LBC Basics

Linear Codes basics & property with example

Defination - A Block code is said to be linear code if its codewords satisfy the condition that the sum of any two codewords gives another codeword.

i.e. $c_p = c_i + c_k$

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i) The all-zero wo ... 0] is always a codeword

ii) Given any three ... such that

$c_p = c_i + c_k$

iii) Minimum distan

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property

i) The all-zero words [0, 0, 0, ... 0] is always a codeword.

ii) Given any three codewords c_i, c_j and c_k such that

$$c_p = c_i + c_k \text{, then } d(c_i, c_j) = W(c_p)$$

iii) Minimum distance of the code

$$d_{min} = W_{min}$$

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iii) Minimum distance of the code

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- (7,4) Hamming Code

$$c_1 = 0\,0\,0\,1\,0\,1\,1 \quad \boxed{3}$$

$$c_{10} = 1\,0\,1\,0\,0\,1\,1 \quad \boxed{4} \qquad c_{11} = c_1 + c_{10}$$

$$c_{11} = 1\,0\,1\,1\,0\,0\,0$$

- $c_0 = [0\ 0\ 0\ 0\ 0\ 0\ 0]$

iii) Minimum distance of the code

$$d_{min} = W_{min}$$

(7,4) Hamming code

$$c_1 = 0001011 \quad \boxed{3}$$
$$c_{10} = 1010011 \quad \boxed{4} \qquad c_{11} = c_1 + c_{10}$$
$$c_{11} = 1011000 \quad \boxed{3}$$

- $c_0 = [0\ 0\ 0\ 0\ 0\ 0\ 0]$

$d(c_1, c_{10}) = 3$ | $d(c_1, c_{10}) = 3 = W(c_{11})$

$W(c_{11}) = 3$

- (7,4) Hamming Code

$$C_1 = 0\,0\,0\,1\,0\,1\,1 \quad \boxed{3}$$
$$C_{10} = 1\,0\,1\,0\,0\,1\,1 \quad \boxed{4} \qquad C_{11} = C_1 + C_{10}$$
$$C_{11} = 1\,0\,1\,1\,0\,0\,0 \quad \boxed{3}$$

- $C_0 = [0\ 0\ 0\ 0\ 0\ 0\ 0]$

$$d(C_1, C_{10}) = 3 \qquad d(C_1, C_{10}) = 3 = W(C_{11})$$
$$W(C_{11}) = 3$$

- $C_{15} = [1\ 1\ 1\ 1\ 1\ 1\ 1]$, $\omega = 7$

other then C_{15} codes are having weight 3 & 4.

$$d_{min} = W_{min}$$

S.No.	m0	m1	m2	m3	p0	p1	p2
0	0	0	0	0	0	0	0
1	0	0	0	1	0	1	1
2	0	0	1	0	1	1	0
3	0	0	1	1	1	0	1
4	0	1	0	0	1	1	1
5	0	1	0	1	1	0	0
6	0	1	1	0	0	0	1
7	0	1	1	1	0	1	0
8	1	0	0	0	1	0	1
9	1	0	0	1	1	1	0
10	1	0	1	0	0	1	1
11	1	0	1	1	0	0	0
12	1	1	0	0	0	1	0
13	1	1	0	1	0	0	1
14	1	1	1	0	1	0	0
15	1	1	1	1	1	1	1

Show that (4, 3) Even-parity code is a linear and (4, 3) odd parity is not linear.

→ (4, 3) even parity code.

C	d_1	d_2	d_3	P
	0	0	0	
	0	0	1	
	0	1	0	

(4,3) odd parity is not linear.

→ (4,3) even parity code.

C	d_1	d_2	d_3	P
C_0	0	0	0	0
C_1	0	0	1	1
C_2	0	1	0	1
C_3	0	1	1	0
C_4	1	0	0	1
C_5	1	0	1	0
C_6	1	1	0	0
C_7	1	1	1	1

		W
0 0 1 1	C_1	2
0 1 0 1	C_2	2
0 1 1 0	$\boxed{C_3}$	2

$d_{min}(C_1, C_2) = 2$

$W(C_3) = 2$

$\boxed{d_{min} = W}$

- Odd parity (4, 3) code

C	d_1	d_2	d_3	P
c_0	0	0	0	1
c_1	0	0	1	0
c_2	0	1	0	0
c_3	0	1	1	1
c_4	1	0	0	0
c_5	1	0	1	1
c_6	1	1	0	1
c_7	1	1	1	0

- Odd parity (4, 3) code

C	d_1	d_2	d_3	P
C_0	0	0	0	1
C_1	0	0	1	0
C_2	0	1	0	0
C_3	0	1	1	1
C_4	1	0	0	0
C_5	1	0	1	1
C_6	1	1	0	1
C_7	1	1	1	0

$C_1 \longrightarrow$ 0 0 1 0 C_1

$C_2 \longrightarrow$ 0 1 0 0 C_2

0 1 1 0

- $C_1 + C_2$ is not present in odd parity (4, 3) code
- It is not linear block code.

Generator Matrix to generate Codeword in LBC

Generator matrics in Linear code
to generate code words

- Using a matrix to generate codewords is a better approach.

$$[c] = [i][G]$$

[c] = Code word
[i] = Information words
[G] = Generator matrix

- The Generator matrix of an (n, K) linear code has 'K' rows and 'n' colomns.

Generator matrix for $(7, 4)$ code is given by

[i] = Information words

[G] = Generator matrix

- The Generator matrix of an (n, k) linear code has 'k' rows and 'n' columns.

- Generator matrix for (7, 4) code is given by

$$[G] = [I : P]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$[G] = [I : P]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

Identity Matrix I_K Parity Matrix

$\rightarrow$ $[C] = [i][G]$

Example - Generate Codeword for $\underline{i = (1110)}$ with (7, 4) generator matrix code.

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$C = [i][G]$$

$$1.1 + 1.0 + 1.0 + 0.0$$

$$= [1\ 1\ 1\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [1$$

— Determine the set of codewords for the (6, 3)

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

→ Message $[i] = [1\ 1\ 1\ 0]$

$$1.1 + 1.0 + 1.0 + 0.0$$

$$C = [i][G]$$

$$= [1\ 1\ 1\ 0] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= [1\ 1\ 1\ 0\ 1\ 0\ 0]$$

Example – Determine the set of codewords for the (6, 3)

Example - Determine the set of codewords for the (6, 3) code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ $n = 6$

$K = 3$

→ Message bits = 3

$m_0 \; m_1 \; m_2$

code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ $n = 6$

$K = 3$

→ Message bits = 3

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$K = 3$

→ Message bits = 3

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

→ $C = [i][G]$

→ $C_0 = [0\ 0\ 0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= [0\ 0\ 0\ 0\ 0\ 0]$

→ $C_1 = [0\ 0\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

$= [0\ 0\ 1\ 1\ 1\ 0]$

	m_0	m_1	m_2	p_0	p_1	p_2
c_0	0	0	0	0	0	0
c_1	0	0	1	1	1	0
c_2	0	1	0	1	0	1
c_3	0	1	1	0	1	1
c_4	1	0	0	0	1	1
c_5	1	0	1	1	0	1
c_6	1	1	0	1	1	0
c_7	1	1	1	0	0	0

$= [0\ 0\ 1\ 1\ 1\ 0]$

... the set of codewords for the (6, 3) code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

→ $n = 6$

$K = 3$

→ Message bits = 3

m_0	m_1	m_2
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

→ $C = [i][G]$

$\boxed{C = [m, P]}$

→ $$C_0 = [0\ 0\ 0]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 0\ 0\ 0\ 0]$$

→ $$C_1 = [0\ 0\ 1]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1\ 0]$$

Generator matrix in Linear code
to generate code words

$[G] = [I : P]$

- Using a matrix to generate codewords is a better approach.

$[c] = [i][G]$

$[c]$ = Codeword

$[i]$ ... ion words

... ator matrix

... rix of an (n, k) linear code

... 'n' columns.

... matrix for $(7, 4)$ code is given

$[G] = [I : P]$

$[c] = [m : P_c]$

$[P_c] = [m][P]$

Systematic Generator Matrix

Systematic Generator matrix in Linear block codes

- A Generator matrix $[G] = [I_k : P]$ is said to be in a Systematic form if it generates the Systematic codewords.

- Here

$$[I_k] \longrightarrow k \times k \text{ matrix}$$

$$[P] \longrightarrow k \times (n-k) \text{ matrix}$$

$$[G] \longrightarrow k \times n \text{ matrix}$$

- In these matrix Information bits are placed together

- Codeword

Systematic Generator matrix in Linear block codes

- A Generator matrix $[G] = [I_k : P]$ is said to be in a Systematic form if it generates the systematic codewords.

$$[C] = [i][G] = [m, P]$$

- Here

$$[I_k] \longrightarrow k \times k \text{ matrix}$$

$$[P] \longrightarrow k \times (n-k) \text{ matrix}$$

$$[G] \longrightarrow k \times n \text{ matrix}$$

- In the [illegible] atrix Inform[illegible] [illegible]ether
- Code[illegible]

- In these matrix Information bits are placed together
- Codeword

$$[C] = [i][G]$$

$$= [i_1, i_2, i_3, i_4]\begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & \vdots & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & \vdots & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & \vdots & 0 & 1 & 1 \end{bmatrix}$$

- So when you get Codeword

$$C = (i_1, i_2, i_3, i_4, P_1, P_2, P_3)$$

- Ident matrix Keeps Information together
- P atrix generates Pa bits

- Codeword

$$[C] = [i][G]$$

- $$= [i_1, i_2, i_3, i_4] \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

- So when you get Codeword

$$C = (i_1, i_2, i_3, i_4, P_1, P_2, P_3)$$

- Identity matrix keeps Information together
- Parity matrix generates Parity bits

$$
- \quad = [i_1, i_2, i_3, i_4]
\begin{array}{cccccccc}
i_1 & i_2 & i_3 & i_4 & & p_1 & p_2 & p_3 \\
\end{array}
$$

$$
\left[\begin{array}{cccc|ccc}
1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}\right]
$$

- So when you get Codeword

$$c = (i_1, i_2, i_3, i_4, p_1, p_2, p_3)$$

- Identity matrix keeps Information together
- Parity matrix generates Parity bits.

$$p_1 = i_1 + i_2 + i_3$$

$$p_2 = i_2 + i_3 + i_4$$

$$p_3 = i_1 + i_2 + i_4$$

The (5,3) linear code has the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [\, I_K : P \,]$$

- Determine systematic form of G.
- generate codeword for Information (011) with Systematic G & Non-systematic G.

The (5,3) linear code has the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} = [\, I_K : P \,]$$

- Determine systematic form of G.
- generate codeword for Information (0 1 1) with Systematic G & Non-systematic G.

$$[G] = \begin{bmatrix} \times & 1 & 0 & 1 & 0 & 0 \\ \checkmark & 0 & 1 & 0 & 0 & 1 \\ \times & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_2 \text{ to } R_3]{\text{Add}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$[G] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_2 \text{ to } R_3]{\text{Add}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Add R_3 to R_1

$$\Big\downarrow$$

$$\begin{bmatrix} \boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}} & \boxed{\begin{matrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{matrix}} \end{bmatrix}$$

$$\quad I \qquad\qquad P$$

$$\boxed{G = [\, I_k : P \,]}$$

$$[G] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_2 \text{ to } R_3]{\text{Add}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\Big\downarrow \text{Add } R_3 \text{ to } R_1$$

$$\Big[\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} \ \underbrace{\begin{matrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{matrix}}_{P} \Big]$$

$$\boxed{G = [I_K : P]}$$

→ For Non Systemetic $[G]$

$$c = [i][G]$$

$$= [0\ 1\ 1] \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1]$$

$$c = [i][G]$$

$$= [\underline{0\ 1\ 1}]\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= [0\ 0\ 1\ 1\ 1]$$

$\downarrow$ R_3 to R_1

$$\left[\ \underbrace{\boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}}_{I}\ \underbrace{\boxed{\begin{matrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{matrix}}}_{P}\ \right]$$

$$\boxed{G = [I_k : P]}$$

→ For Systemetic [G]

$$c = [i][G]$$

$$= [\underline{0\ 1\ 1}]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$= [\underbrace{0\ 1\ 1}_{\text{Information}}\ \underbrace{1\ 0}_{\text{Parity}}]$$

Parity check matrix

Parity Check Matrices in Linear block Codes with Examples.

- From Generator matrix $[G] = [I_k \mid P]$ we can identify parity matrix.
- By taking P^T we can make parity Check Matrix $[H]$

$$[H] = [P^T : I_{n-k}]$$

- Parity check matrix is used at R_x to decode data.

<u>Example</u> - Generate Parity check Matrix for (7,4) code

$$[G] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Example - Generate Parity check Matrix for (7,4) Code

$$[G] = \left[\ \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{\substack{\text{Identity} \\ \text{matrix} \\ I}}\ \middle|\ \underbrace{\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}}_{\substack{\text{Parity} \\ \text{Matrix} \\ P.}}\ \right]$$

$$\left[\ \underset{\substack{\uparrow\\ \text{Identity}\\ \text{matrix}\\ I}}{\boxed{0\quad 0\quad 0\quad 1}}\ \underset{\substack{\uparrow\\ \text{Parity}\\ \text{Matrix}\\ P.}}{\boxed{0\quad 1\quad 1}}\ \right]$$

$$\rightarrow\ P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Identity
Matrix
I

Parity
Matrix
P.

$$\rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow I_{n-k} = I_{7-4} = I_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow I_{n-k} = I_{7-4} = I_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow [H] = [P^T : I_{n-k}]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Prove that GH^T and $CH^T = 0$.

Prove that GH^T and $CH^T = 0$.

$$\Rightarrow \quad GH^T = [\, I_k \mid P \,]\,[\, P^T \mid I_{n-k} \,]^T$$

$$= [\, I_k \mid P \,] \begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix}$$

$$= I_k P + P I_{n-k}$$

$$= P + P$$

$$P \nearrow 0, \quad P \searrow 1 \qquad \text{For Mod-2 add}, \quad \begin{array}{l} 0 + 0 = 0 \\ 1 + 1 = 0 \end{array}$$

$$\Rightarrow \boxed{GH^T = 0}$$

$$= [I_k | P] \begin{bmatrix} P \\ I_{n-k} \end{bmatrix}$$

$$= I_k P + P I_{n-k}$$

$$= P + P$$

$P \to 0$, $P \to 1$ For Mod-2 add, $0 + 0 = 0$, $1 + 1 = 0$

$$\Rightarrow \boxed{G H^T = 0}$$

$$\Rightarrow C H^T = [i][G][H^T]$$

$$= [i]\, 0$$

$$\boxed{C H^T = 0}$$