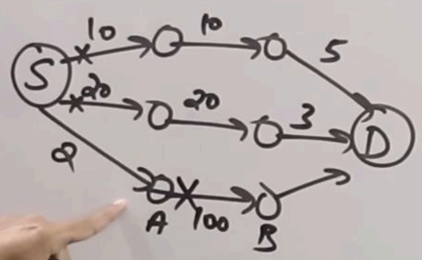
Introduction to Dynamic Programming

Dynamic Programming: It divide the problem into series of overlapping sub-problems.

Two features 1) Optimal Substructure

2) Overlapping Subproblems



Dynamic Programming: It divide the probrem into series of overlapping sub-problems.

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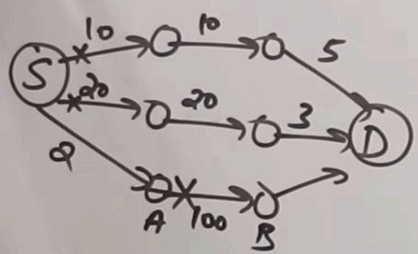
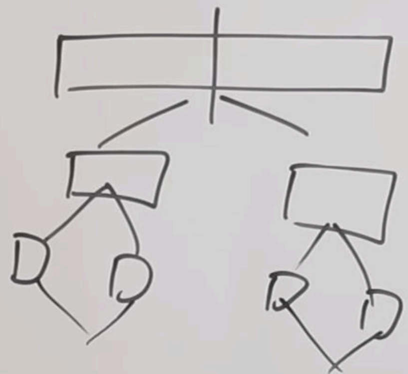
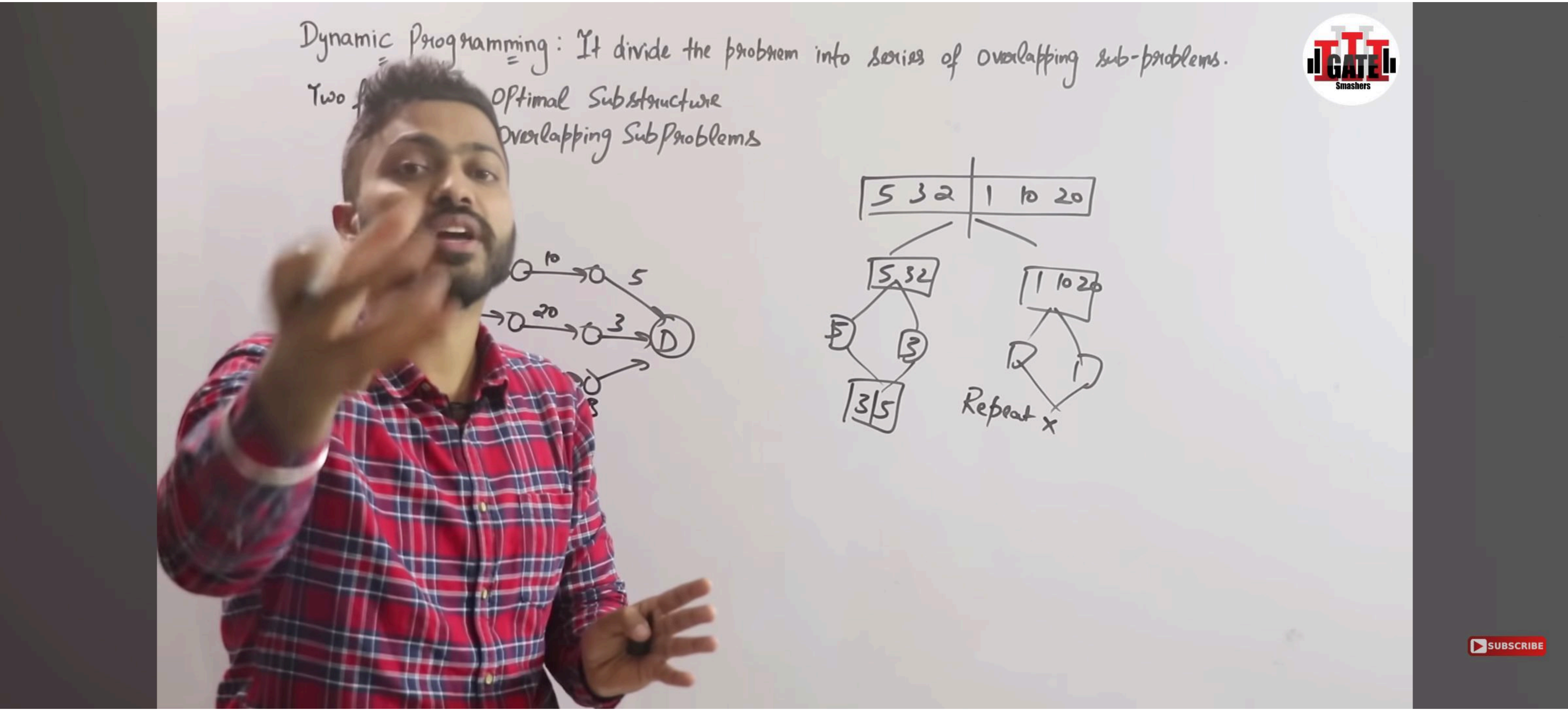
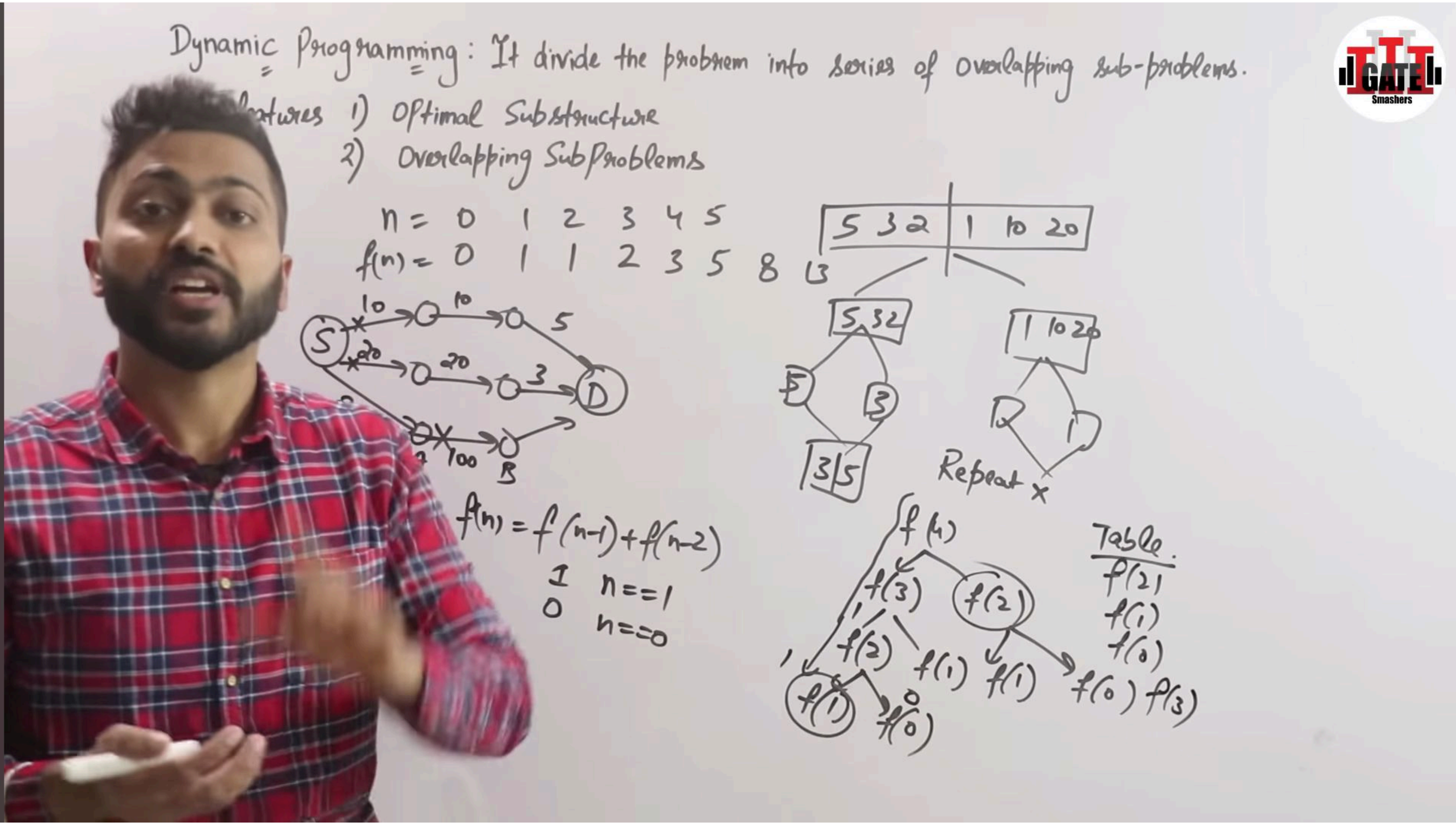
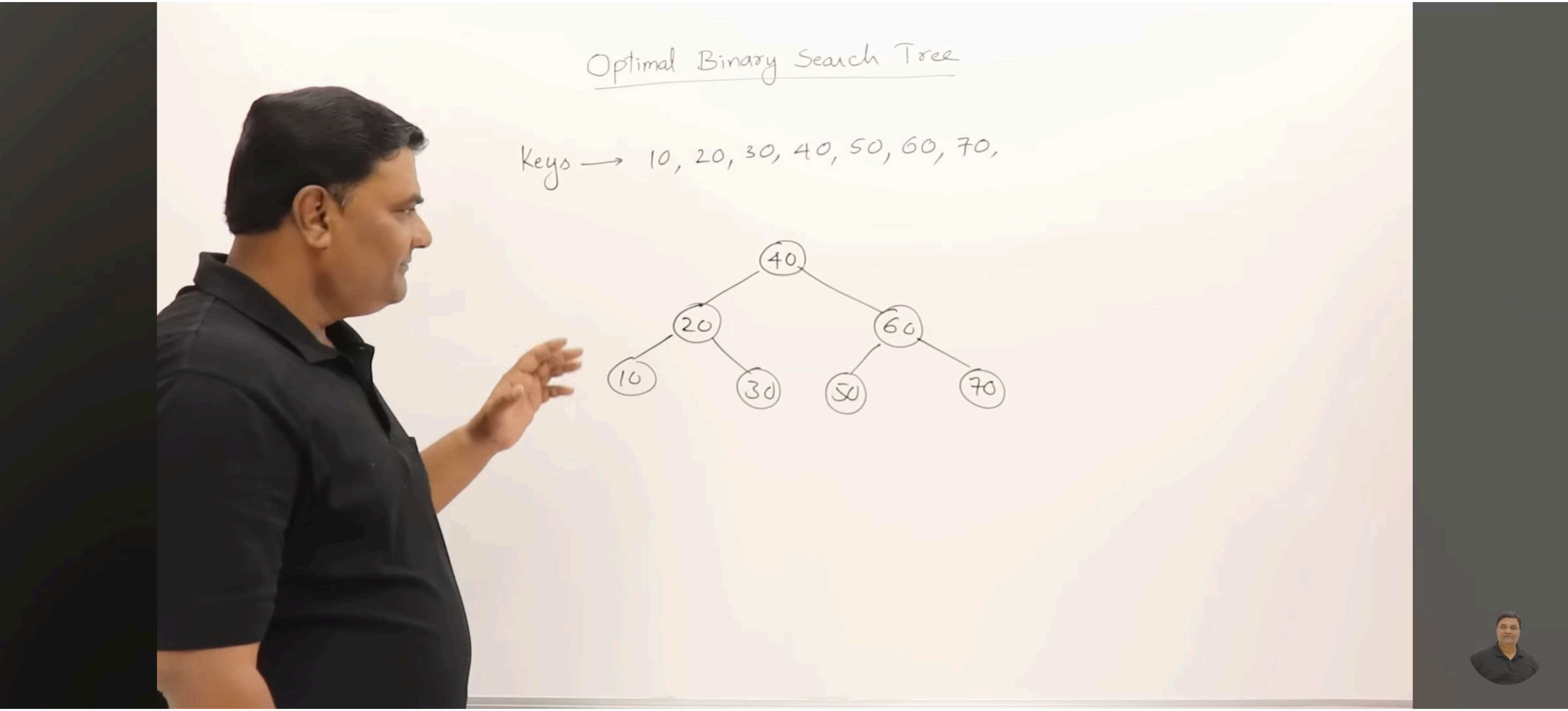




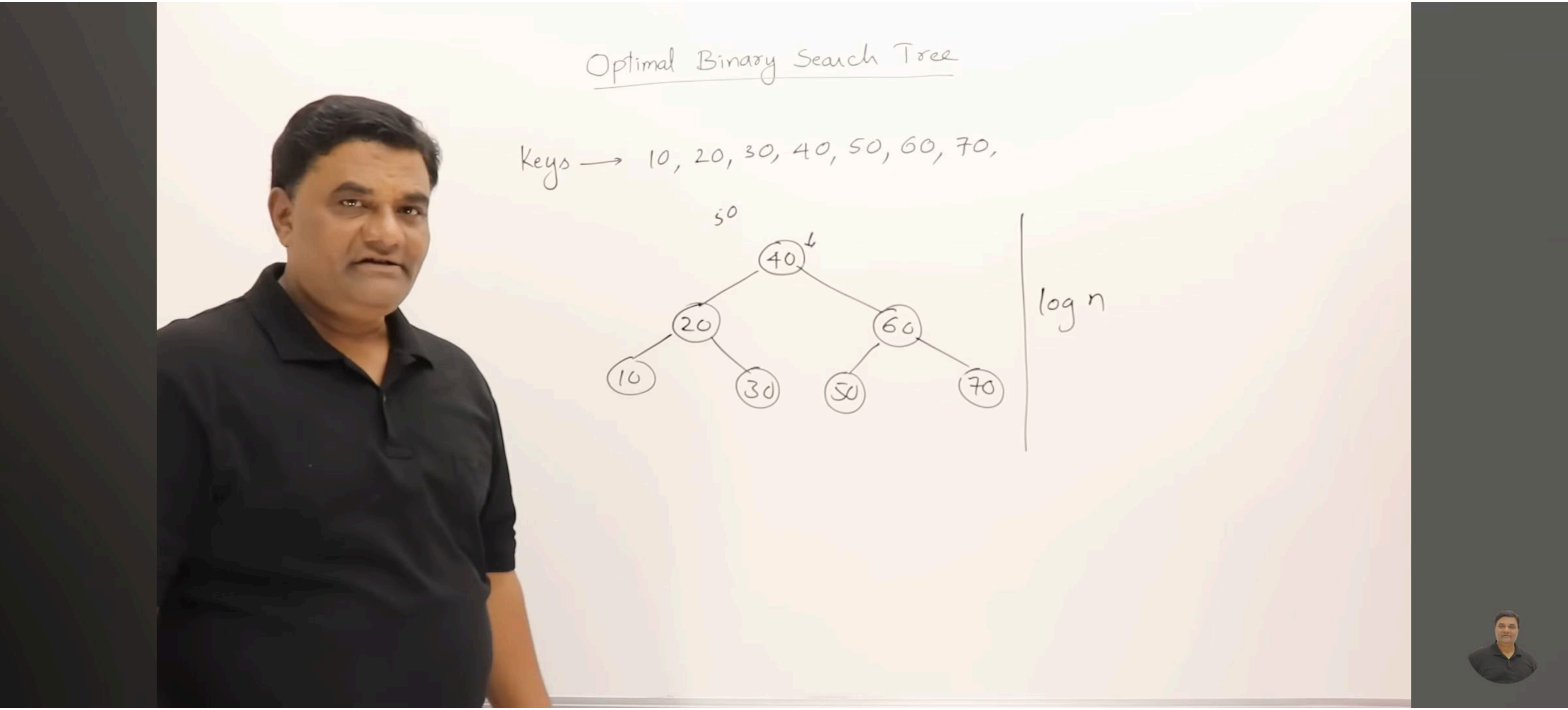

Dynamic Programming : It divide the probrem into series of overlapping sub-problems.
Two f Optimal Substructure
Overlapping SubProblems
5 3 2 | 1 10 20
5 3 2
1 10 20
3 5
Repeat x
D
GATE
Smashers
SUBSCRIBE

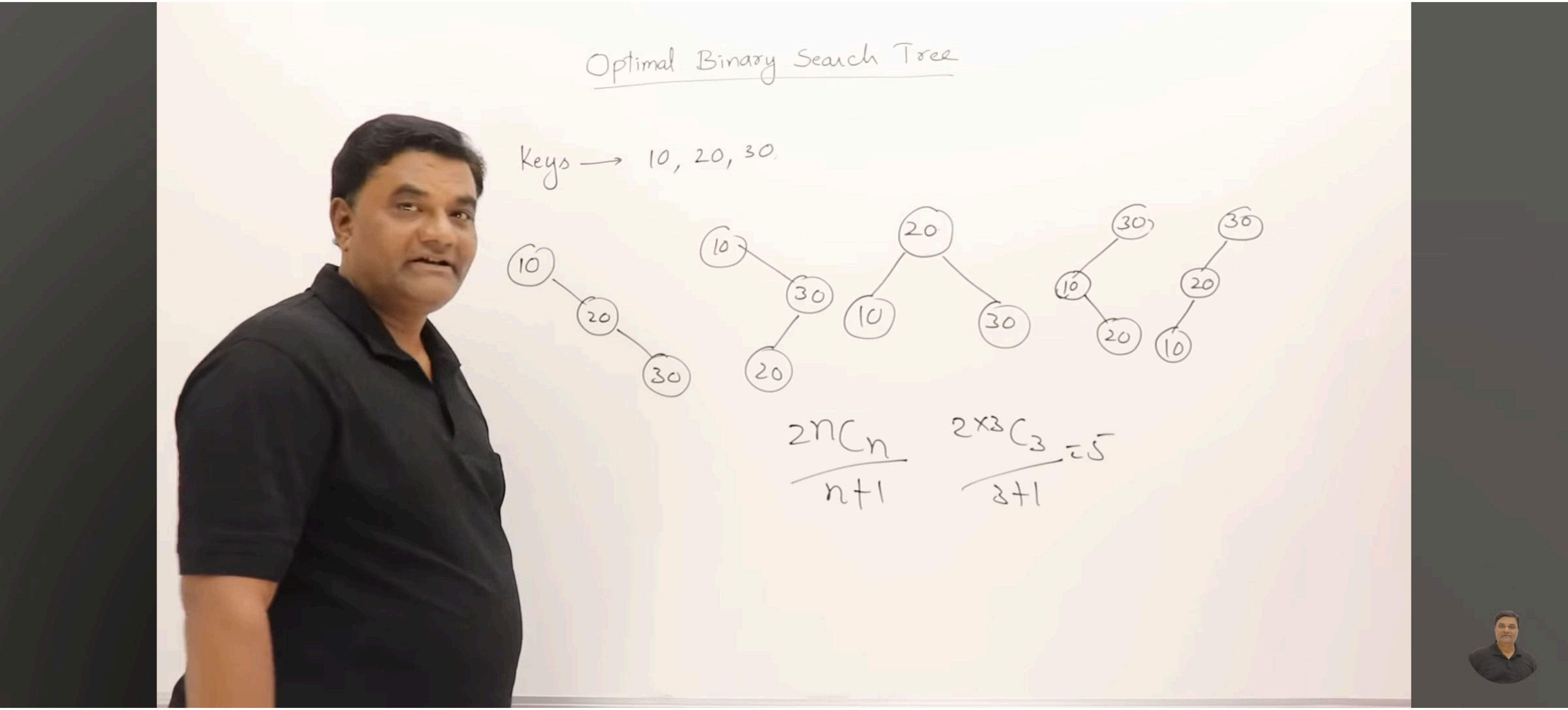

Dynamic Programming : It divide the probrem into series of overlapping sub-problems.
features 1) Optimal Substructure
2) Overlapping SubProblems
n = 0 1 2 3 4 5
f(n) = 0 1 1 2 3 5 8 13
f(n) = f(n-1) + f(n-2)
1 n==1
0 n==0
5 3 2 | 1 10 20
Repeat ×
Table
f(2)
f(1)
f(0)
f(4)
f(3)
f(2)
f(1)
f(0)
GATE Smashers
SUBSCRIBE

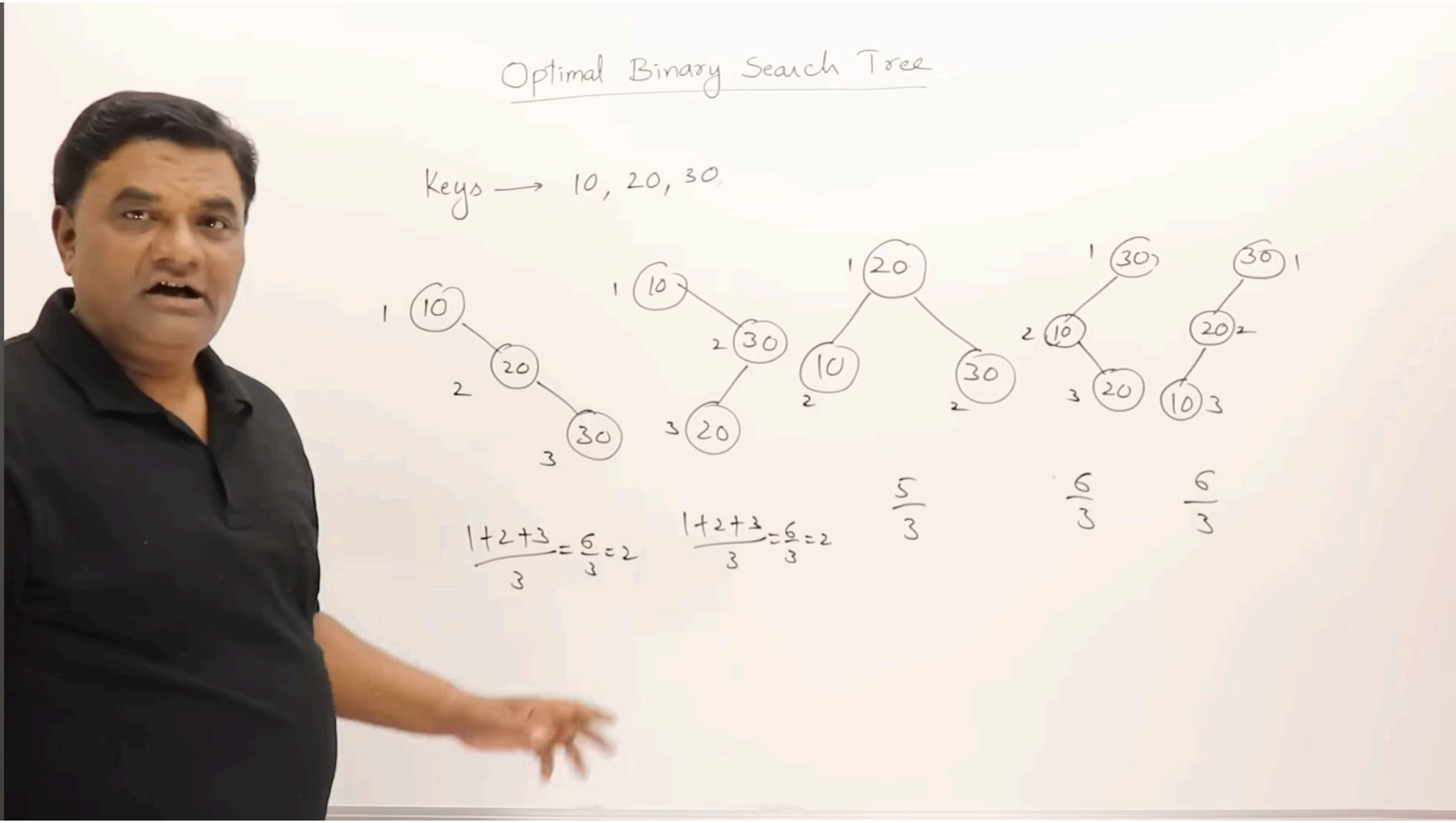
Optimal Binary Search Tree(OBST)

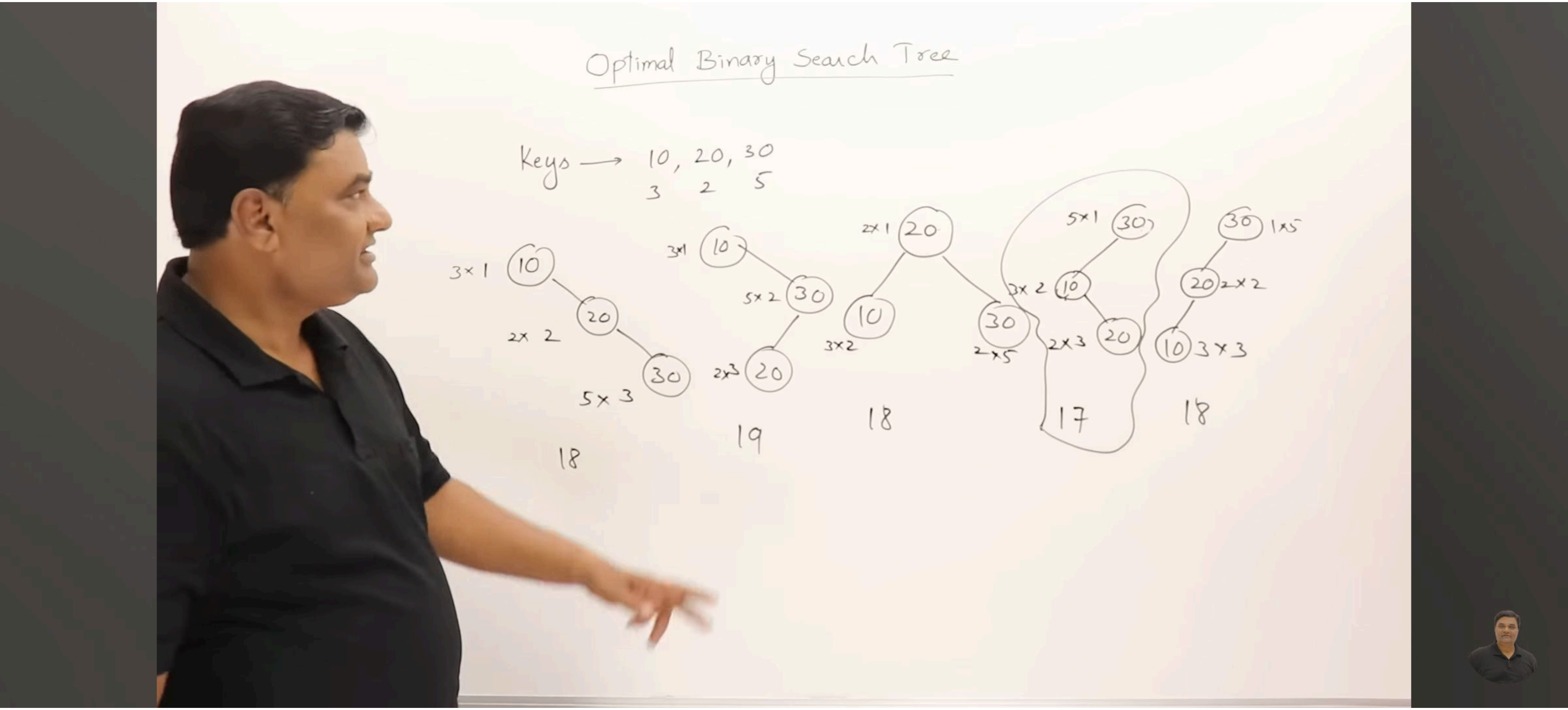


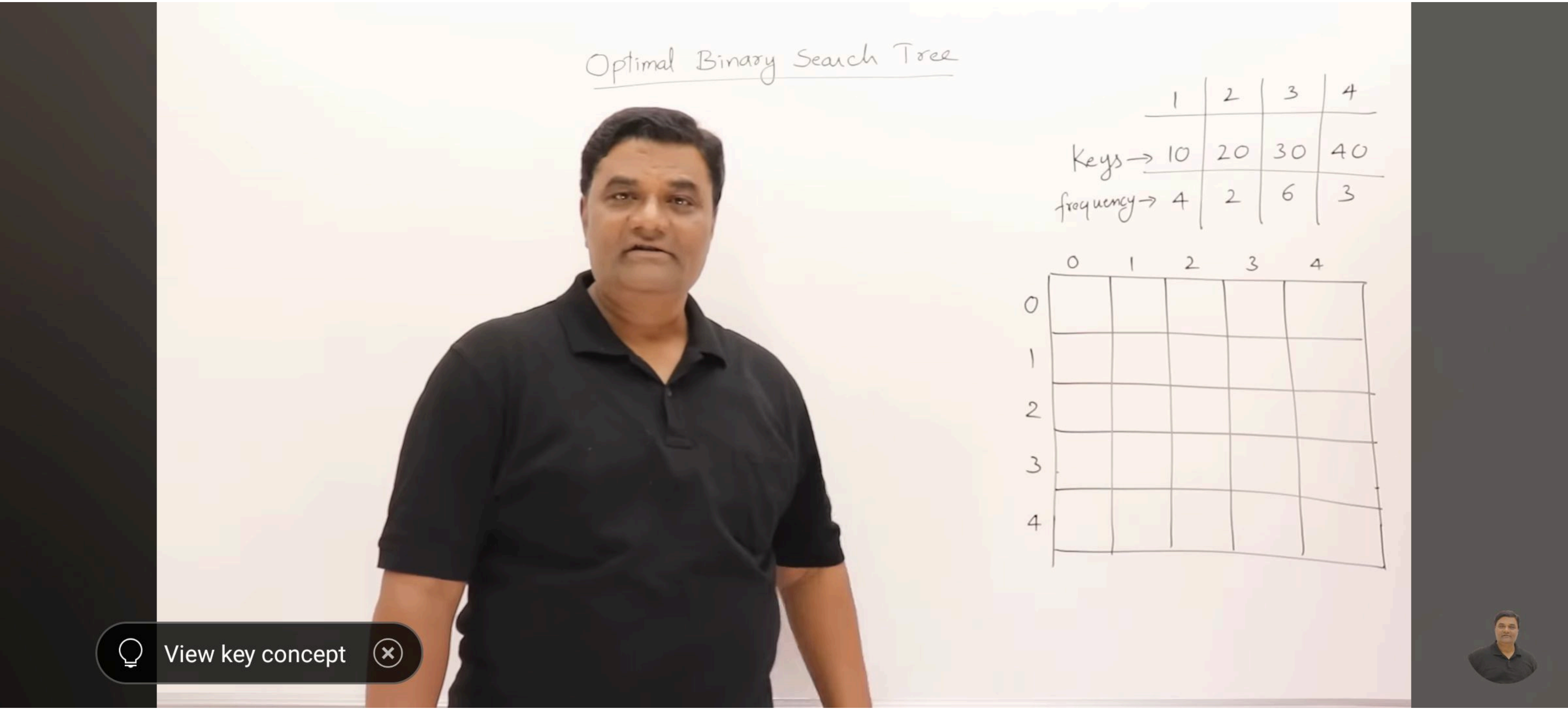
Optimal Binary Search Tree
Keys → 10, 20, 30, 40, 50, 60, 70,
40
20
60
10
30
50
70

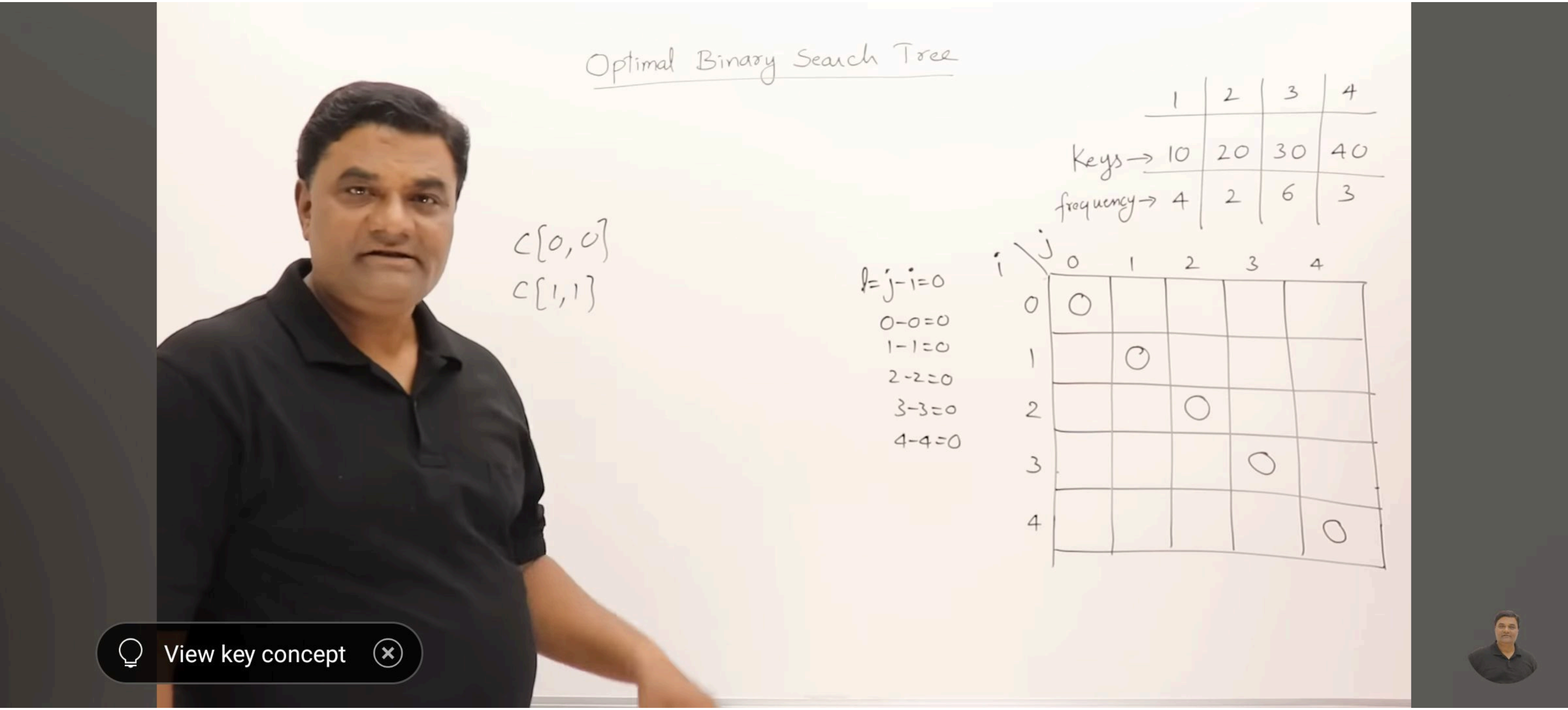

Optimal Binary Search Tree
Keys → 10, 20, 30, 40, 50, 60, 70,
50
40
20
60
10
30
50
70
log n

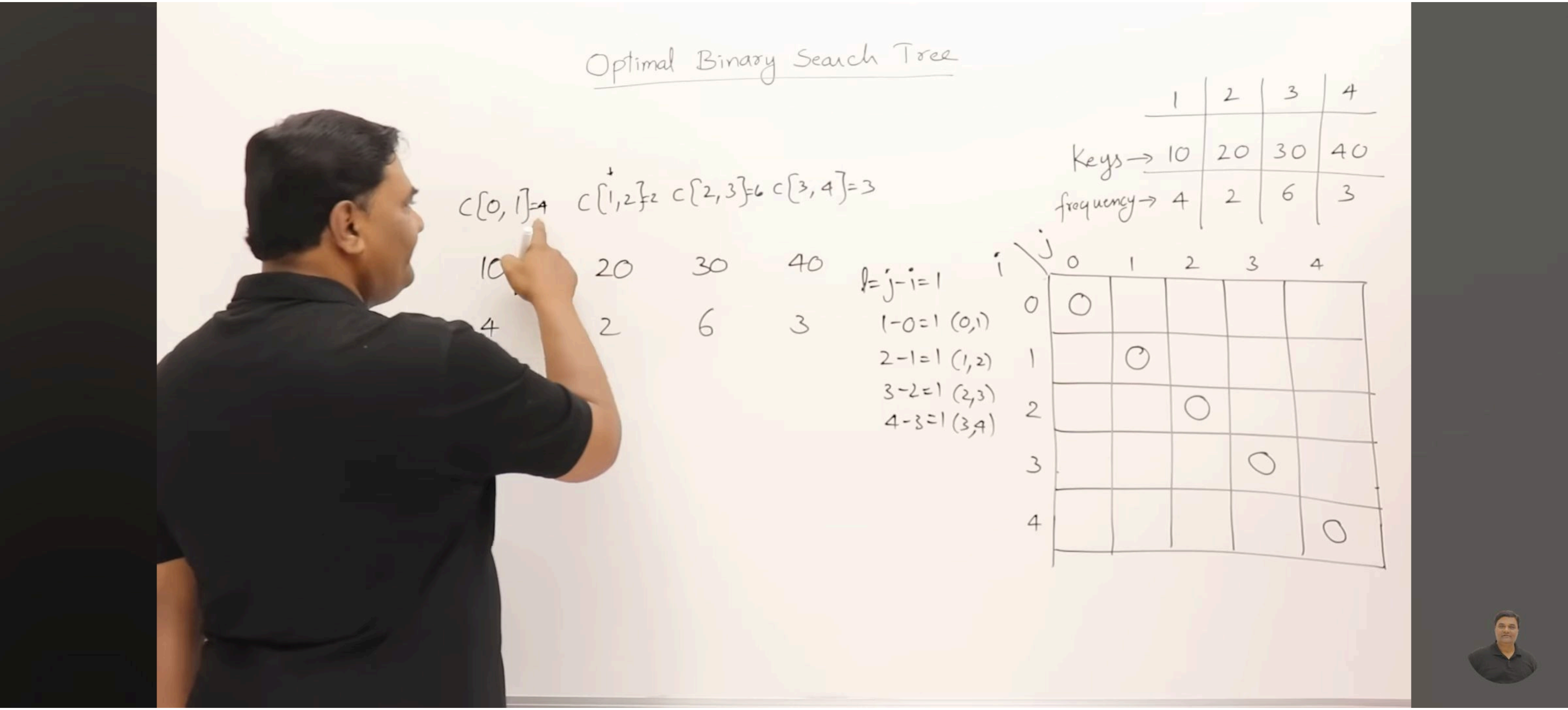

Optimal Binary Search Tree
Keys → 10, 20, 30
10
20
30
10
30
20
20
10
30
30
10
20
30
20
10
2nCn / n+1
2×3C3 / 3+1 = 5

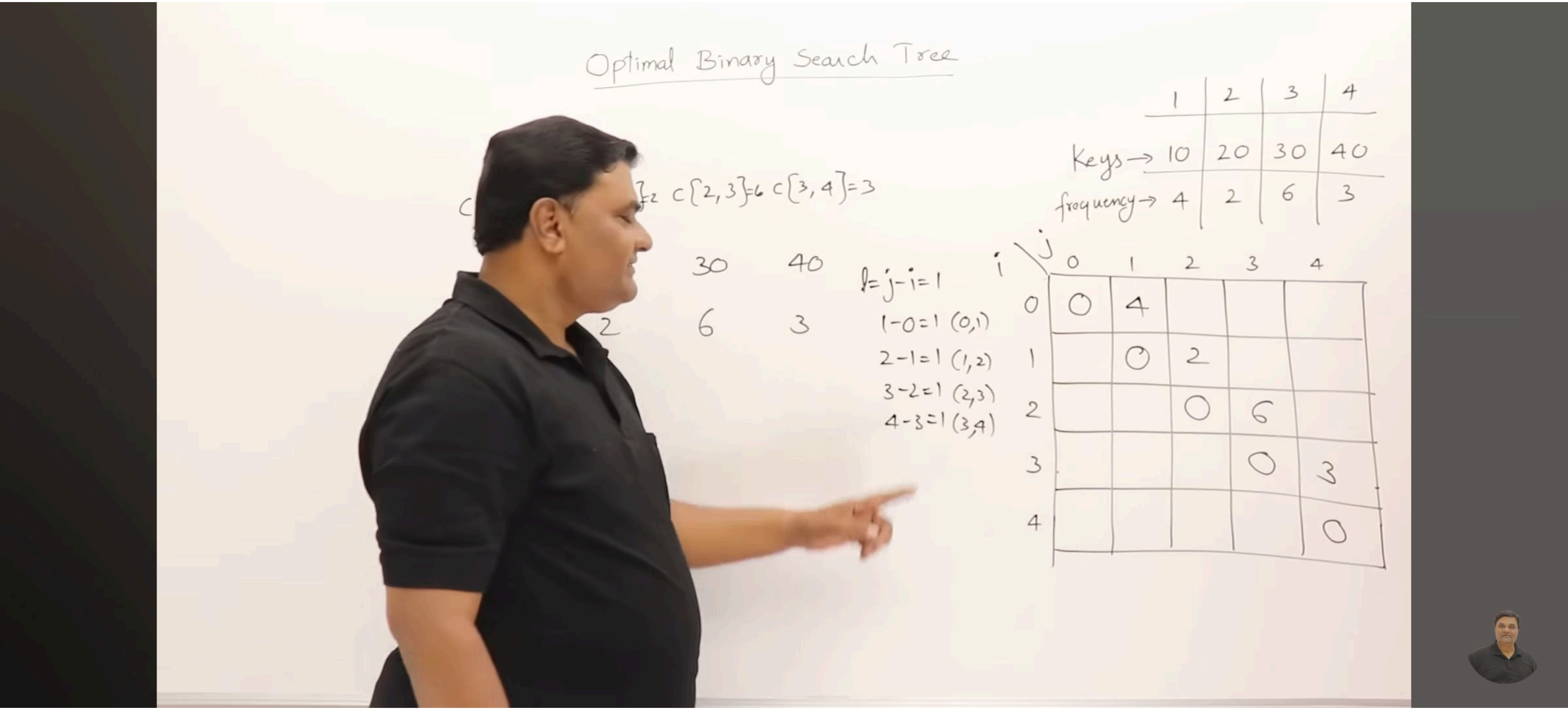

Optimal Binary Search Tree
Keys → 10, 20, 30
10
20
30
1+2+3 / 3 = 6/3 = 2
1+2+3 / 3 = 6/3 = 2
5/3
6/3
6/3

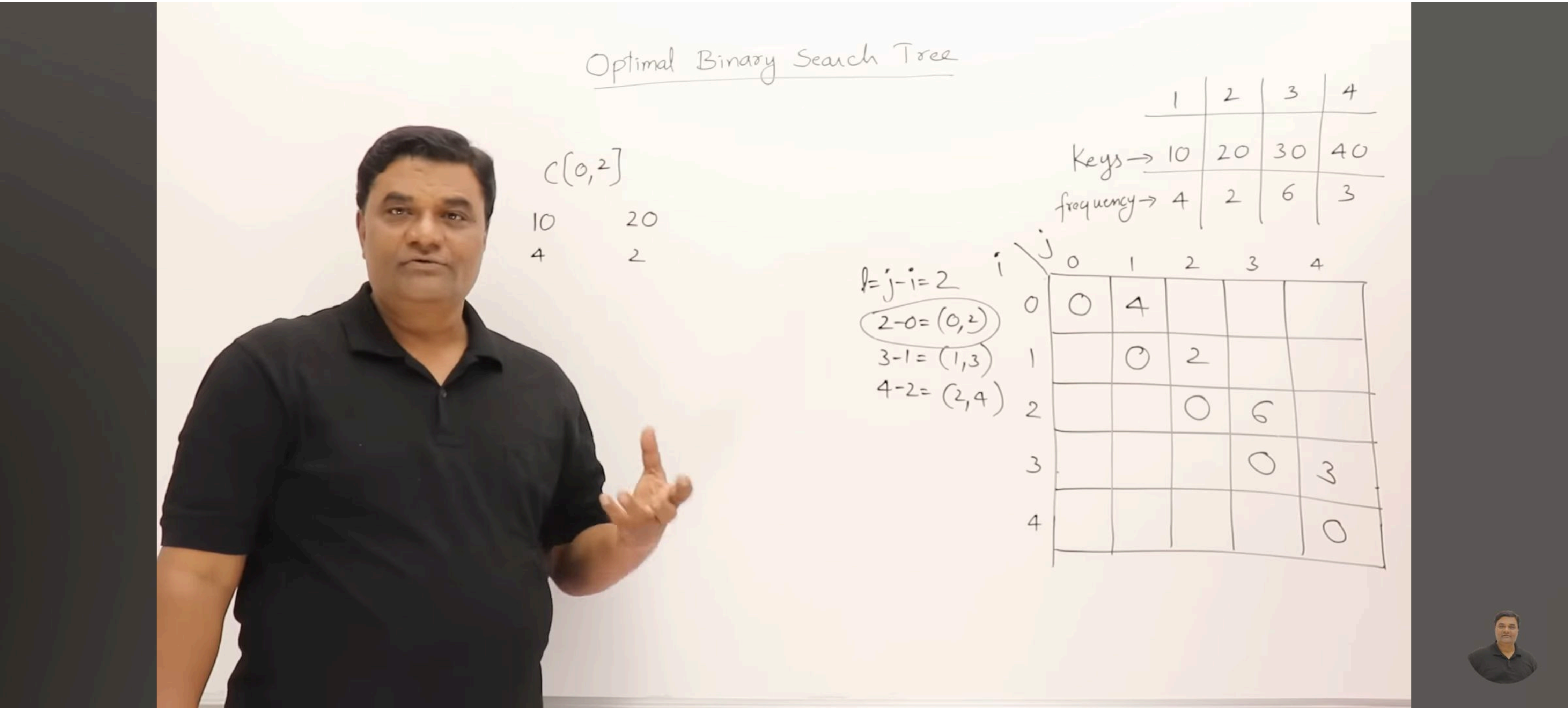

Optimal Binary Search Tree
Keys → 10, 20, 30
3 2 5
3×1 10
2× 2 20
5× 3 30
18
3×1 10
5×2 30
2×3 20
19
2×1 20
3×2 10
2×5 30
18
5×1 30
3×2 10
2×3 20
17
30 1×5
20 2×2
10 3×3
18

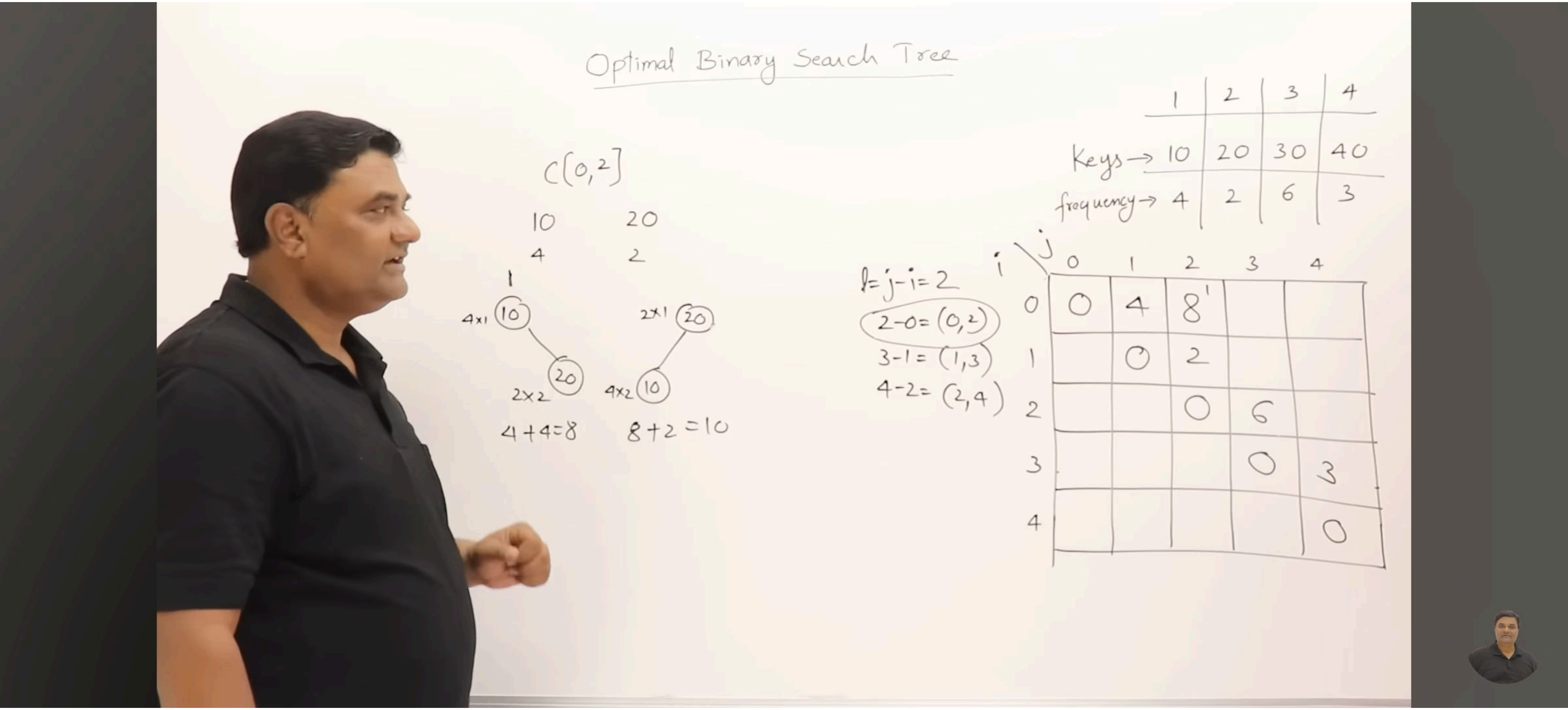

Optimal Binary Search Tree
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
0 1 2 3 4
0
1
2
3
4
View key concept

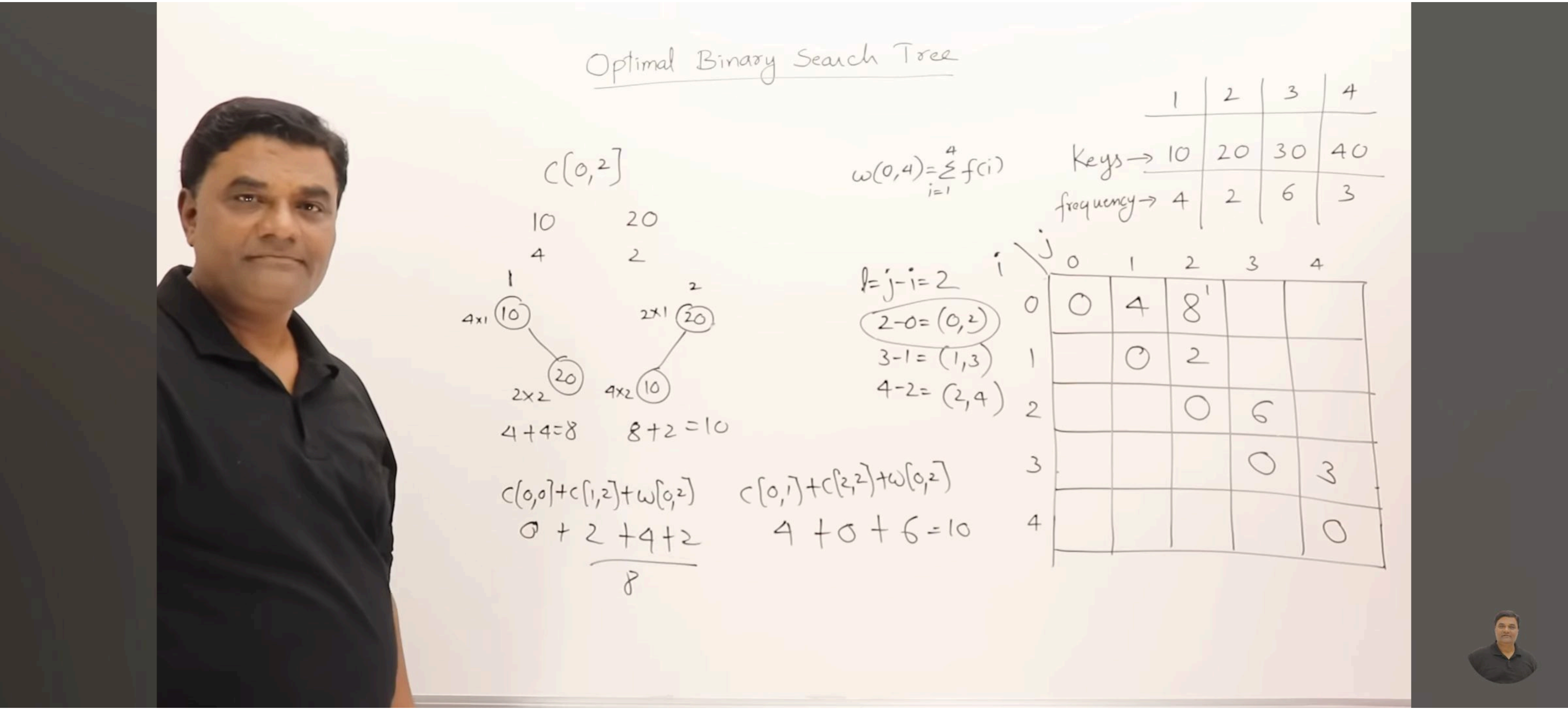

Optimal Binary Search Tree
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
C[0,0]
C[1,1]
l=j-i=0
0-0=0
1-1=0
2-2=0
3-3=0
4-4=0
i \ j
0 1 2 3 4
0 0
1 0
2 0
3 0
4 0
View key concept

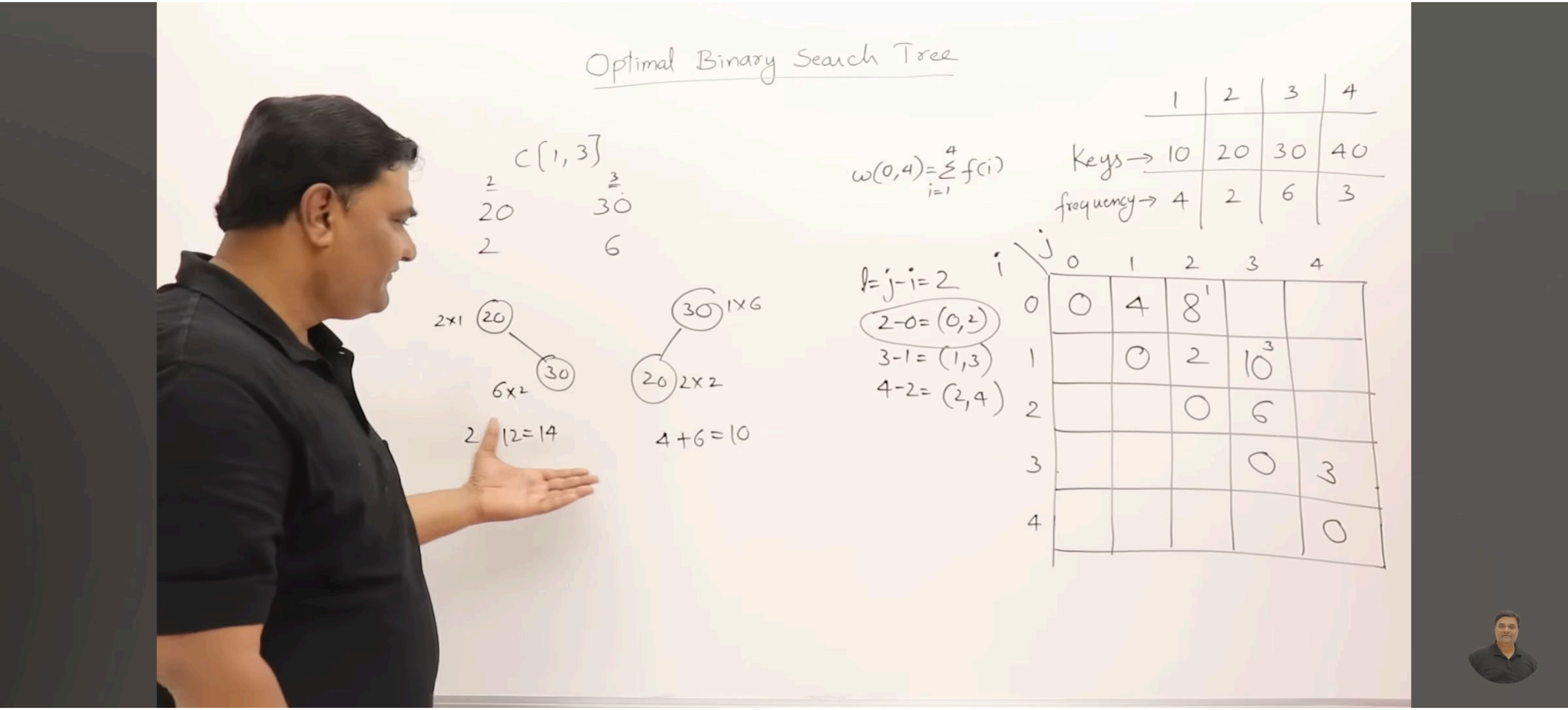

Optimal Binary Search Tree
C[0, 1]=4 C[1,2]=2 C[2,3]=6 C[3,4]=3
10 20 30 40
4 2 6 3
l=j-i=1
1-0=1 (0,1)
2-1=1 (1,2)
3-2=1 (2,3)
4-3=1 (3,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i j
0 1 2 3 4
0 0
1 0
2 0
3 0
4 0


Optimal Binary Search Tree
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
c[2,3]=6 c[3,4]=3
30 40
2 6 3
l=j-i=1
1-0=1 (0,1)
2-1=1 (1,2)
3-2=1 (2,3)
4-3=1 (3,4)
i \ j 0 1 2 3 4
0 0 4
1 0 2
2 0 6
3 0 3
4 0

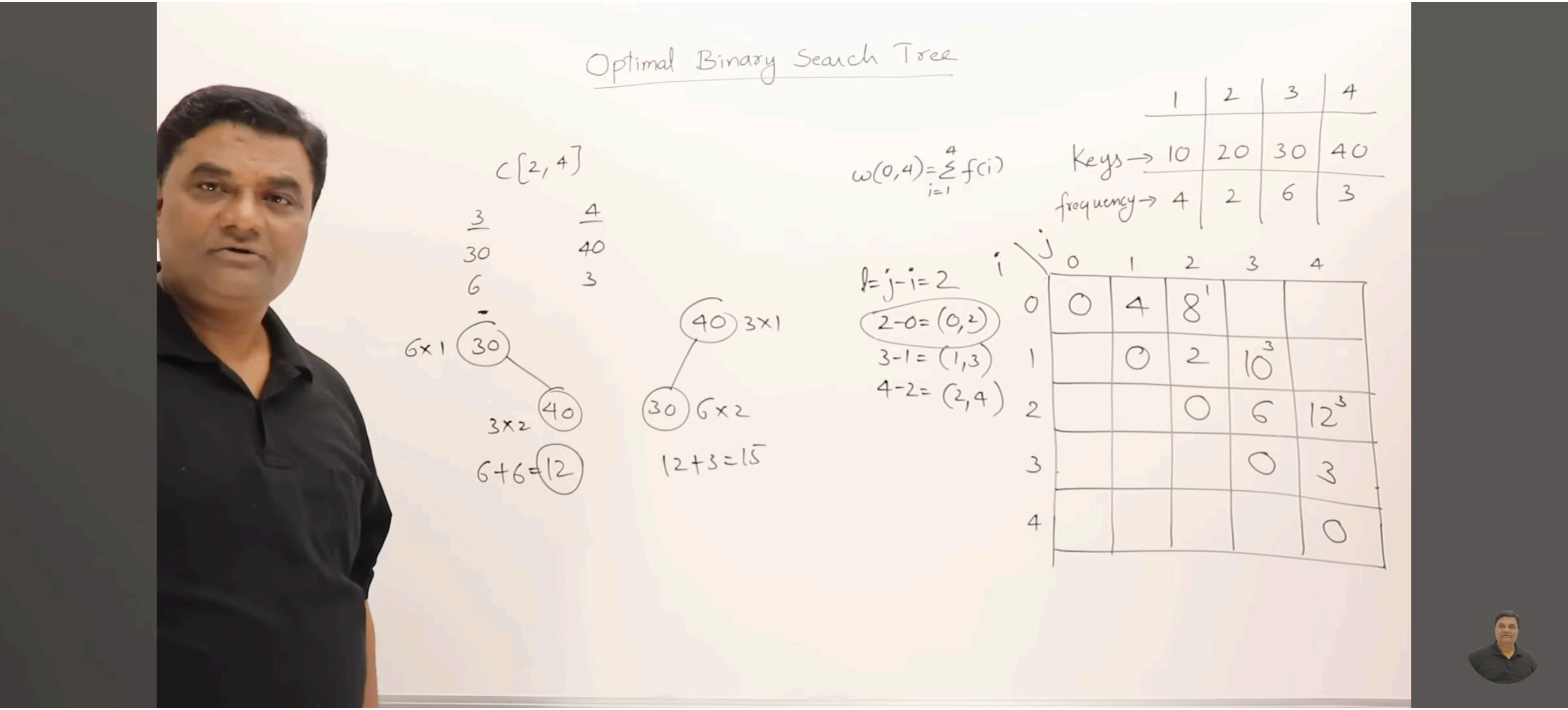

Optimal Binary Search Tree
C[0,2]
10 20
4 2
l=j-i=2
2-0=(0,2)
3-1=(1,3)
4-2=(2,4)
1 2 3 4
Keys→ 10 20 30 40
frequency→ 4 2 6 3
i \ j 0 1 2 3 4
0 0 4
1 0 2
2 0 6
3 0 3
4 0


Optimal Binary Search Tree
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
C[0,2]
10 20
4 2
4x1 10
20 2x2
2x1 20
4x2 10
4+4=8
8+2=10
l=j-i=2
2-0=(0,2)
3-1=(1,3)
4-2=(2,4)
i \ j 0 1 2 3 4
0: 0 4 8
1: 0 2
2: 0 6
3: 0 3
4: 0

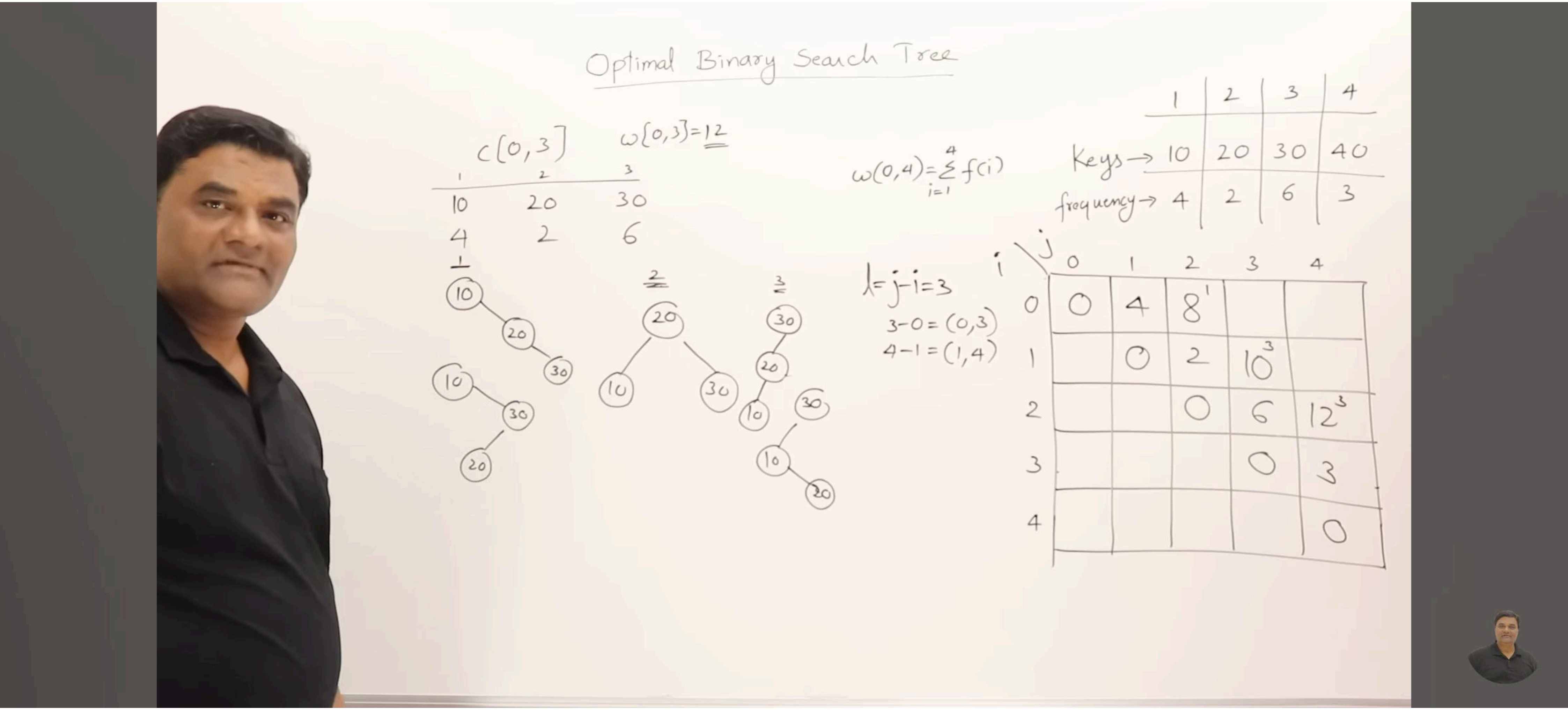

Optimal Binary Search Tree
C[0,2]
10 20
4 2
4x1 10
2x2 20
4+4=8
2x1 20
4x2 10
8+2=10
c[0,0]+c[1,2]+ω[0,2]
0 + 2 + 4 + 2
8
c[0,1]+c[2,2]+ω[0,2]
4 + 0 + 6 = 10
ω(0,4) = Σ_{i=1}^{4} f(i)
l = j - i = 2
2-0 = (0,2)
3-1 = (1,3)
4-2 = (2,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0: 0 4 8
1: 0 2
2: 0 6
3: 0 3
4: 0

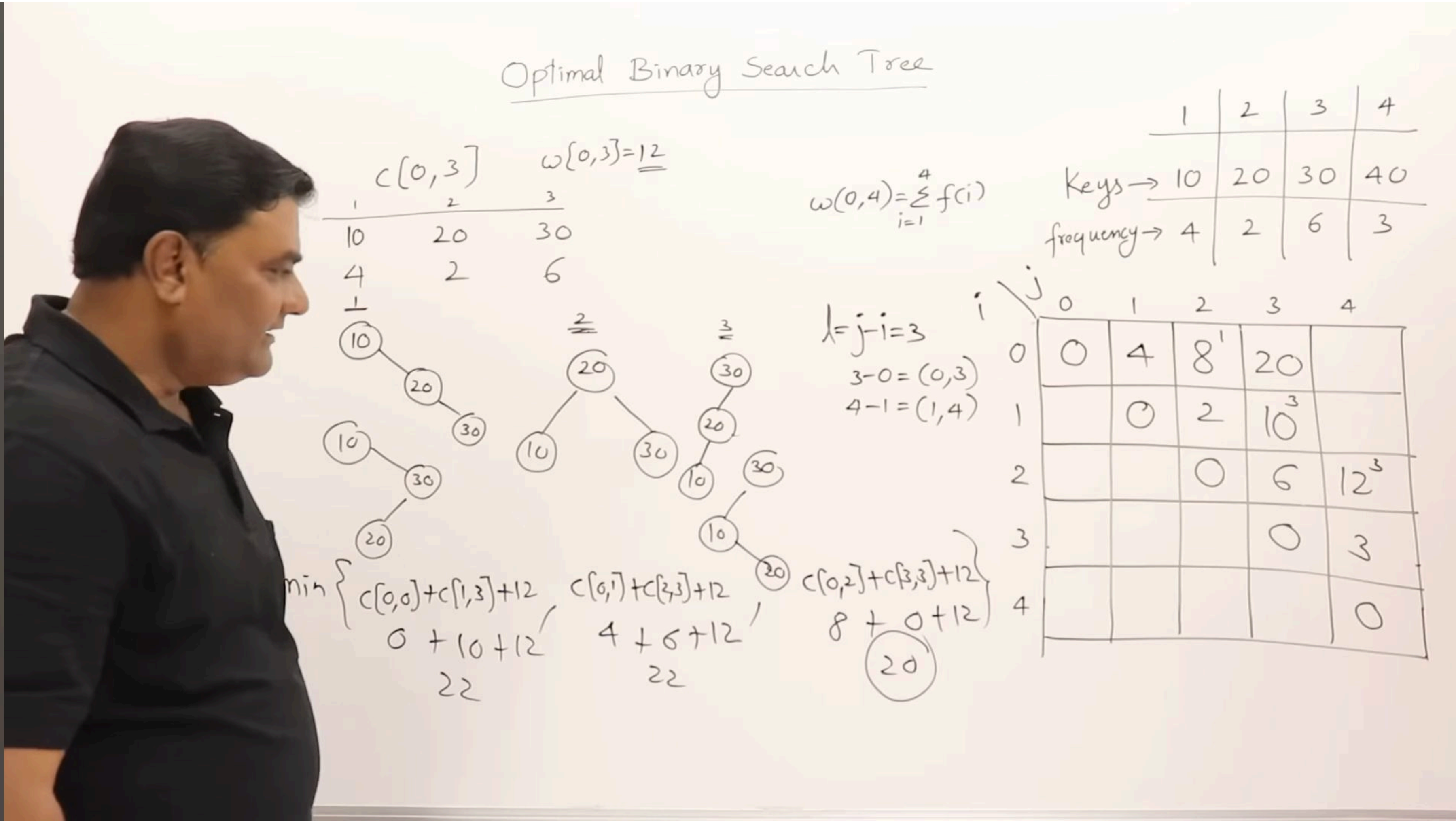


Optimal Binary Search Tree
C[1, 3]
20
30
2
6
2×1
20
30
6×2
1×6
20
2×2
2 12=14
4+6=10
ω(0,4)= Σ f(i)
l=j-i=2
2-0=(0,2)
3-1=(1,3)
4-2=(2,4)
Keys→ 10 20 30 40
frequency→ 4 2 6 3
i
j
0 1 2 3 4
0 4 8
0 2 10
0 6
0 3
0

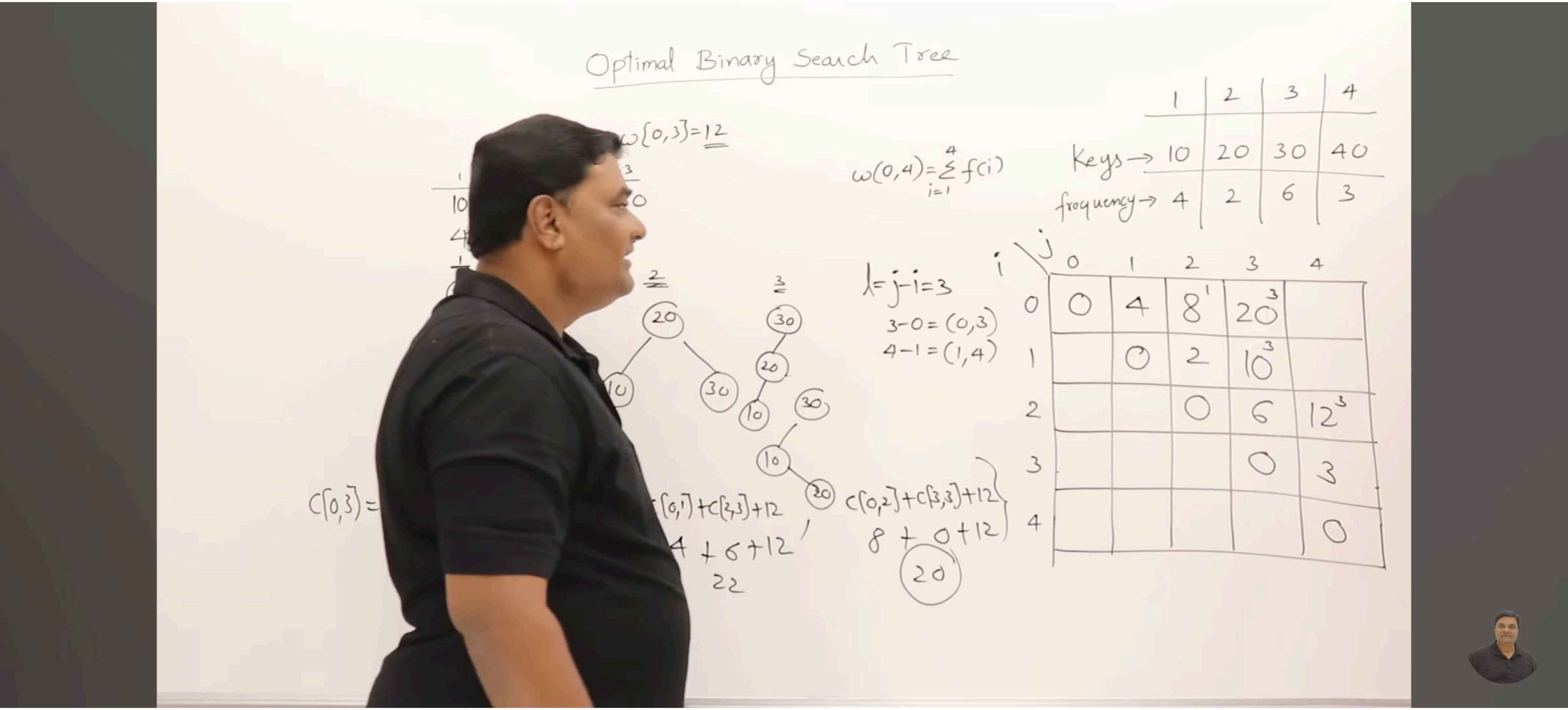


Optimal Binary Search Tree
C[2, 4]
3
30
6
4
40
3
6×1
30
40
3×2
6+6=12
40
3×1
30
6×2
12+3=15
w(0,4)= Σ_{i=1}^{4} f(i)
l=j-i=2
2-0=(0,2)
3-1=(1,3)
4-2=(2,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0: 0 4 8¹
1: 0 2 10³
2: 0 6 12³
3: 0 3
4: 0

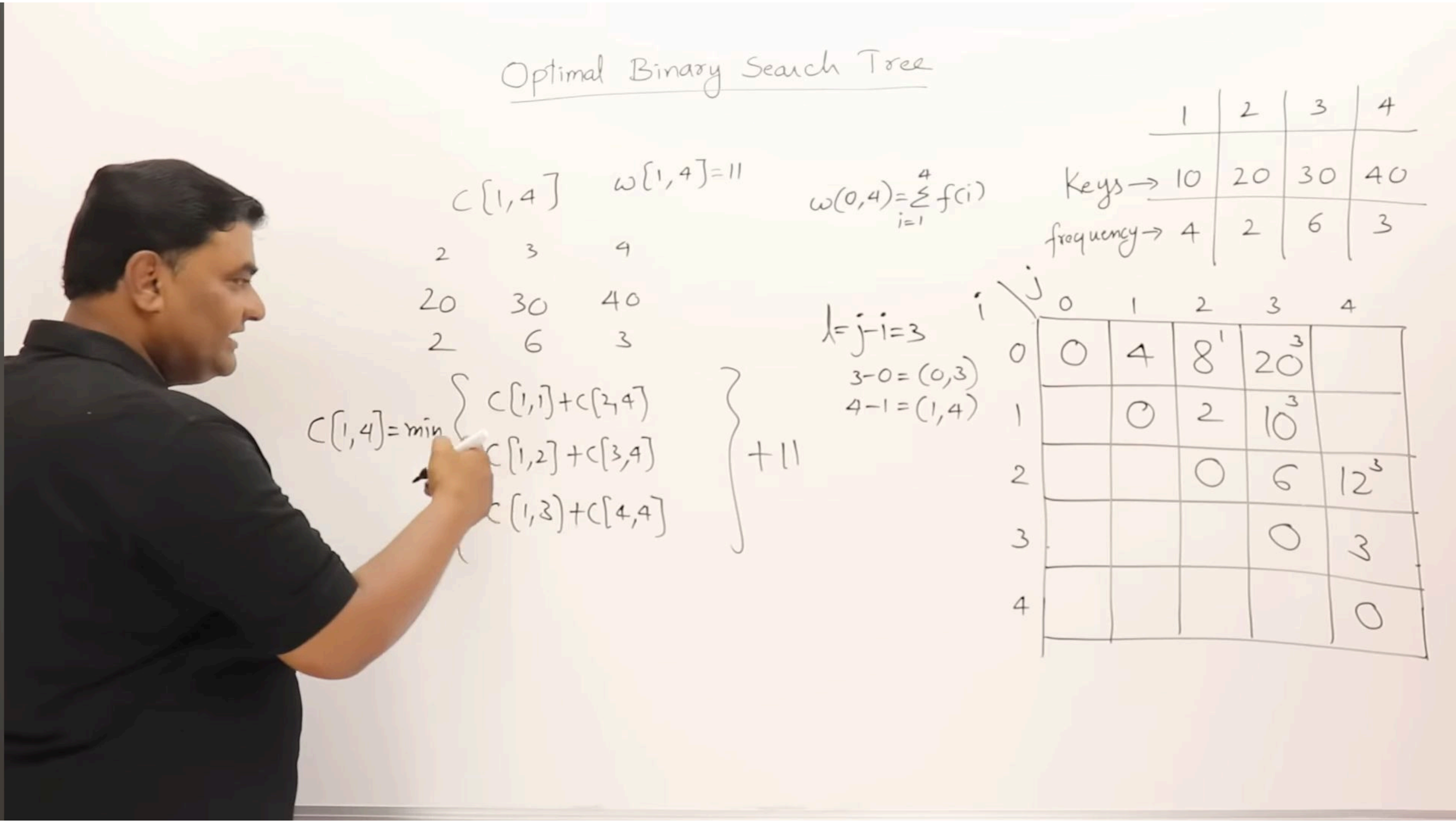

Optimal Binary Search Tree
c[0,3]   w[0,3]=12
1 2 3
10 20 30
4 2 6
w(0,4)= Σ_{i=1}^{4} f(i)
l=j-i=3
3-0=(0,3)
4-1=(1,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0: 0 4 8¹
1: 0 2 10³
2: 0 6 12³
3: 0 3
4: 0



Optimal Binary Search Tree
c[0,3]
w[0,3]=12
1 2 3
10 20 30
4 2 6
1
10
20
30
10
30
20
2
20
10
30
3
30
20
10
30
10
20
min { c[0,0]+c[1,3]+12 c[0,1]+c[2,3]+12 c[0,2]+c[3,3]+12 }
0 + 10 + 12 4 + 6 + 12 8 + 0 + 12
22 22 20
w(0,4)= Σ_{i=1}^{4} f(i)
l=j-i=3
3-0=(0,3)
4-1=(1,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0 0 4 8¹ 20
1 0 2 10³
2 0 6 12³
3 0 3
4 0



Optimal Binary Search Tree
w[0,3]=12
w(0,4)= Σ_{i=1}^{4} f(i)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
l=j-i=3
3-0=(0,3)
4-1=(1,4)
i \ j 0 1 2 3 4
0: 0 4 8 20
1: 0 2 10
2: 0 6 12
3: 0 3
4: 0
20 10 30
30 20 10
30 10 20
C[0,3]=
c[0,1]+c[2,3]+12
4 + 6 + 12
22
c[0,2]+c[3,3]+12
8 + 0 + 12
20


Optimal Binary Search Tree
C[1,4]
w[1,4]=11
w(0,4)= Σ f(i)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
2 3 4
20 30 40
2 6 3
C[1,4]=min
C[1,1]+C[2,4]
C[1,2]+C[3,4]
C[1,3]+C[4,4]
+11
l=j-i=3
3-0=(0,3)
4-1=(1,4)
i j
0 1 2 3 4
0 0 4 8 20
1 0 2 10
2 0 6 12
3 0 3
4 0

Optimal Binary Search Tree

$C[1,4]=16 \quad w[1,4]=11$

2	3	4
20	30	40
2	6	3

$$C[1,4]=\min\begin{cases} C[1,1]+C[2,4] & 0+12 \\ C[1,2]+C[3,4] & 2+3 \\ C[1,3]+C[4,4] & 10+0 \end{cases} +11=16$$

$$w(0,4)=\sum_{i=1}^{4} f(i)$$

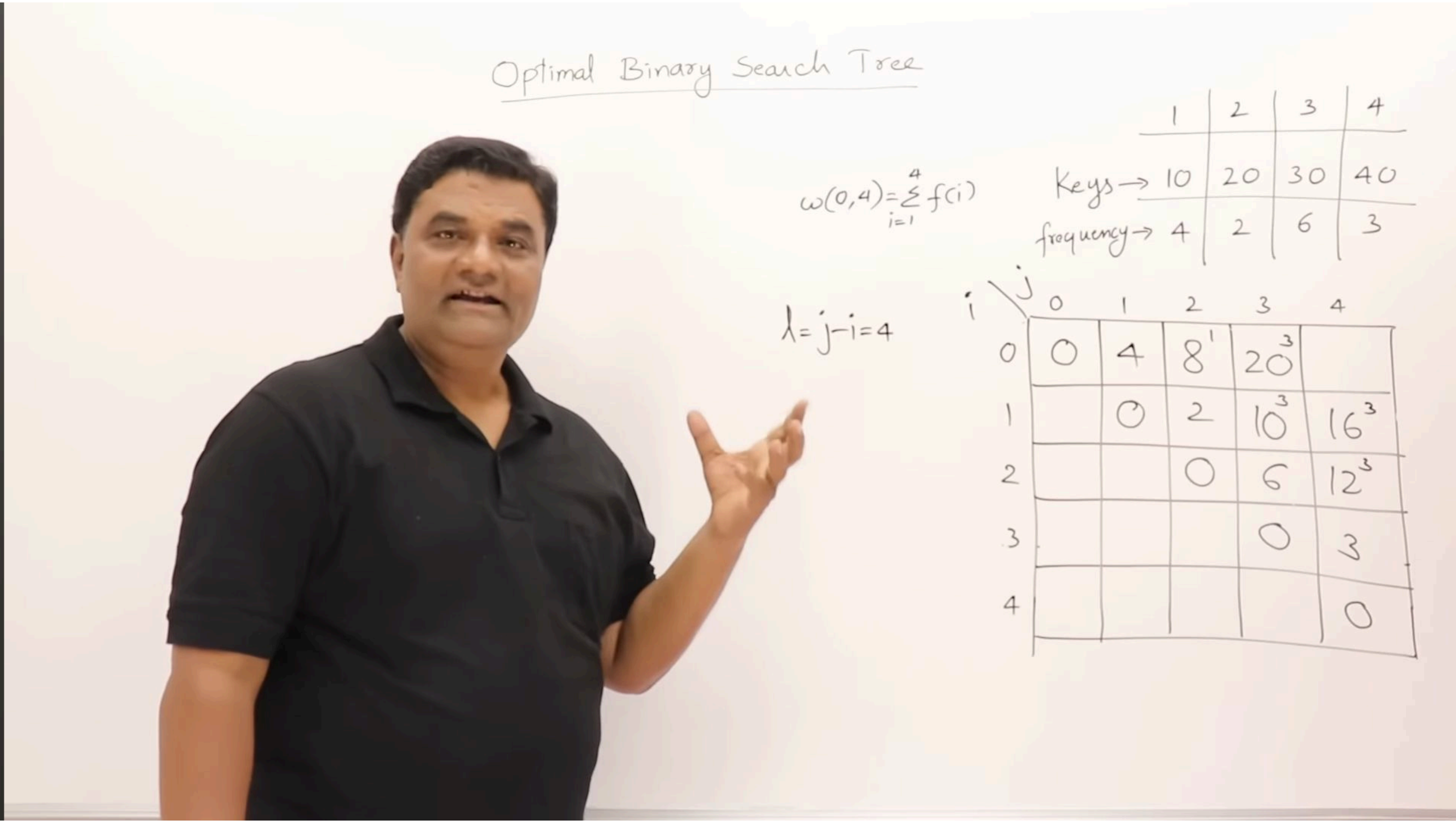
$l = j-i = 3$

$3-0=(0,3)$

$4-1=(1,4)$

	1	2	3	4
Keys →	10	20	30	40
frequency →	4	2	6	3

i \ j	0	1	2	3	4
0	0	4	8^{1}	20^{3}	
1		0	2	10^{3}	
2			0	6	12^{3}
3				0	3
4					0


Optimal Binary Search Tree
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
ω(0,4) = Σ_{i=1}^{4} f(i)
l = j−i = 4
i \ j 0 1 2 3 4
0 0 4 8¹ 20³
1 0 2 10³ 16³
2 0 6 12³
3 0 3
4 0

Optimal Binary Search Tree

$C[0,4]$ $\qquad w[0,4]=15$

1	2	3	4
10	20	30	40
4	2	6	3

$$C[0,4] = \min \left\{ \begin{array}{l} C[0,0] + C[1,4] \\ C[0,1] + C[2,4] \\ C[0,2] + C[3,4] \\ C[0,3] + C[4,4] \end{array} \right\} + 15$$

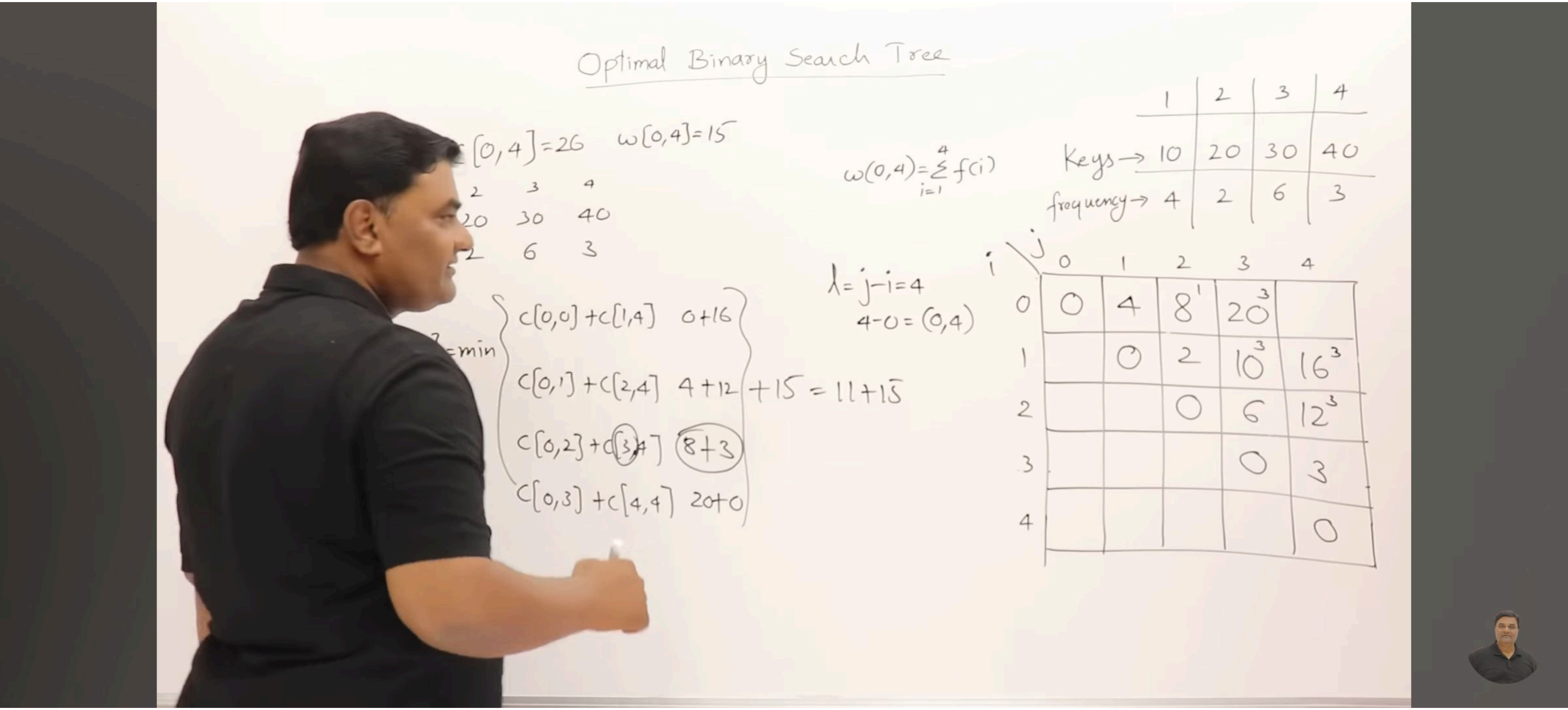
$$w(0,4) = \sum_{i=1}^{4} f(i)$$

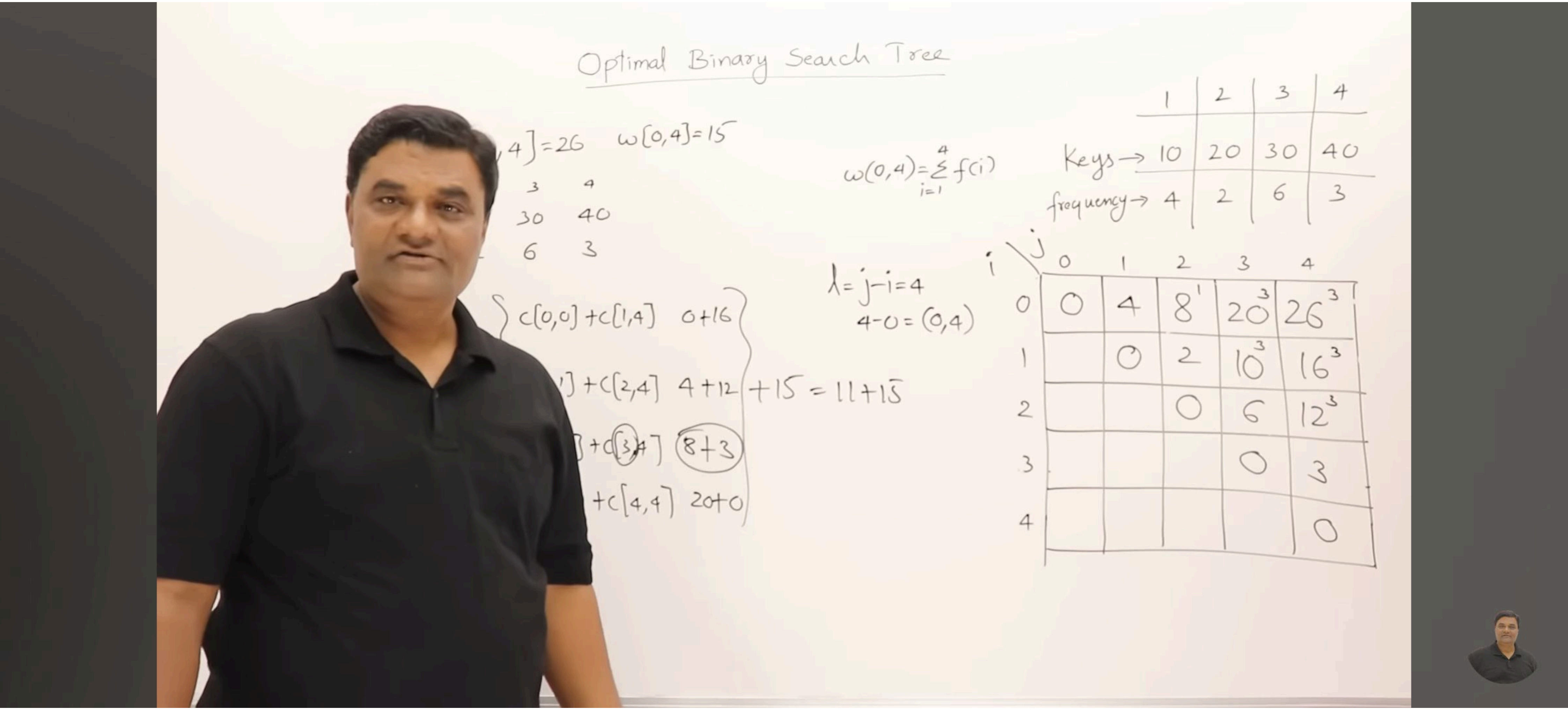
$$l = j - i = 4$$

$$4 - 0 = (0,4)$$

	1	2	3	4
Keys →	10	20	30	40
frequency →	4	2	6	3

i \ j	0	1	2	3	4
0	0	4	8^{1}	20^{3}	
1		0	2	10^{3}	16^{3}
2			0	6	12^{3}
3				0	3
4					0

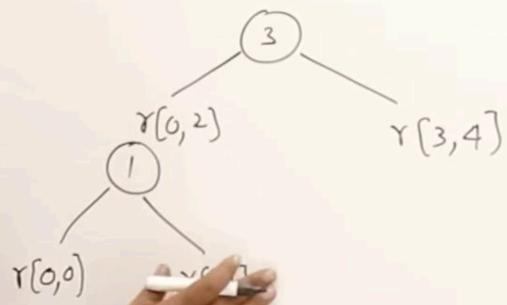

Optimal Binary Search Tree
c[0,4]=26   w[0,4]=15
2 3 4
20 30 40
2 6 3
=min
c[0,0]+c[1,4] 0+16
c[0,1]+c[2,4] 4+12 +15 = 11+15
c[0,2]+c[3,4] 8+3
c[0,3]+c[4,4] 20+0
w(0,4) = Σ f(i), i=1 to 4
l = j-i = 4
4-0 = (0,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i j
0 1 2 3 4
0: 0 4 8 20
1: 0 2 10 16
2: 0 6 12
3: 0 3
4: 0



Optimal Binary Search Tree
,4]=26 ω[0,4]=15
3 4
30 40
6 3
c[0,0]+c[1,4] 0+16
]+c[2,4] 4+12 +15 = 11+15
]+c[3,4] 8+3
+c[4,4] 20+0
ω(0,4)= Σ_{i=1}^{4} f(i)
l = j−i = 4
4−0 = (0,4)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0: 0 4 8¹ 20³ 26³
1: 0 2 10³ 16³
2: 0 6 12³
3: 0 3
4: 0

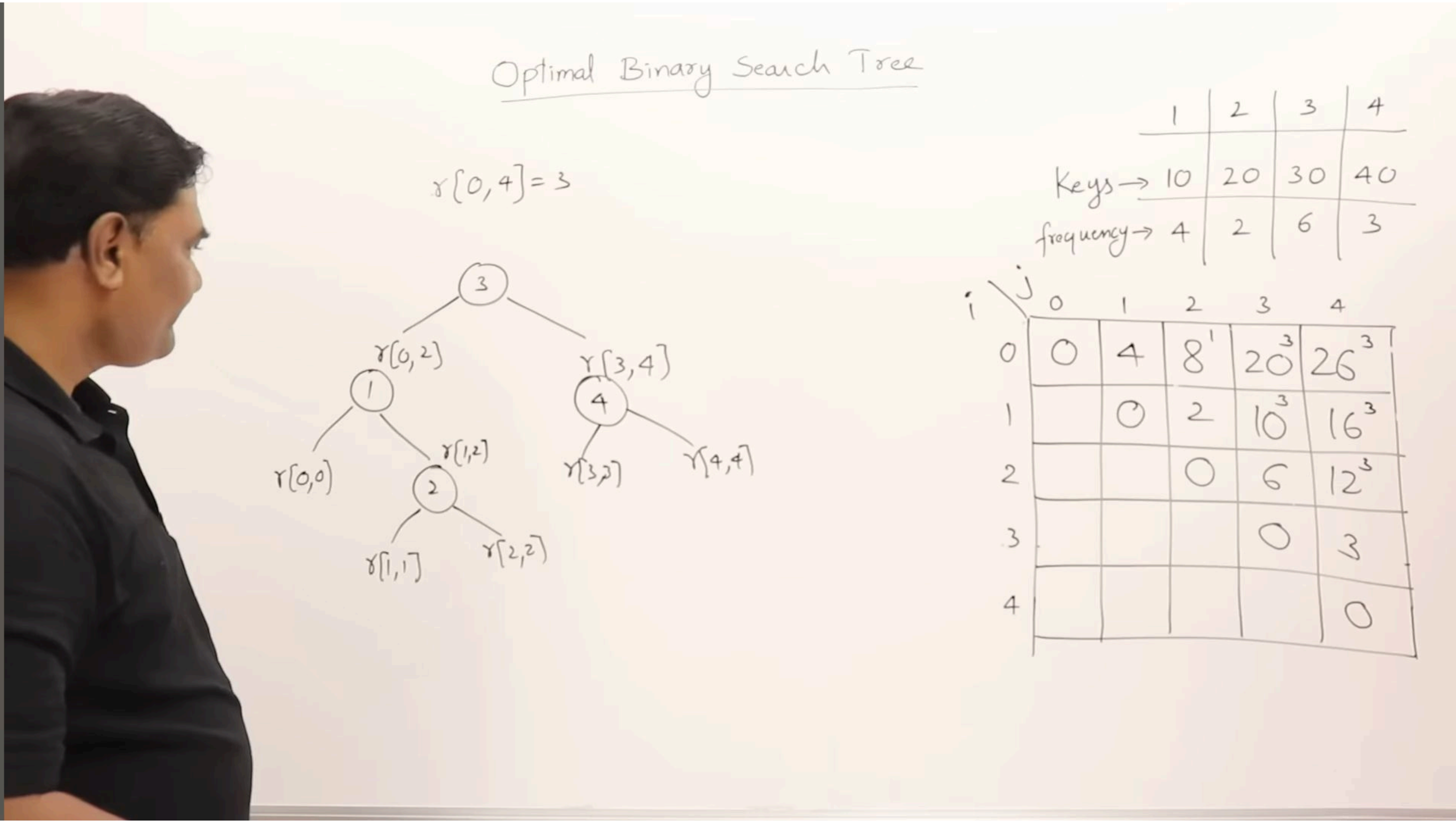
Optimal Binary Search Tree

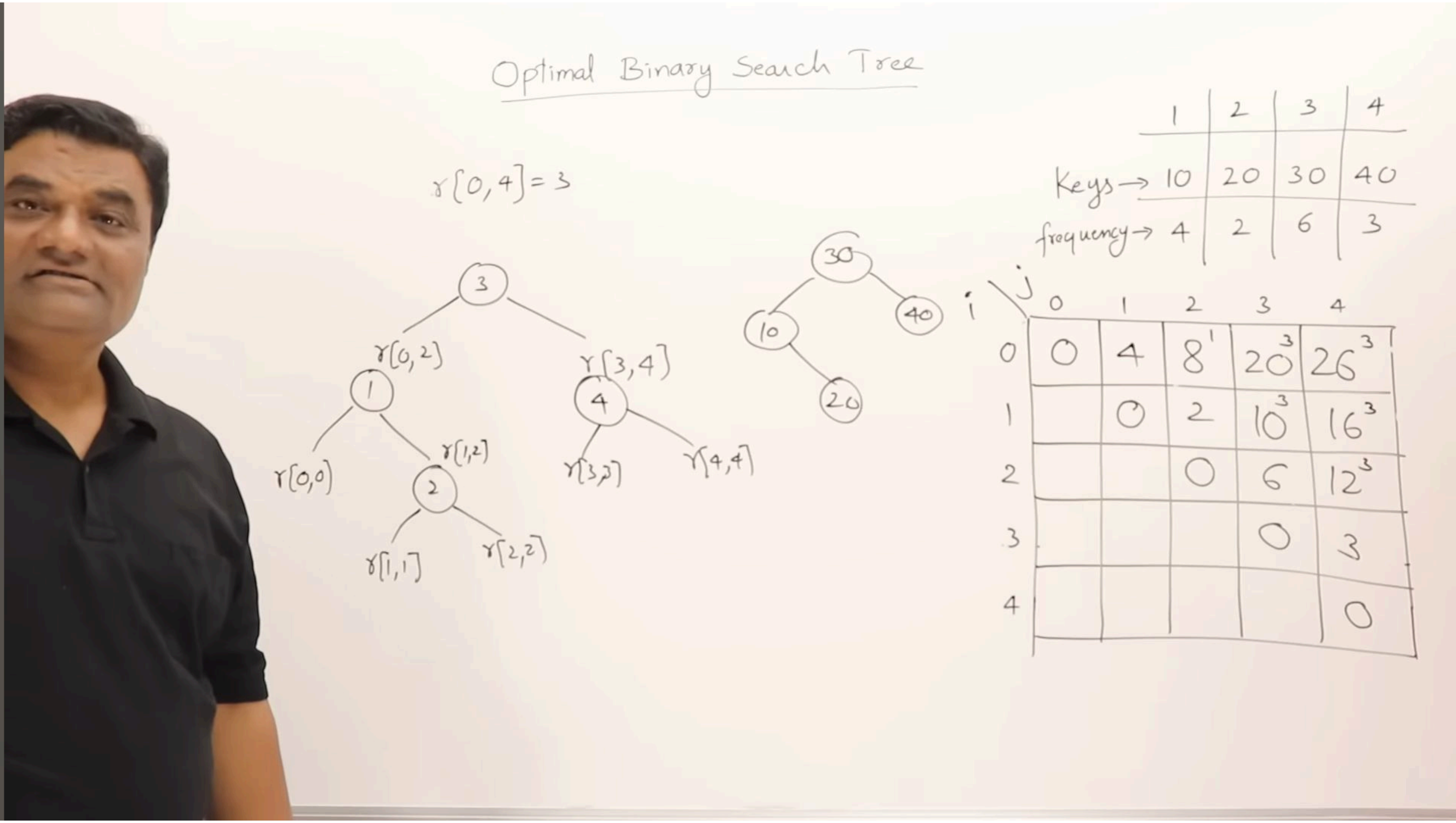
$r[0,4] = 3$



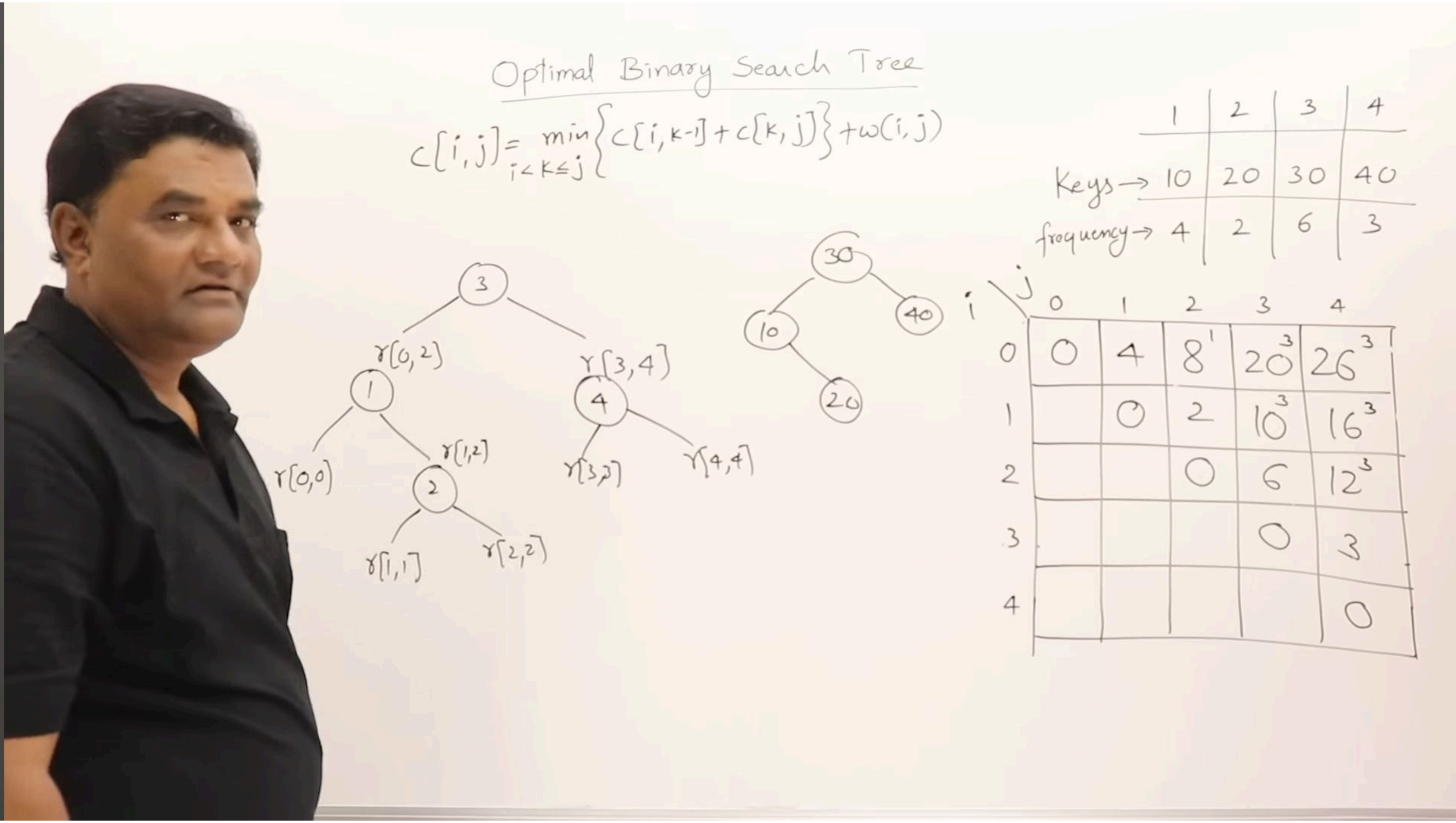
	1	2	3	4
Keys →	10	20	30	40
frequency →	4	2	6	3

i \ j	0	1	2	3	4
0	0	4	8^1	20^3	26^3
1		0	2	10^3	16^3
2			0	6	12^3
3				0	3
4					0

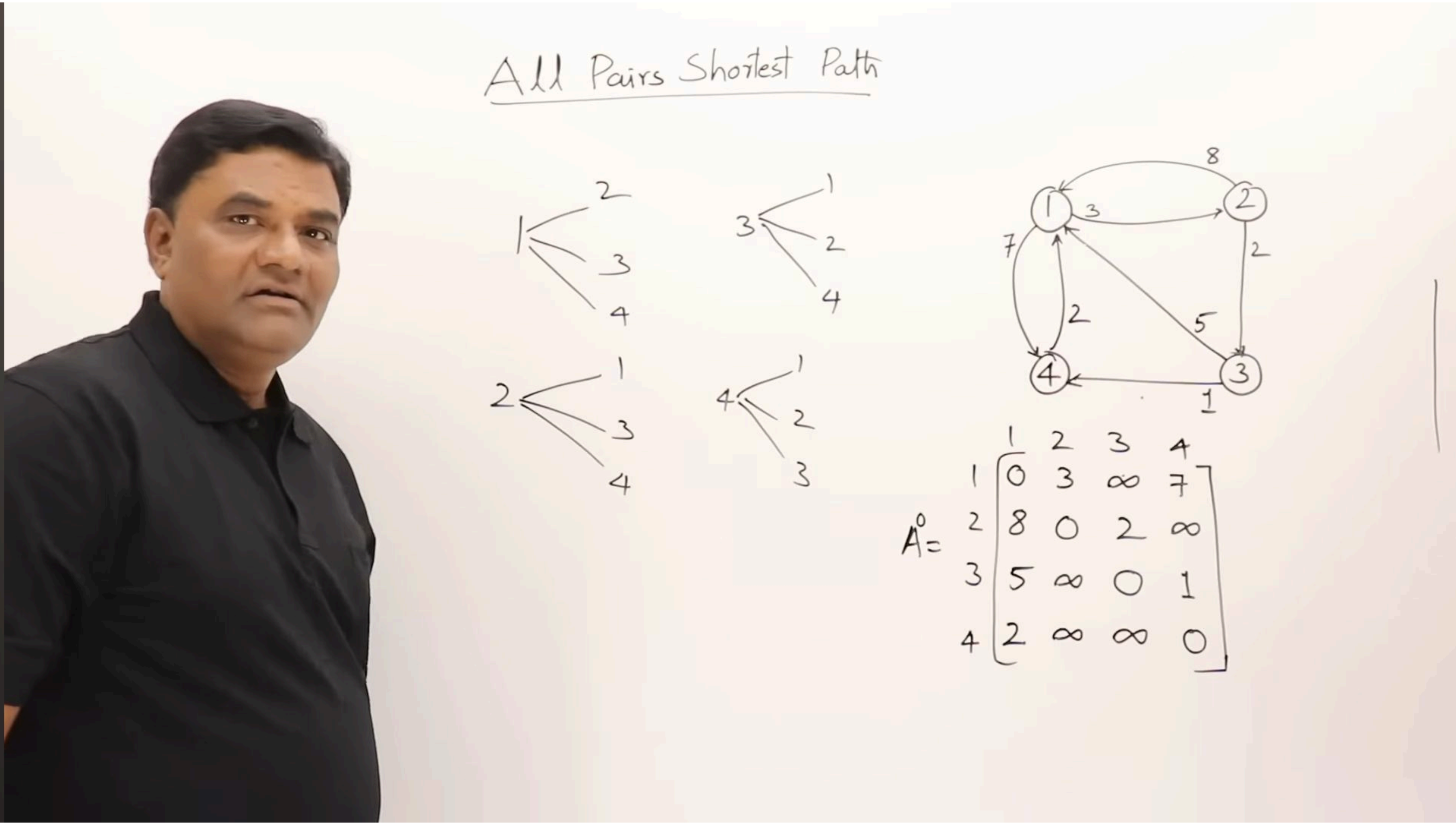

Optimal Binary Search Tree
r[0,4] = 3
3
r[0,2]
r[3,4]
1
4
r[0,0]
r[1,2]
r[3,3]
r[4,4]
2
r[1,1]
r[2,2]
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0 0 4 8 1 20 3 26 3
1 0 2 10 3 16 3
2 0 6 12 3
3 0 3
4 0



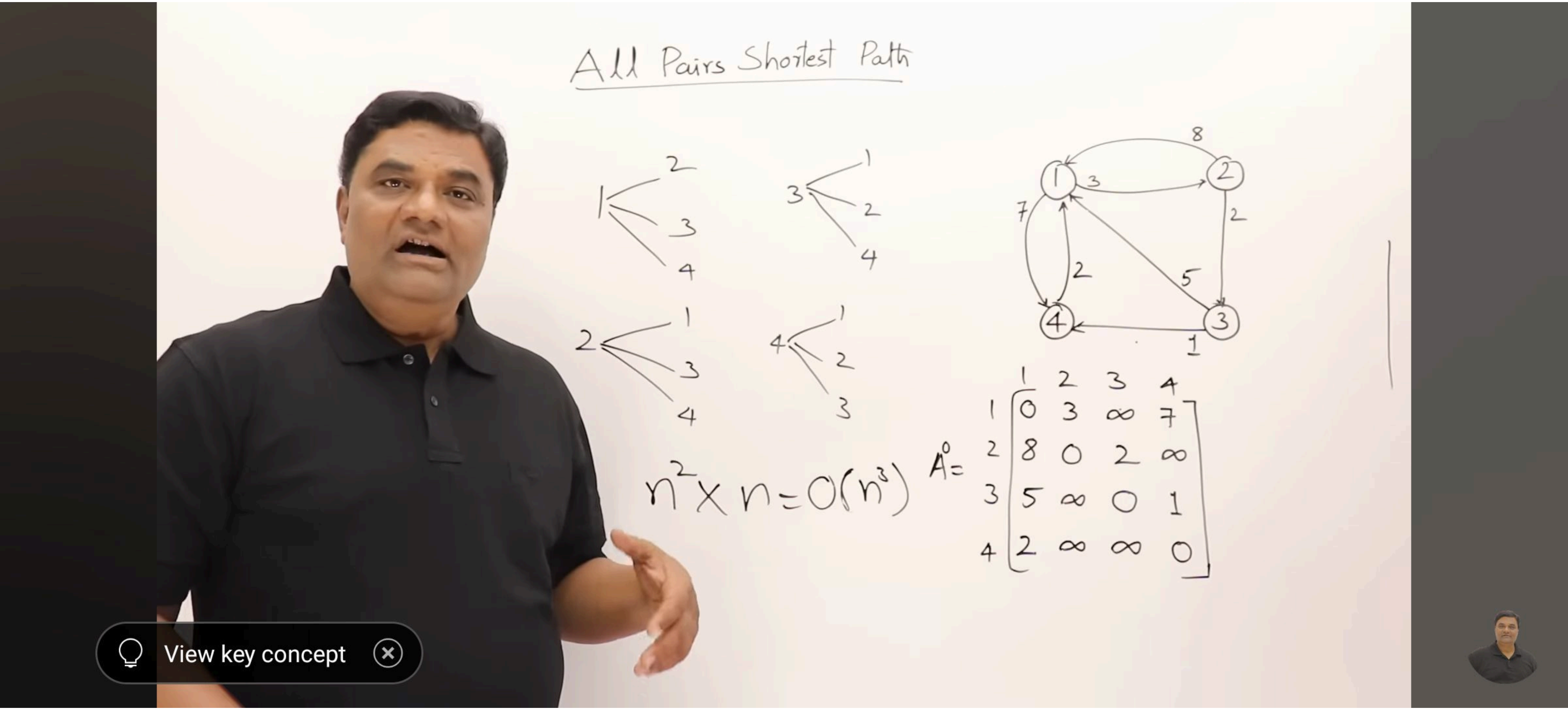
Optimal Binary Search Tree
r[0,4] = 3
3
r[0,2]
r[3,4]
1
4
r[0,0]
r[1,2]
2
r[3,3]
r[4,4]
r[1,1]
r[2,2]
30
10
40
20
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
i \ j 0 1 2 3 4
0: 0 4 8[1] 20[3] 26[3]
1: 0 2 10[3] 16[3]
2: 0 6 12[3]
3: 0 3
4: 0


Optimal Binary Search Tree
c[i,j] = min_{i<k≤j} {c[i,k-1] + c[k,j]} + w(i,j)
1 2 3 4
Keys → 10 20 30 40
frequency → 4 2 6 3
r[0,2]
r[3,4]
r[0,0]
r[1,2]
r[3,3]
r[4,4]
r[1,1]
r[2,2]
i \ j 0 1 2 3 4
0: 0 4 8¹ 20³ 26³
1: 0 2 10³ 16³
2: 0 6 12³
3: 0 3
4: 0

All pair shortest path (Floyd-Warshall algorithm)

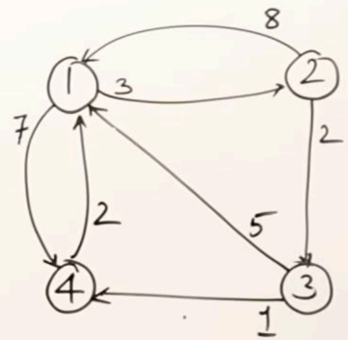


$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

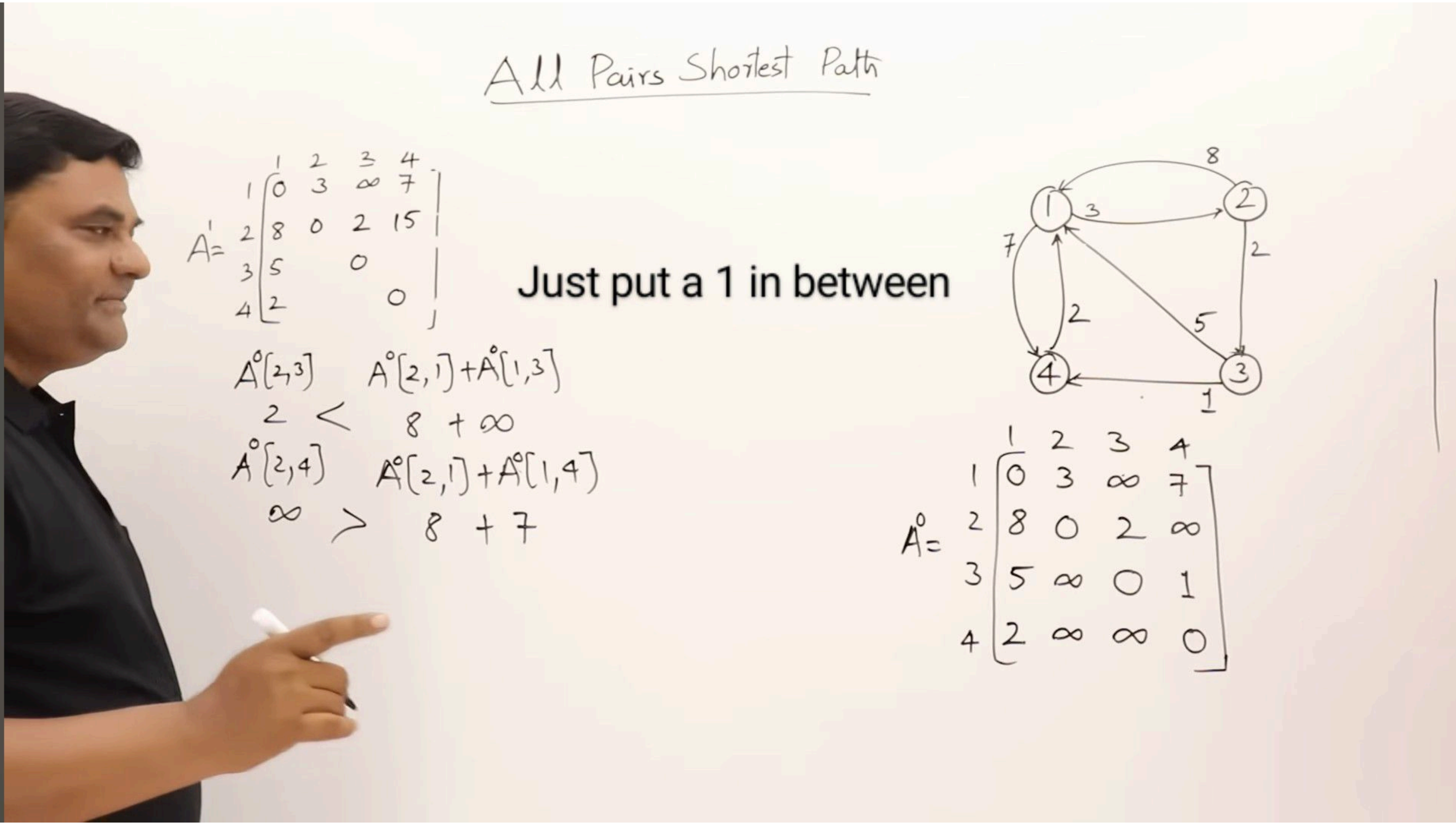

All Pairs Shortest Path
1 2 3 4
2 1 3 4
3 1 2 4
4 1 2 3
8 3 7 2 2 5 1
n² × n = O(n³)
A⁰ =
1 2 3 4
1 0 3 ∞ 7
2 8 0 2 ∞
3 5 ∞ 0 1
4 2 ∞ ∞ 0
View key concept

All Pairs Shortest Path

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & & \\ 3 & 5 & & 0 & \\ 4 & 2 & & & 0 \end{array}$$



$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$



All Pairs Shortest Path
Just put a 1 in between
$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & & 0 & \\ 4 & 2 & & & 0 \end{array}$
$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$
$2 < 8 + \infty$
$A^0[2,4] \quad A^0[2,1] + A^0[1,4]$
$\infty > 8 + 7$
$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$
1
2
3
4
8
3
2
7
2
5
1

All Pairs Shortest Path

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & \\ 4 & 2 & & & 0 \end{array}$$

$A^0[2,3] \quad A^0[2,1] + A^0[1,3]$

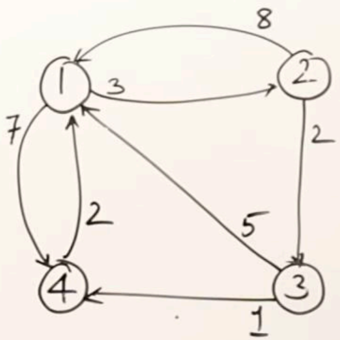
$2 < 8 + \infty$

$A^0[2,4] \quad A^0[2,1] + A^0[1,4]$

$\infty > 8 + 7$

$A^0[3,2] \quad A^0[3,1] + A^0[1,2]$

$\infty > 5 + 3$

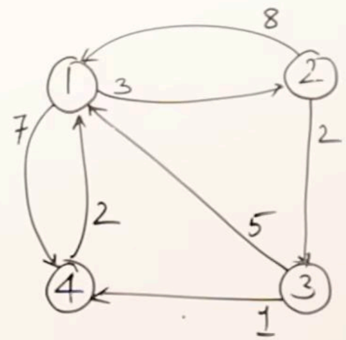


$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

All Pairs Shortest Path

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 8 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & & \\ 8 & 0 & 2 & 15 \\ & 8 & 0 & \\ & 8 & & 0 \end{bmatrix} \end{array}$$



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

All Pairs Shortest Path

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 8 & \infty & 0 \end{array}$$

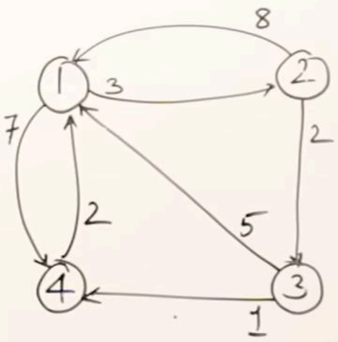
$$A^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & & 8 & 0 & \\ 4 & & 8 & & 0 \end{array}$$

$A^1[1,3] \quad A^1[1,2] + A^1[2,3]$

$\infty > 3 + 2$

$A^1[1,4] \quad A^1[1,2] + A^1[2,4]$

$7 < 3 + 15$



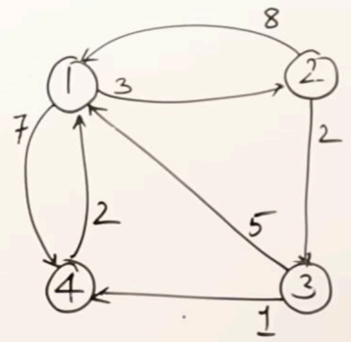
$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

All Pairs Shortest Path

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & & 5 & \\ & 0 & 2 & \\ 5 & 8 & 0 & 1 \\ & & 7 & 0 \end{bmatrix} \end{array}$$



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

All Pairs Shortest Path

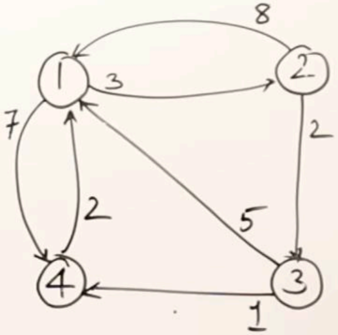
$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

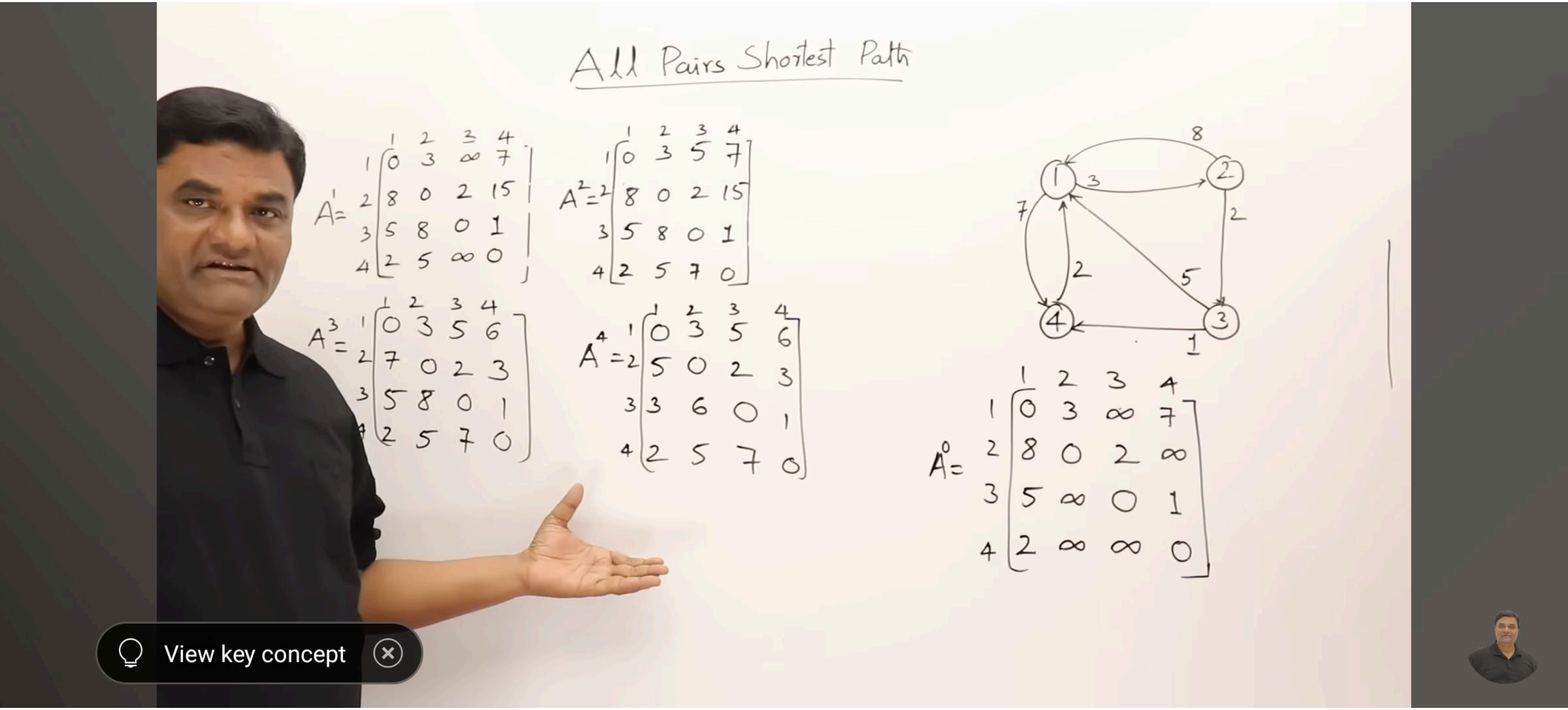
$$A^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & \\ & 0 & 2 & \\ 5 & 8 & 0 & 1 \\ & & 7 & 0 \end{bmatrix} \end{array}$$

$$A^2[1,2] \quad A^2[1,3] + A^2[3,2]$$

$$3 < 5 + 8$$



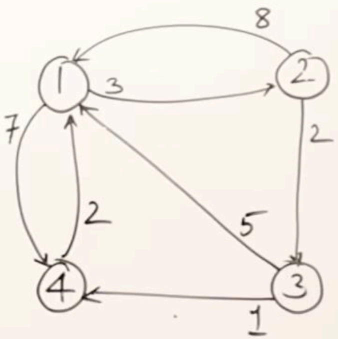
$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$


All Pairs Shortest Path
A¹ = 0 3 ∞ 7 / 8 0 2 15 / 5 8 0 1 / 2 5 ∞ 0
A² = 0 3 5 7 / 8 0 2 15 / 5 8 0 1 / 2 5 7 0
A³ = 0 3 5 6 / 7 0 2 3 / 5 8 0 1 / 2 5 7 0
A⁴ = 0 3 5 6 / 5 0 2 3 / 3 6 0 1 / 2 5 7 0
A⁰ = 0 3 ∞ 7 / 8 0 2 ∞ / 5 ∞ 0 1 / 2 ∞ ∞ 0
1 2 3 4
8 3 2 7 2 5 1
View key concept

All Pairs Shortest Path

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

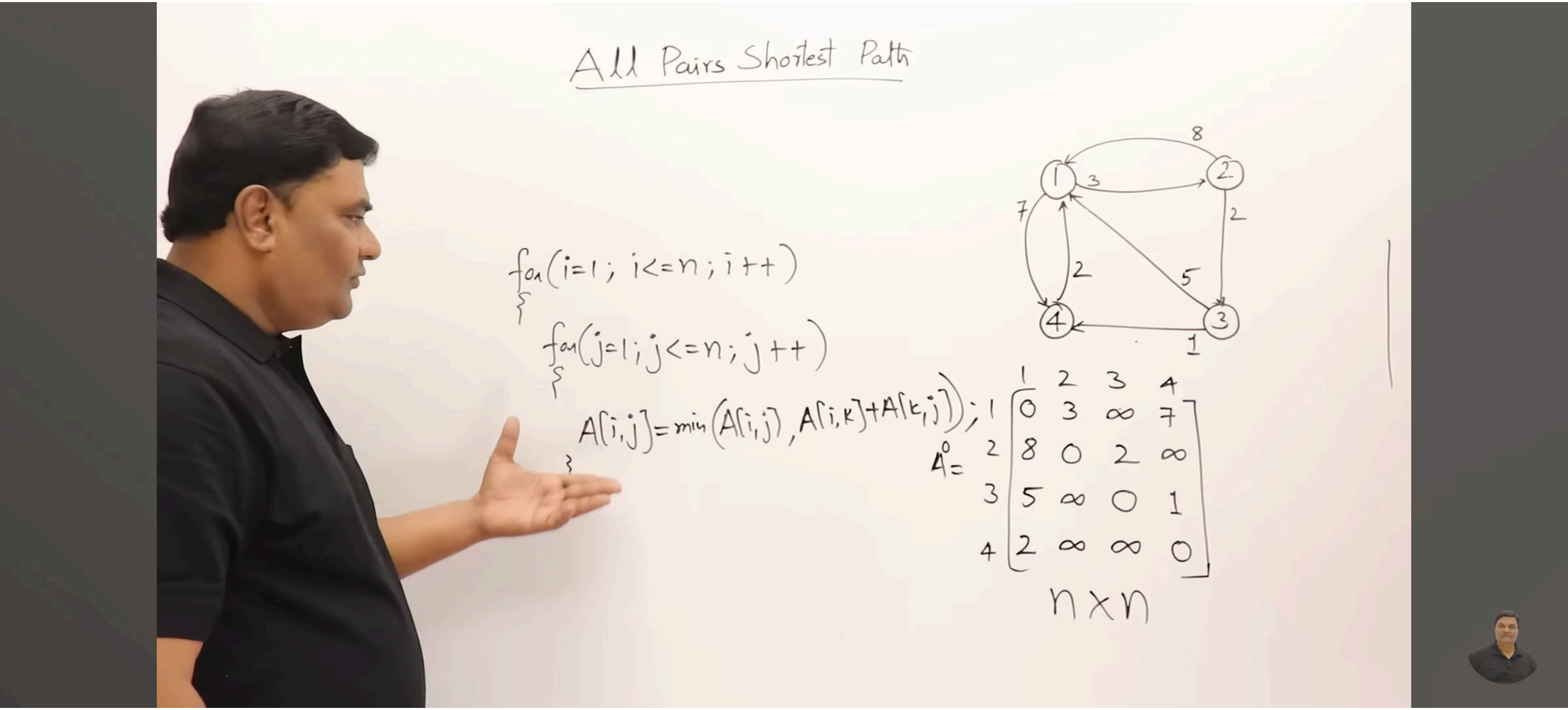


$$= \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

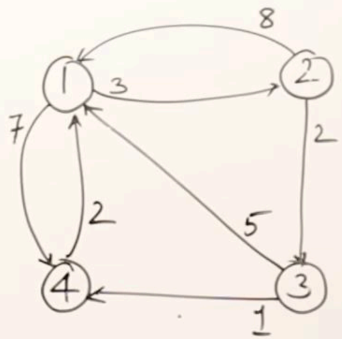
$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^k[i,j] = \min\left\{ \underline{A^{k-1}[i,j]},\ \underline{A^{k-1}[i,k] + A^{k-1}[k,j]} \right\}$$


All Pairs Shortest Path
for(i=1; i<=n; i++)
{
for(j=1; j<=n; j++)
{
A[i,j]=min(A[i,j], A[i,k]+A[k,j]);
}
1 2 3 4
8 3 2 7 2 5 1
1 2 3 4
A0 =
1 0 3 ∞ 7
2 8 0 2 ∞
3 5 ∞ 0 1
4 2 ∞ ∞ 0
n×n

All Pairs Shortest Path

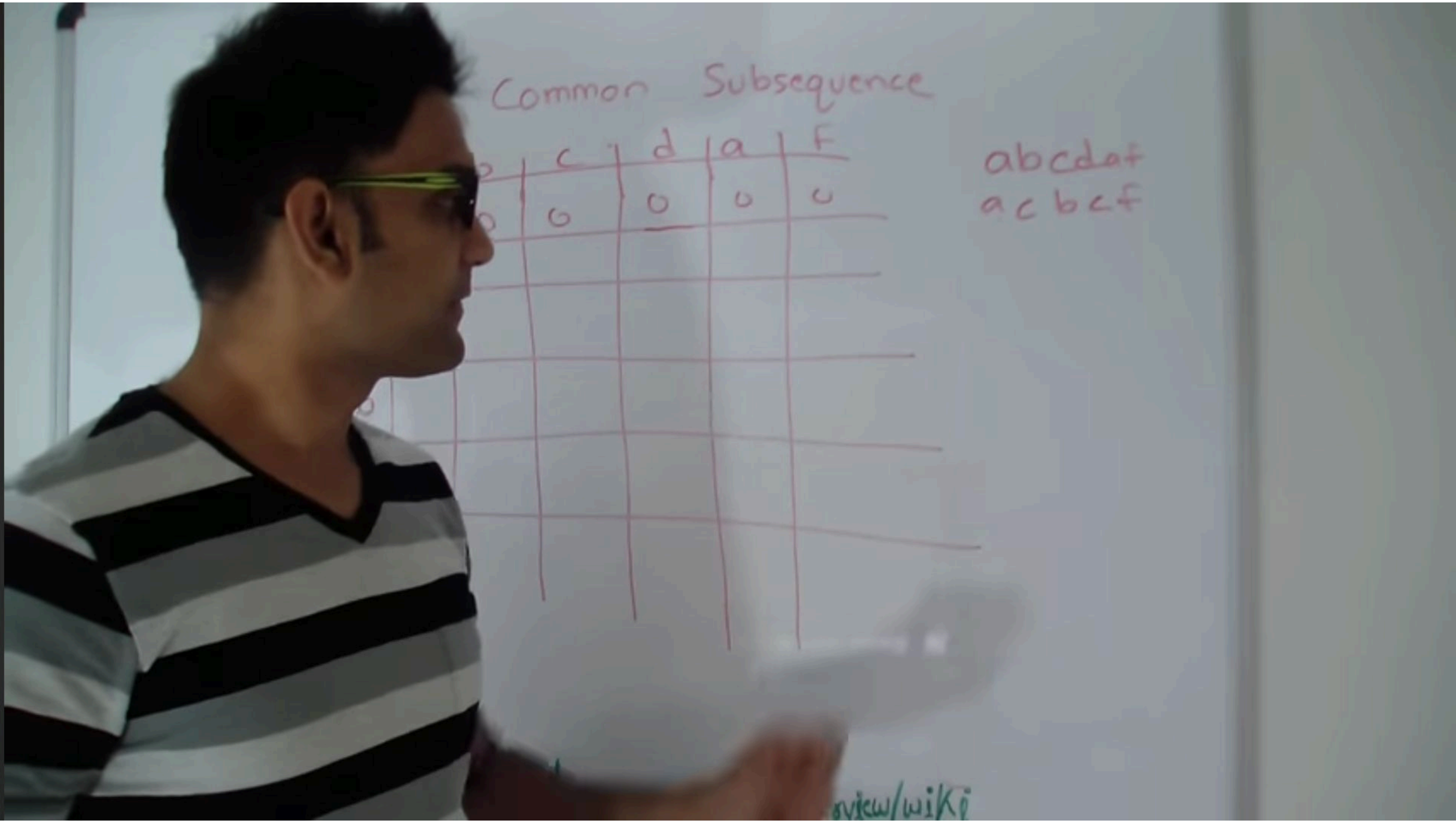
```
for(k=1; k<=n; k++)
{
    for(i=1; i<=n; i++)
    {
        for(j=1; j<=n; j++)
        {
            A[i,j]=min(A[i,j], A[i,k]+A[k,j]);
        }
    }
}
```

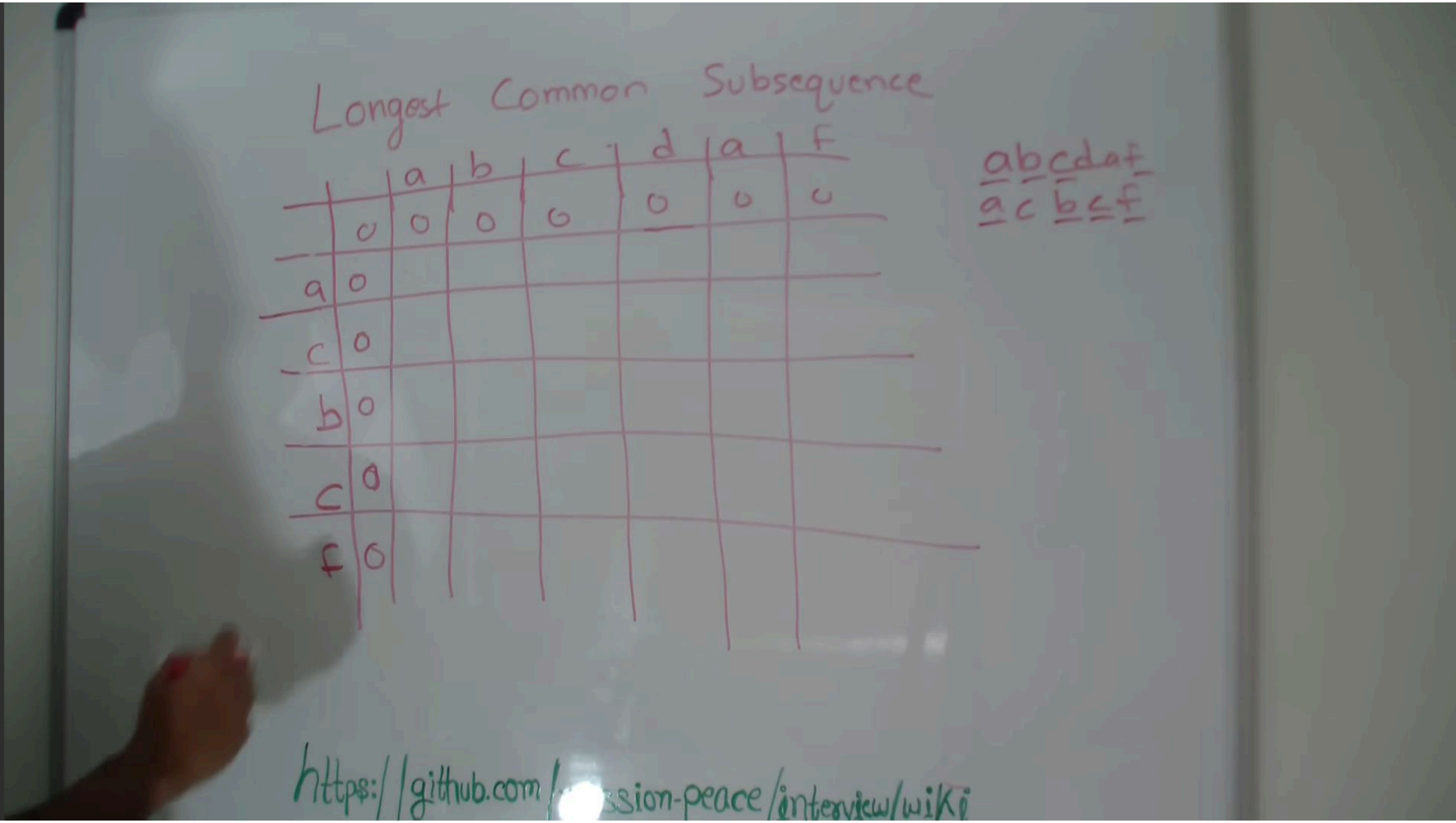


$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

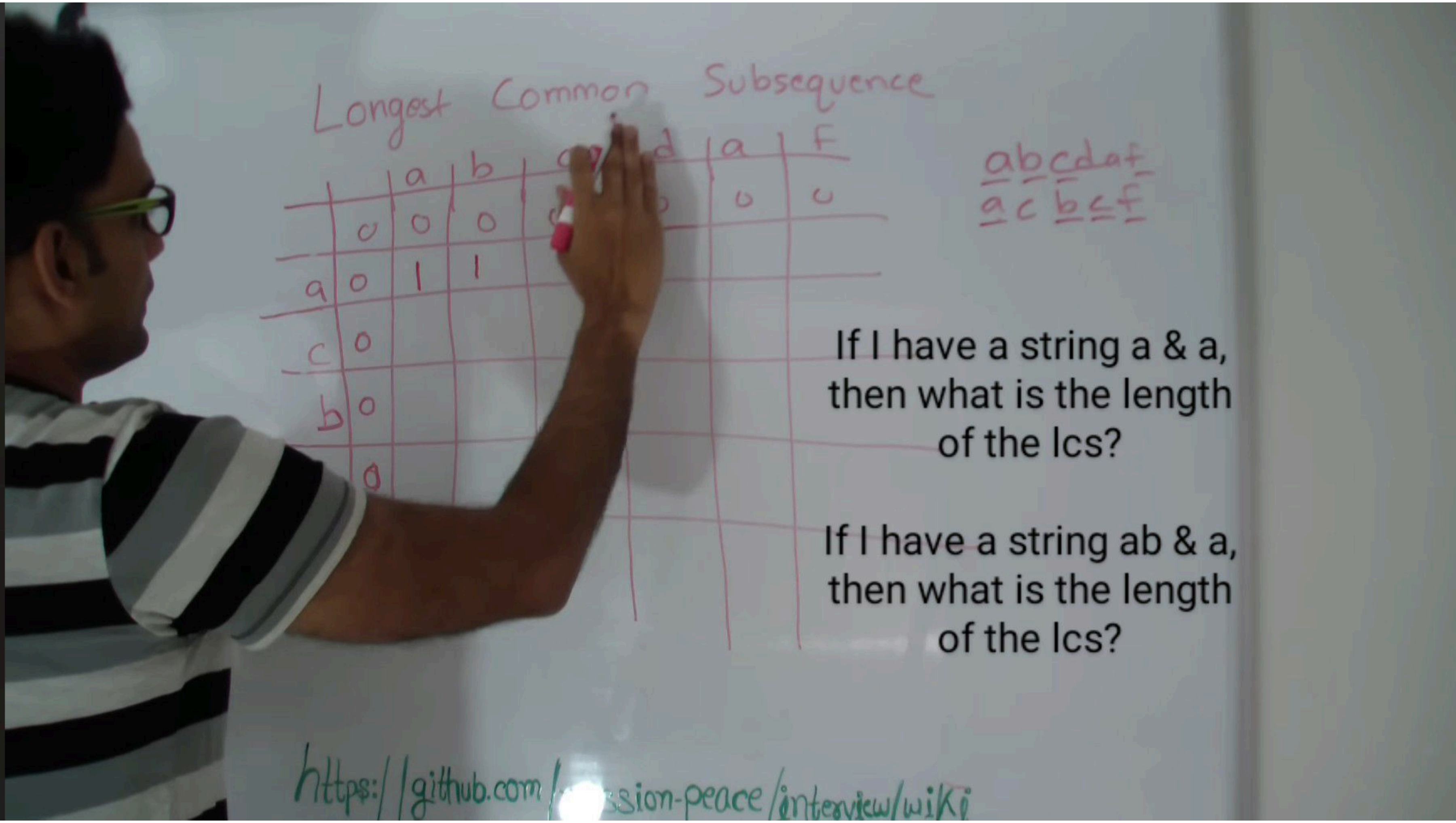
$n \times n$

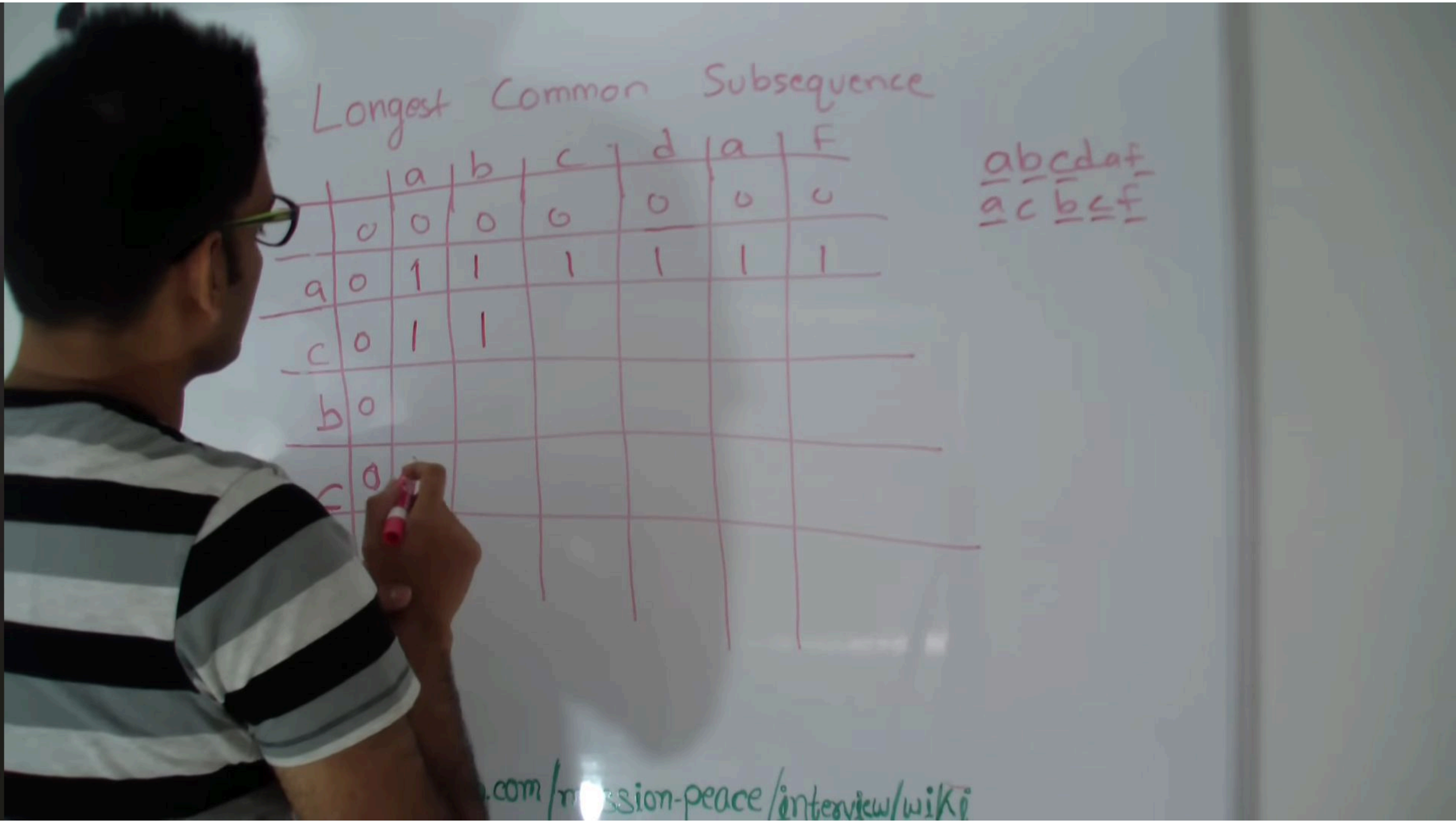
Longest common subsequence (LCS)

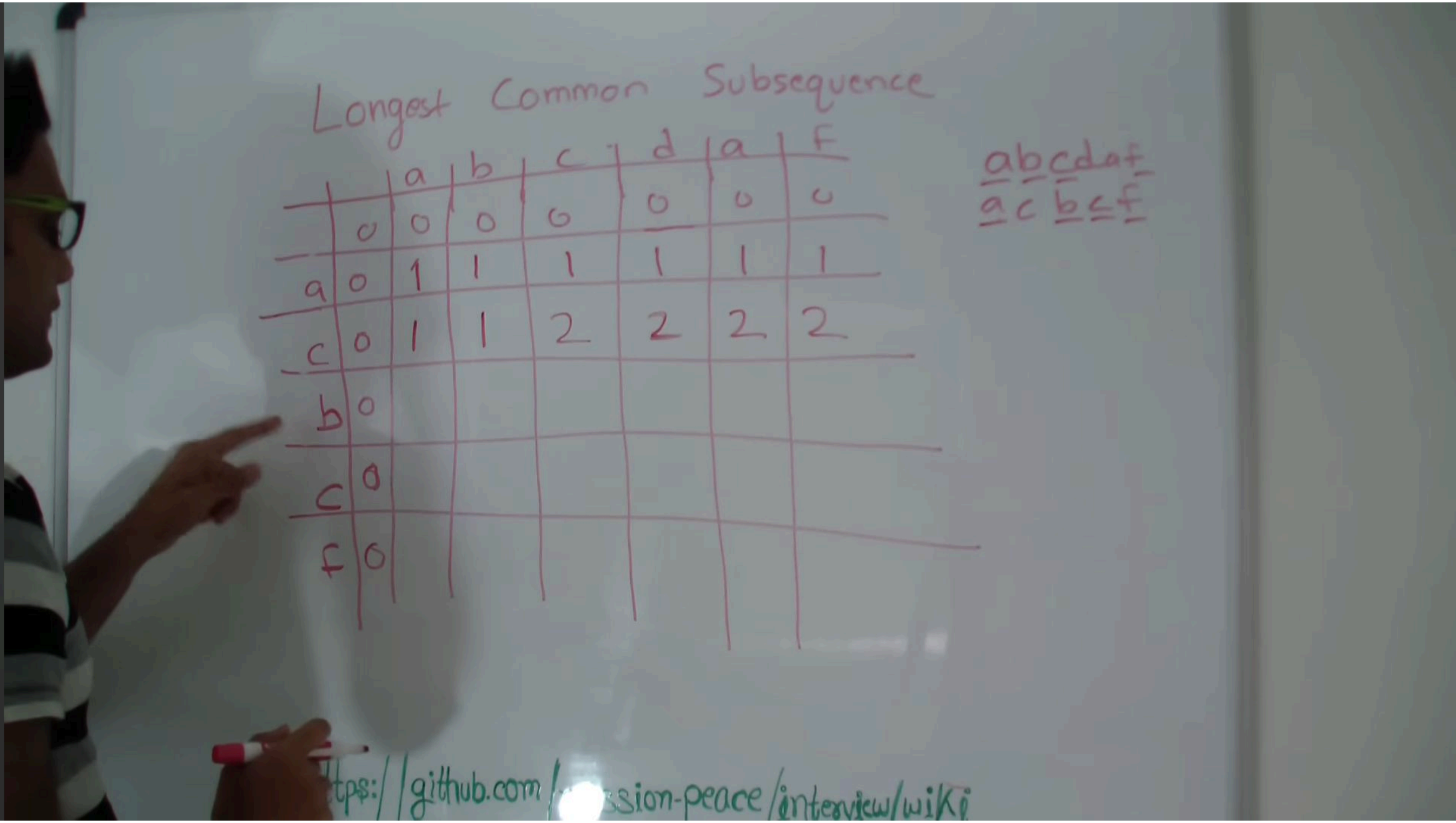

Common Subsequence
abcdaf
acbcf

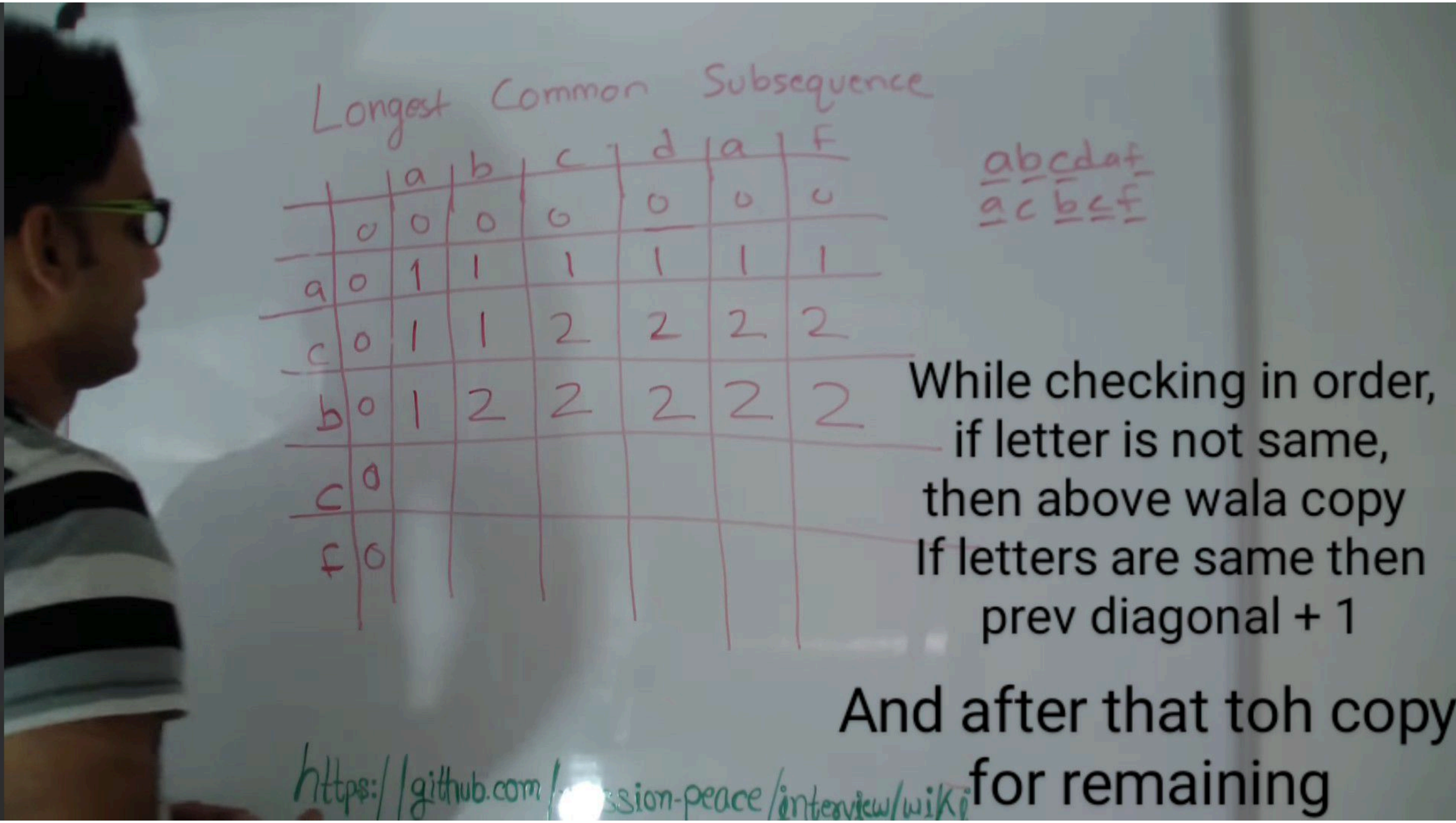


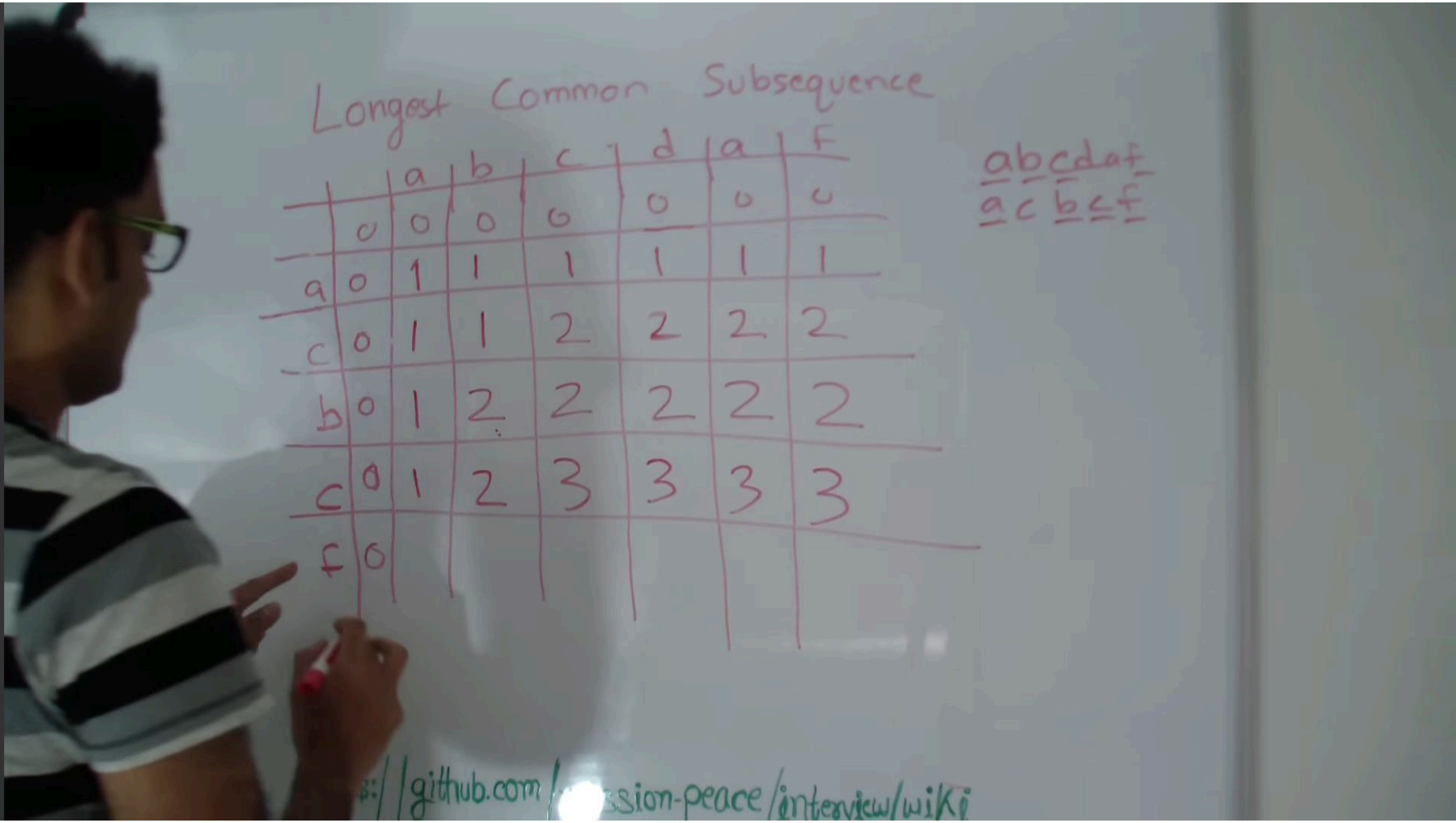
Longest Common Subsequence
a b c d a f
a c b c f
0 0 0 0 0 0 0
a 0
c 0
b 0
c 0
f 0
abcdaf
acbcf
https://github.com/
ssion-peace/interview/wiki

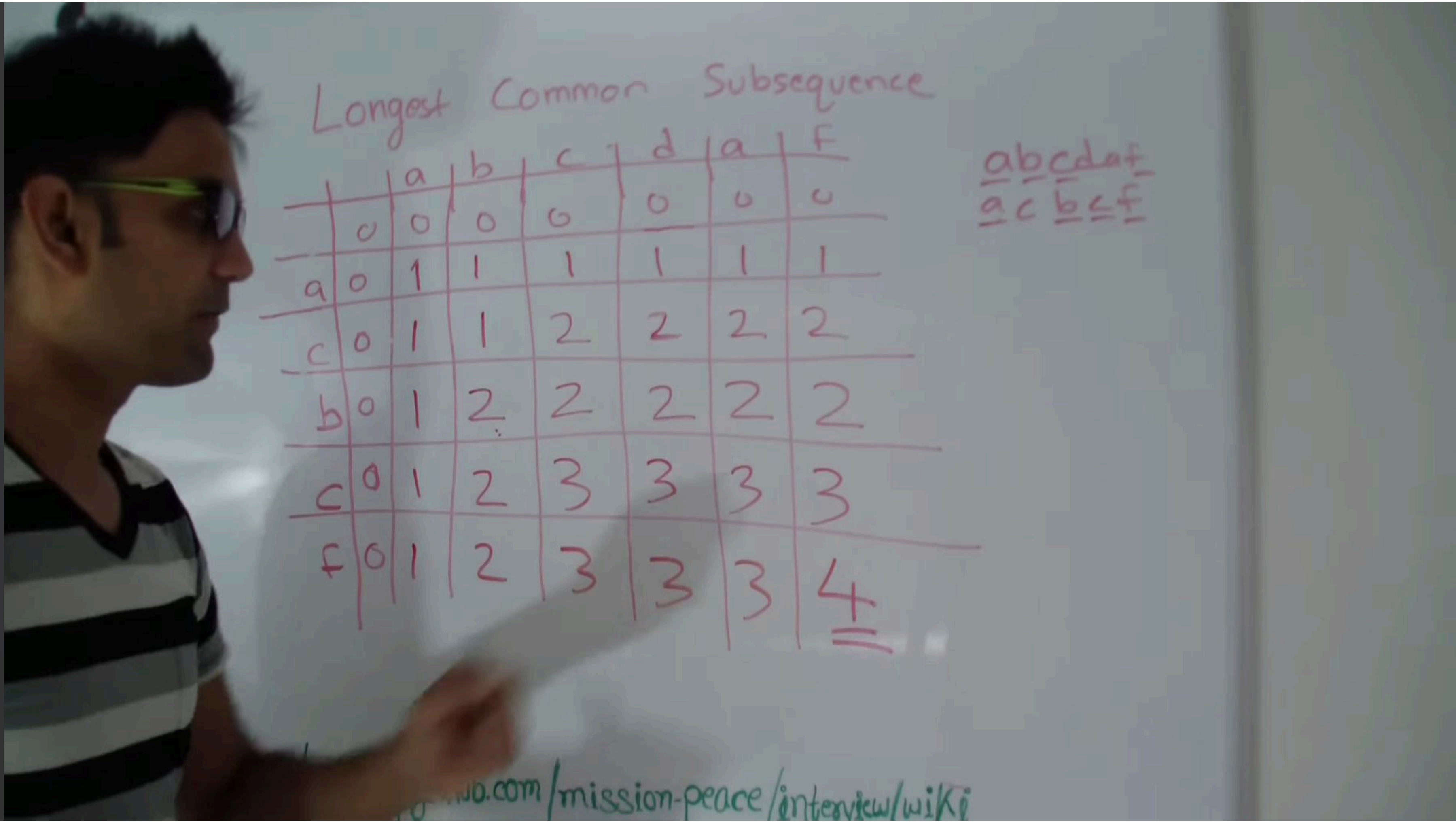

Longest Common Subsequence
abcdaf
acbcf
If I have a string a & a,
then what is the length
of the lcs?
If I have a string ab & a,
then what is the length
of the lcs?
https://github.com/

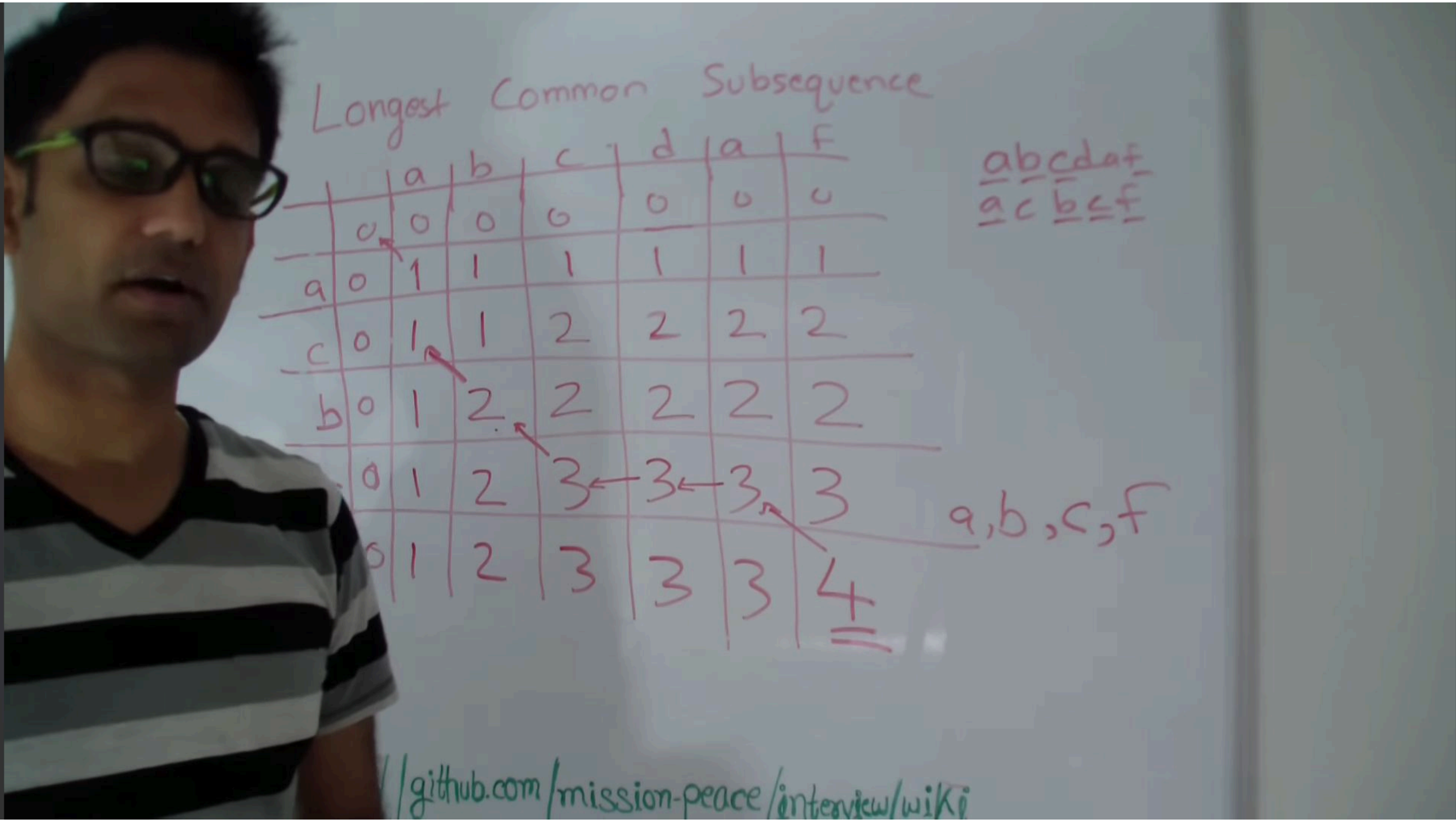

Longest Common Subsequence
abcdaf
acbcf

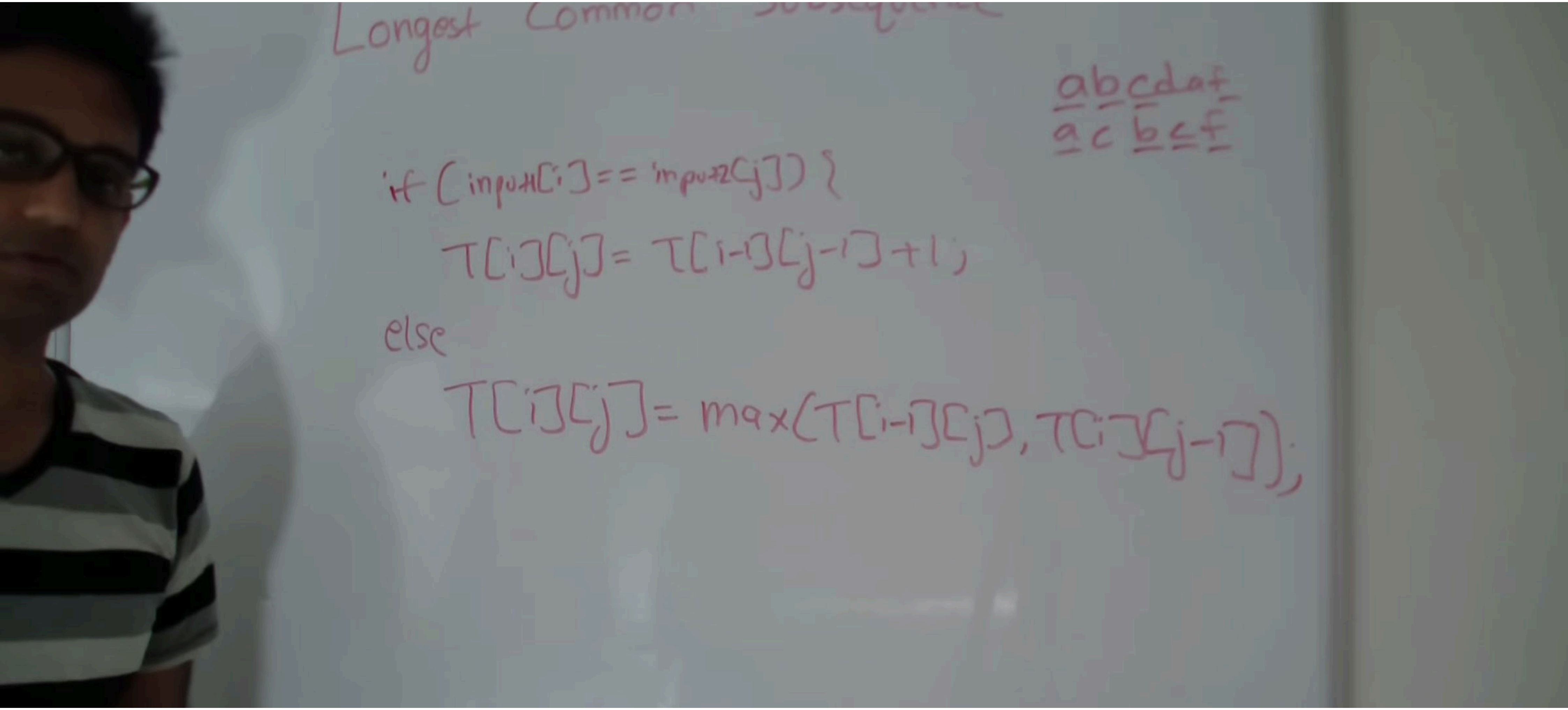

Longest Common Subsequence
a b c d a f
0 0 0 0 0 0 0
a 0 1 1 1 1 1 1
c 0 1 1 2 2 2 2
b 0
c 0
f 0
abcdaf
acbcf
tps://github.com/
ssion-peace/interview/wiki


Longest Common Subsequence
a b c d a f
0 0 0 0 0 0 0
a 0 1 1 1 1 1 1
c 0 1 1 2 2 2 2
b 0 1 2 2 2 2 2
c 0
f 0
abcdaf
acbcf
While checking in order,
if letter is not same,
then above wala copy
If letters are same then
prev diagonal + 1
And after that toh copy
for remaining
https://github.com/
interview/wiki


Longest Common Subsequence
a b c d a f
0 0 0 0 0 0 0
a 0 1 1 1 1 1 1
c 0 1 1 2 2 2 2
b 0 1 2 2 2 2 2
c 0 1 2 3 3 3 3
f 0
abcdaf
acbcf
github.com
interview/wiki

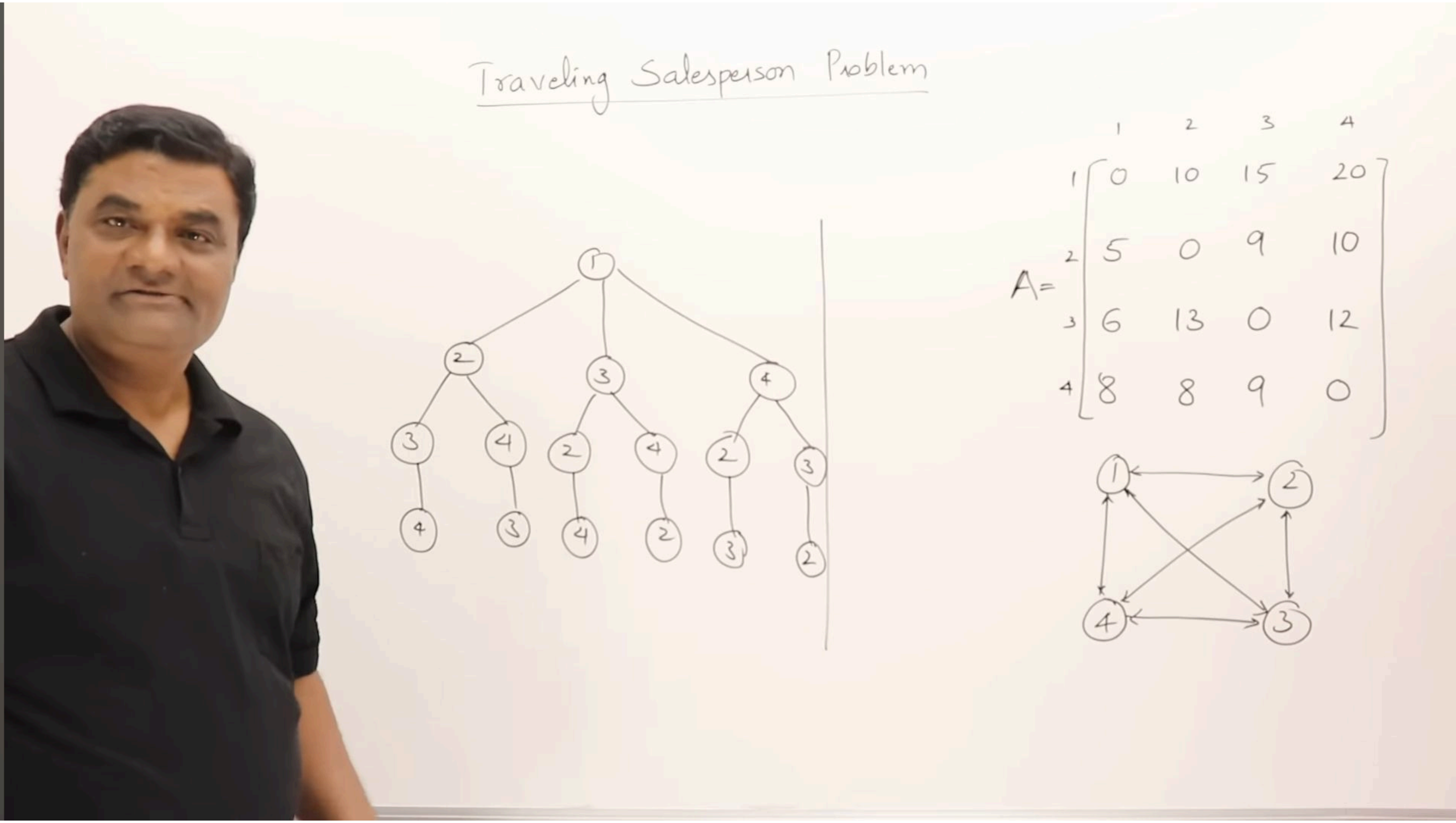

Longest Common Subsequence
a b c d a f
0 0 0 0 0 0 0
a 0 1 1 1 1 1 1
c 0 1 1 2 2 2 2
b 0 1 2 2 2 2 2
c 0 1 2 3 3 3 3
f 0 1 2 3 3 3 4
abcdaf
acbcf
.com/mission-peace/interview/wiki

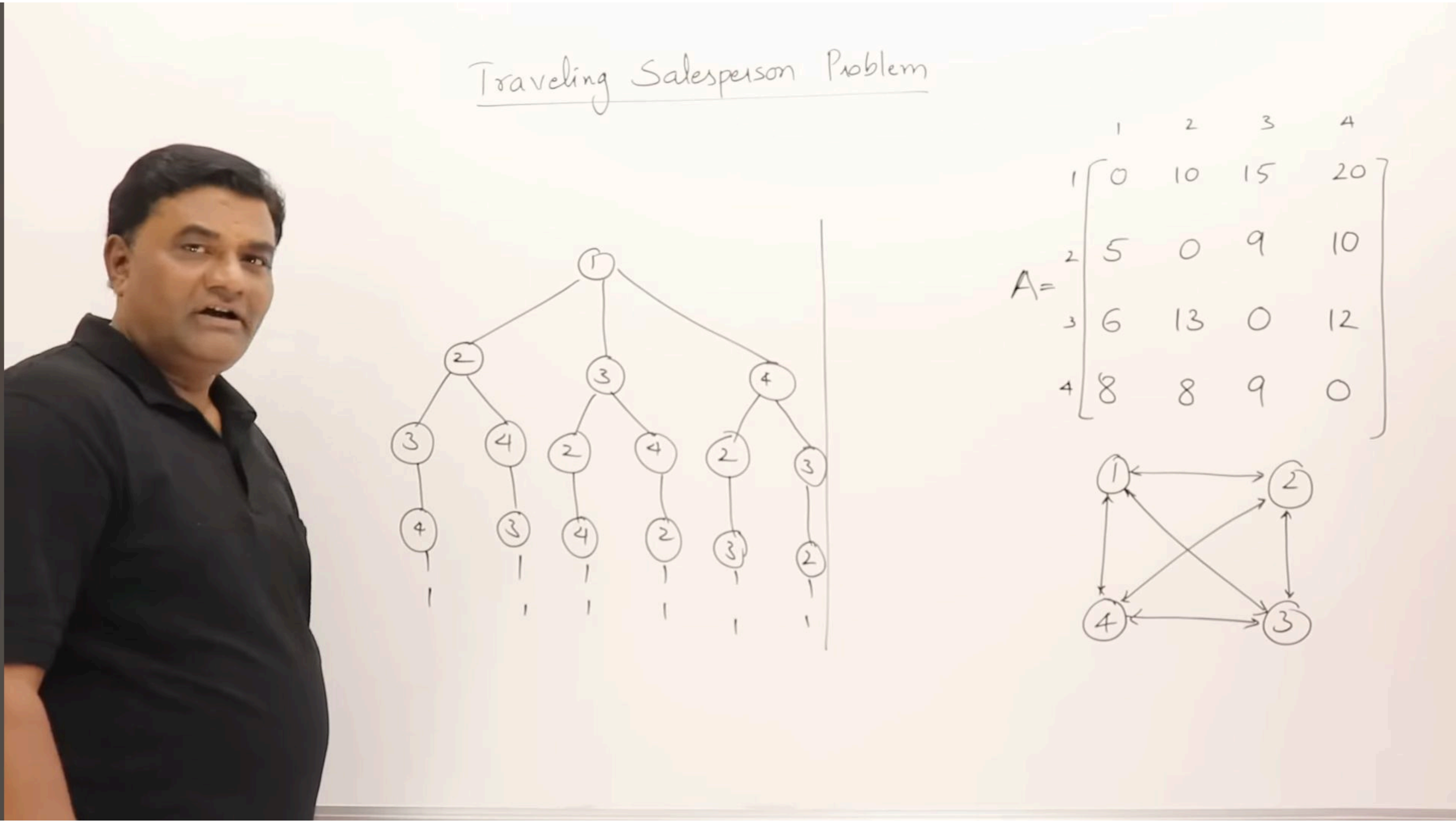

Longest Common Subsequence
abcdaf
acbcf
a,b,c,f
/github.com/mission-peace/interview/wiki

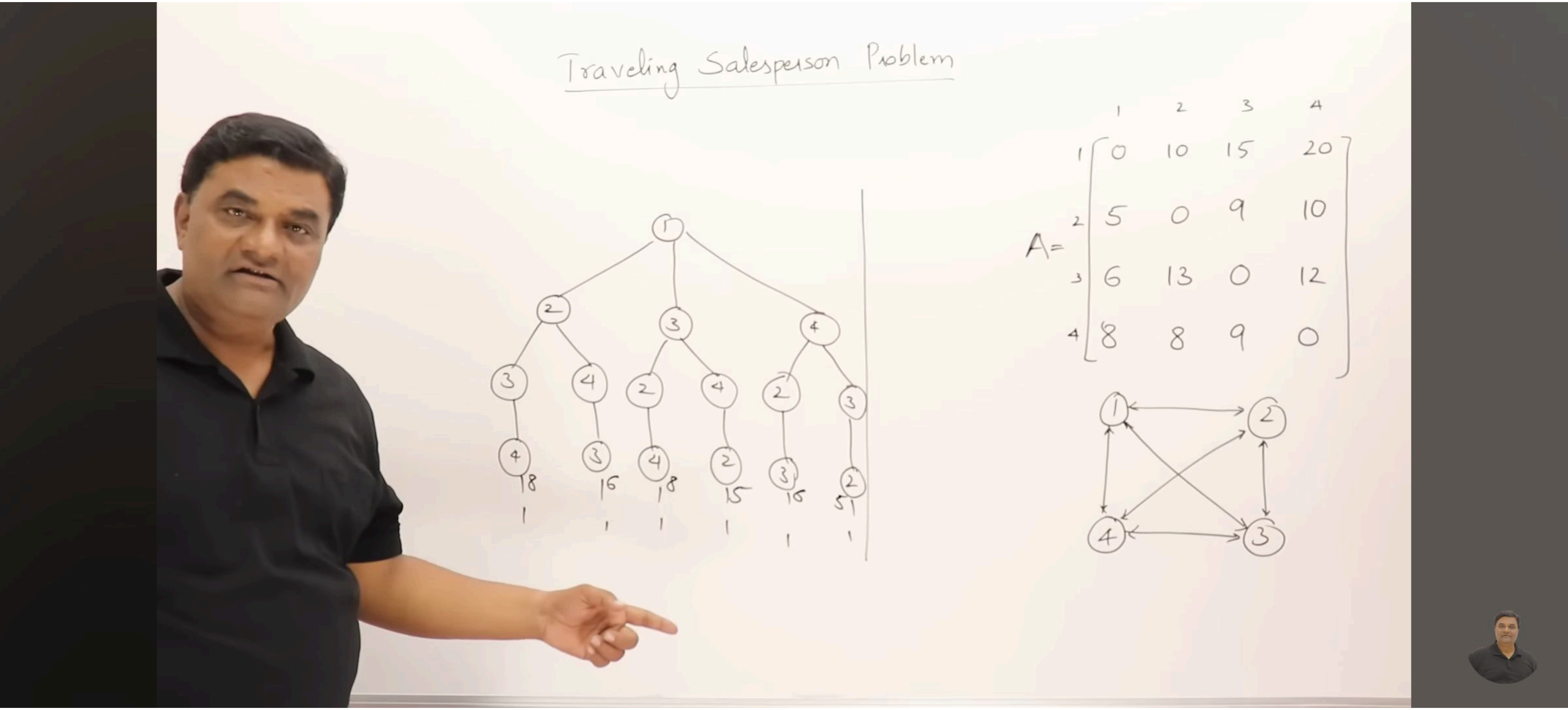

Longest Common Subsequence
abcdaf
acbcf
if (input1[i] == input2[j]) {
T[i][j] = T[i-1][j-1] + 1;
else
T[i][j] = max(T[i-1][j], T[i][j-1]);

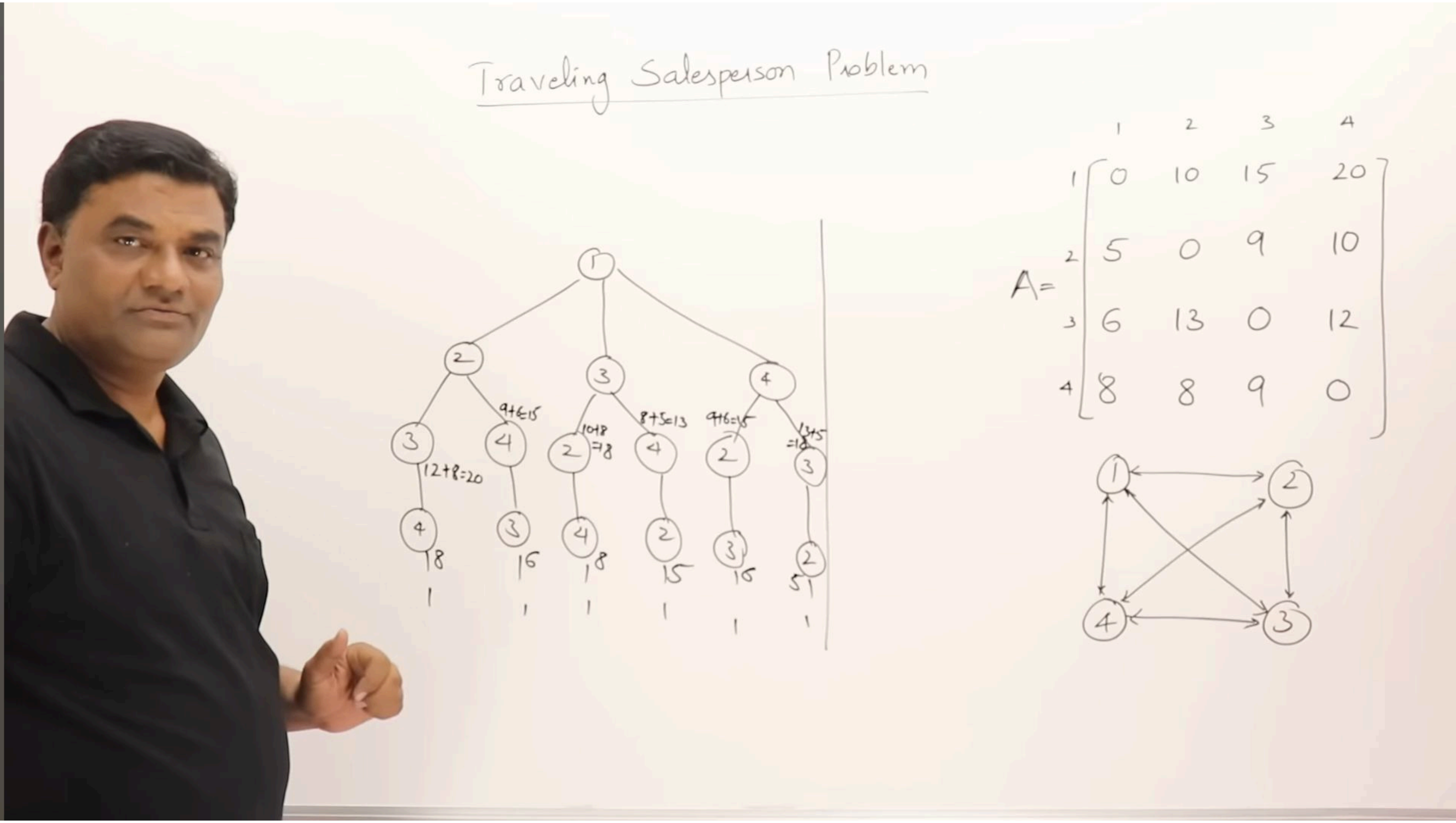
Traveling Salesman Problem (using DP)

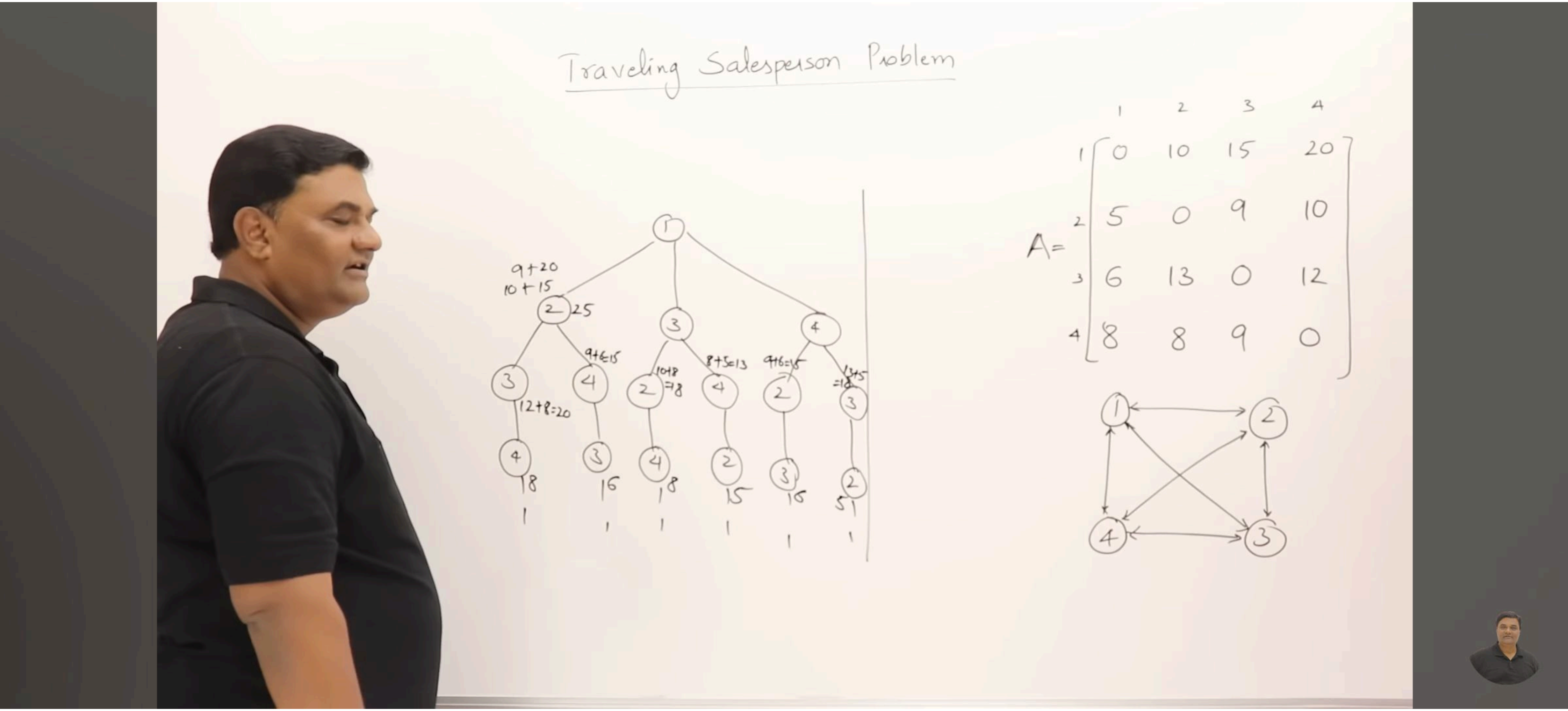

Need to find a path such that it is going through all vertices and returning to the starting vertex
Traveling Salesperson Problem
A=
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
1
2
3
4

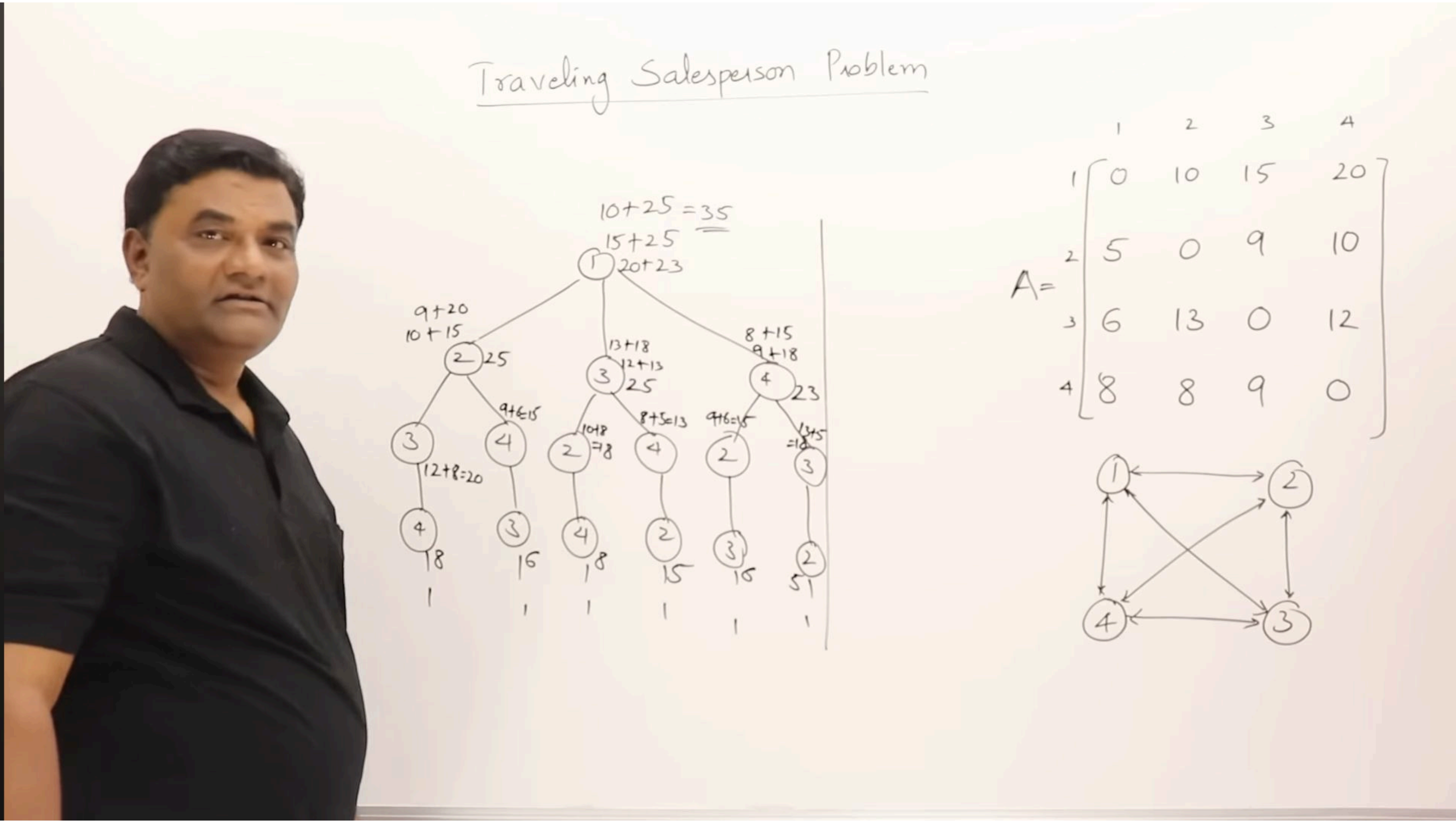

Traveling Salesperson Problem
1 2 3 4
A=
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0

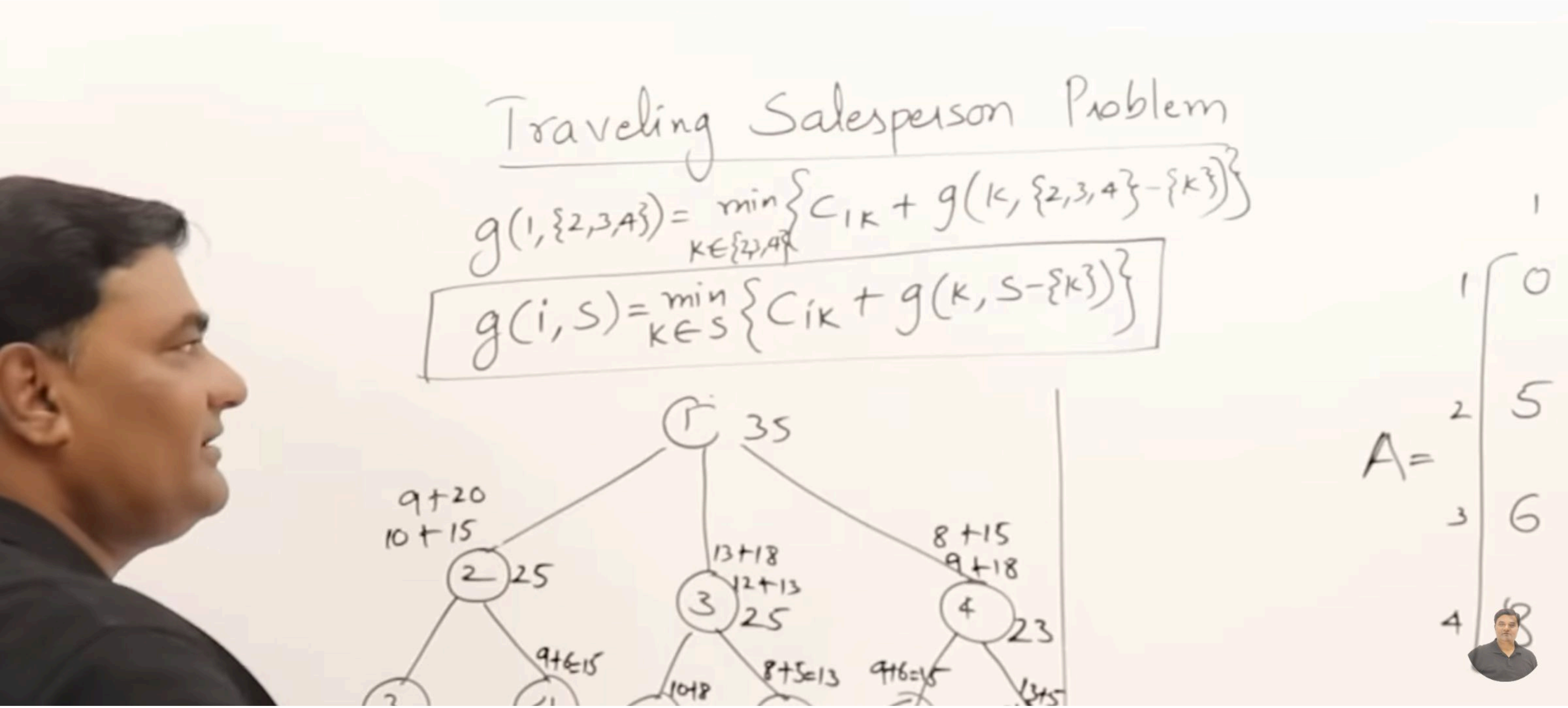

Traveling Salesperson Problem
A=
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0


Traveling Salesperson Problem
A=
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
18 16 18 15 16 51

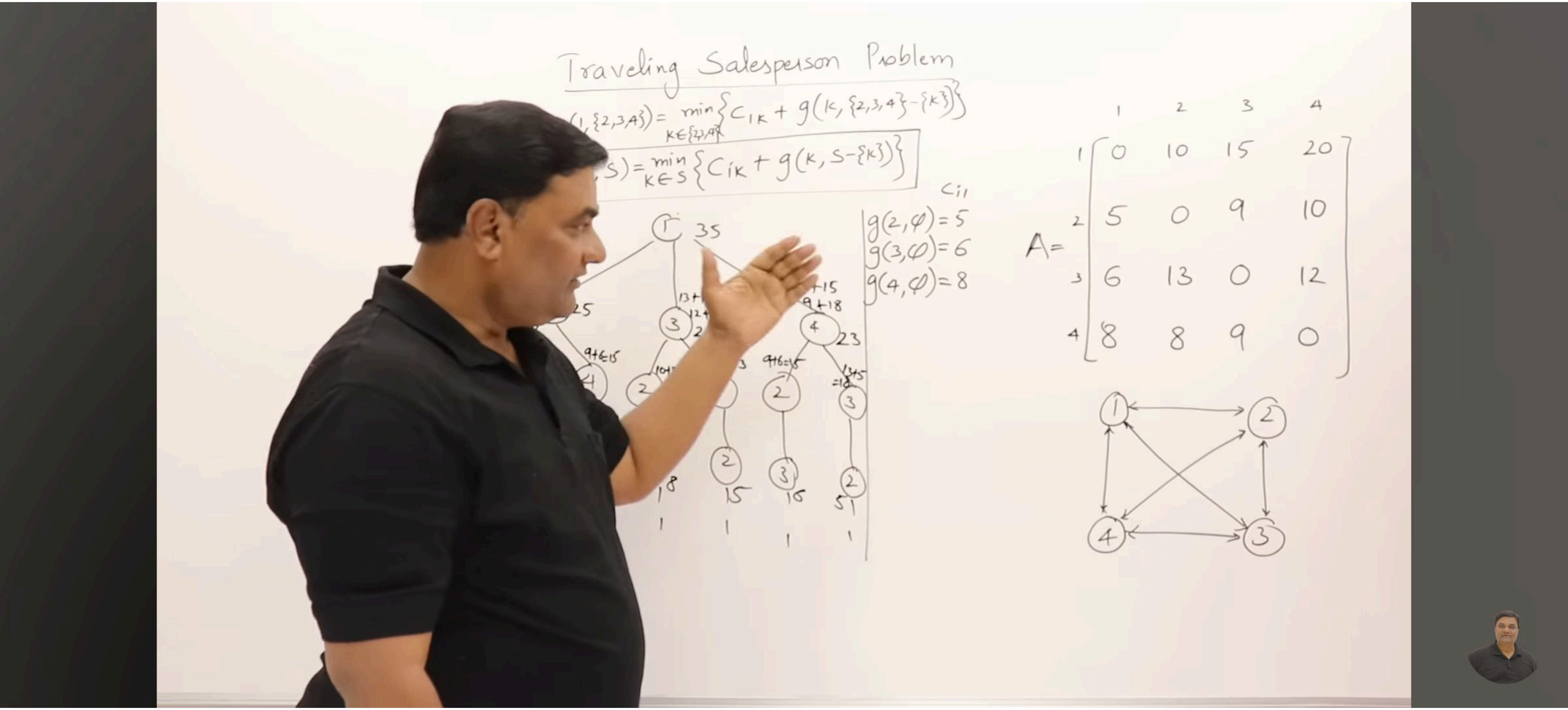

Traveling Salesperson Problem
A=
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
12+8=20
9+6=15
8+5=13
9+6=15

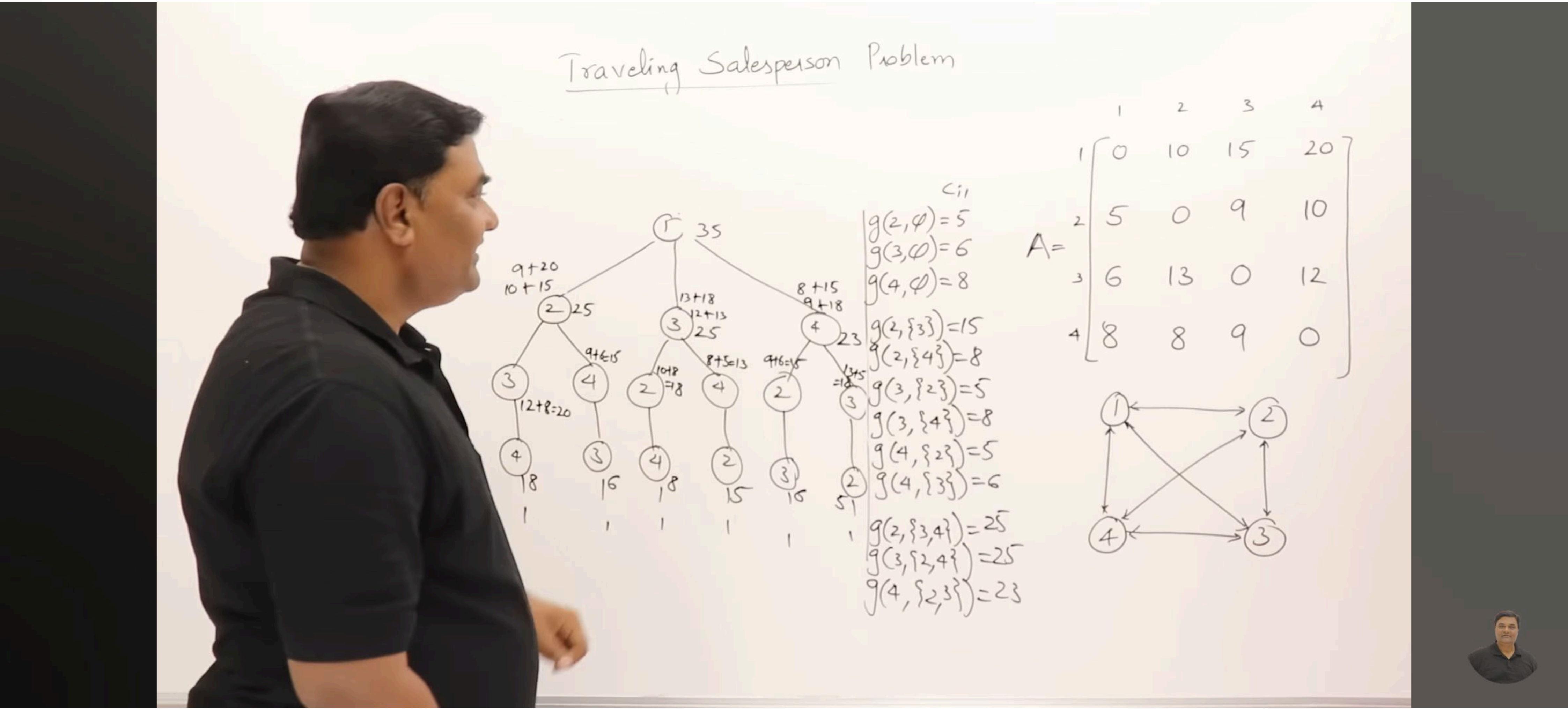

Traveling Salesperson Problem
A=
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
9+20
10+15
25
9+6=15
12+8=20
8+5=13
9+6=15
18
16
18
15
16
1
1
2
3
4


Traveling Salesperson Problem
10+25=35
15+25
20+23
9+20
10+15
25
13+18
12+13
25
8+15
9+18
23
9+6=15
8+5=13
9+6=15
12+8=20
10+8
=18
13+5
=18
18
16
18
15
16
1
1
1
1
1
1
1 2 3 4
A=
1 [0 10 15 20]
2 [5 0 9 10]
3 [6 13 0 12]
4 [8 8 9 0]
1
2
3
4

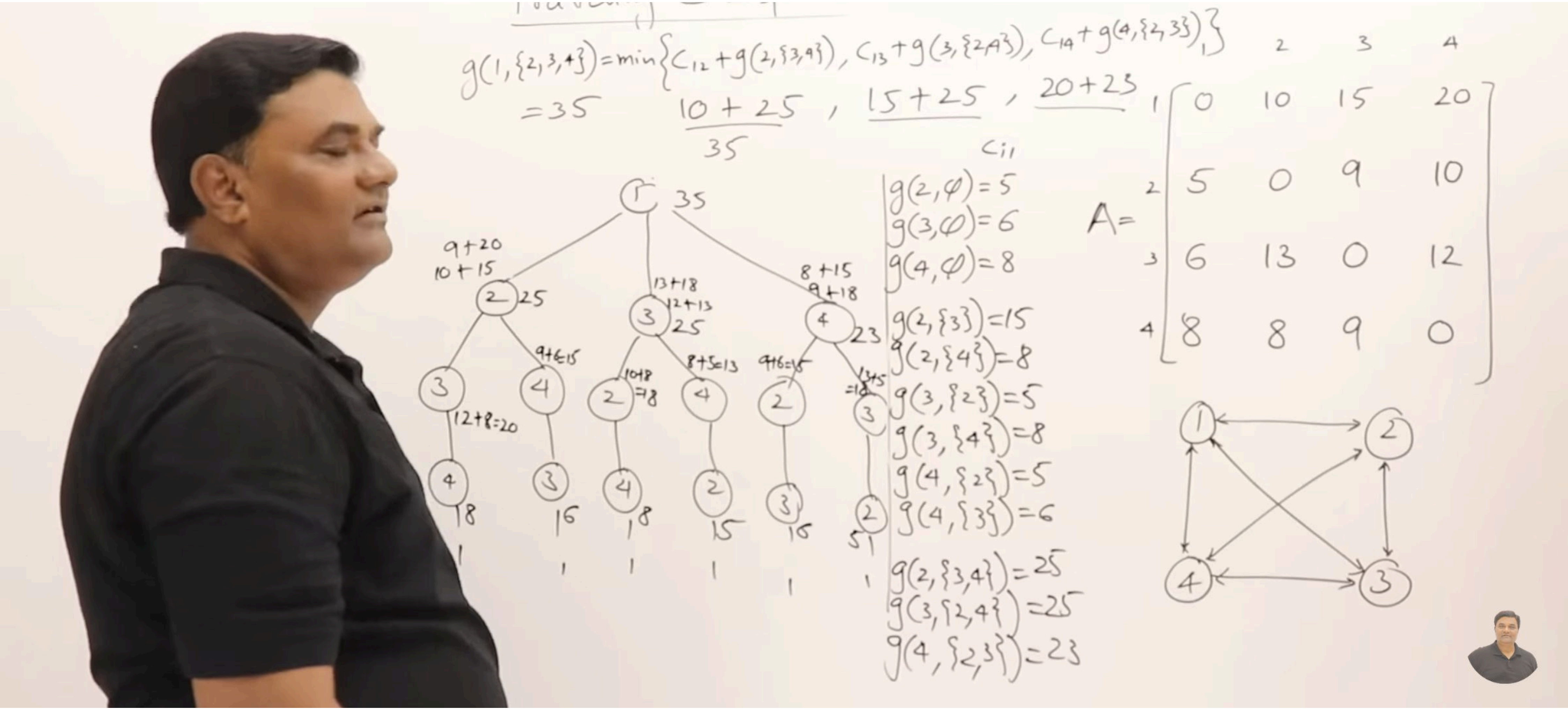


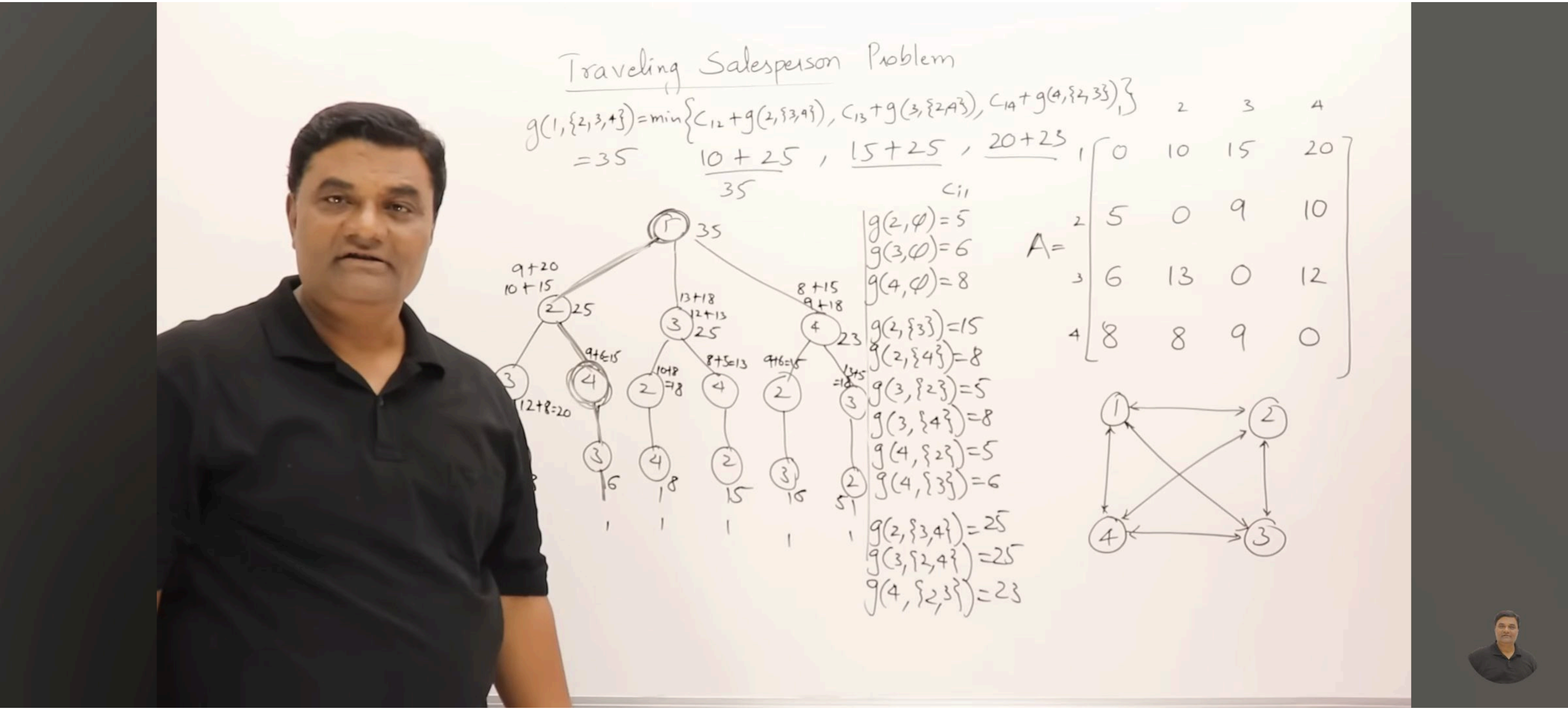
Traveling Salesperson Problem
g(1,{2,3,4}) = min k∈{2,3,4} {c1k + g(k,{2,3,4}-{k})}
g(i,S) = min k∈S {Cik + g(k,S-{k})}
1
35
9+20
10+15
2
25
13+18
12+13
3
25
8+15
9+18
4
23
9+6=15
10+8
8+5=13
9+6=15
13+5
A=
1
0
5
6
8


Traveling Salesperson Problem
(1, {2,3,4}) = min k∈{2,3,4} { C1k + g(k, {2,3,4} - {k}) }
S) = min k∈S { Cik + g(k, S-{k}) }
Ci1
g(2,φ)=5
g(3,φ)=6
g(4,φ)=8
35
25
23
1
2
3
4
0
10
15
20
5
0
9
10
A=
6
13
0
12
8
8
9
0



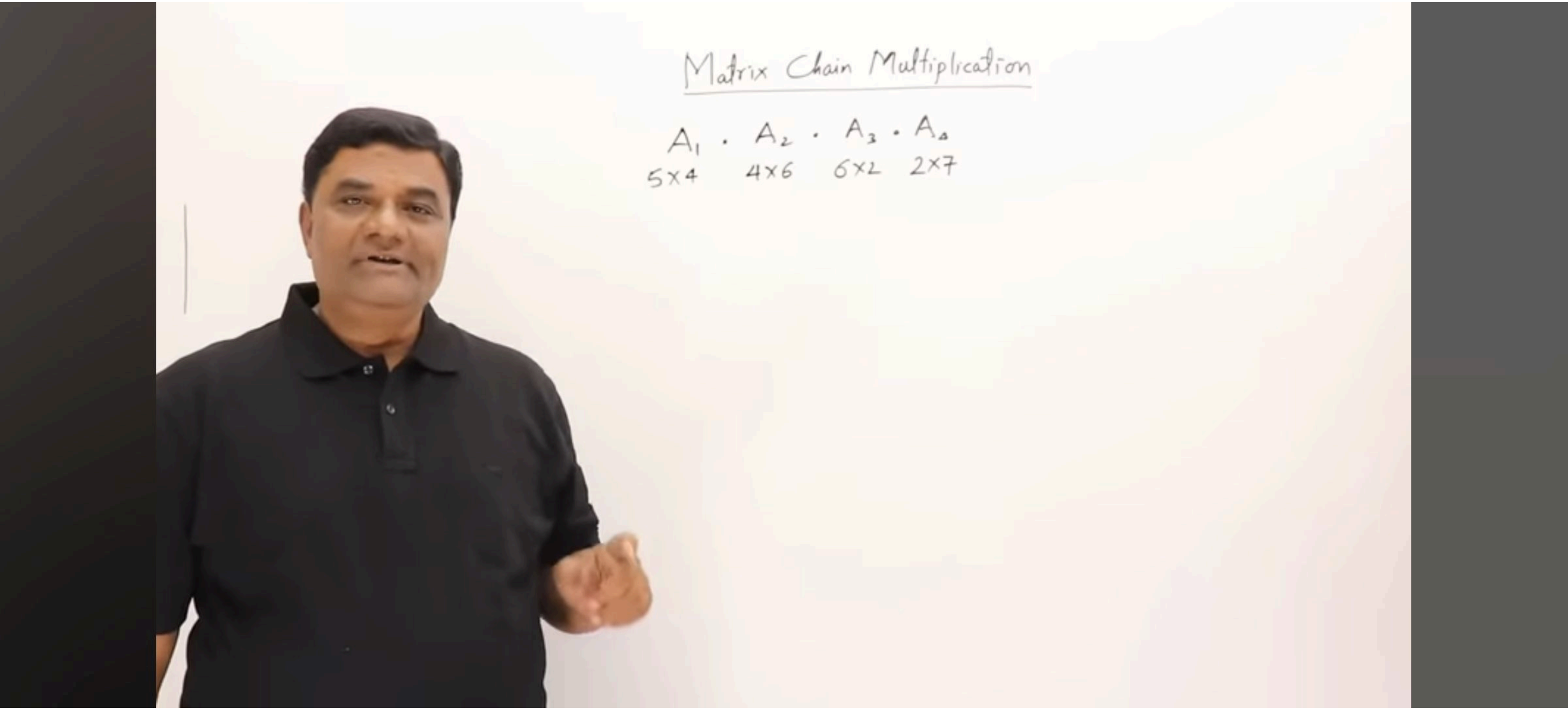
Traveling Salesperson Problem
A=
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
c_i1
g(2,φ)=5
g(3,φ)=6
g(4,φ)=8
g(2,{3})=15
g(2,{4})=8
g(3,{2})=5
g(3,{4})=8
g(4,{2})=5
g(4,{3})=6
g(2,{3,4})=25
g(3,{2,4})=25
g(4,{2,3})=23
35
25
25
23
9+20
10+15
13+18
12+13
8+15
9+18
12+8=20

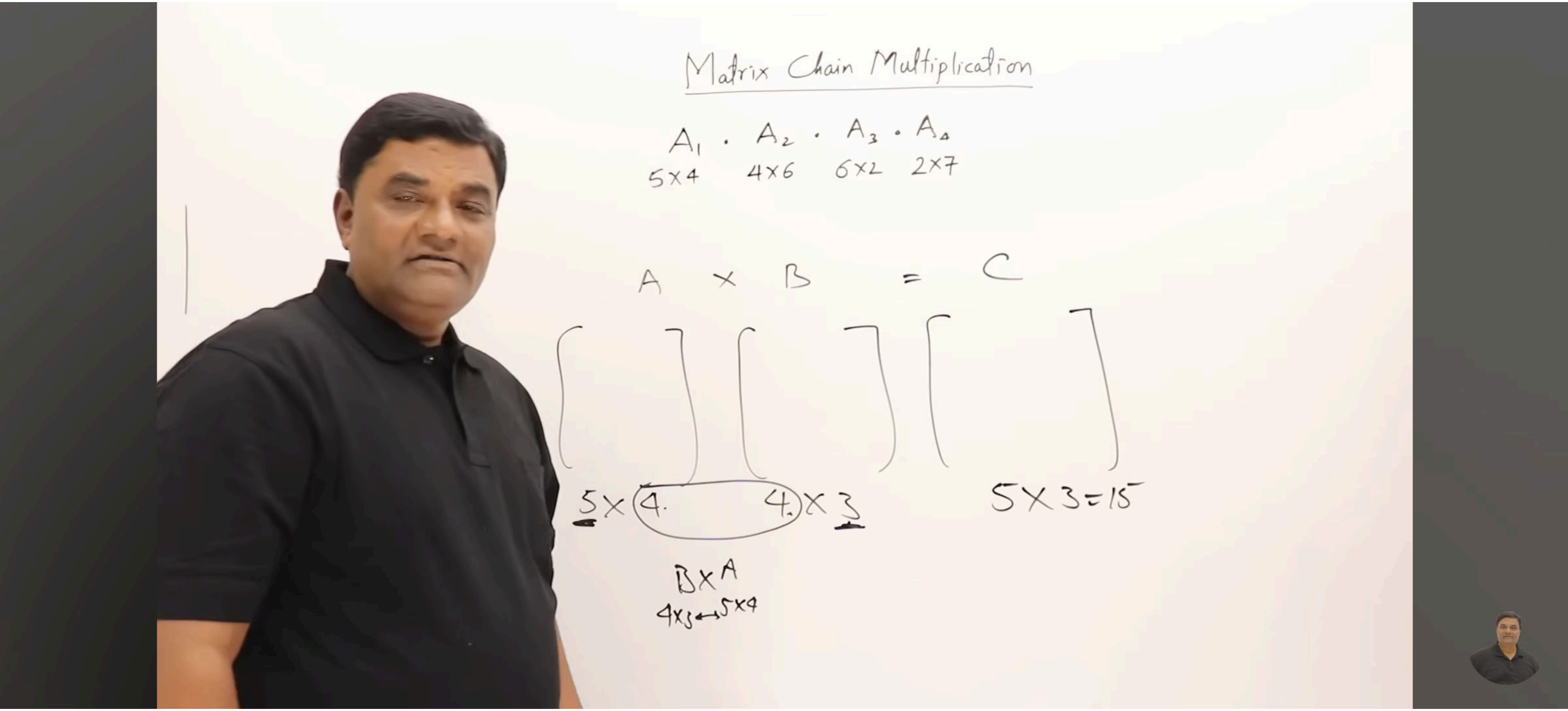

g(1,{2,3,4}) = min{c12 + g(2,{3,4}), c13 + g(3,{2,4}), c14 + g(4,{2,3})}
= 35
10 + 25 , 15 + 25 , 20 + 23
35
ci1
g(2,φ) = 5
g(3,φ) = 6
g(4,φ) = 8
g(2,{3}) = 15
g(2,{4}) = 8
g(3,{2}) = 5
g(3,{4}) = 8
g(4,{2}) = 5
g(4,{3}) = 6
g(2,{3,4}) = 25
g(3,{2,4}) = 25
g(4,{2,3}) = 23
A =
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
35
9+20
10+15
25
13+18
12+13
25
8+15
9+18
23
9+6=15
12+8=20
10+8
=18
8+5=13
9+6=15
13+5
=18
18
16
18
15
16
5

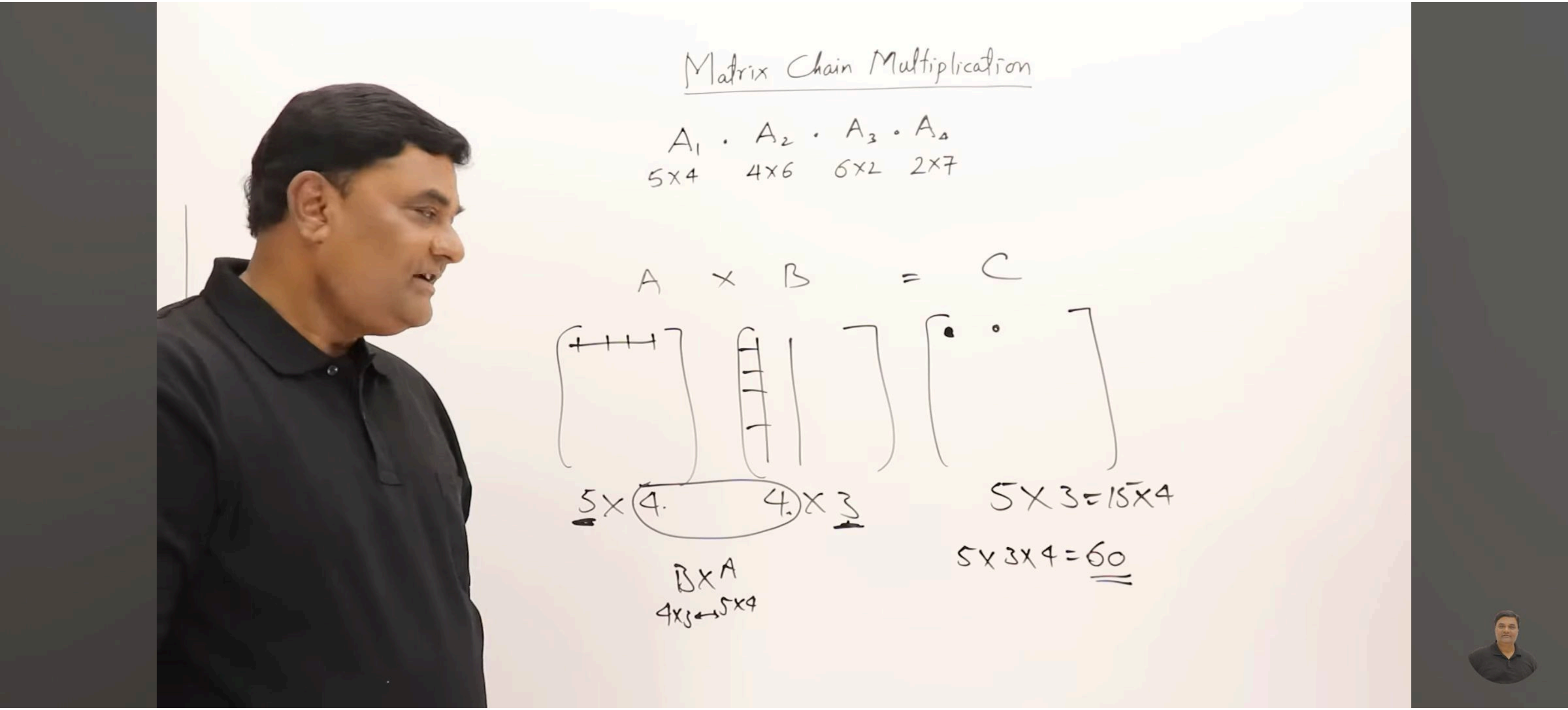


Traveling Salesperson Problem
g(1, {2,3,4}) = min {c12 + g(2, {3,4}), c13 + g(3, {2,4}), c14 + g(4, {2,3})}
= 35
10 + 25 , 15 + 25 , 20 + 23
35
1 2 3 4
1 0 10 15 20
2 5 0 9 10
3 6 13 0 12
4 8 8 9 0
A =
ci1
g(2, φ) = 5
g(3, φ) = 6
g(4, φ) = 8
g(2, {3}) = 15
g(2, {4}) = 8
g(3, {2}) = 5
g(3, {4}) = 8
g(4, {2}) = 5
g(4, {3}) = 6
g(2, {3,4}) = 25
g(3, {2,4}) = 25
g(4, {2,3}) = 23
35
9+20
10+15
25
13+18
12+13
25
8+15
9+18
23
9+6=15
12+8=20
10+8=18
8+5=13
9+6=15
13+5=18
6
8
15
18

Matrix chain multiplication


Matrix Chain Multiplication
$A_1 \cdot A_2 \cdot A_3 \cdot A_4$
$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
A × B = C
5×4 4×3
5×3=15
B×A
4×3↔5×4



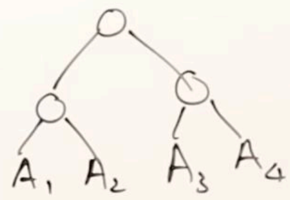
Matrix Chain Multiplication
A1 · A2 · A3 · A4
5x4 4x6 6x2 2x7
A x B = C
5x 4. 4. x 3
5x3=15x4
5x3x4=60
BxA
4x3↔5x4

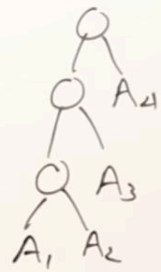
Matrix Chain Multiplication

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4$$
$$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$$

$$((A_1 \cdot A_2) \cdot A_3) \cdot A_4$$

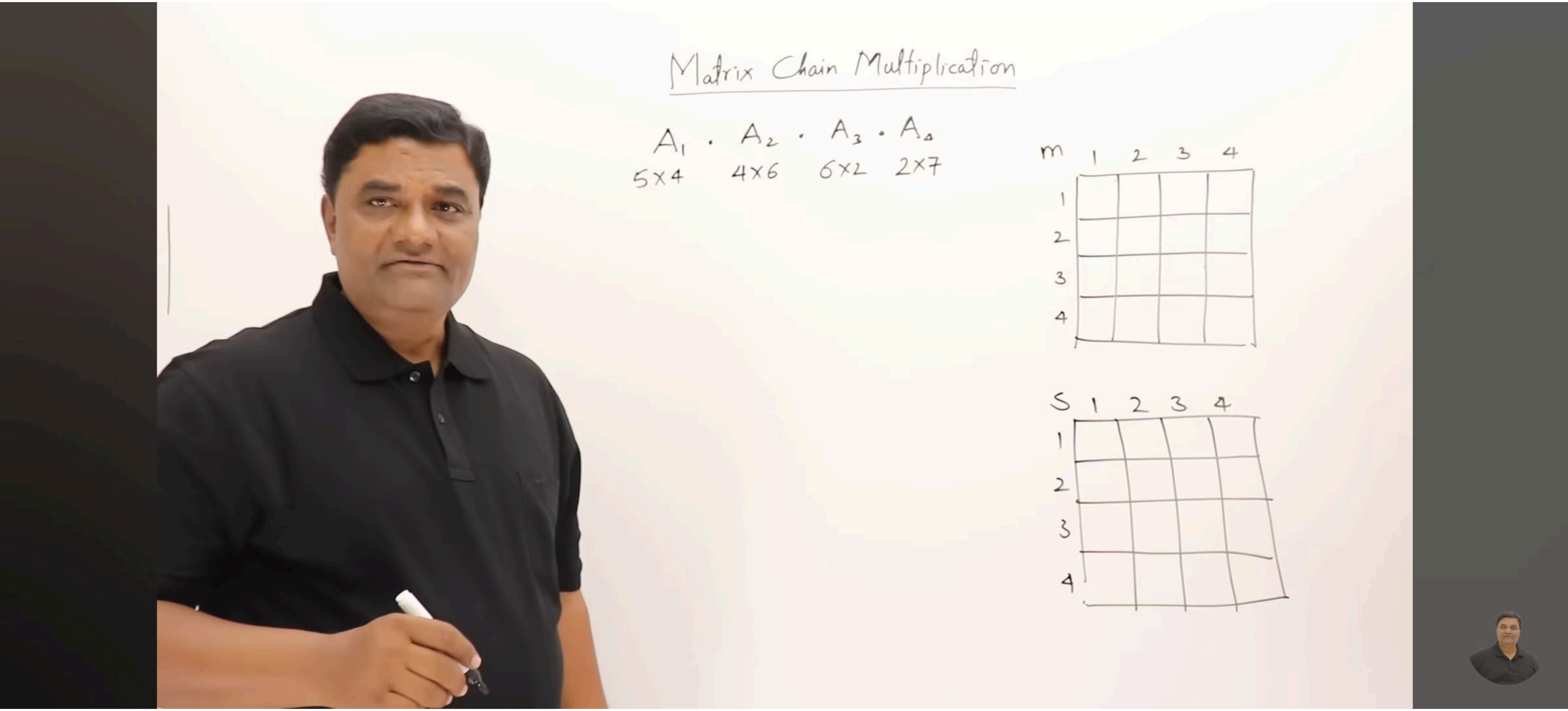
$$(A_1 \cdot A_2) \cdot (A_3 \cdot A_4)$$

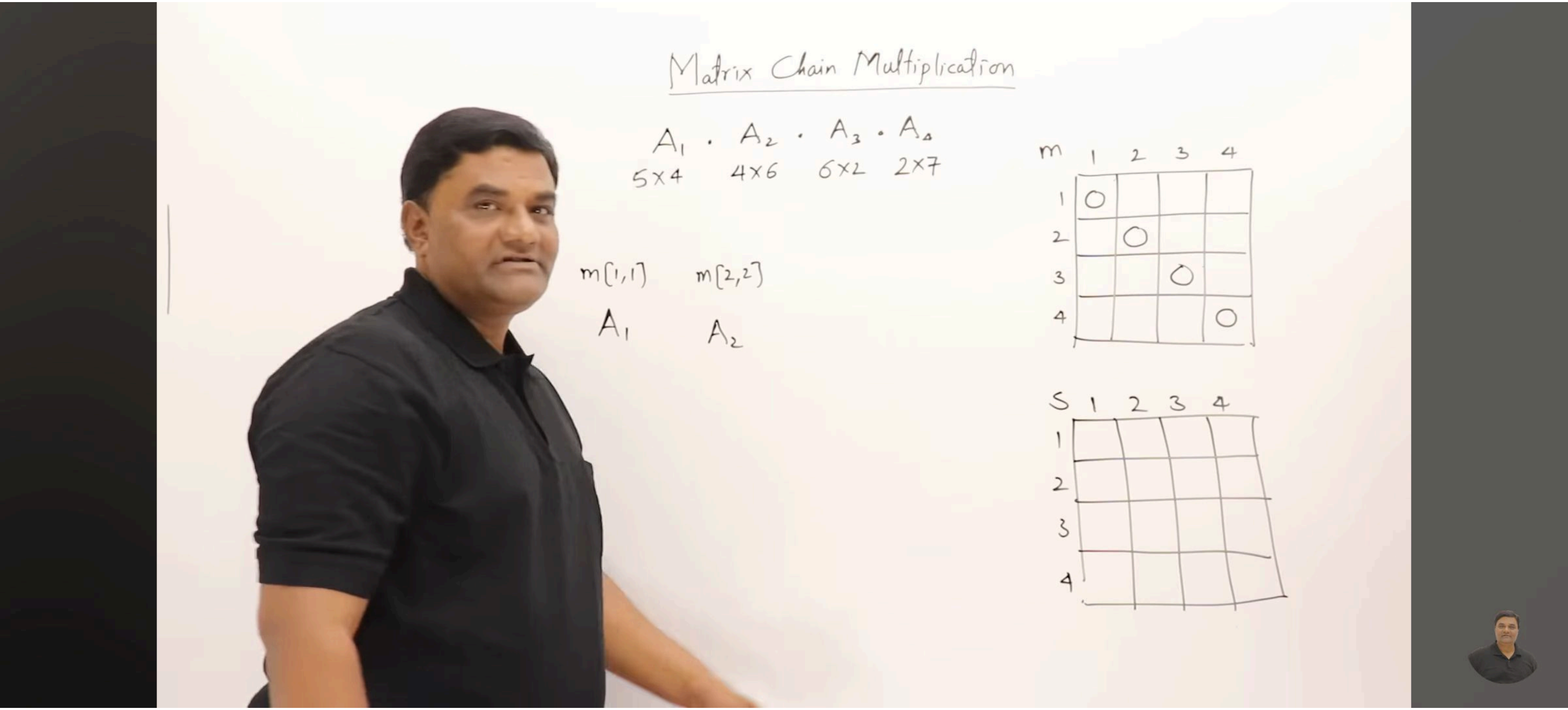


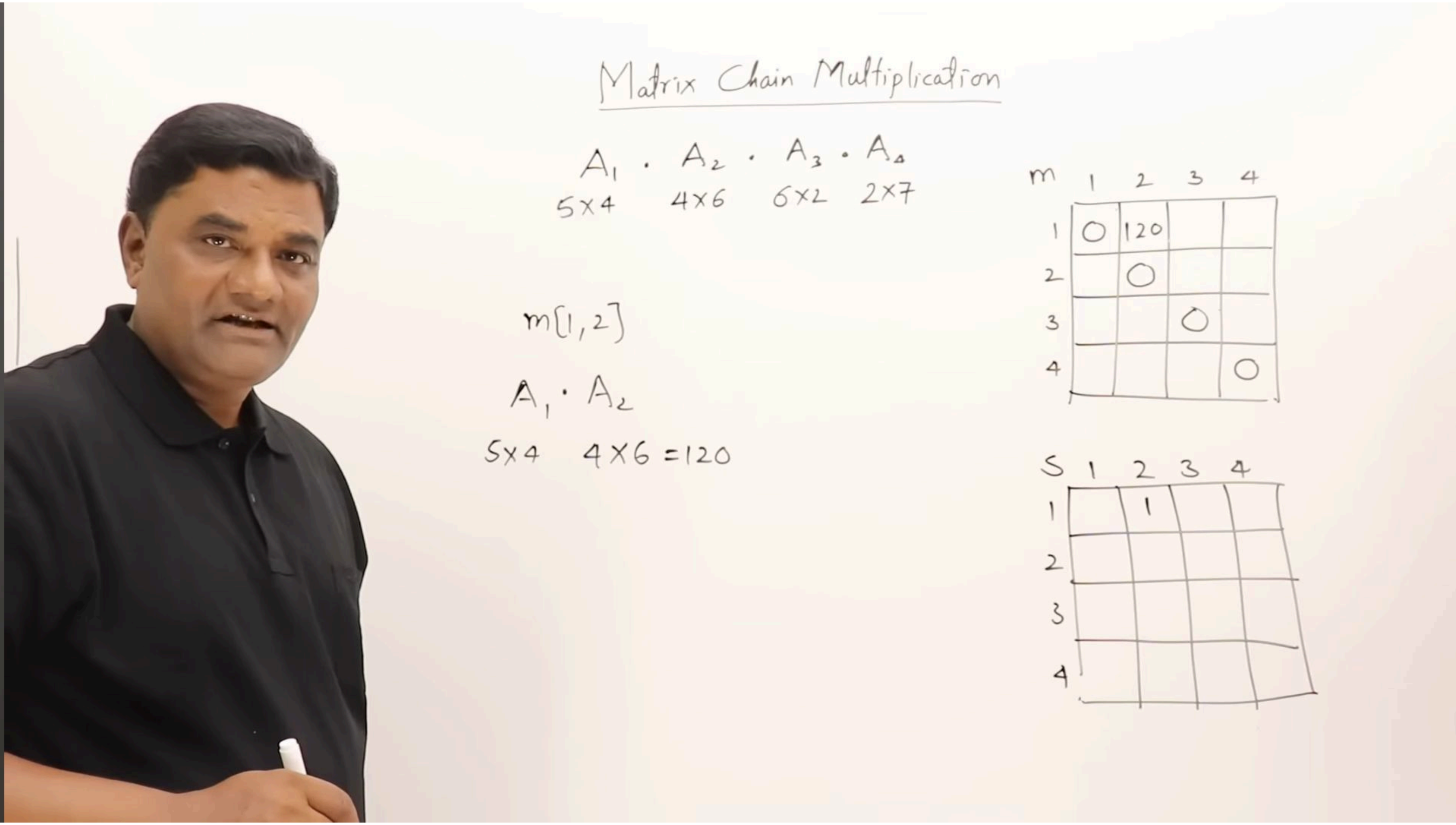


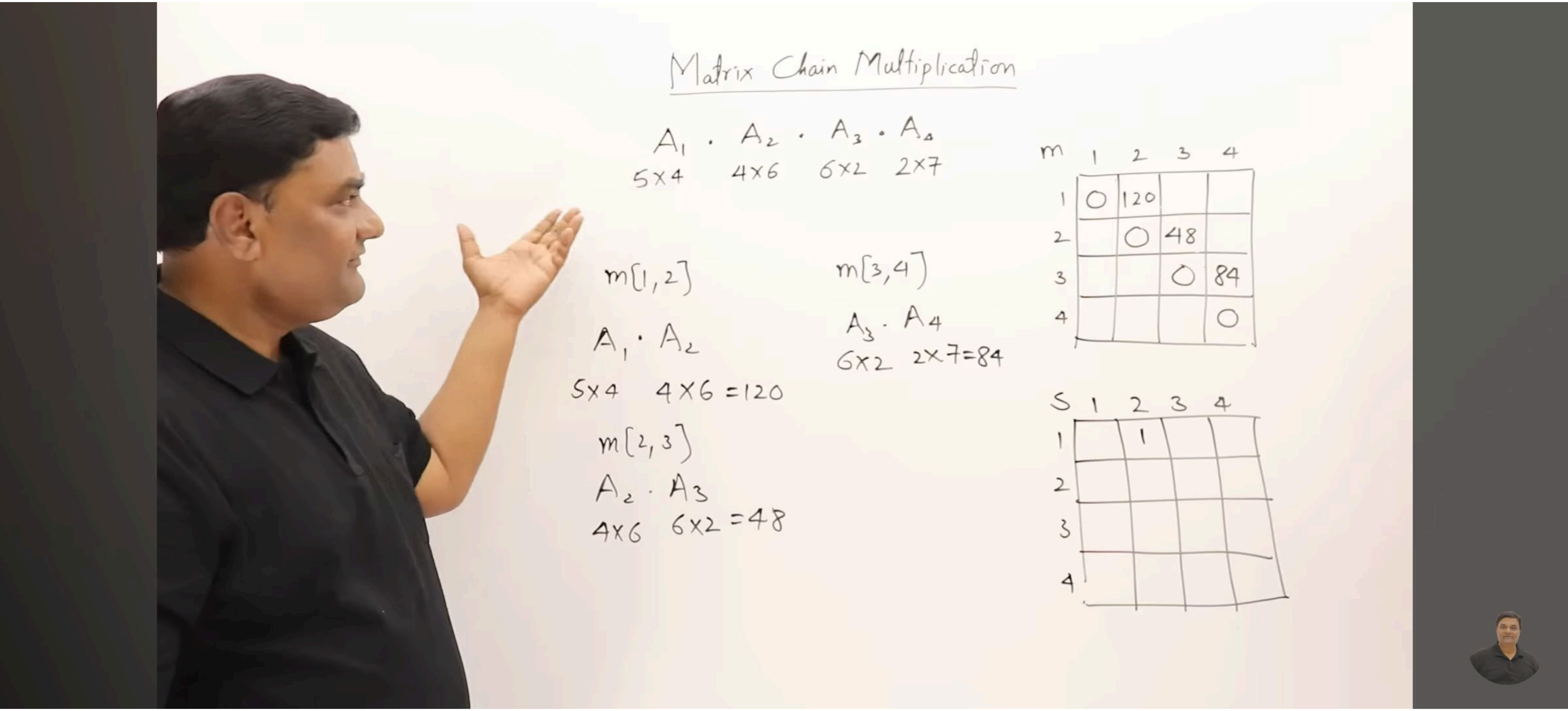
$$T(n) = \frac{{}^{2n}C_n}{n+1}$$

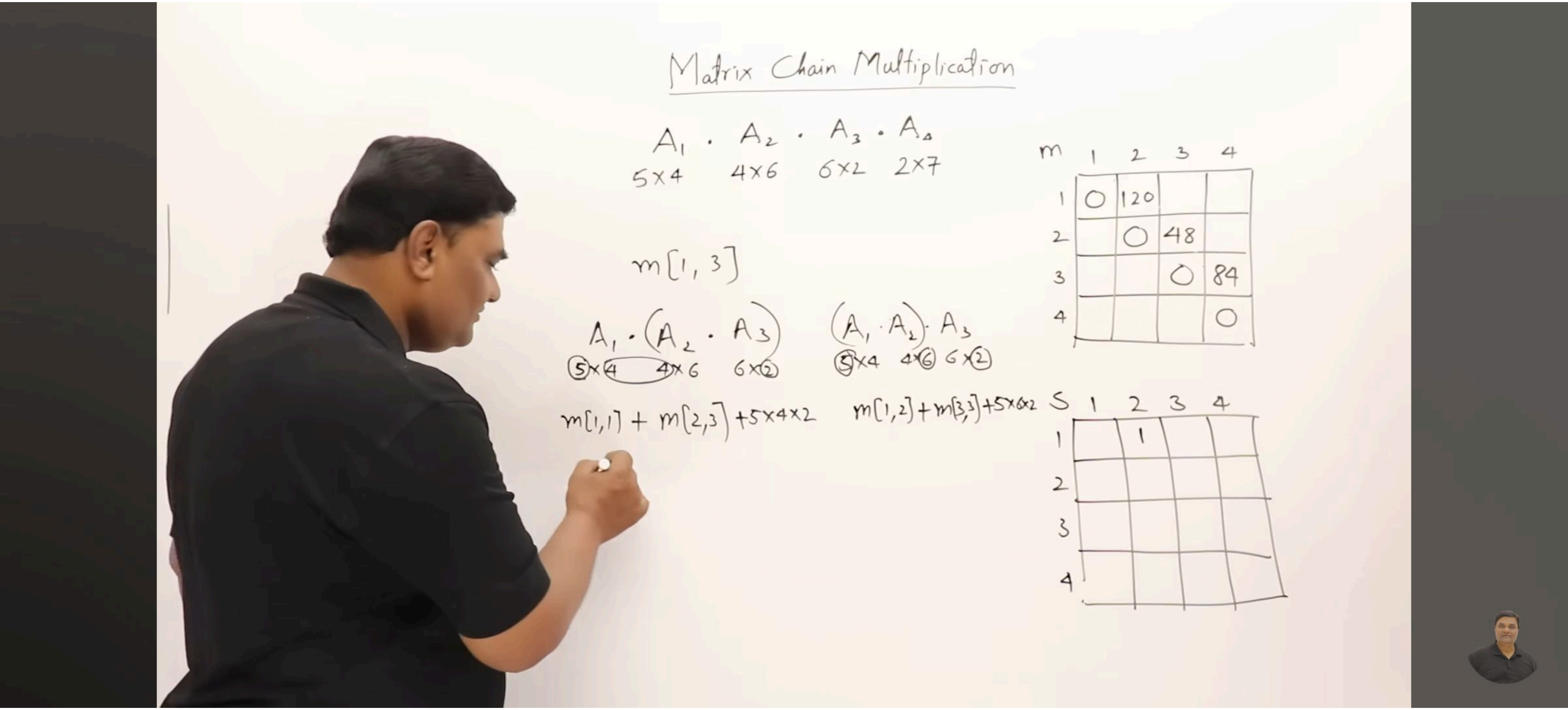
$$T(3) = \underline{5}$$


Matrix Chain Multiplication
$A_1 \cdot A_2 \cdot A_3 \cdot A_4$
5×4 4×6 6×2 2×7
m 1 2 3 4
1 2 3 4
S 1 2 3 4
1 2 3 4


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[1,1] m[2,2]
A1 A2
m 1 2 3 4
1 0
2 0
3 0
4 0
S 1 2 3 4
1
2
3
4


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[1,2]
A1 · A2
5×4 4×6 =120
m 1 2 3 4
1 0 120
2 0
3 0
4 0
S 1 2 3 4
1 1
2
3
4


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5x4 4x6 6x2 2x7
m[1,2]
A1 · A2
5x4 4x6 =120
m[2,3]
A2 · A3
4x6 6x2 =48
m[3,4]
A3 · A4
6x2 2x7=84
m 1 2 3 4
1 0 120
2 0 48
3 0 84
4 0
S 1 2 3 4
1 1
2
3
4


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[1, 3]
A1 · (A2 · A3)
5×4 4×6 6×2
m[1,1] + m[2,3] + 5×4×2
(A1 · A2) · A3
5×4 4×6 6×2
m[1,2] + m[3,3] + 5×6×2
m 1 2 3 4
1 0 120
2 0 48
3 0 84
4 0
S 1 2 3 4
1 1
2
3
4

Matrix Chain Multiplication

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4$$
$$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$$

m	1	2	3	4
1	0	120	88	
2		0	48	
3			0	84
4				0

$m[1,3] = 88$

$A_1 \cdot (A_2 \cdot A_3)$

$5\times4 \quad 4\times6 \quad 6\times2$

$m[1,1] + m[2,3] + 5\times4\times2$

$0 + 48 + 40$

$\underline{88}$

$(A_1 \cdot A_2) \cdot A_3$

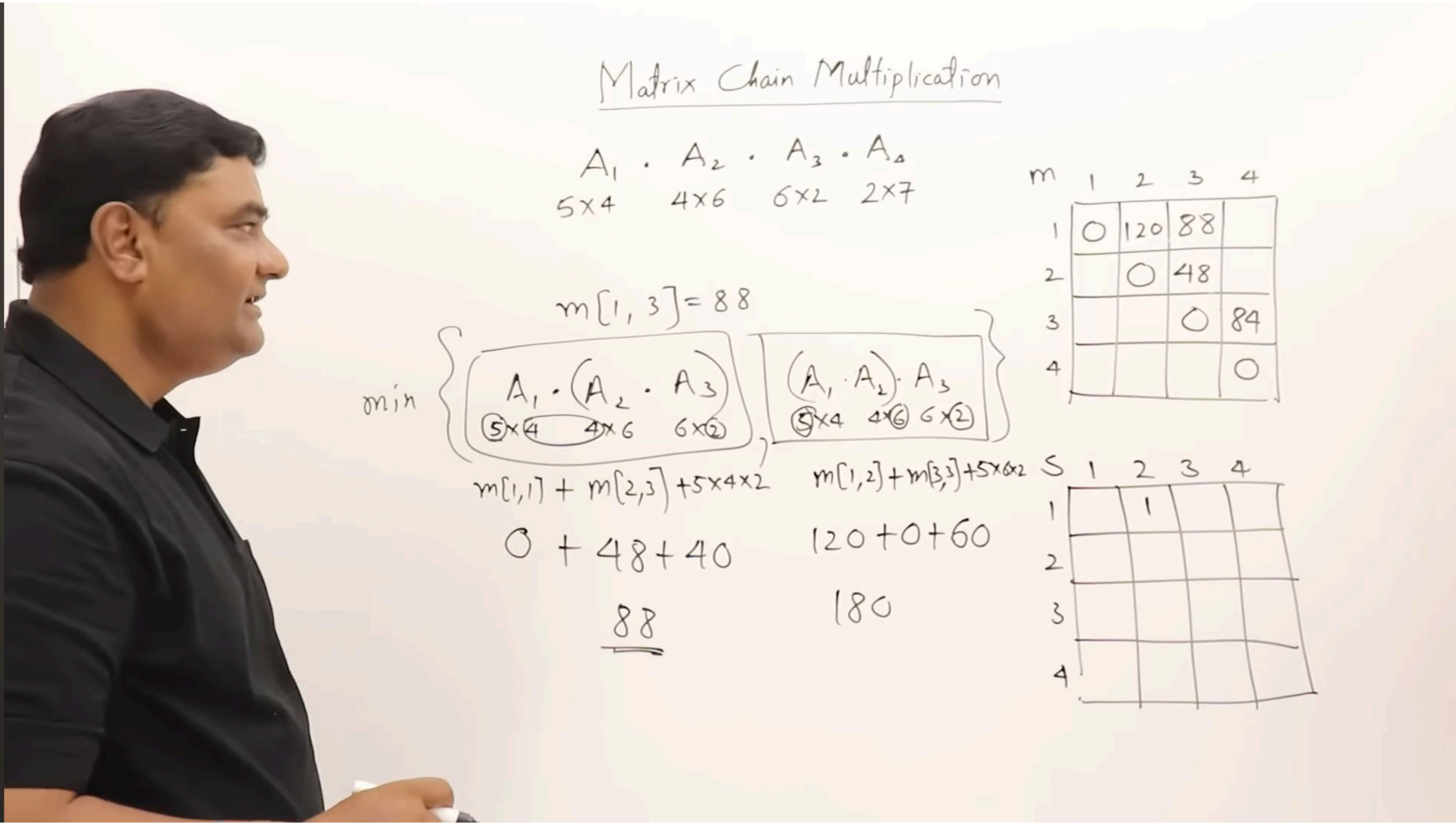
$5\times4 \quad 4\times6 \quad 6\times2$

$m[1,2] + m[3,3] + 5\times6\times2$

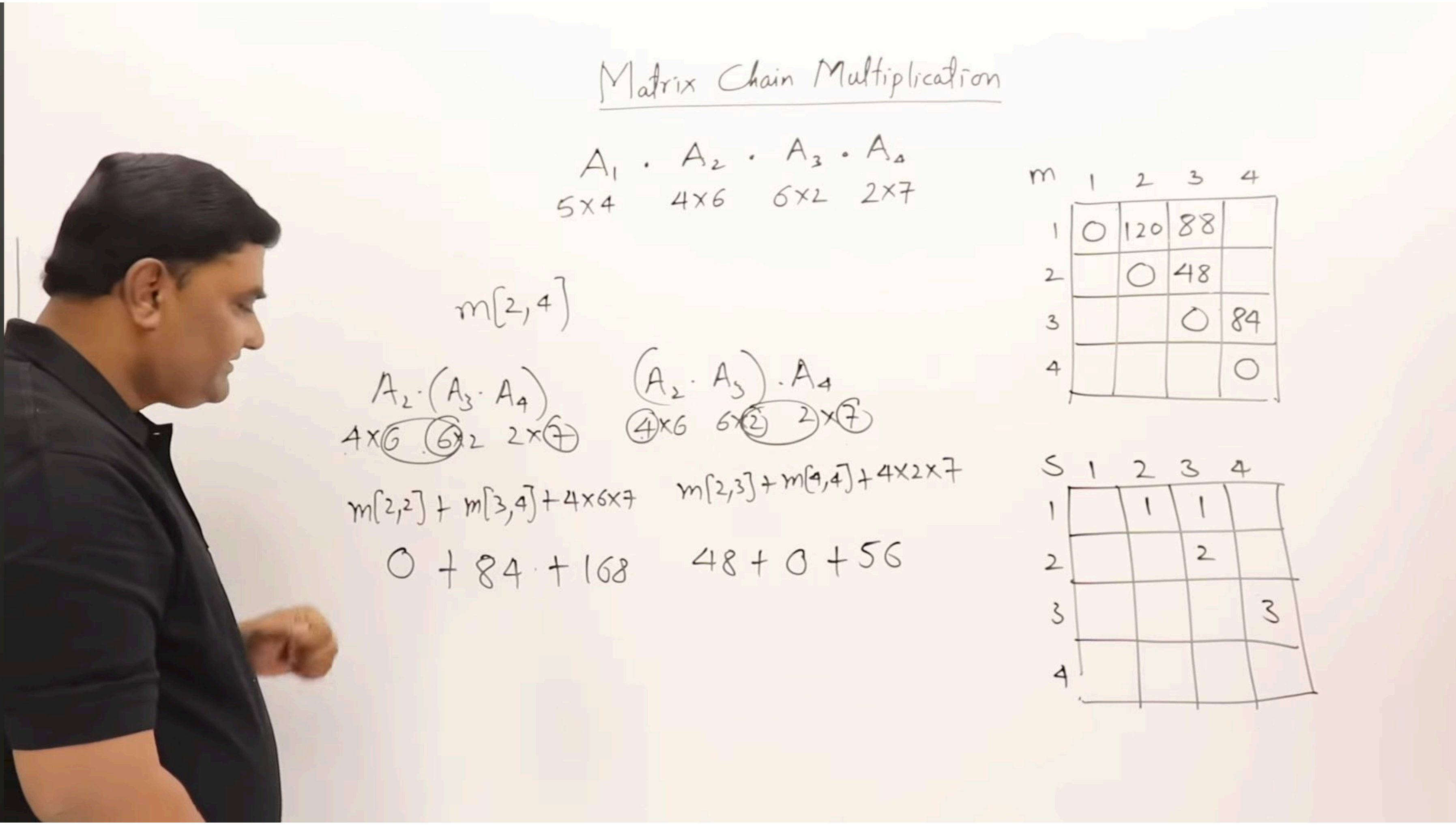
$120 + 0 + 60$

180

s	1	2	3	4
1		1		
2				
3				
4				


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[1, 3] = 88
min
A1 · (A2 · A3)
5×4 4×6 6×2
(A1 · A2) · A3
5×4 4×6 6×2
m[1,1] + m[2,3] + 5×4×2
0 + 48 + 40
88
m[1,2] + m[3,3] + 5×6×2
120 + 0 + 60
180
m 1 2 3 4
1 0 120 88
2 0 48
3 0 84
4 0
S 1 2 3 4
1 1
2
3
4

Matrix Chain Multiplication
A_1
A_4
5×4
2×7
m 1 2 3 4
1 0 120 88
2 0
3 84
4 0
1 2 3 4
1 1 1
2 2
3 3
4



Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[2,4]
A2 · (A3 · A4)
4×6 6×2 2×7
(A2 · A3) · A4
4×6 6×2 2×7
m[2,2] + m[3,4] + 4×6×7
m[2,3] + m[4,4] + 4×2×7
0 + 84 + 168
48 + 0 + 56
m 1 2 3 4
1 0 120 88
2 0 48
3 0 84
4 0
S 1 2 3 4
1 1 1
2 2
3 3
4

Matrix Chain Multiplication

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4$$
$$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$$

$$m[2,4] = 104$$

$$A_2 \cdot (A_3 \cdot A_4) \qquad (A_2 \cdot A_3) \cdot A_4$$
$$4\times6 \quad 6\times2 \quad 2\times7 \qquad 4\times6 \quad 6\times2 \quad 2\times7$$

$$m[2,2] + m[3,4] + 4\times6\times7 \qquad m[2,3] + m[4,4] + 4\times2\times7$$

$$0 + 84 + 168 \qquad 48 + 0 + 56$$

$$252 \qquad 104$$

m	1	2	3	4
1	0	120	88	
2		0	48	104
3			0	84
4				0

s	1	2	3	4
1		1	1	
2			2	3
3				3
4				

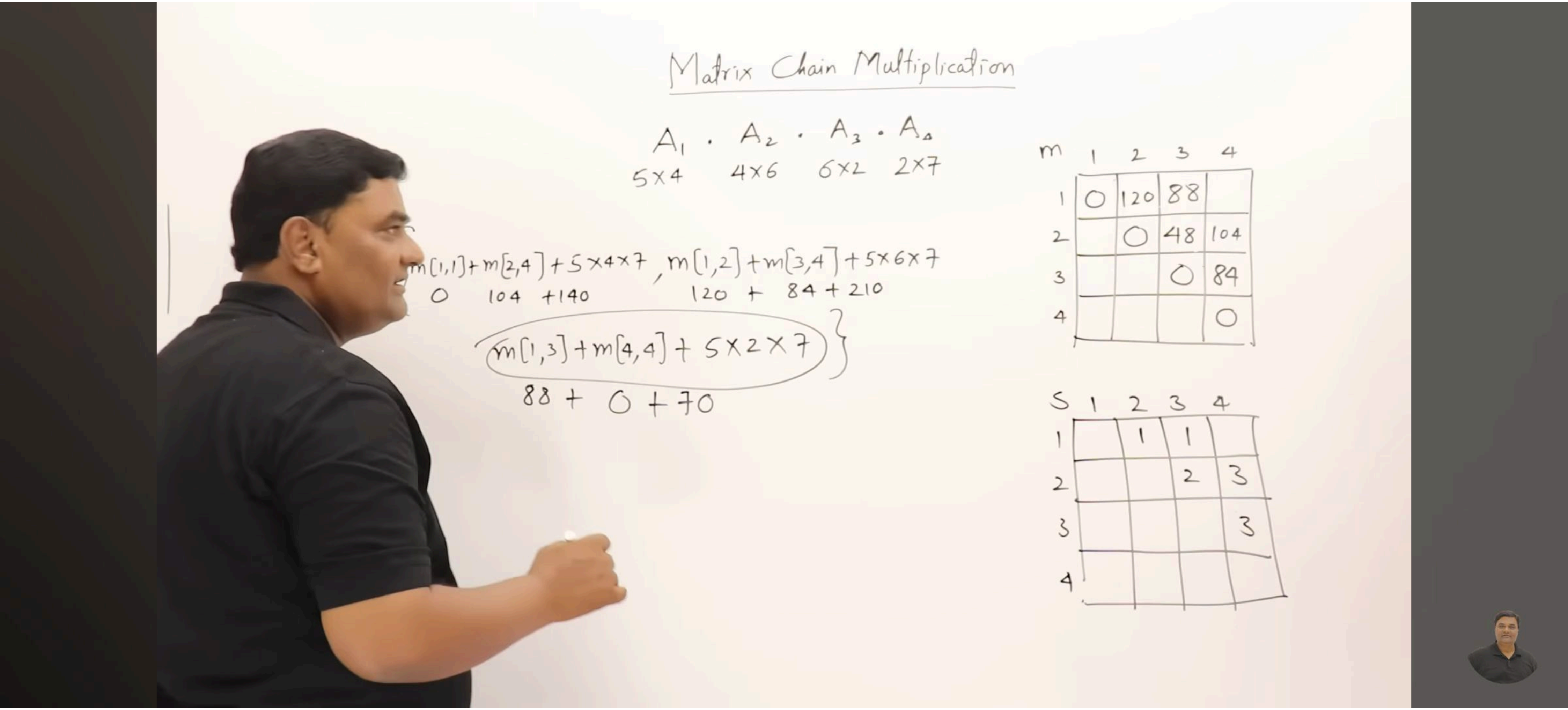
Matrix Chain Multiplication

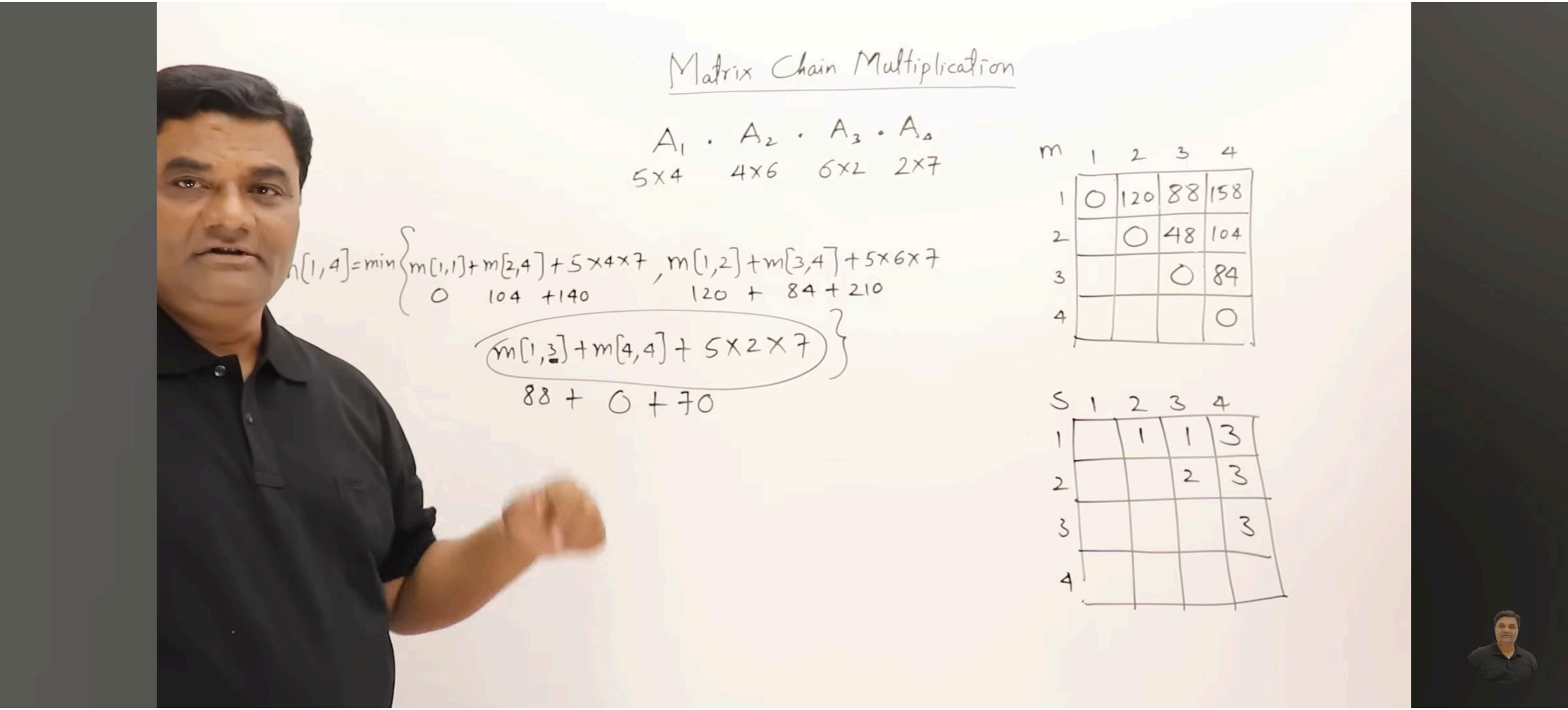
$$A_1 \cdot A_2 \cdot A_3 \cdot A_4$$
$$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$$

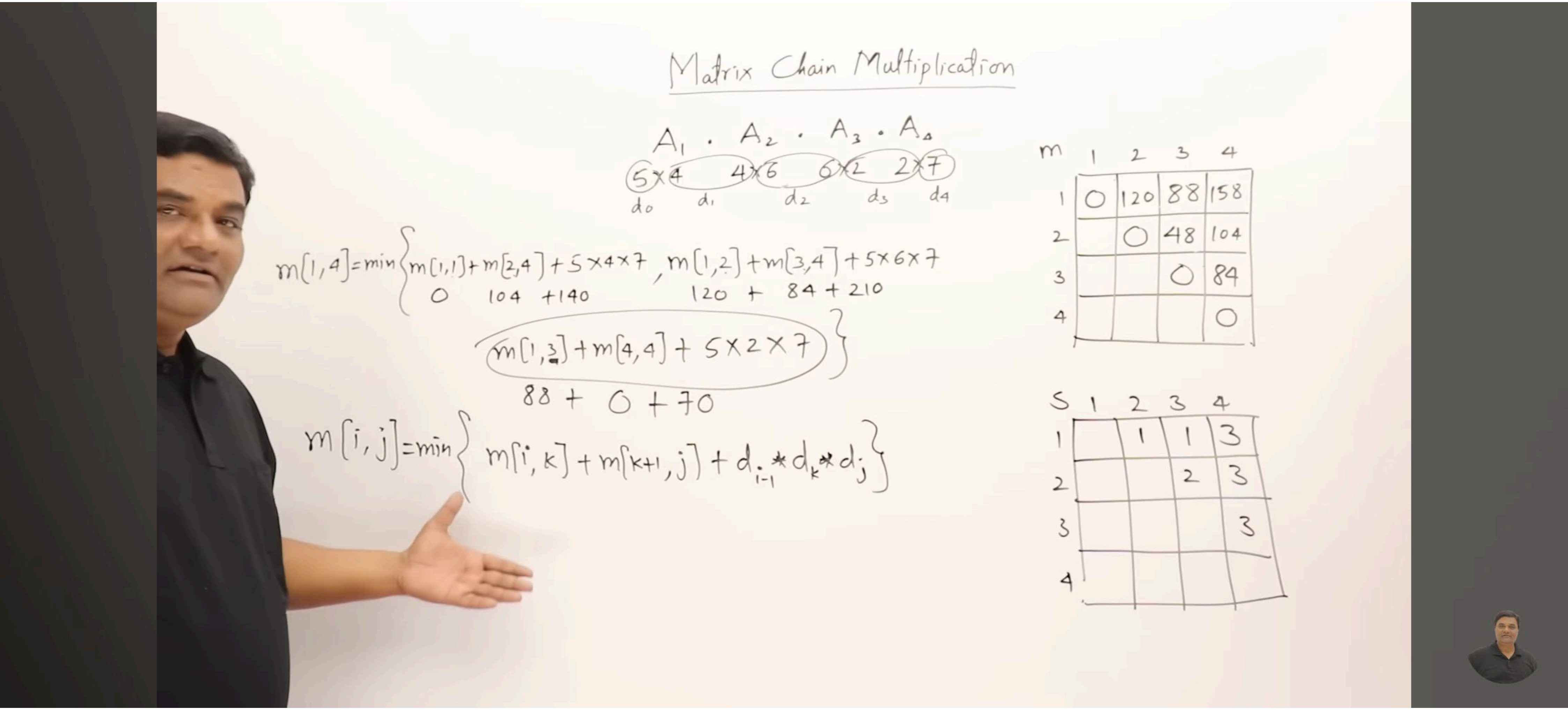
$$m[1,4] = \min\left\{ m[1,1] + m[2,4] + 5\times4\times7,\ m[1,2] + m[3,4] + 5\times6\times7,\ m[1,3] + m[4,4] + 5\times2\times7 \right\}$$

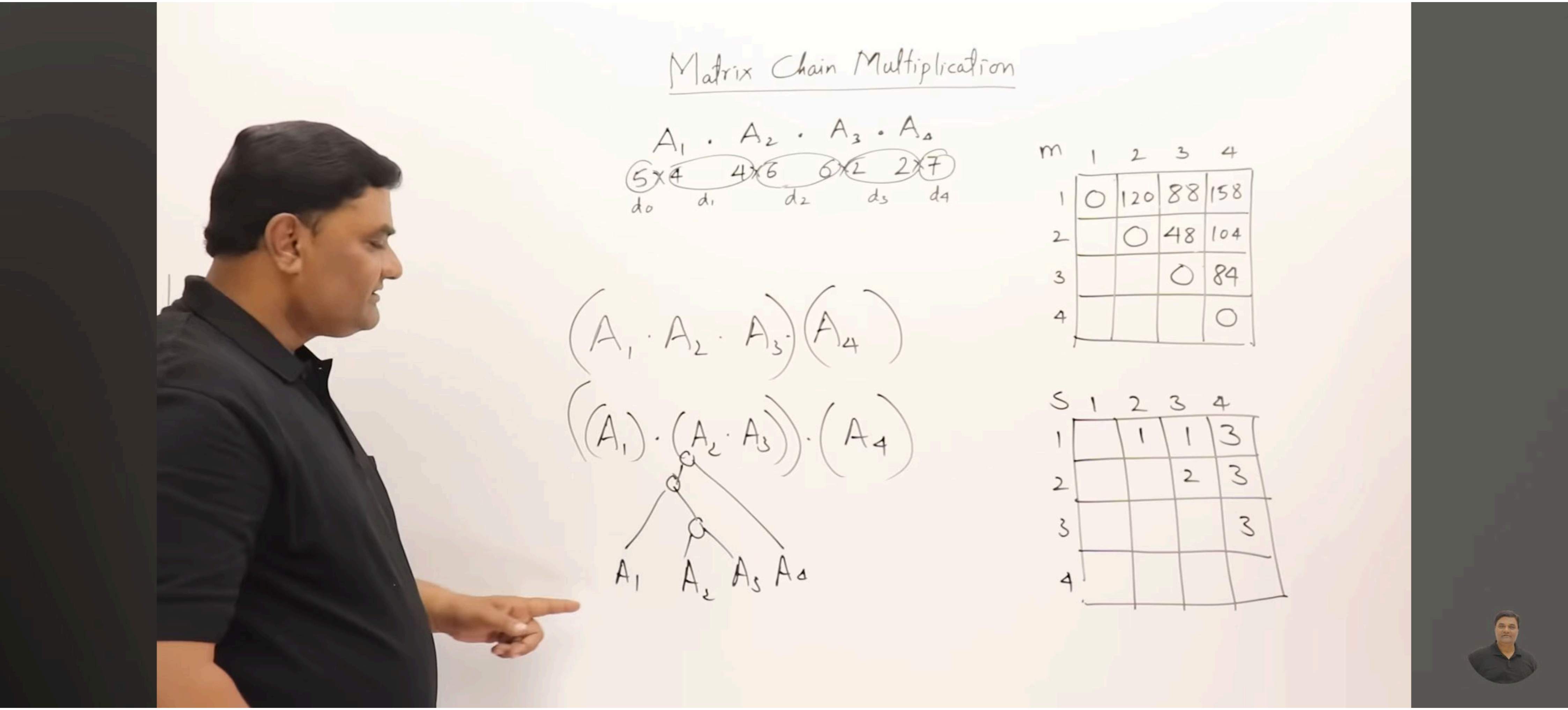
m	1	2	3	4
1	0	120	88	
2		0	48	104
3			0	84
4				0

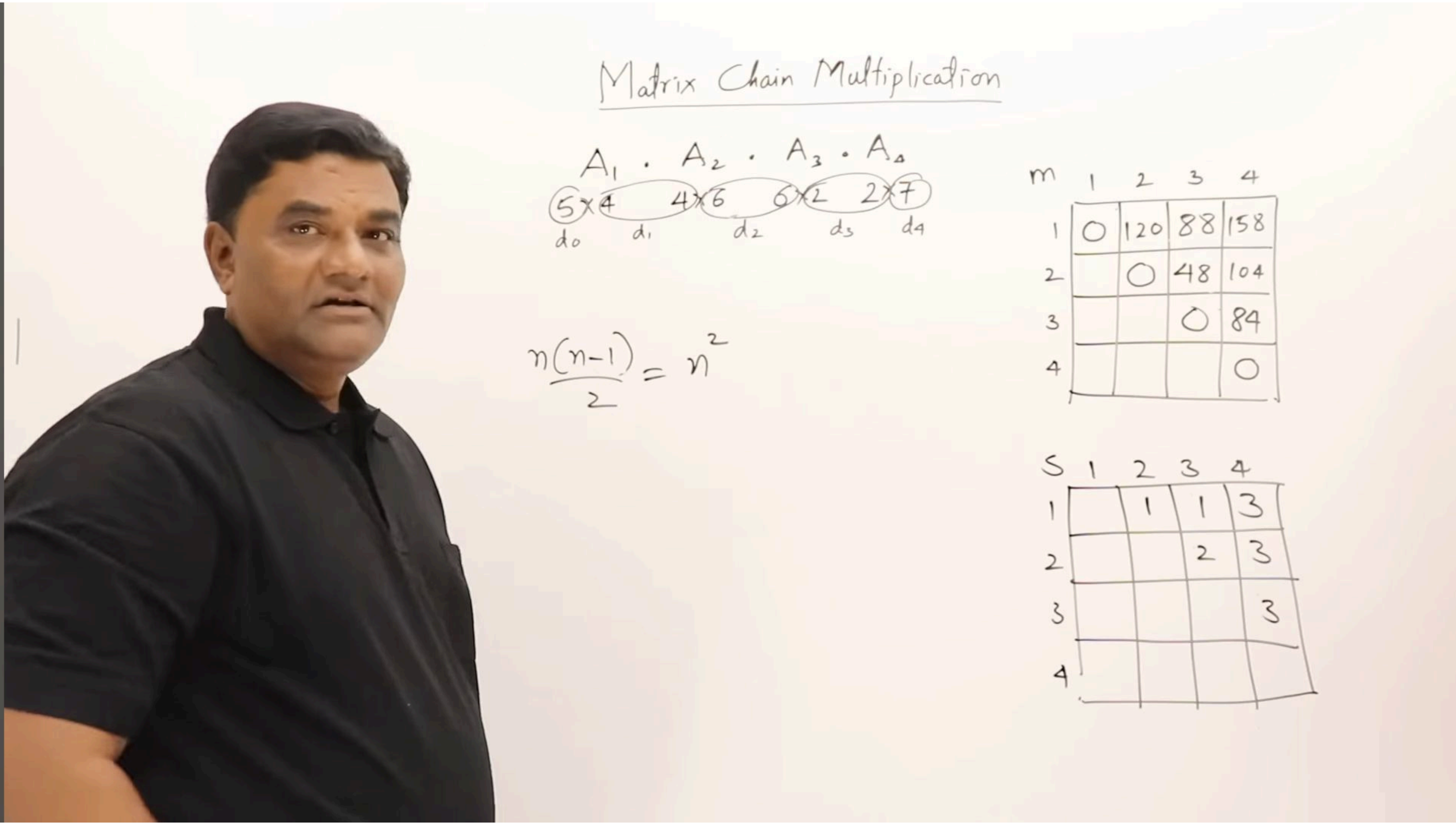
s	1	2	3	4
1		1	1	
2			2	3
3				3
4				


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[1,1]+m[2,4]+5×4×7 , m[1,2]+m[3,4]+5×6×7
0 104 +140 120 + 84 + 210
m[1,3]+m[4,4]+5×2×7
88 + 0 + 70
m 1 2 3 4
1 0 120 88
2 0 48 104
3 0 84
4 0
s 1 2 3 4
1 1 1
2 2 3
3 3
4

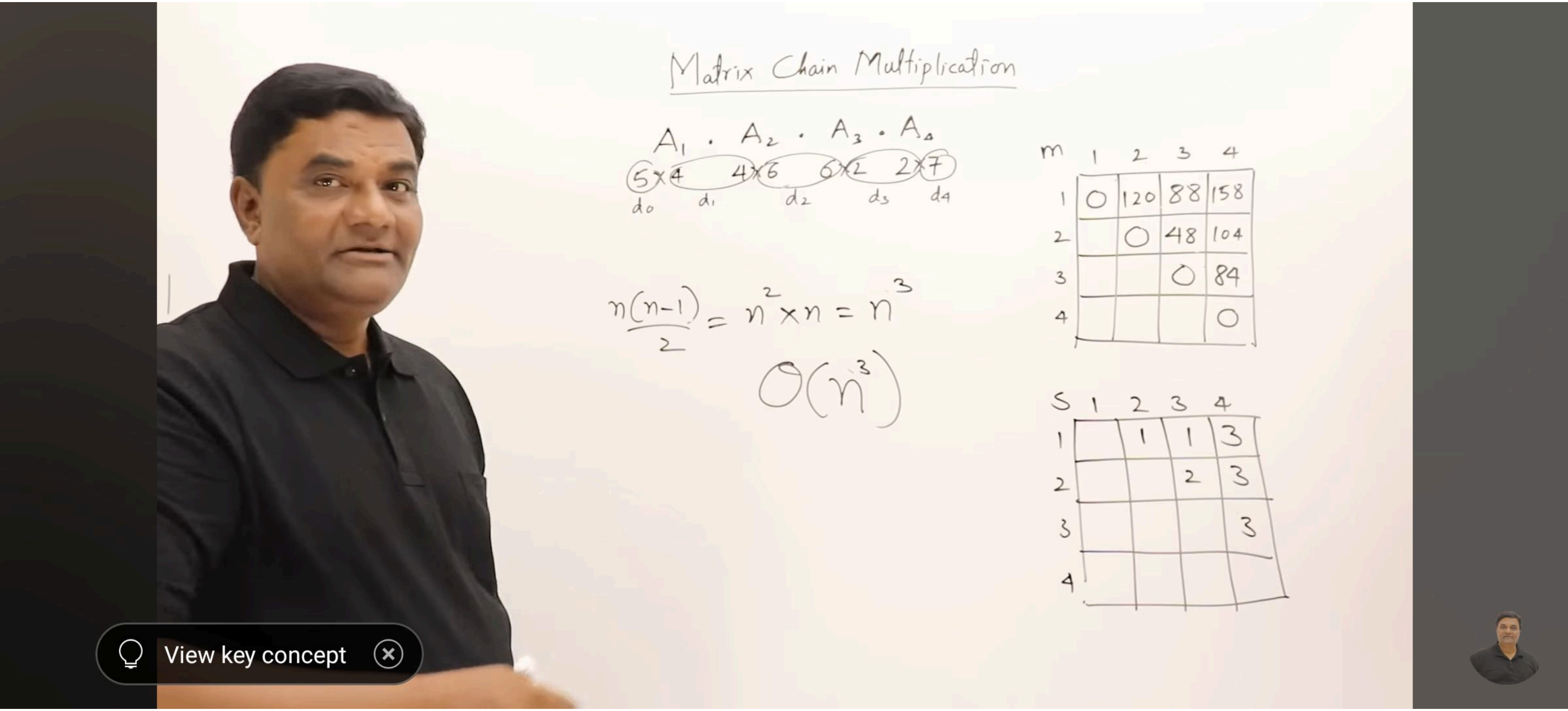

Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
m[1,4] = min { m[1,1] + m[2,4] + 5×4×7 , m[1,2] + m[3,4] + 5×6×7
0 104 +140 120 + 84 + 210
m[1,3] + m[4,4] + 5×2×7 }
88 + 0 + 70
m 1 2 3 4
1 0 120 88 158
2 0 48 104
3 0 84
4 0
s 1 2 3 4
1 1 1 3
2 2 3
3 3
4

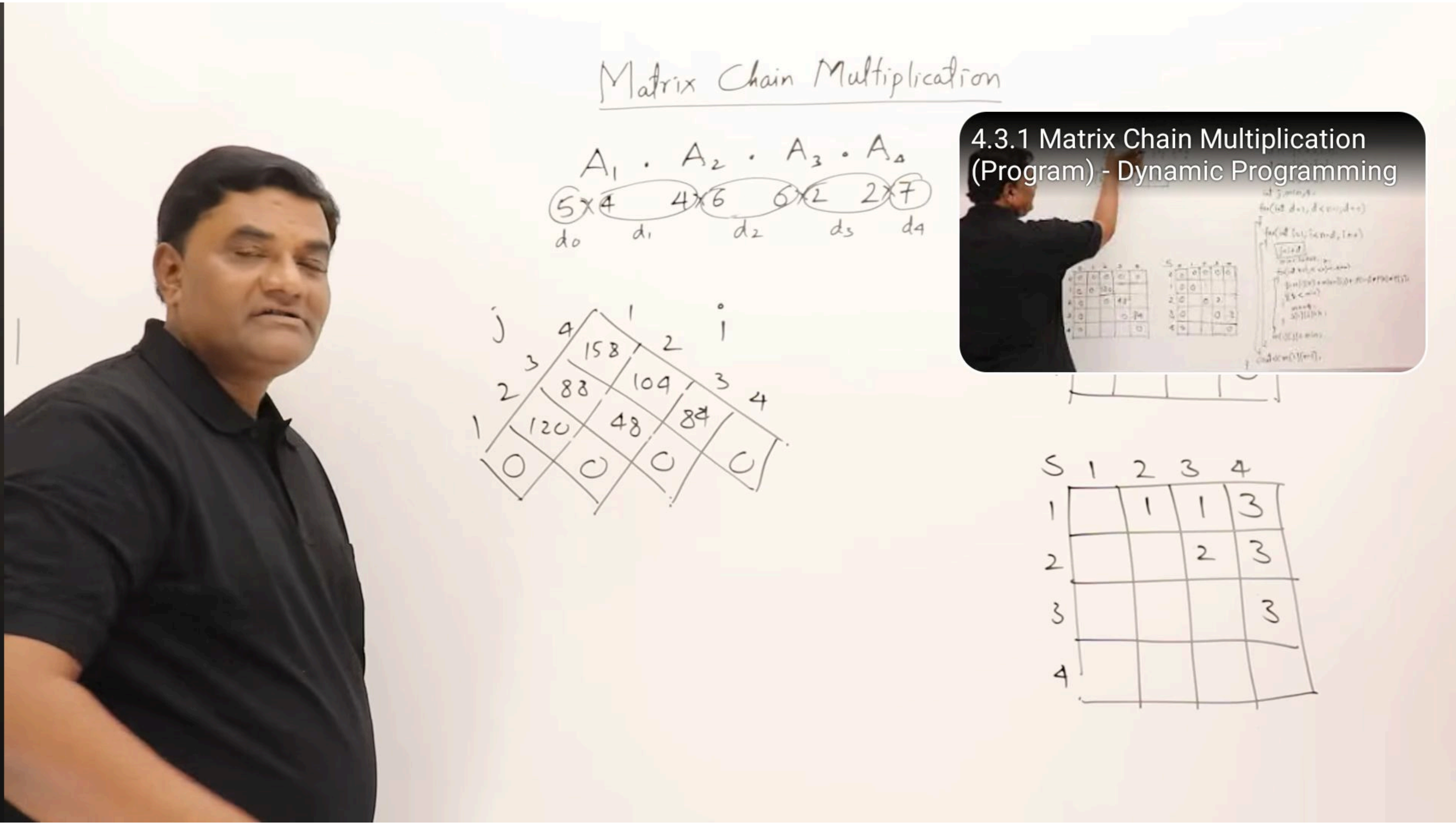

Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
d0 d1 d2 d3 d4
m[1,4] = min { m[1,1] + m[2,4] + 5×4×7 , m[1,2] + m[3,4] + 5×6×7
0 104 +140 120 + 84 + 210
m[1,3] + m[4,4] + 5×2×7 }
88 + 0 + 70
m[i, j] = min { m[i, k] + m[k+1, j] + d_{i-1} * d_k * d_j }
m 1 2 3 4
1 0 120 88 158
2 0 48 104
3 0 84
4 0
S 1 2 3 4
1 1 1 3
2 2 3
3 3
4


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
d0 d1 d2 d3 d4
m 1 2 3 4
1 0 120 88 158
2 0 48 104
3 0 84
4 0
(A1 · A2 · A3) · (A4)
((A1) · (A2 · A3)) · (A4)
A1 A2 A3 A4
S 1 2 3 4
1 1 1 3
2 2 3
3 3
4



Matrix Chain Multiplication
A1 . A2 . A3 . A4
5×4 4×6 6×2 2×7
d0 d1 d2 d3 d4
n(n-1)/2 = n^2
m 1 2 3 4
1 0 120 88 158
2 0 48 104
3 0 84
4 0
S 1 2 3 4
1 1 1 3
2 2 3
3 3
4


Matrix Chain Multiplication
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
d0 d1 d2 d3 d4
n(n-1)/2 = n² × n = n³
O(n³)
m 1 2 3 4
1 0 120 88 158
2 0 48 104
3 0 84
4 0
S 1 2 3 4
1 1 1 3
2 2 3
3 3
4
View key concept


Matrix Chain Multiplication
4.3.1 Matrix Chain Multiplication (Program) - Dynamic Programming
A1 · A2 · A3 · A4
5×4 4×6 6×2 2×7
d0 d1 d2 d3 d4
j
i
158
88 104
120 48 84
0 0 0 0
S 1 2 3 4
1 1 1 3
2 2 3
3 3
4

V

@saturns01 • 4 yr ago (edited)

For those who are still confused with the table S:

It is filled with the min. value of K (K is introduced at 17:50).

Example:

for M[2,4] we have selected -> (A2*A3)*A4 which gives

-> M[2,3]+M[4,4]+4*2*7

compare this with :

-> M[i][j] = min { M[i,K]+ M[K+1,j] + ... }

we get K=3

Put the minimum value of K into the table S