Mod 1 : Probability & Probability Distribution

→ Conditional Probability

$$P\left(\frac{B}{A}\right) = \frac{m_{12}}{m_1} = \frac{n(A \cap B)}{n(A)} \quad \text{if } = P(B)$$

$$P\left(\frac{A}{B}\right) = \frac{m_{12}}{m_1} = \frac{n(A \cap B)}{n(B)} \quad \text{if } = P(A)$$

} A & B are said to be independent events.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

→ Baye's Theorem

$$P(E_i / A) = \frac{P(E_i) \cdot P(A / E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A / E_i)} \longrightarrow P(A)$$

Steps

① Get $P(E_1), P(E_2) \ldots$ Normal Probability

② Get $P(A|E_1), P(A|E_2) \ldots$ Probability of smth happening if something already happens

③ Get $P(E_n / A)$ as asked in Qn.

④ Sometimes might ask not specific to $E_1, E_2 \ldots$ then Just find $P(A)$

Note :

Mutually exclusive $P(\ \cap\ ) = 0$

Mutually exhaustive $P(\ \cup\ ) = 1$

→ Discrete random variable

① Given X
$P(X)$ | $\Sigma P(x) = 1$

② $E(x) = \Sigma x \cdot P(x)$

③ $V(x) = E(x^2) - [E(x)]^2$

$E(x^2) = \Sigma x^2 \cdot P(x)$

⊛ If some eqⁿ given in question of x & Y
Use it to find Y on different X's
Create new table Y
$P(Y)$ → use related x values as probabilities

→ Continuous random variable

$$\int_a^b f(x) \cdot dx = 1$$

$$\text{Mean} = E(x) = \int_a^b x \cdot f(x) \cdot dx$$

$$E(x^2) = \int_a^b x^2 \cdot f(x) \cdot dx$$

$$\text{Variance} = Var(x) = E(x^2) - [E(x)]^2$$

$$\int_c^M f(x) \cdot dx = \frac{1}{2}$$ ← Median Q's

→ Common for both

$E(X_1 \pm X_2) = E(X_1) \pm E(X_2)$

$E(X \cdot Y) = E(X) \cdot E(Y)$

$Var(X) = E(X^2) - [E(x)]^2$

$E(ax) = a E(x)$

$Var(aX_1 \pm bX_2) = a^2 \, var(X_1) + b^2 \, var(X_2)$

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* Joint & Marginal (pmf)

① Use given eqn to make table
② Table must contain all permut & comb, Total
③ Get marginal by splitting two parts of totals

In balls question,
consider x & y
check how much they go
use combination (${}^nC_x$) to solve / total possible (C_x)
Make table finally
If conditional asked given ($Y=1$)

$= P(X=0|Y=1) + \ldots P(X=1|Y=1) \ldots$

$= \frac{P(X=0, Y=1)}{P(Y=1)} + \frac{P(X=1; Y=1)}{P(Y=1)} \ldots$

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→ **Binomial Distribution**

$P(X=x) = {}^nC_x\, p^x q^{n-x}$

$n \rightarrow$ no. of trials

n repeated N times $\rightarrow$ frequency

Mean (Expectation) $= np$

Variance $= npq$

$p+q = 1$ (p ↑ success, q ↑ failure)

Mode $= (n+1)p$ $\xrightarrow{\text{int } (k)}$ k & $k-1$; $\xrightarrow{\text{non int}}$ integral part

Recurrence relation

$P(X \geq 2) = 1 - P(X<2)$

$$p(x+1) = \frac{n-x}{x+1} \cdot \frac{p}{q}\, p(x)$$

For frequency, $f(x) = N \times p(x)$

$$f(x+1) = \frac{n-x}{x+1} \cdot \frac{p}{q}\, f(x)$$

→ **Poisson's Distribution** (Mean = variance = m)

$$P(X=x) = \frac{e^{-m} m^x}{x!} \qquad m>0$$

Mean $= m$

$SD = \sqrt{var} = \sqrt{\sigma^2}$

Variance $= m$

Mode $= m$ $\xrightarrow{\text{not int}}$ int b/w $m-1$ & m; $\xrightarrow{\text{int}}$ m & $m-1$

Recurrence relation

$$P(x+1) = \frac{m}{x+1} \cdot P(x)$$

Remember

$var(ax_1 + bx_2) = a^2\, var(x_1) + b^2\, var(x_2)$

$m = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$ ← Used in 'fit' questions

Also $F = N \times P$

N ↑ Total freq Σf_i

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→ Uniform Distribution

$E(x) = \frac{n+1}{2}$ (Mean)

$E(x^2) = \frac{n^2-1}{12}$ (Var)

* Continuous uniform distribution

$f(x) = \frac{1}{b-a}$, $a \le x \le b$

0 , otherwise

Mean

$E(x) = \frac{a+b}{2}$ $E(x^2) = \frac{b^2+ab+a^2}{3}$

Variance

$var(x) = \frac{(b-a)^2}{12}$

① Get interval, determine density
② Find probability by * range of $\int$ to density
③ Apply find condⁿ (if any)

→ Exponential distribution

$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

$e^{-\infty} = 0$
$e^{\infty} = \infty$

Mean : $E(x) = \frac{1}{\lambda}$ | Variance : $var(x) = \frac{1}{\lambda^2}$

① Find $\mu \rightarrow \frac{1}{\lambda} \rightarrow$ Get $\lambda \rightarrow$ form $f(x)$
② Find probabilities by given range (integrating actually)
③ Use binomial distribution, if anything other than time is said

→ **Normal Distribution** $\sigma = S.D = \sqrt{var}$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

Quartile Deviation

$$Q_1 = m - \frac{2}{3}\sigma$$

$$Q_3 = m + \frac{2}{3}\sigma$$

Inter Quartile Range

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{2}{3}\sigma$$

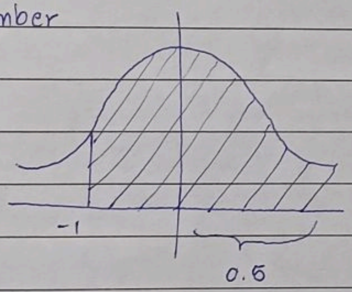
Mean Deviation $= \frac{4}{5}\sigma$

Inter Quartile Range $= Q_3 - Q_1$

* **For range Q's** (Convert $x \to z$)

$$z = \frac{x-m}{\sigma} = \frac{x - np}{\sqrt{npq}}$$

\# Remember



From Q5 - Q13

If c or unknown x involved

Then ① convert $x \to z$

② If $>$ given → Right of 0

$<$ given → Left of 0

exception if $<$ given but value is bigger than 0.5 → Right of 0

~~similarly opposite also exception~~

③ Bring to $0 \gtrless z \gtrless z_1$ form & find z_1

④ Get x_1 from z_1 by reconverting

For table Q's
make cumulative column too

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* Quartiles (3 Values → Split into 4)

$Q_1 = \left[\frac{N+1}{4}\right]$ th term ; $\frac{N}{4}$

$Q_2 = \left[\frac{N+1}{2}\right]$ th term ; $\frac{N}{2}$

$Q_3 = \left[\frac{3(N+1)}{4}\right]$ th term ; $\frac{3N}{4}$

for odd ; for even

For Range Q's

$Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times C_1$

l_1 : lower bound of class
m_1 : cumulative of prev class
f_1 : freq of class
C_1 : length of class

* Deciles (9 values → Split into 10)

$D_1 = \left[\frac{N+1}{10}\right]$ th term

$D_2 = \left[\frac{2(N+1)}{10}\right]$ th term

$D_9 = \left[\frac{9(N+1)}{10}\right]$ th term

* Percentile (100 equal parts)

$P_1 = \frac{N+1}{100}$ th term

$P_2 = \frac{2(N+1)}{100}$ th term

$P_{99} = \frac{99(N+1)}{100}$ th term