Symbolic manipulation

In [1]:
```
f(x) = x^2+x-2
show(f(x))
```

$x^2 + x - 2$

In [2]:
```
f
```

Out[2]:
```
x |--> x^2 + x - 2
```

In [3]:
```
type(f)
```

Out[3]:
```
<class 'sage.symbolic.expression.Expression'>
```

In [4]:
```
f(5)
```

Out[4]:
```
28
```

In [5]:
```
f(-pi)
```

Out[5]:
```
-pi + pi^2 - 2
```

In [6]:
```
f(-pi).n()
```

Out[6]:
```
4.72801174749956
```

In [7]:
```
solve(f(x)==0,x)
```
Out[7]:
```
[x == 1, x == -2]
```
In [8]:
```
solve(f(x)==0,x,solution_dict=True)
```
Out[8]:
```
[{x: 1}, {x: -2}]
```
In [9]:
```
solve(f(x)==0,x,solution_dict=False)
```
Out[9]:
```
[x == 1, x == -2]
```
In [10]:
```
f.roots()
```
Out[10]:
```
[(1, 1), (-2, 1)]
```
In [11]:
```
a = (x^2+2*x+1).roots()
a
```
Out[11]:
```
[(-1, 2)]
```
In [12]:
```
a = (x^2+x+1).roots()
a
```
Out[12]:
```
[(-1/2*I*sqrt(3) - 1/2, 1), (1/2*I*sqrt(3) - 1/2, 1)]
```
In [13]:
```
show(a)
```
$$\left[\left(-\frac{1}{2} i \sqrt{3} - \frac{1}{2}, 1\right), \left(\frac{1}{2} i \sqrt{3} - \frac{1}{2}, 1\right)\right]$$

In [14]:
```
show((x^5+x+1).roots())
```

$$\left[\left(-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(3\sqrt{23}\sqrt{3}-25\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-\frac{\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(-i\sqrt{3}+1\right)}{3\left(3\sqrt{23}\sqrt{3}-25\right)^{\frac{1}{3}}}+\frac{1}{3},1\right),\left(-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(3\sqrt{23}\sqrt{3}-25\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-\frac{\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(i\sqrt{3}+1\right)}{3\left(3\sqrt{23}\sqrt{3}-25\right)^{\frac{1}{3}}}+\frac{1}{3},1\right),\left(\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(3\sqrt{23}\sqrt{3}-25\right)^{\frac{1}{3}}+\frac{2\left(\frac{1}{2}\right)^{\frac{2}{3}}}{3\left(3\sqrt{23}\sqrt{3}-25\right)^{\frac{1}{3}}}+\frac{1}{3},1\right),\left(-\frac{1}{2}i\sqrt{3}-\frac{1}{2},1\right),\left(\frac{1}{2}i\sqrt{3}-\frac{1}{2},1\right)\right]$$

In [15]:
```
var('x,y')
solve([x+y==6,x-y==4],[x,y])
```

Out[15]:
```
[[x == 5, y == 1]]
```

In [16]:
```
var('x,y')
solve([x+y==6,x+y==7],[x,y])
```

Out[16]:
```
[]
```

In [17]:
```
solve([x+y==5],[x,y])
```

Out[17]:
```
[[x == -r1 + 5, y == r1]]
```

Solving a system of non linear equations

In [18]:
```
s = solve([x^2+y^2==1, x*y==1/4],[x,y],solution_dict=True)
show(s)
```

$$\left[\left\{x : -\frac{1}{2}\,\sqrt{\sqrt{3}+2}, y : \frac{1}{2}\,\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)\right\}, \left\{x : \frac{1}{2}\,\sqrt{\sqrt{3}+2}, y : -\frac{1}{2}\,\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)\right\}, \left\{x : -\frac{1}{2}\,\sqrt{-\sqrt{3}+2}, y : -\frac{1}{4}\,\sqrt{3}\sqrt{2}-\frac{1}{4}\,\sqrt{2}\right\}, \left\{x : \frac{1}{2}\,\sqrt{-\sqrt{3}+2}, y : \frac{1}{4}\,\sqrt{3}\sqrt{2}+\frac{1}{4}\,\sqrt{2}\right\}\right]$$

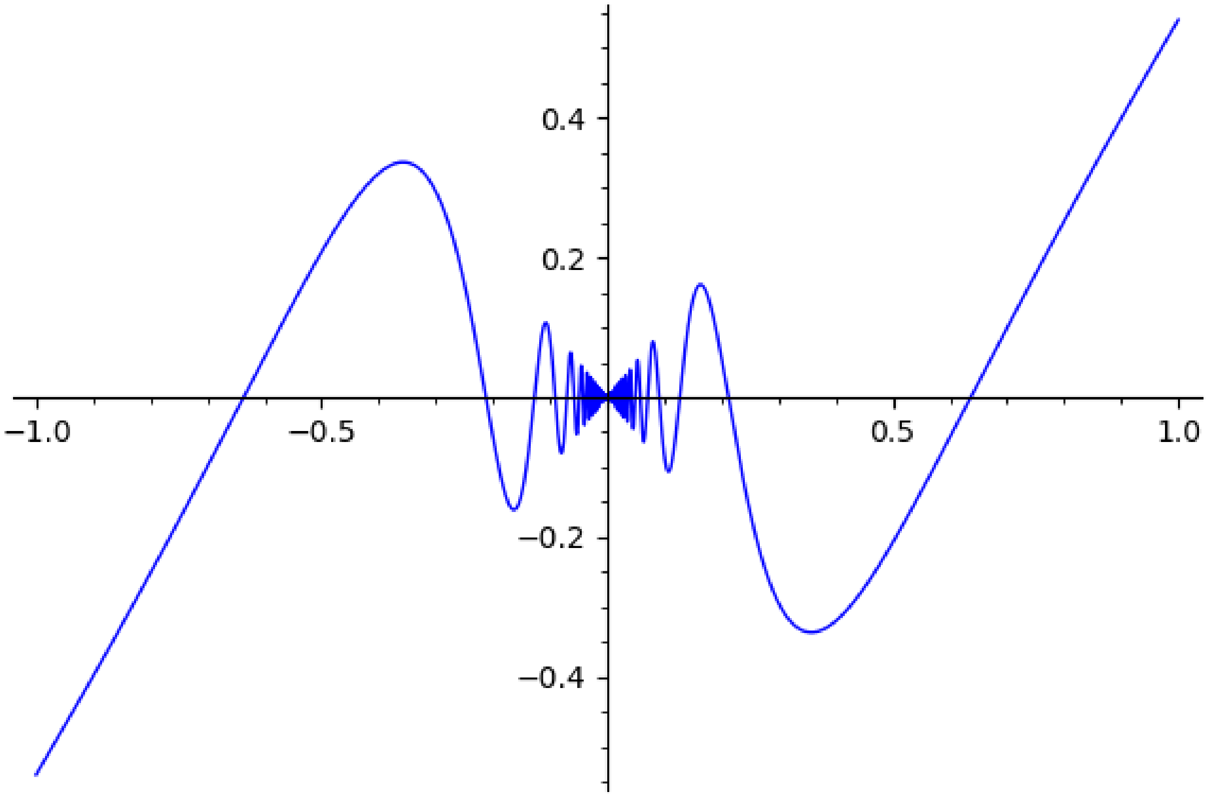
In [ ]:
```
show( s[0])
```

Graph of explicit functions

In [19]:

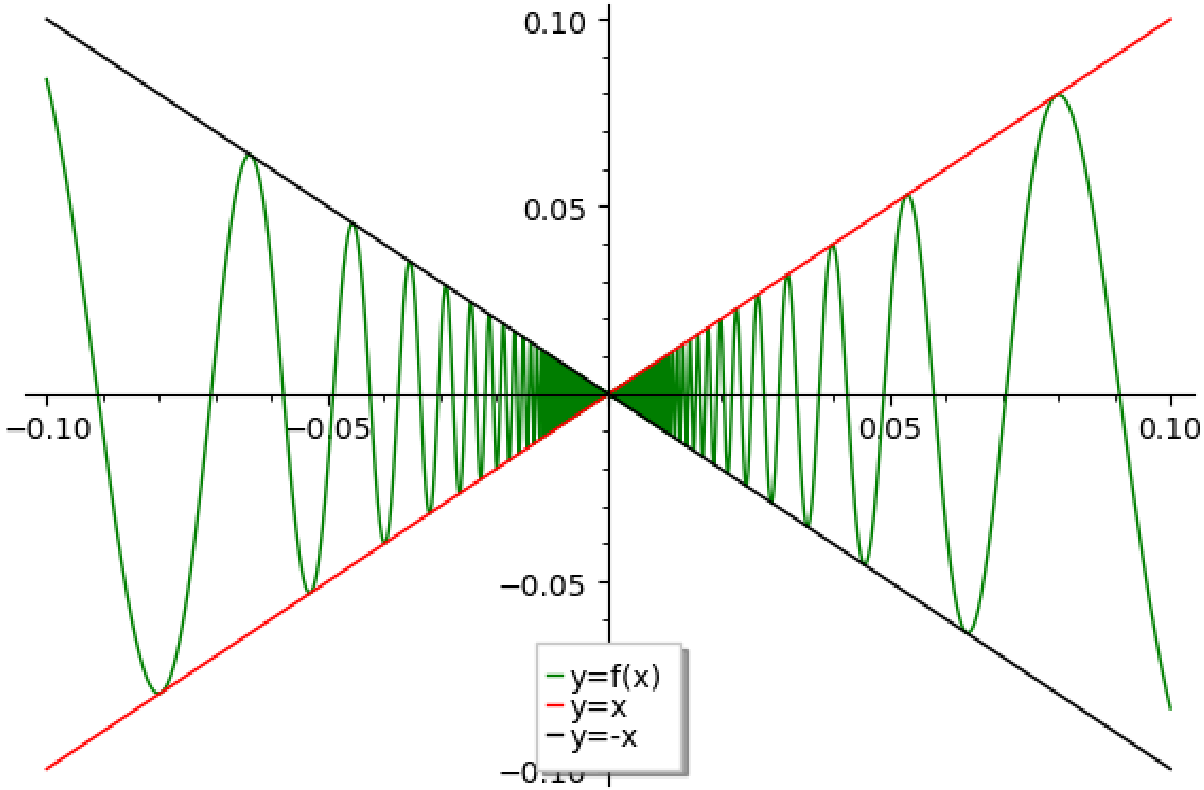
```
var('x')
f(x) = x*cos(1/x)
f.plot()
```

Out[19]:



Plotting multiple graphs together Example: Plot the graph of y = xcos1/x,y = x and y = −x together.

In [20]:
```
p = plot(f, (x, -0.1,0.1),figsize = 6,color = 'green',legend_label='y=f(x)')
p1 = plot(x, -0.1,0.1, color='red',legend_label='y=x')
p2 = plot(-x, -0.1,0.1, color='black',legend_label='y=-x')
show(p+p1+p2,figsize=6)
```



One Variable Calculus with SageMath

In [21]:
```
f(x)=sin(x)
```

In [22]:
```
a = 0
f.limit(x=a)
```

Out[22]:
```
x |--> 0
```

In [23]:
```
f(x)=sin(x)/x
a = 0
f.limit(x=a)
```

Out[23]:
```
x |--> 1
```

In [24]:
```
f.limit(x=a,dir='-')
```

Out[24]:
```
x |--> 1
```

In [25]:
```
f.limit(x=a,dir='+')
```

Out[25]:
```
x |--> 1
```

In [ ]:
```
f.plot((x,-10,10))
```

In [26]:
```
f(x)=sin(x)
df = f.diff()
show(df)
```

$x \mapsto \cos(x)$

In [27]:
```
f(x)=sin(x)/x
df = f.diff()
show(df)
```

$$x \mapsto \frac{\cos(x)}{x} - \frac{\sin(x)}{x^2}$$

In [28]:
```
d2f = f.diff(2)
show(d2f)
```

$$x \mapsto -\frac{\sin(x)}{x} - \frac{2\,\cos(x)}{x^2} + \frac{2\,\sin(x)}{x^3}$$

In [29]:
```
d2f = f.diff(20)
show(d2f)
```

$$x \mapsto \frac{\sin(x)}{x} + \frac{20\,\cos(x)}{x^2} - \frac{380\,\sin(x)}{x^3} - \frac{6840\,\cos(x)}{x^4} + \frac{116280\,\sin(x)}{x^5} + \frac{1860480\,\cos(x)}{x^6} - \frac{27907200\,\sin(x)}{x^7} -$$

$$+ \frac{5079110400\,\sin(x)}{x^9} + \frac{60949324800\,\cos(x)}{x^{10}} - \frac{670442572800\,\sin(x)}{x^{11}} - \frac{6704425728000\,\cos(x)}{x^{12}} + 6033983$$

$$+ \frac{482718652416000\,\cos(x)}{x^{14}} - \frac{3379030566912000\,\sin(x)}{x^{15}} - \frac{20274183401472000\,\cos(x)}{x^{16}} + \frac{101370917007}{x^1}$$

$$+ \frac{405483668029440000\,\cos(x)}{x^{18}} - \frac{1216451004088320000\,\sin(x)}{x^{19}} - \frac{2432902008176640000\,\cos(x)}{x^{20}} + 24329020$$

In [30]:
```
f(x)=sin(x)
show(f.integral(x))
```

$$x \mapsto -\cos(x)$$

In [31]:
```
f.integral(x,0,1)
```
Out[31]:
```
-cos(1) + 1
```
In [32]:
```
f(x) = x^2*sin(2*x)+x^2*exp(-x)+x^2-x+3
show(f(x))
```
$$x^2 e^{(-x)} + x^2 \sin(2\,x) + x^2 - x + 3$$

In [ ]:
```
a = oo
f.limit(x=a)
```
In [ ]:
```
limit(f(x),x=1).n()
```
In [33]:
```
df = f.diff()
show(df)
```
$$x \mapsto 2\,x^2 \cos(2\,x) - x^2 e^{(-x)} + 2\,xe^{(-x)} + 2\,x \sin(2\,x) + 2\,x - 1$$

In [34]:
```
f.derivative()
```
Out[34]:
```
x |--> 2*x^2*cos(2*x) - x^2*e^(-x) + 2*x*e^(-x) + 2*x*sin(2*x) + 2*x - 1
```
In [35]:
```
show(f.derivative())
```
$$x \mapsto 2\,x^2 \cos(2\,x) - x^2 e^{(-x)} + 2\,xe^{(-x)} + 2\,x \sin(2\,x) + 2\,x - 1$$

In [36]:
```
show(f.integral(x))
```
$$x \mapsto \frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 - \frac{1}{4}\,\left(2\,x^2 - 1\right)\cos(2\,x) - \left(x^2 + 2\,x + 2\right)e^{(-x)} + \frac{1}{2}\,x \sin(2\,x) + 3\,x$$

In [37]:
```
f.
```

Out[37]: 3.30262155002575

Laplace transform of standard Functions and piecewise functions

In [38]:
```
# Laplace tranform of a constant function f(t)= c
t,s,c=var('t,s,c')
f(t)= c
f.laplace(t, s)
show(f.laplace(t, s))
```

$$t \mapsto \frac{c}{s}$$

In [39]:
```
var('n')
var('t')
var('s')
laplace(t^n, t, s, algorithm='sympy')
```

Out[39]: (gamma(n + 1)/(s*s^n), 0, re(n) > -1)

In [40]:
```
show(laplace(t^n, t, s, algorithm='sympy'))
```

$$\left(\frac{\Gamma\left(n+1\right)}{s s^{n}}, 0, \mathtt{re(n)} > \text{-1}\right)$$

In [41]:
```
## Laplace tranform of f(t)=sin(t)
f=sin(4*t)
f.laplace(t,s)
show(f.laplace(t,s))
```

$$\frac{4}{s^2 + 16}$$

In [42]:
```
## Laplace tranform of a piecewise defined function
t,s=var('t,s')
f = piecewise([[(0,2),1],[(2,4),t],[(4,infinity),exp(-2*t)]])
show(f)
Fs=f.laplace(t,s)
show(Fs)
```

$$piecewise\left(\left(\left((0,\ 2), 1\right), \left((2,\ 4), t\right), \left((4,\ +oo), e^{(-2\,t)}\right)\right), t\right)$$

$$\frac{(2\,s + 1)e^{(-2\,s)}}{s^2} - \frac{e^{(-2\,s)}}{s} + \frac{e^{(-4\,s)}}{se^8 + 2\,e^8} - \frac{(4\,s + 1)e^{(-4\,s)}}{s^2} + \frac{1}{s}$$

Laplace transform of derivative and Integral

In [43]:
```
## Laplace tranform of 1st derivative
var('t')
f = function('f')(t)
laplace(diff(f,t),t,s)
```

Out[43]:
```
s*laplace(f(t), t, s) - f(0)
```

In [44]:
```
show(laplace(diff(f,t),t,s))
```

$$s\mathcal{L}\left(f\left(t\right), t, s\right) - f\left(0\right)$$

In [45]:
```
f(t) = t*sin(t)
show(f(t))
var('a')
g = f.integral(t)
show(g)
show(g.laplace(t,s))
h = g.laplace(t,s)
show(h.full_simplify())
```

$$t \sin(t)$$

$$t \mapsto -t\cos(t) + \sin(t)$$

$$t \mapsto -\frac{2\,s^2}{\left(s^2+1\right)^2} + \frac{2}{s^2+1}$$

$$\frac{2}{s^4 + 2\,s^2 + 1}$$

Inverse Laplace Transform

In [46]:
```
F(s) = 1/s^11*factorial(10)
inverse_laplace(F(s),s,t)
```

Out[46]:
```
t^10
```

In [47]:
```
show(inverse_laplace(F(s),s,t))
```

t^{10}

In [48]:
```
F(s) = s/(s^3+s^2+s+1)
show(F(s))
show(inverse_laplace(F(s),s,t))
```

$$\frac{s}{s^3 + s^2 + s + 1}$$

$$\frac{1}{2} \cos(t) - \frac{1}{2} e^{(-t)} + \frac{1}{2} \sin(t)$$

Solving ODE using Laplace Transform

Example: Solve x'(t) + x(t) = cos(2t), x(0) = 2 using the Laplace transform.

In [49]:
```
s,t = var('s,t')
x = function('x')(t)
de = diff(x,t) + x == cos(2*t)
```

In [50]:
```
desolve_laplace(de,x,ics=[0,2])
```

Out[50]:
```
1/5*cos(2*t) + 9/5*e^(-t) + 2/5*sin(2*t)
```

In [51]:
```
show(desolve_laplace(de,x,ics=[0,2]))
```

$$\frac{1}{5} \cos(2\,t) + \frac{9}{5} e^{(-t)} + \frac{2}{5} \sin(2\,t)$$

Example Solve the 2nd order initial value problem

x''(t) + 2x'(t) + 2x = e^(-2t), x(0) = 0, x'(0) = 0

In [ ]:
```
s,t = var('s,t')
x = function('x')(t)
de = diff(x,t,t)+2*diff(x,t)+2*x==exp(-2*t)
```

In [52]:
```
show(desolve_laplace(de,x,ics=[0,0,0]))
```

$$\frac{1}{5} \cos(2\,t) - \frac{1}{5}\,e^{(-t)} + \frac{2}{5}\sin(2\,t)$$

In [ ]:
```

```