Euler's Totient Function

Cryptography and Network Security

Euler's Totient Function
(Phi Function)

Euler's Totient Function

- Denoted as $\Phi(n)$.
- $\Phi(n)$ = Number of positive integers less than 'n' that are relatively prime to n.

Euler's Totient Function

Example 1: Find $\Phi(5)$.

Solution:

Here $n=5$.

Numbers less than 5 are 1, 2, 3 and 4.

GCD	Relatively Prime?
GCD (1, 5) = 1	✓
GCD (2, 5) = 1	✓
GCD (3, 5) = 1	✓
GCD (4, 5) = 1	✓

$\therefore \Phi(5) = 4.$

Euler's Totient Function

Example 2: Find $\Phi(11)$.

Solution:

Here $n=11$.

Numbers less than 11 are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

GCD	Relatively Prime?
GCD (1, 11) = 1	✓
GCD (2, 11) = 1	✓
GCD (3, 11) = 1	✓
GCD (4, 11) = 1	✓
GCD (5, 11) = 1	✓

GCD	Relatively Prime?
GCD (6, 11) = 1	✓
GCD (7, 11) = 1	✓
GCD (8, 11) = 1	✓
GCD (9, 11) = 1	✓
GCD (10, 11) = 1	✓

$\therefore \Phi(11) = 10.$

Euler's Totient Function

Example 3: Find $\Phi(8)$.

Solution:

Here n=8.

Numbers less than 8 are 1, 2, 3, 4, 5, 6, and 7.

GCD	Relatively Prime?
GCD (1, 8) = 1	✓
GCD (2, 8) = 2	✗
GCD (3, 8) = 1	✓
GCD (4, 8) = 4	✗

GCD	Relatively Prime?
GCD (5, 8) = 1	✓
GCD (6, 8) = 2	✗
GCD (7, 8) = 1	✓

$\therefore \Phi(8) = 4.$

Euler's Totient Function

	Criteria of 'n'	Formula
$\Phi(n)$	'n' is prime.	$\Phi(n) = (n-1)$
	$n = p \times q$. 'p' and 'q' are primes.	$\Phi(n) = (p-1) \times (q-1)$
	$n = a \times b$. Either 'a' or 'b' is composite. Both 'a' and 'b' are composite.	$\Phi(n) = n \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \dots$ where $p_1, p_2, \dots$ are distinct primes.

Euler's Totient Function

Example 1: Find $\Phi(5)$.

Solution:

Here n=5.

'n' is a prime number.

$\Phi(n) = (n-1)$

$\Phi(5) = (5-1)$

$\Phi(5) = 4$

So, there are 4 numbers that are lesser than 5 and relatively prime to 5.

Euler's Totient Function

Example 2: Find $\Phi(31)$.

Solution:

Here $n=31$.

'n' is a prime number.

$\Phi(n) \quad = (n-1)$

$\Phi(31) \quad = (31-1)$

$\Phi(31) \quad = 30$

So, there are 30 numbers that are lesser than 31 and relatively prime to 31.

Euler's Totient Function

Example 3: Find $\Phi(35)$.

Solution:

Here n=35.

'n' is a product of two prime numbers 5 and 7.

Let us assign p=5 and q=7.

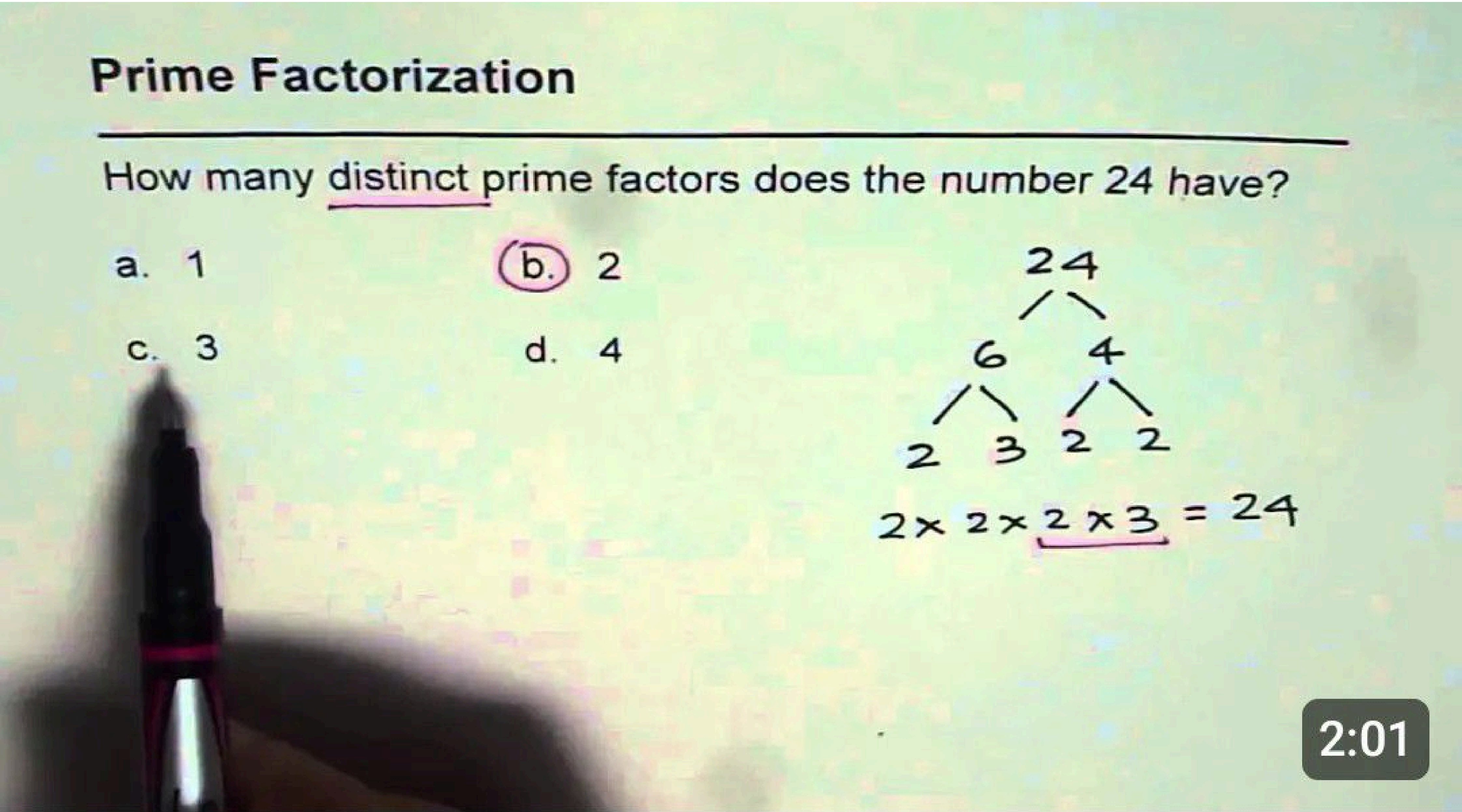
$\Phi(n) \quad = (p-1) \times (q-1)$

$\Phi(35) \quad = (5-1) \times (7-1)$

$\Phi(35) \quad = 4 \times 6$

$\Phi(35) \quad = 24$

So, there are 24 numbers that are lesser than 35 and relatively prime to 35.



01 Distinct Prime Factors for 24

Euler's Totient Function

Example 5: Find $\Phi(7000)$.

Solution:

Here $n = 7000 = 2^3 \times 5^3 \times 7^1$

Distinct prime factors are 2, 5 and 7.

$$\Phi(n) = n \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\left(1 - \frac{1}{p_3}\right)\ldots.$$

$$\Phi(7000) = 7000 \times \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right)$$

$$\Phi(7000) = 7000 \times \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)$$

$$\Phi(7000) = 2400$$

Find phi(999)
Using Eulers totient function

To find **φ(999)** using **Euler's Totient Function**, we follow these steps:

Step 1: Prime Factorization of 999

$999 = 3 \times 333$

$= 3 \times 3 \times 111$

$= 3 \times 3 \times 3 \times 37$

$= \mathbf{3^3 \times 37}$

Step 2: Euler's Totient Function Formula

If:

$\mathbf{n = p_1\text{^}a \times p_2\text{^}b \times \ldots \times p_k\text{^}z,}$

1. φ(369)

Step 1: Prime factorization of 369

$369 = 3 \times 123$

$= 3 \times 3 \times 41$

$= \mathbf{3^2 \times 41}$

Step 2: Apply Euler's formula

$\varphi(369) = 369 \times (1 - 1/3) \times (1 - 1/41)$

$= 369 \times (2/3) \times (40/41)$

Step 3: Calculate

- $369 \times (2/3) = 246$
- $246 \times (40/41) =$ **9840 / 41 = 240**

So, φ(369) = 240

2. φ(372)

Step 1: Prime factorization of 372

$372 = 2 \times 186$

$= 2 \times 2 \times 93$

$= 2^2 \times 3 \times 31$

$= \mathbf{2^2 \times 3 \times 31}$

Step 2: Apply Euler's formula

$\varphi(372) = 372 \times (1 - 1/2) \times (1 - 1/3) \times (1 - 1/31)$

$= 372 \times (1/2) \times (2/3) \times (30/31)$

Step 3: Calculate step by step

- $372 \times (1/2) = 186$
- $186 \times (2/3) = 124$
- $124 \times (30/31) = \mathbf{3720 / 31 = 120}$

So, φ(372) = 120