Tutorial Number: 6

Probability, Statistics, and Optimization Techniques

Module 3: Estimation and Sampling

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Batch: SY-IT(B3)

```
# Import necessary libraries
import math
import numpy as np
from scipy.stats import norm, t
```

Question 1

A random sample of 200 observations has a mean of 6.5 cm.
Can it be a random sample from a population whose mean is 7 cm and variance is 8.5 cm?

```
# Given data
sample_size = 200
sample_mean = 6.5
population_mean = 7
population_variance = 8.5
population_std = math.sqrt(population_variance)

# Compute the z-score
z_calculated = abs((sample_mean - population_mean) / (population_std / math.sqrt(sample_size)))

# Critical z-value for 95% confidence level (two-tailed test)
z_critical = norm.ppf(0.975)

# Print results
print("Calculated Z-score:", round(z_calculated, 4))
print("Critical Z-value:", round(z_critical, 4))

# Hypothesis test conclusion
if z_calculated > z_critical:
    print("Reject the null hypothesis: The sample is unlikely from the given population.")
else:
    print("Fail to reject the null hypothesis: The sample could be from the population.")

print("\nRitesh Jha, Roll No: 16010423076, Q1")
```

```
Calculated Z-score: 2.4254
Critical Z-value: 1.96
Reject the null hypothesis: The sample is unlikely from the given population.

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```

Question 2

An examination of the weight of 9 apples provided the following data:
150, 152, 149, 151, 148, 152, 150, 151, and 153 grams.
Investigate whether the average weight of the apples can be assumed to be 151 grams.

```
# Given data
sample_data = np.array([150, 152, 149, 151, 148, 152, 150, 151, 153])
hypothesized_mean = 151
sample_size = len(sample_data)

# Compute sample statistics
sample_mean = np.mean(sample_data)
sample_std = np.std(sample_data, ddof=1)  # Unbiased standard deviation

# Compute t-score
```

```
t_score = (sample_mean - hypothesized_mean) / (sample_std / np.sqrt(sample_size))

# Critical t-value for 95% confidence level (two-tailed test)
t_critical = t.ppf(0.975, df=sample_size-1)

# Print results
print("Sample Mean:", round(sample_mean, 4))
print("Sample Standard Deviation:", round(sample_std, 4))
print("Computed T-score:", round(t_score, 4))
print("Critical T-value:", round(t_critical, 4))

# Hypothesis test conclusion
if abs(t_score) > t_critical:
    print("Reject the null hypothesis: The mean apple weight is significantly different from 151g.")
else:
    print("Fail to reject the null hypothesis: The mean apple weight can be assumed to be 151g.")

print("\nRitesh Jha, Roll No: 16010423076, Q2")
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```
Sample Mean: 150.6667
Sample Standard Deviation: 1.5811
Computed T-score: -0.6325
Critical T-value: 2.306
Fail to reject the null hypothesis: The mean apple weight can be assumed to be 151g.

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```

Question 3

Twenty students participated in a mathematics competition. They were provided with additional tutoring sessions for a month before participating in another similar competition. The scores of each student in both competitions were recorded.
Test if the scores provided below indicate that the tutoring sessions had a positive impact on the students' performance.

Scores in Competition 1:

85, 78, 72, 90, 93, 65, 79, 81, 70, 75, 87, 69, 82, 74, 86, 88, 91, 73, 77, 84

Scores in Competition 2:

88, 80, 75, 91, 95, 68, 82, 84, 73, 79, 89, 71, 85, 77, 90, 92, 94, 76, 78, 83

```
# Scores before and after tutoring
scores_before = np.array([85, 78, 72, 90, 93, 65, 79, 81, 70, 75, 87, 69, 82, 74, 86, 88, 91, 73, 77, 84])
scores_after = np.array([88, 80, 75, 91, 95, 68, 82, 84, 73, 79, 89, 71, 85, 77, 90, 92, 94, 76, 78, 83])

# Compute differences
score_differences = scores_after - scores_before

# Compute sample statistics
mean_diff = np.mean(score_differences)
std_diff = np.std(score_differences, ddof=1)
n = len(score_differences)

# Compute t-score
t_score = mean_diff / (std_diff / np.sqrt(n))

# Critical t-value for one-tailed test at 95% confidence
t_critical = t.ppf(0.95, df=n-1)

# Print results
print("Mean Difference:", round(mean_diff, 4))
print("Standard Deviation of Differences:", round(std_diff, 4))
print("Computed T-score:", round(t_score, 4))
print("Critical T-value:", round(t_critical, 4))

# Hypothesis test conclusion
if t_score > t_critical:
    print("Reject the null hypothesis: The tutoring sessions significantly improved student performance.")
else:
    print("Fail to reject the null hypothesis: No significant evidence of improvement.")

print("\nRitesh Jha, Roll No: 16010423076, Q3")
```

```
Mean Difference: 2.55
Standard Deviation of Differences: 1.191
Computed T-score: 9.5753
Critical T-value: 1.7291
Reject the null hypothesis: The tutoring sessions significantly improved student performance.

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```

Question 4

Test the significance of the difference between the means of two normal populations with the same standard deviation from the following data.

Sample	Size	Mean	Standard Deviation
Sample 1	1000	25	5
Sample 2	2000	23	7

```
# Given data
n1, mean1, std1 = 1000, 25, 5
n2, mean2, std2 = 2000, 23, 7

# Compute standard error
se = math.sqrt((std1**2 / n1) + (std2**2 / n2))

# Compute z-score
z_calculated = abs((mean1 - mean2) / se)

# Critical z-value for 95% confidence level (two-tailed test)
z_critical = norm.ppf(0.975)

# Print results
print("Computed Z-score:", round(z_calculated, 4))
print("Critical Z-value:", round(z_critical, 4))

# Hypothesis test conclusion
if z_calculated > z_critical:
    print("Reject the null hypothesis: The means of the two populations are significantly different.")
else:
    print("Fail to reject the null hypothesis: No significant difference between the means.")

print("\nRitesh Jha, Roll No: 16010423076, Q4")
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```
Computed Z-score: 8.9893
Critical Z-value: 1.96
Reject the null hypothesis: The means of the two populations are significantly different.

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Question 5

The weights of eight randomly selected athletes are recorded in kilograms:
70, 75, 78, 80, 82, 85, 87, 90.

The weights of twelve randomly selected basketball players are recorded in kilograms:
72, 74, 76, 78, 79, 80, 82, 83, 84, 85, 87, 88.

Can it be concluded that basketball players, on average, weigh more than athletes?

```
# Given data
athletes_weights = np.array([70, 75, 78, 80, 82, 85, 87, 90])
basketball_weights = np.array([72, 74, 76, 78, 79, 80, 82, 83, 84, 85, 87, 88])

# Compute sample statistics
mean_athletes = np.mean(athletes_weights)
mean_basketball = np.mean(basketball_weights)
std_athletes = np.std(athletes_weights, ddof=1)
std_basketball = np.std(basketball_weights, ddof=1)
n1, n2 = len(athletes_weights), len(basketball_weights)

# Compute pooled standard deviation
sp = np.sqrt(((n1 - 1) * std_athletes**2 + (n2 - 1) * std_basketball**2) / (n1 + n2 - 2))

# Compute t-score
t_score = (mean_basketball - mean_athletes) / (sp * np.sqrt(1/n1 + 1/n2))

# Degrees of freedom
df = n1 + n2 - 2

# Critical t-value for one-tailed test at 95% confidence level
t_critical = t.ppf(0.95, df)

# Print results
print("Mean Weight of Athletes:", round(mean_athletes, 4))
```

```
print("Mean Weight of Basketball Players:", round(mean_basketball, 4))
print("Computed T-score:", round(t_score, 4))
print("Critical T-value:", round(t_critical, 4))

# Hypothesis test conclusion
if t_score > t_critical:
    print("Reject the null hypothesis: Basketball players weigh significantly more than athletes on average.")
else:
    print("Fail to reject the null hypothesis: No significant evidence that basketball players weigh more.")

print("\nRitesh Jha, Roll No: 16010423076, Q5")
```

```
Mean Weight of Athletes: 80.875
Mean Weight of Basketball Players: 80.6667
Computed T-score: -0.0802
Critical T-value: 1.7341
Fail to reject the null hypothesis: No significant evidence that basketball players weigh more.

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```