

Mod 5 : Computability theory

NP-Hard and NP-Complete

Polynomial Time

Linear Search — n

Binary Search — $\log n$

Insertion Sort — n^2

Merge Sort — $n \log n$

Matrix Multiplication — n^3

Exponential Time

0/1 Knapsack — 2^n

Traveling SP — 2^n

Sum of Subsets — 2^n

Graph Coloring — 2^n

Hamiltonian Cycle — 2^n



Objectives

1 : relate the problems atleast

So that if one problem is solved we can easily solve the others

2 : If we are not able to write polynomial time deterministic algorithms for them, why don't we write polynomial time non-deterministic algorithms for them.

NP-Hard and NP-Complete

Nondeterministic

```
Algorithm NSearch(A, n, key)
{
    j = choice();
    if (key = A[j])
    {
        write(j);
        Success();
    }
    write(0);
    Failure();
}
```

Exponential Time

0/1 Knapsack — 2^n

Traveling SP — 2^n

Sum of Subsets — 2^n

Graph Coloring — 2^n

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NP-Hard and NP-complete

Nondeterministic

Algorithm NSearch(A, n, key)

```
{
    j = choice(); — 1
    if (key = A[j])
    {
        write(j);
        Success(); — 1
    }
    write(0);
    Failure(); — 1
}
```

0(1)

Exponential Time

0/1 Knapsack — 2^n

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NP-Hard and NP-complete

Nondeterministic

Algorithm NSearch(A, n, key)

```
{  
  j = choice(); — 1  
  if (key = A[j])  
  {  
    write(j);  
    Success(); — 1  
  }  
  write(0);  
  Failure(); — 1  
}
```

A

10	8	6	9	4	2
1	2	3	4	5	6

 $O(1)$
key = 9

Exponential Time

0/1 Knapsack — 2^n

Traveling SP — 2^n

Sum of Subsets — 2^n

Graph Coloring — 2^n

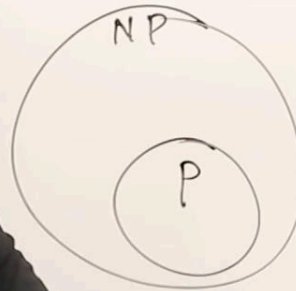
Hamiltonian Cycle — 2^n



NP-Hard and NP-Complete

P

NP



Exponential Time

0/1 Knapsack — 2^n

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P - set of deterministic algorithms which are taking polynomial time

NP - Non deterministic but they take polynomial time

NP-Hard and NP-Complete

CNF-Satisfiability.

$$x_i = \{x_1, x_2, x_3\}$$

$$CNF = (\underbrace{x_1 \vee \bar{x}_2 \vee x_3}_{C_1}) \wedge (\underbrace{\bar{x}_1 \vee x_2 \vee \bar{x}_3}_{C_2})$$

x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Exponential Time

0/1 Knapsack — 2^n

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NP-Hard and NP-Complete

CNF-Satisfiability — 2^n

$$x_i = \{x_1, x_2, x_3\}$$

$$\text{CNF} = (\underbrace{x_1 \vee \bar{x}_2 \vee x_3}_{c_1}) \wedge (\underbrace{\bar{x}_1 \vee x_2 \vee \bar{x}_3}_{c_2})$$

$$2^3 \Rightarrow 2^n$$

x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Exponential Time

0/1 Knapsack — 2^n

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NP-Hard and NP-Complete

CNF-Satisfiability — 2^n

0/1 Knapsack

$$p = \{10, 8, 12\} \quad n=3$$

$$w = \{5, 4, 3\} \quad m=8$$

Exponential Time

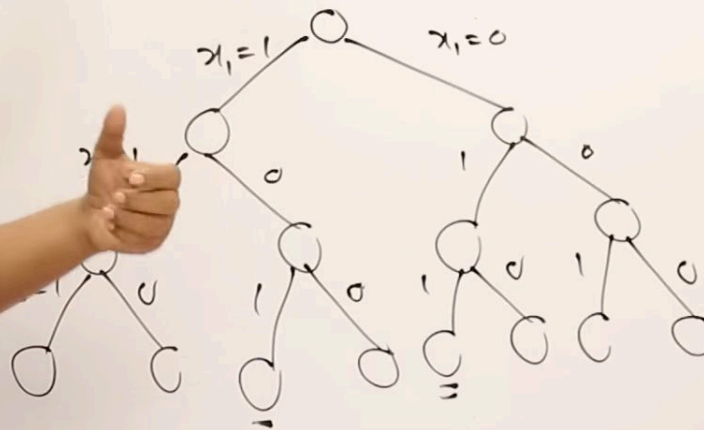
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NP-Hard and NP-Complete

CNF-Satisfiability — 2^n

0/1 Knapsack

$$p = \{10, 8, 12\} \quad n=3$$

$$w = \{5, 4, 3\} \quad m=8$$

$$x_i = \{0/1, 0/1, 0/1\}$$

$$\begin{matrix} x_1 & x_2 & x_3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 2^3 & \Rightarrow & 2^n \end{matrix}$$

Exponential Time

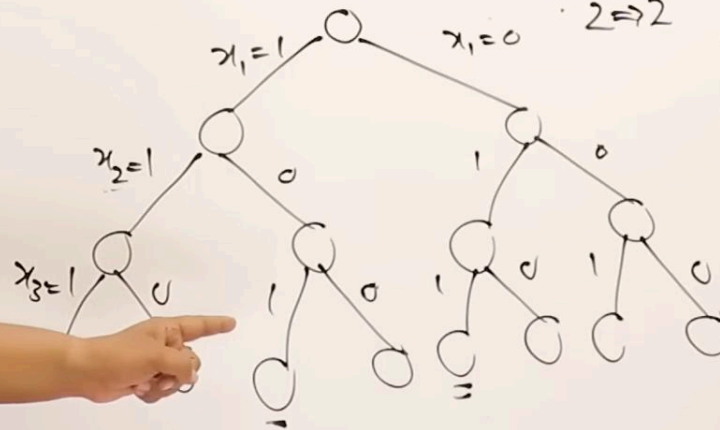
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NP-Hard and NP-Complete

CNF-Satisfiability — 2^n

Exponential Time

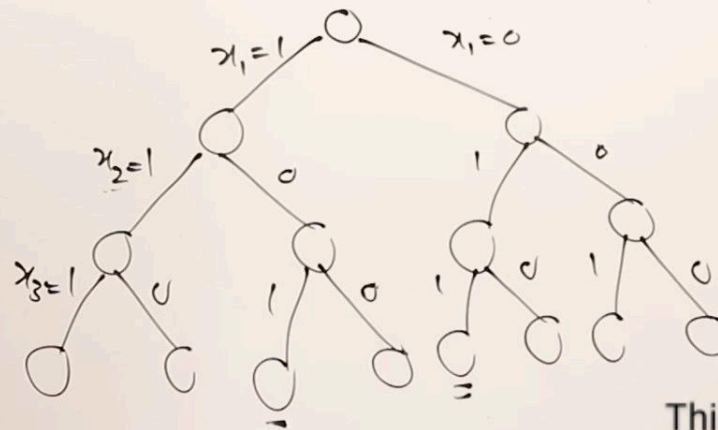
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This state space tree is
our relator 😊



NP-Hard and NP-Complete

NP-hard Satisfiability $- 2^n$

Exponential Time

0/1 Knapsack $- 2^n$

Traveling SP $- 2^n$

Sum of Subsets $- 2^n$

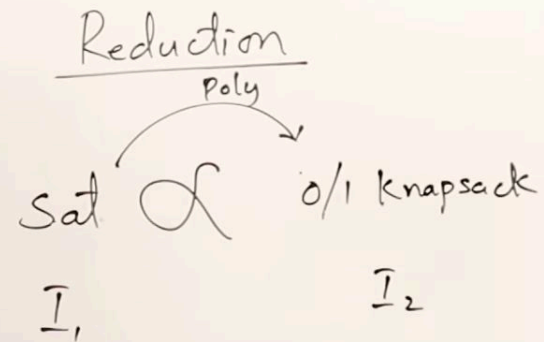
Graph Coloring $- 2^n$

Hamiltonian Cycle $- 2^n$

Hard



NP-Hard and NP-Complete



Satisfiability $- 2^n$

Exponential Time

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Traveling SP $- 2^n$

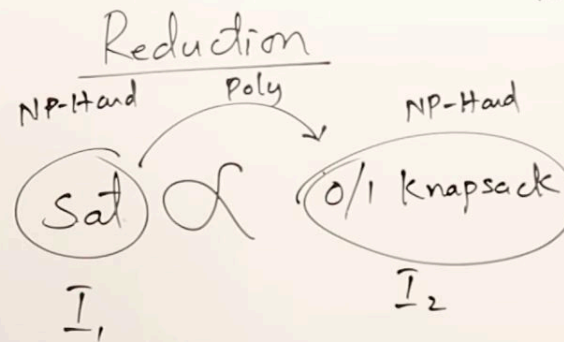
Sum of Subsets $- 2^n$

Graph Coloring $- 2^n$

Hamiltonian Cycle $- 2^n$



NP-Hard and NP-Complete



NP-Hard \rightarrow Satisfiability $- 2^n$

Exponential Time

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NP-Hard and NP-Complete

NP-Hard \rightarrow Satisfiability $- 2^n$

Exponential Time

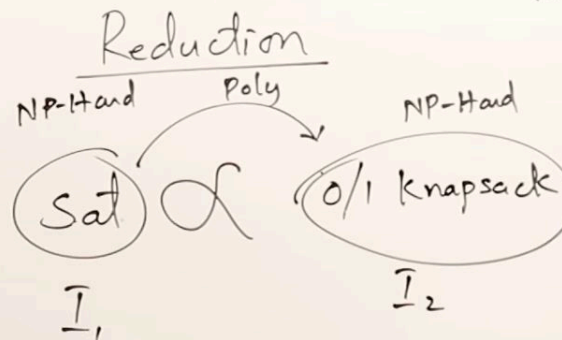
0/1 Knapsack $- 2^n$

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Sat $\propto L$

\uparrow

NP-Hard

Sat $\propto L_1$ $L_1 \propto L_2$

NP-Hard and NP-Complete

$\text{Sat} \propto L$
 \uparrow
NP

NP-Hard \rightarrow Satisfiability $- 2^n$
NP-complete
Exponential Time

0/1 Knapsack $- 2^n$

Traveling SP $- 2^n$

Sum of Subsets $- 2^n$

Graph Coloring $- 2^n$

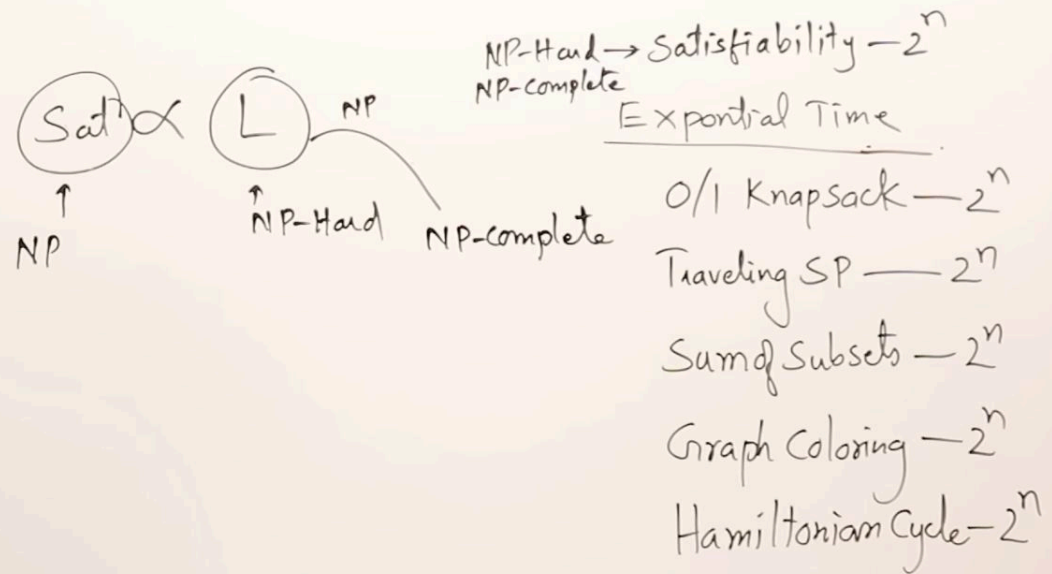
Hamiltonian Cycle $- 2^n$



View key concept



NP-Hard and NP-Complete



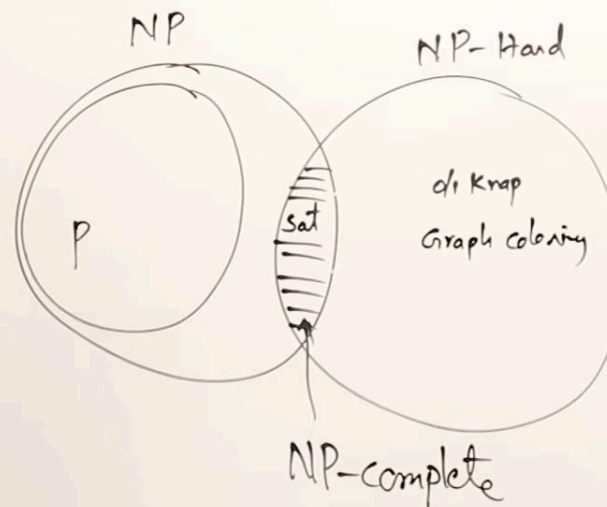
If your research,

1) Your problem is directly or indirectly related to satisfiability -> NP hard class

2) You make a Non deterministic algorithm for solving that problem -> NP Complete class

Both these together -> Research work complete

NP-Hard and NP-Complete



Satisfiability $- 2^n$

Exponential Time

0/1 Knapsack $- 2^n$

Traveling SP $- 2^n$

Sum of Subsets $- 2^n$

Graph Coloring $- 2^n$

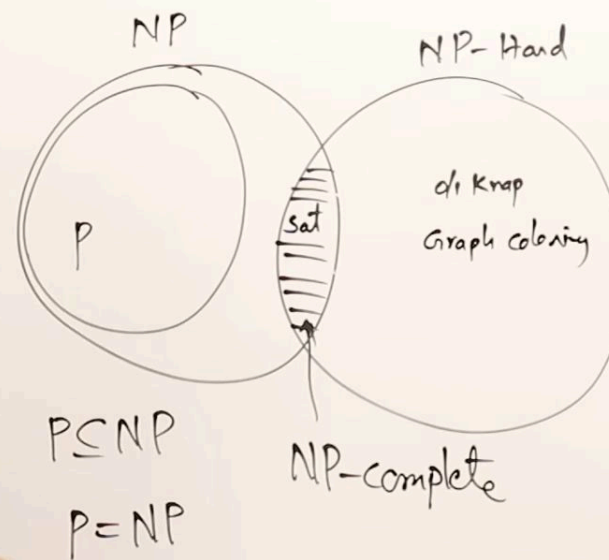
Hamiltonian Cycle $- 2^n$

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Only, if we are able to prove that $P=NP$
Then our research is valid because what is
the non deterministic never goes on to
become deterministic with time?

NP-Hard and NP-Complete



Satisfiability $- 2^n$

Exponential Time

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Traveling SP $- 2^n$

Sum of Subsets $- 2^n$

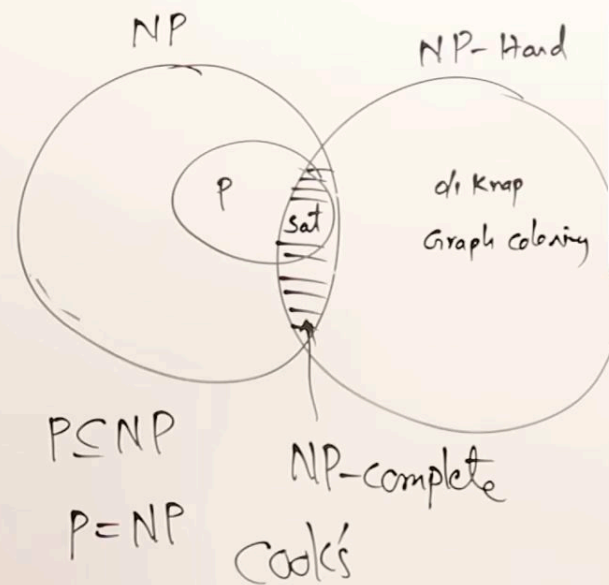
Graph Coloring $- 2^n$

Hamiltonian Cycle $- 2^n$

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NP-Hard and NP-Complete



Algorithms



⇒ 84 videos

Sum of Subsets — 2^n

Graph Coloring — 2^n

Hamiltonian Cycle — 2^n

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