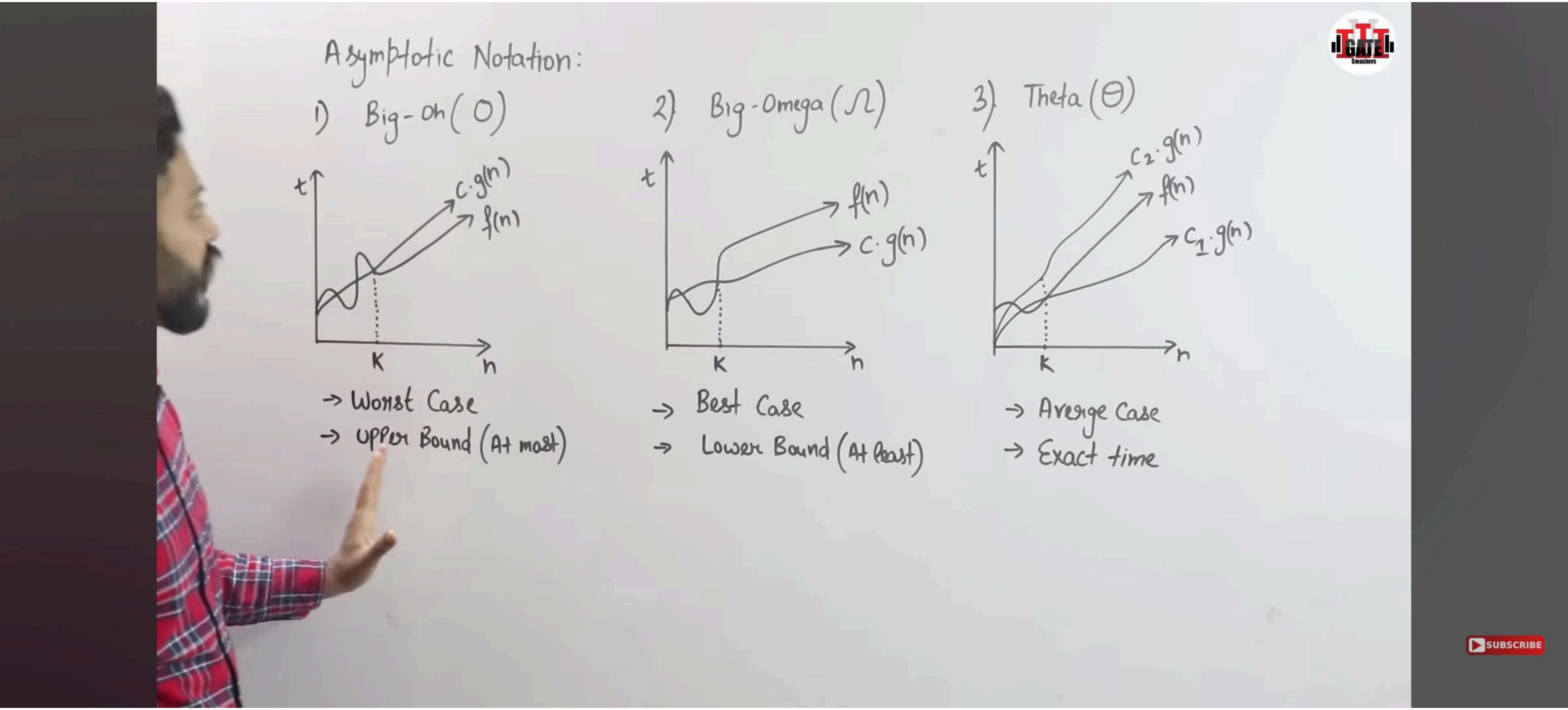
Asymptotic notations
Big O, omega, theta


Asymptotic Notation:
1) Big-Oh (O)
t
c.g(n)
f(n)
K
n
→ Worst Case
→ Upper Bound (At most)
2) Big-Omega (Ω)
t
f(n)
c.g(n)
K
n
→ Best Case
→ Lower Bound (At least)
3) Theta (Θ)
t
c2.g(n)
f(n)
c1.g(n)
K
n
→ Averge Case
→ Exact time
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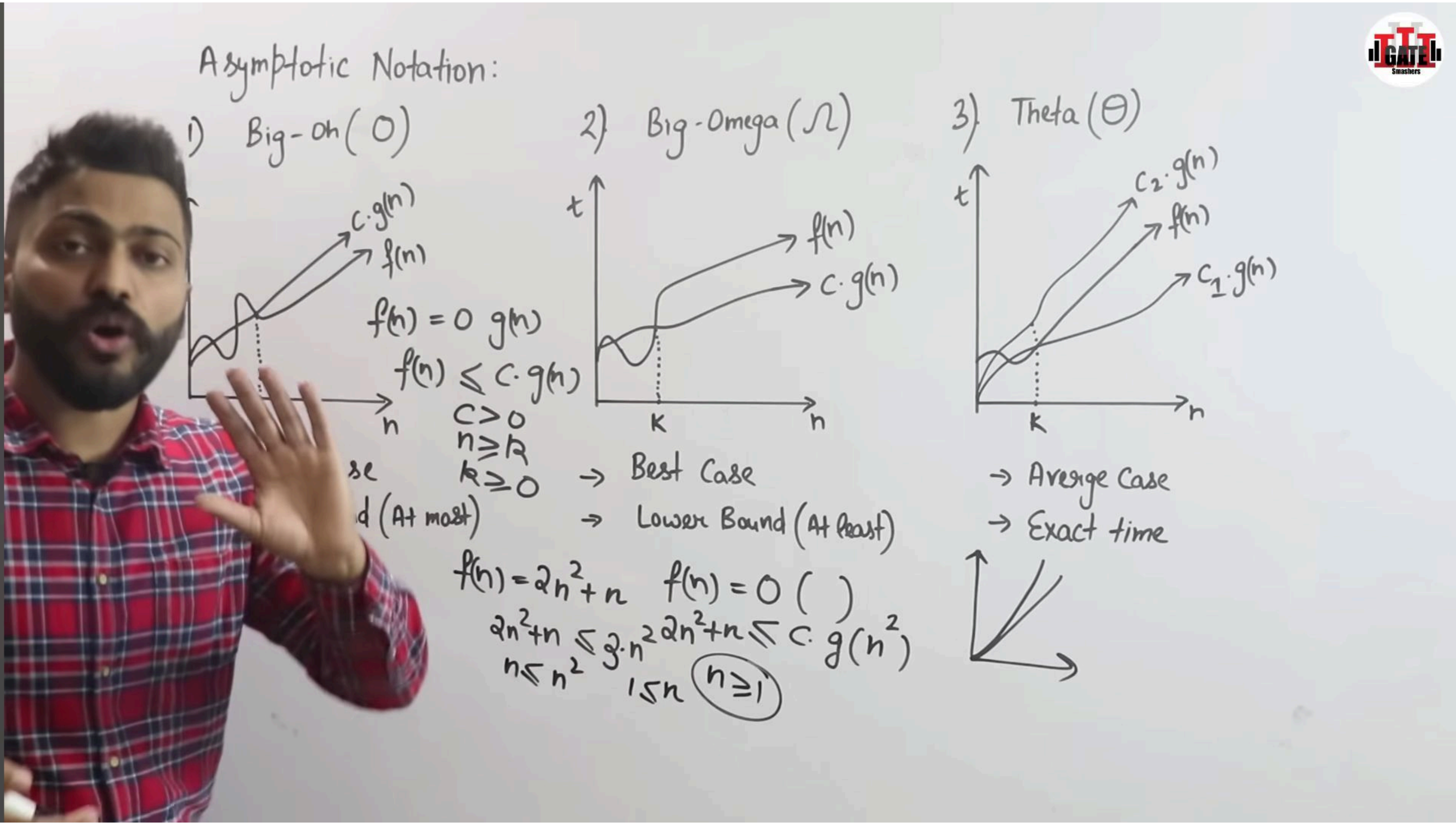
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Asymptotic Notation:
1) Big-Oh (O)
2) Big-Omega (Ω)
3) Theta (Θ)
c·g(n)
f(n)
n
t
f(n)
c·g(n)
k
n
t
c2·g(n)
f(n)
c1·g(n)
k
n
f(n) = O g(n)
f(n) ≤ C·g(n)
C > 0
n ≥ k
k ≥ 0
se
d (At most)
→ Best Case
→ Lower Bound (At least)
→ Averge Case
→ Exact time
f(n) = 2n² + n
f(n) = O ( )
2n² + n ≤ 3·n²
2n² + n ≤ C·g(n²)
n ≤ n²
1 ≤ n
n ≥ 1
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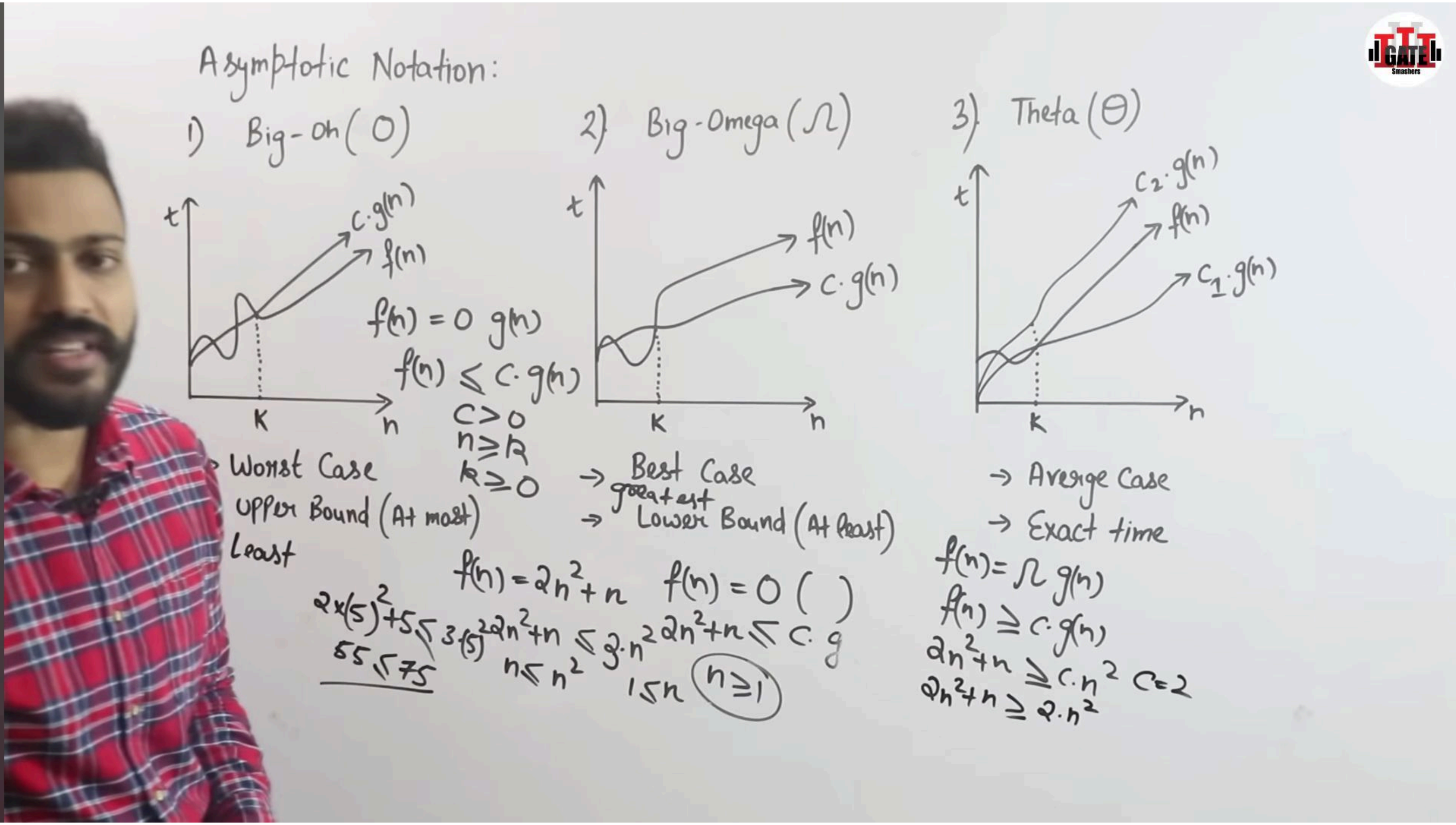

Asymptotic Notation:
1) Big-Oh (O)
t
c.g(n)
f(n)
K
n
f(n) = O g(n)
f(n) ≤ c.g(n)
c > 0
n ≥ k
k ≥ 0
→ Worst Case
Upper Bound (At most)
Least
2) Big-Omega (Ω)
t
f(n)
c.g(n)
K
n
→ Best Case
→ Lower Bound (At least)
greatest
3) Theta (Θ)
t
c2.g(n)
f(n)
c1.g(n)
K
n
→ Average Case
→ Exact time
f(n) = 2n^2 + n
f(n) = O ( )
2n^2 + n ≤ c. g
2x(5)^2+5 ≤ 3(5)^2
55 ≤ 75
2n^2+n ≤ 3.n^2
n ≤ n^2
1 ≤ n
n ≥ 1
f(n) = Ω g(n)
f(n) ≥ c.g(n)
2n^2+n ≥ c.n^2
c=2
2n^2+n ≥ 2.n^2
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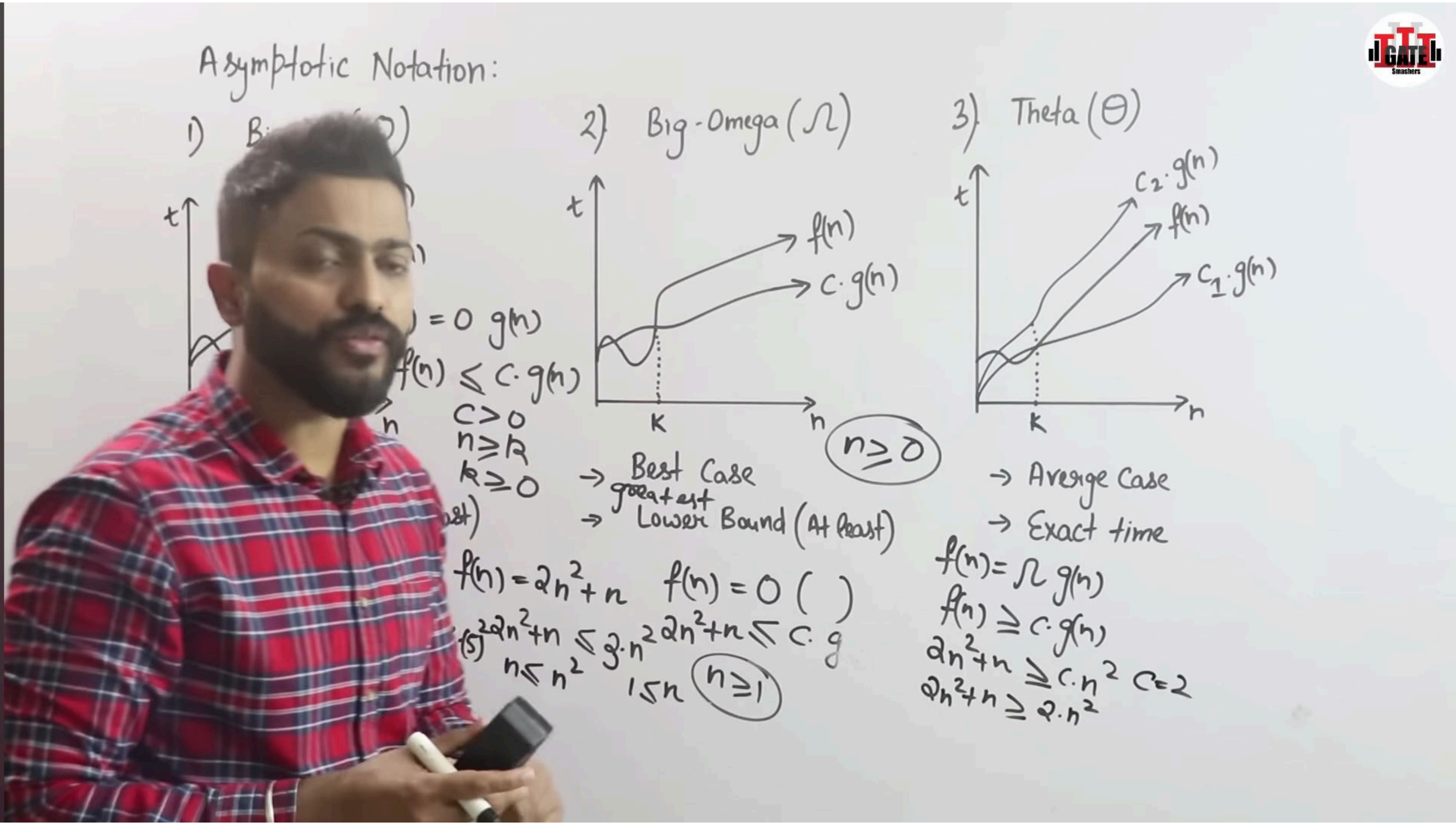

Asymptotic Notation:
1)
2) Big-Omega (Ω)
3) Theta (Θ)
t
n
K
f(n)
c·g(n)
c2·g(n)
c1·g(n)
= O g(n)
f(n) ≤ c·g(n)
c > 0
n ≥ k
k ≥ 0
n ≥ 0
→ Best Case
→ greatest Lower Bound (At least)
→ Averge Case
→ Exact time
f(n) = 2n² + n
f(n) = O ( )
2n² + n ≤ 3·n²
2n² + n ≤ c·g
n ≤ n²
1 ≤ n
n ≥ 1
f(n) = Ω g(n)
f(n) ≥ c·g(n)
2n² + n ≥ c·n²
c = 2
2n² + n ≥ 2·n²
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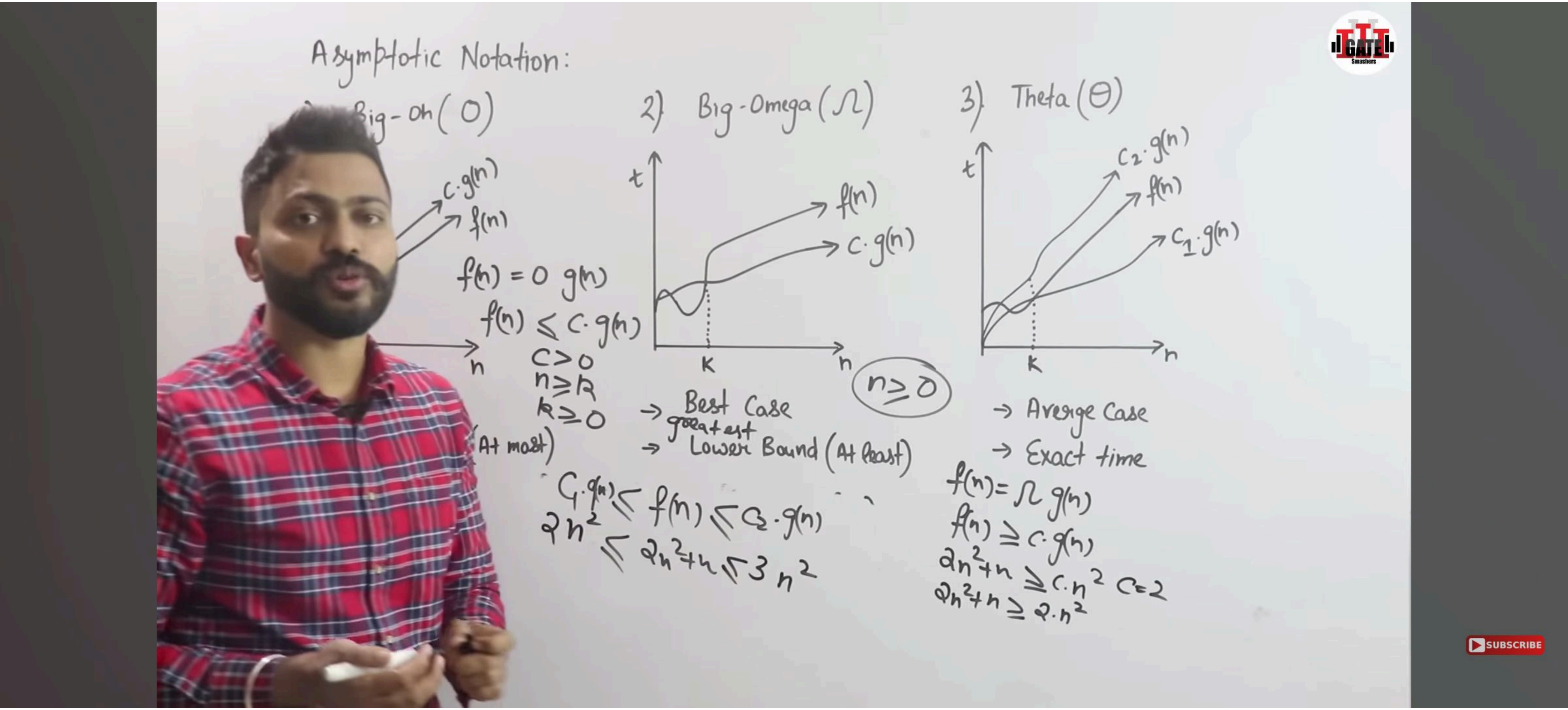
Asymptotic Notation:
Big-Oh (O)
c·g(n)
f(n)
f(n) = O g(n)
f(n) ≤ c·g(n)
n
c > 0
n ≥ k
k ≥ 0
(At most)
2) Big-Omega (Ω)
t
f(n)
c·g(n)
k
n
n ≥ 0
→ Best Case
greatest
→ Lower Bound (At least)
3) Theta (Θ)
t
c2·g(n)
f(n)
c1·g(n)
k
n
→ Averge Case
→ Exact time
f(n) = Ω g(n)
f(n) ≥ c·g(n)
2n²+n ≥ c·n² c=2
2n²+n ≥ 2·n²
c1·g(n) ≤ f(n) ≤ c2·g(n)
2n² ≤ 2n²+n ≤ 3n²
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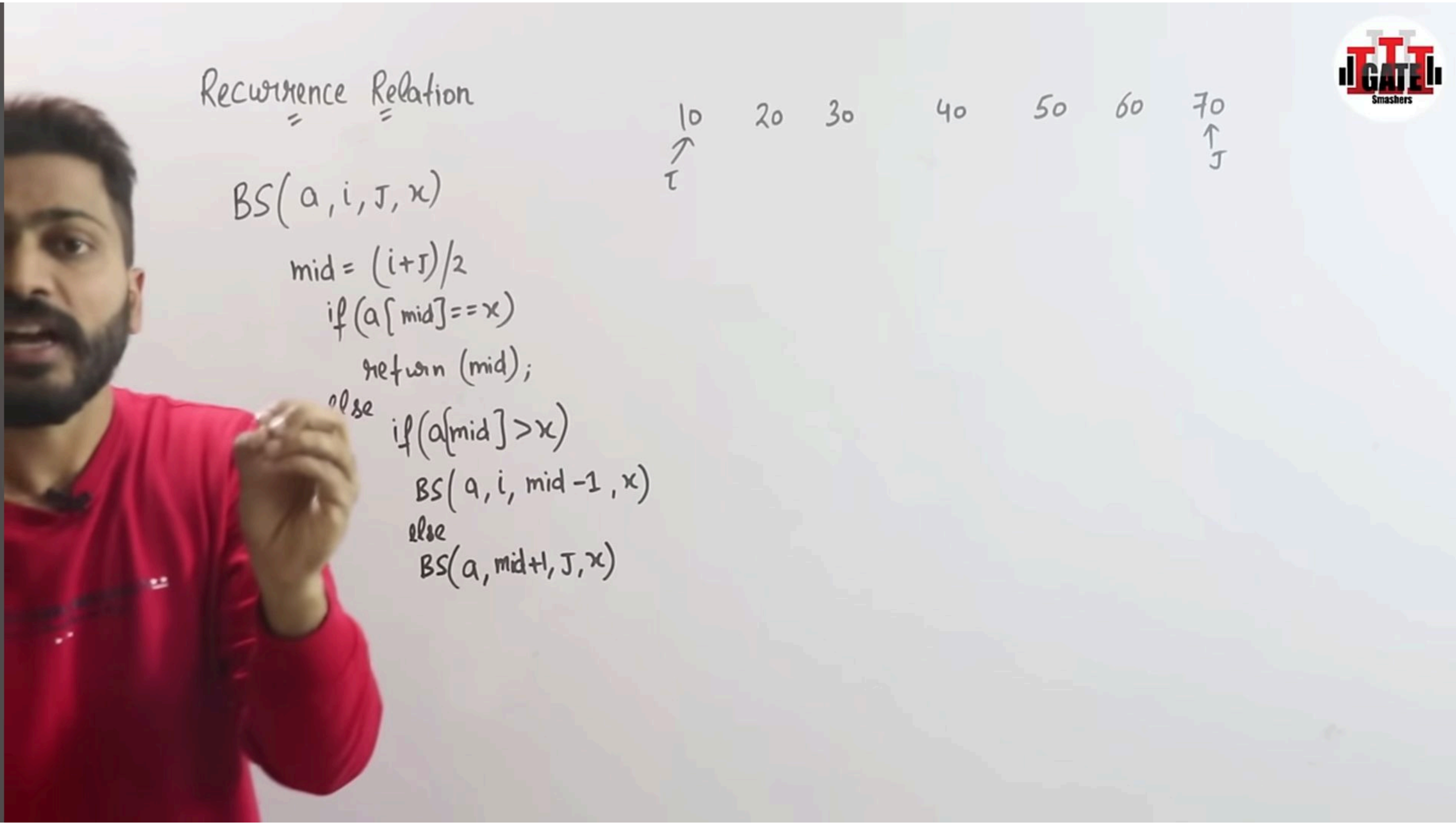
Recurrence relation


Recurrence Relation
10 20 30 40 50 60 70
i
J
BS(a, i, J, x)
mid = (i+J)/2
if (a[mid] == x)
return (mid);
else
if (a[mid] > x)
BS(a, i, mid-1, x)
else
BS(a, mid+1, J, x)
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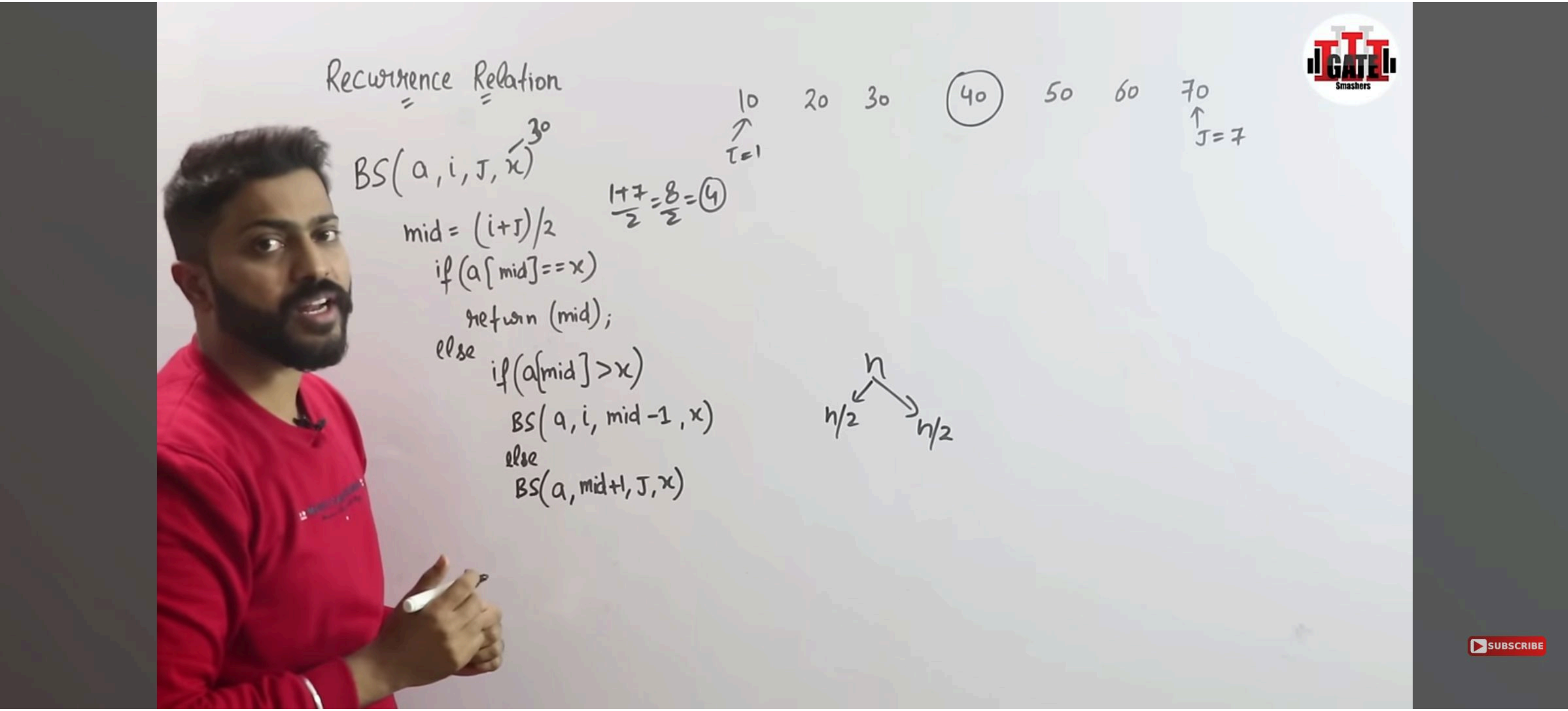


GATE Smashers
Recurrence Relation
10 20 30 40 50 60 70
I=1
J=7
30
BS(a, i, J, x)
1+7/2 = 8/2 = 4
mid = (i+J)/2
if(a[mid]==x)
return (mid);
else
if(a[mid]>x)
BS(a, i, mid-1, x)
else
BS(a, mid+1, J, x)
n
n/2
n/2
SUBSCRIBE

Recurrence Relation

$$BS(a, i, J, \overset{30}{x})$$

$$mid = (i+J)/2 \qquad \frac{1+7}{2} = \frac{8}{2} = 4$$

$$\text{if } (a[\underset{40}{mid}] == \underset{40}{x}) \qquad O(c)$$

return (mid);

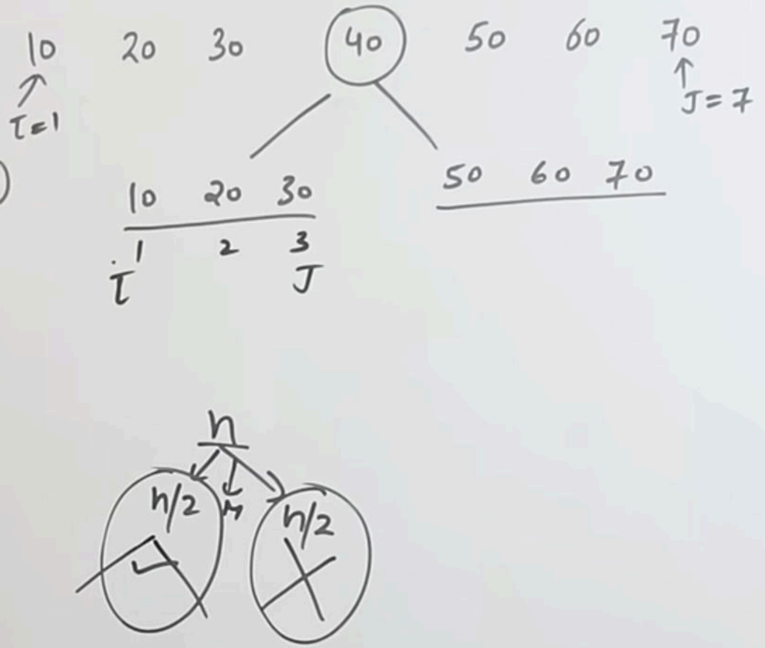
else

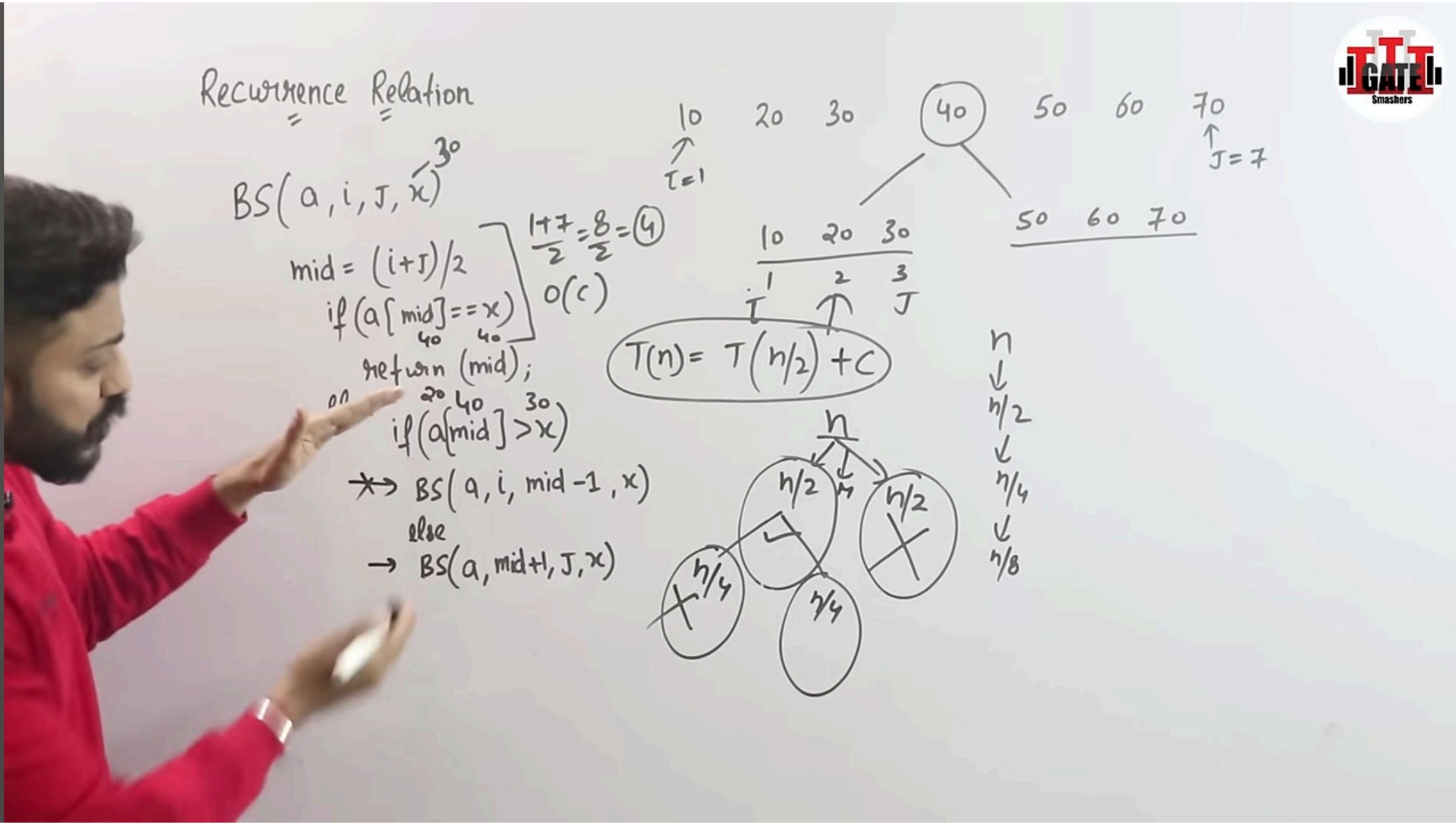
$$\text{if } (a[\overset{40}{mid}] > \overset{30}{x})$$

→ BS(a, i, mid-1, x)

else

→ BS(a, mid+1, J, x)





Recurrence Relation
BS(a, i, J, x)
mid = (i+J)/2
if (a[mid] == x)
return (mid);
if (a[mid] > x)
BS(a, i, mid-1, x)
else
BS(a, mid+1, J, x)
(1+7)/2 = 8/2 = 4
O(c)
10 20 30 40 50 60 70
I=1
J=7
10 20 30
1 2 3
I J
50 60 70
T(n) = T(n/2) + C
n
n/2
n/4
n/8
GATE Smashers
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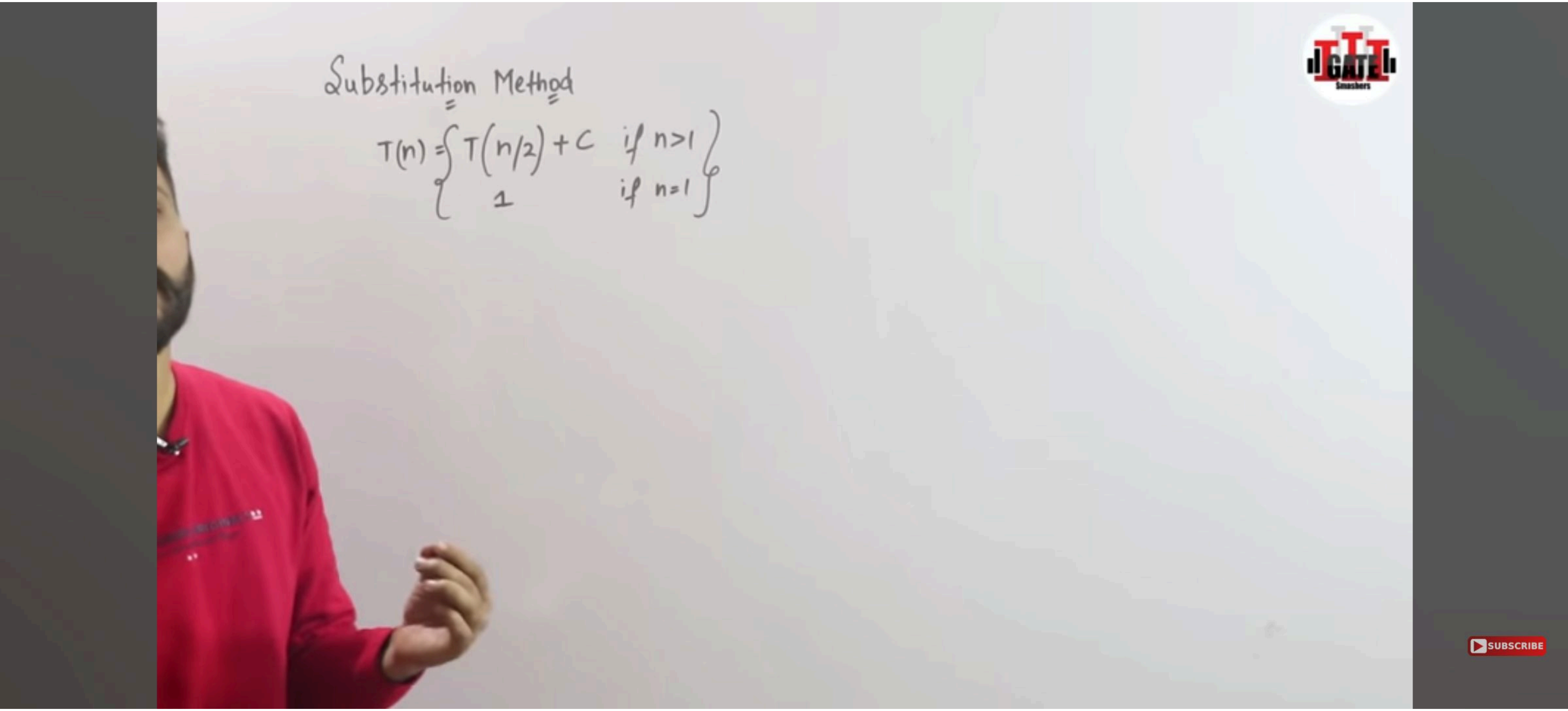
Substitution method


Substitution Method
T(n) = { T(n/2) + C if n>1
1 if n=1 }
GATE Smashers
SUBSCRIBE

Substitution Method

$$T(n) = \begin{cases} T(n/2) + C & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

$T(n) = T(n/2) + C$ — ①

$T(n/2) = T(n/4) + C$ — ②

$T(n/4) = T(n/8) + C$ — ③

$n = 2^k$

$\log n = \log 2^k$

$= k \log 2$

$T(n) = T(n/4) + C + C$

$= T(n/2^2) + 2C$

$= T(n/8) + C + 2C$

$= T(n/2^3) + 3C$

$= T(n/2^4) + 4C$

$T(n/2^5) + 5C$

⋮

k times

$T(n/2^k) + kC$

$T(n/n) + kC$

$T(1) + kC$

$1 + kC$

Substitution Method

$$T(n) = \begin{cases} T(n/2) + c & \text{if } n>1 \\ 1 & \text{if } n=1 \end{cases}$$

$T(n) = T(n/2) + c$ — ①

$T(n/2) = T(n/4) + c$ — ②

$T(n/4) = T(n/8) + c$ — ③

$T(n) = T(n/4) + c + c$

$= T(n/2^2) + 2c$ —

$= T(n/8) + c + 2c$

$= T(n/2^3) + 3c$ —

$= T(n/2^4) + 4c$ —

$T(n/2^5) + 5c$ -

⋮

k times

$T(n/2^k) + kc$

$n = 2^k$

$\log n = \log 2^k$

$= k \log 2$

$T(\cancel{n}/n) + kc$

$T(1) + kc$

$1 + kc$

$1 + \log n \cdot c$

$O(\log_2 n)$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

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$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

$n/2 \times 2$

$n/4 \times 2$

$$T(n/2) = 2T(n/4) + n/2$$

$$T(n/4) = 2T(n/8) + n/4$$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

$n/2 \times 2$ $\quad$ $n/4 \times 2$ $\quad$ ①

$$T(n/2) = 2T(n/4) + n/2 \quad \text{— } 2$$

$$T(n/4) = 2T(n/8) + n/4 \quad \text{— } 3$$

$$\cancel{2}\left[2T(n/4) + n/\cancel{2}\right] + n$$

$$2^2 T(n/2^2) + n + n$$

$$2^2 T(n/2^2) + 2n$$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases} \quad (1)$$

$n/2 \times 2$ $\quad n/4 \times 2$

$$T(n/2) = 2T(n/4) + n/2 \quad — 2$$

$$T(n/4) = 2T(n/8) + n/4 \quad — 3$$

$$\cancel{2}\left[2T(n/4) + n/\cancel{2}\right] + n$$

$$2^2 T(n/2^2) + n + n$$

$$2^2 T(n/2^2) + 2n$$

SUBSCRIBE

$$\begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

$n/2\times 2$ $\quad$ $n/4\times 2$ $\quad$ ①

$$= 2T(n/4) + n/2 \quad — 2$$

$$(n/4) = \underbrace{2T(n/8) + n/4}\quad — 3$$

$$\cancel{2}\left[2T(n/4) + \cancel{n/2}\right] + n$$

$$2^2\,T(n/2^2) + n + n$$

$$2^2\,\boxed{T(n/2^2)} + 2n$$

$$\cancel{2}^2\left[2T(n/8) + \cancel{n/4}\right] + 2n$$

$$2^3\,T(n/2^3) + 3n$$

$$\vdots$$

$$2^4\,T(n/2^4) + 4n, \quad 2^5\,T(n/2^5) + 5n$$

$$\vdots$$

k times

$$2^k\,T(n/2^k) + kn$$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

$n/2 \times 2$ $\quad$ $n/4 \times 2$ $\quad$ 1

$$T(n/2) = 2T(n/4) + n/2 \quad - 2$$

$$T(n/4) = \underbrace{2T(n/8) + n/4}\quad - 3$$

$$2\left[2T(n/4) + n/2\right] + n$$

$$2^2\, T(n/2^2) + n + n$$

$$2^2\, \boxed{T(n/2^2)} + 2n$$

$$2^2\left[2T(n/8) + n/4\right] + 2n$$

$$2^3\, T(n/2^3) + 3n$$

$$2^4\, T(n/2^4) + 4n, \quad 2^5\, T(n/2^5) + 5n$$

k times

$$2^k\, T(\underbrace{n/2^k}_{T(1)}) + kn \qquad T(1) = 1$$

$$n/2^k = 1$$

$$n = 2^k$$

$$\log n = \log_2 2^k$$

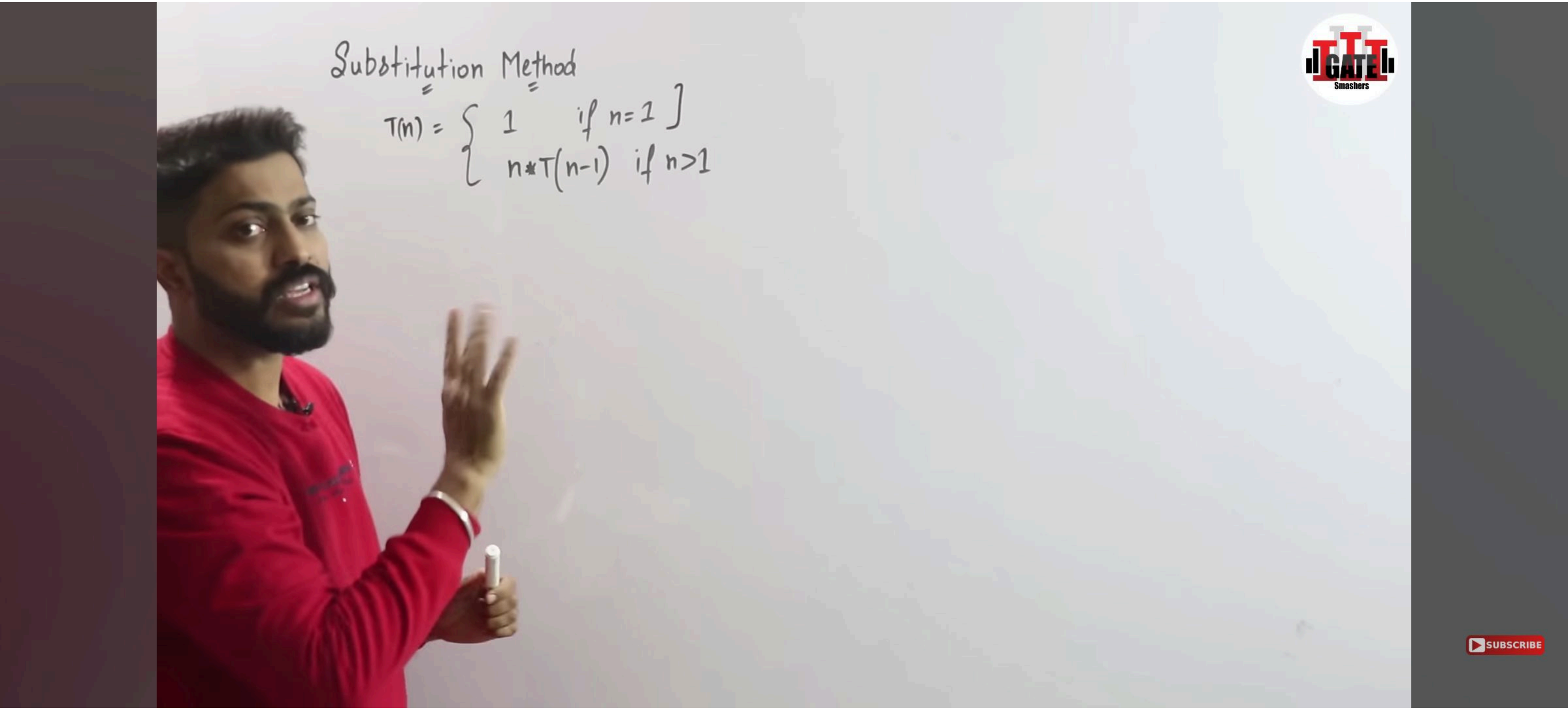
$$\log n = k.1$$

$$2^k\, T(1) + kn$$

$$n \cdot 1 + n \cdot \log n$$

$$n + n \log n$$

$$O(n \log n)$$


Substitution Method
T(n) = { 1 if n=1]
{ n*T(n-1) if n>1
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Substitution Method

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ n * T(n-1) & \text{if } n > 1 \end{cases}$$

$$T(n) = n * T(n-1) \quad \text{— (1)}$$

$$T(n-1) = (n-1) * T((n-1)-1)$$

$$= (n-1) * T(n-2) \quad \text{— (2)}$$

$$T(n-2) = (n-2) * T(n-3) \quad \text{— (3)}$$

$$T(n) = n * (n-1) * T(n-2)$$

$$= n * (n-1) * (n-2) * T(n-3)$$

$$\vdots$$

Substitution Method

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ n * T(n-1) & \text{if } n > 1 \end{cases}$$

$$T(n) = n * T(n-1) \quad \text{— } 1$$

$$T(n-1) = (n-1) * T((n-1)-1)$$

$$= (n-1) * T(n-2) \quad \text{— } 2$$

$$T(n-2) = (n-2) * T(n-3) \quad \text{— } 3$$

$$T(n) = n * (n-1) * T(n-2)$$

$$= n * (n-1) * (n-2) * T(n-3)$$

$$=$$

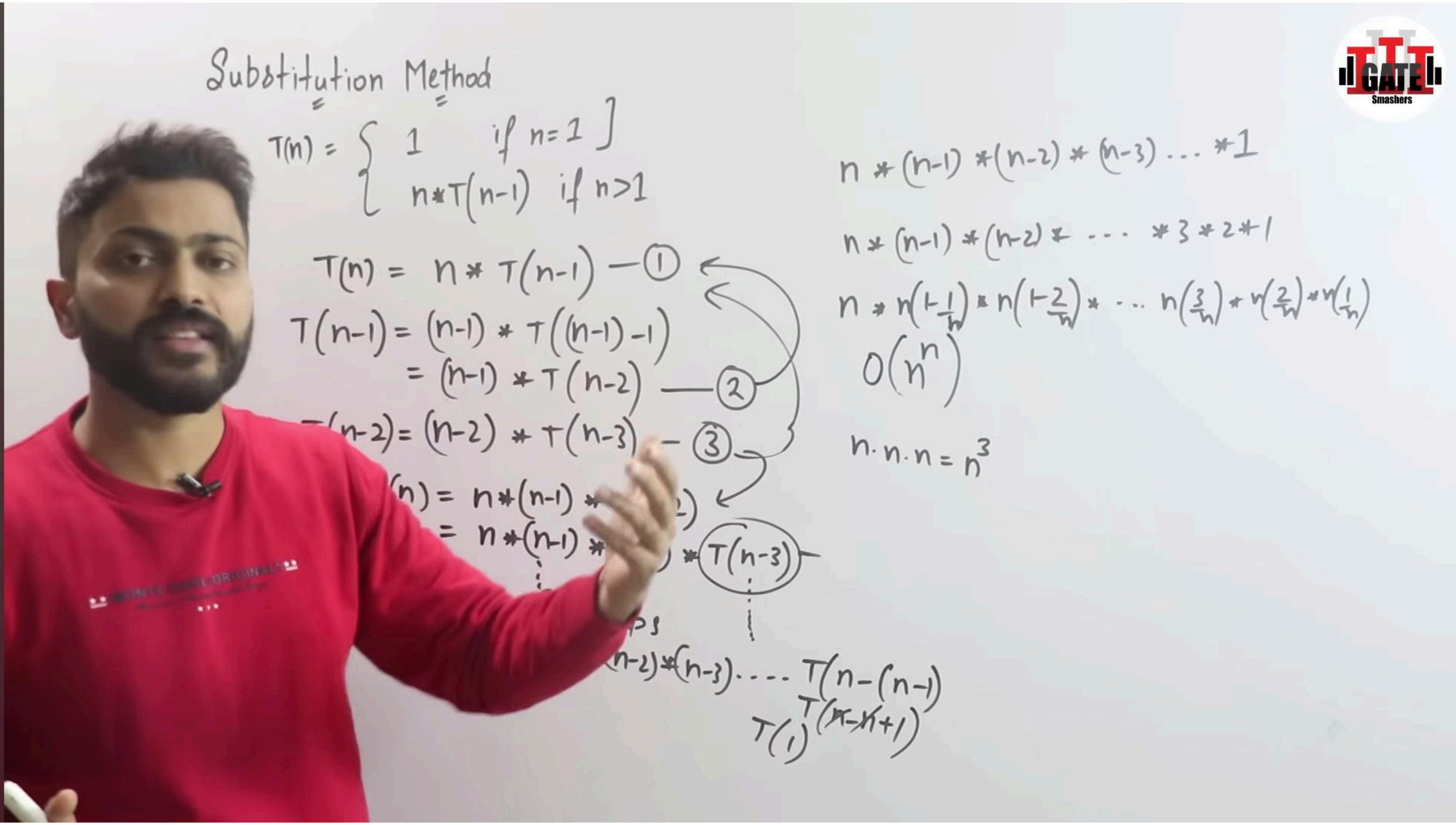
(n-1) steps

$$n * (n-1) * (n-2) * (n-3) \ldots T(n-(n-1))$$

$$T(\cancel{n}-\cancel{n}+1)$$

$$T(1)$$

$$n * (n-1) * (n-2) * (n-3) \ldots *$$


Substitution Method
T(n) = 1 if n=1
n*T(n-1) if n>1
T(n) = n * T(n-1) — 1
T(n-1) = (n-1) * T((n-1)-1)
= (n-1) * T(n-2) — 2
(n-2) = (n-2) * T(n-3) — 3
n * (n-1) * (n-2) * (n-3) . . . * 1
n * (n-1) * (n-2) * . . . * 3 * 2 * 1
n * n(1-1/n) * n(1-2/n) * . . . n(3/n) * n(2/n) * n(1/n)
O(n^n)
n.n.n = n^3
(n-2)*(n-3) . . . . T(n-(n-1))
T(1)
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$$T(n) = \begin{cases} 1 & , \text{if } n = 1 \\ T(n-1) + \log n & , \text{if } n > 1 \end{cases} \quad \text{— } 1$$

$$T(n-1) = T(n-2) + \log(n-1) \quad \text{— } 2$$

$$T(n-2) = T(n-3) + \log(n-2) \quad \text{— } 3$$

$$T(n) = T(n-2) + \log(n-1) + \log n$$

$$T(n-3) + \log(n-2) + \log(n-1) + \log n$$

$$T(n-4) + \log(n-3) + \log(n-2) + \log(n-1) + \log n$$

$$\vdots$$

$$T(n) = \begin{cases} 1 & , \text{if } n = 1 \\ T(n-1) + \log n & , \text{if } n > 1 \end{cases} \quad - (1)$$

$$T(n-1) = T(n-2) + \log(n-1) \quad - (2)$$

$$T(n-2) = T(n-3) + \log(n-2) \quad - (3)$$

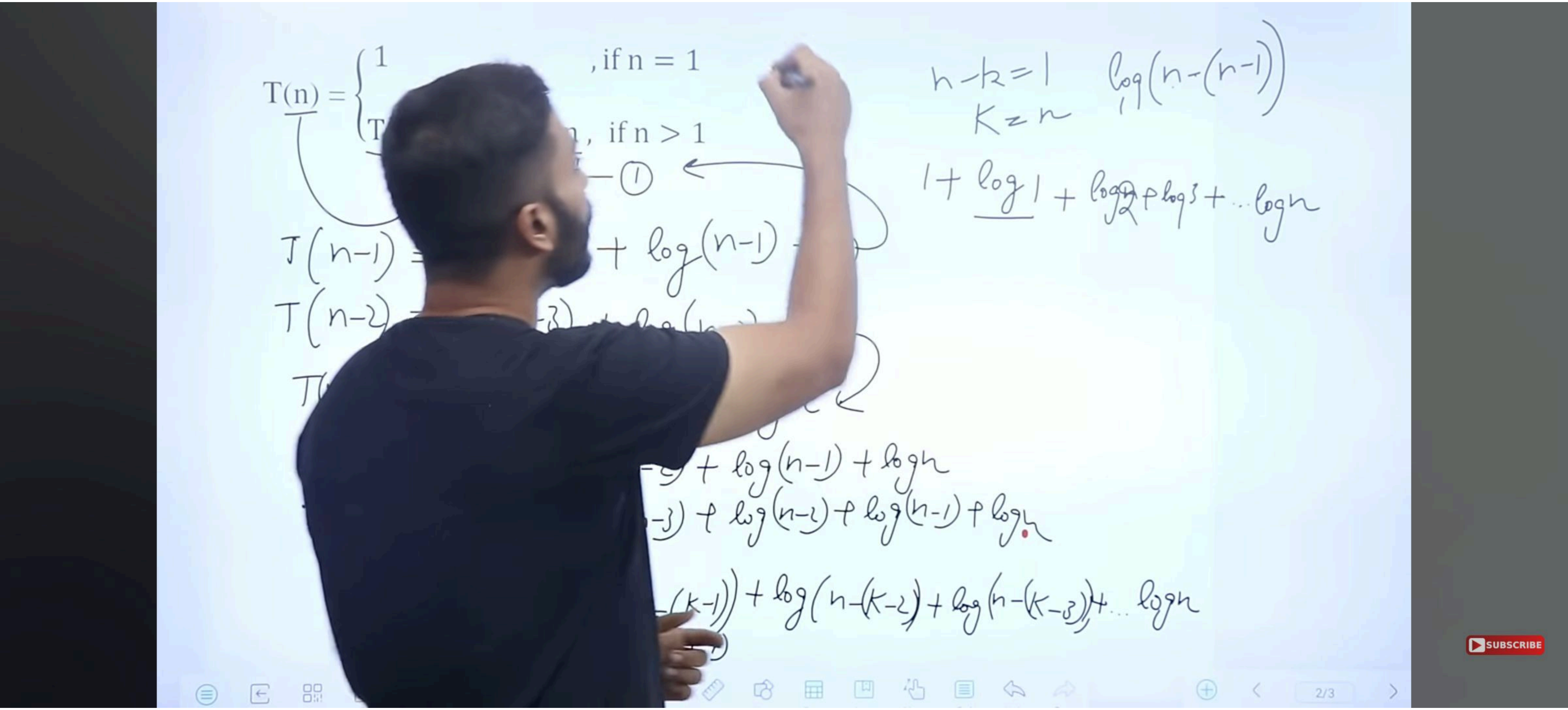
$$T(n) = T(n-2) + \log(n-1) + \log n$$

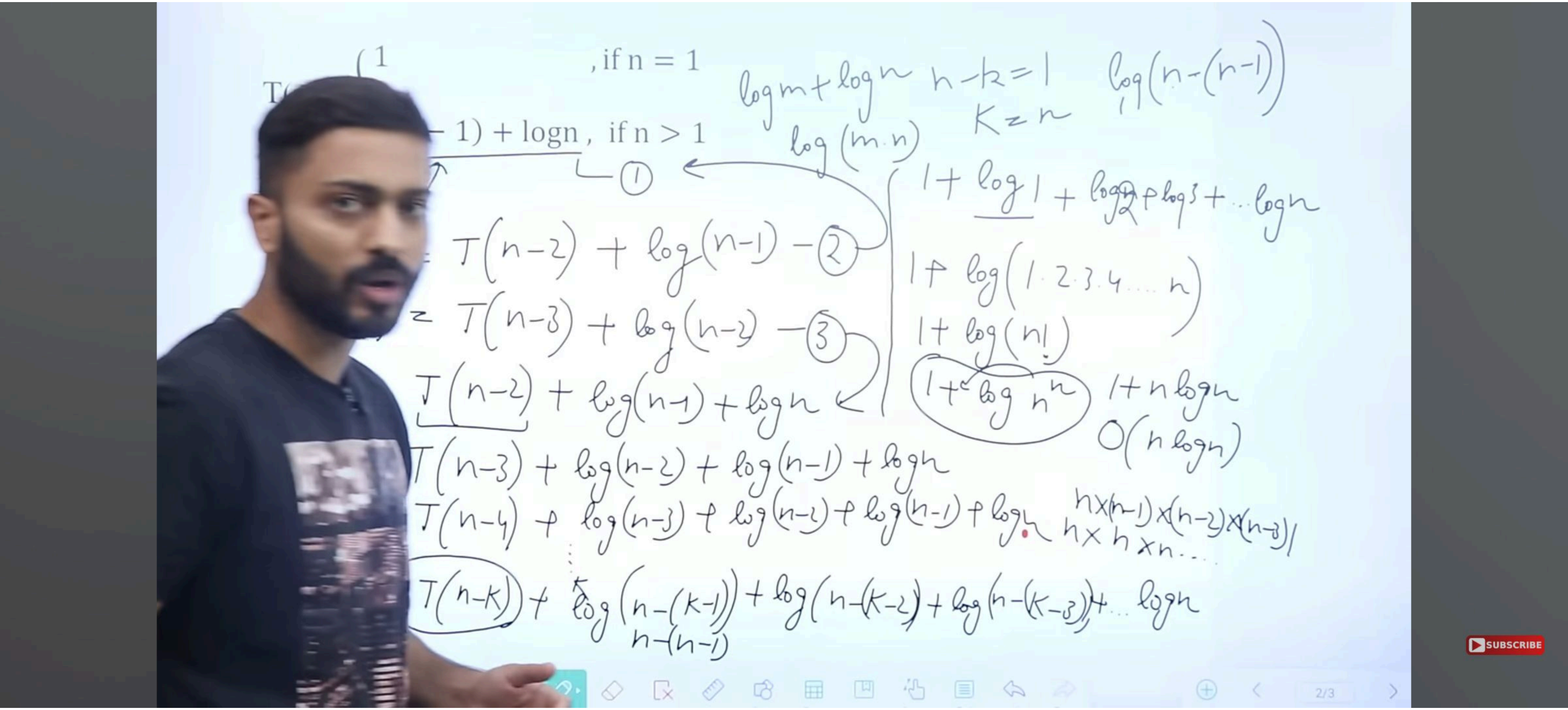
$$T(n-3) + \log(n-2) + \log(n-1) + \log n$$

$$T(n-4) + \log(n-3) + \log(n-2) + \log(n-1) + \log n$$

$$\vdots$$

$$T(n-k) + \log(n-(k-1)) + \log(n-(k-2)) + \log(n-(k-3)) + \dots \log n$$


T(n) = { 1 , if n = 1
n, if n > 1
— 1
n − k = 1
K = n
log(n − (n − 1))
1 + log 1 + log 2 + log 3 + … log n
T(n−1) =
+ log(n−1)
T(n−2)
+ log(n−1) + log n
−3) + log(n−2) + log(n−1) + log n
−(K−1)) + log(n−(K−2)) + log(n−(K−3)) + … log n
2/3
SUBSCRIBE


1 , if n = 1
1) + logn , if n > 1
log m + log n
log (m·n)
n − k = 1
K = n
log(n − (n−1))
= T(n−2) + log(n−1) — 2
= T(n−3) + log(n−2) — 3
T(n−2) + log(n−1) + log n
T(n−3) + log(n−2) + log(n−1) + log n
T(n−4) + log(n−3) + log(n−2) + log(n−1) + log n
T(n−K) + log(n−(K−1)) + log(n−(K−2)) + log(n−(K−3)) + … log n
n−(n−1)
1 + log 1 + log 2 + log 3 + … log n
1 + log(1·2·3·4 …. n)
1 + log(n!)
1 + c log n^n
1 + n log n
O(n log n)
n×(n−1)×(n−2)×(n−3)
n × n × n …
2/3
SUBSCRIBE

Master Theorem/ Method

* Master Method (Theorem)

→ $T(n) = aT(n/b) + f(n)$

$a \geq 1 \; , \; b > 1$

→ Solution is:

$$T(n) = n^{\log_b a}\left[U(n)\right]$$

→ $U(n)$ depends on $h(n)$

→ $h(n) = \frac{f(n)}{n^{\log_b a}}$

→ Relation between $h(n)$ and $U(n)$ is:

if $h(n)$	$U(n)$
$n^r, r>0$	$O(n^r)$
$n^r, r<0$	$O(1)$
$(\log n)^i, i\geq 0$	$\frac{(\log_2 n)^{i+1}}{i+1}$

1) $T(n) = 8T(n/2) + n^2$

2) $T(n) = T(n/2) + C$

* Master Method (Theorem)

→ $T(n) = aT(n/b) + f(n)$

$\boxed{a \geq 1}$, $\boxed{b > 1}$

→ Solution is:

$$T(n) = n^{\log_b a}\left[U(n)\right]$$

→ $U(n)$ depends on $h(n)$

→ $h(n) = \dfrac{f(n)}{n^{\log_b a}}$

→ Relation between $h(n)$ and $U(n)$ is:

if $h(n)$	$U(n)$
$n^r, r>0$	$O(n^r)$
$n^r, r<0$	$O(1)$
$(\log n)^i, i \geq 0$	$\dfrac{(\log_2 n)^{i+1}}{i+1}$

1) $T(n) = 8T(n/2) + n^2$

2) $T(n) = T(n/2) + C$

$\cancel{T(n)} = 1.T\left(\frac{n-1}{1}\right) + 1$

* Master Method (Theorem)

... $= aT(n/b) + f(n)$

$\boxed{a \geq 1}$, $\boxed{b > 1}$

... is:

... $= n^{\log_b a} [U(n)]$ ✓

... depends on $h(n)$

$h(n) = \dfrac{f(n)}{n^{\log_b a}}$

... between $h(n)$ and ... is:

$h(n)$	$U(n)$
$n^r, r>0$	$O(n^r)$
$n^r, r<0$	$O(1)$
$(\log n)^i, i \geq 0$	$\dfrac{(\log_2 n)^{i+1}}{i+1}$

$T(\cancel{n}) = 1.T\left(\frac{n-1}{1}\right) + 1$

1) $T(n) = 8T(n/2) + n^2$

2) $T(n) = T(n/2) + c$

$T(n) = 8T(n/2) + n^2$

$a = 8 \quad b = 2 \quad f(n) = n^2$

$T(n) = n^{\log_b a} U(n)$

$= n^{\log_2 8} U(n)$

$= n^3 U(n)$

* Master Method (Theorem)

→ $T(n) = a\,T(n/b) + f(n)$

$\boxed{a \geq 1}\,,\ \boxed{b > 1}$

$\cancel{T(n)} = 1.T\left(\frac{n-1}{1}\right) + 1$

→ Solution is:

$$T(n) = n^{\log_b a}\left[U(n)\right]$$ ✓

→ $U(n)$ depends on $h(n)$

→ $h(n) = \dfrac{f(n)}{n^{\log_b a}} = \dfrac{n^2}{n^{\log_2 8}} = \dfrac{n^2}{n^3} = \dfrac{1}{n} = n^{-1}$

→ Relation between $h(n)$ and $U(n)$ is:

if $h(n)$	$U(n)$
→ $n^r,\ r>0$	$O(n^r)$
→ $n^r,\ r<0$	$O(1)$
→ $(\log n)^i,\ i\geq 0$	$\dfrac{(\log_2 n)^{i+1}}{i+1}$

1) $T(n) = 8T(n/2) + n^2$

2) $T(n) = T(n/2) + C$

$T(n) = 8T(n/2) + n^2$

$a = 8 \quad b = 2 \quad f(n) = n^2$

$$T(n) = n^{\log_b a}\, U(n)$$
$$= n^{\log_2 8}\, U(n)$$
$$= n^3\, U(n)$$

* Master Method (Theorem)

$T(n) = aT(n/b) + f(n)$

$\boxed{a \geq 1}$, $\boxed{b > 1}$

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...on is:

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$a=8 \quad b=2 \quad f(n)=n^2$

$T(n) = n^{\log_b a}\, U(n)$

$= n^{\log_2 8}\, U(n)$

$= n^3\, \frac{U(n)}{1}$

$n^3 \cdot 1 = O(n^3)$

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$\boxed{a \geq 1}$, $\boxed{b > 1}$

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$T(n) = T(n/2) + c$

$a = 1 \quad b = 2 \quad f(n) = c$

$T(n) = n^{\log_b a}\, U(n)$

$= n^{\log_2 1}\, U(n) \Rightarrow n^0\, U(n) \Rightarrow U(n)$

* Master Method (Theorem)

→ $T(n) = aT(n/b) + f(n)$

$\boxed{a \geq 1}$, $\boxed{b > 1}$

→ Solution is:

$$T(n) = n^{\log_b a}[U(n)]$$

→ $U(n)$ depends on $h(n)$

→ $h(n) = \frac{f(n)}{n^{\log_b a}} = \frac{C}{n^{\log_2 1}} = \frac{C}{n^0} = \frac{C}{1} = C$

→ Relation between $h(n)$ and $U(n)$ is:

if $h(n)$	$U(n)$
$n^r, r>0$	$O(n^r)$
$n^r, r<0$	$O(1)$
$(\log n)^i, i \geq 0$	$\frac{(\log_2 n)^{i+1}}{i+1}$

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$a = 1 \quad b = 2 \quad f(n) = C$

$T(n) = n^{\log_b a} \; U(n)$

$= n^{\log_2 1} \; U(n) \Rightarrow n^0 \; U(n) \Rightarrow U(n)$

$h(n) = \frac{f(n)}{n^{\log_b a}}$

* Master Method (Theorem)

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$\boxed{a \geq 1}$, $\boxed{b > 1}$

...on is:

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...) depends on $h(n)$

$h(n) = \frac{f(n)}{n^{\log_b a}} = \frac{C}{n^{\log_2 1}} = \frac{c}{n^0} = \frac{c}{1} = c$

...ion between $h(n)$ and $U(n)$ is:

if $h(n)$	$U(n)$
$n^r, r>0$	$O(n^r)$
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$(\log n)^i, i \geq 0$	$\frac{(\log_2 n)^{i+1}}{i+1}$

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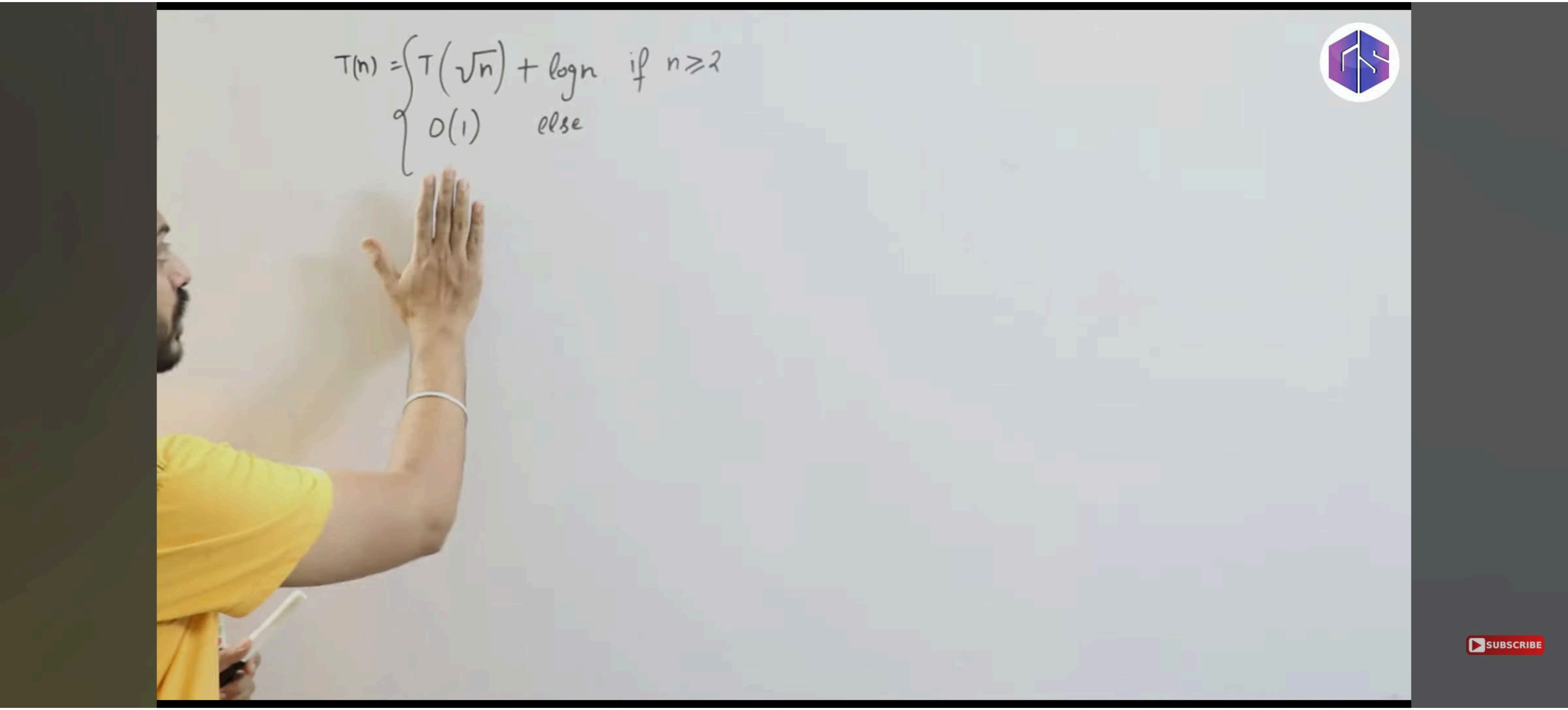
$= n^{\log_2 1} \; U(n) \Rightarrow n^0 \; U(n) \Rightarrow U(n)$

$h(n) = \frac{f(n)}{n^{\log_b a}} = (\log_2 n)^0 \cdot c$


$T(n) = \begin{cases} T(\sqrt{n}) + \log n & \text{if } n \geq 2 \\ O(1) & \text{else} \end{cases}$
SUBSCRIBE

$$T(n) = \begin{cases} T(\sqrt{n}) + \log n & \text{if } n \geq 2 \\ O(1) & \text{else} \end{cases}$$

$$T(n) = T(\sqrt{n}) + \log n \qquad T(n) = aT\left(\frac{n}{b}\right) + n^k \log^p n$$

$$n = 2^m \qquad (n)^{1/2} \qquad \log 2^m$$

$$T(2^m) = T(2^{m/2}) + m \qquad (2^m)^{1/2}$$

$$2^{m/2} \qquad m \log_2 2$$

$$m$$

$$T(n) = \begin{cases} T(\sqrt{n}) + \log n & \text{if } n \geq 2 \\ O(1) & \text{else} \end{cases}$$

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$$(n)^{1/2}$$

$$(2^m)^{1/2}$$

$$2^{m/2}$$

$$\log 2^m$$

$$m \log_2 2$$

$$m$$

$$T(2^m) = T(2^{m/2}) + m$$

$$T(2^m) = S(m)$$

$$S(m) = S(m/2) + m$$

$$T(n) = \begin{cases} T(\sqrt{n}) + \log n & \text{if } n \geq 2 \\ O(1) & \text{else} \end{cases}$$

$$\log n = \log 2^m$$

$$T(n) = T(\sqrt{n}) + \log n \qquad T(n) = aT\left(\frac{n}{b}\right) + n^k \log^p n$$

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$$(n)^{1/2} \qquad \log 2^m$$

$$(2^m)^{1/2}$$

$$T(2^m) = T(2^{m/2}) + m$$

$$2^{m/2} \qquad m \log_2 2$$

$$m$$

$$T(2^m) = S(m)$$

$$S(m) = S(m/2) + m \qquad a = 1 \quad b = 2 \quad k = 1 \quad p = 0$$

$$= n^k \log^p n$$

$$= m^1 \log^{\cancel{0}} n$$

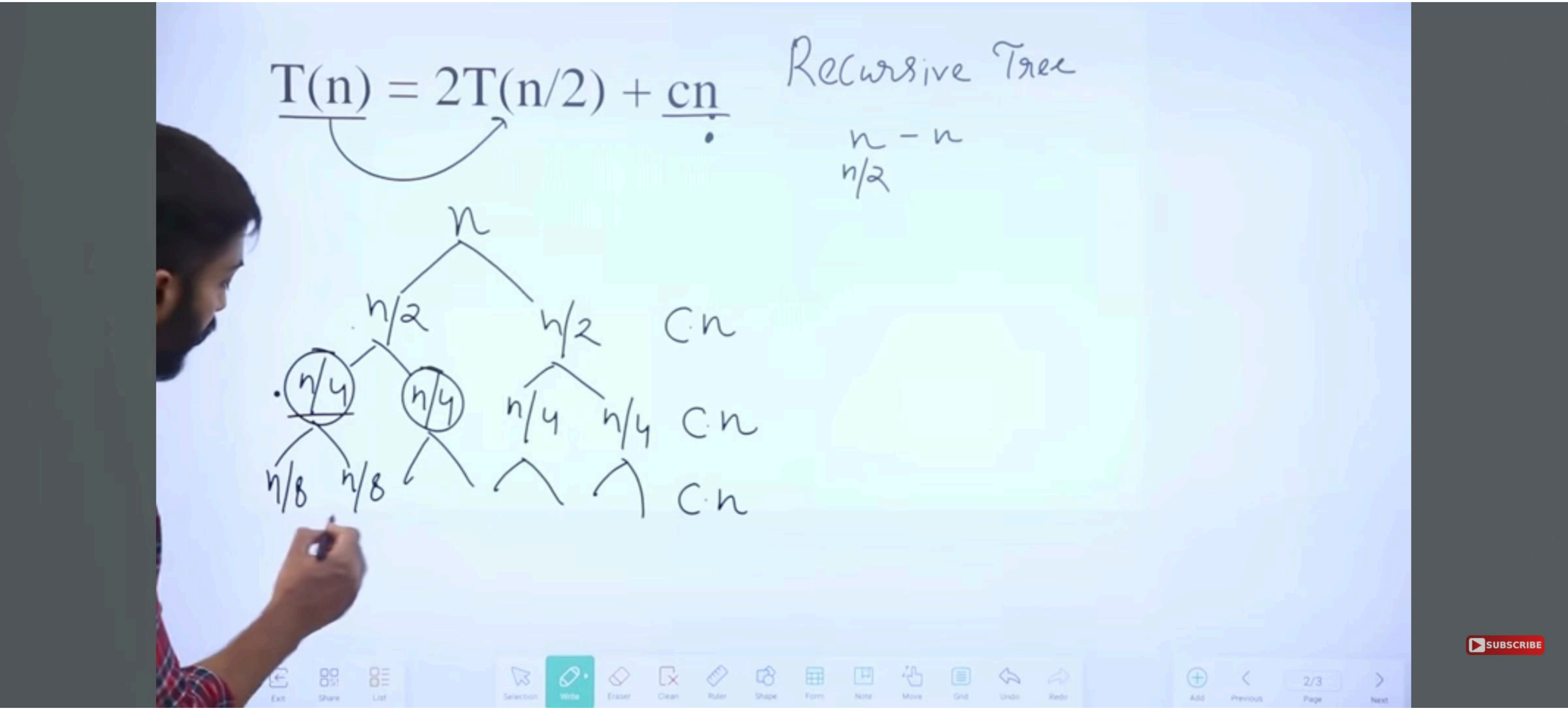
$$m \qquad \log n$$

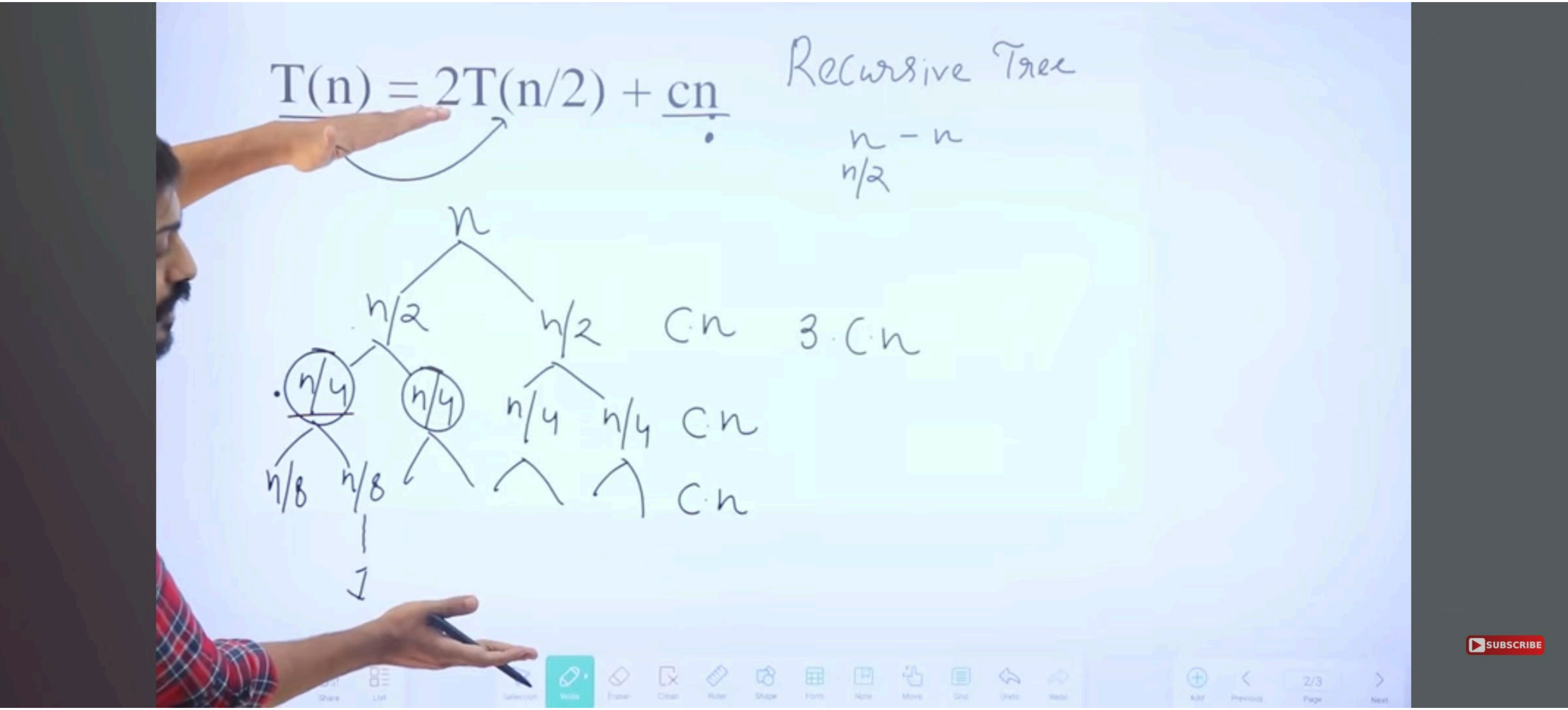
$$a < b^k$$

$$1 < 2^1 \quad 1 < 2$$

Recursive tree method

$$T(n) = 2T(n/2) + cn$$


$T(n) = 2T(n/2) + cn$
Recursive Tree
n - n
n/2
n
n/2 n/2 C·n
n/4 n/4 n/4 n/4 C·n
n/8 n/8 C·n
SUBSCRIBE


T(n) = 2T(n/2) + cn
Recursive Tree
n - n
n/2
n
n/2
n/2
C·n
3·C·n
n/4
n/4
n/4
n/4
C·n
n/8
n/8
C·n
1
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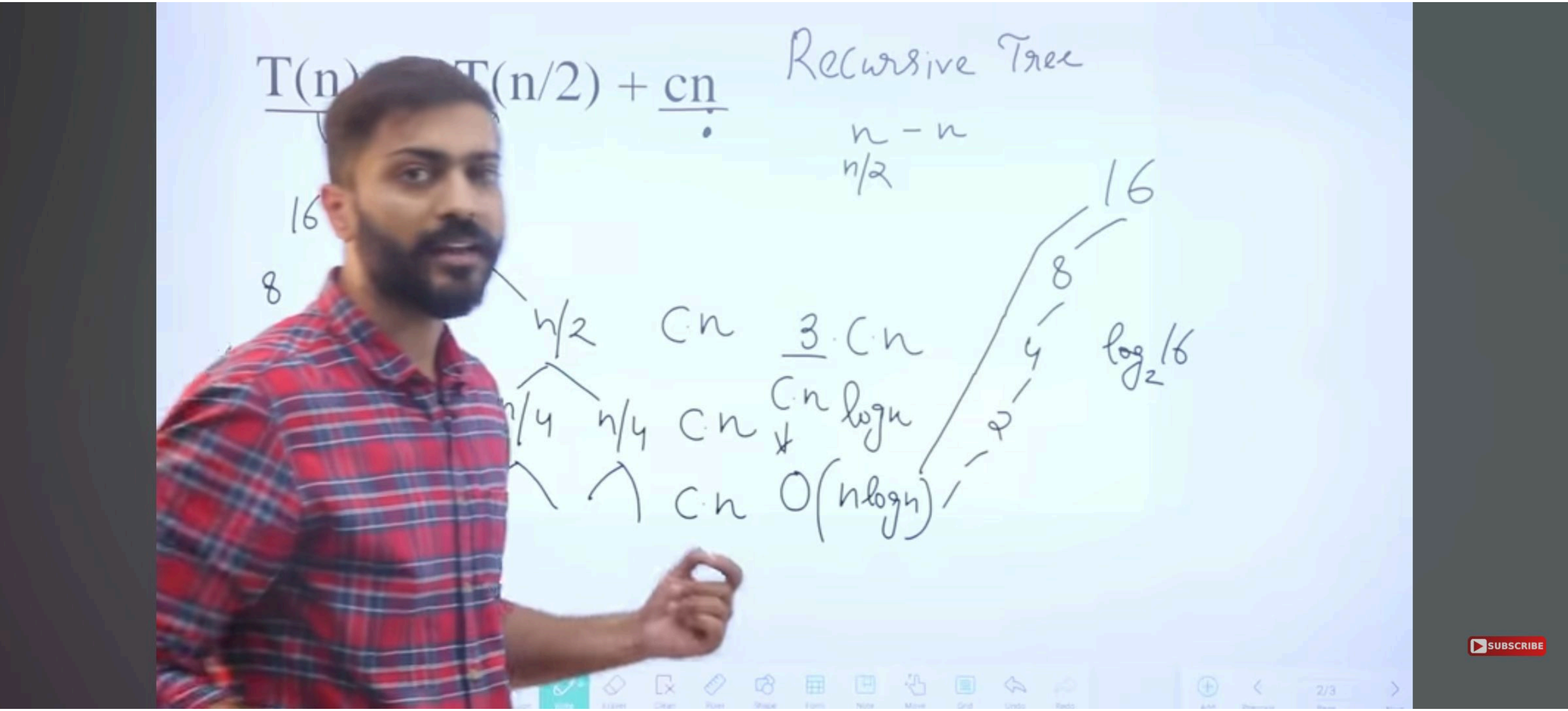
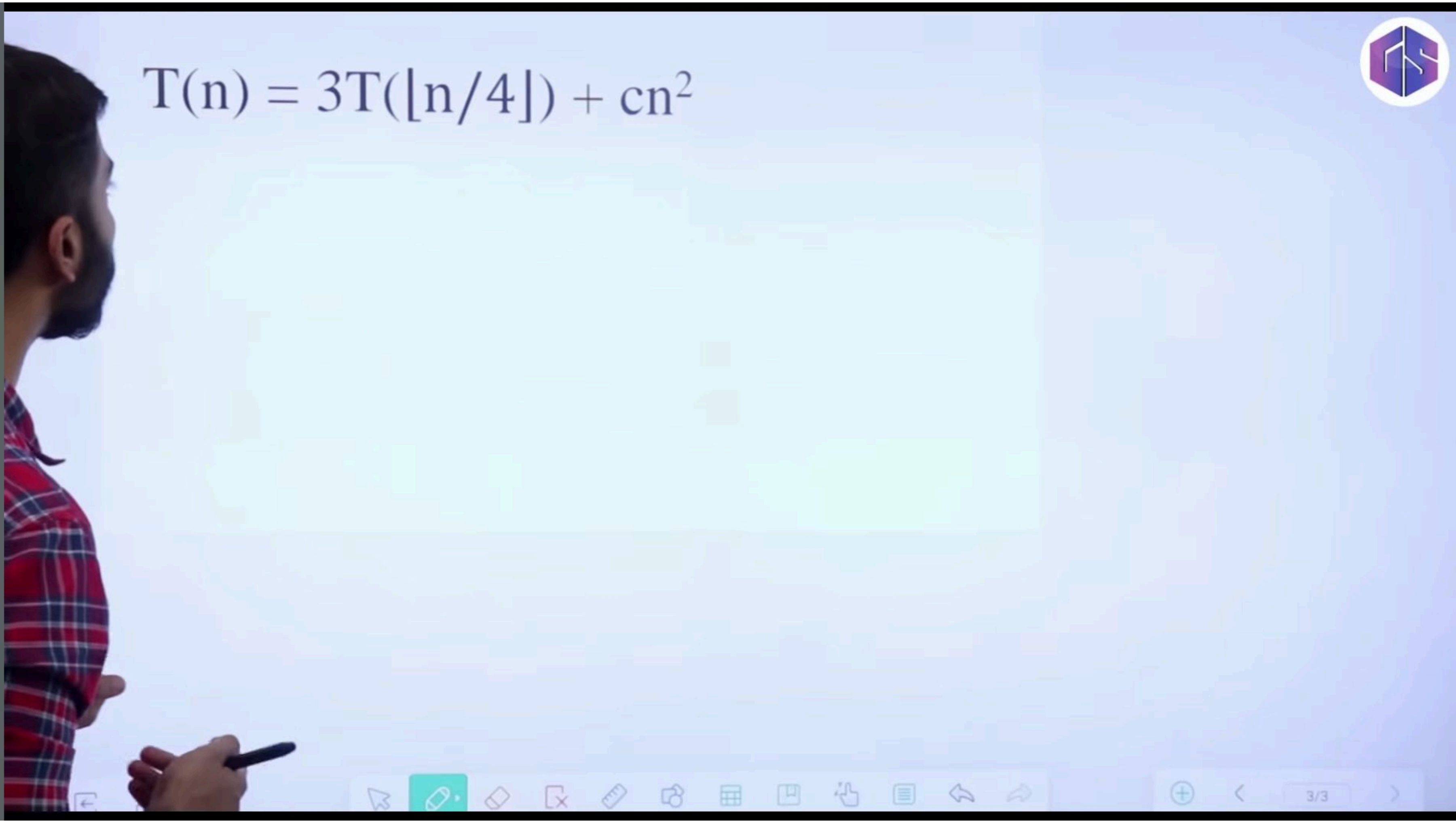
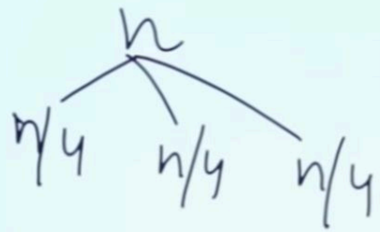

T(n/2) + cn
Recursive Tree
n - n
n/2
16
8
n/2
cn
3 · cn
cn logn
n/4
n/4
cn
cn
O(nlogn)
16
8
4
2
log2 16
SUBSCRIBE


$T(n) = 3T(\lfloor n/4 \rfloor) + cn^2$
3/3
SUBSCRIBE

$$T(n) = 3T(\lfloor n/4 \rfloor) + cn^2$$

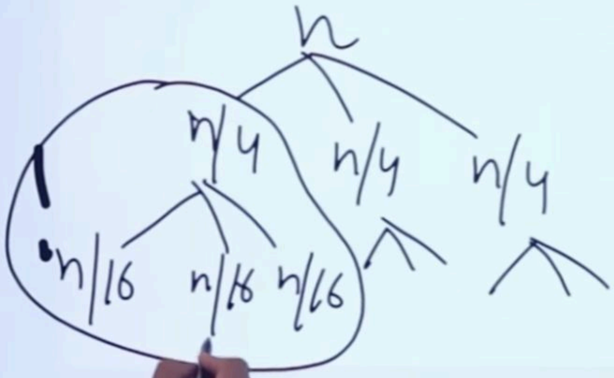


$$T(n) = 3T(\lfloor n/4 \rfloor) + cn^2$$

$n/4$ $\quad$ $n/4 \times \frac{1}{4}$ $\quad$ $n/16$

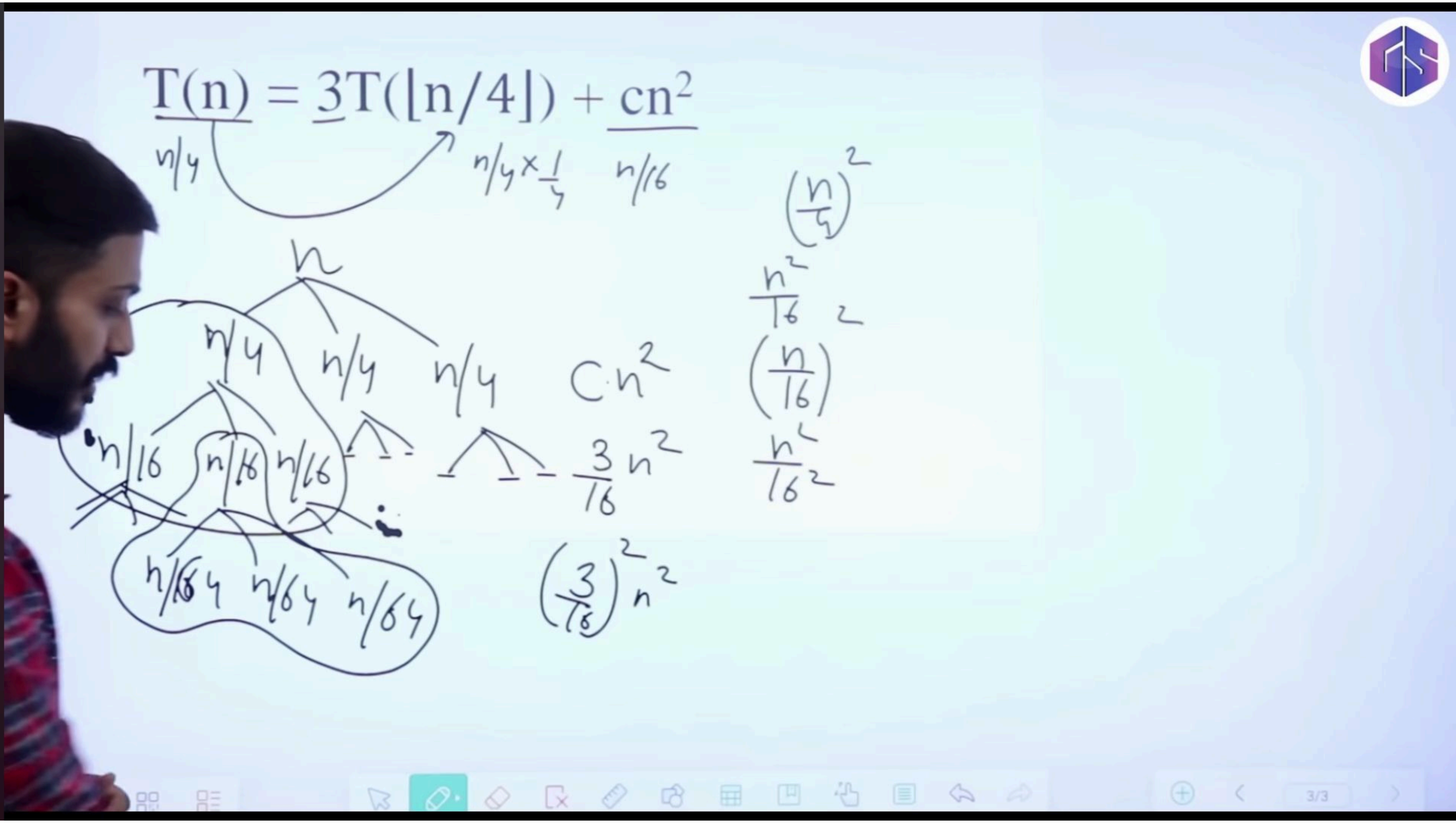
$\left(\frac{n}{4}\right)^2$

$\frac{n^2}{16}$

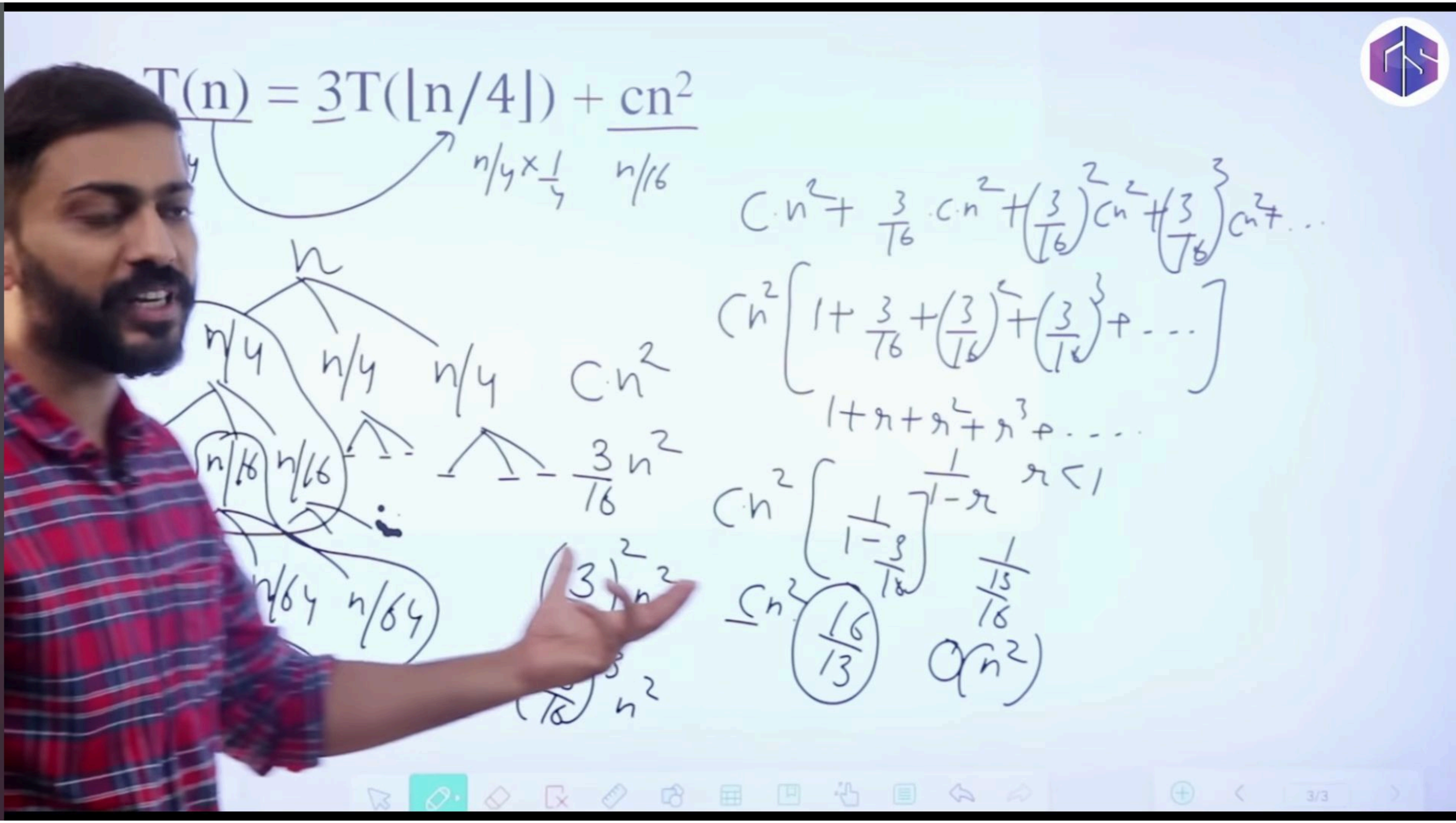


$C \cdot n^2$

$\frac{3}{16} n^2$



T(n) = 3T(⌊n/4⌋) + cn²
n/4
n/4 × 1/4
n/16
(n/4)²
n²/16²
(n/16)
n²/16²
n
n/4 n/4 n/4
C·n²
n/16 n/16 n/16
3/16 n²
n/64 n/64 n/64
(3/16)² n²
SUBSCRIBE
3/3


T(n) = 3T(⌊n/4⌋) + cn^2
n/4 × 1/4
n/16
n
n/4 n/4 n/4
C·n^2
n/16 n/16
3/16 n^2
n/64 n/64
(3/16)^2 n^2
(3/16)^3 n^2
C·n^2 + 3/16 ·c·n^2 + (3/16)^2 cn^2 + (3/16)^3 cn^2 + ...
Cn^2 [1 + 3/16 + (3/16)^2 + (3/16)^3 + ...]
1 + r + r^2 + r^3 + ...
1/(1-r)
r < 1
Cn^2 [1/(1 - 3/16)]
1/(13/16)
Cn^2 16/13
O(n^2)
SUBSCRIBE
3/3