Convolution Code basics

Convolutional codes basics, parameters & designing

- In Convolutional codes, block of 'n' code digits generated by the encoder in time unit depends on not only block of 'k' message digits with in that time unit but also on the preceding (m-1) blocks of message digits.

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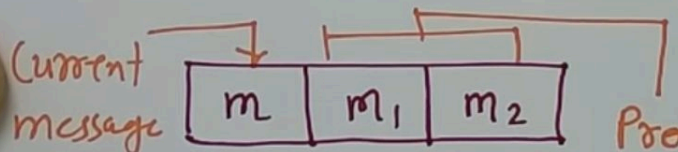
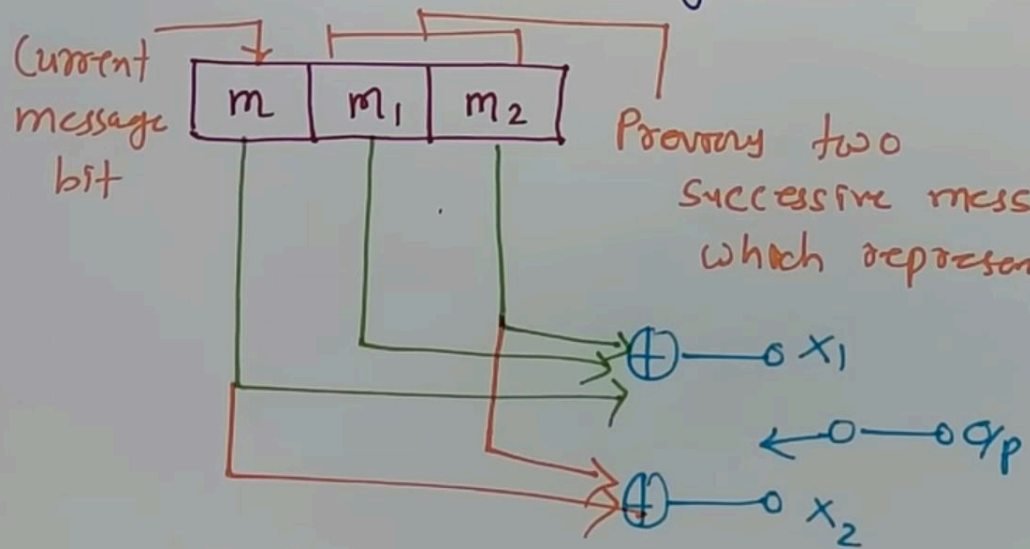


also on the preceding (m-1) blocks of message digits



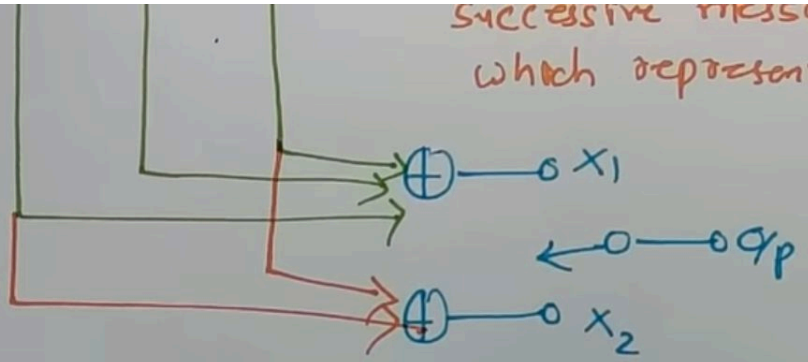
$X_1 = m \oplus m_1 \oplus m_2$

$X_2 = m \oplus m_2$

m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

bit

successive message bit
which represent state of convolution code



$X_1 = m_1 \oplus m_2 \oplus m_2$

$X_2 = m \oplus m_2$

m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

- k = no of message bits

  n = no of encoded o/p bits

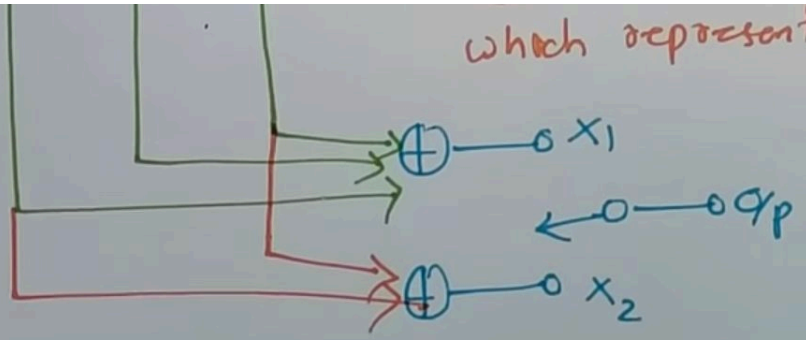
  K = Constrain Length

- Here o/p will switch in bit n

bit

$K = 1$

which represent state of



m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

$X_1 = m_1 \oplus m_2 \oplus m_2$

$X_2 = m \oplus m_2$

- k = no of message bits = 1

  n = no of encoded o/p bits = 2

  K = Constrain Length = 3

- Here o/p will switch in bet.^{n} x_1 & x_2 so o/p will be

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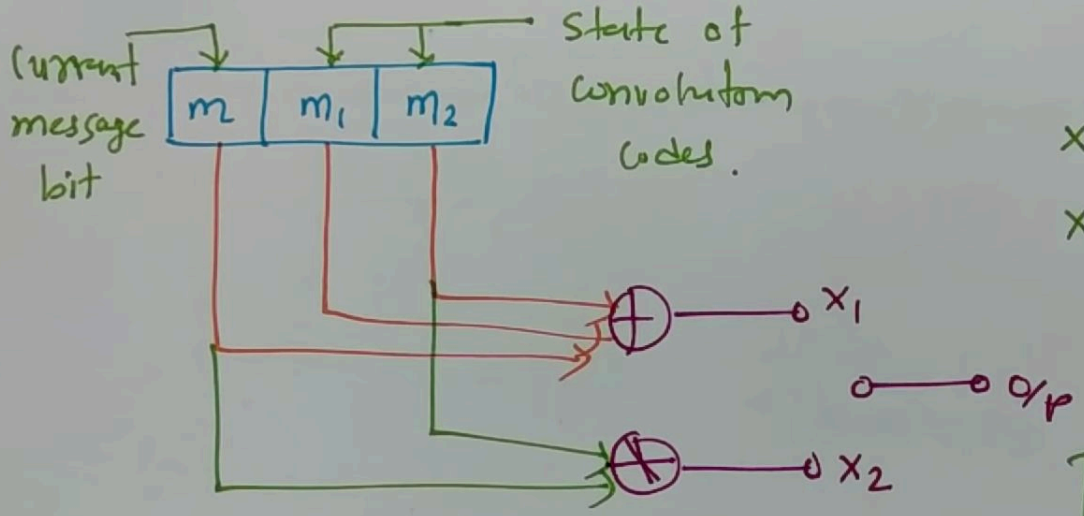
  n = no of encoded o/p bits = 2
  K = Constrain Length = 3
- Here o/p will switch in bet.^{n} X_1 & X_2 so o/p will be

$$X = X_1 X_2 X_1 X_2 \ldots\ldots$$

- Code rate $r = \frac{k}{n} = \frac{1}{2}$
- Constraint Length (K)
  - Single message bit influences encoder o/p for different successive shift.
- Code dimensions $(n, k) = (2, 1)$

Convolution Codes States & Code tree.



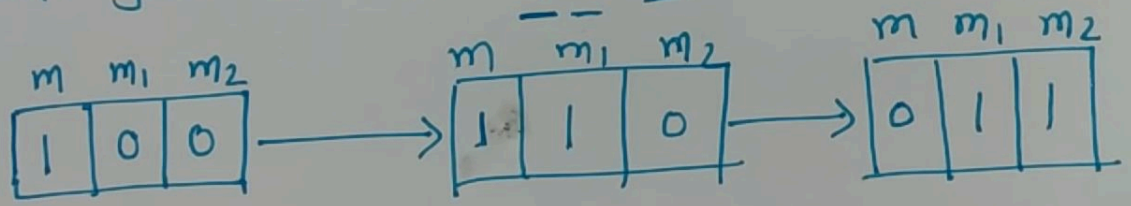
$$x_1 = m \oplus m_1 \oplus m_2$$

$$x_2 = m \oplus m_2$$

m_1	m_2	States
0	0	a
0	1	b
1	0	c
1	1	d

Code tree

- each branch of tree represent an Vp symbol with the corresponding pair of o/p binary symbols indicating on the branch
- lets give Vp = 1 1 0



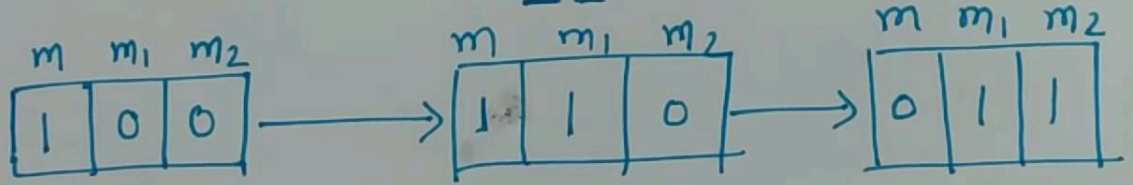
Code tree

- each branch of tree represent an Vp symbol with the corresponding pair of o/p binary symbols indicating on the branch

$x_1 = m \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

- lets give Vp = 1 1 0



$x_1 = 1$, $x_2 = 1$ $\Rightarrow x_1x_2 = 11$

$x_1 = 0$, $x_2 = 1$ $\Rightarrow x_1x_2 = 01$

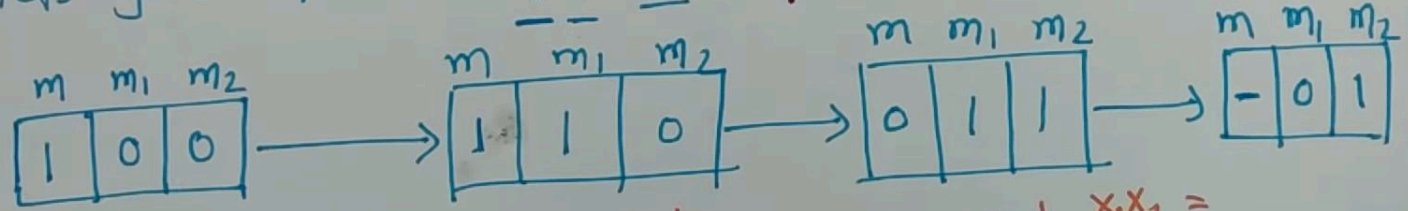
$x_1 = 0$, $x_2 = 1$ $\Rightarrow x_1x_2 = 01$

the corresponding pair of o/p binary symbols indicating on the branch

$x_1 = m \oplus m_1 \oplus m_2$

$x_2 = m \oplus m_2$

– lets give I/p = 1 1 0



$x_1 = 1$, $x_2 = 1$ $\Rightarrow x_1x_2 = 11$ — State – a

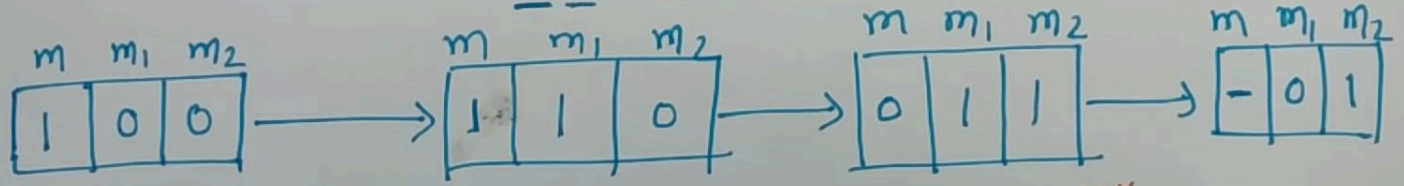
$x_1 = 0$, $x_2 = 1$ $\Rightarrow x_1x_2 = 01$ — State – c

$x_1 = 0$, $x_2 = 1$ $\Rightarrow x_1x_2 = 01$ — State – d

State – b

– lets give V_p = 1 1 0

$x_2 = m \oplus m_2$

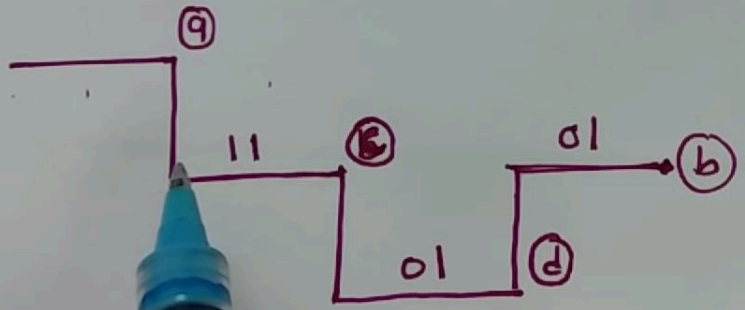


$x_1 = 1$, $x_2 = 1$ } $x_1x_2 = 11$ — State – a

$x_1 = 0$, $x_2 = 1$ } $x_1x_2 = 01$ — State – c

$x_1 = 0$, $x_2 = 1$ } $x_1x_2 = 01$ — State – d

State – b

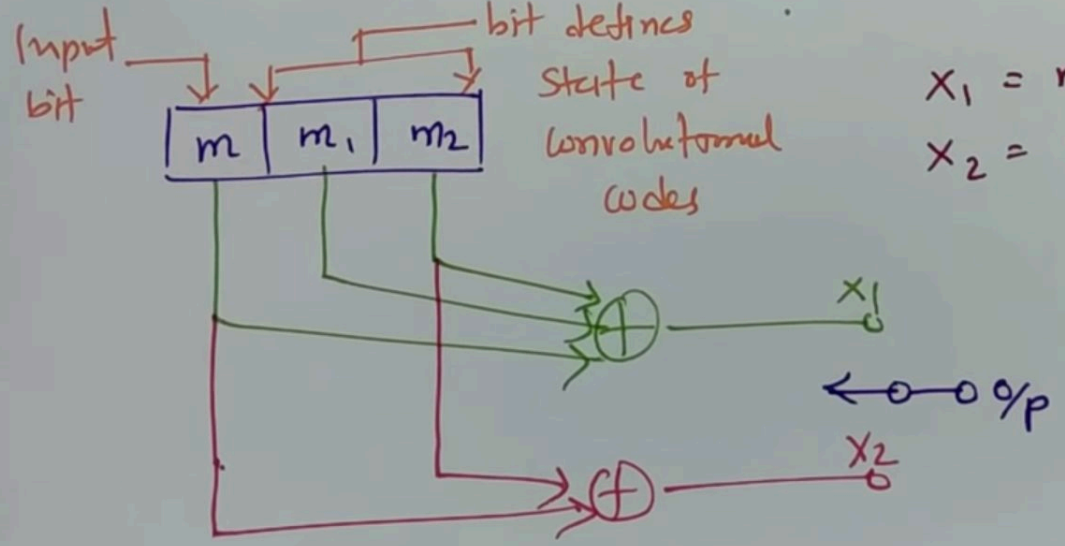


– down step means $V_p = 1$

– up step means $V_p = 0$

Code trellis and State Diagram

Code trellis & State diagram of Convolutional Codes



$$X_1 = m \oplus m_1 \oplus m_2$$

$$X_2 = m \oplus m_2$$

m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

m	m_1	m_2	X_1	X_2	Current State	Next state
	0	0			a	

m_2	m_1	m_2	X_1	X_2	current state	Next state
	0	0			a	
	0	0			a	
	0	1			b	
	0	1			b	
	1	0			c	
	1	0			c	
	1	1			d	
	1	1			d	

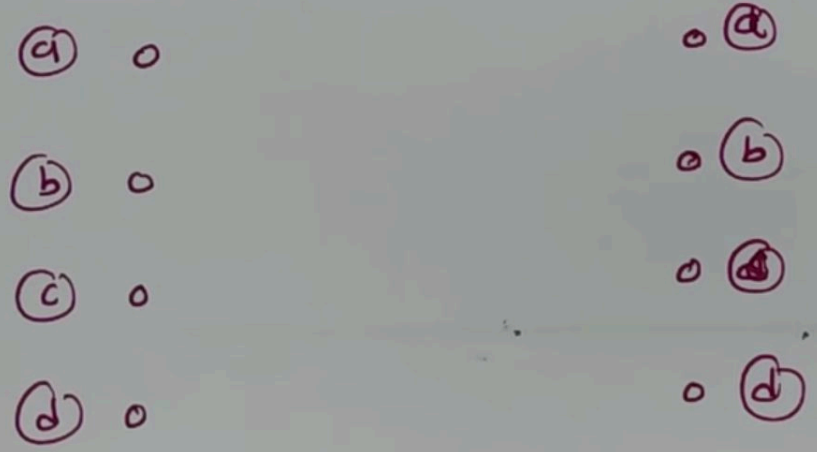
m_2	m_1 →	m_2 →	X_1	X_2	current state	Next state
0	0	0	0	0	a	a
1	0	0	1	1	a	c
0	0	1	1	1	b	a
1	0	1	0	0	b	c
0	1	0	1	0	c	b
1	1	0	0	1	c	d
0	1	1	0	1	d	b
1	1	1	1	0	d	d

$X_1 = m \oplus m_1 \oplus m_2$

$X_2 = m \oplus m_2$

0	1	1	0	1	d	b
1	1	1	1	0	d	d

Code trellis

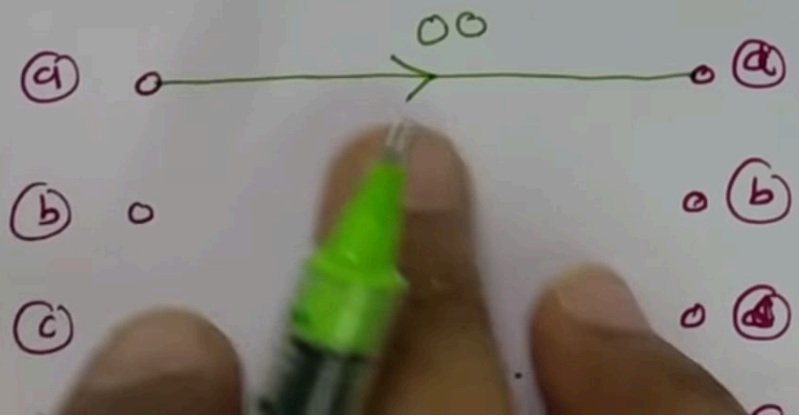


——— Input bit 0

– – ––– Input bit 1

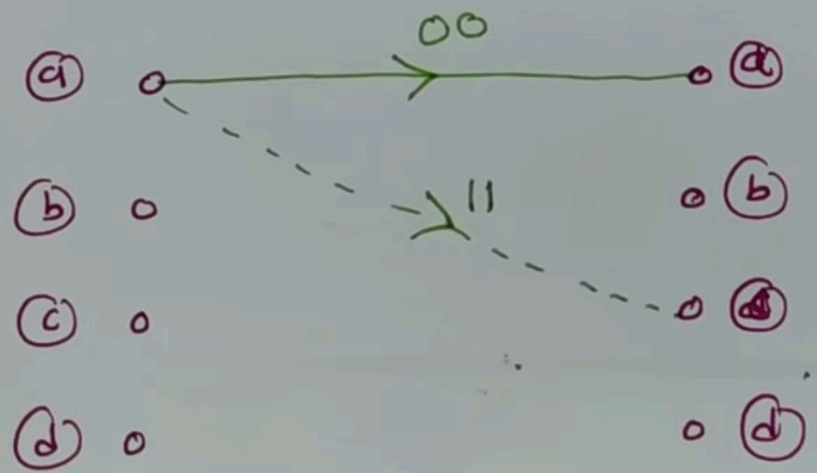
1	0	1	0	0	b	c
0	1	0	1	0	c	b
1	1	0	0	1	c	d
0	1	1	0	1	d	b
1	1	1	1	0	d	d

Code trellis



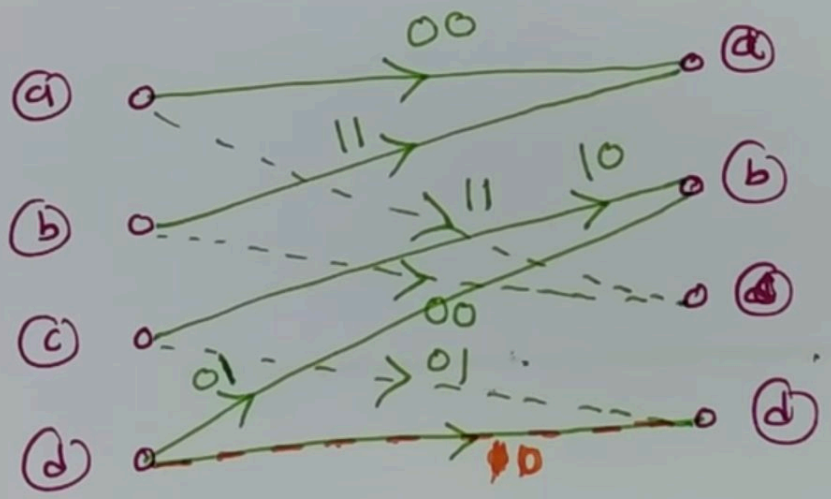
0	1	1	0	1	d	b
1	1	1	1	0	d	d

Code trellis



0	1	1	0	1	d	b
1	1	1	1	0	d	d

Code trellis

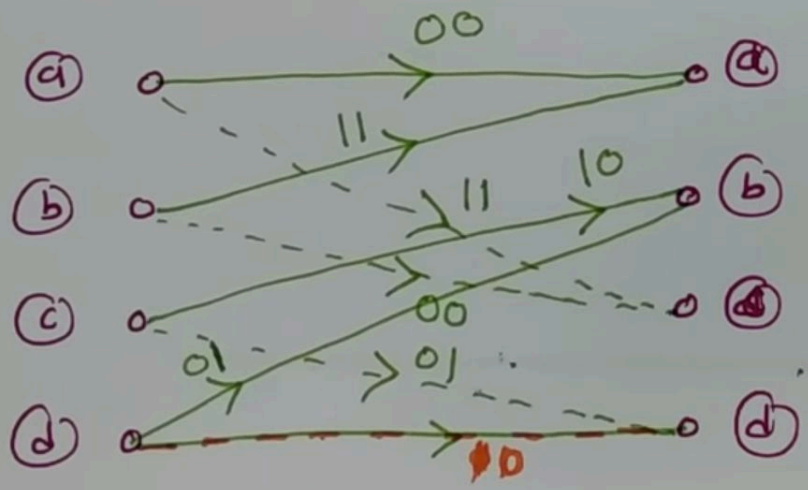


Input bit 0

Input bit 1

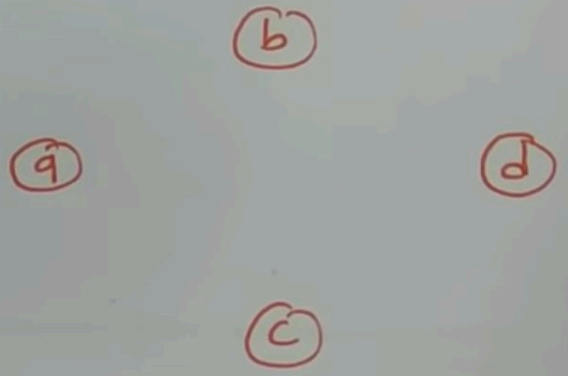
0	1	1	0	1	d	b
1	1	1	1	0	d	d

Code trellis



Input bit 0

Input bit 1

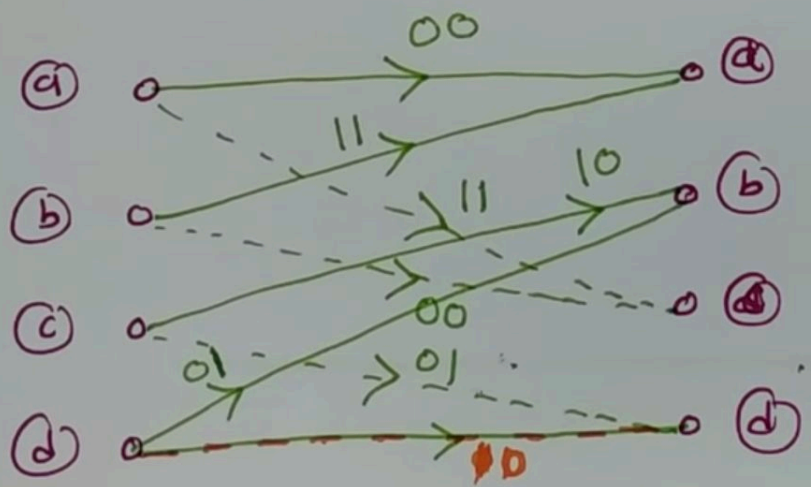


Input 0 bit

Input 1 bit

0	1	1	0	1	d	b
1	1	1	1	0	d	d

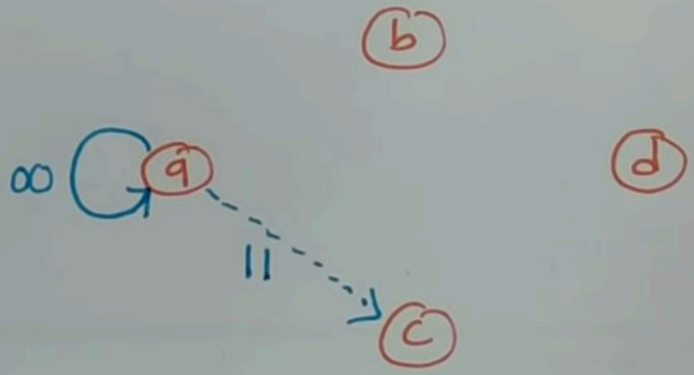
Code trellis



Input bit 0

Input bit 1

state diagram

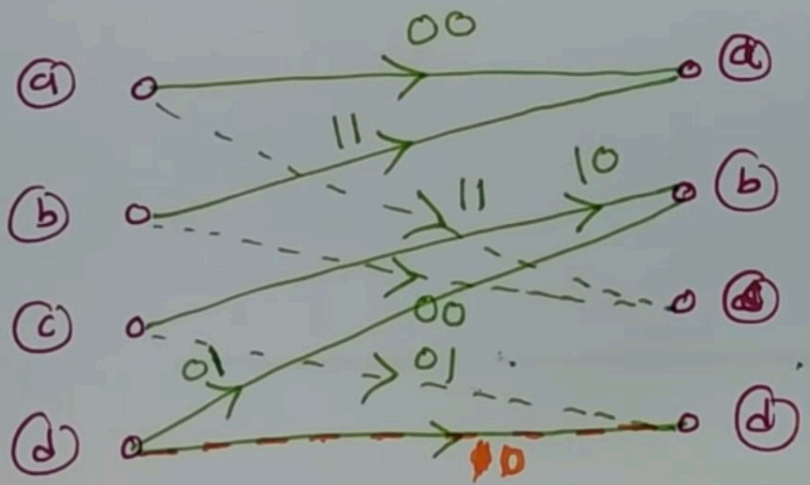


Input 0 bit

Input 1 bit

0	1	1	0	1	d	b
1	1	1	1	0	d	d

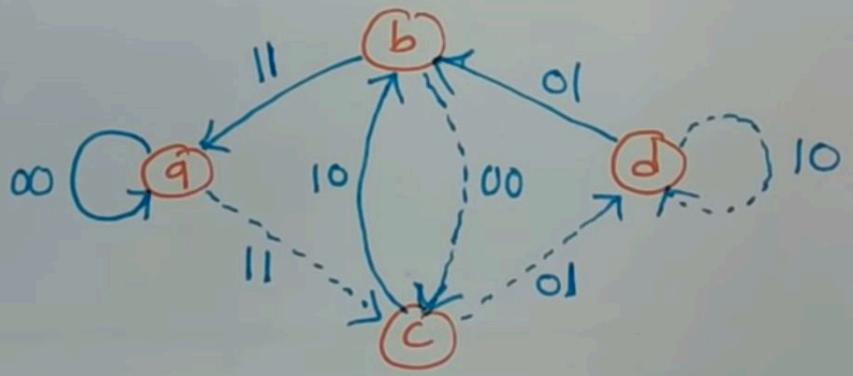
Code trellis



Input bit 0

Input bit 1

state diagram



Input 0 bit

Input 1 bit

Applications of Convolutional Codes:

1. **Satellite Communication:**
   - Used to protect data transmitted over long distances where signal quality can degrade.
2. **Wireless Communication (Mobile Networks):**
   - Helps correct errors caused by noise, fading, and interference in mobile phone networks (like 4G and older systems).
3. **Deep Space Communication:**
   - Essential for transmitting data from spacecraft to Earth with very low error rates (e.g., NASA missions).
4. **Voice Transmission (Telephony):**
   - Used in cellular voice transmission (like GSM) to maintain call quality even with weak signals.
5. **Digital TV Broadcasting:**
   - Protects video and audio data in digital television standards (like DVB).
6. **Wi-Fi and Bluetooth:**
   - Helps in error correction in wireless data transfer technologies (especially older Wi-Fi versions and Bluetooth).
7. **Optical Communication:**
   - Used in fiber-optic communication to correct data errors during high-speed transmission.
8. **Military Communication Systems:**
   - Ensures secure and reliable data transmission even under harsh and noisy conditions.
9. **Data Storage Devices:**
   - Applied in hard drives and CDs/DVDs to correct read errors.
10. **Internet Data Transfer (TCP/IP Layer Protection):**
    - Sometimes used in network protocols for enhanced data reliability, especially over noisy links.