Tutorial 5

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Q1)i)

In [6]:
```
var('x')
var('n')
assume(n,'integer')

f(x)= ((pi - x)/2)^2

L = pi
an=(1/L)*integrate(f*cos(n*pi*x/L),x,0,2*L)
a0=(1/L)*integrate(f,x,0,2*L)
bn=(1/L)*integrate(f*sin(n*pi*x/L),x,0,2*L)
s_10 =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,10)

show("The Value of a0 is : ", a0)
show("The Value of an is : ", an)
show("The Value of bn is :  ", bn)
show("Fourier series of 10 terms is : ",s_10)

plot(f,0,2*L,color = "darkblue",legend_label="f(x)") + plot(s_10,0,2*L,color = "red",legend_label="Fourier Series(10terms)")
```

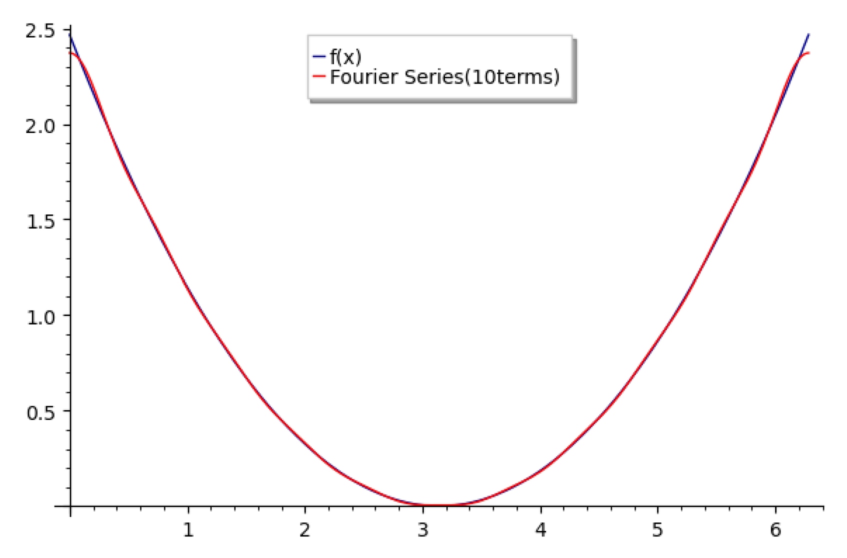
The Value of a0 is : $\frac{1}{6}\pi^2$

The Value of an is : $\frac{1}{n^2}$

The Value of bn is : 0

Fourier series of 10 terms is : $\frac{1}{12}\pi^2 + \frac{1}{100}\cos(10x) + \frac{1}{81}\cos(9x) + \frac{1}{64}\cos(8x) + \frac{1}{49}\cos(7x) + \frac{1}{36}\cos(6x) + \frac{1}{25}\cos(5x) + \frac{1}{16}\cos(4x) + \frac{1}{9}\cos(3x) + \frac{1}{4}\cos(2x) + \cos(x)$

Out[6]:



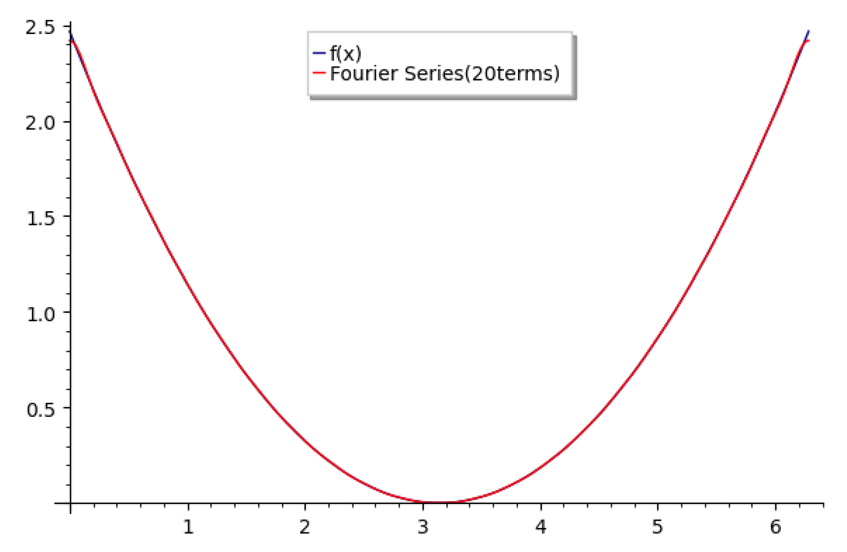
In [7]:
```
s_20 =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,20)

show("\n Fourier series of 20 terms is : \n",s_20)

plot(f,0,2*L,color = "darkblue",legend_label="f(x)") + plot(s_20,0,2*L,color = "red",legend_label="Fourier Series(20terms)")
```

Fourier series of 20 terms is : $\frac{1}{12}\pi^2 + \frac{1}{400}\cos(20x) + \frac{1}{361}\cos(19x) + \frac{1}{324}\cos(18x) + \frac{1}{289}\cos(17x) + \frac{1}{256}\cos(16x) + \frac{1}{225}\cos(15x) + \frac{1}{196}\cos(14x) + \frac{1}{169}\cos(13x) + \frac{1}{144}\cos(12x) + \frac{1}{121}\cos(11x) + \frac{1}{100}\cos(10x) + \frac{1}{81}\cos(9x) + \frac{1}{64}\cos(8x) + \frac{1}{49}\cos(7x) + \frac{1}{36}\cos(6x) + \frac{1}{25}\cos(5x) + \frac{1}{16}\cos(4x) + \frac{1}{9}\cos(3x) + \frac{1}{4}\cos(2x) + \cos(x)$

Out[7]:



Q1)ii)

In [8]:
```
f(x)= x^5

L = pi

an=(1/L)*integrate(f*cos(n*pi*x/L),x,-L,L)
a0=(1/L)*integrate(f,x,-L,L)
bn=(1/L)*integrate(f*sin(n*pi*x/L),x,-L,L)
s_5 =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,5)

show("Value of a0 is: ", a0)
show("Value of an is: ", an)
show("Value of bn is: ", bn)
show("Fourier series of 5 terms: ",s_5)

print("\n")

plot(f,-L,L,color = "darkblue",legend_label="f(x)") + plot(s_5,-L,L,color = "red",legend_label="Fourier Series(5terms)")
```

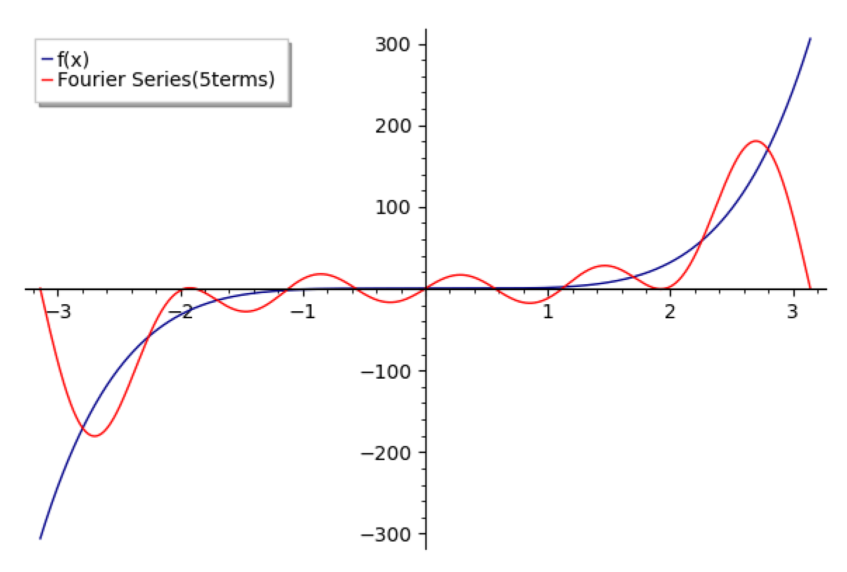
Value of a0 is: 0

Value of an is: 0

Value of bn is: $-\frac{2\left(120\pi + \pi^5 n^4 - 20\pi^3 n^2\right)(-1)^n}{\pi n^5}$

Fourier series of 5 terms: $\frac{2}{625}\left(125\pi^4 - 100\pi^2 + 24\right)\sin(5x) - \frac{1}{64}\left(32\pi^4 - 40\pi^2 + 15\right)\sin(4x) + \frac{2}{81}\left(27\pi^4 - 60\pi^2 + 40\right)\sin(3x) - \frac{1}{2}\left(2\pi^4 - 10\pi^2 + 15\right)\sin(2x) + 2\left(\pi^4 - 20\pi^2 + 120\right)\sin(x)$

Out[8]:



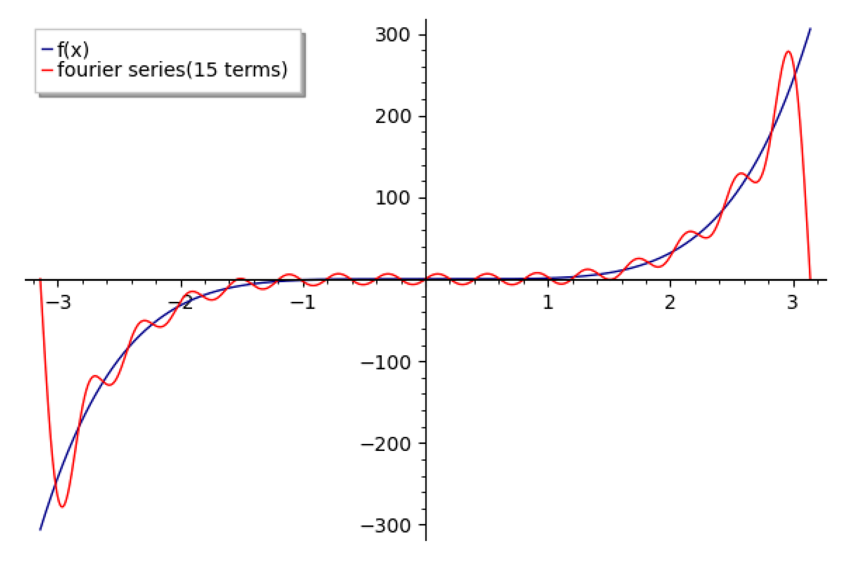
In [9]:
```
s_15 =a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)

show("Fourier series of 15 terms is : ",s_15)

plot(f,-L,L,color = "darkblue",legend_label="f(x)") + plot(s_15,-L,L,color = "red",legend_label="fourier series(15 terms)")
```

Fourier series of 15 terms is : $\frac{2}{50625}\left(3375\pi^4 - 300\pi^2 + 8\right)\sin(15x) - \frac{1}{33614}\left(4802\pi^4 - 490\pi^2 + 15\right)\sin(14x) + \frac{2}{371293}\left(28561\pi^4 - 3380\pi^2 + 120\right)\sin(13x) - \frac{1}{5184}\left(864\pi^4 - 120\pi^2 + 5\right)\sin(12x) + \frac{2}{161051}\left(14641\pi^4 - 2420\pi^2 + 120\right)\sin(11x) - \frac{1}{1250}\left(250\pi^4 - 50\pi^2 + 3\right)\sin(10x) + \frac{2}{19683}\left(2187\pi^4 - 540\pi^2 + 40\right)\sin(9x) - \frac{1}{2048}\left(512\pi^4 - 160\pi^2 + 15\right)\sin(8x) + \frac{2}{16807}\left(2401\pi^4 - 980\pi^2 + 120\right)\sin(7x) - \frac{1}{162}\left(54\pi^4 - 30\pi^2 + 5\right)\sin(6x) + \frac{2}{625}\left(125\pi^4 - 100\pi^2 + 24\right)\sin(5x) - \frac{1}{64}\left(32\pi^4 - 40\pi^2 + 15\right)\sin(4x) + \frac{2}{81}\left(27\pi^4 - 60\pi^2 + 40\right)\sin(3x) - \frac{1}{2}\left(2\pi^4 - 10\pi^2 + 15\right)\sin(2x) + 2\left(\pi^4 - 20\pi^2 + 120\right)\sin(x)$

Out[9]:



Q2)

In [10]:
```
f(x) = x

L = 2

an=(2/L)*integrate(f(x)*cos(n*pi*x/L),x,0,L)
a0=(2/L)*integrate(f(x),x,0,L)
s =a0/2+sum(an*cos(n*pi*x/L),n,1,20)

show("Value of a0 is : ", a0)
show("Value of an is : ", an)
show("Cosine series for n=20 is : \n",s)

plot(f,0,L,color = "darkblue",legend_label="f(x)") + plot(s,0,L,color = "red",legend_label="Half-Range Cosine Series(n=20) ")
```

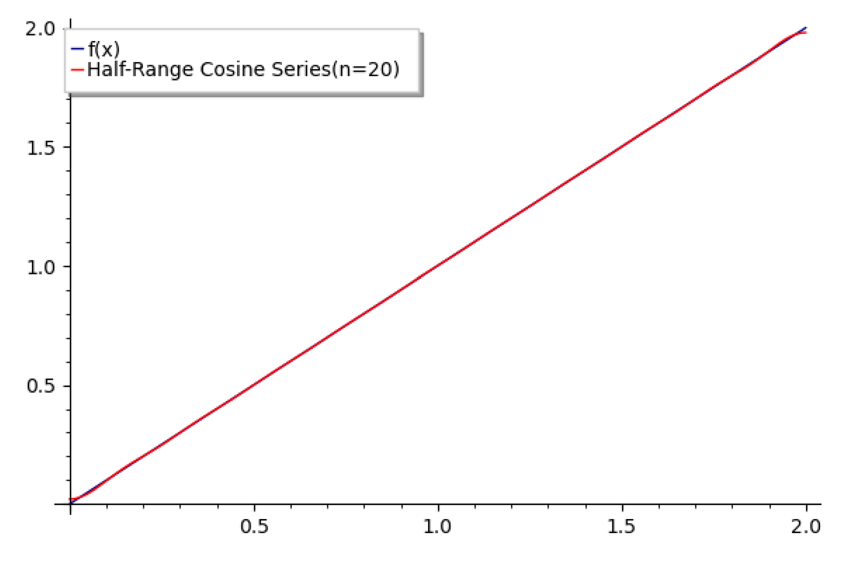
Value of a0 is : 2

Value of an is : $\frac{4(-1)^n}{\pi^2 n^2} - \frac{4}{\pi^2 n^2}$

Cosine series for n=20 is :

$$-\frac{8\left(586396035225\cos\left(\frac{19}{2}\pi x\right) + 732487781025\cos\left(\frac{17}{2}\pi x\right) + 940839860961\cos\left(\frac{15}{2}\pi x\right) + 1252597448025\cos\left(\frac{13}{2}\pi x\right) + 1749495609225\cos\left(\frac{11}{2}\pi x\right) + 2613444058225\cos\left(\frac{9}{2}\pi x\right) + 4320183035025\cos\left(\frac{7}{2}\pi x\right) + 8467558748649\cos\left(\frac{5}{2}\pi x\right) + 23520996524025\cos\left(\frac{3}{2}\pi x\right) + 211688968716225\cos\left(\frac{1}{2}\pi x\right)\right)}{211688968716225\pi^2} + 1$$

Out[10]:



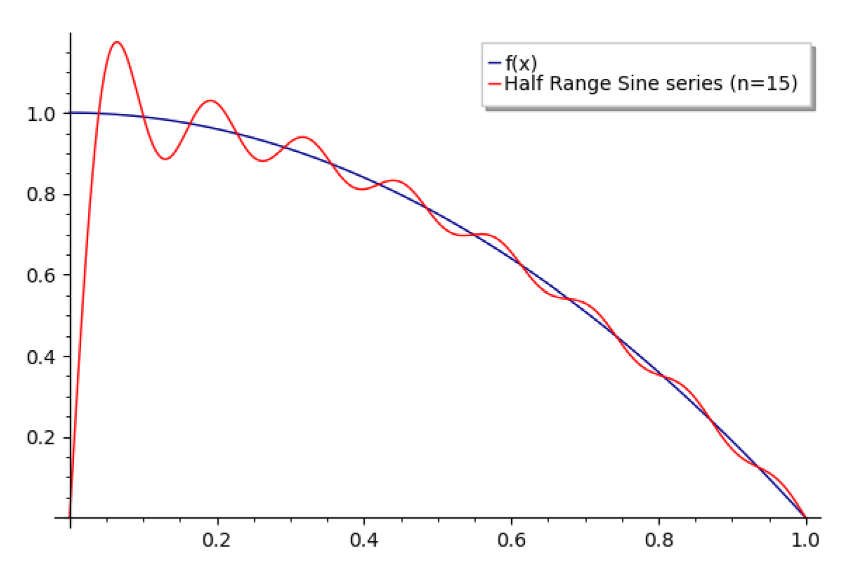
Q3)

In [11]:
```
f(x) = 1 - x^2
L = 1
bn=(2/L)*integrate(f(x)*sin(n*pi*x/L),x,0,L)
s =sum(bn*sin(n*pi*x/L),n,1,15)
show("value of bn is: ", bn)
show("\nThe Half-Range sine series for n=15 is: \n",s)
print("\n")
plot(f,0,L,color = "darkblue",legend_label="f(x)") + plot(s,0,L,color = "red",legend_label="Half Range Sine series (n=15)")
```

value of bn is: $\frac{2\left(\pi^2 n^2 + 2\right)}{\pi^3 n^3} - \frac{4(-1)^n}{\pi^3 n^3}$

The Half-Range sine series for n=15 is: $\frac{52227799123500\pi^2\sin(14\pi x) + 60932432310750\pi^2\sin(12\pi x) + 73118918772900\pi^2 \text{si} \ldots + 121864864621500\pi^2\sin(6\pi x) + 182797296932250\pi^2\sin(4\pi x) + 365594593864500\pi \ldots (15\pi x) + 332812557000\left(169\pi^2 + 4\right)\sin(13\pi x) + 549353259000\left(121\pi^2 + 4\right)\sin(11\pi \ldots + 2131746903000\left(49\pi^2 + 4\right)\sin(7\pi x) + 5849513501832\left(25\pi^2 + 4\right)\sin(5\pi x) + 27081 \ldots + 731189187729000\left(\pi^2 + 4\right)\sin(\pi x)}{365594593864500\pi^3}$

Out[11]:



Q4)

In [12]:
```
f(x) = x * (pi - x)

L = pi

an = (1/L) * integrate(f(x)*cos(n*pi*x/L), x, -L, L)
a0 = (1/L) * integrate(f(x), x, -L, L)
bn = (1/L) * integrate(f(x)*sin(n*pi*x/L), x, -L, L)
s = a0/2 + sum(an*cos(n*pi*x/L) + bn*sin(n*pi*x/L), n, 1, 15)
a10 = (1/L) * integrate(f(x)*cos(10*pi*x/L), x, -L, L)
b15 = (1/L) * integrate(f(x)*sin(15*pi*x/L), x, -L, L)

show("Value of a0 is: ", a0)
show("Value of an is: ", an)
show("Value of bn is: ", bn)
show("\nFourier series (n=15) is : ", s)
show("Value of a10 is: ", a10)
show("Value of b15 is: ", b15)
```

Value of a0 is: $-\frac{2}{3}\pi^2$

Value of an is: $-\frac{4(-1)^n}{n^2}$

Value of bn is: $-\frac{2\left(\frac{(\pi^2 n^2 - 1)(-1)^n}{n^3} + \frac{(-1)^n}{n^3}\right)}{\pi}$

Fourier series (n=15) is : $-\frac{1}{3}\pi^2 + \frac{2}{15}\pi\sin(15x) - \frac{1}{7}\pi\sin(14x) + \frac{2}{13}\pi\sin(13x) - \frac{1}{6}\pi\sin(12x) + \frac{2}{11}\pi\sin(11x) - \frac{1}{5}\pi\sin(10x) + \frac{2}{9}\pi\sin(9x) - \frac{1}{4}\pi\sin(8x) + \frac{2}{7}\pi\sin(7x) - \frac{1}{3}\pi\sin(6x) + \frac{2}{5}\pi\sin(5x) - \frac{1}{2}\pi\sin(4x) + \frac{2}{3}\pi\sin(3x) - \pi\sin(2x) + 2\pi\sin(x) + \frac{4}{225}\cos(15x) - \frac{1}{49}\cos(14x) + \frac{4}{169}\cos(13x) - \frac{1}{36}\cos(12x) + \frac{4}{121}\cos(11x) - \frac{1}{25}\cos(10x) + \frac{4}{81}\cos(9x) - \frac{1}{16}\cos(8x) + \frac{4}{49}\cos(7x) - \frac{1}{9}\cos(6x) + \frac{4}{25}\cos(5x) - \frac{1}{4}\cos(4x) + \frac{4}{9}\cos(3x) - \cos(2x) + 4\cos(x)$

Value of a10 is: $-\frac{1}{25}$

Value of b15 is: $\frac{2}{15}\pi$