

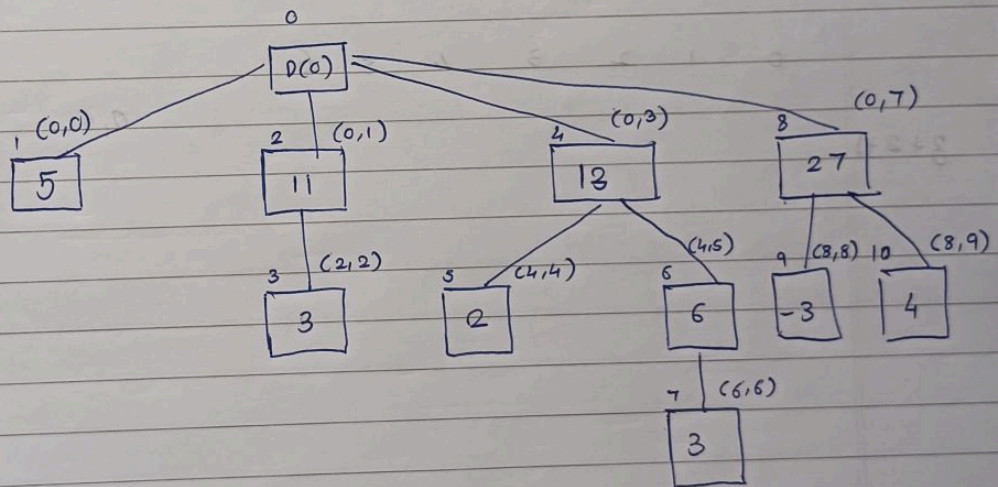
$\downarrow \uparrow \downarrow$
 5 6 3 -1 2 4 3 5 -3 7
 0 1 2 3 4 5 6 7 8 9

Fenwick Tree

* To find Parent Node (Flip Rightmost 1)

Parent

| | | | | | | |
|----|---|------|---|------|---|---|
| 1 | - | 0001 | - | 0000 | - | 0 |
| 2 | - | 0010 | - | 0000 | - | 0 |
| 3 | - | 0011 | - | 0010 | - | 2 |
| 4 | - | 0100 | - | 0000 | - | 0 |
| 5 | - | 0101 | - | 0100 | - | 4 |
| 6 | - | 0110 | - | 0100 | - | 4 |
| 7 | - | 0111 | - | 0110 | - | 6 |
| 8 | - | 1000 | - | 0000 | - | 0 |
| 9 | - | 1001 | - | 1000 | - | 8 |
| 10 | - | 1010 | - | 1000 | - | 8 |



Tree Index

no. of elements to be considered from start index

1 — $0 + 2^0 \rightarrow (0, 0)$

↑

start index in array

- 2 — $0 + 2^1 \rightarrow (0, 1)$
- 3 — $2^1 + 2^0 \rightarrow (2, 2)$
- 4 — $0 + 2^2 \rightarrow (0, 3)$
- 5 — $2^2 + 2^0 \rightarrow (4, 4)$
- 6 — $2^2 + 2^1 \rightarrow (4, 5)$
- 7 — $[2^2 + 2^1] + 2^0 \rightarrow (6, 6)$
- 8 — $0 + 2^3 \rightarrow (0, 7)$
- 9 — $2^3 + 2^0 \rightarrow (8, 8)$
- 10 — $2^3 + 2^1 \rightarrow (8, 9)$

* Prefix Sum

$(0, 3) = 3 + 1 = 4 \rightarrow$ starting tree node no. (Index (4) $\xrightarrow{\text{Parent}}$ Node (0))

$$(0, 3) = 13$$

$$(0, 8) = \text{index}(9) \xrightarrow{\text{Parent}} \text{Node}(8) \xrightarrow{\text{Parent}} \text{Node}(0)$$

$$= -3 + 27$$

$$= 24$$

$$(0, 5) = \cancel{2+13} \text{ index}(6) \rightarrow \text{Node}(4) \rightarrow \text{Node}(0)$$

$$= \cancel{15} 6 + 13$$

$$= 19$$

$$(0, 6) = \text{index}(7) \rightarrow \text{node}(6) \rightarrow \text{Node}(4) \rightarrow \text{Node}(0)$$

$$= 3 + 6 + 13$$

$$= 22$$

5 6 3 -1 2 4 3 5 -3 7
0 1 2 3 4 5 6 7 8 9

- 1) To get Parent
Take Two's complement
AND with OG
Subtract from OG

$$\begin{array}{r}
 \text{Tree index} - 1 \quad - \quad 000001 \\
 + \quad 111110 \quad \text{1st Comp} \\
 \hline
 111111 \\
 \text{AND} \\
 000001 \\
 \hline
 000001 \\
 - \quad 000001 \\
 \hline
 000000 \quad - \quad \boxed{0}
 \end{array}$$

$$\begin{array}{r}
 9 \quad - \quad 001001 \\
 + \quad 110110 \\
 \hline
 110111 \\
 \text{AND} \\
 001001 \\
 \hline
 000001
 \end{array}$$

$$\begin{array}{r}
 001001 \\
 - \quad 000001 \\
 \hline
 001000 \quad - \quad \boxed{8}
 \end{array}$$

$$\begin{array}{r}
 3 \quad - \quad 000011 \\
 + \quad 111100 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 1 \\
 111101 \\
 \text{AND} \\
 000011 \\
 \hline
 000001
 \end{array}$$

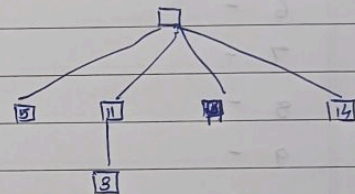
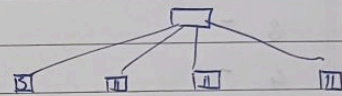
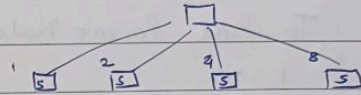
$$\begin{array}{r}
 000011 \\
 000001 \\
 \hline
 000010 \quad - \quad \boxed{2}
 \end{array}$$

Initially in all tree nodes (boxes) 0 is stored

- 2) Get Next
 Take two's comp
 AND with OG
 ADD to OG

if index 0 \rightarrow Value gets added
 if not 0 \rightarrow Not added

$$\begin{array}{r}
 \text{Next}(1) - \quad 0001 \\
 \quad \quad \quad 1000 \quad \quad \quad \left. \begin{array}{l} \text{2's} \\ \text{AND} \end{array} \right\} \\
 \hline
 \quad \quad \quad 1111 \\
 \quad \quad \quad 0001 \\
 \hline
 \quad \quad \quad 0001 \\
 \quad \quad \quad 0001 \\
 \hline
 \quad \quad \quad 0001 \\
 \quad \quad \quad 0001 \\
 \hline
 \quad \quad \quad 0010 \quad \rightarrow \quad \boxed{2}
 \end{array}$$



Q

4 - 0 - 1 6 3 2 - 4 7 - 3 + 6 7 4 2 (1)

0 1 2 3 4 5 6 7 8 9 10 11 12

1) To find Parent Node

1 - - - 0

2 - - - 0

3 - - - 2

4 - - - 0

5 - - - 4

6 - - - 4

7 - - - 6

8 - - - 0

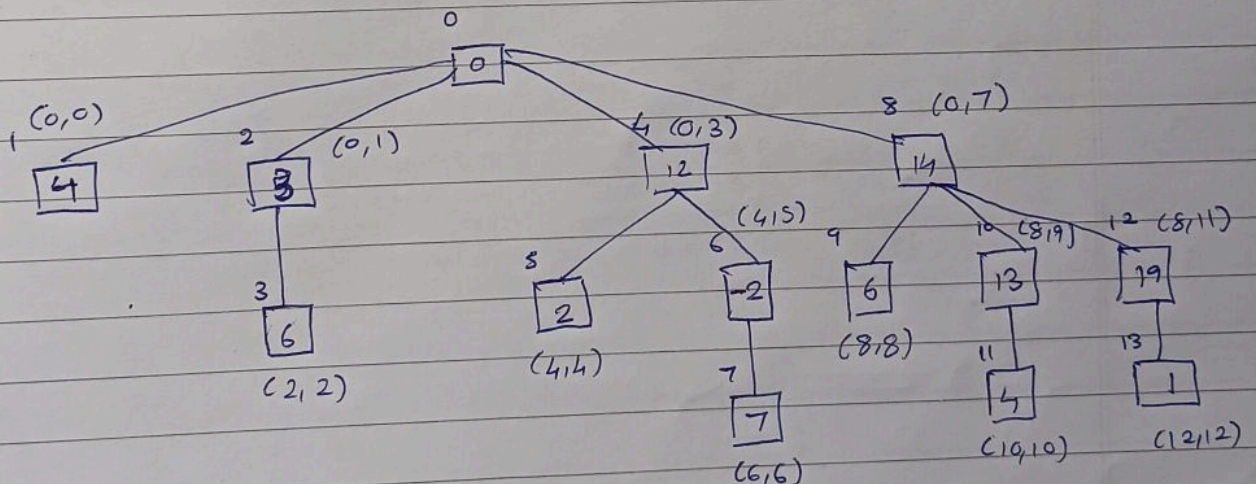
9 - - - 8

10 - - - 8

11 - 1011 - 1010 - 10

12 - 1100 - 1000 - 8

13 - 1101 - 1100 - 12



2) Tree Index

$$1 \quad - \quad (0, 0)$$

$$2 \quad - \quad (0, 1)$$

$$3 \quad - \quad (2, 2)$$

$$4 \quad - \quad (0, 3)$$

$$5 \quad - \quad (4, 4)$$

$$6 \quad - \quad (4, 5)$$

$$7 \quad - \quad (6, 6)$$

$$8 \quad - \quad (0, 7)$$

$$9 \quad - \quad (8, 8)$$

$$10 \quad - \quad (8, 9)$$

$$11 \quad - \quad (2^3 + 2^1) + 2^0 \quad - \quad (10, 10)$$

$$12 \quad - \quad 2^3 + 2^0 \quad - \quad (8, 11)$$

$$13 \quad - \quad (2^3 + 2^2) + 2^0 \quad - \quad (12, 12)$$

3) Prefix Sum

pattern 1 : a b c a g a b

| | | | | | | | |
|--------------------|---|---|---|---|---|---|---|
| Index | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Pattern | a | b | c | a | g | a | b |
| LPS LPS | 0 | 0 | 0 | 1 | 0 | 1 | 2 |

| | | | | | | | | | | |
|-----|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| | a | a | h | b | c | a | b | a | a | g |
| | <u>a</u> | <u>a</u> | <u>h</u> | <u>b</u> | <u>c</u> | <u>a</u> | <u>b</u> | <u>a</u> | <u>a</u> | <u>g</u> |
| | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| | 0 | 1 | 0 | 0 | 0 | 1 | 0 | | | |
| | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 2 | 0 |
| LPS | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 2 | 0 |

| | | | | | | | | | | | |
|-----|---|---|---|---|---|---|---|---|---|---|---|
| | a | b | h | g | a | b | c | a | b | h | a |
| LPS | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 2 | 3 | 1 |

Test : onionionsgr
pattern : onions