Mod 3 (ITC) : LBC

→ Basics

Def^n : $C_p = C_i + C_k$ [sum of any two CW gives CW]

Property : ① All 0 is also CW
② $C_p = C_i + C_k$ then $d(C_i, C_k) = W(C_p)$
③ $d_{min} = W_{min}$

Q Show (n,k) is LBC

① Make table with k data (message) & n total

② Adjust the extra / parity $(n-k)$ bits for odd / even parity

③ Take any two C & XOR
if new C is present in matrix table
then check $d_{min}(C, C) = W(\text{new } C)$ → LBC
else if new C is not present → Not LBC

→ Generator matrix to generate CW in LBC

$[C] = [i][G]$

C ↑ CW, i ↑ infor words, G ↑ Generator Matrix

GM of (n,k) Linear code has k rows & n columns

$[G] = [I : P]$

Q G & i given just multiply to get a one row CW
Σ(i full row × G full column) → Given 1 bit

Q If only (n,k) & G given in Q
k = message bits → create table $m_0\ m_1\ m_2 \ldots$
Take one row & perform like last Q & similarly
repeat for all combos of i $m_0\ m_1\ m_2\ p_0\ p_1 \ldots$
Create a new table of C_0
C_1
C_2

* Systematic Generator matrix

$$\underset{k \times n}{[G]} = [\underset{k \times k}{I} : \underset{k \times (n-k)}{P}]$$

$C = (i_1 \; i_2 \; i_3 \; i_4 \; P_1 \; P_2 \; P_3)$

$P_1 = i_1 \oplus i_2 \oplus i_3$

$P_2 = i_2 \oplus i_3 \oplus i_4$

$P_3 = i_1 \oplus i_2 \oplus i_4$

$\cancel{P_0 = i_0 \oplus i_2}$

$\cancel{P_1 = i_1 \oplus i_2}$

$\cancel{P_2 = i_0 \oplus i_1}$

~~smaller (a,b)~~

Basically nothing but bring to form of [I : P] (I = identity) & multiply i/ to get CW

→ Parity Check Matrix

$[G] = [I_k : P]$

P = Parity Matrix

$[H] = [P^T : I_{n-k}]$

[H] = Parity check matrix

P^T = Transpose of Parity matrix

→ LBC Error Detection & correction

Detection	Correction
$d_{min} \geq s + 1$	$d_{min} \geq 2t + 1$
s = error detection capacity	t = error correction capacity

* Error Syndromes & Error correction

$[Y] \leftarrow$ received CW

Syndrome :

$[S] = [Y][H^T]$

Corrected CW :

$[Y] = [C] + [E]$

steps

① Multiply received CW & H^T

② Get new sequence ⟶ find that sequence appears in which row of H^T (1, 2, 3 ...)

③ XOR a new sequence with all 0's except 1 at error row no, with received CW (Y).

classmate

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LBC Complete Example

Given (n,k) & G

ⓐ Corresponding code vectors?

$G = [I : P]$

$[C] = [i][G]$

$[P_c] = [i_m][P]$

$[P_0 \ P_1 \ P_2] = [i_0 \ i_1 \ i_2][P]$

Here, $P_0 = i_0 \oplus i_2$

$P_1 = i_1 \oplus i_2$

$P_2 = i_0 \oplus i_1$

Create Table

	i_0	i_1	i_2	P_0	P_1	P_2
C_1						
C_2						
C_3						
⋮						
C_8						

ⓑ Minimum Hamming distance d_{min}? → min weight from C's

ⓒ Error detection & correction?

$d_{min} \geq s+1$ $\quad$ $d_{min} \geq 2t+1$

ⓓ Parity Check Matrix $[H]$?

$H = [P^T : I_{n-k}]$

ⓔ Error if received code is (_ _ _ _ . . .)

$S = [Y][H^T]$

$Y = C + e$ or $[X] = [Y] + [e]$