

Cyclic codes (Basics & Properties)

Cyclic Codes basics & Properties with example

- Cyclic Codes are subset of linear block codes
- It follows following properties.

① Linearity Property

- If we have two code words c_i & c_j then

$$c_p = c_i + c_j$$

where, c_p should be a code word.

② Cyclic shifting

$$\text{code word} = (c_1, c_2, c_3, \dots, c_n)$$

- After shifting left or right by any no of bits,

Properties of Cyclic codes.

① Linearity property

- If we have two code words c_i & c_j then

$$c_p = c_i + c_j$$

where, c_p should be a code word.

② Cyclic shifting

$$\text{code word} = (c_1, c_2, c_3, \dots, c_n)$$

- After shifting left or right by any no of bits, resultant code should be a code word.
- If code words follow above two property then only that codes will be cyclic codes.

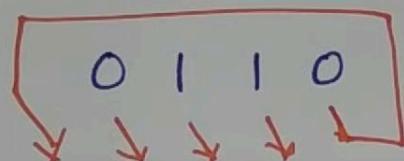
e.g. $\{0000, \underline{0110}, 1001, 1111\}$, is it cyclic codes?

→ Check Property of linearity

$$\begin{array}{r} 0110 \\ 1001 \\ \hline \checkmark 1111 \end{array} \quad \begin{array}{r} 0110 \\ 1111 \\ \hline \checkmark 1001 \end{array} \quad \begin{array}{r} 1001 \\ 1111 \\ \hline \checkmark 0110 \end{array}$$

— So it follows property of linearity

→ Check Property of shifting



0 0 1 1 ← It is not a code word.

→ So above codes are not cyclic codes.

* $\{0000, 0101, 1010, 1111\}$, is it a code?

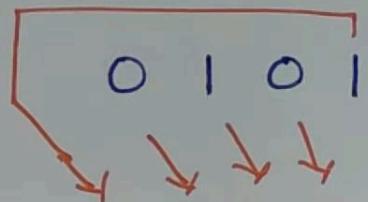
→ Check property of linearity

↓ GCD

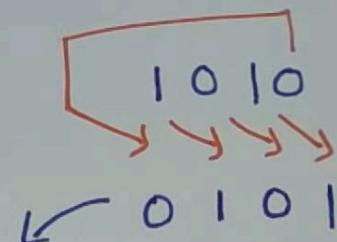
$$\begin{array}{r} 0101 \\ 1010 \\ \hline \checkmark 1111 \end{array} \quad \begin{array}{r} 0101 \\ 1111 \\ \hline \checkmark 1010 \end{array} \quad \begin{array}{r} 1010 \\ 1111 \\ \hline \checkmark 0101 \end{array}$$

- It follows property of linearity.

→ Check shifting propertj.



1010 & It is code word.



→ It follows shifting propertj.

Generator Polynomial (Non-systematic codes)

Cyclic Codes for Non-Systematic Codeword

* Code word for non-systematic codeword is given by

$$c(x) = m(x) \cdot g(x)$$

where, $c(x)$ = code word polynomial

$m(x)$ = message polynomial

$g(x)$ = generator polynomial

Cyclic Codes for Non-Systematic Codeword

- * Code word for non-systematic codeword is given by

$$c(x) = m(x) \cdot g(x)$$

where, $c(x)$ = code word polynomial

$m(x)$ = Message polynomial

$g(x)$ = Generator polynomial

$$\rightarrow \boxed{C = [\begin{matrix} \text{message}, & \text{Parity} \end{matrix}]} \rightarrow \begin{matrix} | 0 & 1 & 1 \\ x^3 & x^2 & x^1 & x^0 \\ = x^3 \cdot 1 + x^2 \cdot 0 + x^1 \cdot 1 + x^0 \cdot 1 \\ = x^3 + x + 1 \end{matrix}$$

* Construct Non-systematic Cyclic codes (7, 4), using generator Polynomial $g(x) = x^3 + x^2 + 1$, with message $(1 \ 0 \ 1 \ 0)$.

$$\rightarrow m = [1 \ 0 \ 1 \ 0]$$

$$m \begin{matrix} \downarrow \\ x^3 \end{matrix} \begin{matrix} \downarrow \\ x^2 \end{matrix} \begin{matrix} \downarrow \\ x^1 \end{matrix} \begin{matrix} \downarrow \\ x^0 \end{matrix}$$

$$\rightarrow m(x) = x^3 + x$$

$$g(x) = x^3 + x^2 + 1$$

$$\begin{aligned} \rightarrow c(x) &= m(x) g(x) \\ &= (x^3 + x)(x^3 + x^2 + 1) \end{aligned}$$

$$\rightarrow m = [1 \ 0 \ 1 \ 0]$$

m ↓ ↓ ↓ ↓
 x^3 x^2 x^1 x^0

$$\rightarrow m(x) = x^3 + x$$

$$g(x) = x^3 + x^2 + 1$$

$$\rightarrow c(x) = m(x) g(x)$$

$$\begin{aligned} &= (x^3 + x)(x^3 + x^2 + 1) \\ &= x^6 + x^5 + \cancel{x^3} + x^4 + \cancel{x^2} + x \\ &= x^6 + x^5 + x^4 + x \end{aligned}$$

$$\boxed{c = [1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0]}$$

While XOR ing ->
Same term becomes 0

Generator Polynomial (Systematic Codes)

Cyclic-Codes for Systematic Codeword

- Codeword for systematic codeword is given by

$$c(x) = x^{n-k} m(x) + p(x)$$

where, $p(x) \rightarrow \text{Rem} \left[\frac{x^{n-k} m(x)}{g(x)} \right]$

, $c(x)$ = Codeword polynomial

$m(x)$ = message polynomial

$g(x)$ = Generator polynomial

$$C = [\xrightarrow{\text{message}}, \xrightarrow{\text{Parity}}]$$

* Construct a cyclic code (7,4), using generator systematic

Polynomial $g(x) = x^3 + x^2 + 1$, with message
(1 0 1 0).

→ (7,4) code

$n=7$, $K=4$

→ $m = (1 0 1 0)$

$$\begin{matrix} & \downarrow & \downarrow & \downarrow & \downarrow \\ x^3 & x^2 & x^1 & x^0 \end{matrix}$$

$$m(x) = x^3 + x$$

* Construct a cyclic code (7,4), using generator systematic

Polynomial $g(x) = x^3 + x^2 + 1$, with message
 $(1 \ 0 \ 1 \ 0)$.

$\rightarrow (7,4)$ code

$$n=7, k=4$$

$\rightarrow m = (1 \ 0 \ 1 \ 0)$

$$m(x) = x^3 + x$$

\downarrow \downarrow \downarrow \downarrow
 x^3 x^2 x^1 x^0

$$\rightarrow c(x) = x^{n-k} m(x) + p(x)$$

$$p(x) = \text{Rem} \left[\frac{x^{n-k} m(x)}{g(x)} \right]$$

$$= \text{Rem} \left[\frac{x^3(x^3+x)}{x^3+x^2+1} \right]$$

$$= \text{Rem} \left[\frac{x^6+x^4}{x^3+x^2+1} \right]$$

$$n=7, k=4$$

$$\rightarrow m = (1 \ 0 \ 1 \ 0)$$

$$\begin{array}{cccc} & \downarrow & \downarrow & \downarrow \\ x^3 & x^2 & x^1 & x^0 \end{array}$$

$$m(x) = x^3 + x$$

$$\text{PGO} = \text{Rem} \left[\frac{x^m}{g(x)} \right]$$

$$= \text{Rem} \left[\frac{x^3(x^3+x)}{x^3+x^2+1} \right]$$

$$= \text{Rem} \left[\frac{x^6+x^4}{x^3+x^2+1} \right] = 1$$

$$\begin{array}{r} x^3+x^2+1 \quad | \quad \overline{x^6+x^4} \\ \underline{x^6+x^5+x^3} \\ \underline{x^8+x^6+x^2} \\ \underline{x^8+x^7+x^5} \\ \underline{x^3+x^2} \\ \underline{x^3+x^2+1} \\ \rightarrow \boxed{1} \end{array}$$

$\rightarrow (7, 4)$ code

$$n = 7, k = 4$$

$\rightarrow m = [1 \ 0 \ 1 \ 0]$

$$\begin{matrix} & \downarrow & \downarrow & \downarrow & \downarrow \\ x^3 & x^2 & x^1 & x^0 \end{matrix}$$

$$m(x) = x^3 + x$$

$$\rightarrow c(x) = x^{n-k} m(x) + p(x)$$

$$= x^3(x^3 + x) + 1$$

$$= x^6 + x^4 + 1$$

$$\rightarrow c = [1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1]$$

$$c(x) = x^n m(x) + p(x)$$

$$p(x) = \text{Rem} \left[\frac{x^{n-k} m(x)}{g(x)} \right]$$

$$= \text{Rem} \left[\frac{x^3(x^3 + x)}{x^3 + x^2 + 1} \right]$$

$$= \text{Rem} \left[\frac{x^6 + x^4}{x^3 + x^2 + 1} \right] = 1$$

$$\begin{array}{r} x^3 + x^2 + 1 \\ \hline x^6 + x^4 \\ \underline{x^6 + x^5 + x^3} \\ x^8 + x^5 + x^3 \end{array}$$

$$\begin{array}{r} x^8 + x^4 + x^3 \\ \hline x^8 + x^4 + x^2 \end{array}$$

$$\begin{array}{r} x^3 + x^2 \\ \hline x^3 + x^2 + 1 \end{array}$$

$L(g(x))$

, $c(x) = \text{Code word polynomial}$

$m(x) = \text{message Polynomial}$

$g(x) = \text{Generator Polynomial}$

$$C = [\underbrace{\text{message}}_{m(x)}, \underbrace{\text{Parity}}_{L(g(x))}]$$

* Construct a cyclic code $(7,4)$, using generator

sys
Polynomial with message

$(1 \ 0 \ 1)$

$\rightarrow (7$

$) + p(x)$

$n-k m(x)]$

$$\rightarrow m = \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}$$

$x^3 \quad x^2 \quad x^1 \quad x^0$

$$m(x) = x^3 + x$$

$$\begin{aligned}\rightarrow C(x) &= x^{n-k} m(x) + p(x) \\ &= x^3 (x^3 + x) + 1 \\ &= x^6 + x^4 + 1\end{aligned}$$

$$\rightarrow C = \left[\begin{array}{|ccc|c|} \hline 1 & 0 & 1 & 0 \\ \hline \pi & & & 1 \\ \hline \text{Message} & & \text{Parity} & \\ \hline \end{array} \right]$$

$$\begin{aligned}&= \text{Rem} \left[\frac{x^3(x^3+x)}{x^3+x^2+1} \right] \\ &= \text{Rem} \left[\frac{x^6+x^4}{x^3+x^2+1} \right] = \\ &\quad \overline{x^3+x^2+1} \quad \overline{x^6+x^4} \\ &\quad \overline{x^8+x^5+x^3} \\ &\quad \overline{x^8+x^4+x^2} \\ &\quad \overline{x^3+x^2} \\ &\quad \overline{x^3+x^2} \\ &\rightarrow \boxed{1}\end{aligned}$$



Cyclic code for Generator Matrix

Generator matrix of cyclic code.

- Generator matrix

$$[G] = [I : P]$$

$\nearrow \quad \nwarrow$

Identity Matrix Parity Matrix

- If cyclic code is (n, k) , then generator matrix G is having n columns & k rows.
- Identity matrix is known to us.
- So Row of parity matrix based on generator

- Identity matrix is known to us.
- So Row of parity matrix based on generator Polynomial $g(x)$ is given by.

$$1^{\text{st}} \text{ row : } \text{Rem} \left[\frac{x^{n-1}}{g(x)} \right]$$

$$2^{\text{nd}} \text{ row : } \text{Rem} \left[\frac{x^{n-2}}{g(x)} \right]$$

$$\vdots$$

$$k^{\text{th}} \text{ row : } \text{Rem} \left[\frac{x^{n-k}}{g(x)} \right]$$

* If generator polynomial of cyclic code $\overset{(7,4)}{\text{is}}$ given by
 $g(x) = x^3 + x + 1$.

then construct generator matrix.

→ (7, 4) code

$$\boxed{7=n}, \boxed{K=4}$$

$$\rightarrow G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - & - \\ 0 & 0 & 0 & 1 & - & - & - \end{array} \right]$$

- 1st Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-1}}{g(x)} \right], \text{Rem} \left[\frac{x^6}{x^3+x+1} \right]$$

- 2nd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-2}}{g(x)} \right] = \text{Rem} \left[\frac{x^5}{x^3+x+1} \right]$$

- 3rd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right]$$

- 4th Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-4}}{g(x)} \right] = \text{Rem} \left[\frac{x^3}{x^3+x+1} \right]$$

- 3rd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right] =$$

- 4th Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-4}}{g(x)} \right] = \text{Rem} \left[\frac{x^3}{x^3+x+1} \right] =$$

$$\begin{array}{r} x^3+x+1 \\ \hline x^3+x+1 \quad | \quad x^6 \\ \quad x^6+x^4+x^3 \\ \hline \quad x^4+x^3 \\ \quad x^4+x^2+x \\ \hline \quad x^3+x^2+x \\ \quad x^3+x+1 \\ \hline \quad x^2+1 \end{array}$$

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & - & - & - \\ 0 & 1 & 0 & 0 & - & - & - \\ 0 & 0 & 1 & 0 & - & - & - \\ 0 & 0 & 0 & 1 & - & - & - \end{array} \right]$$

- 1st Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-1}}{g(x)} \right], \text{Rem} \left[\frac{x^6}{x^3+x+1} \right] = x^2 + 1$$

- 2nd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-2}}{g(x)} \right] = \text{Rem} \left[\frac{x^5}{x^3+x+1} \right] =$$

- 3rd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right] =$$

$$= \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right] =$$

- 4th Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-4}}{g(x)} \right] = \text{Rem} \left[\frac{x^3}{x^3+x+1} \right] =$$

$$\begin{array}{r} x^3+x+1 \\ \overline{x^6} \\ x^6+x^4+x^3 \\ \hline x^4+x^3 \\ \overline{x^4+x^2+x} \\ x^3+x^2+x \\ \hline x^2+x+1 \\ \hline x^2+1 \end{array}$$

$$\begin{array}{r} x^3+x+1 \\ \overline{x^5} \\ x^5+x^3+x^2 \\ \hline x^3+x^2 \\ \overline{x^3+x+1} \\ x^2+x+1 \\ \hline x^2+1 \end{array}$$

- 2nd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-2}}{g(x)} \right] = \text{Rem} \left[\frac{x^5}{x^3+x+1} \right] = x^2+x+1$$

- 3rd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right] = x^2+1$$

- 4th Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-4}}{g(x)} \right] = \text{Rem} \left[\frac{x^3}{x^3+x+1} \right] = x+1$$

$$\begin{array}{r} x^3+x+1 \\ +x+1 \end{array} \overline{\overline{x^6}} \\ \underline{x^6+x^4+x^3} \\ x^4+x^3$$

$$\begin{array}{r} x^2+1 \\ x^3+x+1 \end{array} \overline{\overline{x^5}} \\ \underline{x^5+x^3+x^2} \\ x^2+x^2$$

$$\begin{array}{r} x \\ x^3+x+1 \end{array} \overline{\overline{x^4}} \\ \underline{x^4+x^2+1} \\ x^2+1$$

$$= \text{Rem} \left[\frac{x^1}{g(x)} \right], \text{Rem} \left[\frac{x^2}{x^3+x+1} \right] = x^2 + 1 = [101]$$

- 2nd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-2}}{g(x)} \right] = \text{Rem} \left[\frac{x^5}{x^3+x+1} \right] = x^2 + x + 1 = [111]$$

- 3rd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-3}}{g(x)} \right] = \text{Rem} \left[\frac{x^4}{x^3+x+1} \right] = x^2 + 1 = [101]$$

- 4th Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-4}}{g(x)} \right] = \text{Rem} \left[\frac{x^3}{x^3+x+1} \right] = x+1 = [011]$$

$$x^3 + x + 1 \overline{)x^6 + x^4 + x^3}$$

$$x^3 + x + 1 \overline{)x^5 + x^3 + x^2}$$

$$x^3 + x + 1 \overline{)x^4 + 2}$$



→ (7, 4) code

$$\boxed{7 = n}, \boxed{K = 4}$$

$$\rightarrow G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array} \right]$$

- 1st Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-1}}{g(x)} \right], \text{Rem} \left[\frac{x^6}{x^3+x+1} \right] = x^2+1 = [101]$$

- 2nd Row of Parity Matrix

$$= \text{Rem} \left[\frac{x^{7-2}}{g(x)} \right] = \text{Rem} \left[\frac{x^5}{x^3+x+1} \right] = x^2+x+1 = [111]$$

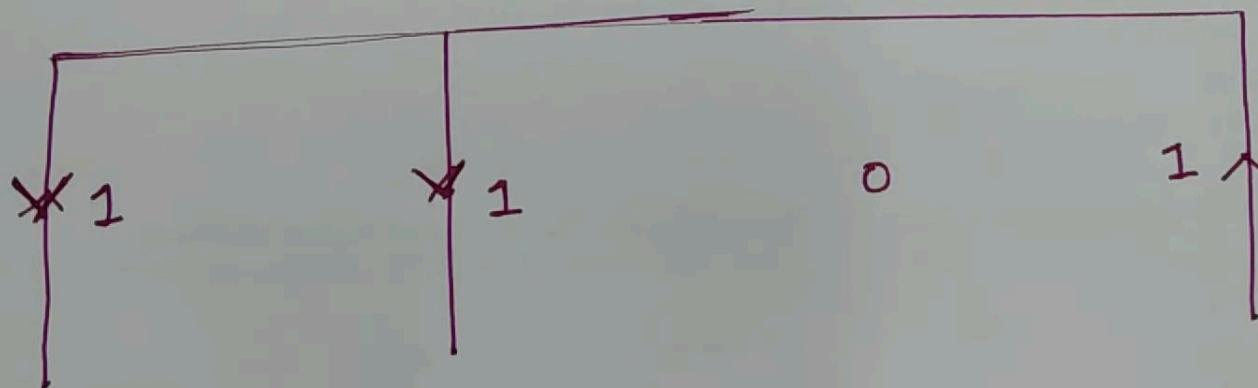
Encoder designing for cyclic codes

Cyclic Encoder design

- If generator Polynomial is $g(x) = 1 + x + x^3$
calculate
① Cyclic encoder
② codeword if message is (1110)

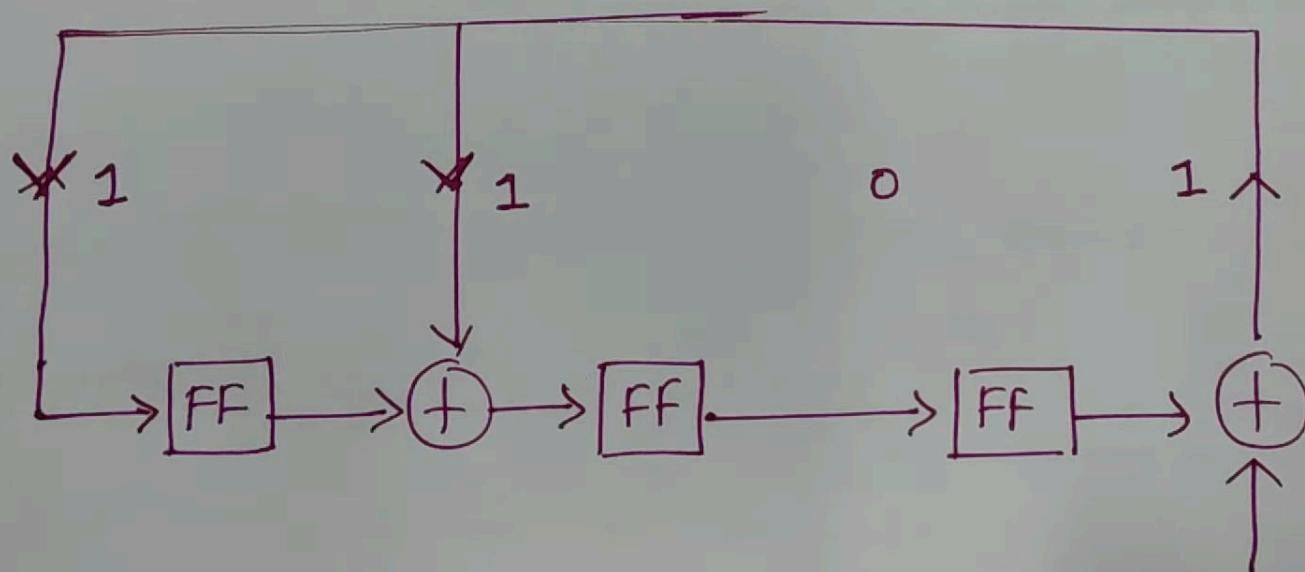
② codeword If message is (1110)

$$\begin{aligned}g(x) &= 1 + x + x^3 \\&= \underline{1} + \underline{x} + \underline{0}x^2 + \underline{x}^3\end{aligned}$$



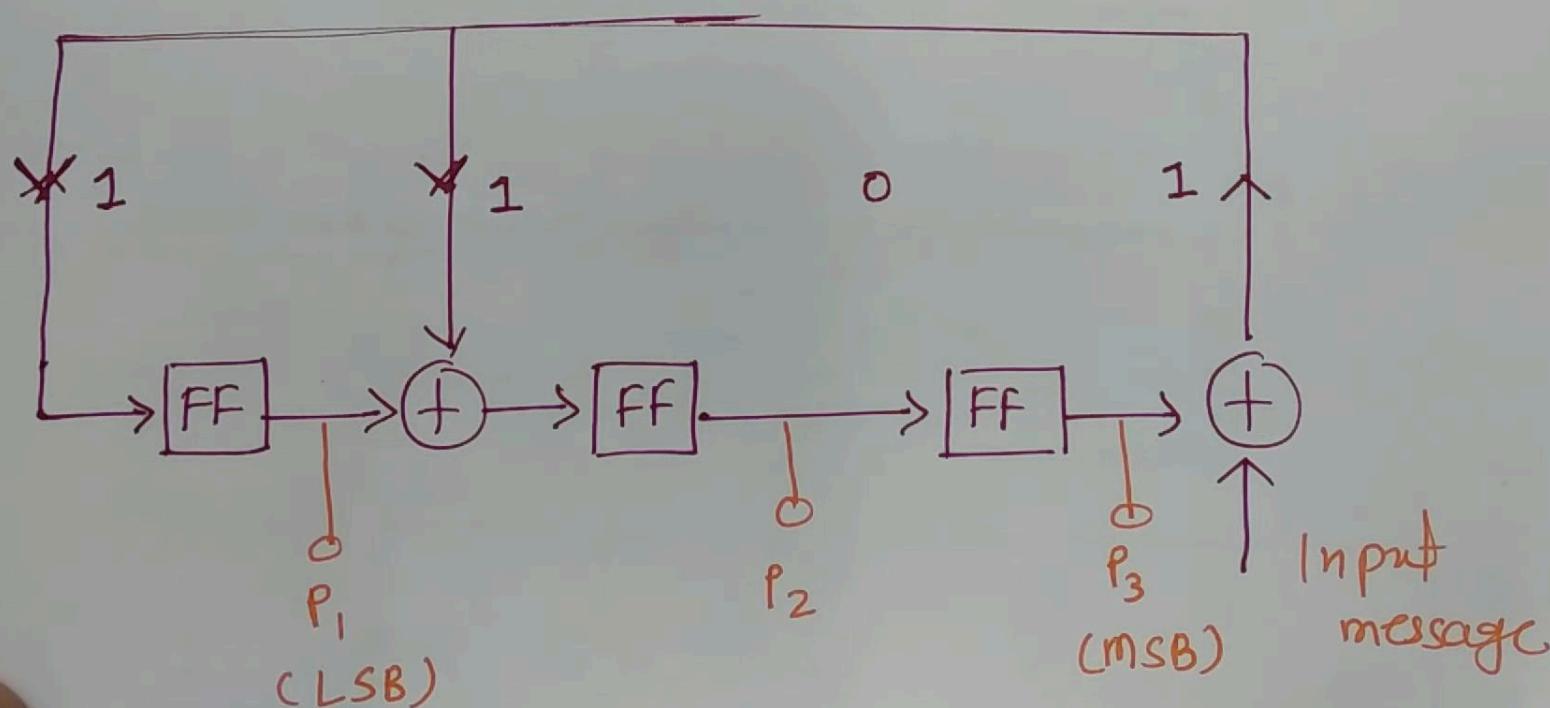
② Codeword if message is (1110)

$$\begin{aligned}g(x) &= 1 + x + x^3 \\&= \underline{1} + \underline{x} + \underline{0}x^2 + \underline{x}^3\end{aligned}$$



Q) What would be message if message is (1110)

$$\begin{aligned}g(x) &= 1 + x + x^3 \\&= \underline{1} + \underline{x} + \underline{0}x^2 + \underline{x^3}\end{aligned}$$



P_1
(LSB)

P_2

P_3
(MSB)

Input
message

| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| | 0 | 0 | 0 |

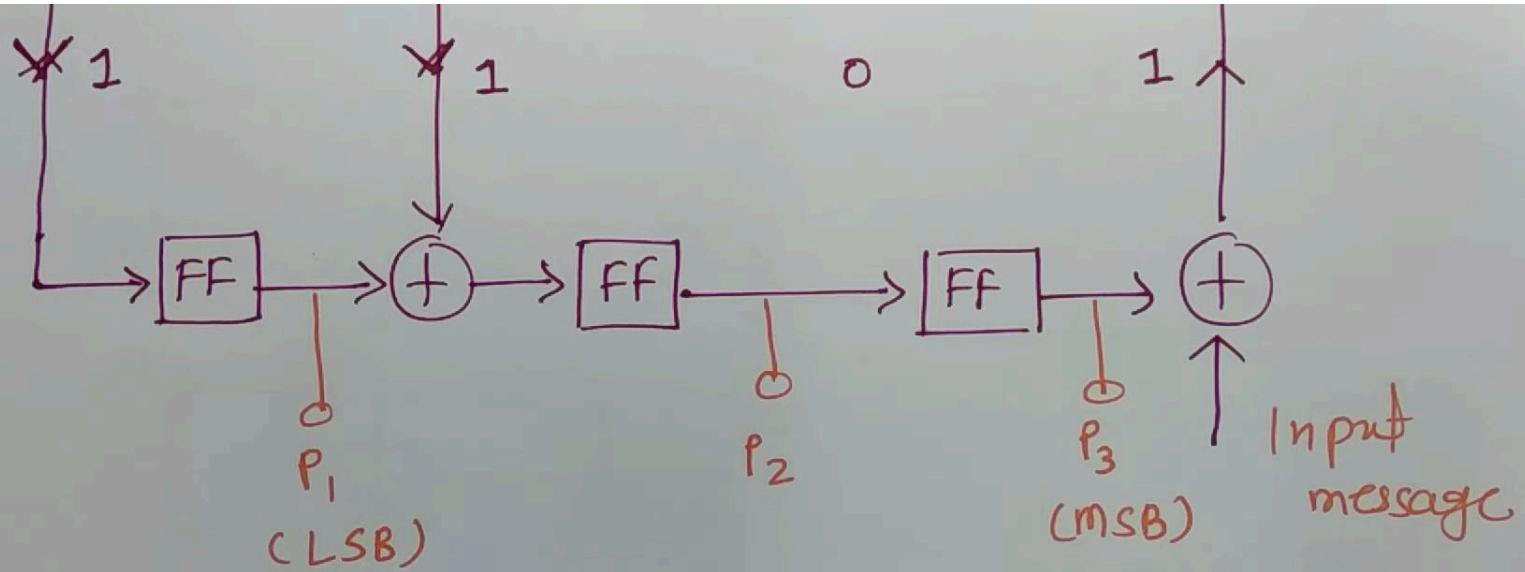
P_1
(LSB)

P_2

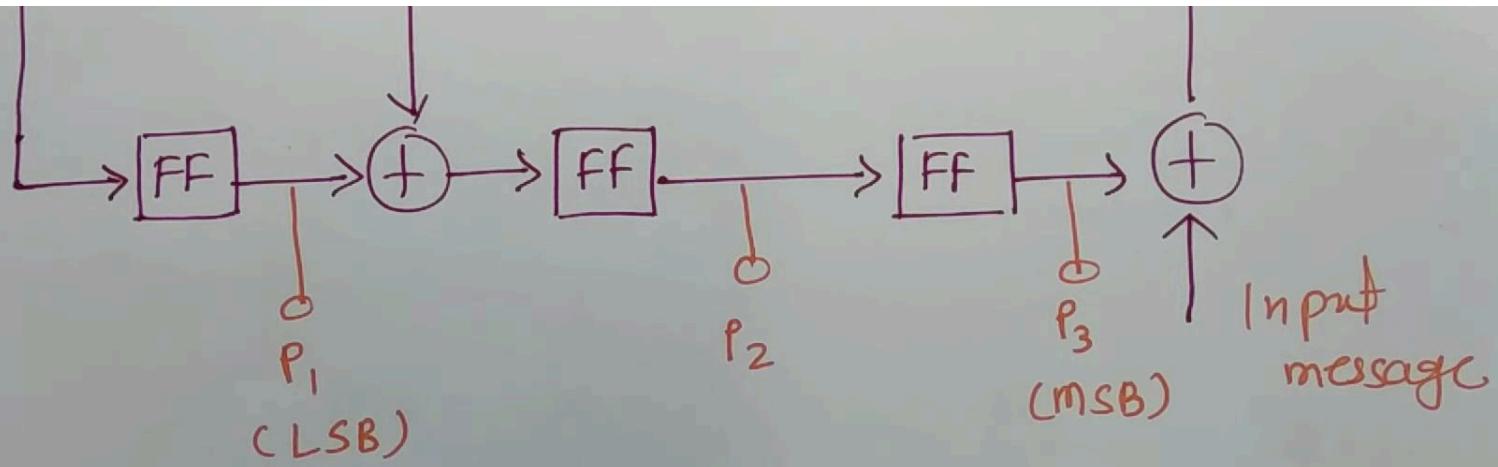
P_3
(MSB)

Input
message

| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 0 | 0 | 0 |
| 1 | | | |
| 1 | | | |
| 0 | | | |



| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |



| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |

P_1
(LSB)

P_2

P_3
(MSB)

Input
message

| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | | 1 | 0 |

P_1
(LSB)

P_2

P_3
(MSB)

T

Input
message

| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |

| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| | | | 1 |

↓ ↑

LSB. MSB

| m | P_1 | P_2 | P_3 |
|-----|-------|-------|-------|
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

↓ ↑
LSB. MSB

$$\begin{aligned}
 \text{Codeword} &= [\text{Message}, \text{Parity}] \\
 &= [1110, 100]
 \end{aligned}$$

Syndrome Calculation Circuit

SYNDROME CALCULATOR FOR CYCLIC CODE

Ques:- Design a Syndrome calculator for a (7,4) Cyclic Hamming code generated by the polynomial $G(P) = P^3 + P + 1$. Calculate Syndrome for $y = (1001101)$

 View key concept 



generated by "the polynomial"
Syndrome for $y = (1001101)$

$$g(p) = p^{n-k} + \sum_{i=1}^{n-k-1} g_i p^i + 1$$

$$(n, k) = (7, 4) \quad \text{Number of FF} = h - k = 3$$

$$\underline{n=7} \quad \underline{k=4}$$

$$g(p) = p^{7-4} + \sum_{i=1}^{7-4-1} g_i p^i + 1$$
$$= p^3 + \sum_{i=1}^2 g_i p^i + 1$$

$$g(p) = p^3 + g_1 p^1 + g_2 p^2 + 1$$

$$g(p) = p^3 + g_2 p^2 + g_1 p + 1$$

Given $G(p) = p^3 + p + 1$

$$g_2 = 0 \quad g_1 = 1$$



generated by the polynomial
Syndrome for $Y = (1001101)$

$$g(p) = p^{n-k} + \sum_{i=1}^{n-k-1} g_i p^i + 1$$

$$(n, k) = (7, 4) \quad \text{Number of FF} = n-k = 3$$

$$\underline{n=7} \quad \underline{k=4}$$

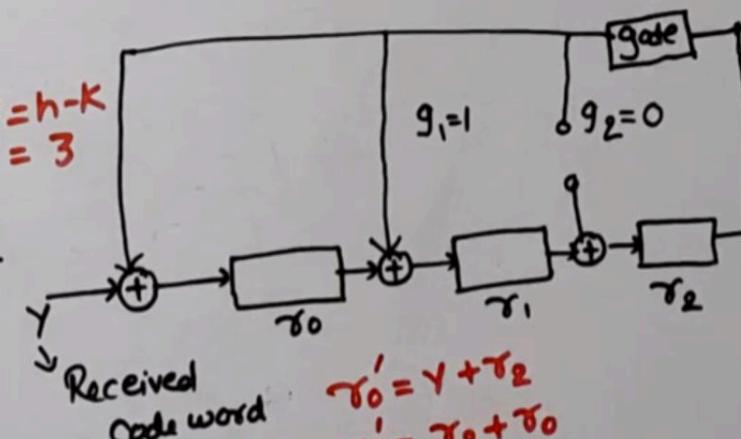
$$g(p) = p^{7-4} + \sum_{i=1}^{7-4-1} g_i p^i + 1$$

$$= p^3 + \sum_{i=1}^2 g_i p^i + 1$$

$$g(p) = p^3 + g_1 p^1 + g_2 p^2 + 1$$

$$g(p) = p^3 + g_2 p^2 + g_1 p + 1$$

Given $G(p) = p^3 + p + 1$
 $g_2 = 0 \quad g_1 = 1$



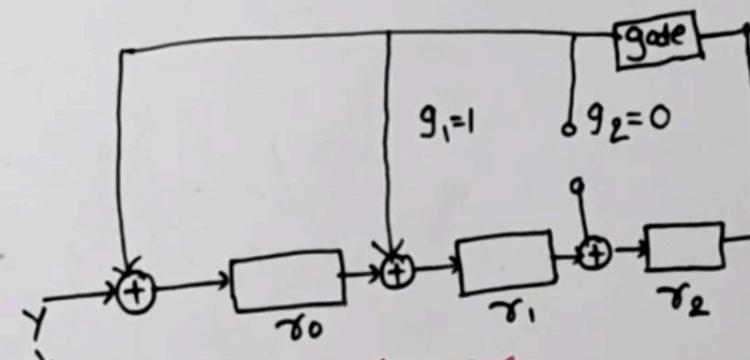
$$\begin{aligned} r'_0 &= Y + r_2 \\ r'_1 &= r_2 + r_0 \\ r'_2 &= r_1 \end{aligned}$$



generated by the matrix
Syndrome for $y = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

| No. of Shift | y | τ_0' | τ_1' | τ_2' |
|--------------|-----|---------------------|--------------------------|-----------|
| | | $(y \oplus \tau_2)$ | $(\tau_2 \oplus \tau_0)$ | τ_1 |
| 0 | 0 | 0 | 0 | 0 |

| | |
|---|---|
| 1 | 1 |
| 2 | 0 |
| 3 | 0 |
| 4 | 1 |
| 5 | 1 |
| 6 | 0 |
| 7 | 1 |



Received
Code word

$$\tau_0' = y \oplus \tau_2$$

$$\tau_1' = \tau_2 \oplus \tau_0$$

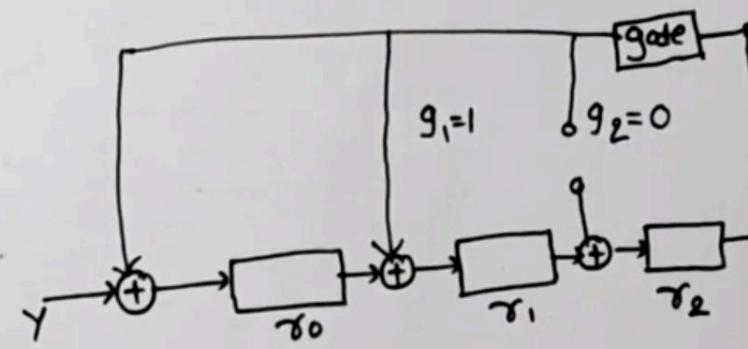
$$\tau_2' = \tau_1$$

$$S = [\tau_2' \ \tau_1' \ \tau_0']$$



generated by the matrix
Syndrome for $y = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

| No. of Shift | y | τ_0' | τ_1' | τ_2' |
|--------------|-------------------|---------------------|--------------------------|-----------|
| | | $(y \oplus \tau_2)$ | $(\tau_2 \oplus \tau_0)$ | τ_1 |
| 1 | $1 \rightarrow 1$ | 0 | 0 | 0 |
| 2 | $0 \rightarrow 0$ | 1 | 0 | 0 |
| 3 | $0 \rightarrow 0$ | 0 | 1 | 0 |
| 4 | $1 \rightarrow 0$ | 1 | 0 | 0 |
| 5 | $1 \rightarrow 1$ | 0 | 1 | 0 |
| 6 | $0 \rightarrow 1$ | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |



Received
Code word

$$\tau_0' = y \oplus \tau_2$$

$$\tau_1' = \tau_2 \oplus \tau_0$$

$$\tau_2' = \tau_1$$

$$S = [\tau_2' \ \tau_1' \ \tau_0']$$



generated by the polynomial
Syndrome for $y = (1001101)$

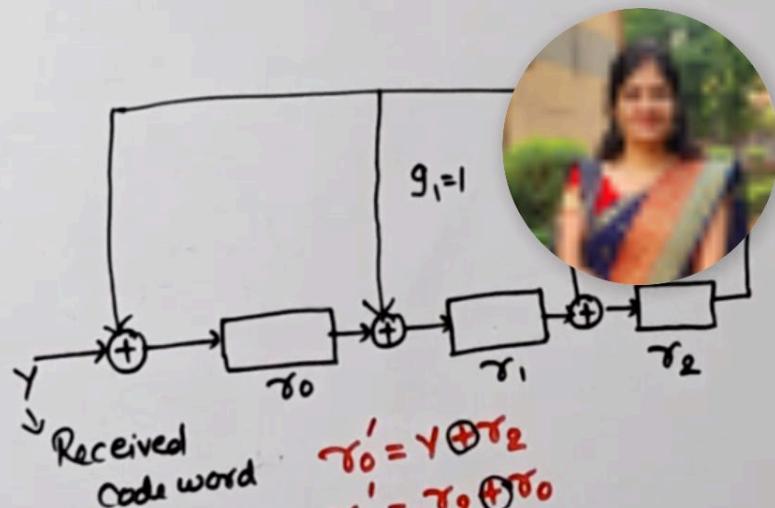
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(a) τ_0'
0
0
0
1
0
0
 τ_1'
0
0
1
0
0
0
 τ_2'



Cyclic Redundancy Test

❖ Basics of Cyclic Redundancy Check

- ❑ CRC uses a Divisor with L bits. {Divisor may be given in Galois fields.}
- ❑ Here we append L – 1 bits with data bits.
- ❑ Here we perform binary division for CRC generation.
- ❑ Receiver and Transmitter should be agreed upon by the same Divisor.

❖ Example of Cyclic Redundancy Check

- ❑ Data = 01010001, Divisor = 1001. Find transmitted codeword using CRC.

$$\begin{array}{r} 01010001 \text{ } 000 \\ 1001 \mid \\ \hline 001100 \\ 1001 \mid \\ \hline 01011 \\ 1001 \mid \\ \hline 001000 \\ 1001 \mid \\ \hline 00010 \end{array}$$

Tx Codeword = 01010001 010
data crc

← crc



❖ Example of Cyclic Redundancy Check

❖ Example of Cyclic Redundancy Check

- Data = 1110010101, Divisor = $X^3 + X^2 + 1$. Find transmitted codeword using CRC.

= 1101

$$\begin{array}{r} 1110010101 \text{ | } 000 \\ 1101 \\ \hline 001101 \\ 1101 \\ \hline 000001010 \\ 1101 \\ \hline 01110 \\ 1101 \\ \hline 00 \boxed{110} \leftarrow \text{CRC} \end{array}$$

$$\text{Tx Codeword} = \frac{\text{1110010101}}{\text{data}} \frac{\text{110}}{\text{CRC}}$$



Examples on CRC – Cyclic Redundancy Check

Example 1 – The message 11001001 is to be transmitted using the CRC polynomial $X^3 + 1$ to protect it from errors. The message that should be transmitted is If the received codeword at the receiver is 11011001011 then how the receiver will take decision?



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$$\rightsquigarrow \text{Divisor} = X^3 + 1 = 1001 \leftarrow L = 4 \text{ bits}$$

$$\begin{array}{r}
 11001001000 \\
 \underline{1001} \\
 01011 \\
 1001 \\
 \hline 001000 \\
 1001 \\
 \hline 0001100 \\
 1001 \\
 \hline 01010 \\
 1001 \\
 \hline 0\boxed{011} \rightarrow \text{CRC}
 \end{array}$$

$$T_x = \frac{11001001}{\text{data}} \frac{011}{\text{CRC}}$$



→ 11011001011 , D = 1001

$$\begin{array}{r} 1001 \\ \hline 01001 \\ 1001 \\ \hline 0000001011 \\ 1001 \\ \hline 0010 \end{array} \leftarrow R_x \neq 0$$

Error

