10/01/25 ITC Practice Tut

Q Flip a coin 3 times

h, t, h

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

→ Self Information

$I_i(p) = \log_2\left(\frac{1}{p}\right) = -\log_2 p$

→ Entropy (Avg. uncertainity element in the source)

$H(s) = -p \cdot \log_2 p$

Uncertainity ↑ Compression ↓

Entropy ↑

$H(s) = -\sum p_i \cdot \log_2 p_i$

Q A DMS has 4 symbols $S = \{s_1, s_2, s_3, s_4\}$ with probabilities $p_1 = 0.5$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.1$

Sol^n $H(s) = -\sum_{i=1}^{4} p_i \cdot \log_2 p_i$

$\log_2 a = \frac{\log_{10} a}{\log_{10} 2}$

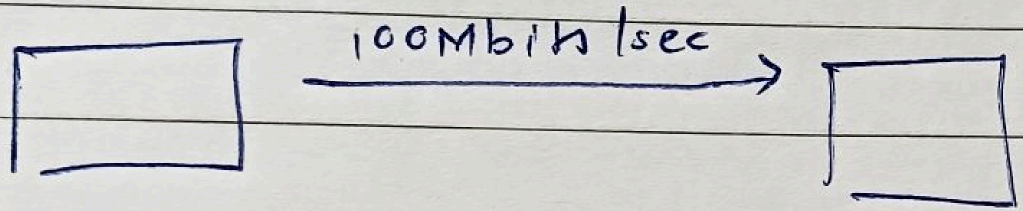
$= -\left(0.5 \log_2 0.5 + 0.2 \log_2 0.2 + 0.2 \log_2 0.2 + 0.1 \log_2 0.1\right)$

$= -\left(0.5 \frac{\log_{10} 0.5}{\log_{10} 2} + 0.2 \frac{\log_{10} 0.2}{\log_{10} 2} + 0.2 \frac{\log_{10} 0.2}{\log_{10} 2} + 0.1 \frac{\log_{10} 0.1}{\log_{10} 2}\right)$

$H(S) = -1.76$ bits/symbol

$\log_{10} 2 = 0.301$

→ Rate of Information



Symbol 'r' = 100 M bits/sec

$= 100 \times 10^6$ symbols/sec

Information Rate $R = r \cdot H(S)$

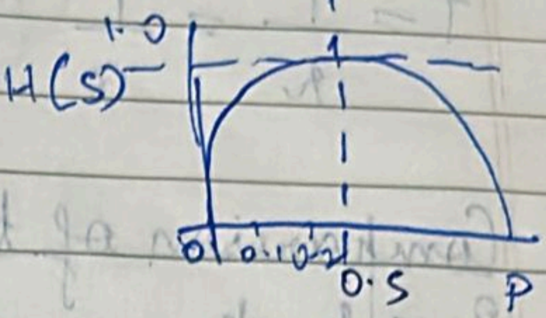
$= 100 \times 10^6 \times 1.760$

$= 176 \times 10^6$ bits/sec

$= 1.76 \times 10^6$ bits/sec

Q. $S = \{S_1, S_2\}$ with probabilities P_1, P_2. Calculate $H(S)$.

P_1	P_2	$H(S)$
0.1	0.9	
0.3	0.7	
0.5	0.5	1.0



$P_1 + P_2 = 1$

$P_2 = 1 - P_1$

$P, 1 - P$

$P = 0.1$

$H(S) = 0.1 \log_2 0.1 + 0.9 \log_2 0.9$

$= -(-0.332 - 0.136)$

$= 0.466$

$P = 0.3$

$H(S) = -0.52 - 0.36$

$= -(0.88) = 0.88$

$P = 0.5$

$H(S) = -(-0.5 - 0.5)$

$= 1$

Entropy max^m when $P = 0.5$

OpenAI

ITC Question Solutions

Uploaded image
I am attaching an Image which has three questions from a subject which I have this semester called, information theory and coding(ITC).
Solve those three questions in simple and explanatory way.
Format : First write the question then solution.

Also I want you to solve a similar question after each of the three questions by changing some concepts or values (basically similar formulas etc but new). Which will make the total count of questions 6.

Solve all the questions.

Question 1:

There are 20 balls in a basket; 5 are red, 10 are blue, and 5 are green. If three balls are selected at random, what is the probability that the first ball is red and the next two are green?

Solution:

- Total number of balls = 20
- Probability of selecting a red ball first = $\frac{5}{20} = \frac{1}{4}$
- After the first ball is chosen, the total number of balls reduces to 19. Now there are still 5 green balls left.
  - Probability of selecting a green ball second = $\frac{5}{19}$
  - Probability of selecting a green ball third = $\frac{4}{18} = \frac{2}{9}$

The required probability is:

$$P(\text{red, green, green}) = \frac{1}{4} \cdot \frac{5}{19} \cdot \frac{2}{9} = \frac{10}{684} = \frac{5}{342}.$$

Similar Question:

There are 15 balls in a basket; 4 are red, 6 are blue, and 5 are green. If three balls are selected at random, what is the probability that the first ball is blue and the next two are red?

Solution:

- Total number of balls = 15
- Probability of selecting a blue ball first = $\frac{6}{15} = \frac{2}{5}$

- After the first ball is chosen, the total number of balls reduces to 14. Now there are still 4 red balls left.
  - Probability of selecting a red ball second = $\frac{4}{14} = \frac{2}{7}$
  - Probability of selecting a red ball third = $\frac{3}{13}$

The required probability is:

$$P(\text{blue, red, red}) = \frac{2}{5} \cdot \frac{2}{7} \cdot \frac{3}{13} = \frac{12}{455}.$$

Question 2:

A message consists of 4 symbols $S = \{s_1, s_2, s_3, s_4\}$ with probabilities $\frac{1}{8}, \frac{2}{8}, \frac{2}{8}, \frac{3}{8}$ respectively. Calculate the self-information of each symbol and the source entropy. If the speed of transmission is 1500 symbols/sec, what is the information rate?

Solution:

- Self-information of a symbol s_i:

$$I(s_i) = -\log_2(P(s_i)).$$

For each symbol:

1. $I(s_1) = -\log_2\left(\frac{1}{8}\right) = 3$ bits
2. $I(s_2) = I(s_3) = -\log_2\left(\frac{2}{8}\right) = -\log_2\left(\frac{1}{4}\right) = 2$ bits
3. $I(s_4) = -\log_2\left(\frac{3}{8}\right) \approx 1.415$ bits

- Source entropy $H(S)$:

$$H(S) = \sum_{i=1}^{4} P(s_i) \cdot I(s_i) = \left(\frac{1}{8} \cdot 3\right) + \left(\frac{2}{8} \cdot 2\right) + \left(\frac{2}{8} \cdot 2\right) + \left(\frac{3}{8} \cdot 1.415\right).$$

$$H(S) \approx 0.375 + 0.5 + 0.5 + 0.531 = 1.906 \,\text{bits/symbol}.$$

- Information rate:

$$R = H(S) \cdot \text{transmission speed} = 1.906 \cdot 1500 = 2859 \,\text{bits/sec}.$$

Similar Question:

A message consists of 3 symbols $S = \{s_1, s_2, s_3\}$ with probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. Calculate the self-information of each symbol and the source entropy. If the speed of transmission is 1200 symbols/sec, what is the information rate?

Solution:

- Self-information for each symbol:

1. $I(s_1) = -\log_2\left(\frac{1}{4}\right) = 2$ bits

2. $I(s_2) = -\log_2\left(\frac{1}{2}\right) = 1$ bit
3. $I(s_3) = -\log_2\left(\frac{1}{4}\right) = 2$ bits

- Source entropy:

$$H(S) = \left(\frac{1}{4} \cdot 2\right) + \left(\frac{1}{2} \cdot 1\right) + \left(\frac{1}{4} \cdot 2\right) = 0.5 + 0.5 + 0.5 = 1.5\,\text{bits/symbol.}$$

- Information rate:

$$R = H(S) \cdot \text{transmission speed} = 1.5 \cdot 1200 = 1800\,\text{bits/sec.}$$

Question 3:

An information source S transmits two symbols $\{x, y\}$. If p and $(1-p)$ are the probabilities of x and y respectively, show (by gradually increasing values of p from 0 to 1) that the source entropy is maximum when $p = (1-p) = \frac{1}{2}$.

Solution:

- Source entropy:

$$H(S) = -\left[p\log_2(p) + (1-p)\log_2(1-p)\right].$$

For different values of p:

1. $p = 0$: $H(S) = 0$
2. $p = 0.25$:

$$H(S) = -\left[0.25\log_2(0.25) + 0.75\log_2(0.75)\right] = -(0.25 \cdot -2 + 0.75 \cdot -0.415) = 1.061\text{ bits.}$$

3. $p = 0.5$:

$$H(S) = -\left[0.5\log_2(0.5) + 0.5\log_2(0.5)\right] = 1\text{ bit.}$$

4. $p = 0.75$: Similar to $p = 0.25$, $H(S) = 1.061$ bits.
5. $p = 1$: $H(S) = 0$.

Thus, the entropy is maximum at $p = 0.5$.

Similar Question:

An information source S transmits three symbols $\{x, y, z\}$ with probabilities $p, q, (1-p-q)$. Show that the entropy $H(S)$ is maximum when $p = q = \frac{1}{3}$.

Solution:

- Source entropy:

$$H(S) = -\left[p\log_2(p) + q\log_2(q) + (1-p-q)\log_2(1-p-q)\right].$$

- Test with values $p = q = \frac{1}{3}$:

$$H(S) = -\left[\frac{1}{3}\log_2\left(\frac{1}{3}\right) + \frac{1}{3}\log_2\left(\frac{1}{3}\right) + \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right] = 1.585\,\text{bits}.$$

The entropy is maximum when the probabilities are equal.