

### Mod 1

1.1

Intro.

Binary

: number systems



Octal

Hexadecimal



Binary code

: codes

BCD code

Excess - 3 code

Gray code

ASCII code

1.2 Binary arithmetic

Binary addition &amp; subtraction (1's complement &amp; 2's complement).

1.3 Gates: NOT, AND, OR, NAND, NOR,  
XOR, XNOR.

1.4 Theorems &amp; properties of Boolean Algebra.

SOP &amp; POS form

K-maps (2, 3, 4) — simplification.

NAND - NOR realization.

Binary → 2

Decimal → 10

Octal → 8

Hexadecimal → 16

~~(base)~~ ( $\div$ )

~~(base) ( $\times 4 +$ )~~

### # CONVERSION OF DECIMAL $\frac{4}{2}$ TO DECIMAL.

# To convert anything → Decimal  $(n, \dots, 2, 1, 0) \leftarrow$  powers

→ for eg: we are given  $101_2$ , now we have to multiply the powers of 2 to each digit of  $101$ . & add them.

→ for eg: we are given  $64_8$ , now we have to multiply the power of 8 to each digit of  $64$  & add them

↳ (descending order)

$$\rightarrow ① 101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 4 + 0 + 1 = \underline{\underline{5}} = 101$$

$$\rightarrow ② 63_8 = 6 \times 8^1 + 3 \times 8^0 \\ = 48 + 3 = \underline{\underline{51}} \text{ (in octal } 63_8 = \underline{\underline{51}} \text{ in decimal)}$$

$$\rightarrow ③ (110)_6 = 1 \times 6^2 + 1 \times 6^1 + 0 \times 6^0 \quad \times 1 \times 6^1 + 1 \times 6^0 \\ (110)_6 = 36 + 6 = \underline{\underline{42}} = 16 + 10 = \underline{\underline{26}} = 14$$

# To convert decimal → binary  $(\div \text{ by } 2) \rightarrow$  (remainder = Binary code)

→ For going decimal → binary → divide the given decimal number by 2 & (remainder is the binary code)

→ For eg: we are given 16 eg: we are given 17.

$$\begin{array}{r} 16 \\ \downarrow \\ 2 \quad 8 \quad 0 \\ \downarrow \\ 2 \quad 4 \quad 0 \\ \downarrow \\ 2 \quad 2 \quad 0 \\ \downarrow \\ 1 \quad 0 \end{array}$$

$$= (10000)_2 = 16$$

$$\begin{array}{r} 17 \\ \downarrow \\ 2 \quad 8 \quad 1 \\ \downarrow \\ 2 \quad 4 \quad 0 \\ \downarrow \\ 2 \quad 2 \quad 0 \\ \downarrow \\ 1 \quad 0 \end{array}$$

$$(10001)_2 = 17$$

\* decimal  $\rightarrow$  octal  $\rightarrow$  binary  $\div$  by 2  
 $\rightarrow$  binary  $\div$  by 16 } remainder  
 $\rightarrow$  hexa  $\div$  by 16 } = ans.

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\* To convert decimal to octal ( $\div$  by 8) (remainder = octal code)

→ For going from decimal to octal, divide the decimal value by 8, the remainder we get is the octal code.

for eg: we are given  $(26)_{10}$ .

$$\therefore \begin{array}{r} 8 \\ \underline{\quad} \\ 26 \end{array}$$

$$\begin{array}{r} 8 \\ \underline{2} \\ 2 \end{array}$$

$$(26)_8 = (26)_{10}$$

eg: we are given  $(54)_{10}$ .

$$8 \quad 54$$

$$\begin{array}{r} 5 \\ \underline{4} \\ 4 \end{array}$$

$$(54)_8 = (54)_{10}$$

\* To convert decimal to hexadecimal ( $\div$  by 16) (remainder = hex code)

→ For going from decimal to hexadecimal, divide decimal by 16, the remainder we get is hexadecimal value.

for eg: we are given  $(26)_{10}$ .

$$\therefore \begin{array}{r} 16 \\ \underline{\quad} \\ 26 \end{array}$$

$$1 \quad \begin{array}{r} 10 \\ \rightarrow A \end{array} = \underline{1A}$$

$$= (110)_{16}$$

\* now if we get values in remainder b/wn (10-15), we write them as (A-f)

for eg :  $16 \quad 46$

$$2 \quad \begin{array}{r} 14 \\ \rightarrow E \end{array} \quad \underline{2E}$$

# Binary → octal } Separate binary no. into parts  
→ Hexa } of 3 digits & 4 digits  
(Octal)      (Hexa)

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# Convert Binary to octal : (3 digits)

- ① Take the binary code.
- ② Divide the binary code into parts of 3 digits.
- ③ convert the parts of 3 digits to numbers.
- ④ combine numbers.

for example : 1 1001110001

① separating binary into part of 3 digits.

→ 10, 001, 110, 001      8 4 2 1.

② convert 3 digit parts to numbers.

→ 2, 3, 6, 1

③ combine numbers → (2361)<sub>8</sub>

# Convert binary to Hexadecimal (4 digits)

\* Similar to octal conversion

① Take binary code.

② Divide binary code into parts of 4 digits

③ Convert parts of 4 digits to numbers.

④ combine.

For eg 110011110001

→ Separating into parts of 4 digits

→ 100, 1111, 0001

→ Converting to number → 4, F, 1

= 4F1//

① separate into 4 digits.

② convert separated 4 digits into numbers.

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Octal      → Binary }      ① Convert to decimal  
 4                  }  
 Hexa decimal      }      ② Decimal → binary.

### Conversion of Octal to Binary

① Convert Octal → Decimal.

i.e. multiply by powers of 8 in descending order.  
+ add digits.

② Take the decimal number & make its binary.

→ by  $(\frac{\text{?}}{2} \text{ by } 2)$  → remainder binary

eg:  $(63)_8$

① Convert to decimal.  $(8^n)$ .

$$6 \times 8^1 + 3 \times 8^0 = (51)$$

② decimal → binary. ( $\div$  by 2)

2 51

$$\begin{array}{r} 2 25 1 \\ 2 12 1 \\ 2 6 0 \\ 2 3 0 \\ 1 1 \end{array} = \underline{(110011)_2}$$

① hexa → decimal

### Conversion of Hexa decimal → Binary.      ② Decimal → binary.

① Convert hexa decimal → Decimal.

→ multiply by powers of  $(16)^n$  in descending order  
+ then add.

eg:  $(63)_{16}$

② Decimal to Binary.

→ Divide by (2)

$$6 \times 16^1 + 3 \times 16^0 = (99)_{10}$$

$$\begin{array}{r} 2 99 \\ 2 49 \\ 2 24 \\ 2 12 \\ 2 6 \\ 2 3 \\ 2 1 \end{array} \quad \underline{(11000011)_2}$$

# Octal to Hexa + Hexa to octal

① convert octal to  
Decimal

$\rightarrow \times$  by powers of (8) +  $\oplus$

① convert hexa to Decimal

$\rightarrow \times$  by powers of (16) +  $\oplus$

② take decimal + convert  
to hexa  
( $\div$  by 16)

② Take decimal + convert  
to octal  
( $\div$  by 8)

eg:  $(76)_8$

$$\begin{aligned} & \text{① convert octal to decimal} \\ & = 7 \times 8^1 + 6 \times 8^0 \\ & = 56 + 6 = \underline{\underline{(62)}_{10}} \end{aligned}$$

eg:  $(1A)_{16}$   $A=10$

$$\begin{aligned} & \text{① convert hexa to decimal} \\ & = 1 \times 16^1 + 10 \times 16^0 \\ & = 16 + 10 = \underline{\underline{(26)}_{10}} \end{aligned}$$

② Decimal to hexadecimel.  
( $\div$  by 16)

$$\begin{array}{r} 16 \quad 62 \\ \underline{\underline{3}} \quad \underline{\underline{14}} \xrightarrow{\text{F}} \end{array}$$

$(BE)_{16}$

② Decimal to octal  
( $\div$  by 8)

$$\begin{array}{r} 8 \quad 26 \\ \underline{\underline{3}} \quad \underline{\underline{2}} \xrightarrow{\text{4}} \end{array}$$

$(32)_8$

# I Decimal to Binary

Q  $(12.375)_{10} \rightarrow \text{Binary}$ .

~~Divide 2 18~~

$$\begin{array}{r}
 2 \cdot 6 \cdot 1 \\
 2 \cdot 3 \cdot 0 \\
 \hline
 1 \cdot 1
 \end{array}
 \quad \begin{array}{l}
 \underline{(1101)} \\
 = \underline{(1101 \cdot 011)}
 \end{array}$$

$$\begin{aligned}
 875 &\rightarrow 0.375 \times 2 = 0.750 \rightarrow 0 \\
 &\rightarrow 0.750 \times 2 = \underline{1.500} \rightarrow 1 \\
 &\rightarrow 0.500 \times 2 = \underline{1.00} \rightarrow 1 \\
 &= \underline{011}
 \end{aligned}$$

\* Keep multiplying by 2, till we get 0.

## # Sign Magnitude Method

- \* 1 bit is reserved for sign of the number. i.e. (MSB)
  - \* 0 = +ve
  - \* 1 = -ve
- \* Remaining bits are → the number.

Ex (+ve 4)      (-ve 4)  
= 0100      = ①100  
                ↳ signbit.

B9      4      0100      4      0100      ∵ 0100  
    + 3      0011      - 3      ④ 1011      - 0011  
    \underline{+}      0111      \underline{\neq} 1111      0001

compare & subtract → -4 = 1100  
→ 3      ⊖ 0011  
- 1      10001 = -1.

#

1's complement Method: (Invert all bits of +ve no = -ve no.)

\* For +ve same as sign magnitude method.

$$\text{i.e. } +13 = \underline{0 \ 1101}$$

\* For -ve

① need to invert all bits of same +ve no.

e.g.:  $-13$  in  $1^{\text{st}}$  complement

= invert all bits of  $+13$

$$\text{i.e. } = \underline{0 \ 1101} = +13$$

now invert all bits

$$\therefore \underline{(-13) = 1 \ 0010}$$

#	Invert
1 + 0	
of +ve	
$\equiv -ve$	

# 2's complement Method ( $1^{\text{st}}$  complement + 1)

\* For +ve no. same as sign magnitude method

$$\text{i.e. } +13 = \underline{0 \ 1101}$$

\* For negative numbers

① Take  $1^{\text{st}}$  complement

(i.e. invert bits of same +ve no.)

$$\Rightarrow +13 = 01101$$

② Add (+1) to  $1^{\text{st}}$  complement.

$\therefore 1^{\text{st}}$  complement

$$= -13 = 10010$$

e.g.:  $-13$  in  $2^{\text{nd}}$  complement

in  $2^{\text{nd}}$  complement

① Take  $1^{\text{st}}$  complement (Invert bits of +13)

$\Rightarrow 1^{\text{st}}$  complement + 1

$$= 01101 = +13$$

$$= 10010 + 1 = 10011$$

$$\therefore -13 \text{ in } 1^{\text{st}} \text{ complement} = 10010$$

$$\therefore -13 \text{ in } 2^{\text{nd}} \text{ complement} = (1^{\text{st}} \text{ complement} + 1)$$

$$= 10010 + 1 = \underline{(10011)} = -13 \text{ } 2^{\text{nd}} \text{ complement}$$

W Pines For addition

A	+ B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

141 E 0 C0444 ①.

## Rules for Subtraction

A	=	B	diff	borrow carry
0		0	0	0
0		1	1	1
1		0	1	0
-1		1	0	0

$$(0-1) = 1 \quad \text{but } r_0 \approx 1$$

$$85 - 28 = 85 + (-28)$$

- 28 - 25 complement or 1s complement

### # 1<sup>st</sup> complement addition (add carry to answer)

(eg) :  $85 \rightarrow (-28)$

- ① Take 1s complement of (-28) .
  - ② Add 85 + 1s complement of (-28)
  - # ③ If carry is generated , add that to answer )

# 2<sup>3</sup> complement addition (ignore if carry is generated)

$$\text{eg: } 25 + (-28)$$

- ① take 2's complement of (28).
  - ② Add 35 + 2's complement of (28)
  - ③ (If carry is generated, ignore)

## # Classification of codes

- \* Weighted codes → Binary, 8421, 2421, etc.  
→ each position represent some value.
  - \* Non weighted codes → XS-3 code, Gray code.
  - \* Alphanumeric code → ASCII code.

# Binary coded Decimal : (binary code for each digit) (ubit)

\* ① Each decimal digit is represented by a 4bit binary number.

# Basically, write binary code for each digit.

Decimal	BCD (same as binary just 4 bits)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

### # BCD to Decimal :-

- ① Take decimal no.
- ② write the binary code for (each digit) of decimal.

eg:  $(227)_{10}$

$$\begin{aligned}
 &= 2 = 0010 \quad \rightarrow (\underline{\underline{0010}} \underline{\underline{0010}} \underline{\underline{0111}}) \\
 &2 = 0010 \\
 &7 = 0111
 \end{aligned}$$

### BCD to decimal :- (group BCD into 4 bits) 4 (then calc. decimal value)

similar  
→ to  
binary to  
hexa

- ① Group the ~~→~~ BCD into 4 bit group.
- ② Take decimal value of 4 bit groups.

eg:  $\underline{\underline{0010}} \underline{\underline{0010}} \underline{\underline{0111}}$

① → 2      ② → 5      ③ → 7

$$\begin{aligned}
 &\textcircled{1} \rightarrow 2 \\
 &\textcircled{2} \rightarrow 5 \quad = \underline{(227)_{10}}
 \end{aligned}$$

$$\textcircled{3} \rightarrow 7$$

### # 10 in binary

$$\underline{10 = 1010}$$

### 10 in BCD

$$10 = 1 + 0.$$

$$= \underline{\underline{(0001 \ 0000)}}$$

## (self complementing)

# Add  
8 to BCDExcess-8 code ( $X_8 - 8$  code) Add 8 to BCD\* convert Decimal to  $X_8 - 8$  code.

# ① convert Decimal to BCD.

# ② Add  $(0011)_B$  to BCD.=  $X_8 - 8$  code

$$\text{eg: } 5 \rightarrow \text{BCD} = 0101 \rightarrow \text{BCD} + 8 = \begin{array}{r} 0101 \\ + 0011 \\ \hline \end{array}$$

$$X_8 - 8 \text{ code} = \begin{array}{r} 1000 \\ \hline \end{array} = 8 \text{ for } 5$$

<u>Decimal</u>	<u>BCD</u>	<u>BCD + 8</u>
0	0000	$0000 + 0011 = 0011$
1	0001	$0001 + 0011 = 0100$
2	0010	$\dots = 0101$
3	0011	$\dots = 0110$
4	0100	$\dots = 0111$
5	0101	$\dots = 1000$
6	0110	$\dots = 1001$
7	0111	$\dots = 1010$
8	1000	$\dots = 1011$
9	1001	$\dots = 1100$

#  $\therefore X_8 - 8$  code for 24.

(1) convert 24 to BCD.

(2) Add 0000 0011 to BCD 24.

$$\rightarrow \text{BCD } 24 = 00100100$$

$$\rightarrow \text{BCD } 24 + 8 = 00100100$$

$$\begin{array}{r} 00100100 \\ + 00000011 \\ \hline 00100111 \end{array} = X_8 - 8 \text{ code for } 24$$

\* X8-3 code for digits above 9.

① Take the BCD of the number.

② Add 3 to each BCD value of the number.

Eg. X8-3 code for 24.

① BCD of 24 = 2 4.

(0010 0100)

② Add (0011) to 2 BCD + (0011) to 4 BCD.

$$\begin{array}{r}
 = 0010 \qquad \qquad 0100 \\
 + 0011 \qquad \qquad + 0011 \\
 \hline
 0101 \qquad \qquad 0111 \\
 \\ 
 = \underline{\underline{(01010111)}}
 \end{array}$$

Eg. X8-3 code for 156.

① BCD of 156 = 1 5 6.

0001 0101 00110

② Add 3 to each digit BCD.

$$\begin{array}{r}
 = 0001 \qquad 0101 \qquad 0110 \\
 + 0011 \qquad + 0011 \qquad + 0011 \\
 \hline
 0100 \qquad 1000 \qquad 10001 \\
 \\ 
 = \underline{\underline{010010001001}}
 \end{array}$$

\* Binary  $\rightarrow$  Gray } MSB remains  
 Gray  $\rightarrow$  Binary } the same.

# Graycode:  $Y = AB \rightarrow AR$  (XOR)

- ## • Reflected Binary Code (RBC).

\* two successive value differs in only one bit.

\* Binary number is converted to Gray code to reduce switching operation.

Binary Gray.

$B_3 \quad B_2 \quad B_1 \quad B_0$

$G_{13}$     $G_{12}$     $G_{11}$     $G_{10}$

9. 0 0 1 1  
4. 0 1 0 0

$$\begin{array}{ccccc} 0 & \textcircled{0} & 1 & 0 \\ 0 & \textcircled{1} & 1 & 0 \end{array}$$

$\therefore$  to change from 8 to 4,  
we need to change 8 bits.

$\therefore$  to change from 3 to 4  
in Gray code we need to  
change only one bit

## Binary to Graycode Conversion

① Record MSB as it is

② Add MSB to next bit, record the sum, neglect carry.

③ Repeat. ② -

\* Eg convert 1011 to Gray code

$\rightarrow \begin{array}{l} 1 \\ 0 \\ 1 \\ 1 \end{array}$   
MSB

① Record MSB as it is. = ① .

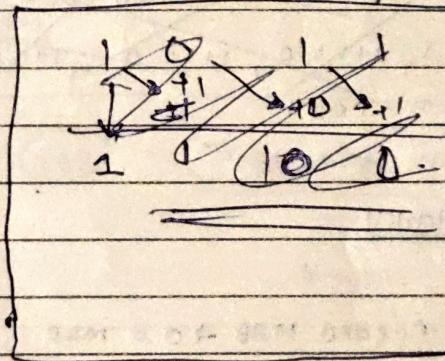
② Add MSB  $\rightarrow$  next bit =  $1 + 0 = 1$  .  
(record sum, neglect carry).

repeat process

$$= 0 + 1 = 1 \quad = \underline{(1110)}$$

$$= 0 + 1 = 0$$

# # Binary to Gray:



① Record MSB

② Add ①<sup>st</sup> & 2<sup>nd</sup> pos

Add 2<sup>nd</sup> & 3<sup>rd</sup> pos

Add 3<sup>rd</sup> & 4<sup>th</sup> pos

# Record sum ignore  
carry.

-- continue till last element

Eg: 1110

MSB = 1.

$$1 + 1 = 0.$$

$$1 + 1 = 0 \quad = \underline{(1001)}$$

$$1 + 0 = 1$$

1 1 1 0

$\downarrow \rightarrow 1 \rightarrow 1 \rightarrow 1$

1 0 0 1

$\downarrow \rightarrow 1 \rightarrow 0 \rightarrow 1$

1 1 1 0



Binary  $\rightarrow$  Gray code  $\rightarrow$  add 1<sup>st</sup> & 2<sup>nd</sup>, 2<sup>nd</sup> & 3<sup>rd</sup>, 3<sup>rd</sup> & 4<sup>th</sup>

Gray  $\rightarrow$  Binary  $\rightarrow$  add sum to next position.

$\Rightarrow$  add previous sum to next position.

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# Gray code  $\rightarrow$  Binary Conversion: [same as ~~Binary  $\rightarrow$  Gray~~]

① Record MSB as it is.

② Add MSB to next bit of gray code.

Basically

① Record MSB

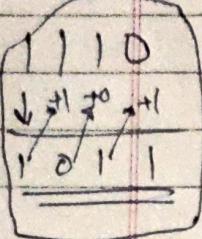
② Add 1<sup>st</sup> & 2<sup>nd</sup> position

③ record sum

Add 2<sup>nd</sup> & 3<sup>rd</sup> position

④ Ignore carry

Add 3<sup>rd</sup> & 4<sup>th</sup> position



Eg: ~~1110~~ ~~Gray code~~

$$= 1, 1+0=0, 1+1=1, 1+0=1$$

$$B_1 = G_1$$

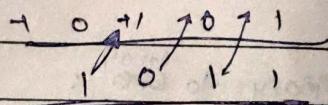
$$B_2 = G_2 \oplus B_1$$

$$B_3 = G_3 \oplus B_2 \quad \text{Basically}$$

$$B_4 = G_4 \oplus B_3$$

Eg: 1110 ~~Gray code~~

$$= 1 \ 1 \ 1 \ 0$$

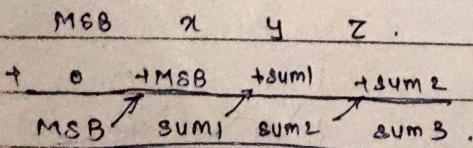


① Record MSB  $\rightarrow$  0 = MSB.

② Add MSB + 0 = MSB  $\rightarrow$  next bit = sum ①

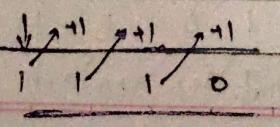
Add sum ① to next bit = sum ②.

Add sum ② to next bit = sum ③.



# Basically add result of previous sum to next bit

Eg: 1 0 0 1 (Gray code) if 1001 = binary to Gray



$$\therefore 1, 1+0, 0+0, 0+1 = (1 \ 1 \ 0 \ 1)$$

## # ASCII - codes

(Group 7 bits in binary).

\*  $48 - 57 = 0 - 9$ .

\*  $65 - 90 = A - Z$  capital.

$97 - 122 = a - z$  small

### ASCII

Binary  $\rightarrow$  Decimal  $\rightarrow$  ASCII

# Group 7 bits in binary.

\* convert each group to decimal.

10100011001011

81

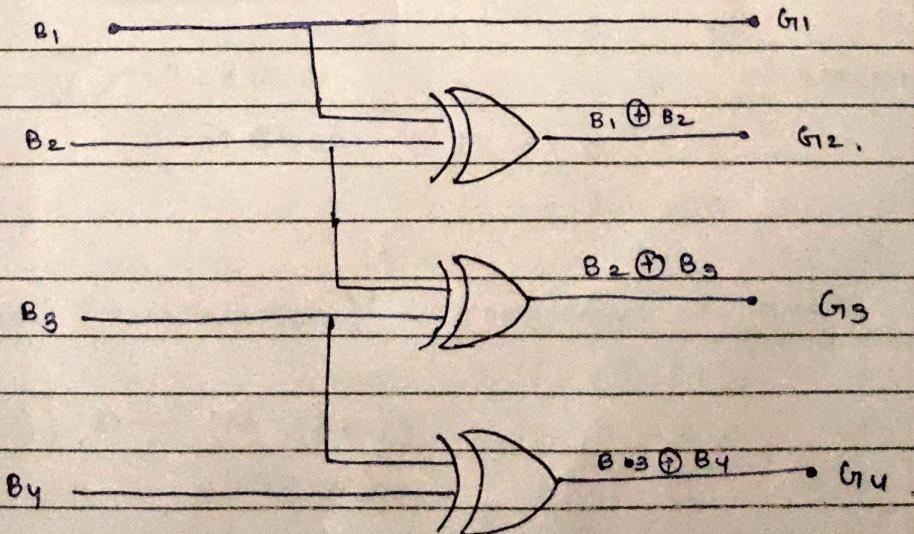
75

$\therefore$  from 65-90, A-Z capital.

# Binary  $\rightarrow$  Gray code = XOR . (Just apply XOR)

Binary Bits.

Gray bits.



#### 4) Gray code to binary (Karnaugh)

Gray code

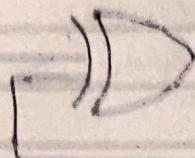
Binary

$G_4 G_3 G_2 G_1$

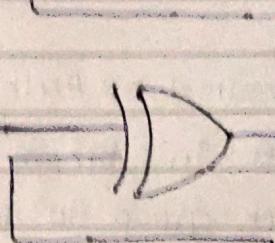
$G_3$

$G_2$

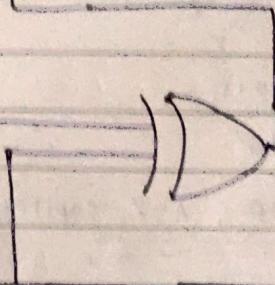
$G_1$



$$B_4 = G_4 \oplus G_3$$



$$B_3 = G_3 \oplus G_2 \oplus G_1$$



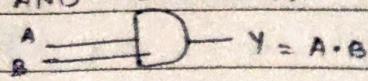
$$B_2 = G_2 \oplus G_1$$

$$B_1 = G_1$$

#### 5) BCD to RS to code

## # Basic Logic Gates

① AND (MULTIPLY)

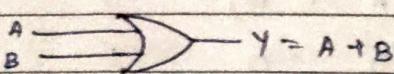


A · B = Y

0	0	0
0	1	0
1	0	0
1	1	1

AND (x)

② OR (ADD)



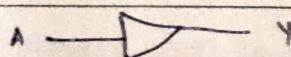
A + B = Y

0	0	0
0	1	1
1	0	1
1	1	1

OR (y)

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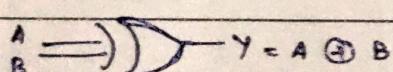
③ NOT (complement)



A → Y

1	0	NOT (complement)
0	1	

④ XOR (PURE ADDITION)



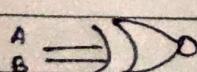
A ⊕ B = Y

$$= \bar{A}B + A\bar{B}$$

0	0	0
0	1	1
1	0	1
1	1	0

XOR (pure addition)

⑤ X-NOR (Pure addition)' = complement of (XOR)'



(A ⊕ B)' = Y

$$= \bar{A}\bar{B} + AB$$

0	0	1
0	1	0
1	0	0
1	1	1

(XOR)' = X-NOR

⑥ NAND  $X = (A \cdot B)'$  (complement of AND)

$$(A \cdot B)' \quad Y$$

$\Rightarrow$	$D$	$y = (AB)'$	0	0	1
			0	1	1
			1	0	1
			1	1	0

⑦ NOR  $X = (A + B)'$  (complement of OR)

$$(A + B)' \quad Y$$

$$OR = \overline{(1+1=1)}$$

$\stackrel{A}{\Rightarrow}$	$\stackrel{B}{\Rightarrow}$	$y = (A+B)'$	0	0	1
			0	1	0
			1	0	0
			1	1	0

serial  $\rightarrow$  use AND  
parallel  $\rightarrow$  use OR.

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## # Boolean Algebra

\* Commutative law

$$\textcircled{10} \cdot A + B = B + A$$

$$\textcircled{11} \cdot A \cdot B = B \cdot A$$

\* Associative law

$$\textcircled{12} \cdot (A + B) + C = A + (B + C)$$

$$\textcircled{13} \cdot (A \cdot B) \cdot C = A \cdot (B \cdot C)$$

\* Distributive law

$$\textcircled{14} \cdot A \cdot (B + C) = AB + AC$$

$$\textcircled{15} \cdot A + (B \cdot C) = (A + B) \cdot (A + C)$$

## # Duality theorem

① change each OR to AND

② change each AND to OR

③ complement each 0 & 1

## # Fundamental laws

\* AND laws

$$x=1 \quad x'=0.$$

$$\textcircled{1} \quad x \cdot x = x.$$

$$\textcircled{2} \quad x \cdot 1 = x$$

$$\textcircled{3} \quad x \cdot 0 = 0 \quad \text{AND}$$

$$\textcircled{4} \quad x \cdot \bar{x} = 0 \quad \text{Complement}$$

\* OR laws

$$A = 01, \bar{A} + A' = 0$$

$$\textcircled{5} \quad A + 0 = A$$

$$\textcircled{6} \quad A + 1 = 1 \quad 1+1=1 \quad \text{OR}$$

$$\textcircled{7} \quad A + A' = 1 \quad \text{Complement}$$

$$\textcircled{8} \quad A + \bar{A} = 1 \quad 1+0=1 \quad \text{OR}$$

because  $x=1$ ,

$$\bar{x}=0.$$

$$0+1=1$$

$$\bar{\bar{x}} = x$$

# Dual of Distributive law

$$\textcircled{10} \quad \text{top of page}$$

$$\textcircled{15} \# (x + (y z)) = (\underline{x+y}) \cdot (\underline{x+z})$$

$$\text{DeMorgan's} = \overline{A \cdot B} = \overline{A} + \overline{B}$$

$$A \cdot B = \overline{\overline{A} + \overline{B}}$$

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### \* DeMorgan's Theorem

$$\begin{aligned} * \quad \overline{A + B} &= \overline{A} \cdot \overline{B} \\ * \quad \overline{A \cdot B} &= \overline{A} + \overline{B} \end{aligned} \quad \left. \begin{array}{l} \text{converting AND to OR} \\ \text{OR to AND} \end{array} \right\}$$

### \* Identity Theorem:

$$* \quad x \rightarrow (\overline{x} \cdot y) = (x + y)$$

$$* \quad \underline{x \cdot (\overline{x} + y)} = (x \cdot y)$$

$$\rightarrow x \rightarrow (\overline{x} \cdot y)$$

$$= (x + \overline{x}) \cdot \underline{(x + y)} = (x + y)$$

$$x \geq 1, \therefore \overline{x} \geq 0$$

$$= 1 \cdot (x + y) = x + y \quad x + \overline{x} = 1 \quad \underline{x \geq 0, \overline{x} \geq 1}$$

$$\therefore \underline{\underline{x \rightarrow \overline{x}}} = 1$$

$$\rightarrow x(\overline{x} + y) = x \cdot y$$

$$(x \cdot \overline{x}) + (y \cdot x) = \underline{(x \cdot y)}$$

$$0 + (y \cdot x) = \underline{(x \cdot y)}$$

1 - any input = 1.

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A=1, A=0, B=1, B=0

(q1)  $y = \bar{A}B + A\bar{B} + AB$ .

Take A common.

$$\bar{A}B + A(\bar{B} + B) \quad \dots \quad \bar{B} + B = 1.$$

$$\bar{A}B + A \cdot$$

$$\therefore (C+AD) = (C+A) \cdot (C+D)$$

$$\therefore (A+\bar{A}) \cdot (A+B)$$

$$\therefore \underline{\underline{(A+\bar{A})}} = 1$$

$$\therefore \underline{\underline{(A+B)}}.$$

(q2)  $y = \underline{\underline{\bar{ABC}}} + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \underline{\underline{A\bar{B}C}}$

Take  $\bar{B}\bar{C}$  common.

$$y = \underline{\underline{\bar{A}\bar{B}\bar{C}}} + A\bar{B}\bar{C} + \bar{B}\bar{C}(\bar{A} + A).$$

$$\therefore (\bar{A} + A) = 1.$$

$$= \bar{A}\bar{B}\bar{C} + \underline{\underline{A\bar{B}\bar{C}}} + \bar{B}\bar{C}$$

Take  $\bar{B}$  common.

$$= \bar{A}\bar{B}\bar{C} + \bar{B}(\bar{A}\bar{C} + C).$$

~~cancel out B~~

$$= \bar{A}\bar{B}\bar{C} + \bar{B}[(\bar{A}\bar{C} + C)]$$

$$(\bar{C} + \bar{C}) = 1.$$

$$= \bar{A}\bar{B}\bar{C} + \bar{B}[A + \bar{C}]$$

$$= \bar{A}\bar{B}\bar{C} + \underline{\underline{AB}} + \underline{\underline{AC}}$$

(q3)  $F = \bar{y}\bar{z} + \bar{w}\bar{x}\bar{z} + \underline{\underline{\bar{w}\bar{x}\bar{y}\bar{z}}} + y\bar{z}$

Take  $\bar{y}\bar{z}$  common.

$$= \bar{y}\bar{z} + (1 + \bar{w}\bar{x}) + \bar{w}\bar{x}\bar{z} + y\bar{z})$$

$$(x+1) = x.$$

$$= \underline{\underline{\bar{y}\bar{z}}} + (1 + \bar{w}\bar{x}) + \underline{\underline{y\bar{z}}}$$

Take  $\bar{z}$  common.

$$\bar{z}(\bar{y} + y) + \bar{w}\bar{x}\bar{z} = \bar{z} + \underline{\underline{\bar{w}\bar{x}\bar{z}}}.$$

$$(\bar{y} + y) = 1.$$

$$\bar{z} + \bar{w}\bar{x}\bar{z}$$

take  $\bar{z}$  common.

$$\bar{z}(1 + \bar{w}\bar{x}) =$$

$$(x+1) = 1 \quad \therefore x=1$$

$$= \underline{\bar{z}}$$

# Solving XNOR.

$$\rightarrow \bar{A}\bar{B} + AB$$

$$\rightarrow \overline{\bar{A}B + A\bar{B}}$$

$$= \overline{x+y} = \bar{x} \cdot \bar{y} \text{ demorgans.}$$

$$\overline{\bar{A} \cdot \bar{B}} + \overline{A \cdot \bar{B}}$$

= Again demorgans.

$$= (\bar{A} + \bar{B}) \cdot (\bar{A} + B)$$

$$= (A + \bar{B}) \cdot (\bar{A} + B)$$

$$= A\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}$$

$$\therefore A\bar{A} + B\bar{B} = 0$$

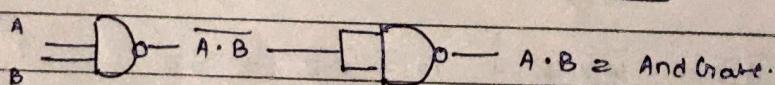
$$= \underline{(AB + \bar{A}\bar{B})}$$

#

Construct AND using NAND.

$$\therefore (A \cdot B)' \rightarrow A \cdot B. \quad (2 \text{ nand gates})$$

2 nand gates  
series

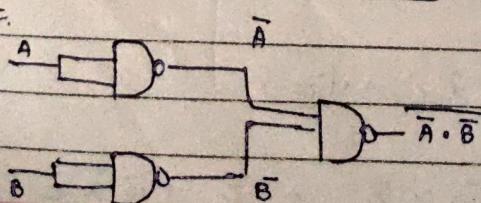


# Construct OR using NAND.  $(A + B)' \leftarrow (A \cdot B)'$

short input

2 nand  
gates  
parallel

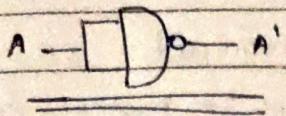
again  
nand gate



APPLY demorgan  $\rightarrow \bar{A} \cdot \bar{B}$

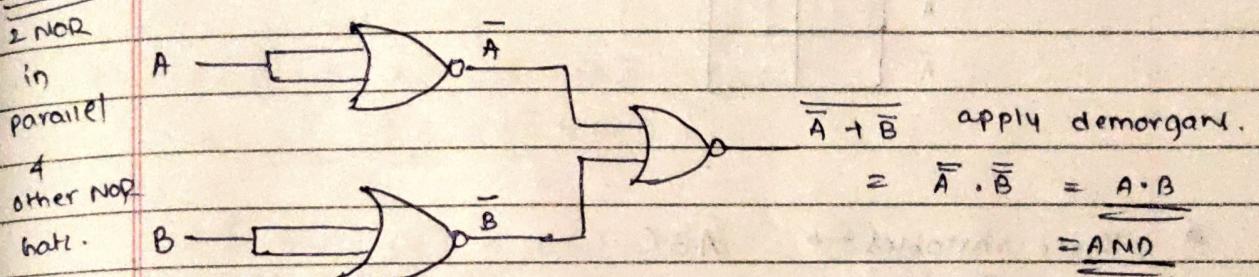
$$= \bar{A} + \bar{B} = \underline{(A + B)}$$

# NOT using NAND :  $(A \cdot B)' \rightarrow A'$

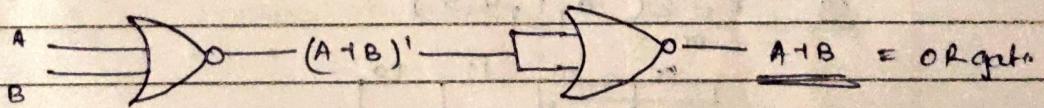


# Construction of AND using NOR

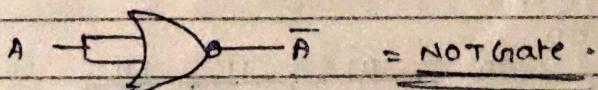
$$(A + B)' \rightarrow A \cdot B$$



# OR Gate using NOR  $(A + B)' \rightarrow A + B$



# NOT Gate using NOR



$\bar{A} = 0$     $A = 1$

# K-map is always made for Output

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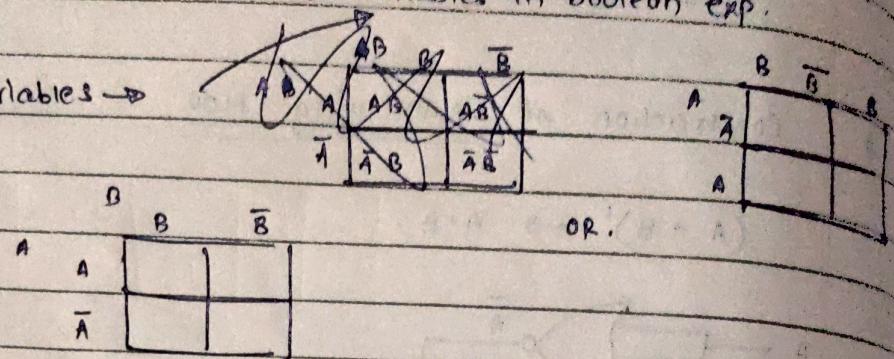
### K-maps

$N = 2^n$ ,  $N$  = no. of cells required

$n$  = no. of variables in boolean exp.

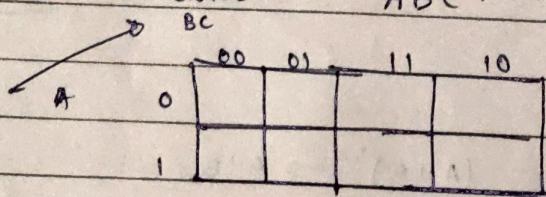
#

Two variables  $\rightarrow$



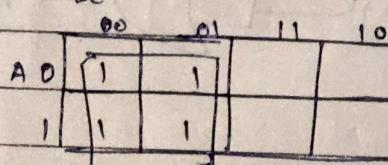
#

Three variables  $\rightarrow ABC$ .



eg:  $f = \Sigma(0, 1, 4, 5)$

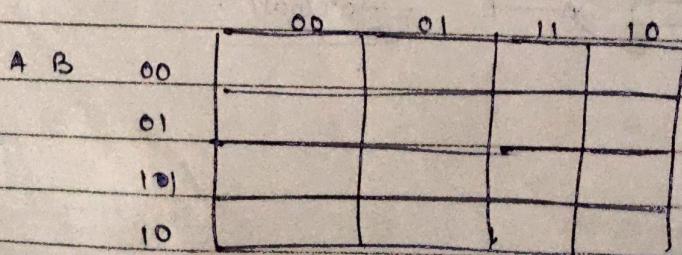
BC.



#

4 variable K-map.

CD.



1 group ①      (SOP)

KMap for Minterm

①.

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Ex. Minterm  $AB + A\bar{B} + BC$ . (Three variable,

BC

A	00	01	11	10
00	0	1	1	0
01	1	0	0	1

\* All spots where

$A \neq B$  are 1

\* All spots where

$A = 1 \neq B = 0$ .

$A + \bar{B}C$

\* All spots where

$B \neq C = 1$ .

#  $\bar{A}\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$ .

$A(\bar{B}\bar{C} + \bar{B}C + B\bar{C} + BC)$ .

$A(\bar{B}(\bar{C}+C) + B(\bar{C}+C))$        $\bar{C}+C = 1$ .

$A(\bar{B}+B) = \underline{\underline{A}}$        $\bar{B}+B = 1$ .

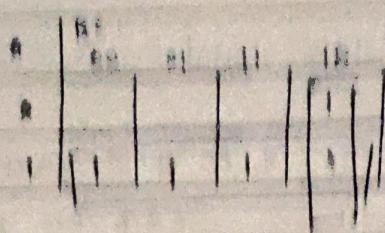
#  $F = A + \bar{B}C$  Three Variables.

$2^3 = 8$       prepare truthtable for  $A + \bar{B}C$ .

A	B	C	F	$F = (\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + ABC + ABC + ABC + ABC + ABC)$
0	0	0	0	$\checkmark$
0	0	1	0	= (for which $F=1$ )
0	1	0	1	
0	1	1	1	$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + ABC + ABC + ABC + ABC + ABC$
1	0	0	1	$= \bar{A}\bar{B}\bar{C}(B+\bar{B}) + ABC(\bar{C}+C) + ABC$
1	0	1	1	$= \bar{A}\bar{C} + ABC + ABC$
1	1	0	1	$= \bar{A}\bar{C} + ABC(1+C)$
1	1	1	1	$= \bar{A}\bar{C} + ABC$ . -- $1+0=1$

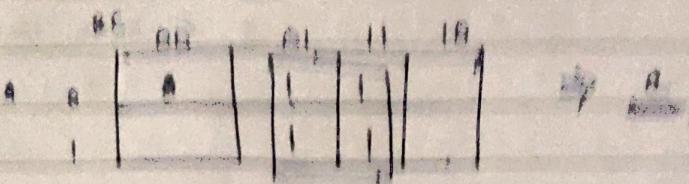
A + B = C

A + C = B



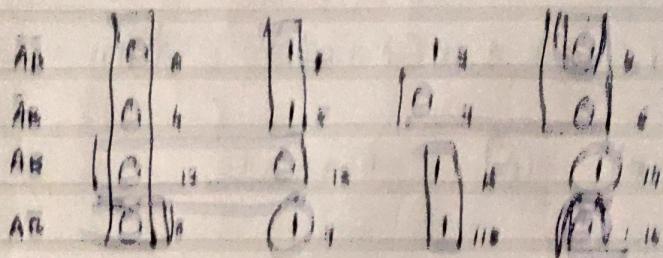
A + B = C

B) (A+B+C) + (D+E+F)



AB AB AB AB AB

AB



AB + AB + AB + AB + AB

C) EP + ABC + ABD + ABE + ACD + ADE + ABE + ACD

(A+D) + (A+B+C) + (A+B+E) + (A+C+D)  
+ (A+C+E+D)

group 0

[POS]

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Kmap for max-term ①.

eg

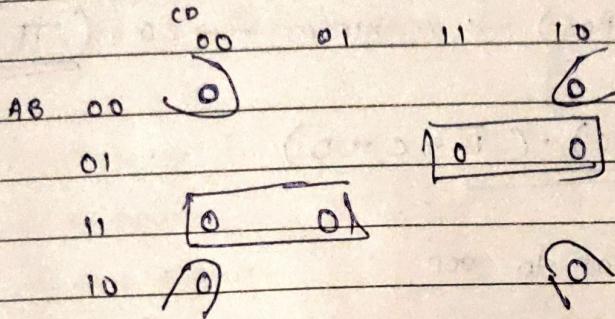
$$F(A, B, C, D) = \overline{\Sigma m}(1, 3, 4, 5, 9, 11, 14, 15).$$

↳ min-terms (1)

$$= \Pi m(0, 2, 6, 7, 8, 10, 12, 13).$$

↳ max-terms (0)

$$\therefore 2^4 = 16.$$



$$(\bar{B}\bar{D}) + AB\bar{C} + \bar{A}BC$$

4. Write it as, invert everything | complement everything.

$$\Rightarrow (\bar{B} + D) \cdot (\bar{A} + \bar{B} + C) \cdot (\bar{A} + \bar{B} + \bar{C}).$$

① change  $A' \rightarrow \bar{A}$

4 change  $(\cdot \rightarrow +) + (+ \rightarrow \cdot)$

#

Standard forms

#

Sum of products $(SOP) \rightarrow \text{minterms} \rightarrow 0 \cdot (\Sigma) / (2)$ 

$$SOP \text{ eqn} = \bullet A \bullet B \bullet C + \bullet D \bullet E \bullet R$$

#

Product of sum $(POS) \rightarrow \text{maxterm} \rightarrow 0 \cdot (\Pi)$ 

$$POS \text{ eqn} = \underline{(A + B + C)} \cdot \underline{(C \bar{B} + C \rightarrow D)}$$

$\rightarrow$  solved similar to SOP,

① plot '0' in the K map instead of 1.

② group '0' instead of 1

③ get can in the form of SOP.

④ convert SOP  $\rightarrow$  POS.

$\rightarrow$  change  $(\bullet)$  to  $(+)$

+ viceversa.

$\rightarrow$  change  $\bar{A} \rightarrow A$  + viceversa.

#

SOP

(minterms)

POS (maxterms)

