Prove that

$$H(X,Y) = H(X/Y) + H(Y)$$

$$H(X,Y) = H(Y/X) + H(X)$$

UNIT 2

Prove that

$$H(XY) = H(X/Y) + H(Y)$$

we know that $P(x_i, y_j) = P(x_i/y_j)\, P(y_j)$

$$\sum_{i=1} P(x_i, y_j) = P(y_j)$$

$$H(X, Y) = \sum_{i=1} \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(x_i, y_j)}$$

$$= \sum_{i=1} \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(x_i/y_j) \cdot P(y_j)}$$

$$= \sum_{i=1} \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(x_i/y_j)} + \sum_{i=1} \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(y_j)}$$

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$$H(X,Y) = \sum_{i=1} \sum_{j=1} p(x_i, y_j) \log \frac{1}{p(x_i, y_j)}$$

$$= \sum_{i=1} \sum_{j=1} p(x_i, y_j) \log \frac{1}{p(x_i/y_j) \cdot p(y_j)}$$

$$= \underbrace{\sum_{i=1} \sum_{j=1} p(x_i, y_j) \log \frac{1}{p(x_i/y_j)}} + \sum_{i=1} \sum_{j=1} p(x_i\, y_j) \log \frac{1}{p(y_j)}$$

$$= H(X/Y) + \sum_{j=1} \left\{ \sum_{i=1} p(x_i, y_j) \log \frac{1}{p(y_j)} \right\}$$

$$= H(X/Y) + \sum_{j=1} p(y_j) \log \frac{1}{p(y_j)}$$

$$\boxed{H(XY) = H(X/Y) + H(Y)}$$

$$H(XY) = H(Y/X) + H(X)$$

$$P(x_i, y_j) = P(y_j/x_i) \cdot P(x_i)$$

$$\sum_{j=1} P(x_i, y_j) = P(x_i)$$

$$H(XY) = \underline{\sum_{i=1}} \underline{\sum_{j=1}} P(x_i, y_j) \log \frac{1}{P(x_i, y_j)}$$

$$= \sum_{i=1} \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(y_j/x_i) P(x_i)}$$

$$= \sum_{i=1} \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(y_j/x_i)} + \sum_{i=1} \left\{ \sum_{j=1} P(x_i, y_j) \log \frac{1}{P(x_i)} \right\}$$

$$= H(Y/X) + \sum_{i=1} P(x_i) \log \frac{1}{P(x_i)}$$

$$= H(Y/X) + H(X)$$

Proofs of mutual information properties

Proofs of Mutual Information Properties:-

Prop. 1:- (i) $I(X;Y) = I(Y;X)$

From Probability theory.

J.P
$$\left[\begin{array}{ll} P(x_i y_j) = P(x_i/y_j)\,P(y_j) & \text{--- (i)} \\ P(x_i y_j) = P(y_j/x_i)\,P(x_i) & \text{--- (ii)} \end{array}\right.$$

Proofs of Mutual Information Properties:-

Prop. 1:- (i) $I(X;Y) = I(Y;X)$

From Probability theory.

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$$\begin{cases} P(x_i y_j) = P(x_i/y_j)\,P(y_j) & \text{— (i)} \\ P(x_i y_j) = P(y_j/x_i)\,P(x_i) & \text{— (ii)} \end{cases}$$

$$P(x_i/y_j)\,P(y_j) = P(y_j/x_i)\,P(x_i)$$

$$\frac{P(x_i/y_j)}{P(x_i)} = \frac{P(y_j/x_i)}{P(y_j)} \quad \text{— (iii)}$$

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 \frac{P(x_i/y_j)}{P(x_i)} \quad \text{— (iv)}$$

$$I(Y;X) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2 \frac{P(y_j/x_i)}{P(y_j)} \quad \text{— (v)}$$

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Proofs of Mutual Information Properties:-

Prop. 1:- (i) $I(X;Y) = I(Y;X)$

From Probability theory.

J.P
$$\begin{cases} P(x_i y_j) = P(x_i/y_j)\,P(y_j) & \text{— (i)} \\ P(x_i y_j) = P(y_j/x_i)\,P(x_i) & \text{— (ii)} \end{cases}$$

$$P(x_i/y_j)\,P(y_j) = P(y_j/x_i)\,P(x_i)$$

$$\frac{P(x_i/y_j)}{P(x_i)} = \frac{P(y_j/x_i)}{P(y_j)} \quad \text{— (iii)}$$

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2 \frac{P(x_i/y_j)}{P(x_i)} \quad \text{— (iv)}$$

$$I(Y;X) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2 \frac{P(y_j/x_i)}{P(y_j)} \quad \text{— (v)}$$

$$I(Y;X) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2 \frac{P(x_i/y_j)}{P(x_i)} \quad \text{— (vi)}$$

From (iv) and (vi)

$$\boxed{I(X;Y) = I(Y;X)}$$

Proved.

~~[illegible]~~ Mutual Information

Ques) Show that ~~[illegible]~~.

$$I(X;Y) = H(X) - H(X|Y)$$

$$= H(Y) - H(Y|X)$$

$$H(X|Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j) \log_2 \left(\frac{1}{P(x_i/y_j)}\right)$$

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i|y_j)}{P(x_i)}$$

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Mutual Information

Ques) Show that

$$I(X;Y) = H(X) - H(X|Y)$$
$$= H(Y) - H(Y|X)$$

$$H(X|Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2\left(\frac{1}{P(x_i/y_j)}\right)$$

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2\frac{P(x_i|y_j)}{P(x_i)}$$

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2\left(\frac{1}{P(x_i)}\right) - \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2\left(\frac{1}{P(x_i|y_j)}\right)}_{H(X|Y)}$$

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2\left(\frac{1}{P(x_i)}\right) - H(X|Y)$$

$$\sum_{j=1}^{m} P(x_i, y_j) = P(x_i)$$

$$I(X;Y) = \underbrace{\sum_{i=1}^{n} P(x_i)\log_2\left(\frac{1}{P(x_i)}\right)}_{H(X)} - H(X|Y)$$

$$\boxed{I(X;Y) = H(X) - H(X|Y)}$$

Proved.

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Mutual Information

Prove:- Mutual Info^n is always +ve.

$I(X;Y) \geq 0$

$$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i|y_j)}{P(x_i)}$$

we know that, $P(x_i|y_j) = \dfrac{P(x_i, y_j)}{P(y_j)}$

$$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i, y_j)}{P(x_i) P(y_j)}$$

$$I(X;Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i) P(y_j)}{P(x_i, y_j)}$$

$$-I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i) P(y_j)}{P(x_i, y_j)}$$

$$\boxed{\sum_{K=1}^{m} P_K \log_2 \left(\frac{q_K}{P_K}\right) \leq 0}$$

$-I(X;Y) \leq 0$

or $I(X;Y) \geq 0$

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Mutual Information:-

Prove:- $I(X;Y) = H(X) + H(Y) - H(X,Y)$

$H(X,Y) = H(X|Y) + H(Y)$ — (i)

and $I(X;Y) = H(X) - H(X|Y)$ — (ii)

From (i)

$H(X|Y) = H(X,Y) - H(Y)$ — (iii)

Putting (iii) in (ii)

$$\boxed{I(X;Y) = H(X) + H(Y) - H(X,Y)}$$

Entropy proof

$$H = \sum_{i=1}^{M} P_i I_i$$

$$I_i = -\log P_i$$

$$H = \sum_{i=1}^{M} P_i \left(-\log (P_i)\right)$$

$$\boxed{H = -\sum_{i=1}^{M} P_i \cdot \log (P_i)}$$

8:00

1)ITC_Module...

b. Entropy (H)

1. Entropy is average information per message when many messages are transferred through a channel.

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Entropy Derivation

2. Consider M number of different and independent messages $m_1, m_2, m_3 \ldots m_M$ having probablities $p_1, p_2, p_3 \ldots p_M$. The source generates sequence of L independent messages over a long period of time and $L >>>> M$.

The number of messages m_1 with L messages is $p_1 \cdot L$. Similarly for m_M it is $p_M \cdot L$

The amount of information in each m_1 is

$I_1 = -\log_b P_1$

The total information in all m_1 messages is

$m_1 \; I_{1\,total} = -P_1 L \log_b P_1$

Therefore for all m_M messages

$I_{M\,total} = -P_M L \log_b P_M$

Total amount of messages in L message is

$$I_{total} = I_{1\,total} + I_{2\,total} + \ldots\ldots + I_{M\,total}$$

$$I_{total} = -P_1 L \log P_1 - P_2 L \log P_2 - \ldots\ldots - P_M L \log P_M$$

Average information $= \dfrac{I_{total}}{L} = \dfrac{-P_1 L \log P_1 - \ldots\ldots - P_M L \log P_M}{L}$

$$\text{Entropy } (H) = \frac{I_{total}}{L} = \sum_{i=1}^{M} P_i \log \frac{1}{P_i} = -\sum_{i=1}^{M} P_i \log P_i$$

Comparison of two sources

classmate

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* Comparision of two sources

$R_1 = r_1 . h$

$R_2 = r_2 . h$

If $r_1 > r_2$

$R_1 > R_2$

The first source generates more information per se-cond.

Verify the equation,

$I(n_i x_j) = I(x_i) + (x_j)$

If x_i & x_j are independent

I)

$I(x, y)$ — mutual information

$I(x), I(y)$ — Information measures for x & y

$p(x, y) = p(x) . p(y) \longrightarrow x, y$ independent

$$I(x, y) = \log \frac{1}{p(x, y)}$$

$$= \log \frac{1}{p(x) . p(y)}$$

$$\log(ab) = \log a + \log b$$

$$\log \frac{1}{p(x) . p(y)} = \log \frac{1}{p(x)} + \log \frac{1}{p(y)}$$

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$$\therefore I(x,y) = \log \frac{1}{p(x)} + \log \frac{1}{p(y)}$$

$$\log \frac{1}{p(x)} = I(x)$$

$$\log \frac{1}{p(y)} = I(y)$$

$$\boxed{\therefore I(x,y) = I(x) + I(y)}$$

Mutual Information measures dependence b/w 2 random variables

If x & y are independent, $I(x,y) = 0$