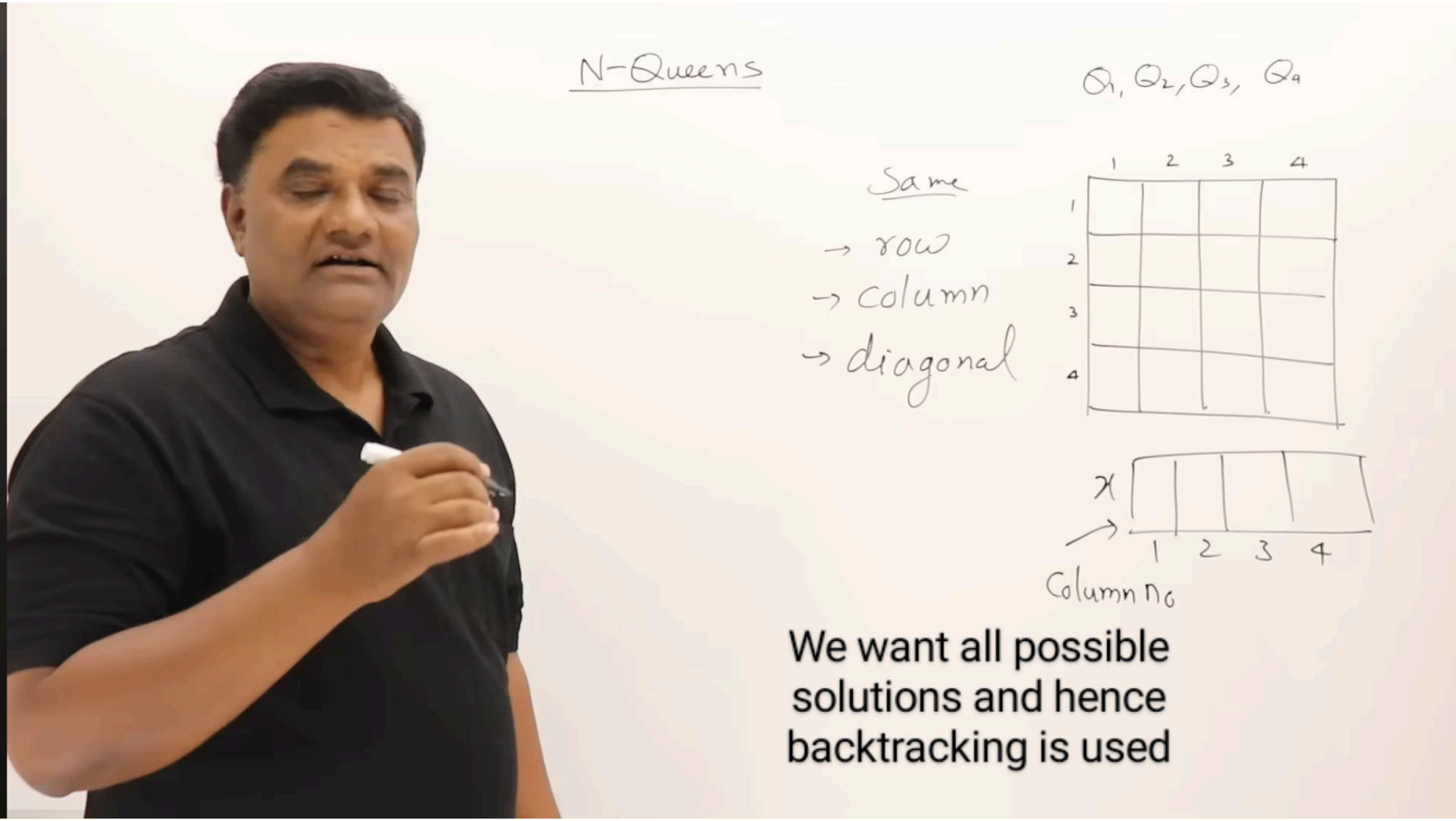

BackTracking
Brute force Approach
B1, B2 G1
B1 G1 B2
State Space Tree
B1
B2
G1
G1
B2
n=3 3!


BackTracking
Brute force Approach
B1, B2 G1
B2 B1 G1
State Space Tree
n=3 3!
B1
B2
G1
B2
G1
B1
G1
B
B2
G1
B2
G1
B1
B2
B1


BackTracking
Brute force Approach
B1, B2 G1
B2 B1 G1
n=3 3!
State Space Tree
B1
B2
G1
Killed
Bounding function
Constraint - girl should not sit in the middle chair

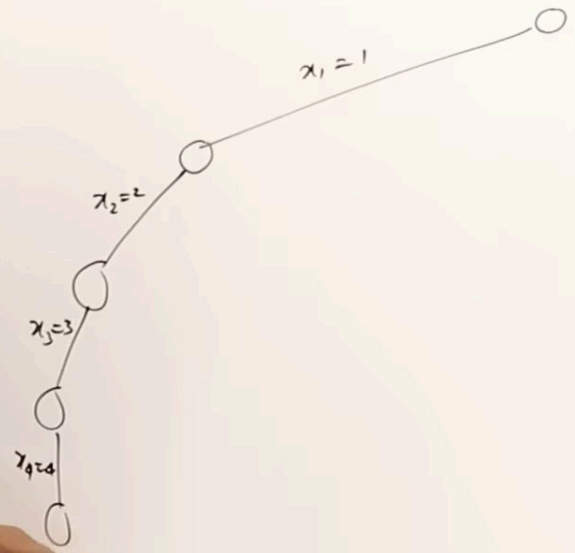

BackTracking
DFS
Branch and Bound
BFS
B1 B2 G1
B2 B1 G1
State Space Tree
n=3 3!
B1
B2
G1
B2
G1
B1
G1
B1
B2
G1
B1
G1
B1
B2
B1

N-Queens problem (using backtracking)

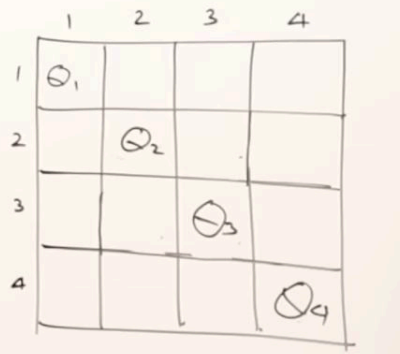

N-Queens
Q_1, Q_2, Q_3, Q_4
Same
→ row
→ column
→ diagonal
1 2 3 4
1
2
3
4
x
1 2 3 4
Column no
We want all possible solutions and hence backtracking is used

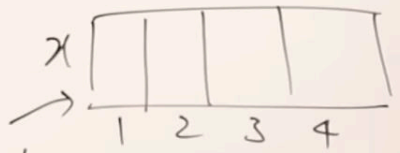
N-Queens

State space Tree

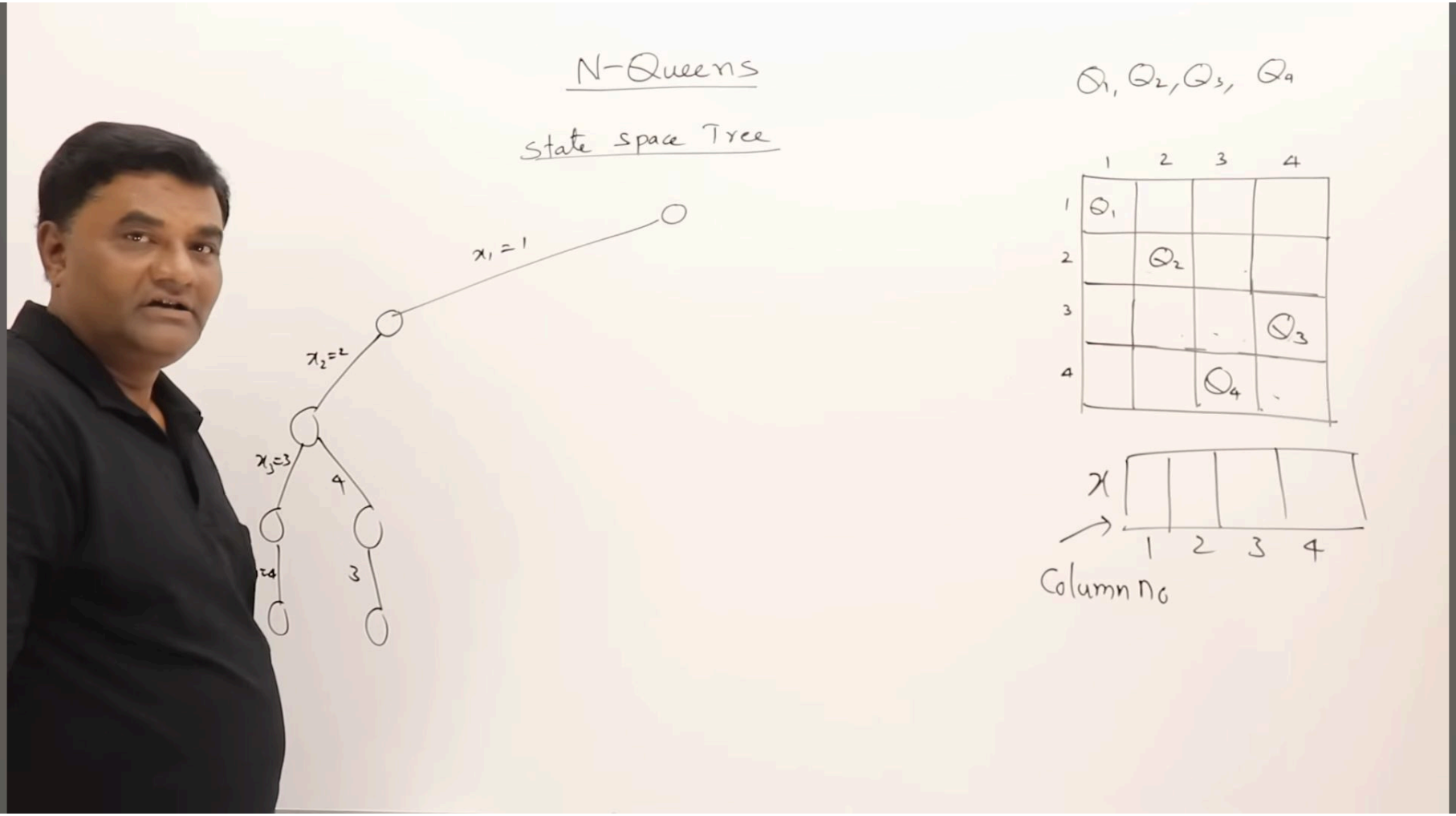


Q_1, Q_2, Q_3, Q_4



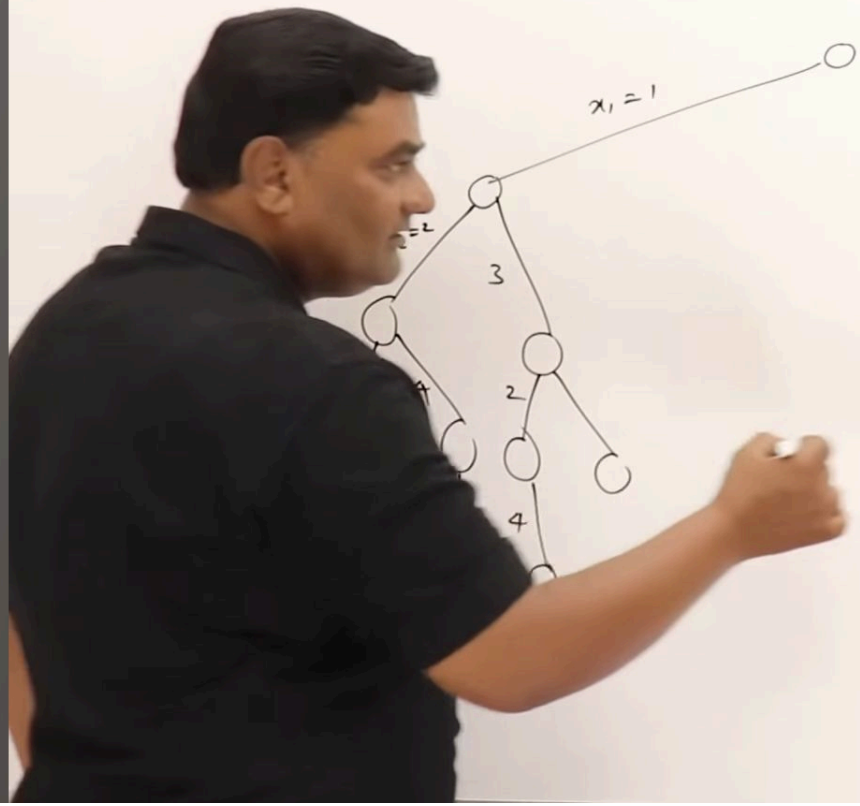


Column no

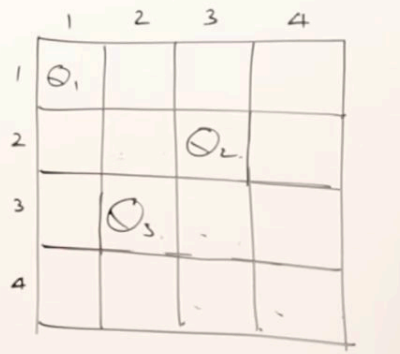

N-Queens
State space Tree
$x_1 = 1$
$x_2 = 2$
$x_3 = 3$
4
3
Q_1, Q_2, Q_3, Q_4
1 2 3 4
Q_1
Q_2
Q_3
Q_4
x
1 2 3 4
Column no

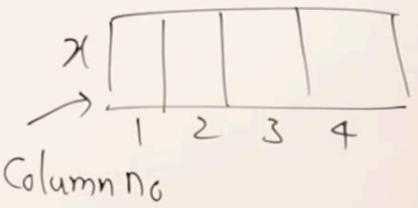
N-Queens

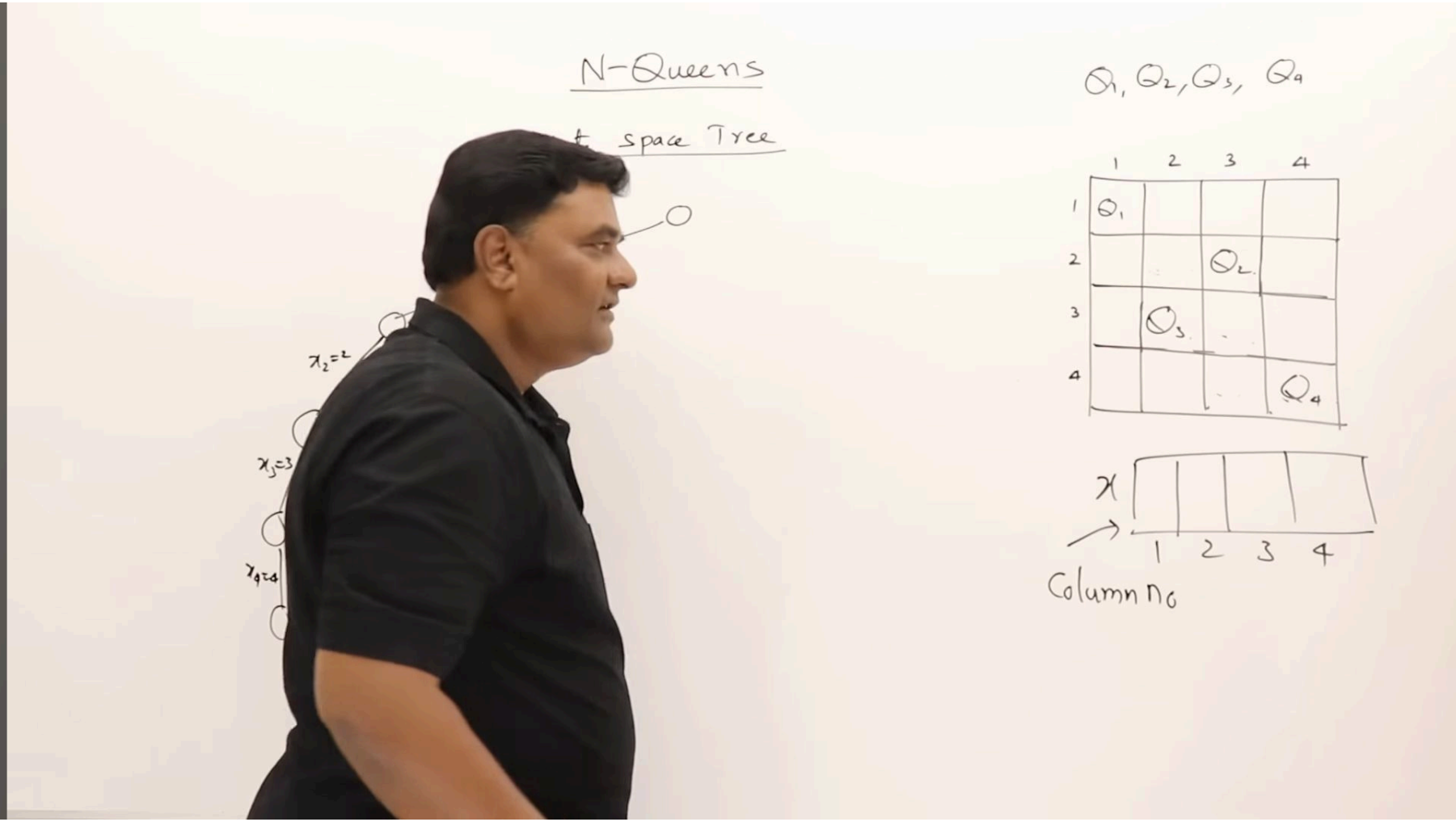
State space Tree



Q_1, Q_2, Q_3, Q_4

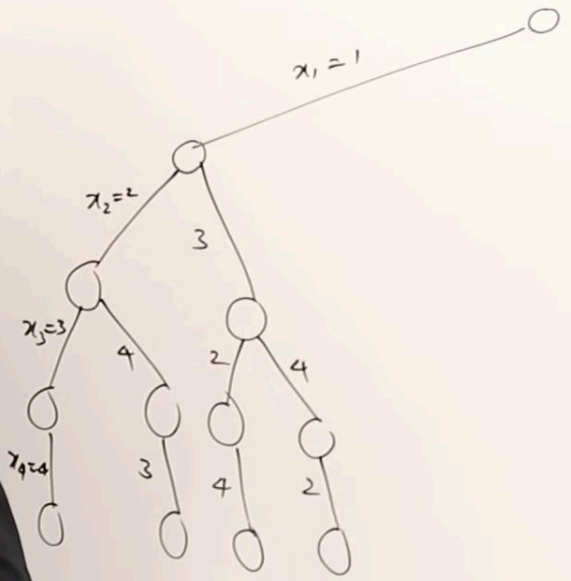




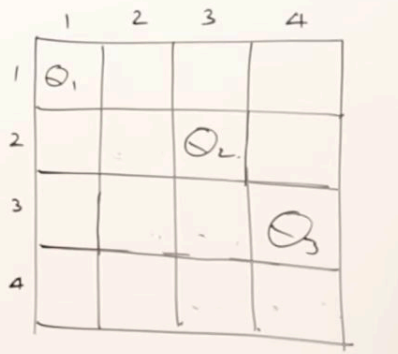

N-Queens
t space Tree
$x_2=2$
$x_3=3$
$x_4=4$
Q_1, Q_2, Q_3, Q_4
1 2 3 4
Q_1
Q_2
Q_3
Q_4
x
1 2 3 4
Column no

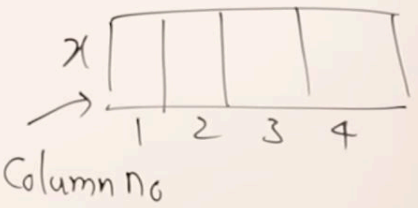
N-Queens

State space Tree

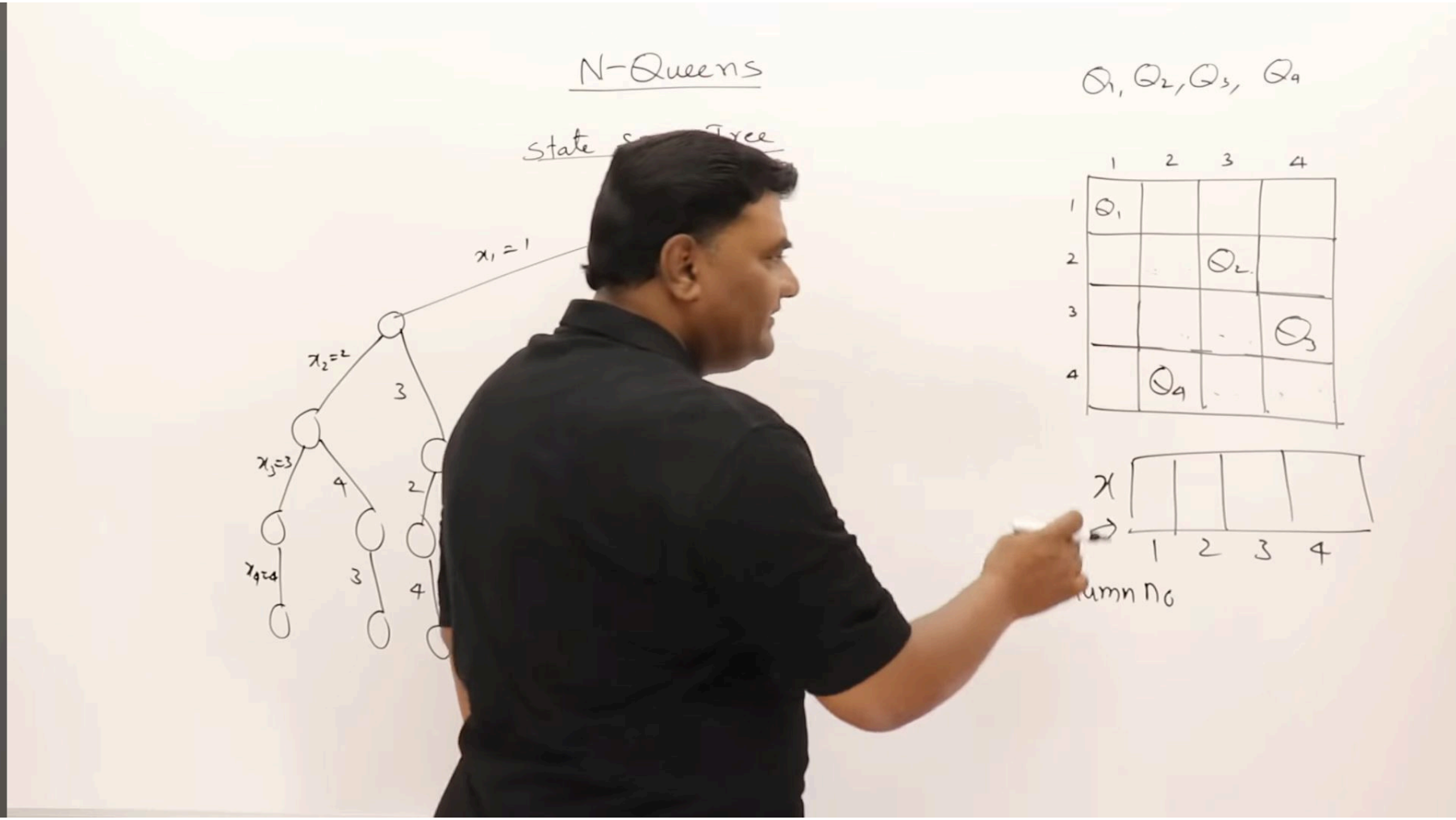


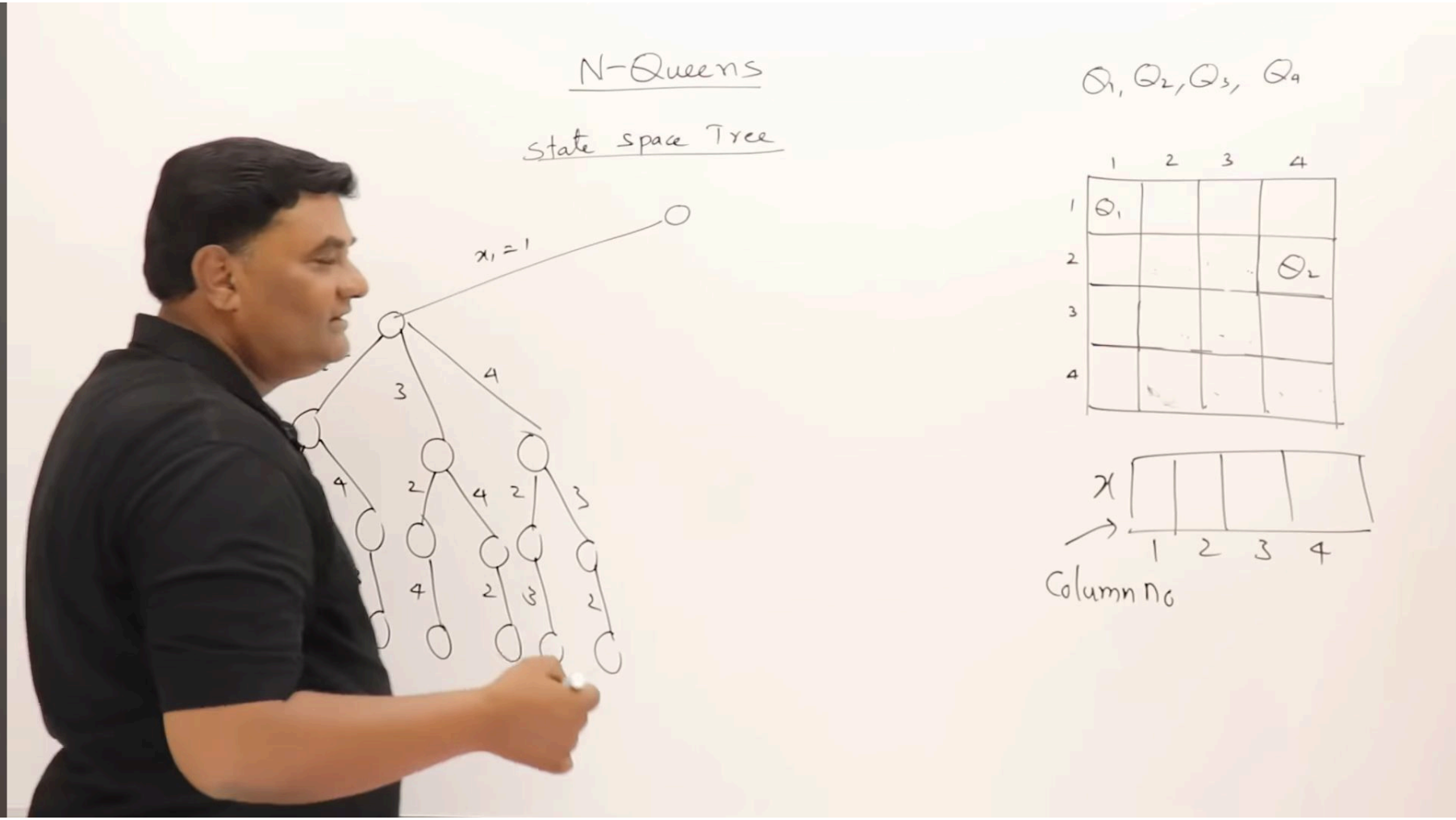
Q_1, Q_2, Q_3, Q_4





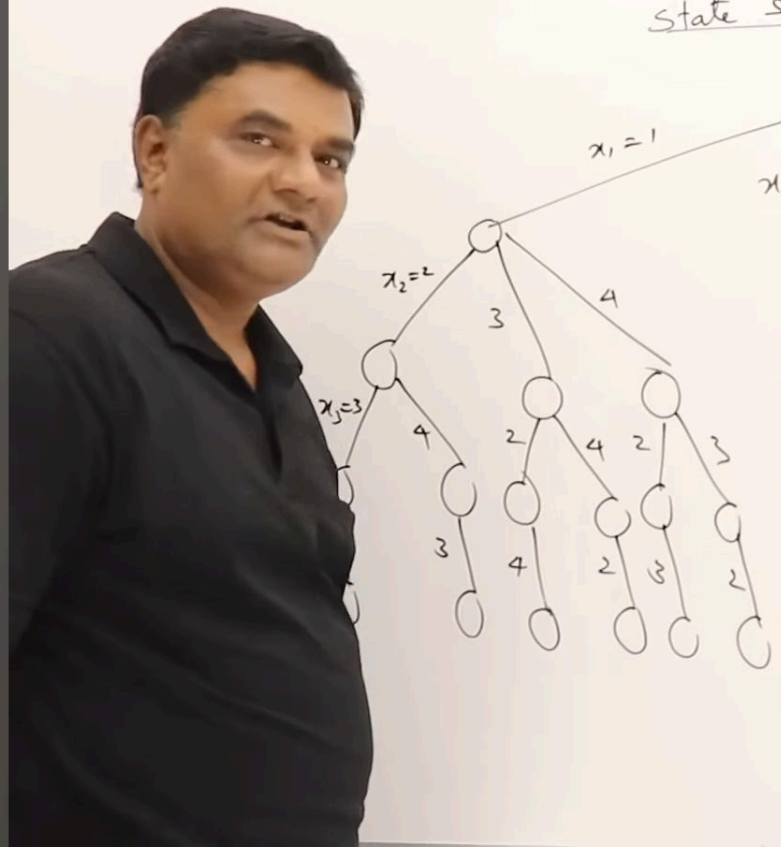
Column no


N-Queens
State
Tree
Q_1, Q_2, Q_3, Q_4
$x_1 = 1$
$x_2 = 2$
3
$x_3 = 3$
4
2
$x_4 = 4$
3
4
1 2 3 4
1
2
3
4
Q_1
Q_2
Q_3
Q_4
x
1 2 3 4
umn n_6

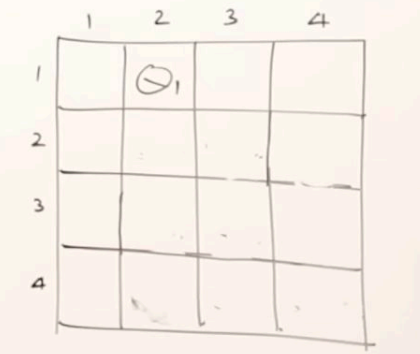

N-Queens
State space Tree
$x_1 = 1$
3
4
4
2
4
2
3
4
2
3
2
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1
2
3
4
Q_1
Q_2
x
1 2 3 4
Column no

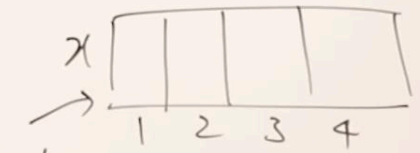
N-Queens

State space Tree

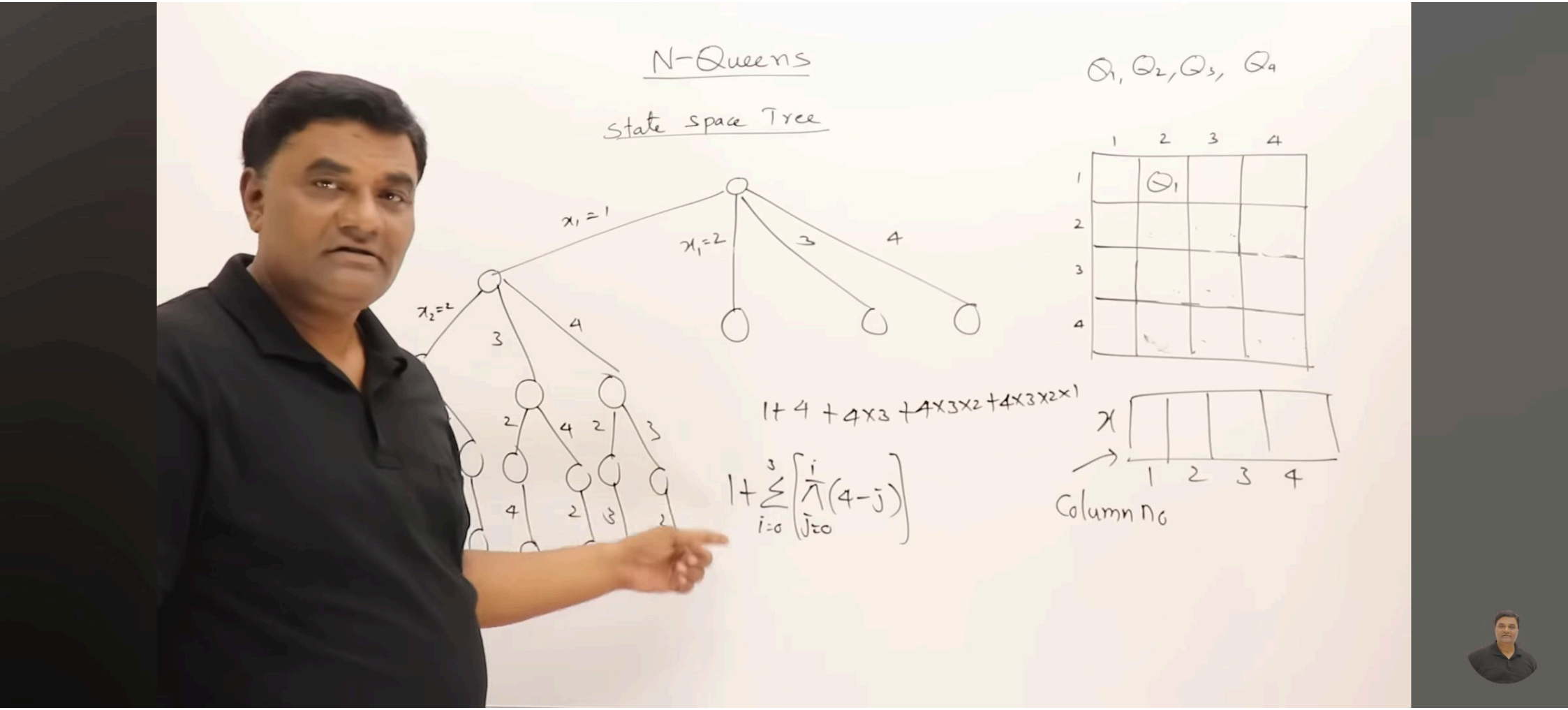


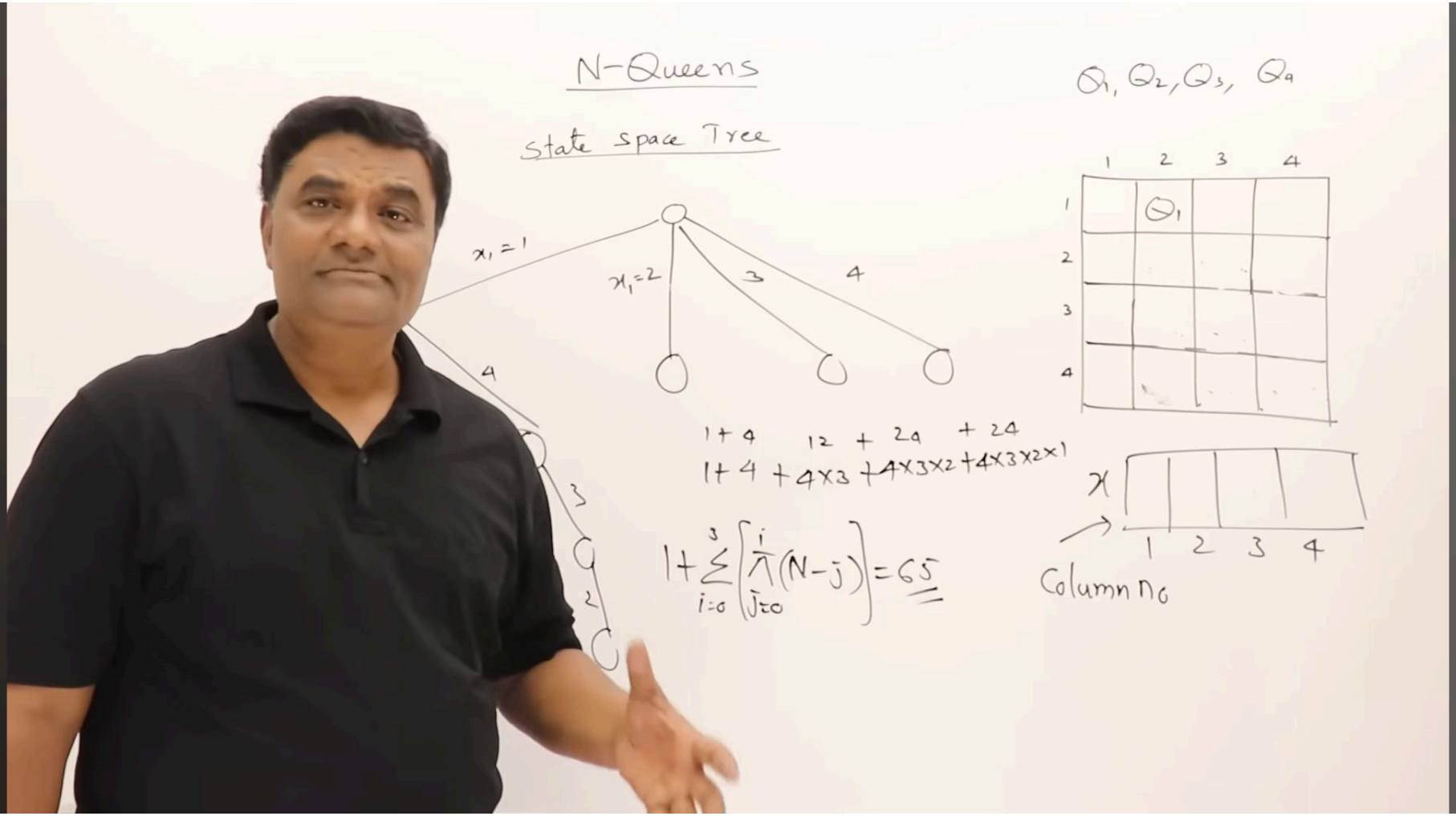
Q_1, Q_2, Q_3, Q_4

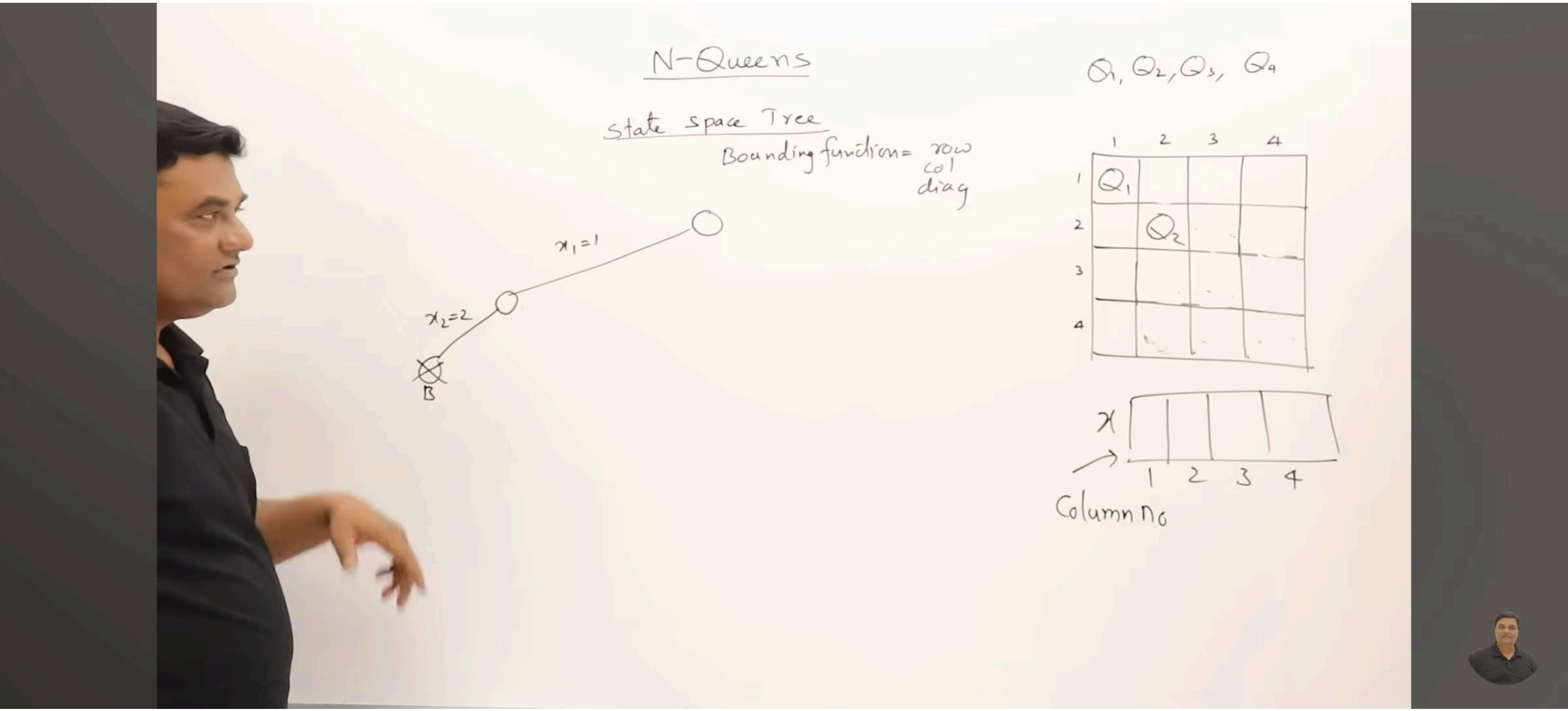


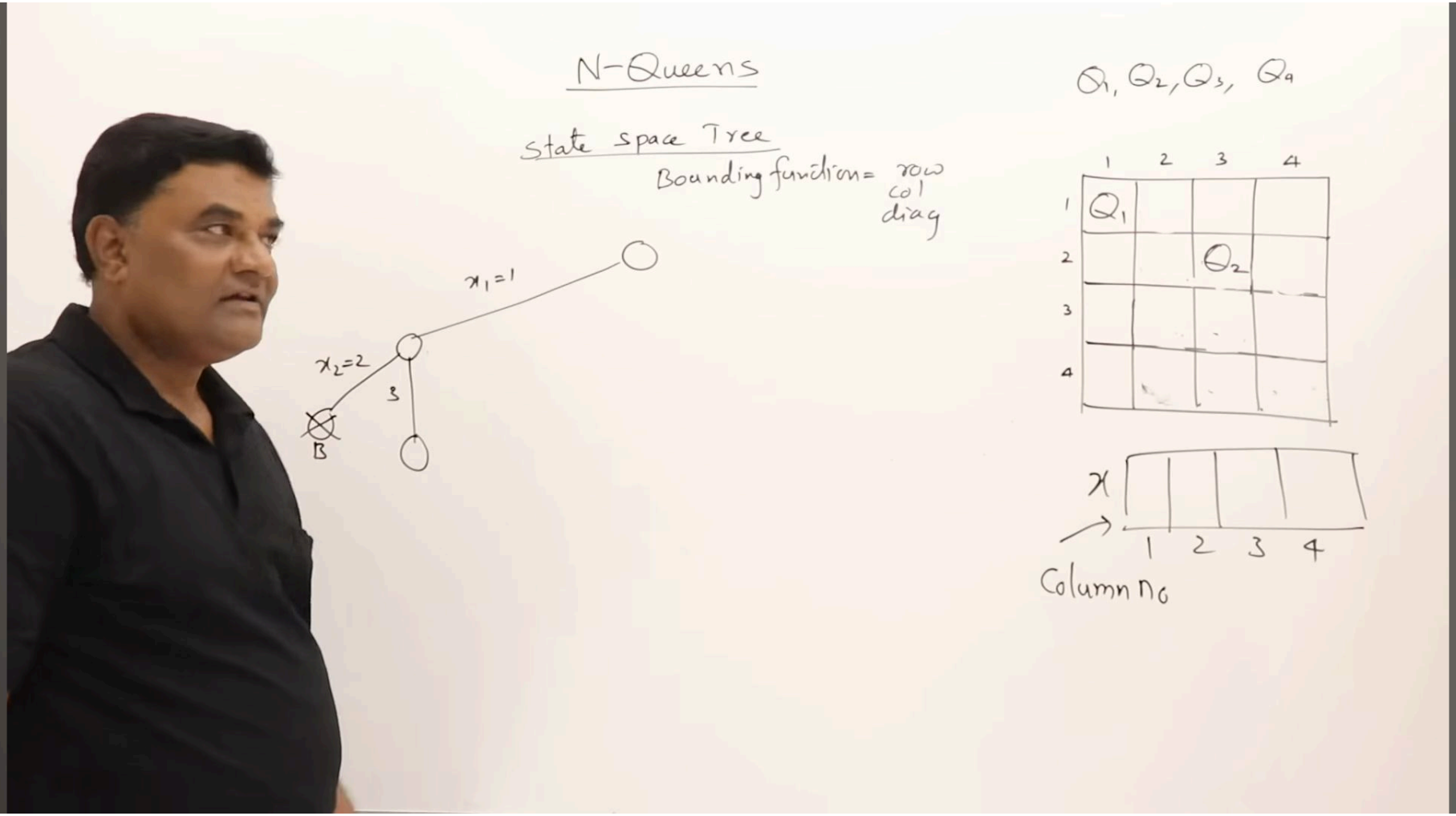


Column no


N-Queens
State space Tree
Q_1, Q_2, Q_3, Q_4
$x_1 = 1$
$x_1 = 2$
3
4
$x_2 = 2$
3
4
1 2 3 4
1
2
3
4
Q_1
$1 + 4 + 4\times3 + 4\times3\times2 + 4\times3\times2\times1$
$1 + \sum_{i=0}^{3} \left[\prod_{j=0}^{i} (4-j) \right]$
x
1 2 3 4
Column no


N-Queens
State space Tree
Q_1, Q_2, Q_3, Q_4
$x_1 = 1$
$x_1 = 2$
3
4
4
3
2
1 2 3 4
1
2
3
4
Q_1
$1 + 4 \quad 12 + 24 + 24$
$1 + 4 + 4 \times 3 + 4 \times 3 \times 2 + 4 \times 3 \times 2 \times 1$
$1 + \sum_{i=0}^{3} \left[\prod_{j=0}^{i} (N - j) \right] = 65$
x
1 2 3 4
Column no


N-Queens
State space Tree
Bounding function= row
col
diag
$x_1=1$
$x_2=2$
B
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1
2
3
4
Q_1
Q_2
x
1 2 3 4
Column no

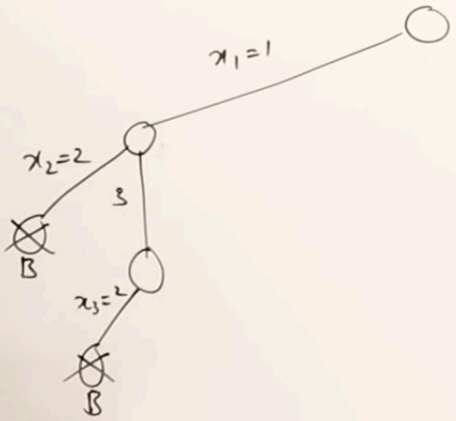

N-Queens
State space Tree
Bounding function= row
col
diag
$x_1=1$
$x_2=2$
3
B
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1
2
3
4
Q_1
Q_2
x
1 2 3 4
Column no

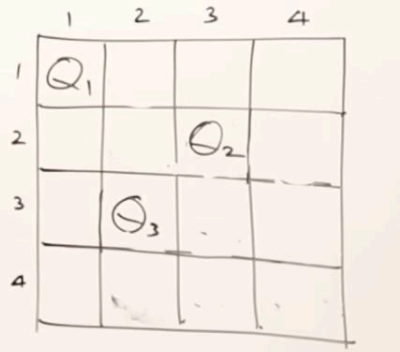
N-Queens

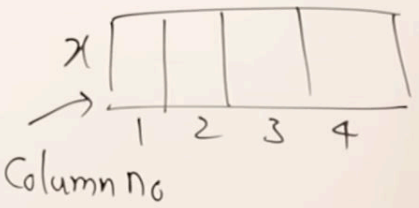
Q_1, Q_2, Q_3, Q_4

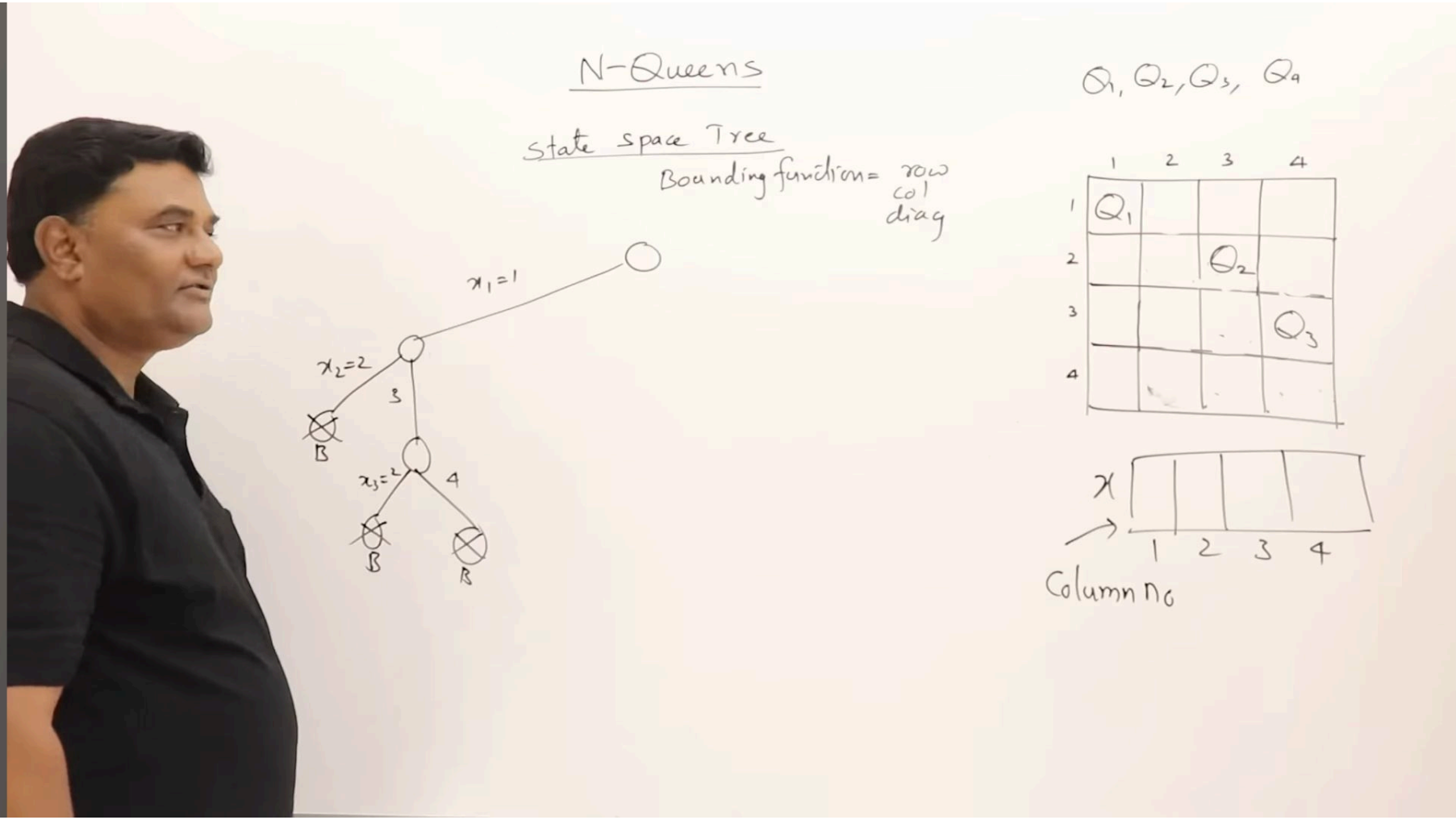
State space Tree

Bounding function = row
col
diag







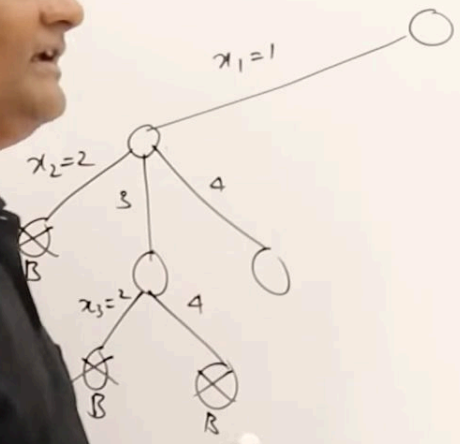

N-Queens
State space Tree
Bounding function= row
col
diag
$x_1=1$
$x_2=2$
3
B
$x_3=2$
4
B
B
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1 Q_1
2 Q_2
3 Q_3
4
x
1 2 3 4
Column no

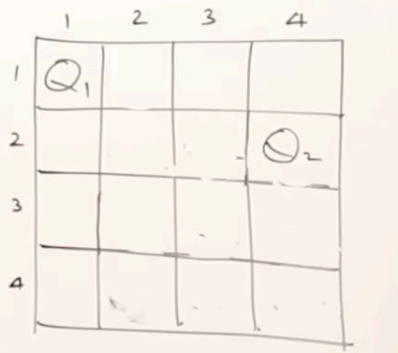
N-Queens

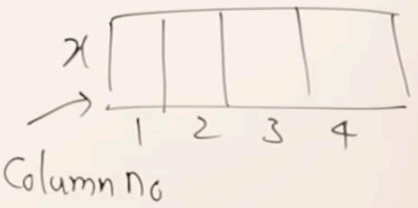
Q_1, Q_2, Q_3, Q_4

State space Tree

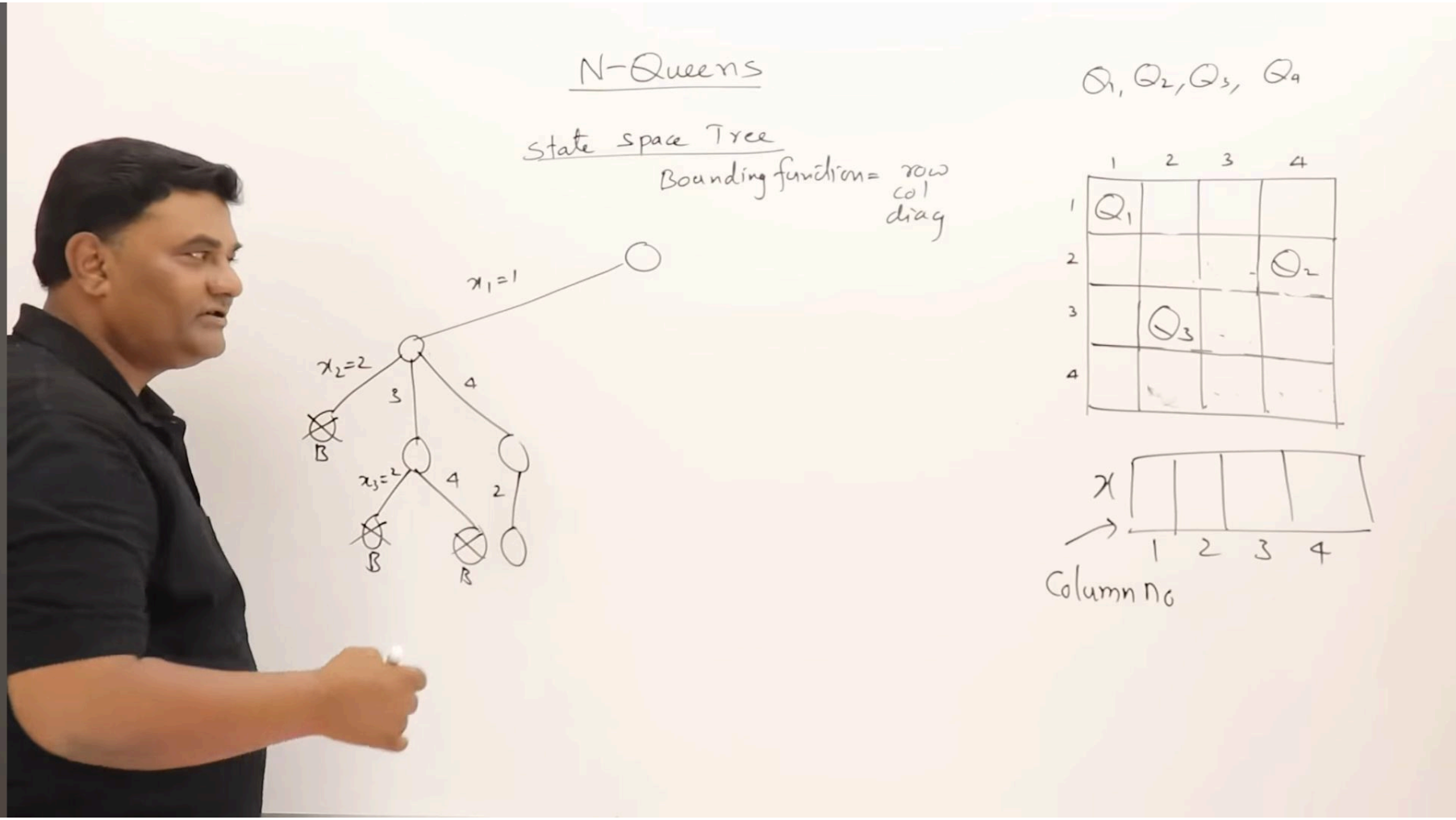
Bounding function = row
col
diag

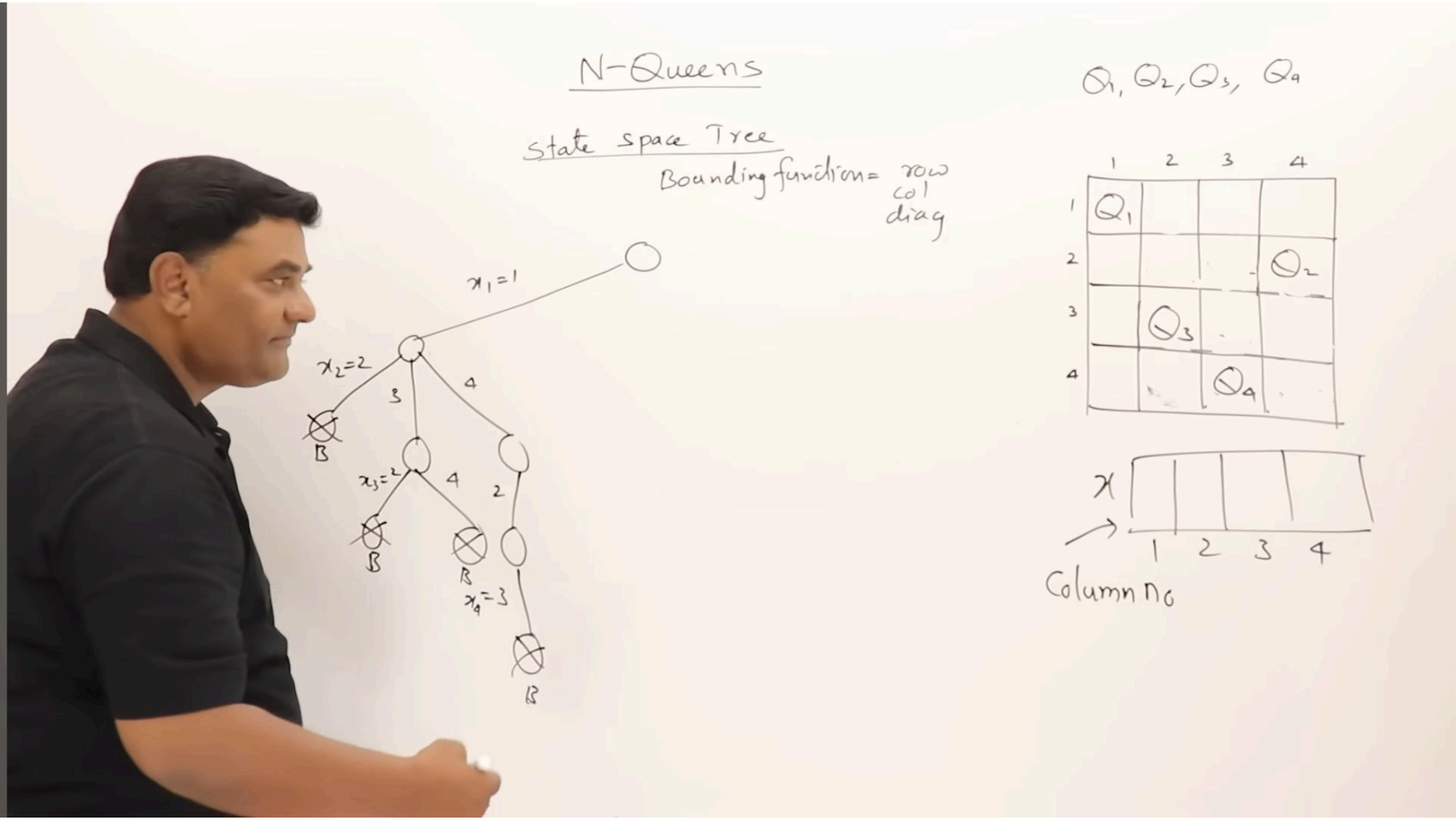






Column no

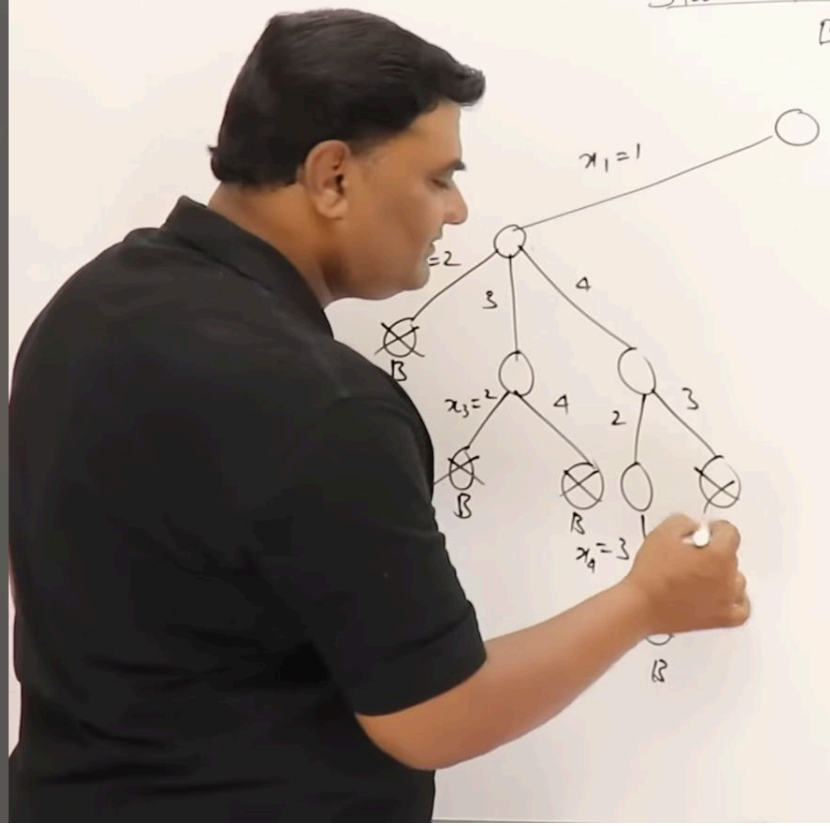

N-Queens
State space Tree
Bounding function= row
col
diag
$x_1=1$
$x_2=2$
3
4
B
$x_3=3$
4
2
B
B
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1
2
3
4
Q_1
Q_2
Q_3
x
1 2 3 4
Column no


N-Queens
State space Tree
Bounding function= row
col
diag
$x_1=1$
$x_2=2$
3
4
B
$x_3=2$
4
2
B
B
$x_4=3$
B
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1 Q_1
2 Q_2
3 Q_3
4 Q_4
x
1 2 3 4
Column no

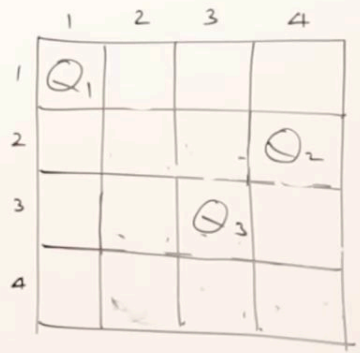
N-Queens

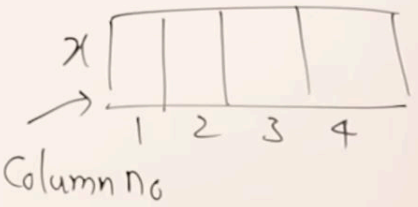
State space Tree

Bounding function= row
col
diag



Q_1, Q_2, Q_3, Q_4





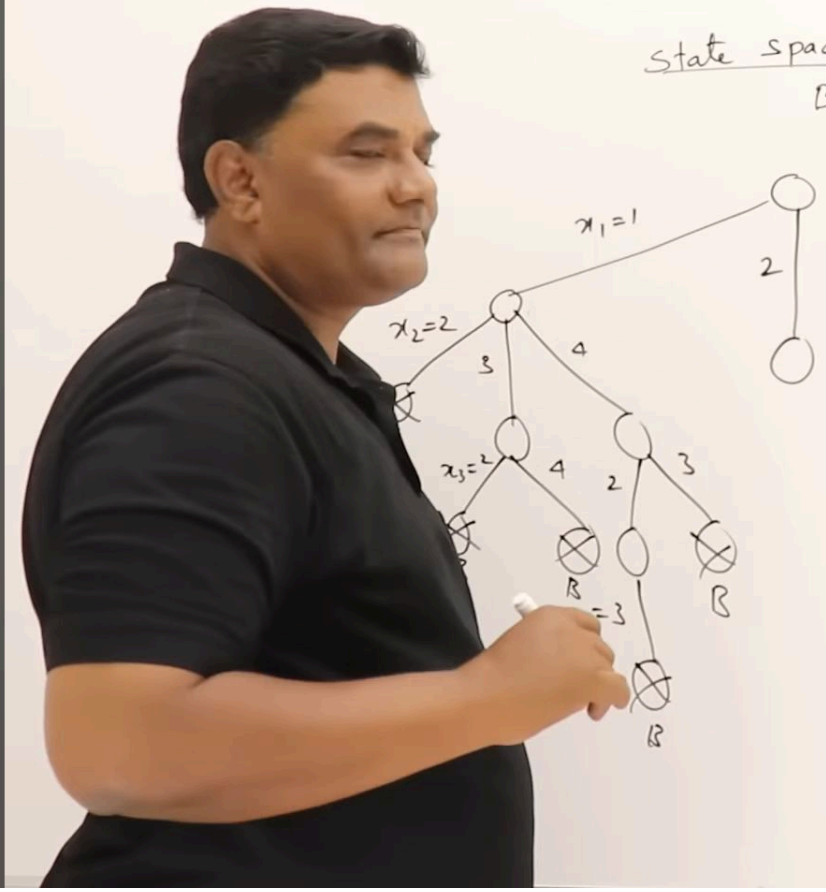
Column no

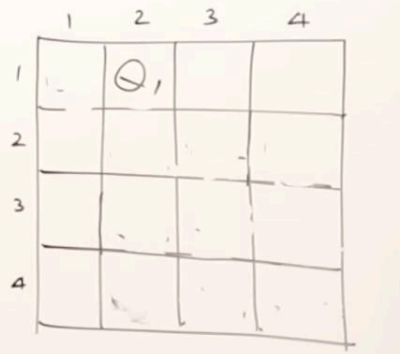
N-Queens

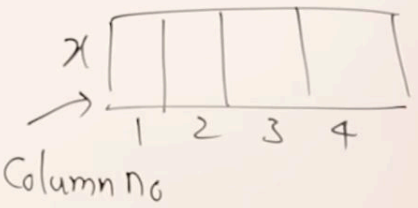
Q_1, Q_2, Q_3, Q_4

State space Tree

Bounding function = row col diag







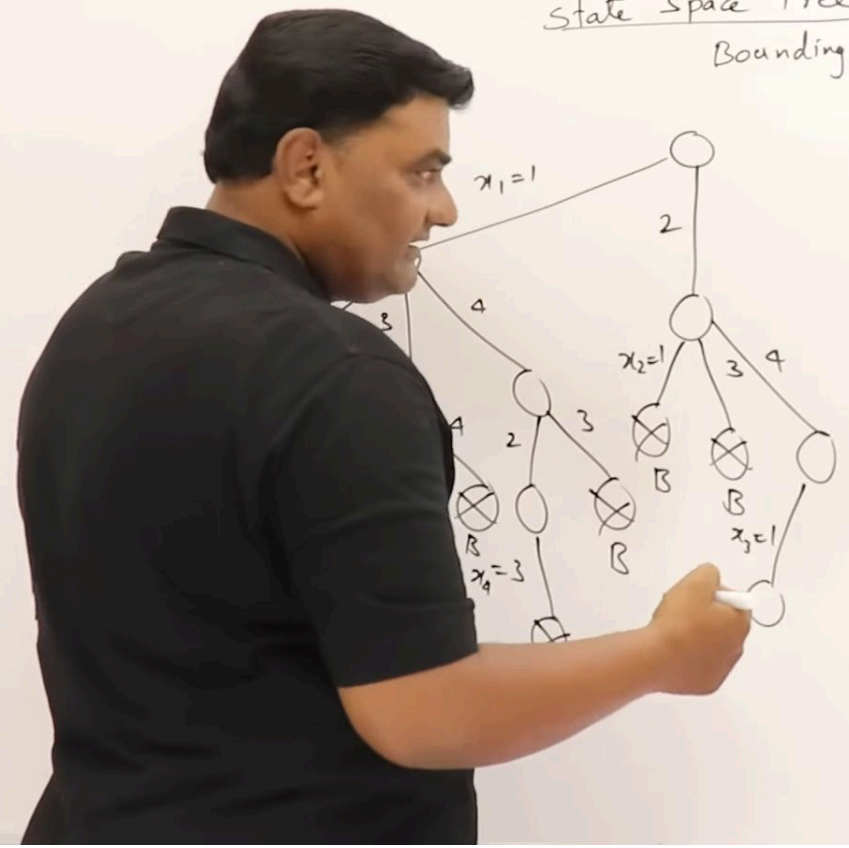
Column no

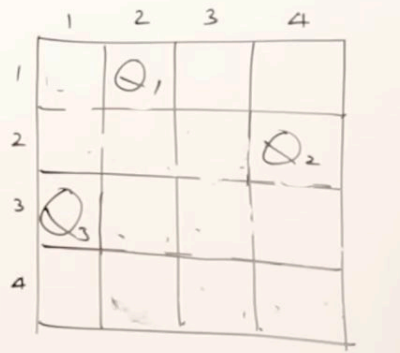
N-Queens

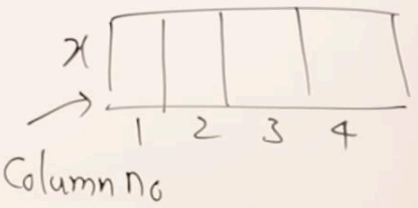
Q_1, Q_2, Q_3, Q_4

State space Tree

Bounding function = row
col
diag





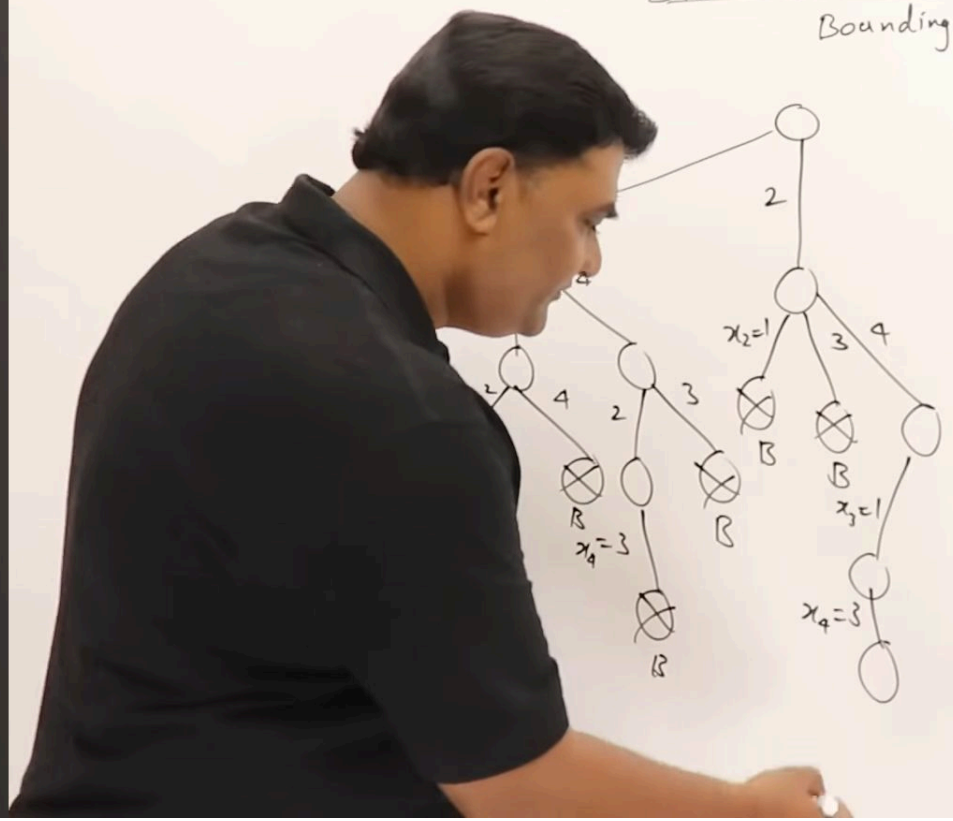


Column no

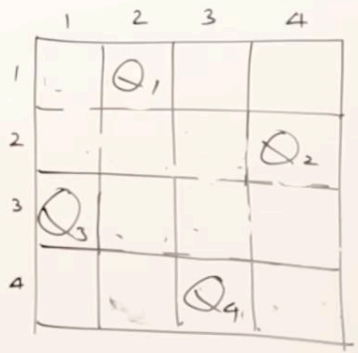
N-Queens

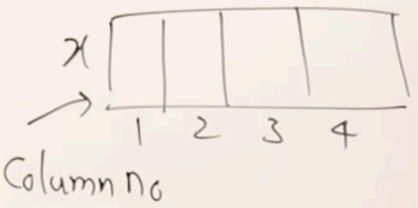
State space Tree

Bounding function= row
col
diag

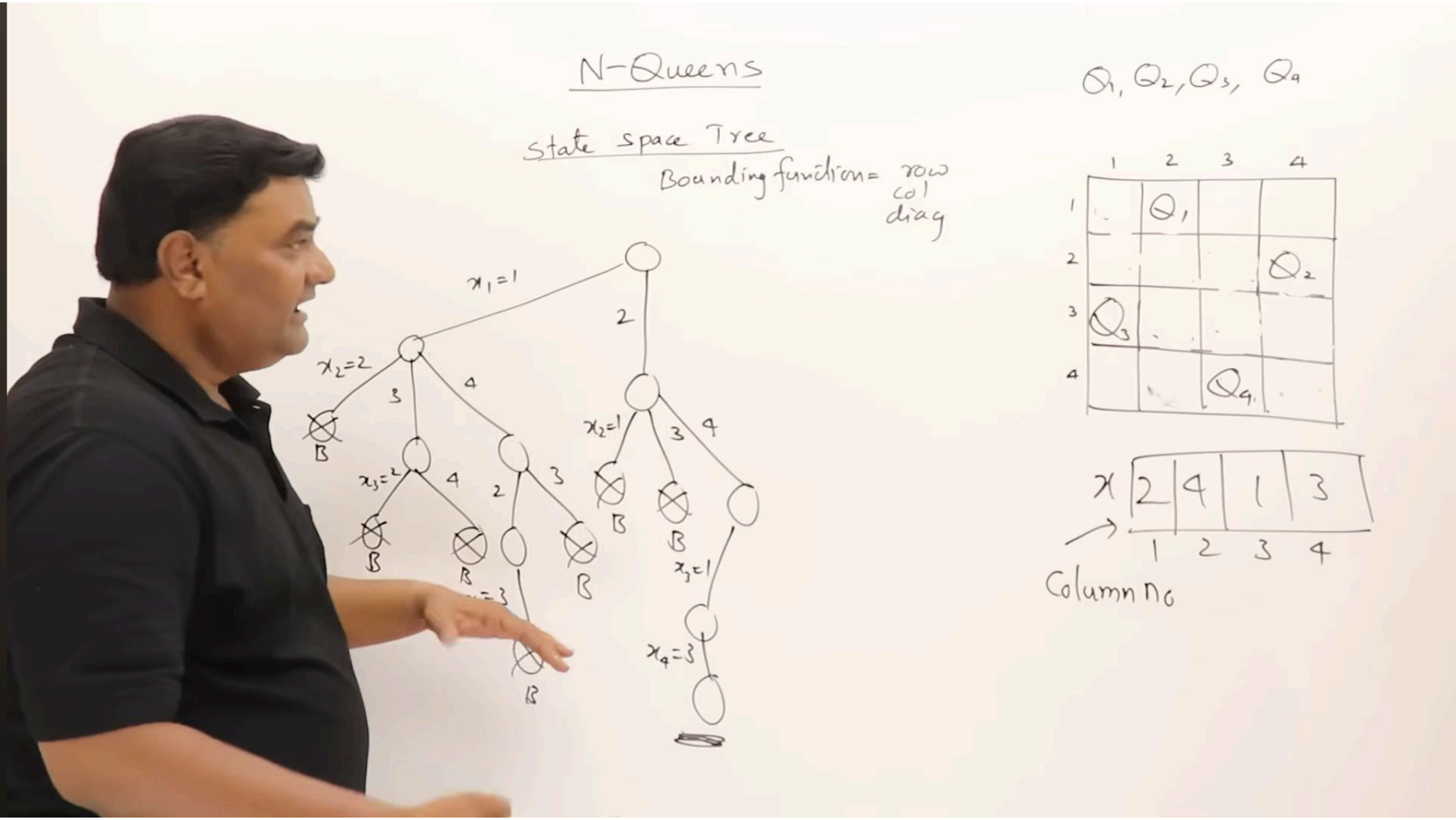


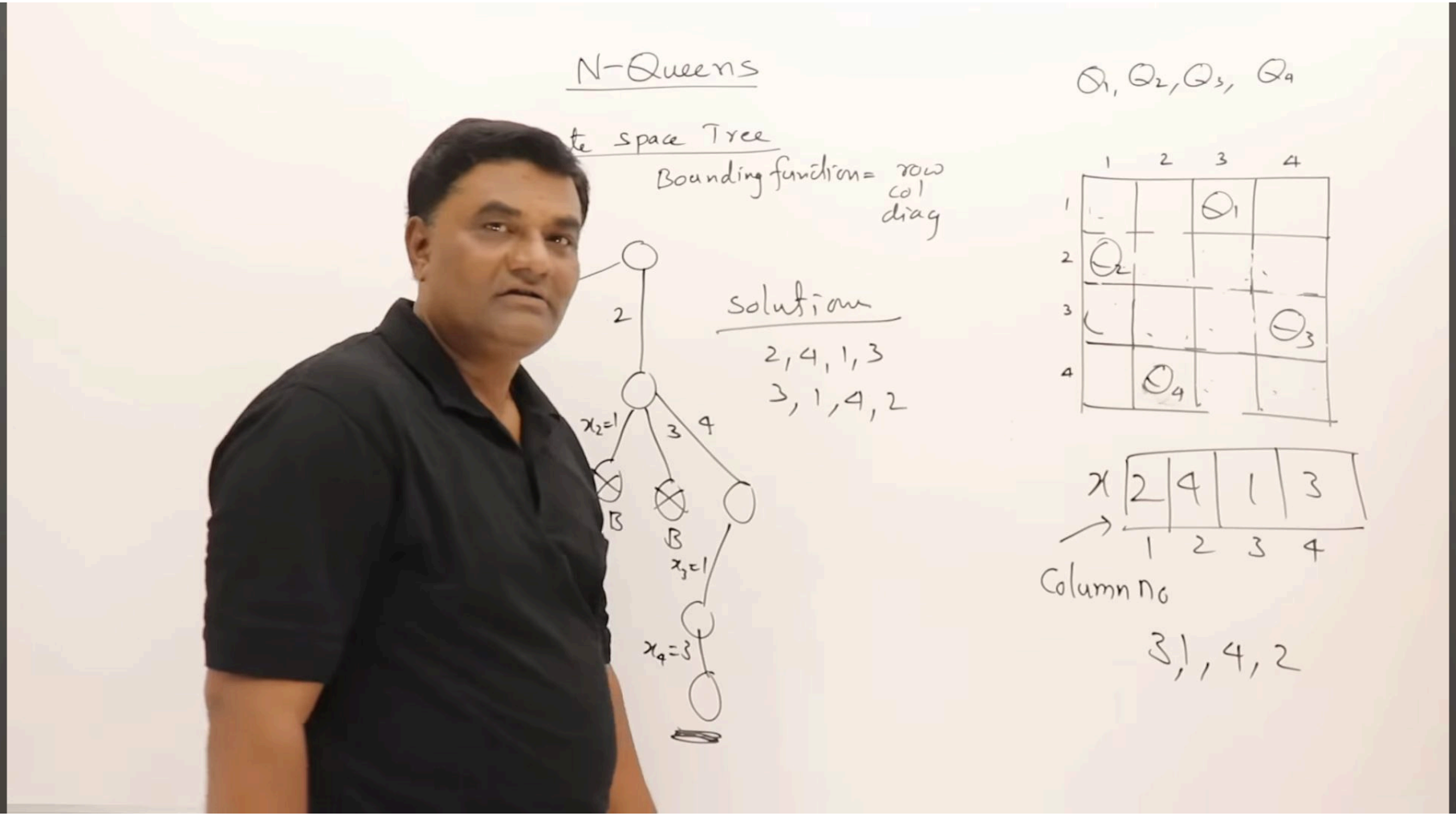
Q_1, Q_2, Q_3, Q_4





Column no


N-Queens
State space Tree
Bounding function= row col diag
$x_1=1$
2
$x_2=2$
3
4
B
$x_3=2$
4
2
3
B
B
B
B
$x_2=1$
3
4
B
B
$x_3=1$
$x_4=3$
B
Q_1, Q_2, Q_3, Q_4
1 2 3 4
1 Q_1
2 Q_2
3 Q_3
4 Q_4
x 2 4 1 3
1 2 3 4
Column no


N-Queens
te space Tree
Bounding function= row
col
diag
2
$x_2=1$
3
4
B
B
$x_3=1$
$x_4=3$
solution
2,4,1,3
3,1,4,2
Q_1, Q_2, Q_3, Q_4
1 2 3 4
Q_1
Q_2
Q_3
Q_4
x 2 4 1 3
1 2 3 4
Column no
3,1,4,2

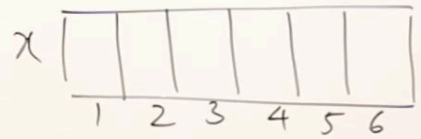
Sum of subsets problem

Sum of Subsets

$w[1:6] = \{5, 10, 12, 13, 15, 18\}$

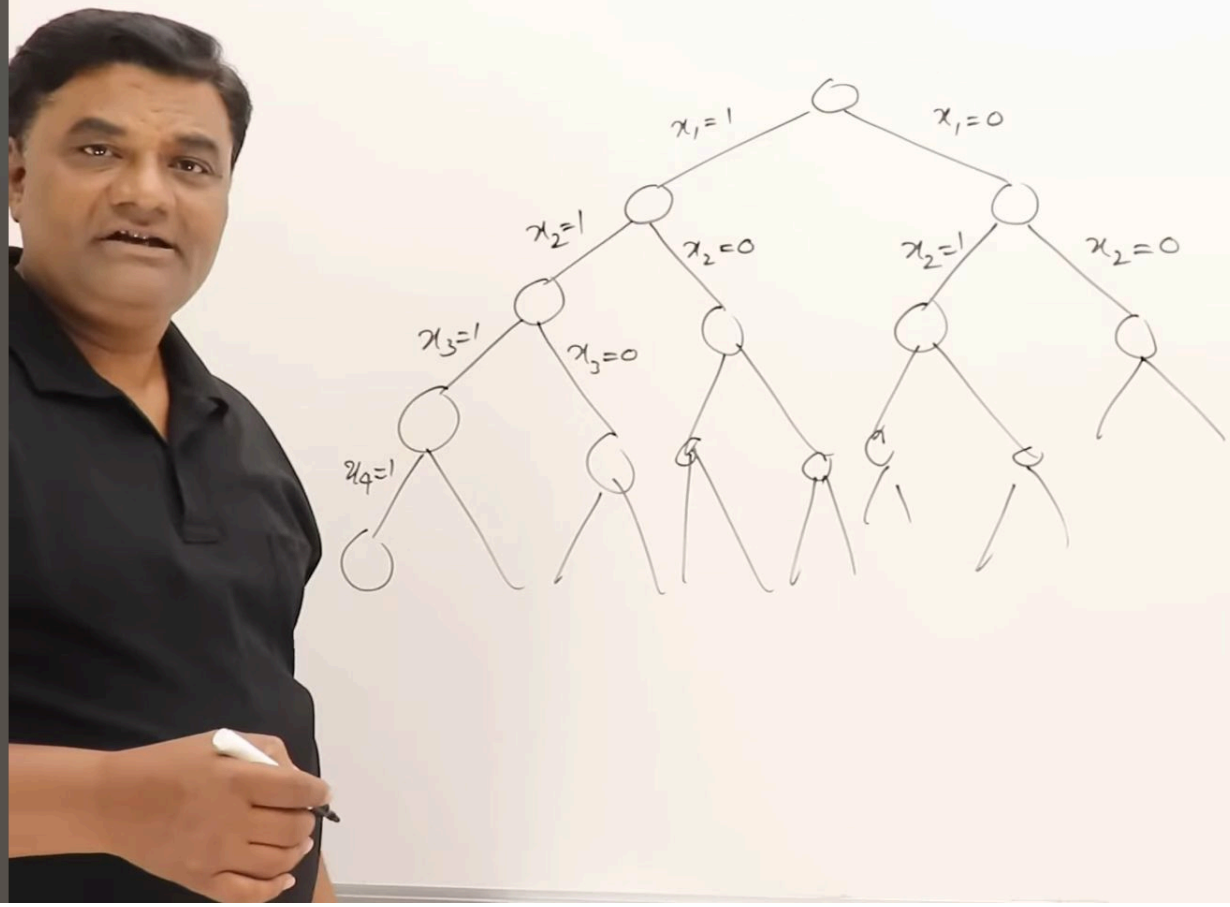
(1 2 3 4 5 6)

$n = 6 \quad m = 30$



$x_i = 0/1$

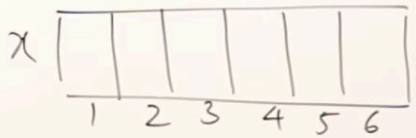
Sum of Subsets



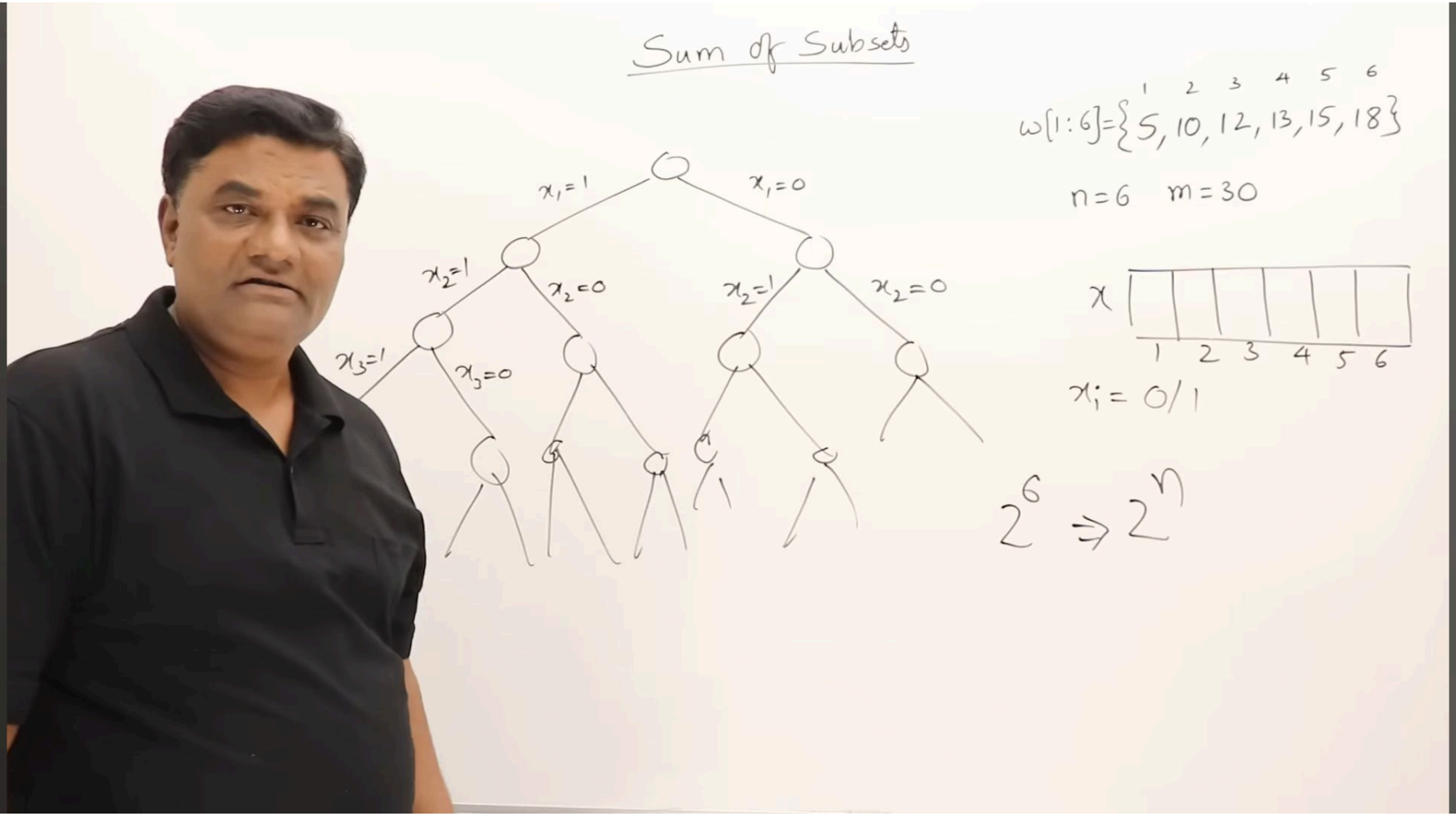
$w[1:6]=\{5, 10, 12, 13, 15, 18\}$

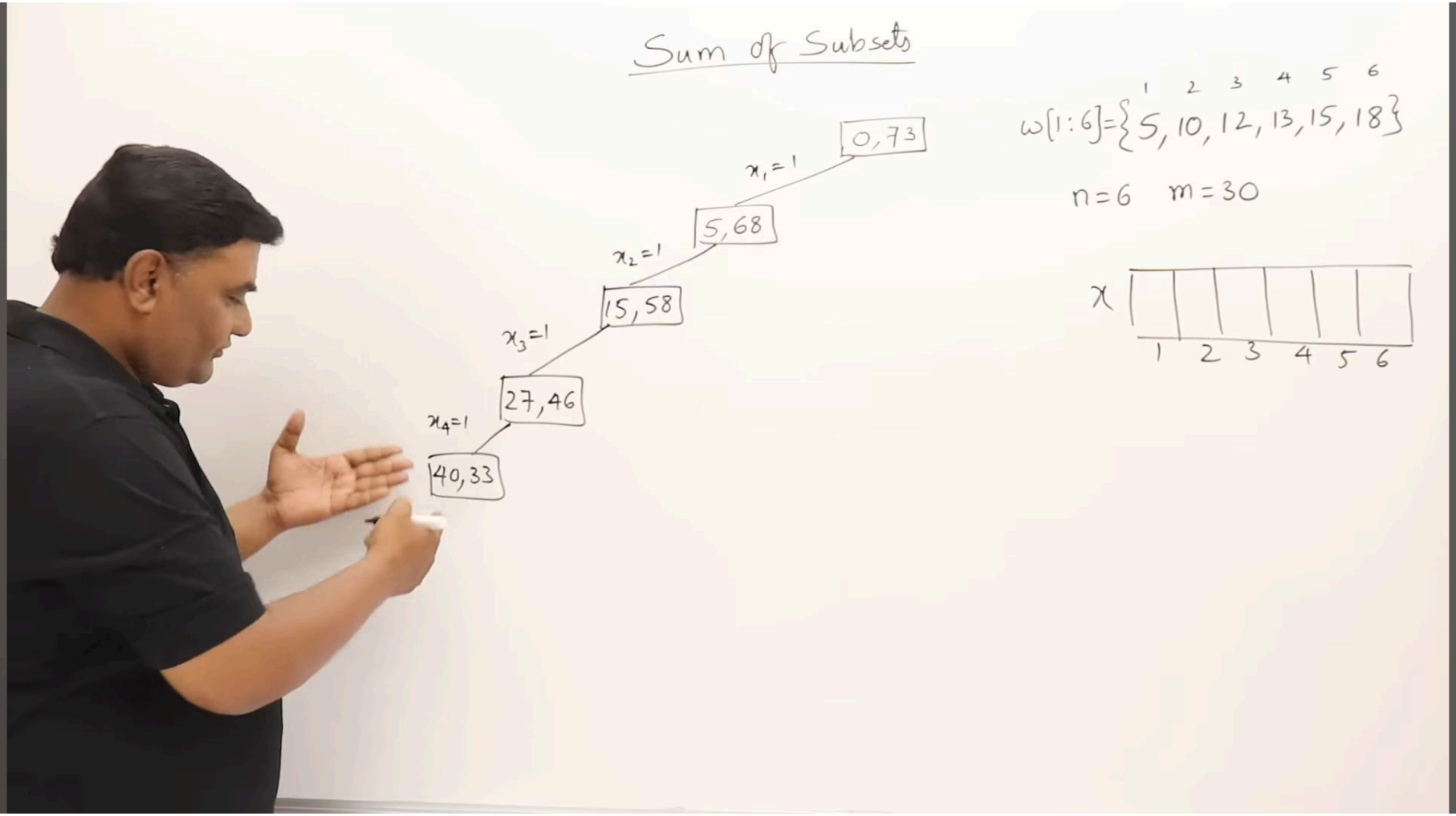
(1 2 3 4 5 6)

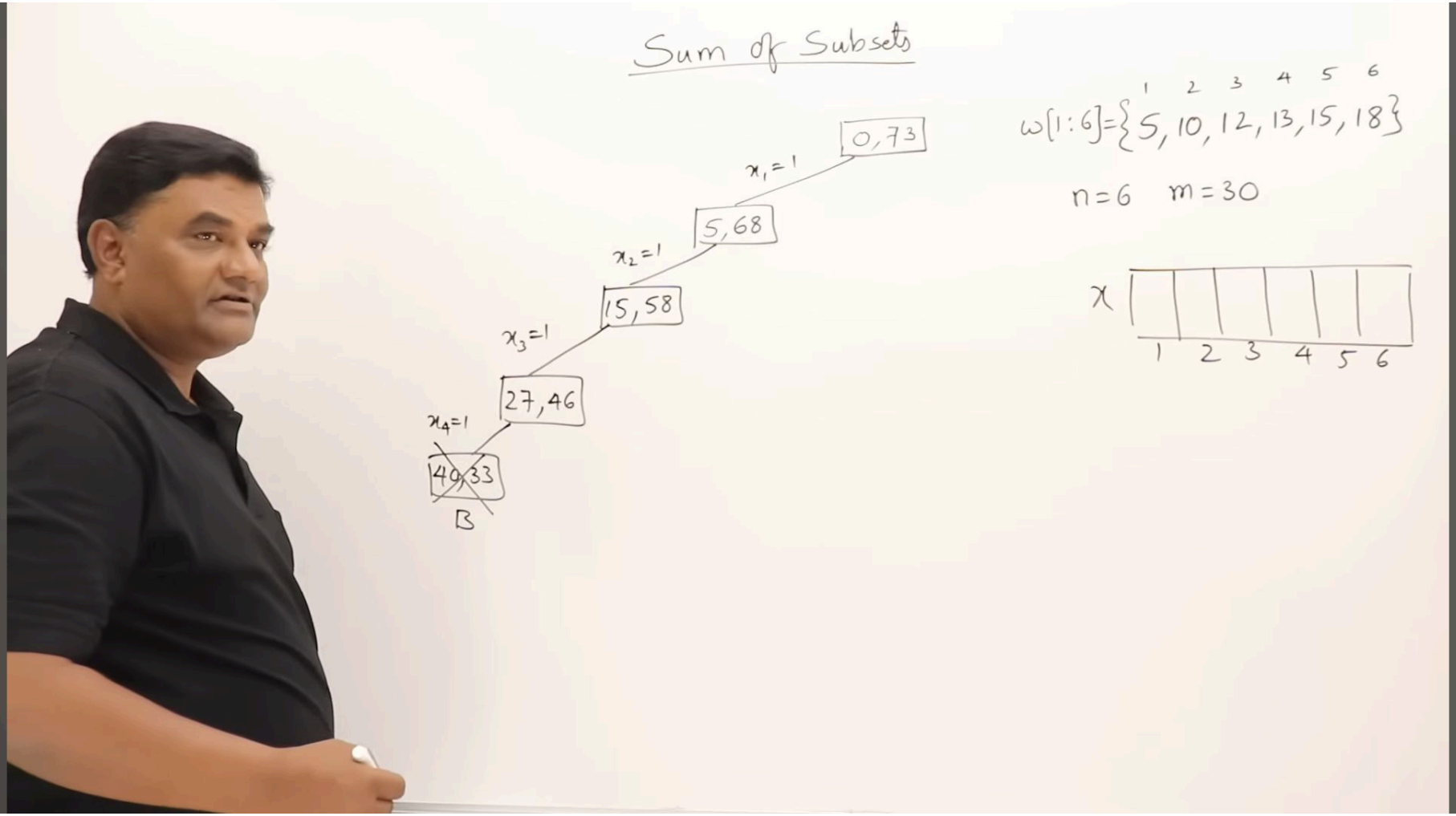
$n=6 \quad m=30$

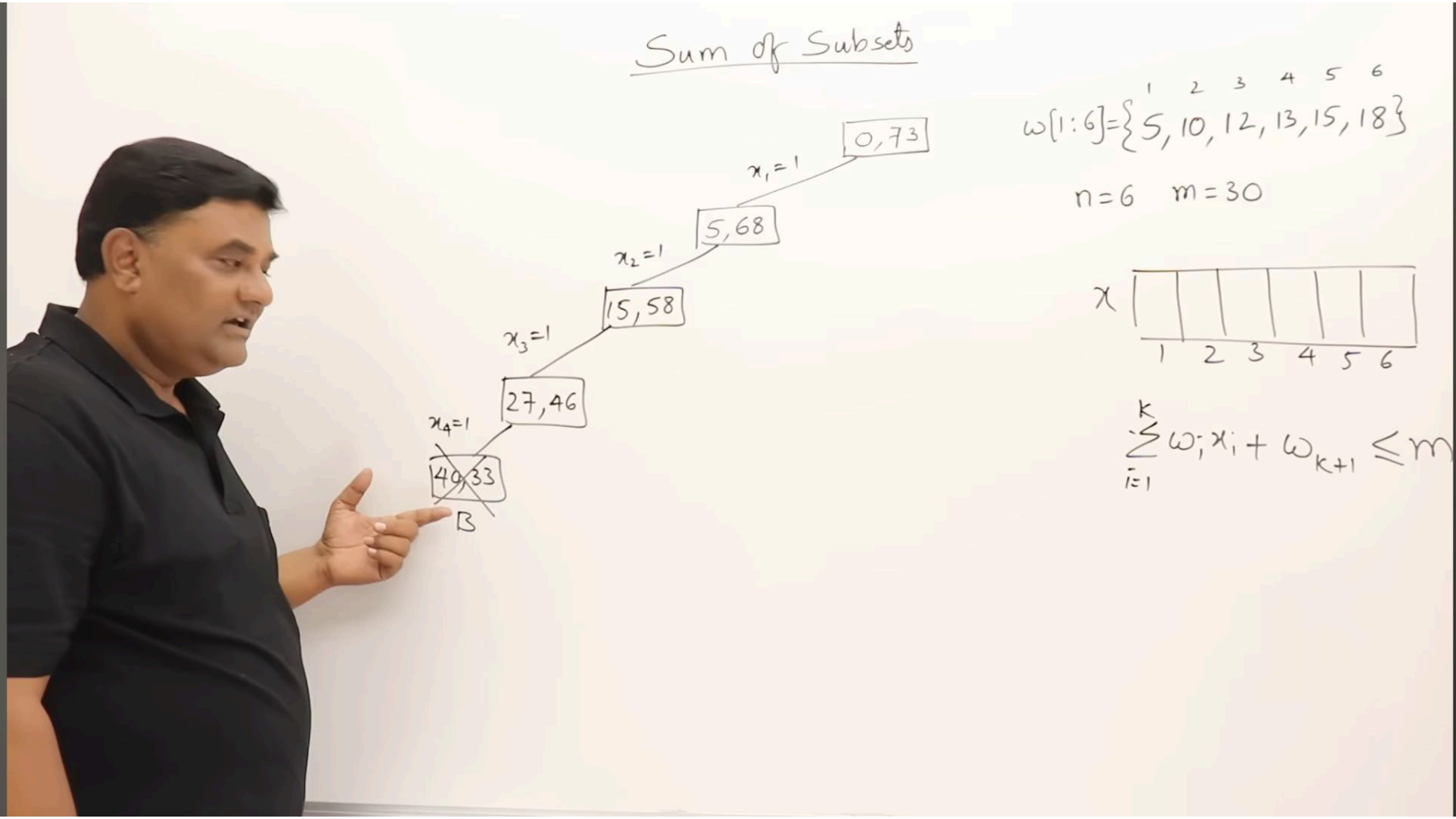


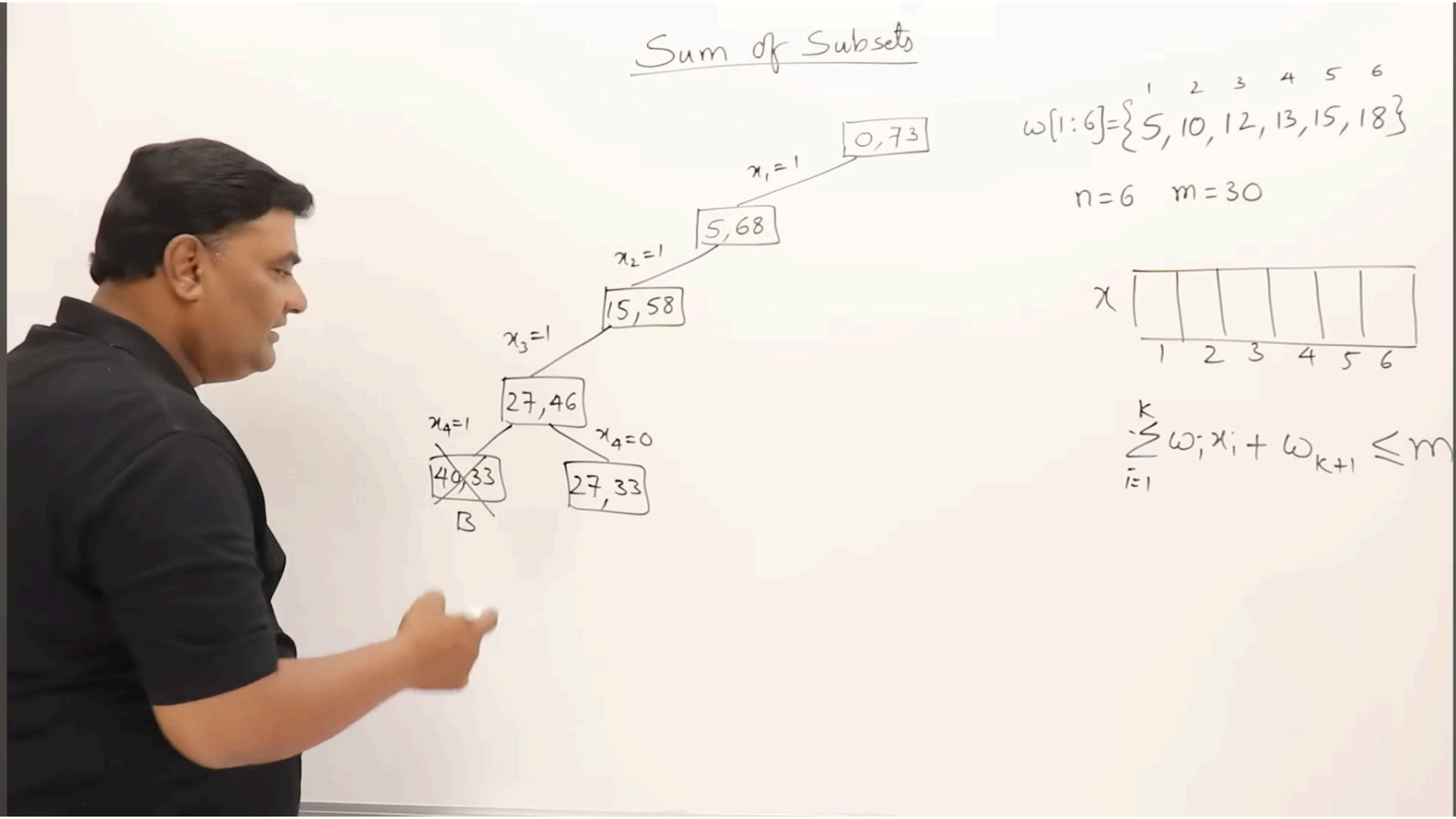
$x_i = 0/1$

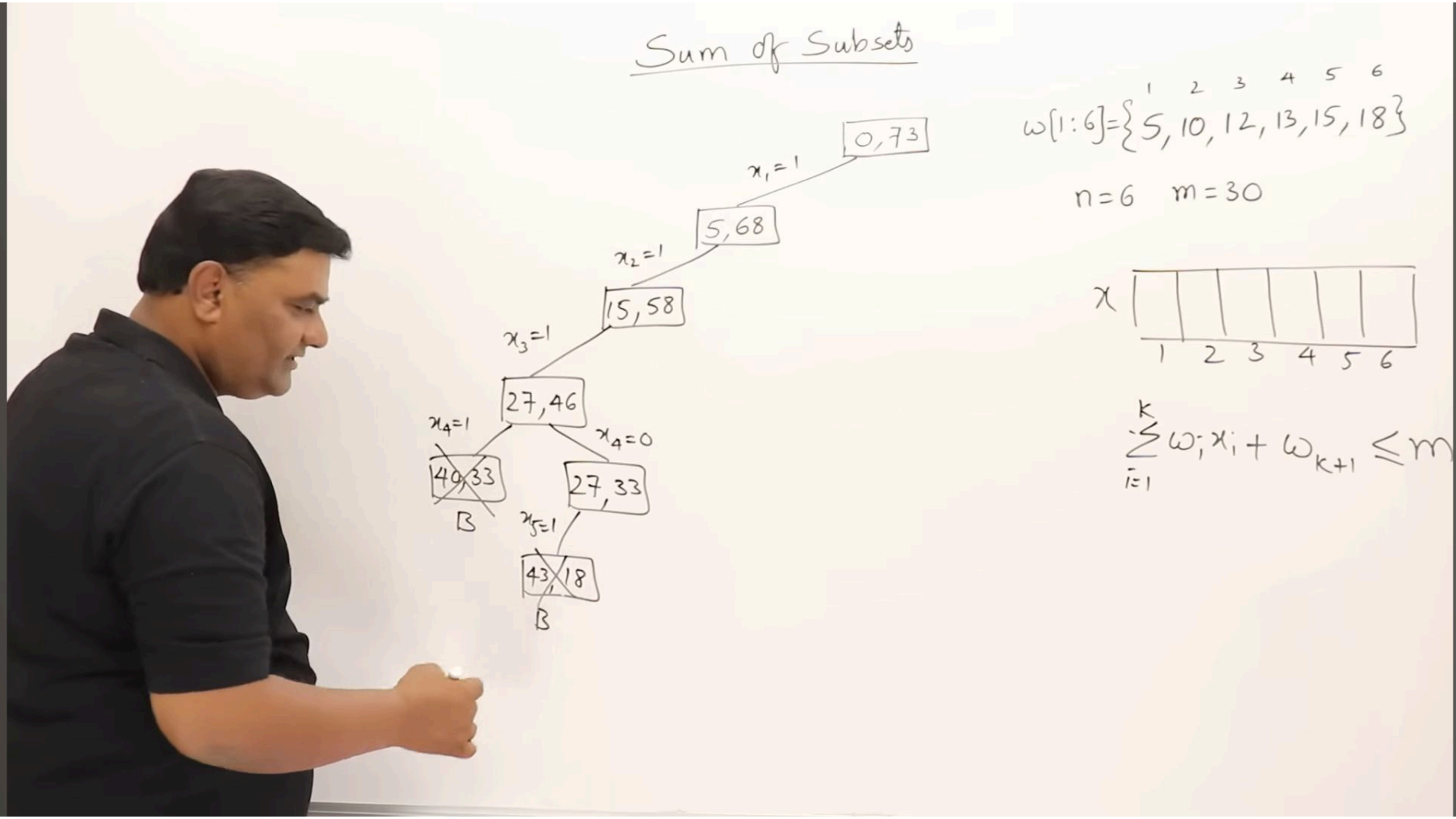

Sum of Subsets
$x_1=1$
$x_1=0$
$x_2=1$
$x_2=0$
$x_2=1$
$x_2=0$
$x_3=1$
$x_3=0$
$w[1:6]=\{5, 10, 12, 13, 15, 18\}$
1 2 3 4 5 6
$n=6 \quad m=30$
x
1 2 3 4 5 6
$x_i = 0/1$
$2^6 \Rightarrow 2^n$

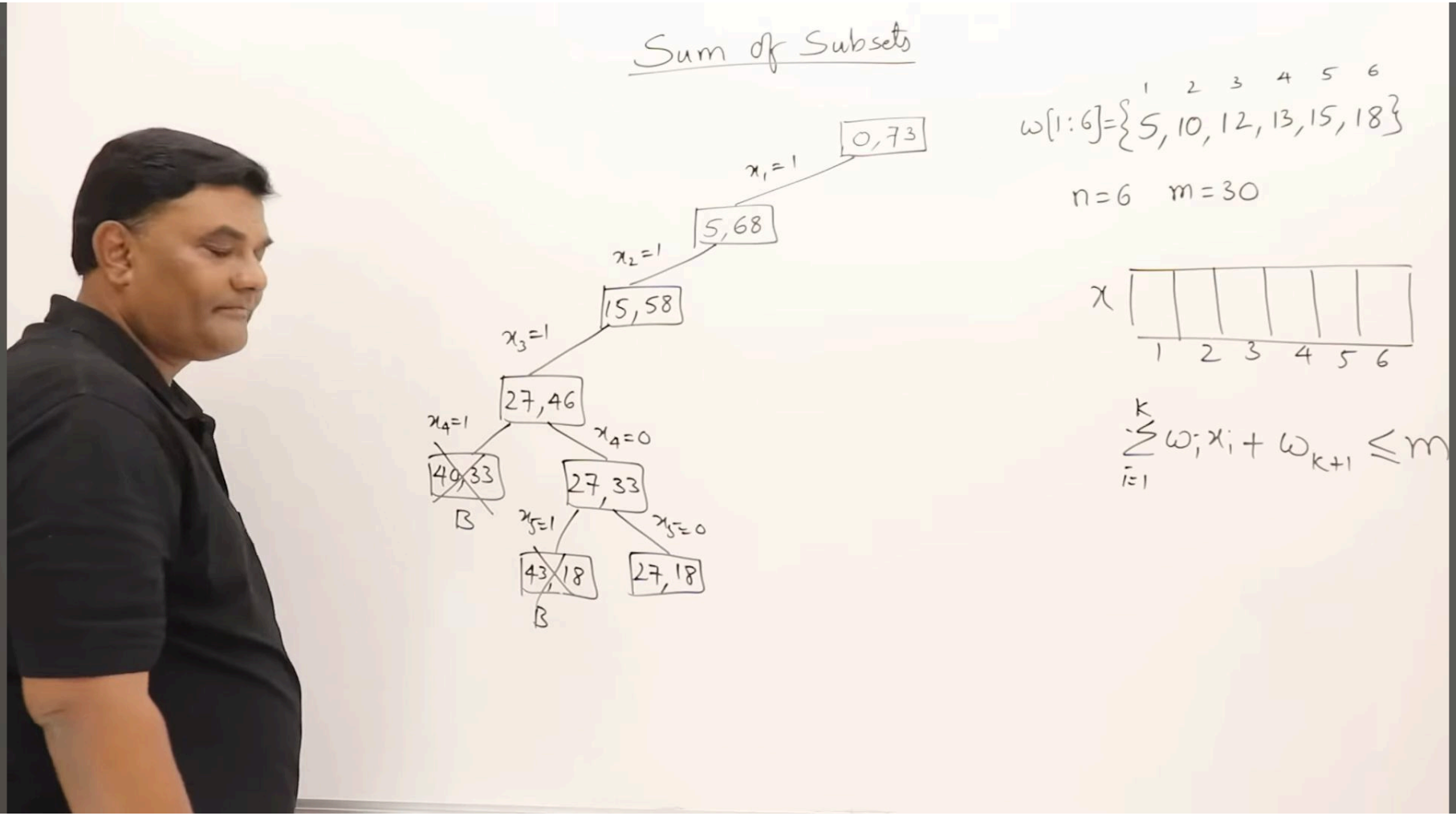

Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
40,33
w[1:6]={5, 10, 12, 13, 15, 18}
1 2 3 4 5 6
n=6 m=30
x
1 2 3 4 5 6

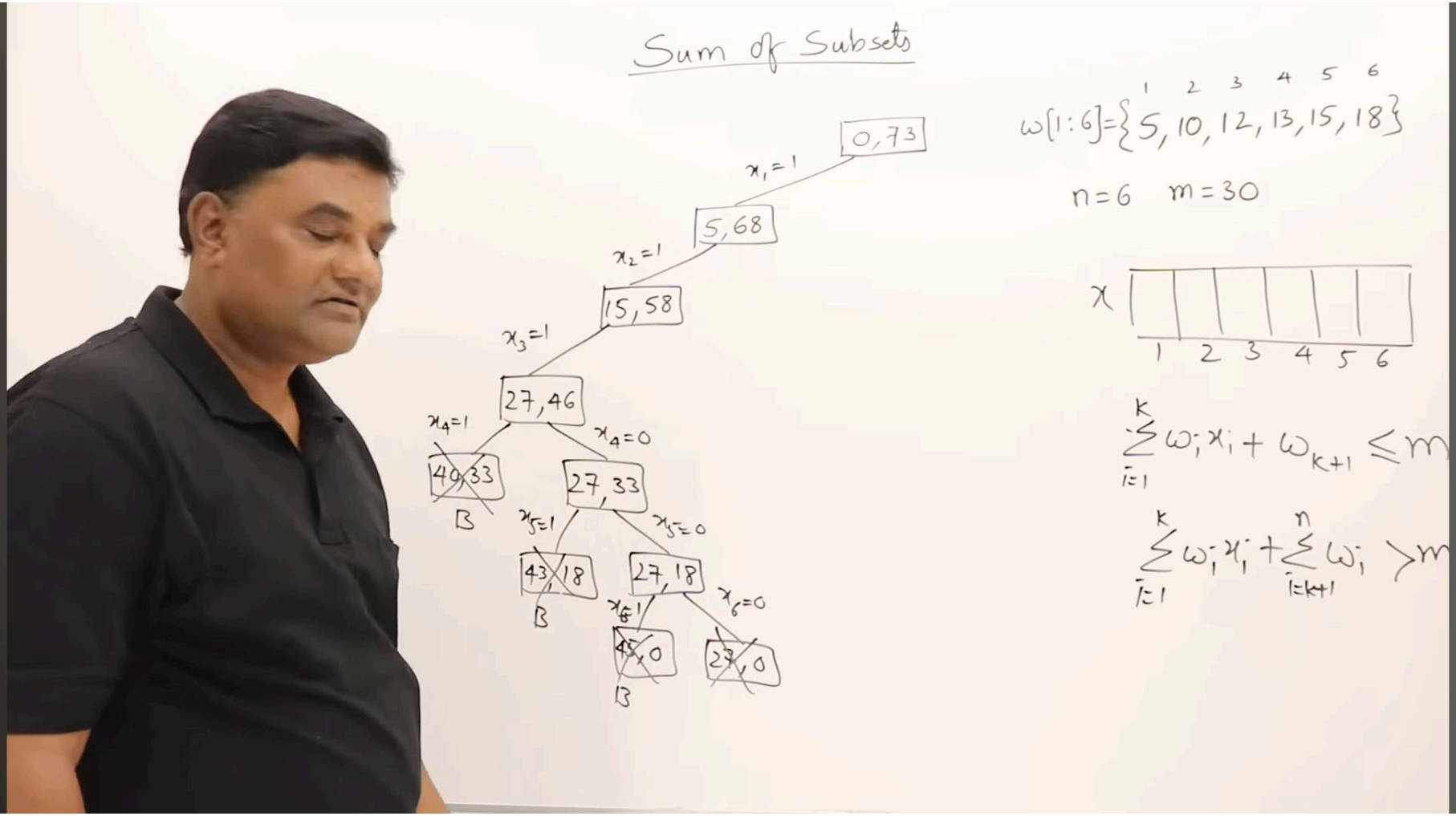

Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
40,33
B
w[1:6]={5, 10, 12, 13, 15, 18}
1 2 3 4 5 6
n=6 m=30
x
1 2 3 4 5 6

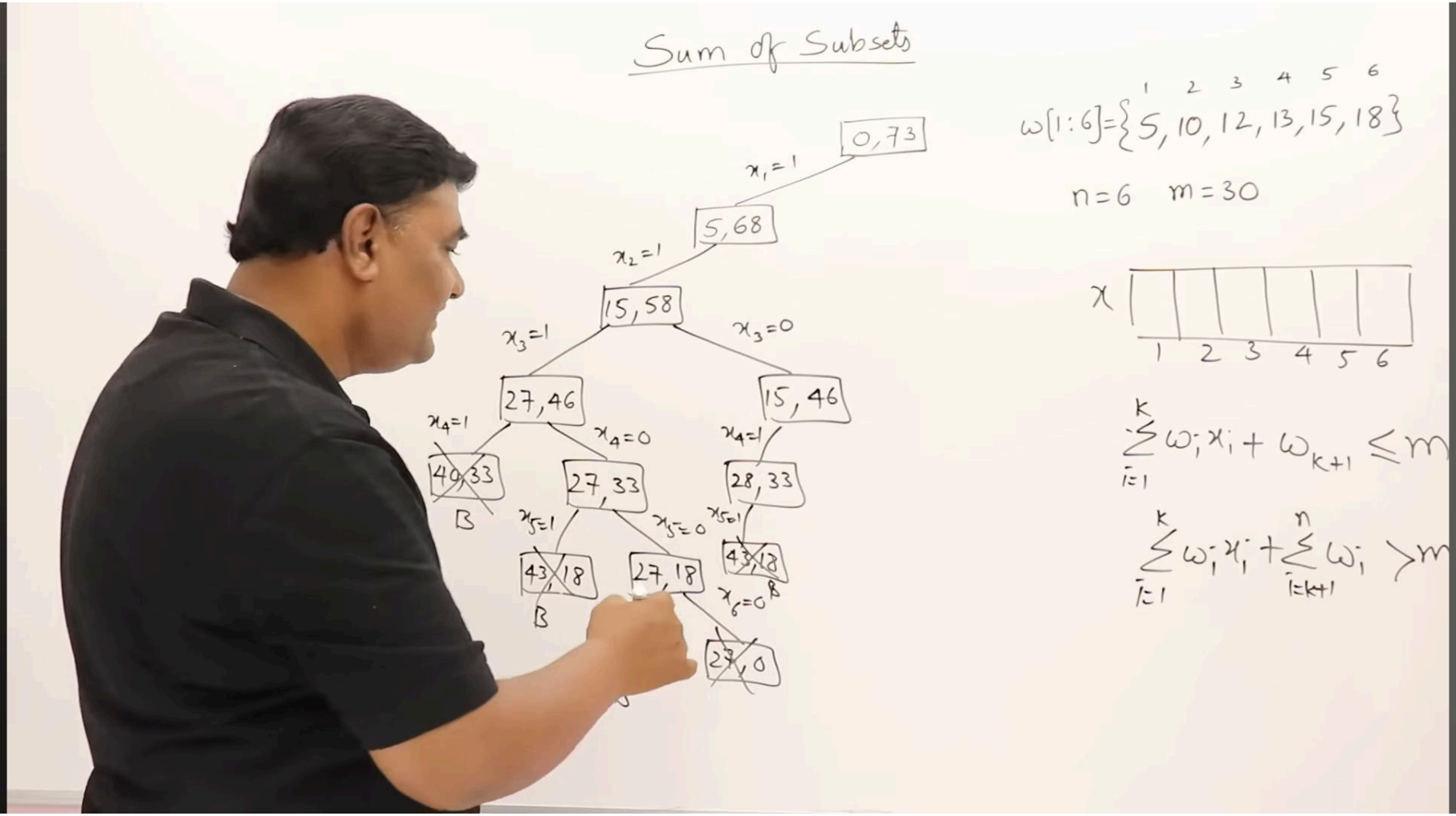

Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
40,33
B
$w[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
n=6 m=30
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$

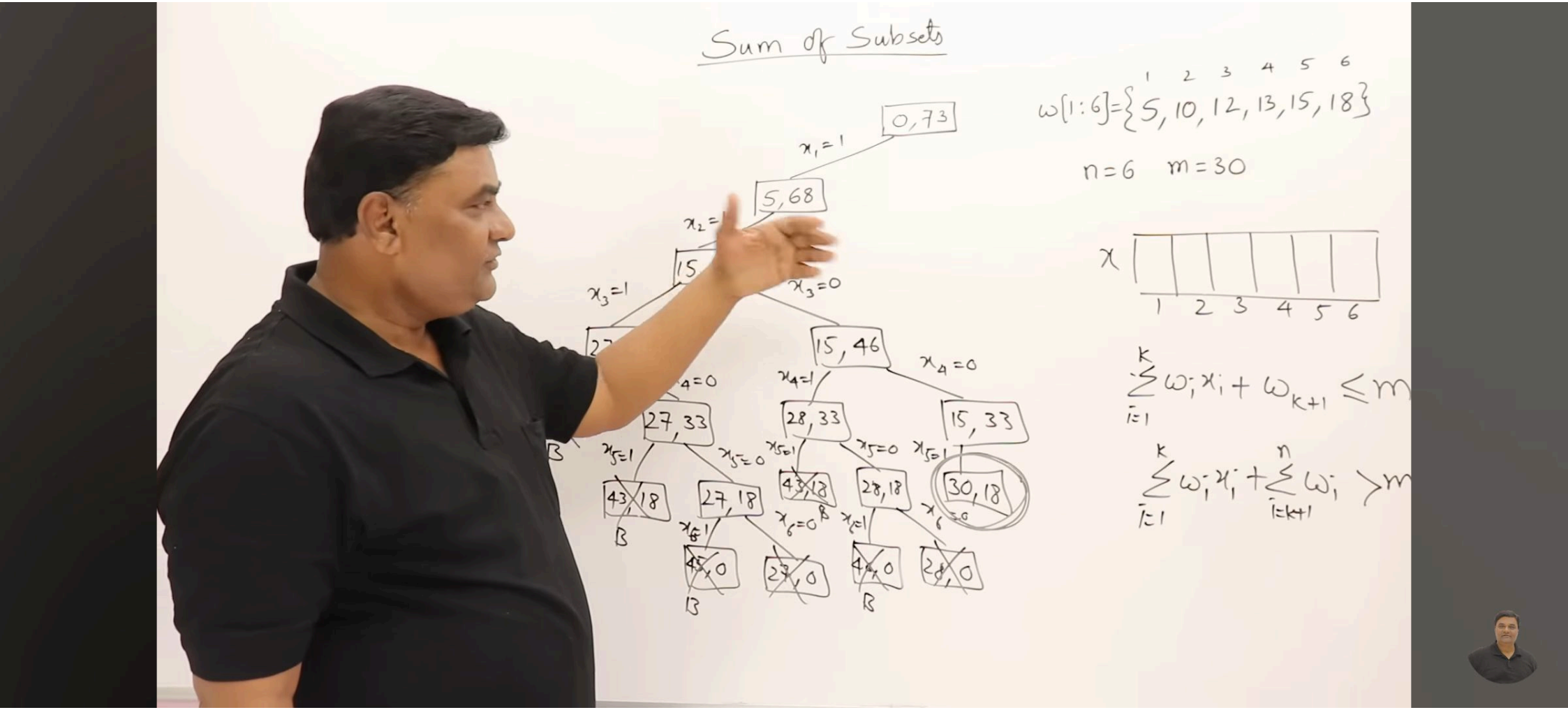

Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
$x_4=0$
40,33
27,33
B
$w[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
n=6 m=30
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$

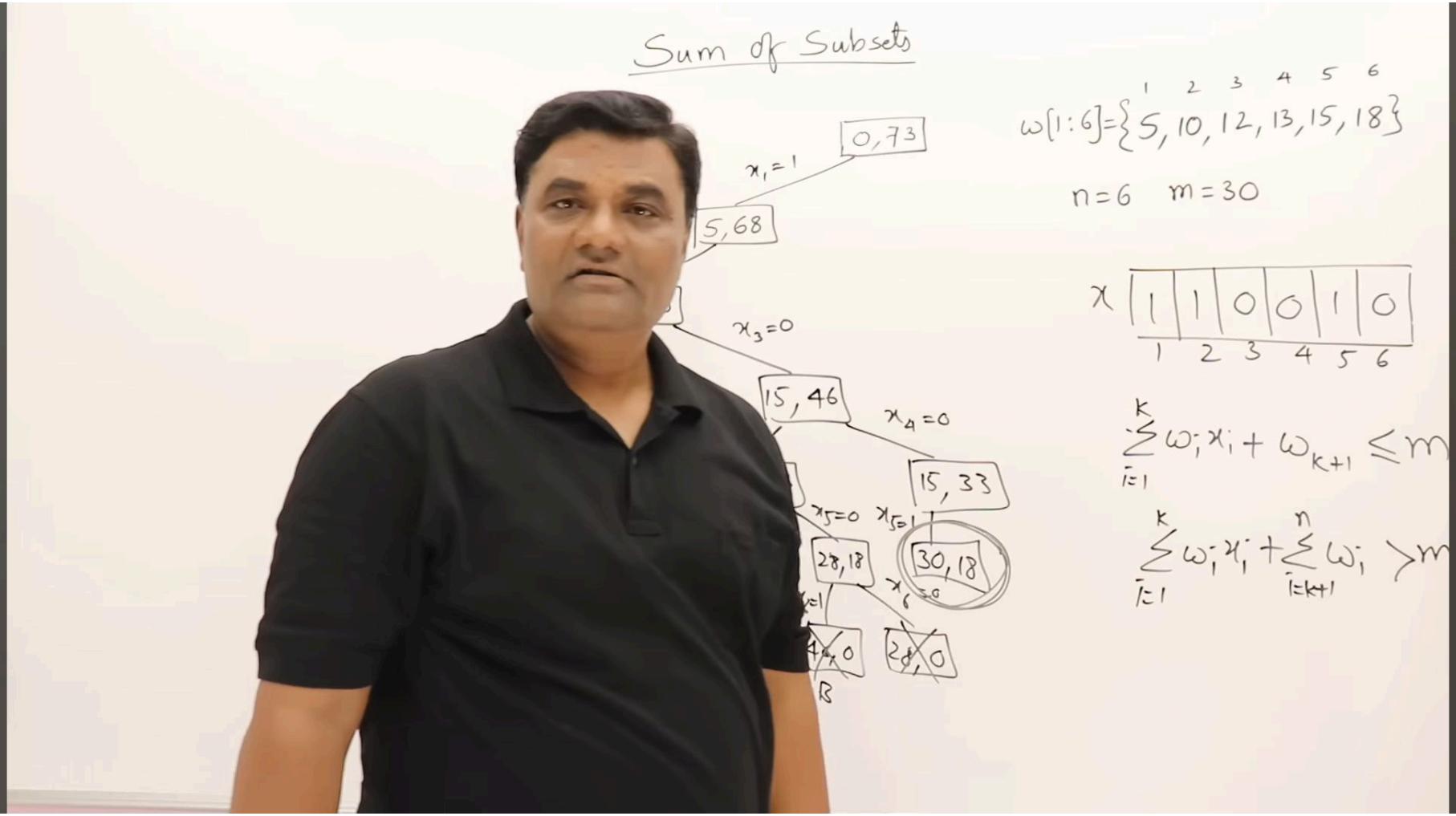

Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
40,33
B
$x_4=0$
27,33
$x_5=1$
43,18
B
$w[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
$n=6 \quad m=30$
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$

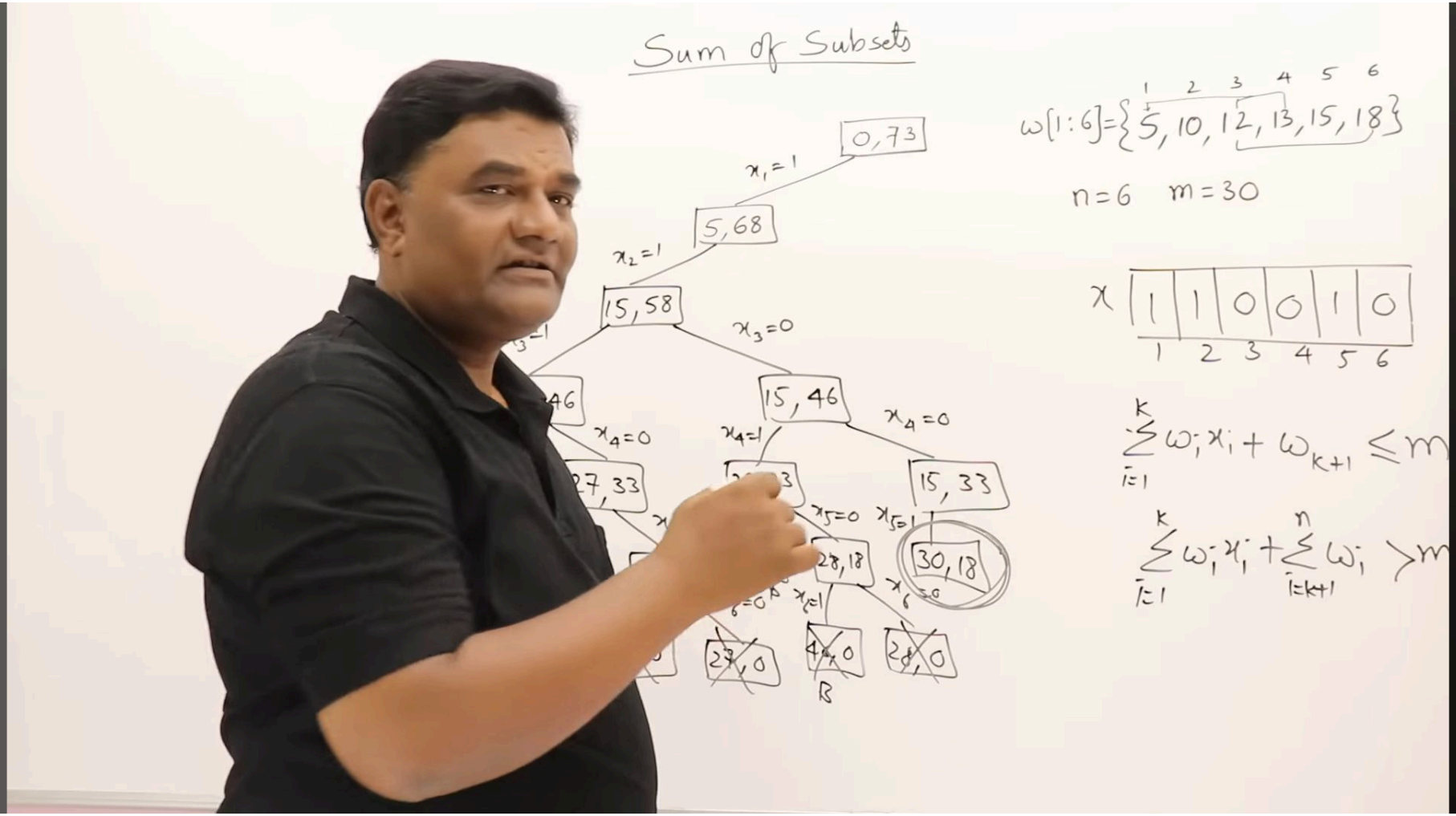

Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
$x_4=0$
40,33
B
27,33
$x_5=1$
$x_5=0$
43,18
B
27,18
$w[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
$n=6 \quad m=30$
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$


Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
27,46
$x_4=1$
40,33
B
$x_4=0$
27,33
$x_5=1$
43,18
B
$x_5=0$
27,18
$x_6=1$
45,0
B
$x_6=0$
23,0
$w[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
$n=6 \quad m=30$
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$
$\sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} w_i > m$


Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=1$
$x_3=0$
27,46
15,46
$x_4=1$
$x_4=0$
$x_4=1$
40,33
B
27,33
28,33
$x_5=1$
$x_5=0$
$x_5=1$
43,18
B
27,18
43,18
B
$x_6=0$
23,0
$w[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
$n=6$ $m=30$
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$
$\sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} w_i > m$


Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15
$x_3=1$
$x_3=0$
15,46
$x_4=0$
$x_4=1$
$x_4=0$
27,33
28,33
15,33
$x_5=1$
$x_5=0$
$x_5=1$
$x_5=0$
$x_5=1$
43,18
27,18
43,18
28,18
30,18
B
$x_6=1$
$x_6=0$
$x_6=1$
$x_6=0$
45,0
27,0
44,0
28,0
B
B
B
1 2 3 4 5 6
$w[1:6]=\{5, 10, 12, 13, 15, 18\}$
$n=6 \quad m=30$
x
1 2 3 4 5 6
$\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m$
$\sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} w_i > m$


Sum of Subsets
0,73
$x_1=1$
5,68
$x_3=0$
15,46
$x_4=0$
15,33
$x_5=0$
$x_5=1$
28,18
30,18
$x_6=0$
$\omega[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
$n=6 \quad m=30$
x | 1 | 1 | 0 | 0 | 1 | 0 |
1 2 3 4 5 6
$\sum_{i=1}^{k} \omega_i x_i + \omega_{k+1} \leq m$
$\sum_{i=1}^{k} \omega_i x_i + \sum_{i=k+1}^{n} \omega_i > m$


Sum of Subsets
0,73
$x_1=1$
5,68
$x_2=1$
15,58
$x_3=0$
15,46
$x_4=0$
15,33
$x_5=0$
$x_5=1$
28,18
30,18
$x_6=0$
$x_6=1$
$\omega[1:6]=\{5,10,12,13,15,18\}$
1 2 3 4 5 6
$n=6 \quad m=30$
x | 1 | 1 | 0 | 0 | 1 | 0 |
1 2 3 4 5 6
$\sum_{i=1}^{k} \omega_i x_i + \omega_{k+1} \leq m$
$\sum_{i=1}^{k} \omega_i x_i + \sum_{i=k+1}^{n} \omega_i > m$